Graphs

Introduction to Graphs:

Graph G is a pair (V, E), where V is a finite set of vertices and E is a finite set of edges. We will often denote n = |V|, e = |E|.

A graph is generally displayed as figure 6.5.1, in which the vertices are represented by circles and the edges by lines.

An edge with an orientation (i.e., arrow head) is a directed edge, while an edge with no orientation is our undirected edge.

If all the edges in a graph are undirected, then the graph is an undirected graph. The graph in figure 6.5.1(a) is an undirected graph. If all the edges are directed; then the graph is a directed graph. The graph of figure 6.5.1(b) is a directed graph. A directed graph is also called as digraph. A graph G is connected if and only if there is a simple path between any two nodes in G.

A directed graph G is said to be connected, or strongly connected, if for each pair (u, v) for nodes in G there is a path from u to v and also a path from v to u. On the other hand, G is said to be unilaterally connected if for any pair (u, v) of nodes in G there is a path from u to v or a path from v to u. For example, the digraph shown in figure 6.5.1 (e) is strongly connected.

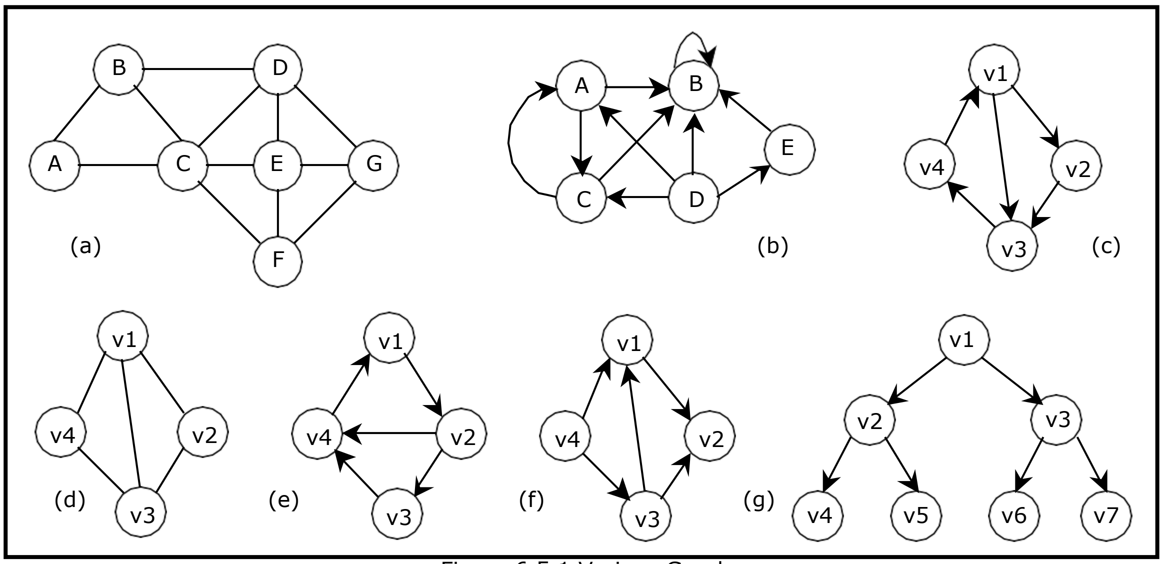


Figure 6.5.1 Various Graphs

We can assign weight function to the edges: $w_G(e)$ is a weight of edge $e \in E$. The graph which has such function assigned is called weighted graph.

The number of incoming edges to a vertex v is called in–degree of the vertex (denote indeg(v)). The number of outgoing edges from a vertex is called out-degree (denote outdeg(v)). For example, let us consider the digraph shown in figure 6.5.1(f),

