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FIR filter

Filter → Impulse response is finite.

o/p → Depends only on present or past values.

Application → where linear phase is important

Ex: Data transmission, Speech processing, correlation processing, interpolation.

Advantages:

1. Stable
2. Can be realized in both recursive and non-recursive
3. Exact linear phase
4. Flexible
5. Low sensitivity to quantization noise.
6. Efficiently realized in hardware

Disadvantages:

1. Complex
2. Requires more filter co-efficients to be stored
3. Long duration impulse response requires large amount of processing time

Difference eqn representation of FIR system:-

$$y(n) = \sum_{k=0}^{N-1} b_k x(n-k)$$

$$Y(z) = \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N-1} h z^{-k}$$

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-(N-1)}$$

$$\& \; h(n) = \{b_0, b_1, \dots b_{N-1}\}$$

Linear phase FIR system

FIR filter has linear phase if its impulse response satisfy following condition $h(n) = \pm h(N-1-n)$

Properties of FIR filters

- Inherently stable
- Linear phase can be acheived
- All zero filters (No poles)

Comparision of FIR and IIR filters:

FIR filter design methods:-

Windowing

$H_d(\omega) \xrightarrow{F^{-1}} h_d(n) \xrightarrow{w(n)} h(n) \xrightarrow{Z} H(z)$

Frequency Sampling

$H_d(\omega) \xrightarrow{\text{Sample}} H(k) \xrightarrow{IDFT} h(n) \xrightarrow{Z} H(z)$

FIR filter design using windows:

Different type of windows:

1. Rectangular
2. Bartlett (triangular
3. Blackman
4. Hanning
5. Hamming
6. Kaiser

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Rectangular window:

1. $w_R(n) = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$

2 Hanning window:

$0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$

$$w(n) = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

3 Hamming window:

$$w(n) = \begin{cases} 0.54 - 0.46 \cos \frac{2\pi n}{N-1} & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

Attenuation and transition width of windows.

window | Transition width of main lobe | Relative amplitude of sidelobe

window	Transition width of main lobe	Min stopband attenuation	Relative amplitude of sidelobe
Rectangular	$\frac{4\pi}{N+1}$	-21dB	-13dB
Hanning	$\frac{8\pi}{N}$	-44dB	-31dB
Hamming	$\frac{8\pi}{N}$	-53dB	-41dB

Design of linear phase FIR filters using windows:-

The desired frequency response of the filter is $H_d(\omega)$ is given by

$H_d(\omega) = \sum_{n=0}^{N-1} h_d(n) e^{-j\omega n}$ and $h_d(n)$ is obtained by taking inverse fourier transform of $H_d(\omega)$.

$$hd(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Hd(\omega)\, e^{j\omega n}\, d\omega$$

$$Hd(\omega) = \begin{cases} e^{-j\omega\tau} & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \end{cases}$$

τ = Slope of linear phase

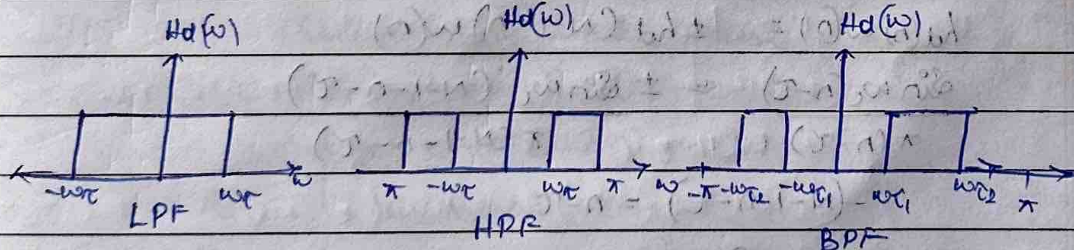
$$hd(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\tau)}\, d\omega.$$

