20EC510

__/__/__

1) Show that the matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 3 & 2 \\ 0 & 6 & 4 \end{bmatrix}$ are equilent using Elemen-tary row operation.

Elementary row operation of A

so, $R_2 \to R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_1 \to C_1 - \frac{1}{2} C_2$

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \longrightarrow R_2 + 2R_1$$

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \end{bmatrix}$$

2) Reduce the following matrices to Hchelon and hence find their rank

a) $$A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$$

interchange R_3 with R_1

$$A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \end{bmatrix} \quad R_3 \longrightarrow R_3 - 5R_1$$

$$A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix} \quad R_3 \longrightarrow R_3 + 8R_1$$

$$A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & -4 \end{bmatrix}$$

$\therefore$ The Rank is 3

$\rho(A) = 3$

because of no. of nonzero

b) $B = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

$R_3 \to R_3 + R_1$

$B = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$ $\quad \rho(B) = 3$

c) $C = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_4 \leftrightarrow R_1$

$C = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ -2 & -1 & -3 & -1 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$C = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_2 \to R_2 + 2R_1$ $\quad R_3 \to R_3 - R_1$

$C = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 4 & 5 & -3 \\ & & & \\ & & & \end{bmatrix}$

$R_2 \to R_2 + R_3$

II By using elementry row transformation find the inverse of the following matrices

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$A = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$

$R_2 \leftarrow R_2 - 3R_1$

$A = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$ $\quad R_1 \leftarrow R_1 - 2R_2$

$A = \left[\begin{array}{cc:cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$ $\quad \boxed{AI = I^{-1}A}$

$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

2) $B = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$B = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right]$ $\quad R_2 \leftarrow R_2 - 2R_1$

$B = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array}\right]$ $\quad R_2 \leftarrow -\frac{1}{5} R_2$

$B = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 2/5 & -1/5 \end{array}\right]$ $\quad R_1 \leftarrow R_1 - 2R_2$

$B = \left[\begin{array}{cc:cc} 1 & 0 & 1/5 & 2/5 \\ 0 & 1 & 2/5 & -1/5 \end{array}\right]$

$B^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

III Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

$A = \left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$R_2 \leftarrow R_2 - 3R_1$ & $R_3 \leftarrow R_3 - R_1$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -4 & 0 & -3 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] \quad R_2 \leftarrow \frac{-1}{4} R_2$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] \quad R_3 \leftarrow R_3 + R_2$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 \end{array}\right] \quad R_1 \leftarrow R_1 - 2R_2$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -2/4 & 2/4 & 0 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 1 \end{array}\right] \quad R_1 \leftarrow R_1 - R_3$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 3/4 & -1 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1/4 & 3/4 & -1 \\ 3/4 & -1/4 & 0 \\ -1/4 & -1/4 & 1 \end{bmatrix}$$

IV using elementary transformation find the inverse of the following

1) $$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$ & $R_3 \leftarrow R_3 + 2R_1$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \quad R_2 \leftarrow R_2 - R_3$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 - 3R_3$ & $R_1 \leftarrow$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 0 & -2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 - 2R_2$

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

2) Find inverse of matrix $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$

$$A = \left[\begin{array}{cccc|cccc} 0 & 1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$A = \left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$ $R_3 \leftarrow R_3 - 2R_1$

$$A = \left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -3 & 0 & -2 & 1 & 0 \\ 0 & 1 & -1 & -3 & 0 & -2 & 0 & 1 \end{array}\right]$$

$R_1 \rightarrow R_1 - R_2$, $R_4 \rightarrow R_4 - R_2$

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -3 & 0 & -2 & 1 & 0 \\ 0 & 0 & -3 & -5 & -1 & -2 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 + R_3$ $R_3 \leftarrow \cancel{R} \frac{1}{2} R_2$

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -2 & 3/2 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -3 & -5 & -1 & -2 & 0 & 1 \end{array}\right]$$

_/_/_

Guassian Elimination method :-

a) $2x_1 + 4x_2 + x_3 = 3$ — ①

$3x_1 + 2x_2 - 2x_3 = 2$ — ②

$x_1 - x_2 + x_3 = 6$ — ③

multiply eq ① by 3/2

$\cancel{3x_1} + 6x_2 + \frac{3}{2}x_3 = 9/2$

$\underline{-\cancel{3x_1} \pm 2x_2 \mp 2x_3 = \_2}$

$4x^2 + 7/2\, x_3 = 5/2$ — ⑥

multiply eq ③ by ②

$2x_1 - 2x_2 + 2x_3 = 6$ — ④

$\cancel{2x_1} + 4x_2 + x_3 = 3$

$\underline{-\cancel{2x_1} \mp 2x_2 \pm 2x_3 = \_6}$

$8x_2 - x_3 = -3$ — ⑤

b) slove by the guassian elimination method

$2x + y + 4z = 12$ — ①

$8x - 3y + 2z = 23$ — ②

$4x + 11y - z = 33$ — ③

Multiply eq ① by 2

$4\cancel{x} + 2y + 8z = 24$

$\underline{-\cancel{4x} \pm 11y \mp z = \_33}$

$-9y + 9z = -9$ — ④

multiply eq ② by $\frac{1}{2}$

$\cancel{4x} - 3/2\, y + z = 23/2$

$\underline{-\cancel{4x} \pm 11y \mp z = \_33}$

$+ Q z$

_/_/_

* Echelon matrices and row canonical form of a matrix :-

→ Echelon :- matrix A is called echelon matrix if following two conditions satifies :-

- All zero rows, if any, are at the bottom of the matrix
- Non zero entry in a row, right of the leading non zero preceding row.

→ Row canonical form :- A matrix is said to be row canonical if it is an Echelon matrix that is it satisfies by above two properties and it satisfies following addition two properties

- Each pivot [leading non zero element in nonzero] = 1
- Each pivot element is the only non zerodentry in its coloumn.

problem :-

I) find the solution for given linear equation

i) $x + y - 6z = 0$

$-3x + y + 2z = 0$

$x - y + 2z = 0$

$$A = \begin{bmatrix} 1 & 1 & -6 \\ -3 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix} \quad \begin{matrix} R_2 \leftarrow R_2 + 3R_1 \\ R_3 \leftarrow R_3 - R_1 \end{matrix}$$

_/_/_

$$A = \begin{bmatrix} 1 & 1 & -6 \\ 0 & 4 & -16 \\ 0 & -2 & 8 \end{bmatrix} \quad R_3 \leftarrow R_3 + \frac{1}{2}R_2$$

$$A = \begin{bmatrix} 1 & 1 & -6 \\ 0 & 4 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y - 6z = 0$

$4y - 16z = 0$

$z = 0$ } $z = c$ [Take an orbitary value]

$4y - 16c = 0 \rightarrow 4y = 16c$

$\boxed{y = 4c}$

$x + 4c - 6c = 0$

$\boxed{x = 2c}$

2) $x + 2y + z = 3$

$2x + 5y - z = -4$

$3x - 2y - z = 5$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 3R_1 \end{array}$$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & -8 & -4 & -4 \end{array}\right] \quad R_3 \leftarrow R_3 + 8R_2$$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right] \quad R_1 \leftarrow R_1 - 2R_2$$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 7 & 23 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right] \quad R_1 \leftarrow 4R_1 + R_3 \quad \times$$

12/09/23

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 7 & 23 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right] \quad R_3 \leftarrow \frac{-1}{28} R_3$$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 7 & 23 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & 0 & 3 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 + 3R_3 \\ R_1 \leftarrow R_1 - R_3 \end{array}$$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \leftarrow R_1 - 2R_2$$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 2$
$y = -1$
$c = 3$
} row canonical form
$\rho(M) = 3$ [rank of augmented]
rank of $(A) = 3$ [$r(A) = 3$]
no of varible $= 3$ [$n = 3$]

If the rank of $r(A)$ = no of varibles 'n' then there will be unique solution.

(or)

The solution is unique if and only if $r(A) = r(M) = n$

~~[illegible]~~

1) $x + 2y + z = 3$

$2x + 5y - z = -4$

$3x - 2y - z = 5$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

$$M = \left[\begin{array}{cccc} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & -8 & -4 & -4 \end{array}\right]$$

$R_3 \leftarrow R_3 + 8R_2$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right]$$

$R_3 \leftarrow -\frac{1}{28} R_3$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ arguemented matrix

$R_1 \leftarrow R_1 - R_3$

$R_2 \leftarrow R_2 + 3R_3$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ $R_1 \leftarrow R_1 - 2R_2$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

rank (A) = rank (M) = 3 [solution is there]

$\rho(A) = \rho(M) = N = 3$ [unique solution]

N = no. of varibles in simplified matrix

Gauss - Jordan method

1) $$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{array}\right]$$ $R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

$$M = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & -8 & -4 & -4 \end{array}\right]$$ $R_3 \leftarrow R_3 + 8R_2$

$R_1 \leftarrow R_1 - 2R_2$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 7 & 23 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right]$$ $R_3 \leftarrow -1/28 R_3$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 7 & 23 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ $R_1 \leftarrow R_1 - 7R_3$

$R_2 \leftarrow R_2 + 3R_3$

$$M = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\rho(M) = \rho(A) = N = 3$ [unique solution]

2) $x_1 + x_2 - 2x_3 + 4x_4 = 5$

$2x_1 + 2x_2 - 3x_3 + x_4 = 3$

$3x_1 + 3x_2 - 4x_3 - 2x_4 = 1$

$$M = \left[\begin{array}{cccc:c} 1 & 1 & -2 & 4 & 5 \\ 2 & 2 & -3 & 1 & 3 \\ 3 & 3 & -4 & -2 & 1 \end{array}\right]$$ $R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

$$M = \left[\begin{array}{cccc:c} 1 & 1 & -2 & 4 & 5 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 2 & -14 & -14 \end{array}\right]$$ $R_3 \leftarrow R_3 - 2R_2$

$$M = \left[\begin{array}{cccc:c} 1 & 1 & -2 & 4 & 5 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(M) = 2$

$\rho(A) = \rho(M) \neq N$ [infinite solution]

simplified Eq $x_1 + x_2 - 2x_3 + 4x_4 = 5$

$x_3 - 7x_4 = -7$

Observe that x_1 & x_3 are piote varibles and x_2 & x_4 are free varibles by considering orbitary values to the free varibles we can obtain infinite solution for given non-homogeneous Eq.

