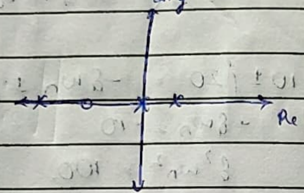
Find e_{ss} with step & ramp i/p with & w/o controller. Compare the results and comment. In which case error performance is better

13/11/23

1. Plant tr fn of a system is $G = \frac{s+1}{s(s-1)(s+6)}$ a. Locate poles & zeros of G. b. Is the plant stable

c. The proportional controller with k is used with unity f/b. By tuning k (k>0) can the cl system be made stable. Find the range of K for the stability of cl system

→ $G = \frac{s+1}{s(s-1)(s+6)}$



Zeros: -1

poles: $0, 1, -6$.

The plant is unstable

$L(s) = DGH$ $\quad CE = 1 + DGH$

$= \frac{K(s+1)}{s(s-1)(s+6)}$ $\quad = 1 + \frac{K(s+1)}{s(s-1)(s+6)}$

$= \frac{s(s-1)(s+6) + K(s+1)}{s(s-1)(s+6)}$

$CE = s(s-1)(s+6) + Ks + K = 0.$

$= (s^2 - s)(s+6) + Ks + K$

$= s^3 + 6s^2 - s^2 - 6s + Ks + K$

$= s^3 + 5s^2 + (K-6)s + K$

s^3	1	$K-6$
s^2	5	K
s^1	$\frac{4K-30}{5}$	0
s^0	K	

$\frac{5K-30-K}{5}$ $\quad \frac{4K-30}{5}$

(ii) $\frac{4K-30}{5} > 0.$

⇒ (i) $K > 0$ $\quad$, $4K > 30$

$K > \frac{30}{4} = 7.5$

2. A 2nd order plant $G = \frac{A}{s(\tau s+1)}$ use proportional controller with gain K_p & unity f/b.

a. Obtain CL tr fn.

b. Obtain the expression for K_p in terms of ξ & ω_n.

→ $CE = 1 + DGH$ $\qquad \frac{Y(s)}{R(s)} = \frac{GD}{1+GDH}$

$= 1 + \frac{K_pA}{s(\tau s+1)}$ $\qquad = \frac{\frac{K_pA}{s(\tau s+1)}}{\frac{s(\tau s+1) + K_pA}{s(\tau s+1)}} = \frac{K_pA}{\tau s^2 + s + K_pA}$

$= s^2 + \frac{s}{\tau} + \frac{K_pA}{\tau}$ $\qquad = \frac{A/\tau\, K_p}{s^2 + \frac{1}{\tau}s + \frac{K_pA}{\tau}}$

$= s^2 + 2\xi\omega_n s + \omega_n^2$

⇒ $\omega_n = \sqrt{\frac{K_pA}{\tau}}$ → $K_p = \frac{\tau\omega_n^2}{A}$

$2\xi\omega_n = \frac{1}{\tau}$ ⇒ $\tau = \frac{1}{2\xi\omega_n}$

$\boxed{K_p = \frac{\omega_n}{2\xi A}}$

3. A 1st order plant $G = A/(\tau s+1)$ uses PI controller & unity f/b.

a. Obtain CL tr fn.

b. Obtain the expressions for K_p & K_I in terms of ξ & ω_n.

→ $L(s) = DGH$

$= \left(K_p + \frac{K_I}{s}\right)\frac{A}{\tau s+1}$

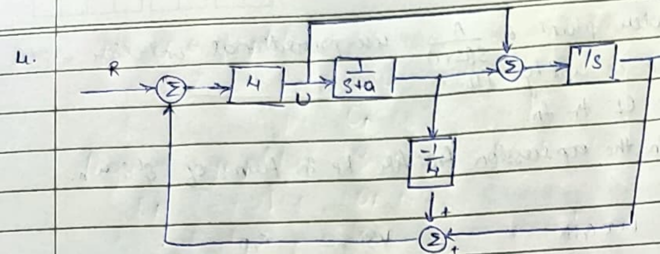
$\frac{Y(s)}{R(s)} = \frac{DG}{1+DGH} = \frac{\left(K_p + \frac{K_I}{s}\right)A/(\tau s+1)}{1 + \left(K_p + \frac{K_I}{s}\right)\frac{A}{\tau s+1}} = \frac{(K_psA + K_IA)}{s(\tau s + 1 + K_psA + K_IA)}$

