Problem 1

Construct the Nyquist's plot of a system having $G(s)H(s) = \frac{K}{s(s+1)(s+2)}$ and find the range of values of K for which the closed loop system is stable.

Solution: $G(s)H(s) = \frac{K}{s(s+1)(s+2)}$.

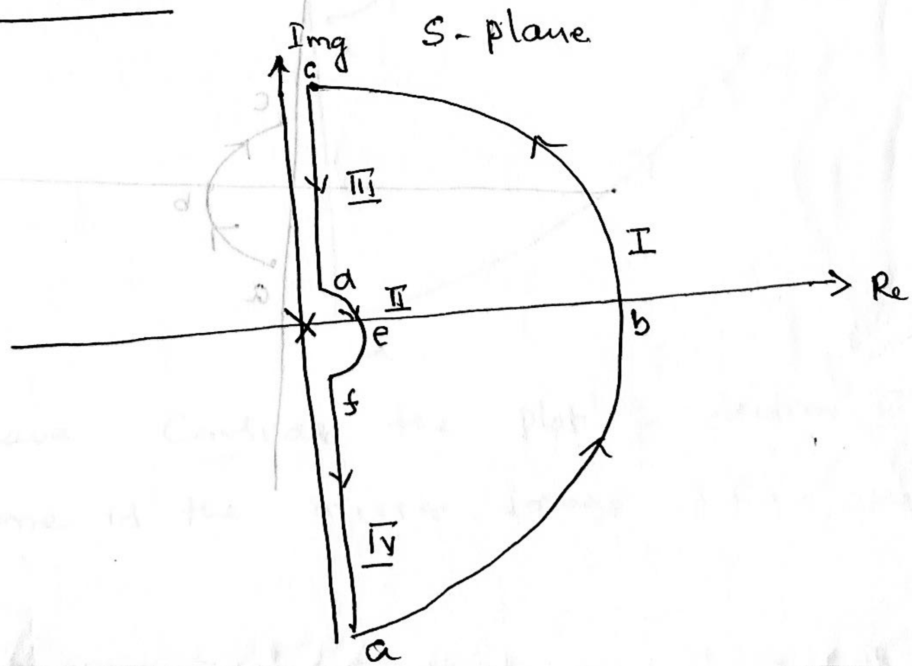
Open loop poles: $s = 0, -1, -2$

All poles lie in the LH of S-plane.

$\Rightarrow$ openloop system is stable.

Also $\boxed{P = 0}$ $\because$ $G(s)H(s)$ has No poles in R-H of S-plane.

Nyquist's path



Nyquist's plot

Section - I

Path : $a \to b \to c$

$S = R e^{j\theta}$; $R \to \infty$.

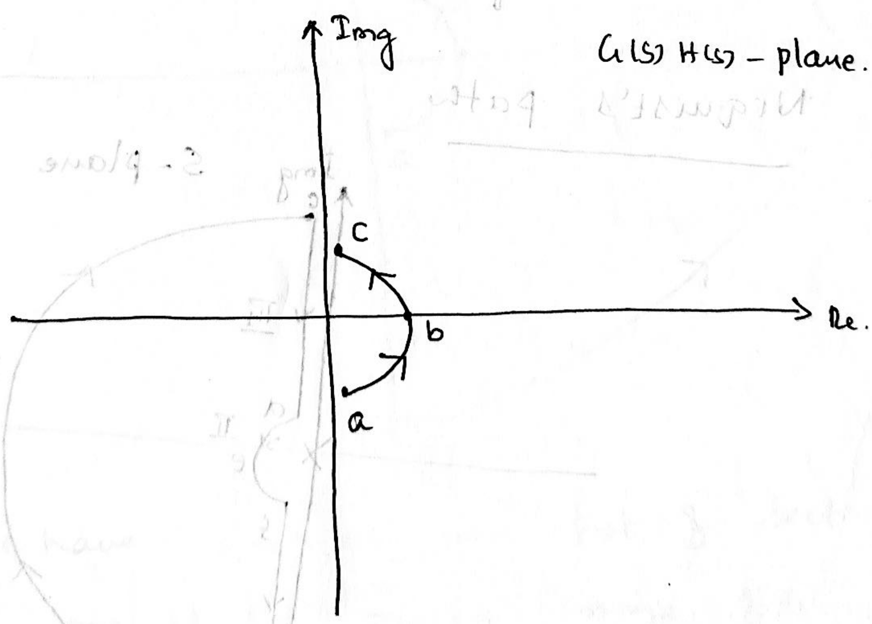
$$G(s)H(s) = \frac{k}{S(S+1)(S+2)} = \frac{k}{Re^{j\theta}(Re^{j\theta}+1)(Re^{j\theta}+2)}$$

$$= \frac{k}{\infty\, e^{j3\theta}} = 0\, e^{-j3\theta}.$$

Point a : $\theta = -90^\circ \Rightarrow G(s)H(s) = 0\, e^{j270^\circ} = 0 \angle 270^\circ$