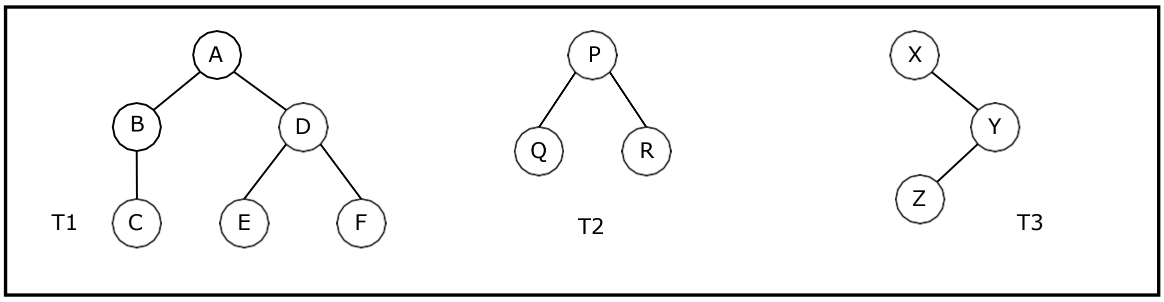
indegree(v_1) = 2	outdegree(v_1) = 1
indegree(v_2) = 2	outdegree(v_2) = 0

A path is a sequence of vertices (v_1, v_2, , v_k), where for all i, (v_i, v_{i+1}) ε E. A path is simple if all vertices in the path are distinct. If there is a path containing one or more edges which starts from a vertex V_i and terminates into the same vertex then the path is known as a **cycle.** For example, there is a cycle in figure 6.5.1(a), figure 6.5.1(c) and figure 6.5.1(d).

If a graph (digraph) does not have any cycle then it is called **acyclic graph**. For example, the graphs of figure 6.5.1 (f) and figure 6.5.1 (g) are acyclic graphs.

A graph G′ = (V′, E′) is a sub-graph of graph G = (V, E) iff V′ $\subseteq$ V and E′ $\subseteq$ E.

A **Forest** is a set of disjoint trees. If we remove the root node of a given tree then it becomes forest. The following figure shows a forest F that consists of three trees T1, T2 and T3.



A Forest F

A graph that has either self loop or parallel edges or both is called **multi-graph**.

Tree is a connected acyclic graph (there aren't any sequences of edges that go around in a loop).

A **spanning tree** of a graph G = (V, E) is a tree that contains all vertices of V and is a subgraph of G. A single graph can have **multiple spanning trees.**

Let T be a spanning tree of a graph G. Then

1. *Any two vertices in T are connected by a unique simple path.*
2. *If any edge is removed from T, then T becomes disconnected.*
3. *If we add any edge into T, then the new graph will contain a cycle.*
4. *Number of edges in T is n-1.*

Representation of Graphs:

There are two ways of representing digraphs. They are:

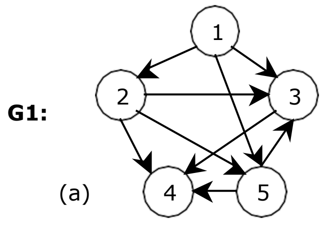
- Adjacency matrix.
- Adjacency List.
- Incidence matrix.

Adjacency matrix:

In this representation, the adjacency matrix of a graph G is a two dimensional n x n matrix, say A = ($a_{i,j}$), where

$$a_{i,j} = \begin{cases} 1 & \textit{if there is an edge from } v_i \textit{ to } v_j \\ 0 & \textit{otherwise} \end{cases}$$

The matrix is symmetric in case of undirected graph, while it may be asymmetric if the graph is directed. This matrix is also called as Boolean matrix or bit matrix.



(a)

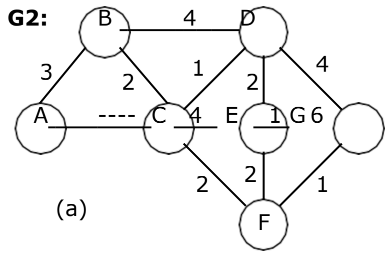
(b)

	1	**2**	**3**	**4**	**5**
1	0	1	1	0	1
2	0	0	1	1	1
3	0	0	0	1	0
4	0	0	0	0	0
5	0	0	1	1	0

Figure 6.5.2. A graph and its Adjacency matrix

Figure 6.5.2(b) shows the adjacency matrix representation of the graph G1 shown in figure 6.5.2(a). The adjacency matrix is also useful to store multigraph as well as weighted graph. In case of multigraph representation, instead of entry 0 or 1, the entry will be between number of edges between two vertices.

