Flash Card

Definitions

`Rise time (` t_r`) :` It is the time taken by the step response to rise from 10% to 90% of final value.

$$t_r = \frac{1.8}{\omega_n}$$

`Peak Overshoot (` M_p`) :` Largest deviation of step response above the step input.

$$M_p = e^{\frac{-\pi\epsilon}{\sqrt{1-\epsilon^2}}}$$

`Peak time (` t_p `):` The time at which peak overshoot occurs.

$$t_p = \frac{\pi}{\omega_d}$$

`Settling time (` t_s `):` The time taken by the step response to rise and stay at 1% tolerance.

$$t_s = \frac{4.6}{\epsilon\,\omega_n}$$

`Poles of a second order system:`

$$S = -\sigma \ \pm \ j\ \omega_d$$

$$\sigma = \epsilon\,\omega_n$$
$$\omega_d = \omega_n\sqrt{1-\epsilon^2}$$

Standard form of sytstems

`1st order:`

$$\frac{Y(S)}{R(S)} = \frac{1}{S+\sigma}$$

`2nd Order:`

$$\frac{Y(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\epsilon\omega\ S\ +\ 1}$$

`Derivation:` For the derivation of the above parameters:

- $y(t) = 1 - e^{-\sigma\ t} cos\omega_d\ t\ -\ \frac{\sigma}{\omega_d} sin\omega_d t$
- $a\ cos\alpha\ + b\ sin\alpha = \sqrt{a^b+b^2}\ cos(\alpha-\beta)\ or\ \frac{1}{\sqrt{a^b+b^2}}\ sin(\alpha+\beta)$
  $\beta = tan^-1\ (\frac{b}{a})$
- t_p :
  - $\frac{d\ y(t)}{dt} = 0$
  - Equate $\omega_d\ t_p = m\pi$
- `% Overshoot:`
  - $M_p = \frac{y_{max}-y_{ss}}{y_{ss}}\ ;\ y_{ss} = 1\ and\ y_{max} =\ y(t_p)$
- t_s :
  - $e^{-\sigma\ t}$ is called damping coefficient. Equate this to the requirement.
  - Consider damping ratio to be negligible
  - Consider damping frequency to be equal to 1

`Note:` $\sigma = damping\ ratio$
$\omega_d =\ damping\ frequency$

Root Locus

- Find the no. of poles and zeros.
- Find the region where the root locus exists.
- Find the centroid $\sigma = \frac{\sum Re(poles)\ -\sum Re(zeroes)}{P\ -Z}$
- Find the angle of asymptotes $\theta_q = \frac{(2q+1)180}{P-Z}$

- Find the break-away or break-in point/s using CE: $1 + kL(s) = 0$ and $\frac{dK}{dS} = 0$, for **TWO ADJACENT POLES OR ZEROS**
- Find the intersection with imaginary axis using RH criteria using the CE equation.
- Find the angle of departure = $180 - \theta_p + \theta_z$ or the angle of arrival = $180 - \theta_z + \theta_p$ in the case of **COMPLEX ZEROS**.

Types of systems and its stability:

`Type 0:` No poles at s = 0

`Type 1:` 1 pole at s = 0

`Type 2:` 2 poles at s = 0

D → Controller gain

G → Plant gain

H → Feedback gain

$$K_p = \lim_{s \to 0} DGH \; ; \; Step : \; e_{ss} = \frac{1}{1 + K_p}$$

$$K_v = \lim_{s \to 0} SDGH ; \; Ramp : \; e_{ss} = \frac{1}{K_v}$$

$$K_u = \lim_{s \to 0} S^2 DGH ; \; Parabola : \; e_{ss} = \frac{1}{K_u}$$

	`Step`	`Ramp`	`Parabola`
`Type 0`	$\frac{1}{1+K_p}$	∞	∞
`Type 1`	0	$\frac{1}{K_v}$	∞
`Type 2`	0	0	$\frac{1}{K_u}$

Design of Controllers

`Controller Signal:` It is obtained by multiplying the error signal with a constant K_p

`Closed Loop Transfer Function:`

$$T(s) = \frac{GD}{1 + GDH}$$

`Open Loop Transfer Function:`

$$T(s) = GDH$$

1. `Proportional Controller:`
    - $D = K_P$
    - Find the characteristic equation $CE : 1 + DGH = 0$
    - Compare with standard second order equation $s^2 + 2\,\epsilon\,\omega_n\,s + \,\omega_n^2 = 0$
    - Find the required values.
    - Find the error constant e_{ss}

2. `Proportional Integral Controller:`
    - $D = K_P + \frac{K_I}{s}$
    - Follow the procedure given above, but just change the value of D.

