Parabolic.

$$\lim_{s\to 0} s \frac{s^2}{s^2+11s+18}\frac{1}{s^3} = \frac{1}{8} \rightarrow \text{Type 2}$$

b). a + 1+ Sa.

5. DC motor $\dot{y} + 60y = 600V_a - 1500w,$

$V_a = -\left(K_p e + K_I \int_0 e\,dt\right)$

$e = r - y$

$$(s+60)Y(s) = -600\left[K_p Y(s) + \frac{K_I}{s} Y(s)\right] - 1500\,W(s)$$

$$\frac{Y(s)}{W(s)} = \frac{-1500s}{s^2+60(1+10K_p)s+600K_I}$$

$-60 \pm j60$

$s^2 + 2\xi\omega_n s + \omega_n^2$

$(s+60+j60)(s+60-j60)$

$(s+60)^2 + 60^2 = s^2 + 120s + 7200$

$\Rightarrow \omega_n^2 = 7200.$

$\omega_n = 60\sqrt{2}$

$\xi = \frac{120}{2\omega_n} = \frac{1}{\sqrt{2}}$

$\omega_n^2 = 600K_I$

$K_I = \frac{600}{\omega_n^2} = 12$

$60(1+10K_p) = 2\xi\omega_n$

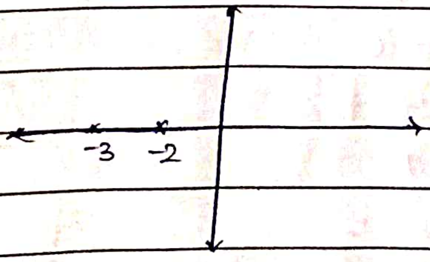
$\boxed{K_p = 0.1}$

ant—

$$L(s) = \frac{K}{s(s+2)(s+3)}$$

Open loop poles:- $s = 0, -2, -3$ $\quad P = 3$

Open loop zeros: nil $\quad Z = 0$



Root locus on real axis lies b/w $s=0$ and $s=-2$

$s=-3$ and $s=-\infty$

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No. of asymptotes,

$P - Z = 3$

Angle of asymptotes,

$$\theta_n = \frac{(2q+1)180}{P-Z}$$

where $q = 0, 1, 2 \ldots$

$q = 0$, $\theta_n = 60^\circ$

$q = 1$, $\theta_n = 180^\circ$

$q = 2$, $\theta_n = 300^\circ$

$$\sigma = \frac{\Sigma \underbrace{R.P}_{\text{real part}} \text{ of poles} - \Sigma R.P \text{ of zeros}}{P-Z}$$

$$= \frac{(-2-3) - 0}{3}$$

$$= -5/3$$

$$= -1.66$$

Breakaway points:

$$\frac{dK}{ds} = 0.$$

$CE: 1 + L(s) = 0$

$$1 + \frac{K}{s(s+2)(s+3)} = 0$$

$$K = -s(s+2)(s+3)$$

$$K = -s(s^2+5s+6)$$

$$K = -(s^3+5s^2+6s.)$$

$$\frac{dK}{ds} = -(3s^2+10s+6) = 0$$

$$s_1, s_2 = -0.71, \ -2.54$$

$\checkmark$ $\quad$ $\times$

$\frac{12 \times 6}{72}$

$$\frac{-10 \pm \sqrt{100-72}}{6}$$

$$= \frac{-10 \pm \sqrt{28}}{6}$$

Value of K at breakaway point:

$CE = 1 + L(s) = 0$

$$1 + \frac{K}{s(s+2)(s+3)} = 0$$

$$\frac{K}{s(s+2)(s+3)} = -1$$

$$\left| \frac{K}{s(s+2)(s+3)} \right| = 1$$

Every point on root locus must satisfy this.

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$K = |s(s+2)(s+3)|$

$K_s = -0.71 = |s||s+2||s+3|$

$= 0.71 \times 1.29 \times 2.29$

$= 2.21$

Intersection of root locus with imaginary axis.

This can be obtained using RH criteria.

CE: $1 + \frac{K}{s(s+2)(s+3)} = 0 \rightarrow s^3 + 5s^2 + 6s + K = 0$

s^3	1	6
s^2	5	K
s	$\frac{30-K}{5}$	0
s^0	K	

For the system to be stable, all the elements in 1st column must be positive

(i) $K > 0$

(ii) $\frac{30-K}{5} > 0 \Rightarrow 0 < K < 30$

Consider s^2 row from which K_{marg} is obtained.

$A(s) = 5s^2 + K = 0$

$5s^2 + 30 = 0$

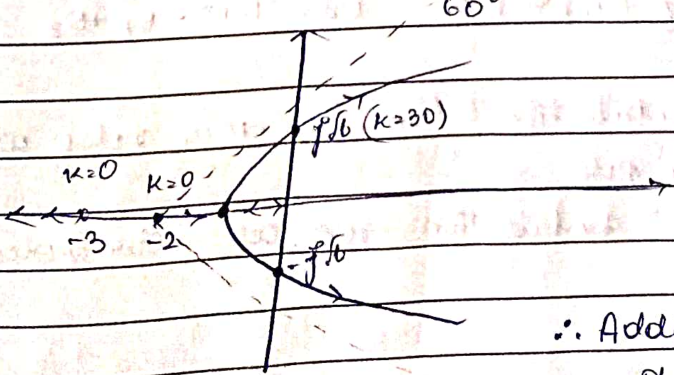
$s^2 = -6$

$s = \pm j\sqrt{6} = \pm j\omega$

$\therefore \omega = \sqrt{6}$ rad/sec

- when $K = 30$ branches of root locus cuts $j\omega$ axis at $\pm j\sqrt{6}$ and the system becomes marginally stable.

- $K > 30$, The root locus enters into right half of s plane and the system becomes unstable.



$L(s) = \frac{K(s+1)}{s(s+2)(s+3)} \xleftarrow[\text{one zero}]{\text{Added}} \frac{K}{s(s+2)(s+3)}$

$0 < K < \infty$ no restriction on value of K

$0 < K < 30$

$\therefore$ Additional zero improves stability of system

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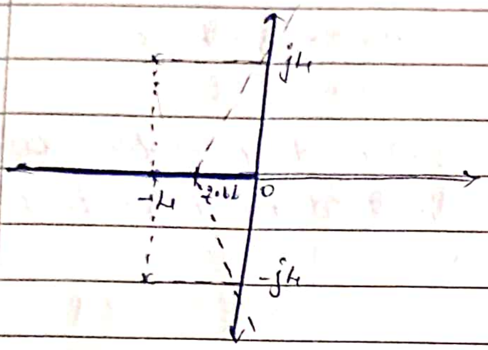
Complex poles:-

$$L(s) = \frac{K}{s((s+4)^2+16)} = \frac{K}{s(s^2+8s+32)}$$

$-4+j4,\ -4-j4$ $\rightarrow$ $\frac{-8 \pm \sqrt{64-128}}{2}$

Open loop poles: $s=0$ $s=-4\pm j4$ $= -4 \pm j4$



Root locus on real axis lies b/w $s=0$ & $s=-\infty$

No. of asymptotes: $P-Z=3$.