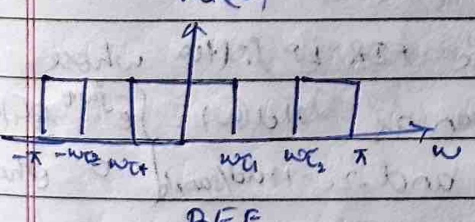
$$= \begin{cases} \dfrac{\sin \omega_c(n-\tau)}{\pi(n-\tau)} & \text{for } n \ne \tau \\ \dfrac{\omega_c}{\pi} & \text{for } n = \tau \end{cases}$$

for $n = \tau$

$$hd(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e\, d\omega = \frac{\omega_c}{\pi}$$

$$h(n) = hd(n) \cdot \omega(n).$$

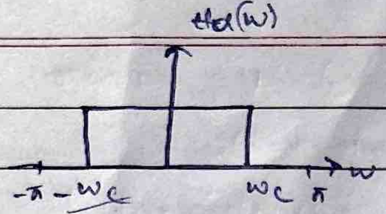




Examples of FIR filter design using windows:

To design a LP FIR filter cut-off frequency Ω_c and phase delay τ. The desired freq. response is given by

$$Hd(\omega) = \begin{cases} e^{j\omega\tau} & -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$



$$h_d(n) = \frac{1}{2\pi}\int_{-w_c}^{w_c} H_d(w)\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-w_c}^{+w_c} e^{-jw\tau}\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-w_c}^{+w_c} e^{jw(n-\tau)}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw_c(n-\tau)} - e^{-jw_c(n-\tau)}}{j(n-\tau)}\right]$$

$$= \frac{1}{\pi(n-\tau)}\left[\frac{e^{jw_c(n-\tau)} - e^{-jw_c(n-\tau)}}{2j}\right]$$

$$= \frac{1}{\pi(n-\tau)}\left[\sin w_c(n-\tau)\right] \quad \text{if } n \neq \tau$$

for $n = \tau$

$$h_d(n) = \frac{1}{2\pi}\int_{-w_c}^{w_c} dw = \frac{w_c}{\pi}$$

To achieve linear phase symmetry, conditions have to satisfied:-

$$h(n) = \pm h(N-1-n)$$

$$h_d(n)\,w(n) = \pm h_d(N-1-n)\,w(n)$$

$$\frac{\sin w_c(n-\tau)}{\pi(n-\tau)} = \pm \frac{\sin w_c(N-1-n-\tau)}{\pi(N-1-n-\tau)}$$

$$-(N-1-n-\tau) = n-\tau$$

$$\tau = \frac{N-1}{2}$$

1. Design a linear phase / symmetric FIR LP filter whose desired freq. response is given as $H_d(w) = \begin{cases} e^{-jw\tau} & \text{for } |w| < w_c \\ 0 & \text{otherwise} \end{cases}$ Length of filter should be 7 and $\Omega_c = 1$ rad/sample use a. rectangular b. Hanning c. Hamming window.

→ a. The desired impulse response is given by

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw.$$

$$H_d(w) = \frac{1}{2\pi}\begin{cases} e^{-jw\tau} & \text{for } -1 < w < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$hd(n) = \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega\tau} e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(n-\tau)}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\tau)}}{j\omega(n-\tau)}\right]_{-1}^{1}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{j(n-\tau)}\right]$$

$$= \frac{1}{\pi(n-\tau)}\left[\frac{e^{j\omega(n-\tau)} - e^{-j(n-\tau)}}{2j}\right]$$

$$hd(n) = \frac{\sin(n-\tau)}{\pi(n-\tau)} \quad \text{for } n \neq \tau$$

If $n = \tau$

$$hd(n) = \frac{1}{2\pi}\int_{-1}^{1} d\omega = \frac{1}{2\pi}\cdot 2 = \frac{1}{\pi} \quad \text{for } n=\tau.$$

$$hd(n) = \begin{cases} \dfrac{\sin(n-\tau)}{\pi(n-\tau)} & \text{for } n \neq \tau \\ 1/\pi & \text{for } n = \tau \end{cases}$$

