Prayas Notes

INDEX

Name : Adi Sameer Inamdar

Std. : ________ Div. : ________ Roll No. : ________

Sub. : Physics

School / College : ________

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Units & Measurements

Physical Quantities : Anything / Quantity that can be measured by Physical Instruments.

length → Scale Current → Ammeter
Temp → Thermometer Voltage → Voltmeter

Classification of P.Q.

1). Based on their directional properties :-

Vector	Scalar
i). Have magnitude	— " —
ii). Have direction	May have direction (current)
iii). Follow Vector law of Addition, based on θ betn them.	Direct Addition

{Both are Tensors}

2). Based on their dependency

Fundamental PQ Derived PQ

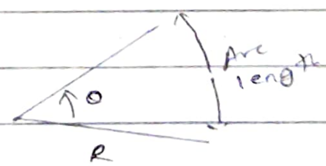
* Units of Physical Quantities

MKS + Supplementary units = S.I.

	FPS	MKS	CGS
Mass	Pound	Kg	gm
length	Foot	meter	cm
Time	Sec	Sec	sec.

Supplementary units :

i) Plane Angle :



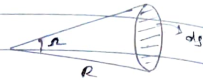
$$\theta = \text{Plane Angle} = \frac{\text{Arc length}}{\text{Radius}}$$

$$180° = \pi$$

Unit : Radian
dimension : $[M^0L^0T^0]$

iii) Solid Angle (Ω)

$$\Omega = \frac{ds}{R^2}$$



unit : sterian

dimension : $[M^0 L^0 T^0]$

a) Sphere : $\Omega = 4\pi$ str.

b). Hemisphere : $\Omega = 2\pi$ str

* Conversion of units

let n = magnitude & u = unit.

1 m = 100 cm

$n_1 \; u_1 \quad n_2 \; u_2$

$n \propto 1/u$

$\therefore \boxed{nu = \text{constant}}$

* Dimensions : Dimensional formula of any Physical quantity is that expression which represent how & which of the base quantities are included in that quantity.

	Fundamental Quantity	unit	Dimension
1)	Mass	M	kg
2)	length	L	m
3).	Time	T	sec
4)	Current	A / I	Ampere
5).	Temperature	K	Kelvin
6)	Amount of Substance	mol	mole
7)	Luminous intensity	Cd	candela

Some Important Dimensions

1). Distance, Displacement, Radius of Gyration, Light year, Parsec, Astronomical unit, Mean free path — $[L^1]$

2). Speed, Velocity, Avg. Speed, Avg. Velocity, Terminal velocity, critical velocity, velocity of light, Escape velocity, Drift velocity, orbital velocity — $[LT^{-1}]$

3). Acceleration, Avg. Acceleration, Instantaneous Acc., Acc. due to gravity, Intensity of gravitational field, Centrifugal Acc., Centripetal Acc. — $[LT^{-2}]$

4). Momentum, impulse

$(mv) = [MLT^{-1}]$

Power $= \frac{\text{work}}{\text{time}} = \frac{ML^2T^{-2}}{T}$

$= [ML^2T^{-3}]$

5). Mass Density

$[ML^{-3}]$

Density $= \frac{m}{v} = [ML^{-3}]$

charge density $[L^{-3}AT]$

current density $[AL^{-2}]$

$\frac{\text{current}}{\text{Area.}}$

Energy Density $= \frac{\text{Energy}}{\text{Vol.}} = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

u_B : Magnetic Energy Density $= \frac{1}{2}\frac{B^2}{\mu_0}$

work = all Energies.

u_E : Electric — " — $= \frac{1}{2}\varepsilon_0 E^2$

6) Force
Friction
Thrust
Viscous drag
Spring force
Magnetic —"—
Radiation —"—

$[MLT^{-2}]$

Tension
Normal Rxn
Weight
Lorentz force
Restoring force
All forces

7). work
Energy
K.E / P.E
Heat Energy
Vibrational Energy
Imp
Moment of force
Torque / couple
Force × disp.
$[ML^2T^{-2}]$
Strain Energy

$[ML^2T^{-2}]$

8). Pressure
Stress
Young modulus
Bulk —"—
Shear —"—
Modulus of rigidity
Energy density

$[ML^{-1}T^{-2}]$

9) Permittivity

$f = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$\therefore \varepsilon_0 = \frac{q^2}{fr^2} = \frac{A^2T^2}{MLT^{-2} \times L^2}$

$= [M^{-1}L^{-3}T^4A^2]$

Permeability.

$\therefore c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$= LT^{-1} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\therefore \mu_0 = \frac{1}{\varepsilon_0 \times L^2T^{-2}}$

Electric field

$\therefore F = qE.$

$\therefore E = \frac{F}{q} = \frac{MLT^{-2}}{AT} = MLT^{-3}A$

Magnetic field.

$\therefore E = Bc \quad \therefore B = \frac{E}{c} = \frac{MLT^{-3}A}{LT^{-1}} = [MT^{-2}A]$

* Principle of Homogeneity

→ we can add or subtract same Physical Quantity

$L + L = L \quad \& \quad L - L = L$

1). Trignometric Ratio ⇒ uske ander $M^0L^0T^0$
uska result $M^0L^0T^0$

2). Kisi bhi number ki power ⇒ $M^0L^0T^0$
uska result ⇒ $M^0L^0T^0$

3) Log ⇒ uske ander $M^0L^0T^0$
uska result $M^0L^0T^0$

* Dimensional Correctness of formula

If any formula has
Dimension = Dimension
L.H.S R.H.S

$\therefore$ It is dimensionally correct

* Derivation of formula :-

by considering the power → variable

Ex. If time depends on mass, length & ~~Period~~ g.

$\therefore T \propto m^x$
$\propto l^y$
$\propto g^z$

$\therefore T = k\, m^x l^y g^z$

$\therefore M^0L^0T^1 = [M]^x[L]^y[LT^{-2}]^z$

$M^0L^0T^1 = M^x L^{y+z} T^{-2z}$

$\therefore x = 0$

$y + z = 0 \qquad \therefore z = -1/2$

$-2z = 1 \qquad y = 1/2$

$\therefore T = k\sqrt{l/g}$

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$\therefore T = k\, m^x l^y g^z$

$\therefore M^0L^0T^1 = [M]^x[L]^y[LT^{-2}]^z$

$M^0L^0T^1 = M^x L^{y+z} T^{-2z}$

$\therefore x = 0$

$y + z = 0 \qquad \therefore z = -1/2$

$-2z = 1 \qquad y = 1/2$

$\therefore T = k\sqrt{l/g}$

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* Significant Digit rules.

1). All non Zero digit are significant
2). All zero betn two non Zero digit are Significant
3). All zero to the right of a non Zero digit but to left of understood decimal point are not significant.

* Error Analysis :-

1). Taking mean of every reading ($\bar{T}_{avg}$)
2). finding error in each reading (ΔT_n)
3). Avg. value of Error Report ($\Delta \bar{T}_{avg}$)

Report $= \bar{T} \pm \Delta\bar{T} = T \pm \Delta T$

fractional Error : $\frac{\Delta T}{T}$

** Question on Lec 1

Basic Maths

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* Unit of Angle

$\theta = \frac{\text{arc length}}{\text{Radius}}$ unit = radian, Dim : $M^0L^0T^0$

Radian (S.I.) | degree | min / sec (small Angle)

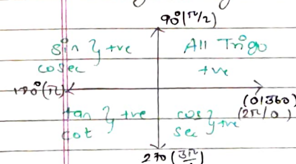
$1^\circ = 60' \rightarrow$ Minute
$1' = 60'' \rightarrow$ Second

* Trignometric Identities :

1). $\sin^2\theta + \cos^2\theta = 1$
2). $1 + \tan^2\theta = \sec^2\theta$
3). $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4) $\sin(A+B) = \sin A\cos B + \cos A\sin B$
5) $\sin(A-B) = \sin A\cos B - \cos A\sin B$
6) $\cos(A+B) = \cos A\cos B - \sin A\sin B$
7) $\cos(A-B) = \cos A\cos B + \sin A\sin B$
8) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
9) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

* Trignometry function change :-



Angle $\Rightarrow (90 \times n \pm \theta)$

n = Even	n = Odd
sin → sin	sin ⇌ cos
cos → cos	tan ⇌ cot
tan → tan	sec ⇌ cosec

Howork :

1). $\sin(135) = \sin(90\times1+45)$

$= +\cos 45$

$= \underline{\underline{1/\sqrt{2}}}$

2). $\sin(120) = \sin(90\times1+30)$

$= +\cos 30$

$= \underline{\underline{\sqrt{3}/2}}$

3). $\cos(330^\circ) = \cos(90\times3+60)$

$= +\sin 60$

$= \underline{\underline{\sqrt{3}/2}}$

* Small Angle Approximation

1). $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\boxed{\begin{array}{l}\sin\theta \approx \theta \\ \cos\theta \approx 1 \\ \tan\theta \approx \theta\end{array}}$

2). $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

3). $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots$

Higher terms gets neglected.

* 4). $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

Binomial Expansion :-

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

$\therefore$ for SG : $(1+x)^n$

Very small

$x << 1$

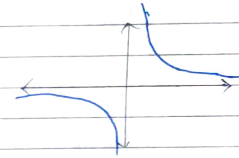
* Important Graphs :-

1). Rectangular Hyperbola :-

$yx = \text{Constant}$

$\therefore y = \frac{c}{x}$ $\therefore x \to 0 \quad y \to \infty$

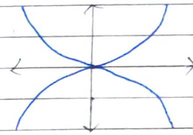
$x \to \infty \quad y \to 0$



2). Parabola :-

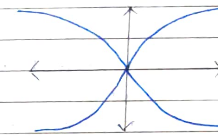
$y = x^2$

y: +ve; x: +ve, −ve



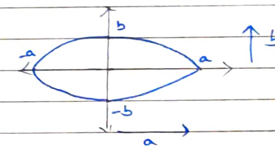
$x = y^2$

x: +ve; y: +ve, −ve



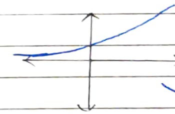
3). Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

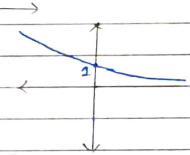


4) Exponent function :

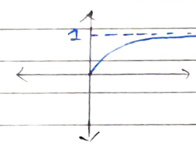
a). $y = e^x$



b). $y = e^{-x}$



c). $y = 1 - e^{-x}$



Motion in a Dimension.

Rest :- If the position of object with respect to observer is not changing. (motion is vice-versa)

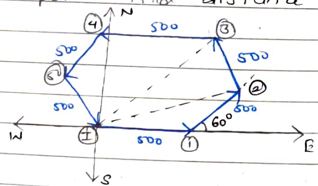
Motion & Rest, both are Relative terms. w.r.t to observer.

* Distance & Displacement.

Distance	Displacement
It is the total Path covered.	It is the difference betn initial & final position.
It is scalar.	→ follow vector rule.

Q) A person starts moving towards East, moves 500 m & take left turn at 60°. He keeps on doing it till he reaches the same point. Find distance & displacement.

Turns	Dist.	Displ.
1	500	500
2	1000	$500\sqrt{3}$
3	1500	1000
4	2000	$500\sqrt{3}$
5	2500	500
6	3000	0



i). If position of particle is $x = t^2 - 6t + 4$. Find distance & displacement from $t=0$ to $t = 5$ sec.

→ $x = t^2 - 6t + 4$

$v = \frac{dx}{dt} = 2t - 6$

∴ $v = 0 = 2t - 6$

∴ at $t = 3$, $v = 0$

x at $t = 5$

$x = 25 - 30 + 4$

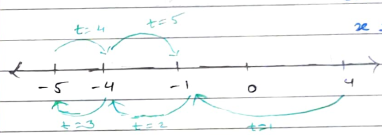
$= -1$

at $t = 0$

$x = 4$

at $t = 3$

$x = 9 - 18 + 4 = -5$



∴ displacement $= 5\hat{i}$

distance = 13 unit

Concept : 1). Average Velocity $= \frac{\text{Total Displacement}}{\text{Total Time}}$

2) Average Speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

3). Instantaneous Velocity $= v = \left.\frac{dx}{dt}\right|_{t=t_1}$

— " — Speed = |instantaneous velocity|

* Acceleration :- Rate of change of velocity w.r.t time

∴ $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

Average acceleration

Avg acc $= \frac{\Delta v}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_2 - t_1}$

Instantaneous Velocity.

$a = \left.\frac{dv}{dt}\right|_{t=t_1}$

* Calculus in Kinematics :

x = Position $\underset{\int dx = \int v\,dt}{\overset{dx/dt = v}{\rightleftharpoons}}$ Velocity $\underset{\int dv = \int a\,dt}{\overset{dv/dt = a}{\rightleftharpoons}}$ Acceleration

Lec-03

* Advance level Questions

Terminal Velocity Concept

Q) The motion of a body falling from rest in resisting medium is described by the equation $\frac{dv}{dt} = a' - bv$, where a' and b are constant. The velocity at any time t is ____.

→ ∴ $t = 0,\ u = 0$

$\frac{dv}{dt} = a' - bv \Rightarrow \int_0^v \frac{dv}{(a' - bv)} = \int_0^t dt$

∴ $\frac{1}{-b}\left[\ln(a' - bv)\right]_0^v = t$

∴ $\frac{-1}{b}\left[\ln(a' - bv) - \ln(a')\right] = t$

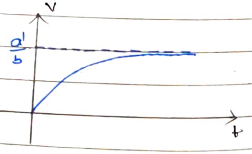
$\ln\left(\frac{a' - bv}{a'}\right) = -bt$

$\frac{a' - bv}{a'} = 1 - \frac{bv}{a'} = e^{-bt}$

$\frac{bv}{a'} = 1 - e^{-bt}$

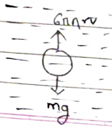
$v = \frac{a'}{b}\left(1 - e^{-bt}\right)$

Graph



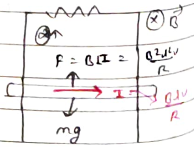
→ EMI

concept → Fluid



∴ $mg - 6\pi\eta rv = ma$

$a = g - \frac{6\pi\eta rv}{m}$



$a = g - \frac{B^2 l^2 v}{mR}$

Q) Starting from rest, a particle moves in a straight line with acceleration.

$a = (25 - t^2)^{1/2}$ m/s² for $0 \le t \le 5s$

$a = \frac{3\pi}{8}$ m/s² for $t > 5s$

The velocity of particle at $t = 7s$ is :

→ ∴ $a = \frac{dv}{dt}$ ∴ $\int dv = \int a\,dt$

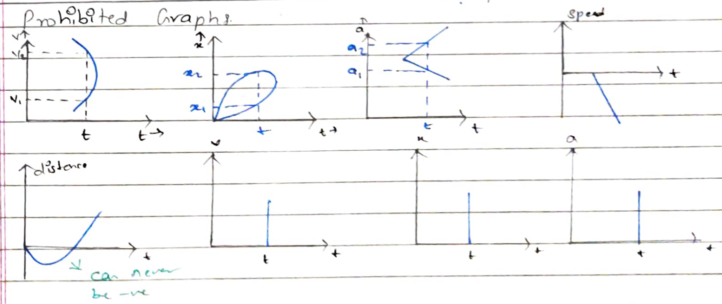
∴ $\int_0^v dv = \int_0^5 \sqrt{(25 - t^2)}\,dt + \int_5^7 \frac{3\pi}{8}\,dt$

$v = \left[\frac{t}{2}\sqrt{25 - t^2} + \frac{25}{2}\sin^{-1}\left(\frac{t}{5}\right)\right]_0^5 + \frac{3\pi}{8}[t]_5^7$

$v = \left[\cancel{\frac{5}{2}\sqrt{25 - 25}} + \frac{25}{2}\sin^{-1}(1)\right] - \left(\cancel{\frac{0}{2}\sqrt{25 - 0}} + \cancel{\frac{25}{2}\sin^{-1}(0)}\right) + \frac{3\pi}{8} \times 2$

$v = \frac{25}{2} \cdot \frac{\pi}{2} + \frac{3\pi}{4} = \frac{28\pi}{4} = 21.98 \simeq 22$ m/s

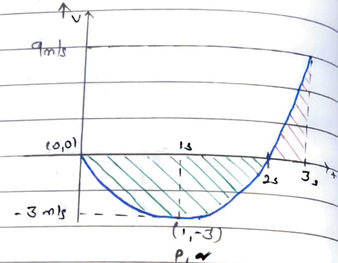
* Prohibited Graphs



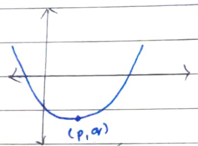
* Comprehension

Q) A particle moves along x-axis. Its velocity time graphs is given below :-

Adv.



Equation of Parabola :



$y = a(x-p)^2 + q$

$\therefore \quad y = a(x-1)^2 - 3$

$\therefore$ (0,0) is satisfied from graph

$\therefore \quad 0 = a(1) - 3$

$a = 3$

$\therefore \quad v = 3(t-1)^2 - 3$

Q) displacement from 0 to 3 sec

$\therefore \quad v = 3(t-1)^2 - 3$

$\int dx = \int v\,dt = \int 3(t-1)^2 - 3 = \int 3t^2 - 6t$

$= \cancel{3}\left[\frac{t^3}{\cancel{3}}\right]_0^3 - \cancel{6}^{3}\left[\frac{t^2}{\cancel{2}}\right]_0^3$

$= (27 - 0) - 3(3^2 - 0)$

$= 27 - 27 = 0$

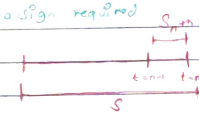
* Kinematics Equations :- Only when acc. is constant

1). $\vec{v} = \vec{u} + \vec{a}t$ } used with vector sign

2). $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

3). $v^2 = u^2 + 2aS$ } → Dot product, ∴ No sign required

4). $S_{n^{th}} = u + \frac{a}{2}(2n-1)$ } $= S_n - S_{n-1} =$



* Concept of Retardation :-

Acc. againts the motion, to decrease the speed

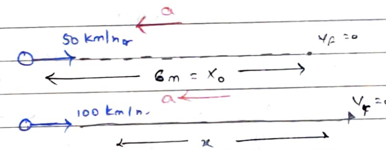
Q) A car moving with a speed of 50 km/hr, can be stopped by brakes after atleast 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is.

A. 2003

→ M-I ∴ $v^2 = u^2 + 2as$

$0 = 50^2 - 2a\,x_0$ —(i)

$0 = 100^2 - 2a\,x$ —(ii)



∴ from (i) & (ii)

$\frac{\cancel{100}^2 \times \cancel{100}^2}{\cancel{50} \times \cancel{50}} = \frac{\cancel{2a}\,x}{\cancel{2a}\,x_0}$

$x = 4x_0 = 6 \times 4 = 24$ m

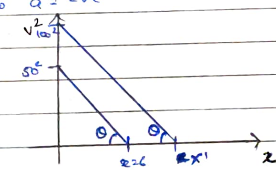
M-II :- ∴ $v^2 = u^2 + 2ax$

$y = c + m\,x$ ∴ $a = -ve$

∴ $\tan\theta = \frac{50^2}{6} = \frac{100^2}{x'}$

∴ $x' = \frac{\cancel{100}^2 \times \cancel{100} \times 6}{\cancel{50} \times \cancel{50}}$

$x' = 24$ m



Q) An engine of a train, moving with uniform acc, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

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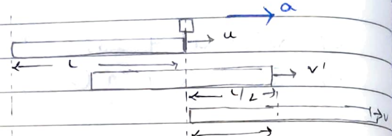
→

$\therefore v'^2 = u^2 + 2a(L/2)$ —(i)

$v^2 = v'^2 + 2a(L/2)$ —(ii)



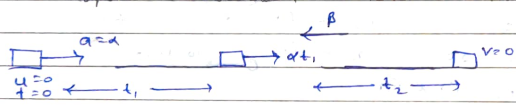
$v'^2 - u^2 = v^2 - v'^2$

$2v'^2 = v^2 + u^2$

$v' = \sqrt{\dfrac{v^2+u^2}{2}}$

Q) A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is.

∴



$\therefore v_1 = \cancel{u} + \alpha t_1$

$v_1 = \alpha t_1$, after some time.

$v_2 = \alpha t_1 + (-\beta) t_2$

$0 = \alpha t_1 - \beta t_2$

$\therefore \alpha t_1 = \beta t_2$

$t_1 = \dfrac{\beta t_2}{\alpha}$

$\therefore t_1 + t_2 = t$

$\dfrac{\beta t_2}{\alpha} + t_2 = t$

$\left(\dfrac{\beta+\alpha}{\alpha}\right) t_2 = t$

$t_2 = \dfrac{\alpha t}{\beta+\alpha}$ & $t_1 = \dfrac{\beta t}{(\alpha+\beta)}$

(i) $S = \cancel{ut}^{0} + \frac{1}{2}at^2$

$\therefore S_1 = \frac{1}{2}\alpha t_1^2 = \frac{1}{2}\alpha\left(\dfrac{\beta^2 t^2}{(\alpha+\beta)^2}\right)$

(ii) $S = ut + \frac{1}{2}at^2$

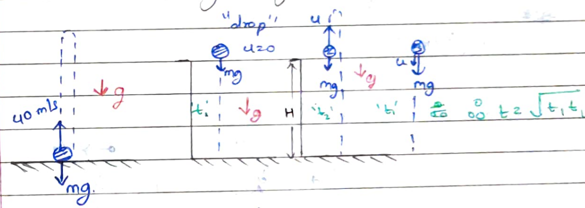
$S_2 = \beta t_2^2 + \frac{1}{2}(-\beta)t_2^2$

$= \beta t_2^2 - \frac{1}{2}\beta t_2^2 = \frac{1}{2}\beta t_2^2 = \frac{1}{2}\beta\left(\dfrac{\alpha^2 t^2}{(\alpha+\beta)^2}\right)$

$\therefore S = S_1 + S_2 = \dfrac{1}{2}\dfrac{\alpha\beta^2 t^2}{(\alpha+\beta)^2} + \dfrac{1}{2}\dfrac{\beta(\alpha^2 t^2)}{(\alpha+\beta)^2}$

$= \dfrac{1}{2}\dfrac{\alpha\beta t^2(\alpha+\beta)}{(\alpha+\beta)^2} = \dfrac{1}{2}\dfrac{\alpha\beta t^2}{(\alpha+\beta)^2}$

* Motion under gravity. (1D)



1) Time of ascent = Time of descent = $\dfrac{u}{g}$

2) Total Time = $\dfrac{2u}{g}$

3) Maximum Height = $\dfrac{u^2}{2g}$

4) Speed of projection = Speed of landing (at Same level)

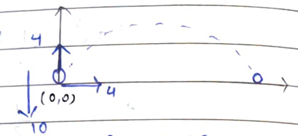
5) "In drop Case" Time of flight = $T = \sqrt{2H/g}$

Velocity with which it hits ground = $\sqrt{2gH}$

Q) The initial velocity of a particle of mass 2 kg is $(4\hat{i} + 4\hat{j})\,ms^{-1}$. A constant force of $-20\hat{j}$ N is applied on the particle. Initially the particle was at (0,0). Find the x-coordinate of the point where its y-coordinate is again zero.

→ ∴ $F = ma$

∴ $\vec{a} = \frac{-20\hat{j}}{2} = -10\hat{j}$



~~$S_y = u_y t + \frac{1}{2} a t^2$~~

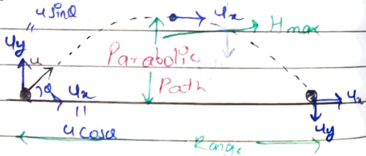
Time of flight $= \frac{2u}{g} = \frac{2\times4}{10} = 8/10$

∴ $0 = 4(8/10) - \frac{1}{2}(10)\left(\frac{8}{10}\right)^2$

$S_x = u_x t + \frac{1}{2} a_x t^2$ ∴ at $x = 3.2m$

$x = 4(8/10) = 3.2$

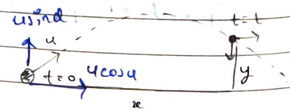
* Ground to Ground Projection :-



- $V_{\text{Top of projectile}} = V\cos\theta\,\hat{i} + 0\hat{j}$
- $a_x = 0$, $a_y = -g$
- at top point $\vec{v} \perp \vec{a}$
  ∴ $\vec{v}\cdot\vec{a} = 0$
- Same Horizontal level pe same speed.

Coordinate at any time.

∴ $S_x = u_x t + \frac{1}{2} a_x t^2$
$x = u\cos\theta\, t$ —— (i)



$S_y = u_y t + \frac{1}{2} a_y t^2$

∴ $y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$ —— (ii)

Equation of trajectory :

from (i) & (ii)

$x = u\cos\theta\, t$, $y = u\sin\theta\, t - \frac{1}{2} g t^2$

$t = \frac{x}{u\cos\theta}$ put here to eliminate 't'

∴ $y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g\left(\frac{x^2}{u^2\cos^2\theta}\right)$

∴ $\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$ ***

take $x\tan\theta$ comon

∴ $y = x\tan\theta\left(1 - \frac{g x^2}{2u^2\cos^2\theta \times \frac{\sin\theta}{\cos\theta}}\right)$

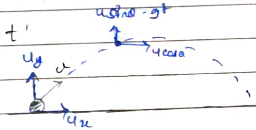
∴ $y = x\tan\theta\left(1 - \frac{g x}{2u^2\cos\theta\sin\theta}\right)$ → $\sin 2\theta$ & $R = u^2\sin 2\theta / g$

Another form

∴ $\boxed{y = x\tan\theta\left(1 - \frac{x}{R}\right)}$ ***

R = Range

* Velocity of particle at time 't'



- At any time the velocity along x-axis remains constant $u_x = u\cos\theta$
- for velocity at y-axis $V_y = u\sin\theta - gt$

1). Time of flight $= \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$ → $T_{ascent} = u\sin\theta/g$, $T_{decent} = u\sin\theta/g$

2) Maximum Height $= u_y^2/2g = \frac{u^2\sin^2\theta}{2g}$

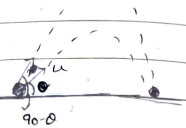
3). Range $=$ ~~2u~~ $u^2\sin 2\theta/g$

→ displacement in x, in time $= \frac{2u\sin\theta}{g}$

* Comparison of two projectile of Equal Range.

∴ for Complementary angle.

object lands at same place.



∴ let θ be first angle.

∴ $R_\theta = \frac{u^2 \sin 2\theta}{g}$, $T_\theta = \frac{2u\sin\theta}{g}$, $H_\theta = \frac{u^2 \sin^2\theta}{2g}$

Same

∴ Complementary Angle be $90-\theta$

∴ $R_{90-\theta} = \frac{u^2 \sin 2(90-\theta)}{g} = \frac{u^2 \sin(180-2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$

$H_{90-\theta} = \frac{u^2 \cos^2\theta}{2g}$, $T_{90-\theta} = \frac{2u\cos\theta}{g}$

***

∴ $\frac{T_\theta}{T_{90-\theta}} = \frac{2u\sin\theta/g}{2u\cos\theta/g} = \tan\theta$

$\frac{H_\theta}{H_{90-\theta}} = \frac{u^2\sin^2\theta/2g}{u^2\cos^2\theta/2g} = \tan^2\theta$

$H_\theta + H_{90-\theta} = \frac{u^2(\sin^2\theta + \cos^2\theta)}{2g}$

$H_\theta + H_{90-\theta} = u^2/2g$

$H_\theta + H_{90-\theta} = H_{max}$ of vertical projection.

$\frac{R_\theta}{R_{90-\theta}} = 1$

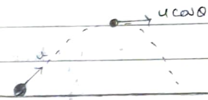
Range is maximum at $\theta = 45°$

& when $\theta = 45°$ $R_{max} = u^2/g$

& $H_{max} = u^2/4g = \frac{R_{max}}{4}$

* Kinetic Energy of a projectile.

K.E is minimum at minimum velocity i.e at top point



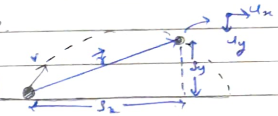
∴ $V_{min} = u\cos\theta$

∴ $KE_{min} = \frac{1}{2} m u^2 \cos^2\theta = K.E_i \cos^2\theta$

* Angular momentum of projectile.

∴ Angular momentum $= \vec{L} = \vec{r} \times \vec{p}$

$= m(\vec{r} \times \vec{v})$



∴ 1) $\vec{r} = S_x \hat{i} + S_y \hat{j}$

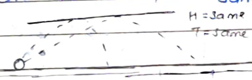
$= (u_x t)\hat{i} + (u_y t - \frac{1}{2} g t^2)\hat{j}$

2) $\vec{v} = V_x \hat{i} + V_y \hat{j}$

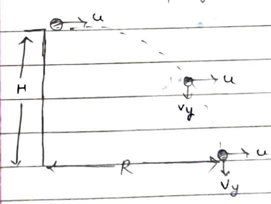
$= (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

* Important points

- $T = \frac{2u_y}{g}$, $H = \frac{u_y^2}{2g}$, $R = \frac{2u_x u_y}{g}$
- If time of flight of two projectile is same, ∴ H_{same}



* Horizontal projectile



acc. of the body $= -g\hat{j}$

along x - axis $F_x = 0$

$a_x = 0$

Velocity along x = constant

- due to acc. along y, with time $velocity_y$ ↑

$u_x = u$ $a_x = 0$

$u_y = 0$ $a_y = -g$

$S_x = u_x t + \frac{1}{2} a_x t^2$ — (i)

$x = ut$

$S_y = u_y t - \frac{1}{2} g t^2$

$y = -\frac{1}{2} g t^2$ — (ii)

from (i) & (ii)

$\boxed{y = -\frac{1}{2} g \frac{x^2}{u^2}}$ Parabolic Path.

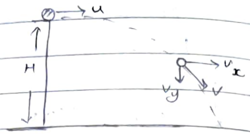
I). Velocity

at any time :-

$\vec{V}_t = u_x\hat{i} + V_y\hat{j}$

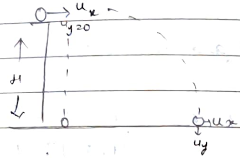
$= (u_x + \cancel{a_x t})\hat{i} + (\cancel{u_y} + a_y t)\hat{j}$

∴ $\vec{V} = u\hat{i} - gt\hat{j}$



II) Time of flight

$\vec{S}_y = \vec{u}_y t + \frac{1}{2}a_y t^2$



∴ $-H = -\frac{1}{2}gt^2$

∴ $\boxed{t = \sqrt{\frac{2H}{g}}}$

at $t = \sqrt{\frac{2H}{g}}$

∴ $\vec{v} = u\hat{i} - g\sqrt{\frac{2h}{g}}\hat{j}$

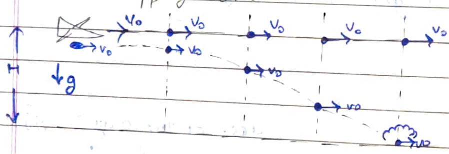
$\vec{v} = u\hat{i} - \sqrt{2gH}\hat{j}$

III) Horizontal Range

∴ $S_x = u_x t + \cancel{\frac{1}{2}a_x t^2}$

$R = u\sqrt{\frac{2H}{g}}$

* Plane dropping bomb.



$T = \sqrt{\frac{2H}{g}}$

Velocity of bomb, at time of drop = V_0

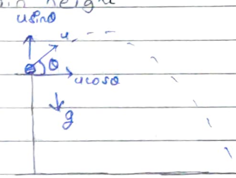
Agar plane const. speed mei raha toh, bomb uske neeche fatega

∴ Bomb neeche na explode ho isliye, plane → acc

↳ or change dir.

* Oblique Projection from a Certain height

i) from a height at θ angle.



∴ $u_x = u\cos\theta$ $\quad a_x = 0$

$u_y = u\sin\theta$ $\quad a_y = -g$.

∴ $V_x = u_x + a_x t$

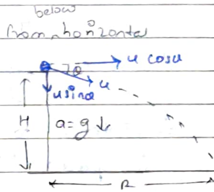
$S_x = u_x t + \frac{1}{2}a_x t^2$ } use with sign

$V_x^2 = u_x^2 + 2a_x S_x$

ii) Projection from a height at θ angle below from horizontal

∴ $u_x = u\cos\theta$ $\quad a_x = 0$

$u_y = -u\sin\theta$ $\quad a_y = -g$.



Q) The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed V_0, then

M 2019

a) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = 5/3\ ms^{-1}$

b) $\theta_0 = \cos^{-1}(2/\sqrt{5})$ and $v_0 = 3/5\ ms^{-1}$

c) ✓ $\theta_0 = \cos^{-1}(1/\sqrt{5})$ and $v_0 = 5/3\ ms^{-1}$

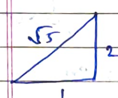
d) $\theta_0 = \sin^{-1}(2/\sqrt{5})$ and $v_0 = 3/5\ ms^{-1}$

$y = 2x - 9x^2$

∴ $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

∴ $\tan\theta = 2$

$\frac{g}{2u^2\cos^2\theta} = 9$



∴ $\tan\theta = \frac{P}{B} = \frac{2}{1}$

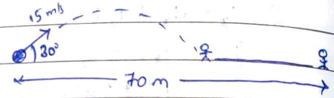
∴ $\cos\theta = 1/\sqrt{5}$

$\sin\theta = 2/\sqrt{5}$

$\frac{10}{2v^2(1/5)} = 9$

$\frac{25}{v^2} = 9$, ∴ $v = \frac{5}{3}$

Q) A batsman hits a ball at an angle of 30° to the horizontal with an initial speed of 15 m/s. A fielder 70 m away in the direction of the ball hit starts immediately to catch the ball. The speed with which the fielder should run so as to catch the ball just before it touches the ground is.



Time of flight of ball :

$T = \frac{2u\sin\theta}{g} = \frac{2 \times 15 \times 1/2}{10}$

$T = 3/2$ sec

Range $= \frac{u^2 \sin 2\theta}{g} = \frac{15 \times 15 \times \sqrt{3}}{10 \times 2} = \frac{45\sqrt{3}}{4} = 19.46$

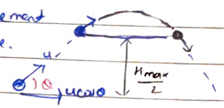
∴ distance to travel $= 70 - 19.46 = 50.54$

∴ Speed $= \frac{50.54}{3/2} = 33$ m/s

Q) Find the average velocity of a projectile betⁿ the instant it crosses half the maximum height. It is projected with a speed u at an angle θ with the horizontal.

HCV

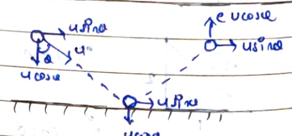
Average Velocity = $\frac{\text{Avg displacement}}{\text{Total time}}$



∴ disp $= (u\cos\theta) \cdot t$

∴ $V_{avg} = \frac{u\cos\theta\, t}{t} = u\cos\theta$

Collision Concept



e = coefficient of restitution

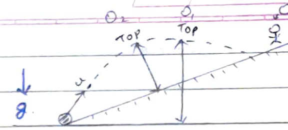
$V_{\parallel}$ to surface = Same

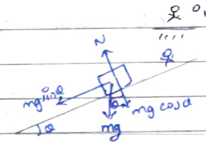
$V_{\perp}$ to surface = $eV_{\perp\,hit}$

* Incline motion

$T = \frac{2u_y}{a_y}$

$H = \frac{u_y^2}{2a_y}$ $\qquad R = \frac{2u_x u_y}{a_y}$





∴ w.r.t to inclined observer

$a_y = -g\cos\alpha$, $a_x = -g\sin\alpha$

α = angle of incline

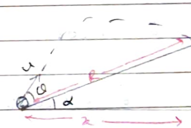
θ = angle of projection

∴ $\boxed{t_a = \frac{u\sin\theta}{g\cos\alpha}}$, $\boxed{T = \frac{2u\sin\theta}{g\cos\alpha}}$ $\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g\cos\alpha}}$

$R = S_x = u_x t + \frac{1}{2} a_x t^2$

$\boxed{R = u\cos\theta (T) - \frac{1}{2} g\sin\alpha (T)^2}$

Alternate :



observe from ground :

$u_x = u\cos(\theta + \alpha)$, $a_x = 0$

∴ $S_x = u_x t + \frac{1}{2} a_x t^2$

$x = u\cos(\theta+\alpha)\left(\frac{2u\sin\theta}{g\cos\alpha}\right) = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos\alpha}$

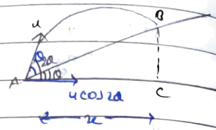
∴ $\cos\alpha = \frac{B}{H} = \frac{x}{R}$ ∴ $\boxed{R = \frac{x}{\cos\alpha} = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}}$

To find θ for max. Range ⇒ $\frac{dR}{d\theta} = 0$

- Jis line ke along acc. lag rahi hai, uske opp. mein jaye.
- angle betⁿ surface & acc opp direction find karye
- ∴ $\theta_{\text{for max. Range}} = \frac{\text{Angle betⁿ surface \& acc. opp.}}{2}$

Q1. A particle is projected from point A on plane AB, so that AB = $(2u^2 \tan\theta)/g\sqrt{3}$ in the figure as shown. If u is velocity find θ.

Adv



$\therefore AB = \text{Range} \cancel{\frac{2u_x u_y}{g}}$

$\therefore T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g\cos\theta}$

$\therefore x = u\cos(2\theta)(T)$ $\qquad \therefore \cos\theta = \frac{x}{R}$

$x_{ground} = \frac{u\cos(2\theta) \cdot 2u\sin\theta}{g\cos\theta}$ $\qquad \therefore R = \frac{2u^2\tan\theta \cdot \cos(2\theta)}{g \cdot \cos\theta}$

$\therefore R = \frac{2u^2\tan\theta}{g\sqrt{3}} = \frac{2u^2\tan\theta \cdot \cos 2\theta}{g\cos\theta}$

$\therefore \cos 2\theta = \frac{\cos\theta}{\sqrt{3}} \Rightarrow \cos^2\theta - \sin^2\theta = \frac{\cos\theta}{\sqrt{3}}$

$= \cos^2\theta(1 - \cos^2\theta) = \frac{\cos\theta}{\sqrt{3}}$

$\therefore 2\cos^2\theta - 1 = \frac{\cos\theta}{\sqrt{3}}$

$\therefore 2\sqrt{3}\cos^2\theta - \cos\theta - \sqrt{3} = 0$, put $\cos\theta = x$

$2\sqrt{3}x^2 - x - \sqrt{3} = 0$, $\qquad \therefore x = \frac{1 \pm \sqrt{1 - 4(2\sqrt{3})(-\sqrt{3})}}{2 \cdot 2\sqrt{3}}$

$x = \frac{1 \pm \sqrt{25}}{4\sqrt{3}} = \frac{1 \pm 5}{4\sqrt{3}} \Rightarrow x = \frac{1+5}{4\sqrt{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore \theta = 30^\circ$

*** Adv Ques. in Lec-07

* Relative Velocity :- Velocity w.r.t observer

↳ Jisko observer banana hai, uski velocity (-ve) karke sabko dedo.

& uski acc. ko bhi -ve karke sabko dedo.

, use Vectors.

A → V_A $\qquad$ B → V_B $\qquad \therefore \vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

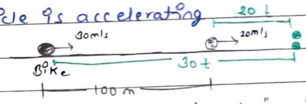
Velocity of A w.r.t B

*** $V_{\text{observer w.r.t himself}} = 0$

$acc._{\text{observer w.r.t himself}} = 0$

* Concept of Catching or Overtaking.

Case 1 : when no particle is accelerating



1). w.r.t. ground :- In time t, to catch car

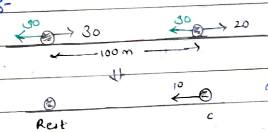
distance by bike = Gap + dist. by car

$\therefore 30t = 100 + 10t$

$\therefore \underline{t = 10\,s}$

Same time pe Same position

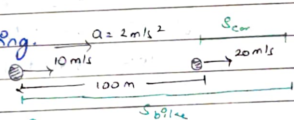
11). w.r.t Bike :-



$\therefore S = ut$

$t = 100/10 = 10\,sec$

Case II : when one particle is accelerating.



