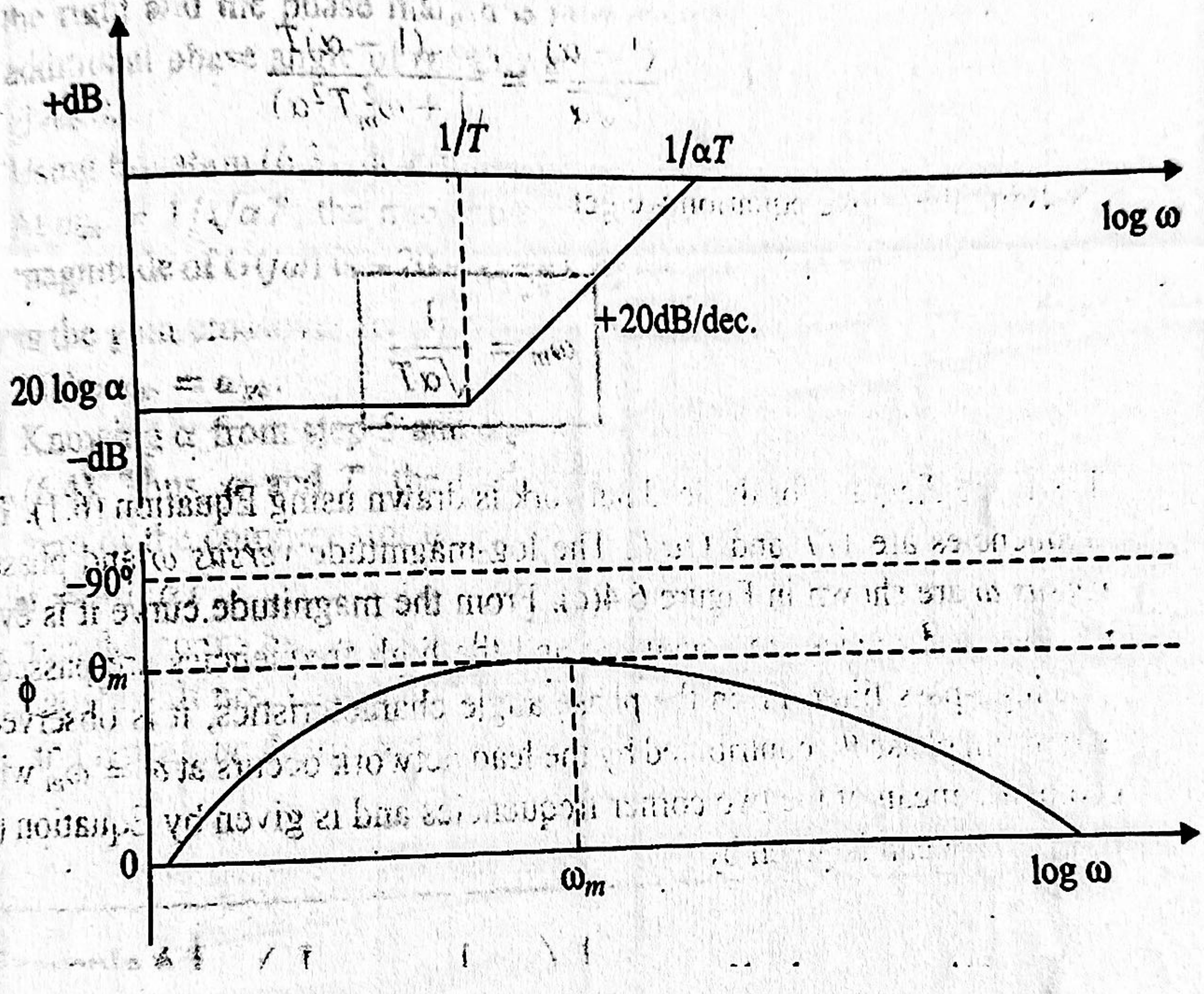
Design of Lead Compensator using Bode-plot Method.