Angles, $\frac{(2q+1)180}{P-Z} \rightarrow 60°, 180°, 300°$

centroid

$$\sigma = \frac{(0-4-4)-0}{3} = -2.66$$

Angle of departure :-

For complex poles, it is necessary to find the angle of departure

Angle of departure is given by $\boxed{\theta_d = 180° - \theta_p + \theta_z}$

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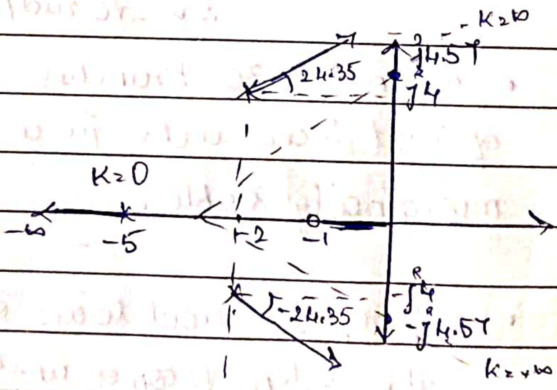
$$L(s) = \frac{s+1}{s(s+5)(s^2+4s+8)}$$

$P-Z=3$ $\theta_A = 60°, 180°, 300°$

$$\sigma = \frac{0-4-4}{3} = -2.66$$



Angle of departure of complex poles: $\theta_d = 180° - \theta_p + \theta_z$

$\theta_p = \Sigma$ angle contribution from all other poles at the complex pole

$\theta_z = \Sigma$ angle contribution from all other poles at zero

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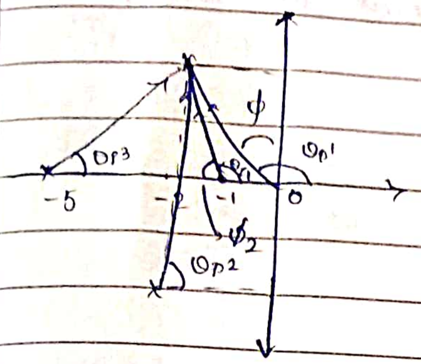

$\theta p = \theta p_1 + \theta p_2 + \theta p_3$

$\theta_z = \theta z_1$

$\phi_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56°$

$\theta p_1 = 90 + 26.56 = 116.56°$

$\theta p_2 = 90°$

$\theta p_3 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

$\theta p = 116.56 + 90 + 53.13 = 259.69°$

$\theta z_1 = \cancel{\theta z}\ 180 - \phi_2 \qquad \because \theta z_1 + \phi_2 = 180°$

$= 180° - \cancel{t}\tan^{-1}\left(\frac{4}{1}\right)$

$= 180° - 75.96$

$\theta z_1 = 104.04°$

$\theta d = 180° - 259.69 + 104.04$

$\boxed{\theta d = 24.35°}$

Intersection of root locus with imaginary axis:-

$L(s) = \dfrac{K(s+1)}{s(s+5)(s^2+4s+8)}$

$1 + L(s) = s(s+5)(s^2+4s+8) + Ks + K = 0.$

$= (s^2+5s)(s^2+4s+8) + Ks + K = 0$

$= s^4 + 4s^3 + 8s^2 + 5s^3 + 20s^2 + 40s + Ks + K = 0$

$= s^4 + \cancel{4s^3}\ 9s^3 + 28s^2 + (40+K)s + K = 0$

s^4	1	28	K
s^3	9	K+40	0
s^2	$\frac{252-K-40}{9}$	K	
s^1	h	0	
s^0	K		

$\dfrac{\frac{212-K}{9} \times (K+40) - 9K}{\frac{212-K}{9}}$

$\dfrac{212K + 8480 - K^2 - 40K - 81K}{\cancel{9} \times \frac{212-K}{9}}$

$h = \dfrac{8480 + \cancel{163}\ 91K - K^2}{212-K}$

$212 - K > 0$

$\boxed{K < 212}$

$h > 0$

$8480 + 91K - K^2 > 0$

K

Sri Sai Sridhar

For the system to be stable, all elements in Routh's table I column must be positive.

$-K^2 + 91K + 18480 > 0$

$(K-K_1)(K-K_2) > 0 \rightarrow (4)$

$K_1, K_2 = \dfrac{-91 \pm \sqrt{91^2 - 4(18480}}{2}$

$K_1 = 148.21$ $\qquad$ $K < 148.21$

$K_2 = -57.21$

$(K - 148.21)(K+57.21) > 0 \rightarrow (4)$

Substitute 150 for K (4) is +ve.

Substitute −60 to K (4) is +ve.

(i) $K > 0$

(ii) $K < 212$

(iii) $K < 148.2$ ✓

(iv) $K > -57.12 \rightarrow \times$

~~148.2 < K < 212~~ $\qquad 0 < K < 148.2 \Rightarrow K_{marg} = 148.2$

If we substitute $K = 148.2$ in eq^n of s'

$A(s) = \dfrac{212 - 148.2}{9} s^2 + 148.2 = 0$

$s = \pm 4.57j = \pm j\omega$

$\omega = 4.57$

→ Solve quadratic K

→ Choosing proper range of K

→ Complex pole

→ angle of departure

27/4/23

1. Construct root locus for $L(s) = \dfrac{K}{s((s+4)^2+16)}$

$L(s) = \dfrac{1}{s(s^2+3s+10)}$ $\qquad$ Angle of departure:

→ $Z = 0$ $\qquad$ $s = 0, -1.5+2.78j, -1.5, -2.78j$

$P = 3$

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$\theta d = 180° - \theta p + \theta z$

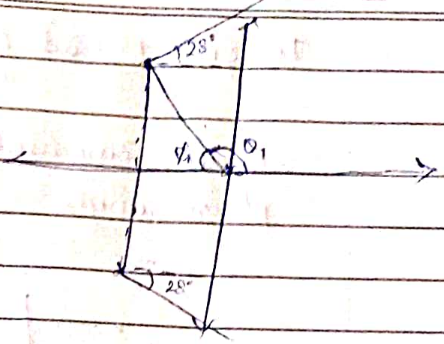
$\theta_1 = 180° - \phi = 180° - \tan^{-1}\left(\frac{2.78}{1.5}\right)$

$= 118.36°$

$\theta d = 180° - \theta p$

$= 180° - [118° + 90]$

$\theta d = \pm 28°$



$L(s) = \frac{1}{(s^2+3s+10)}$ $\rightarrow$ $a = -3$ $\theta_A = \pm 90°$ $\theta d = \pm 90°$ $\omega_0 >$ none

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Design of Compensators:

The controller gain K_P, K_I, K_D are selected to obtain the desired steady state error.

→ Controller improves steady state response

e_{ss} (step) $= \frac{1}{1+K_P}$ → controller gain

$e_{ss} \downarrow \Rightarrow K_P \uparrow$

- If gain is ↑ to decrease ss error transient response will be poor effect M_P, t_r
- Compensator → Add poles and zeros to improve steady state response along with transient response
- A well designed control system incorporates both controller and compensator.

Design of compensator:

- Time domain method → Root locus
- Freq domain method → Bode plot

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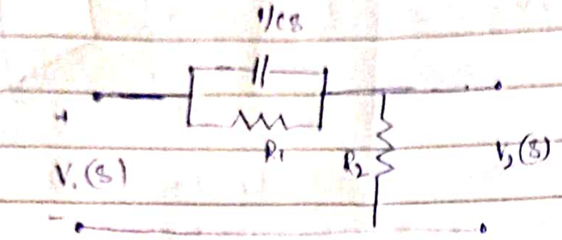
Design of lead compensator using R-L technique