In case of weighted graph, the entries are weights of the edges between the vertices. The adjacency matrix for a weighted graph is called as cost adjacency matrix. Figure 6.5.3(b) shows the cost adjacency matrix representation of the graph G2 shown in figure 6.5.3(a).



(a)

(b)

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	0	3	6	∝	∝	∝	∝
B	3	0	2	4	∝	∝	∝
C	6	2	0	1	4	2	∝
D	∝	4	1	0	2	∝	4
E	∝	∝	4	2	0	2	1
F	∝	∝	2	∝	2	0	1
G	∝	∝	∝	4	1	1	0

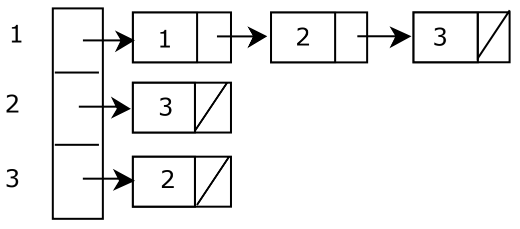
Figure 6.5.3 Weighted graph and its Cost adjacency matrix

Adjacency List:

In this representation, the n rows of the adjacency matrix are represented as n linked lists. An array Adj[1, 2, ... n] of pointers where for $1 \leq v \leq n$, Adj[v] points to a linked list containing the vertices which are adjacent to v (i.e. the vertices that can be reached from v by a single edge). If the edges have weights then these weights may also be stored in the linked list elements. For the graph G in figure 6.5.4(a), the adjacency list in shown in figure 6.5.4 (b).

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 \end{array}$$

(a) Adjacency Matrix



(b) Adjacency List

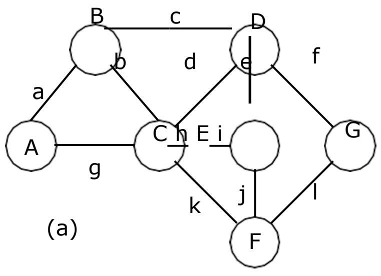
Figure 6.5.4 Adjacency matrix and adjacency list

Incidence Matrix:

In this representation, if G is a graph with n vertices, e edges and no self loops, then incidence matrix A is defined as an n by e matrix, say A = ($a_{i,j}$), where

$$a_{i,j} = \begin{cases} 1 & \text{if there is an edge } j \text{ incident to } v_i \\ 0 & \text{otherwise} \end{cases}$$

Here, n rows correspond to n vertices and e columns correspond to e edges. Such a matrix is called as vertex-edge incidence matrix or simply incidence matrix.



(a)

	a	b	c	d	e	f	g	h	i	j	k	l
A	1	0	0	0	0	0	1	0	0	0	0	0
B	1	1	1	0	0	0	0	0	0	0	0	0
C	0	1	0	1	0	0	1	1	0	0	1	0
D	0	0	1	1	1	1	0	0	0	0	0	0
E	0	0	0	0	1	0	0	1	1	1	0	0
F	0	0	0	0	0	0	0	0	0	1	1	1
G	0	0	0	0	0	1	0	0	1	0	0	1

(b)

Figure 6.5.4 Graph and its incidence matrix

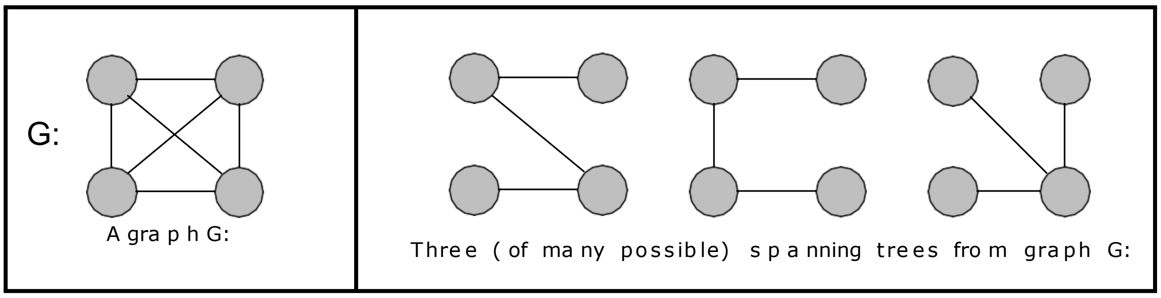
Figure 6.5.4(b) shows the incidence matrix representation of the graph G1 shown in figure 6.5.4(a).