1). w.r.t ground

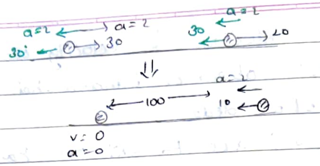
$\therefore S_{bike} = \text{Gap} + S_{car}$

$ut + \frac{1}{2}at^2 = 100 + (ut + \cancel{\frac{1}{2}at^2})$

$10t + \frac{1}{2} \cdot 2 \cdot t^2 = 100 + (2t)$

$= 10t + \frac{1}{2}t^2 = 100 + 20t \qquad \therefore t^2 - 10 - 100 = 0$

11). w.r.t bike



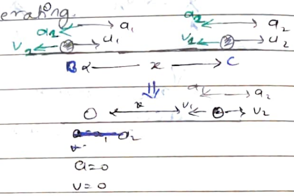
$\therefore S = ut + \frac{1}{2}at^2$

$\therefore$

$-100 = -10t - \frac{1}{2}(2)t^2 \Rightarrow \therefore t^2 - 10 - 100 = 0$

Case 3 : when both particle are accelerating.

1). w.r.t to bike



$\therefore a_{rel} = a_2 - a_1$

$V_{rel} = V_2 - V_1$

$\therefore$

$V_{rel} = u_{rel} + a_{rel} t$

$S_{rel} = u_{rel} t + \frac{1}{2} a_{rel} t^2$

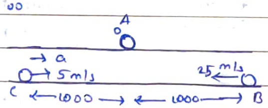
$V^2_{rel} - u^2_{rel} = 2a_{rel} s_{rel}$

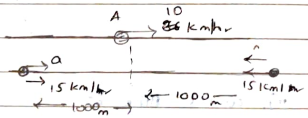
Q) Two lane road car A with speed ~~36~~ 10 km/hr, car B & C approaches A in opposite dirn. Minimum acc. C needed to take over A before B.

NCERT

$\therefore$ putting car A at rest

$\therefore$





$\therefore$ time for B to reach A $= \frac{1000}{25}$ $= 40$ sec

$\therefore$ C has to overtake in 40 sec.

$\therefore S = ut + \frac{1}{2}at^2$

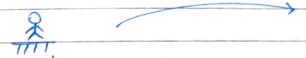
$1000 = 5t + \frac{1}{2}at^2 \Rightarrow 1000 = 200 + \frac{1}{2}a(\cancel{1600}\ 800)$

$\therefore$

$800 = 800a$

$a = 1 m/s^2$

* Concept of Relative Velocity in Motion under gravity

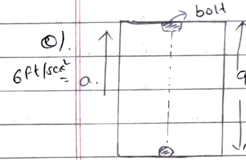


observer = always at rest.

$a_{relative}$ of two particle under gravity $= 0$

cond? : find relative velo. & acc. of all w.r.t observer

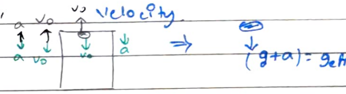
Q). find time for bolt to hit ground



at time of drop both will have same velocity

1 m = 3.2 feet

$g = 32$ ft/s²



$(g+a) = 9.6ft$

$\therefore S = \cancel{ut}^0 + \frac{1}{2}at^2$

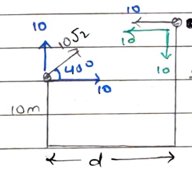
$S = \frac{1}{2} \times 32 \times t^2$

9.5

$\therefore t = \sqrt{\frac{9.5 \times 2}{32}}$

$t = \frac{1}{\sqrt{2}}$

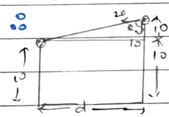
Q)



what should be d so that they collide

let A be observer

$\therefore$



$\therefore \tan\theta = \frac{d}{10} = \frac{20}{10}$

$\therefore d = 20$

* Rain - Man Problems

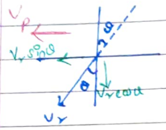
$\vec{V}_r = \vec{V}_{rain\ w.r.t\ gnd.}$

$\vec{V}_p = \vec{V}_{Person\ —"—}$

$\vec{V}_{rp} = \vec{V}_r - \vec{V}_p$

Jis direction mein rain hongi uske opposite dir mei chata honga.

Q) If rain is already falling at an angle θ with the vertical and an observer is moving horizontally with a velocity $\vec{V}$, and an observer is moving horizontally with speed $\vec{V}_m$. Find that the rain drop are hitting on his head vertically downwards in which condition.



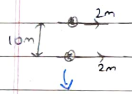
∴ for rain to be hit vertically down the x component of rain should be cancelled.

∴ V_p bhi $V_r \sin\theta$ ke equal rahna chahiye (direction bhi same), take uska aur rain ke x component ke beech relative motion zero ho.

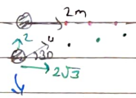
∴ $V_p = V_r \sin\theta \, (-\hat{i})$

* Concept of Collision :

Same time par
Same position



∴ time of collision, $t \to \infty$



yaha bhi, collision kabhi nhi honga.
dono sirf cross karenge.

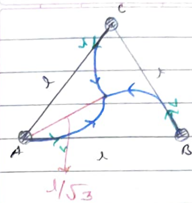
Time for crossing = $\dfrac{d_{approach}}{V_{approach}} = \dfrac{10}{2} = 5$ sec

∴ for collision
Parallel component should be same.

$V_{\text{relative } \parallel^n \text{ dir.}} = 0$ ← Same dir bhi rehna chahiye

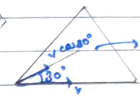
Velocity of approach > 0

Q)



A → B ko approach karega
B → C
C → A

∴ $t = \dfrac{d_{app.}}{V_{app.}} = \dfrac{(l/\sqrt{3})}{\left(\dfrac{V\sqrt{3}}{2}\right)} = \dfrac{2l}{3V}$

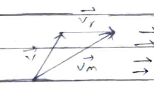


∴ $V_{app} = V\left(\dfrac{\sqrt{3}}{2}\right)$

* River - Boat (Problems)

If man is crossing the river non-collinearly, then use vector

∴ $\vec{V}_m = \vec{V} + \vec{V}_r$



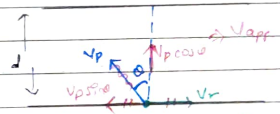
- Use vector addition
- (or) using $\hat{i}, \hat{j}, \hat{k}$

- for shortest path
for shortest distance.

$V_{net \text{ along } x} = 0$

∴ $V_p \sin\theta = V_r$

$\sin\theta = V_r / V_p$



time to cross River $= t = \dfrac{d_{app}}{V_{app}}$

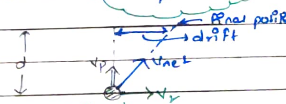
$t = d / V_p \cos\theta = \dfrac{d}{V_p\sqrt{1-\sin^2\theta}} = \dfrac{d}{V_p\sqrt{1-\frac{V_r^2}{V_p^2}}}$

$t = \dfrac{d}{\sqrt{V_p^2 - V_r^2}}$

- for minimum time

\# from previous code

$t_{min} = \dfrac{d}{V_p \cos\theta}$ ∴ $\theta = 0$ (vertically)
↳ max. (or) $\theta = 90°$ (Horizontally / ~~parallel~~ w.r.t river).



∴ drift = motion along River in total time.

$S_x = u_x t + \frac{1}{2} a_x t^2$

$\boxed{\text{drift} = V_r \left(\dfrac{d}{V_p}\right)}$

Q). A boat sails at a speed $v_1 = (3\hat{i} + 4\hat{j})$ km/hr relative to water. If the water flows with a speed $v_2 = \hat{i}$ km/hr. If the river width $\vec{d} = 100\,m\hat{j}$ then.

a). Trajectory of boat is straight line

b). Time of crossing the river is 2.5 min.

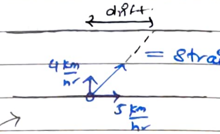
c). Angle of heading is $\tan^{-1}(4/3)$ with dir^n of flow, relative to water.

d). Drift of boat in the direction of flow is 200 m.

$\rightarrow \quad \vec{V}_{BW} = \vec{V}_B - \vec{V}_\omega$

$3\hat{i} + 4\hat{j} = \vec{V}_B - (\hat{i})$

$4\hat{i} + 4\hat{j} = \vec{V}_B$



river ka $\hat{i}$

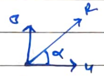
w.r.t ground

a) Velocity of boat = $5\hat{i} + 4\hat{j}$

b) Velocity of approach = 4 km, dist. = 100 m

$\therefore \quad t = \dfrac{100}{4 \times \frac{5}{18} \times 60} \quad \dfrac{\cancel{18} \times \cancel{100}}{\cancel{20} \times \cancel{60}} = \dfrac{3}{2} = 1.5\ min$

c) with respect to water, $\vec{V} = 3\hat{i} + 4\hat{j}$



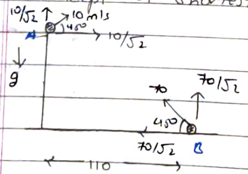
$\therefore \tan\alpha = \left(\dfrac{4}{3}\right)$

d) drift is due to x component

$\therefore S_x = u_x t + \cancel{\frac{1}{2} a_x t^2}^{\,0} \qquad t = 1.5\ min = 90\ sec.$

$x = (5)(90)\left(\dfrac{5}{18}\right) = 125\ m$

*Q). Concept of Shortest distance :

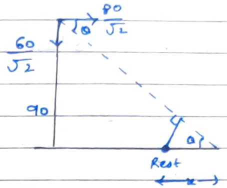


find closest dist. of approach.

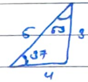
let B be observer

$\therefore V_{AB} = 10/\sqrt{2}\,\hat{i} + 10/\sqrt{2}\,\hat{j} - (-70/\sqrt{2}\,\hat{i} + 70/\sqrt{2}\,\hat{j})$

$= \dfrac{80}{\sqrt{2}}\hat{i} - \dfrac{60}{\sqrt{2}}\hat{j}$



$\therefore \tan\theta = \dfrac{80}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{60} = \dfrac{3}{4}$



$\tan\theta = \dfrac{90}{110 + x} = \dfrac{3}{4}$

$x = 10\ m.$

$\therefore \sin\theta = \dfrac{r_\perp}{x} = \sin 37 = \dfrac{r_\perp}{10}$

$r_\perp = 10 \times \dfrac{3}{5} = 6$

Newton laws of Motion.

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* Inertia.
"Capacity of a body to maintain its state of Rest, Motion & direction".

Inertia of Rest | Inertia of Motion | Inertia of direction

[Inertia $\propto$ mass]

* force : push or pull, which can change the state of a body.

⊙ four types of ~~body~~ forces
- → Electromagnetic force (Tension, spring, friction, Normal)
- → Gravitational force
- → Strong Nuclear force (Inside Nucleus)
- → weak Nuclear force (interaction betn α & β)

* Momentum : The total quantity of motion possessed by a moving body is known as the momentum of body.
$\vec{p} = m\vec{v}$

· follows vector Addition.

* Newton's first law (Galileo's law of inertia)
A body will maintain its state of Rest / Motion / direction until & unless net external force Acts on it.

* Newton's Second law.
$\vec{F}_{net\ external} = \frac{d\vec{P}}{dt} \Rightarrow F_{ext} = \frac{d(mv)}{dt}$

$F_{ext} = m\frac{dv}{dt} + v\frac{dm}{dt}$

· m = const. (Block Problems)
$\therefore \frac{dm}{dt} = 0, \quad F_{ext} = m\frac{dv}{dt}$
$F = ma$

· v = constant (Fluid)
$\therefore \frac{dv}{dt} = 0, \quad F = v\frac{dm}{dt}$

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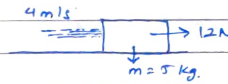
$F_{avg} = \frac{P_f - P_i}{t_f - t_i}$ = Area of graph, $F_{inst} = \frac{dP}{dt}\Big|_{t=t_1}$ = Slope of graph

Q) A 5 kg block is resting on a frictionless plane. It is struck by a jet, releasing water at the rate of 3 kg/s emerging with a speed of 4 m/s. calculate the initial acceleration of the block.



→ for water $\frac{dm}{dt} = 3\ \frac{kg}{s}$

$\therefore F = \frac{dm}{dt} \cdot v = 3 \times 4 = 12\,N$

$\therefore a = \frac{F}{m} = \frac{12}{5} = 2.4\ m/s^2$

* Impulse Momentum Theorm.
when $F_{net\ ext}$ acts on body for a certain time, momentum of body changes

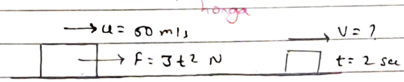
$F = \frac{dP}{dt}$

$\int F\,dt = \int dP$

$\int_{t_1}^{t_2} F\,dt = P_f - P_i$

$\| \vec{P_i} + \int \vec{F}dt = \vec{P_f} \|$

(+) Jab speed inc. honga   (−) Jab speed decrease honga



Q)

$\therefore a = \frac{F}{m} = \frac{3t^2}{2}$

$\frac{dv}{dt} = \frac{3t^2}{2}$

$\int_{50}^{v} dv = \int_0^2 \frac{3t^2}{2}$

$v - 50 = \frac{\cancel{3}}{2}\left(\frac{t^3}{\cancel{3}}\right)_0^2 \Rightarrow v = 50 + 4 = 54$

Impulse Theorm

$P_i + \int_0^2 F\,dt = P_f$

$2 \times 50 + \int_0^2 3t^2 = 2v_f$

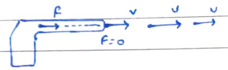
$100 + \cancel{3}\left(\frac{t^3}{\cancel{3}}\right)_0^2 = 2v_f$

$100 + 8 = 2v_f$

$\frac{108}{2} = v_f = 54$

Q). A bullet is fired from a gun. The force on the bullet is given by $F = 600 - 2\times10^5 t$, where F is newton and t in second. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?

*** Momentum, jab tak lagenga, jab tak force lagenga.

→ 

$\therefore F = 600 - 2\times10^5 t$

$0 = 600 - 2\times10^5 t$

$\dfrac{600}{2\times10^5} = t = \underline{3\times10^{-3} s}$

$\therefore \Delta P = P_f - P_i = \int F\,dt = \int_0^{3\times10^{-3}} (600 - 2\times10^5 t)\,dt$

$\Delta P = \int_0^{3ms} 600\,dt - 2\times10^5 \int_0^{3ms} t\,dt$

$= 600 \times 3\times10^{-3} - \cancel{2}\times10^5 \left(\dfrac{t^2}{\cancel{2}}\right)_0^{3ms}$

$= 600 \times 3\times10^{-3} - 9\times10^5\times10^{-6}$

$= 1.8 - 0.9$

$= \underline{0.9\ Ns}$

---

* Newton's third law of Motion.

- For every action, there is equal & opposite Reaction. These action - Reaction always acts on diff. bodies.

- Internal forces : "when Action / Reaction are inside System".
  They cancel out each other, they cannot move / ~~shift~~ the System.
- External Forces : only $F_{net\ ext}$ can move the System.

* free body diagram.

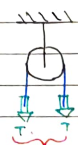
weight | Normal Rxn | String force | Spring force.

1). weight : weight of body is due to gravitational pull from planet, $W = mg$ ← always fix
- Weighing machine, normal force measure karti.
- always vertically down.

2). Normal Rxn : • Contact force
- Always $\perp^r$ to surface.
- It is push force bet^n surfaces (Dv koi wali force)

3). Tension force :
Thread → Inextensible (l = const) & ~~massless~~ massless (mass = 0)

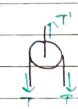
- Tension is a pull force



IR Pulley is massless & frictionless, ∴ Pulley is not rotating

- 1 String 1 tension.

IR massless :



∴ Rest / const. velocity / Acc. } $T' = 2T$

4) Spring force : = Coiled wire

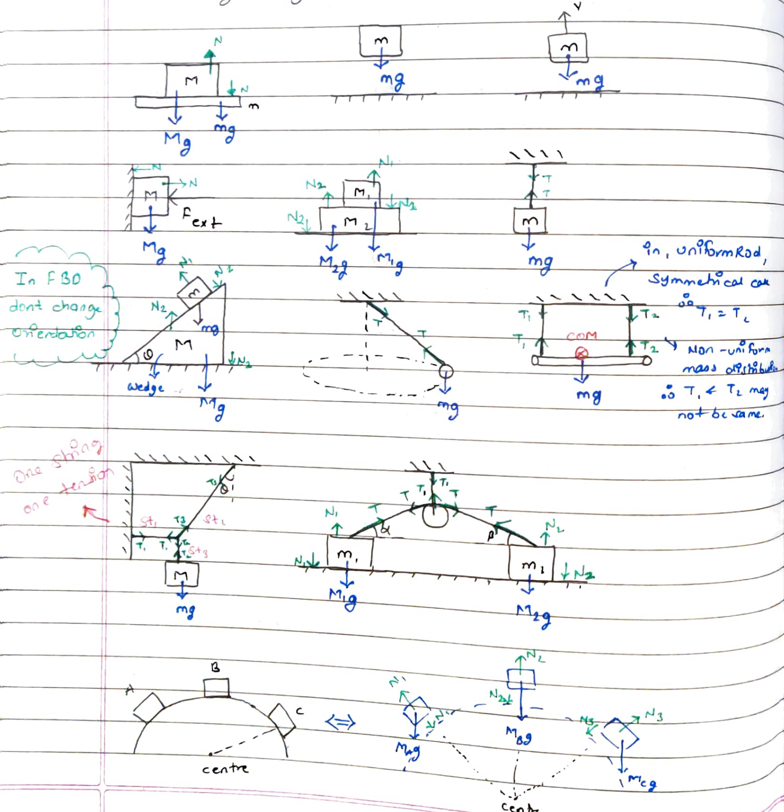
when spring is in it's natural length, $\therefore F_s = 0$

when compressed or elongated

$\therefore F_s \propto -x$ → Always opposite to 'x'!

$F_s = -Kx$

* Free body Diagrams :-

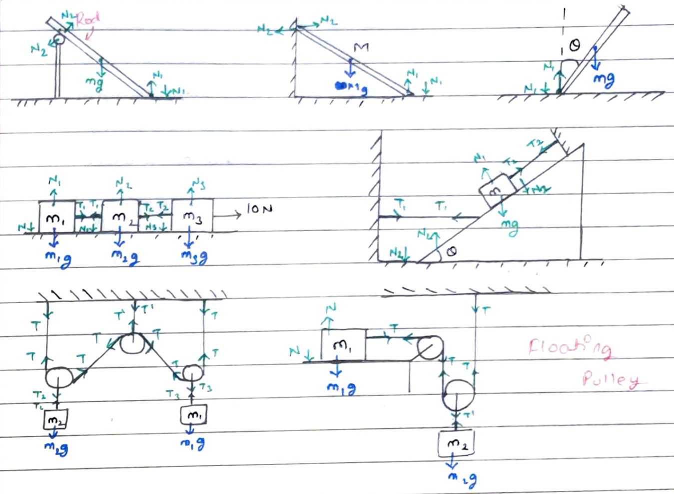


In FBD dont change orientation

in uniform Rod, symmetrical case $\therefore T_1 = T_2$

Non-uniform mass distribution $\therefore T_1 \neq T_2$ may not be same.

One string one tension



* Equilibrium :-

State of a body in which body is either at Rest or moving with Constant velocity || acc. = 0 || $\therefore \vec{F}_{net\,ext} = 0$ ***

Lec-03

1). $m = 0.3$ kg

$F = -Kx = -15x$

$x = 20$ cm

$F = -15(20) \times 10^{-2} = -300 \times 10^{-2}$

$\therefore ma = -3$

$a = -3/0.3 = -10\,m/s^2$

Q). which of the following sets of concurrent forces may be in equilibrium ?

Vectors

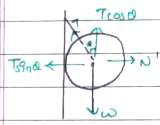
a). $f_1 = 3N$, $f_2 = 5N$, $f_3 = 9N$ — closed polygon $R = 0$

b). $f_1 = 3N$, $f_2 = 5N$, $f_3 = 1N$ — 3 side Polygon = Triangle

c). $f_1 = 3N$, $f_2 = 5N$, $f_3 = 15N$ — || Sum of 2 sides $\geq$ Third side ||

d). $f_1 = 3N$, $f_2 = 5N$, $f_3 = 6N$ — for all pairs ***

Q).

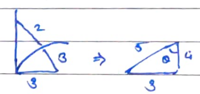


is in equilibrium, find tension.

$R = 3m$, $l = 2m$, $w = w$

$\therefore T\sin\theta = N$

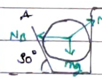
$T\cos\theta = w$



$\therefore T(4/5) = w$

$T = 5w/4$

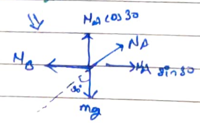
Q).



$m = 50$ kg, find contact force at A and B

$\therefore N_B = N_A \sin 30$

& $N_A \cos 30 = mg = 500$



$N_A = 500 \times \left(\frac{2}{\sqrt3}\right)$, $\therefore N_A = \frac{1000}{\sqrt3}$

$N_B = \left(\frac{1000}{\sqrt3}\right)\left(\frac{1}{2}\right) = \frac{500}{\sqrt3}$

Q). As shown in the figure, two equal masses each of 2kg are suspended from a spring balance. The reading of the spring balance will be.

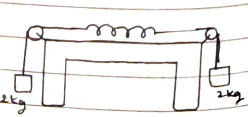
a) Zero

b) 2kg

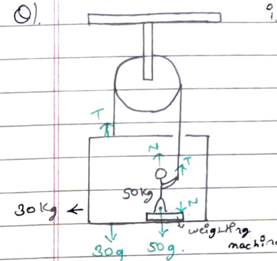
c) 4 kg

d) betn 0 & 2kg

A spring balance shows the reading applied at both the ends equally, $\therefore$ to keep the balance at equilibrium $\therefore$ It will show 2kg.



Q).



is the box is at rest, the weight shown by machine.

$\therefore$ w.r.t to men.

$T + N = 500$

$N + 300 = T$

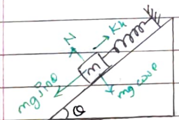
$\therefore 300 + N + N = 500$

$2N = 200$

$N = 100\,N$

Spring Question

Q).

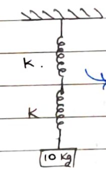


find max. Elongation

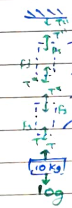
At equilibrium $Kx = mg\sin\theta$

$x = \frac{mg\sin\theta}{K}$

Q).



Reading = ?



$T'' = 10g$

$F_s = 10g$

$T' = 10g$

$F_s = 10g$

$T = 10g$

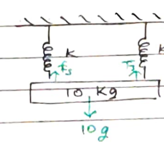
$\therefore$ both will show Same Reading.

$F = 100N$ or 10 kg.

If spring identical $\therefore x \rightarrow$ both same

— " — different $\therefore x \rightarrow$ different.

Q).



$\therefore 2F_s = 10$

$F_s = 5N$

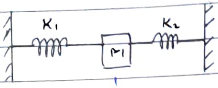
$\therefore$ both will repeat 5kg.

$\therefore$ for identical spring in Parallel, is k is same

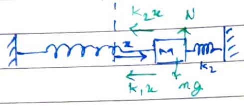
$\therefore$ reading is same

Reading = $\frac{\text{applied load}}{\text{no. of balance}}$

(Q) HCV



Both springs are at Natural length. If block is shifted by distance x towards Right & Released. Find initial acc.



$\therefore$ $ma = (K_1 + K_2)x$

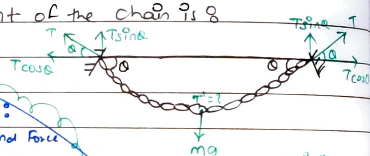
$a = \frac{(K_1 + K_2)x}{m}$

Advance level :

(Q). A flexible chain of mass m hangs betⁿ two fixed point A and B at the same level. The inclination of the chain with the horizontal at the two point of support is θ. The Tension at the mid point of the chain is :

$\Sigma f_y = 0$, $\Sigma f_x = 0$

$\therefore T\sin\theta = \frac{M}{2}g$

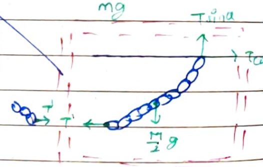


$T\cos\theta = T'$

$\therefore \tan\theta = \frac{Mg}{2T'}$

$\therefore \boxed{T' = \frac{Mg}{2\tan\theta}}$ Ans.

concept : kabhi internal force nikalna hai, to System ko half karke equation banao warna sab cancel honga



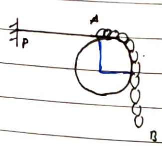
(Q) A chain of mass per unit length λ and length 1.5 m rests on a fixed smooth sphere of radius R = 2/π m. Such that A end of chain is at the Top of sphere while the other end is hanging freely. Chain is held stationary by a horizontal thread PA. Then Tension in thread is :

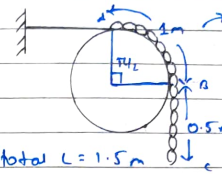
(a) $\lambda g(1/2 + 2/\pi)$

b) $\lambda g(\pi/2 + 2/\pi)$

c) $\lambda g(2/\pi)$

d) None of these





$\rightarrow \frac{\text{Arclength}}{R} = \theta$ $\therefore$ Arc. length $= \frac{\pi}{2} \cdot \frac{2}{\pi} = 1m$

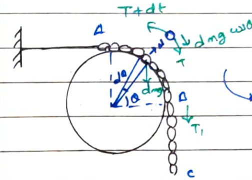
total L = 1.5 m

λ = linear mass density

$\therefore dM = \lambda\, dx$

mass of length BC $= \lambda(1/2)$

$= \lambda/2$



$\therefore T_1 =$ weight of hanging $= \lambda g/2$

$\therefore \cancel{T} + dt = \cancel{T} + dmg\cos\theta$

$dt = dmg\cos\theta$

$\int_{T_1}^{T_2} dt = \int_{0}^{\pi/2} \frac{\lambda 2 g\, d\theta}{\pi} \cos\theta$

$R\,d\theta = \frac{2}{\pi}d\theta$

$dm = \lambda R\, d\theta = \frac{\lambda 2 d\theta}{\pi}$

$T_2 - T_1 = \frac{2\lambda g}{\pi}[\sin\theta]_0^{\pi/2}$

$T_2 = T_1 + \frac{2\lambda g}{\pi}$

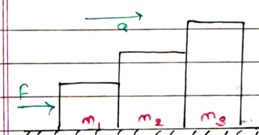
$T_2 = \frac{\lambda g}{2} + \frac{2\lambda g}{\pi} = \lambda g\left[1/2 + 2/\pi\right]$

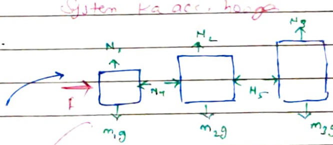
* Problems on Motion :-

whenever body is in motion (acc.)

$\therefore \vec{F}_{net\,ext} = m.\vec{a}$ → Jidhar net ext. hoga, udhar hi System ka acc. hoga

Three bodies in contact.





$F - N_4 = m_1 a$

$N_4 - N_5 = m_2 a$

$N_5 - 0 = m_3 a$

$F = (m_1 + m_2 + m_3)a$

$\therefore F = (m_1 + m_2 + m_3)a$

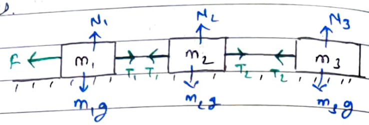
$a = \frac{F}{m_1 + m_2 + m_3}$ Common Acc

Normal force

$\| N = F_{net}$ (mass of body which is pushed in same dirⁿ).

M_T

* Three Connected Bodies.



$\therefore$ $F_1 - T_1 = m_1 a$

$T_1 - T_2 = m_2 a$

$T_2 - 0 = m_3 a$

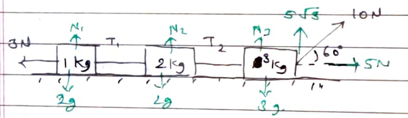
$F = (m_1 + m_2 + m_3)a$

$T_2 = m_3 a = \dfrac{m_3 F}{m_1 + m_2 + m_3}$, $T_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$

String :

$\| T = \dfrac{F_{net} \text{ (mass Jisko pull kiya hai in dir. of motion)}}{\text{Mass Total}} \|$

Q).



find T_1 & T_2

$\therefore$ $1g = N_1$

$2g = N_2$

$3g = N_3 + 5\sqrt{3}$

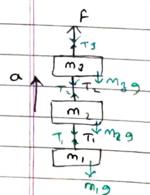
$N_3 = 5\sqrt{3} - 3g$

$a = \dfrac{5-3}{6} = \dfrac{2}{6} = \dfrac{1}{3}$ m/s²

$T_1 = \dfrac{F_{net}(m_{pull})}{M_T} = \dfrac{2(1)}{6} = \dfrac{1}{3}$ N

$T_2 = \dfrac{F_{net}(m_{pull})}{M_T} = \dfrac{2(3)}{6} = 1$ N

* Bodies Accelerating Vertically Upwards.



$F - T_2 - m_3 g = m_3 a$

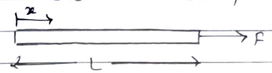
$T_2 - T_1 - m_2 g = m_2 a$

$T_1 - m_1 g = m_1 a$

$F - (m_1 + m_2 + m_3)g = (m_1 + m_2 + m_3) a$

$\therefore$ $a = \dfrac{F - (m_1 + m_2 + m_3)g}{m_1 + m_2 + m_3}$

* Tension in a Rod | Heavy rod. | Heave rope.



Total mass = M

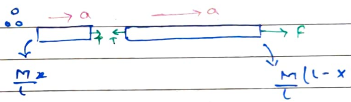
find tension in rod at a distance = x from other end.

Concept : whenever you have to find Tension inside wire, Rod, break the system.

$\therefore$ L length → M

1 → M/L

x → M/L.(x)



$\therefore$ $F - T = \dfrac{M(L-x)a}{L}$

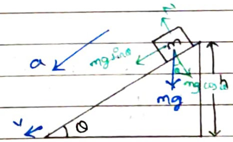
$T = \dfrac{M(x) \cdot a}{L}$ $\therefore$ $T = \dfrac{Mx}{L} \cdot \dfrac{F}{M} = \dfrac{Fx}{L}$

$F = Ma$

$a = \dfrac{F}{M}$

* Wedge Problems

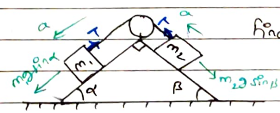
Acc. of block = $g\sin\theta$



$\therefore$ ⊥ is in equili. $\therefore$ $N = mg\cos\theta$

If $u = 0$, $\therefore$ $v = \sqrt{2gh}$

Q).



find acc. of Block.

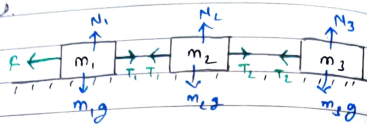
$\therefore$ $T - m_2 g\sin\beta = m_2 a$

$m_1 g\sin\alpha - T = m_1 a$

$m_1 g\sin\alpha - m_2 g\sin\beta = (m_1 + m_2)a$

$a = \dfrac{m_1 g\sin\alpha - m_2 g\sin\beta}{(m_1 + m_2)}$ $\left(\dfrac{F_{helping} - F_{opposing}}{M_T}\right)$

* Three Connected Bodies.



$\therefore$ $F_1 - \cancel{T_1} = m_1 a$

$\cancel{T_1} - \cancel{T_2} = m_2 a$

$\cancel{T_2} - 0 = m_3 a$

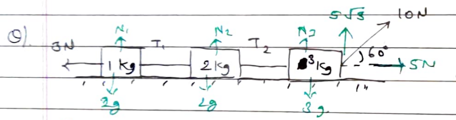
$F = (m_1 + m_2 + m_3) a$

$T_2 = m_3 a = \dfrac{m_3 F}{m_1 + m_2 + m_3}$, $T_1 = \dfrac{(m_2 + m_3) F}{m_1 + m_2 + m_3}$

String :-

|| $T = \dfrac{F_{net}\ (\text{mass Jisko pull Kiya hai in dir. of motion})}{\text{Mass}_{Total}}$ ||

Q1.



find T_1 & T_2

$\therefore$ $1g = N_1$ $\quad a = \dfrac{5-3}{6} = \dfrac{2}{6} = \dfrac{1}{3}$ m/s

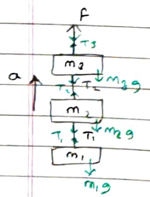
$2g = N_2$

$3g = N_3 + 5\sqrt{3}$ $\quad T_1 = \dfrac{F_{net}\ (m_{pull})}{M_T} = \dfrac{2(1)}{6} = \dfrac{1}{3}$ N

$N_3 = 5\sqrt{3} - 3g$

$T_2 = \dfrac{F_{net}\ (m_{pull})}{M_T} = \dfrac{2(3)}{6} = 1$ N

* Bodies Accelerating Vertically Upwards.



$F = \cancel{T_2} - m_3 g = m_3 a$

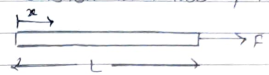
$\cancel{T_2} - \cancel{T_1} - m_2 g = m_2 a$

$\cancel{T_1} - m_1 g = m_1 a$

$F - (m_1 + m_2 + m_3) g = (m_1 + m_2 + m_3) a$

$\therefore$ $a = \dfrac{F - (m_1 + m_2 + m_3) g}{m_1 + m_2 + m_3}$

* Tension in a Rod / Heavy rod. / Heavy rope.



Find tension in rod at a distance x from other end.

Total mass = M

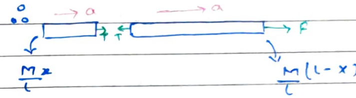
Concept :- whenever you have to find Tension inside wire, Rod, break the system.

$\therefore$ L length $\rightarrow$ M

$1 \rightarrow M/L$

$x \rightarrow M/L.(x)$

$\therefore$



$\therefore$ $F - T = \dfrac{M}{L}(L-x)\,a$

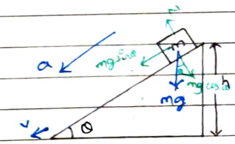
$T = \dfrac{M}{L}(x).a$ $\quad \therefore T = \dfrac{Mx}{L}.\dfrac{F}{M} = \dfrac{Fx}{L}$

$F = Ma$

$a = \dfrac{F}{M}$

* Wedge Problems

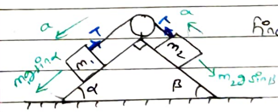
Acc. of block $= g\sin\alpha$



$\therefore$ $\perp$ is in equili. $\therefore$ $N = mg\cos\alpha$

If $u = 0$, $\therefore$ $v = \sqrt{2gh}$

Q1.



Find acc. of Block.

$\therefore$ $T - m_2 g\sin\beta = m_2 a$

$m_1 g\sin\alpha - T = m_1 a$

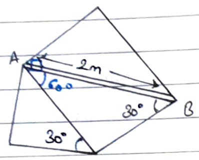
$m_1 g\sin\alpha - m_2 g\sin\beta = (m_1 + m_2) a$

$a = \dfrac{m_1 g\sin\alpha - m_2 g\sin\beta}{(m_1 + m_2)}$ $\left(\dfrac{F_{rejane\ wala} - F_{rokne\ wala}}{M_T}\right)$

Q) Smooth diagonal groove, find time to reach B.

$u=0$

level up



$(g\sin 30)\cos 60$ = lejane wala acc

$\therefore a = g\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{g}{4} = \frac{5}{2}$

from $s = ut + \frac{1}{2}at^2$

$2 = \frac{1}{2}\left(\frac{5}{2}\right)t^2$

$t = \sqrt{\frac{8}{5}}$ sec

* Atwood Machine problems (Pulley).

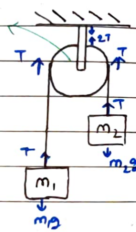
massless & frictionless

$m_1 g - T = m_1 a$

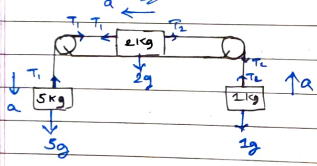
$T - m_2 g = m_2 a$

$m_1 g - m_2 g = (m_1 + m_2)a$

$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{F_{net}}{M_{net}}$



Q)



$\therefore 50 - T_1 = 5a$

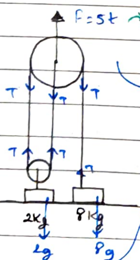
$T_1 - T_2 = 2a$

$T_2 - 10 = 1a$

$4a = 8a$

$a = 5\ m/s$

Q)



Time when 2kg mass will lift up.

$\therefore 5t = 3T$

block will leave contact $\therefore N = 0$

$\therefore 2T = 20$

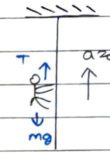
$T = 10$

$\therefore 5t = 30$

$t = 6$ sec

• Monkey problems

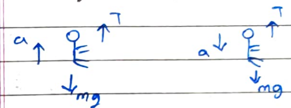
i). Monkey moving up/down with constant velocity



$\therefore T - Mg = M(0)$

$\therefore T = Mg$

ii). Monkey acc. up or down or free fall



at free fall

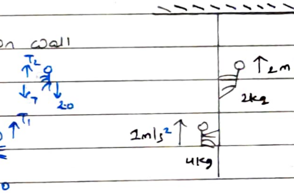
$a = g$.

$\therefore T = 0$

$\therefore T - mg = ma$ $\therefore Mg - T = ma$

$T = m(a+g)$ $M(g-a) = T$

Q) find force exerted by rope on wall



$\therefore$

$T_2 - (T_1 + 20) = 2 \cdot 2$ —①

$T_1 - 40 = 4 \cdot 1$

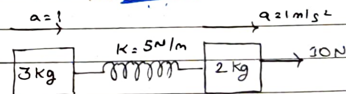
$T_1 = 44\ N$

$\therefore T_2 - (44 + 20) = 4$

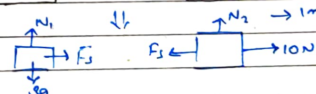
$T_2 = 4 + 64$

$T_2 = 68\ N$

Q)



find a = ?



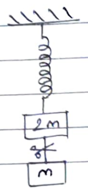
$\therefore 10 - F_s = 2 \cdot 1$

$F_s = 8\ N$

$\therefore$ for 3 kg block

$F_s = a \cdot 3$

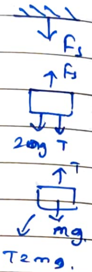
$a = \frac{8}{3}\ m/s^2$

Q) 

System was in equilibrium and at rest. Find acc. of mass 2m just after the string is cut will be

Cutting of string :
1) Draw FBD.
ii) When string is cut draw FBD, where T=0
iii) Analyse force and cal. acc.

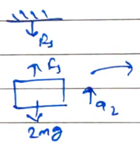
F.B.D at Equili.



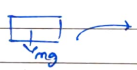
$\therefore F_s = 2mg + mg$
$F_s = 3mg$

$T = mg$.

F.B.D, string cut



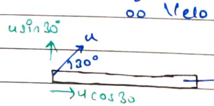
$\therefore 3mg - 2mg = 2m \cdot a$
$mg = 2m \cdot a_2$
$a_2 = g/2$.



$a_1 = g$

* Constraint Motion

Concept : length of string / rod remains constant
$\therefore$ Velocity along length is same.

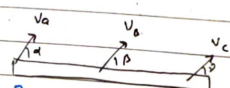


$\therefore u \cos 30 = 10$

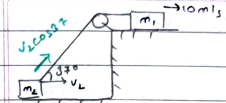
$\frac{u\sqrt{3}}{2} = 10$

$u = 20/\sqrt{3}$

$\therefore$ Rod =

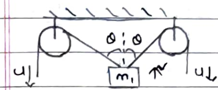


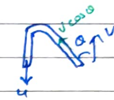
$\therefore V_a \cos\alpha = V_B \cos\beta = V_c \cdot \cos\gamma$

Q) 

$\therefore V_2 \cos 37 = 10$

$V_2 = \frac{10 \times 5}{4} = 12.5$ m/s.

Q) 



$\therefore v\cos\theta = u$

$v = \frac{u}{\cos\theta}$

* Velocity of pulley problems.

$\therefore l =$ const.

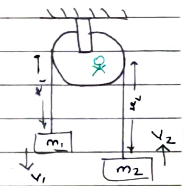
$\therefore x_1 + x_2 + \pi R = l$

diff w.r.t time.