- The filter should have linear phase

$$h(n) = \pm h(N-1-n).\, w(n)$$

$$hd(n)\, w(n) = \pm hd(N-1-n)\, w(n)$$

$$hd(n) = \pm hd(N-1-n)$$

$$\frac{\sin(n-\tau)}{\pi(n-\tau)} = \frac{\sin(N-1-n-\tau)}{\pi(N-1-n-\tau)}$$

$$\frac{-\sin(n-\tau)}{-\pi(n-\tau)} = \pm\frac{\sin(N-1-n-\tau)}{(N-1-n-\tau)}$$

$$-(n-\tau) = N-1-n-\tau$$

$$-n+\tau = N-1-n-\tau$$

$$\tau = \frac{N-1}{2} \Rightarrow \frac{7-1}{2} = 3.$$

$$hd(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & \text{for } n \neq 3 \\ \dfrac{1}{\pi} & \text{for } n = 3 \end{cases}$$

$$w(n) = \begin{cases} 1 & \text{for } 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Rectangular:

n	$h_d(n)$	$w(n)$	$h(n)$
0	0.014	1	0.014
1	0.144	1	0.144
2	0.267	1	0.267
3	$1/\pi$	1	0.318
4	0.267	1	0.267
5	0.144	1	0.144
6	0.014	1	0.014

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$= 0.014 + 0.144z^{-1} + 0.267z^{-2} + 0.318z^{-3} + 0.267z^{-4} + 0.144z^{-5} + 0.014z^{-6}$$

Frequency response:

$$H(\omega) = H(z)\big|_{z=e^{j\omega}}$$

$$= 0.014 + 0.144e^{-j\omega} + 0.267e^{-2j\omega} + 0.318e^{-3j\omega} + 0.267e^{-4j\omega} + 0.144e^{-5j\omega} + 0.014e^{-6j\omega}$$

$$= 0.014(1 + e^{-j6\omega}) + 0.144(e^{-j\omega} + e^{-5j\omega}) + 0.267(e^{-2j\omega} + e^{-4j\omega}) + 0.318(e^{-3j\omega})$$

$$= e^{-j3\omega}\left[0.014(e^{3j\omega} + e^{-3j\omega}) + 0.144(e^{2j\omega} + e^{-2j\omega}) + 0.267(e^{j\omega} + e^{-j\omega}) + 0.318\right]$$

$$= e^{-j3\omega}\left[0.318 + 2 \times 0.267\cos\omega + 2 \times 0.144\cos 2\omega + 2 \times 0.014\cos 3\omega\right]$$

$$|H(\omega)| = 0.318 + 2 \times 0.267\cos\omega + 2 \times 0.144\cos 2\omega + 2 \times 0.014\cos 3\omega$$

$$\phi(\omega) = -3\omega$$

Hanning:

n	hd(n)	w(n)	h(n)
0	0.014	0	0
1	0.144	0.25	0.036
2	0.267	0.75	0.2
3	0.318	1	0.318
4	0.267	0.75	0.2
5	0.144	0.25	0.036
6	0.014	0	0

$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$

$= \sum_{n=0}^{6} h(n) z^{-n}$

$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6}$

$= 0.014 + 0.144 z^{-1} + 0.267 z^{-2} + 0.318 z^{-3} + 0.267 z^{-4} + 0.144 z^{-5}$
$+ 0.014 z^{-6}$

Frequency response:-

$H(\omega) = H(z) |_{z = e^{j\omega}}$

$= 0 \cancel{0.014} + 0.036 e^{-j\omega} + 0.2 e^{-2j\omega} + 0.318 e^{-3j\omega} + 0.2 e^{-4j\omega} + 0.036 e^{-5j\omega}$
$+ \cancel{0.014} e^{-6j\omega} 0.$

$= 0.036 (e^{-j\omega} + e^{-5j\omega}) + 0.2 (e^{-j2\omega} + e^{-4j\omega}) + 0.318 e^{-3j\omega}$

$= e^{-3j\omega} [0.036 (e^{2j\omega} + e^{-2j\omega}) + 0.2 (e^{j\omega} + e^{-j\omega}) + 0.318]$