Point b : $\theta = 0 \Rightarrow G(s)H(s) = 0\, e^{j0} = 0 \angle 0^\circ$

point c $\theta = 90^\circ \Rightarrow G(s)H(s) = 0\, e^{-j270^\circ} = 0 \angle -270^\circ$



Section - II

path: $d \to e \to f$

$s = r e^{j\phi}$; $r \to 0$

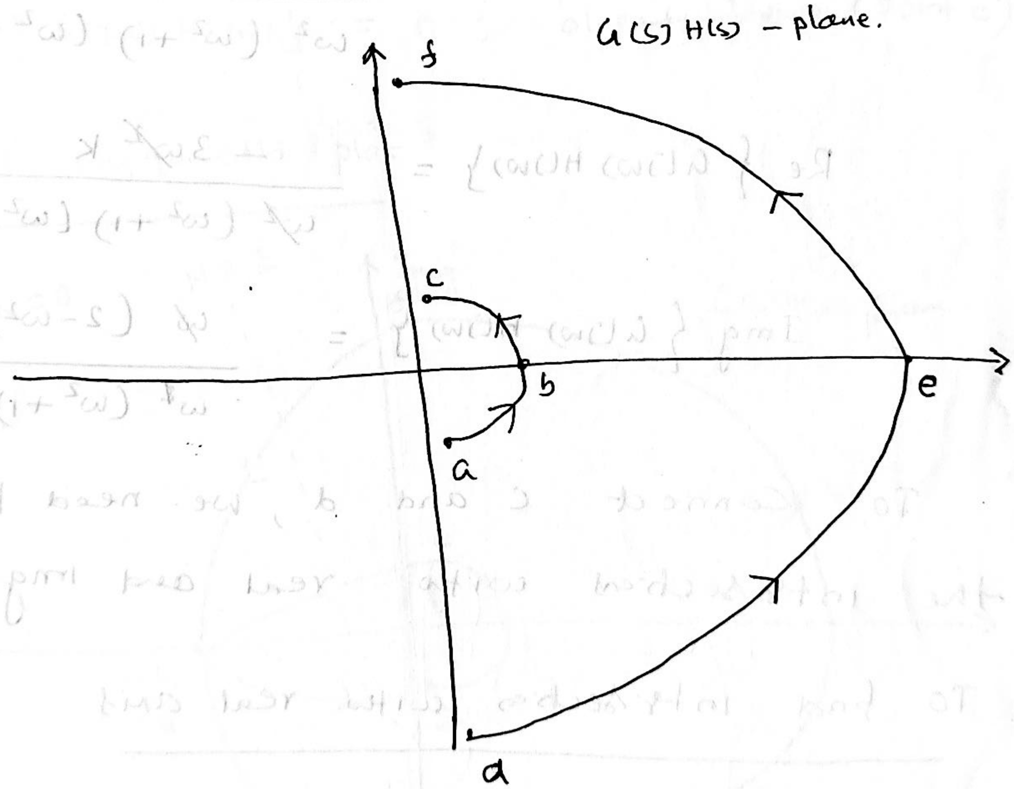
$$G(s)H(s) = \frac{K}{s(s+1)(s+2)} = \frac{k}{re^{j\phi}(re^{j\phi}+1)(re^{j\phi}+2)}$$

$$= \infty e^{-j\phi}$$

point d: $\phi = 90^\circ \Rightarrow G(s)H(s) = \infty e^{-j90^\circ} = \infty \angle -90^\circ$

point e: $\phi = 0 \Rightarrow G(s)H(s) = \infty \angle 0^\circ$

point f: $\phi = -90^\circ \Rightarrow G(s)H(s) = \infty \angle 90^\circ$



Now we have Consider the plot of Section III or IV, $\because$ one is the mirror image of the other.

Section - III

path: c $\rightarrow$ d.