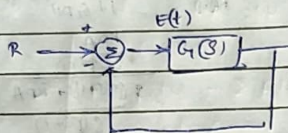
$= \frac{(K_ps + K_I)A/\tau}{s^2 + \left(\frac{1+AK_p}{\tau}\right)s + \frac{AK_I}{\tau}}$

$\omega_n^2 = \frac{1+K_pA}{\tau}$ ⇒ $\frac{AK_I}{\tau}$ ⇒ $K_I = \frac{\tau\omega_n^2}{A}$

$2\xi\omega_n = \frac{1+K_pA}{\tau}$ ⇒ $K_p = \frac{2\xi\omega_n\tau - 1}{A}$

4.





Apply superposition theorem

$\to Y(s) = \frac{1}{s}\left[1+\frac{1}{s+a}\right]U(s)$

$U(s) = 4\left[R(s) - \left[Y(s) - \frac{1}{4}\frac{1}{s+a}\right]\right]$

$= \frac{4(s+a)[R(s)-Y(s)]}{s+a-1}$

$Y(s) = \frac{1}{s}\left(\frac{s+a-1}{s+a}\right)\frac{4(s+a)}{s+a-1}(R(s)+Y(s))$

$= \frac{4(s+a+1)(R(s)-Y(s))}{s(s+a-1)}$

$G(s) = \frac{4(s+a+1)}{s(s+a-1)}$

b. $a=1$

$\frac{4(s+2)}{s^2}$

$\Rightarrow k_p, k_v, k_a$

$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{s^2}{s^2+4s+8}$

$e_{ss} = \lim_{s\to0} s\cdot\frac{s^2}{s^2+4s+8}\cdot\frac{1}{s} = \infty \to k_p \to e_{ss}=0$ should be

Ramp. $\lim_{s\to0} s\cdot\frac{s^2}{s^2+4s+8}\cdot\frac{1}{s^2} \to \infty \to k_v \to e_{ss}=0$ should be

Parabolic.

$\lim_{s\to0} s\cdot\frac{s^2}{s^2+4s+8}\cdot\frac{1}{s^3} = \frac{1}{8}$ → Type 2

b). $a \leftarrow 1+5a$.

5. DC motor $\dot{y} + 60y = 600V_a - 1500w$,

$V_a = -(K_p e + K_I\int e\,dt)$

$e = -y$

$(s+60)Y(s) = -600\left[K_pY(s) + K_I\frac{Y(s)}{s}\right] - 1500W(s)$

$\frac{Y(s)}{W(s)} = \frac{-1500s}{s^2+60(1+10K_p)s+600K_I}$

$-60\pm j60$,

$s^2+2\xi\omega_n s+\omega_n^2$

$(s+60+j60)(s+60-j60)$

$(s+60)^2+60^2 = s^2+120s+7200$

$\Rightarrow \omega_n^2 = 7200$

$\omega_n = 60\sqrt{2}$

$\xi = \frac{120}{2\omega_n} = \frac{1}{\sqrt{2}}$

$\omega_n^2 = 600K_I$

$K_I = \frac{600}{\omega_n^2} = 12$

$60(1+10K_p) = 2\xi\omega_n$

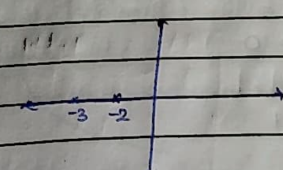
$\boxed{K_p = 0.1}$

— Absent —

$L(s) = \frac{K}{s(s+2)(s+3)}$

Open-loop poles: $s=0,-2,-3$ $P=3$

Open loop zeros: nil $Z=0$



Root locus on real axis lies on b/w $s=0$ and $s=-2$, $s=-3$ and $s=-\infty$

DATE

No. of asymptotes,

$P - Z = 3$

$q = 0$, $\theta_A = 60^\circ$

Angle of asymptotes.