$|H(\omega)| = 0.318 + 0.2 \times 2 \cos\omega + 0.036 \times 2 (\cos 2\omega)$

$\phi(\omega) = -3\omega$

Hamming:-

n	hd(n)	w(n)	h(n)
0	0.014	0.08	1.12×10^{-3}
1	0.144	0.31	0.044
2	0.267	0.77	0.205
3	0.318	~~0.861~~ 1	0.318
4	0.267	0.77	0.205
5	0.144	0.31	0.044
6	0.014	0.08	1.12×10^{-3}

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$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$
$$= \sum_{n=0}^{6} h(n) z^{-n}$$
$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$
$$= 1.12\times10^{-3} + 0.044z^{-1} + 0.205z^{-2} + 0.318z^{-3} + 0.205z^{-4} + 0.044z^{-5} + 1.12\times10^{-6}$$
$$= 1.12\times10^{-3} + 0.044e^{-j\omega} + 0.205e^{-2j\omega} + 0.318e^{-3j\omega} + 0.205e^{-4j\omega} + 0.044e^{-5j\omega} + 1.12\times10^{-6j\omega}$$
$$= 1.12\times10^{-3}(1+e^{-6j\omega}) + 0.044(e^{-j\omega}+e^{-5j\omega}) + 0.205(e^{-2j\omega}+e^{-4j\omega}) + 0.318e^{-3j\omega}$$
$$= e^{-3j\omega}\left(1.12\times10^{-3}(e^{3j\omega}+e^{-3j\omega}) + 0.044(e^{2j\omega}+e^{-2j\omega}) + 0.205(e^{j\omega}+e^{-j\omega})\right)$$
$$= e^{-3j\omega}(0.318 + 0.205\cos\omega + 0.044\cos 2\omega + 1.12\times10^{-3}\cos 3\omega)$$
$$|H(\omega)| = 0.318 + 0.205\cos\omega + 0.044\cos2\omega + 1.12\times10^{-3}\cos3\omega$$
$$\phi(\omega) = -3\omega$$

2. Design a BR linear phase FIR filter having a cut-off frequency $\omega_{c1} = \pi/8$, $\omega_{c2} = \ldots$ r/s. Obtain unit sample response using the following window $w(n) = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$. Also find the magnitude of frequency response.

→
$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & \text{otherwise} \end{cases}$$

$$\tau = \frac{N-1}{2} = \frac{6}{2} = 3.$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_{c2}}^{\omega_{c2}} e^{-j\omega\tau} \cdot e^{j\omega n}\, d\omega$$
$$= \frac{1}{2\pi}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\omega\tau} e^{j\omega n}\, d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{-j\omega\tau} e^{j\omega n}\, d\omega\right]$$
$$= \frac{1}{2\pi}\left\{\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-\omega_{c2}}^{-\omega_{c1}} + \left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{\omega_{c1}}^{\omega_{c2}}\right\}$$

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$$h_d(n) = \frac{\sin w_{c2}(n-\tau) - \sin w_{c1}(n-\tau)}{\pi(n-\tau)} \quad \text{for } n \neq \tau$$

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-w_{c2}}^{-w_{c1}} dw + \int_{w_{c1}}^{w_{c2}} dw\right] \quad \text{for } n = \tau$$

$$h_d(n) = \frac{w_{c2} - w_{c1}}{\pi} = \frac{1}{\pi} \quad n = \tau$$

$$h_d(n) = \begin{cases} \dfrac{\sin 2(n-3) - \sin(n-3)}{\pi(n-3)} & n \neq \tau \\ \dfrac{1}{\pi} & n = \tau \end{cases}$$

n	$h_d(n)$	$w(n)$	$h(n)$
0	-0.044	1	-0.044
1	-0.265	1	-0.265
2	0.021	1	0.021
3	0.318	1	0.318
4	0.021	1	0.021
5	-0.265	1	-0.265
6	-0.044	1	-0.044

$$H(Z) = \sum_{n=0}^{N-1} h(n) Z^{-n}$$
$$= \sum_{n=0}^{6} h(n) Z^{-n}$$
$$= -0.044 - 0.265 Z^{-1} + 0.021 Z^{-2} + 0.318 Z^{-3} + 0.021 Z^{-4} - 0.265 Z^{-5} - 0.044 Z^{-6}$$