- The transfer function of Lead Compensator is

$$D(s) = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}$$

- The Bode plot of lead Compensator is shown below.



θ_m : Maximum phase lead of the Compensator

ω_m : Frequency at which θ_m occurs.

$$\sin\theta_m = \frac{1-\alpha}{1+\alpha} \Rightarrow \alpha = \frac{1-\sin\theta_m}{1+\sin\theta_m}$$

Gain at $\omega_m = 20\log\sqrt{\alpha}$.

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}}$$

Procedure to design lead Compensator

→ Given the TF of un compensated system, Construct the Bode plot and Compute phase margin.

→ If the phase margin is inadequate to meet the specified phase margin, determine required phase angle of lead Compensator, ϕ.

→ When the lead Compensator is inserted into the system, ~~Gain of~~ gain crossover frequency of the Composite system (original system + Compensator) is shifted to the right and the phase margin is decreased. To account for this, add an additional phase angle of 7° to 12°.

$$\theta_m = \phi + \text{Additional phase.}$$

→ The frequency Corresponding to this is ω_m.

→ $$\alpha = \frac{1 - \sin\theta_m}{1 + \sin\theta_m}$$

→ At ω_m, $M_{dB} = 20\log\sqrt{\alpha}$

Using this gain, find ω_m from the plot.

→ $$\tau = \frac{1}{\omega_m\sqrt{\alpha}}$$

Problem: 01

The openloop TF of an uncompensated unity f/b system is, $G(s)H(s) = \frac{5}{s(s+2)}$

Design a lead compensator for the system so that $K_v \geq 20\,s^{-1}$, the phase margin is at least $55°$

Soln: $G(s)H(s) = \frac{5}{s(s+2)}$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \frac{5s}{s(s+2)} = 2.5$$

But required K_v is 20.

Hence an amplifier with a gain of $\frac{20}{2.5} = 8$ is required to be connected in cascade.

$$G(s)H(s) = \frac{40}{s(s+2)} = \frac{20}{s(1+s/2)}$$

Construct the bode plot of $G(s)H(s)$

From the Bode plot (see next page)

$\omega_{gc} = 6.32$ r/s

$PM = 17.6°$

But the required phase margin is $55°$.