$q = 1$, $\theta_A = 180^\circ$

$$\theta_A = \frac{(2q+1)180}{P-Z}$$

$q = 2$, $\theta_A = 300^\circ$

where $q = 0, 1, 2 \ldots$

$$\sigma = \frac{\Sigma \text{ R.P of poles} - \Sigma \text{ R.P of zeros}}{P - Z}$$

(R.P: real part)

$$= \frac{(-2-3) - 0}{3}$$

$$= -5/3$$

$$= -1.66$$

Breakaway points:

$$\frac{dK}{ds} = 0.$$

$CE: 1 + L(s) = 0$

$$1 + \frac{K}{s(s+2)(s+3)} = 0$$

$$K = -s(s+2)(s+3)$$

$$K = -s(s^2+5s+6)$$

$$K = -(s^3+5s^2+6s)$$

$$\frac{dK}{ds} = -(3s^2+10s+6) = 0$$

$$\frac{-10 \pm \sqrt{100-72}}{6} = \frac{-10 \pm \sqrt{28}}{6}$$

$s_1, s_2 = -0.71, \cancel{-2.54}$

Value of k at breakaway points

$CE = 1 + L(s) = 0$

$$1 + \frac{K}{s(s+2)(s+3)} = 0$$

$$\frac{k}{s(s+2)(s+3)} = -1$$

$$\left| \frac{K}{s(s+2)(s+3)} \right| = 1$$

Every point on root locus must satisfy this.

DATE

$$K = |s(s+2)(s+3)|$$

$$K_{s_1 = -0.71} = |s||s+2||s+3|$$

$$= 0.71 \times 1.29 \times 2.29$$

$$= 2.21$$

Intersection of root locus with imaginary axis.

This can be obtained using RH criteria.

$$CE: 1 + \frac{K}{s(s+2)(s+3)} = 0 \rightarrow s^3 + 5s^2 + 6s + K = 0$$

s^3	1	6
s^2	5	K
s^1	$\frac{30-K}{5}$	0
s^0	K	

For the system to be stable, all the elements in 1 column must be positive

(i) $K > 0$

(ii) $\frac{30-K}{5} > 0 \Rightarrow 0 < K < 30$

Consider s^2 row from which K marg is obtained.

$A(s) = 5s^2 + K = 0$

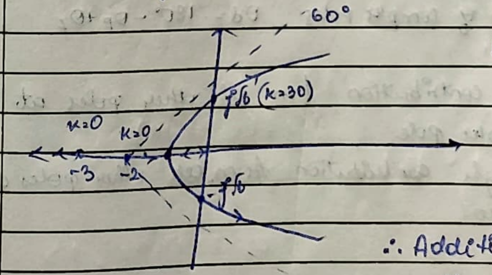
$5s^2 + 30 = 0$

$s^2 = -6$

$s = \pm j\sqrt{6} = \pm j\omega$

$\therefore \omega = \sqrt{6}$ rad/sec

- when $K = 30$ branches of root locus cuts $j\omega$ axis at $\pm j\sqrt{6}$ and the system becomes marginally stable.
- $K > 30$, The root locus enters into right half of s plane and the system becomes unstable



$$L(s) = \frac{K(s+1)}{s(s+2)(s+3)} \xleftarrow{\text{Added one zero}} \frac{K}{s(s+2)(s+3)}$$

$0 < K < \infty$ — No restriction on value of K

$0 < K < 30$

$\therefore$ Additional zero improves stability of system

DATE

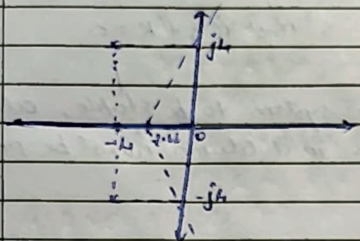
Complex poles:-

$$L(s) = \frac{K}{s(\underbrace{(s+4)^2+16}_{-4+j4,\ -4-j4})} = \frac{K}{s(s^2+8s+32)}$$

$\rightarrow \frac{-8 \pm \sqrt{64-128}}{2}$

$= \underline{-4 \pm j4}$

Open loop poles: $s=0$ $s = -4 \pm j4$



Root locus on real axis lies b/w $s=0$ & $s=-\infty$

No. of asymptotes: $P-Z = 3$.

Angles: $\frac{(2q+1)180}{P-Z} \rightarrow 60°, 180°, 300°$

centroid

$$\sigma = \frac{(0-4-4)-0}{3} = -2.66$$

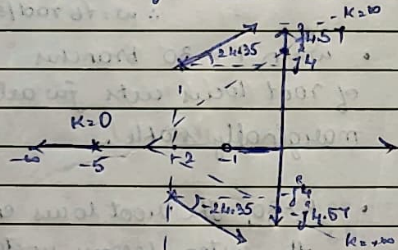
Angle of departure:-

For complex poles, it is necessary to find the angle of departure

Angle of departure is given by $\boxed{\theta_d = 180° - \theta_P + \theta_Z}$

22/11/23

$$L(s) = \frac{s+1}{s(s+5)(s^2+4s+8)}$$



$P-Z=3$ $\theta_A = 60°, 180°, 300°$

$$\sigma = \frac{0-4-4}{3} = -2.66$$

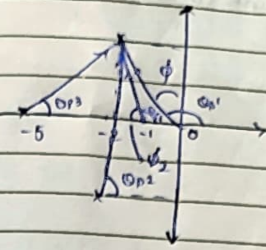
Angle of departure of complex poles $\theta_d = 180° - \theta_P + \theta_Z$

