

## Bode plot

(01)

- It is the plot of  $|G(j\omega)H(j\omega)|_{dB}$  and  $\angle G(j\omega)H(j\omega)$  as a function of  $\omega$
- The plot of  $|G(j\omega)H(j\omega)|_{dB}$  as a function of  $\omega$  is called the magnitude plot
- The plot of  $\angle G(j\omega)H(j\omega)$  as a function of  $\omega$  is called the phase plot.
- Both are plotted on the same semilog sheet
- The stability of closed loop system is determined by measuring Gain margin (GM) and phase margin (PM) from the plot.
- The critical point is frequency domain is  $(-1 + j0)$  point which is equivalent to  $1 \angle -180^\circ$  or  $0dB \angle -180^\circ$ .
- For example, if the gain of the system is  $-40dB$  and phase  $-180^\circ$  at same  $\omega$ , then there is a margin of  $40dB$ . i.e. if an additional gain of  $40dB$  is added to the system, then the gain becomes  $-40 + 40 = 0$  and phase  $-180^\circ$  and the system becomes critically stable.  
 $\therefore$  Gain margin is  $40dB$  in this case.
- on the other hand, if the gain of the system is  $0dB$  and phase  $-130^\circ$ , then the phase margin is  $50^\circ$ . why?

→ The frequency at which  $|G(j\omega)H(j\omega)|_{dB} = 0$  is called 'Gain cross over frequency' denoted by  $\omega_{gc}$ .

→ The frequency at which  $\angle G(j\omega)H(j\omega) = -180^\circ$  is called 'phase cross over frequency' denoted by  $\omega_{pc}$ .

→ If  $\omega_{pc} = \omega_{gc}$  the system is Critically stable.

→ The system is stable for  $\omega_{pc} < \omega_{gc}$

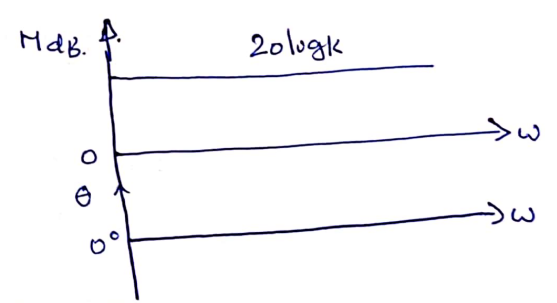
→ The system is unstable for  $\omega_{pc} > \omega_{gc}$ .

### Bode plot for various factors

#### Constant term K

$$|M(j\omega)|_{dB} = |G(j\omega)H(j\omega)|_{dB} = 20 \log K.$$

$$\theta = \angle G(j\omega)H(j\omega) = 0$$



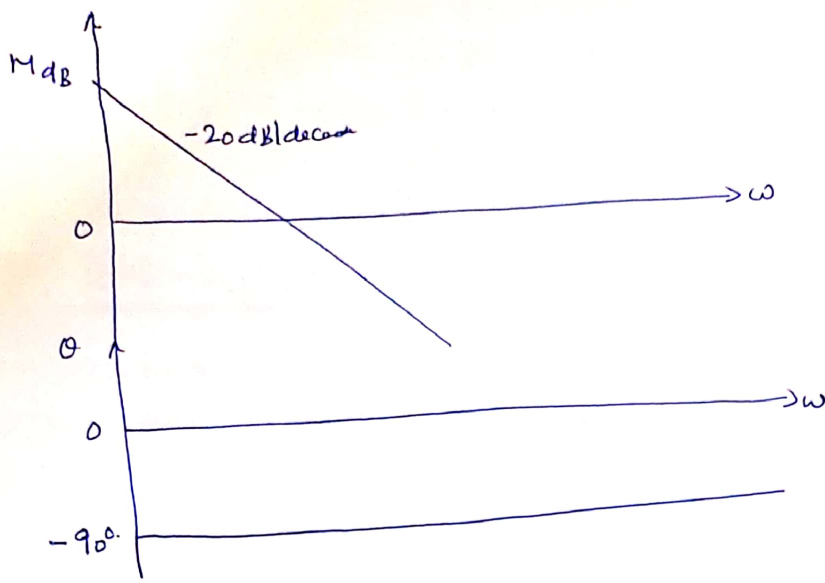
Bode plot of  $G(s)H(s) = 1/s$  i.e. pole at  $s = 0$