Lead compensator adds +ve phase angle to the system phase angle



$$Z_1 = R_1 \parallel 1/CS$$
$$= \frac{R_1 \frac{1}{CS}}{R + \frac{1}{CS}}$$
$$Z_1 = \frac{R_1}{1+R_1CS}$$

$$V_2 = \frac{V_1 Z_2}{Z_1+Z_2}$$

$$\frac{V_2}{V_1} = \frac{R_2}{\frac{R_1}{1+R_1CS}+R_2}$$

$$= \frac{R_2(1+R_1CS)}{(R_1+R_2+R_1R_2CS)}$$

$$= \underbrace{\frac{R_2}{(R_1+R_2)}}_{\alpha} \frac{1+R_1CS}{1+\frac{R_1R_2}{R_1+R_2}CS}$$

$$= \alpha = \frac{R_2}{R_1+R_2} < 1 \quad [\because \text{Den is greater than Num}]$$

$$\frac{V_2}{V_1} = \alpha \frac{1+\tau S}{1+\alpha\tau S}$$

Zero : $1+\tau S = 0 \Rightarrow S = -1/\tau = S_1$

Poles : $1+\alpha\tau S = 0 \Rightarrow S = -1/\alpha\tau = S_2$

$$|S_2| > |S_1| \quad [\because \alpha \text{ is less than one}]$$

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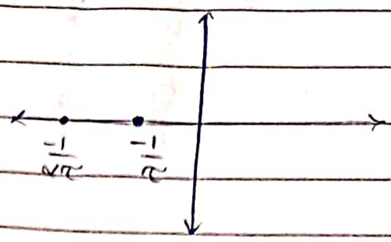
Freq response:-

Sinusoidal i/p

$s = j\omega$

$$\frac{V_2(j\omega)}{V_1(j\omega)} = \alpha \frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$



$$\angle \frac{V_2}{V_1} = \phi \ (+ve)$$

$= \angle N^r - \angle D^r$

$= \tan^{-1}(\omega\tau) - \tan^{-1}(\alpha\omega\tau)$

$\Rightarrow \tan^{-1}(\omega\tau) > \tan^{-1}(\alpha\omega\tau)$

$\omega\tau > \alpha\omega\tau$

$\tau > \alpha\tau$

$$\boxed{\frac{1}{\tau} < \frac{1}{\alpha\tau}}$$

1 The plant tr. fn of unity f/b control system is given by $G(s) = \frac{K}{s(s+1)}$. Design a lead compensator using root locus technique to meet the following specifications.

Damping ratio. $\xi = 0.7$, $t_s = 1.4$ sec (2% tolerance) $K_v \geq 2\ sec^{-1}$

→ Step 1: using the specifications calculate the desired pole locations (to meet the given specs)

$\xi = 0.7 \rightarrow$ underdamped $\Rightarrow$ Desired poles are complex conjugates

$S_d = -\xi\omega_n \pm j\omega_d$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$t_s = \frac{3.91}{\xi\omega_n}$ [2% tolerance]

$\omega_n = \frac{3.91}{0.7 \times 1.4} = 4$ rad/sec

$S_d = -2.8 \pm j2.8$

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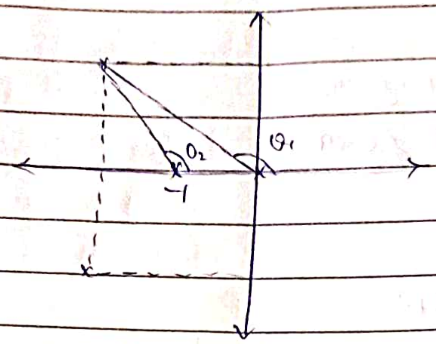
Step 2: Angle requirement from lead compensator



• To obtain the desired spec S_{d_1} & S_{d_2} must lie on root locus.

Root locus satisfies $1+L(s)=0$

$G(s) = -1 = 1\angle 180°$

• For every point on root locus, $|G|=1$ and $\angle G = -180°$

$\theta_1 = \tan^{-1}\frac{2.8}{2.8} = 180-45 = 135°$

$\theta_2 = 180° - \tan^{-1}\frac{2.8}{1.8} = 122.73$

$\theta = \theta_1 + \theta_2 = -257.73$

$\frac{\angle N_r}{\angle D_r} = \underset{Z}{\angle N_r} - \underset{P}{\angle D_2}$ ∵ Angle contributed by poles will be negative

$= -135 - 122.73$

$= -257.73$

$-257.73 + \phi = -180°$

$\boxed{\phi = 77.7°}$

+ve → lead controller

A standard form of lead controller is given by

$D(s) = \frac{s+Z}{s+P}$ $s+Z=0 \Rightarrow s=-Z$, Zero of compensator

$s+P=0 \Rightarrow s=-P$, Pole of compensator

* where to fix pole and zero of compensator.

$Z = \frac{\omega_n \sin\gamma}{\sin(\theta+\gamma)}$ $\gamma = \frac{1}{2}[180-\theta-\phi]$

$P = \frac{\omega_n \sin(\gamma+\phi)}{\sin(\theta+\gamma+\phi)}$

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$\theta = \cos^{-1} \xi$

ϕ = compensator angle

$\xi = 0.7$

$\theta = 45.57°$

$\mu = 28.365°$

$Z = 1.977$

$P = 8.090$

Transfer function of lead compensator

$$D(s) = \frac{s+Z}{s+P} = \frac{s+1.977}{s+8.090}$$

Required gain at P

$$|D(s)\ G(s)|_{at\ P} = 1$$

$$\left| \frac{s+Z}{s+P} \cdot \frac{K}{s(s+1)} \right|_{s=-2.8+j2.8} = 1$$

$$\left| \frac{-2.8+j2.8+1.977\quad K}{(s+8.09)\,(s\,(s+1))} \right| = 1$$

$$K(-0.823+j2.8) = s(s+1)(s+8.09)$$

$$K = -27+0.41j.$$

$$K = 27.01$$

$\frac{73.89}{2.91}$

Gain requirement based on kv

$$k_v = \lim_{s\to 0} s\ G(s) = 2 \quad [\text{Given}]$$

Additional gain can be adjusted using amplifier.

$25 = 2 \times$ amplifier gain

Amplifier gain $= 12.5$

$$R(s) \xrightarrow{+} (\Sigma) \rightarrow [\text{Amplifier}] \rightarrow [D(s)] \rightarrow [G(s)] \rightarrow C(s)$$

Calculate actual kv:

$$k_v = \lim_{s\to 0} \text{Amplifier gain} \times D(s) \times G(s)$$

$$= \lim_{s\to 0} s \cdot 12.5 \times \frac{s+Z}{s+P} \times \frac{2}{s(s+1)} = 6.109$$

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1. $G(S) = \frac{K}{S(S+1)(S+4)}$ $\xi = 0.5$ $\omega_n = 2$ rad/sec

$K_v > 1.5$

→ Desired poles (Sd)

$Sd = -\xi\omega_n \pm j\omega_d$

System phase angle at P

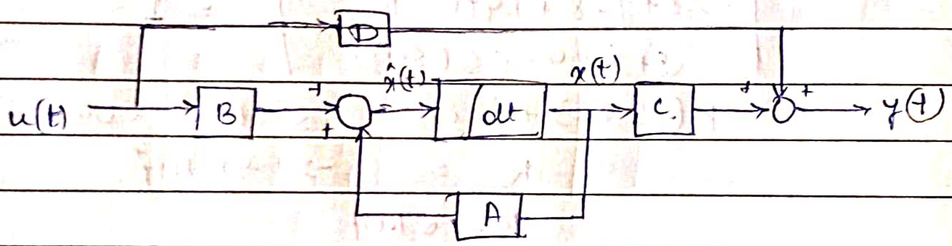
$\theta_1 = 90 + \phi_1$

$\theta_2 =$

4/12/23

State space analysis:-

The state of a system which is defined as minimum no. of interconnections that must be specified at any initial time to so that the complete dynamic behaviour of a system at any time $t > t_0$ is determined when the i/p $u(t)$ is known



1. $\dot{x}(t) = A\,x(t) + B\,u(t) \rightarrow ①$
2. $y(t) = C\,x(t) + D\,u(t) \rightarrow ②$

$x(t)$ State vector $= \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n \times 1}$ $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}_{p \times 1}$ $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}_{r \times 1}$