$\theta_P = \Sigma$ angle contribution from all other poles at the complex pole

$\theta_Z = \Sigma$ angle contribution from all other poles at zero

classmate PAGE

DATE



$\theta_P = \theta_{P1} + \theta_{P2} + \theta_{P3}$

$\theta_Z = \theta_{Z1}$

$\phi_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56°$

$\theta_{P1} = 90 + 26.56 = 116.56°$

$\theta_{P2} = 90°$

$\theta_{P3} = \tan^{-1}\left(\frac{4}{3}\right) = \underline{53.13°}$

$\theta_P = 116.56 + 90 + 53.13 = 259.69°$

$\theta_{Z1} = \theta_Z = 180 - \phi_2$ $\because \theta_{Z1} + \phi_2 = 180°$

$= 180° - \tan^{-1}\left(\frac{4}{1}\right)$

$= 180° - 75.96$

$\theta_{Z1} = 104.04°$

$\theta_d = 180° - 259.69 + 104.04$

$\boxed{\theta_d = 24.35°}$

Intersection of root locus with imaginary axis:-

$$L(s) = \frac{K(s+1)}{s(s+5)(s^2+4s+8)}$$

$1 + L(s) = s(s+5)(s^2+4s+8) + Ks + K = 0$

$= (s^2+5s)(s^2+4s+8) + Ks + K = 0$

$= s^4 + 4s^3 + 8s^2 + 5s^3 + 20s^2 + 40s + Ks + K = 0$

$= s^4 + \underset{9s^3}{\cancel{9}s^3} + 28s^2 + (40+K)s + K = 0$

s^4	1	28	K
s^3	9	K+40	0
s^2	$\frac{252-K-40}{9}$	K	
s^1	h	0	
s^0	K		

$$\frac{\frac{212-K}{9}(K+40) - 9K}{\frac{212-K}{9}}$$

$$\frac{212K + 8480 - K^2 - 40K - 81K}{9 \cdot \frac{212-K}{9}}$$

$$h = \frac{8480 + 91K - K^2}{\frac{212-K}{9}}$$

$212 - K > 0$

$\boxed{K < 212}$

$h > 0$

$8480 + 91K - K^2 > 0$

classmate PAGE

Sri Sai Srihari

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For the system to be stable, all elements in Routh's table - I column must be positive.

$-K^2 + 91K + 8480 > 0$

$(K-K_1)(K-K_2) > 0 \rightarrow (a)$

$K_1, K_2 = \dfrac{-91 \pm \sqrt{91^2 - 4(8480)}}{2}$

$K_1 = 148.21$ $\quad K \not< 148.21$

$K_2 = -57.21$

$(K - 148.21)(K + 57.21) > 0 \rightarrow (4)$

Substitute 150 for K (4) is +ve.

Substitute -60 to K (4) is +ve

(i) $K > 0$

(ii) $K < 212$

(iii) $K \not< 148.2$ ✓

(iv) $K \not> -57.12 \rightarrow \times$

~~148.2 < K < 212~~ $\quad 0 < K < 148.2 \Rightarrow K_{marg} = 148.2$

If we substitute $K = 148.2$ in eqn of S^2

$A(S) = \dfrac{212 - 148.2}{9} S^2 + 148.2 = 0$

$S = \pm 4.57 j = \pm j\omega$

$\omega = 4.57$

→ Solve quadratic eqn

→ Choosing proper range of K

→ Complex pole

→ Angle of departure

27/11/23

1. Construct root locus for $L(S) = \dfrac{K}{S((S+1.5)^2 + 16)}$

$L(S) = \dfrac{1}{S(S^2 + 3S + 10)}$ $\quad$ Angle of departure:

→ $Z = 0$ $\quad S = 0, -1.5 \pm 2.78j, -1.5, -2.78j$

$P = 3$

classmate PAGE

DATE

$\theta_d = 180° - \theta_P + \theta_Z$

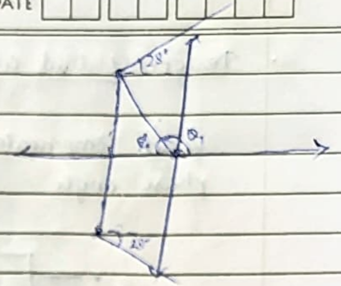
$\theta_1 = 180° - \phi = 180° - \tan^{-1}\left(\dfrac{2.78}{1.5}\right)$

$= 118.34'$

$\theta_d = 180° - \theta_P$

$= 180° - [118° + 90]$

$\theta_d = \pm 28°$



2. $L(S) = \dfrac{1}{(S^2 + 3S + 10)}$ $\rightarrow \sigma = -3 \quad \theta_a = \pm 90° \quad \theta_d = \pm 90° \quad \omega \rightarrow$ none

29/11/23

Design of Compensators:

The controller gain K_P, K_I, K_D are selected to obtain the desired steady state error.

→ Controller improves steady state response

$e_{ss}(\text{step}) = \dfrac{1}{1 + K_P}$ → Controller gain

$e_{ss} \downarrow \Rightarrow K_P \uparrow$

- If gain is ↑ to decrease ss error transient response will be poor effect M_P, t_r

- Compensator → Add poles and zeros to improve steady state response along with transient response

- A well designed control system incorporates both controller and compensator.

Design of compensator:

- Time domain method ↓ Root locus
- Freq domain method ↓ Bode plot

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OP → cp.999

1 place

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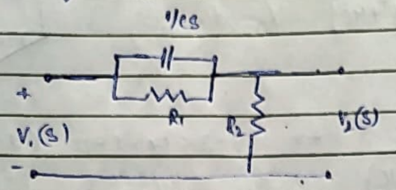
Design of lead compensator using R-L technique

Lead compensator adds +ve phase angle to the system phase angle



$Z_1 = R_1 \parallel 1/cs$

$= \dfrac{R_1 \frac{1}{CS}}{R + \frac{1}{CS}}$

$Z_1 = \dfrac{R_1}{1+R_1 CS}$

$V_2 = \dfrac{V_1 Z_2}{Z_1 + Z_2}$

$\dfrac{V_2}{V_1} = \dfrac{R_2}{\frac{R_1}{1+R_1C} + R_2}$

$= \dfrac{R_2(1+R_1CS)}{(R_1+R_2+R_1R_2CS)}$

$= \underbrace{\dfrac{R_2}{(R_1+R_2)}}_{\alpha} \; \dfrac{1+R_1CS}{1+\frac{R_1R_2}{R_1+R_2}CS}$

$= \alpha = \dfrac{R_2}{R_1+R_2} < 1$ [∵ Den is greater than Num]

$\dfrac{V_2}{V_1} = \alpha \dfrac{1+\tau S}{1+\alpha\tau S}$

Zero: $1+\tau S = 0 \Rightarrow S = -1/\tau = S_1$

Poles: $1+\alpha\tau S = 0 \Rightarrow S = -1/\alpha\tau = S_2$

$|S_2| > |S_1|$ [∵ α is less than one]

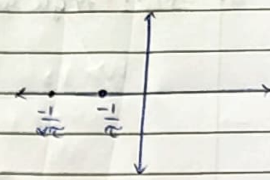
DATE

Freq response

Sinusoidal i/p

$S = j\omega$

$\dfrac{V_2(j\omega)}{V_1(j\omega)} = \alpha \dfrac{1+j\omega\tau}{1+j\omega\alpha\tau}$



$\angle \dfrac{V_2}{V_1} = \phi \; (+ve)$

$= \angle N^r - \angle D^r$

$= \tan^{-1}(\omega\tau) - \tan^{-1}(\alpha\omega\tau)$

$\Rightarrow \tan^{-1}(\omega\tau) > \tan^{-1}(\alpha\omega\tau)$

$\omega\tau > \alpha\omega\tau$

$\tau > \alpha\tau$

$\boxed{\dfrac{1}{\tau} < \dfrac{1}{\alpha\tau}}$

1. The plant tr. fn of unity f/b control system is given by $G(S) = \dfrac{K}{S(S+1)}$. Design a lead compensator using root locus technique to meet the following specifications. Damping ratio: $\xi = 0.7$, $t_s = 1.4$ sec, $k_v \geq 2\ sec^{-1}$ (2% tolerance)