$$M(j\omega) = 1/j\omega \Rightarrow |M(j\omega)| = 1/\omega ; \angle M(j\omega) = \theta = -90^\circ$$

$$MdB = 20 \log (1/\omega) = -20 \log \omega, \omega > 0.$$

$\omega$	MdB
1	0
10	-20
} one decade.	
} 20dB fall in gain.	

Slope of  $20 \log 1/\omega$  is  $-20 \text{ dB/decade}$ .



### Homework

Bode  $\gamma$   $G(s)H(s) = s$  Zero at  $s = 0$

$$\theta = +90^\circ$$

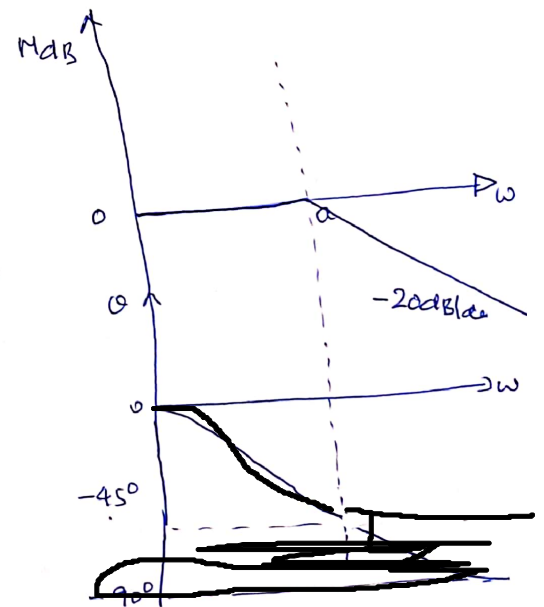
Slope:  $+20 \text{ dB/decade}$ .

Bode plot of  $G(s)H(s) = \frac{1}{1+s/a}$

i.e. pole at  $s = -a$ .

$$M_{dB} = \begin{cases} 0 & \text{for } \omega < a \\ -20 \log(\omega/a) & \text{for } \omega > a \end{cases}$$

$$\theta = \begin{cases} 0 & \text{for } \omega < a \\ -\tan^{-1}(\omega/a) & \text{for } \omega > a \end{cases}$$

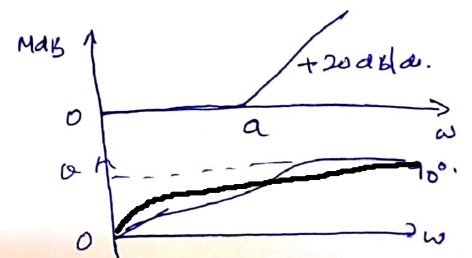


Bode plot of  $G(s)H(s) = (1+s/a)$

i.e. Zero at  $s = -a$ .

$$M_{dB} = \begin{cases} 0 & ; \omega < a \\ 20 \log(\omega/a) & ; \text{for } \omega > a \end{cases}$$

$$\theta = \tan^{-1}(\omega/a) \text{ for all } \omega.$$



## Bode plot of Complex poles.

(04)

$$G(s)H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{(s/\omega_n)^2 + 2\xi(s/\omega_n) + 1}$$

$$M_{dB} = \begin{cases} 0 & ; \omega < \omega_n \\ -40 \log(\omega/\omega_n) & ; \omega > \omega_n. \end{cases}$$

$$\theta = \begin{cases} -\tan^{-1} \left[ \frac{2\xi(\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \right] & ; \omega < \omega_n \\ -90^\circ & ; \omega = \omega_n \\ -180 + \tan^{-1} \left[ \frac{2\xi(\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \right] & ; \omega > \omega_n. \end{cases}$$

## problem

$$G(s)H(s) = \frac{27(s+2)}{s(s+6)(s^2+4s+9)}$$

Complex pole.

