Realization of Digital filters

IIR	FIR
1) Direct form-I	1) Direct form
2) Direct form-II	2) Lattice form
3) Transposed Direct form	
4) Cascade	
5) parallel	
6) Lattice-Ladder	

Realization of IIR filters

1) Direct form-I

The transfer function $H(z)$ of IIR or Recursive digital filters can be written as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M-1} a_k z^{-k}}{\sum_{k=0}^{N-1} b_k z^{-k}}$$

(2)

$$H(z) = \frac{\sum_{k=0}^{M-1} a_k z^{-k}}{1 + \sum_{k=1}^{N-1} b_k z^{-k}} = \frac{Y(z)}{X(z)}$$

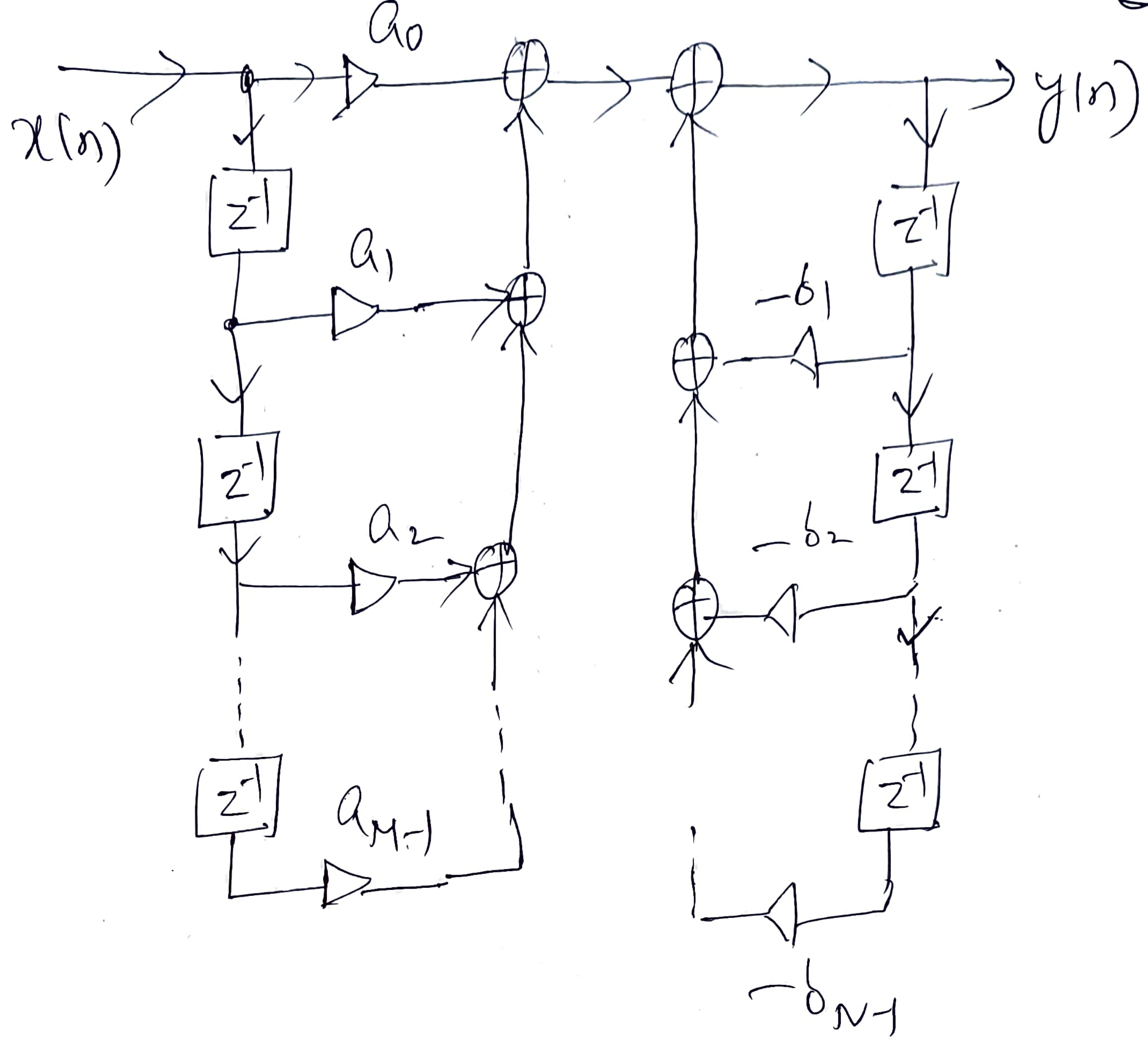
$$\Rightarrow \quad Y(z) + \sum_{k=1}^{N-1} b_k z^{-k} Y(z) = \sum_{k=0}^{M-1} a_k z^{-k} X(z)$$

Taking Inverse Z-transforms on both sides,

$$y(n) + \sum_{k=1}^{N-1} b_k\, y(n-k) = \sum_{k=0}^{M-1} a_k\, x(n-k)$$

$$\Rightarrow \quad y(n) = \sum_{k=0}^{M-1} a_k\, x(n-k) - \sum_{k=1}^{N-1} b_k\, y(n-k) \qquad \text{———} *$$

Realization of filter using Eqn $*$ is called Direct form-I realization



2) <u>Direct from - II</u>

$$H(z) = \frac{Y(z)}{X(z)} = H_1(z) \cdot H_2(z)$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$= \sum_{k=0}^{M-1} a_k z^{-k} \cdot \frac{1}{1 + \sum_{k=1}^{N-1} b_k z^{-k}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N-1} b_k z^{-k}}$$

