DSP

20EC540

Chapter - 01

signal processing :- processing means operating in some fashion to the signal to extract some useful information
Ex :- we use our ears as i/p and then auditory path ways in brain to extract the information the signal
The signal processor may be electric, mechanic or even it may be computer program.

Discrete time system :-

$$x(n) \longrightarrow \boxed{T(\ )} \longrightarrow y(n)$$

$$y(n) = T[x(n)]$$

let us consider a discreate time signal $x(n)$ having $0 \le n \le N-1$, the DTFT of the signal is

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n} \quad — ①$$

$X(\omega)$ using a total of 'n' equally spaced samples in the range of $\omega \in (0, 2\pi)$ so sampling interval is $2\pi/n$ that is we sample $X(\omega)$ using freq

$$\omega = \omega_k = 2\pi k/n \qquad 0 \le k \le N-1$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega_k n}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k n}{n}} \quad — ②$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$X(k) = DFT\{x(n)\} = \sum_{N=0}^{N-1} x(n)\, W_N^{kn} \qquad 0 \le k \le N-1$$

Inverse DFT :-

The DFT $X(k)$ value unity defines to the inverse DFT formula

$$x(n) = IDFT\{X(k)\} = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-kN} \qquad 0 \le n \le N-1$$

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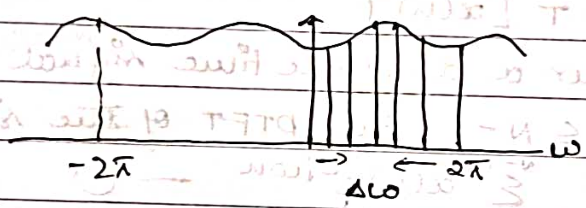
Frequency domain sampling & reconstruction of DTS

dù [DTS] is converted to equilent freq domain using fourier transform. $x(n) \rightarrow X(e^{j\omega})$ hence fourier transform is samples to obtain a frequency domain $X(K) \rightarrow$ discreate fourier tranform [DFT].

Non periodic discreate time signal $x(n) \rightarrow X(e^{j\omega})$

$$X(e^{j\omega}) = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \quad \text{--- (1)}$$

periodic $\rightarrow$ fundamental freq range $0 < \omega < 2\pi$ rad



N equidistant sample $0 \leq \omega < 2\pi$

$$\Delta\omega = \frac{2\pi}{N} k \quad ; \quad 0 \leq K \leq N-1$$

$k=0 \rightarrow 0$, $K=1 \rightarrow 2\pi/N$, $k=2 \rightarrow 4\pi/N$

$$X\left(\frac{2\pi}{N} k\right) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j 2\pi/N\, kn} \; ; \; 0 \leq K \leq N-1 \quad \text{--- (2)}$$

$x(n) \rightarrow$ infinite no. of summations

$$X\left(\frac{2\pi}{N}\right) = \sum_{n=-N}^{-1} x(n)\, e^{-j\frac{2\pi}{N} kn} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N} kn} + \sum_{n=N}^{2N-1} x(n)\, e^{-j\frac{2\pi}{N} kn} \quad \text{--- (3)}$$

$$(3) \Rightarrow X\left(\frac{2\pi}{N} k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi}{N} kn} \quad \text{--- (4)}$$

$$n = n - lN \Rightarrow lN = n - n$$

$$\boxed{lN = 0}$$

$$X\left[\frac{2\pi}{N} k\right] = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N} \cdot kn}$$

$$X\left[\frac{2\pi}{N} k\right] = \sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n-lN)}_{\text{periodic signal } N}\, e^{-j\frac{2\pi}{N} kn}$$

$$X\left[\frac{2\pi}{N}k\right] = \sum_{n=0}^{N-1}\sum_{l=-\infty}^{\infty} x(-lN)\, e^{-j\frac{2\pi}{N}kn}$$

$$\boxed{X\left[\frac{2\pi}{N}k\right] = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}kn}} \quad \text{--- (5)}$$

Fourier series

$$x_p(n) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn} \quad 0 \le n \le N-1 \quad \text{--- (6)}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi kn}{N}} \quad 0 \le k \le N-1 \quad \text{--- (7)}$$

Fourier cofficient

compare eq (7) & Eq (6)

$$a_k = \frac{1}{N}\left[\frac{2\pi}{N}k\right]; \quad 0 \le k \le N-1 \quad \text{--- (8)}$$

put eq (8) in (6)

$$x_p(n) = \sum_{n=0}^{N-1}\frac{1}{N}\times\left[\frac{2\pi}{N}k\right] e^{j\frac{2\pi}{N}kn}; \quad 0 \le n \le N-1$$

$$\boxed{x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} x\left[\frac{2\pi}{N}k\right] e^{j\frac{2\pi}{N}kn}} \quad \text{--- (9)} \quad 0 \le n \le N-1$$

DFT & IDFT Equation

DFT → Discreate fourier transfor

IDFT → Inverse discreate fourier transform

DFT → $x(n)$; $0 \le n \le N-1$

$$\boxed{DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}} \quad 0 \le k \le N-1$$

$$x\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$\boxed{DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}} \quad 0 \le k \le N-1$$

$W_N = e^{-j\frac{2\pi}{N}k}$ → phase factor of Twiddle factor

$X(k)$ = seq in freq domine units → N

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IDFT

$$\boxed{\text{IDFT}\{X(k)\} = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}} \quad 0 \le n \le N-1$$

Eq (9) $$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left[\frac{2\pi}{N}k\right] e^{j\frac{2\pi}{N}kn}$$

$$\boxed{\text{IDFT}\{X(k)\} = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, \omega_N^{-kn} \cdot 0 \le n \le N-1}$$

$$\omega_N = e^{j\frac{2\pi}{N}}$$

The DFT has a linear transformation :-

* The formula for DFT & IDFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{kn} \quad k = 0, 1, \cdots N-1 \quad — (4)$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, \omega_N^{-kn} \quad n = 0, 1, \cdots N-1 \rightarrow (5)$$

$$\omega_N = e^{-j\frac{2\pi}{N}}$$

which is an nth root of unity

we know that computation of each point of DFT can be accomplished by n complex multiplication n-1.

hence n point DFT values can be computed in N^2 complex multiplication of N-1 complex addition.

it is instructive to view the DFT and IDFT has linear transformation $\{x(n)\}$ & $\{X(k)\}$.

- Let us define n point vector X_N of the signal sequence X_N $X(n) \cdot n = 0, 1 \cdots N-1$ when N point vector $x(n)$ of frequency samples & N×N matrix

$$X_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \qquad X_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ X(N-1) \end{bmatrix}$$

$$W_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \quad \text{—— (6)}$$

The N point DFT may be expressed in matrix form

$$X_N = W_N x_N \quad \text{—— (7)}$$

- where W_N is the matrix of the linear transformation & symmetric matrix
- if we assume that the inverse of W_N exist then Eq (7) can be inverted by ~~W_N^{-1}~~ premultiply both side by W_N^{-1}

$$x_N = W_N^{-1} X_N \quad \text{—— (8)}$$

The IDFT has given by in Eq (5) can be expressed in matrix form $x_N = \frac{1}{N} W_N^* X_N$ —— (9)

where $W_N^* \rightarrow$ complex conjugate of the matrix W_N

comparision of eq (8) & (9)

$$W_N^{-1} = \frac{1}{N} W_N^* \quad \text{—— (10)}$$

which in terms $\boxed{W_N W_N^* = N I_N}$

where $I_N \Rightarrow N \times N$ identity matrix

Theorefore matrix W_N is the transformation orthagonal

- The DFT & IDFT's are computational tools that play an important role in many digital applications such as freq analysis, specturm power estimation & linear filtering.