$\therefore \frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$

$\vec{V}_{1p} + \vec{V}_{2p} = 0$

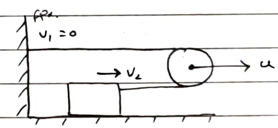
$\vec{V}_1 - \vec{V}_p + \vec{V}_2 - \vec{V}_p = 0$



$m_1 > m_2$

$\therefore \vec{V}_p = \frac{\vec{V}_1 + \vec{V}_2}{2}$

put V_p, V_1, V_2 with sign

Q) 

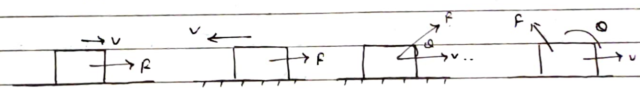
$\therefore \vec{V}_p = \frac{0 + v}{2}$

$\therefore \vec{V}_p = \frac{v}{2}$

* Concept of Internal Force.

Action / Reaction pairs are inside system.

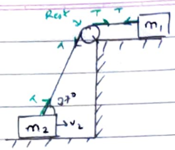
$W_{o_{int}}$ Force $= 0$ = Power = $F \cdot v \cdot \cos\theta$ → Angle betn F and v.



$\theta = 0°$

$P = Fv$ | $P = -Fv$ | $P = F \cdot v \cos\theta$, θ: acute, P = +ve | $P = |F||v| \cos\theta$, θ: obtuse, P = -ve

Q1



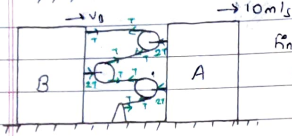
$\therefore \sum P_{int} = 0$

$\therefore P_1 + P_2 = 0$

$(\cancel{T})(10)\cos 180 + (\cancel{T})v_2 \cos 37° = 0$

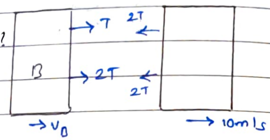
$v = \frac{10 \times 5}{4} = 12.5 \text{ m/s}$

Q2



Find v_B?

$\Rightarrow$

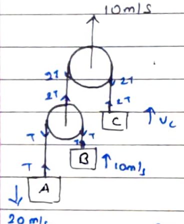


$\therefore \sum P_{int} = 0 \quad \therefore (3T)(v_B)\cos 0 + (4T)(10)\cos 180 = 0$

$4\cancel{T} \times 10 = 3\cancel{T} v_B$

$v_B = \frac{40}{3} \text{ m/s}$

Q



Find v_c?

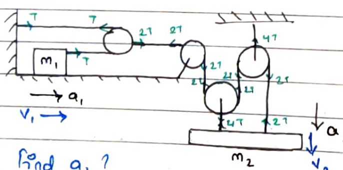
$\therefore \sum P_i = 0$

$\therefore (\cancel{T})(20)\cos 180 + (\cancel{T})(10)\cos 0 + (2\cancel{T})(v_c)\cos 0 + (4\cancel{T})(10)\cos 180 = 0$

$-20 \quad 2v_c - 40 + 10 =$

$\therefore v_c = 50/2 = 25 \text{ m/s}$

Q.



Find a_1?

$\therefore \sum P_i = 0$

$\therefore (\cancel{T})(v_1)\cos 0 + (6\cancel{T})(v_2)\cos 180 = 0$

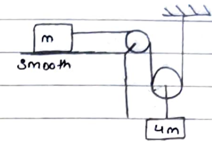
$\therefore v_1 = 6v_2$

. diff w.r.t time

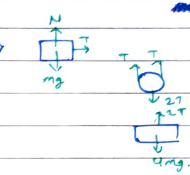
$\therefore a_1 = 6a_2$

Concept : Block ~~ke~~ per Tension alag ho, toh acc. alag hogi.

Q.



$\Rightarrow$



$\therefore 4mg - 2T = 4ma_1$

$T = ma_2$

from constraint Relation $\sum P_i = 0$

$\therefore (T)(v_2)\cos 0 + (2T)(v_1)\cos 180 = 0$

$\therefore v_2 = 2v_1$

$a_2 = 2a_1$

$\therefore$ Putting $a_2 = 2a_1$

$4mg - 2T = 4ma_1 \quad - (1)$

$T = 2ma_1 \times 2$

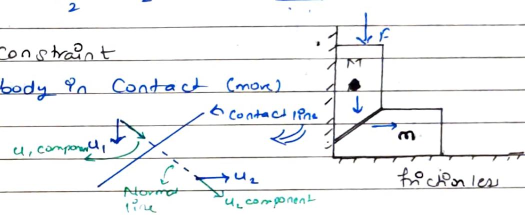
$2T = 4ma_1 \quad - (2)$

add ~~Subtract~~ $(1)+(2)$ $\therefore$ $4mg = 8ma_1$

$\frac{g}{2} = a_2 = 2a_1 \quad \therefore a_1 = g$

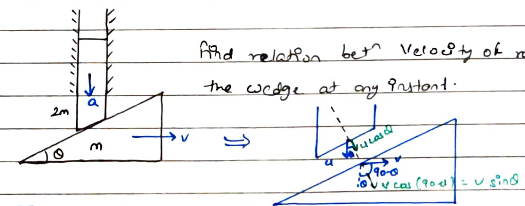
* Wedge constraint

Rigid body in contact (mov.)



$\therefore$ Velocities along Normal $\Rightarrow$ Same

Q



Find relation betn velocity of rod and that of the wedge at any instant.

$\therefore$ along Normal, velocity should be same

$\therefore u\cos\theta = v\sin\theta$

$\therefore u = v\tan\theta$

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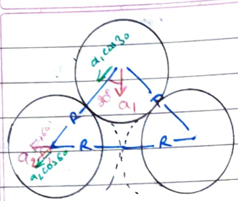
Q).



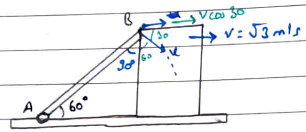
$\therefore a_1 \cos 30 = a_2 \cos 60$

$a_1 \frac{\sqrt{3}}{2} = a_2 \frac{1}{2}$

$\therefore a_1 = \frac{a_2}{\sqrt{3}}$

Q)

level up

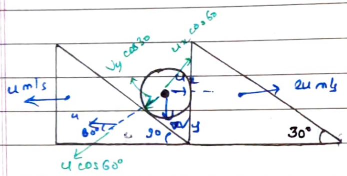


$\therefore u \cos 30° = v$

$u = \frac{v}{\sqrt{3}/2} = \frac{\sqrt{3} \cdot 2}{\sqrt{3}} = 2$

Q)

Advance



find velocity of cylinder

$\therefore u \cos 60° = v_y \cos 30 - v_x \cos 60$

$v_x = 2u$

$\frac{u}{2} = \frac{v_y\sqrt{3}}{2} - \frac{v_x}{2}$

$\therefore \frac{u}{2} = \frac{v_y\sqrt{3}}{2} - \frac{2u}{2}$

$\frac{3u}{2} = \frac{v_y\sqrt{3}}{2}$

$\therefore v_y = \sqrt{3}u$

Friction

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Friction is a contact force, which opposes the tendency of Relative Motion, Relative Motion betn two Surfaces

friction are of Two Types → Static friction, Kinetic friction

- It depend upon the area of contact particle of contact surface
- $f \propto N$

Static friction

- Opposes tendency of Relative slipping.
- Self adjusting nature
- we can cal. max. value of static friction (limiting friction)

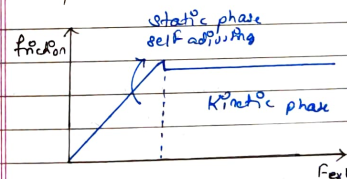
$\therefore f_{s_{max}} = \mu_s N$

Kinetic friction

- Opposes relative sliding and has a constant value

generally $\mu_s, \mu_k < 1$ & $\mu_s > \mu_k$

* Graph betn friction and Fext

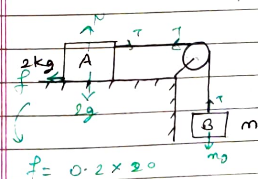


$F_{ext} \leq$ friction

if $||friction| = |F_{ext}||$

static case

Q).



$\mu_s = 0.2$, find m, so as the two block does not move

$f = 0.2 \times 20 = 4N$

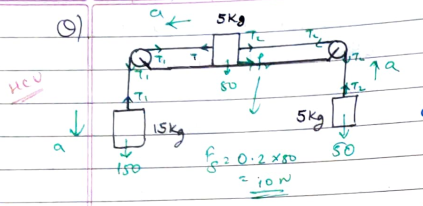
$\therefore T = f_r$

$\therefore mg = 4$

$m = 0.4$ kg

Q) $\mu_s = 0.2$, ~~find tension in the two strings~~

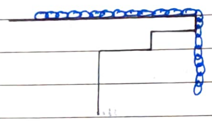
Find acc.



$\therefore a_{dir}, \frac{F_{lejane\ wala} - F_{rokne\ wala}}{M_T}$

$$a = \frac{150 - (10 + 50)}{25} = \frac{90}{25}$$

Q)



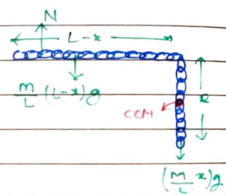
length of chain is L and coefficient of friction is μ. Calculate the max. length of chain, which can hange from the table without sliding.

$\therefore$ Mass $= M$

length $= L$

$\therefore$ 1 length mass $= \frac{M}{L}$

$x = \frac{M}{L}x$



for chain to Remain at Rest

$\therefore$ weight hanging = fsmax of part at table.

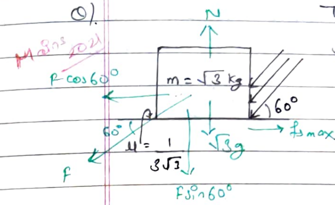
$\therefore (\frac{M}{L}x) g = \mu(\frac{M}{L}(L-x)g)$

$x = \mu L - \mu x$

$x(1+\mu) = \mu L$

$x = \frac{\mu L}{1+\mu}$

Q)



The critical force, applied on the vertical surface is at horizontal angle 60°, such that it does not move, will be 3x. $\therefore$ x is?

$\therefore$ Block is at rest

$\therefore F\cos 60° = f_s max$

$F \sin 60 + \sqrt{3}g = N$

$\frac{F}{2} = \mu\left(\frac{F\sqrt{3}}{2} + \sqrt{3}g\right)$

$\frac{F}{2} = \frac{F\mu\sqrt{3}}{2} + 10\mu\sqrt{3}$

$F\left(\frac{1}{2} - \frac{\mu\sqrt{3}}{2}\right) = 10\mu\sqrt{3}$

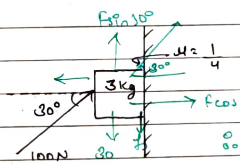
$F\left(\frac{1}{2} - \frac{1}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{2}\right) = 10 \cdot \frac{1}{3\sqrt{3}} \cdot \sqrt{3}$

$F\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{10}{3}$

$F\left(\frac{4}{12}\right) = \frac{10}{3}$

$\therefore F = 10 = 3x$ $\quad x = 3.33$

Q)



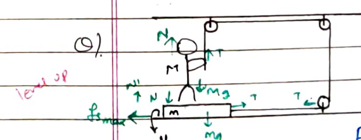
the frictional force acting on block is.

$\therefore F\sin 30° = 30 + f_s$

$\frac{100}{2} = 30 + f_s$

$f_s = 20N$ downwards

Q)



max. force that the man can exert on the rope so that the board does not slip on the floor

for board not to slip

$T = f_s max$ — (1)

$N + T = Mg$

$N' = N + mg$

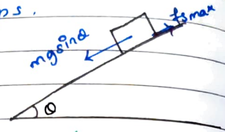
$T = \mu N' = \mu(N + mg) = \mu(Mg - T + mg)$

$T = \mu Mg + \mu mg - T\mu$

$T(1+\mu) = \mu g(M+m)$

$T = \frac{\mu g(M+m)}{(1+\mu)}$

* Angle of repose = is that angle of Incline at which block Just slips.



At time of slipping

$mg\sin\theta = f_{s\,max}$

$mg\sin\theta = \mu_s N$

$\cancel{mg}\sin\theta = \mu_s \cancel{mg}\cos\theta$

$\tan\theta = \mu_s$

$\| \; \theta = \tan^{-1}(\mu_s) \; \|$

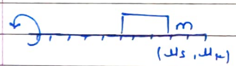
i). $\theta < \tan^{-1}(\mu)$

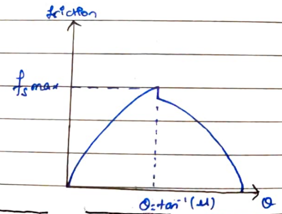
No slipping, $f = mg\sin\theta$

ii) $\theta > \tan^{-1}(\mu)$

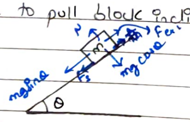
will slipp, and will be in motion

Q) A body of mass m is placed on a Rough Surface, one end of surface is rotated, draw friction vs θ graph.





Force to pull block incline

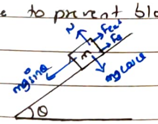


min force req. to pull block upward

$F_{ext} = mg\sin\theta + \mu_s mg\cos\theta$

$\| \; a = g\sin\theta + \mu_k g\cos\theta \; \|$

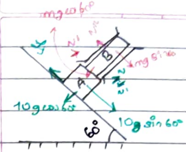
Force to prevent block to slide down



$F_{ext} = mg\sin\theta - \mu_s mg\cos\theta$

$\| \; a = g\sin\theta - \mu_k g\cos\theta \; \|$

Q). $A = 10\,kg$, $\mu = 1/\sqrt{3}$. find minimum mass of rod B so that block A does not slide on incline



$\longrightarrow$ At Equilibrium of A

$f_s = 10g\sin 60° = \frac{100\sqrt{3}}{2}$ — (i)

At Equilibrium of B

$mg\cos 60° = N'$

$mg\sin 60° = N''$

$N = N' + 10g\cos 60$ — (ii)

$N = mg\cos 60° + \frac{100}{2}$

$\therefore f_s = \mu N = \frac{1}{\sqrt{3}}\left(mg\frac{1}{2} + 50\right)$

$50\sqrt{3} = \frac{1}{\sqrt{3}}(5m + 50)$

$50 \times 3 = 5m + 50$

$5m = 100$

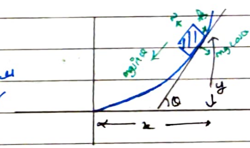
$m = 20$

Q) A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If $\mu = 0.5$, the max. height above the ground at which the block can be placed without slipping is.

$\rightarrow$ Block slip at angle repose

$\theta = \tan^{-1}(\mu)$, $\tan\theta = \mu$

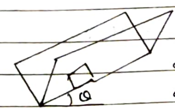
$\therefore$ trajectory $\Rightarrow$ $y = \frac{x^3}{6}$



$\therefore dy/dx = \frac{3x^2}{6} = \frac{x^2}{2} = \frac{1}{2}$ $\quad \therefore x = 1$

If $x = 1$, $y = 1/6 = h_{max}$

Q). A block of mass m slides in an inclined right angle trough as shown in fig. If $\mu = \mu$, then the acc. of block will be.

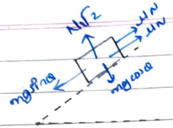


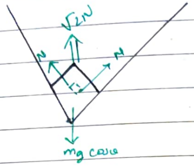
a). $(\cos\theta - \mu\sin\theta)g$

b) $(\sin\theta - \mu\sin\theta)g$

c). $(\sin\theta - \mu\cos\theta)g$

d). $(\sin\theta - \sqrt{2}\mu\cos\theta)g$



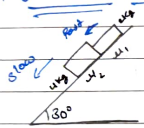


$N\sqrt{2} = mg\cos\theta$

∴ $mg\sin\theta - 2\mu N = ma$

$\cancel{m}g\sin\theta - 2\mu \frac{\cancel{m}g\cos\theta}{\sqrt{2}} = \cancel{m}a$

|| $g\sin\theta - \sqrt{2}\mu\cos\theta = a$ ||

Q) 

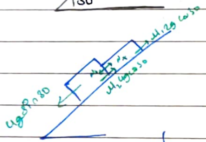
Cal. acc. of 2.0 kg block if $\mu_1 = 0.2$ & $\mu_2 = 0.3$

Note : individual acc. is independent of mass.

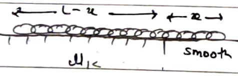
$\mu\uparrow,\ a\downarrow$

∴ $acc_{4kg} < acc_{2kg}$

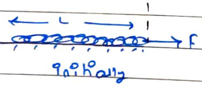
2kg will stick 4kg



$(4g\sin 30 + 2g\sin 30) - (\mu_1 2g\cos 30 + \mu_2 4g\cos 30) = (4+2)a$

Q) 

mass per unit length = ρ, pulled by const. F. Initially at rest with $x = 0$, determine the velocity when $x = L$.



$F - f_k = ma$

$F - \mu_k(\rho(L-x)g) = \rho L a$ (at any x)

∴ $a = \frac{F - \mu_k \rho g (L-x)}{\rho L} = \frac{F}{\rho L} - \mu_k g + \frac{\mu_k g x}{L}$

$\int v \frac{dv}{dx} = \int \frac{F}{\rho L} - \mu_k g + \frac{\mu_k g x}{L}$

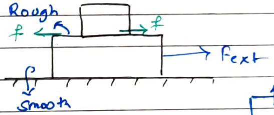
$\int_0^v v\,dv = \int_0^L \frac{F}{\rho L}dx - \int \mu_k g\,dx + \int \frac{\mu_k g x}{L}dx$

$\left[\frac{v^2}{2}\right]_0^v = \frac{F}{\rho L}(x)_0^L - \mu_k g[x]_0^L + \frac{\mu_k g}{L}\left[\frac{x^2}{2}\right]_0^L$

$\frac{v^2}{2} = \frac{F}{\rho} - \mu_k g L + \frac{\mu_k g L}{2}$

* Two Block system :-

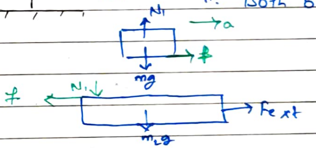
Direction of friction.



Case I : force act on below block

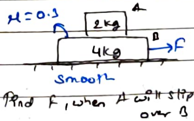
1. when Fext is applied (>0)

ii. Both block move as a system.

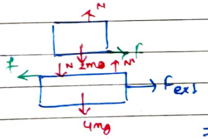


1) FBD

2) Fext = $f_{s\,max}$, then system will move.

Q) 

Find F, when A will slip over B



∴ $N = 20$

$N' = N + 40 = 60$

$f_{s\,max} = \mu N = 0.3 \times 20 = 6N$

Fext such that both move as system

$F_{ext} - f = 4a$

$f = 2a$

$F = 6a$

$a_{com} = F/6$

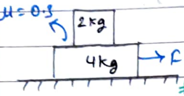
∴ friction $= 2a = 2 \cdot \frac{F}{6} = \frac{F}{3}$

∴ $f_{limit} = 6N$

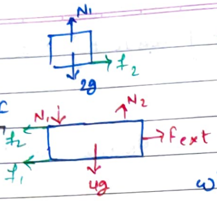
∴ $F = 6 \times 3 = 18N$

∴ A will slip over B when $F_{ext} \geq 18N$

Q). $\mu = 0.3$



$\mu = 0.6$



$\therefore f_1 = \mu N_2 = 0.6 \times 60$
$= 36N$

$f_2 = \mu N_1 = 0.3 \times 20$
$= \underline{6N}$

$\therefore N_1 = 20$
$N_2 = 60$

when $F_{ext} = 36N$, then lower block will slip on ground & both block will move with common acc.

$F - 36 - f_2 = 4a$
$f_2 = 2a$
$F - 36 = 6a$
$a = \frac{F-36}{6}$

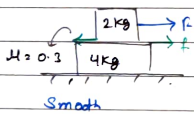
$\therefore f = 2a = 2\left(\frac{F-36}{6}\right)$

$f = F - 36/3$

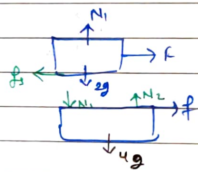
$\therefore f_{limit} = 6$

$6 = \frac{F-36}{3}$ $\therefore \underline{F = 54N}$

Q)



Smooth



$\therefore N_1 = 20, \; N_2 = 60$

$F_{s\,max} = 0.3 \times 20 = 6N$

If $F > 0$, both will move

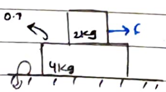
$F - f = 2a$
$f = 4a$
$F = 6a, \; a = F/6$

$\therefore f = 4F/6 = \frac{2F}{3}$

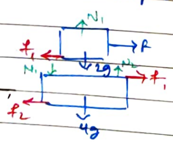
$\therefore f_{max} = 6 = \frac{2F}{3}$ $\therefore \underline{F = 9N}$

$\therefore$ Min force to move A & B together $= 0^+ N$
Max force upto which A & B move together $= 9N$
Min force for A to slip over B $= 9N$

Q) $\mu = 0.8$



$\mu = 0.2$



$N_1 = 20, \; N_2 = 60$
$f_1 = 0.8 \times 20 = 16N$
$f_2 = 0.2 \times 60 = 12N$

Min force for both block to move together $= 12^+ N$

$\therefore F - f_1 = 2a$
$f_1 - 12 = 4a$
$F - 12 = 6a$
$a = \frac{F-12}{6}$

$\therefore f = 4\left(\frac{F-12}{6}\right) + 12$

$\therefore f_{max} = 16$

$\therefore 16 = 2\left(\frac{F-12}{3}\right) + 12$

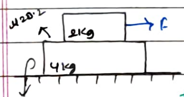
$\frac{4 \times 3}{2} + 12 = F$

$\therefore \underline{F = 18N}$

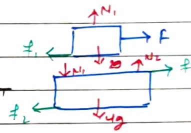
$\therefore F_{min}$ for A to slip B $= 18^+ N$

F_{max} for which both move together $= \underline{18N}$

Q). $\mu = 0.2$

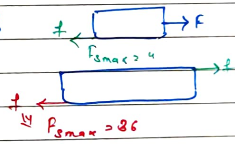


$\mu = 0.6$



$\therefore N_1 = 20, \; N_2 = 60$
$f_{1\,max} = 0.2 \times 20 = 4N$
$f_{2\,max} = 0.6 \times 60 = \underline{36N}$

$\therefore$



$\therefore F_{ext}$ is 4^+, then A will move
B will never move

Circular Motion

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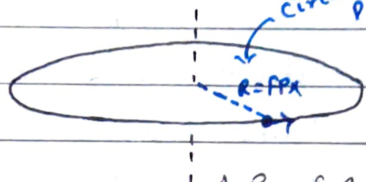
Circular Motion :

when a point mass is moving on a circular path of fixed radius.



circular motion → Clockwise, ↳ Anticlockwise

1). Angular displacement (Radians) = Angle Rotated (Dimension less)
↳ Vector Qty = Axial vector

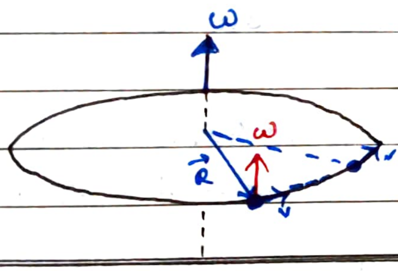
Direction = jis direction mei particle ghum raha hai, Right hand curl.

Angular displacement = 'θ'

11). Angular Velocity = 'ω'. Rate of change of Angular disp.

111). Angular Acc. = 'α' Rate of change of Angular Velocity.

* Relation betn angular Velocity and tangential Velocity.



This 'v' and 'ω' are related by $\boxed{\vec{v} = \vec{\omega} \times \vec{R}}$

$|v| = \omega R \sin 90°$

$\dot{v} = R\dot{\omega}$ (Particle chalenga toh 'ω' banega

Kinematic eqn are valid.

$x \to \theta$, $v \to \omega$, $a \to \alpha$

$\omega = \dfrac{d\theta}{dt} = \dfrac{\theta_f - \theta_i}{T_T}$

$\alpha = \dfrac{d\omega}{dt} = \dfrac{\omega_f - \omega_i}{T_T}$

If α = const.

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta_{n^{th}} = \omega_i + \frac{\alpha}{2}(2n-1)$

If Tangential speed changes with time, there is tangential acc.

$\therefore$ v → change, ω → change

$\therefore \vec{v} = \vec{\omega} \times \vec{R}$

diff $\Rightarrow \boxed{\vec{a}_t = \vec{\alpha} \times \vec{R}}$

$a_t = R\alpha$

$\dfrac{dv}{dt} = a_t$

$\dfrac{d\omega}{dt} = \alpha$

* Centripedal Acceleration
- There is a requirement of force to change the direction.
- Force towards centre, change dir
- Acc due to centripedal force is Centripedal Acc.

$\boxed{a_{centripedal} = \frac{v^2}{R} = R\omega^2}$ (ye sirf direction change karti hai)

- Bina a_{centri} circular motion possible nhi.

*). Uniform Circular motion
- Jab equal angle in equal time chale.

$v =$ constant , $\omega =$ const

$a_t = 0$, $\alpha = 0$

tanjent ke along speed kabhi change nhi hongi

$a_{net} = a_{centripedal}$ (only acc.)

*). Non-Uniform Circular motion

Circular motion mei, speed up ya slow down ho

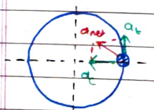
$\therefore \omega = f(t)$

$v_t = f(t)$

$v = R\omega$ — change, change

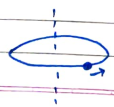
$\boxed{a_t = R\alpha}$ tangential speed changes with time $\therefore a_t = R\alpha = \frac{d|v|}{dt}$

$a_{net} = \sqrt{a^2_{centri} + a^2_{tangential}}$



Q) A solid body rotates about a stationary axis with an angular retardation $\alpha = k\sqrt{\omega}$, where ω is the angular velocity of body. Find the time after which body will come to rest if at $t = 0$, angular velocity of body was ω_0.

$\Rightarrow$



$\alpha = -k\sqrt{\omega}$ (retardation)

$\frac{d\omega}{dt} = -k\sqrt{\omega}$

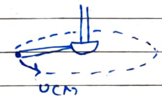
$\int_{\omega_0}^{0} \frac{d\omega}{\omega^{1/2}} = -k\int_0^t dt$

$\int_{\omega_0}^{0} \omega^{-1/2}\, d\omega = -k\int_0^t dt$

$\left[2\sqrt{\omega}\right]_{\omega_0}^{0} = -k[t]_0^t \Rightarrow -2\sqrt{\omega_0} = -kt$

$\therefore t = \frac{2\sqrt{\omega_0}}{k}$

Q) An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acc. of a point on the tip of the blade is. 2014



$r = 30$ cm $= 30 \times 10^{-2}$ m

freq. $= 120$ rpm $= \frac{120}{60}$ rps $= 2$ rps

$\therefore \omega = 2\pi\nu = 2 \times \pi \times 2 = 4\pi$ rad/s

$\therefore a_{centri} = R\omega^2 = 30 \times 10^{-2} \times 16\pi^2 = \frac{30}{100} \times 16 \times \pi^2$

$= 47.4\ ms^{-2}$

Q) The speed of a particle moving in a circle of radius $r = 2$m varies with time t as $v = t^2$, the net acc at $t = 2$s is. 2012

$\therefore v = t^2$

$\therefore a_t = 2t$, $a_{centri} = \frac{v^2}{R} = \frac{t^4}{2}$

at $t = 2$

$a_t = 4$ $a_c = \frac{2^4}{2} = 8$ m/s $\therefore a_{net} = \sqrt{4^2 + 8^2} = \sqrt{16 + 64}$

$= \sqrt{80}\ m/s^2$

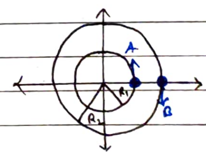
Q) Two particle A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their position and direction of motion are shown in the fig. The rel. velocity $V_A - V_B$ at $t = \frac{\pi}{2\omega}$ is given by :

$\therefore v = \omega R$

$\therefore$ ~~$v_A = \omega R_1$~~, ~~$v_B = \omega R_2$~~

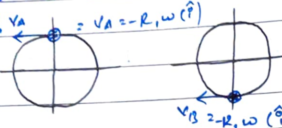
$\therefore$ 'ω' is const.

$\therefore \theta = \omega t = \omega \cdot \frac{\pi}{2\omega} = \frac{\pi}{2}$



$\therefore$ at $t = \pi/2\omega$, both will cover $\pi/2$ dist.

$\therefore v_A = -R_1\omega(\hat{i})$



$\therefore v_A - v_B = -R_1\omega - (-R_2\omega)$

$= \omega(R_2 - R_1)\hat{i}$

Q) A particle moves in a circular path of radius 1m with an angular speed $\omega = 2t^2 + 1$ rad/sec. Find angle betn total acc. and normal acc. at $t = 1$ sec

$\Rightarrow$ $R = 1m$, $\omega = 2t^2 + 1$ (NUCM)

$\therefore a_{cen.} = \frac{v^2}{R} = R\omega^2$

$= (1)(2t^2+1)^2$

$= (2t^2+1)^2$

$a_{tang} = \frac{d|v|}{dt} = R\alpha = (1)(4t) = 4t$

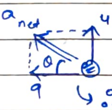
$\alpha = \frac{d\omega}{dt} = 4t$

$\therefore t = 1$

$a_c = (2(1)+1)^2 = (3)^2 = 9$

$a_t = 4t = 4$

$a_{net} = \sqrt{9^2+4^2} = \sqrt{81+16} = \sqrt{97}$



$\therefore \tan\theta = 4/9$

$\therefore \theta = \tan^{-1}(4/9)$

Q) For a particle moving along cir. path, the radial acc. $= a_r$ is ~~__~~ directly proportional t, if a_t is acc. tangential, $\therefore$ which of the following is independent of time?

a) a_t

b) $a_r \cdot a_t$

c) a_r / a_t

d) $a_r (a_t)^2$ ✓

$\therefore a_c \propto t$, $a_c = kt$

$\frac{v^2}{R} = kt$

$v^2 = ktR$

diff. $v = \sqrt{ktR}$

$a_t = \sqrt{kR} \cdot \frac{1}{2\sqrt{t}}$ $\Rightarrow$ $\therefore a_t \propto \frac{1}{\sqrt{t}}$

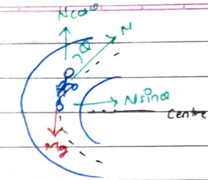
$a^2 \propto \frac{1}{t}$

$\therefore a_r \cdot a_t^2 \propto \cancel{t} \cdot \frac{1}{\cancel{t}} = $ const.

$\therefore a_r \cdot a_t^2$ is independent of time

* Circular Dynamics

1) Bending of Cyclist



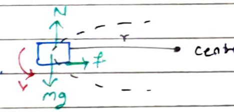
Along tanget (Equilibrium) $\quad N\cos\theta = Mg$ — (i)

Along radius. $\quad F_{net\ centre} = \frac{mv^2}{r}$

$\therefore N\sin\theta = mv^2/r$ — (ii)

$\therefore$ (i)/(ii) $\quad \therefore \tan\theta = \frac{v^2}{rg}$ $\quad$ θ = angle bend by person, r = radius of turn

ii) Turning of car on levelled road.



Car has tendency to slip out

Static friction will oppose the tendency.

- Along Tangent $\Rightarrow$ $N = mg$

$f = \frac{mv^2}{r} = \mu N = \mu mg$

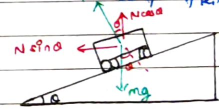
$\therefore v_{max} = \sqrt{\mu_s r g}$

v_{max} of turn without slipping

$\therefore v_{car} > v_{max}$ Car slips out

$\therefore f \to f_{kinetic}$

iii) Banking of Road.



One side of Road is lifted.

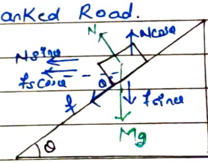
Case I : when Car is turning without use of friction

(Car does not have tendency of slipping).

$\therefore N\sin\theta = \frac{mv^2}{r}$ $\quad N\cos\theta = mg$

$\therefore \tan\theta = \frac{v^2}{rg}$

Case II : V_{max} for turning on Banked Road.



Tendency of car = outside

f = inside

$\therefore N\sin\theta + f_s\cos\theta = \frac{mv^2}{r}$

$N\cos\theta = Mg + f_s\sin\theta$

at V_{max}, $f_{s\,max} = \mu N$

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r}$$

$$N\cos\theta = mg + \mu N\sin\theta \Rightarrow N(\cos\theta - \mu\sin\theta) = mg$$

$$N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$N(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$$

$$\cancel{m}g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) = \frac{\cancel{m}v^2}{r}$$

$\div\cos\theta$

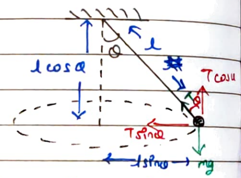
$$g\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right) = \frac{v^2}{r}$$

$$\therefore \boxed{v_{max} = \sqrt{\frac{(\tan\theta + \mu)\,rg}{(1 - \mu\tan\theta)}}} \qquad \boxed{v_{min} = \sqrt{\frac{rg\,(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}}$$

If $\mu = 0$ $\therefore v = \sqrt{rg\tan\theta}$ = case I

iv) Conical Pendulum

constant 'ω' $= \frac{2\pi}{T}$



$$T\sin\theta = mr\omega^2$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{r\omega^2}{g}$$

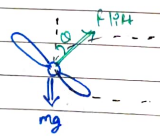
$$\omega = \sqrt{\frac{g\tan\theta}{r}} = \frac{2\pi}{T}$$

$\therefore$ Time period for conic. pend.

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{l\sin\theta}{g\frac{\sin\theta}{\cos\theta}}}$$

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

v) Banking of aeroplane



$$\tan\theta = \frac{v^2}{rg}$$

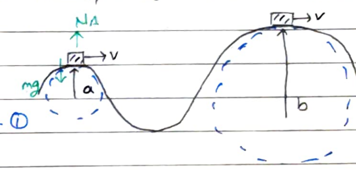
- Banking of aeroplane
- train
- Turning of car on Banked road $\mu = 0$
- Bending of cyclist

Q) A car moves at a const speed on a road as shown. The normal force by the road on the car is N_A and N_B when it is at the points A and B respectively.

For A

$$mg - N_A = \frac{mv^2}{a}$$

$$\therefore N_A = mg - \frac{mv^2}{a} \quad \text{— (1)}$$

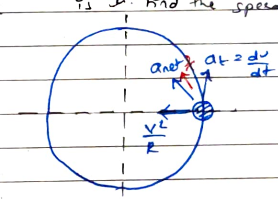


For B

$$mg - N_B = \frac{mv^2}{b} \Rightarrow N_B = mg - \frac{mv^2}{b}$$

$$\therefore N_B > N_A$$

Q) A car goes on a horizontal circ. road of Radius R, the speed increasing at a constant rate $\frac{dv}{dt}$. The friction coefficient betn the road and the tyre is μ. Find the speed at which the car will skid.



$$\Rightarrow m a_{net} = F_{net} = f_{s\,max}$$

$$\therefore a_{net} = \sqrt{a_c^2 + a_t^2}$$

$$\therefore \cancel{m}\sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2} = \mu_s \cancel{m}g$$

$$\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2 = \mu_s^2 g^2$$

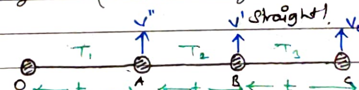
$$\frac{v^4}{R^2} = \mu_s^2 g^2 - \left(\frac{dv}{dt}\right)^2$$

$$v^4 = R^2\left(\mu_s^2 g^2 - \left(\frac{dv}{dt}\right)^2\right)$$

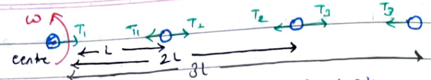
$$v = \left[R^2\left(\mu_s^2 g^2 - \left(\frac{dv}{dt}\right)^2\right)\right]^{1/4}$$

Q) Three identical particles are joined together. If speed of outermost particle is v_0, then the ratio of tensions in the three sections of the string is: (Assume string remains straight)

$\Rightarrow$ All three particle will not have same velocity but will have same 'ω'.



$$\omega = \text{same} = \frac{\text{dist. travel}}{\text{time}}$$

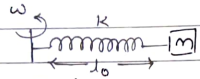


$\therefore$ $T_3 = m(3l)\omega^2$ $\Rightarrow$ $T_3 = 3ml\omega^2$

$T_2 - T_3 = m(2l)\omega^2$ $\quad T_2 = 5ml\omega^2$

$T_1 - T_2 = ml\omega^2$ $\quad T_1 = 6ml\omega^2$

$\therefore$ $T_1 : T_2 : T_3 = \underline{6 : 5 : 3}$

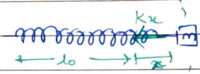
Q) 

· gravity free space – , elongation = ?

Mains 2020

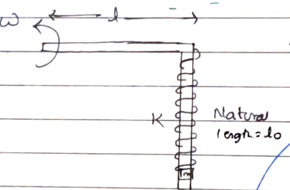
$\therefore$ $Kx = m(l_0 + x)\omega^2$

$Kx = ml_0\omega^2 + mx\omega^2$



$x(K - m\omega^2) = ml_0\omega^2$

$x = \dfrac{ml_0\omega^2}{K - m\omega^2}$

Q) 

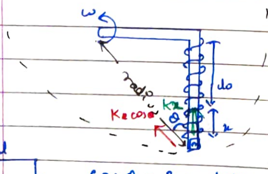
Subjective Adv.

· frictionless & gravity free space.

$\therefore$ radius $= \sqrt{l^2 + (l_0 + x)^2}$

$\therefore$ $Kx\cos\theta = m\left(\sqrt{l^2 + (l_0+x)^2}\right)\omega^2$

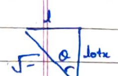
$Kx \dfrac{(l_0 + x)}{\sqrt{l^2 + (l_0+x^2)}} = m\left(\sqrt{-}\right)\omega^2$



$\therefore$

$Kxl_0 + Kx^2 = m(l^2 + (l_0+x)^2)\omega^2$

$Kxl_0 + Kx^2 = ml^2\omega^2 + ml_0^2\omega^2 + mx^2\omega^2 + m.2l_0x\omega^2$



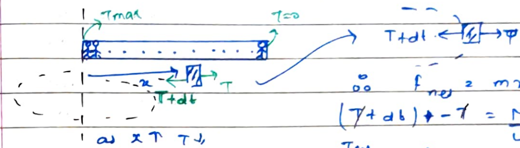
$\cos\theta = \dfrac{B}{H} = \dfrac{l_0 + x}{\sqrt{l^2 + (l_0+x)^2}}$

$\therefore$ $\underbrace{(K - m\omega^2)}_{a}x^2 + \underbrace{(Kl_0 - 2l_0m\omega^2)}_{b}x \underbrace{- m\omega^2(l^2 + l_0^2)}_{c} = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Q) A uniform rod of length l is being rotated in a horizontal plane with a const angular speed, about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a dist. x from the axis, then which of the following graph depicts it most closely.

Mains 2019

$\Rightarrow$ 

$\therefore$ $f_{net} = mr\omega^2$

$(T + dt) - T = \dfrac{M}{L}dx \cdot x \cdot \omega^2$

$L \to M$

$\therefore dx \to \dfrac{M}{L}dx = dm$

$\displaystyle\int_{T_{max}}^{T(x)} dT = \int_0^x \frac{M}{L}\omega^2 \cdot x \cdot dx$

$\therefore$ $T_{(x)} - T_{max} = \dfrac{M}{L}\omega^2\left(\dfrac{x^2}{2}\right)_0^x$

$\displaystyle\int_{T_{max}}^{0} dT = \int_0^L \frac{M}{L}\omega^2 x . dx$

$T_{(x)} - T_{max} = \dfrac{M}{2L}\omega^2 x^2$

$\therefore$ $-T_{max} = \dfrac{M}{L}\omega^2\left(\dfrac{x^2}{2}\right)_0^L$

$\therefore$ $T_{(x)} + \dfrac{M}{2L}\omega^2 L^2 = \dfrac{M}{2L}\omega^2 x^2$

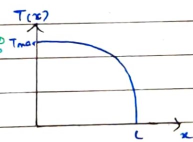
$T_{max} = \dfrac{-M\omega^2 . L^2}{2L}$

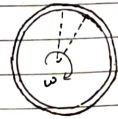
$T_{(x)} = \dfrac{M}{2L}\omega^2(x^2 - L^2)$

$\therefore$ $x = 0$ $\quad T = max$

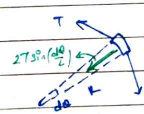
$x = L$ $\quad T = 0$

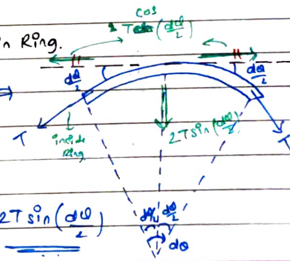
$\therefore$ graph $\therefore$



Q) 

what would be tension in Ring.