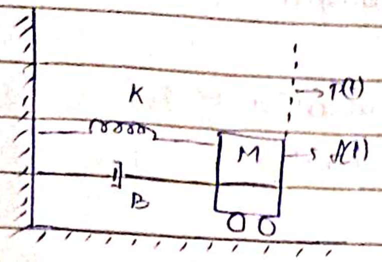
A → state mat $n \times n$

B → i/p mat $n \times r$

C → o/p mat $p \times n$

D → Direct transmission mat $p \times r$

The system is described by the 2nd order DE & consists of 2 state variables



$$M \frac{d^2 p(t)}{dt^2} + B \frac{dp(t)}{dt} + K p(t) = f(t) \rightarrow ①$$

Let $x_1(t) = p(t)$

$$\dot{x}_1(t) = \frac{dp(t)}{dt} \longrightarrow ②$$

Let $x_2(t) = \frac{dp(t)}{dt} = \dot{x}_1(t)$

from eq^n ① $\dot{x}_2(t) = \frac{d^2 p(t)}{dt^2}$

$$M \dot{x}_2(t) + B x_2(t) + K x_1(t) = u(t)$$

$$\Rightarrow \cancel{\dot{x}_2(t) + \frac{B}{M} x_2(t) + \frac{K}{M} x_1(t)}$$

$$\Rightarrow \dot{x}_2(t) = -\frac{K}{M} x_1(t) - \frac{B}{M} x_2(t) + \frac{1}{M} u(t) \longrightarrow ③$$

In matrix form eq^n ② & eq^n ③

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix} u(t)$$

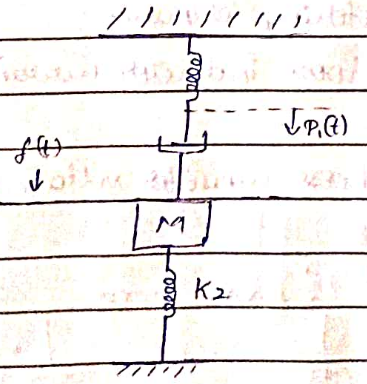
$$\dot{x}(t) = A x(t) + B x(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ \frac{-K}{M} & \frac{B}{M} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = C x(t)$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Problem:-



$$M\ddot{P}_2 + B(\dot{P}_2 - \dot{P}_1) + K_2 P_2 = f(t) \longrightarrow ①$$

$$B(\dot{P}_1 - \dot{P}_2) + K_1 P_1 = 0 \longrightarrow ②$$

mode = 'min'

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2^{nd} order DE $\Rightarrow$ Two State variables

$x_1 = P_2$

$\dot{x}_1 = \dot{P}_2 = x_2 \longrightarrow (3)$ $\boxed{2^{nd} \text{ derivative of } P_2 \text{ exist}}$

$\dot{x}_2 = \ddot{P}_2$

$x_3 = P_1$

$M\dot{x}_2 + K_1 x_3 + K_2 x_1 = u(t)$

$\dot{x}_2 = -\frac{K_2}{M} x_1 - \frac{K_1}{M} x_3 + \frac{1}{M} u(t) \longrightarrow (4)$

$B\dot{x}_3 - Bx_2 + K_1 x_3 = 0$

$\dot{x}_3 = x_2 - \frac{K_1}{B} x_3 \longrightarrow (5)$

In matrix form.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{K_2}{M} & 0 & -\frac{K_1}{M} \\ 0 & 1 & -\frac{K_1}{B} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \\ 0 \end{bmatrix} u(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -K_2/M & 0 & -\frac{K_1}{M} \\ 0 & 1 & -\frac{K_1}{B} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/M \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

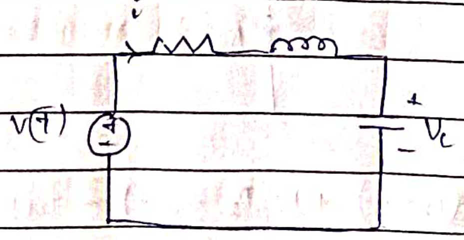
State equations for electrical circuits:

L & C are energy storing elements

Inductor: $\frac{1}{2} LI^2$: choose inductor current as state variable

Capacitor: $\frac{1}{2} CV^2$: choose capacitor voltage as state variable

$L\frac{di}{dt} + Ri + V_c$



x_1 = ind current

x_2 = cap vtg

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$L\frac{di}{dt} + Ri + V_c = V(t)$ from KVL

$L\dot{x}_1 + Rx_1 + x_2 = u(t)$

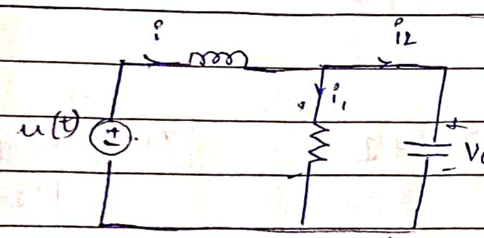
$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u(t) \longrightarrow ①$

$x_2 = \frac{1}{C}\int i\,dt = \frac{1}{C}\int x_1\,dt$

$\dot{x}_2 = \frac{1}{C}x_1 \longrightarrow ②$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-R/L & -\frac{1}{L}\\ 1/C & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1/L\\ 0\end{bmatrix}u(t)$$

Q.



$x_1 = i$

$x_2 = V_c$

$L\frac{di}{dt} + V_c = u(t) \rightarrow L\dot{x}_1 + x_2 = u(t)$

$\dot{x}_1 = -\frac{1}{L}x_2 + \frac{1}{L}u(t) \longrightarrow ①$

$x_2 = \frac{1}{C}\int i_2\,dt$

$= \frac{1}{C}\int i - i_1\,dt$

$= \frac{1}{C}\int x_1 - \frac{x_2}{R}$

$i_1 = \frac{V_C}{R} \rightarrow x_2$

$\dot{x}_2 = \frac{1}{C}(x_1 - x_2/R) \rightarrow ②$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & -1/L\\ 1/C & -1/RC\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix}1/L\\ 0\end{bmatrix}}_{B}u(t)$$

Transfer function from state equation:

$\dot{x}(t) = Ax(t) + Bu(t) \rightarrow ①$

$y(t) = Cx(t) + Du(t) \longrightarrow ②$

$\dot{x}(t) = \frac{dx(t)}{dt}$

Taking Laplace transform on both sides of eqn ① we have

$s\,X(s) - x(0) = A\,X(s) + B\,U(s)$

To obtain tr. fn, initial conditions = 0.

$sX(s) = A\,X(s) + B\,U(s)$

$X(s)[sI - A] = BU(s)$ $\quad I =$ identity matrix

$X(s) = \dfrac{B\,U(s)}{sI - A} \rightarrow X(s) = [sI-A]^{-1} B\,U(s) \rightarrow$ ③

Taking LT for eqn ②.

$Y(s) = \cancel{CB}\ C\,X(s) + D\,U(s)$

$Y(s) = C[sI-A]^{-1} B\,U(s) + D\,U(s)$

$Y(s) = (C[sI-A]^{-1}B + D)\,U s$

$\therefore$ Transfer function :-

$\dfrac{Y(s)}{U(s)} = C[sI-A]^{-1}B + D$ $\qquad [sI-A]^{-1} = \dfrac{1}{|sI-A|}\,\text{adj}\,[sI-A]$

Consider the following state equations!