Relationship of DFT to other transforms

(i) Relationship to the fourier series co-efficients of a periodic signals :-

- A periodic sequence $\{x_p(n)\}$ with fundamental period of N can be represented

$$x_p(n) = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi nk}{N}} \cdot \quad -\infty < n < \infty \quad \text{—— (11)}$$

where Fourier series co-efficient

$$CK = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi nk/N} \quad K=0,1, \dots N-1 \quad \text{—} (12)$$

we compare eq (11) & (12) with (4) & (5)

$X(n) = x_p(n) \cdot 0 \le n \le N-1$ the DFT of the sequence is simply $X(K) = NCk$ — (13)

ii) Relationship to the fourier transform of a periodic sequence :-

$$X(\omega) \quad \omega k = \frac{2\pi K}{N} \quad K = 0, 1 \dots N-1$$

$$X(K) = X(\omega)\Big|_{\omega = 2\pi K/N} = \sum_{n=\infty}^{\infty} x(n) e^{-j\frac{2\pi K}{N}} \quad K=0,1, .N-1 \quad (14)$$

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN) \quad \text{—} (15)$$

$x_p(n)$ is obtained by aliasing $\{x(n)\}$ over the interval $0 \le n \le N-1$

The finite duration of the sequence

$$\hat{x}(n) = \begin{cases} x_p(n) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{—} (16)$$

no resemblance to the original sequence $\{x(n)\}$ unless $x(n)$ is of infinite duration & length $L \le N$

$$x(n) = \hat{x}(n) \quad 0 \le n \le N-1 \quad (17)$$

iii) Relationship to the z transform :-

Let us consider sequence $x(n)$ having the z transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \longleftarrow (12)$$

with an ROC that includes the unit circle

if $X(z)$ is sampled at n equally spaced points on the unit circle $Z_k = e^{2\pi K/N}$ $K = 0, 1 \dots N-1$

$$X(K) = X(z)\Big|_{z = e^{j\frac{2\pi nk}{N}}} \quad K = 0, 1 \dots N-1$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi}{N}nk} \quad \text{— (19)}$$

The Eq (19)

consequently $X(z)$ can be expressed as a function of DFT as

$$X(z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}$$

$$X(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi K n}{N}} \right] z^{-n}$$

$$X(z) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \sum_{n=0}^{N-1} \left[e^{j\frac{2\pi K}{N}} z^{-1} \right]^{n} \quad \text{— (20)}$$

$$X(z) = \frac{1 - z^{-N}}{N} \sum_{K=0}^{N-1} \frac{X(K)}{1 - e^{j2\pi K/N}\, z^{-1}}$$

$$X(\omega) = \frac{1 - e^{-j\omega N}}{N} \sum_{K=0}^{N-1} \frac{X(K)}{1 - e^{-j\left(\omega - \frac{2\pi K}{N}\right)}} \quad \text{— (21)}$$

when evaluated on unit circle the eq (20) hence $X(\omega)$ in terms of DFT

<u>problems</u>

1) compute 4. DFT of the sequence $x(n) = (1, 2, 1, 0)$

given :- $N = 4$

$$W_N = e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = -j$$

$$X = W_N x$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$W_4^0 = e^{-j\frac{2\pi}{4}\times 0} = e^0 = 1$$

$$W_4^1 = e^{-j\frac{2\pi}{4}} = -j$$

$$W_4^2 = e^{-j\pi} = -1 \qquad W_4^9 = e^{-j\frac{18\pi}{4}} =$$

$$W_4^3 = e^{-j\frac{6\pi}{4}} = j$$

$$W_4^4 = e^{-j\frac{8\pi}{4}} = 1$$

$$W_4^6 = e^{-j\frac{12\pi}{4}} = -1$$

$$\omega_4^4 = \omega_4^0 = 1$$
$$\omega_4^6 = \omega_4^2 = -1$$
$$\omega_4^9 = \omega_4^1 = -j$$

$$= \begin{bmatrix} \omega_4^0 & \omega_4^0 & \omega_4^0 & \omega_4^0 \\ \omega_4^0 & \omega_4^1 & \omega_4^2 & \omega_4^3 \\ \omega_4^0 & \omega_4^2 & \omega_4^0 & \omega_4^2 \\ \omega_4^0 & \omega_4^3 & \omega_4^2 & \omega_4^1 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

2) compute 4. DFT of the sequence $x(n) = (1,2,1,0)$ also find $y(K)$ if $y(n) = x((-n))_N$ $\quad 0 \le k \le 3$

sol $N = 4$

$$\omega_N = e^{-j\frac{2\pi}{N}}$$

$\omega_4^0 = 1 \qquad \omega_4^3 = j \qquad \omega_4^9 = -j$

$\omega_4^1 = -j \qquad \omega_4^4 = 1$

$\omega_4^2 = -1 \qquad \omega_4^6 = -1$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{kn}$$

$$X(K) = \sum_{n=0}^{3} x(n)\, \omega_4^{kn}$$

$$X(K) = x(0)\omega_4^0 + x(1)\omega_4^{1k} + x(2)\omega_4^{2k} + x(3)\omega_4^{3k} \quad \text{--- } (1)$$

$$X(0) = 1 + 2(1) + 1(1) = 4$$
$$X(1) = 1 + 2(-j) + 1(-1) = -2j$$
$$X(2) = 1 + 2(-1) + 1(1) = 0$$
$$X(3) = 1 + 2(j) + 1(-1) = 2j$$

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$X(k) = X^*(N-k)$ [symmetric property]

$y(n) = x((-n))_N$

$Y(k) = X((-k))_N$

$Y(k) = \{4, 2j, 0, -2j\}$

3) The DFT of the sequence $x(n) = 1$ $0 \le n \le 2$ for $N=4$ sketch the magnitude and phase spectrum.

sol $N = 4$ $x(n) = \{1, 1, 1, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}$$

$W_N = e^{-j\frac{2\pi}{N}}$

$W_4^0 = 1$ $\quad W_4^2 = -1$ $\quad \cancel{W_4^4 \neq 1}$ $\quad W_4^3 = j$

$W_4^1 = -j$ $\quad \cancel{W_4^3 \neq N}$

$$X(k) = \sum_{n=0}^{3} x(n) \cdot W_4^{kn}$$

$X(k) = x(0) W_4^0 + x(1) W_4^{1k} + x(2) W_4^{2k}$ — (1)

$+ x(3) W_4^{3k}$

$X(0) = 1 + 1 + 1 = 3$

$X(1) = 1 - j - 1 = -j$

$X(2) = 1 - 1 + 1 = 1$

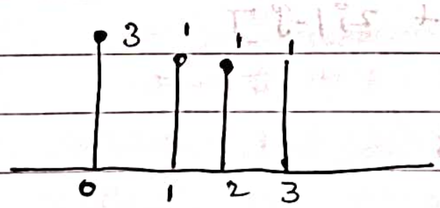
$X(3) = 1 + j - 1 = j$

$X(k) = \{3, -j, 1, j\}$

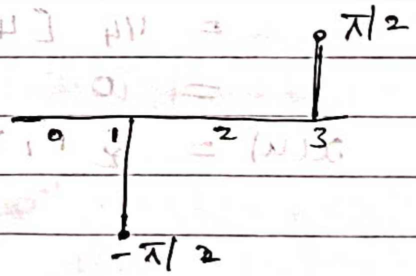
$|X(k)| = \{3, 1, 1, 1\}$

$\angle X(k) = \{0, -\pi/2, 0, \pi/2\}$

$|X(k)|$



$\angle X(k)$



4) Find IDFT of the 4. sequence $X(k) = \{4, -2j, 0, 2j\}$

a) using defining equation of IDFT

b) using DFT

a) $$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-kn} \qquad n = 0, 1, 2 \cdots N-1$$

$N = 4$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, W_4^{-kn}$$

$$= \frac{1}{4}\left[X(0) W_4^0 + X(1) W_4^{-n} + X(2) W_4^{-2n} + X(3) W_4^{-3n} \right]$$