$\therefore$ $f_{net\ toward\ centre} = \cancel{2T\sin(\frac{d\theta}{2})}$ $2T\sin\left(\dfrac{d\theta}{2}\right)$

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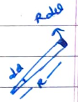


For dm.

$\therefore$ $2\pi R \rightarrow M$

$\therefore$ $Rd\theta \rightarrow \frac{M}{2\pi R} \cdot Rd\theta$

$= \frac{Md\theta}{2\pi}$

$\therefore$ $2T \sin\left(\frac{d\theta}{2}\right) = dm \cdot R \cdot \omega^2$

$2T \sin\left(\frac{d\theta}{2}\right) = \frac{Md\theta}{2\pi} \cdot R\omega^2$

$d\theta \rightarrow 0$ $\therefore$ $\sin\theta \simeq \theta$

$\therefore$ $2T\left(\frac{\cancel{d\theta}}{\cancel{2}}\right) = \frac{M\cancel{d\theta}}{\cancel{2}\pi} \cdot R\omega^2$

$$\underline{T_{\text{inside ring}} = \frac{MR\omega^2}{2\pi}}$$

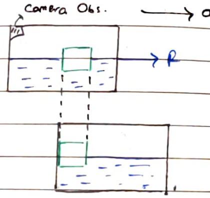
* Pseudo force in Non - Inertial frame. (NIF)

from Camera observer

box is in motion without force.

$\therefore$ Assume Pseudo force.

$\vec{P} = -m_{body}\,\vec{a}_{NIF}$



If our Observer is in NIF "acc."

$\therefore$ 1. FBD

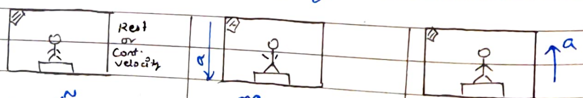
2. Nakli force banao. $\vec{P} = m_{body}\, a_{NIF}$

dir. opp. to acc of NIF

3. Camera $\rightarrow$ Equilibrium $\therefore$ $\Sigma f_{net} = 0$

motion $\therefore$ $\Sigma f_{net} = ma$

Elevator problems : weighing machine measures "Normal Rxn".



$\therefore$ N, mg, $a = 0$

$\therefore$ $\underline{N = mg}$

$\therefore$ ma, N, mg

$\therefore$ $N + ma = mg$

$\underline{N = m(g-a)}$

$g_{eff} = g - a$

$\therefore$ N, mg, ma

$\therefore$ $N = mg + ma$

$\underline{N = m(g+a)}$

$g_{eff} = g + a$

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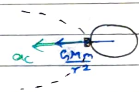
Circular motion from non-inertial from.

Camera = obs. moving with earth = acc.



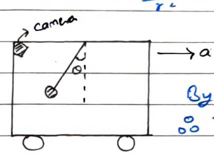
Centrifugal force = Pseudo force.

$\therefore$ Camera =

$\frac{GMm}{r^2} \leftarrow \bigcirc \rightarrow \frac{mv^2}{r}$

$\therefore$ $\frac{GMm}{r^2} = \frac{mv^2}{r}$

Q1)



Find θ.

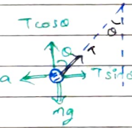
By camera = Rest

$\therefore$ $T\sin\theta = ma$

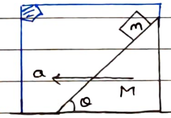
$\underline{T\cos\theta = mg}$

$\tan\theta = a/g$

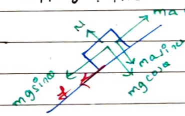
$\underline{\theta = \tan^{-1}(a/g)}$



Q1

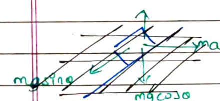


what should be a, so that there is no relative slipping. friction coefficient = μ.



If $a = g\tan\theta$, $\therefore$ No need of friction.

If $a > g\tan\theta$, $\therefore$ body tends to move up.



$\therefore$ $mg\sin\theta + f_{smax} = ma\cos\theta$

$N = ma\sin\theta + mg\cos\theta$

$\therefore$ $\cancel{m}g\sin\theta + \mu(\cancel{m}a\sin\theta) + \mu(\cancel{m}g\cos\theta) = \cancel{m}a\cos\theta$

$a(\mu\sin\theta - \cos\theta) = -\mu g\cos\theta - g\sin\theta$

$$\left\|\frac{\mu g\cos\theta + g\sin\theta}{(\cos\theta - \mu\sin\theta)}\right\| = a_{max}$$

$$\left\|\frac{g(\tan\theta + \mu)}{1 - \mu\tan\theta} = a_{max}.\right\|$$

$$\frac{g(\tan - \mu)}{1 + \mu\tan\theta} = a_{min}$$

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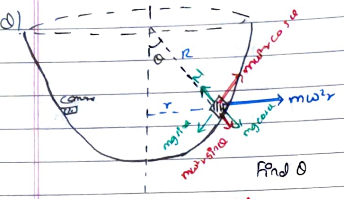
Q) 

Camera = Rotating body.

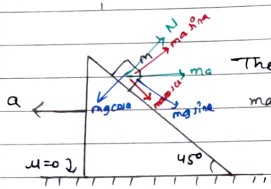
Seing at rest

$\therefore$ $\cancel{m}g\sin\theta = \cancel{m}\omega^2 r\cos\theta$

$N = mg\cos\theta + m\omega^2 r\sin\theta$

Find θ

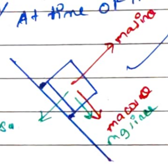
$\tan\theta = \frac{\omega^2 r}{g} = \frac{a}{g}$

Q) 

The minimum value of 'a' for which, the mass falls freely

$N = 0$

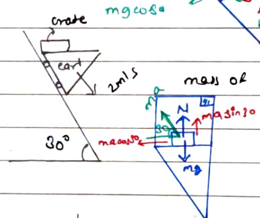
$\Downarrow$ At time of free fall



$\cancel{m}g\cos\theta = \cancel{m}a\sin\theta$

$\therefore$ $g\,\cancel{\cos 45^\circ} = a\,\cancel{\sin 45^\circ}$

$a = g$

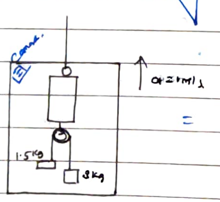
Q) 

mass of crate = 10kg, the Normal force exerted by cart on the crate.

$\therefore$ $N + ma\sin 30 = mg$

$N + 10 \cdot 2 \cdot \frac{1}{2} = 10 \cdot 10$

$N = 100 - 10 = 90N$

Q) 

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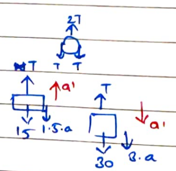

$\therefore$ $30 + 3 - T = 3 \cdot a'$

$T - 15 - 1.5 = 1.5a'$

$33 - 16.5 = 4.5a'$

$a' = \frac{16.5}{4.5}$

$\therefore$ $T = 33 - 3 \cdot a'$

Advance question at lectures

Work Energy Power

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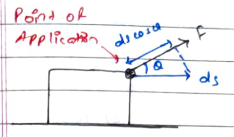


work is defined as the Product of projection of disp. of point of application of force along direction of force and magnitude of force

$\vec{F} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$

$$dW = F(ds\cos\theta)$$
$$dW = \vec{F}\cdot\vec{ds}$$

$\therefore$ $W_T = F_x \cdot S_x + F_y \cdot S_y + F_z \cdot S_z$

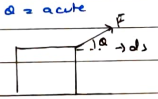
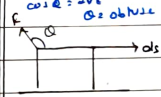
* If force is varying.

$\therefore$ $W_{Total} = \int \vec{F}\cdot\vec{ds}$

Sign of work:

Angle b/w $\vec{F}$ & $\vec{S}$

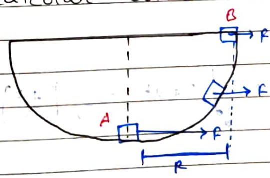
$\therefore$ $W = |F||S|\cos\theta$

W = +ve $\cos\theta$ = +ve θ = acute	W = −ve $\cos\theta$ = −ve θ = obtuse	work done = 0
		$F_{net} = 0$, $W = 0$ / $ds = 0$, $W = 0$ / $\theta = 90^\circ$, $\cos 90 = 0$, $W = 0$
W = +ve (K.E. ↑)	W = −ve (K.E. ↓)	$W = 0$, (K.E = const)

~~Case 1~~

* Calculate Work done from A to B

Case 1:

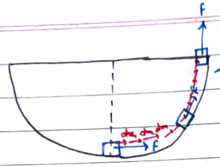


"Slowly"

Blocked is moved from A to B by applying const. force

$\therefore$ $Work_{A \to B} = +F(R)$

Case 2:

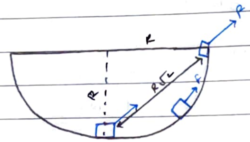


Net disp. along force throughout motion

Work = Force × disp. of point of app. of force along force.

$$W = F\left(\frac{\pi R}{2}\right)$$

Case 3:



Force is applied on block of const. Mag. F always directing Parallel to line AB.

$$W = F(R\sqrt{2})$$

Q) A force $F = (10 + 0.5x)$ N acts on a particle in x-dir; where x is in meter. Find work for $x = 0$ to $x = 2$

AIEEE 2009

$\Rightarrow$ $F = 10 + 0.5x$

$\therefore dW = F \cdot dx$

$\int dW = \int_0^2 \left(10 + \frac{x}{2}\right) dx$ $\quad \therefore W = 10(x)_0^2 + \frac{1}{2}\left(\frac{x^2}{2}\right)_0^2$

$W = 10 \times 2 + \frac{1}{4} \cdot 4 = \underline{\underline{21}}$

Q) Mass = 6 Kg, $x = \frac{t^2}{4}$, find work done for 2 sec.

$\Rightarrow$ $m = 6$ Kg

$x = \frac{t^2}{4}$ $\quad \therefore v = \frac{2t}{4} = \frac{t}{2}$ $\quad \therefore v = \frac{dx}{dt} = \frac{t}{2}$

$\therefore dx = \frac{t\,dt}{2}$

$\therefore a = \underline{\underline{1/2}}$ $\quad \therefore F = m.a = 6 \times \frac{1}{2} = \underline{\underline{3N}}$

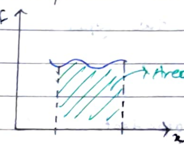
$\therefore dW = F.dx$

$\int dW = \int_0^2 F.\frac{t}{2}dt = \int_0^2 \frac{3t\,dt}{2} = \frac{3}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{3 \cdot 4}{4}$

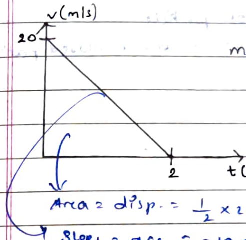
$= \underline{\underline{3J}}$

* Work done from Graph. Work done $= \int F \cdot dx$

$\therefore$ graphically:



Q).



mass of particle = 2Kg.

work done = ?

$\therefore F_1 = m.a = 2 \times -10 = -20$ N

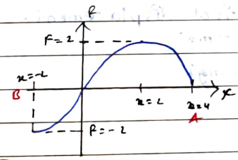
$\therefore W = F.S = -20 \times 20 = -400$

$\therefore$ F ← □ → s = 20 m

Area = disp. = $\frac{1}{2} \times 2 \times 20 = 20$

Slope = acc. = -10 m/s

Q)



find work done from A → B

$\therefore W = \int F \cdot dx$ = Area of graph

But $\cos\theta = +/-$ = Angle bet^n F & S

(I) from A → O

Force is +ve

& s is decrease

$\therefore$ → F

s ← □ $\quad \therefore \cos\theta = 180$

$\therefore W_{A \to O} = A_1 = \frac{\pi(2)^2}{2} = \underline{-2\pi}$ −ve.

(II) from O → B, F = −ve & s = −ve

$\therefore$ F ←

s ← □ $\quad \therefore \cos\theta = +ve$

$\therefore W_{O \to B} = \frac{\pi(2)^2}{4}$

$= \underline{\underline{\pi}}$

$\therefore W_T = -2\pi + \pi = \underline{\underline{-\pi}}$

* Conservative and Non-Conservative Force.

Forces are of two types :

① Conservative forces	② Non-Conservative forces
• Work done is independent of path.	• Work done depends on path of particle
• Work done depends on initial & final point	• Work done in closed path $\neq 0$
• Work done closed path $= 0$	Friction, viscous force.
Gravitational force, spring, Electrostatic force.	

* Work done by Gravity :

- We have to see only vertical displacement.
- Particle → down ⇒ $W_g = +mgh$
  → up ⇒ $W_g = -mgh$
- $W_{\text{closed path gravity}} = 0$

* Work done by friction :

↳ depends on Path taken

Static

⇒ $W = +ve$

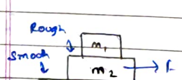
$W = -ve$

$W_{Total} = 0$

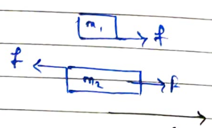
Kinetic

- Kinetic friction always opp. to Relative motion betn two surface

∴ $W_{Kinetic} = -ve$.



∴ $f > 0$, both will move as system

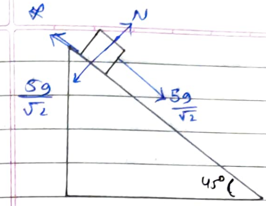


∴ $W_{\text{friction on } m_1} = +ve$

$W_{\text{friction on } m_2} = -ve$

$W_{\text{Total static friction}} = 0$

Q).



block mass $= 5\,kg$, $\mu = 0.20$,

when block slides $10\,m$, Work done by friction $= ?$

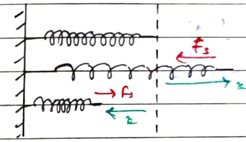
∴ $f = \mu N = 0.2 \times \frac{5 \times 10}{\sqrt{2}} = 5\sqrt{2}$

∴ $W_f = |f||s| \cos 180 = -5\sqrt{2} \times 10$

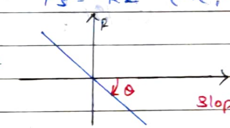
$= -50\sqrt{2}$

* Work done by Spring :

Concept of force due to spring.

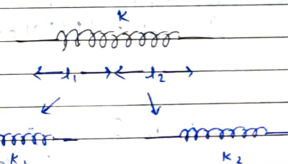


$F_s = -Kx$ (−ve, = Restoring force)



Slope $= \tan\theta = -K$

cutting of spring :

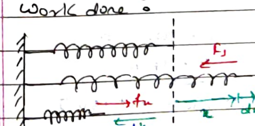


$$\boxed{\begin{aligned} K_1 &= K\left(1 + \frac{l_2}{l_1}\right) \\ K_2 &= K\left(1 + \frac{l_1}{l_2}\right) \end{aligned}}$$

$K \propto \frac{1}{\text{length}}$ K is material property

Work done :

Case I :



Elongation / Compression.

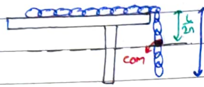
∴ $dW = F_s\, dx$

$dW = -Kx\, dx$

$\int dW = \int -Kx\, dx = -K\left(\frac{x^2}{2}\right)_0^{x_f}$

∴ $W_s = \frac{-K x_f^2}{2}$

$W_s = -\frac{K}{2}\left(x_f^2 - x_i^2\right)$

Q) 

uniform cable of mass M and length 'L' is placed such that its $(1/n)^{th}$ part is hanging. To lift the hanging part, find work done should be.

=> · Work done is against gravity

· To lift the cable on Table, we need to pull the com above by $\left(\frac{L}{2n}\right)$

mass of hanging part =>

∴ $M \to L$

∴ $L/n \to \frac{M}{L}\cdot\frac{L}{n}$

$= M/n$

∴ $W_{gravity} = (-)\,mgh$

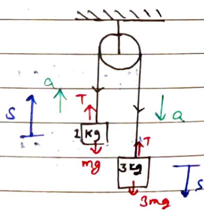
∴ $W = -\left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

* Work done by tension and normal reaction

∴ we have calc. T, N in NLM

∴ $|W| = |F||S|\cos\theta$

find work done by tension in 2 sec on 1kg, 3kg & system.



∴ FBD: $3g - T = 3a$

$T - 1g = 1a$

$2g = 4a$

∴ $a = 5\ m/s$

$u = 0$

∴ $S = \frac{1}{2}at^2$

$= \frac{1}{2}\times 5\times 2^2 = 10\,m$

∴ $T - 10 = a$

$T = 15\,N$

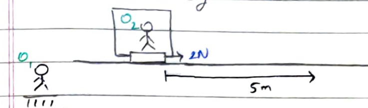
∴ $Work_T$ on 1kg = $15\times 10\cos 0 = +150\,J$

— " — 3kg = $15\times 10\cos 180 = -150\,J$

— " — on system = $+150 - 150 = 0\,J$ => always

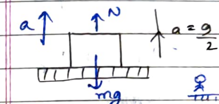
* Work done by Pseudo force

$W = F\cdot ds\cos\theta$



→ w.r.t O_2  but no disp. ∴ $W = 0$

↳ Work done by $F_{ext} = 2\times 5\times\cos 0 = 10\,J$ (w.r.t O_1)

Q) 

M.2019

$u = 0$, work done by normal w.r.t gnd. on block in time t is.

∴ FBD: $N - mg = ma$

$N = m\left(\frac{g}{2} + g\right) = \frac{3}{2}mg$

∴ $S = ut + \frac{1}{2}at^2$

Lec-03 level up.

$S = \frac{1}{2}\cdot\frac{g}{2}t^2 = \frac{gt^2}{4}$

∴ $W_{Normal} = \left(\frac{3}{2}mg\right)\left(\frac{gt^2}{4}\right)\cos 0$

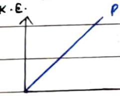
$= \frac{3mg^2t^2}{8}$

* Kinetic energy

Energy possessed by body by virtue of its motion

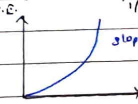
$KE = \frac{1}{2}mv_{rel}^2$ K.E is ref. dependent.

$K.E. = \frac{p^2}{2m} = \frac{\vec{p}\cdot\vec{p}}{2m}$

a) 

$p = const.$

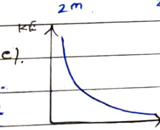
$K.E. = \frac{p^2}{2}\left(1/m\right)$

b) 

slope = $\frac{1}{2m}$

$K.E = \frac{p^2}{2m}$

$y = kx^2$

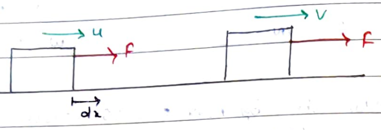
c) 

$P = const.$

$KE = \frac{p^2}{2m}$

$y = c/x$ ∴ $xy = const.$

* Work Energy theorm :

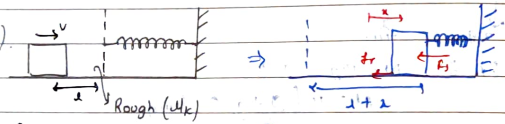


$dW = F \cdot dx$

$W_T = \int m a \, dx = \int_u^v m \cdot v \cdot \frac{dv}{dx} \, dx$ ∴ $W_{total} = m\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$

of all forces ← $W_T = K.E_f - K.E_i$

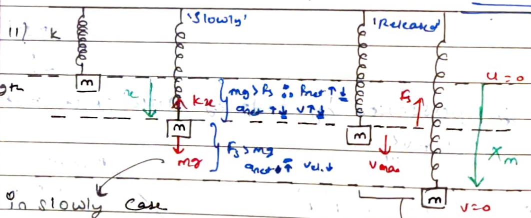
e.g. 1)



find max compression. ∴ $W_f + W_s = K_f - K_i$

$-f_k(l+x) + (-)\frac{1}{2}Kx^2 = 0 - \frac{1}{2}mv^2$

II)



In slowly case

$Kx = mg$

∴ $x = \frac{mg}{k}$ = max Elongation

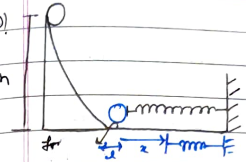
for released case.

$W_g + W_s = K_f - K_i$

$mg(X_m) + (-)\frac{1}{2}K X_m^2 = 0 - 0$

∴ $X_m = \frac{2mg}{k}$ = max elongation

Q) HCV



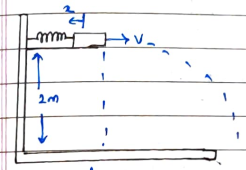
find max comp.

∴ Acc. to WET

$W_g + W_f + W_s = K_f - K_i$

∴ $mgh + f_s(l+x) + \left(-\frac{1}{2}kx^2\right) = 0 - 0$

Q) HCV



mass = 100g, compressed spring = 50cm.

k = 100 N/m, find Range.

∴ Acc. WET

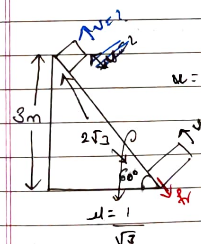
∴ $\cancel{W_g}^{\theta = 90^\circ} + W_s = K_f - K_i$

$+\frac{1}{2}Kx^2 = \frac{1}{2}mv^2 - 0$

∴ Range $= v\sqrt{\frac{2H}{g}}$ $v = \sqrt{\frac{K}{m}}\,x$

$= \sqrt{\frac{K}{m}} \cdot x \cdot \sqrt{\frac{2H}{g}} = \sqrt{\frac{100 \times 2 \times 2}{\frac{1}{10} \times 10}} \cdot \frac{1}{2} = 10\,m$

Q)



u = 10 m/s

∴ $W_g + W_f = K_f - K_i$

∴ $-mg(3) + (-\mu mg \cos 60 \cdot 2\sqrt{3}) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= -30 - \frac{1}{\sqrt{3}} \cdot 10 \cdot \frac{1}{2} \cdot 2\sqrt{3} = \frac{1}{2}v^2 - \frac{1}{2} \cdot 10^2$

$-40 \times 2 + 100 = v^2$

$20 = v^2$ ∴ $v = \sqrt{20}$

Q) A particle of mass m moves on the x-axis under the influence of a force of attraction towards the origin is given by $\vec{F} = -K/x^2\,\hat{i}$. If the particle starts from rest at $x = a$, then the speed it will attain to reach the point $x = x$ will be.

↳ $F = K/x^2(-\hat{i})$.

a) $\sqrt{\frac{2K}{m}}\left[\frac{a-x}{ax}\right]^{1/2}$ b) $\sqrt{\frac{2K}{m}}\left[\frac{a+x}{ax}\right]^{1/2}$ c) $\sqrt{\frac{K}{m}}\left[\frac{ax}{a-x}\right]$ d) $\sqrt{\frac{m}{2K}}\left[\frac{a-x}{ax}\right]^{1/2}$

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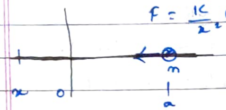


$F = \frac{K}{x^2}(-\hat{i})$ ∴ Acc. WET

$W_F = \frac{1}{2}mv^2 - \cancel{\frac{1}{2}mu^2}$

∴ $F = K/x^2$ ∴ $dW = |F||dx| \cos 180°$

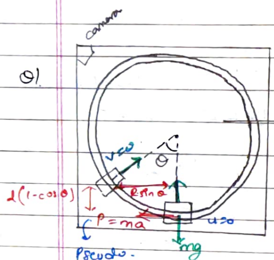
$= -\frac{K}{x^2}dx$

∴ $W = \int_a^x -\frac{K}{x^2}dx = \frac{1}{2}mv^2$

$-K\left[-\frac{1}{x}\right]_a^x = \frac{1}{2}mv^2 = K\left[\frac{1}{x} - \frac{1}{a}\right]$

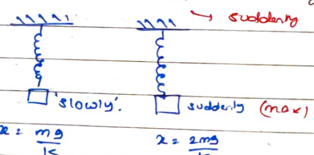
$v = \sqrt{\frac{2K}{m}\left[\frac{a-x}{xa}\right]}$

Q)



$u = 0$, const. acc.

find θ_{max} reached by bead



WET :

∴ $W_g + W_p = 0 - 0$

$-g\cancel{m}(\cancel{R}(1-\cos\theta)) + \cancel{m}a\cancel{R}\sin\theta = 0$

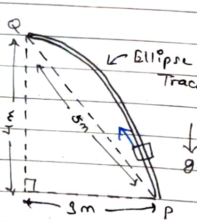
∴ $g(1-\cos\theta) = a\sin\theta$

$g \cdot \cancel{2}\sin^2\frac{\theta}{2} = a\,\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\tan\frac{\theta}{2} = \frac{a}{g}$ ∴ $\frac{\theta}{2} = \tan^{-1}(a/g)$

$\theta = 2\tan^{-1}(a/g)$ Suddenly case.

Q. Adv. 2014



Block mass = 1 Kg, block pulled up with force = 18 N, $\parallel^{m}$ to PQ, K.E of block $g = 10 m/s$ at Q is $(n \times 10)$, find n?

∴ WET :

$W_g + W_{Fext} = K.E_f - K.E_i$

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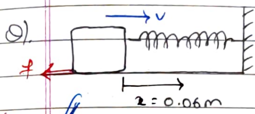
∴ $-mg(4) + F_{ext}(5) = K.E_f$

$-1 \cdot 10 \cdot 4 + 18 \cdot 5 = K.E$

$-40 + 90 = KE$

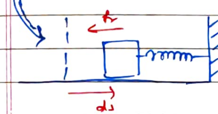
$K.E = 50 = 5 \times 10$ ∴ $n = 5$

Q) Adv.



mass = 0.18 kg, K = 2 N/m, μ = 0.1 initially at rest, an impulse is given to the block as shown. The block slides 0.06 m and comes to rest for the first time. Find v.

∴ WET



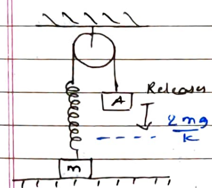
$W_s + W_f = K_f - K_i$

∴ $-\frac{1}{2}Kx^2 + \mu mgx = 0 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 2 \times 0.06^2 + 0.1 \times 0.18 \times 10 \times 0.06 = \frac{1}{2} \times 0.18 \times u^2$

$v = 0.4$

Q) 2016

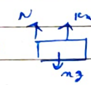


initially at rest, find mass A, for which on releasing it, block mass m, break of ground.

∴ max. elongation $= \frac{2mg}{K}$

∴ $f_s = Kx$.

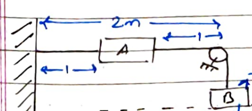
at time of release m, leave ground. ∴



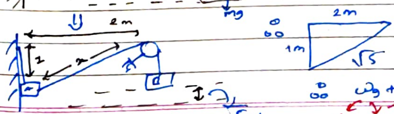
⇒ N = 0, ∴ $Kx = mg$

∴ $K\left(\frac{2m_A g}{K}\right) = mg$ ∴ $m_A = \frac{m}{2}$

Q) Adv.



A = 2 Kg, B : 0.5 Kg, if system is released. find speed of A, will hit the wall.



∴ extra length $= \sqrt{5} - 1$ m.

internal $W_T = 0$

∴ $W_g + W_T = K_f - K_i$

∴ $2g(1) - (0.5g)(\sqrt{5}-1) = \frac{1}{2} \cdot 2v^2 + \frac{1}{2} 0.5u^2$

∴ $u = v\frac{2}{\sqrt{5}}$

* Potential Energy :

Concept : 3d space, in which F can be exerted on body.

- P.E is conservative field
- It is reference dependent

$\therefore W_{gravity} = +mg(h_2 - h_1)$

$$\boxed{W_{cons} = -(U_f - U_i)}$$

$W_{gravity}$ = +ve ; P.E. ↓

— " — = −ve ; P.E ↑

independent of velocity

$$\boxed{W_{ext} = U_f - U_i}$$ slowly.

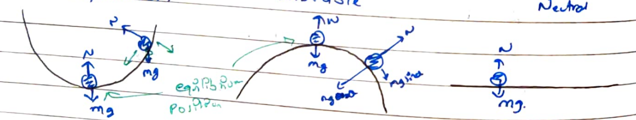
$\vec{F}\cdot\vec{dr} = -dU$

$$\vec{F}\cdot\vec{dr} = -dU \qquad \vec{F} = -\frac{dU}{dr}$$

Concept of Equilibrium :

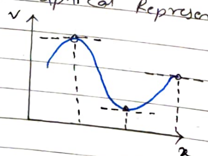
state of body → rest, constant velocity

Stable Equilibrium — Unstable — Neutral



- P.E = minimum.
- Regains position.
- P.E. is maximum
- does not regain.
- P.E is const

* Graphical Representation :



$\therefore U = f(x)$, $F = -\frac{dU}{dx}$, $F = -$slope of U/x graph

$\therefore$ equilibrium point ⇒ slope = 0

$P.E_{max}$ = unstable ⇒ $\frac{d^2U}{dx^2} < 0$

$P.E_{min}$ = stable ⇒ $\frac{d^2U}{dx^2} > 0$

$P.E_{const}$ = neutral ⇒ $d^2U = 0$

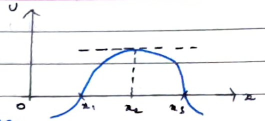
Q) Adv. 2009

a) x_1 is in stable eq.

b) x_2 is in stable eq.

c) x_3 is in stable eq.

d) None of this ✓



$\therefore$ Force = 0 = slope of U/x

Slope is 0 at x_2

but have max. P.E. $\therefore$ unstable.

Q) If the P.E. of two molecules is given by $U = \frac{A}{r^{12}} - \frac{B}{r^6}$, then at equilibrium, the p.E. is equal to.

⇒ $\therefore U = \frac{A}{r^{12}} - \frac{B}{r^6}$ at equil. $|F| = 0 = \left|\frac{dU}{dr}\right|$

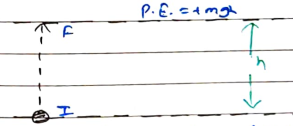
$\therefore |F| = \frac{-12A}{r^{13}} + \frac{6B}{r^7} = 0$, $\therefore \frac{2A}{r^{13}} = \frac{B}{r^7}$

$\frac{2A}{B} = r^{13-7}$, $\therefore r = \left(\frac{2A}{B}\right)^{1/6}$

$\therefore P.E = \frac{A}{\left(\frac{2A}{B}\right)^2} - \frac{B}{\left(\frac{2A}{B}\right)}$ → $\frac{AB^2}{4A^2} - \frac{B^2}{2A}$ = $\frac{B^2 - 2B^2}{4A} = -\frac{B^2}{4A}$

* P.E. due to gravity :

Concept :



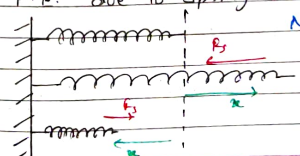
$U_i = 0$

$W_g = -mgh$, $W_c = -(U_f - U_i)$

$+mgh = +(U_f - U_i)$

$U_f = mgh$

* P.E. due to spring :



Natural length $F_s = 0$, $U_i = 0$

$W_{con} = -(U_f - U_i)$

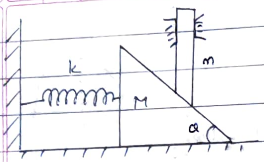
$+\frac{1}{2}kx^2 = +(U_f - 0)$

$\therefore P.E._{elongation/compression} = \frac{1}{2}kx^2$

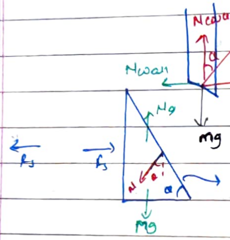
*** lec-06 pe

Q) Adv 2019

The system is in equilibrium the potential energy stored in spring.



$N = \frac{mg}{\cos\alpha}$



equilibrium ∴ $N\cos\theta = mg$

$N\sin\theta = N_{wall}$

∴ $f_s = N\sin\theta$.

$Kx = N\sin\alpha = \frac{mg\sin\alpha}{\cos\alpha}$

$N_g = Mg + N\cos\theta$

$Kx = mg\tan\alpha$

∴ $P.E = \frac{1}{2}Kx^2$

$PE = \frac{1}{2}K\left(\frac{mg\tan\alpha}{k}\right)^2 = \frac{m^2g^2\tan^2\alpha}{2k}$

* Mechanical Energy Conservation :-

Energy initial = Energy final.

∴ $K.E_i + P.E_i = K.E_f + P.E_f$

Non conservative force absent ho tab use karna

* Power :- Rate of doing work

$P = \frac{dW}{dt}$

Avg. Power : $P_{avg} = \frac{W_{Total}}{\text{Time Total}} = \frac{K.E_f - K.E_i}{\text{Time}}$

Instantaneous Power : $P = \frac{dW}{dt} = \frac{F.dx}{dt} = \vec{F}\cdot\vec{v}$

Q) A pump ejects 12000 kg of water of speed of 4 m/s in 40 second. Find the average rate at which the pump is working.

⇒ Avg. Power = $\frac{W_{Total}}{\text{Total Time}} = \frac{K.E_f - K.E_i^{\to 0}}{\text{Time}} = \frac{\frac{1}{2}mv^2}{T} = \frac{\frac{1}{2}\times 12000 \times 16}{40}$

$= 2400$ watt

$= 2.4$ kW

Q) A box is moved along a straight line by a machine delivering constant power. The dist. moved by body in time 't' is proportional to.

⇒ ∴ P = constant

$F.v$ = cont

$m.a.v = c$

$av = c'$

$\frac{dv}{dt}.v = c'$

$\int v.dv = \int c'dt$

$\frac{v^2}{2} \propto t$

∴ $v \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$\int dx \propto \int t^{1/2}dt$

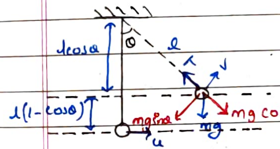
∴ $x \propto \frac{t^{3/2}}{3/2}$

∴ $x \propto t^{3/2}$

* Vertical Circular motion.

Case I : VCM with thread.

a) Velocity at any angle.



∴ Acc. to WET

$-\cancel{m}gl(1-\cos\theta) = \frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}u^2$

$-2gl(1-\cos\theta) = v^2 - u^2$

∴ $v = \sqrt{u^2 - 2gl(1-\cos\theta)}$

b). Tension at any angle

∴ $T - mg\cos\theta = \frac{mv^2}{r}$

$T - mg\cos\theta = \frac{mu^2}{l} - 2g(1-\cos\theta)m$

∴ $T = \frac{mu^2}{l} - 2mg + 3mg\cos\theta$

c). Angular Amplitude

↳ max θ, where v=0

∴ $v = \sqrt{u^2 - 2gl(1-\cos\theta)}$

∴ $\theta \to \alpha$, $v = 0$

$0 = u^2 + 2gl(1-\cos\alpha)$

$u^2 = +2gl \mp 2gl\cos\alpha$

$\cos\alpha = \frac{u^2 \mp 2gl}{-2gl} = \frac{2gl - u^2}{2gl}$

$\alpha = \cos^{-1}\left(\frac{2gl - u^2}{2gl}\right)$

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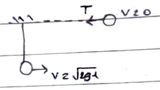
•) If v is $\sqrt{2gl}$ ∴ $\theta = \cos^{-1}\left(\frac{2gl-2gl}{2gl}\right) = \cos^{-1}(0) = \frac{\pi}{2}$

∴ max rotation is $\frac{\pi}{2}$ and v is 0 "at horizontal level".



at extreme point $T = 3mg\cos\frac{\pi}{2} + \frac{m(2gl)}{l} - 2mg$

∴ $T = 0$ → ∴ String will slag for a moment

but just after that, again tight

•) If $v < \sqrt{2gl}$

body will oscilate

•) If $v = \sqrt{gl}$,

∴ $\theta = 30°$

→ ∴ $\alpha = \cos^{-1}\left(\frac{2gl-4gl}{2gl}\right) = \cos^{-1}(-1)$

$\alpha = \pi$

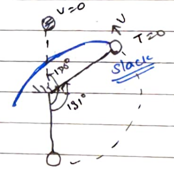
•) If $u = \sqrt{4gl}$ → but isse phele he tension zero ho jayenga

∴ $T = 3mg\cos\theta - 2mg + \frac{mu^2}{l}$ Put $T=0$, $u = \sqrt{4gl}$

$0 = 3mg\cos\theta - 2mg + 4mg$

$-2mg = 3mg\cos\theta$

$\theta = \cos^{-1}\left(-\frac{2}{3}\right)$ ∴ $\theta = 132°$



→ ∴ Ang. Amp. $\alpha = \cos^{-1}\left(\frac{2g-5gl}{2gl}\right)$

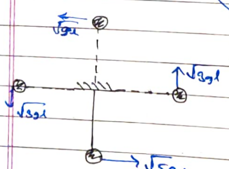
$\alpha = \cos^{-1}(-3/2)$ → -1.5, n.p.

∴ No. Angle, where $v = 0$

• If $u = \sqrt{5gl}$

v_{min} for complete VCM

→ $T = 0$



∴ $0 = 3mg\cos\theta - 2mg + \frac{mu^2}{l}$ → 5mg

$3mg\cos\theta = -3mg$.

$\cos\theta = -1$, ∴ $\theta = \pi$

∴ $T = 0$ at higher point

∴ $T_H - T_L = 6mg$.

Tension is highest at lowest point.

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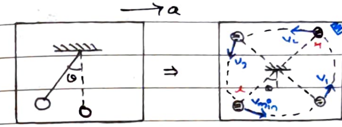
* VCM in case of rod

∴ Rod can never slack

∴ For complete rotation $v_{min} = \sqrt{4gl}$

* VCM in NIF.



∴ $v_{min} = \sqrt{5 g_{eff} l}$

∴ $v_1 = \sqrt{3gl}$

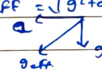
$v_2 = \sqrt{gl}$

$v_3 = \sqrt{3gl}$

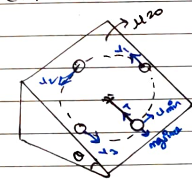
$g = g_{eff}$ & Rest same

String case

$g_{eff} = \sqrt{g^2 + a^2}$



* VCM in incline



∴ a along wedge = $g\sin\theta$

∴ $g_{eff} = g\sin\theta$.

∴ $u_{min} = \sqrt{5g\sin\theta\, l}$

$v_1 = \sqrt{3g\sin\theta\, l}$

$v_2 = \sqrt{gl\sin\theta}$

$v_3 = v_1$

Rest Same

* VCM on Spherical surface.

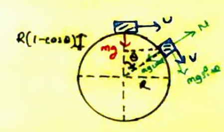
∴ Acc. WET

$+ mgR(1-\cos\theta) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

∴ $v = \sqrt{u^2 + 2gR(1-\cos\theta)}$

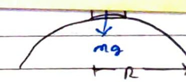
For N. ∴ $mg\cos\theta - N = \frac{mv^2}{R}$

∴ $N = mg\cos\theta - \frac{mu^2}{R} - 2mg$



(1) u_{min} so the m leaves the contact.

∴ $Mg - \cancel{N} = \frac{mv^2}{R}$ ∴ $\boxed{v_{min} = \sqrt{gR}}$



Q. By Sligth touch, at what angle from Vertical, Surface leaves.

$\therefore$ Acc. WET

$+mgR(1-\cos\theta) = \frac{1}{2}mv^2 - 0$

$\therefore \boxed{v^2 = 2gR(1-\cos\theta)}$

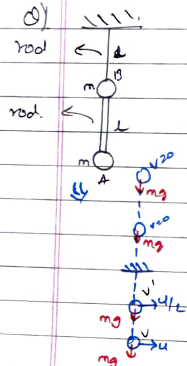
$\therefore mg\cos\theta - \cancel{N}^{0} = \frac{mv^2}{R}$

$g\cos\theta = \frac{2g(1-\cos\theta)}{R}$

$\cos\theta = 2 - 2\cos\theta$

$\cos\theta = \frac{2}{3}$ $\therefore \theta = \cos^{-1}(2/3)$

Q)

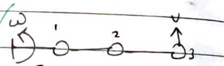


find min. speed req. with which A mass to be projected horizontaly, so that system complete VCM.