3) $x_1 + x_2 - 2x_3 + 3x_4 = 4$

$2x_1 + 3x_2 + 3x_3 - x_4 = 3$

$5x_1 + 7x_2 + 4x_3 + x_4 = 5$

$$M = \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & 1 & 5 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 5R_1 \end{array}$$

$$M = \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 2 & 14 & -14 & -15 \end{array}\right] \quad R_3 \leftarrow R_3 - 2R_2$$

$$M = \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right]$$

$\rho(M) = 3$

$\rho(A) = 2$

$\rho(M) \neq \rho(A)$ [No solution]

3rd row is called has degenerate Equation

LDU Factorization [Decomposition]

1) suppose $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 13 \\ 2 & 1 & -5 \end{bmatrix}$ reduce the given co-efficient matrix has LDU Factorization.

$R_2 \leftarrow R_2 + 3R_1$

$R_3 \leftarrow R_3 - 2R_1$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & -3 & 1 \end{bmatrix} \quad R_3 \leftarrow R_3 + 3/2\, R_2$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Echelon matrix $= A = U = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}$ Upper triangular valuy

13/09/23

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -3/2 & 1 \end{bmatrix}$$ Low triangular

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$ Diagonal matrix

I 1) Find the LDU Factorization of

a) $A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 + R_1$

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & -3 \\ 0 & -5 & 6 \end{bmatrix} \quad R_3 \leftarrow R_3 + \frac{5}{2} R_2$$

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & -3 \\ 0 & 0 & -3/2 \end{bmatrix} = U \text{ [Upper triangular matrix]}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ +2 & 1 & 0 \\ -1 & -5/2 & 1 \end{bmatrix} \text{ [lower triangular matrix]}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3/2 \end{bmatrix}$$

b) $B = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 8 & 1 \\ -5 & -9 & 7 \end{bmatrix}$ $\quad R_2 \leftarrow R_2 - 2R_1$, $R_3 \leftarrow R_3 + 5R_1$

$$B = \begin{bmatrix} 1 & 4 & -3 \\ 0 & 0 & 7 \\ 0 & 11 & -8 \end{bmatrix}$$ ~~$R_2 \leftrightarrow R_3$~~

observed that the 2nd diagonal entry is zero thus the above triangular form without row interchange forms

II slove the following Equations by LUD composition method and obtain the solution for the unknows

1) $2x_1 + x_2 + x_3 = 2$

$x_1 + 3x_2 + 2x_3 = 2$

$3x_1 + x_2 + 2x_3 = 2$ $\quad Ax = B$

$$A = \left[\begin{array}{ccc:c} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 3 & 1 & 2 & 2 \end{array}\right] \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

To find L & U consider only co-efficient matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} \quad \begin{array}{l} R_2 \leftarrow 2R_2 - R_1 \\ R_3 \leftarrow \frac{2R_3}{3} - R_1 \end{array}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 5 & 3 \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \quad R_3 \leftarrow -3R_3$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 5 & 3 \\ 0 & 1 & -1 \end{bmatrix} \quad R_3 \leftarrow 5R_3 - R_2$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 5 & 3 \\ 0 & 0 & -8 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 3/2 \\ 0 & 0 & 4/5 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -1/5 & 1 \end{bmatrix} \Big\} \text{sir's}$$

$LY = B$ — 1

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -1/5 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$Y_1 = 2$

$\frac{1}{2} Y_1 + Y_2 = 2$

$\boxed{Y_2 = 1}$

$\boxed{Y_3 = -4/5}$

$UX = Y$ — 2

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 3/2 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4/5 \end{bmatrix}$$

$\boxed{x_3 = -1}$

$5/2x_2 + 3/2x_3 = 1$

$5/2x_2 + 12/10 = 1$

$5/2x_2 = -1/5$

$\boxed{x_2 = 1}$

$\boxed{x_1 = -1}$

2) $2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$R_2 \leftarrow R_2 - \frac{1}{2}R_1$

$R_3 \leftarrow R_3 - \frac{3}{2}R_1$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & -7/2 & 1/2 \end{bmatrix} \quad R_3 \leftarrow R_3 + 7R_2$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$Y_1 = 9$

$Y_2 = 3/2$

$Y_3 = 5$

$UX = Y$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 5 \end{bmatrix}$$

$z = 5/18$

$y = 29/18$

$x = 35/18$

3) $2x - 3y + 10z = 3$

$-x + 4y + 2z = 20$

$5x + 2y + z = -12$

$$A = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

$R_2 \leftarrow R_2 + \frac{1}{2}R_1$

$R_3 \leftarrow R_3 - 5/2 R_1$

$$A = \begin{bmatrix} 2 & -3 & 10 \\ 0 & 5/2 & 7 \\ 0 & 19/2 & -24 \end{bmatrix} \quad R_3 \leftarrow R_3 - \frac{19}{5}R_2$$

$$A = \begin{bmatrix} 2 & -3 & 10 \\ 0 & 5/2 & 7 \\ 0 & 0 & -253/5 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 5/2 & 19/5 & 1 \end{bmatrix}$$

$X = -4 \quad Y = 3 \quad z = 2$

16/9/23

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 5/2 & 19/5 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

$Y_1 = 3$

$Y_2 = 43/2$

$Y_3 = -431/5$

$UX = Y$

$$\begin{bmatrix} 2 & -3 & 10 \\ 0 & 5/2 & 7 \\ 0 & 0 & -253/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

$z =$

$y =$

$x =$

Vector spaces :-

In order to discuss vector space we use set of vectors & scalars. To define a vector space we need a field 'F' and elements of F is scalar. In addition to that we need two binary operation vector addition and scalar multiplication. this defined using Internal & External composition.

- Internal composition :- let R be any set
  $a * b \in R \quad \forall \; a, b \in R$
  and $a*b$ is unique and this $*$ is known has Internal composition.
- External composition :- let V be the set of vectors and F be a field. then a binary operation defined b/w the vectors and scalars is called external composition.
  if $a \circ \alpha \in V \quad \forall \alpha \in V \;\&\; a \in F$
  $a\circ\alpha$ is unique

Introduction to vectors spaces :-

Let G be a non-empty set & * be a binary operation defined on it, then structure $(G,*)$ is said to be a group if the following axioms are satisfied:

→ Axioms :-

(i) closure property :- $a*b \in G \ \forall a,b \in G$

(ii) associative :- $a*(b*c) = (a*b)*c \ \forall a,b, \in G$

(iii) Existance identity :- where existant an element $e \in G$ such that $a*e = e*a = a \ \forall a \in G$

(iv) Existance of inverse :- for each element $a \in G$ there exist an element $b \in G$ such that $a*b = b*a = e$ where element b is called inverse of element a.
$b = a^{-1}$

Abelian (or) commutative group :-

A group $(G,*)$ is said to abelian or commutative if $a*b = b*a \ \forall a,b \in G$

the group which are not Abelian called Non abelian or non-commutative group

→ Finite and infinite group :-

if the group contains finite no. of elements then it is finite group. if it has infinite no. of elements then it is called infinite group.

order of group :-

• The no. of elements in finite group is called order of group.

→ Definition of field :- Let F be an any Non empty set equiped with two binary operation addition and multiplication $a, b \in F$, $a+b \in F$ & $a \cdot b \in F$.

The alzebric structure $(F,+,\cdot)$ is called field if it satifies the following condition.

• addition is associative
$(a+b)+c = a+(b+c) \ \forall a,b,c \in F$

$\in$ = belongs

• addition is commutative.
$a+b = b+a \ \forall a,b \in F$

• There exits an identity element 0 in F such that $a+0 = a = 0+a \ \forall a \in F$

• To each element $a \in F$ their exits
$a+(-a) = 0$

• multiplication is commutative
$a \cdot b = b \cdot a \ \forall a,b \in F$

• multiplication is associative
$(a \cdot b) \cdot c = a \cdot (b \cdot c) \ \forall a,b,c \in F$

• There exist an Non zero element in F such that
$a \cdot 1 = a = 1 \cdot a$ where $a \in F$

• There will be a^{-1} for every non-zero element $a \in F$ such that $a \cdot a^{-1} = 1$

• for multiplication there exist distibutive property
$a \cdot (b+c) = a \cdot b + a \cdot c \ \forall a,b,c \in F$

→ sub field :-

Let F be field. an non empty sub set K of F is said to sub field of F. if K is closed wrt addition & multiplication in F & K.

→ Vector space :- [definition] Let V be a non empty set of vectors and F be a field then an alzebric structure $(V,+,\cdot)$ together with two binary operation vector addition & scalar Multi is said to vector space over F. if this structure satifies the following condition :-

(i) $(V,+)$ is a abelian group

(ii) $a(\alpha+\beta) = a\alpha + a\beta \ \forall \alpha,\beta \in V$ & $a \in F$

(iii) $(a+b)\alpha = a\alpha + b\alpha \ \forall \alpha \in V$ & $a,b \in F$

(iv) $(ab)\alpha = a(b\alpha) \ \forall \alpha \in V$ & $a,b \in F$

(v) $1 \cdot \alpha = \alpha \ \forall \alpha \in V$ & $1 \in F$

This vector space V over F is denoted by $V(F)$

Examples :-

1) show that a field K can be regarded as a vector space over any subfield F of K

sol we know that $F \subset K$ [$\subset$ = subset]

Now K contains set of vectors and F contains set of scalar values.

we need to verify different properties to prove F is subset of K using two binary operation.

consider $a\alpha \in K$ $\forall a \in F$ & $\alpha \in K$

if 1 is the unity element of K then 1 is also unity element of subset F.