1. $\dot{x} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$

$\rightarrow$ $A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad D = 0$

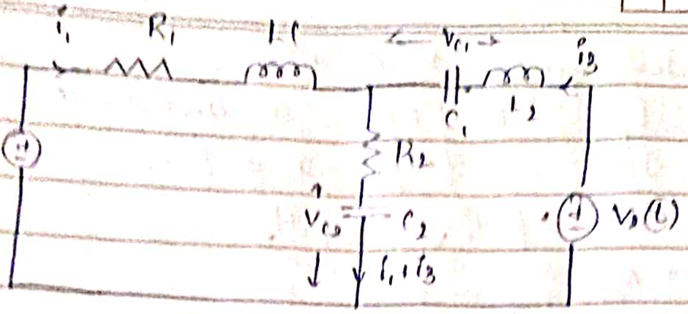
$\dfrac{Y(s)}{U(s)} = C[sI-A]^{-1}B$

$[sI - A] = \begin{bmatrix} s+3 & 1 \\ +2 & s \end{bmatrix} \rightarrow \text{adj} = \begin{bmatrix} s & -1 \\ -2 & s+3 \end{bmatrix}$ $\qquad s^2+3s+2$

$\dfrac{Y(s)}{U(s)} = \begin{bmatrix} 0 & 1 \end{bmatrix} \dfrac{1}{s^2+3s+2} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \dfrac{1}{s^2+3s+2} \quad -2 \quad \cancel{s+3}$

$= \dfrac{-2}{s^2+3s+2} = \dfrac{-2}{(s+1)(s+2)}$



$x_1 = i_1$ $\qquad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$

$x_2 = i_2$

$x_3 = V_{C_1}$

$x_4 = V_{C_2}$

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Solution of state eqns:

Solving for $x_1(t)$, $x_2(t)$ and $y(t)$

State variable ... o/p

Methods
- LT method
- Time domain method

$\dot{x}(t) = Ax(t) + Bu(t)$

Taking LT on bs.

$sX(s) - x(0) = AX(s) + BU(s)$

$sX(s) - AX(s) = x(0) + BU(s)$

$X(s)(sI - A) = x(0) + BU(s)$

Multiplying bs by $[sI-A]^{-1}$

$X(s) = [sI-A]^{-1}x(0) + [sI-A]^{-1}BU(s)$

Taking inverse laplace transform on both sides, we have

$x(t) = L^{-1}\{[sI-A]^{-1}x(0)\} + L^{-1}\{[sI-A]^{-1}BU(s)\}$

→ mat in TD → mat in TD

$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \end{bmatrix}$

initial condion → Free response, natural response, homogeneous response

$x_n(t)$

i/p $u(t)$ → ~~forcing fn~~ → Forced response

$x_f(t)$

$$\boxed{\begin{array}{l} x(t) = x_n(t) + x_f(t) \\ \downarrow \\ \text{~~Total response~~, Complete soln} \\ x_n(t) = L^{-1}\{[sI-A]^{-1}x(0)\} \\ x_f(t) = L^{-1}\{[sI-A]^{-1}BU(s)\} \\ y(t) = Cx(t) + Du(t) \end{array}}$$

$\phi = [sI-A]^{-1}$ → State transition matrix. (FD)

$L^{-1}\{\phi(s)\} = \phi(t)$ — (TD)

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1. A system is described by $\dot{x} = Ax + Bu$

$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ u unit step i/p

$x_1(0) = 1 \quad x_2(0) = -1$

a. STM b. $x(t)$, $c = [0\ 1]$ c. $y(t)$

→ $\phi(s) = [sI - A]^{-1}$

$= \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix} = \frac{1}{|sI - A|} \operatorname{adj}(sI - A)$

$= \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} \frac{1}{s^2 + 3s + 2}$

b. $x_n(t) = L^{-1}\{\phi(s) \cdot x(0)\}$

$= L^{-1}\left\{ \frac{1}{s^2+3s+2} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

$= L^{-1}\left\{ \frac{1}{(s+1)(s+2)} \begin{bmatrix} s-1 \\ -2-s-3 \end{bmatrix} \right\}$

$x_n(t) = L^{-1}\left\{ \begin{bmatrix} \frac{s-1}{(s+1)(s+2)} \\ \frac{-(s+5)}{(s+1)(s+2)} \end{bmatrix} \right\}$

(i) $\frac{s-1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$

$s - 1 = A(s+2) + B(s+1)$

$A + B = 1$

$2A + B = -1$

$-A = 2 \rightarrow \boxed{A = -2} \quad \boxed{B = 3}$

(ii) $\frac{-(s+5)}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$

$-(s+5) = A(s+2) + B(s+1)$

$A + B = -1$

$2A + B = -5 \quad \boxed{B = 3}$

$\boxed{A = -4}$

$x_n(t) = L^{-1}\left\{ \begin{bmatrix} \frac{-2}{s+1} + \frac{3}{s+2} \\ \frac{-4}{s+1} + \frac{3}{s+2} \end{bmatrix} \right\} = \begin{bmatrix} -2e^{-t} + 3e^{-2t} \\ -4e^{-t} + 3e^{-2t} \end{bmatrix}$

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$$x_f(t) = L^{-1}\{\phi(s)BU(s)\}$$

$$= L^{-1}\left\{\frac{1}{(s+1)(s+2)}\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}\right\}.$$

$$= L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\begin{bmatrix} s \\ -2 \end{bmatrix}\right\}$$

$$= L^{-1}\left\{\begin{bmatrix} \frac{1}{(s+1)(s+2)} \\ \frac{-2}{s(s+1)(s+2)} \end{bmatrix}\right\}$$

(i) $$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$1 = A(s+2) + B(s+1)$$

$$A+B=0$$

$$2A+B=1$$

$$\boxed{A=1} \qquad \boxed{B=-1}$$

(ii) $$\frac{-2}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$-2 = A(s+1)(s+2) + B(s)(s+2) + C(s)(s+1)$$

Put $s=0$ $\quad -2 = 2A \Rightarrow \boxed{A=-1}$

Put $s=-1$ $\quad -2 = -B \Rightarrow \boxed{B=2}$

Put $s=-2$ $\quad -2 = 2C \Rightarrow \boxed{C=-1}$

$$\rightarrow x_f(t) = L^{-1}\left\{\begin{bmatrix} \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-1}{s} + \frac{2}{s+1} - \frac{1}{s+2} \end{bmatrix}\right\}$$

$$x_f(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -1 + 2e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = x_n(t) + x_f(t)$$

$$= \begin{bmatrix} -2e^{-t} + 3e^{-2t} \\ -4e^{-t} + 3e^{-2t} \end{bmatrix} + \begin{bmatrix} e^{-t} - e^{-2t} \\ -1 + 2e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} -e^{-t} + 2e^{-2t} \\ -1 - 2e^{-t} + 2e^{-2t} \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad \begin{aligned} x_1(t) &= -e^{-t} + 2e^{-2t} \\ x_2(t) &= -1 - 2e^{-t} + 2e^{-2t} \end{aligned}$$

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$\therefore y(t) = C x(t) + \overset{0}{D} u(t)$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} + 2e^{-2t} \\ -1 \quad 2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$\rightarrow y(t) = -1 - 2e^{-t} + 2e^{-2t}$

HW

2 Determine the response for a system described by $\dot{x}_1 = -x_1$, $\dot{x}_2 = x_1 - x_2 + u(t)$, $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ i/p is unit step. Also find $y(t)$ $C = [-1 \quad 1]$

$\rightarrow$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t)$$

12/12/23 Cannonical forms:-

Minimal components for the realization of transfer function

controllable cannonical form | observable cannonical form

Controllability }
Observability } control system

- If a system is fully controllable it can be realized in controllable cannonical form.
- If a system is fully observable it can be realized in observable cannonical form

* Controllable cannonical form:

$$\frac{Y(s)}{U(s)} = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{1.s^3 + a_2 s^2 + a_1 s + a_0} \qquad N^r \text{ degree} = D^r \text{ degree}$$

$$\frac{Y(s)}{X_1(s)} \frac{X_1(s)}{U(s)} = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{1} \cdot \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\frac{Y(s)}{X_1(s)} = b_3 s^3 + b_2 s^2 + b_1 s + b_0 \rightarrow (1)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0} \rightarrow (2)$$

