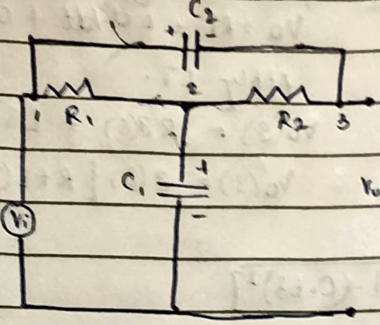
4/4/23

20EC530

Control Systems

5/4/23 Equation for bridge Tee circuit

Determine the TF for the circuit shown



Apply KCL

$V_1 = V_i$; $\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} + C_1\frac{dV_2}{dt}$ → ① → ②

at node 3.

③ ← $-\frac{V_2 - V_3}{R_2} - C_2\frac{d(V_1 - V_3)}{dt} = 0$.

Capacitor vtgs : V_{c1}, V_{c2}. → $V_1 = V_i$ $V_{c1} = V_2$; $V_1 - V_3 = V_{c2}$

$V_3 = V_1 - V_{c2}$

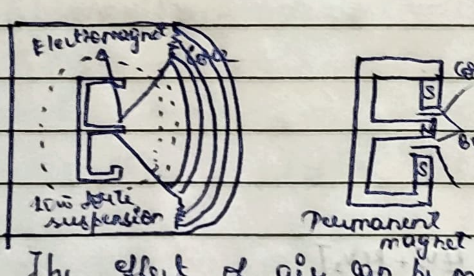
Eq^n ② → $\frac{V_{c1} - V_i}{R_1} + \frac{-V_1 + V_{c2} + V_{c1}}{R_2} + C_1\frac{d(V_{c1})}{dt}$:

$$\frac{d(V_{c1})}{dt} = -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_{c1} - \frac{1}{C_1}\left(\frac{1}{R_2}\right)V_{c2} + \frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_i = 0$$

Eq^n ③

$$\frac{d}{dt}(V_{c2}) = -\frac{V_{c1}}{C_2R_2} - \frac{V_{c2}}{C_2R_2} + \frac{V_i}{C_2R_2}$$

Loud speaker:



The permanent magnet establishes a radial field in the cyl. gap b/w the poles of the magnet. The force on the conductor wound on the bobbin causes the voice coil to move, producing sound.

The effect of air can be modelled as if the cone had an equivalent mass 'M' & viscous ~~force~~ friction 'v'. The force is given by:

$F = Bil$ Newton

$l = N\pi d$ → diameter of bobbin

No of turns

Assume $N = 20$, $d = 2cm$, $B = 0.5T$

$F = Bil$ ← $l = 20 \times \pi \times 2 \times 10^{-2}$

$= \underline{0.628i}$

$F = M\frac{d^2(x)}{dt^2} + b\frac{dx}{dt}$

$0.628i = M\frac{d^2x}{dt^2} + b\frac{dx}{dt}$

$0.628I(S) = X(S)[MS^2 + bS]$

$$\frac{X(S)}{I(S)} = \frac{0.628}{MS^2 + bS} = \frac{\frac{0.628}{M}}{S^2 + \frac{b}{M}S} = \frac{\frac{0.628}{M}}{S\left(S + \frac{b}{M}\right)}$$

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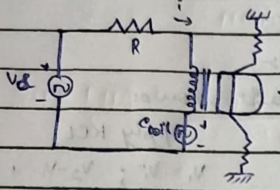
Loud speaker with Circuit:-



$e_{coil} = Blv = Bl\frac{dx}{dt} = 0.628\frac{dx}{dt}$

Apply KVL:

$V_a = Ri + L\frac{di}{dt} + e_{coil}$

$V_a = Ri + L\,di/dt + 0.63\,dx/dt$

Apply LT.

$V_a(s) = RI(s) + LsI(s) + 0.63s\,X(s)$

$V_a(s) = I(s)[R + L(s)] + 0.63s\,X(s)$

$$\frac{X(s)}{V_a(s)} = \frac{0.63}{s[(Ms+b)(R+Ls) + (0.63)^2]}$$

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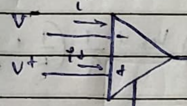
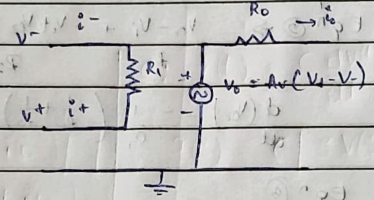
Modelling of Circuits:

Op-Amp :-

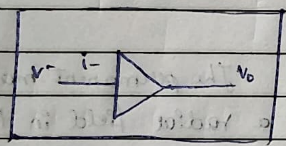


$\Rightarrow$



$V^+ \to$ Connected to gnd

$\Downarrow$

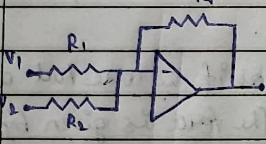


ideal: $R_i = \infty$, $R_o = 0$, $A_v = \infty$

$i = i_+ = 0 \to (1)$

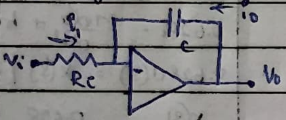
$V_+ = V_- = 0 \to (2)$

Summer:



$V_o = -\left[\frac{R_f V_1}{R_1} + \frac{R_f V_2}{R_2}\right]$

Integrator:-



$i_1 + i_o = 0$

$\frac{V_i}{R_i} + C\frac{dV_o}{dt} = 0$

$V_o = -\frac{1}{RC}\int V_i\,dt$

$V_o = -\frac{1}{R_iC}\int_0^t V_i(\tau)\,d\tau + V_o(0)$

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Replacing $\frac{d}{dt} = s$; $\frac{V_o(s)}{V_i(s)} = \frac{1}{s}\frac{1}{RiC}$

* Standard form
* Properties of Laplace Transform
* ILT
* Differential form
* Fourier Series
* Properties.

Course Outcomes:

CO1: ~~Displ~~ Explain the physical systems as

CO2: Analyze various properties of control sys. in time domain & freq. domain using appropriate tools.

CO3: Design & test controllers for transfer fn & state space models.

CO4: Evaluate the state space models of systems using appropriate tools

CO5: Demonstrate the performance of controllers using modern tools.

Textbook:

1. G.F. Franklin, G.D Powell, A.E. "Feedback Control of Dynamic systems", Pearson Ed. 5^{th} Edition, 2002
2. M. Gopal, "Control systems: Principles & Design, Tata McGraw Hill

Laplace Transforms:

$f(t) \to F(s)$

$s = \sigma + j\omega$

$$L\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

$\delta(t)$	1	t^n	$\frac{n!}{s^{n+1}}$
$u(t)$	$\frac{1}{s}$		
e^{at}	$\frac{1}{s-a}$	te^{at}	$\frac{1}{(s-a)^2}$
e^{-at}	$\frac{1}{s+a}$	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
t	$1/s^2$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$

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- $\sin\omega(t) = \dfrac{\omega}{s^2+\omega^2}$; $\cos\omega t \;\; \dfrac{s}{s^2+\omega^2}$

- $\sin(\omega t+\theta) \;\; \dfrac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$; $\cos(\omega t-\theta) \;\; \dfrac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$

- $e^{-at}\sin\omega t = \dfrac{\omega}{(s+a)^2+\omega^2}$; $e^{-at}\cos\omega t \;\; \dfrac{s}{(s+a)^2+\omega^2}$

RLC:-

$V(t) = R\,i(t)$

$V(S) = R\,I(S)$

L $\quad V(t) = L\dfrac{di}{dt}$

$\boxed{V(S) = L[S I(S) - i(0^-)]}$

$V(S) = LSI(S) - L\,i(0^-)$

$\boxed{I(S) = \dfrac{V(S) + i(0^-)}{LS}}$

C: $\; i(t) = C\dfrac{dV(t)}{dt}$

$\boxed{I(S) = C[S V(S) - V(0^-)]}$

$V(S) = \dfrac{I(S)}{CS} + V(0^-)$

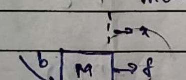
11/9/23 Types of Control systems:-

1. Electrical systems $\rightarrow$ R, L, C, active elements
2. Mechanical systems $\rightarrow$ Mass, spring, friction
3. Electromechanical systems $\rightarrow$ Combo of elec. & mechanical
4. Pneumatic systems.