$$s^2 + 4s + 9 = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\Rightarrow \omega_n = 3 \text{ rad/s} \quad \xi = 0.66.$$

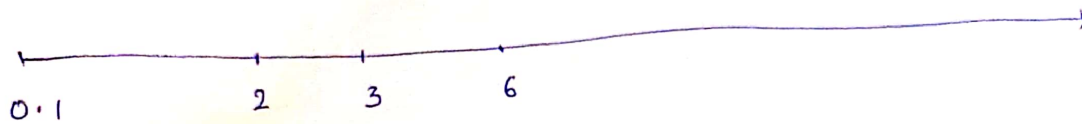
$$G(s)H(s) = \frac{27 \cdot 2 \cdot (1 + s/2)}{s \cdot 6 \cdot (1 + s/6) \cdot 9 \left[ \left(\frac{s}{3}\right)^2 + \frac{4}{3}\left(\frac{s}{3}\right) + 1 \right]}$$

$$G(j\omega)H(j\omega) = M(j\omega) = \frac{1 + j\omega/2}{j\omega (1 + j\omega/6) + [(j\omega/3)^2 + 1.33(j\omega/3) + 1]}$$

## Cross frequencies

$$\omega = 2 \text{ rad/s}, 3 \text{ rad/s} \text{ \& } 6 \text{ rad/s}.$$

$$M_{dB} = \underbrace{20 \log(\omega/2)}_{\omega \gg 2} - \underbrace{20 \log(\omega)}_{\omega \gg 0} - \underbrace{20 \log(\omega/6)}_{\omega \gg 6} - \underbrace{40 \log(\omega/3)}_{\omega \gg 3}$$



$$\underline{0.1 < \omega < 2}$$

$$M_{dB} = -20 \log(\omega).$$

$$\text{at } \omega = 0.1, \quad M_{dB} =$$

$$\text{at } \omega = 2 \quad M_{dB} =$$

$$\text{slope of } M_{dB} \text{ plot: } -20 \text{ dB/dec.}$$

$$\underline{2 < \omega < 3}$$

$$M_{dB} = -20 \log \omega - 20 \log \omega/2$$

$$\omega = 3 \quad M_{dB} =$$

$$\text{slope: } -40 \text{ dB/dec}$$

$$\underline{3 < \omega < 6}$$

$$M_{dB} = -20 \log \omega - 20 \log \omega/2 - 40 \log \omega/3$$

$$\omega = 6, \quad M_{dB} =$$

$$\text{slope: } -80 \text{ dB/dec.}$$

$$\underline{\omega > 6}$$

$$M_{dB} = 20 \log \omega/2 - 20 \log \omega - 20 \log(\omega/6) - 40 \log(\omega/3).$$

$$\omega = 10 \text{ or } 100$$

$$M_{dB} =$$

$$\text{slope: } -60 \text{ dB/decade.}$$





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$\omega$	Mag	slope dB/dec
0.1		-20
2		
3		-40
6		
		-80
10 & 100		
		-60

phase angle

$$M(j\omega) = \frac{1 + j\omega/2}{j\omega (1 + j\omega/6) ((j\omega/3)^2 + 1.33(j\omega/3) + 1)}$$

$$\theta = \underbrace{\tan^{-1}(\omega/2)}_{\theta_1} - \underbrace{90}_{\theta_2} - \underbrace{\tan^{-1}(\omega/6)}_{\theta_3} - \underbrace{\tan^{-1} \left[ \frac{1.33(\omega/3)}{1 - (\omega/3)^2} \right]}_{\theta_4}$$

$$= \theta_1 + \theta_2 + \theta_3 + \theta_4$$

$$* \theta_4 = \begin{cases} -\tan^{-1} \left[ \frac{1.33(\omega/3)}{1 - (\omega/3)^2} \right] & ; \omega < 3 \\ -90^\circ & ; \omega = 3 \\ -180 + \tan^{-1} \left[ \frac{1.33(\omega/3)}{1 - (\omega/3)^2} \right] & ; \omega > 3 \end{cases}$$

$$\theta_1 = \tan^{-1} \omega/2$$

$$\theta_2 = -90$$

$$\theta_3 = -\tan^{-1}(\omega/6)$$

$$\theta_4 = -\tan^{-1}(\downarrow)$$

$\omega$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta$
0.1					
2					
3				-90°	
6					
10 & 100					