$$\Rightarrow W(z) + \sum_{k=1}^{N-1} b_k z^{-k} W(z) = X(z)$$

$$W(z) = X(z) - \sum_{k=1}^{N-1} b_k z^{-k} W(z)$$

Taking IZT

$$\Rightarrow w(n) = x(n) - \sum_{k=1}^{N-1} b_k w(n-k) \longrightarrow (1)$$

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M-1} a_k z^{-k}$$

$$\Rightarrow Y(z) = \sum_{k=0}^{M-1} a_k z^{-k} W(z)$$

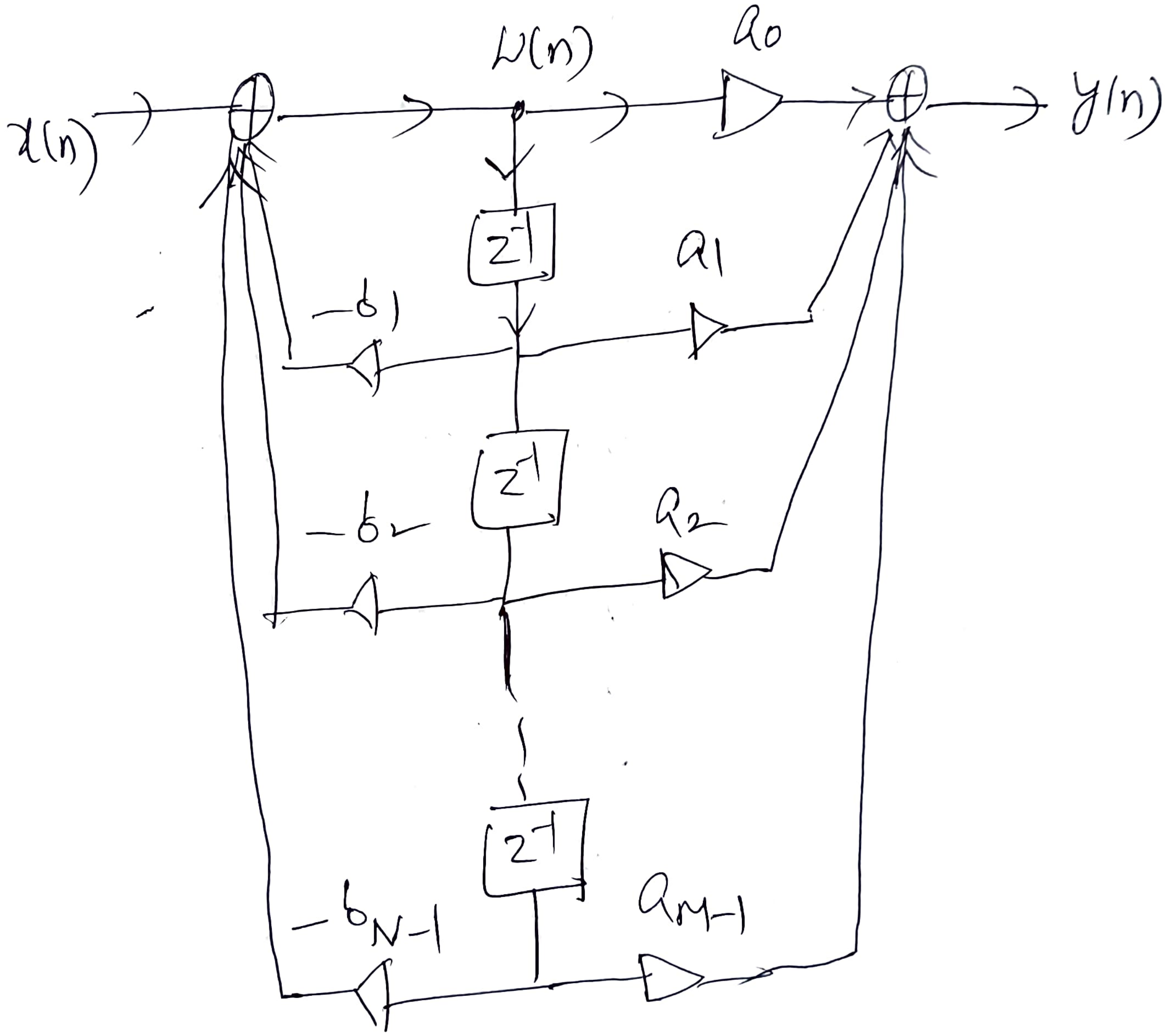
Taking IZT

$$y(n) = \sum_{k=0}^{M-1} a_k w(n-k) \longrightarrow (2)$$

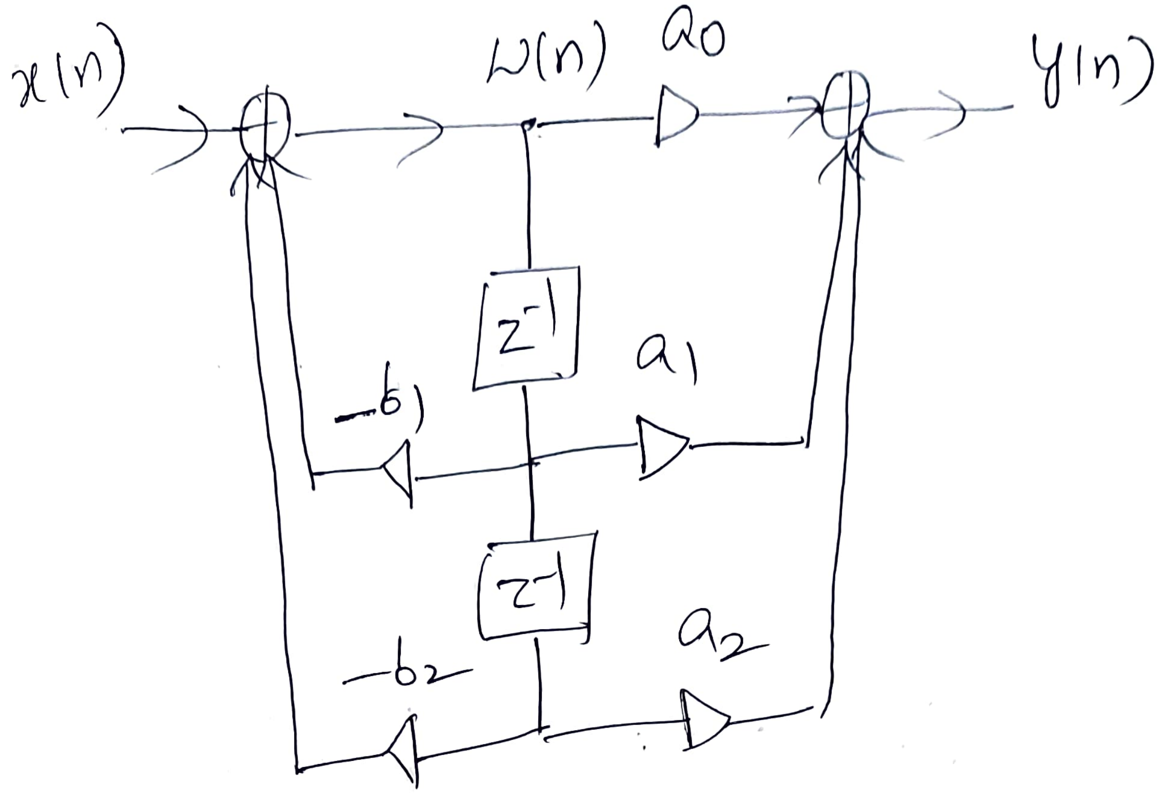
* Writing structures using eq^{ns} 1 and 2 gives Direct form-II structure.

* DF-II realization is also known as canonical realization.

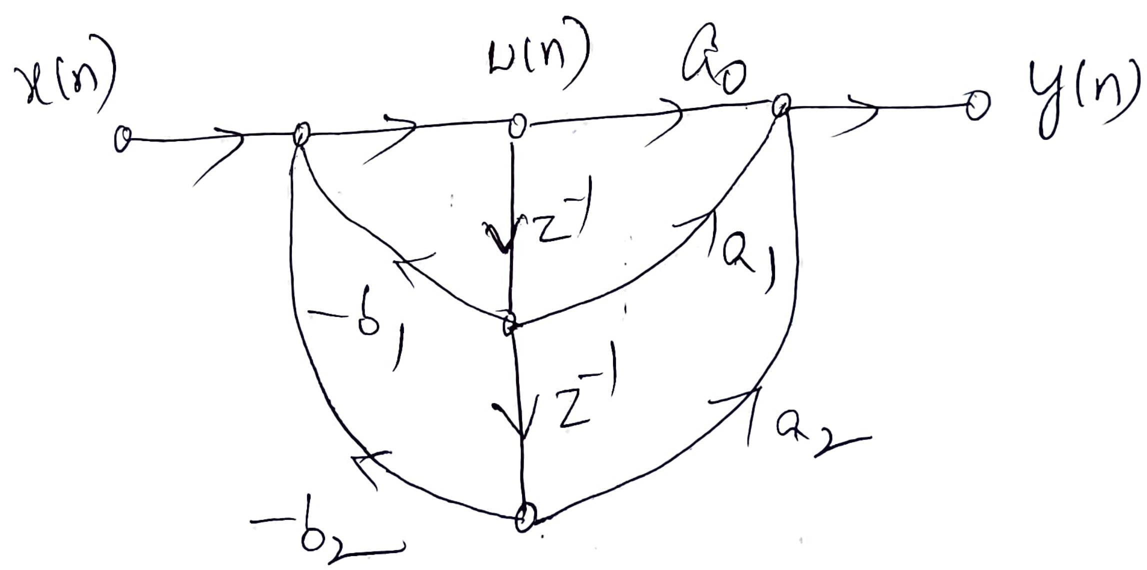
  → uses minimum number of delay elements



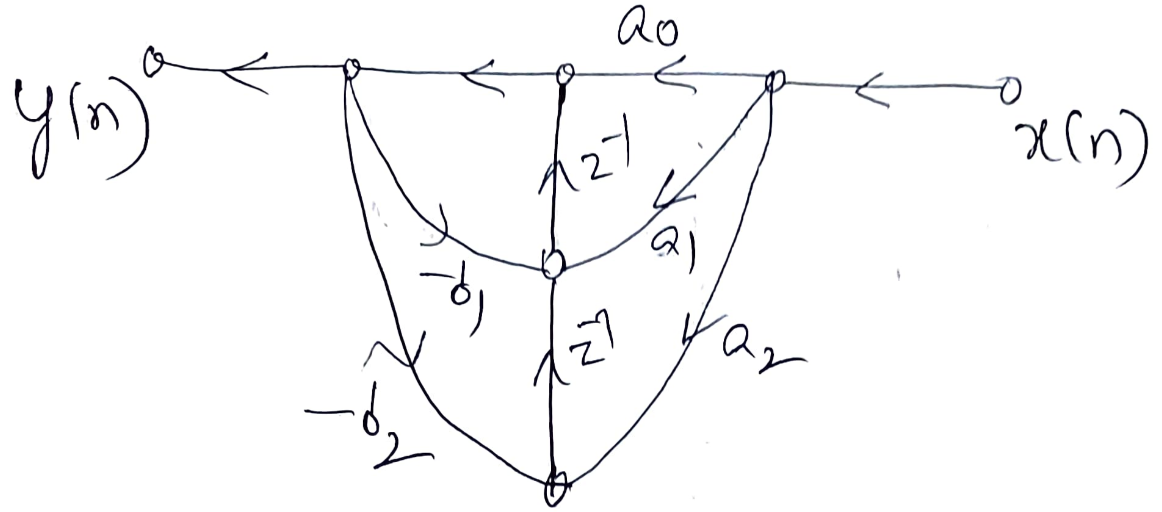
3) Transposed Direct form-II Structure



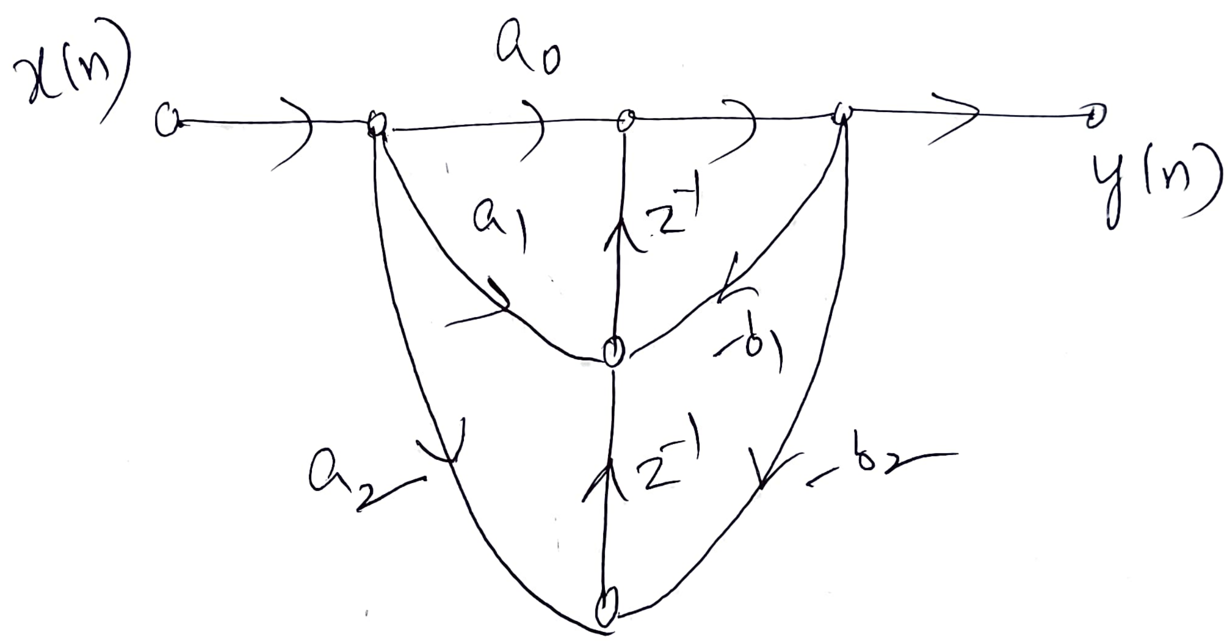
Signal flow graph for the above Structure is written as



Applying transposition theorem

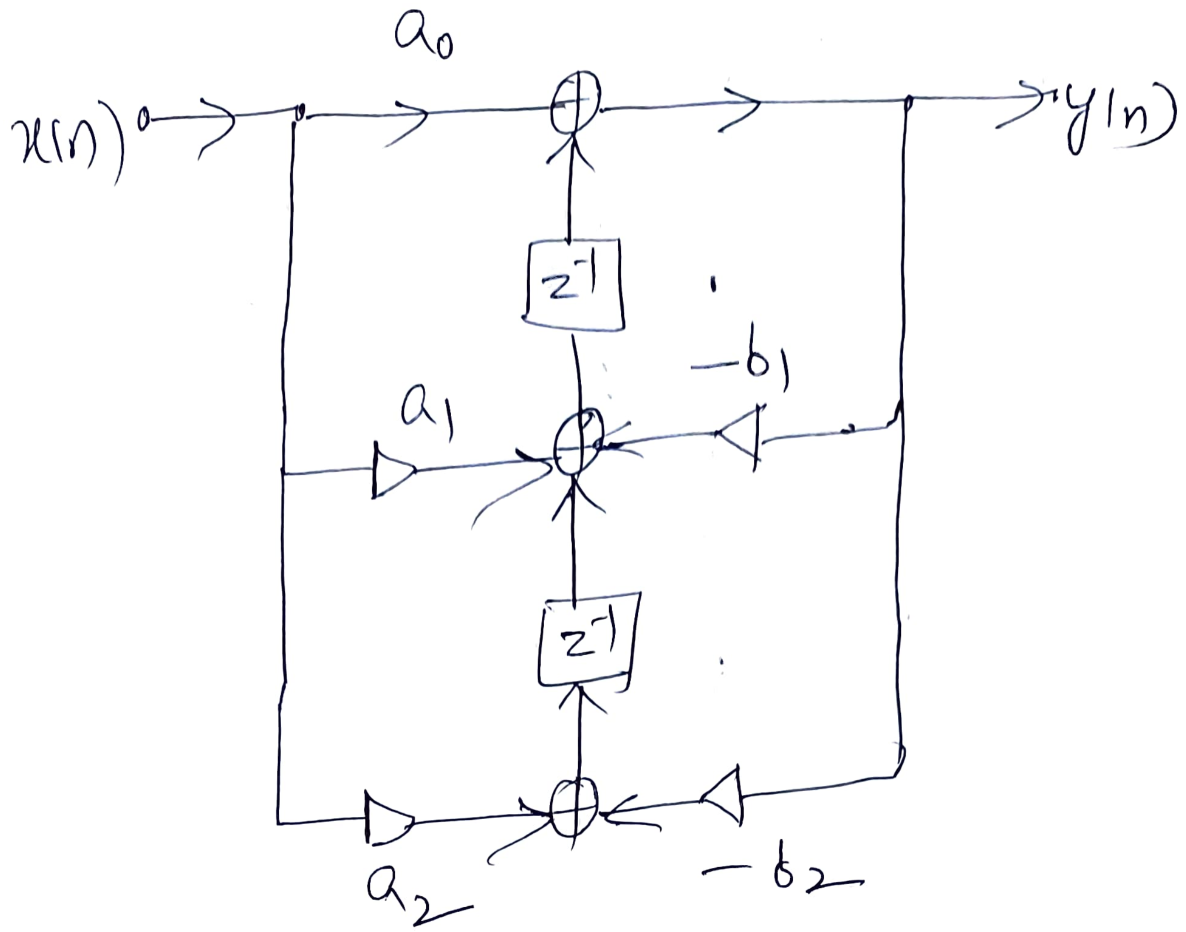


Rewriting the structure gives



Writing the block diagram representation for the SFG gives

...