$\Rightarrow \quad \phi + 17.6° = 55 \Rightarrow \phi = 37.4°$

ϕ = phase angle contribution by lead compensator

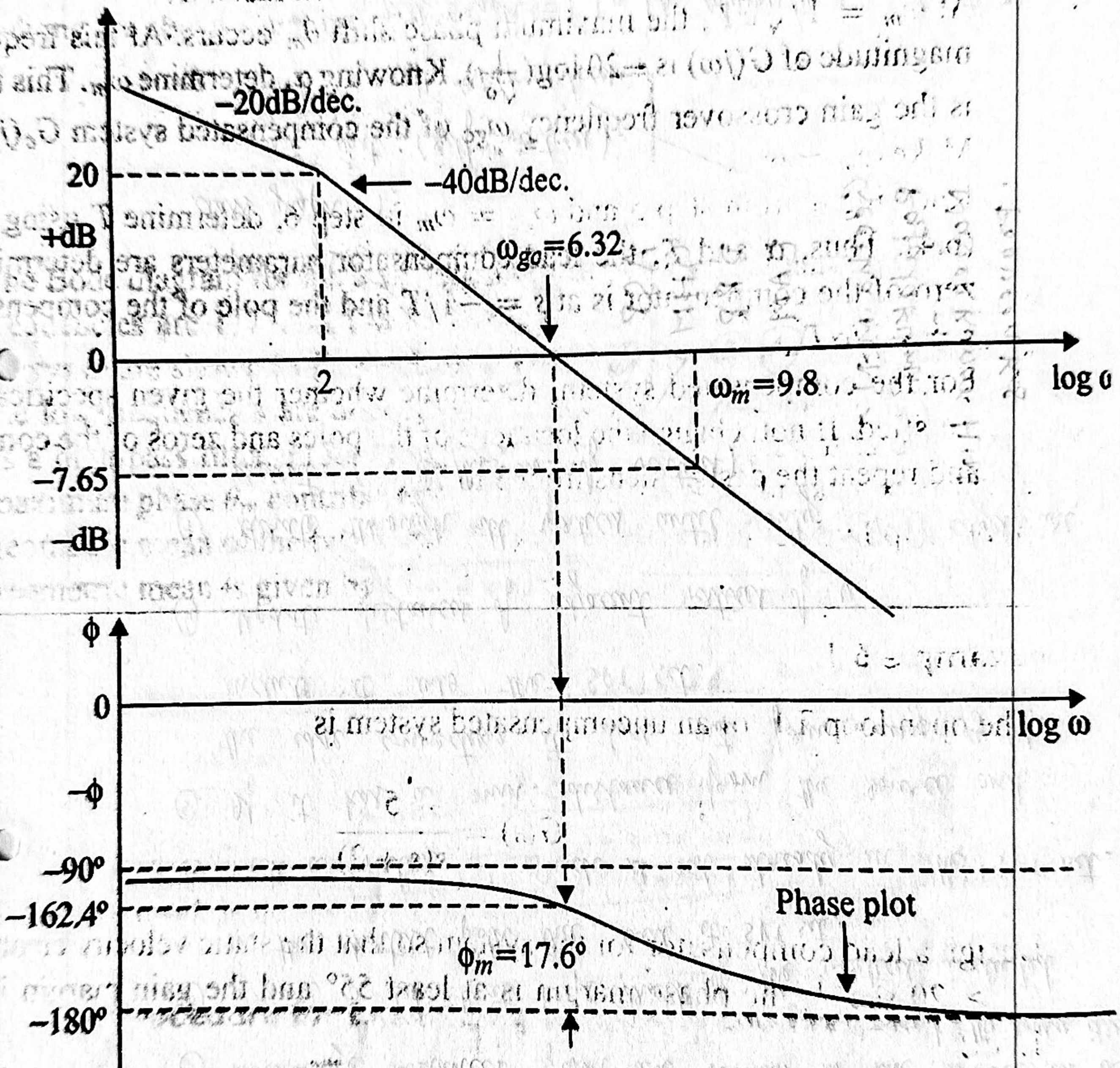
Adding another $7.6°$ (any value between $7°$ & $12°$)

we obtain, $\theta_m = \phi + 7.6° = 45°$.

* $7.6°$ is chosen to get round figure for θ_m.

Bode plot of

$$G(s)H(s) = \frac{20}{s(1+s/2)}$$



$$\alpha = \frac{1 - \sin\theta_m}{1 + \sin\theta_m} = 0.172$$

Gain corresponding to ω_m is

$$20 \log \sqrt{\alpha} = -7.65 \text{ dB.}$$

From the magnitude plot, corresponding to -7.65 dB,

$$\omega_m = 9.8 \text{ r/s.}$$

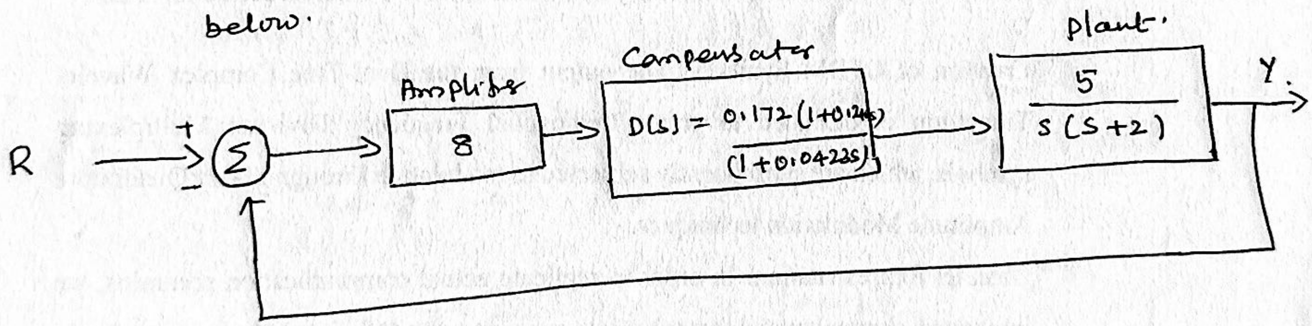
$$\tau = \frac{1}{\omega_m \sqrt{\alpha}} = 0.246.$$