Objective of mechanical sys $\rightarrow$ Motion

along a straight line $\downarrow$ Translational mech sys — Mass, spring, friction

about an axis $\downarrow$ Rotational mech sys. — Moment of Inertia, spring action, friction



$x \rightarrow$ displacement

$\dfrac{dx}{dt} \rightarrow$ velocity

$\dfrac{d^2x}{dt^2} \rightarrow$ acceleration

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mass : force $\propto$ acceleration

$f = m\dfrac{d^2x}{dt^2}$

Friction: $f \propto$ velocity

$f = b\dfrac{dx}{dt}$ — b: Friction coeff

Spring: $f \propto x$

$f = kx$

Applied force should overcome — Inertia $\rightarrow m\dfrac{d^2x}{dt^2}$; Friction $\rightarrow b\dfrac{dx}{dt}$

$f = M\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt}$

$f(t) \longrightarrow \boxed{\text{Mech sys}} \longrightarrow x(t)$

$F(S) \qquad\qquad X(S)$

$T\{F(S)\} = X(S)$

Transfer fn: $\dfrac{X(S)}{F(S)}$

- Transfer fn is applicable for LTI (Linear Time invariant) systems.
- Starts from rest.
- No initial condition

$L\left\{\dfrac{dx(t)}{dt}\right\} = Sx(S) - \cancel{x(0)}^{\,0} = Sx(S)$

$L\left\{\dfrac{d^2x(t)}{dt^2}\right\} = S^2x(S)$

$L\left\{\dfrac{d^nx(t)}{dt^n}\right\} = S^nx(S)$

$f(t) = M\dfrac{d^2x(t)}{dt^2} + b\dfrac{d}{dt}x(t) \rightarrow$ Performance eq^n — always Time-domain; Integro-differential eq^n

$\Downarrow$ LT

$F(S) = MS^2x(S) + bSx(S) = x(S)[MS^2+bS]$

$\dfrac{X(S)}{F(S)} = \dfrac{1}{[MS^2+bS]} = \dfrac{1/M}{S^2+\frac{b}{M}S} = \dfrac{1/M}{S[S+\frac{b}{M}]} = \dfrac{A}{S} + \dfrac{B}{S+\frac{b}{M}}$

$f(t) = \delta(t)$

$F(S) = 1$

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$$x(t) = A + Be^{-(b/M)t}$$
$$= A + Be^{-t/\tau} \quad \boxed{\because \tau = M/b}$$

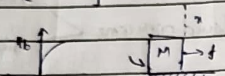
$$V_c(t) = V[1 - e^{-t/RC}]$$
$$= V[1 - e^{-t/\tau}]$$
$\tau = RC =$ time constant [electrical]

$$1/M = A(s + b/M) + B(s)$$

$s = 0 \quad 1/M = Ab/M \rightarrow A = 1/b$

$s = -b/M \quad 1/M = 0 - \frac{bB}{M} \Rightarrow B = -1/b$

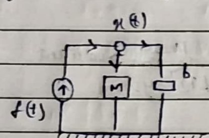
$$\therefore x(s) = \frac{1}{b}\left[\frac{1}{s} - \frac{1}{s + b/M}\right]$$
$$x(t) = 1/b\left[1 - e^{-(b/M)t}\right]$$



$$f(t) \longrightarrow \boxed{M, b} \longrightarrow x(t)$$

Mechanical equivalent circuit:

No. of nodes = no. of displacements

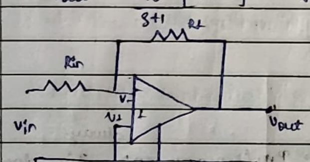


$$f(t) = \frac{Md^2x(t)}{dt^2} + \frac{bdx}{dt}.$$

12/9/23

1. First step follower realistic model of op-amp is given by following eqn & it is shown in fig 1.

$$V_{out} = \frac{10^7}{s+1}[V_+ - V_-] \qquad i_+ = i_- = 0$$



Find the transfer fn of the simple amplification circuit shown using this model.

Apply KCL

$$\frac{V_{in} - V_-}{R_{in}} + \frac{V_{out} - V_-}{R_f} = 0.$$

$$V_- = \frac{R_f}{R_{in}+R_f}V_{in} + \frac{R_{in}}{R_{in}+R_f}V_{out}$$

$$V_{out} = \frac{10^7}{s+1}\left[0 - \left(\frac{R_f}{R_{in}+R_f}V_{in} + \frac{R_{in}}{R_{in}+R_f}V_{out}\right)\right]$$

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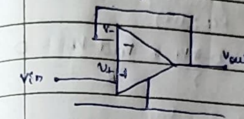
$$\frac{10^7}{s+1}V_{out}\left(1 + \frac{R_{in}}{R_{in}+R_f}\right) = \frac{-10^7}{s+1}\frac{R_f}{R_{in}+R_f}V_{in}.$$

$$\frac{V_{out}}{V_{in}} = \frac{-10^7\frac{R_f}{R_{in}+R_f}}{s+1+10^7\frac{R_{in}}{R_{in}+R_f}}$$

2. ST op-amp connection shown in fig 1 results in $V_{out} = V_{in}$. If the op amp is ideal. Give the tr. fn if the op-amp has non-ideal tr. fn of problem 1.



ideal

$V_{in} = V_+$

$V_{out} = V_-$

$V_+ = V_- \rightarrow$ ideal

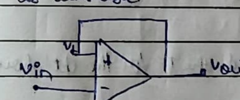
$V_+ \neq V_- \rightarrow$ Non-ideal

$$V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$$
$$= \frac{10^7}{s+1}[V_{in} - V_{out}]$$

$$V_{out}\left(1 + \frac{10^7}{s+1}\right) = \frac{10^7}{s+1}V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{10^7}{s+1}}{1 + \frac{10^7}{s+1}} = \frac{10^7}{s+1+10^7} \approx \frac{10^7}{s+10^7}$$

3. ST the non-ideal tr fn of problem 1 op amp shown in fig. 3 is unstable.



$$V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$$

$V_+ = V_{out}$; $V_- = V_{in}$

$$V_{out} = \frac{10^7}{s+1}[V_{out} - V_{in}]$$

NOTE: Tr fn has a d^r with $s - 10^7$ & the -ve sign means exp time fn is increasing, which means that it an unstable one

$$V_{out}\left(1 - \frac{10^7}{s+1}\right) = \frac{-10^7 V_{in}}{s+1}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{-10^7}{s+1}}{1 - \frac{10^7}{s+1}} = \frac{-10^7}{s+1-10^7} \approx \frac{-10^7}{s-10^7}$$

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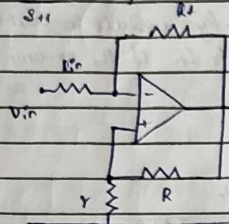
4. An op-amp connection with f/b to both -ve & +ve terminals is shown in fig. 4. If the op-amp has non-ideal dc. In even in problem 1 give the maximum value possible for the +ve f/b ratio $P = \frac{r}{r+R}$ -ve f/b ratio $N = \frac{R_{in}}{R_{in}+R_f}$ for the ckt to remain stable.

→ $V_{out} = \frac{10^7}{S+1}[V_+ - V_-]$



KCL (-ve)

$$\frac{V_{in} - V_-}{R_{in}} + \frac{V_{out} - V_-}{R_f} = 0 \rightarrow (1)$$

KCL: (+ve)

$$\frac{0 - V_+}{r} + \frac{V_{out} - V_+}{R} = 0 \rightarrow (2)$$

From (1),

$$\frac{V_-}{R_{in}} + \frac{V_-}{R_f} = \frac{V_{in}}{R_{in}} + \frac{V_{out}}{R_f}$$

$$V_- = \frac{R_f V_{in}}{R_{in}+R_f} + \frac{R_{in} V_{out}}{R_{in}+R_f}$$

From (2),

$$V_+ = \frac{r\, V_{out}}{r+R} = P.V_{out}$$

$N = \frac{R_{in}}{R_{in}+R_f}$ $\quad V_- = (1-N)V_{in} + N.V_{out}$

Now,

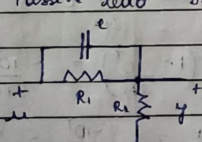
$$V_{out} = \frac{10^7}{S+1}[V_+ - V_-]$$

$$= \frac{10^7}{S+1}[P.V_{out} - (1-N)V_{in} - N.V_{out}]$$

$$\frac{V_{out}}{V_{in}} = \frac{-10^7(1-N)}{S+1-10^7P+10^7N}$$

5. Write the dynamic eq^n and find the TF for the ckt shown in fig 5

a. Passive lead b. Active lead c. Passive lag d. Active lag



$$\frac{V_u - V_y}{R_1} + C\frac{d}{dt}(V_u - V_y) - \frac{V_y}{R_2} = 0$$

$$\frac{V_u}{R_1} - \frac{V_y}{R_1} + \frac{Cd(V_u)}{dt} - \frac{Cd(V_y)}{dt} - \frac{V_y}{R_2}$$

$$C.\dot{V}_y + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_y = C.\dot{V}_u + V_u$$

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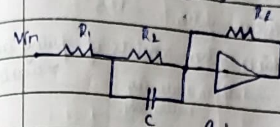
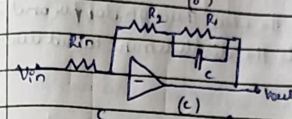
Applying L.T.

$$\frac{V_y(s)}{V_u(s)} = \frac{SC + \frac{1}{R_1}}{CS + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$



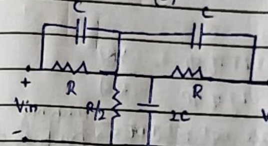
(b)

$$\frac{V_{out}}{V_{in}} = \frac{-R_f}{2}\ \frac{S + \frac{1}{CR_1}}{S + \frac{1}{R_1C} + \frac{1}{R_2C}}$$



(c)

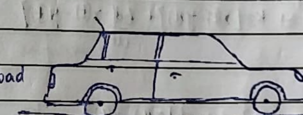
$$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_{in}}\ \frac{S + \frac{1}{R_2C} + \frac{1}{R_1C}}{S + \frac{1}{R_1C}}$$



(d)

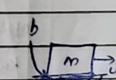
14/9/23 Cruise control system:-

Ex. Car.