$W_4^0 = 1$ $\qquad W_4^{-3} = -j$

$W_4^{-1} = j$

$W_4^{-2} = -1$

$$x(0) = \frac{1}{4}\left[4 + (-2j)\cdot 1 + 0 \cdot 1 + 2j \cdot 1 \right]$$

$= 1$

$$x(1) = \frac{1}{4}\left[4 W_4^0 + (-2j) W_4^{-1} + 0 + (2j) W_4^{-3} \right]$$

$$= \frac{1}{4}\left[4 + (-2j)(-j) + 0 + 2j \right]$$

$= 2$

$$x(2) = 1/4 \left[4 W_4^0 + (-2j) W_4^{-2} + 0 + (2j) W_4^{-6} \right]$$

$= 1/4\, [4 + (-2j)(-j) + 0 + 2j]$

$= 1/4\, [4 + 2j - 2j]$

$= 1$

$$x(3) = 1/4 \left[4 W_4^0 + (-2j) W_4^{-3} + 0 + (2j) W_4^{-9} \right]$$

$= 1/4\, [4 + -2j \cdot 1 + 2j \cdot -j]$

$= 0$

$x(n) = \{1, 2, 1, 0\}$

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b) $x^*(k) = \{4, 2j, 0, -2j\}$ → conjugate

~~x(k)~~ #

$$DFT\{x^*(k)\} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 2j \\ 0 \\ -2j \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 4 \\ 0 \end{bmatrix}$$

$$x(n) = IDFT[X(k)] = \frac{1}{N}[DFT\{x^*(k)\}]$$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 4 \\ 0 \end{bmatrix} = \{1, 2, 1, 0\}$$

5) Find IDFT $\{7, -2-j, 1, -2+j\}$ using
a) defining eq of IDFT
b) using DFT

a) $$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, W_N^{-Kn} \qquad n = 0, 1, 2 \ldots N-1$$

$\boxed{N=4}$

$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, W_4^{-Kn}$$

$$= \frac{1}{4}\left[X(0)W_4^0 + X(1)W_4^{-n} + X(2)W_4^{-2n} + X(3)W_4^{-3n}\right]$$

$W_N = e^{-j\frac{2\pi}{N}}$

$W_4^0 = 1 \qquad W_4^{-2} = -1 \qquad W_4^{-4} = 1 \qquad W_4^{-9} = j$

$W_4^{-1} = j \qquad W_4^{-3} = -j \qquad W_4^{-6} = -1$

~~x(k)~~ #

$$x(0) = \frac{1}{4}[7 + (-2-j) + 1 + (-2+j)]$$

$$= \frac{1}{4}[7 - 2 - j + 1 - 2 + j]$$

$$= 1$$

$$x(1) = \frac{1}{4}\left[X(0)W_4^0 + X(1)W_4^{-1} + X(2)W_4^{-2} + X(3)W_4^{-3}\right]$$

$$= \frac{1}{4}[7 + (-2-j)\cdot j + (1)\cdot -1 + (-2+j)\cdot -j]$$

$= \frac{1}{4}\left[7 - \cancel{2j} + \cancel{1} - \cancel{1} + \cancel{2j} + 1\right]$

$= 2$

$X(2) = \frac{1}{4}\left[x_0 W_4^0 + X(1) W_4^{-2} + X(2) W_4^{-4} + X(3) W_4^{-6}\right]$

$= \frac{1}{4}\left[7 + (-2-j)\cdot -1 + (1)\cdot 1 + (-2+j) -1\right]$

$= \frac{1}{4}\left[7 + 2 + \cancel{j} + 1 + 2 - \cancel{j}\right]$

$= 12/4 = 3$

$X(3) =$

b) $X^*(k) = \{7, -2+j, 1, -2-j\}$

$$x(n) = DFT\{X^*(k)\} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 7 \\ -2+j \\ 1 \\ -2-j \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 4 \end{bmatrix}$$

$x(n) = IDFT\{X^*(k)\} = \frac{1}{N}\{DFT\{X^*(k)\}\}$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 12 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

6) Find 4. sequence for $x(n) = \{1, 2, 3, 4\}$

7) Compute 8. DFT of the sequence given below

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$X(K) = \sum_{n=0}^{7} x(n)\, \omega_8^{kn}$$

$\omega_N = e^{-j\frac{2\pi}{8}}$

$\omega_8^0 = 1$ $\qquad$ $\omega_8^3 = -\frac{1}{\sqrt2} - \frac{1}{\sqrt2} j$ $\qquad$ $\omega_8^9 = \frac{1}{\sqrt2} - \frac{1}{\sqrt2} j$

$\omega_8^1 = \frac{1}{\sqrt2} - \frac{1}{\sqrt2} j$ $\qquad$ $\omega_8^4 = -1$

$\omega_8^2 = -j$ $\qquad$ $\omega_8^6 = \cancel{-\frac{1}{\sqrt2}} \cancel{+} \cancel{\frac{1}{\sqrt2}} j$

$$X(k) = \cancel{\Sigma} \left[x(0)\omega_8^0 + x(1)\,\omega_8^n + x(2)\,\omega_8^{2n} + x(3)\,\omega_8^{3n} + x(4)\,\omega_8^{4n} + x(5)\,\omega_8^{5n} + x(6)\,\omega_8^{6n} + x(7)\,\omega_8^{7n}\right]$$

$$X(0) = [1+1+1+1] = 4$$

$$X(1) = \left[1 + \cancel{\bullet}1.\left(\frac{1}{\sqrt2} - \frac{1}{\sqrt2}j\right) + 1.(-j) + 1.\left(-\frac{1}{\sqrt2} - \frac{1}{\sqrt2}j\right)\right]$$

$$= \left[1 + \cancel{\frac{1}{\sqrt2}} - \frac{1}{\sqrt2}j - j - \cancel{\frac{1}{\sqrt2}} - \frac{1}{\sqrt2}j\right]$$

$$= 1 - (1+\sqrt2)j$$

$$X(2) = [1 + \omega_8^2 + \omega_8^4 + \omega_8^6]$$

$$= \cancel{\Sigma}\, 1 - j - 1 + j = 0$$

$$X(3) = 1 + \omega_8^3 + \omega_8^6 + \omega_8^9$$

$$= 1 - \frac{1}{\sqrt2} - j\frac{1}{\sqrt2} + j + \frac{1}{\sqrt2} - j\frac{1}{\sqrt2}$$

$$= 1 - 0.414j$$

__/__/__

properties of DFT

1) conjucate symmetry :- if $x(n)$ is real then DFT $X(k) = X^*(N-k)$

proof :- $X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad K = 0, 1, 2, \dots N-1$

$$X^*(k) = \left\{ \sum_{n=0}^{N-1} x(n) W_N^{kn} \right\}^*$$

$$= \sum_{n=0}^{N-1} x^*(n) W_N^{-kn}$$

$$= \sum_{n=0}^{N-1} x^*(n) W_N^{-kn} \cdot W_N^{Nn} \quad \{ \text{since } W_N^{Nn} = 1 \}$$

$$= \sum_{n=0}^{N-1} x^*(n) W_N^{(N-k)n}$$

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{(N-k)} = X^*(N-k)$$

1) compute 5. DFT of the sequence $x(n) = \{1, 0, 1, 0, 1\}$ and hence verify symmetric property

sol
$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$X(K) = \sum_{n=0}^{4} x(n) W_5^{kn}$$