(i) $a(\alpha+\beta) = a\alpha + a\beta$ $\forall a \in F$ & $\alpha, \beta \in K$

(ii) $(a+b)\alpha = a\alpha + b\alpha$ $\forall a, b \in F$ & $\alpha \in K$

(iii) $(ab)\alpha = a(b\alpha)$ $\forall a, b \in F$ & $\alpha \in K$

iv) $1\alpha = \alpha$ $\forall \alpha \in K$, $1 \in F$

By the above observation K is a vector space over the field F which is denoted as K(F)

2) show that the set of all ordered n-tuples forms a vector space over a field F.

sol $R^n = \{(a_1, a_2, \dots a_n)\}$; $a_i \in R$

if $a_1, a_2, a_3, \dots a_n$ are n elements of field f then an ordered set $\alpha = (a_1, a_2, a_3, \dots a_n)$ is called as an n-tuples over F

Now we shall show that V is vector space wrt addition composition and scalar multiplication.

(i) closure property :-

for all $\alpha = (a_1, a_2, \dots a_n) \in V$

$\beta = (b_1, b_2, \dots b_n) \in V$

$\alpha + \beta = (a_1+b_1, a_2+b_2 + \dots a_n+b_n)$

since $a_1+b_1, a_2+b_2 \dots a_n+b_n$ are all elements of F so that $\alpha+\beta \in V$ $\forall \alpha, \beta \in V$

hence V is closed for addition of tuples.

ii) associativity of addition in V :-

$\alpha = (a_1, a_2, \dots a_n)$

$\beta = (b_1, b_2, \dots b_n)$

$\gamma = (c_1, c_2, \dots c_n)$ of V

$\alpha + (\beta+\gamma) = (\alpha+\beta)+\gamma$

$= (a_1, a_2, \dots a_n) + [(b_1, b_2 \dots b_n) + (c_1, c_2, \dots c_n)]$

$= [a_1+(b_1+c_1), a_2+(b_2+c_2) \dots, a_n+(b_n+c_n)]$

$= [(a_1+b_1)+c_1, (a_2+b_2)+c_2 \dots, (a_n+b_n)+c_n]$

$= [a_1+b_1, a_2+b_2, \dots a_n+b_n] (c_1, c_2 \dots c_n)$

$= [(a_1, a_2 \dots a_n) + (b_1, b_2 \dots b_n)] (c_1, c_2 \dots c_n)$

$= [\alpha+\beta]+\gamma$

iii) Existence of additive identity in V :-

consider $\alpha = (a_1, a_2, \dots a_n) \in V$

$\alpha + 0 = (a_1, a_2, \dots a_n) + (0, 0, \dots 0)$

$= (a_1+0, a_2+0, \dots a_n+0)$

$= (a_1, a_2 \dots a_n)$

$\alpha + 0 = \alpha$

iv) Existence of additive inverse in V

$\alpha = (a_1, a_2, \dots a_n) \in V$

$-\alpha = (-a_1, -a_2, \dots -a_n) \in V$

$\alpha + (-\alpha) = (a_1, a_2, \dots a_n) + (-a_1, -a_2, \dots -a_n) \in V$

$= [a_1+(-a_1), a_2+(-a_2) + \dots a_n+(-a_n)] \in V$

$\alpha + (-\alpha) = 0$ $\forall$ $0 \in V$

v) commutativity of addition in V :-

$\alpha = (a_1, a_2, \dots a_n) \in V$

$\beta = (b_1, b_2, \dots b_n) \in V$

$\alpha + \beta = (a_1, a_2, \dots a_n) + (b_1, b_2 \dots b_n)$

$= (a_1+b_1, a_2+b_2, \dots a_n+b_n)$

$= (b_1+a_1, b_2+a_2, \dots b_n+a_n)$

$= (b_1, b_2, \dots b_n) + (a_1, a_2, \dots a_n)$

$\alpha + \beta = \beta + \alpha$

Now we observe that $a(\alpha+\beta) = a\alpha + a\beta$ where $\alpha = a_1 a_2 \cdots a_n \in V$ & $\beta = b_1, b_2 \cdots b_n \in V$ $\forall a = F$ & $\alpha, \beta = V$

(i) $a(\alpha+\beta) = a[(a_1, a_2 \cdots a_n] + (b_1, b_2, \cdots b_n)]$

$= a[(a_1+b_2, a_2+b_2 + \cdots a_n+b_n)]$

$= a[(b_1+a_1, b_2+a_2 + \cdots b_n+a_n)]$

$= (ab_1+aa_1, ab_2+aa_2 + \cdots ab_n+aa_n)$

$= a(b_1, b_2 + \cdots b_n) + a(a_1, a_2, \cdots a_n)$

$= a\alpha + a\beta$

(ii) $\alpha(a+b) = \alpha a + \alpha\beta$ $a, b \in F$ $\alpha = (a_1, a_2, \cdots a_n) \in V$

$\alpha(a+b) = (a_1, a_2, \cdots a_n)(a+b)$

$= (a+b)a_1, (a+b)a_2, \cdots (a+b)a_n$

$= (aa_1+ba_1), (aa_2+ba_2) \cdots (aa_n+ba_n)$

$= (aa_1, aa_2, \cdots a_n) + (a_1b, a_2b \cdots a_nb)$

$= (a_1, a_2 \cdots a_n)a + (a_1, a_2, \cdots a_n)b$

$= \alpha a + \alpha b$

(iii) $(ab)\alpha = a(b\alpha)$

$(ab)\alpha = (ab)(a_1, a_2, \cdots a_n)$

$= [(ab)a_1, (ab)a_2, \cdots (ab)a_n]$

$= [a(ba_1), a(ba_2) \cdots a(ba_n)]$

$= a[ba_1, ba_2, \cdots ba_n]$

$= a[b(a_1, a_2 \cdots a_n)]$

$= a(b\alpha)$

(iv) if 1 is unity element of F and $\alpha = (a_1, a_2, \cdots a_n) = b$ then $1\alpha = \alpha$

$1\alpha = 1(a_1, a_2, \cdots a_n)$

$1\alpha = [(1 \cdot a_1), (1 a_2) \cdots (1 a_n)]$

$1\alpha = [a_1, a_2, \cdots a_n]$

$1\alpha = \alpha$

by the above observation b is a vector space over a field F and can be denoted as $V_n(F)$

19/09/23

3) show that the set of all $m \times n$ matrices with their elements as real no's is vector space over the field F of real no's wrt to addition of matrices has addition of vectors and multiplication of matrix by scalar has scalar multiplication.

sol Let $M_{mn} = \{A, B, C \cdots\}$ with the set of all $m \times n$. we shall show that $M_{mn}(F)$ will form abilean group under addition.

(i) closure property :-

For all $A, B \in M_{mn}$ we have $A+B \in M_{mn}$

ii) Associative property :-

For all $A, B, C \in M_{mn}$ we have $(A+B)+C = A+(B+C)$

iii) Existance of identity :-

If O be null matrix of order $m \times n$ then $O \in M_{mn}$ and also $A \in M_{mn}$ then we have $A+O = O+A = A$

here O is additive identity in the given vector space.

iv) Existance of inverse :-

$A \in M_{mn}$, $-A \in M_{mn}$ $\forall A \in M_{mn}$ we have

$A+(-A) = 0 = (-A)+A$

here $-A$ is the additive inverse of A

v) Commutativity :-

For all $A, B \in M_{mn}$ we have

$A+B = B+A$

If $a \in F$ and $A = [a_{ij}]_{m\times n} \in M_{mn}$

Now we observe that :-

For all $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ in M_{mn} & $a \in F$

then $a(A+B) = aA + aB$

$= a\left[[a_{ij}]_{m\times n} + [b_{ij}]_{m\times n}\right]$

$= a[(a_{ij}+b_{ij})_{m\times n}]$

$= (aa_{ij} + ab_{ij})_{m\times n}$

$= [aa_{ij}]_{m\times n} + [ab_{ij}]_{m\times n}$

$a(A+B) = aA + aB$

- For $a, b \in F$ and $A = [a_{ij}] \in M_{m \times n}$

$$\begin{aligned}(a+b)A &= aA + bA \\ &= (a+b)[a_{ij}]_{m\times n} \\ &= [(a+b)\cdot a_{ij}]_{m\times n} \\ &= [(a a_{ij} + b a_{ij})]_{m\times n} \\ &= [a a_{ij}]_{m\times n} + [b a_{ij}]_{m\times n} \\ &= a[a_{ij}]_{m\times n} + b[a_{ij}]_{m\times n} \\ (a+b)A &= aA + bA\end{aligned}$$

- For all $a, b \in F$ and $A = [a_{ij}]_{m\times n} \in M_{mn}$

$$\begin{aligned}(ab)A &= a(bA) \\ &= (ab)[a_{ij}]_{m\times n} \\ &= [(ab)a_{ij}]_{m\times n} \\ &= [a(ba_{ij})]_{m\times n} \\ &= a[ba_{ij}]_{m\times n} \\ (ab)A &= a(bA)\end{aligned}$$

- ~~For~~ since $1 \in F$ and $A = [a_{ij}]_{m\times n} \in M_{mn}$

$$\begin{aligned}1.A &= A \\ &= 1\cdot[a_{ij}]_{m\times n} \\ &= [1\cdot a_{ij}]_{m\times n} \\ &= [a_{ij}]_{m\times n} \\ 1.A &= A\end{aligned}$$

Vector subspaces [vector space within a vector space]:-
let W be an non empty subset of V, where V is a vector space over a field 'F'. then W is said to be a vector subspace of $V(F)$ if W is itself a vector space over F wrt to the same operations as defined on V.
For Ex:- The set $W = \{(a, b, 0) : a, b \in R\}$, subspace $R^3(R)$

Elementary properties of vector subspaces:-
→ Theorem 1:- The necessary & suffient condition for a non Empty subset W of $V(F)$ to be a subspace are that

20/9/23

(i) $\alpha \in W$, $\beta \in W \Rightarrow \alpha - \beta \in W$
(ii) $a \in F$, $\alpha \in W \Rightarrow a\alpha \in W$

proof:- suppose W is a subspace of vector space $V(F)$. then $\beta \in W \Rightarrow -\beta \in W$ [using inverse]
Therefore $\alpha \in W, -\beta \in W \Rightarrow \alpha + (-\beta) \in W$
$\Rightarrow \alpha - \beta \in W$

ii) $a \in F$, $\alpha \in W \Rightarrow a\alpha \in W$
conversly, suppose W is subset of V and
(i) $\alpha \in W$, $\beta \in W \Rightarrow \alpha - \beta \in W$
(ii) $a \in F$, $\alpha \in W \Rightarrow a\alpha \in W$
Now we have to show that W is a subset for the purpose we proced as follows:-
$\alpha \in W$, $-\alpha \in W \Rightarrow \alpha - \alpha \in W$
$\Rightarrow 0 \in W$
and $0 \in W$, $\alpha \in W \Rightarrow 0 - \alpha \in W$
$= -\alpha \in W$
Now, $\alpha \in W$, $-\beta \in W \Rightarrow \alpha - (-\beta) \in W$
$= \alpha + \beta \in W$
This proofs that W is a vector space sub of $V(F)$ & W satisfie vector addition having associative and commutative to form abelian.