3. `Proportional Integrator Differentiator Controller:`
    - $D = K_P + \frac{K_I}{s} + s\,K_U$
    - Follow the procedure given above, but just change the value of D.

`NOTE:` If the order of the system is ≥ 2 then use R-H criteria to find the value of K, using the **characteristic equation**.

`Design of` **`Lead Compensator`** `using root locus:`

- angle of poles - angle of zeros = 180

- Calculate the angle of poles and zeros using the requirements given in the question
- $\theta = cos^{-1}(\epsilon)$
- $r = \frac{1}{2}[180 \ - \ \theta \ - \phi]$
- $Z = \frac{\omega_n sinr}{sin(\theta + \phi)}$
- $P = \frac{\omega_n sin(r + \phi)}{sin(r + \theta + \phi)}$
- $D(s) = \frac{s + z}{s + p}$

`State Space Analysis:` The state of a system which is defined as minimum no. of interconnection that must be specified at any initial time t_c so that the complete dynamic behavior of a system at any time $t > t_o$ is determined when the i/p u(t) is known

$$\frac{y(s)}{u(s)} = C\ [SI - A]^{-1}\ B\ + D$$

$$\phi(s) = [SI - A]^{-1}$$

$$x_n(t) = L^{-1}[\phi(s)X(0)]$$

$$x_f(t) = L^{-1}[\phi(s)BU(s)]$$

$$x(t) = x_n(t) + x_f(t)$$

Canonical Forms

`Controllable Form:`

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [U]$$

$$y = \begin{bmatrix} b_0' & b_1' & b_2' \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_3 U$$

- To `check` if a system is controllable:
  - Make sure the matrices are in **controller canonical form.**

$$Q = \begin{bmatrix} B & AB & .... & A^{n-1}B \end{bmatrix}$$

$$|Q| \neq 0, then\ the\ sysytem\ is\ controllable$$

- To `design` a controller using pole-placement technique:
  - Compare the below two equations to get the required gains, for the given poles.

$$|\lambda I - (A - BK)| = 0$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2) = 0$$

`Observable Form:`

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -a_2 & 1 & 0 \\ -a_1 & 0 & 1 \\ -a_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_2' \\ b_1' \\ b_0' \end{bmatrix} [U]$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_3 U$$

- To `check` if a system is observable:
  - Make sure the matrices are in **observer canonical form.**

$$S = \begin{bmatrix} C^T & A^T C^T & .... & (A^T)^{n-1} C^T \end{bmatrix}$$

$$|s| \neq 0, then\ the\ sysytem\ is\ observable$$

- To `design` a observer using pole-placement technique:
  - Compare the below two equations to get the required gains, for the given poles.

$$|\lambda I - (A - LC)| = 0$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2) = 0$$

`Similarity Transformation:`

$$A_w = P^{-1}AP\,; B_w = P^{-1}B$$
$$C_w = CP\,;\ D_w = D$$

Bode Plot

- Plot the bode plot using the transfer function.
- Find ω_{gc} and the corresponding angle to ω_{gc}
- Find the required angle to obtain the given phase. Add 7-12 degrees to make it a whole number. This is θ_m

$$D(S) = \frac{\alpha(1 +\ \tau S)}{(1\ + \alpha\tau S)}$$

$$\alpha = \frac{1 - sin\theta_m}{1 + sin\theta_m}$$
$$At\ \omega_m,\ M_{db} = 20log(\sqrt{\alpha})$$
$$\tau = \frac{1}{\omega_m\sqrt{\alpha}}$$

Mathematical modelling

`Speaker:`

$$F = M\frac{d^2x}{dt^2} + b\frac{dx}{dt}$$
$$F = BIL$$
$$L = N\pi d$$
$$V = Ri + L\frac{di}{dt} + e_{coil};\ e_{coil} = Blv$$

`Cruise Control:`

$$U = m\frac{d^2x}{dt^2} + b\frac{dx}{dt}$$

`DC motor:`

$$e_a = i_a R_a + L_a\frac{di_a}{dt} + e_b$$
$$e_b = K_f\frac{d\theta}{dt}$$
$$T_a = k_t i_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt}$$