6.2. Minimum Spanning Tree (MST):

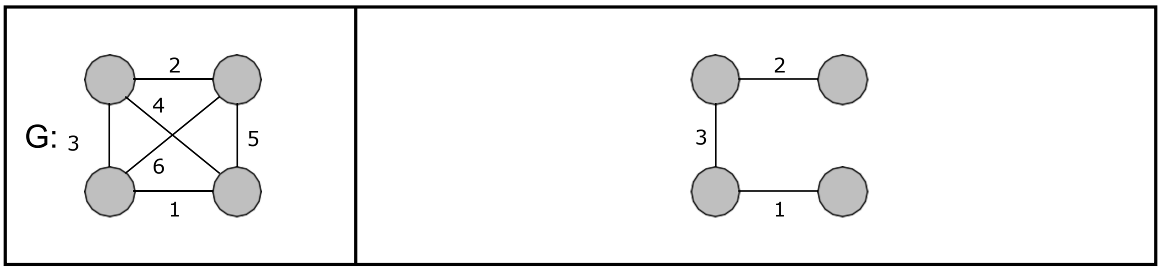
A spanning tree for a connected graph is a tree whose vertex set is the same as the vertex set of the given graph, and whose edge set is a subset of the edge set of the given graph. i.e., any connected graph will have a spanning tree.

Weight of a spanning tree w(T) is the sum of weights of all edges in T. Minimum spanning tree (MST) is a spanning tree with the smallest possible weight.

Example:



A graph G: Three (of many possible) spanning trees from graph G:



A weighted graph G: The minimal spanning tree from weighted graph G:

Let's consider a couple of real-world examples on minimum spanning tree:

- One practical application of a MST would be in the design of a network. For instance, a group of individuals, who are separated by varying distances, wish to be connected together in a telephone network. Although MST cannot do anything about the distance from one connection to another, it can be used to determine the least cost paths with no cycles in this network, thereby connecting everyone at a minimum cost.

- Another useful application of MST would be finding airline routes. The vertices of the graph would represent cities, and the edges would represent routes between the cities. MST can be applied to optimize airline routes by finding the least costly paths with no cycles.

Minimum spanning tree, can be constructed using any of the following two algorithms:

1. Kruskal's algorithm and
2. Prim's algorithm.

Both algorithms differ in their methodology, but both eventually end up with the MST. *Kruskal's algorithm uses edges, and Prim's algorithm uses vertex connections* in determining the MST. In *Prim's algorithm at any instance of output it represents tree* whereas in *Kruskal's algorithm at any instance of output it may represent tree or not*.

6.3.1. Kruskal's Algorithm

This is a greedy algorithm. A greedy algorithm chooses some local optimum (i.e. picking an edge with the least weight in a MST).

Kruskal's algorithm works as follows: Take a graph with 'n' vertices, keep on adding the shortest (least cost) edge, while avoiding the creation of cycles, until (n - 1) edges have been added. Sometimes two or more edges may have the same cost.The order in which the edges are chosen, in this case, does not matter. Different MST's may result, but they will all have the same total cost, which will always be the minimum cost.

Kruskal's Algorithm for minimal spanning tree is as follows:

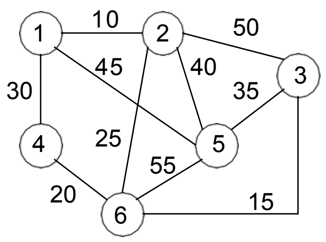
1. Make the tree T empty.
2. Repeat the steps 3, 4 and 5 as long as T contains less than n - 1 edges and E is not empty otherwise, proceed to step 6.
3. Choose an edge (v, w) from E of lowest cost.
4. Delete (v, w) from E.
5. If (v, w) does not create a cycle in T

   then Add (v, w) to T

   else discard (v, w)
6. If T contains fewer than n - 1 edges then print no spanning tree.