→ Theorem 2:- The nessary & suffient condition for non Emty set of W of a vector space $V(F)$ To be a subspace of V
$a, b \in F$, $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$
proof:- suppose W is a subspace of vector space $V(F)$ Then W is ~~called~~ closed under vector addition & Multi therefore we have $a \in F$, $\alpha \in W \Rightarrow a\alpha \in W$
$b \in F$, $\beta \in W \Rightarrow b\beta \in W$
$a\alpha \in W$, $b\beta \in W \Rightarrow a\alpha + b\beta \in W$
conversly suppose W is a subset of $V(F)$ satisfying the above condition then we have to show that W is a

subset of $V(F)$ by performing vector add & scalar multiplication

- Now taking $a=1, b=1$ then

$1 \in F, \alpha, \beta \in W \Rightarrow 1\alpha + 1\beta \in W$

$\alpha + \beta \in W$

W is closed under the vector addition

- Now taking $a=0, b=-1$ then

$a\alpha + b\beta \in W$

$0\alpha + (-1)\beta \in W$

$-\beta \in W$

therefore additive inverse exist in W

- Now taking $a=0, b=0$ then

$a\alpha + b\beta \in W$

$\alpha \mp 0\beta \in W$

$0 \in W$

therefore exist of identity in W

since $W \subseteq V$ therefore vector add is associative & commutative then W is abelian group under vector addition

- Now taking $\beta = 0$ we have

$a\alpha + b\beta \in W$

$a\alpha + 0\beta \in W$

$a\alpha \in W$

Here W is closed under scalar multiplication hence W is a vector space and consequently W is vector space of $V(F)$

~~Sloved Examples~~

1) show that the set $W = \{(a,b,c) : a-3b+4c=0\}$ is a subspace of the 3-tuple space $R^3(R)$.

sol Let $\alpha = (a_1, b_1, c_1)$

$\beta = (a_2, b_2, c_2)$ be any two elements of W such that

$a_1 - 3b_1 + 4c_1 = 0$ & $a_2 - 3b_2 + 4c_2 = 0$ —— (1)

For $a, b \in R$ we have ~~$a\alpha + b\beta =$~~

$a\alpha + b\beta = a(a_1, b_1, c_1) + b(a_2, b_2, c_2)$

$= (aa_1, ab_1, ac_1) + (ba_2 + bb_2 + bc_2)$

$= (aa_1 + ba_2, ab_1 + bb_2, ac_1 + bc_2)$

$= (aa_1 + ba_2) - 3(ab_1 + bb_2) + 4(ac_1 + bc_2)$

$= (aa_1 + ba_2), (-3ab_1 - 3bb_2) + (4ac_1 + 4bc_2)$

$= (aa_1 - 3ab_1 + 4ac_1) + (ba_2 - 3bb_2 + 4bc_2)$

$= a(a_1 - 3b_1 + 4c_1) + b(a_2 - 3b_2 + 4c_2)$

$= a(0) + b(0)$ —— from eq (1)

$= 0$ there $a\alpha + b\beta \in W$ then $\alpha \in W, \beta \in W$ & $a\alpha + b\beta \in W$

$\forall\, a, b \in R$ therefore W is subspace of $R^3(R)$

2) show that the set $W = \{(a_1, a_2, 0) : a_1, a_2 \in F\}$ is a subspace of $V_3(F)$

sol Let $\alpha, \beta \in W$ then,

$\alpha = (a_1, a_2, 0)$ & $\beta = (b_1, b_2, 0)$ $\forall\, a_1, a_2, b_1, b_2 \in W$

$a\alpha + b\beta = a(a_1, a_2, 0) + b(b_1, b_2, 0)$

$= (aa_1, aa_2, a0) + (bb_1, bb_2, b0)$

$= (aa_1 + bb_1, aa_2 + bb_2, 0+0)$

since $aa_1 + bb_1, aa_2 + bb_2 \in F$ i.e $a\alpha + b\beta \in W$

hence W is subspace of $V_3(F)$

3) Let W be the collection of all elements from the space $M_2(F)$ of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ show that W is a subspace of $M_2(F)$. where W is given as $W = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in F \right\}$

sol Let $\alpha, \beta \in W$ then

$\alpha = \begin{bmatrix} a_1 & b_1 \\ -b_1 & a_1 \end{bmatrix}$ & $\beta = \begin{bmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{bmatrix}$ $\forall\, a_1, b_1, b_2, a_2 \in F$

$a\alpha + b\beta = a \begin{bmatrix} a_1 & b_1 \\ -b_1 & a_1 \end{bmatrix} + b \begin{bmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{bmatrix}$

$= \begin{bmatrix} aa_1 & ab_1 \\ -ab_1 & aa_1 \end{bmatrix} + \begin{bmatrix} ba_2 & bb_2 \\ -bb_2 & ba_2 \end{bmatrix}$

$$= \begin{bmatrix} aa_1 + ba_2 & ab_1 + bb_2 \\ -(ab_1 + bb_2) & aa_1 + ba_2 \end{bmatrix}$$

$(aa_1 + ba_2), (ab_1 + ba_2) \in F$

hence W is a subspace of $M_2(F)$

4) If a_1, a_2, a_3 are fixed elements of a field F, then the set W of all ordered triads $[x_1, x_2, x_3]$ of elements of field F such that $[a_1x_1 + a_2x_2 + a_3x_3] = 0$ is a subspace of $V_3(F)$

sol let $W = \{(x_1, x_2, x_3) : a_1x_1 + a_2x_2 + a_3x_3 = 0\}$, $a_1a_2a_3$ are fixed

consider $\alpha, \beta \in W$ then

$\alpha = (x_1, x_2, x_3)$

$\beta = (y_1, y_2, y_3)$

$a_1(x_1) + a_2x_2 + a_3x_3 = 0$ then $a_1y_1 + a_2y_2 + a_3y_3 = 0$

$a\alpha + b\beta = a(x_1, x_2, x_3) + b(y_1, y_2, y_3)$

$= (ax_1, ax_2, ax_3) + (by_1, by_2, by_3)$

$= (ax_1 + by_1, ax_2 + by_2, ax_3 + by_3)$

$= a_1(ax_1 + by_1), a_2(ax_2 + by_2), a_3(ax_3 + by_3)$

$= a(a_1x_1, a_2x_2, a_3x_3) + b(a_1y_1 + a_2y_2 + a_3y_3)$

$= a(0) + b(0) = 0$ hence W is subspace of $V_3(F)$

5) Which of the following sets of vectors $\alpha = (a_1, a_2, a_3 \cdots a_n) \in R^n$ are subspaces of R^n $(n \geq 3)$

(i) all α such that $a_1 \leq 0$

(ii) all α such that a_3 is an integer

(iii) all α such that $a_2 + 4a_3 = 0$

(iv) all α such that $a_1 + a_2 + \cdots + a_n = k$

sol let $W = \{(a_1, a_2, a_3 \cdots a_n) : a_1 \leq 0\}$

(i) let $\alpha, \beta \in W$ then

$\alpha = (a_1, a_2, a_3 \cdots a_n)$

$\beta = (b_1, b_2, b_3 \cdots b_n)$

$a\alpha + b\beta = a(a_1, a_2, \cdots a_n) + b(b_1, b_2 \cdots b_n)$

$= (aa_1, aa_2 \cdots aa_n) + (bb_1, bb_2, \cdots bb_n)$

$= (aa_1 + bb_1, aa_2 + bb_2 \cdots, aa_n + bb_n)$

since $a_1 \leq 0$ & $b_1 \leq 0$ and if $a < 0$ & $b < 0$ then $aa_1 > 0$ & $bb_1 > 0$

so that $aa_1 + bb_1 > 0$ thus $a\alpha + b\beta$ does not belong W

hence W is not a subspace of $R^n(R)$

(ii) let $\{(a_1, a_2, \cdots a_n) : a_3$ is an integer$\}$

let $\alpha = (a_1, a_2, a_4 \cdots a_n) \in W$

consider $a = 1/3 \in R$

Now $a\alpha = \frac{1}{3}(a_1, a_2, 2, a_4 \cdots a_n)$

$a\alpha = a_1/3, a_2/3, 2/3, a_4/3 \cdots a_n/3$

$a\alpha \notin W$

hence W is not a subspace of R^n

iii) let $W = \{(a_1, a_2, a_3 \cdots a_n) : a_2 + 4a_3 = 0\}$

consider $\alpha = (a_1, a_2, a_3 \cdots a_n)$ & $\beta = (b_1, b_2, b_3 \cdots b_n)$