$$= \{ x(0) W_5^0 + x(1) W^k + x(2) W^{2k} + x(3) W^{3k} + x(4) W^{4k} \}$$

$$W_N^{kn} = e^{-j\frac{2\pi k}{N}} = e^{-j\frac{2\pi k}{5}}$$

$W_5^0 = 1$

$W_5^1 = 0.3 - 0.95j$

$W_5^2 = -0.8 - 0.58j$

$W_5^3 = -0.8 + 0.58j$

$W_5^4 = 0.3 + 0.95j$

$W_5^8 = -0.8 + 0.58j$

$x(0) = \{1 + 1 + 1\}$

$= 3$

$x(1) = \{$

$x(2) = \{1 + 1 \cdot w_5^4 + 1 \cdot w_5^8\}$

$= 1 + 1(0.3 + 0.95j) + 1 \cdot (-0.8 + 0.58j)$

$= 1 + 0.3 + 0.95j + 1 - 0.8 + 0.58j$

$= 0.5 + 1.53j$

$x(3) =$

verify :-

$x^*(k) = x(N-k)$

$x^*(k) = x(5-k)$

$x^*(1) = x^*(4)$

$x^*(2) = x^*(3)$

~~x(n) = {1, 2, 3, 4, 5}~~

2) compute 5. DFT of the sequence $x(n) = \{1, 2, 3, 4, 5\}$ hence verify conjugate symmetry property

sol $X(k) = \sum_{n=0}^{N-1} x(n)\, w_N^{kn}$

$x(k) = \sum_{n=0}^{4} x(n)\, w_5^{kn}$

$= \{ x(0) W_5^0 + x(1) W_5^k + x(2) W_5^{2k} + x(3) W_5^{3k} + x(4) W_5^{4k} \}$

$W_N^{kn} = e^{-j\frac{2\pi k}{N}} = e^{-j\frac{2\pi k}{5}}$

$X(0) = 1 + 2 + 3 + 4 + 5$

$= 15$

$$\left.\begin{aligned} X(1) &= -2.5 + 3.44j \\ X(2) &= -2.5 + 0.81j \\ X(3) &= -2.5 - 0.81j \\ X(4) &= -2.5 - 3.44j \end{aligned}\right]$$

Verify.

$X^*(k) = X(N-k)$

$X^*(k) = X(5-k)$

$X^*(1) = X(4)$

$X^*(2) = X(3)$

2) Linearity :- DFT of $\{ax_1 n + b x_2 n\} = \{a X_1(k) + b X_2(k)\}$

proof :- we know that

$DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

letting $x(n) = a x_1(n) + b x_2(n)$

$DFT\{\underbrace{a x_1(n) + b x_2(n)}_{x(n)}\} = \sum_{n=0}^{N-1} [a x_1(n) + b x_2(n)] W_N^{kn}$

$= a \sum_{n=0}^{N-1} x_1(n) W_N^{kn} + b \sum_{n=0}^{N-1} x_2(n) W_N^{kn}$

$= a X_1(k) + b X_2(k)$

a) Find 4. DFT of sequence $x(n) = \cos\left[\frac{\pi}{4} n\right] + \sin\left[\frac{\pi}{4} n\right]$ using linearity property.

sol $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

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$x(n) = x_1(n) + x_2(n)$

$x_1(n) = \cos\left[\frac{\pi}{4}n\right] = \left\{1, \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right\}$

$x_2(n) = \sin\left[\frac{\pi n}{4}\right] = \left\{0, \frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}\right\}$

$X_1(k) = \text{DFT}\{x_1(n)\}$

$X_2(k) = \text{DFT}\{x_2(n)\}$

$$X_1(k) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1-0.44j \\ 1 \\ 1+j0.44 \end{bmatrix}$$

$$X_2(k) = \begin{bmatrix} x_2(0) \\ x_2(0) \\ x_2(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1 \\ j1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2.414 \\ -1 \\ -0.414 \\ -1 \end{bmatrix}$$

$X_1(k) + X_2(k) = \{3.414, -j1.414, 0.586, j1.414\}$

3) circular time shift :-

if DFT of $\{x(n)\} = X(k)$

then DFT $\{x(n-m)\} = W_N^{mk} X(k)$ $0 \le k \le N-1$

proof :-

$$x(n) \longrightarrow \boxed{\text{DFT}\{\}} \longrightarrow X(k)$$

$n = 0, 1, \cdots N-1$ $\quad$ $k = 0, 1, \cdots N-1$

From the defination of IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-kn}$$

$$x(n-m) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-k(n-m)}$$

since it is shift is circular

$$x((n-m))_N = \frac{1}{N}\sum_{k=0}^{n-1}\left[X(k)\, W_N^{km}\right] W_N^{-kn}$$

$$x((n-m))_N = IDFT\left[X(k)\, W_N^{km}\right]$$

(or)

$$DFT\{x((n-m))_N\} = W_N^{km} X(k)$$

4) circular frequency shift :-

if DFT of $\{x(n)\} = X(k)$ then

$$DFT\{W_N^{-ln} x(n)\} = X((k-l))_N$$

proof :-

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$X(k-l) = \sum_{n=0}^{N-1} x(n)\, W_N^{(k-l)n}$$

since the shift in frequency is circular

$$X((k-l))_N = \sum_{n=0}^{N-l} \left[x(n)\, W_N^{-ln}\right] \cdot W_N^{kn}$$

$$DFT\{x(n)\, W_N^{-ln}\} = X((k-l))_N$$

5) symmetry a real value sequence :-

* Find 4. DFT of the sequence $x(n) = \{1,-1,1,-1\}$. also using time shift property find the DFT of the sequence $y(n) = x((n-2))_4$

sol $N=4$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$X(k) = \sum_{n=0}^{3} x(n)\, W_4^{kn}$$

$$X(k) = x(0)W_4^0 + x(1)W_4^k + x(2)W_4^{2k} + x(3)W_4^{3k}$$

$$W_N^{kn} = e^{-j\frac{2\pi k}{N}}$$

$X(0) = 0$

$X(1) = 0$

$X(2) = 4$

$X(3) = 0$

$X(K) = \{0, 0, 4, 0\}$

Apply circular time shift

$y(n) = x((n-2))_4$

$Y(K)$ ~~=~~ ~~to~~ $DFT\{x((n-m))_N\} = W_N^{km} X(K)$

$Y(K) = W_4^{2K} X(K)$

$Y(0) = W_4^0 X(0) = 0$

$Y(1) = W_4^2 X(1) = 0$

$Y(2) = W_4^4 X(2) = 4$

$Y(3) = W_4^6 X(3) = 0$

$Y(K) = \{0, 0, 4, 0\}$

* suppose $x(n)$ is a sequence defined on 0-7 only has (0, 1, 2, 3, 4, 5, 6, 7)

a) Illustrate $x((n-2))_8$

b) if $DFT\{x(n)\} = X(K)$ what is the $DFT\{x((n-2))_8\}$

sol $N = 8$

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$X(K) = \sum_{n=0}^{7} x(n) W_8^{kn}$$

$$X(K) = x(0)W_8^0 + x(1)W_8^K + x(2)W_8^{2K} + x(3)W_8^{3K} + x(4)W_8^{4K} + x(5)W_8^{5K} + x(6)W_8^{6K} + x(7)W_8^{7K}$$

$W_N^{kn} = e^{-j\frac{2\pi k}{N}}$

$X(0) = 6$ $\qquad X(7) =$

$X(1) =$

$X(2) =$

$X(3) =$

$X(4) =$

$X(5) =$

$X(6) =$

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* compute 4. DFT $x(n) = \{1, 0, 1, 0\}$ also find $y(n)$
if $Y(K) = x((K-2))_4$

sol $X(K) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{kn}$

$= \sum_{n=0}^{3} x(n)\, \omega_4^{kn}$

$X(K) = x(0)\,\omega_4^0 + x(1)\,\omega_4^k + x(2)\,\omega_4^{2k} + x(3)\,\omega_4^{3k}$