$\therefore$ We know, for rod $V_{min} = \sqrt{4gl}$

$\therefore$ Acc. WET

$+mg(2l) + mg(4l) = 0 + \left(\frac{1}{2}mu^2 + \frac{1}{2}mv^2\right)$ ($v^2/4$)



$\therefore \omega = \omega_1 = \omega_2 = \omega_L$

$\therefore v = R\omega$ $\therefore u = 2l\omega$

$\frac{u}{2} = l\omega$

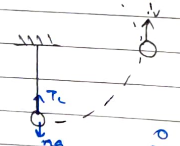
$\therefore v' = \frac{u}{2}$

$6mgl = \frac{1}{2}\cdot\frac{3v^2}{4}$

$\therefore v = 4\sqrt{gl}$

and for $v' = 2\sqrt{gl}$

Q)



weight = 0.5kg, $l = 0.5m$, $T_L = 45N$.

To what height will stone rise, if rope break when velocity is directed upwards

$\therefore$ at lowest point.

$\therefore T_L - mg = \frac{mu^2}{l}$ $\therefore 45 - 5 = \frac{1}{2}\frac{u^2}{1/2}$

$\therefore$ At time of rope break

$\therefore$ WET $\therefore$

$-mgl = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$\therefore u^2 = 40$

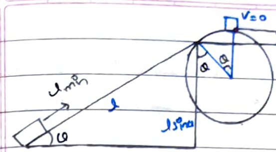
$u = \sqrt{40}$

$u^2 - 2gl = v^2$

$40 - 2\times10\times\frac{1}{2} = v^2$ $\therefore v^2 = 30$

$\therefore H_{max} = \frac{v^2}{2g} = \frac{30}{2\times10} = 1.5m$

Q)



V_{min} to reach at top.

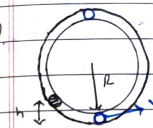
$R(1-\cos\theta)$

$\therefore$ WET $\therefore$

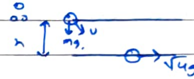
$-mg[l\sin\theta + R(1-\cos\theta)] = 0 + \frac{1}{2}mu^2_{min}$

$\therefore u_{min} = \sqrt{2g(l\sin\theta + R(1-\cos\theta))}$

Q)



find V_{min} so that ball reaches top from initially at h.



$\therefore + mgh = \frac{1}{2}m(4gR) - \frac{1}{2}mu^2$

$u^2 = 4gR - 2gh$

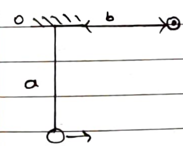
$\therefore u = \sqrt{2g(2R-h)}$

Q) Advance question

in lec-02 of collision

Q)

Adv.

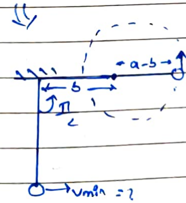


at 'b' their is nail. in horizontal line from O.

The V_{min} req. in order that it may make a complete revolution round the nail without string becoming slack.

$v = \sqrt{u^2 - 2gl(1-\cos\theta)}$

$\therefore$ v at any angle.



$V_{min} = \sqrt{5g(a-b)}$ $\therefore \sqrt{5g(a-b)} = \sqrt{u_{min}^2 - 2gl(1-\cos\frac{\pi}{2})}$

$5g(a-b) = U_{min}^2 - 2gl(1-0)$

$5g(a-b) + 2g\cdot a = U_{min}^2$

$\sqrt{5ga - 5gb + 2ga} = U_{min}$

$\sqrt{7ga - 5gb} = U_{min}$

$\sqrt{g(7a - 5b)} = U_{min}$

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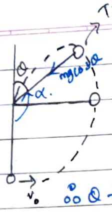
Q1. Adv.



Find v_0.

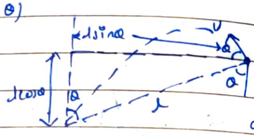
$\therefore v = \sqrt{v_0^2 - 2gl(1-\cos(\pi-\theta))}$
$= \sqrt{v_0^2 - 2gl(1+\cos\theta)}$

$\therefore \theta + \alpha = \pi$
$\alpha = \pi - \theta$

$\therefore mg\cos\theta + \cancel{T}^{0} = \frac{mv^2}{r}$

$\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{r}$ $\quad \therefore v^2 = gl\cos\theta$

$\therefore gl\cos\theta = v_0^2 - 2gl(1+\cos\theta)$
$v_0^2 = gl\cos\theta + 2gl + 2gl\cos\theta$
$v_0 = \sqrt{3gl\cos\theta + 2gl}$



$a_x = 0$
$a_y = -g$

$\therefore S_x = u_x t + \frac{1}{2}a t^2$
$-l\sin\alpha = -v\cos\alpha\, t$
$\therefore t = \frac{l\sin\alpha}{v\cos\alpha}$.

$\therefore S_y = u_y t + \frac{1}{2}at^2$
$-\cancel{l}\cos\alpha = \cancel{v}\sin\alpha.\left(\frac{\cancel{l}\sin\alpha}{\cancel{v}\cos\alpha}\right) - \frac{1}{2}g\left(\frac{l\sin\alpha}{v\cos\alpha}\right)^2$

$= -\cos\alpha = \frac{\sin^2\alpha}{\cos\alpha} - \frac{1}{2}g\frac{l\sin^2\alpha}{v^2\cos^2\alpha}$ $\quad \therefore v^2 = gl\cos\alpha$

$-\cos\alpha = \frac{\sin^2\alpha}{\cos\alpha} - \frac{1}{2}\cancel{g}\frac{\cancel{l}\sin^2\alpha}{\cancel{gl}\cos^3\alpha}$ $\Rightarrow -\cos\alpha = \frac{2\cos^2\alpha\sin^2\alpha - \sin^2\alpha}{2\cos^3\alpha}$

$+2 = \frac{\sin^2\alpha - 2\cos^2\alpha\sin^2\alpha}{\cos^4\alpha}$ $\Rightarrow \frac{\sin^2\alpha}{\cos^2\alpha}\frac{(1-2\cos^2\alpha)}{\cos^2\alpha} = 2$

$2 = \tan^2\alpha(\sec^2\alpha - 2)$
$2 = (\sec^2\alpha - 1)(\sec^2\alpha - 2)$
let $\sec^2\alpha = x$
$2 = (x-1)(x-2)$
$\therefore x = 3$ $\quad \therefore \sec^2\alpha = 3$
$\cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore v_0 = \sqrt{3gl\cos\left(\frac{1}{\sqrt{3}}\right) + 2gl}$

$v_0 = \sqrt{3gl\left(\frac{1}{\sqrt{3}}\right) + 2gl}$

$v_0 = \sqrt{gl(2+\sqrt{3})}$

Centre of Mass & Collisions.

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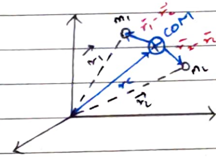
① Mass moment : product of mass of point object and it's position vector.

$\vec{Z} = m\vec{R}$

② Defination of COM in term on mass moment :

· Centre of mass is a point in space where net mass moment = 0.

$\therefore \sum m_i \vec{r_i} = 0$



$\therefore m_1(\bar{r}_1 - \bar{r}_c) + m_2(\bar{r}_2 - \bar{r}_c) = 0$

$m_1\bar{r}_1 + m_2\bar{r}_2 = (m_1+m_2)\bar{r}_c$

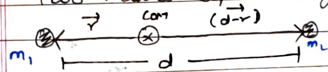
$$\boxed{\therefore \vec{r}_c = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1+m_2}}$$ $\Rightarrow$ multiple :

$\vec{R}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n}{m_1+m_2+\dots+m_n}$

$\therefore X_{com}, Y_{com}, Z_{com}$.

$$\boxed{X_{com} = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1+m_2+\dots+m_n}}$$

Two masses seprated by dist. d



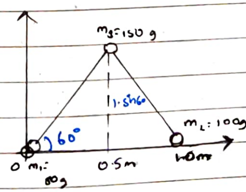
$\rightarrow \therefore \sum m_i\bar{r}_i = 0$

$\therefore m_1(-\bar{r}) + m_2(\vec{d} - \vec{r}) = 0$

$-m_1\bar{r} + m_2\bar{d} - m_2\bar{r} = 0$

$\therefore \frac{m_2\bar{d}}{m_1+m_2} = \bar{r}$ } Result

Q) M.2019



$\rightarrow \therefore x_c = \frac{m_1(0) + 2m_2(1) + 3m_3(1/2)}{6m}$

$= \frac{2+3/2}{6} = 7/12$

$y_c = \frac{\cancel{m}(0) + 2\cancel{m}(0) + 3\cancel{m}(\sqrt{3}/2)}{6\cancel{m}}$

$= \sqrt{3}/4$

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③ Centre of Mass of Continous Distribution of Mass.

∴ when mass is distributed
→ length → Area → Volume

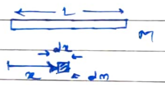
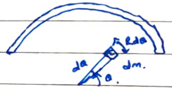
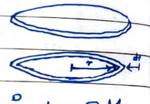
e.g. wire, thread, string | disc, square plate | sphere, cone, cylinder

"λ" $\left(\frac{kg}{m}\right)$ | "σ" (kg/m^2) | "ρ" (kg/m^3)

∴ $x_{com} = \dfrac{\int x\,dm}{\int dm}$

How to take element :

· Rod. · Angular Arc. · Disc.

∴ $dm = \lambda\,dx = \left(\frac{M}{L}\right)dx$ (if uniform)

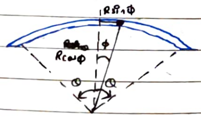
∴ $dm = \lambda(d\theta\,R)$

∴ $dm = \sigma\,dA = \sigma(2\pi r\,dr)$

$dA = 2\pi r\,dr$

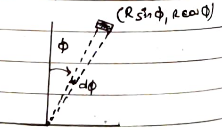
④ Centre of mass of Angular Ring.

com will lie in Y axis



∴ $Y_{com} = \dfrac{\int y\,dm}{\int dm}$ → Y coordinate of element

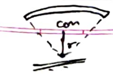
$= \dfrac{\int R\cos\phi\,\lambda R\,d\phi}{\int \lambda R\,d\phi} = \dfrac{R^2\lambda\int\cos\phi\,d\phi}{R\lambda\int d\phi}$



$= \dfrac{R\int_{-\theta}^{\theta}\cos\phi\,d\phi}{\int_{-\theta}^{\theta}d\phi}$

$= \dfrac{R[\sin\phi]_{-\theta}^{\theta}}{[\phi]_{-\theta}^{\theta}} = \dfrac{2R\sin\theta}{2\theta} = \dfrac{R\sin\theta}{\theta}$

∴ General formula for Arc $= \dfrac{R\sin\theta}{\theta} = r$ (com to origin dist.)



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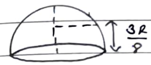
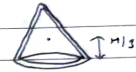
* COM of Symmetrical Bodies :

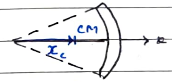
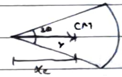
1). Semicircular ring | Semicircular disc | Hemisphere shell







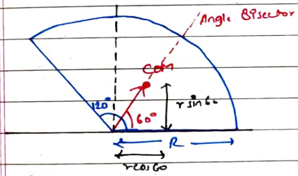
Solid Hemisphere | Solid cone | Hollow cone

$x_c = \dfrac{r\sin\theta}{\theta}$ | $x_c = \dfrac{2r\sin\theta}{3\theta}$

circular Arc | Sector of circular plate

Q)



∴ $r = \dfrac{2R\sin\theta}{3\theta} = \dfrac{2R\sin(\pi/3)}{3(\pi/3)}$

$= \dfrac{2R}{\pi}\cdot\dfrac{\sqrt{3}}{2} = \dfrac{R\sqrt{3}}{\pi}$

$2\theta = \dfrac{2\pi}{3}$, $\theta = \dfrac{\pi}{3}$

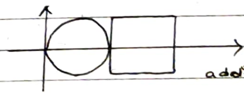
∴ Coordinate $(r\cos 60, r\sin 60)$

$= \left(\dfrac{R\sqrt{3}}{\pi}\cdot\dfrac{1}{2}, \dfrac{R\sqrt{3}}{\pi}\cdot\dfrac{\sqrt{3}}{2}\right)$

$= \left(\dfrac{R\sqrt{3}}{2\pi}, \dfrac{3R}{2\pi}\right)$

⑤ Additive Systems of Area based problems

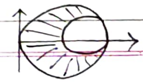
$X_{com} = \dfrac{A_1x_1 + A_2x_2}{A_1+A_2}$



$Y_{com} = \dfrac{A_1y_1 + A_2y_2}{A_1+A_2}$

Negative System of Area based problems.

$X_{com} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$



$Y_{com} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$

(6.) Volume based :

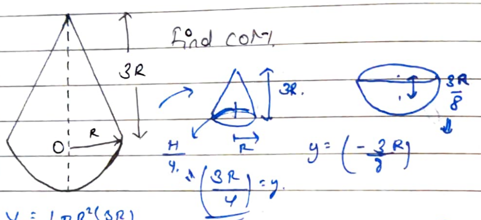
a) Additive System.

$$X_c = \frac{V_1 x_1 + V_2 x_2}{V_1 + V_2} \qquad Y_c = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

b) Negative System.

$$X_c = \frac{V_1 x_1 - V_2 x_2}{V_1 - V_2} \qquad Y_c = \frac{V_1 y_1 - V_2 y_2}{V_1 - V_2}$$

Q. find COM.



$\frac{H}{4} = \left(\frac{3R}{4}\right) = y$ $\qquad y = \left(-\frac{3R}{8}\right)$

$V_1 = \frac{1}{3}\pi R^2(3R) = \pi R^3$

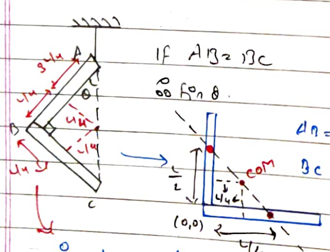
$V_2 = \frac{2}{3}\pi R^3$

$$\therefore X_c = \frac{\pi R^3(0) + \frac{2}{3}\pi R^3(0)}{\pi R^3 + \frac{2}{3}\pi R^3} = 0$$

$$Y_c = \frac{\pi R^3(3R/4) + \frac{2}{3}\pi R^3\left(-\frac{3}{8}R\right)}{\pi R^3 + \frac{2}{3}\pi R^3}$$

$$\therefore Y_c = \frac{\frac{3R}{4} + \left(-\frac{1}{4}R\right)}{1 + \frac{2}{3}} = \frac{2R}{4}\cdot\frac{3}{5} = \frac{3R}{10}$$

Q. If AB = BC, find θ. (M. 2019)

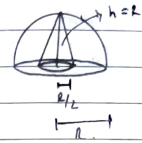


$AB = m(\ell/2, 0)$ $\qquad X_c = \frac{m(\ell/2) + m(0)}{2m} = \frac{\ell}{4}$

$BC = m(0, \ell/2)$ $\qquad Y_c = \frac{m(0) + m(\ell/2)}{2m} = \frac{\ell}{4}$

$$\therefore \tan\theta = \frac{\frac{\ell}{4}}{\frac{3\ell}{4}} = \frac{1}{3} \qquad \therefore \theta = \tan^{-1}(1/3)$$

Q.



$\therefore V_H = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi R^3 \quad \left(0, \frac{3R}{8}\right)$

$V_c = \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 R = \frac{\pi R^3}{6} \quad \left(0, \frac{R}{4}\right)$

$$\therefore X_c = 0, \quad Y_c = \frac{\frac{2}{3}\pi R^3\left(\frac{3R}{8}\right) + \frac{\pi R^3}{6}\left(\frac{R}{4}\right)}{\frac{2}{3}\pi R^3 + \frac{\pi R^3}{6}}$$

$$\therefore Y_c = \frac{\frac{7R}{24}}{5/6} = \frac{7R}{20}$$

* Motion of COM :

(1.) Shifting of COM $\quad \Delta\vec{R}_{com} = \frac{m_1 \Delta\vec{R}_1 + m_2 \Delta\vec{R}_2}{m_1 + m_2}$ $\quad$ change in position $\quad \Delta R = R_f - R_i$

(2.) Velocity of COM $\quad \vec{V}_{com} = \frac{d\vec{R}_{com}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

(3.) Acceleration of COM $\quad \vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

Q. A body ~~of~~ A, mass 1kg is placed at (1, -2) and another body B of mass 2 kg is placed at (2, 3). If A is displaced to (2, 0) where B should be moved so that there is no shifting of COM.

A (1kg) $\quad (1, -2) \rightarrow (2, 0)$

B (2kg) $\quad (2, 3) \rightarrow (x, y)$

$\therefore \Delta\vec{R}_{com} = 0$

$\therefore \Delta\vec{R}_1 = (2-1, 0-(-2)) = (1, 2)$

$\Delta\vec{R}_2 = (x-2, y-3)$

$\therefore \Delta\vec{R}_{com} = \frac{m_1\Delta\vec{R}_1 + m_2\Delta\vec{R}_2}{m_1 + m_2}$

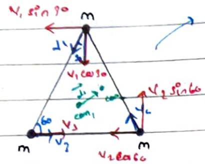
$\therefore 0\hat{i} + 0\hat{j} = \frac{1(1\hat{i} + 2\hat{j}) + 2(x-2, y-3)}{3}$

$\therefore 2\hat{i} + (2x-4)\hat{i} = 0$

$2\hat{j} + (y-3)2\hat{j} = 0$

$x = 3/2 \qquad y = 2$

Q)



Initially com at Centroid

$A = m = \left(-\frac{v_1}{2}\hat{i} - \frac{v_1\sqrt{3}}{2}\hat{j}\right)$

$B = m = \left(v_3\hat{i} + 0\hat{j}\right)$

find disp of com

$C: m: \left(-\frac{v_2}{2}\hat{i} + \frac{v_2\sqrt{3}}{2}\hat{j}\right)$

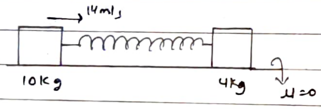
$\therefore \vec{V}_{com} = \frac{m\vec{V}_1 + m\vec{V}_2 + m\vec{V}_3}{3m}$

$\therefore \vec{V}_{com} = \frac{\vec{V}_1 + \vec{V}_2 + \vec{V}_3}{3} = \frac{\left(V_3 - \left(\frac{V_1+V_2}{2}\right)\right)\hat{i} + \frac{\sqrt{3}}{2}(V_2 - V_1)\hat{j}}{3}$

$\therefore \vec{S}_{com} = \vec{V}_{com} \times t$

$d = \frac{\left\{\left(V_3 - \left(\frac{V_1+V_2}{2}\right)\right)\hat{i} + \frac{\sqrt{3}}{2}(V_2-V_1)\hat{j}\right\}}{3} t$

Q)



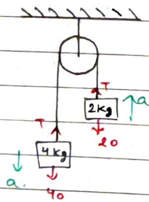
find Velocity of COM

$\vec{V}_{com} = \frac{10(14\hat{i}) + 4(0)}{14}$

At the time when 10 kg block is given $V = 14\hat{i}$, 4kg block is at Rest

$= \underline{10\hat{i}}$

Q)



Calculate Acc. of COM.

$\therefore 40 - T = 4a$

$T - 20 = 2a$

$20 = 6a$

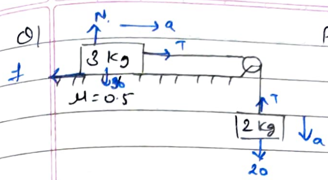
$a = 10/3$

$\therefore \vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2}$

$\vec{a}_{com} = \frac{4(-10/3) + 2(10/3)}{6}$

$= \frac{-20}{3\times 6} = \frac{-10}{9}\hat{j}$

Q)



find a_{com}.

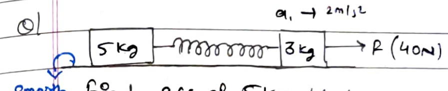
$\therefore 20 - T = 2a$

$T - \mu(30) = 3a$

$20 - \frac{1}{2}(30) = 5a$, $a = 1\ m/s$.

$\therefore \vec{a}_{com} = \frac{(3\times 1)\hat{i} + (2\times -1)\hat{j}}{5} = \frac{3}{5}\hat{i} - \frac{2}{5}\hat{j}$

Q)



Smooth find acc of 5kg block.

$\therefore$

System.

$\therefore a_{com} = 40/8$

$\therefore a_{com} = \frac{5(a) + 3(2)}{8} = \frac{40}{8} \Rightarrow 5a = 34$

$\therefore a = 34/5$

* Concept of $F_{ext} = 0$ along a certain direction.

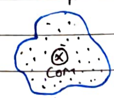
If $\vec{F}_{net} = 0$, along any axis,

$\therefore$ for that axis, momentum is conserved along that axis.

$\therefore \vec{P}_{initial\ net} = \vec{P}_{final\ net}$.

Motion of COM

Concept: for complete System

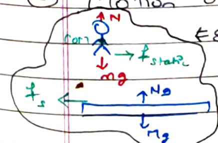


until & unless net Fext acts on body COM state remain same.

i) Initially COM at rest, $\Sigma\vec{F}_{net} = 0$, COM Rest

ii) Initially COM at motion, $\Sigma\vec{F}_{net} = 0$, COM Motion.

(1) Motion of person on planck or boat



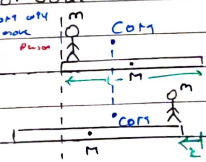
Σ System

Person along x-axis $\Sigma F_{net_x} \neq 0$

$\therefore$ COM will move

Planck along y-axis $\Sigma F_{net_y} = 0$

along x-axis $\Sigma F_{net_x} \neq 0$, $\therefore$ COM will move



System along x axis

$\Sigma F_{net} = 0$

COM of system is at rest.

∴ Sol^n : Along X axis, System $\Sigma F_{net\,x} = 0$

∴ $\boxed{\Delta \vec{R}_{com} = 0}$

∴ $\Delta \vec{R}_{com} = \dfrac{m_1 \Delta \vec{r}_1 + m_2 \Delta \vec{r}_2}{m_1 + m_2}$

∴ Person — mass $= m$ — dist travelled $= (L - x)$

Planck — mass $= M$ — dist. travelled $= x$

$0 = \dfrac{m(L-x) + M(-x)}{M+m}$

$0 = mL - mx - Mx$

∴ $x = \dfrac{mL}{M+m}$ — Relative length bet^n Person/Planck; M total.

Result : Shift of boat $\| x = \dfrac{mL}{M+m} \|$

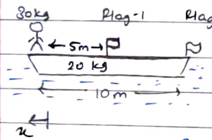
Shift of Planck $= \dfrac{M_{\text{other body}} \times \text{Relative length}}{M_T}$ — mass of other body

Cond^n : ① Two body system

② Initially com at rest

Q) find shift of boat.

①.



→ ∴ for flag 2

$\Delta \vec{R}_{com} = 0 = \dfrac{30(5-x) + 20(-x)}{50} = 150 - 50x$

$x = 3$

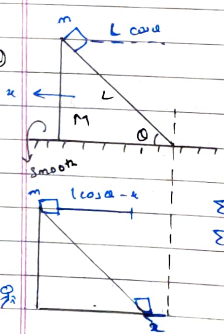
(+) that mean, whatever sign is consider is Right

∴ for Flag - 2

$\Delta R_{com} = 0 = \dfrac{30(10-x) + 20(-x)}{50} = 300 - 50x$

$x = +6m$

②



∴ $\Delta \vec{R}_{com} = 0 = \dfrac{M(-x) + m(l\cos\theta - x)}{M+m}$

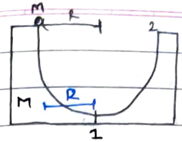
$\Sigma F_x = 0$

$\Sigma F_y \neq 0$ (weight along y)

$0 = -Mx + ml\cos\theta - mx$

$x = \dfrac{ml\cos\theta}{M+m}$

③.

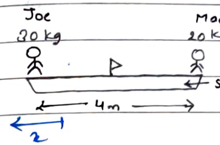


Find shift, when ball reaches 1 & 2.

1) For Point 1 $\quad x = \dfrac{mR}{M+m}$ (Direct Result).

2) For Point 2 $\quad x = \dfrac{m\,2R}{M+m}$

④. HCV



find shift of boat when they meet.

	disp.
∴ 30 kg	$2-x$
20 kg	$-(2+x)$
50 kg	$-x$

Method - 1

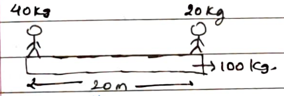
For multiple people

∴ $\Delta \vec{R}_{com} = 0 = \dfrac{30(2-x) + 20(-(2+x)) + 50(-x)}{100}$

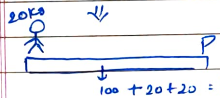
$0 = 60 - 30x - 40 - 20x - 50x$

$20 = 100x$, $\quad x = \dfrac{20}{100} = 0.2$ left

Q)

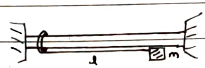


Find disp. by boat.



∴ $x = \dfrac{20(20)}{140+20} = \dfrac{5}{2}$ m

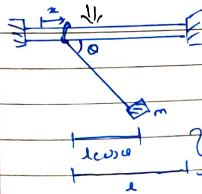
Q)



Adv.

Find shift of ring if block m is released.

∴ $\Sigma \vec{F}_{net\,x} = 0$, ∴ $\Delta \vec{R}_{com\,x} = 0$

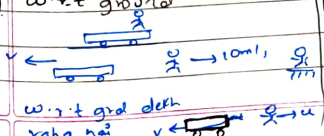


$x_{ring} = \dfrac{m \cdot l(1-\cos\theta)}{M+m}$

along x $= l(1-\cos\theta)$ chala

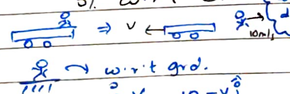
2. Jumping w.r.t cart / ground.

a) w.r.t ground



w.r.t grd dekh raha hai

b) w.r.t cart.



$V_p = 10$ m/s w.r.t cart de raha hai.

→ w.r.t grd.

∴ $V_p = 10 - v\hat{i}$

$V_c = -v\hat{i}$

$V_{p/c} = V_p - V_c = 10 - v - (-v) = 10$

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Case 1: $V_{\text{person w.r.t ground}} = u$.

$\therefore$ M (cart), m (person)

initially Rest.

$\Sigma F_{net\,x} = 0$, $\therefore \vec{P_{i_x}} = \vec{P_{f_x}}$

$v \leftarrow$ cart, person $\rightarrow u$

$\therefore 0 = mu\hat{i} - Mv\hat{i}$

$$\therefore \boxed{v = \frac{mu}{M}}$$ Recoil velocity of cart

Case 2: If person jump with u relative to cart

$\therefore$ initially at Rest

$v \leftarrow$ cart, person $\rightarrow (u-v)$

$\therefore P_{i_x} = P_{f_x}$

$0 = m(u-v)\hat{i} - Mv\hat{i}$

$$\therefore \boxed{v = \frac{mu}{M+m}}$$ v_{recoil}

(3) Recoil of gun.

Case 1: V of bullet w.r.t grd $= u$.

M (gun), m (bullet)

$v \leftarrow$ gun, bullet $\rightarrow u$

$\therefore P_{i_x} = P_{f_x}$

$0 = mu\hat{i} - Mv\hat{i}$

$$\boxed{v = \frac{mu}{M}}$$

Case 2: V of bullet w.r.t gun $= u$. (Muzzle speed)

M (gun), m (bullet)

$v \leftarrow$ gun, bullet $\rightarrow (u-v)$

$P_{i_x} = P_{f_x}$ $\therefore 0 = m(u-v)\hat{i} - Mv\hat{i}$

$$\therefore \boxed{v = \frac{mu}{M+m}}$$

Q) Diff cases, let $M = 10$ kg, $m = 2$ kg, find recoil of cart

(1) Kid jump with 6 m/s w.r.t grd, initially at rest.

$\therefore \vec{P_{i_x}} = \vec{P_{f_x}}$ $\therefore 0 = m(6) - M(v)$

$\therefore v = \frac{2 \times 6}{10} = 1.2$

(+), so dir. Socha hai vo dahi hai

(2) Kid jump with 6 m/s w.r.t cart. $(6-v)$

$\therefore v = \frac{m(6)}{M+m} = \frac{2 \times 6}{12} = 1$ m/s

(3) Kid jump with 6 m/s w.r.t. grd, initially cart moving with $\rightarrow 1$ m/s

$\therefore \vec{P_{i_x}} = \vec{P_{f_x}}$ $\therefore (m+M)(1) = m(6) + M(v)$

Case 1: Kid jump $\parallel$ to initial velocity

$12 = 12 - 10(v)$

$\therefore v = 0$

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Case 2: Antiparallel

$\therefore P_{i_x} = P_{f_x}$

$\therefore (M+m)(1) = m(-u) + M(v)$

$\therefore v = \frac{(M+m) + mu}{M} = \frac{12 + 12}{10} = 2.4$ towards right

(4) Kid jump with 6 m/s w.r.t cart. (cart moving $\rightarrow 1$ m/s)

Case (1): Parallel

$\therefore P_{i_x} = P_{f_x}$

$(m+M)(1) = (m)(6-v) + M(-v)$

$12 = 12 - \cancel{2v} + 10(-v)$

$\therefore v = 0$

Case (2): Antiparallel

$\therefore (M+m)(1) = m(6-v) + M(v)$

$\therefore v = \frac{12 + 12}{M+m} = \frac{24}{12} = 2$ Right

(5) 6 kg, 2 kg, 10 kg; 4 m/s (g) $\leftarrow$, $\rightarrow$ 2 m/s (g)

Find Recoil of cart, initially at rest

$\therefore 0 = 6(-4) + 2(2) + 10(v)$

$24 - 4 = 10v$

$v = 2$ w.r.t ground. (Right)

(6) m, u (c). 1) Rest, 2). Find velocity of Cart 2.

$v \leftarrow$ cart, person $\rightarrow (u-v)$

$\therefore$ For cart (1): $0 = m(u-v) - (M)(v)$

$\therefore v = \frac{mu}{M+m}$ = velocity of Cart 1

For cart (2)

person $\rightarrow (u-v)$, M

$V_p = u - V_{cart}$

$= u - \frac{mu}{M+m} = \frac{mu + Mu - mu}{M+m}$

$$V_{p/g} = \frac{Mu}{M+m}$$

$\therefore \vec{P_{i_x}} = m\left(\frac{Mu}{M+m}\right)\hat{i} + 0$

$\vec{P_{f_x}} = (M+m)V_c$

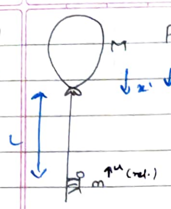
$P_{i_x} = P_{f_x}$

$\therefore \frac{Mmu}{M+m} = (M+m)v_c$

$$\therefore V_c = \frac{Mmu}{(M+m)^2}$$

Q) Find the ~~speed~~ x' with which balloon will move

HCV.



$\therefore \Delta R_{com} = 0 = M(-x') + m(L - x')$

$\therefore x' = \dfrac{mL}{M+m}$

Speed of balloon

$\therefore P_{iy} = P_{fy} \therefore 0 = -Mv + m(u \mp v)$

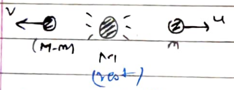
$v_{m/b} = v_m - v_b$

$-u = v_m - (v)$

$v_m = u \mp v$

$\therefore v = \dfrac{mu}{M+m}$

(4) Explosion of Bomb. at rest.



At time of explosion (Instantly)

$\Sigma F_{ext} = 0$

$\vec{P_i} = \vec{P_f}$ Just before & After explosion

Work done by internal force on rigid body = 0 (Always)

" on non rigid / Explosion / collision = Can be Non zero

$\therefore P_{ix} = P_{fx}$

~~$0 = m(u) + (M-m)(-v)$~~

$0 = m_1 v_1 - m_2 v_2$

$P = \therefore m_1 v_1 = m_2 v_2$

work done by Internal : $\dfrac{1}{2} m_1 v_1^2 + \dfrac{1}{2} m_2 v_2^2 - 0$

⇓

Energy stored in form of chemical energy $= \dfrac{P^2}{2m_1} + \dfrac{P^2}{2m_2} = \dfrac{P^2}{2}\left[\dfrac{1}{m_1} + \dfrac{1}{m_2}\right]$

$= \dfrac{P^2}{2\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)}$

** Energy stored $= \dfrac{P^2}{2\mu}$

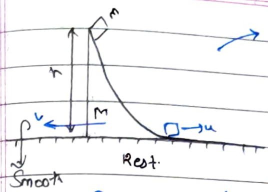
$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ (Reduced mass).

Explosion problems mei

$\vec{P_i} = \vec{P_f}$ Karna hai

Just Before & After explosion.

Q)



→ Explosion problem

$E_{int} = mgh$

$E_f = K.E_M + K.E_m$.

$\therefore$ along x-axis $\vec{P_{ix}} = \vec{P_{fx}}$

$\therefore 0 = M(-v) + mu$

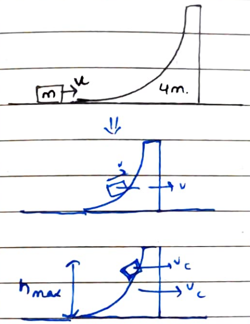
$\therefore mu = Mv = P$.

$E_{int} = \dfrac{P^2}{2\mu} = \dfrac{P^2}{2\frac{Mm}{M+m}}$

$\cancel{m}gh = \dfrac{(\cancel{m}u)^2 (M+m)}{2M\cancel{m}}$

$\therefore u = \sqrt{\dfrac{2Mgh}{M+m}}$

Q)



$\therefore \vec{P_{ix}} = \vec{P_{fx}}$

$m(u) + 0 = 4m(v_c) + m(v_c)$

$\therefore \dfrac{\cancel{m}u}{5\cancel{m}} = v_c$

$\therefore$ WET

$-mgh = \dfrac{1}{2} 5m (v_c)^2 - \dfrac{1}{2} m(u)^2$

$-\cancel{m}gh = \dfrac{\cancel{5}}{2}\cancel{m}\left(\dfrac{u}{\cancel{5}}\right)^2 - \dfrac{1}{2}\cancel{m}u^2$

$-gh = \dfrac{u^2}{10} - \dfrac{u^2}{2} \Rightarrow h = \dfrac{u^2}{g}\left[\dfrac{1}{2} - \dfrac{1}{10}\right]$

$h = \dfrac{u^2}{g}\left[\dfrac{4}{10}\right] = \dfrac{2u^2}{5g}$

* Conservation of linear Momentum

Block - Bullet System :

a) when bullet remains embedded in the block

At time of collision, Along x

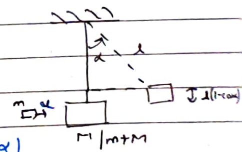
$\vec{P_{ix}} = \vec{P_{fx}}$

$m(u) = (M+m)v$.

$\therefore v = \dfrac{mu}{M+m}$

$\alpha = \cos^{-1}\left(\dfrac{2gl - v^2}{2gl}\right)$

Height risk $= l(1 - \cos\alpha)$



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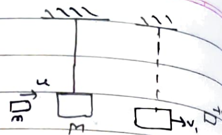
b). If bullet emerges out of block



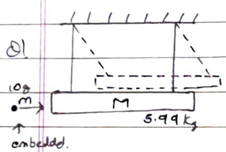
$\therefore \sum F_{net} = 0$, $\vec{P_{ix}} = \vec{P_{fx}}$

$mu = Mv_1 + mv_2$

$\alpha = \cos^{-1}\left(\frac{2gl - v_1}{2gl}\right)$

$h = l(1-\cos\alpha)$

Q)



find speed of bullet before collision $(g = 9.9)$

$m_{bullet} = \frac{10}{1000} = 0.01\ kg$

Along x-axis $\qquad h = l(1-\cos\alpha) = 9.9\ cm$

$mu = (M+m)V$

$V = \frac{mu}{M+m} = \frac{10^{-2}u}{6}$ $\rightarrow$ WET (Along height rise)

$\cancel{0 - (M+m)gh} - \frac{1}{2}(M+m)v^2 = -\frac{1}{2}mu^2$

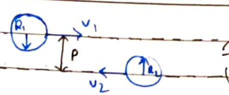
$\left(\frac{10^{-2}u}{6}\right)^2 = 2gh$ $\qquad 0 + (\cancel{M+m})gh = 0 + \frac{1}{2}(\cancel{M+m})v^2$

$u = \sqrt{\frac{36 \times 2 \times g \times h}{10^{-4}}}$ $\qquad v^2 = 2gh$

$u = \sqrt{\frac{2 \times 36 \times 9.9 \times 9.9}{10^{-4} \times 100}} = \frac{9.9 \times 6 \times 10^{-1}\sqrt{2}}{10^{-2}} = \underline{831.4}$

* Collision :-

① Impact Parameter : (P)



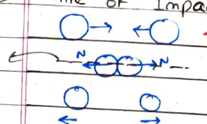
$P > (R_1+R_2)$ No collision

$P = (R_1+R_2)$ Grazing collision

$P < (R_1+R_2)$ collision will occur

$P = 0$ → Head on $\qquad 0 < P < R_1 + R_2$ → oblique collision.

② Line of Impact



→ Just before collision

→ At time of collision

→ Just After collision

Line of impact (LoI).

① Along LoI, If we consider individual bodies there momentum changes

② Along LoI, If we consider whole system, Momentum is conserved.

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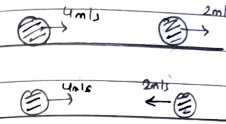
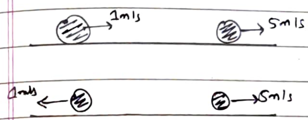
③. Velocity of approach :- Relative Velocity.



→ Velocity of approach = 4 − 2 = 2 !

→ —— " —— = 4 − (−2) = 6 !

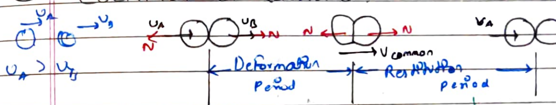
④ Velocity of Separation :- Relative Velocity



→ Velocity of separation = 5 − 1 = 4 !

→ —— " —— = 5 − (−1) = 6 !

⑤ Coefficient of Restitution



$V_B > V_A$

$u_A > u_B$

Deformation Period

Restitution period

Max. Deformation

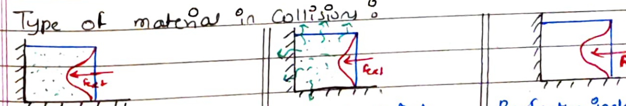
when bodies regain their shape (diff for All material)

- Time interval is very small
- If both bodies are in system , $\sum F_{net} = 0$
- $\vec{P_i} = \vec{P_{A_f}} = \vec{P_f}$

e (coefficient of Restitution) $= \frac{\text{Velocity of Sep. along LoI}}{\text{Velocity of Approach along LoI}}$

e is always cal. along LoI

* Type of material in Collision :



Perfectly Elastic

- Body will deform
- No loss of Energy
- All Energy is stored in P.E.
- When $F_{ext} = 0$, This P.E. is used to Regain Back

Inelastic Body

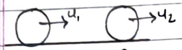
- Body will deform
- Some of energy is in P.E. & some is lost
- when $F_{ext} = 0$, body will not regain properly

Perfectly inelastic body

- All energy is lost & No Regaining After $F_{ext} = 0$

* Conservation of Momentum during impact

Just before collision



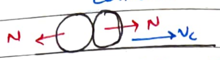
$u_1 > u_2$

$\vec{P}_i = m_1\vec{u}_1 + m_2\vec{u}_2$

$K.E_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$\vec{V}_{app} = \vec{u}_1 - \vec{u}_2$

AT time of collision



Max. deformation

$P_{At} = (m_1+m_2)\vec{V}_c$

$KE_a = \frac{1}{2}(m_1+m_2)V_c^2$

e = 1, Perfectly Elastic

"Some energy is stored in form of P.E."