$a_2 + 4a_3 = 0$ & $b_2 + 4b_3 = 0$

$a\alpha + b\beta = a(a_1, a_2, a_3 \cdots a_n) + b(b_1, b_2, b_3 \cdots b_n)$

$= (aa_1, aa_2, aa_3, \cdots aa_n) + (bb_1, bb_2, bb_3, \cdots bb_n)$

$= (aa_1 + bb_1, aa_2 + bb_2, aa_3 + bb_3 \cdots aa_n + bb_n)$

Now according to the given function $(aa_2 + bb_2) + 4(aa_3 + bb_3)$

$aa_2 + 4aa_3, bb_2 + 4bb_3 = 0$

$a(a_2 + 4a_3), b(b_2 + 4b_3) = 0$

$a(0), b(0) = 0$

since $(aa_1 + bb_1 \cdots aa_n + bb_n)$ therefore $a\alpha + b\beta \in W$

hence W is subspace of R^n

iv) let $W = \{(a_1, a_2 \cdots a_n) : a_1 + a_2 \cdots a_n = k\}$

if $k = 0$ then W is a subspace of $R^n(R)$, but if $k \neq 0$ then W is not a subspace

6) let V be the vector space of all 2×2 matrices over the field R. show that W is not a subspace of V where W contains all 2×2 matrices with 0 determinants.

sol let $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$ where $a, b \in R$

22/09/23

and $a \neq 0$ & $b \neq 0$

$$A+B = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$|A+B| = ab \neq 0$

hence W is not a subspace of $V(R)$

Algebra of subspace

→ Theorem-1 :- Intersection of any two subspace of a vector space is a subspace

proof :- Let $V(F)$ be a vector space over F and W_1, W_2 be two subspaces of $V(F)$. then we have to show that intersection of $W_1 \cap W_2$ is subspace of $V(F)$

Let $\alpha, \beta \in W_1 \cap W_2$ which also indicates $\alpha, \beta \in W_1$ & $\alpha, \beta \in W_2$

Since $W_1 W_2$ are subspaces of V we have $a, b \in W$, $\alpha, \beta \in W_1$

which indicates $a\alpha + b\beta \in W_1$ — (1)

Similarly $a, b \in F$ & $\alpha, \beta \in W_2$ — (2)

which implies $a\alpha + b\beta \in W_2$

from (1) & (2) if $a, b \in F$ & $\alpha, \beta \in W_1 \cap W_2$ then

$a\alpha + b\beta \in W_1 \cap W_2$

hence $W_1 \cap W_2$ is subspace of $V(F)$

→ Theorem-2 :-

The intersection of orbitary collection of subspaces of a vector space is also a subspace.

proof :- Let $\{W_\lambda : \lambda \in x\}$ be an orbitary collection of a vector space V. then we have to show that $\cancel{a\alpha+b\beta}$ $\cap \{W_\lambda : \lambda \in x\}$ is subspace of V.

Let us consider $\alpha, \beta \in W_\lambda$ & $a, b \in F$ therefore $\alpha, \beta \in W_\lambda$ for each $\lambda \in x$, $a\alpha + b\beta \in W_\lambda$ for each $\lambda \in x$

$a\alpha + b\beta \in \cap \{W_\lambda : \lambda \in x\}$

hence $\cap\{W_\lambda : \lambda \in x\}$ is subspace of V

→ Theorem-3 :

The union of two spaces of a vector space is not nessarly a subspace

⊆ improper subspace

proof :- Let $W_1 W_2$ be two subspaces of vector space V where

$W_1 = \{(a_1, a_2, 0) : a_1, a_2 \in F\}$

$W_2 = \{(a_1, 0, a_3) : a_1, a_3 \in F\}$

By observing above values we can say that W_1 & W_2 are ~~spa~~ subspace of a vector space. $R^3(R)$.

Now if you consider the elements for the given subspaces and assign the numerical value such that $\alpha = (1,2,0)$ & $\beta = (3,0,5)$ $\in W_1 \cup W_2$ then for scalar's $a=1$ & $b=2$ & substitute in the subspace equation $a\alpha + b\beta = 1(1,2,0) + 2(3,0,5)$

$= (1,2,0) + (6,0,10)$

$= (7,2,10) \notin W_1 \cup W_2$

which may not exist in union W_1 & W_2

Thus if $\alpha \in W_1 \cup W_2$ & $\beta \in W_1 \cup W_2$ then it is not nessarly imply that $a\alpha + b\beta \in W_1 \cup W_2$ for some $a, b \in F$

→ Theorem-4 :-

The union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

proof :- Let $V(F)$ be a vector space & W_1, W_2 subspace of V

- Suppose $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. then we have to show that $W_1 \cup W_2$ is a subspace of V
- Suppose $W_1 \cup W_2 = W_2$ if $W_1 \subseteq W_2$ & W_2 is subspace of V. also $W_1 \cup W_2 = W_1$. if $W_2 \subseteq W_1$ & W_1 is subspace of V.
- Let $a, b \in F$, $\alpha, \beta \in W_1 \cup W_2 \Rightarrow a\alpha + b\beta \in W_1 \cup W_2$

Now taking $a=1$ & $b=1$ we have

$1\alpha + 1\beta \in W_1 \cup W_2$

$\alpha + \beta \in W_1 \cup W_2$

$\alpha + \beta \in W_1$ or $\alpha + \beta \in W_2$

Suppose $\alpha + \beta = W_1$ & $\alpha \in W_1$ then $(\alpha+\beta) - \alpha$ also $\in W_1$ because W_1 is a subspace of V. therefore $\beta \in W_1$ similarly if $\beta \in W_2$ then $(\alpha+\beta) - \beta \in W_2$ where $\alpha \in W_2$ because W_2 is subspace of V

26/9/23

Linear combination of vectors

Let V be a vector space over a field F and $\alpha_1, \alpha_2 \dots \alpha_n \in V$ then any vector $\alpha \in V$ can be expressed as below.

$\alpha = a_1\alpha_1 + a_2\alpha_2 + \dots a_n\alpha_n$

When $a_1, a_2 \dots a_n \in F$ is said to be a linear combination of vectors $\alpha_1, \alpha_2 \dots \alpha_n$. (or)

Let V(F) be a vector space over F and S be any non empty subset of V then the set of all linear combination of finite element is called as it is denoted as $L(S) = \{a_1\alpha_1 + a_2\alpha_2 \dots a_n\alpha_n : a_1, a_2, \dots a_n \in F$ and $\alpha_1, \alpha_2 \dots \alpha_n \in S\}$

* Theorem-1 :- The linear span L(S) of a non empty subset 'S' of a vector space V(F) is the smallest subspace of V containing S. –

proof :- We have $L(S) = \{a_1\alpha_1 + a_2\alpha_2 \dots a_n\alpha_n : a_1, a_2, \dots a_n \in F$ & $\alpha_1, \alpha_2 \dots \alpha_n \in S$ $a_i \in V\}$

Let $\alpha \in S$ then $\alpha = 1\alpha$, $1 \in F$, so $\alpha \in L(S)$

$S \le$ [improper subset] $L(S)$

Let α, β be any two arbitary elements of L(S) then

$\alpha = a_1\alpha_1 + a_2\alpha_2 \dots + a_n\alpha_n$; $\alpha_1, \alpha_2 \dots \alpha_n \in S$ and

$\beta = b_1\beta_1 + b_2\beta_2 \dots b_m\beta_m$; $\beta_1, \beta_2 \dots \beta_m \in S$ also $a, b \in F$

$a\alpha + b\beta = a(a_1\alpha_1 + a_2\alpha_2 \dots a_n\alpha_n) + b(b_1\beta_1 + b_2\beta_2 + \dots b_m\beta_m)$

$= (aa_1)\alpha_1 + (aa_2)\alpha_2 + \dots (aa_n)\alpha_n + (bb_1)\beta_1 + (bb_2)\beta_2 \dots + (bb_m)\beta_m \Rightarrow a\alpha + b\beta$ is a linear combination of finite no. combinations

$a\alpha + b\beta \in L(S)$ hence L(S) is a subspace of V

hence L(S) is the smallest subspace of V containing S

* Theorem-2 :- If S, T are two subsets of a vectorspace of V. Then

(i) $S \subseteq T \Rightarrow L(S) \le L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L[L(S)] = L(S)$

proof :- Let α be a arbitary element L(S) then $\alpha \in L(S) = a_1\alpha_1 + a_2\alpha_2 + \dots a_n\alpha_n$ where $a_1, a_2 \dots a_n \in F$

i) since $S \subseteq T$, so that $\alpha_1, \alpha_2 \dots \alpha_n \in T$ Therefore α is also the linear combination of finite elements of T.

$\Rightarrow \alpha \in L(T)$ hence $S \subseteq T$ and also $L(S) \le L(T)$

ii) since $S \le S \cup T$ & $T \le S \cup T$ then we have

$\left.\begin{array}{l} L(S) \le (S \cup T) \\ L(T) \le (S \cup T) \end{array}\right\}$ ~~$L(S) + L(T) \le L(S \cup T)$~~ — (1)

Let α be an arbitary element of $L(S \cup T)$ then α is linear combination of finite elements of $S \cup T$ therefore we can state that $\alpha_i \in S$ also $\alpha_j \in T$ Therefore $\alpha \in L(S) + L(T)$

Then $L(S \cup T) \le L(T) + L(S)$ — (2)

$L(S \cup T) = L(S) + L(T)$

(iii) since S improper subset of T(S) then $L(S) \le L[L(S)]$

proof :- we have $\alpha = b_1\beta_1 + b_2\beta_2 + b_3\beta_3 + \dots b_n\beta_n$

$= \sum_{i=1}^{n} b_i\beta_i$

where each $\beta_i \in L(S)$. also each β_i is a linear combination finite element of S. so that $\beta_1 = a_{11}\alpha_1 + a_{12}\alpha_2 + a_{13}\alpha_3 + \dots + a_{1n}\alpha_n$

$\beta_2 = a_{21}\alpha_1 + a_{22}\alpha_2 + \dots a_{2n}\alpha_n$

on substituting β_1 & β_2 values in α eq we see that α is a linear combination of finite element of S.

$L[L(S)] = L(S)$

* Theorem 3 :- The linear sum of two subspaces W_1 & W_2 of a vector space V(F) is generated by their union.

i.e $W_1 + W_2 = L(W_1 \cup W_2)$

proof :- We have already prove that linear sum of two subspaces is also a subspace & linear span of subset of a subspace is also a subspace. Therefore $W_1 + W_2$ & $L(W_1 \cup W_2)$ are subspaces of V(F).