$s = j\omega$, since the path is along $j\omega$ axis.

$$G(j\omega)\,H(j\omega) = \frac{K}{j\omega\,(j\omega+1)\,(j\omega+2)}$$

Rationalise the denominator.

$$G(j\omega)\,H(j\omega) = \frac{K\,(-j\omega)\,(-j\omega+1)\,(-j\omega+2)}{\omega^2\,(\omega^2+1)\,(\omega^2+4)}$$

$$= \frac{-3\omega^2 + j\omega\,(2-\omega^2)}{\omega^2\,(\omega^2+1)\,(\omega^2+4)}\,K$$

$$Re\,\{G(j\omega)\,H(j\omega)\} = \frac{-3\cancel{\omega^2}\,K}{\cancel{\omega^2}\,(\omega^2+1)\,(\omega^2+4)} \quad - (1)$$

$$Img\,\{G(j\omega)\,H(j\omega)\} = \frac{\cancel{\omega}\,(2-\omega^2)\,K}{\omega^{\cancel{2}}\,(\omega^2+1)\,(\omega^2+4)} \quad - (2)$$

To connect c and d, we need to find the intersections with real and img axes.

To find intersection with real axis

$$Img\,\{G(j\omega)\,H(j\omega)\} = 0$$

$$\frac{\cancel{\omega}\,(2-\omega^2)\,K}{\omega^{\cancel{2}}\,(\omega^2+1)\,(\omega^2+4)} = 0 \Rightarrow \omega = \infty \text{ and}$$

$$2-\omega^2 = 0$$

$$\Rightarrow \omega = \sqrt{2}.$$

Section - III

path: c → d.

$s = j\omega$, Since the path is along $j\omega$ axis.

$$G(j\omega)\,H(j\omega) = \frac{k}{j\omega\,(j\omega+1)\,(j\omega+2)}$$

Rationalise the denominator.

$$G(j\omega)\,H(j\omega) = \frac{k\,(-j\omega)\,(-j\omega+1)\,(-j\omega+2)}{\omega^2\,(\omega^2+1)\,(\omega^2+4)}$$

$$= \frac{-3\omega^2 + j\omega\,(2-\omega^2)}{\omega^2\,(\omega^2+1)\,(\omega^2+4)}\,k$$

$$Re\left\{G(j\omega)\,H(j\omega)\right\} = \frac{-3\cancel{\omega^2}\,k}{\cancel{\omega^2}\,(\omega^2+1)\,(\omega^2+4)} \quad -(1)$$

$$Img\left\{G(j\omega)\,H(j\omega)\right\} = \frac{\cancel{\omega}\,(2-\omega^2)\,k}{\omega^{\cancel{2}}\,(\omega^2+1)\,(\omega^2+4)} \quad -(2)$$

To connect c and d, we need to find the intersections with real and img axes.

To find intersection with real axis

$$Img\left\{G(j\omega)\,H(j\omega)\right\} = 0$$

$$\frac{\cancel{\omega}\,(2-\omega^2)\,k}{\omega^{\cancel{2}}\,(\omega^2+1)\,(\omega^2+4)} = 0 \Rightarrow \omega = \infty \text{ and}$$

$$2-\omega^2 = 0$$

$$\Rightarrow \omega = \sqrt{2}.$$

when $\omega = \infty$

$Re\{G(j\omega)H(j\omega)\} = 0$; already shown (point c)