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$Y(s) = X_1(s)\,[b_3 s^3 + b_2 s^2 + b_1 s + b_0]$

$Y(s) = b_3 s^3 X_1(s) + b_2 s^2 X_1(s) + b_0 X_1(s)$

$y = b_3 \dddot{x}_1 + b_2 \ddot{x}_1 + b_1 \dot{x}_1 + b_0 x_1$ $x_1 \rightarrow$ state variable

Order of the system = 3

$\rightarrow$ Three state variables

x_1

$x_2 = \dot{x}_1$

$x_3 = \dot{x}_2 = \ddot{x}_1$

$\dot{x}_3 = \dddot{x}_1$

Substituting these in previous eqn, we have

Time	Freq
$x(t)$	$X(s)$
$y(t)$	$Y(s)$
$\frac{d}{dt}$ $\rightarrow$	s
$\frac{d^2}{dt^2}$ $\rightarrow$	s^2
$\frac{dx}{dt} = \dot{x}$;	$\frac{d^2x}{dt^2} = \ddot{x}$

$y = b_3 \dot{x}_3 + b_2 x_3 + b_1 x_2 + b_0 x_1 \rightarrow (3) \rightarrow$ o/p eqn

Consider eqn (2). Crossmultiply, $U(s) = X_1(s)\,[s^3 + a_2 s^2 + a_1 s + a_0]$

$U(s) = s^3 X_1(s) + a_2 s^2 X_1(s) + a_1 s X_1(s) + a_0 X_1(s)$

$u = \dot{x}_3 + a_2 x_3 + a_1 x_2 + a_0 x_1$

$\dot{x}_3 = u - a_2 x_3 - a_1 x_2 - a_0 x_1 \rightarrow (4)$

Substituting (4) in eqn (3)

$y = b_3(u - a_2 x_3 - a_1 x_2 - a_0 x_1) + b_2 x_3 + b_1 x_2 + b_0 x_1$

$y = (b_2 - a_2 b_3) x_3 + (b_1 - a_1 b_3) x_2 + (b_0 - a_0 b_3) x_1 + b_3 u$

$y = \bar{b}_0 x_1 + \bar{b}_1 x_2 + \bar{b}_2 x_3 + b_3 u \rightarrow (5)$

$\boxed{\begin{aligned}\bar{b}_0 &= b_0 - a_0 b_3\\ \bar{b}_1 &= b_1 - a_1 b_3\\ \bar{b}_2 &= b_2 - a_2 b_3\end{aligned}}$

$\dot{x}_1 = x_2 \rightarrow (6)$

$\dot{x}_2 = x_3 \rightarrow (7)$

$\dot{x}_3 = -a_0 x_1 - a_1 x_2 - a_2 x_3 + u \rightarrow (8)$

Let us represent state eqns in matrix form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_0 & -a_1 & -a_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} u$$

$$y = \begin{bmatrix}\bar{b}_0 & \bar{b}_1 & \bar{b}_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + b_3 u$$

Realization:

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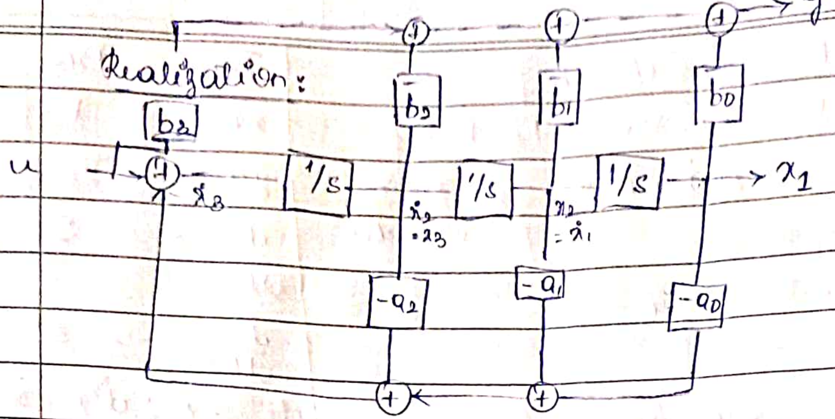
Realization:



Example: $\frac{Y(s)}{U(s)} = \frac{s^3+12s^2+44s+48}{s^3+9s^2+23s+15} = \frac{b_3s^3+b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}$

$a_3=1;\ a_2=9;\ a_1=23;\ a_0=15$

$b_3=1;\ b_2=12;\ b_1=44;\ b_0=48$

$\cancel{\frac{X(s)}{X_1(s)} = s^3+12s^2+44s+48}$

$\cancel{Y(s) = s^3X_1(s)+12s^2X_1(s)+44sX_1(s)+48X_1(s)}$

$\cancel{y = x_3}$

$\bar{b}_0 = b_0 - a_0 b_3 = 33$

$\bar{b}_1 = b_1 - a_1 b_3 = 21$

$\bar{b}_2 = b_2 - a_2 b_3 = 3$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -15 & -23 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}33 & 21 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + u$$

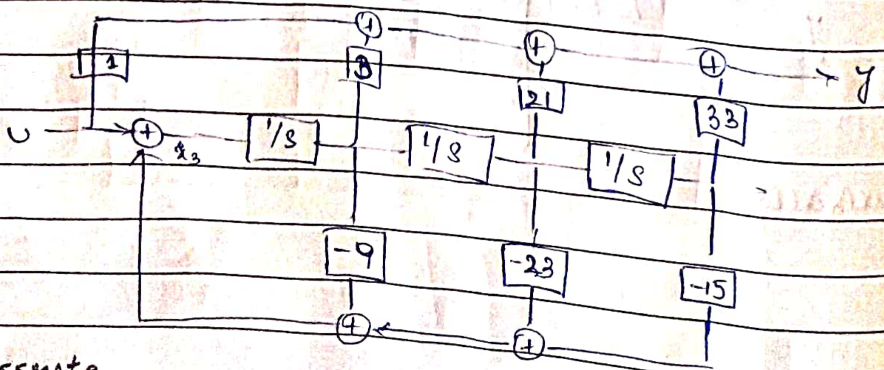
State eqns.

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = \cancel{(a_0x)} - 15x_1 - 23x_2 - 9x_3 + u$

$y = 33x_1 + 21x_2 + 3x_3 + u$



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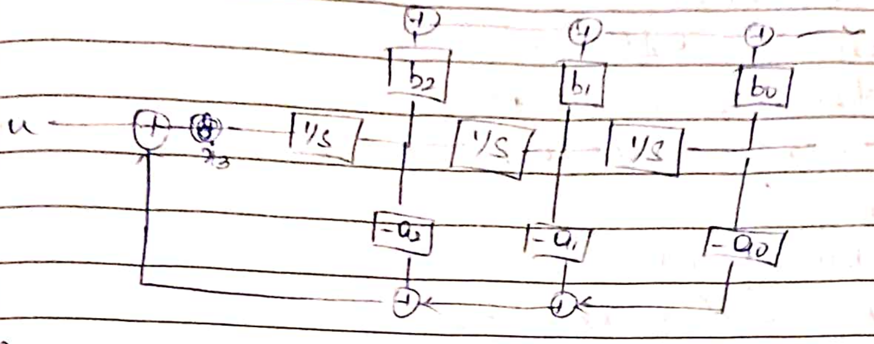
$$Y(s) = \frac{12s^2+44s+48}{s^3+9s^2+23s+15} \qquad b_3 = 0 \Rightarrow \begin{array}{l} \bar{b}_0 = b_0 \\ \bar{b}_1 = b_1 \\ \bar{b}_2 = b_2 \end{array}$$