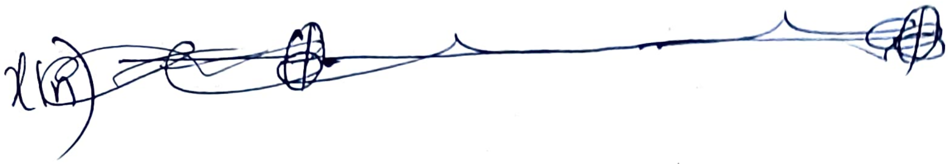
Problem

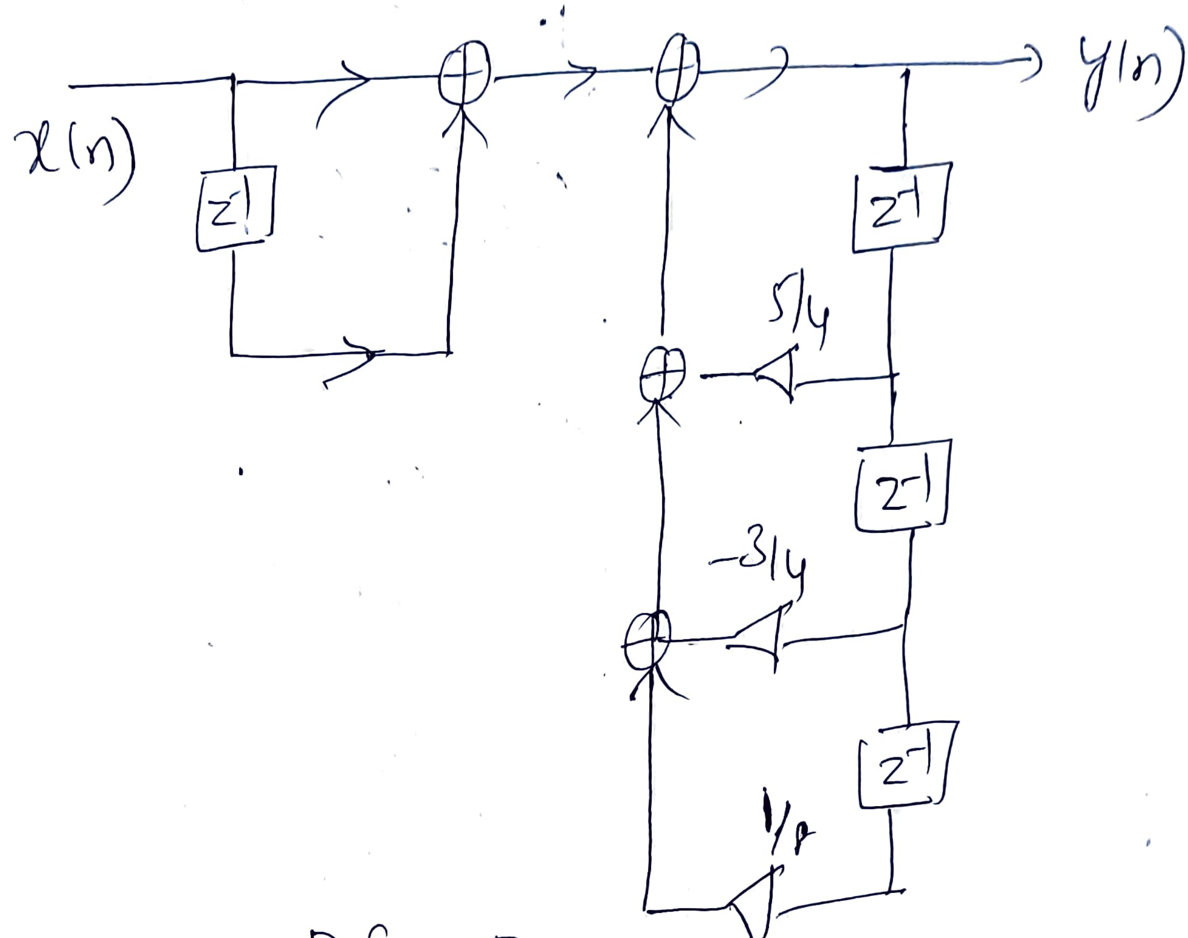
Obtain DF-I, DF-II and Transposed DF-II realization for the function

$$H(z) = \frac{1+z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)\left(1-z^{-1}+\frac{1}{2}z^{-2}\right)}$$

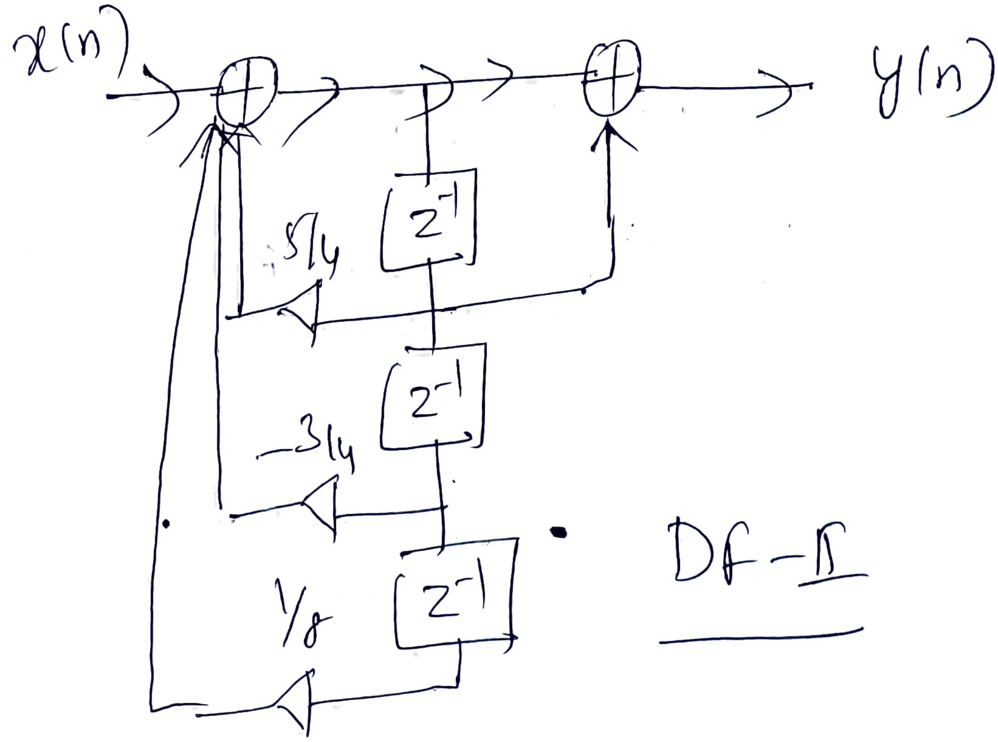
Sol^n

$$H(z) = \frac{1+z^{-1}}{1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}}$$

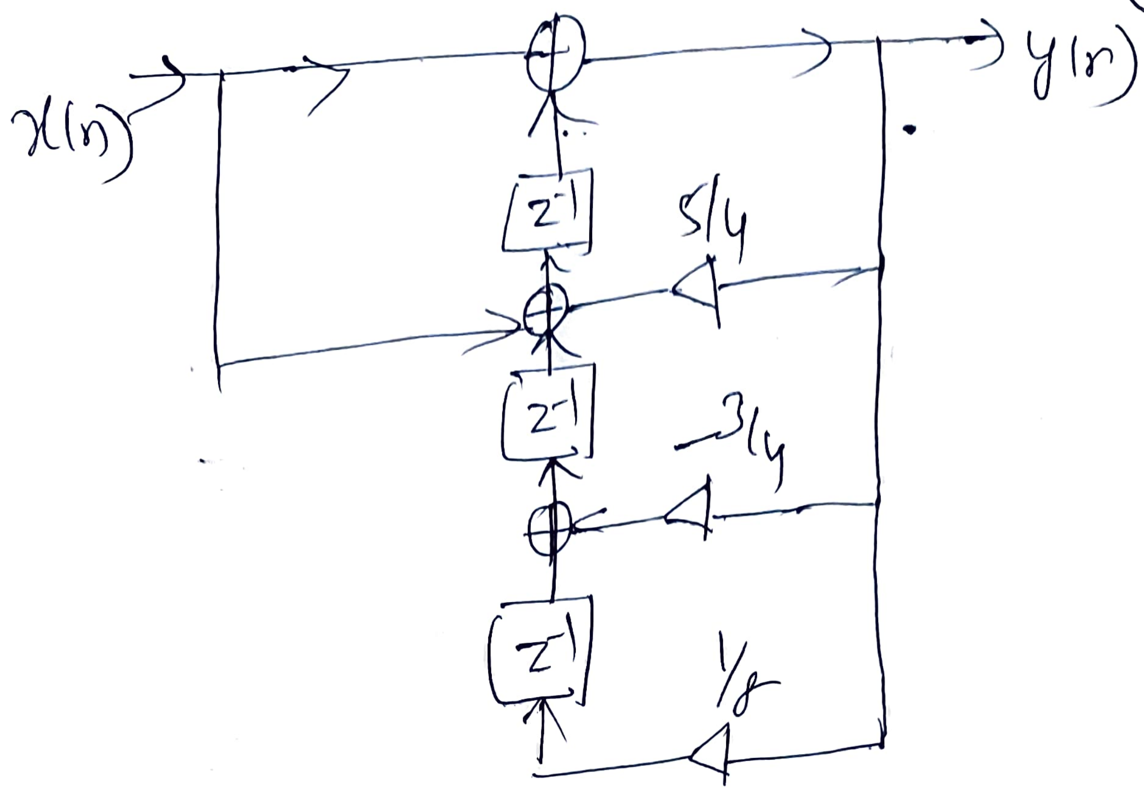




Df – I



Df – II



Transposed DF-II

Prob 2 Realize the following system using a) DF-I b) DF-II and Transposed DF-II

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2)$$
$$= x(n) + \frac{1}{3}x(n-1) + \frac{1}{6}x(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Prob 3 Draw the block diagram representation of the following filter using a) DF-I b) DF-II and c) Transposed DF-II forms.

$$H(z) = \frac{2z^2+z-2}{4z^2-4z-3}$$

Also write the SFG representation of the system.