→ Step 1: Using the specifications calculate the desired pole locations (to meet the given spec)

$\xi = 0.7 \rightarrow$ under damped ⇒ Desired poles are complex conjugates

$S_d = -\xi\omega_n \pm j\omega_d$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

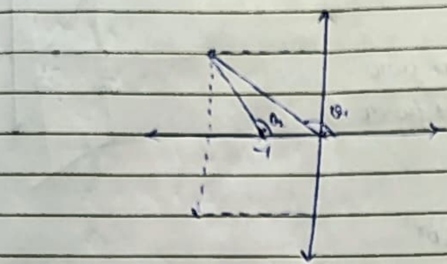
$t_s = \dfrac{3.91}{\xi\omega_n}$ [2% tolerance]

$\omega_n = \dfrac{3.91}{0.7 \times 1.4} = 4$ rad/sec

$S_d = -2.8 \pm j2.8$

DATE

Step 2: Angle requirement from lead compensator



- To obtain the desired spec S_{d_1} & S_{d_2} must lie on root locus

Root locus satisfies $1+L(s)=0$

$G(s) = -1 = 1\angle 180^\circ$

- For every point on root locus, $|G|=1$ and $\angle G = -180^\circ$

$\theta_1 = \tan^{-1}\frac{2.8}{2.8} = 180-45 = 135^\circ$

$\theta_2 = 180^\circ - \tan^{-1}\frac{2.8}{1.8} = 122.73^\circ$

$\theta = \theta_1 + \theta_2 = -257.73$

$\frac{\angle Nr}{\angle Dr} = \underset{Z}{\angle Nr} - \underset{P}{\angle Dr}$ ∵ Angle contributed by poles will be negative.

$= -135 - 122.73$

$= -257.73$

$-257.73 + \phi = -180^\circ$

$\boxed{\phi = 77.7^\circ}$

+ve → lead controller.

A standard form of lead controller is given by

$D(s) = \frac{s+Z}{s+P}$ $\quad s+Z=0 \Rightarrow s=-Z$, zero of compensator

$s+P=0 \Rightarrow s=-P$, pole of compensator

∴ Where to fix pole and zero of compensator.

$Z = \frac{\omega_n \sin\gamma}{\sin(\theta+\gamma)}$ $\qquad \gamma = \frac{1}{2}[180-\theta-\phi]$

$P = \frac{\omega_n \sin(\gamma+\phi)}{\sin(\theta+\gamma+\phi)}$

7/2/23

DATE

$\theta = \cos^{-1}\xi$

ϕ = compensator angle

$\xi = 0.7$

$\theta = 45.57^\circ$

$\gamma = 28.365^\circ$

$Z = 1.977$

$P = 8.090$

Transfer function of lead compensator

$D(s) = \frac{s+Z}{s+P} = \frac{s+1.977}{s+8.090}$

Required gain at P

$|D(s)\,G(s)|_{at\,P} = 1$

$\left|\frac{s+Z}{s+P}\,\frac{K}{s(s+1)}\right|_{s=-2.8+j2.8} = 1$

$\left|\frac{-2.8+j2.8+1.977 \quad K}{(s+8.09)(s(s+1))}\right| = 1$

$K(-0.823+j2.8) = s(s+1)(s+8.09)$ $\qquad \frac{75.89}{2.91}$

$K = -27 + 0.41j$

$K = 27.01$

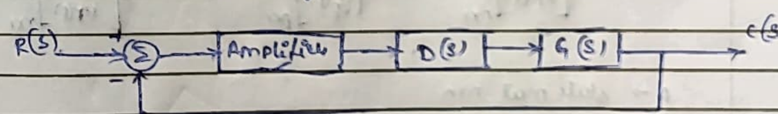
Gain requirement based on k_v

$k_v = \lim_{s\to 0} s\,G(s) = 2$ [Given]

Additional gain can be adjusted using amplifier

$25 = 2 \times$ amplifier gain

Amplifier gain $= 12.5$



Calculate actual k_v:

$k_v = \lim_{s\to 0}$ Amplifier gain $\times D(s) \times G(s)$

$= \lim_{s\to 0} s \cdot 12.5 \times \frac{s+Z}{s+P} \times \frac{2}{s(s+1)} = 6.109$

a. $G(S) = \frac{K}{S(S+1)(S+4)}$ $\quad \xi = 0.5 \quad \omega_n = 2$ rad/sec

$K_v \geq 1.5$

→ Desired poles (Sd)