Velocity of separation $= \vec{v}_2 - \vec{v}_1$

$\vec{P}_f = m_1\vec{v}_1 + m_2\vec{v}_2$

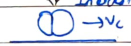
$K.E_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

1. $\vec{P}_i = \vec{P}_a = \vec{P}_f$
2. $KE_i \neq KE_a$
3. $KE_f \neq KE_a$
4. $KE_i = KE_f$

$T.E_i = T.E_a = T.E_f$

$P.E_{stored\,max} = K.E_i - K.E_a$

e = 0 Perfectly Inelastic



$v_1 = v_2 = v_c$

$\vec{P}_f = (m_1+m_2)\vec{V}_c$

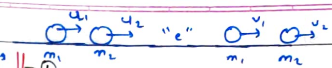
- $\vec{P}_i = \vec{P}_a = \vec{P}_f$
- $KE_i \neq KE_a$

TE is not conserved, There will be loss

~~Ka ≠ K~~

- $KE_a = KE_f$
- loss $= KE_i - KE_a$

General collision Case:



- $\boxed{m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2}$ — (1)
- $\boxed{e = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}}$

∴ $e\vec{u}_1 - e\vec{u}_2 = \vec{v}_2 - \vec{v}_1$

$\vec{v}_1 + e\vec{u}_1 = \vec{v}_2 + e\vec{u}_2$

$\vec{v}_2 = \vec{v}_1 + e\vec{u}_1 - e\vec{u}_2$ — (II)

Put in (1) → we get,

$$\boxed{\vec{v}_1 = \frac{(m_1 - em_2)\vec{u}_1 + (1+e)m_2\vec{u}_2}{m_1+m_2}}$$

$$\boxed{\vec{v}_2 = \frac{(m_2 - em_1)\vec{u}_2 + (1+e)m_1\vec{u}_1}{m_1+m_2}}$$

- loss of K.E. $= K.E_i - K.E_f$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right]$$

loss of K.E. $= \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2(1-e^2)$

For e = 1

1. loss of K.E. = 0
2. If masses are same & 2 is at rest
   ∴ Velocity get interchanged along LOI.
3. $m_1 >>> m_2$ & $e=1$

$$v_1 = \frac{(m_1 - e\cancel{m_2})\vec{u}_1 + (1+e)\cancel{m_2}\vec{u}_2}{m_1+\cancel{m_2}} = \frac{m_1\vec{u}_1}{m_1} = \underline{\vec{u}_1}$$

$$v_2 = \frac{(\cancel{m_2} - em_1)\vec{u}_2 + (1+e)m_1\vec{u}_1}{m_1+\cancel{m_2}} = \frac{-m_1\vec{u}_2 + 2m_1u_1}{m_1} = \underline{2\vec{u}_1 - \vec{u}_2}$$

For e = 0

1. loss of K.E. $= \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(\vec{u}_1 - \vec{u}_2)^2$
2. $\vec{v}_1 = \dfrac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$

   $\vec{v}_2 = \dfrac{m_2\vec{u}_2 + m_1\vec{u}_1}{m_1+m_2}$

   } Both stick to each other along LOI

Q). Find final velocities and Max. P.E. stored for :

1). 6 m/s → 1 kg, 3 m/s → 2 kg , for $e = 1, \frac{1}{2}, 0$

a) b) c)

At time $1\times6 + 2\times3 = (1+2)v_c$

$v_c = \frac{12}{3} = 4$

Method I : a) $(1)(6) + (2)(3) = (1)v_1 + (2)v_2$

$\therefore 12 = v_1 + 2v_2$

$e = 1 = \frac{v_2 - v_1}{3}$ $\therefore 3 = v_2 - v_1$

$\therefore \cancel{v_1} + 2v_2 = 12$

$v_2 - \cancel{v_1} = 3$

$3v_2 = 15$

$\underline{v_2 = 5}$

$\therefore 5 - v_1 = 3$

$\underline{v_1 = 2}$

$P.E_{stored} = K.E_i - K.E_f$

$= 27 - 24 = \underline{3J}$

$KE_i = \frac{1}{2}(1)(6)^2 + \frac{1}{2}\times2\times3^2 = 18 + 9 = 27$

$KE_f = \frac{1}{2}(1+2)(4)^2 = \underline{24}$

b) Method II : direct Formula $e = 1/2$

$v_1 = \frac{(1 - \frac{1}{2}\cdot2)6 + (1+\frac{1}{2})2\cdot3}{3}$ $v_2 = \frac{(2 - \frac{1}{2}\times1)3 + (1+\frac{1}{2})(1)\cdot(6)}{3}$

$v_1 = \frac{(3/2)6}{3} = 3$ $= \underline{9/2}$

c) $e = 0$ Method I : $(1)\times6 + 2\times3 = 1\times v_c + 2\times v_c$

$e = \frac{v_2 - v_1}{3} = 0$ $12 = 3v_c$ $\therefore \underline{v_c = 4}$

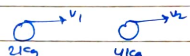
$\therefore v_1 = v_2 = v_c$

$\therefore K.E._{loss} = \frac{1}{2}\times1\times6^2 + \frac{1}{2}\times2\times3^2 - \frac{1}{2}(1+2)4^2$

$= 27 - 24 = \underline{3J}$

For P.E. stored in e=1 is same as that of loss of K.E for e=0

1). 7 m/s → 2 kg, ← 1 m/s 4 kg a) for $e = 1$



$\therefore \vec{P}_{cons} \Rightarrow 2(7) + 4(-1) = 2(v_1) + 4(v_2)$

$\therefore 12 = 2v_1 + 4v_2$ $e = 1 = \frac{v_2 - v_1}{9}$

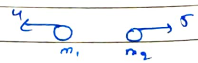
$\underline{6 = v_1 + 2v_2}$

$\therefore \cancel{v_1} + 2v_2 = 6$ $\therefore v_2 - v_1 = 9$

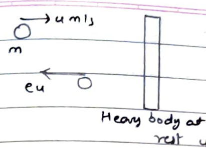
$v_2 - \cancel{v_1} = 9$ $\Rightarrow 5 - v_1 = 9$

$3v_2 = 15$ $\underline{v_1 = -4}$

$\underline{v_2 = 5}$



3).

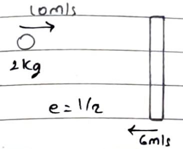


If $e = e' = \frac{v_2 - v_1}{u_1 - u_2}$

$\therefore eu_1 = -v_1$

$\therefore \underline{v_1 = -eu_1}$

4).



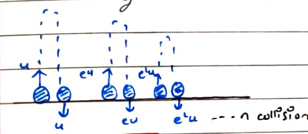
$m_2 \gg m$. $\therefore v_1 = \frac{(m_1 - em_2)u_1 + (1+e)m_2u_2}{m_1 + m_2}$

$v_1 = -eu_1 + (1+e)u_2$

$v_1 = -\frac{1}{2}\cdot10 + (1+\frac{1}{2})(-6)$

$v_1 = -5 + (-9) = \underline{-14 \text{ m/s}}$

* Bouncing of ball



$T_0 = \frac{2u}{g}$ $H_0 = \frac{u^2}{2g}$

$v_1 = eu$, $T_1 = \frac{2eu}{g} = eT_0$

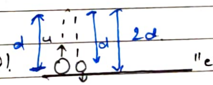
$H_1 = \frac{(eu)^2}{2g} = e^2H_0$

$\therefore v_n = e^n v_0$

$T_n = e^n T_0$

$H_n = e^{2n} H_0$

Q).



Find time taken for ball to stop.

$\therefore T = \frac{2u}{g} + \frac{2eu}{g} + \frac{2e^2u}{g} \ldots$

$= \frac{2u}{g}(1 + e + e^2 + e^3 + \ldots)$

$T = \frac{2u}{g}\left(\frac{1}{1-e}\right)$

Total distance travelled by ball

$\therefore$ Total distance in 2 collision $= 2d = 2\left(\frac{u^2}{2g}\right)$

$2d = \frac{u^2}{g}$

$\therefore$ distance $= \frac{u^2}{g} + \frac{e^2u^2}{g} + \frac{e^4u^2}{g} + \ldots$

$\frac{u^2}{g}(1 + e^2 + e^4 \ldots) \Rightarrow \frac{u^2}{g}\left(\frac{1}{1-e^2}\right)$

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Average Speed = $\dfrac{\text{Total dist}}{\text{Total time}} = \dfrac{\frac{u^2}{g}\left(\frac{1}{1-e^2}\right)}{\frac{2u}{g}\left(\frac{1}{1-e}\right)} = \dfrac{u}{2}\left(\dfrac{1}{1+e}\right)$

Q! HCV. m —— l —— l —— $2m$, $e=1$. Find velocities of balls.

$\therefore$ $\vec{P}$ con.

$m(\sqrt{2gl}) + 2m(-\sqrt{2gl}) = m(v_1) + 2m(v_2)$

by WET

$mgl = \frac{1}{2}mv^2 \therefore u_1 = u_2 = \sqrt{2gl}$.

$\therefore v_1 + 2v_2 = -\sqrt{2gl}$ —(1)

& $e = 1 = \dfrac{v_2 - v_1}{u_1 - u_2}$ $(-u)$ $= v_2 - v_1 = 2u$

$\therefore \cancel{v_1} + v_2 = -u$

$2v_2 - \cancel{v_1} = 2u$

$3v_2 = u$

$v_2 = u/3$

$\therefore \dfrac{u}{3} - v_1 = 2u$

$v_1 = \dfrac{u}{3} - 2u = -\dfrac{5u}{3}$

Q! "e=1" 2kg → 1.0 m/s, 4kg; $\mu = 0.2$; 16 cm

Find the separation betn the two bodies when they come to rest.

Wet For block I

$-\mu 2g\left(\frac{16}{100}\right) = \frac{1}{\cancel{2}}\cancel{2}v^2 - \frac{1}{\cancel{2}}\times\cancel{2}\times 1^2$

$-0.2\times 2\times 10\times \frac{16}{100} = v^2 - 1$

$1 - \frac{64}{100} = v^2 \quad \therefore v = 0.6$

0.6 m/s → 2kg, 4kg, $u_2 = 0$, $e=1$

$\therefore v_1 = \dfrac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$

$= \dfrac{(2-4)\times 0.6}{6} = -0.2$ m/s

$v_2 = \dfrac{(m_2 - m_1)u_2 + (2m_1u_1)}{6}$

$= \dfrac{2\times 2\times 0.6}{6} = 0.4$ m/s

0.2 ← → 0.4; x_1, x_2

$\therefore$ WET

$-\mu 2g x_1 = -\frac{1}{2}\times 2\times(0.2)^2$

$x_1 =$ ___

$-\mu 4g x_2 = -\frac{1}{2}4\times(0.4)^2$

x_2 ___

Ans $= x_1 + x_2$

---

lec-06. M.2019

Q! A: $m \to v$, B: m, C: M; $e=0$, $e=0$

5/6th of K.E. is lost in whole process then value of $M/m = ?$

(I) For A & B collision

$e = 0$, $\therefore m(v) + m(0) = (m+m)v_c$

$\therefore v_c = \dfrac{mv}{2m} = \dfrac{v}{2}$.

(II) $2m \xrightarrow{v/2}$ M, $e=0$

$\therefore 2m(v/2) = (M+2m)v_f$

$\therefore v_f = \dfrac{mv}{M+2m}$

(III) $\therefore$ KE loss $= \frac{5}{6}K_i$

$K.E_i - KE_f$

$\therefore \frac{1}{2}mv^2 - \frac{1}{2}(M+2m)\left(\dfrac{mv}{M+2m}\right)^2 = \frac{5}{6}\cdot\frac{1}{2}mv^2$

$= \cancel{\frac{1}{2}mv^2} - \cancel{\frac{1}{2}}\dfrac{(M+2m)\,m^2v^2}{(M+2m)^2} = \frac{5}{6}\cancel{\frac{1}{2}mv^2}$

$1 - \dfrac{m}{M+2m} = \dfrac{5}{6} \Rightarrow \dfrac{M+\cancel{2m}-\cancel{m}}{M+2m} = \dfrac{5}{6}$

$\therefore 6M + 6m = 5M + 10m$

$M = 4m \quad \therefore \dfrac{M}{m} = 4$

Q! M.2021 C: $m \to v$; A: m —k— B: m; l

The maximum compression in the spring is.

$\therefore e = 1$

JBC

ATC

Possible if A moves with total v.

$\therefore e=1$

$\therefore P_i = P_f$

$\therefore m(v) = mv_c + mv_c$

$\therefore v_c = \dfrac{v}{2}$

$\therefore$ WET

$-\frac{1}{2}kx^2 = \frac{1}{2}(2m)v_c^2 - \frac{1}{2}mv^2$

$\therefore kx^2 = mv^2 - 2mv_c^2$

$x^2 = \dfrac{mv^2 - 2m(v/2)^2}{k}$

$x = \sqrt{\dfrac{mv^2}{2k}} = \sqrt{\dfrac{m}{2k}}\,v$

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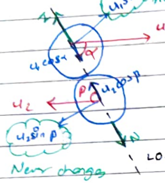
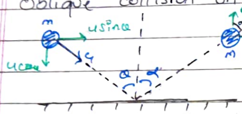
* Oblique Collisions :



$\therefore$ along LoI

$\vec{P_i}_{LoI} = \vec{P_f}_{LoI}$

- Oblique collision on horizontal ground.

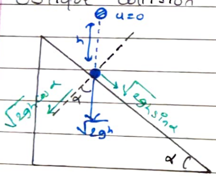


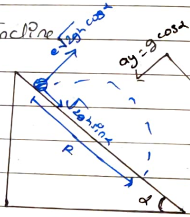
$v_f = \sqrt{e^2u^2\cos^2\theta + u^2\sin^2\theta}$

$v_f = u\sqrt{e^2\cos^2\theta + \sin^2\theta}$

$\tan\alpha = \frac{P}{B} = \frac{u\sin\theta}{eu\cos\theta} = \frac{\tan\theta}{e}$

- Oblique collision on Incline



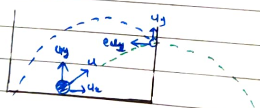


$S_x = u_x t + \frac{1}{2}a_x t^2$

$T = \frac{2u_y}{a_y}$

$T = \frac{2e\sqrt{2gh}\cos\alpha}{g\cos\alpha}$

- Projectile motion and Collision with wall

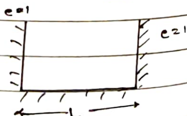


$H_m = \frac{u_y^2}{2g}$

$H_{max} = \frac{e^2 2gh\cos^2\alpha}{2g\cos\alpha}$

- Range is impacted by e.
  T & H remains same

↳ Summary :



if $L = \frac{R}{2}$ & $e=1$ for both wall

whatever may be position of person it will always land on his position

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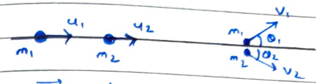
- Oblique collision for point masses



If bodies are point masses

$\therefore$ we cannot draw LoI

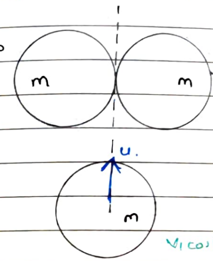
$\therefore \sum \vec{P_i}_x = \sum \vec{P_f}_x$

$\sum \vec{P_i}_y = \sum \vec{P_f}_y$

$\vec{P_i}_x = \vec{P_f}_x \Rightarrow m_1u_1 + m_2u_2 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$

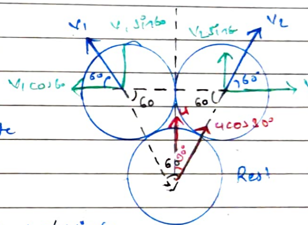
$\vec{P_i}_y = \vec{P_f}_y = m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2$

Q) $u=0$ $u=0$, After collision, the third ball come to rest, find coefficient of restitution betn the balls.



$\therefore$ symmetry

$\therefore v_1 = v_2$



$\therefore$ ~~$u\cos30° = v$~~

$\therefore$ For complete system.

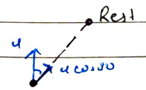
$\vec{P_i}_y = \vec{P_f}_y$

$\cancel{m}u = \cancel{m}v\sin60 + \cancel{m}v\sin60$

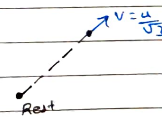
$u = 2v\sin60$ $\therefore u = v\sqrt{3}$ & $v = u/\sqrt{3}$

$\therefore$

Before Collision



After Collision



$\therefore e = \frac{v_2 - v_1}{\vec{u_1} - u_2}$ along LoI

$\therefore e = \frac{u\sqrt{3} - 0}{\frac{u\sqrt{3}}{2} - 0}$

$e = \frac{2}{3}$

~~$\vec{P_i} = \vec{P_f}$ along LoI~~

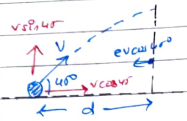
$\therefore$ ~~$m \cdot u\cos30°$~~ ~~$= m \cdot v$~~

$\therefore u_1 = u\cos30 = \frac{u\sqrt{3}}{2}$ $v_1 = 0$

$u_2 = 0$ $v_2 = u/\sqrt{3}$

4- Q) lec - 11. The ball rebounds to her hand. what is 'e'.

$\therefore T = \dfrac{2v\sin 45^\circ}{g}$



$t_{\text{gone}} = \dfrac{d}{v\cos 45}$ $\qquad t_{\text{come}} = \dfrac{d}{ev\cos 45}$

$\therefore T = t_{\text{gone}} + t_{\text{come}}$

$\dfrac{2v\sin 45^\circ}{g} = \dfrac{d}{v(1/\sqrt{2})} + \dfrac{d}{ev(1/\sqrt{2})}$

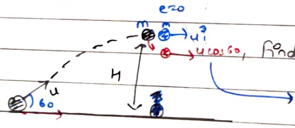
$\dfrac{\cancel{\sqrt{2}}\,2v(1)}{g\,\cancel{\sqrt{2}}} = \dfrac{d\sqrt{2}}{v} + \dfrac{d\sqrt{2}}{ev}$ $\qquad \therefore \dfrac{v}{g} = \dfrac{d}{v}\left[1+\dfrac{1}{e}\right]$

$\dfrac{v^2}{gd} = 1 + \dfrac{1}{e}$

$\dfrac{1}{e} = \dfrac{v^2}{gd} - 1 = \dfrac{v^2 - gd}{gd}$

$e = \dfrac{gd}{v^2 - gd}$

5- Q) find Range.

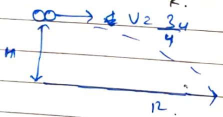


$\therefore \vec{P_i} = \vec{P_f}$

$\cancel{m}(u\cos 60) + \cancel{m}u \cdot \varepsilon = (2\cancel{m})v$

$\therefore v = \dfrac{u(1/2) + u}{2} = \dfrac{3u}{4}$

$\therefore$



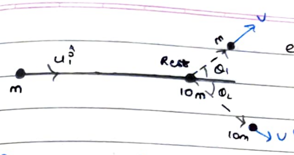
$H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2 \cdot (3/4)}{2 \cdot 10}$

$H = \dfrac{3u^2}{80}$

$\therefore R = v\sqrt{\dfrac{2H}{g}} = \dfrac{3u}{4}\sqrt{\dfrac{2 \cdot 3u^2}{10 \cdot 80}}$

$= \dfrac{3\sqrt{3}u^2}{8g}$

Q) $e = 1$. m_1 moves with half its initial K.E. If $\sin\theta_1 = \sqrt{n}\,\sin\theta_2$ find n.



For m, $KE_f = \frac{1}{2}KE_i$ $\therefore \frac{1}{\cancel{2}}\cancel{m}v^2 = \frac{1}{\cancel{2}} \cdot \frac{1}{2}\cancel{m}u^2$

$\therefore v = u/\sqrt{2}$

$\vec{P_{ix}} = \vec{P_{fx}}$

$\therefore m \cdot u = m(v\cos\theta_1) + 10m(v'\cos\theta_2)$

$\vec{P_{iy}} = \vec{P_{fy}}$

$\therefore 0 = m(v\sin\theta_1) + 10m(-v'\sin\theta_2)$

$\therefore v\sin\theta_1 = 10v'\sin\theta_2$

$\dfrac{u}{\sqrt{2}}\sin\theta_1 = 10v'\sin\theta_2$

$\dfrac{\cancel{u}}{\cancel{\sqrt{2}}}\sin\theta_1 = \cancel{10}^{\sqrt{10}}\dfrac{\cancel{u}}{\cancel{\sqrt{2}}\cancel{\sqrt{10}}}\sin\theta_2$

$\therefore \sqrt{n} = \sqrt{10}$ $\quad \therefore n = 10$

Since energy is cons. $e=1$

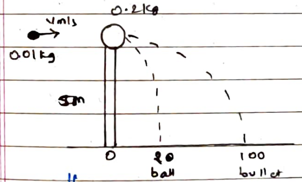
$\therefore K.E_i = KE_f$

$\frac{1}{2}mu^2 = \frac{1}{2}\cdot\frac{1}{2}mu^2 + \frac{1}{2}(10m)v'^2$

$\frac{1}{2}\cdot\frac{1}{\cancel{2}}\cancel{m}u^2 = \frac{1}{\cancel{2}}10\cancel{m}v'^2$

$\therefore v'^2 = \dfrac{u}{\sqrt{2}\sqrt{10}}$

Q) 2011 Adv. Find initial v.



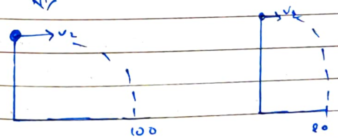
$\therefore$ P cons.

$\therefore (0.01)v = 0.2v_1 + 0.01v_2$

$\dfrac{v}{100} = \dfrac{2}{10} \cdot 20 + \dfrac{1}{100} \cdot 100$

$v = 400 + 100$

$v = 500$ m/s



$\therefore R = v\sqrt{\dfrac{2H}{g}}$ $\qquad \therefore 20 = v_1\sqrt{\dfrac{2 \cdot 5}{10}}$

$100 = v_2\sqrt{\dfrac{2 \cdot 5}{10}}$ $\qquad v_1 = 20$

$v_2 = 100$

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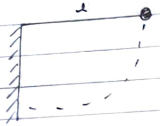
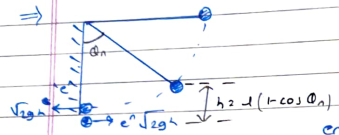
Q). $e = \frac{2}{\sqrt5}$, what is the minimum no. of collision after which the Amplitude of oscillations becomes less than 60° ?



$v_{\text{hit } 1^{st} \text{ time}} = \sqrt{2gl}$

$v_{\text{rebound}} = e\sqrt{2gl}$

$v_{\text{rebound after } n \text{ time collision}} = e^n\sqrt{2gl}$



$h = l(1-\cos\theta_n)$ ⇐ After n^{th} collision, height rise

energy consr.

$\frac{1}{2}m(e^n\sqrt{2gl})^2 = mgl(1-\cos\theta_n)$

$= \frac{1}{2}e^{2n}\cdot 2gl = gl(1-\cos\theta_n)$

$\therefore e^{2n} = 1-\cos\theta_n$

$\left(\frac{2}{\sqrt5}\right)^{2n} = 1-\cos\theta_n$

$\therefore \left(\frac{4}{5}\right)^n = 1-\cos\theta_n$

$\therefore \cos 60 = 1/2$

$\therefore$ if $\theta < 60°$

$\cos 60 > 1/2$

$\therefore \left(\frac{4}{5}\right)^n < 1-\frac{1}{2}$

$\left(\frac{4}{5}\right)^n < 0.5$

$(0.8)^n < 0.5$

$\therefore (0.8)^2 = 0.64 > 0.5$

$(0.8)^3 = 0.6\ldots > 0.5$

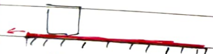
$(0.8)^4 = 0.5\ldots > 0.5$

$(0.8)^5 = 0.4\ldots < 0.5$

$\therefore$ max. at power 4 $\therefore n = 4$

$\therefore$ 4 collision will happen.

Q). $m = 0.4\,kg$



oil layer

$u = 0$

An impulse of 1.0 Ns is applied on block at $t = 0$, $v(t) = v_0 e^{-t/\tau}$, where v_0 = Const & $\tau = 4s$. The disp. of block when $t = \infty$ is

(Take $e^{-1} = 0.37$)

$\therefore v(t) = v_0 e^{-t/\tau}$

at $t = 0$, $v = v_0 = 2.5$

impulse $= m.u_i$

$\therefore u_i = \frac{1}{0.4} = \frac{10}{4} = 2.5$

$\therefore v = 2.5e^{-t/\tau}$

$\frac{dx}{dt} = 2.5e^{-t/\tau}$

$\int_{x_i}^{x_f} dx = \int_0^4 2.5e^{-t/4}dt$

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$\therefore \Delta x = 2.5 \cdot \left[\frac{e^{-t/4}}{\frac{d(-t/4)}{dt}}\right]_0^4 = 2.5\left[\frac{e^{-t/4}}{-1/4}\right]_0^4$

$\Delta x = 2.5\left[4e^{-t/4}\right]_4^0 = 4\times 2.5\left[0.27 - 0.37\right]$

Rotational Motion

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1. Moment of Inertia (I)

↳ depends on Mass of body, Mass distribution of body about the axis

$\boxed{I = mr^2}$ → radius of rot / $\perp^{r}$ dist from axis

for discrete particle

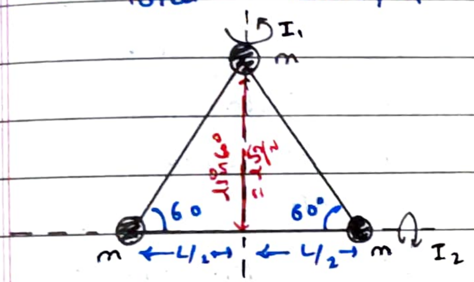
$\therefore I_{Total} = \sum m_i r_i^2$

for continuous body

$I = \int dm\, r^2$

$dm = \lambda\, dx$
$= \sigma\, dx$
$= \rho\, dx$

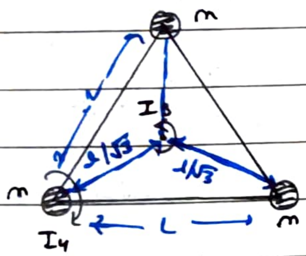
Q) i)



For $I_1 = m(0)^2 + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2$

$= \frac{ml^2}{2}$

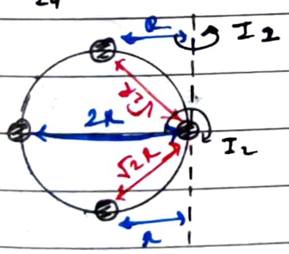
$I_2 = m(0)^2 + m(0)^2 + m\left(\frac{l\sqrt{3}}{2}\right)^2 = \frac{3ml^2}{4}$



$I_3 = 3m\left(l/\sqrt{3}\right)^2 = ml^2$

$I_4 = m(0)^2 + 2m(l^2) = 2ml^2$

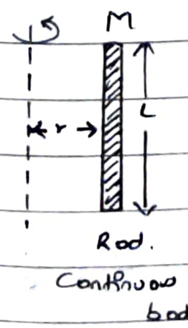
ii)

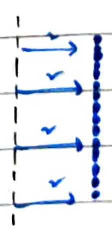


$I_2 = m(0) + m(R)^2 + m(R^2) + m(2R)^2$
$= 6mR^2$

$I_2 = m(0) + m(2R)^2 + 2(m(\sqrt{2}R)^2)$
$= 8mR^2$

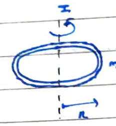
Note: If all mass is at fix dist from axis, then finding MOI.





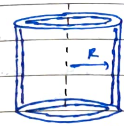
$\therefore I = \int dm r^2$
$= r^2 \int dm$
$I = mr^2$

Applied concept :



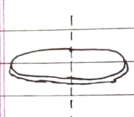
$I = mR^2$

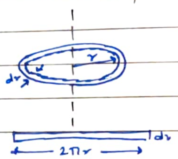
$I = mR^2$



$I = mR^2$

Q) find IOM of disc , passing $\perp^r$ to plane of ring.





$dA = 2\pi r\, dr$

$\therefore\ I = \int dm\, r^2$

$dm = \sigma \cdot dA$

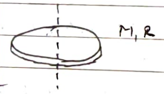
$dm = \dfrac{M}{\pi R^2} \cdot 2\pi r\, dr$

$\therefore\ I = \int \dfrac{M \cdot 2 r\, dr\, r^2}{R^2}$

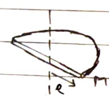
$I = \dfrac{2M}{R^2}\left[\dfrac{r^4}{4}\right]_0^R = \dfrac{MR^2}{2}$

Cutting of disc and semi disc and angular disc

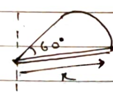
original mass he m. hai



$I = \frac{1}{2}MR^2$

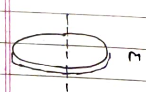


$I = \frac{1}{2}MR^2$

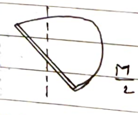


$I = \frac{1}{2}MR^2$

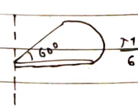
cutting :



$I = \dfrac{mR^2}{2}$



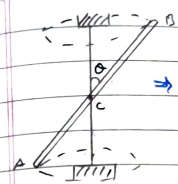
$I = \frac{1}{2}\left(\frac{M}{2}\right)R^2 = \dfrac{MR^2}{4}$



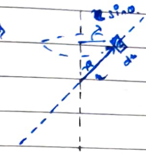
$I = \frac{1}{2}\left(\frac{M}{6}\right)R^2$

$I = \dfrac{MR^2}{12}$

Q) find MOI



$\Rightarrow$



$\therefore\ I = \int dm\, r^2$

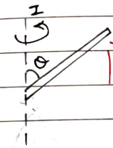
$dm = \lambda\, dx$

$= \dfrac{M}{L}\, dx.$

$\therefore\ I = \int \dfrac{M}{L}\, dx \cdot (x \sin\theta)^2$

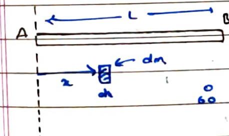
$\therefore\ I = \dfrac{M}{L}\sin^2\theta \left[\dfrac{x^3}{3}\right]_{-L/2}^{+L/2}$

General formula :- $\boxed{I = \frac{1}{12}ML^2\sin^2\theta.}$



$\boxed{I = \frac{1}{3}ML^2\sin^2\theta}$

Q. $\lambda(x) = \lambda_0\left(1 + \dfrac{x}{L}\right)$, find IOM



$\therefore\ dm = \lambda\, dx = \lambda_0\left(1 + \frac{x}{L}\right)dx$

$\therefore$

$dI = dm\, x^2 = \lambda_0\left(1 + \frac{x}{L}\right)dx \cdot x^2$

$\therefore\ I = \int dm\, x^2 = \int_0^L \lambda_0\left(x^2 + \frac{x^3}{L}\right)dx$

$I = \lambda_0\left[\left[\frac{x^3}{3}\right]_0^L + \frac{1}{L}\left[\frac{x^4}{4}\right]_0^L\right] = \lambda_0\left[\frac{L^3}{3} + \frac{L^4}{4}\right]$

$= \dfrac{7}{12}(\lambda_0 L^3)$ — ①

$M = \int dm = \int_0^L \lambda_0\left(1 + \frac{x}{L}\right)dx$

$= \dfrac{7}{12}(\lambda_0 L)L^2$

$M = \lambda_0\left[L + \dfrac{L^2}{2L}\right]$

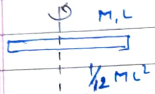
$M = \dfrac{3\lambda_0 L}{2}$

$= \dfrac{7}{12}\left(\dfrac{2M}{3}\right)L^2 = \dfrac{7ML^2}{18}$

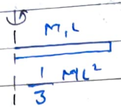
$\lambda_0 L = \left(\dfrac{M2}{3}\right)$

* List of MOI

1) Rod.

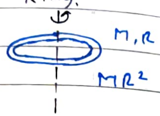


$\frac{1}{12}ML^2$



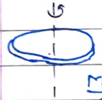
$\frac{1}{3}ML^2$

2) Ring.



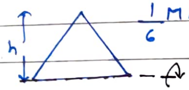
MR^2

3). Disc



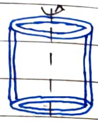
$\frac{MR^2}{2}$

4). Triangular plate



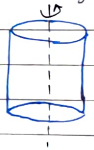
$\frac{1}{6}Mh^2$

5). Hollow cylinder



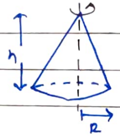
MR^2

6) Solid cylinder



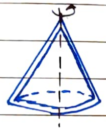
$\frac{1}{2}MR^2$

7). Solid Cone



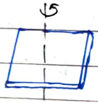
$\frac{3}{10}MR^2$

8). Hollow cone



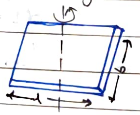
$\frac{1}{2}MR^2$

9). Square Plate



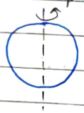
$\frac{1}{6}Ma^2$

10) Rectangular plate



$\frac{1}{12}M(a^2+b^2)$

11). Solid Sphere



$\frac{2}{5}MR^2$

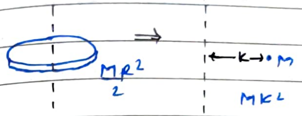
12). Hollow sphere

$\frac{2}{3}MR^2$

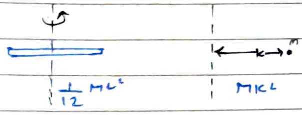
* Radius of Gyration (K)

It is that dist from axis of rotation where whole mass the body is kept as point mass without changing MoI.

1).



$\therefore K = \frac{R}{\sqrt{2}}$



$K = \frac{L}{\sqrt{12}}$

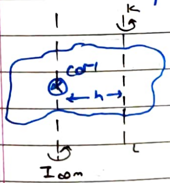
* Shifting of Moment of inertia

$\parallel^{el}$ axis theorm

- new axis is $\parallel^{el}$ to axis, passing through COM.
- for all types of body.

~~$I_z = I_x + I_y$~~

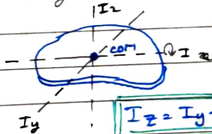
- ~~axis must pass from~~ COM



$\therefore I_{KL} = I_{com} + M_{body}h^2$

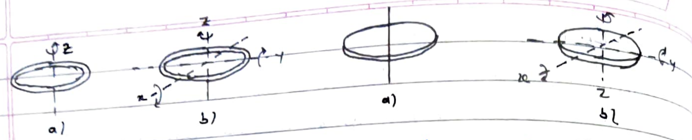
$\perp^r$ axis theorm.

- new axis is $\perp$ to Axis passing Through COM.
- Only for 2D object.



$\boxed{I_z = I_y + I_x}$

*** • axis must pass from COM

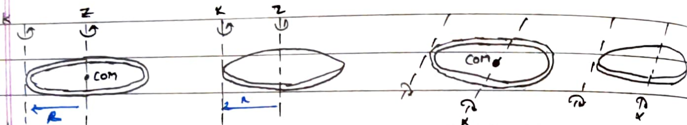


a) $I_z = MR^2$

b) $\perp^{r}$ axis theorm
$I_z = I_y + I_x$
$I_x \,\&\, I_y = I_{dia}$
$\therefore$
$I_z = MR^2 = 2I_{dia}$
$\therefore I_{dia} = \frac{MR^2}{2}$

a) $I_z = \frac{MR^2}{2}$

b) By $\perp^{r}$ axis theorm
$I_z = 2I_{dia}$
$\therefore I_{dia} = \frac{MR^2}{4}$



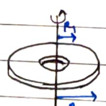
$\therefore \parallel^{r}$ axis th.
$I_{KL} = I_{com} + MR^2$
$= MR^2 + MR^2$
$\boxed{I_{KL} = 2MR^2}$

$I_{KL} = I_{com} + MR^2$
$= \frac{MR^2}{2} + MR^2$
$\boxed{I_{KL} = \frac{3}{2}MR^2}$

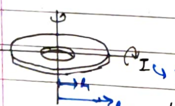
$\parallel^{r}$ axis
$\therefore I_{dia} = \frac{MR^2}{2}$
$\therefore I_{KL} = I_{dia} + MR^2$
$= \frac{MR^2}{2} + MR^2$
$\boxed{I_{KL} = \frac{3}{2}MR^2}$

$\parallel^{r}$ axis
$\therefore I_{dia} = \frac{MR^2}{4}$
$\therefore I_{KL} = \frac{MR^2}{4} + MR^2$
$\boxed{I_{KL} = \frac{5MR^2}{4}}$

1) Annular Disc :

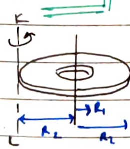


$\boxed{\frac{1}{2}M(R_1^2 + R_2^2)}$



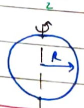
$I_{\perp} = \boxed{\frac{1}{4}M(R_1^2 + R_2^2)}$ (Dia)

$I_z = I_y + I_x$, $I_z = 2I_D$
$\therefore I_D = \frac{I_z}{2} = \frac{\frac{1}{2}M(R_1^2+R_2^2)}{2} = \frac{1}{4}M(R_1^2+R_2^2)$

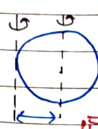


$I_{KL} = I_c + MR_2^2$
$= \frac{1}{2}M(R_1^2+R_2^2) + MR_2^2$
$\boxed{= \frac{M}{2}(3R_2^2 + R_1^2)}$

2) Solid Sphere :

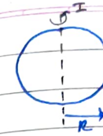


$\boxed{\frac{2}{5}MR^2}$

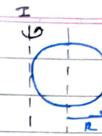


$I_{new} = I_c + MR^2$
$= \frac{2}{5}MR^2 + MR^2$
$\boxed{I_{new} = \frac{7}{5}MR^2}$

3).



$\boxed{I = \frac{2}{3}MR^2}$



$\boxed{I = \frac{5}{3}MR^2}$

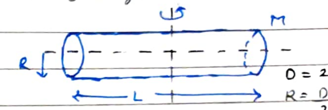
Thin Hollow Sphere

***

Q). MoI of a solid cylinder of length L and diameter D, about an axis passing through its centre of gravity & $\perp^{r}$ to its geometric axis is.

M. Repeating
V.V.imp

a) $M(D^2/4 + L^2/12)$
b). $M(L^2/16 + D^2/8)$
c) $M(D^2/4 + L^2/6)$
✓d) $M(L^2/12 + D^2/16)$



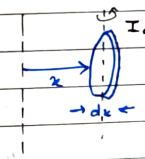
$D = 2R$
$R = \frac{D}{2}$

M.I : $R \to 0$ $\therefore$ thin rod $\Rightarrow I = \frac{1}{12}ML^2$

$L \to 0$ $\therefore$ Disc $\Rightarrow I_{dia} = \frac{MR^2}{4} = \frac{MD^2}{16}$

$\therefore$ Total $I = \left(\frac{1}{12}ML^2 + \frac{MD^2}{16}\right)$

M.II :



$I_{dia} = \frac{1}{4}dmR^2$
$= \frac{1}{4}\frac{M}{L}dx R^2$

Parallel axis theorm
MoI of element disc about com axis.

$\therefore dI = dI_{com} + dMx^2$
$dI = \frac{1}{4}\frac{M}{L}dx R^2 + \frac{M}{L}dx\, x^2$

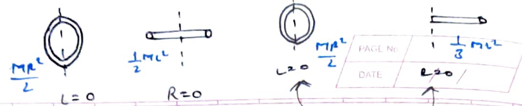
$dm = \rho\, dV$
$= \frac{M}{\pi R^2 L} \cdot \pi R^2 dx = \frac{M}{L}dx$

$\therefore I = \int dI = \int_{-L/2}^{+L/2} \frac{1}{4}\frac{M}{L}R^2 dx + \int_{-L/2}^{+L/2} \frac{M}{L}x^2 dx$

$I = \frac{1}{4}\frac{M}{L}R^2\left[\frac{L}{2} + \frac{L}{2}\right] + \frac{M}{L}\cdot\frac{1}{12}L^3$

$= \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ $\quad\therefore 2R = D$

$\therefore \frac{MD^2}{16} + \frac{ML^2}{12}$



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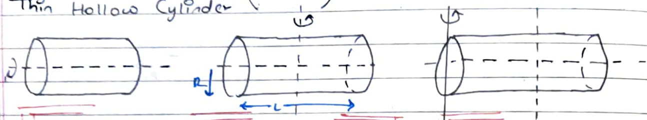
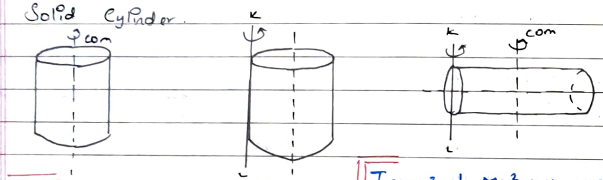
4). Thin Hollow Cylinder



$\boxed{MR^2}$ $\quad$ $\boxed{\frac{1}{12}ML^2 + \frac{1}{2}MR^2}$ $\quad$ $\boxed{\frac{1}{3}ML^2 + \frac{1}{2}MR^2}$

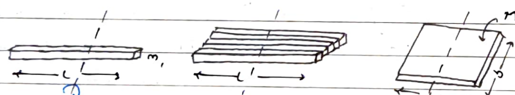
5). Solid Cylinder.