The applied force has to overcome:

1. Inertia of the car $- m\frac{d^2x}{dt^2}$
2. Friction between Road surface and Fuel $- b\frac{dx}{dt}$



$$u = m\frac{d^2x}{dt^2} + b\frac{dx}{dt}$$

i/p ← $u = m\frac{dv}{dt} + bv \rightarrow$ o/p

$$U(s) = mSV(s) + bV(s)$$

$$U(s) = V(s)(mS+b)$$

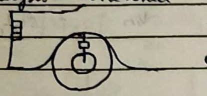
$$\Rightarrow \frac{V(s)}{U(s)} = \frac{1}{mS+b} = \frac{1/m}{S + b/m}$$

Automobile suspension system:-

* To reduce the impact of road irregularities on the rider / passenger

→ Car → Four wheels. Fig: suspension is shown for two wheels.

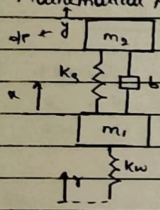
* For analysis → One wheel → Quarter car system



* Suspension system consists of spring and damper. → helps in reducing oscillations. → cushion effect

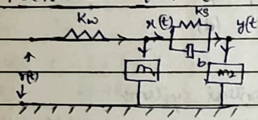
* Body of the car is connected to the wheel through damper and spring
* Input to the suspension system is the displacement caused due to irregularity of the road

Mathematical model:



Transfer fn = $\frac{\text{LT of o/p}}{\text{LT of i/p}} = \frac{Y(S)}{R(S)}$

Performance eq^n :- DE ⇆ Mech ntw, Free body diagram



* Displacement for mass is always independent.

Node $x(t)$:

$$K_w[r(t)-x(t)] = m_1\frac{d^2x(t)}{dt^2} + K_s[x(t)-y(t)] + b\frac{d}{dt}[x(t)-y(t)] \rightarrow ①$$

Node $y(t)$:

$$m_2\frac{d^2y(t)}{dt^2} = K_s[x(t)-y(t)] + b\frac{d}{dt}[x(t)-y(t)] \rightarrow ②$$

Apply LT:

$$K_w[R(S)-X(S)] = m_1 S^2 X(S) + K_s[X(S)-Y(S)] + bS[X(S)-Y(S)] \rightarrow ③$$

$$m_2 S^2 Y(S) = K_s[X(S)-Y(S)] + bS[X(S)-Y(S)]$$

Required: $\frac{Y(S)}{R(S)}$ → eliminate $X(S)$

$$\therefore \frac{Y(S)}{R(S)} = \frac{\frac{K_w b}{m_1 m_2}\left(S+\frac{K_s}{b}\right)}{S^4 + \left(\frac{b}{m_1}+\frac{b}{m_2}\right)S^3 + \left(\frac{K_s}{m_1}+\frac{K_s}{m_2}+\frac{K_w}{m_1}\right)S^2 + \left(\frac{K_w b}{m_1 m_2}\right)S + \frac{K_w K_s}{m_1 m_2}}$$

Electromechanical system

I/P → Electric vtg/current   O/P → Mech. displacement < Translational / Rotational

Ex: DC motor → i/p → vtg ; o/p → rotational displacement

Application: Need to control speed of DC motor

e_a → Armature controlled DC motor → $\theta(t)$, $\frac{d\theta(t)}{dt}$

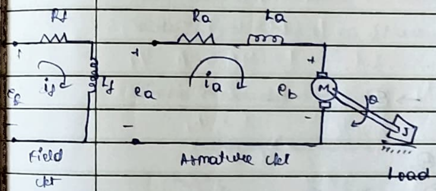
Armature vtg control | Field current control

Motor speed ∝ $\frac{\text{Armature vtg}}{\text{Field flux}}$ → Arm vtg ↑ speed ↑; field flux ↓ speed ↑

In armature control, we keep field flux constant ⇒ speed ∝ arm. vtg.

∴ Tr. fn = $\frac{\text{LT of o/p}}{\text{LT of i/p}} = \frac{\theta(S)}{E_a(S)}$ (or) $\frac{S\theta(S)}{E_a(S)}$

Armature controlled DC motor:-



In armature controlled DC motor, the field flux is kept constant and the armature voltage is varied to vary the speed.

KVL - Armature ckt:

$$e_a = i_a R_a + L_a\frac{di_a}{dt} + e_b \rightarrow ①$$

e_a → Applied armature vtg; i_a → Armature current; L_a → Armature winding inductance; R_a → Armature winding resistance; e_b → back emf

Taking LT, we have

$$E_a(S) = I_a(S)R_a + L_a S I_a(S) + E_b(S) \rightarrow ②$$

→ Back emf ∝ speed of the armature

$$e_b \propto \frac{d\theta}{dt}$$

$$e_b = K_b\frac{d\theta}{dt} \rightarrow ③$$

$$E_b(S) = K_b S\theta(S) \rightarrow ④$$

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Torque developed by the motor is proportional to the product of field flux and armature current

$T_a \propto \phi_f\, i_a \Rightarrow T_a \propto i_a$

constant $T_a = k_t i_a$

$T_a(s) = k_t I_a(s) \rightarrow (5)$

T_a < Load torque, Friction

$M\frac{d^2x}{dt^2}$ - Mass

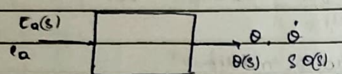
$T_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} \rightarrow (6)$

J - Moment of Inertia

$b\frac{dx}{dt}$ → Friction

kx = Spring

$T_a(s) = Js^2\theta(s) + bs\theta(s) \rightarrow (7)$



T. Fn → $\frac{\theta(s)}{E_a(s)}$

$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + L_a s} \rightarrow (3)$

$E_b(s) = K_e s\,\theta(s) \rightarrow (4)$

$T_a(s) = K_t I_a(s) \rightarrow (5)$

$T_a(s) = \theta(s)[Js^2 + bs] \rightarrow (7)$

$K_t I_a(s) = \theta(s).[Js^2+bs]$

$K_t\left[\frac{E_a(s) - E_b(s)}{R_a + sL_a}\right] = \theta(s)[Js^2+bs]$

$K_t\left[\frac{E_a(s) - K_e s\,\theta(s)}{R_a + L_a s}\right] = \theta(s)[Js^2+bs]$

$\frac{K_t E_a(s)}{R_a + sL_a} - \frac{K_t K_e s\,\theta(s)}{R_a + L_a s} = \theta(s)[Js^2+bs]$

$\frac{K_t E_a(s)}{R_a + L_a s} = \theta(s)\left[\frac{Js^2+bs + K_t K_e s}{R_a + L_a s}\right]$

$\frac{K_t E_a(s)}{\cancel{R_a + L_a s}} = \theta(s)\left[\frac{Js^2+bs\,(R_a+L_a s) + K_t K_e s}{\cancel{R_a+L_a s}}\right]$

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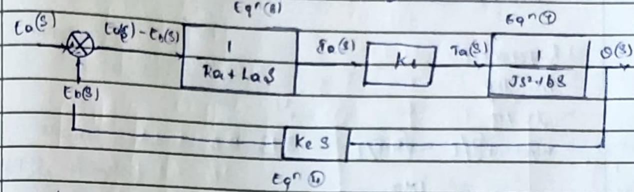
$\frac{\theta(s)}{E_a(s)} = \frac{K_t}{(Js^2+bs)(R_a+L_a s) + K_t K_e s}$

or

$\frac{s\,\theta(s)}{E_a(s)} = \frac{K_t}{(Js+b)(R_a+L_a s) + K_t K_e}$

K_t not present in char eqn

Block diagram: Interconnection of several blocks



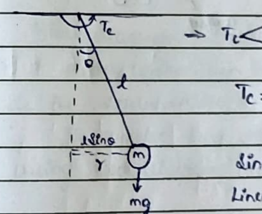
Observation:-

Armature controlled DC motor is a closed loop system with f/b.

Stability: Stable, as it uses f/b.