Linear depedence & independence of vectors

* Linear dependence :- Let $V(F)$ be a vector space over a field F then a finite set $\{\alpha_1 + \alpha_2 - - \alpha_n\}$ of vectors of V is said to be linearly dependend if there exist scalar $a_1\ a_2\ --\ a_n$ not of all of them equal to zero such that,

$a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n = 0$

* Linear independent :- Let $V(F)$ be a vector space over a F then a finite set $\{\alpha_1 + \alpha_2 - - \alpha_n\}$ of V is said to be linearly independent if for every expression of type $a\alpha_1 + a_2\alpha_2 + - a_n\alpha_n$, $-- a_1, a_2, - a_n \in F$ & also $a_1 = 0$ & $a_2 = a_3 = a_n$

problem :- [Linear combi]

1) Is the vector $(2,-5,3)$ in subspace of R^3 spand by the vectors $(1,-3,2)$, $(2,-4,-1)$, $(1,-5,7)$.

sol Let $\alpha = (2,-5,3)$ & $\alpha_1 = (1,-3,2)$ & $\alpha_2(2,-4,-1)$,

$\alpha_3 = (1,-5,7)$.

$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$

$(2,-5,3) = a_1(1,-3,2) + a_2(2,-4,-1) + a_3(1,-5,7)$

$= (a_1, -3a_1, 2a_1) + (2a_2, -4a_2, -a_2) + (a_3, -5a_3, 7a_3)$

$= (a_1 + 2a_2 + a_3), (-3a_1 - 4a_2 - 5a_3), (2a_1 - a_2 + 7a_3)$

From above

$a_1 + 2a_2 + a_3 = 2$ — ①

$-3a_1 - 4a_2 - 5a_3 = -5$ — ②

$2a_1 - a_2 + 7a_3 = 3$ — ③

Eliminate a_2 by considering eq ① & eq ②

$2(a_1 + 2a_2 + a_3 = 2)$

$2a_1 + 4a_2 + 2a_3 = 4$

$-3a_1 - 4a_2 - 5a_3 = -5$

$-a_1 - 3a_3 = -1$ — ④

again eliminate a_2 in eq ② & ③

$-4(2a_1 - a_3 + 7a_3 = 3)$

$-8a_1 + 4a_3 - 28a_3 = -21$ $\quad -5 = -3a_1 - 4a_2 - 5a_3$

$-3a_1 - 4a_2 - 5a_3 = -5$ $\quad -12 = -8a_1 + 4a_2 - 28a_3$

$-11a_1 - 33a_3 = -17$ — ⑤

$11a_1 + 33a_3 = +11$

$-11a_1 - 33a_3 = -17$

$= -6$

since no of a_3 & a_1 will satisfy Eq ④ & ⑤ Eq^n ① ② & ③ does not have any solution hen α cannot be expressed as LC of $\alpha_1, \alpha_2, \alpha_3$. hence the vector $(2,-5,3)$ is not spaned by the vector $(1,-3,2)$, $(2,-4,1)$, $(1,-5,7)$

2) In vector space R^3 express the vectors $(1,-2,5)$ as a LC of vectors $(1,1,1)$ $(1,2,3)$ & $(2,-1,1)$

sol Let $\alpha = (1,-2,5)$ & $\alpha_1 = (1,1,1)$ & $\alpha_2(1,2,3)$

$\alpha_3 = (2,-1,1)$

$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$

$(1,-2,5) = a_1(1,1,1) + a_2(1,2,3) + a_3(2,-1,1)$

$= (a_1, a_1, a_1) + (a_2, 2a_2, 3a_2) + (2a_3, -a_3, a_3)$

$= (a_1 + a_2 + 2a_3), (a_1 + 2a_2 - a_3), (a_1 + 3a_2 + a_3)$

From above Eq

$a_1 + a_2 + 2a_3 = 1$ — ①

$a_1 + 2a_2 - a_3 = -2$ — ②

$a_1 + 3a_2 + a_3 = 5$ — ③

$a_1 + a_2 + 2a_3 = 1$

$-a_1 \pm 2a_2 \mp a_3 = \mp 2$

$-a_2 + 3a_3 = 3$ — ④

simply ⑤ & ③

$a_1 + 2a_2 - a_3 = -2$

$-a_1 \pm 3a_2 \pm a_3 = -5$

$-a_2 - 2a_3 = -7$ — ⑤

$-a_2 + 3a_3 = 3$

$\mp a_2 \mp 2a_3 = \mp 7$

$5a_3 = 10$ $\quad \boxed{a_3 = 2}$

$-a_2 - 4 = -7$

$\boxed{a_2 = 3}$

$a_1 + 9 + 2 = 5$

$\boxed{a_1 = -6}$

apply a_1, a_2, a_3 in ①

$-6 + 3 + 6 = 1$

27/09/23

3) For what value of m the vector $(m,3,1)$ is a LC of the vectors $(3,2,1)$ & $(2,1,0)$

sol Let $\alpha = (m,3,1)$ & $\alpha_1 (3,2,1)$ & $\alpha_2 (2,1,0)$

$\alpha = a_1\alpha_1 + a_2\alpha_2$

$(m,3,1) = a_1(3,2,1) + a_2(2,1,0)$

$= (3a_1, 2a_1, a_1) + (2a_2, a_2, 0)$

$= (3a_1+2a_2), (2a_1+a_2), (a_1+0)$

$3a_1 + 2a_2 = m$ — ①

$2a_1 + a_2 = 3$ — ②

$\boxed{a_1 = 1}$ — ③

$2 + a_2 = 3$

$\boxed{a_2 = 1}$

$3 + 2 = m$

$\boxed{m = 5}$

4) In the vector space R^4 determine whether or not the vector $(3,9,-4,-2)$ is a linear combination of the vectors $(1,-2,0,3)$, $(2,3,0,-1)$ & $(2,-1,2,1)$

sol Let $\alpha = (3,9,-4,-2)$ & $\alpha_1 (1,-2,0,3)$ & $\alpha_2 = (2,3,0,-1)$

$\alpha_3 = (2,-1,2,1)$

$\alpha = a\alpha_1 + a_2\alpha_2 + a_3\alpha_3$

$(3,9,-4,-2) = a_1(1,-2,0,3) + a_2(2,3,0,-1) + a_3(2,-1,2,1)$

$= (a_1, -2a_1, 0a_1, 3a_1) + (2a_2, 3a_2, 0a_2, -a_2) + (2a_3, -a_3, +2a_3, a_3)$

$= (a_1+2a_2+2a_3), (-2a_1+3a_2-a_3), (0a_1+0a_2+2a_3)$
$(3a_1 - a_2 + a_3)$

$a_1 + 2a_2 + 2a_3 = 3$ — ①

$-2a_1 + 3a_2 - a_3 = 9$ — ②

$2a_3 = -4$ — ③

$3a_1 - a_2 + a_3 = -2$ — ④

From ③ $\boxed{a_3 = -2}$

$3a_1 + 6a_2 = 21$ — ⑤ $\quad a_2 = \frac{21}{7} = 3$

$3a_1 - a_2 = 0$ — ⑥

$7a_2 = 21$

$3a_1 - a_2 = 0$

$\boxed{a_1 = 1}$

5) write the polynomial $f(x) = x^2 + 4x - 3$ over R as a linear combination of the polynomial

$f_1(x) = x^2 - 2x + 5$, $f_2(x) = 2x^2 - 3x$, $f_3(x) = x + 3$

sol $\alpha = (1,4,-3)$ & $\alpha_1 = (1,-2,5)$

$\alpha_2 = (2,-3,0)$

$\alpha_3 = (0,1,3)$

$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$

$(1,4,-3) = a_1(1,-2,5) + a_2(2,-3,0) + a_3(0,1,3)$

$= (a_1, -2a_1, 5a_1) + (2a_2, -3a_2, 0a_2) + (0a_3, a_3, 3a_3)$

$= (a_1+2a_2+0a_3), (-2a_1-3a_2+a_3), (5a_1+0a_2+3a_3)$

$a_1 + 2a_2 = 1$ — ①

$-2a_1 - 3a_2 + a_3 = 4$ — ②

$5a_1 + 3a_3 = -3$ — ③

$a_1 = -3$, $a_2 = 2$, $a_3 = 4$

6) In the vector space R^3, let $\alpha = (1,2,1)$ & $\beta (3,1,5)$ $\gamma = (3,-4,7)$, show that the subspace spanned by $S = \{\alpha, \beta\}$ & $T = \{\alpha, \beta, \gamma\}$ are the same

sol $L(S) = L(T)$ $\quad [S \leq T \Rightarrow L(S) \leq L(T)]$

$\gamma = a\alpha + b\beta$

$(3,-4,7) = a(1,2,1) + b(3,1,5)$

$= (a, 2a, a) + (3b, b, 5b)$

$= (a+3b), (2a+b), (a+5b)$

$a + 3b = 3$ — ①

$2a + b = -4$ — ②

$a + 5b = 7$ — ③

γ can be expressed has LC of $\alpha\beta$

$2a + 6b = 6$ $\quad$ $\cancel{b = 7/5}$

$2a + b = -4$

$5b = 10$ $\quad$ $\boxed{b = 2}$ $\quad$ $2a + 12 = 6$ $\quad$ $\boxed{a = -3}$

Then $\gamma = -3\alpha + 2\beta$ Now let us consider an orbitary value $\delta \in L(T)$ then δ can be Expressed LC of α, β & γ where γ can be replaced by $-3\alpha+2\beta$.
Then $\delta \in L(S)$ therefore
$$L(T) \subseteq L(S) \Rightarrow L(T) = L(S)$$

7) find a convective condition of a, b, c such that $\alpha=(a,b,c)$ is LC of vectors $(1,-3,2)$ & $(2,-1,1)$

sol
$\alpha = a_1\alpha_1 + a_2\alpha_2$
$(a,b,c) = a_1(1,-3,2) + a_2(2,-1,1)$
$= (a_1-3a_1, 2a_1) + (2a_2, -a_2, a_2)$
$= (a_1+2a_2), (-3a_1-a_2), (2a_1+a_2)$