$\boxed{\frac{1}{2}MR^2 = I_{com}}$ $\quad$ $\boxed{\frac{3}{2}MR^2 = I_{KL}}$ $\quad$ $\boxed{I_{com} = \frac{1}{12}ML^2 + \frac{1}{4}MR^2}$

$\boxed{I_{KL} = \frac{1}{3}ML^2 + \frac{1}{4}MR^2}$

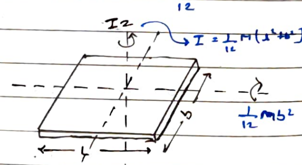
- MOI about an axis is independent of length ∥'' to axis.



$I = \frac{1}{12}ML^2$ $\quad$ $I_T = \frac{1}{12}m_1L^2 + \frac{1}{12}m_2L^2$ $\quad$ $I = \frac{1}{12}ML^2$

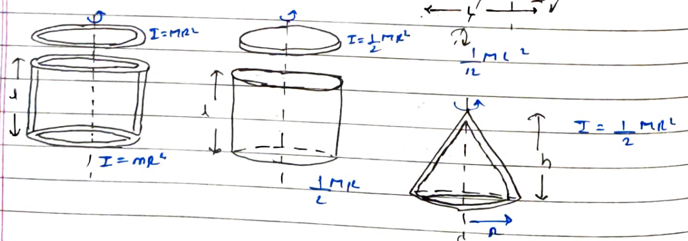
$\dots + \frac{1}{12}m_nL^2$

$\Sigma \therefore I_T = \frac{1}{12}(m_T)L^2$



$\frac{1}{12}ML^2$

*



$I = \frac{1}{2}MR^2$

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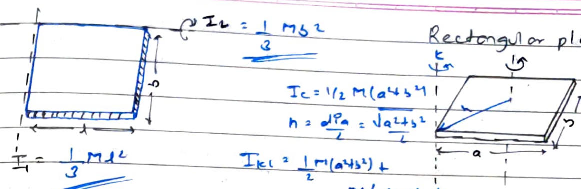


$I_2 = \frac{1}{3}Mb^2$

Rectangular plate

$I_1 = \frac{1}{3}ML^2$

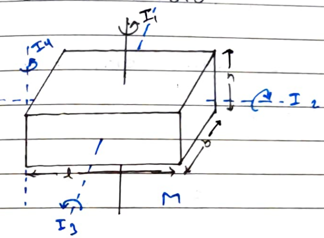
$I_C = \frac{1}{12}M(a^2+b^2)$

$h = \frac{dia}{2} = \frac{\sqrt{a^2+b^2}}{2}$

$I_{KL} = \frac{1}{12}M(a^2+b^2) + \frac{M(a^2+b^2)}{4}$

$I_{KL} = \frac{1}{3}M(a^2+b^2)$

6) MOI of Cuboid



$I_1 = \frac{1}{12}M(L^2+b^2)$

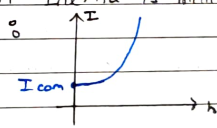
$I_2 = \frac{1}{12}M(h^2+b^2)$

$I_3 = \frac{1}{12}M(L^2+h^2)$

$I_4 = \frac{1}{3}M(L^2+b^2)$

* Moment of Inertia is minimum at COM

Graphically :



$I_{new} = I_{com} + Mh^2$

$I_{new} > I_{com}$

Q) IF MOI of body varies with x as $x^2 - 4x + 8$. find location of com

$I = f(x) = x^2 - 4x + 8$

$\frac{dI}{dx} = 2x - 4 = 0$

$x = 2 > 0$ Minima

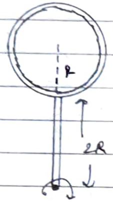
$I_{com} = 2^2 - 4(2) + 8$

$= 4 - 8 + 8 = 4\ kgm^2$

* Additive and Negative Systems

we can add / subtract MOI after shifting to common axis of Rotation.

Q!



∘∘ Additive system

1)  $I = \frac{1}{3} M(2R)^2$

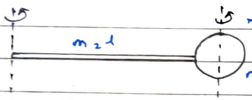
2)  $\cancel{I_{total}}\ I = MR^2$

∘∘ $I_{comm.} = MR^2 + M(3R)^2$

$= 10MR^2$

∘∘ $I_{Total} = 10MR^2 + \frac{4MR^2}{3}$

Q!



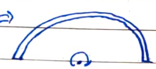
$\frac{2}{5}m_1r_1^2$; $m_2 l$; m_1, r_1

∘∘ $I_{Total} = \frac{1}{3}m_2l^2 + \left(\frac{2}{5}m_1r_1^2 + Mh^2\right)$

$I_{total} = \frac{1}{3}m_2l^2 + \frac{2}{5}m_1r_1^2 + m_1(r_1+l)^2$

Q! A uniform wire of length l and mass M is bent in the ~~sh~~ shape of a semicircle of radius r as shown in fig. Cal. moi about axis xx'.

⇒ 



l, M

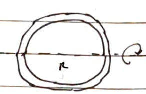
$l = \pi r$

$r = l/\pi$

$I = MR^2$

$= \frac{Ml^2}{\pi^2}$

Again:



$I_{dia} = \frac{1}{2}(2M)R^2$

$= MR^2$

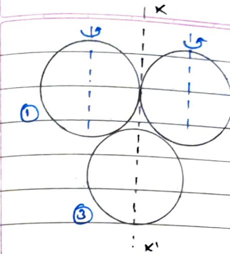
Half :

mass = half

$I = \frac{1}{2}\left(\frac{M}{\cancel{2}}\right)R^2, \frac{MR^2}{2}$

$= \frac{Ml^2}{2\pi^2}$

Q!



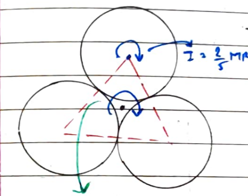
Rings, M, R, find MoI along xx'

$I_T = I_1 + I_2 + I_3$

$= \left(\frac{MR^2}{2} + MR^2\right)\left(\frac{MR^2}{2} + MR^2\right) + \frac{MR^2}{2}$

$= \frac{3}{\cancel{2}}MR^2 + \frac{MR^2}{2} = \frac{7MR^2}{2}$

Q!

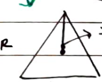


Identical sphere, M, R

$I = \frac{2}{5}MR^2$

$I = 3\left[I_c + M\left(\frac{2R}{\sqrt{3}}\right)^2\right]$

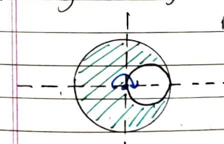
$= 3\left[\frac{2}{5}MR^2 + \frac{4MR^2}{3}\right]$



$l = 2R$

$\frac{l}{\sqrt{3}} = \frac{2R}{\sqrt{3}}$

Q! Negative system :



Disc of mass M and radius R, a smaller disc of radius R/2 is removed. Find MoI along z axis.

$M_{smaller} = \sigma \cdot A$

$= \frac{M}{\pi R^2} \cdot \pi r^2$

$r = R/2$

∘∘ $I_{c_{smaller}} : \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$ $M_s = \frac{M}{\pi R^2} \cdot \frac{\pi R^2}{4} = \frac{M}{4}$

$= \frac{MR^2}{32}$

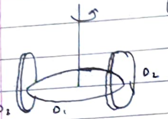
$I_{remaining} = I_{bigger} - I_{smaller}$ | common axis

$= \frac{1}{2}MR^2 - \left(\frac{MR^2}{32} + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2\right)$

→ mass of body

$= \frac{MR^2}{2} - \frac{MR^2}{32} - \frac{MR^2}{16} = \frac{13MR^2}{32}$

Q) Disc, find MOI



$\therefore I = I_{O_1} + I_{O_2} + I_{O_3}$

$= \frac{1}{2}MR^2 + 2\left(\frac{1}{4}MR^2 + MR^2\right)$

$= \frac{MR^2}{2} + \frac{5MR^2}{2} = \underline{\underline{3MR^2}}$

Q) Adv 2016 A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If its MOI about the tangential axis is equal to I, then value of r is equal to.

a) $\frac{2}{\sqrt{15}}R$

b) $\frac{2}{\sqrt{5}}R$

c) $\frac{3}{\sqrt{15}}R$

d) $\frac{\sqrt{3}}{\sqrt{15}}R$

M, R

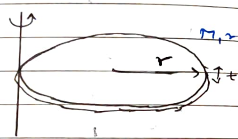
$I_s = \frac{2}{5}MR^2$

$\therefore I_D = I_s$

$\therefore \frac{3}{2}Mr^2 = \frac{2}{5}MR^2$

$r^2 = \frac{4}{15}R^2$

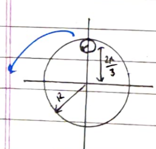
$r = \underline{\underline{\frac{2}{\sqrt{15}}R}}$



$I = \frac{1}{2}Mr^2 + Mr^2$

$I_D = \frac{3}{2}Mr^2$

Q) M. 2019 disc, R, 9M, small disc ⇒ radius = R/3, MOI of remaining part about axis ⊥ʳ to plane of disc passing through centre.



$r = \frac{R}{3}$

$A_r = \frac{\pi R^2}{9}$

$\therefore I_{bigger} = \frac{1}{2}(9M)(R^2)$

$9M \longrightarrow \pi R^2$

radius 1 $\longrightarrow 9M/\pi R^2$

$\frac{\pi R^2}{9} \longrightarrow \frac{9M}{\pi R^2} \times \frac{\pi R^2}{9} = M$

$I_{smaller/common\ axis} = I_{com} + Mh^2$

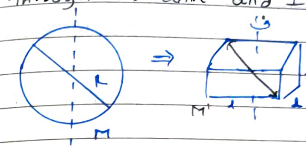
$= \frac{1}{2}M\left(\frac{R}{3}\right)^2 + (M)\left(\frac{2R}{3}\right)^2$

$I_{small} = \frac{MR^2}{18} + \frac{4MR^2}{3}$

$I_{Total} = \frac{9MR^2}{2} - \frac{MR^2}{18} - \frac{4MR^2}{3} = \underline{\underline{4MR^2}}$

Q) M. 2015 From a solid sphere of mass M, radius R, a cube of maximum possible volume is cut. MOI of cube about an axis passing through its centre and ⊥ʳ to one of through face is

⇒



For maximum volume

$\therefore$ diag $= 2R$ = max. length of cube

$\therefore \sqrt{3}\,l = 2R$

$l = \frac{2R}{\sqrt{3}}$

$\frac{4}{3}\pi R^3 \rightarrow M$

$1 \longrightarrow \frac{M3}{4\pi R^3}$

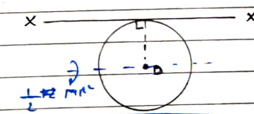
Vol. of cube $= l^3 = \left(\frac{2R}{\sqrt{3}}\right)^3 = \frac{8R^3}{3\sqrt{3}}$

$\frac{8R^3}{3\sqrt{3}} \rightarrow \frac{M3}{4\pi R^3} \times \frac{8R^3}{3\sqrt{3}} = \underline{\underline{\frac{2M}{\pi\sqrt{3}}}}$

$\therefore I = \frac{1}{12}M'(l^2 + l^2) = \frac{1}{6}M'l^2 = \frac{1}{3}\left(\frac{2M}{\pi\sqrt{3}}\right)\left(\frac{2R}{\sqrt{3}}\right)^2$

$I = \underline{\frac{4MR^2}{9\sqrt{3}\pi}}$

Q) Adv. 2000 A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The MOI of loop about XX' is.



⇒ l, ρ

$\therefore l = 2\pi R$

$R = l/2\pi$

$\boxed{M = \rho l}$

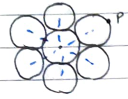
$I_{Ans} = I_c + Mh^2$

$= \frac{1}{2}MR^2 + M\cdot(R)^2 = \frac{3}{2}MR^2 = \frac{3}{2}(\rho l)\left(\frac{l}{2\pi}\right)^2$

$I_{Ans} = \underline{\underline{\frac{3\rho l^3}{8\pi^2}}}$

Q) Symmetrical disc, MOI normal to plane passing through P.

M. 2019



M, R

∴ First bring I_{all} to com.

∴ $I_{com} = \frac{1}{2}MR^2 + 6\left(\frac{1}{2}MR^2 + M(2R)^2\right)$

$= \frac{1}{2}MR^2 + 6\left(\frac{9MR^2}{2}\right) = MR^2\left(\frac{1}{2} + \frac{54}{2}\right)$

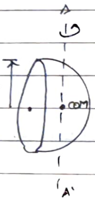
$= \frac{55}{2}MR^2$.

∴ $I_{at\ P\ of\ all} = \frac{55MR^2}{2} + (7M)(3R^2)$

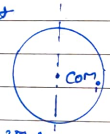
$= \frac{191}{2}MR^2$

***

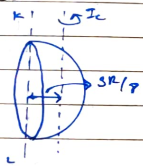
Q) Solid Hemisphere, Mass M. Find MOI along AA', passing through com.



∴ let



2M, R



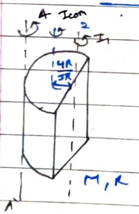
∴ $I = \frac{2}{5}(2M)R^2$

$\boxed{I_{com} = \frac{2MR^2}{5} - \frac{9MR^2}{64}}$

∴ $I_{KL} = I_{com} + Mh^2$

∴ $I_{com} = \frac{2}{5}MR^2 - M\left(\frac{3R}{8}\right)^2$

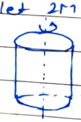
Q) Find MOI along AA'



M, R

$R - \frac{4R}{3\pi}$

⇒ let 2M



⇒

$I = \frac{1}{2}(M2)R^2$ $\quad I = \frac{1}{2}MR^2$

∴ $I_1 = I_{com} + Mh^2$

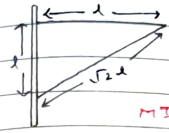
$\frac{1}{2}MR^2 = I_{com} + M\left(\frac{4R}{3\pi}\right)^2$

∴ $I_{com} = \frac{1}{2}MR^2 - \frac{16MR^2}{9\pi^2}$

∴ $I_{AA'} = I_{com} + M\left(R - \frac{4R}{3\pi}\right)^2$

∴ $I_{AA'} = \frac{1}{2}MR^2 - \frac{16MR^2}{9\pi^2} + M\left(R - \frac{4R}{3\pi}\right)^2$

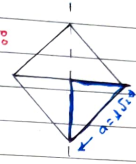
Q) Triangular lamina : $I = \frac{1}{6}Mh^2$



MI :

$= \frac{1}{6}Ml^2$

MII :



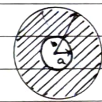
∴ I_{diag} of square $= \frac{1}{12}(4M)a^2$

$I = \frac{1}{3}M2l^2 = \frac{2}{3}Ml^2$.

∴ For $\frac{1}{4}$th part $= \frac{1}{4}\cdot\frac{2}{3}Ml^2 = \frac{1}{6}Ml^2$

***

Q) A circular disc of radius b has a hole of radius 'a' at its centre. If the mass per unit area of disc varies as (σ_0/r) then the radius of gyration of the disc about its axis passing through the centre is :



⇒ $\sigma = \sigma_0/r$, take element ring.

$dI = dm\,r^2$

$dI = (\sigma \cdot 2\pi r\,dr)r^2$

$dI = \frac{\sigma_0}{\cancel{r}} \cdot 2\pi \cancel{r} \cdot r^2 dr$

$I = \int dI = \int_a^b 2\pi\sigma_0 r^2 dr \Rightarrow \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

$M = \int dm = \int_a^b \frac{\sigma_0}{r} \cdot 2\pi r\,dr = 2\pi\sigma_0(b-a)$

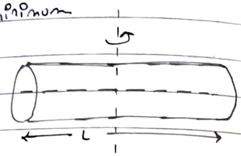
∴ $I = MK^2 = 2\pi\sigma_0(b-a)K^2$

∴ $\frac{\cancel{2\pi\sigma_0}(b^3-a^3)}{3} = \cancel{2\pi\sigma_0}(b-a)\cdot K^2$

$K = \sqrt{\frac{(b^2+a^2+ab)}{3}}$

Q1 The moment of inertia of uniform cyclinder of length l and radius R about its $\perp^r$ bisector is I. what is the ratio l/R such that MOI is minimum

$\Rightarrow$ $\therefore$ $I = \frac{1}{12}Ml^2 + \frac{1}{4}MR^2$



Concept: dono l & R ko I term mei laake minima dhundho.

$V = \pi R^2 l$ = const.

$\therefore R^2 = \frac{V}{\pi l}$ $\qquad$ $\therefore I = \frac{1}{12}Ml^2 + \frac{1}{4}M\left(\frac{V}{\pi l}\right)$

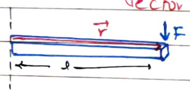
$\therefore \frac{dI}{dL} = 0 = \frac{Ml}{6} - \frac{VM}{4\pi L^2}$ $\qquad$ $\therefore \frac{L}{6} = \frac{V}{4\pi L^2} = \frac{\pi R^2 L}{4\pi L^2}$

$\therefore \frac{L}{6} = \frac{R^2}{4L}$ $\qquad$ $\therefore \frac{L^2}{R^2} = \frac{6}{4} = \frac{3}{2}$ $\qquad$ $\therefore \frac{L}{R} = \sqrt{\frac{2}{3}}$

* Torque: The moment of force about axis of rotation is called Torque.

Vector Qty (Axial Vector)

(dir se add karenge. τ $\parallel^{r}$ to axis he rotate karega)



$\therefore \vec{\tau} = \vec{r} \times \vec{F}$

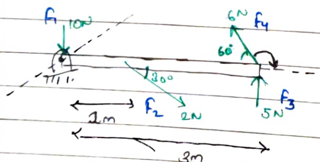
$\tau = rF\sin\theta$ $\qquad$ $r = r_1\hat{i} + r_2\hat{j} + r_3\hat{k}$, $F = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

$\tau_{min} = 0$

$\tau_{max} = \pm rF$

$\therefore \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & r & - \\ - & F & - \end{vmatrix}$

Q1



$\therefore \tau_{\text{due to } F_1} \Rightarrow (0)(10)\sin\theta = 0$

$\tau_{F_2} \Rightarrow (1)(2)\sin 90 = 2\ (\hat{k})$

$\tau_{F_3} = (3)(5)\sin 90 = 15\ (\hat{k})$

$\tau_{F_4} = 3 \times 6 \times \sin 120 = 18 \times \sin(90 \times 2 - 60) = 18\sin 60 = 9\sqrt{3}\ (+\hat{k})$

$\therefore \tau_T = 15 + 9\sqrt{3} - 1 = 14 + 9\sqrt{3}\ \hat{k}$

* Calculation of Torque:

Meth. 1) $\vec{\tau} = \vec{r} \times \vec{F}$ $\quad$ $\therefore |\tau| = rF\sin\theta$

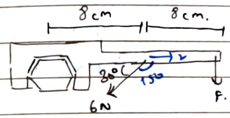
Meth. 2) · Draw a line of force.
· Find $r_\perp$ to line of force.
· $|\tau| = F \cdot r_\perp$

dir. $\Rightarrow$ Right hand thumb rule

Meth. 3) · Resolve the forces $\parallel^{r}$ & $\perp$ to $\vec{r}$.
· $|\tau| = F_\perp \cdot r$

lec. 05

Q) 1) MCV



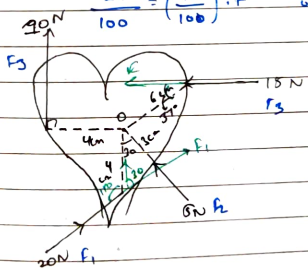
$\therefore$ (1) $\tau = rF\sin\theta = \left(\frac{8}{100}\right)(6)\sin 150 = \frac{48}{100} \times \sin(90 \times 1 + 60) = \frac{48}{100} \cdot \cos 60 = \frac{24}{100}$

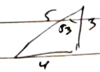
$\therefore \tau = 0.24$ Nm.

$\therefore$ case (2): $\tau = rF\sin\theta$ (90)

$\frac{24}{100} = \left(\frac{16}{100}\right) \cdot F$ $\qquad$ $\therefore F = \frac{24}{16} = 1.5$ N

Q2)





$\therefore \tau_{F_1} = rF_1\sin 30 = \frac{4}{100} \times 20 \times \frac{1}{2} = 0.4$ Nm $(+\hat{k})$

$\tau_{F_2} = \theta = \pi$, $\tau = 0$

$\tau_{F_3} = \left(\frac{6}{100}\right)(15)\sin 37 = \frac{6}{100} \times 15 \times \frac{3}{5} = 0.54\ (+\hat{k})$

$\tau_{F_4} = \frac{4}{100} \times 10 \times (1) = 0.4\ (-\hat{k})$

$\therefore \tau_{Total} = 0.4 + 0.54 - 0.4 = 0.54\ \hat{k}$

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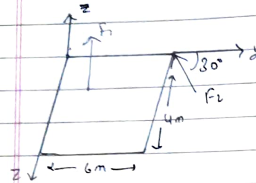
Q) A slab is subjected to two forces $\vec{F_1}$ & $\vec{F_2}$ of same magn.

M. 2020 F_1 acts at $(2\hat{i} + 3\hat{j})$. Find moment of force about O.

y Torque



$\therefore \tau_1 = \vec{r_1} \times \vec{F_1}$

$\vec{r_1} = 2\hat{i} + 3\hat{j} + 0\hat{k}$

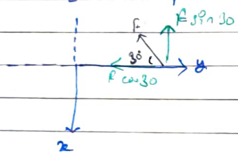
$\vec{F_1} = 0\hat{i} + 0\hat{j} + F\hat{k}$

$$\therefore \tau_1 = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$\therefore \tau_1 = \hat{i}(3F) - \hat{j}(2F - 0) + \hat{k}(0 - 0)$

$\vec{\tau_1} = 3F\hat{i} - 2F\hat{j}$

For F_2 : Top view =)



$\therefore \vec{F_2} = -F\sin 30\,\hat{i} - F\cos 30\,\hat{j}$

$\vec{r_2} = 0\hat{i} + 6\hat{j} + 0\hat{k}$

$$\therefore \tau_2 = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 0 & 6 & 0 \\ -\frac{F}{2} & -\frac{F\sqrt{3}}{2} & 0 \end{vmatrix}$$

$\vec{\tau_2} = +\hat{k}\left(\frac{F}{2} \cdot 6\right) = 3F\hat{k}$

$\therefore \tau_T = 3F\hat{i} - 2F\hat{j} + 3F\hat{k}$

Q) Trajectory of particle m is : $x = x_0 + a\cos(\omega_1 t)$

Adv. 2011 Find τ on particle $y = y_0 + b\sin(\omega_2 t)$

about origin, at $t = 0$.

$\Rightarrow$ $\vec{\tau} = \vec{r} \times \vec{F}$ at $t = 0$, $\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = (x_0 + a)\hat{i} + (y_0)\hat{j}$

$\vec{r}(t) = (x_0 + a\cos(\omega_1 t))\hat{i} + (y_0 + b\sin(\omega_2 t))\hat{j}$

$\vec{v}(t) = (-a\omega_1 \sin(\omega_1 t))\hat{i} + (b\omega_2 \cos(\omega_2 t))\hat{j}$

$\vec{a}(t) = (-a\omega_1^2 \cos(\omega_1 t))\hat{i} + (-b\omega_2^2 \sin(\omega_2 t))\hat{j}$

$\therefore$

$F = m.\vec{a} = (-ma\omega_1^2 \cos\omega_1 t)\hat{i} - (mb\omega_2^2 \sin(\omega_2 t))\hat{j}$

at $t = 0$

$F = -ma\omega_1^2\,\hat{i}$

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$\therefore \tau = \vec{r} \times \vec{F}$

$$\therefore \tau = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ x_0 + a & y_0 & 0 \\ -ma\omega_1^2 & 0 & 0 \end{vmatrix}$$

$\therefore \tau = +(ma\omega_1^2 y_0)\hat{k}$

* Rotational Equilibrium

State of body $\Longrightarrow$ Rest or Const. Velocity

Equilibrium

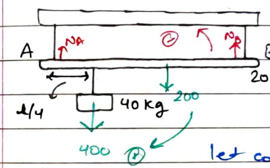
Translational Eq. — $\Sigma F_{net} = 0$

Rotational Eq. — $\Sigma \tau_{net\ about\ any\ point} = 0$

Clockwise : -ve

Anticlockwise : +ve

Q)



Find tension in threads

$\therefore N_A + N_B = 600$ (Trans. Eq.)

Rotational eq. :-

$\therefore \Sigma \tau_{net} = 0$

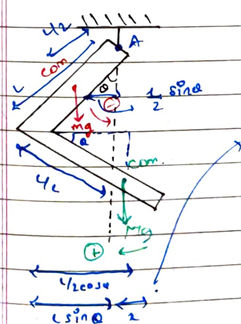
let consider point A as axis.

$\therefore T_A(0) + \cancel{400}^{100}\left(\frac{\cancel{l}}{\cancel{4}}\right) + \cancel{200}^{100}\left(\frac{\cancel{l}}{\cancel{2}}\right) - T_B(\cancel{l}) = 0$

$\therefore T_B = 200$

$\therefore T_A = 400$

Q)



Find θ.

$\therefore x = \frac{L}{2}\cos\theta - L\sin\theta$.

$\therefore$ Rotational Eq.

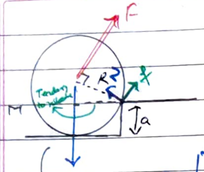
Take point A as axis.

$\therefore 0 = +Mg\left(\frac{L}{2}\sin\theta\right) - Mg\left[\frac{L}{2}\cos\theta - L\sin\theta\right] =$

$0 = \frac{\sin\theta}{2} - \frac{\cos\theta}{2} + \sin\theta$

$\therefore \frac{\cos\theta}{\cancel{2}} = \frac{3\sin\theta}{\cancel{2}}$ $\Rightarrow \therefore \tan\theta = \frac{1}{3}$

Q1 Ad. 2020



Find minimum force required to pull up.

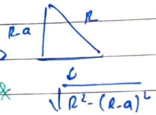
$\therefore N_{grd} = 0 \quad \therefore f_{grd} = 0$

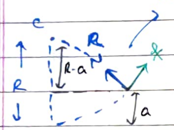
consider point on edge

$\therefore \tau_{edge}$ due to f & $N = 0$

$\tau_F = -FR$

$\tau_{mg} = +(Mg)(\sqrt{R^2-(R-a)^2})$



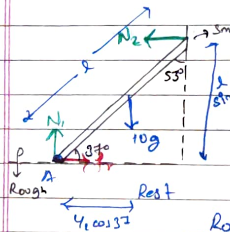


$\therefore$ To lift cylinder

$FR \geq Mg\sqrt{R^2-(R-a)^2}$

$\therefore F \geq Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

Q1



Ladder Problem.

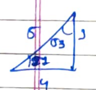
Find N & f_r force exerted by floor on ladder

$\therefore$ Trans eq. :-

$\therefore N_2 = f_r$ & $N_1 = 10g = 100$

Rotational eq. :-

$\therefore \tau_{net\ about\ A} = 0$



$\therefore \cos 37 = \frac{4}{5}$

$\sin 37 = \frac{3}{5}$

$\therefore 0 = -10g\left(\frac{l}{2}\cos 37\right) + N_2(l\sin 37)$

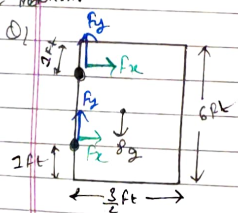
$0 = -\cancel{100}^{20}\left(\frac{l}{2} \times \frac{4^2}{5}\right) + N_2\left(l \cdot \frac{3}{5}\right)$

$0 = -40l + N_2 \frac{l \cdot 3}{5}$

$\therefore N_2 = \frac{40 \times 5}{3} = \frac{200}{3}$ & $f_r = \left(\frac{200}{3}\right)$

door Problems

Q1



If the mag. of forces exerted by the hinges on the door are equal, find the mag.

$\therefore$ Trans. equi. :

$\therefore f_y + f_y = 8g$ (along y)

$\therefore 2f_y = 80$

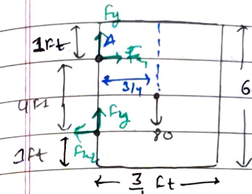
$f_y = 40$ Newton

along x-axis :

$f_{x_1} + f_{x_2} = 0$ → opp in dir

$\therefore f_{x_1} = -f_{x_2}$

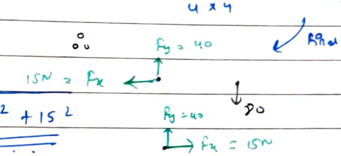
Rotational equi. :-



τ_{net} about $A = 0$

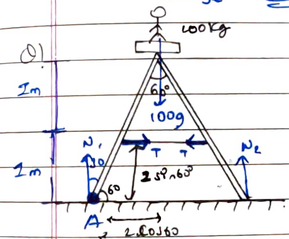
$-8g\left(\frac{3}{4}ft\right) - f_{x_2}(4ft) = 0$

$f_x = -\frac{80 \times 3}{4 \times 4} = -15N$



$\therefore f_{net}$ on hinge $= \sqrt{40^2 + 15^2}$

Q1



Find Tension & Normal from grd.

$\therefore$ Trans. eq.

$\Sigma F_y \quad N_1 + N_2 = 100g$ -(i) (By symmetry $N_1 = N_2$)

$\Sigma F_x \quad T = T$ -(ii)

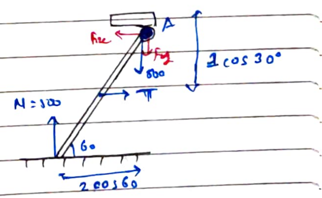
$\therefore$ Rotational eq. about A

$\therefore N_1(0) - \cancel{T\sin 60} + \cancel{T\sin 60} - 100g(2\cos 60) + N_2(4\cos 60) = 0$

$\therefore N_2 = \frac{\cancel{100}^{50} \times 10 \times \cancel{2} \times \frac{1}{2}}{\cancel{4} \times \frac{1}{2}}$

$N_2 = 500$, $\therefore N_1 = 500$

• For Tension Take half part



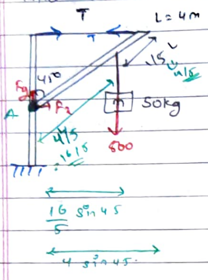
$\therefore \tau$ about A

$\therefore T\, 2\cos 30 - 500(2\cos 60) = 0$

$T\frac{\sqrt{3}}{2} = 500 \times 2 \times \frac{1}{2}$

$T = \frac{1000}{\sqrt{3}}$

Q| Find Tension in Cable.



∴ Trans eq. (Taking only Rod forces)

$f_x = T \qquad f_y = 500$

Rot. eq. abt A (only Rod)

∴ $f_y(0) + f_x(0) - 500\left(\frac{16}{5}\sin 45\right) + T(4\sin 45)$

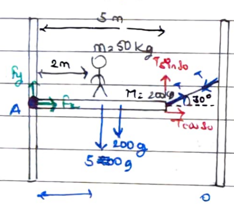
∴ $T \cdot \cancel{4}^{\,1} \cdot \cancel{\sin 45} = \cancel{500}^{\,100} \times \frac{\cancel{16}^{\,4}}{\cancel{5}_{\,1}} \cdot \cancel{\sin 45}$

∴ $\underline{T = 400\ N}$

∴ $f_x = 400 \quad , \quad f_y = 500$

$F_{net\ on\ nut} = \sqrt{400^2 + 500^2}$

Q|



Find Tension

$\Sigma F_x = 0 \qquad \Sigma F_y = 0$

$f_x + T\cos 30 = 0 \qquad f_y + T\sin 30 = 2500$

$f_x = -T\cos 30 \qquad f_y + \frac{T}{2} = 2500$

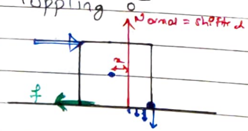
$\vec{\tau}_{about\ A} = 0$

∴ $-2500(2) + T\cos 30(x) + T\sin 30(5) = \frac{2000(5/2)}{20}$

~~∴ $T = \frac{\cancel{2500}^{500} \times 2}{5 \times (1/2)} = 500 \times 4 = 2000$~~

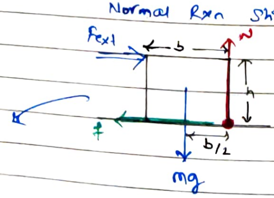
∴ $T = \frac{\cancel{2000}^{1000}(5/2) + 1000}{5 \times (1/2)} = \frac{\cancel{6000}^{1200} \times 2}{\cancel{5}_1} = \underline{2400}$

* Toppling :-



when F_{ext} ↑

Normal Rxn shift



$\tau_{N} \quad \tau_{friction\ about\ A} = 0$

τ_{Fext} will try to rotate body CW $= F_{ext} \cdot h$

τ_{weight} will — " — " — ACW $: mg \times b/2$

body will topple :-

$\tau_{Fext} \geq \tau_{weight}$ about A

∴ $F_{ext}\, h \geq \frac{mgb}{2}$ ∴ $\boxed{F_{ext} \geq \frac{mgb}{2h}}$

Q| Find the minimum value of F for the block to topple about an edge before slipping.

⇒ Same of Theory $F = \frac{mgb}{2h}$

Toppling before Slipping

$F_{Topp} < F_{min\ slip}$

∴ $\frac{Mgb}{2h} < \mu_s Mg$

∴ $\boxed{\frac{a}{2b} < \mu_s}$

Slipping before Toppling.

$\mu_s mg < \frac{Mga}{2b}$

$\boxed{∴ \mu_s < \frac{a}{2b}}$

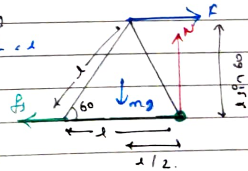
Q| Min. F to topple before slipping

∴ equi Plate, mass = M, length = l

∴ at toppling condⁿ

$\tau_{Fext} = \tau_{weight}$

$F(l\sin 60) = mg\left(\frac{l}{2}\right)$



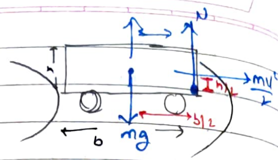
∴ $F = \underline{\frac{Mg}{\sqrt{3}}}$

$F_{Toppling} < \mu_s N$

∴ $\frac{\cancel{Mg}}{\sqrt{3}} < \mu_s \cancel{Mg}$

$\underline{\frac{1}{\sqrt{3}} < \mu_s}$

Q) Max speed at car take turn so that it does not topple



$\Rightarrow \tau_{centrifugal} = \frac{mv^2}{r}\left(\frac{h}{2}\right)$

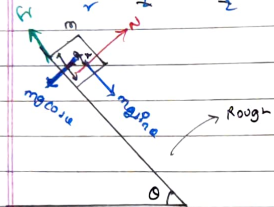
$\tau_{weight} = mg(b/2)$

$\therefore$

$\frac{mv^2}{r}\cdot\frac{h}{2} \geq \frac{mgb}{2}$

$\therefore v > \sqrt{\frac{gbr}{h}}$

Q)



find torque of the normal force acting on the block about its centre.

uniform speed $\Rightarrow f = mg\sin\theta$

$N = mg\cos\theta$

about com

$\tau_{friction} = f \times \frac{a}{2} = mg\sin\theta\,\frac{a}{2}\ (-\hat{k})$

$\tau_{Normal} = N x\ (+\hat{k})$

at eq.

$\Sigma\tau_{net} = 0$ $\therefore mg\sin\theta\,\frac{a}{2} = Nx$

$\therefore x = \frac{\cancel{mg}\sin\theta\, a}{\cancel{mg}\cos\theta\ 2} = \frac{\tan\theta \cdot a}{2}$

* Fixed axis Rotation

when there is torque acting on body, there is an acc. motion

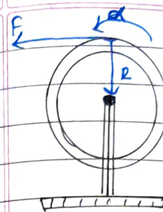
Rotational motion:

1) FBD
2) Cal. τ_{net} acting on body
3) $\vec{\tau}_{net} = I_{about\ axis}\,\vec{\alpha}$
4) α = kinematics eqⁿ.

$a_c = \frac{v^2}{R} = R\omega^2$, $v = R\omega$

$a_t = R\alpha = \frac{d|v|}{dt}$

Q)



$I = 0.20\ kg\,m^2$, $R = 20\ cm$

initially at rest, $F = 20N$, find angular velocity of wheel after 5 s.

$\therefore \tau = I\alpha$

$\tau = F(R) = I\alpha$

$\therefore (20)\left(\frac{20}{100}\right) = \left(\frac{20}{100}\right)\cdot\alpha$

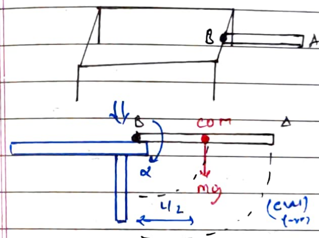
$\therefore \alpha = 20\ rad/sec^2$

$\frac{d\omega}{dt} = 20$ $\therefore \int_{0}^{\omega_f} d\omega = \int_0^5 20\,dt$

Rest

$\therefore \omega_f = 20\times 5 = 100\ rad/s$

Q)



uniform stick of length L cm, hinged at B, point A is suddenly released. The acc. of com of stick at time of release is.

↳ Rod just Rotate $\therefore \omega = 0$

$\therefore \tau = -(Mg)\left(\frac{L}{2}\right) = I_{axis}\,\alpha$

$-\frac{\cancel{M}g\cancel{L}}{2} = \frac{1}{3}\cancel{M}L^{\cancel{2}}\cdot\alpha$

$\therefore \alpha = -\frac{3}{2}\frac{g}{L}$

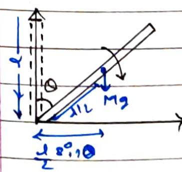
at time of Release

$\omega = 0$, $a_c = R\omega^2 = 0$

$a_t = R\alpha = R\left(-\frac{3}{2}\frac{g}{L}\right) = \left(\frac{L}{2}\right)\left(-\frac{3}{2}\frac{g}{L}\right)$

$= -\frac{3g}{4}$

Q)



M, L, no friction.

if it is released, The angular acc. of rod when it make angle θ is

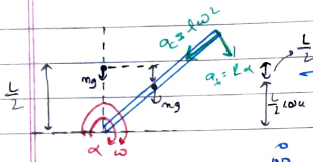
$\therefore \tau = Mg\left(\frac{L}{2}\sin\theta\right) = I_{axis}\,\alpha$

$= \frac{\cancel{M}g\cancel{L}\sin\theta}{2} = \frac{1}{3}\cancel{M}L^{\cancel{2}}\cdot\alpha$

$\alpha = \frac{3g\sin\theta}{2L}$

Additional: acc. of point A at angle θ with vertical

$\therefore a_t = L \cdot \left(\frac{3g\sin\theta}{2L}\right) = \frac{3g\sin\theta}{2}$



To find ω.
use WET.

$\therefore W_g = K.E_f - K.E_i$

$+ mg\left(\frac{L}{2}(1-\cos\theta)\right) = \left(\frac{1}{2} I_{axis}\omega^2 - 0\right)$

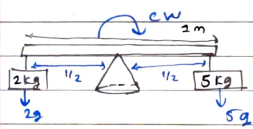
$\therefore \omega^2 = \frac{mgL(1-\cos\theta)}{I}$

$\therefore a_c = L\cdot\omega^2$
$= \frac{mgL^2(1-\cos\theta)}{I}$

(i) Pure Translational
$KE = \frac{1}{2}mv^2$

(ii) Pure Rotational
$KE = \frac{1}{2} I_{axis}\omega^2$

(iii) CRTM
$KE = \frac{1}{2}mv_c^2 + \frac{1}{2}I_{axis}\omega^2$

Q) 

Find initial ang. acc. of rod. assuming it was horizontal in the beginning. (rod massless)

$\tau_{net} = I\alpha = -5g\left(\frac{L}{2}\right) + 2g\left(\frac{L}{2}\right) = I_{axis}\alpha$

$= -3g\frac{L}{2} = I_{axis}\alpha$

$-\frac{3gL}{2} = \left(5\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2\right)\alpha$

$-15 = \left(\frac{5}{4} + \frac{2}{4}\right)\alpha, \quad \therefore \alpha = -\frac{15\times4}{7} = -\frac{60}{7}$ rad/s²

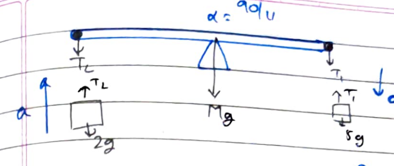
Q) Suppose rod has mass 1 kg, find initial α.