when $\omega = \sqrt{2}$

$$Re\{G(j\omega)H(j\omega)\} = \frac{-3K}{(2+1)(2+4)} = -\frac{K}{6}.$$

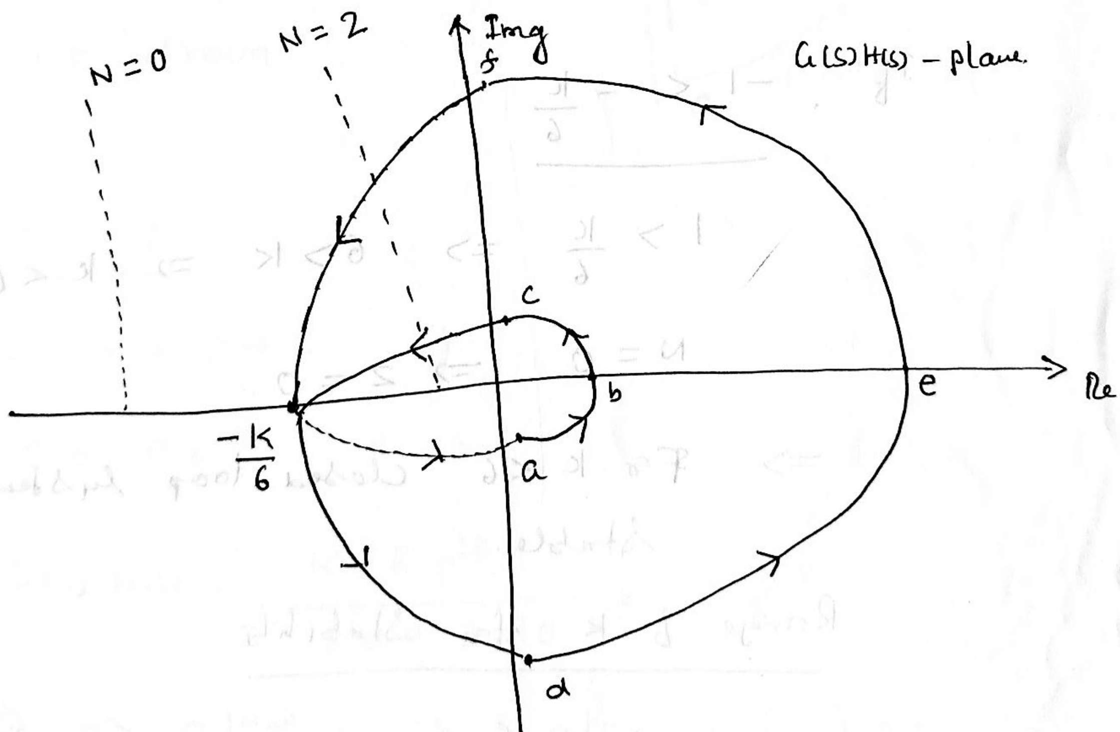
To find intersection with Img axis

$$Re\{G(j\omega)H(j\omega)\} = 0$$

$$\frac{-3K}{(\omega^2+1)(\omega^2+4)} = 0 \Rightarrow \omega = \infty.$$

$Img\{G(j\omega)H(j\omega)\} = 0$; already shown (point c)

Complete Nyquist plot



Stability of closed loop system

If $-1 > -\frac{K}{6}$

$1 < \frac{K}{6} \Rightarrow 6 < K \Rightarrow K > 6$

$N = 2$

But $N = Z - P$

$P = 0$ (No open loop Poles on R-H of S-plane)

$2 = Z - 0 \Rightarrow \boxed{Z = 2}$

$\Rightarrow$ For $K > 6$, $1 + G(s)H(s)$ has Two zeros on the RH of S-plane or $\frac{G(s)}{1+G(s)H(s)}$ has two poles on RH of S-plane.

Hence the closed loop system is unstable for $K > 6$

If $-1 < -\frac{K}{6}$

$1 > \frac{K}{6} \Rightarrow 6 > K \Rightarrow K < 6$

$N = 0 \Rightarrow Z = 0.$

$\Rightarrow$ For $K < 6$ closed loop system is Stable.

Range of K for Stability

$0 < K < 6.$

Problem 2 $\quad G(s)H(s) = \dfrac{k(s+2)}{(s+1)(s-1)}$

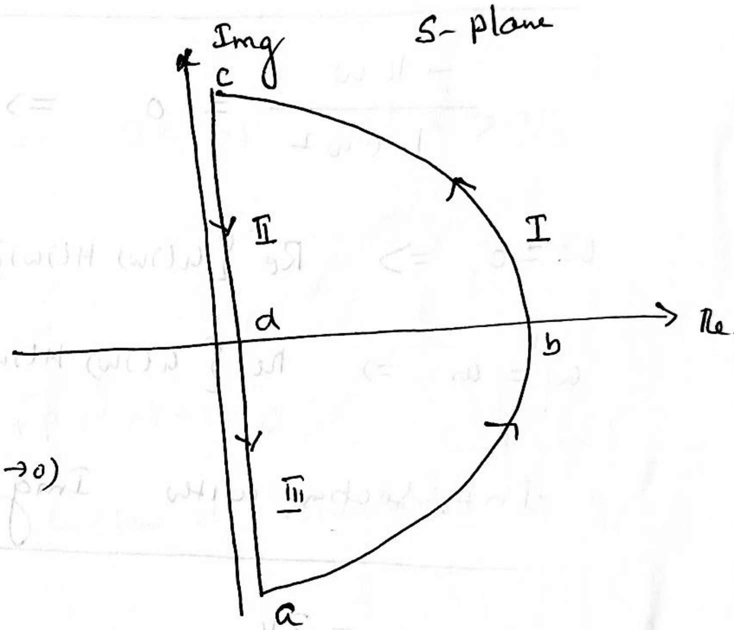
Solution:

open loop poles $\quad s = -1, 1.$

$G(s)H(s)$ has one pole in R-H of S plane.