$$H(w) = H(Z)|_{Z \to e^{jw}}$$
$$= -0.044 - 0.265 e^{-jw} + 0.021 e^{-2jw} + 0.318 e^{-3jw} + 0.021 e^{-4jw} - 0.265 e^{-5jw} - 0.044 e^{-6jw}$$
$$= -0.044(1 + e^{-6jw}) - 0.265(e^{-jw} + e^{-5jw}) + 0.021(e^{-2jw} + e^{-4jw}) + 0.318 e^{-3jw}$$
$$= e^{-3jw}\left[-0.044(e^{3jw} + e^{-3jw}) - 0.265(e^{+2jw} + e^{-2jw}) + 0.318\right]$$

$$|H(w)| = 0.318 - 0.265\underbrace{(e^{2jw} + e^{-2jw})}_{\cos 2w} - 0.044\underbrace{(e^{3jw} + e^{-3jw})}_{\cos 3w}$$

$$|H(w)| = 0.318 - 0.53 \cos 2w - 0.088 \cos 3w + 0.02 \cos w$$

$$\phi(w) = -3w$$

3. Design a normalized linear phase FIR filter having phase delay $\tau = 4$ and atleast 40dB attn in the stop band. Also obtain the magnitude of freq. response of the filter

$\tau = \frac{N-1}{2}$ $\quad \omega_c = 1$ rad/sec.

$4 = \frac{N-1}{2} \Rightarrow N = 9$

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & \omega_{c1} \leq \omega \leq \omega_{c2} \\ 0 & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_{c1}}^{\omega_{c2}} H_d(\omega) \cdot e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(n-\tau)} \cdot d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-1}^{1}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n-\tau)} - e^{-j(n-\tau)}}{j(n-\tau)}\right]$$

$$= \frac{\sin(n-\tau)}{\pi(n-\tau)}$$

$\tau = 4$

$$= \frac{\sin(n-4)}{\pi(n-4)} \quad n \neq \tau$$

When $n = \tau$

$$h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} d\omega = \frac{1}{\pi}$$

$$h_d(n) = \begin{cases} \frac{\sin(n-4)}{\pi(n-4)} & n \neq \tau \\ \frac{1}{\pi} & n = \tau \end{cases}$$

n	hd(n)	w(n)	h(n)
0	-0.06	0	0
1	0.0149	0.146	2.17×10^{-3}
2	0.1447	0.5	0.072
3	0.2678	0.8535	0.228
4	0.318	1	0.318
5	0.2678	0.8535	0.228
6	0.1447	0.5	0.072
7	0.0149	0.1464	2.17×10^{-3}
8	-0.06	0	0

$$H(z) = \sum_{n=0}^{8} h(n) z^{-n}$$

$$= \sum_{n=0}^{8} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-n} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8}$$

$$= 2.17\times10^{-3} z^{-1} + 0.072 z^{-2} + 0.228 z^{-3} + 0.318 z^{-4} + 0.228 z^{-5} + 0.072\times10^{-6} + 2.17\times10^{-7}$$

$$= 2.17\times10^{-3} e^{-j\omega} + 0.072 e^{-2j\omega} + 0.228 e^{-3j\omega} + 0.318 e^{-4j\omega} + 0.228 e^{-5j\omega} + 0.072 e^{-6j\omega} + 2.17\times10^{-3} e^{-7j\omega}$$

$$= 2.17\times10^{-3}(e^{-j\omega} + e^{-7j\omega}) + 0.072(e^{-2j\omega} + e^{-6j\omega}) + 0.228(e^{-5j\omega} + e^{-3j\omega}) + 0.318 e^{-4j\omega}$$

$$= e^{-4j\omega}\left[2.17\times10^{-3}(e^{3j\omega} + e^{-3j\omega}) + 0.072(e^{2j\omega} + e^{-2j\omega}) + 0.228 (e^{j\omega} + e^{-j\omega}) + 0.318 e^{-4j\omega}\right]$$

$$|H(\omega)| = 0.318 + 2\times0.228\cos\omega + 0.072\times2\cos2\omega + 2\times2.17\times10^{-3}\cos3\omega$$

$$\phi(\omega) = -4\omega.$$

4. The freq. response of a linear phase FIR filter is given by

$$H(\omega) = e^{j3\omega}\left[2 + 1.8\cos3\omega + 1.2\cos2\omega + 0.5\cos\omega\right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(n - \frac{N-1}{2}\right)\right]$$

→ $N = 7$.