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Observable cannonical form:-

$$\frac{Y(s)}{U(s)} = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{1 \cdot s^3 + a_2 s^2 + a_1 s + a_0}$$

Cross multiply:-

$$Y(s)\left[s^3 + a_2 s^2 + a_1 s + a_0\right] = U(s)\left[b_3 s^3 + b_2 s^2 + b_1 s + b_0\right]$$

$$s^3 Y + a_2 s^2 Y + a_1 s Y + a_0 Y = b_3 s^3 U + b_2 s^2 U + b_1 s U + b_0 U$$

associated with Y

Keep highest power of s, on LHS

$$s^3 Y = b_3 s^3 U + b_2 s^2 U + b_1 s U + b_0 U - a_2 s^2 Y - a_1 s Y - a_0 Y$$

$$s^3 Y = b_3 s^3 U + \left[b_2 U - a_2 Y\right] s^2 + \left[b_1 U - a_1 Y\right] s + \left[b_0 U - a_0 Y\right]$$

Dividing throughout by s^3 we have

$$Y = b_3 U + \left[b_2 U - a_2 Y\right] s^{-1} + \left[b_1 U - a_1 Y\right] s^{-2} + \left[b_0 U - a_0 Y\right] s^{-3}$$

Let $Y = b_3 U + X_1 \rightarrow (1) \Rightarrow$ ILT $\rightarrow \boxed{y = b_3 u + x_1}$ (*)

$$X_1 = \left[b_2 U - a_2 Y\right] s^{-1} + \left[b_1 U - a_1 Y\right] s^{-2} + \left[b_0 U - a_0 Y\right] s^{-3} \qquad \times \text{ by } s.$$

$$sX_1 = \left[b_2 U - a_2 Y\right] + s^{-1}\left[b_1 U - a_1 Y\right] + s^{-2}\left[b_0 U - a_0 Y\right]$$

$$sX_1 = b_2 U - a_2 Y + X_2 \rightarrow \text{ILT} \rightarrow \boxed{\dot{x}_1 = b_2 u - a_2 y + x_2} \quad (*)$$

where

$$X_2 = s^{-1}\left[b_1 U - a_1 Y\right] + s^{-2}\left[b_0 U - a_0 Y\right] \qquad \times \text{ by } s.$$

$$sX_2 = b_1 U - a_1 Y + s^{-1}\left[b_0 U - a_0 Y\right]$$

$$sX_2 = b_1 U - a_1 Y + X_3 \rightarrow \boxed{\dot{x}_2 = b_1 u - a_1 y + x_3} \quad (*)$$

where

$$X_3 = s^{-1}\left[b_0 U - a_0 Y\right] \qquad \times \text{ by } s.$$

$$sX_3 = b_0 U - a_0 Y \Rightarrow \boxed{\dot{x}_3 = b_0 u - a_0 y} \quad (*)$$

what was ... pointed giving letter to HOD?

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Consider,

$\dot{x}_1 = b_2 u - a_2 y + x_2$

Let us substitute for y : $y = b_3 u + x_1$

$\dot{x}_1 = b_2 u + x_2 - a_2(b_3 u + x_1)$

$= b_2 u + x_2 - a_2 b_3 u - a_2 x_1$

$\dot{x}_1 = -a_2 x_1 + x_2 + (b_2 - a_2 b_3) u$

Now consider,

$\dot{x}_2 = b_1 u - a_1 y + x_3$

$\dot{x}_2 = b_1 u - a_1 (b_3 u + x_1) + x_3$

$= b_1 u - a_1 b_3 u - a_1 x_1 + x_3$

$= -a_1 x_1 + x_3 + (b_1 - a_1 b_3) u$

$\dot{x}_3 = b_0 u - a_0 y$

$= b_0 u - a_0 (b_3 u + x_1)$

$= b_0 u - a_0 b_3 u - a_0 x_1$

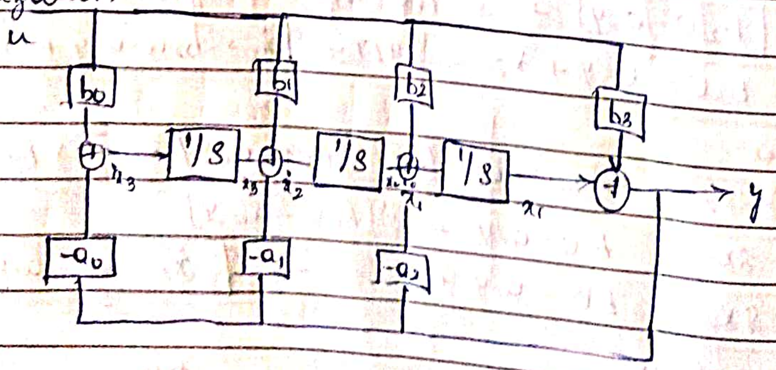
$= -a_0 x_1 + (b_0 - a_0 b_3) u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_2 & 1 & 0 \\ -a_1 & 0 & 1 \\ -a_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_2 - a_2 b_3 \\ b_1 - a_1 b_3 \\ b_0 - a_0 b_3 \end{bmatrix} u$$

$y = x_1 + b_3 u$

$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_3 u$

Realization



1. Repeat the procedure by taking $b_3 = 0 \Rightarrow \bar{b}_2 = b_2$; $\bar{b}_1 = b_1$; $\bar{b}_0 = b_0$; $D = 0$

2. Given that $\frac{Y(s)}{U(s)} = \frac{s^3 + 14s^2 + 50s + 55}{s^3 + 12s^2 + 30s + 15}$

$\rightarrow$ $b_3 = 1$ $\quad b_2 = 14$ $\quad b_1 = 50$ $\quad b_0 = 55$

$a_3 = 1$ $\quad a_2 = 12$ $\quad a_1 = 30$ $\quad a_0 = 15$

$\rightarrow$ $\bar{b}_2 = 2$

$\bar{b}_1 = 20$

$\bar{b}_0 = 40$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -12 & 1 & 0 \\ -30 & 0 & 1 \\ -15 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 20 \\ 40 \end{bmatrix}$$

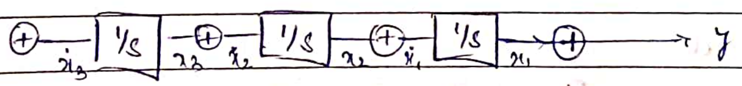
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u.$$

$\dot{x}_1 = -12x_1 + x_2 + 2u$

$\dot{x}_2 = -30x_1 + x_3 + 20u$

$\dot{x}_3 = -15x_1 + 40u \Rightarrow \dot{x}_3 = -15(y-u) + 40u = -15y + 55u$

$y = x_1 + u \Rightarrow x_1 = y - u$

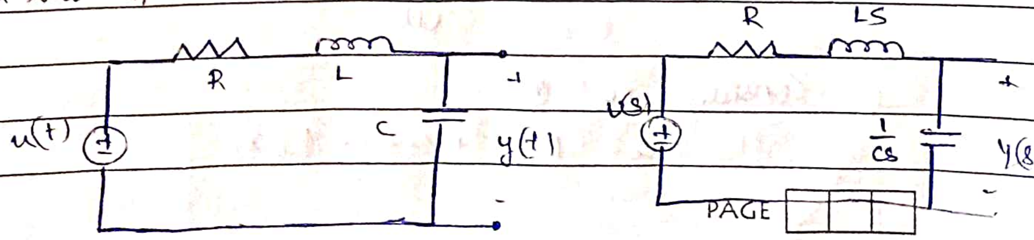


3. $\frac{Y(s)}{U(s)} = \frac{14s^2 + 50s + 55}{s^3 + 12s^2 + 30s + 15}$

4. $\frac{Y(s)}{U(s)} = \frac{s^2 + 6}{s^3 + 9s^2 + 23s + 15}$ $\quad b_3 = 0$

$b_1 = 0$

5. For the ckt shown, obtain tf $Y(s)/U(s)$



classmate

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$$V(s) = \frac{U(s)\, Z_2}{Z_1 + Z_2}$$

$$= \frac{1/Cs}{R + LS + \frac{1}{Cs}} = \frac{1/C}{RS + LS^2 + 1/C} = \frac{1/LC}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$

$$\frac{V(s)}{U(s)} = \frac{b_2 S^2 + b_1 S + b_0}{S^2 + a_1 S + a_0}$$

$b_2 = 0 \quad b_1 = 0 \quad b_0 = 1/LC$

$a_2 = 1 \quad a_1 = R/L \quad a_0 = 1/LC$

Realize in both cannonical forms

15/12/23

State model of Electromechanical system

DC motor

Vtg $\rightarrow$ | DCM | $\rightarrow$ speed

$$\frac{d\theta}{dt} = \omega$$

Obtain the state model of DC motor taking armature current motor speed as state variables.