Soln

$$H(z) = \frac{2+z^{-1}-2z^{-2}}{4-4z^{-1}-3z^{-2}}$$

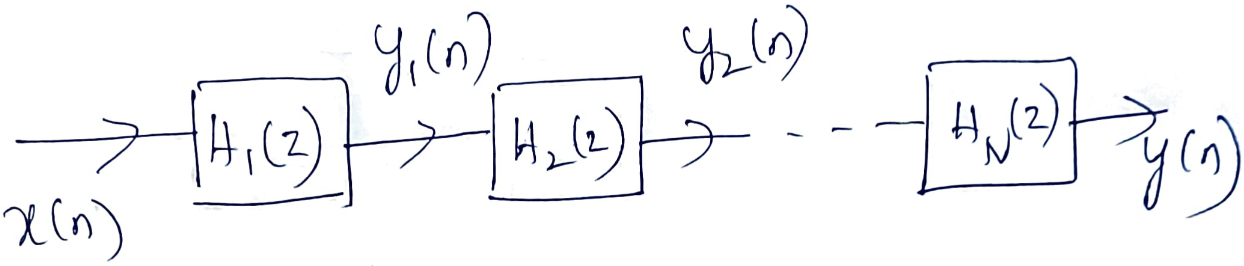
$$= \frac{0.5+0.25z^{-1}-0.5z^{-2}}{1-z^{-1}-0.75z^{-2}}$$

Cascade Realization of IIR filters

In cascade realization, the system function $H(z)$ is written as product of transfer functions $H_1(z) \cdot H_2(z) \cdots H_N(z)$. i.e.,

$$H(z) = H_1(z) \cdot H_2(z) \cdots H_N(z) \quad \text{—— (1)}$$

Eqn (1) is represented by a block diagram shown below



Parallel Realization of IIR filters

In parallel realization, $H(z)$ is written as sum of transfer functions $H_1(z), H_2(z) \cdots H_N(z)$

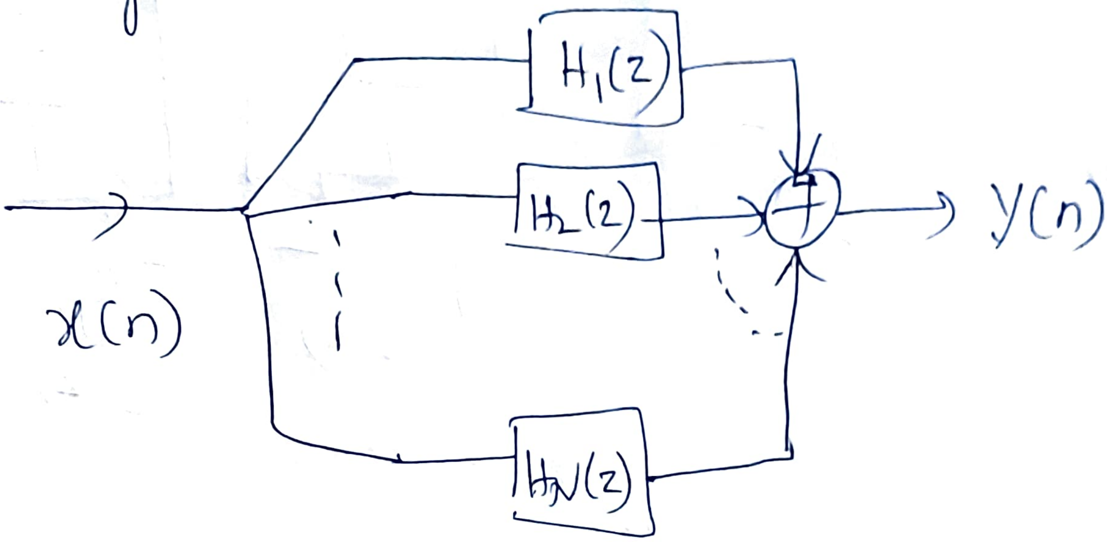
i.e., $$H(z) = H_1(z) + H_2(z) + \cdots H_N(z) \quad \text{—— (2)}$$

using partial fraction expansion.

$$\frac{Y(z)}{X(z)} = H(z)$$

$$\Rightarrow Y(z) = H_1(z)\,X(z) + H_2(z)\,X(z) + \cdots H_N(z)\,X(z)$$

Eq^n (2) is realized using a block diagram shown below



Prob:

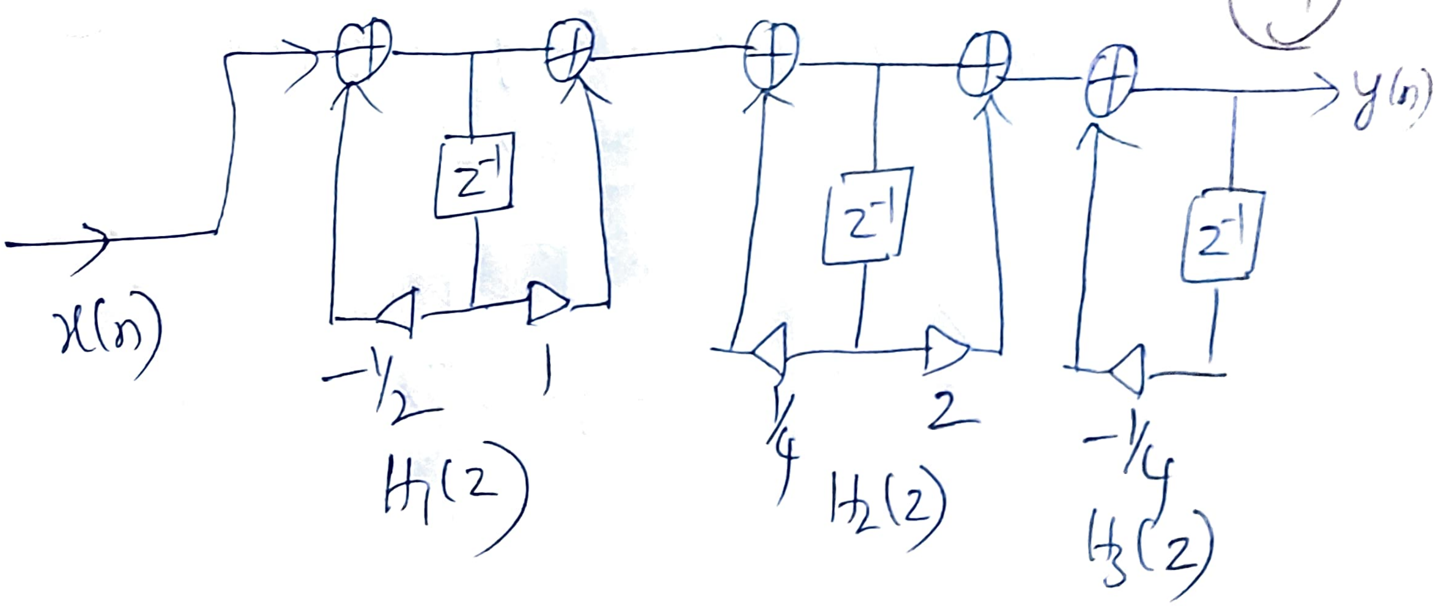
Obtain cascade and parallel realization of the following system.

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{4}z^{-1})}$$

Soln

a) Cascade realization

$$H(z) = \frac{(1+z^{-1})}{(1+\frac{1}{2}z^{-1})} \cdot \frac{(1+2z^{-1})}{(1-\frac{1}{4}z^{-1})} \cdot \frac{1}{(1+\frac{1}{4}z^{-1})}$$



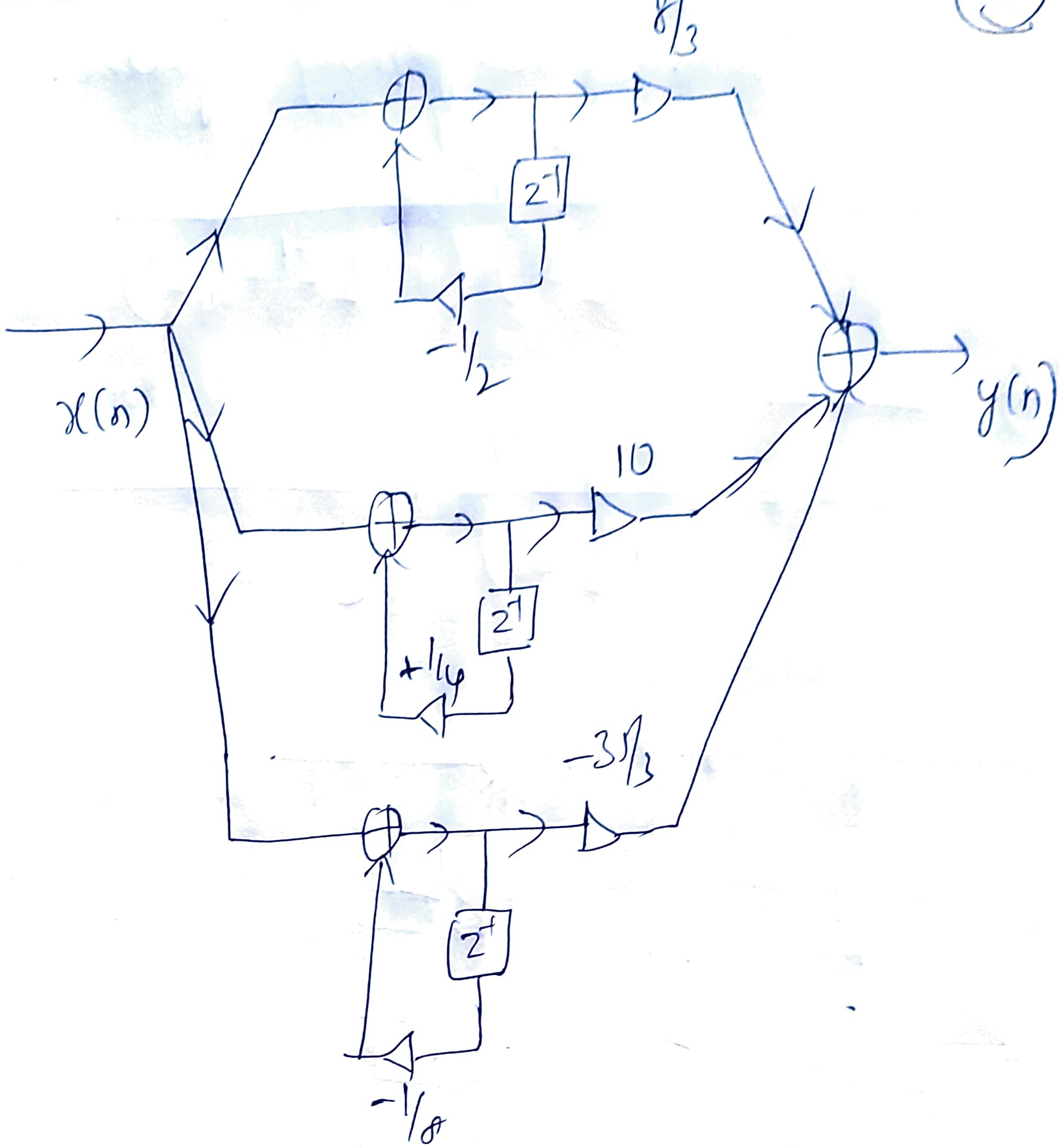
b) <u>Parallel Realization</u>

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{8}z^{-1})}$$

$$= \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}} + \frac{C}{1+\frac{1}{8}z^{-1}}$$

$$A = 8/3 \qquad B = 10 \qquad C = \frac{-35}{3}$$

$$H(z) = \frac{8/3}{1+\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{4}z^{-1}} + \frac{-35/3}{1+\frac{1}{8}z^{-1}}$$