$X(0) = 2$ $\quad X(2) = 2$

$X(1) = 0$ $\quad X(3) = 0$

$X(K) = \{2, 0, 2, 0\}$

$DFT\{\omega_N^{-kn} x(n)\} = X((K-\ell))_N$

$y(n) = \omega_N^{-kn} x(n) \xleftrightarrow{DFT} Y(K) = X((K-\ell))_N$

$y(n) = \omega_4^{-2n} x(n)$

$y(0) = \omega_4^{-0} x(0) = 1$

$y(1) = \omega_4^{-2} x(1) = 0$

$y(2) = \omega_4^{-4} x(2) = 1$

$y(3) = \omega_4^{-6} x(3) = 0$

$y(n) = \{1, 0, 1, 0\}$

property

5) symmetry real valued sequence :-

In the sequence $x(n) = n = 0, 1 \ldots N-1$ is real

then its DFT is such that $X(K) = X^*(N-K)$ $\quad K = 0, 1, \ldots N-1$

proof :-

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$X(k) = \text{DFT}\{x(n)\}$

$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad k = 0, 1, \ldots N-1$

Taking conjugate on both side

$X^*(k) = \sum_{n=0}^{N-1} x^*(n) W_N^{-kn}$

since $x(n)$ is real we have $x^*(n) = x(n)$

$X^*(k) = \sum_{n=0}^{N-1} x(n) W_N^{-kn}$

$X^*(k) = \sum_{n=0}^{N-1} x(n) W_N^{-kn} W_N^{Nn} \left[W_N^{Nn} = 1\right]$

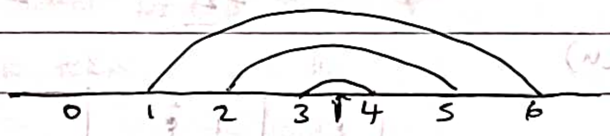
$X^*(k) = \sum_{n=0}^{N-1} x(n) W_N^{(N-k)n}$

$X^*(k) = X(N-k)$

if N is odd the conjugate symmetry is about N/2
if the index $k = N/2$ is called the folding index

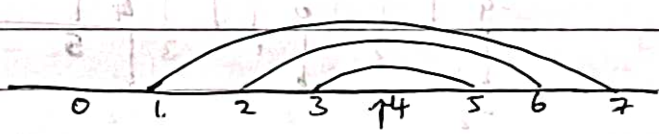
* N = 7

$7/2 = 3.5$



* N = 8

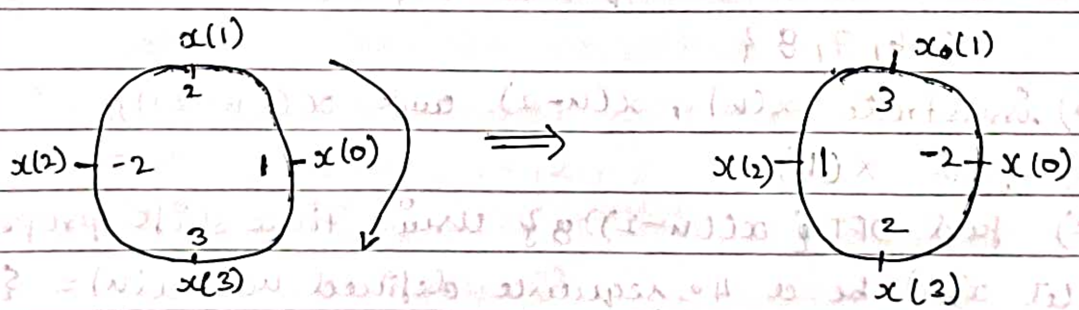
$\frac{8}{2} = 4$



I) 1) Let $x(n) = \{1, 2, -2, 3\}$ find $x_1(n) = x((n+2))_4$;
$0 \le n \le 3$

Sol $N = 4$

a)



$x_1(n) = \{-2, 3, 1, 2\}$

b) $x_1(n) = x(4+n+2)$

$x_1(0) = x(4+0+2) = x(6-4) = x(2) = -2$

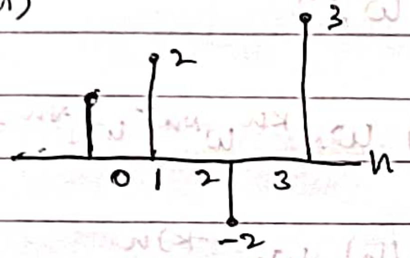
$x_1(1) = x(4+1+2) = x(7-4) = x(3) = 3$

$x_1(2) = x(4+2+2) = x(8-4) = x(4-4) = x(0) = 1$

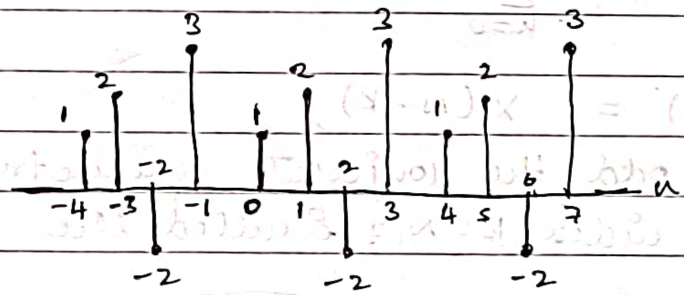
$x_1(3) = x(4+3+2) = x(9-4) = x(5-4) = x(1) = 2$

$x_1(n) = \{-2, 3, 1, 2\}$

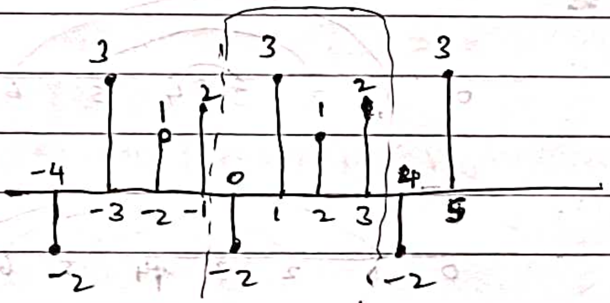
c) $x(n)$



$x_p(n)$



$x_p(n)$



$x_1(n) = \{-2, 3, 1, 2\}$

H.W

2) Let $x(n)$ be a 8 sequence defined has $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

a) illustrate $x(n)$, $x(n-2)$ and $x((n-2))_8$

b) find $X(k)$

c) find DFT $\{x((n-2))_8\}$ using time shift property

3) Let $x(n)$ be a 4 sequence defined has $x(n) = \{4, 5, -6, 7\}$
represent the following graphically

a) $x(n)$

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b) $x(n-3)$ (d) find $X(k)$

c) $x((n-3))_4$

property

6) DFT of a complex conjugate sequence

let $x(n)$ be a complex sequence if $DFT\{x(n)\} = X(k)$

then $DFT\{x^*(k)\} = X^*((-k))_N = X^*(N-k)$

proof :-

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad 0 \le k \le N-1$$

$$X^*(k) = \left\{\sum_{n=0}^{N-1} x(n)\, W_N^{kn}\right\}^*$$

$$X^*(k) = \sum_{n=0}^{N-1} x^*(n)\, W_N^{-kn} \quad \text{— (A)}$$

changing k to −k

$$X^*(-k) = \sum_{n=0}^{N-1} x^*(n)\, W_N^{kn}$$

$$X^*(-k) = DFT\{x^*(n)\}$$

changing k to N−k in (A)