Armature ckt:-

$$e_a = L\frac{di_a}{dt} + i_a R + e_b \rightarrow \text{back emf} \rightarrow ①$$

$$e_b = K_b \frac{d\theta}{dt} = K_b \omega \rightarrow ②$$

o/p side:

$$T_a = K_t i_a \rightarrow ③$$

$$T_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt}$$

$$T_a = J\frac{d\omega}{dt} + B\omega \rightarrow ④$$

$x_1 = \omega \Rightarrow \dot{x}_1 = \frac{d\omega}{dt}$

$x_2 = i_a \Rightarrow \dot{x}_2 = \frac{di_a}{dt}$

Consider eq^n ①

$$e_a = L\dot{x}_2 + x_2 R + K_b x_1$$

$$\boxed{\dot{x}_2 = \frac{e_a}{L} - \frac{K_b x_1}{L} - \frac{R x_2}{L}}$$

Consider eqn (1):- $T_a = J\dot{x}_1 + B x_1$

$$K_t x_2 = J\dot{x}_1 + B x_1$$

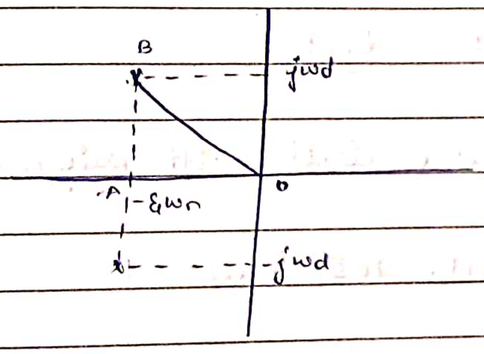
$$\boxed{\dot{x}_1 = \frac{K_t x_2}{J} - \frac{B x_1}{J}}$$

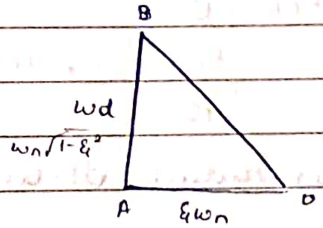
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -B/J & \frac{K_t}{J} \\ \frac{-K_b}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} e_a$$

Design of proportional controller using root-locus method:-

1. $G(s) = \frac{1}{(s+3)(s+4)(s+5)}$. Design a a P-controller so that

$\xi = 0.5$ $\omega_n = 3.5$. Use Root-locus method





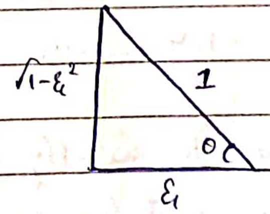
$OB = \sqrt{\omega_d^2 + (\xi\omega_n)^2}$

$OB = \sqrt{\omega_d^2 + \xi^2\omega_n^2}$

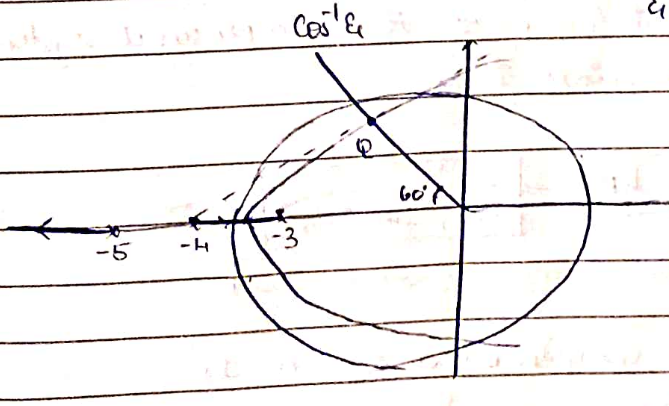
$\omega_n^2(1-\xi^2) + \xi^2\omega_n^2$

$OB = \sqrt{\omega_n^2} = \omega_n$

$\Downarrow$



$\cos\theta = \frac{\xi}{1} \rightarrow \theta = \cos^{-1}\xi$



no. asymptotes = 3

angle = 60, 180, 300

$\sigma = \frac{-3-4-5-0}{3} = -4$

BAP:

$1 + KG(s) = 0$

$K = -(s+3)(s+4)(s+5) = -(s^2+4s+3s+12)(s+5)$

$= -(s^3+7s^2+12s+5s^2+35s+60)$

$= -(s^3+12s^2+47s+60)$

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$$\frac{dK}{ds} = 0 = 3s^2 + 24s + 47$$

$$= -3.42, \; -4.57$$

Intersection of root locus with img axis

$1 + KH(s) = 0.$

$$1 + \frac{K}{(s+3)(s+4)(s+5)} = 0$$

$$(s+3)(s+4)(s+5) + K = 0$$

$$s^3 + 12s^2 + 47s + 60 + K = 0$$

s^3	1	47	$\rightarrow$	$60+K > 0$
s^2	12	$60+K$		$\boxed{K > -60}$
s	$\frac{504-K}{12}$	0		$504 - K > 0$
s^0	$60+K$			$\boxed{K < 504}$

$$0 < K < 504$$

$$12s^2 + 564 = 0$$

$$s^2 = \frac{-564}{12} = \sqrt{-47} = 6.85j$$

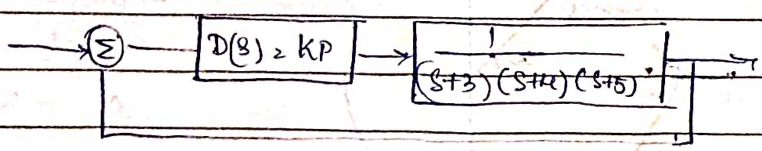
with 3.5 as radius, draw a circle with origin as centre

The point where circle cuts root locus $\rightarrow$ P

$$P = -2.5 + j2.25$$

$$Q = -1.9 + j3.4 \quad \longrightarrow \quad \cos^{-1}\xi$$

For the system to satisfy the requirements, $\xi = 0.5$ & $\omega_n = 3.5$, the root locus of the compensated system shd pass through the points P & Q



The OL tr fn of compensated system is

$$L_1(s) = DG = \frac{K_P}{(s+3)(s+4)(s+5)}$$

For point P to be on root locus, $\left|\frac{K_P}{(s+3)(s+4)(s+5)}\right|_{s=-2.5+j2.25} = 1 \Rightarrow K_{P_1} = 21$

For point Q to be on root locus

$$\left| \frac{K_P}{(s+3)(s+4)(s+5)} \right|_{s=-1.4+j3.4} = 1 \rightarrow K_{P2} = 65.7$$

$\therefore$ we can use the controller gain in the range $K_{P1} < K_P < K_{P2}$

$21 < K_P < 65.7$