$a_1 + 2a_2 = a$ — ①
$-3a_1 - a_2 = b$ — ②
$2a_1 + a_2 = c$ — ③

$2a_1 + 4a_2 = 2a$
$-2a_1 \pm a_2 = -c$
$3a_2 = 2a - c$
$a_2 = \frac{2a-c}{3}$

$2a_1 + 4\left[\frac{2a-c}{3}\right] = 2a$
$2a_1 + \frac{8a}{3} - \frac{4c}{3} = 2a$
$2a_1 = -\frac{2a}{3} + \frac{4c}{3}$
$a_1 = \frac{-4a+8c}{3}$

sirls $a_2 = \frac{1}{5}(3a+b)$
$a_1 = -\frac{1}{5}(a+2b)$
$\cancel{-3\left[\frac{1}{5}(a+2b)\right]}$ substitute to Eq③
$-2\left[\frac{1}{5}(a+2b)\right] + \left[\frac{1}{5}(3a+b)\right] = c$
$= a - 3b - 5c = 0$
so $\alpha(1,-3,-5)$ it act has a linear set so then only the given vectors acts has LC. α is LC of $(1,-3,2)$ & $(2,-1,1)$ if $a-3b-5c=0$

8) show that $(1,1,1), (0,1,1)$ & $(0,1,-1)$ generate R^3 ~~re~~ ex.

sol we have to show that any vector of R^3 is LC of $(1,1,1)$ $(0,1,1)$, $(0,1,-1)$
let $\alpha = (a,b,c)$
$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$
$(a,b,c) = a_1(1,1,1) + a_2(0,1,1) + a_3(0,1,-1)$
$= (a_1 + 0a_2 + 0a_3), (a_1+a_2+a_3), (a_1+a_2-a_3)$

$a_1 = a$ — ①
$a_1 + a_2 + a_3 = b$ — ②
$a_1 + a_2 - a_3 = c$ — ③

$a_2 + a_3 = b - a$
$a_2 - a_3 = c - a$
$a_2 = \frac{-2a+(b+c)}{2}$

$\boxed{a_3 = \frac{b-c}{2}}$

$\cancel{c-a+\frac{2ab+2ac}{2}}$
$\cancel{-a}+\frac{b}{2}+\frac{c}{2} - a_3 = c\cancel{-a}$
$-a_3 = \frac{b}{2} - \frac{c}{2}$
$\boxed{a_3 = \frac{b-c}{2}}$

There exist a LC of vectors as the values of $a_1 a_2$ & a_3 are non zero.

9) write the matrix $E = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as a LC of matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ & $C = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$

sol let $E = xA + yB + zC$
$$\begin{bmatrix} 3 & 1 \\ 1 & -a \end{bmatrix} = x\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + y\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + z\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} x & x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ y & y \end{bmatrix} + \begin{bmatrix} 0 & 2z \\ 0 & -z \end{bmatrix}$$
$$\begin{bmatrix} 3 & 1 \\ 1 & -a \end{bmatrix} = \begin{bmatrix} x & x+2z \\ x+y & y-z \end{bmatrix}$$
$x = 3$
$x + 2z = 1$
$x + y = 1$
$y - z = -1$

$\boxed{z = -1}$
$\boxed{y = -2}$

Linear dependence & independence :-

1) show that the system of 3 vectors $(1,3,2),(1,-7,-8),(2,1,-1)$ of $V_3(R)$ is linearly dependent.

sol let $a,b,c \in R$ such that

$a(1,3,2)+b(1,-7,-8)+c(2,1,-1)=(0,0,0)$

$(a+b+2c),(3a-7b+c),(2a-8b-c)=(0,0,0)$

$a+b+2c=0$ — ①

$3a-7b+c=0$ — ②

$2a-8b-c=0$ — ③

①⇒ $A=\begin{bmatrix}1&1&2\\3&-7&1\\2&-8&-1\end{bmatrix}$ $|A|=\begin{vmatrix}1&1&2\\3&-7&1\\2&-8&-1\end{vmatrix}$

$|A|=1[7+8]-1[-3-2]+2[-24+14]$

$=15+5-20$

$=0$

$B=\begin{bmatrix}1&1&2\\3&-7&1\\2&-8&-1\end{bmatrix}$ $R_2 \Rightarrow R_2-3R_1$, $R_3 \Rightarrow R_3-2R_1$

$B=\begin{bmatrix}1&1&2\\0&-10&-5\\0&-10&-5\end{bmatrix}$ $R_3 \leftarrow R_3-R_2$

$B=\begin{bmatrix}1&1&2\\0&-10&-5\\0&0&0\end{bmatrix}$ $\rho(B)=2$ $[n=3]$

$a+b+2=0$ $\rho(B)<n$

$-10b-5c=0$ Therefore set of homogenous Eq has non zero solution hence the system of given vectors are linearly dependent

2) show that $S=\{(1,2,4),(1,0,0),(0,1,0),(0,0,1)\}$ are linearly subset of vector space $V_3(R)$ where R is the field of real numbers.

3/10/23

we know that the set $(1,0,0)(0,1,0)(0,0,1)$ is linearly independent of $V_3(R)$ $[a=0;b=0;c=0]$ now

$(1,2,4)=a(1,0,0)+b(0,1,0)+c(0,0,1)$

$=(a+b+c)$

$a=1, b=2, c=4$

since we obtain non zero solution for coefficient the given vectors are really dependent.

3) if α,β,γ are linearly independent vectors F is any field of complex no.'s the so also $\alpha+\beta, \beta+\gamma, \gamma+\alpha$.

sol let abc be scalar such that $a(\alpha+\beta)+b(\beta+\gamma)+c(\gamma+\alpha)=0$

arrange according to coefficient values

$(a+c)\alpha+(a+b)\beta+(b+c)\gamma=0$

$a+c=0$

$a+b=0$ $A=\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$ R_2-R_1

$b+c=0$

$A=\begin{bmatrix}1&0&1\\0&1&-1\\0&1&1\end{bmatrix}$ $=R_3 \Rightarrow R_3-R_2$

$A=\begin{bmatrix}1&0&1\\0&1&-1\\0&0&2\end{bmatrix}$ $R_3=\frac{1}{2}R_3$

The $\rho(A)=3=n$ $[n=3\ (a,b,c)]$ ⇒ The system of has zero solution. $a=0, b=0, c=0$

therefore α,β,γ is linearly independent.

4) if $V_3(R)$, where R is the field of real numbers Examining Each of the following sets of vectors for linear dependence.

(i) $\{(1,3,2),(1,-7,-8),(2,1,-1)\}$

(ii) $\{(0,2,-4),(1,-2,-1),(1,-4,3)\}$

(iii) $\{(1,2,0),(0,3,1),(-1,0,1)\}$

(iv) $\{(-1,2,1),(3,0,-1),(-5,4,3)\}$

(v) $\{(2,3,5)\ (4,9,25)\}$

(vi) $\{(2,1,2)\ (8,4,8)\}$

sol

(i) $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & -7 & -8 \\ 2 & 1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 2 \\ 3 & -7 & 1 \\ 2 & -8 & -1 \end{bmatrix}$

$a(1,3,2) + b(1,-7,-8) + c(2,1,-1)$

$a+3a+2a+b-7b+8b+2c+c-c$

$|A| = 1[7+8] - 1[-3-2] + 2[24+14]$

$= +15+5-20 = 0$

$\rho(A) < 3\ [a,b,c] \Rightarrow$ Thus the given vector values are linearly dependent.

ii) $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & -2 & -4 \\ -4 & -1 & 3 \end{bmatrix}$

$|A| = 0(0) - 1(6-16) + 1(-2-8)$

$= 10 - 10 = 0$ $\rho(A) < 3$ Thus it is linearly dependent

iii) $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$|A| = 1(3) - 0(0) - 1(2)$

$= 3 - 2 = 1$ Thus it is linearly independent

iv) $A = \begin{bmatrix} -1 & 3 & -5 \\ 2 & 0 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$|A| = -1(0+4) - 3(6-4) - 5(-2)$

$= -4 - 6 + 10 = 0$ $|A| = 0$ linearly dependent

v) $A = \begin{bmatrix} 2 & 4 \\ 3 & 9 \\ 5 & 25 \end{bmatrix}$ $R_2 \Rightarrow 2R_2 - 3R_1$

$R_3 \Rightarrow 2R_3 - 5R_1$

$A = \begin{bmatrix} 2 & 4 \\ 0 & 6 \\ 0 & 30 \end{bmatrix}$ $R_3 \Rightarrow R_3 - 5R_2$

$A = \begin{bmatrix} 2 & 4 \\ 0 & 6 \\ 0 & 0 \end{bmatrix}$

$\rho(A) =$ No of varibles $= 2$ so the given is linerly independent

(vi) $\begin{bmatrix} 2 & 8 \\ 1 & 4 \\ 2 & 8 \end{bmatrix}$ $R_2 \leftarrow 2R_2 - R_1$

$R_3 \leftarrow R_3 - R_1$

$= \begin{bmatrix} 2 & 8 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\rho(A) = 1$

$\rho(A) < 2$ [No of varibles] linearly dependent

5) prove that $R(x)$ the vector space of all polynomials in x over R. the polynomial eqn $p(x) = 1 + x + 2x^2$, $q(x) = 2 - x + x^2$, $r(x) = -4 + 5x + x^2$ is linearly dependent

sol $a, b, c \in R$

$a\,p(x) + b\,q(x) + c\,r(x) = 0$

$a(1+x+2x^2) + b(2-x+x^2) + c[-4+5x+x^2] = 0 + 0x + 0x^2$

$a + 2b - 4c = 0$

$ax - bx + 5cx = 0x$

$ax^2 + bx^2 + cx^2 = 0x^2$

$A = \begin{bmatrix} 1 & 2 & -4 \\ 1 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

$|A| = 1(-1-5) - 2(1-10) - 4(1+2)$

$= -6 + 18 - 12 = 0$

For abc Non zero solutions which indicates linearly dependent

$\rightarrow$ Basis of vector :- let V be a vector space over a field F and let S be any non empty subset of V. then S is said to be a basis of V if.