$$X^*(N-k) = \sum_{n=0}^{N-1} x^*(n)\, W_N^{-(N-k)n}$$

$$= \sum_{n=0}^{N-1} x^*(n)\, \cancel{W_N^{-Nn}}\, W_N^{kn}$$

$$X^*(N-k) = \sum_{n=0}^{N-1} x^*(n)\, W_N^{kn}$$

$$X^*(N-k) = DFT\{x^*(n)\} = X^*((-k))_N$$

1) Find the so DFT of complex sequence $x(n)$ is given by

$X(k) = \{j, 1+j, 1+j2, 2+2j, 4+j\}$

find $Y(k)$ if $y(n) = x^*(n)$

sol $Y(k) = DFT\{y(n)\}$

$= DFT\{x^*(n)\}$

$= X^*((-k))_N$

$X^*(N-k)$

$Y(0) = X^*(5-0) = X^*(5-5) = X^*(0) = -j$

$Y(1) = X^*(5-1) = X^*(4) = 4-j$

$Y(2) = X^*(5-2) = X^*(3) = 2-2j$

$Y(3) = X^*(5-3) = X^*(2) = 1-2j$

$Y(4) = X^*(5-4) = X^*(1) = 1-j$

$Y(k) = \{-j, 4-j, 2-2j, 1-2j, 1-j\}$

imp

7) circular convolution in time :-

$y(n) = x(n) * h(n)$

$= x(n) \, N \, h(n)$

$= x(n) \otimes_N h(n)$

$$y(n) = \sum_{m=0}^{N-1} x((m))_N \, h((n-m))_N$$

$Y(k) = X(k) \, H(k)$

proof :- $y(n) = \text{IDFT}\{Y(k)\}$

$= \text{IDFT}\{X(k) H(k)\}$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \, H(k) \, W_N^{-kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \left\{ \sum_{l=0}^{N-1} x(l) W_N^{lk} \right\} \left\{ \sum_{p=0}^{N-1} h(p) W_N^{pk} \right\} W_N^{-kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} x(l) \cdot \sum_{p=0}^{N-1} h(p) \cdot \sum_{k=0}^{N-1} (l+p-n) k$$

$$\sum_{k=0}^{N-1} W^{(l+p-n)k} = N$$

if $l+p-n = 0$ or $p = n-l$

$$= \frac{1}{\cancel{N}} \sum_{l=0}^{N-1} x(l) \, h(n-l) \, \cancel{N}$$

$$y(n) = \sum_{l=0}^{N-1} x((l))_N \, h((n-l))_N$$

$$\boxed{y(n) = x(n) * h(n)}$$

problems

1) find $x_1(n) \circledast x_2(n)$ if $x_1(n) = \{1, 2, 3, 1\}$ and $x_2(n) = \{\underset{\uparrow}{4}, 2, 2, 3\}$

a) using time domain approch

b) using frequency domine approch.

a) $$y(n) = \sum_{m=0}^{N-1} x_1((m))_N \, x_2((n-m))_N \quad 0 \le n \le N-1$$

$N = 4$

(i)

n	$x_1((m))_N$	$x_2((n-m))_N$	$y(n)$
0	(1, 2, 3, 1)	(4, 2, 2, 3)	17
1	(1, 2, 3, 1)	(3, 4, 2, 2)	19
2	(1, 2, 3, 1)	(2, 3, 4, 2)	22
3	(1, 2, 3, 1)	(2, 2, 3, 4)	19

Ex :- $1\times4 + 2\times2 + 3\times2 + 1\times3 = 17$

(ii) matrix approch :-

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

b) $$X_1(k) = \sum_{n=0}^{3} x_1(n) \, W_4^{kn}$$

$X_1(k) = x(0) W_4^0 + x(1) W_4^k + x(2) W_4^{2k} + x(3) W_4^{3k}$

$X_1(0) = 7$ $\quad X(3) = -2 + j$

$X_1(1) = -2 - j$

$X_1(2) = \cancel{-2+j}\ 1$

$$X_2(k) = \sum_{n=0}^{3} x_2(n) \, W_4^{kn}$$

$X_2(k) = x(0) W_4^0 + x(1) W_4^k + x(2) W_4^{2k} + x(3) W_4^{3k}$

$X_2(0) = 11$ $\quad X_2(2) = 1$

$X_2(1) = 2 + j$ $\quad X_2(3) = 2 - j$

_/_/_

*** HW

2) Find circular convolution of the following sequence

a) Time domain approach

b) Frequency domain approach

i) $x_1(n) = \{2, 3, 1, 1\}$

ii) $x_2(n) = \{1, 3, 5, 3\}$

3) Find circular convolution of the following sequence

$x_1(n) = \{1, 2, 3, 4\}$

$x_2(n) = \{5, 6, 7\}$

4) For the sequence $x_1(n) = \{2, 1, 1, 2\}$ & $x_2(n) = \{1, -1, 1, -1\}$

a) compute circular convolution $x_3(n)$

b) compute linear convolution $x_4(n)$

c) compute linear convolution using circular convolution $x_5(n)$

d) compute L.C using C.C $x_6(n)$

sol 2) $N = 4$

$n = 0, 1, 2, 3$

n	$(x_1(m))_4$	$(x_2(n-m))_4$	
0	(2, 3, 1, 1)	(1, 3, 5, 3)	= 19
1	(2, 3, 1, 1)	(3, 1, 3, 5)	= 17
2	(2, 3, 1, 1)	(5, 3, 1, 3)	= 23
3	(2, 3, 1, 1)	(3, 5, 3, 1)	= 25

$y(n) = \{19, 17, 23, 25\}$
$\uparrow$

Frequency domain

$X_1(k) = \sum_{n=0}^{N-1} (x_1(m))_N W_N^{kn}$

$x_1(k) = \sum_{n=0}^{3} (x_1(m))_4 W_4^{kn}$

$= 2 + 3W_4^k + W_4^{2k} + W_4^{3k}$

$X_2(k) = \sum_{n=0}^{N-1} (x_2(m))_N W_N^{kn}$

$= \sum_{n=0}^{3} (x_2(m))_4 W_4^{kn}$

$= 1 + 3W_4^{kn} + 5W_4^{2k} + 3W_4^{3k}$

$Y(k) = (X_1(k))(X_2(k))$

$= (2 + 3W_4^k + W_4^{2k} + W_4^{3k})(1 + 3W_4^k + 5W_4^{2k} + 3W_4^{3k})$

$= 2 + 3W_4^k + W_4^{2k} + W_4^{3k} + 6W_4^k + 9W_4^{2k} + 3W_4^{3k} + 3W_4^{4k} + 10W_4^{2k} + 15W_4^{3k} + 5W_4^{4k} + 5W_4^{5k} + 6W_4^{3k} + 9W_4^{4k} + 3W_4^{5k} + 3W_4^{6k}$

$= 2 + 9W_4^k + 20W_4^{2k} + 25W_4^{3k} + 17W_4^{4k} + 8W_4^{5k} + 3W_4^{6k}$

WKT :- $W_4^0 = W_4^{4k} = 1$

$W_4^k = W_4^{5k} =$

$W_4^{2k} = W_4^{6k} =$

$= \cancel{2+17} \; 19 + 17W_4^k + 23W_4^{2k} + 25W_4^{3k} + 17 + 8W_4^k + 3W_4^{2k}$

$= 19 + 17W_4^k + 23W_4^{2k} + 25W_4^{3k}$

$= 19 + 17\delta(n-1) + 23\delta(n-2) + 25\delta(n-3)$

$y(n) = \{19, 17, 23, 25\}$
$\uparrow$

3) $N = 4$

$n = 0, 1, 2, 3$

n	$(x_1(m))_4$	$(x_2(n-m))_4$	
0	(1, 2, 3, 4)	(5, 6, 7, 0)	38
1	(1, 2, 3, 4)	(0, 5, 6, 7)	56
2	(1, 2, 3, 4)	(7, 0, 5, 6)	46
3	(1, 2, 3, 4)	(6, 7, 0, 5)	40