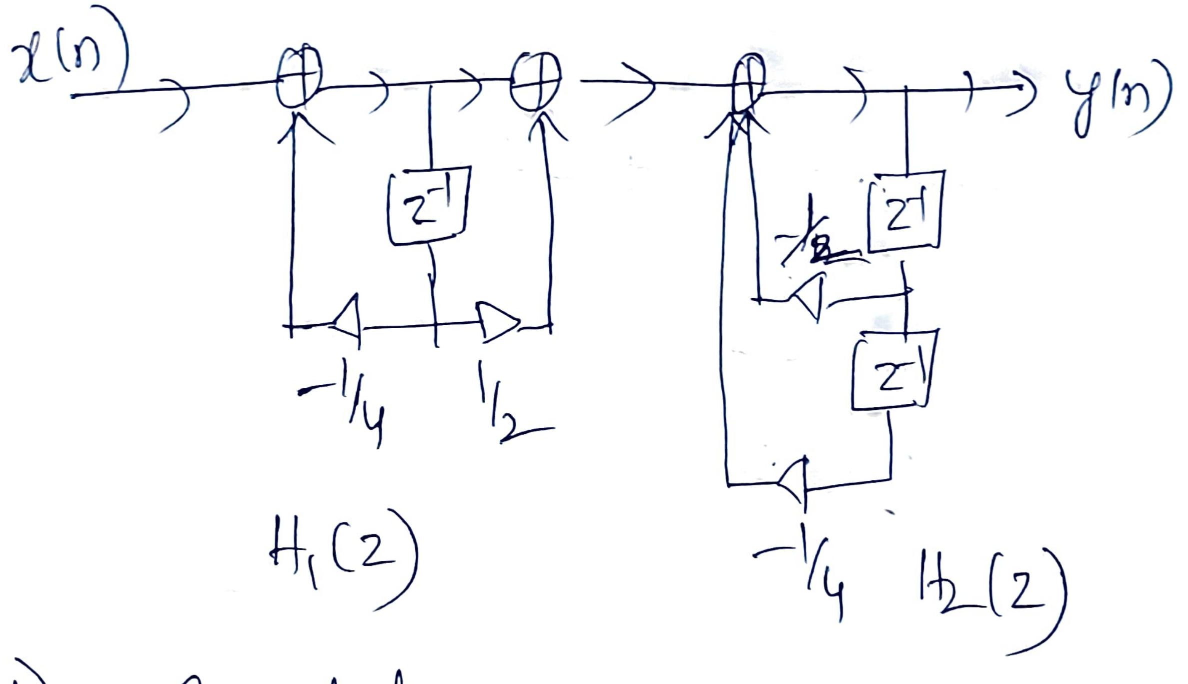

8/3
z^{-1}
-1/2
x(n)
y(n)
10
z^{-1}
+1/4
-31/3
z^{-1}
-1/8

Prob: Obtain cascade and parallel realization of the following system. (16)

$$H(z) = \frac{(1 + \frac{1}{4}z^{-1})}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$

a) Cascade

$$H(z) = \frac{(1+\frac{1}{4}z^{-1})}{(1+\frac{1}{2}z^{-1})} \cdot \frac{1}{(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$



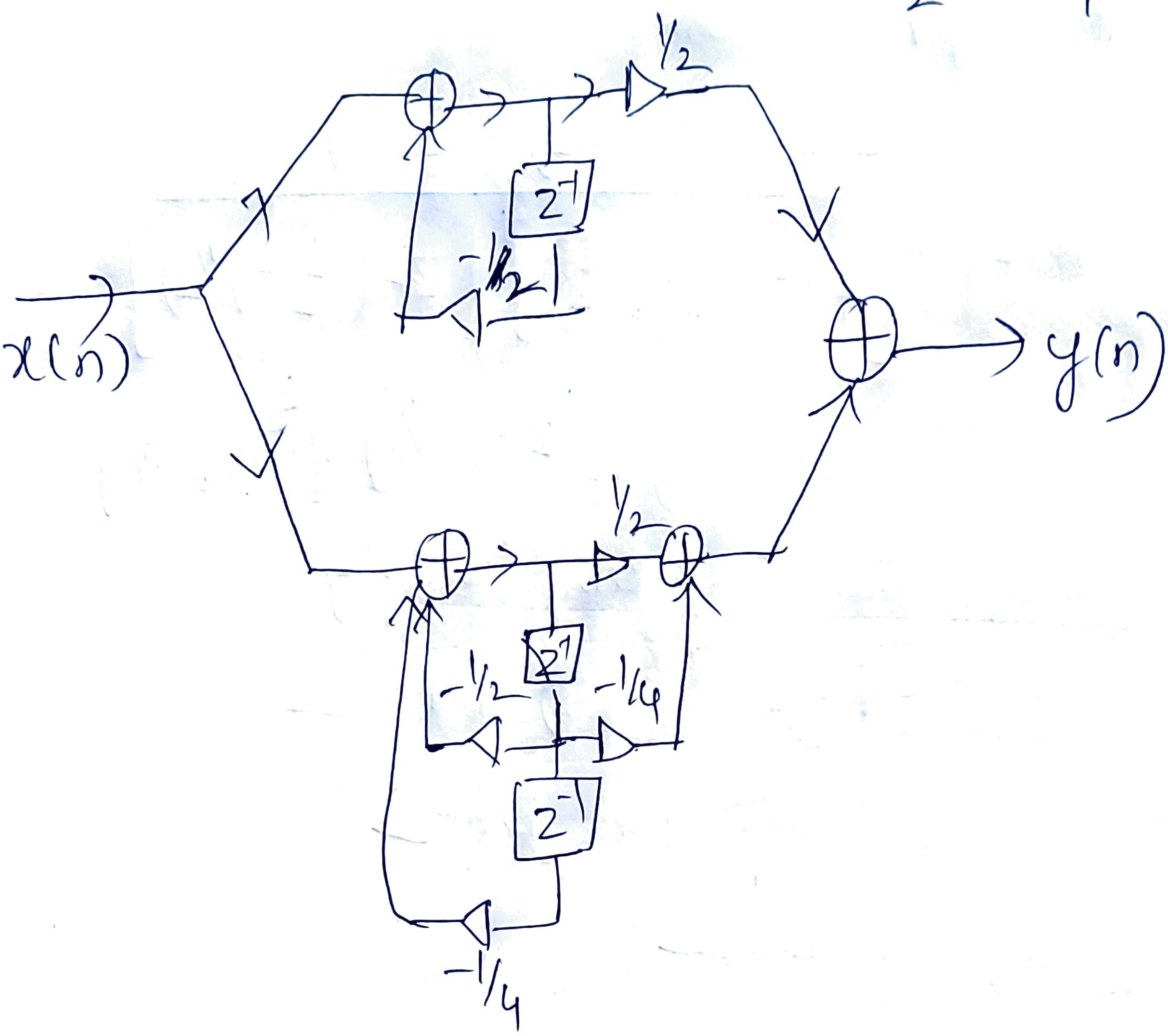
b) Parallel

$$H(z) = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{Bz^{-1}+C}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$A = 1/2$, $B = -1/4$ $C = 1/2$

$$H(z) = \frac{1/2}{1 + 1/2\, z^{-1}} + \frac{-1/4\, z^{-1} + 1/2}{1 + 1/2\, z^{-1} + 1/4\, z^{-2}}$$

17



Prob3 An LTI digital IIR filter is represented by the following transfer function

18

$$H(z) = \frac{z(z-1)(z-2)(z+1)}{\left[z-\left(\frac{1}{2}+j\frac{1}{2}\right)\right]\left[z-\left(\frac{1}{2}-j\frac{1}{2}\right)\right]\left[z-j\frac{1}{4}\right]\left[z+j\frac{1}{4}\right]}$$

Realize the system using

a) DF-I b) DF-II

c) Transposed DF-II

d) Cascade

e) parallel forms.

Also draw SFG for the structures.

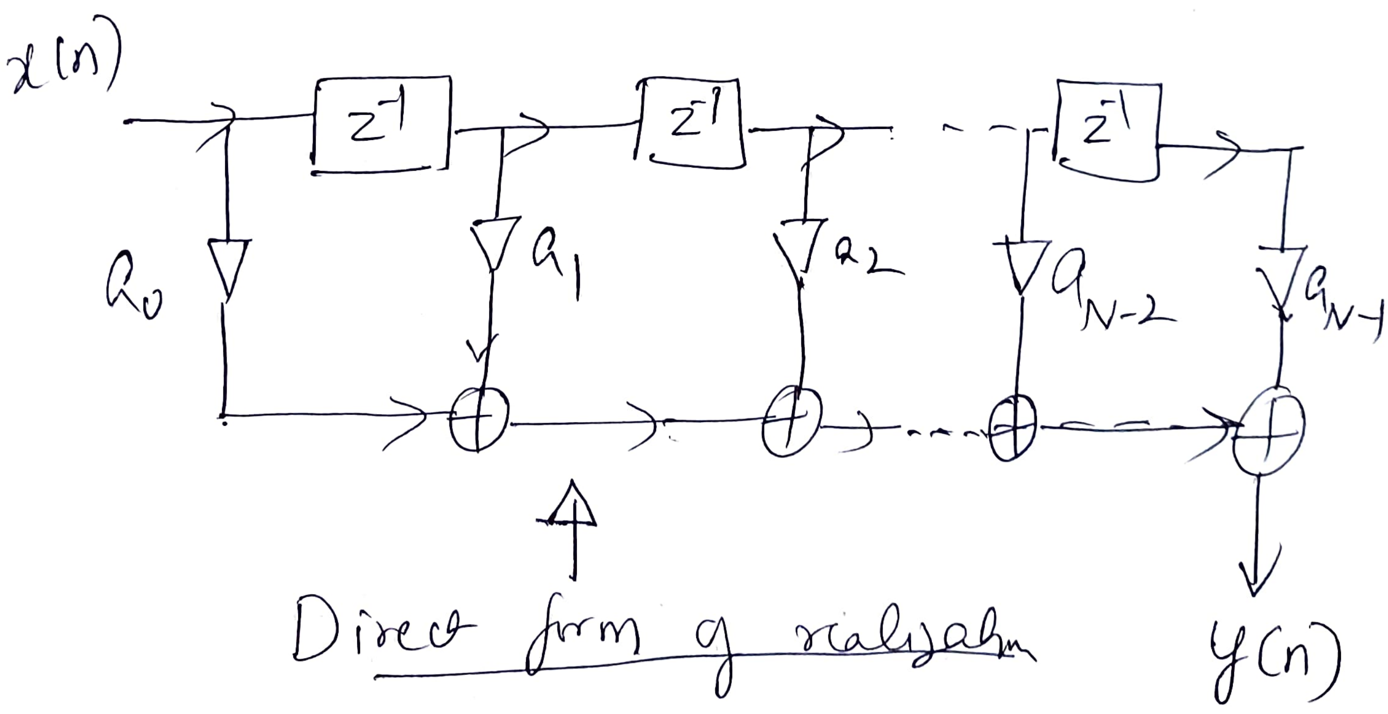
Realization of FIR filters

The transfer function $H(z)$ of a causal FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} a_n z^{-n}$$