Example 1:

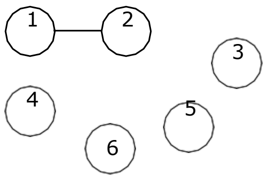
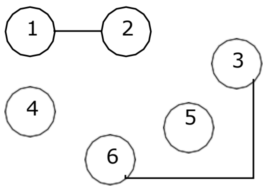
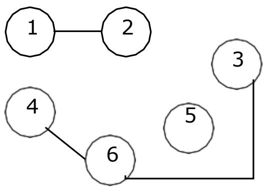
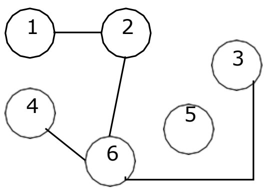
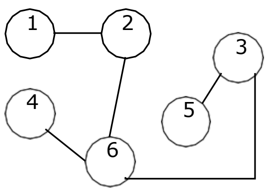
Construct the minimal spanning tree for the graph shown below:



Arrange all the edges in the increasing order of their costs:

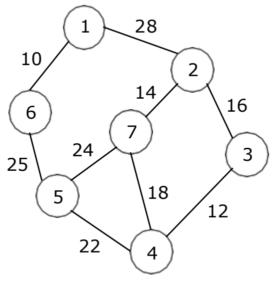
Cost	***10***	***15***	***20***	***25***	***30***	***35***	***40***	***45***	***50***	***55***
Edge	(1, 2)	(3, 6)	(4, 6)	(2, 6)	(1, 4)	(3, 5)	(2, 5)	(1, 5)	(2, 3)	(5, 6)

The stages in Kruskal's algorithm for minimal spanning tree is as follows:

EDGE	**COST**	**STAGES IN KRUSKAL'S ALGORITHM**	**REMARKS**
(1, 2)	10		The edge between vertices 1 and 2 is the first edge selected. It is included in the spanning tree.
(3, 6)	15		Next, the edge between vertices 3 and 6 is selected and included in the tree.
(4, 6)	20		The edge between vertices 4 and 6 is next included in the tree.
(2, 6)	25		The edge between vertices 2 and 6 is considered next and included in the tree.
(1, 4)	30	Reject	The edge between the vertices 1 and 4 is discarded as its inclusion creates a cycle.
(3, 5)	35		Finally, the edge between vertices 3 and 5 is considered and included in the tree built. This completes the tree. The cost of the minimal spanning tree is 105.

Example 2:

Construct the minimal spanning tree for the graph shown below:

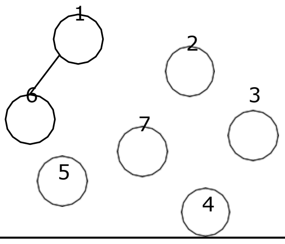
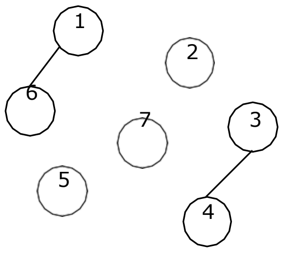
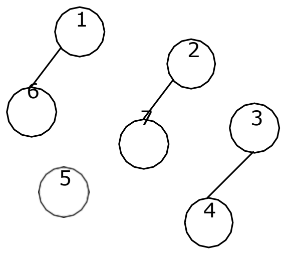


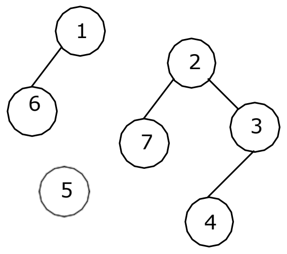
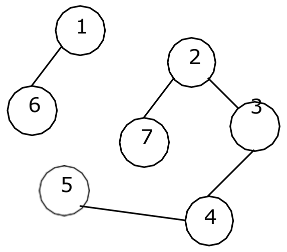
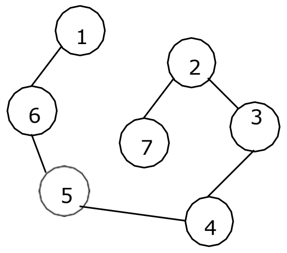
Solution:

Arrange all the edges in the increasing order of their costs:

Cost	***10***	***12***	***14***	***16***	***18***	***22***	***24***	***25***	***28***
Edge	(1, 6)	(3, 4)	(2, 7)	(2, 3)	(4, 7)	(4, 5)	(5, 7)	(5, 6)	(1, 2)

The stages in Kruskal's algorithm for minimal spanning tree is as follows:

EDGE	**COST**	**STAGES IN KRUSKAL'S ALGORITHM**	**REMARKS**
(1, 6)	10		The edge between vertices 1 and 6 is the first edge selected. It is included in the spanning tree.
(3, 4)	12		Next, the edge between vertices 3 and 4 is selected and included in the tree.
(2, 7)	14		The edge between vertices 2 and 7 is next included in the tree.

(2, 3)	16		The edge between vertices 2 and 3 is next included in the tree.
(4, 7)	18	Reject	The edge between the vertices 4 and 7 is discarded as its inclusion creates a cycle.
(4, 5)	22		The edge between vertices 4 and 5 is considered next and included in the tree.
(5, 7)	24	Reject	The edge between the vertices 5 and 7 is discarded as its inclusion creates a cycle.
(5, 6)	25		Finally, the edge between vertices 5 and 6 is considered and included in the tree built. This completes the tree. The cost of the minimal spanning tree is 99.

MINIMUM-COST SPANNING TREES: PRIM'S ALGORITHM

A given graph can have many spanning trees. From these many spanning trees, we have to select a cheapest one. This tree is called as minimal cost spanning tree.

Minimal cost spanning tree is a connected undirected graph G in which each edge is labeled with a number (edge labels may signify lengths, weights other than costs). Minimal cost spanning tree is a spanning tree for which the sum of the edge labels is as small as possible

The slight modification of the spanning tree algorithm yields a very simple algorithm for finding an MST. In the spanning tree algorithm, any vertex not in the tree but connected to it by an edge can be added. To find a Minimal cost spanning tree, we must be selective - we must always add a new vertex for which the cost of the new edge is as small as possible.

This simple modified algorithm of spanning tree is called prim's algorithm for finding an Minimal cost spanning tree. Prim's algorithm is an example of a greedy algorithm.

Prim's Algorithm:

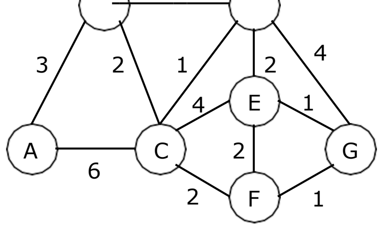
E is the set of edges in G. cost [1:n, 1:n] is the cost adjacency matrix of an n vertex graph such that cost [i, j] is either a positive real number or ∝ if no edge (i, j) exists. A minimum spanning tree is computed and stored as a set of edges in the array t [1:n-1, 1:2]. (t [i, 1], t [i, 2]) is an edge in the minimum-cost spanning tree. The final cost is returned.

Algorithm Prim (E, cost, n, t)

```
{
    Let (k, l) be an edge of minimum cost in E;
    mincost := cost [k, l];
    t [1, 1] := k; t [1, 2] := l;
    for  i :=1 to n do                          // Initialize near
        if (cost [i, l] < cost [i, k]) then near [i] := l;
        else near [i] := k;
    near [k] :=near [l] := 0;
    for  i:=2 to n - 1 do                       // Find n - 2 additional edges for t.
    {
        Let j be an index such that near [j] ≠ 0 and
        cost [j, near [j]] is minimum;
        t [i, 1] := j; t [i, 2] := near [j];
        mincost := mincost + cost [j, near [j]];
        near [j] := 0
        for  k:= 1 to n do                          // Update near[].
            if ((near [k] ≠ 0) and (cost [k, near [k]] > cost [k, j]))
                then near [k] := j;
    }
    return mincost;
}
```

EXAMPLE:

Use Prim's Algorithm to find a minimal spanning tree for the graph shown below starting with the vertex A.