Simple Pendulum:-

↓ harmonic oscillations



⇒ T_c < Inertia of the mass (J/I), Opposing torque

$T_c = J\frac{d^2\theta}{dt^2} + (mg)(l\sin\theta) \rightarrow (1)$

Force distance

$\sin\theta$ → Non linear, term

Linear ⇒ for small θ, $\sin\theta \approx \theta$

$\Rightarrow T_c = J\frac{d^2\theta}{dt^2} + mg(l\theta)$

$J = (m)l^2$

$T_c = ml^2\frac{d^2\theta}{dt^2} + mgl\theta$

$\frac{T_c}{ml^2} = \frac{d^2\theta}{dt^2} + \frac{g}{l}\theta$

But $\omega_n = \sqrt{\frac{g}{l}}$

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$$\Rightarrow \boxed{\frac{T_c}{ml^2} = \frac{d^2\theta}{dt^2} + \omega_n^2\theta} \rightarrow \text{Performance eq}^n.$$

Take LT

$$\frac{T_c(s)}{ml^2} = s^2\theta(s) + \omega_n^2\theta(s) = \theta(s)[s^2+\omega_n^2]$$

$$\text{T. fn} \rightarrow \frac{\theta(s)}{T_c(s)} = \frac{1}{ml^2[s^2+\omega_n^2]} \rightarrow \frac{1/ml^2}{[s^2+\omega_n^2]}$$

Response:

$m = 1kg$

$l = 1m$

$\omega_n = \sqrt{g/l} = \sqrt{9.8/1} = 3.13 \approx 3$

$T_c = u(t)$ — 1Nm

$T_c(s) = 1/s$

$$\theta(s) = \frac{1}{s(s^2+3^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+3^2}$$

$1 = A(s^2+3^2) + (Bs+C)s$

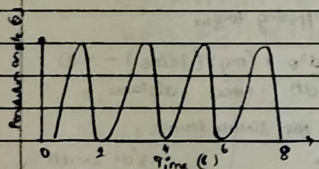
$s=0, \quad 1 = 9A \Rightarrow A = 1/9$

Equate coeff of s^2: $A+B=0 \Rightarrow B = -1/9$

coeff of s: $C=0$

$$\theta(s) = \frac{1}{9}\left[\frac{1}{s} - \frac{s}{s^2+3^2}\right]$$

$$\theta(t) = \frac{1}{9}[1 - \cos 3\omega t]$$



20/9/23

Modelling of Floppy disk drive:

→ Graph → Picture – x –

- Motor develops the torque (T_c / M_c)
- along with this there will be some disturbing torque (T_D/M_D)
- J_1 is the inertia of the motor.
- Motor torque is coupled to head through a flexible shaft which has K, b.
- J_2 is the inertia of the head.

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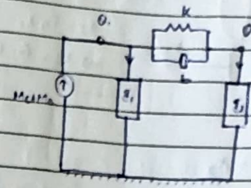
- Motor shaft displacement θ_1 and head displacement is θ_2.



Node θ_1:

$$M_c + M_D = J_1\frac{d^2\theta_1}{dt^2} + K[\theta_1 - \theta_2] + b\frac{d}{dt}(\theta_1 - \theta_2) \rightarrow (1)$$

Node θ_2:-

$$K[\theta_1 - \theta_2] + b\frac{d}{dt}(\theta_1-\theta_2) = J_2\left(\frac{d^2\theta_2}{dt^2}\right)$$

Neglect disturbing torque $\rightarrow M_c = J_1\frac{d^2\theta_1}{dt^2} + K[\theta_1-\theta_2] + b\frac{d}{dt}(\theta_1-\theta_2) \rightarrow (2)$

LT & Simplify

$$\frac{\theta_2(s)}{M_c(s)} = \frac{K}{J_1J_2s^2\left(s^2 + \frac{K}{J_1} + \frac{K}{J_2}\right)}$$

$$\frac{\theta_1(s)}{M_c(s)} = \frac{J_2s^2 + K}{J_1J_2s^2\left(s^2 + \frac{K}{J_1} + \frac{K}{J_2}\right)}$$

Block diagram reduction techniques:-

Every system can be represented by a block diagram which is the interconnection of several blocks.

The complex block diagram can be reduced to single block using block diagram reduction techniques.

Rules for Block diagram reduction:-

$G(s) = G$
$H(s) = H$
$X(s) = X$
} all in s domain

$y(t) = g(t) * x(t)$

↳ Impulse response

$Y(s) = G(s) \cdot X(s)$

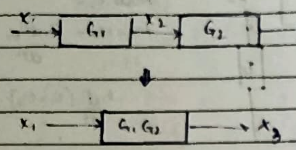
* Convolution in time domain is equal to multiplication in freq. domain

Blocks in cascade :-

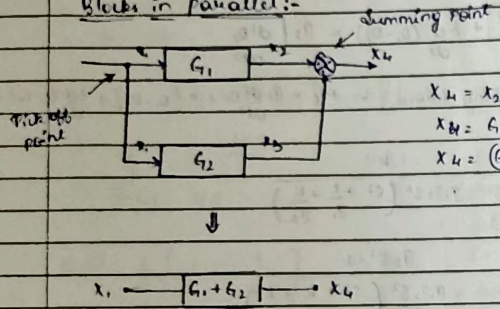
$x_3 = G_2 x_2$

$x_2 = G_1 x_1$

$\Rightarrow x_3 = G_2 G_1 x_1$



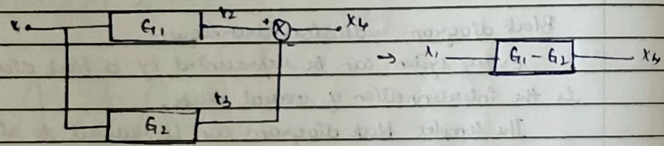
Blocks in parallel :-



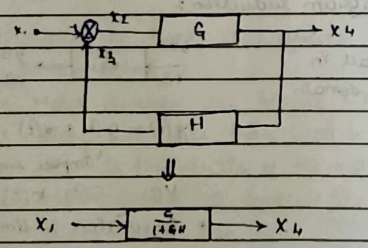
$x_4 = x_2 + x_3$

$x_4 = G_1 x_1 + G_2 x_1$

$x_4 = (G_1 + G_2) x_1$



Feedback loop :-



$x_4 = G x_2$

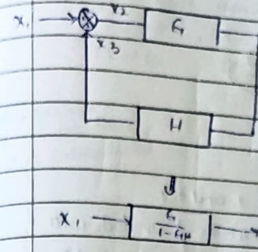
$x_4 = G(x_1 - x_3)$

$x_4 = G(x_1 - H x_4)$

$x_4 [1 + GH] = G x_1$

$\frac{x_4}{x_1} = \frac{G}{1+GH}$

$x_4 = \frac{G}{1+GH} \cdot x_1$



$x_4 = G x_2$

$x_4 = G(x_1 + x_3)$

$x_4 = G(x_1 + H x_4)$

$x_4 (1 - GH) = G x_1$

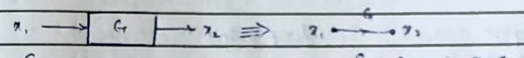
$x_4 = \frac{G}{1 - GH} x_1$

* Absent

27/9/23 Signal Flow Graph [SFG]

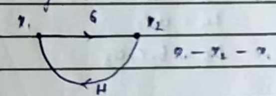
Alternative representation of block diagram.

Terminologies:



$G \rightarrow$ Block gain $\qquad G \rightarrow$ Branch gain

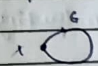
loop: originates and terminates on the same node.



$x_1 - x_2 - x_1$

loop gain $L = GH$

Self loop:



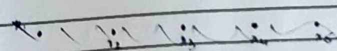
Example: A control system is described by the following set of eq^ns. Construct the signal flow graph

$x_2 = a_1 x_1 + a_2 x_3 + a_3 x_4$

$x_3 = b_1 x_2 + b_2 x_3 + b_3 x_4$

$x_4 = c_1 x_3 + c_2 x_5 + c_3 x_1$

$x_5 = d_1 x_4 + d_2 x_1$



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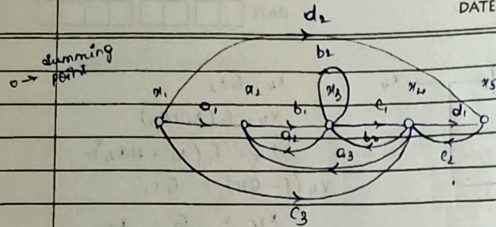

x_1 = i/p
x_5 = o/p

Forward paths b/w x_1 and x_5:

FP1: $x_1 - x_2 - x_3 - x_4 - x_5$
FP2: $x_1 - x_4 - x_5$
FP3: $x_1 - x_5$

} No node will repeat in FP

Gain of forward paths :-

$M_1 = a_1 b_1 c_1 d_1$

$M_2 = c_3 d_1$

$M_3 = d_2$

loops:-

	loop gains
1. $x_2 - x_3 - x_2$	$L_1 = b_1 a_2$
2. $x_3 - x_3$	$L_2 = b_2$
3. $x_3 - x_4 - x_3$	$L_3 = c_1 b_3$
4. $x_4 - x_5 - x_4$	$L_4 = d_1 c_2$
5. $x_2 - x_3 - x_4 - x_2$	$L_5 = b_1 c_1 a_3$

Two non touching loops:-

Two loops are said to be non touching if they do not share a common node

1. L_1 & L_4
2. L_2 & L_4

Gain product of non-touching loops:

$L_1 L_4 = b_1 a_2 d_1 c_2$

$L_2 L_4 = b_2 d_1 c_2$

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Mason's gain formula:-

$$\frac{\text{LT of o/p}}{\text{LT of i/p}} = \frac{C(s)}{R(s)} = \frac{\sum_{k=1}^{n} M_k \Delta_k}{\Delta}$$

$n \rightarrow$ No. of forward paths b/w C(s) & R(s)