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots a_{N-1} z^{-(N-1)} \quad \text{—(1)}$$

The filter structure for the transfer function can be written as follows



Direct form of realization

Problem

Obtain Direct form-I realization for the following FIR filter.

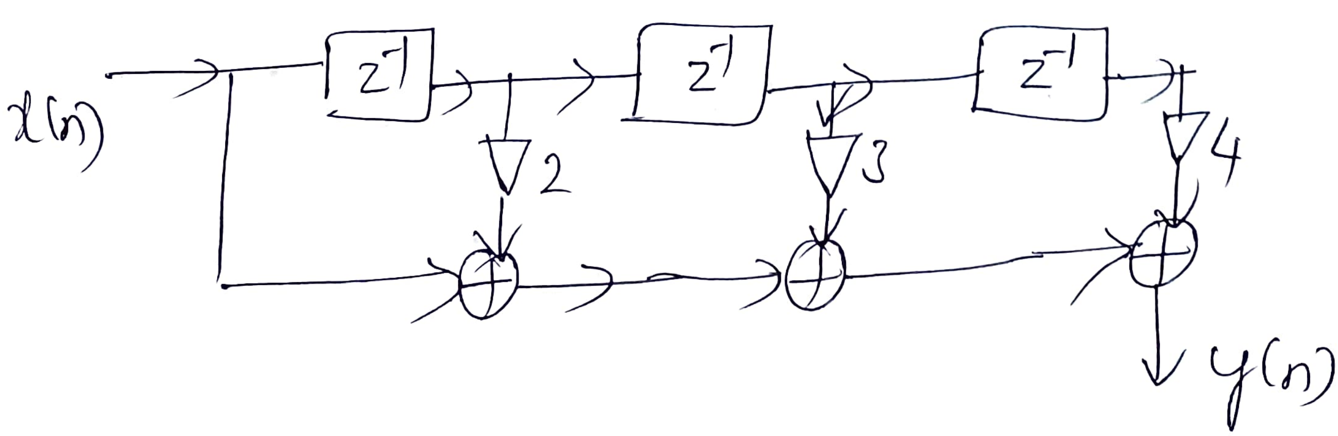
$$y(n) = x(n) + 2x(n-1) + 3x(n-2) + 4x(n-3).$$

Soln: Taking Z-transform on both sides, we get

$$Y(z) = X(z)\left[1 + 2z^{-1} + 3z^{-2} + 4z^{-3}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} \longrightarrow (1)$$

Direct form-I realization of (1) is as follows

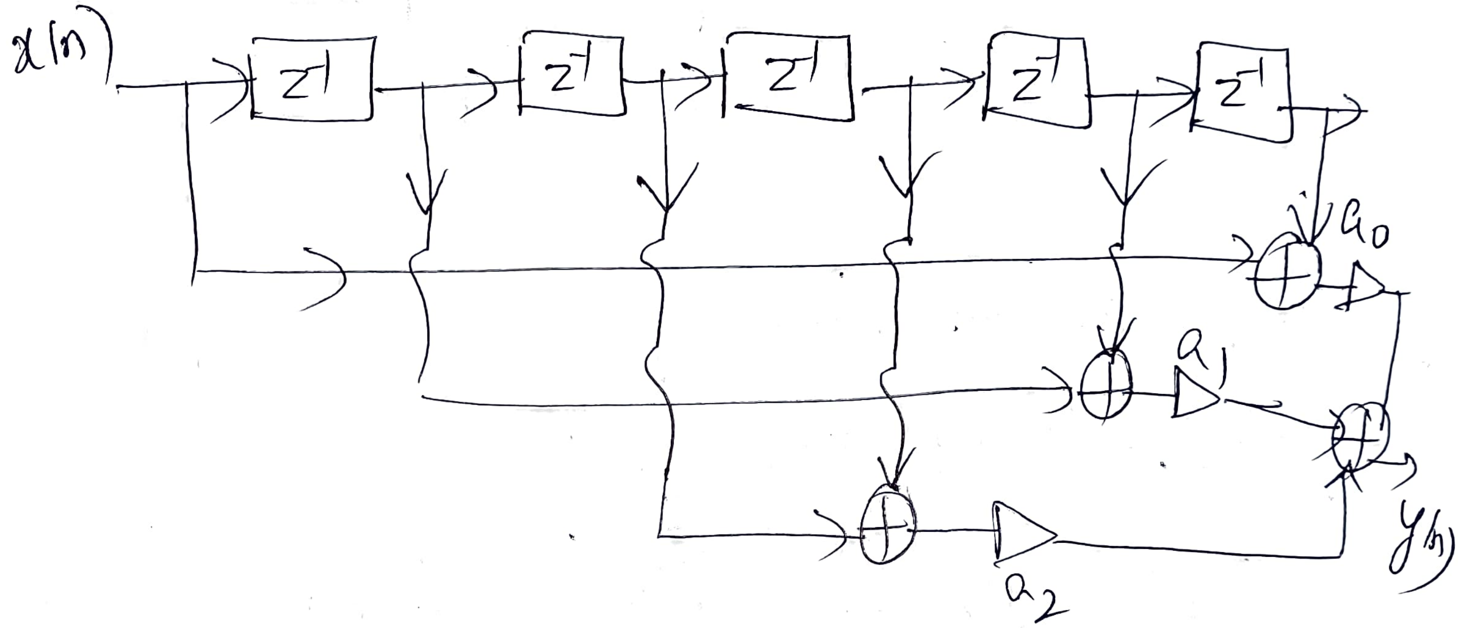


Realization of Linear phase FIR filters (3)

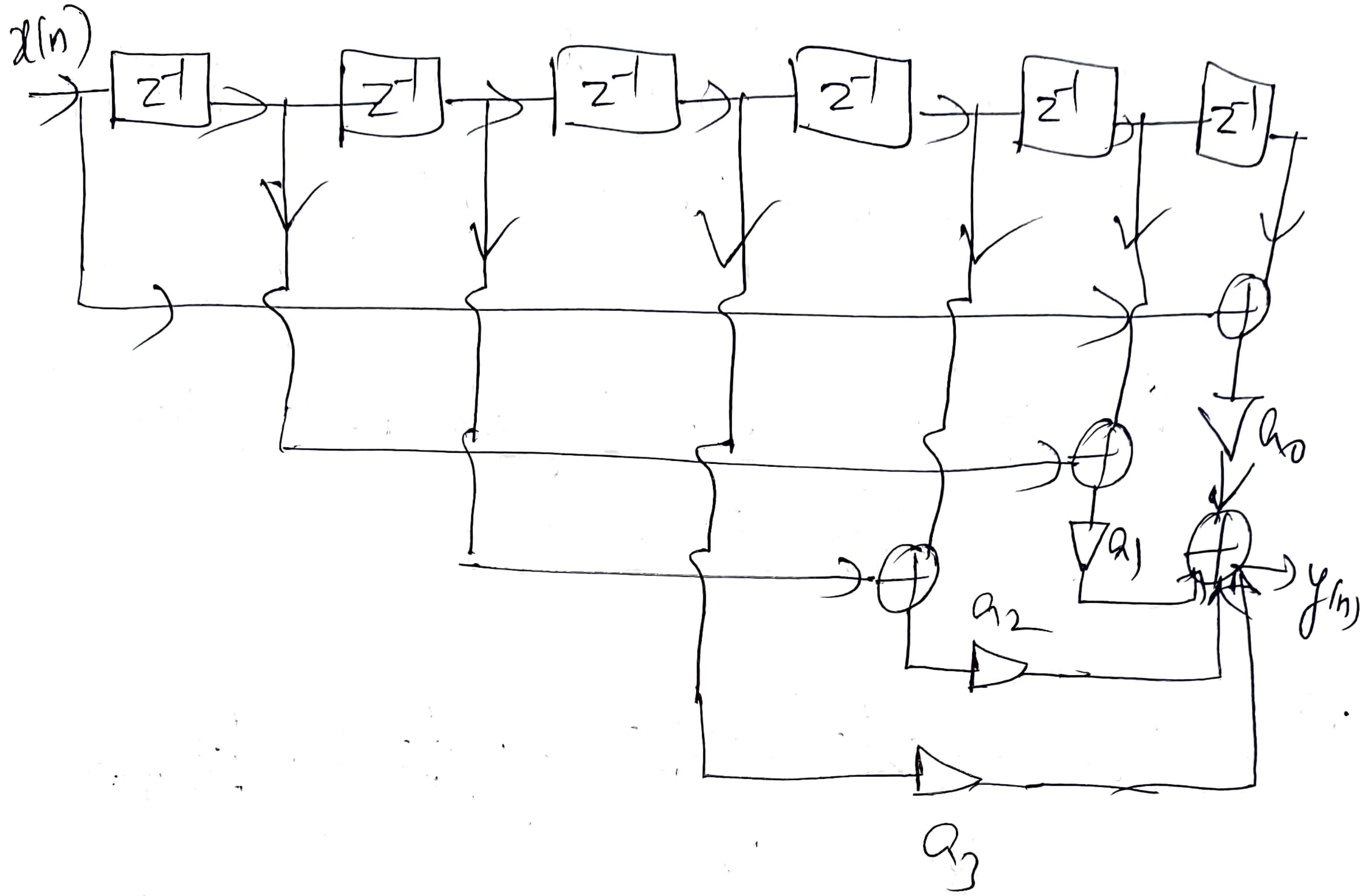
Linear phase can be achieved in FIR filters, if it satisfies symmetry property