## problems

(67)

Q1: The open Loop transfer function of a closed loop system is given by  $G(s)H(s) = \frac{80}{s(s+2)(s+20)}$

Construct the Bodeplot and calculate the following

(i) GM (ii) PM (iii)  $\omega_{gc}$  (iv)  $\omega_{pc}$

Is the closed loop system stable.

Soln:

$$\begin{aligned} G(s)H(s) &= \frac{80}{s(s+2)(s+20)} = \frac{80}{s \cdot 2 \cdot (1+s/2) \cdot 20(1+s/20)} \\ &= \frac{2}{s(1+s/2)(1+s/20)} \end{aligned}$$

Corners frequencies

$$\omega = 2, 4, 20 \text{ rad/s}$$

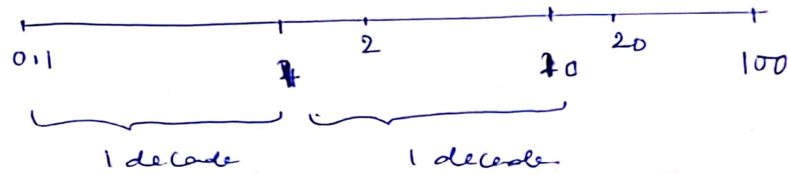
Magnitude plot

$$G(j\omega)H(j\omega) = M(j\omega) = \frac{2}{j\omega(1 + \frac{j\omega}{2})(1 + \frac{j\omega}{20})} \quad \text{--- (1)}$$

$$M_{dB} = \underbrace{20 \log 2}_{\text{all } \omega} - \underbrace{20 \log(\omega)}_{\omega > 0} - \underbrace{20 \log(\omega/2)}_{\omega \geq 2} - \underbrace{20 \log(\omega/20)}_{\omega \geq 20}$$

choose  $\omega \approx 0$  as the origin of  $\omega$  axis. why?

$$\omega = 0.1$$



For  $0.1 \leq \omega \leq 2$  ← First Corner Freq.

$$M_{dB} = 20 \log 2 - 20 \log(\omega). \quad \text{why?}$$

$$\text{at } \omega = 0.1, \quad M_{dB} =$$

$$\text{at } \omega = 2, \quad M_{dB} =$$

slope of  $M_{dB}$  plot:  $-20 \text{ dB/dec.}$  why?

For  $2 \leq \omega \leq 20$

$$M_{dB} = 20 \log 2 - 20 \log \omega - 20 \log \omega/2 - \text{why?}$$

$$\text{at } \omega = 20, \quad M_{dB} =$$

slope of  $M_{dB}$  plot:  $-40 \text{ dB/dec.}$  — why?

For  $\omega > 20$

$$M_{dB} = 20 \log 2 - 20 \log \omega - 20 \log \omega/2 - 20 \log \omega/20 - \text{why?}$$

For  $\omega > 20$  say at  $\omega = 100$

$$M_{dB} =$$

slope:  $-60 \text{ dB/decade}$  — why?



Complete the following table

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$\omega$	MdB	slope
0.1		-20 dB/dec
2		
		-40 dB/dec
20		
		-60 dB/dec.
100		

phase plot

Consider eqn (1)

$$M(j\omega) = \frac{2}{j\omega (1 + j\omega/2) (1 + j\omega/20)}$$

$$\theta = \angle 2 - \angle j\omega - \angle 1 + j\omega/2 - \angle 1 + j\omega/20$$

$$\theta = 0 - 90^\circ - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/20) \quad (2)$$

Complete the following table

$\omega$	$\theta$
0.1	
2	
20	
100	

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Take more points for  $\omega$ , where  $\theta$  crosses  $-180^\circ$