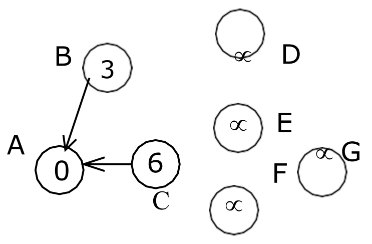
Solution:

The cost adjacency matrix is

$$\begin{matrix} 0 & 3 & 6 & \infty & \infty & \infty & \infty \\ 3 & 0 & 2 & 4 & \infty & \infty & \infty \\ 6 & 2 & 0 & 1 & 4 & 2 & \infty \\ \infty & 4 & 1 & 0 & 2 & \infty & 4 \\ \infty & \infty & 4 & 2 & 0 & 2 & 1 \\ \infty & \infty & 2 & \infty & 2 & 0 & 1 \\ \infty & \infty & \infty & 4 & 1 & 1 & 0 \end{matrix}$$

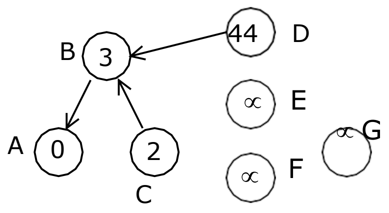
The stepwise progress of the prim’s algorithm is as follows:

Step 1:



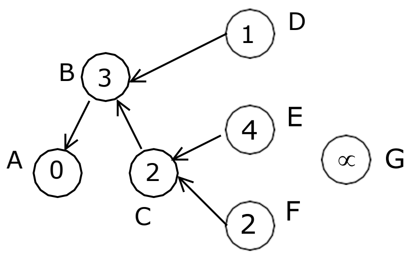
Vertex	A	B	C	D	E	F	G
Status	0	1	1	1	1	1	1
Dist.	0	3	6	∝	∝	∝	∝
Next	*	A	A	A	A	A	A

Step 2:



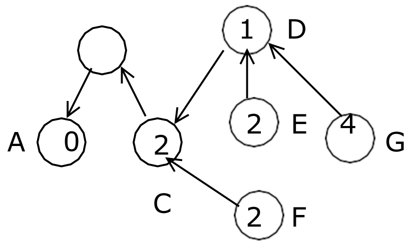
Vertex	A	B	C	D	E	F	G
Status	0	0	1	1	1	1	1
Dist.	0	3	2	4	∝	∝	∝
Next	*	A	B	B	A	A	A

Step 3:



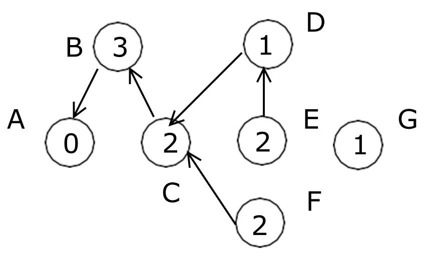
Vertex	A	B	C	D	E	F	G
Status	0	0	0	1	1	1	1
Dist.	0	3	2	1	4	2	∝
Next	*	A	B	C	C	C	A

Step 4:



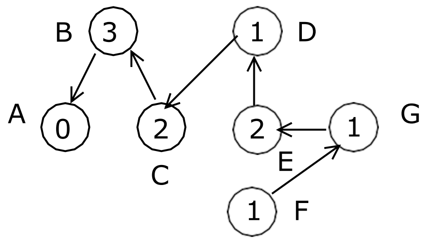
Vertex	A	B	C	D	E	F	G
Status	0	0	0	0	1	1	1
Dist.	0	3	2	1	2	2	4
Next	*	A	B	C	D	C	D

Step 5:



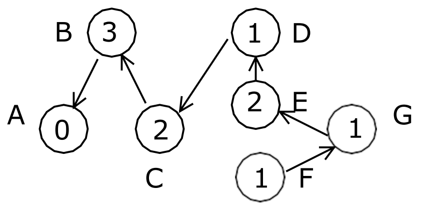
Vertex	A	B	C	D	E	F	G
Status	0	0	0	0	1	0	1
Dist.	0	3	2	1	2	2	1
Next	*	A	B	C	D	C	E

Step 6:



Vertex	A	B	C	D	E	F	G
Status	0	0	0	0	0	1	0
Dist.	0	3	2	1	2	1	1
Next	*	A	B	C	D	G	E

Step 7:



Vertex	A	B	C	D	E	F	G
Status	0	0	0	0	0	0	0
Dist.	0	3	2	1	2	1	1
Next	*	A	B	C	D	G	E

Traversing a Graph

Many graph algorithms require one to systematically examine the nodes and edges of a graph G. There are two standard ways to do this. They are:

- Breadth first traversal (BFT)
- Depth first traversal (DFT)

The BFT will use a queue as an auxiliary structure to hold nodes for future processing and the DFT will use a STACK.