i.e, $h(n) = h(N-1-n)$ $\quad 0 \le n \le N-1$

Realization of Linear phase FIR filters for N even



Realization of Linear phase FIR filters for N odd



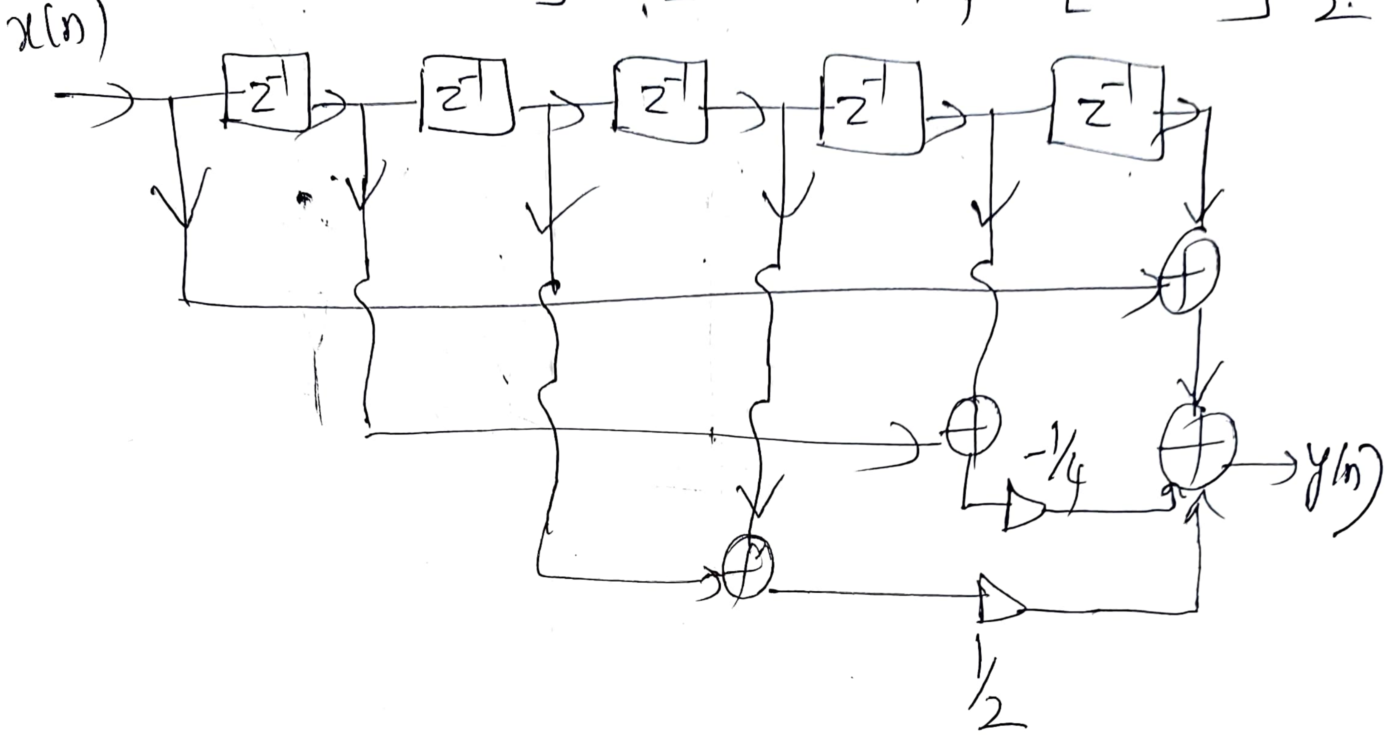
Problem: obtain Linear phase structure for the following FIR filter.

$$h(n) = \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4) + \delta(n-5)$$

Sol^n Taking z- transform, we get (5)

$$H(z) = 1 - \frac{1}{4} z^{-1} + \frac{1}{2} z^{-2} + \frac{1}{2} z^{-3} - \frac{1}{4} z^{-4} + z^{-5}$$

$$= \left(1 + z^{-5}\right) + \left[z^{-1} + z^{-4}\right] \cdot \frac{-1}{4} + \left[z^{-2} + z^{-3}\right] \cdot \frac{1}{2}$$



Prob: Realize the ~~FIR~~ Linear phase FIR filter with impulse response

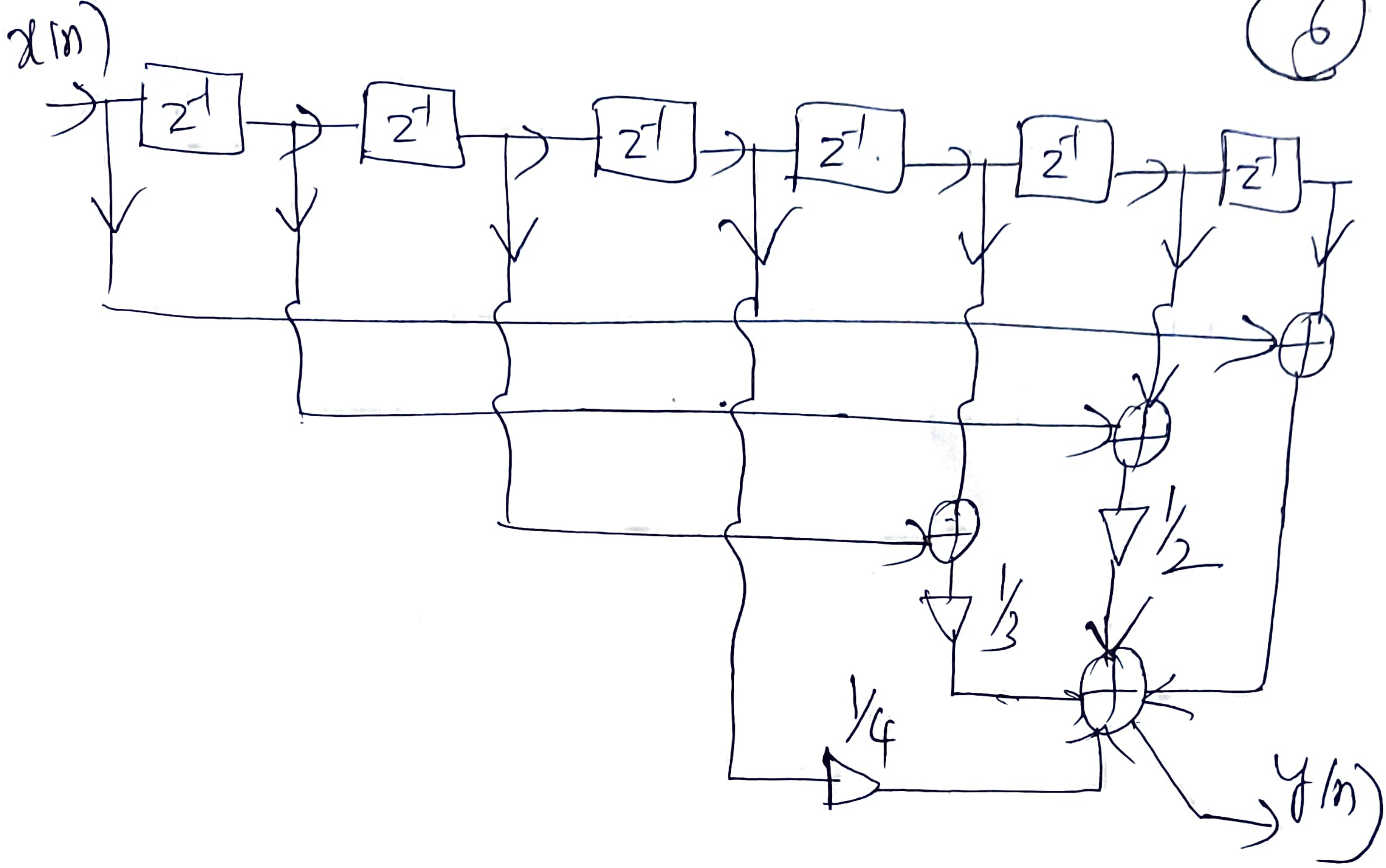
$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{3}\delta(n-2) + \frac{1}{4}\delta(n-3) + \frac{1}{3}\delta(n-4) + \frac{1}{2}\delta(n-5) + \delta(n-6)$$

Sol^n

$$H(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2} + \frac{1}{4} z^{-3} + \frac{1}{3} z^{-4} + \frac{1}{2} z^{-5} + z^{-6}$$

$$= \left[1 + z^{-6}\right] + \frac{1}{2}\left[z^{-1} + z^{-5}\right] + \frac{1}{3}\left[z^{-2} + z^{-4}\right] + \frac{1}{4} z^{-3}$$

6



FIR Lattice Structures

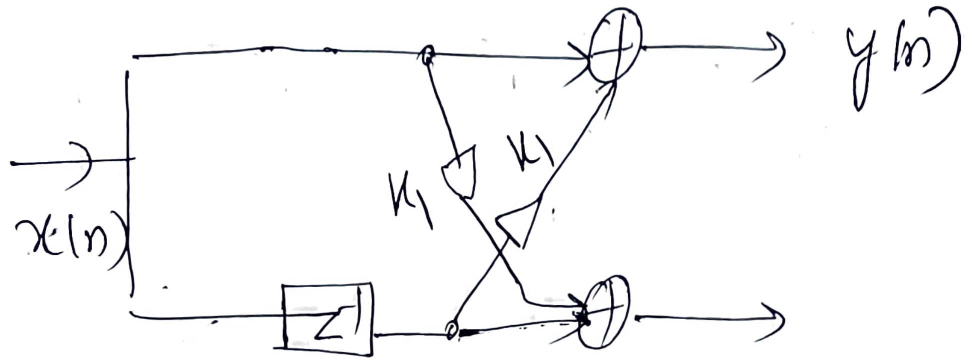
Advantages

a) Recursive from of realization

b) upgrading of filter orders is simple and cheap

c). Computationally very efficient

d) Less sensitive to finite word length effects



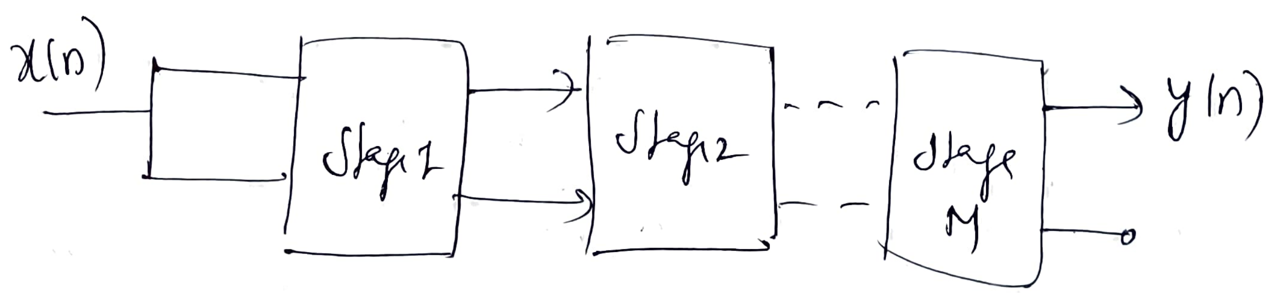
$$A_m(z) = \frac{Y(z)}{X(z)} = 1 + \sum_{i=1}^{m} a_m(i)\, z^{-i}$$

$$A_0(z) = 1$$

$$A_1(z) = 1 + a_1(1)\, z^{-1}$$

$$A_2(z) = 1 + a_2(1)\, z^{-1} + a_2(2)\, z^{-2}$$

$$A_3(z) = 1 + a_3(1)\, z^{-1} + a_3(2)\, z^{-2} + a_3(3)\, z^{-3}$$



formulae for Converting Direct form-I to Lattice

for $m = M, M-1, M-2, \ldots 2, 1$

$$K_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

$$1 \le i \le m-1$$

formulae for Converting Lattice to Direct form-I

for $m = 1, 2, \ldots M$

$$a_m(0) = 1$$

$$a_m(m) = K_m$$

$$a_m(i) = a_{m-1}(i) + a_m(m)\, a_{m-1}(m-i)$$

$$1 \le i \le m-1$$

Problem Obtain Lattice realization for the following FIR filter (9)