$$\alpha\tau = 0.0423.$$

Lead compensator TF is

$$D(s) = \frac{\alpha(1 + \tau s)}{(1 + \alpha\tau s)} = \frac{0.172\,(1 + 0.246 s)}{(1 + 0.0423 s)}$$

Block diagram of compensated system is shown below.



problem 2

Design phase lead compensator for a system with open loop TF is $G(s)H(s) = \frac{10}{s(1+s)}$ to have a PM of $45°$.

Soln:

Construct the Bode plot of $G(s)H(s) = \frac{10}{s(1+s)}$

from the plot, $\omega_{gc} = 3.16$ r/s.

phase margin $= 17.5°$

Required phase margin $= 45°$.

$\therefore \quad 17.5° + \phi = 45° \Rightarrow \phi = 27.5°$

Adding an allowance of $7.5°$

$\theta_m = 27.5° + 7.5° = 35°$.

$$\alpha = \frac{1 - \sin\theta_m}{1 + \sin\theta_m} = 0.271$$

$20 \log \sqrt{\alpha} = -5.67$ dB.

Corresponding to this gain, from the plot

$\omega_m = 4.35$ r/s.

$$\tau = \frac{1}{\omega_m \sqrt{\alpha}} = 0.4386$$

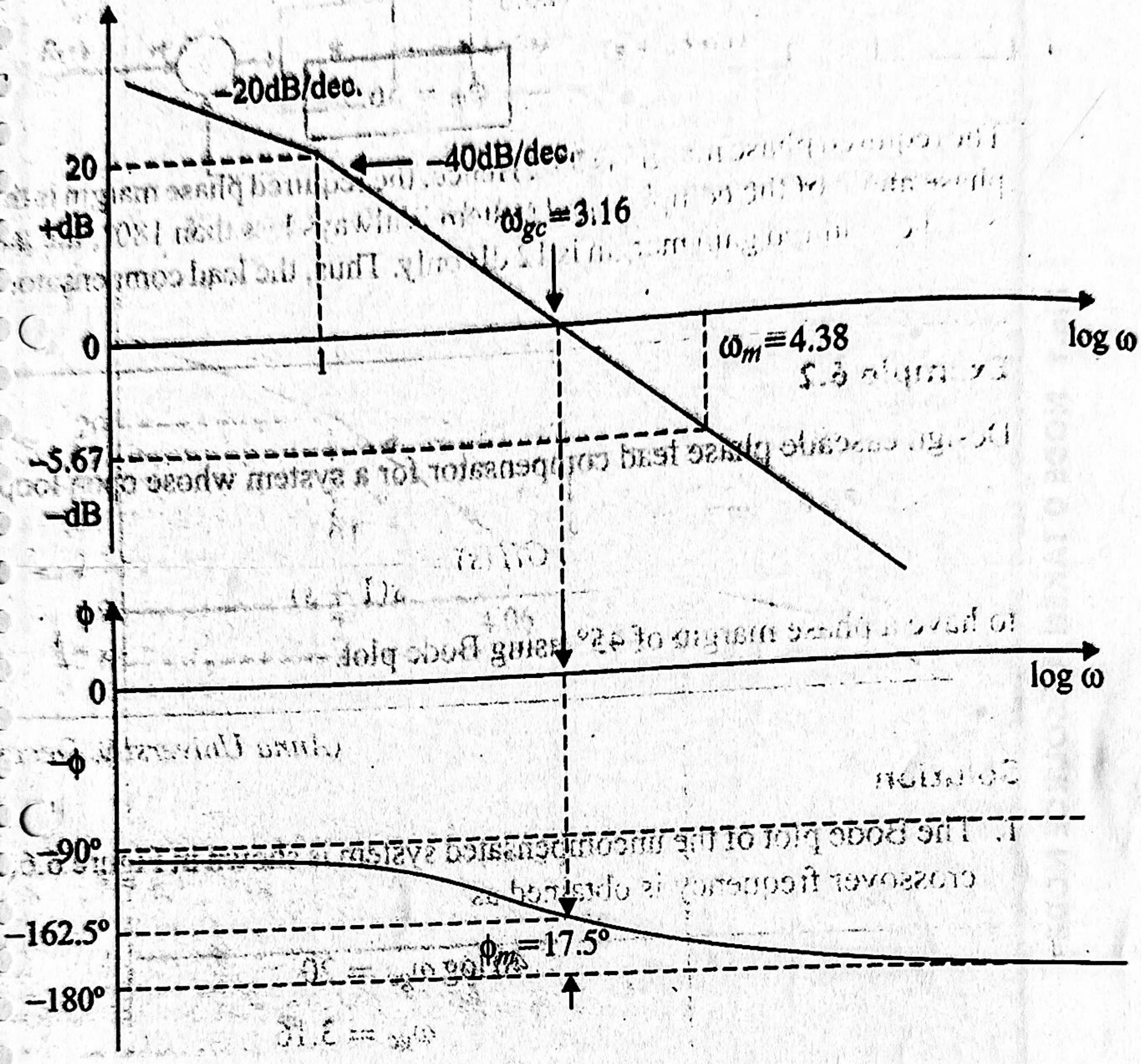
$\alpha\tau = 0.118$

$$D(s) = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)} = \frac{0.271(1+0.4386s)}{(1+0.118)}.$$

The over all system gain reduced by a factor α when the compensator is connected in cascade with the plant.

Bode plot of

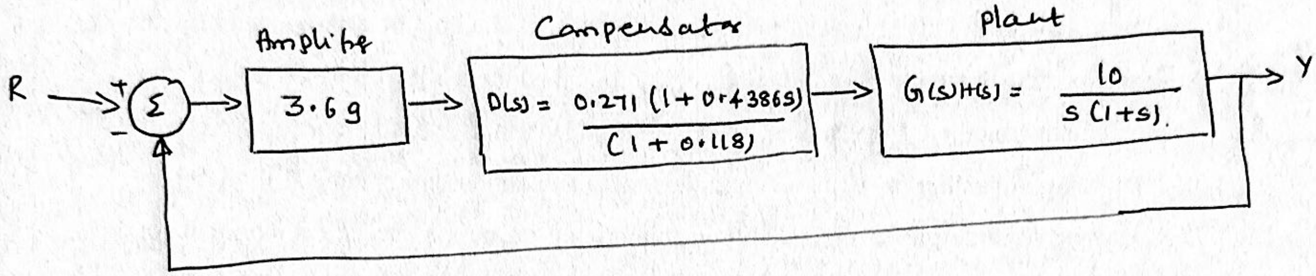
$$G(s)H(s) = \frac{10}{s(1+s)}$$



Hence an amplifier with a gain of

$1/\alpha = 3.69$ is required.

[*** The effect of α can also be considered in problem 01.
additional ampl gain $1/\alpha = 1/0.172 = 5.81$]



Practice problem

$$G(s)H(s) = \frac{K}{s(s+2)}$$

$K_v \geq 12\ s^{-1}$

phase margin : 45^0.

Ans: $K = 24$ to meet K_v.

$\omega_{gc} = 4.9$ r/s

$\phi = 23^0$

$\theta_m = 23^0 + 10^0 = 33^0$

$\alpha = 0.295$

$\omega_m = 6.65$ r/s

$\tau = 0.276$

$\alpha\tau = 0.0814$

Ampl gain $\frac{1}{\alpha} = 3.39$