(i) S is linearly independent

(ii) $L(S) = V$ i.e Every element of V is LC of finite elements of S

4/10/23

→ **Finite dimensional vector space :-**

Let V(F) be a V.F over field 'F' and let S be any non Empty subset of V, the V(F) is said to be finite dimensional if S is ~~subset~~ finite subset of V such that $L(S) = V$.
If there set contains n elements, then the dimension of V is n.

★ **Theorem-1 :-**

If $S = \{\alpha_1, \alpha_2 \dots \alpha_n\}$ is the basis of vector space V(F) then Each Element of V is uniquely expressable as a LC of Elements of S.

proof :- since S is the basis of V.S V(F) then by defination of basis Each Element of V is L.C of Elements of S thus we need to only show the uniqueness

Let us consider two d/f subsets $\{a_1, a_2 \dots a_n\}$ & $\{b_1, b_2 \dots b_n\}$ of scalars corresponding to an element $\alpha \in V$ such that

$\alpha = a_1\alpha_1 + a_2\alpha_2 \dots + a_n\alpha_n$

$\alpha = b_1\alpha_1 + b_2\alpha_2 + \dots + b_n\alpha_2$

$a_1\alpha_1 + a_2\alpha_2 + \dots a_n\alpha_n = b_1\alpha_1 + b_2\alpha_2 + \dots b_n\alpha_n$

$a_1\alpha_1 - b_1\alpha_1 + a_2\alpha_2 - b_2\alpha_2 + \dots a_n\alpha_n - b_n\alpha_n = 0$

$(a_1 - b_1)\alpha_1 + (a_2 - b_2)\alpha_2 + \dots (a_n - b_n)\alpha_n = 0$

since the set S $= \alpha_1 \dots \alpha_n$ is linearly independent

so, $a_1 - b_1 = 0$, $a_2 - b_2 = 0$ & $a_n - b_n = 0$.

$a_1 = b_1$, $a_2 = b_2$ & $a_n = b_n$

For the above demonstration we can say that the given vector form unique solution for L.C of Elements of S.

→ **dimension of subspace of vector space :-**

→ **problems on basis of vectors :-**

1) Let V be the vector space of ordered pairs of complex numbers over real field R i.e let V be the vector space $C^2(R)$ show that the set $S = \{(1,0), (i,0), (0,1), (0,i)\}$ is basis for V.

sol First let us prove that the set S is Linearly independent

Let us consider $a, b, c, d \in R$ such that

$a(1,0) + b(i,0) + c(0,1) + d(0,i) = (0,0)$

$a + ib = 0$

$c + id = 0$

slouing above Equation we get $a=0, b=0, c=0, d=0$ which indicals the given system is Linearly independent

Now, we shall show that $L(S) = V$

Let $(a+ib, c+id)$ be any Element of V where $a, b, c, d \in R$

then $(a+ib, c+id) = a(1,0) + b(i,0) + c(0,1) + d(0,i)$

Theorefore every Element of V can be Expressed has LC of the elements of S. which indicate the given set S is a basis of V.

2) show that the sets $S = \{(1,2)(3,4)\}$ form the basis of R^2

sol since the dimension is R^2 'S' contains two Elements

consider $a, b \in R$

$a(1,2) + b(3,4) = (0,0)$

$a + 3b = 0$

$2a + 4b = 0$ $\Rightarrow A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$|A| = 4 - 6 = -2$

$|A| = -2 \neq 0$

which indicates $a=0$ & $b=0$ so that 'S' is linearly independent.

So, 'S' basis of R^2

3) Let V be vector space of all 2×2 matrices over the field F Prove that V has dimension 4 by Exhibiting a basis for V which has four Elements.

sol Let $S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ where $\alpha_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\alpha_2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

usual basis $(\alpha_1 \alpha_2 \alpha_3 \alpha_4)$

$\alpha_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ & $\alpha_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are the four elements of V.

Now we shall show that s forms the basis of V

let $a, b, c, d \in F$ such that,

$a\alpha_1 + b\alpha_2 + c\alpha_3 + d\alpha_4 = 0$

$a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$a = 0$ & $c = 0$

$b = 0$ & $d = 0$

This concludes s is linearly independent

so, we can say s is basis of V

Next we shall show that $L(S) = V$

let $V = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then,

$a\alpha_1 + b\alpha_2 + c\alpha_3 + d\alpha_4 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

By observing above s forms basis of V which has four elements therefore dimension $V = 4$

4) let $\alpha = (1,2,1)$ $\beta = (2,9,0)$ & $\gamma = (3,3,4)$ show that set $s = \alpha, \beta, \gamma$ is the basis of R^3.

sol let $a, b, c \in R$

$a\alpha + b\beta + c\gamma = 0$

$a(1,2,1) + b(2,9,0) + c(3,3,4) = (0,0,0)$

$a + 2b + 3c = 0$ — ①

$2a + 9b + 3c = 0$ — ②

$a + 0b + 4c = 0$ — ③

let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix}$

$|A| = 1[36-0] - 2[8-3] + 3[0-9]$

$= 36 - 10 - 27$

$= -1$

so, it is linearly independent or $\rho(A) = 3$ [No of values]

$\Rightarrow$ s basis of R^3

5) consider the basis $s = \alpha_1, \alpha_2, \alpha_3$ of R^3 where $\alpha_1 = (1,1,1)$ $\alpha_2 = (1,1,0)$ & $\alpha_3 (1,0,0)$ Express $(2,-3,5)$ in terms of the basis element $\alpha_1, \alpha_2, \alpha_3$

sol consider $a, b, c \in R$

since $s = \alpha_1 \alpha_2 \alpha_3$ forms basis of R^3 then, every element of R^3 can be expressed has LC of $\alpha_1, \alpha_2, \alpha_3$

$L(s) = V$

we have, $a(\alpha_1) + b(\alpha_2) + c(\alpha_3) = (2,-3,5)$

$a(1,1,1) + b(1,1,0) + c(1,0,0) = (2,-3,5)$

$a + b + c = 2$ — ①

$a + b = -3$ — ②

$a = 5$ — ③

$b = -8$ & $c = 5$

$5\alpha_1 - 8\alpha_2 + 5\alpha_3 = (2,-3,5)$

***

6) show that the vectors $\alpha_1 = (1,0,-1)$ $\alpha_2 = (1,2,1)$ $\alpha_3 = (0,-3,2)$ form a basis of R^3. Express each of the standard basis vector of a LC of $\alpha_1, \alpha_2, \alpha_3$

sol consider $a, b, c \in R$

$a(\alpha_1) + b(\alpha_2) + c(\alpha_3) = (0,0,0)$

$a(1,0,-1) + b(1,2,1) + c(0,-3,2) = (0,0,0)$

$a + b = 0$ — ①

$2b - 3c = 0$ — ② $\Rightarrow A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

$-a + b + 2c = 0$ — ③

$|A| = 1[4+3] - 1[-3]$

$= 10$ hence it is linearly independent then

$a = 0$, $b = 0$, $c = 0$ so, s is the basis of R^3

- The standard basis (usual basis) e^3 is

$e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

_/_/_

$\Rightarrow$ let $\alpha = (p, Q, R)$ be any element of R^3. since S forms the basis of R^3. then there exist $x, y, z \in R$ s.t.

$\alpha = x(1,0,-1) + y(1,2,1) + z(0,-3,2)$

$(p,Q,R) = (x, 0x, -x) + (y, 2y, y) + (0z, -3z, 2z)$

$x + y = p$ — ①

$2y - 3z = Q$ — ②

$-x + y + 2z = R$ — ③

$\cancel{x} + y = p$

$-x + y + 2z = R$

$2y + 2z = p + R$ — ④

$-\cancel{2y} \mp 3z = Q$

$5z = p + R - Q$

$z = \dfrac{p+R-Q}{5}$

$q = -2y - \dfrac{3(p+R-Q)}{5}$

$2y = q + \dfrac{3(p+R-Q)}{5}$

$2y = \dfrac{5q + 3p + 3R - 3Q}{5}$

$\boxed{y = \dfrac{3p + 3r + 2q}{10}}$

$p = x + \dfrac{3p+3r+2q}{10}$

$x = p - \dfrac{3p+3r+2q}{10}$

$x = \dfrac{10p - 3p - 3r - 2q}{10}$

$\boxed{x = \dfrac{7p - 2q - 3r}{10}}$

let us consider standard basis

$e_1 = (1,0,0) = (p,q,r)$

$p = 1, \; q = 0, \; r = 0$

$x = \dfrac{7}{10}, \; y = \dfrac{3}{10}, \; z = \dfrac{1}{5}$

so, $e_1 = 7/10\, \alpha_1 + 3/10\, \alpha_2 + 1/5\, \alpha_3$

$\Rightarrow$ ~~let~~ consider standard basis $\varepsilon_2 = (0,1,0) = (p,q,r)$

$x = -\dfrac{2}{10}, \; y = \dfrac{2}{10}, \; z = -\dfrac{1}{5}$

so, $\varepsilon_2 = -\dfrac{2}{10}\alpha_1 + \dfrac{2}{10}\alpha_2 - \dfrac{1}{5}\alpha_3$

$\Rightarrow$ consider standard basis $\varepsilon_3 = (0,0,1) = (p,q,r)$

$x = -3/10, \; y = 3/10, \; z = 1/5$

$\varepsilon_2 = -3/10\, \alpha_1 + 3/10\, \alpha_2 + 1/5\, \alpha_3$

***

7) given that each set S below spans R^3, find the basis of R^3 which is containing in S.

(i) $\{(1,0,2)(0,1,1)(2,1,5)(1,1,3)(1,2,1)\}$

(ii) $\{(2,6,-3)(5,15,-8)(3,9,-5)(1,3,-2)(5,3,-2)\}$