Comparing the two eq^ns we get

$h(3) = 2.$ $h(0) = 0.9$ $h(1) = 0.6$ $h(2) = 0.25$

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Design of linear phase FIR filter using freq. sampling method

Let desired freq. response of FIR filter to be designed be $H_d(\omega)$. Freq. response is sampled uniformly at M points.

Freq. samples are given by

$$\omega_k = \frac{2\pi}{M} k \quad k = 0, 1, 2 \ldots M-1$$

Sampled freq. response is DFT and is denoted by $H(k)$

$$= H_d(\omega)\Big|_{\omega=\omega_k = \frac{2\pi k}{M}}$$

$H(k)$ is M point DFT

- By taking IDFT of $H(k)$, we get $h(n)$.

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(k) e^{\frac{j2\pi nk}{M}} \longrightarrow ① \quad n = 0, 1, \ldots M-1$$

$h(n)$ is impulse response of FIR filter of length M

- For FIR filter to be realizable, the co-efficient $h(n)$ shd be real.
- This is possible if all the complex terms appear in conjugate pairs $\quad H(k) = H^*(M-k) \rightarrow ②$
- using ② in ①, we can write

$$h(n) = \frac{1}{M}\left\{ H(0) + 2\sum_{k=1}^{P} Re\left\{ H(k) e^{\frac{j2\pi kn}{M}} \right\} \right\}$$

where

$$P = \begin{cases} \frac{M-1}{2}, & \text{if } M \text{ is odd} \\ \frac{M}{2} - 1 & \text{if } M \text{ is even} \end{cases}$$

1. Design LP FIR filter using freq. sampling technique having cut-off freq. of $\pi/2$ rad/sample. The filter shd have linear phase and length 17.

→ The desired freq. response of linear phase FIR filter is given by

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{N-1}{2}\right)} & \text{for } |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Considering only +ve range of frequencies, we can write

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{N-1}{2}\right)} & \text{for } 0 \le \omega \le \omega_c \\ 0 & \text{for } \omega_c \le \omega \le \pi \end{cases}$$

HW continue →

→ $h(n) = \{0.9, 0.6, 0.25, \alpha, 0.25, 0.6, 0.9\}$

$N = 17 \quad \frac{N-1}{2} = \frac{17-1}{2} = 8 \; ; \; \omega_c = \pi/2$ rad/samples

$$H_d(\omega) = \begin{cases} e^{-j\omega 8} & \text{for } 0 \le \omega \le \pi/2 \\ 0 & \text{for } \pi/2 \le \omega \le \pi \end{cases}$$

To sample $H_d(\omega)$, put $\omega = \frac{2\pi k}{N}$ $\quad k = 0, 1, \ldots N-1$

$\Rightarrow \quad \omega = \frac{2\pi k}{17} \quad k = 0, 1, \ldots 16$

$H(k) = H_d(\omega) |_{\omega = \frac{2\pi k}{17}}$

$$= \begin{cases} e^{-j\frac{2\pi k}{17} \times 8} & \text{for } 0 \le \frac{2\pi k}{17} \le \pi/2 \\ 0 & \text{for } 17/4 \le \end{cases}$$

$$\begin{cases} e^{-j\frac{16\pi k}{17}}, & \text{for } 0 \le \frac{2\pi k}{17} \le \pi/2 \\ 0, & \text{for } \pi/2 \le \frac{2\pi k}{17} \le \pi \end{cases}$$