$m \rightarrow$ gain of fwd paths

$$= \frac{m_1\Delta_1 + m_2\Delta_2 + \dots + m_n\Delta_n}{\Delta}$$

$\Delta \rightarrow$ Determinant of SFG

$\Delta = 1 - \{$ {Sum of individual loop gains} + {Sum of gain products of two non-touching groups} + {Sum of " — three non touching groups} + ... $\}$

Δ_1 = Det of FP1 , Δ_2 = Det of FP2

$$\frac{x_5}{x_1} = \frac{\sum_{k=1}^{3} m_k \Delta_k}{\Delta} = \frac{m_1\Delta_1 + m_2\Delta_2 + m_3\Delta_3}{\Delta}$$

$\Delta = 1 - \{(L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_4 + L_2 L_4)\}$

$= 1 - \{(b_1 a_2 + b_2 + c_1 b_3 + d_1 c_2 + b_1 c_1 a_3) + (b_1 a_2 d_1 c_2 + b_2 d_1 c_2)\}$

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Mason's gain formula:-

$$\frac{LT \text{ of o/p}}{LT \text{ of i/p}} = \frac{C(S)}{R(S)} = \frac{\sum_{k=1}^{n} M_k \Delta_k}{\Delta}$$

$n \rightarrow$ No. of forward paths b/w $C(S)$ & $R(S)$

$m \rightarrow$ gain of fwd paths

$\Delta \rightarrow$ Determinant of SFG

$$= \frac{m_1\Delta_1 + m_2\Delta_2 + \dots + m_n\Delta_n}{\Delta}$$

$\Delta = 1 - \{$Sum of individual loop gains$\}$ + $\{$Sum of gain products of two non-touching groups$\}$ + $\{$Sum of " — three non touching groups$\}$ + ... $\}$

$\Delta_1 =$ Det of FP1 , $\Delta_2 =$ Det of FP2

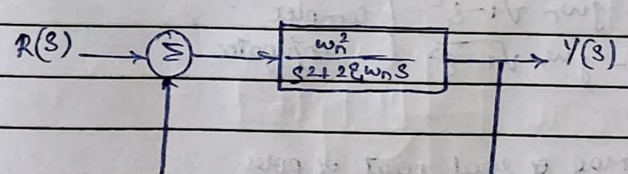
$$\frac{x_5}{x_1} = \frac{\sum_{k=1}^{3} m_k\Delta_k}{\Delta} = \frac{m_1\Delta_1 + m_2\Delta_2 + m_3\Delta_3}{\Delta}$$

$$\Delta = 1 - \{(L_1+L_2+L_3+L_4+L_5) + (L_1L_4 + L_3L_4)\}$$

$$= 1 - \{(b_1a_2 + b_2 + c_1b_3 + d_1c_2 + b_1c_1a_3) + (b_1a_2d_1c_2 + b_2d_1c_2)\}$$

→x— Online class —x—x—x—x—x—

4/10/23 **Standard second order system:**



Single loop unity f/b system.

w_n = Natural frequency (Resonant freq.)

ξ = Damping ratio
⇓
loss in the system

ele R , mech friction

$$\frac{Y(S)}{R(S)} = \frac{\frac{w_n^2}{S^2+2\xi w_n S}}{1+\left(\frac{w_n^2}{S^2+2\xi w_n S}\right)} = \frac{w_n^2}{S^2+2\xi w_n S + w_n^2}$$

- Poles = $S^2 + 2\xi w_n S + w_n^2 = 0$
- Nature of poles (real or complex) is decided by the value of ξ

$$S_1, S_2 = \frac{-2\xi w_n \pm \sqrt{4w_n^2\xi^2 - 4w_n^2}}{2} = -\xi w_n \pm w_n\sqrt{\xi^2-1}$$

Case 1: $\xi = 0$ [No damping]

$S_1, S_2 = \pm w_n\sqrt{-1} = \pm j w_n$ → Roots are complex conjugates & purely img.

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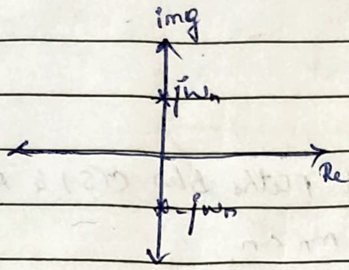

Impulse response

$r(t) = \delta(t)$

$R(S) = 1$

LT^{-1} of $\frac{a}{s^2+a^2} = \sin at$

$$\frac{Y(S)}{R(S)} = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$y(t) = h(t) = \omega_n \sin\omega_n t$

$r(t) = \delta(t) \rightarrow$ [ ] $\rightarrow y(t) = \omega_n \sin\omega_n t$

$\therefore$ Response is oscillatory.

Case II:- $0 < \xi < 1$ underdamped

$\boxed{\xi = 0.8, 0.5}$ $\quad \xi = 0.7 < 1$

$\xi^2 = 0.49 << 1$

$$S_1, S_2 = -\xi\omega_n \pm \omega_n \underbrace{\sqrt{\xi^2-1}}_{-ve}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{-(1-\xi^2)}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$\left.\begin{aligned} S_1 &= -\xi\omega_n + j\omega_n\sqrt{1-\xi^2} \\ S_2 &= -\xi\omega_n - j\omega_n\sqrt{1-\xi^2} \end{aligned}\right\}$$ complex conjugates

$\xi\omega_n = \alpha$ = Mag of real part of poles

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\xi = 0$; $\omega_d = \omega_n$ No damping

$S_1 = -\alpha + j\omega_d$

$S_2 = -\alpha - j\omega_d$

$$\frac{Y(S)}{R(S)} = \frac{\omega_n^2}{(S-S_1)(S-S_2)} = \frac{\omega_n^2}{(S+\alpha-j\omega_d)(S+\alpha+j\omega_d)}$$

$$= \frac{\omega_n^2}{(S+\alpha)^2+\omega_d^2}$$

$(a+ib)(a-ib) = a^2+b^2$

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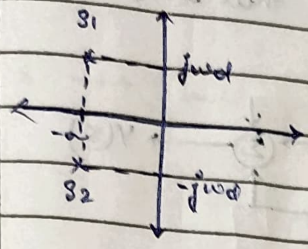


Impulse response : $\omega_d = \omega_n\sqrt{1-\xi^2}$

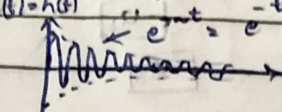
$r(t) = \delta(t)$ $\Rightarrow \left(\frac{\omega_n}{\omega_d}\right)^2 = \frac{1}{1-\xi^2}$

$R(s) = 1$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{(s+\alpha)^2+\omega_d^2} = \frac{\omega_n^2}{\omega_d^2}\frac{\omega_d^2}{(s+\alpha)^2+\omega_d^2}$$

$$y(t) = h(t) = \frac{1}{1-\xi^2} \cdot e^{-\alpha t}\sin\omega_d t \quad , t \geq 0$$

$e^{-\alpha t} = e^{-t/\tau} \Rightarrow \tau = \frac{1}{\alpha}$



under damped response

Ex. $\omega_n = 10$ rad/s. Find s_1 & s_2 for $\xi = 0.707$, $\xi = 0.5$, $\xi = 0.3$

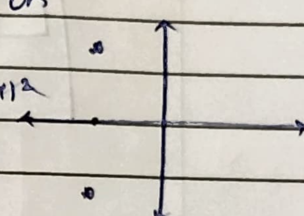
Locate poles in s-plane. Discuss the effect of ξ on

$\Rightarrow$ 1. $\xi = 0.707$

$\Rightarrow \frac{100}{(s-0.707)^2+(0.707)^2}$

$s_1 = -7.07 + j\,7.07$

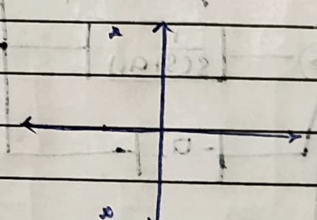
$s_2 = -7.07 - j\,7.07$



2. $\xi = 0.5$

$s_1 = -5 + j\,8.66$

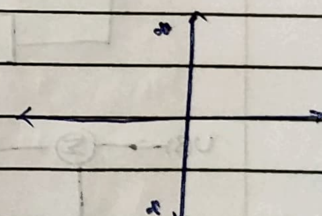
$s_2 = -5 - j\,8.66$



3. $\xi = 0.3$

$s_1 = -3 + j\,9.53$

$s_2 = -3 - j\,9.53$



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Case 3:- $\xi = 1$ Critically damped

$$s_1, s_2 = -\underbrace{\xi\omega_n}_{\alpha} \pm j\underbrace{\omega_n\sqrt{1-\xi^2}}_{0}$$

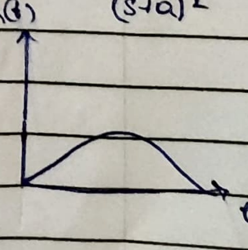
$s_1, s_2 = -\omega_n$ $\rightarrow$ Repeated roots

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s-s_1)(s-s_2)} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$t e^{-at} = \frac{1}{(s+a)^2}$