$\therefore |\tau_{net}| = I_{axis}\alpha$

$15 = \left(5\times\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2 + \underbrace{\frac{1}{2}\times1\times1^2}_{I_{rod}}\right)\alpha$

$\alpha = \frac{90}{11}$

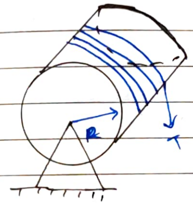
b) Find tension in the support to the block of mass 2kg & 5kg.



$a = R\alpha = \frac{1}{2}\times\frac{90}{11} = \frac{45}{11}$

$\therefore 5g - T_1 = 5a$
$T_1 = 5(g-a) = 5\left(10 - \frac{45}{11}\right)$
$T_1 = 5\left(\frac{65}{11}\right) = \frac{325}{11}$ N

$\therefore T_2 - 2g = 2a$
$T_2 = 2g + 2a$
$T_2 = 2\left(g + \frac{45}{11}\right)$
$T_2 = 2\left(\frac{155}{11}\right)$

Q) 

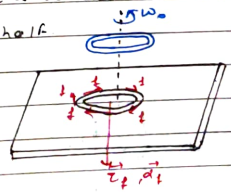
Cylinder mass = 50kg, radius = 0.5m
find Tension that produce ang. acc. of 2 revolution/s² → 1 revol = 2π.
$\therefore \alpha = 4\pi$ rad/sec²

$\therefore \tau = I_{axis}\alpha$

$\therefore T.R = \frac{MR^2}{2}.\alpha$

$\therefore T = \frac{50}{2}\times\frac{1}{2}\times4\pi = 50\pi = 157$ N

Q) A ring of r = R is first rotated with ang. velocity ω_0 & then placed carefully on a rough horizontal surface. The coefficient of friction betⁿ ring & surface is μ. Time after which it's ang. speed is half.



τ due to friction ⇒ will Retard body

$\tau_f = F.r_\perp$
$= (\mu mg).R$

$\therefore -\tau_f = I_{axis}\alpha$

$-\mu mgR = mR^2.\alpha$

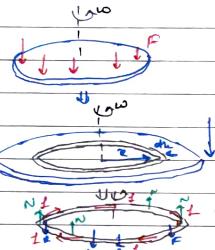
$\therefore \alpha = \boxed{-\frac{\mu g}{R}}$ Retard, const.

$\therefore \omega_f = \omega_0 + \alpha t$

$\frac{\omega_0}{2} = \omega_0 - \frac{\mu g}{R}t$

$\therefore t = \frac{\omega_0 R}{2\mu g}$

Q) To mop clean a floor, a cleaning machine presses a circular mop of radius R vertically down with total force F and rotate it with a const ang. speed about its axis. If the force F is distributed uniformly over the mop & it coeff. of friction is μ, the torque applied by machine on mop is.



$\therefore$ given $\omega = \text{const}$

$\therefore |\tau_{machine}| = |\tau_{friction}|$

$\therefore \alpha_{net} = 0$, $\omega = \text{const}$

$\pi R^2 \rightarrow F$

$1 \rightarrow F/\pi R^2$

Force exerted by ring on ground $\Rightarrow \frac{F}{\pi R^2} \cdot 2\pi x\,dx$

friction on ring:

$f_r = \mu N = \mu\left(\frac{F}{\pi R^2} \cdot 2\pi x\,dx\right)$

τ due to ring $\Rightarrow$

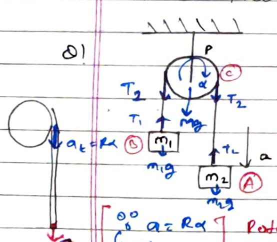
$d\tau = \left(\frac{\mu F\, 2\pi x\,dx}{\pi R^2}\right) x$

$\therefore \tau_{net} = \int d\tau = \int \frac{2\mu F}{R^2} x^2 dx = \frac{2\mu F}{R^2}\left[\frac{R^3}{3}\right]$

$\therefore \tau_{friction} = \frac{2\mu F R}{3}$

* Atwood Problems.

Q) P has radius r and mass m. The string does not slip on the pulley. Find acc. of block & Ang. Acc. of pulley (M.O.I of pulley $= I$).



$\left[\therefore a = R\alpha,\ \alpha = \frac{a}{R}\right]$ Rotational contraint.

$\therefore$ (A) Pure Tran.

$\therefore$ F.B.D. $m_2 g - T_2 = m_2 a$

(B) Pure Tran.

$T_1 - m_1 g = m_1 a$

(C) Pure Rotational

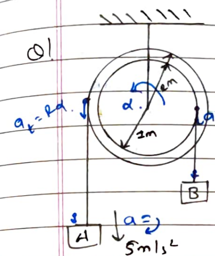
$\tau_{net} = I\alpha \rightarrow T_2 R - T_1 R = I\alpha$.

$\therefore$
$m_2 g - T_2 = m_2 a$
$T_2 R - T_1 R = I\alpha$
$T_1 - m_1 g = m_1 a$

$\Rightarrow$
$m_2 g - \cancel{T_2} = m_2 a$
$\cancel{T_2} - \cancel{T_1} = I a/R^2$
$\cancel{T_1} - m_1 g = m_1 a$

$a = \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{I}{R^2}}$

Q)



Find acc. of B.

$\therefore$ from A $a_t = R\alpha$ [constraint]

$5 = (2)\cdot\alpha$

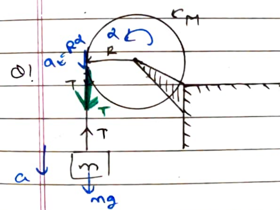
$\therefore \alpha = 5/2$

$\therefore$ smaller bhi usi α se ghumega

$\therefore a_{t\ of\ B} = R\alpha = (1)(5/2)$

$\therefore a_{t\ of\ B} = \frac{5}{2}$ m/s²

Q)



a) find the ang. acc. of disc & tangential acc. of point on the rim.

$\therefore$ Tran. : $mg - T = ma$

Rot : $TR = I_{axis}\,\alpha$

$\therefore$ $mg - \cancel{T} = ma$

$\cancel{T} = \frac{MR^2}{2 \cdot R}\cdot\frac{a_t}{R}$

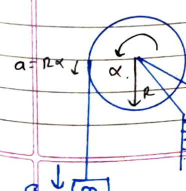
add

$mg = \left(m + \frac{M}{2}\right)a$

$\therefore a = \frac{mg}{\frac{M}{2} + m}$

$\hookrightarrow$ directly: $a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mg}{m + \frac{MR^2}{2R^2}}$

b) at moment $t = 0$, the system is set in motion, find time dependence of the ang velocity of the system.



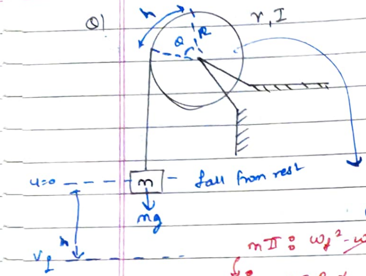
$\therefore \alpha = a/R = \frac{mg}{R\left(m + \frac{M}{2}\right)}$ (const.)

$\therefore$
$\omega_f = \omega_0 + \alpha t$, $t = t$, $\omega_0 = 0$

$\omega_f = \frac{mg\,t}{R\left(m + \frac{M}{2}\right)}$

$\therefore v = \omega R$

$v = \frac{2mgt}{m + M/2}$

Q) 

After falling distance h, the ang. velocity of wheel will be.

$a_{body} = \frac{mg}{m+\frac{I}{R^2}} = \frac{mgr^2}{mR^2+I}$ (const.)

$\theta R = h$

$\theta = h/R$

$\therefore v^2 - \cancel{u^2} = 2as$

$v = \sqrt{\frac{2 \cdot mgR^2 \cdot h}{mR^2+I}}$

$m\text{II} : \omega_f^2 - \cancel{\omega_0^2} = 2\alpha\theta$

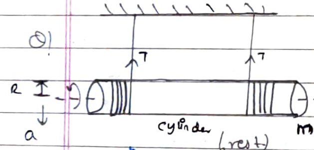
$\therefore a = R\alpha$

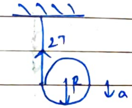
$\therefore \omega = \frac{v}{R} = \sqrt{\frac{2mgh}{mR^2+I}}$

$\therefore \alpha = \frac{a}{R} = \frac{mgR^2}{(mR^2+I)} \cdot \frac{1}{R}$

Same

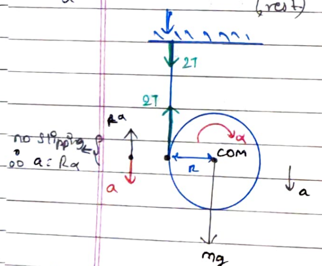
$\therefore \omega_f = \sqrt{2\left(\frac{mg\cancel{R}}{(mR^2+I)}\right)\cdot\left(\frac{h}{\cancel{R}}\right)} = \sqrt{\frac{2mgh}{mR^2+I}}$

Q) 

Side view : 

CRTM

a) Cal. downward linear acc of cylinder



$\therefore T_r : mg - 2T = ma$

$R_r : 2TR = I_{axis}\,\alpha$

$\therefore mg - 2T = ma$ — (I)

$2T = \frac{mR^2}{2} \cdot \alpha$ — (II)

$\therefore a = R\alpha$

$\therefore$ (I) & (II)

$mg - \cancel{2T} = ma$

$\cancel{2T} = \frac{m}{2} \cdot \left(\frac{a}{\cancel{R}}\right)$

(+)

$mg = \frac{3ma}{2}$

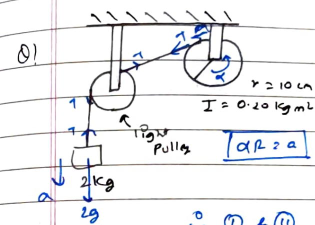
$\therefore a = \frac{2g}{3}$

b) Tension in string :

$\therefore 2T = \frac{ma}{2}$ $\therefore T = \frac{m}{4}\left(\frac{2g}{3}\right) = \frac{mg}{6}$

c) Velocity of cylinder at t = 3 sec.

$v = \cancel{u} + at$ $v = \left(\frac{2g}{3}\right) \cdot 3 = 20$ m/s

Q) 

Find acc. of block.

$\therefore T_r : 2g - T = 2a$ — (I)

$R_r : T r = I \cdot \alpha$

$T = \frac{I}{I \cdot R} \cdot \left(\frac{a}{R}\right)$

$T = \frac{I}{R^2} a$ — (II)

$\therefore$ (I) & (II)

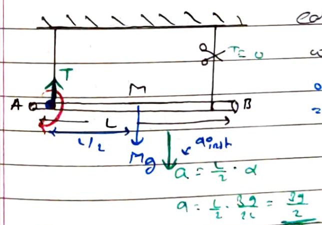
$2g - \cancel{T} = 2a$

$\cancel{T} = \frac{I}{R^2} a$

(+)

$2g = \left(2 + \frac{I}{R^2}\right) a$

$\therefore a = \frac{2g}{\left(2 + I/R^2\right)}$

Q) 

Cal. Tension in left string at the instant when right string snaps.

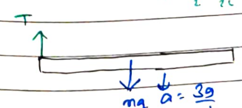
$\therefore \tau = I_{axis}\,\alpha$

$\therefore Mg\left(\frac{L}{2}\right) = \frac{1}{3}ML^2 \cdot \alpha$

$\therefore \alpha = \frac{3g}{2L}$

$a = \frac{L}{2} \cdot \alpha$

$a = \frac{L}{2} \cdot \frac{3g}{2L} = \frac{3g}{4}$



$\therefore Mg - T = Ma$

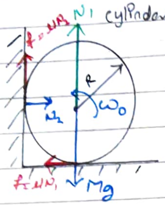
$\therefore T = M(g - a)$

$= M\left[g - \frac{3g}{4}\right]$

$T = \frac{Mg}{4}$

Q| cylinder, initial ω_0, and then placed into a corner the coeff of friction is μ. How many turns will cylinder accomplish before it stops.

Adv.



$\therefore$ Tran. Equili.

$\therefore \Sigma F_y = 0 \qquad \Sigma F_x = 0$

$\mu N_2 + N_1 = Mg \qquad \mu N_1 = N_2$

$\mu^2 N_1 + N_1 = Mg$

$\therefore N_1 = \frac{Mg}{1+\mu^2} \qquad \therefore N_2 = \frac{\mu Mg}{1+\mu^2}$

$\therefore \tau_{f_1} + \tau_{f_2} = \tau_{retard.}$

$f_1 R + f_2 R = I_{axis}\,\alpha$

$[\mu N_1 + \mu N_2] R = \frac{MR^2}{2}.\alpha$

$\left[\frac{\mu Mg}{1+\mu^2} + \frac{\mu^2 Mg}{1+\mu^2}\right] R = \frac{MR^2}{2}.\alpha$

$\therefore \alpha = \frac{2\mu g\,(1+\mu)}{R\,(1+\mu^2)}$

(cont)

$\therefore \omega_f^2 - \omega_0^2 = 2\alpha\theta$ ($\omega_f = 0$)

$-\omega_0^2 = -2\left(\frac{2\mu g(1+\mu)}{R(1+\mu^2)}\right).\theta$ (retard.)

$\therefore \theta = \frac{R\omega_0^2\,(1+\mu^2)}{4g\mu.(1+\mu)}$

$\therefore$ 1 turns = 2π

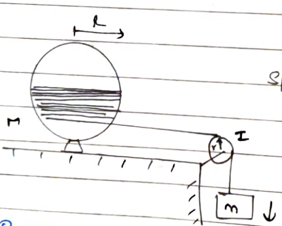
$\therefore n$ turns = $2\pi n$.

$\therefore 2\pi n = \frac{R\omega_0^2\,(1+\mu^2)}{4g\mu\,(1+\mu)}$

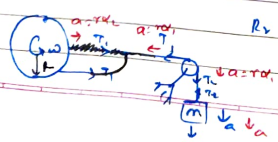
$n = \frac{R\omega_0^2\,(1+\mu^2)}{8\pi g\mu\,(1+\mu)}$

Q| Spherical hollow shell, rotates on frictionless bearing. Find speed of object after it fallen a dist h. from rest.

Adv.



$\therefore$ Top view



Tr: $\therefore mg - T_2 = ma$ — (i)

Rr: For sphere: $T_1 R = I_s \alpha_1$ — (ii)

For pulley: $T_2 r - T_1 r = I\alpha_2$ — (iii)

& $\alpha_2 = a/r$

$\alpha_1 = a/R$

$\therefore$ from (i), (ii), (iii)

$T_1 = \frac{2MR^2}{3R^2}\left(\frac{a}{R}\right)$

$T_2 - T_1 = \frac{I}{r^2}a$

(+) $mg - T_2 = ma$

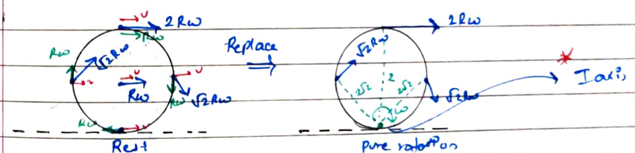
$\left(\frac{mg}{\frac{2}{3}M + \frac{I}{r^2} + m}\right) = a$

$\therefore v^2 = u^2 + 2as$ ($u = 0$)

$v = \sqrt{2\left(\frac{mg}{\frac{2M}{3} + \frac{I}{r^2} + m}\right)h}$

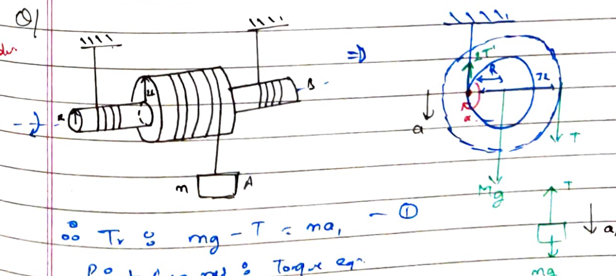
* Instant Axis of Rotation

Point of Contact ke about Rotate hua maan lena & usko Axis consider karna.



Q|

Adv.



$\therefore$ Tr: $mg - T = ma$, — (i)

Point A on my: Torque eqn.

$\therefore Mg(R) + (T)(3R) = I_{axis}\,\alpha$ — (ii)

$MgR + 3TR = (I + MR^2)\,\alpha$

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$\therefore \; mg - T = ma_1 \quad) \times 3$

$Mg + 3T = (I + MR^2)\frac{a_2}{R^2}$

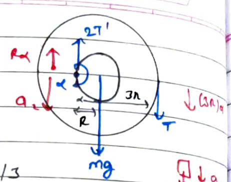
$\oplus$

$3mg + Mg = 3ma_1 + \left(\frac{I}{R^2} + M\right)a_2$

$a_1/3$

$= Mg + 3mg = 3ma_1 + \left(\frac{I}{R^2} + M\right)\frac{a_1}{3}$

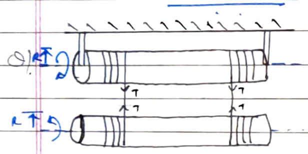
$\therefore \; a_1 = \dfrac{Mg + 3mg}{3m + \frac{I}{3R^2} + \frac{M}{3}}$



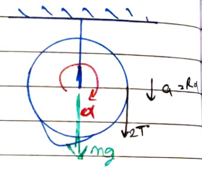
$\downarrow a_1$ mg

$\therefore a_2 = R\alpha \quad \therefore a_1 = 3R\alpha$

$\therefore \alpha = \frac{a_2}{R} \quad \& \quad \frac{a_1}{a_1} = 3$



Q) Adv. Find Tension

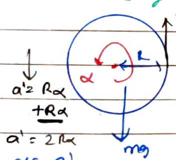


$\therefore \; T_r : \; Mg - 2T = ma' \quad \text{—(i)}$

$R_o : \; 2T(R) = I\alpha$

$2TR = \frac{MR^2}{2}\alpha$

$= 2TR = \frac{MR^2}{2}\left(\frac{a'}{2R}\right) \quad \text{—(ii)}$



$a' = R\alpha + R\alpha$

$a' = 2R\alpha$

$\alpha = \frac{a'}{2R}$

$\therefore$ (i) & (ii)

$Mg - 2T = ma'$

$2T = \frac{ma'}{4}$

$\oplus$

$\cancel{M}g = \frac{5\cancel{m}a'}{4}$

$\therefore a' = \frac{4g}{5}$

From (ii)

$\therefore T = \frac{ma'}{8}$

$T = \frac{m}{\cancel{8}_2}\left(\frac{\cancel{4}g}{5}\right) = \frac{mg}{10}$

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* Kinetic Energy of rotating body about fixed axis.

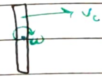
$KE_{body} = \frac{1}{2} I_{axis}\,\omega^2$

a)  Translational $\quad KE = \frac{1}{2} M_{body} V_{com}^2$

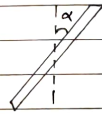
b)  Pure rotational $\quad KE_{Rot} = \frac{1}{2} I_{axis}\,\omega^2$ ← About Axis of rotation

(fixed axis)

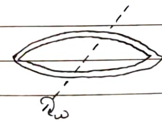
Dono same Ans aata.

c)  Trans + Rot. $\quad K.E_{Total} = \frac{1}{2}MV_c^2 + \frac{1}{2}I_c\omega^2$

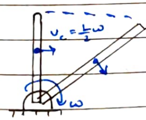
$I_c = I_{com}$

Q) 1). 

$KE = \frac{1}{2} I_{axis}\,\omega^2 = \frac{1}{2}\left(\frac{1}{12}ML^2\sin^2\alpha\right)\omega^2$

2) 

$KE = \frac{1}{2} I_{axis}\,\omega^2 = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2$

3) 

MI : $KE = \frac{1}{2} I_{axis}\,\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{ML^2\omega^2}{6}$

MII : $KE = \frac{1}{2}Mv_c^2 + \frac{1}{2}I_{com}\,\omega^2$

$= \frac{1}{2}M\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\omega^2$

$= \frac{ML^2\omega^2}{8} + \frac{ML^2\omega^2}{24} = \frac{ML^2\omega^2}{6}$

Same

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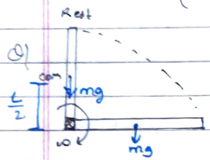
* WET in Rotation

$W_{Total} = KE_f - KE_i$

$(KE_T + KE_R)$ $\quad$ $(KE_T + KE_R)$



Q) Small blow. Find ω. Also find the hinge forces when rod is at vertical position

M I : $W_T = KE_f - KE_i$

$+mg(L/2) = \frac{1}{2} I_{axis}\,\omega^2$

M II : $W_g = (KE_T + KE_R)_f - (KE_T + KE_R)_i$

$\frac{mgL}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$

$\omega = \sqrt{\frac{3g}{L}}$

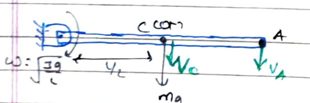
$= mg\left(\frac{L}{2}\right) = \left(\frac{1}{2}Mv_c^2 + \frac{1}{2}I_c\omega^2\right) - 0$

$= \therefore v = R\omega : v = \left(\frac{L}{2}\right)\omega$

$\therefore \frac{mgL}{2} = \frac{1}{2}m\left(\frac{L\omega}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\omega^2$

$\frac{mgL}{2} = \frac{ML^2\omega^2}{8} + \frac{ML^2\omega^2}{24} \Rightarrow g = \frac{4L\omega^2}{12}$

$\therefore \omega = \sqrt{\frac{3g}{L}}$



$v_c = R\omega = \left(\frac{L}{2}\right)\left(\sqrt{\frac{3g}{L}}\right)$

$v_A = R\omega = (L)\left(\sqrt{\frac{3g}{L}}\right)$

$\tau = I\alpha$

$Mg\left(\frac{L}{2}\right) = \frac{1}{3}ML^2\cdot\alpha$

$\therefore a_c = \sqrt{\left(\left(\frac{L}{2}\right)\left(\sqrt{\frac{3g}{L}}\right)^2\right)^2 + \left[\left(\frac{L}{2}\right)\left(\frac{3g}{2L}\right)\right]^2}$

$\therefore \alpha = \frac{3g}{2L}$

$a_A = \sqrt{\left((L)\left(\sqrt{\frac{3g}{L}}\right)^2\right)^2 + \left((L)\left(\frac{3g}{2L}\right)\right)^2}$

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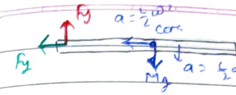


$\therefore a_{com_t} = \frac{L}{2}\alpha = \frac{L}{2}\cdot\frac{3g}{2L} = \frac{3g}{4}$

$a_{com_c} = \frac{L}{2}\omega^2 = \frac{L}{2}\cdot\frac{3g}{L} = \frac{3g}{2}$

$\therefore$ Along y

$Mg - F_y = m\left(\frac{3g}{4}\right)$

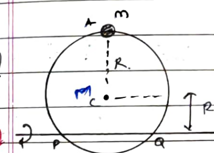
$\therefore F_y = Mg - \frac{3Mg}{4} = \frac{Mg}{4}$

Along X

$F_x = M(r\omega^2) = M\left(\frac{L}{2}\right)\left(\frac{3g}{L}\right) = \frac{3Mg}{2}$

Q) After rotation, find linear speed of particle as it reaches lowest position

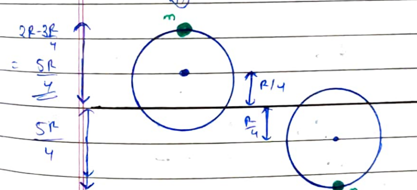
Adv. Physics galaxy



$\therefore$ WET

$\therefore W_g = KE_f - KE_i$

$= +mg\left(\frac{5R}{4}\times 2\right) + Mg\left(\cancel{2}\times\frac{R}{4}\right) = \left(\frac{1}{2}I_{axis}\,\omega^2\right)$



$\frac{5mgR}{2} + \frac{MgR}{2} = \frac{1}{2}\left[m\left(\frac{5R}{4}\right)^2 + \left(\frac{1}{2}MR^2 + M\left(\frac{R}{4}\right)^2\right)\right]\omega^2$

$\therefore \frac{5gR(M+m)}{2} = \frac{1}{2}\left(\frac{25mR^2}{16} + \frac{MR^2}{4} + \frac{MR^2}{16}\right)\omega^2$

$5g(M+m) = \left(\frac{25mR + 4MR + MR}{16}\right)\omega^2$

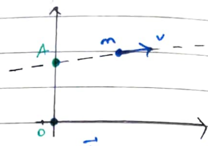
$\therefore \omega = \frac{16\times 5g(M+m)}{5R(5m+M)} = \frac{16g(M+m)}{R(5m+M)}$

$\therefore V_{particle} = R\omega = \left(\frac{5R}{4}\right)\left(\frac{16g(M+m)}{R(5m+M)}\right) = \frac{20g(M+m)}{5m+M}$

* Angular momentum & it's calculation

"Moment of Momentum"

"Rotational impact of Momentum"



$\vec{L} = \vec{r} \times \vec{p}$ $\longrightarrow$ cal. about a point

$\vec{L}_{about\ A} = 0$

$\vec{L}_{-''-\ O} \neq 0$

Axial Vector

$\bar{r}$ = position vector w.r.t reference point

$\bar{p} = m\bar{v}$

$$\bar{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

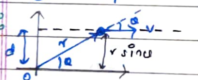
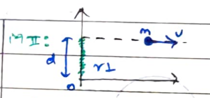
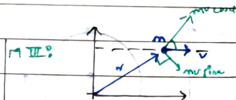
if $\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$

$\vec{p} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$\therefore \vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ - & r & - \\ - & p & - \end{vmatrix}$

Calculations :

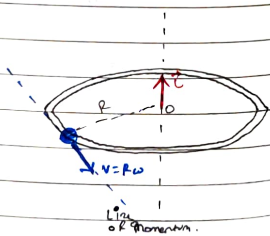
Case I : $\vec{L} = \vec{r} \times \vec{p}$

M I :	M II :	M III :
		
$\vec{L}$ about O	• draw line of $\vec{p}$ & drop a $\perp^r$ to it.	• resolve $\vec{p}$ & $\vec{r}$ which act as $\perp$
$\therefore L = P \cdot r \cdot \sin\theta\ (-\hat{k})$	• $\vec{L} = \lvert\vec{L}\rvert \cdot \hat{L}$	$\therefore L = mvr\sin\theta\ (-\hat{k})$
$L = P(r\sin\theta)(-\hat{k})$	$= mvd\ (-\hat{k})$	$L = mvd\ (-\hat{k})$
$L = P.d\ (-\hat{k}) = mvd$		

Case II : Body is in circular path

$\vec{L}_{about\ o} = mvR\ (+\hat{j})$

$= mR^2\omega = I_{axis}\ \omega$



Case 3 : Body rotating about an axis

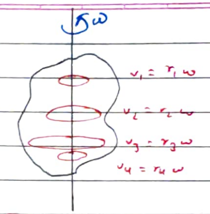
If Rigid body is Rotating

$\therefore \vec{L} = m_1v_1r_1 + m_2v_2r_2 + \ldots$

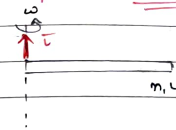
$= m_1r_1^2\omega + m_2r_2^2\omega + \ldots$

$\vec{L} = (\Sigma m_i r_i^2)\,\omega$

$$\boxed{\vec{L} = I_{axis}\ \vec{\omega}}$$ $\rightarrow$ dir. of $\vec{\omega}$ = dir of $\vec{L}$



Q)



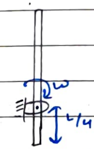
$L = I_{axis} \cdot \omega$

$= \frac{1}{3}ML^2\omega\ (\hat{j})$

Q)

$L = \frac{1}{4}ML^2 \cdot \omega\ (\hat{j})$

Q)



Rod is in vertical plane

$\therefore \vec{L} = I_{axis}\ \omega$

$= \left(\frac{1}{12}ML^2 + m\left(\frac{L}{4}\right)^2\right)\omega\ (-\hat{k})$

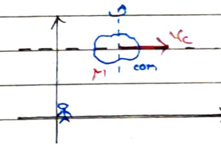
case 4 : Body rotating about an axis and also moving.

~~Bod~~

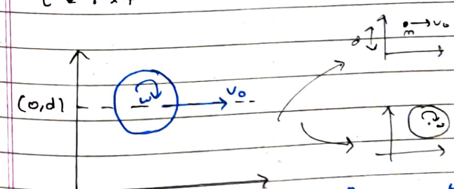
$\therefore \vec{L} = \vec{L}_{com} + \vec{I}_c\,\omega$

Assume particle to be point mass at com, & then use $\vec{L} = \vec{r} \times \vec{p}$

Body ko centre ke about ghumaoge.



Q).

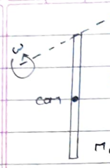


$\therefore \vec{L}_1 = mv_0d\ (-\hat{k})$

$\therefore \vec{L}_2 = \frac{mR^2}{2} \cdot \omega\ (-\hat{k})$

$\therefore L_T = \left(mv_0d + \frac{mR^2\omega}{2}\right)(-\hat{k})$

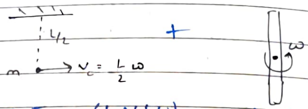
Q)



MI: $\vec{L}_{\text{Rod about axis}} = \vec{I}_{axis}\,\omega$

$= \frac{1}{3}ML^2\cdot\omega$

M II:



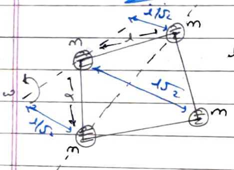
$\therefore L_T = \frac{mL^2\omega}{4} + \frac{mL^2\omega}{12}$

$= \frac{mL^2\omega}{3}$

$\therefore L = m\left(\frac{L}{2}\right)\left(\frac{L}{2}\omega\right) = \frac{mL^2\omega}{4}$

$\therefore L = I_{com}\cdot\omega = \frac{1}{12}ML^2\omega$

Q) M. 2020



Find Ang Momentum.

d, ω

$\Rightarrow \vec{L} = I_{axis}\vec{\omega} = \left[m(0)^2 + m(d\sqrt{2})^2 + 2m\left(\frac{d}{\sqrt{2}}\right)^2\right]\omega$

$L = \left[2md^2 + md^2\right]\omega = 3md^2\omega$

Q) M. 2021 Given: m, moving in time 't', $r = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

The Ang. momentum of part. becomes the same as it was for $t=0$ at time $t =$ ___ sec.

→ $r = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

$v = \frac{dr}{dt} = 20\alpha t\,\hat{i} + 5\beta\hat{j}$

$\therefore p = m(20\alpha t\,\hat{i} + 5\beta\hat{j})$

$\therefore \vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 10\alpha t^2 & 5\beta(t-5) & 0 \\ 20\alpha m t & 5\beta m & 0 \end{vmatrix}$

$\therefore \vec{L} = \hat{k}\left(50\alpha\beta m t^2 - 100\beta\alpha m(t^2-5t)\right)$

$\therefore$ at $t=0 \Rightarrow \vec{L} = 0$

$\therefore$ given $\vec{L}_{at\,t=0} = \vec{L}_{t=t}$

$\therefore 0 = 50\alpha\beta m t^2 - 100\beta\alpha m(t^2-5t)$

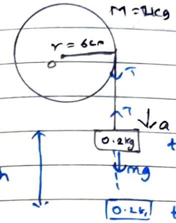
$\therefore \cancel{50}\,\alpha\beta m t^2 = \cancel{100}\,\alpha\beta m t(t-5)$

$t = 2(t-5)$

$t = 2t - 10$

$\therefore t = 10$

Q)



M = 1kg

Cal. Ang. momentum of system about O, after 2 sec from start.

Tr: $\therefore mg - T = ma$

Rr: $\therefore T(R) = I\alpha$.

$\therefore$

$mg - T = ma$

$T = \frac{I\alpha}{R}$

$\oplus$

$mg = ma + \frac{I\alpha}{R}$

$\therefore a = \alpha R$

$\alpha = a/R$

$\therefore 0.2\times10 = 0.2\times a + \frac{MR^2}{2}\left(\frac{a}{R^2}\right)$

$\Rightarrow 2 = a\left(0.2 + \frac{1}{2}\right)$

$\therefore a = \frac{2}{7/10}$

$a = \frac{20}{7}$ (const)

$\therefore v = \cancel{u}^{\,0} + at$

$v = \left(\frac{20}{7}\right)(2) = \frac{40}{7} = 5.7\ m/s$

$\omega = \frac{v}{R} = \frac{40}{7\times6}\cdot100 = \frac{4000}{42} = 95\ rad/s$

$\therefore \vec{L}_{about\,O} = \vec{L}_{Disc} + \vec{L}_{Block}$

$= I\omega + mvr$

$= \left(\frac{MR^2}{2}\cdot95\right) + (1)(5.7)\left(\frac{6}{100}\right)$

Q) AIEEE 2009 A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane. At a time $t < \frac{v_0\sin\theta}{g}$, the ang momentum of particle?

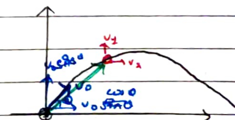
→ $u_x = v_0\cos\theta$ $\quad a_x = 0$

$u_y = v_0\sin\theta$ $\quad a_y = -g$

$\vec{r} = \vec{S_x} + \vec{S_y}$

$= \left(u_x t + \frac{1}{2}a_x t^2\right)\hat{i} + \left(u_y t + \frac{1}{2}a_y t^2\right)\hat{j}$

$\vec{r} = (v_0\cos\theta\, t)\hat{i} + \left(v_0\sin\theta\, t - \frac{1}{2}g t^2\right)\hat{j}$



$\vec{V} = \frac{d\vec{r}}{dt} = (V_0\cos\theta)\hat{i} + (V_0\sin\theta - gt)\hat{j}$

$\therefore$ ~~$\vec{\tau} = \vec{r} \times m\vec{v}$~~ $\quad \vec{p} = mV_0\cos\theta\,\hat{i} + m(V_0\sin\theta - gt)\hat{j}$

$\therefore \vec{\tau} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ V_0\cos\theta t & V_0\sin\theta t - \frac{1}{2}gt^2 & 0 \\ mV_0\cos\theta & m(V_0\sin\theta - gt) & 0 \end{vmatrix}$

$= \hat{k}\left((V_0\cos\theta t)(mV_0\sin\theta - mgt) - (mV_0\cos\theta)\left(V_0\sin\theta t - \frac{1}{2}gt^2\right)\right)$

$= \hat{k}\left(\left(\cancel{mV_0^2\sin\theta\cos\theta t} - mV_0\cos\theta g t^2\right) - \left(\cancel{mV_0^2\cos\theta\sin\theta t} - \frac{mV_0\cos\theta g t^2}{2}\right)\right)$

$\hat{k}\left(-\frac{mV_0}{2}\cos\theta g t^2\right) \quad \therefore \vec{\tau} = \frac{mV_0}{2} g t^2 \cos\theta\,(-\hat{k})$

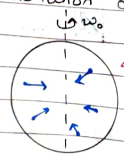
* Relation betⁿ torque & Ang. Momentum & cons. of Ang. Momentum.

Translatory Motion	Rotational Motion
⇒ IInd law : $\vec{F}_{net\,ext} = \frac{d\vec{P}}{dt}$	⇒ IInd law : $\vec{\tau}_{net\,ext} = \frac{d\vec{L}}{dt}$
• if $\Sigma F_{net\,system} = 0 \quad \therefore 0 = \frac{d\vec{P}}{dt}$	Depend on reference • if $\Sigma \tau_{net} = 0 \quad \therefore 0 = \frac{d\vec{L}}{dt}$
$\therefore \vec{P}_i = \vec{P}_f$	$\therefore \vec{L}_i = \vec{L}_f$

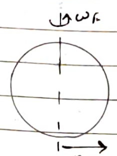
* Ek aisa point dhundo jaha se net $\tau = 0$ aaye, waha se L cons. karna

Applications :-

(1) Rotation of planet and shrinking.



$L_i = I\omega = \frac{2}{5}MR^2\omega_0$



$L_f = \frac{2}{5}M\left(\frac{R}{n}\right)^2\omega_f$

$\therefore L_i = L_f \quad \therefore \cancel{\frac{2}{5}M}R^2\omega_0 = \cancel{\frac{2}{5}M}\frac{\cancel{R^2}}{n^2}\omega_f$

$\therefore \underline{\omega_0 n^2 = \omega_f}$ in term of t : $\frac{2\pi}{T_0}n^2 = \frac{2\pi}{T_f}$

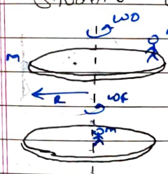
$\therefore T_f n^2 = T_0 \quad \therefore \underline{\frac{T_0}{n^2} = T_f}$

$KE_i = \frac{1}{2}I_0\omega^2 = \frac{1}{2}\frac{I_0\omega^2 I_0}{I_0} = \frac{L^2}{2I_0}$

$\therefore KE_i = \frac{L^2}{2I_i} \quad KE_f = \frac{L^2}{2I_f} \quad \therefore \frac{KE_f}{KE_i} = \frac{I_i}{I_f} = \frac{\cancel{\frac{2}{5}M}R^2}{\cancel{\frac{2}{5}M}\frac{R^2}{n^2}}$

$\therefore \underline{\frac{KE_i}{KE_f} = n^2}$

(2) Student on rotating turntable.



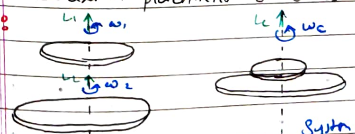
$\vec{L}_{i\,system} = [I_{disc} + I_{person}]\omega_0 = \left[\frac{MR^2}{2} + mR^2\right]\omega_0$

$\vec{L}_{f\,system} = \left[\frac{MR^2}{2} + 0\right]\omega_f$

$\tau_{friction\,net} = 0 \quad \therefore \vec{L}_i = \vec{L}_f \quad \therefore \left[\frac{MR^2}{2} + mR^2\right]\omega_0 = \underline{\frac{MR^2\omega_f}{2}}$

(3) Coaxial placement of discs

case 1 :



System $\tau_{friction\,net} = 0$

$m_1 v_1 \; m_2 v_2 \qquad m_1 m_2 \to v_c$

$\therefore V_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$P_i = P_f$

$\therefore$

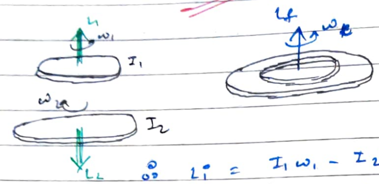
$L_i = L_f$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$\therefore \omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

$$\Delta K = \frac{1}{2}\frac{I_1 I_2}{I_1+I_2}(\omega_1-\omega_2)^2$$

Case 2:

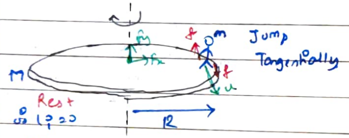


$\therefore L_i = I_1\omega_1 - I_2\omega_2$

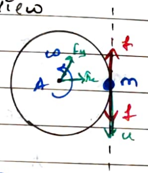
$L_f = (I_1+I_2)\,\omega_c$ $\qquad \because \tau_{net} = 0$

$\therefore \omega_c = \dfrac{I_1\omega_1 - I_2\omega_2}{(I_1+I_2)}$ & $\Delta K = \dfrac{1}{2}\dfrac{I_1 I_2}{I_1+I_2}(\omega_1+\omega_2)^2$

(4) Jumping from Disc



⇒ Top view

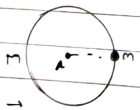


About A $\tau_{hinge} = 0$ & $\tau_f = 0$

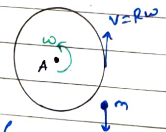
$\therefore L_f$ system $= -muR + I_{axis}\,\omega$ $\qquad \therefore 0 = -muR + I_A\omega$

$\therefore \omega = \dfrac{muR}{MR^2/2} = \dfrac{2mu}{MR}$

what if person jump tangentially relative to disc.

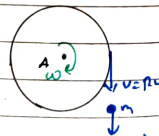


$\vec{L}_i$ about A $= 0$



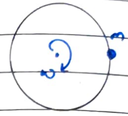
$(u-v) = (u-R\omega)$

$L_{f\,about\,A} = m(u-R\omega)R\,(-\hat{k}) + \dfrac{MR^2\omega}{2}(+\hat{k})$



$(u+v) = (u+R\omega)$

what if disc is Rotating Initially.



$\therefore \vec{L}_i = \left(\dfrac{MR^2}{2} + mR^2\right)\omega\,(-\hat{k})$