Since k is always an integer, range is considered as

$0 \le k \le 4 \quad \& \quad 5 \le k \le 8$

$$H(k) = \begin{cases} e^{-j\frac{16\pi k}{17}} & \text{for } 0 \le k \le 4 \\ 0 & \text{for } 5 \le k \le 8 \end{cases}$$

Now,

$h(n) = \text{IDFT}\{H(k)\}$

$= \sum_{k=0}^{K} H(k) \, e^{j\frac{2\pi kn}{17}} \rightarrow (*)$

Since $h(n)$ is real

$H(k) = H^*(N-k)$

using this in (*), we can write

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \text{Re}\left[H(k)\, e^{j\frac{2\pi kn}{17}}\right]\right]$$

$$h(n) = \frac{1}{17}\left[1 + 2\sum_{k=1}^{8} \text{Re}\left[e^{-j\frac{16\pi k}{17}} \cdot e^{j\frac{2\pi kn}{17}}\right]\right]$$

$$= \frac{1}{17}\left[1 + 2\sum_{k=1}^{8} \text{Re}\, e^{j\frac{2\pi k}{17}\{n-8\}}\right]$$

$$= \frac{1}{17}\left[1 + 2\sum_{k=1}^{8} \cos\left[\frac{2\pi k}{17}(8-n)\right]\right]$$

$n = 0, 1, 2, 3, 4, 5, 6, 7, 8$

$h(n) =$

n	$h(n)$
0	0.03979
1	−0.0488
2	−0.03459
3	0.06598
4	0.03154
5	−0.1074
6	−0.02992
7	0.31876
8	0.5294
9	0.31876
10	−0.0299
11	−0.1079
12	0.03159
13	0.06598
14	−0.03459
15	−0.0488
16	0.03979

$$H(Z) = \sum_{n=0}^{16} h(n) Z^{-n}$$

$$= 0.03979 - 0.0488Z^{-1} - 0.03459Z^{-2} + 0.06598Z^{-3} + 0.03154Z^{-4} - 0.1074Z^{-5} - 0.02992Z^{-6} + 0.31876Z^{-7} + 0.5294Z^{-8} + 0.31876Z^{-9} - 0.0299Z^{-10} - 0.1079Z^{-11} + 0.03159Z^{-12} + 0.06598Z^{-13} - 0.03459Z^{-14} - 0.0488Z^{-15} + 0.03979Z^{-16}$$

$$= 0.03979(1+Z^{-16}) - 0.0488(Z^{-1}+Z^{-15}) - 0.03459(Z^{-2}+Z^{-14}) + 0.065(Z^{-3}+Z^{-13}) + 0.0315(Z^{-4}+Z^{-12}) - 0.107(Z^{-5}+Z^{-11}) - 0.0299(Z^{-6}+Z^{-10}) + 0.31876(Z^{-7}+Z^{-9}) + 0.5294Z^{-8}$$

$$H(\omega) = e^{-j\omega 8}\left[0.03979(e^{j\omega 8}+e^{-j\omega 8}) - 0.0488(e^{j\omega 7}+e^{-j\omega 7}) - 0.03459(e^{j\omega 6}+e^{-j\omega 6}) + 0.065(e^{j\omega 5}+e^{-j\omega 5}) + 0.0315(e^{j\omega 4}+e^{-j\omega 4}) - 0.107(e^{j\omega 3}+e^{-j\omega 3}) - 0.0299(e^{j\omega 2}+e^{-j\omega 2}) + 0.318(e^{j\omega}+e^{-j\omega}) + 0.5294\right]$$

$$|H(\omega)| = 0.5294 + 0.636\cos\omega - 0.0598\cos 2\omega - 0.214\cos 3\omega + 0.063\cos 4\omega + 0.13\cos 5\omega - 0.069\cos 6\omega - 0.0976\cos 7\omega + 0.079\cos 8\omega$$

$$\angle H(\omega) = -8\omega$$

2. Determine the impulse response $h(n)$ of the linear phase FIR filter having desired frequency response $H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{N-1}{2}\right)} & \text{for } 0 \le \omega \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le \omega \le \pi \end{cases}$ Take $N = 7$. Use freq. sampling approach.

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