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$$

Soln

$$A_2(z) = 1 + a_2(1) z^{-1} + a_2(2) z^{-2}$$

$$a_2(1) = 2 \qquad a_2(2) = 1/3$$

$$K_m = a_m(m)$$

$$\boxed{K_2 = a_2(2) = 1/3}.$$

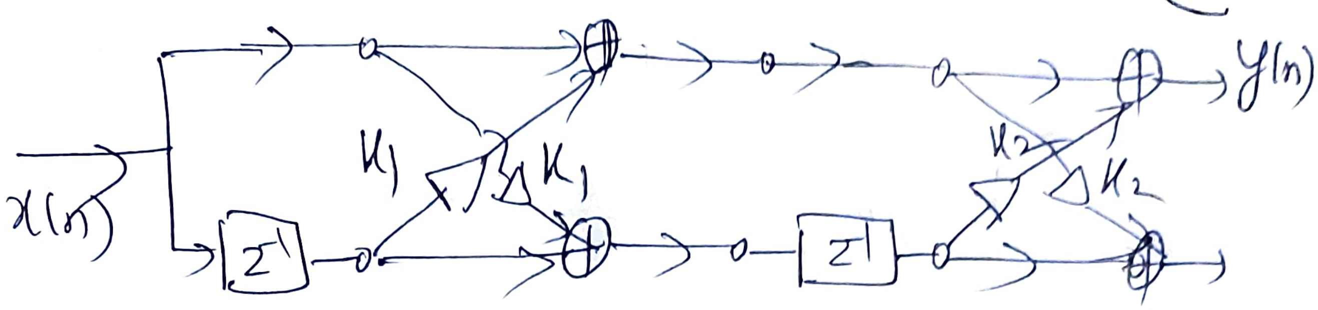
$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

Put $m = 2$ and $i = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - K_2^2} = 3/2$$

$$\boxed{K_1 = a_1(1) = 3/2}$$

(10)



Problem: Obtain Lattice Structure for the following FIR filter

$$H(z) = 2 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{3}z^{-3}$$

$$H(z) = 2\left[1 + \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2} + \frac{1}{6}z^{-3}\right]$$

$$A_3(z) = 1 + a_3(1)z^{-1} + a_3(2)z^{-2} + a_3(3)z^{-3}$$

$$\boxed{K_3 = a_3(3) = 1/6}$$

$$a_2(1) = \frac{a_3(1) - a_3(3)\,a_3(2)}{1 - K_3^2}$$

$$= \frac{1/4 - 1/6 \times 1/8}{1 - (1/6)^2} = \frac{0.229}{0.97} = 0.236$$

(11)

$$a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - K_3^2}$$

$$= \frac{\frac{1}{8} - \frac{1}{6} \times \frac{1}{4}}{1 - \left(\frac{1}{6}\right)^2} = \frac{0.083}{0.97} = \underline{0.086}$$

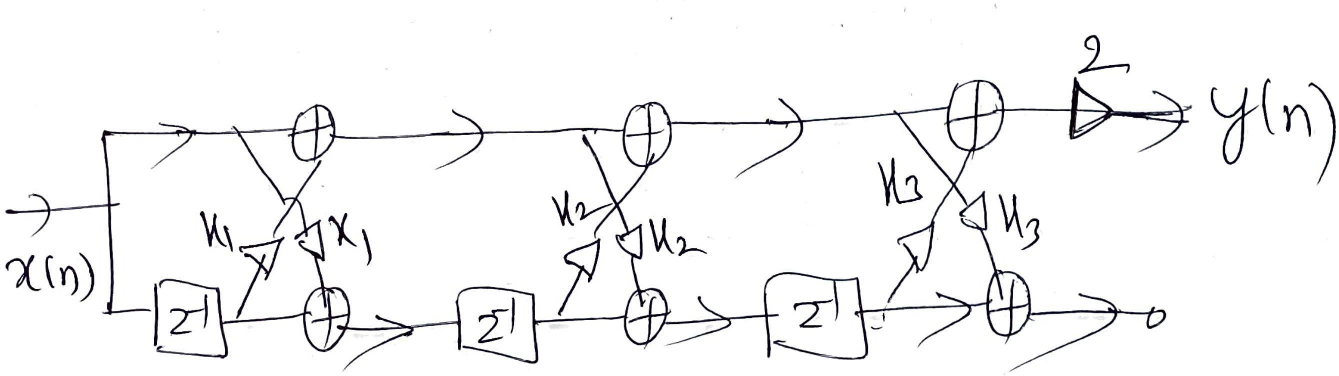
$$\boxed{K_2 = a_2(2) = 0.086}$$

$$a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - K_2^2}$$

$$= \frac{0.236 - 0.086 \times 0.236}{1 - (0.086)^2}$$

$$= \frac{0.215}{0.992} = 0.216$$

$$\boxed{K_1 = a_1(1) = 0.216}$$



Prob: Let the Coefficients of a three Stage FIR Lattice Structure be $K_1 = 0.1$, $K_2 = 0.2$ and $K_3 = 0.3$. Find the Coefficients of direct form-I FIR filter and draw its block diagram. (12)

Soln

$$a_m(0) = 1$$

$$a_m(m) = K_m$$

$$a_m(i) = a_{m-1}(i) + a_m(m)\, a_{m-1}(m-i)$$

$$1 \le i \le m-1$$

m=1:

$$a_1(0) = 1$$

$$a_1(1) = K_1 = 0.1$$

m=2

$$a_2(0) = 1$$

$$a_2(1) = a_1(1) + a_2(2)\, a_1(1)$$

$$= 0.1 + 0.2 \times 0.1 = 0.12$$

$$a_2(2) = K_2 = 0.2$$

m=3

$$a_3(0) = 1$$

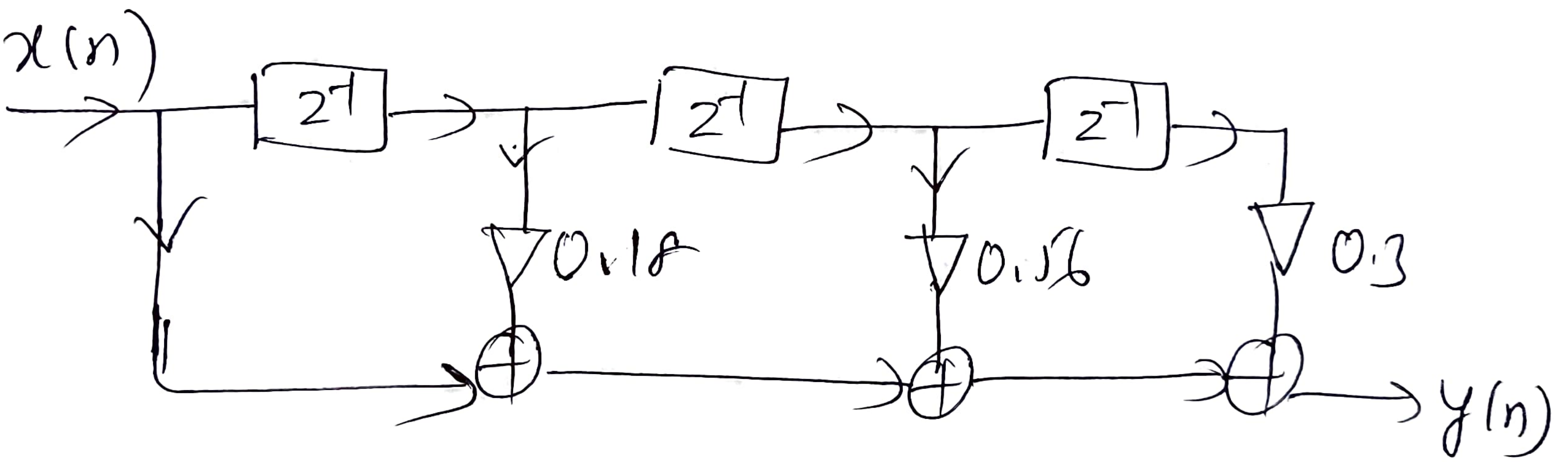
$$a_3(1) = a_2(1) + a_3(3)\, a_2(2)$$

$$= 0.12 + 0.3 \times 0.2 = 0.18$$

$$a_3(2) = a_2(2) + a_3(3)\, a_2(1)$$
$$= 0.2 + 0.3 \times 0.12 = 0.56$$

$$a_3(3) = K_3 = 0.3$$

$$H(z) = A_3(z) = 1 + a_3(1) z^{-1} + a_3(2) z^{-2} + a_3(3) z^{-3}$$
$$= 1 + 0.18 z^{-1} + 0.56 z^{-2} + 0.3 z^{-3}$$



2 Let the Lattice Coefficients of an FIR Lattice filter be $K_1 = 0.84$, $K_2 = 0.4$ and $K_3 = 0.06$. Obtain Direct from-I realization of the filter

Soln $a_m(0) = 1$

(14)

$a_m(m) = k_m$

$$a_m(i) = a_{m-1}(i) + a_m(m)\, a_{m-1}(m-i)$$

$1 \leq i \leq m-1$

$m=1$ $a_1(0) = 1$

$a_1(1) = k_1 = 0.84$

$m=2$ $a_2(0) = 1$

$a_2(1) = a_1(1) + a_1(1) \cdot a_2(2)$

$= 0.84 + 0.84 \times 0.4$

$a_2(1) = 1.176$

$m=3$ $a_3(0) = 1$

$a_3(1) = a_2(1) + a_3(3)\, a_2(2)$

$= 1.176 + 0.06 \times 0.4 = 1.2$

$a_3(2) = a_2(2) + a_3(3)\, a_2(1)$

$= 0.4 + 0.06 \times 1.176$

$= 0.47$

$a_3(3) = k_3 = 0.06$

$$A_3(z) = 1 + a_3(1)z^{-1} + a_3(2)z^{-2} + a_3(3)z^{-3}$$

$$= 1 + 1.2z^{-1} + 0.47z^{-2} + 0.06z^{-3}$$