$r(t) = \delta(t) \Rightarrow R(s) = 1$

$\therefore C(s) = \frac{\omega_n^2}{(s+\omega_n^2)} \overset{LT}{\Longleftrightarrow} \boxed{c(t) = h(t) = \omega_n t e^{-\omega_n t}, \ t \geq 0}$



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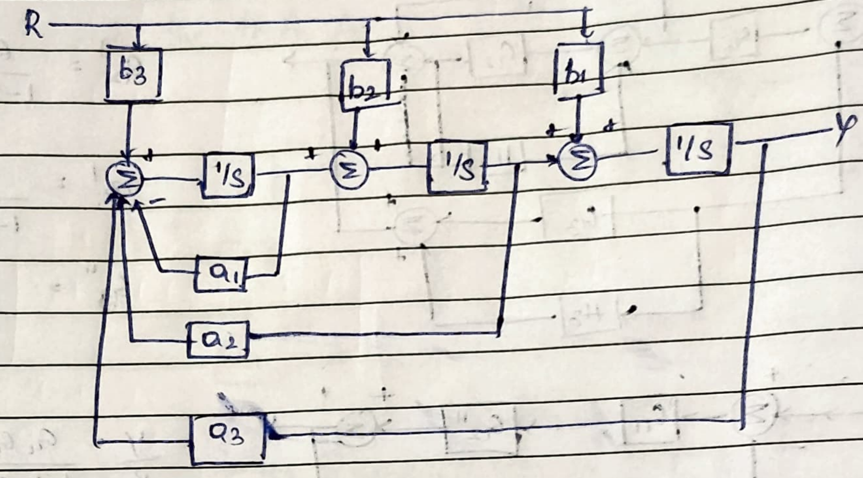
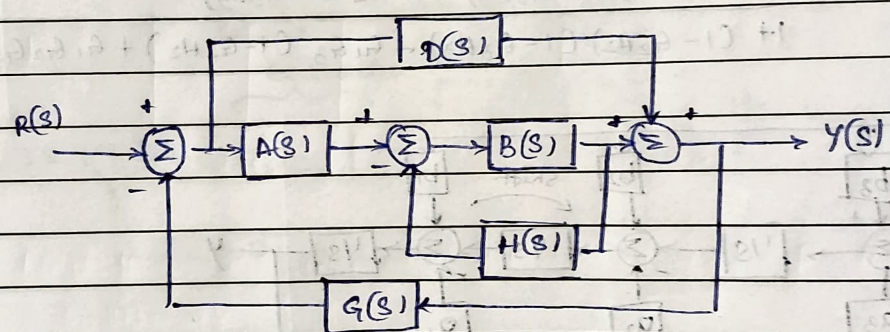
c.



$$Y/R = \frac{b_1 s^2 + (a_1 b_1 + b_2)s + a_1 b_2 + a_2 b_1 + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

d.



$$Y/R = \frac{D(S) + D(S)B(S)H(S) + A(S)B(S)}{1 + H(S)B(S) + G(S)D(S) + G(S)B(S)D(S)H(S) + G(S)A(S)B(S)}$$

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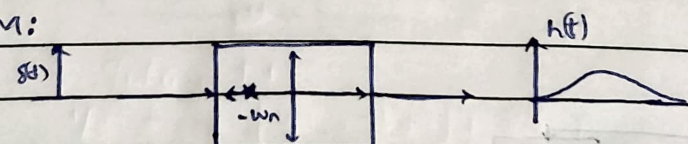
UBM:



Case 4: $\xi > 1$

$S_1, S_2 = -\xi \omega_n \pm j\omega_n \sqrt{1-\xi^2}$ $\hookrightarrow > 1$

$S_1, S_2 = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1}$

Ex: $\omega_n = 10$ $\rightarrow$ $S_1, S_2 = -20 \pm 10\sqrt{3}$

$\xi = 2$

$\therefore S_1 = -20 + 10\sqrt{3} = -2.67$

$S_2 = -20 - 10\sqrt{3} = -37.3$

classmate

Real, -ve & distinct

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$$\frac{C(s)}{R(s)} = \frac{100}{(s+2.679)(s+37.3)}$$

$r(t) = \delta(t)$

$\Rightarrow R(s) = 1$

$$C(s) = \frac{100}{(s+2.679)(s+37.3)} = \frac{A}{(s+2.679)} + \frac{B}{(s+37.3)}$$

$\Rightarrow 100 = A(s+37.3) + B(s+2.679)$

$\Rightarrow s = -2.679 \Rightarrow 100 = A(34.621)$

$$\therefore A = \frac{100}{34.621} = 2.888$$

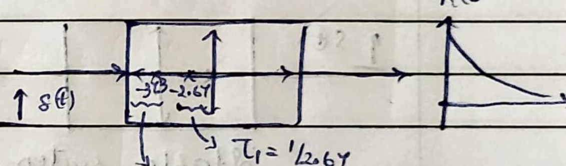
$s = -37.3 \Rightarrow 100 = B(-34.621)$

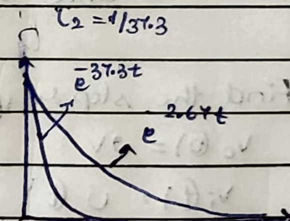
$\therefore B = -2.888$

$$c(t) = 2.88\left[e^{-2.679t} + e^{-37.3t}\right], \quad t \geq 0 = h(t).$$

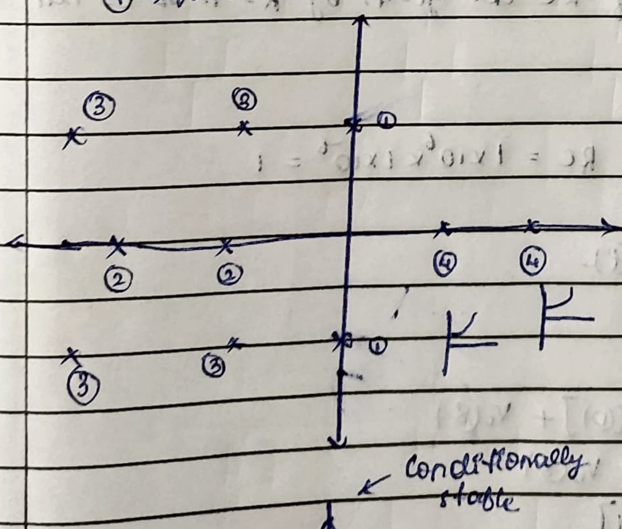
$h(t)|_{t=0} = 0$

$h(t)|_{t=\infty} = 0$





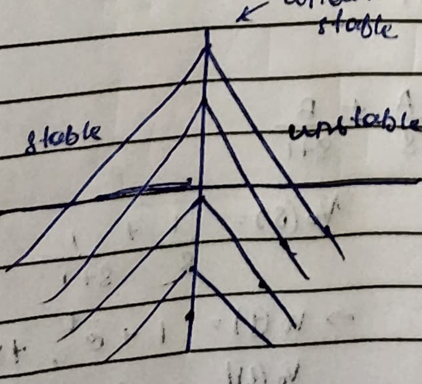
Exercise: write response for poles shown below.
① → question no.



① → Conditionally stable

②, ③ → Stable

④ → unstable



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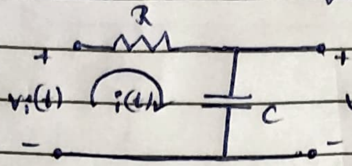
Practical I order systems:



KVL:

$V_i(t) = R\,i(t) + V_o(t) \rightarrow ①$

$V_o(t) = \frac{1}{C}\int i(t)\,dt$

$\frac{dV_o(t)}{dt} = \frac{1}{C}\,i(t)$

$i(t) = C\frac{dV_o}{dt} \rightarrow ②$

Substitute ② in ①

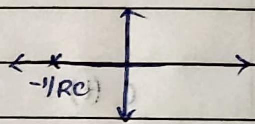
$\boxed{V_i(t) = R.C\frac{dV_o}{dt} + V_o(t)} \rightarrow ③$

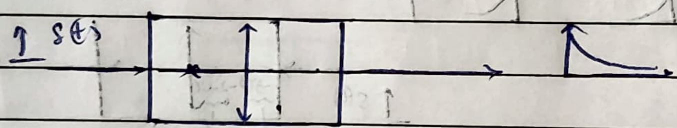
Take LT

$V_i(s) = RCS\,V_o(s) + V_o(s)$ → zero initial condition

$V_o(s)[1+RCS] = V_i(s)$ ∴ TF fn

$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCS} = \frac{1/RC}{S+\frac{1}{RC}}$





Stable system.

Find the step response of RC ckt given by $R = 1M\Omega$ $C = 1\mu F$ $V_c(0) =$

$V_o(0) = 2V$

→ $V_i(t) = u(t)$

∴ $V_i(s) = 1/s$

$RC = 1\times10^6 \times 1\times10^{-6} = 1$

③ → $V_i(t) = \frac{dV_o(t)}{dt} + V_o(t)$.