$\therefore \boxed{P = 1} \Rightarrow$ open loop system is unstable.

Nyquist's path



Observe that, due to the absense of pole at $s = 0$ the semicircle $re^{j\phi}$ $(r \to 0)$ is not drawn.

Section I

Path $a \to b \to c$

$$s = Re^{j\theta} \; ; \; R \to \infty$$

$$G(s)H(s) = \frac{K \cdot Re^{j\theta}}{Re^{j\theta} \; Re^{j\theta}} = 0 \, e^{-j\theta}.$$

$a \Rightarrow 0 \angle 90^\circ \; ; \quad b \Rightarrow 0 \angle 0 \; ; \quad c \Rightarrow 0 \angle -90^\circ$

Section II

path $c \to d$.

$S = j\omega$.

$$Re\{G(j\omega) H(j\omega)\} = \frac{-2k}{1+\omega^2}$$

$$Img\{G(j\omega) H(j\omega)\} = \frac{-k\omega}{(1+\omega^2)}$$

Intersection with real axis

$$\frac{-k\omega}{1+\omega^2} \doteq 0 \Rightarrow \omega = 0 \text{ and } \omega = \infty.$$

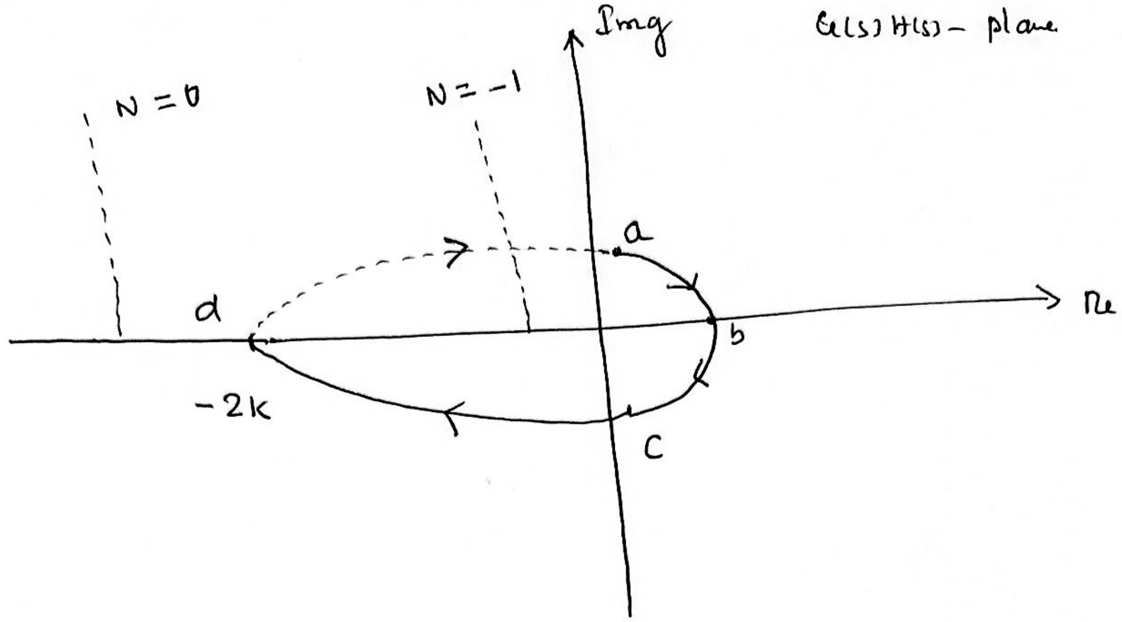
$\omega = 0 \Rightarrow Re\{G(j\omega) H(j\omega)\} = -2k$ (Point d)

$\omega = \infty \Rightarrow Re\{G(j\omega) H(j\omega)\} = 0$ (Point c)

Intersection with Img axis

$$\frac{-2k}{1+\omega^2} = 0 \Rightarrow \omega = \infty$$

$\omega = \infty \Rightarrow Img\{G(j\omega) H(j\omega)\} = 0$ (Point c)



Stability Analysis

For ; $-2K < -1 \Rightarrow 2k > 1 \Rightarrow k > 1/2$

$$N = -1$$

$$N = Z - P.$$

$$Z = N + P = -1 + 1 = 0$$

$\Rightarrow$ closed loop system is stable for $k > 1/2$

$-2k > -1 \Rightarrow 2k < 1 \Rightarrow k < 1/2.$

$$N = 0$$

$$Z = 0 + P = 1$$

$\Rightarrow$ closed loop system is unstable for $k < 1/2$.