$y(n) = \{38, 56, 46, 40\}$

Frequency domain

$$X_1(k) = \sum_{n=0}^{N-1} (x_1(m))_N \, W_N^{kn}$$

$$= \sum_{n=0}^{3} (x_1(m))_4 \, W_4^{kn}$$

$$= 1 + 2W_4^k + 3W_4^{2k} + 4W_4^{3k}$$

$$X_2(k) = \sum_{n=0}^{N-1} (x_2(m))_N \, W_N^{kn}$$

$$= \sum_{n=0}^{3} (x_2(m))_4 \, W_4^{kn}$$

$$= 5 + 6W_4^k + 7W_4^{2k}$$

$$Y(k) = X_1(k) \, X_2(k)$$

$$= (1 + 2W_4^k + 3W_4^{2k} + 4W_4^{3k})(5 + 6W_4^k + 7W_4^{2k})$$

$$= \cancel{5 + 12W_4^k + 21W_4^{2k}} \; 5 + 10W_4^k + 15W_4^k + 20W_4^{3k}$$

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I

1) For the sequence $x_1(n) = \cos\left[\frac{2\pi n}{N}\right]$ & $x_2(n) = \sin\left[\frac{2\pi n}{N}\right]$ $0 \le n \le N-1$

Find the N point circular convolution $x_3(n) = x_1(n) * x_2(n)$

sol

$$x_3(n) = \sum_{m=0}^{N-1} x_1((m))_N \, x_2((n-m))_N$$

$$X_3(k) = X_1(k)\, X_2(k)$$

$$x_3(n) = \text{IDFT}\,\{X_3(k)\}$$

$$x_1(n) = \cos 2\pi n/N$$

$$= \frac{1}{2}\left[e^{j\frac{2\pi n}{N}} + e^{-j\frac{2\pi n}{N}}\right]$$

$$= \frac{1}{2}\left[W_N^{-n} + W_N^{n}\right]$$

$$x_2(n) = \sin 2\pi n/N$$

$$= \frac{1}{2j}\left[e^{j\frac{2\pi n}{N}} - e^{-j\frac{2\pi n}{N}}\right]$$

$$= \frac{1}{2j}\left[W_N^{-n} - W_N^{n}\right]$$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, W_N^{kn}$$

$$= \frac{1}{2}\left\{\sum_{n=0}^{N-1} W_N^{-n} W_N^{kn} + \sum_{n=0}^{N-1} W_N^{n} W_N^{kn}\right\}$$

$$= \frac{1}{2}\left\{\sum_{n=0}^{N-1} W_N^{(k-1)n} + \sum_{n=0}^{N-1} W_N^{(k+1)n}\right\}$$

$$= \frac{1}{2}\left\{N\delta(k-1) + N\delta(N+1)\right\} \quad (1)$$

$$X_2(k) = \frac{1}{2j}\left\{N\delta(k-1) - N\delta(N+1)\right\} \quad (2)$$

$$X_3(k) = X_1(k)\, X_2(k) \qquad \left[\sum_{n=0}^{N-1} W_N^{kn} = N\delta(k)\right]$$

$$X_3(k) = \frac{N^2}{4j}\left\{\delta(k-1) - \delta(k+1)\right\} \quad (3)$$

$$X_3(k) = \frac{N}{2}\left\{\frac{N}{2j}\left[\delta(k-1) - \delta(k+1)\right]\right\} \quad (4)$$

$$x_3(n) = \frac{N}{2}\sin\left[\frac{2\pi n}{N}\right] \quad 0 \le n \le N-1$$

__/__/__

property 8)

Multiplication in time :-

$$DFT\{x_1(n) x_2(n)\} = \frac{1}{N}[X_1(k) \circledast_N X_2(k)]$$

proof :-

$$DFT\{x_1(n) \cdot x_2(n)\} = \sum_{n=0}^{N-1} \{x_1(n) x_2(n)\} W_N^{kn}$$

$$= \sum_{n=0}^{N-1} x_1(n) \left[\frac{1}{N} \sum_{l=0}^{N-1} X_2(l) W_N^{-ln}\right] W_N^{kn}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} X_2(l) \left\{ \sum_{n=0}^{N-1} x_1(n) W_N^{(k-l)n} \right\}$$

$$= \frac{1}{N} \sum_{l=0}^{N-1} x_2((l))_N \, X_1((k-l))_N$$

$$= \frac{1}{N} \{X_1(k) * X_2(k)\}$$

problem :-

1) Find $x_3(n) = x_1(n) \cdot x_2(n)$ if

$$x_1(n) = \{1,1,1,1,1,1,1,1\}$$

$$x_2(n) = \cos(\pi/4\, n) \quad 0 \le n \le 7$$

$$X_1(k) = \sum_{n=0}^{7} x_1(n) W_8^{kn}$$

$$= \sum_{n=0}^{7} 1 \cdot W_8^{kn} = \sum_{n=0}^{7} W_8^{kn}$$

$$X_1(k) = \{\underset{\uparrow}{8},0,0,0,0,0,0,0\}$$

$$\sum_{n=0}^{N-1} W_N^{kn} = \begin{cases} N \Rightarrow k=0 \\ 0 \Rightarrow k \neq 0 \end{cases}$$ identity / Rules

$$X_2(k) = \sum_{n=0}^{7} x_2(n) W_8^{kn} = \sum_{n=0}^{7} \cos(\pi/4\, n) W_8^{kn}$$

$$= \frac{1}{2} \sum_{n=0}^{7} \left[e^{j\pi/4 n} + e^{-j\frac{\pi}{4} n}\right] W_8^{kn}$$

$$= \frac{1}{2} \sum_{n=0}^{7} \left[W_8^{-n} + W_8^{n}\right] \cdot W_8^{kn}$$

$$= \frac{1}{2} \sum_{n=0}^{7} \left\{ W_8^{(k-1)n} + W_8^{(k+1)n} \right\}$$

$$= \frac{1}{2} \left\{ \sum_{n=0}^{7} W_8^{(k-1)n} + \sum_{n=0}^{7} W_8^{(k+1)n} \right\}$$

$$= \frac{1}{2} [8\delta(k-1) + 8\delta(k+1)]$$

$$= 4\delta(k-1) + 4\delta(k+1)$$

$$\left[\sum_{n=0}^{N-1} W_N^{(k-k_0)n} = N\delta(k-k_0) \right]$$

$$= \begin{cases} 4 \rightarrow k=1 \\ 4 \rightarrow k=-1 \\ 0 \rightarrow \text{otherwise} \end{cases}$$

k+N

$-1+8=7$

$X_2(k) = \{0,4,0,0,0,0,0,4\}$

DFT $\{x_1(n)\, x_2(n)\}$

$$X_3(k) = \frac{1}{N}\left[X_1(k) * X_2(k)\right]$$

$$\begin{bmatrix} X_3(0) \\ X_3(1) \\ X_3(2) \\ X_3(3) \\ X_3(4) \\ X_3(5) \\ X_3(6) \\ X_3(7) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 4 \\ 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 32 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 32 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

9) parseval's theorem :- [Inner product]

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X(K)^2|$$

proof :- LHS $\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x^*(n)\, x(n)$

$$= \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{K=0} X(K)\, W_N^{-Kn} \right\}^* x(n)$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left\{ \sum_{K=0}^{N-1} X^*(K)\, W_N^{nk} \right\} x(n)$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K) \left\{ \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \right\}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K)\, X(K)$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

problems :-

1) Find Energy of 4 · sequence

a) using time domain approch

b) using Frequency domain approch

if $x(n) = \sin\left[\frac{2\pi}{N} n\right] \quad 0 \le n \le 3$

a) $x(n) = \sin\left[\frac{2\pi}{N} n\right] = \sin\left[\frac{\pi}{2} n\right] \quad 0 \le n \le 3$

$= \{0, 1, 0, -1\}$

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = 1^2 + 1^2 = 2 \text{ jouls}$$

b) $X(k) = DFT\{x(n)\}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

$$E = \frac{1}{4} \sum_{n=0}^{3} |X(k)|^2 = \frac{1}{4}[4+4] = 2 \text{ jouls}$$

2) verify parsevall's theorem for $x(n) = \{1, 3, 5, 3\}$ (↑ under the 1)

a) Time domain

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = 1^2 + 3^2 + 5^2 + 3^2 = 44 \text{ jouls}$$

b) Frequency domain

$X(k) = DFT\{x(n)\}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 0 \\ -4 \end{bmatrix}$$

$$E = \frac{1}{4} \sum_{n=0}^{3} |X(k)|^2 = \frac{1}{4}[144 + 16 + 0 + 16] = 44 \text{ joules}$$