Taking LT

$1/s = [s[V_o(s) - V_o(0)] + V_o(s)$

$= sV_o(s) - 2 + V_o(s)$

$2 + 1/s = V_o(s)[s+1]$

$V_o(s) = \frac{2s+1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$2s+1 = A(s+1) + B(s)$

$s=0$: $1 = A$ → $\boxed{A=1}$

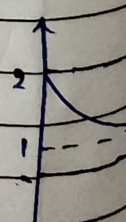
$s=-1$: $-1 = -B$ → $\boxed{B=1}$

$V_o(s) = \frac{1}{s} + \frac{1}{s+1}$

→ $V_o(t) = 1 + e^{-t}$, $t \geq 0$

$V_o(t)|_{t=0} = 2$

$V_o(t)|_{t=\infty} = 1$



$O(s) = \frac{1}{9}\left[\frac{1}{s} - \frac{s}{s^2+3}\right]$

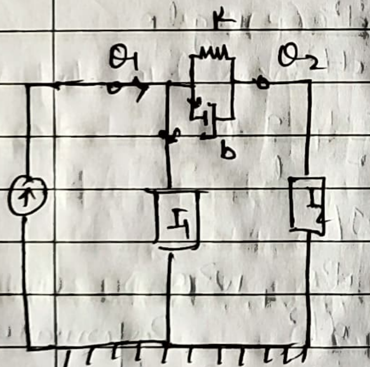
take inverse LT

$O(t) = 1/9\,[1 - \cos 3\omega t]$

→ **Modelling of Floppy disk drive:**

→ Use of disk drive

* Motor develops torque (T_c | M_c)
* along with this there will same disturbing torque (T_D | M_D).
* J_1 | I_1 is inertia of motor.
* Motor torque is coupled to head through a flexible shaft which has K, b
* J_2 | I_2 is the inertia of head
* Motor shaft displacement θ_1 & head displacement is θ_2.



Node θ_1

$$M_c + M_D = I_1 \frac{d^2\theta_1}{dt^2} + K[\theta_1 - \theta_2] + b\frac{d(\theta_1-\theta_2)}{dt} \quad -①$$

Node θ_2

$$K[\theta_1 - \theta_2] + b\frac{d(\theta_1-\theta_2)}{dt} = I_2 \frac{d^2\theta_2}{dt^2} \quad ②$$

take LT

simplify $\frac{\theta_2(s)}{M_c(s)}$, $\frac{\theta_1(s)}{M_c(s)}$

$$\frac{K}{I_1 I_2 s^2\left(s^2 + \frac{K}{I_1} + \frac{K}{I_2}\right)}$$

$$\frac{\theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + K}{I_1 I_2 s^2\left(s^2 + \frac{K}{I_1} + \frac{K}{I_2}\right)}$$

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Tutorials: Problems related Electrical, Mechanical & Electromechanical System.

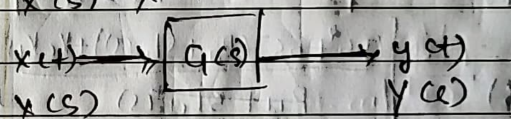
Block Diagram: Reduction Technique

Every system can be represented by block diagram which is the interconnection of several blocks.

The complex b.d. can be reduced to single block using block diagram reduction techniques.

Rules for block diagram Reduction:

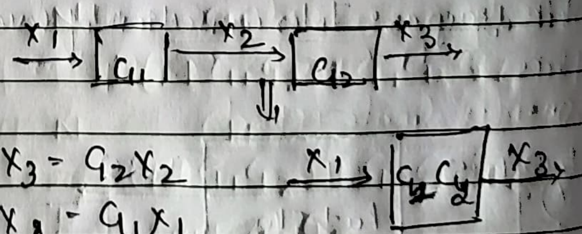
$G(s) = G$
$H(s) = H$ } All in S domain
$X(s) = X$



$y(t) = g(t) * x(t)$ → convolution operator.
I.R.

$Y(s) = G(s) \cdot X(s)$

Blocks in cascade:

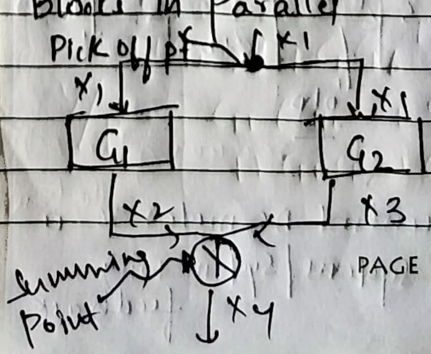


$X_3 = G_2 X_2$

$X_2 = G_1 X_1$

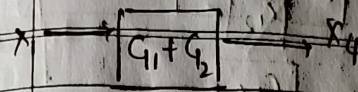
$X_3 = G_2 G_1 X_1$

Blocks in Parallel

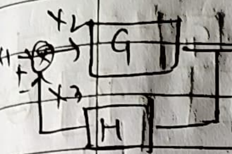


$x_4 = x_2 + x_3$

$x_4 = G_1 x_1 + G_2 x_1 = x_1 (G_1 + G_2)$



→ Feedback loop:



$x_4 = x_2 G$

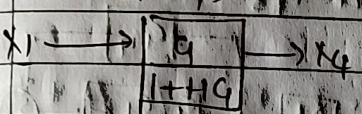
$x_3 = H x_4$

$x_4 = x_2 G$

$= G(x_1 - x_3)$

$x_4 [1 + HG] = G x_1 = G(x_1 - H x_4)$

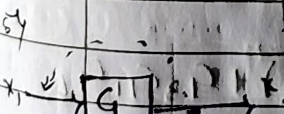
$x_4 = \left(\frac{G}{1+HG}\right) x_1$



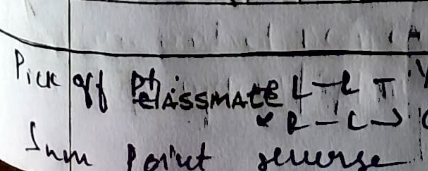
→ Rules for Block Diagram Reduction

1. Blocks in cascade
2. Block in Parallel
3. Feedback loop
4. Shifting pick-off point after a block
5. — " — behind — " —
6. Shifting summing

4) Pick off point



5)

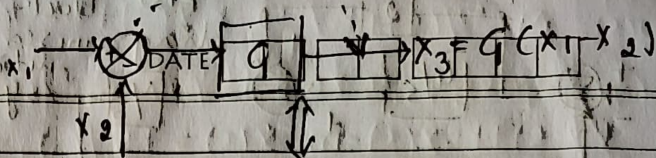


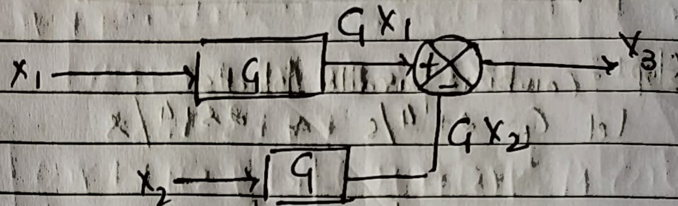
Pick off pt L → R $1/G$

R → L → G

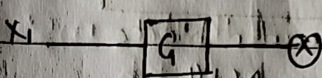
Sum point reverse

6) Shifting summing point after a block.



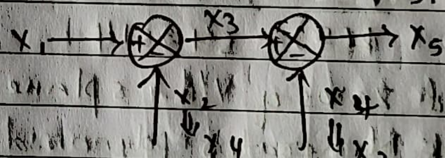


7) Shifting summing point behind a block:



8) Re-arranging the summing points.

$s_2 \leftrightarrow s_1$ $s_2 \rightarrow s_1$



$x_5 = x_3 - x_4$

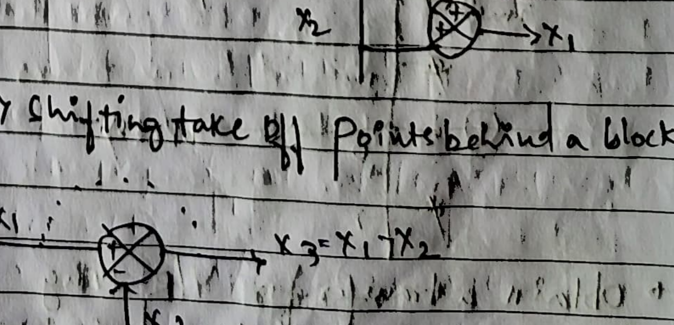
$x_3 = x_1 - x_2$

$x_5 = [x_1 - x_4] - x_2$

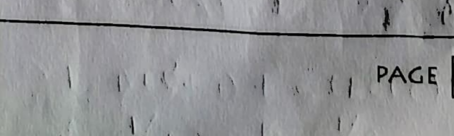
$x_5 = x_1 - x_2 - x_4$

↳ interchange summing pts

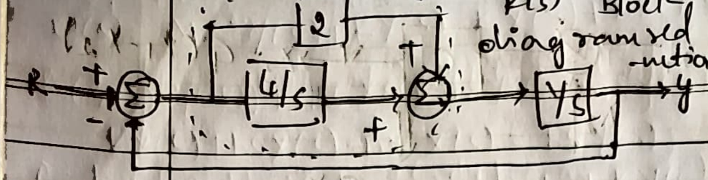
9) Shifting take off points after a block.