$S_d = -\xi\omega_n \pm j\omega_d$

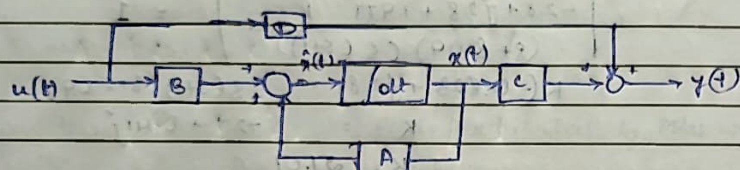
System phase angle at P

$\theta_1 = 90 + \phi.$

$\theta_2 =$

4/10/23 State space analysis:-

The state of a system which is defined as minimum no. of interconnections that must be specified at any initial time to so that the complete dynamic behaviour of a system at any time $t \geq t_0$ is determined when the i/p $u(t)$ is known



1. $\dot{x}(t) = A\,x(t) + B\,u(t) \rightarrow ①$
2. $y(t) = C\,x(t) + D\,u(t) \rightarrow ②$

$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n\times 1}$ $\quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}_{p\times 1}$ $\quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}_{r\times 1}$

State vector

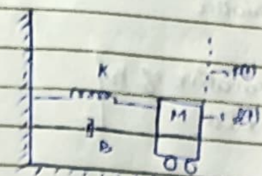
A → state mat n×n

B → i/p mat n×r

C → o/p mat p×n

D → Direct transmission mat p×r

The system is described by the 2nd order DE & consists of 2 state variables



$M\frac{d^2p(t)}{dt^2} + B\frac{dp(t)}{dt} + K\,p(t) = f(t) \rightarrow ①$

Let $x_1(t) = p(t)$

$\dot{x}_1(t) = \frac{dp(t)}{dt} \rightarrow ②$

Let $x_2(t) = \frac{dp(t)}{dt} = \dot{x}_1(t)$

From eqn ① $\dot{x}_2(t) = \frac{d^2p(t)}{dt^2}$

$M\dot{x}_2(t) + B\,x_2(t) + K\,x_1(t) = u(t)$

$\Rightarrow \cancel{\dot{x}_2(t) + \frac{B}{M}x_2(t) + \frac{K}{M}x_1(t)}$

$\Rightarrow \dot{x}_2(t) = -\frac{K}{M}x_1(t) - \frac{B}{M}x_2(t) + \frac{1}{M}u(t) \rightarrow ③$

In matrix form eqn ② & eqn ③

$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix} u(t)$

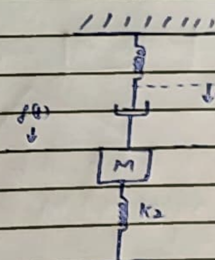
$\dot{x}(t) = A\,x(t) + B\,x(t)$

$A = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$

$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = C\,x(t)$

$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

Problem:-



→ $M\ddot{P}_2 + B(\dot{P}_2 - \dot{P}_1) + K_2 P_2 = f(t) \rightarrow ①$

$B(\dot{P}_1 - \dot{P}_2) + K_1 P_1 = 0 \rightarrow ②$

6 (3)

DATE

2^{nd} order DE $\Rightarrow$ Two State variables

$$x_1 = P_2$$

$$\dot{x}_1 = \dot{P}_2 = x_2 \longrightarrow (3)$$

$$\dot{x}_2 = \ddot{P}_2$$

[2^{nd} derivative of P_2 exist]

$$x_3 = P_1$$

$$M\dot{x}_2 + K_1 x_3 + K_2 x_1 = u(t)$$

$$\dot{x}_2 = \frac{-K_2}{M} x_1 - \frac{K_1}{M} x_3 + \frac{1}{M} u(t) \longrightarrow (4)$$

$$B\dot{x}_3 - Bx_2 + K_1 x_3 = 0$$

$$\dot{x}_3 = x_2 - \frac{K_1}{B} x_3 \longrightarrow (5)$$

In matrix form.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{K_2}{M} & 0 & \frac{-K_1}{M} \\ 0 & 1 & -\frac{K_1}{B} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \\ 0 \end{bmatrix} u(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -K_2/M & 0 & \frac{-K_1}{M} \\ 0 & 1 & \frac{-K_1}{B} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/M \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$