3) if $x(n) = \{1, 2, 0, 3, -2, 4, 7, 5\}$

find following a) $X(0)$ b) $X(4)$ c) $\sum_{k=0}^{7} X(k)$ d) $\sum_{k=0}^{7} |X(k)|^2$

a) Time domain

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = 1 + 4 + 9 + 4 + 16 + 49 + 25 = 108 \text{ Joule}$$

b) Freq domain

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 3 & -2 & 4 & 7 & 5 \\ 5 & 1 & 2 & 0 & 3 & -2 & 4 & 7 \\ & & & & & & & \end{bmatrix}$$

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$X(k) = DFT\{x(n)\}$

$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} = \sum_{n=0}^{7} x(n) W_8^{kn}$

$X(k) = 1 + 2W_8^{k} + 0 + 3W_8^{3k} - 2W_8^{4k} + 4W_8^{5k} + 7W_8^{6k} + 5W_8^{7k}$

$X(0) = 20$

b) $X(4) = 1 + 2W_8^{4} + 3W_8^{12k} - 2W_8^{16k} + 4W_8^{20k} + 7W_8^{24k} + 5W_8^{28k}$

$= 1 + 2(-1) + 3(-1) + (-2)(1) + \cdot$

$= -16 + 8$

$= -8$

property 10) symmetry :- DFT of real even & real odd sequence

Let $x(n)$ be a length 'N' real sequence with a n point DFT given by $X(k)$. if $x(n) = x_e(n) + x_o(n)$ where $x_e(n)$ is the even part & $x_o(n)$ odd part of the sequence $x(n)$ then DFT of $\{x_e(n)\}$ is purely real & DFT $\{x_o(n)\}$ is purely imaginary.

proof :- $x_e(n) = \frac{1}{2}[x(n) + x((-n))_N]$

$DFT\{x_e(n)\} = \frac{1}{2} DFT\{x(n)\} + \frac{1}{2} DFT\{x((-n))_N\}$

$= \frac{1}{2} X(k) + \frac{1}{2} X((-k))_N$

$= \frac{1}{2}[X(k) + X((-k))_N]$

$= \frac{1}{2}[X(K) + X^*(K)]$

$X(K) = A + jB$

$X^*(K) = A - jB$

$DFT\{x_e(n)\} = \frac{1}{2}[A + \cancel{jB} + A - \cancel{jB}]$

$= \frac{1}{2}[2A] = A$

$x_o(n) = \frac{1}{2}[x(n) - x((-n))_N]$

$= \frac{1}{2}[X(K) - X^*(K)]$

$X_o(K) = \frac{1}{2}[\cancel{A} + jB - \cancel{A} + jB]$

$= \frac{1}{2}[\cancel{2}jB] = jB$

property 11) periodicity :-

if $DFT\{x(n)\} = X(K)$ then, $x(n+N) = x(n) \quad \forall n$

$X(K+N) = X(K) \quad \forall K$

proof :- $x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{\frac{j2\pi Kn}{N}} \quad 0 \le n \le N-1$

$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{\frac{2\pi}{N}K(n+N)}$

$= \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{j\frac{2\pi}{N}Kn} \underbrace{\cancel{e^{j\frac{2\pi}{N}KN}}}_{=1}$

$= \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{j\frac{2\pi}{N}Kn}$

$\boxed{x(n+N) = x(n)}$

$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$

$X(K+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(K+N)}$

$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn} \underbrace{\cancel{e^{-j\frac{2\pi}{N}KN}}}_{=1}$

$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$

$\boxed{X(K+N) = X(K)}$

problem:-

1) Find 4. DFT of the sequence $x(n) = \{1, -1, 1, -1\}$ also using time shift property find DFT of the sequence

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$y(n) = x((n-2))_4$

2) Find IDFT of 4-sequence $X(K) = \{4, -2j, 0, 2j\}$ using the DFT

$X^*(k) = \{4, 2j, 0, -2j\}$ [conjugate]

$$DFT\{X^*(k)\} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 2j \\ 0 \\ -2j \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ \\ \\ \end{bmatrix}$$

3) consider a signal of length $= 4$ defined by $x(n) = (1, 2, 3, 1)$

a) compute 4-DFT by solving explicity the 4/4 system of linear equations defined by the inverse DFT

b) verify the result of part-A by finding $X(K)$ using the defining eq of DFT

sol a) $IDFT\{X(K)\} = x(n)$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) W_N^{-Kn}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) e^{j\frac{2\pi}{N}Kn}$$

$$\sum_{K=0}^{N-1} X(K) e^{j\frac{2\pi}{N}Kn} = N x(n)$$

$$\sum_{K=0}^{3} X(K) e^{j\frac{2\pi}{4}Kn} = 4x(n) \quad [n = 0, 1, 2, 3]$$

using linear eq

$X(0) + X(1) + X(2) + X(3) = 4x(0) = 4$

$X(0) + X(1) e^{j\pi/2} + X(2) e^{j\pi} + X(3) e^{j2\pi/2} = 4x(1) = 8$

$X(0) + X(1) e^{j\pi} + X(2) e^{j2\pi} + X(3) e^{j3\pi} = 4x(2) = 12$

$X(0) + X(1) e^{j\frac{3\pi}{2}} + X(2) e^{j3\pi} + X(3) e^{j\frac{9\pi}{2}} = 4x(3) = 4$

Applications of DFT in Linear filtering :-

1) An FIR filter an impulse response of $h(n) = \{\underset{\uparrow}{1}, 2\}$. determine the response of the filter to the input

—/—/—

sequence $x(n) = \{1,2,3\}$ using DFT and IDFT. Verify the result by direct computation of linear.

sol $L_1 = 2 \quad L_2 = 3$

length of the o/p sequence $L = L_1 + L_2 - 1 = 2+3-1 = 4$

$h_1(n) = \{1,2,0,0\}$

$x_1(n) = \{1,2,3,0\}$

$X_1(k) = \sum_{n=0}^{N-1} x_1(n) W_N^{kn}$

$= \sum_{n=0}^{3} x_1(n) W_4^{kn}$

$= 1 + 2W_4^{k} + 3W_4^{2k}$ method -1

$H_1(k) = \sum_{n=0}^{3} h_1(n) W_N^{kn}$

$= 1 + 2W_4^{k}$

$Y(k) = X_1(k) \cdot H_1(k)$

$= (1+2W_4^{k}+3W_4^{2k})(1+2W_4^{k})$

$= 1 + 4W_4^{k} + 7W_4^{2k} + 6W_4^{3k}$

$y(n) = \{\underset{\uparrow}{1}, 4, 7, 6\}$

* direct computational method — method 2

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 6 \end{bmatrix}$$

2) determine the o/p $y(n)$ for the system $h(n) = \{1,2\}$ to an i/p $x(n) = \{1,2,0,4\}$ using a) circular convolution b) DFT & IDFT c) verify the result by direct computation

sol a) $L_1 = 2, \; L_2 = 4$

$L = L_1 + L_2 - 1 = 2+4-1 = 5 \quad [N=5]$

$h_1(n) = \{1,2,0,0,0\}$ & $x_1(n) = \{1,2,0,4,0\}$

n	$h_1((n))_N$	$x_1((n))_N$		
0	(1 2,0,0,0)	(1,2,0,4,0)	=	5
1	(1 2,0,0,0)	(0,1,2,0,4)	=	2
2	(1,2,0,0,0)	(4,0,1,2,0)	=	4
3	(1,2,0,0,0)	(0,4,0,1,2)	=	8