10) Shifting take off points behind a block

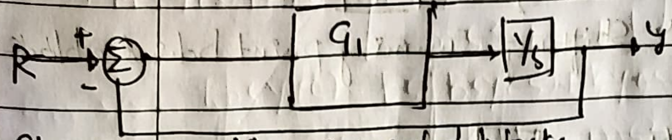


Problems: obtain $\frac{Y(s)}{R(s)}$ using Block diagram reduction



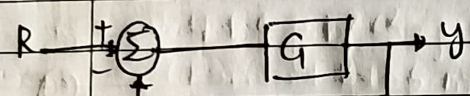
Step 1: combine parallel blocks

Let $G_1 = 2 + 4/s$ & $2s+4/s$



Step 2: combine cascaded blocks

Let $G = \frac{G_1}{s} = \frac{2s+4}{s^2}$



$H = 1$

$$\frac{Y}{R} = \frac{G}{1+GH} = \frac{(2s+4)/s^2}{1+(2s+4)/s^2}$$

$$\frac{Y(s)}{R(s)} = \frac{2s+4}{s^2+2s+4}$$

locate poles and zeros of Y/R in s plane

zero → Root of Numerator polynomial

Pole → Root of Denominator

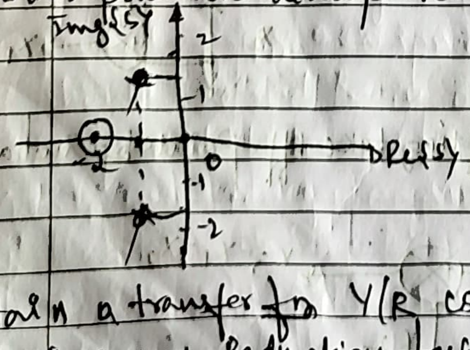
→ $2s+4=0 \Rightarrow s=-2$

→ $s^2+2s+4=0$ $\quad s = -1 \pm \sqrt{3}j$

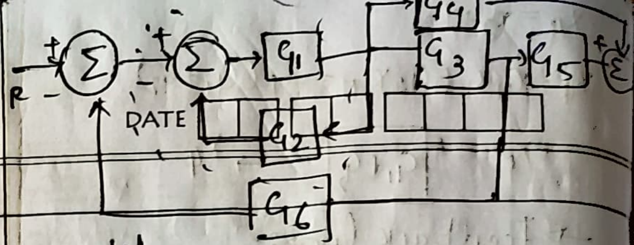
Complex conjugate Poles

* A complex pole/zero must be accompanied by its conjugate.

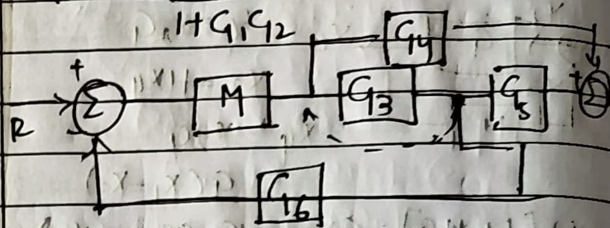
Because the coefficient in N & Denominator poles are always real



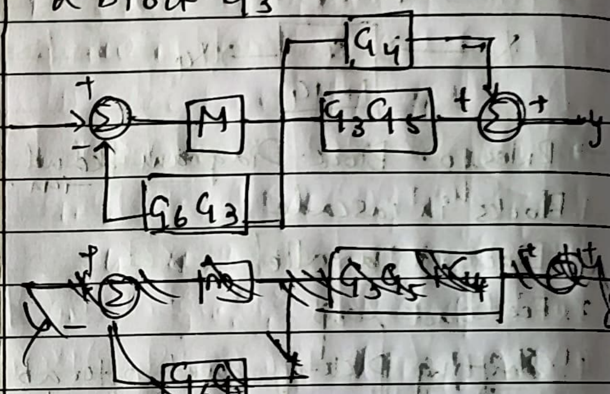
→ obtain a transfer fn Y/R using block diagram reduction technique



Step 1: Let $M = \frac{G_1}{1+G_1G_2}$



Step 2: Shift pick up point behind a block G_3



Step 3: Eliminate feedback loop

$$N = \frac{M}{1+MG_3G_6} = \frac{\frac{G_1}{1+G_1G_2}}{1+\frac{G_1G_3G_6}{1+G_1G_2}}$$

$$N = \frac{G_1}{(1+G_1G_2)+G_1G_3G_6}$$

$$N = \frac{G_1}{(1+G_1G_2)+G_3G_1G_6}$$

$$P = G_4 + G_3G_5$$

$$S = \frac{G_1}{1+G_1G_2+G_3G_1G_6} + G_4G_3G_5$$

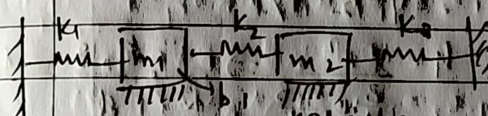
$$Y/R = \frac{G_1(G_4+G_3G_5)}{1+G_1G_2+G_1G_3G_6}$$

$\Sigma f_n = \Sigma f_o$

$k_2(y-x_2) - b_2\frac{dx_2}{dt} = m_2\frac{d^2x_2}{dt^2} + k_2(x_2-x_1)$ → (1)

$k_2(x_2-x_1) = b_1\frac{dx_1}{dt} + k_1x_1 + m_1\frac{d^2x_1}{dt^2}$ → (2)

4)

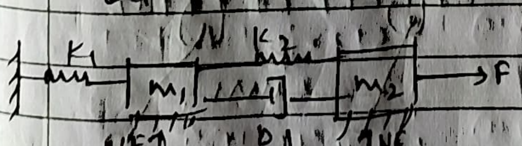


Sol.

Q: $m_1\frac{d^2x_1}{dt^2} = -k_1x_1 - k_2(x_1-x_2) - b_1\frac{dx_1}{dt}$

$k_2(x_2-x_1) = -\left(m_2\frac{d^2x_2}{dt^2} + k_3x_2\right)$

$m_2\frac{d^2x_2}{dt^2} = -k_3x_2 + k_2(x_2-x_1)$



Sol: $m_1\frac{d^2x_1}{dt^2} = -k_1x_1 - k_2(x_1-x_2) - b_1\left[\frac{dx_1}{dt} - \frac{dx_2}{dt}\right]$

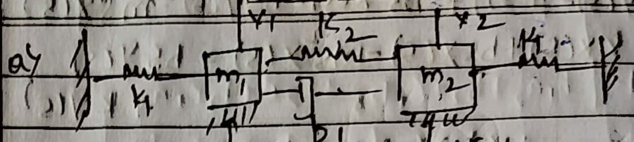
$m_2\frac{d^2x_2}{dt^2} = F - k_2(x_2-x_1) - b_1\left[\frac{dx_2}{dt} - \frac{dx_1}{dt}\right]$

classmate

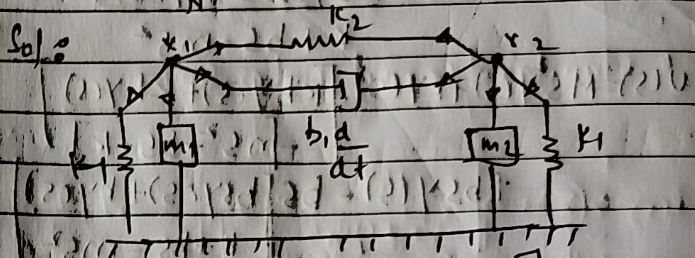
2) Write a differential eqn for mechanical system shown, to given that there both no mass -les any

DATE

a)



Sol:



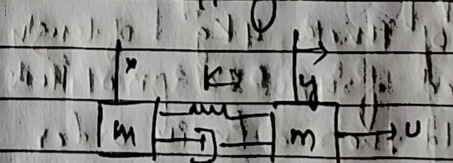
$-k_1x_1 - k_2(x_1-x_2) - b_1\left[\frac{dx_1}{dt} + \frac{dx_2}{dt}\right] = m_1\frac{d^2x_1}{dt^2}$

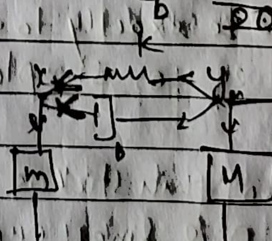
$m_2\frac{d^2x_2}{dt^2} = -k_2(x_2-x_1) - k_1x_2 - b_1\left[\frac{dx_2}{dt} - \frac{dx_1}{dt}\right]$

3) In many mechanical position system there is flexibility b/w one part of system & another. fig Q3 depicts such a situation where a force u has applied to mass M and another mass m is connected to it. The coupling b/w objects is often modelled by spring constant k with damping coefficient b although the actual situation is usually much more complicated than this.

(a) Write the eq. of motion Governing this system.

b) Find the transfer fn b/w control i/p u and o/p y.





Sol: $m\frac{d^2x}{dt^2} = -k(x-y) + b\left[\frac{dx}{dt} - \frac{dy}{dt}\right]$

$M\frac{d^2y}{dt^2} = k(y-x) - b\left[\frac{dy}{dt} - \frac{dx}{dt}\right]$

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