During the execution of these algorithms, each node N of G will be in one of three states, called the *status* of N, as follows:

1. STATUS = 1 (Ready state): The initial state of the node N.

2. STATUS = 2 (Waiting state): The node N is on the QUEUE or STACK, waiting to be processed.

3. STATUS = 3 (Processed state): The node N has been processed.

Both BFS and DFS impose a tree (the BFS/DFS tree) on the structure of graph. So, we can compute a spanning tree in a graph. The computed spanning tree is not a minimum spanning tree. The spanning trees obtained using depth first search are called depth first spanning trees. The spanning trees obtained using breadth first search are called Breadth first spanning trees.

Breadth first search and traversal:

The general idea behind a breadth first traversal beginning at a starting node A is as follows. First we examine the starting node A. Then we examine all the neighbors of A. Then we examine all the neighbors of neighbors of A. And so on. We need to keep track of the neighbors of a node, and we need to guarantee that no node is processed more than once. This is accomplished by using a QUEUE to hold nodes that are waiting to be processed, and by using a field STATUS that tells us the current status of any node. The spanning trees obtained using BFS are called Breadth first spanning trees.

Breadth first traversal algorithm on graph G is as follows:

This algorithm executes a BFT on graph G beginning at a starting node A.

```
Initialize all nodes to the ready state (STATUS = 1).

1.  Put the starting node A in QUEUE and change its status to the waiting
    state (STATUS = 2).

2.  Repeat the following steps until QUEUE is empty:

    a.  Remove the front node N of QUEUE. Process N and change the
        status of N to the processed state (STATUS = 3).

    b.  Add to the rear of QUEUE all the neighbors of N that are in the
        ready state (STATUS = 1), and change their status to the waiting
        state (STATUS = 2).

3.  Exit.
```

Depth first search and traversal:

Depth first search of undirected graph proceeds as follows: First we examine the starting node V. Next an unvisited vertex 'W' adjacent to 'V' is selected and a depth first search from 'W' is initiated. When a vertex 'U' is reached such that all its adjacent vertices have been visited, we back up to the last vertex visited, which has an unvisited vertex 'W' adjacent to it and initiate a depth first search from W. The search terminates when no unvisited vertex can be reached from any of the visited ones.

This algorithm is similar to the inorder traversal of binary tree. DFT algorithm is similar to BFT except now use a STACK instead of the QUEUE. Again field STATUS is used to tell us the current status of a node.

The algorithm for depth first traversal on a graph G is as follows.

This algorithm executes a DFT on graph G beginning at a starting node A.

4. Initialize all nodes to the ready state (STATUS = 1).

5. Push the starting node A into STACK and change its status to the waiting state (STATUS = 2).

6. Repeat the following steps until STACK is empty:

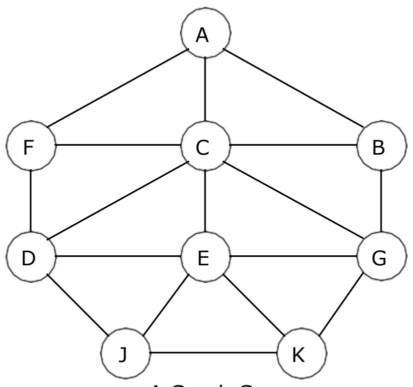
    a. Pop the top node N from STACK. Process N and change the status of N to the processed state (STATUS = 3).

    b. Push all the neighbors of N that are in the ready state (STATUS = 1), and change their status to the waiting state (STATUS = 2).

7. Exit.

Example 1:

Consider the graph shown below. Traverse the graph shown below in breadth first order and depth first order.



A Graph G

Node	***Adjacency List***
A	F, C, B
B	A, C, G
C	A, B, D, E, F, G
D	C, F, E, J
E	C, D, G, J, K
F	A, C, D
G	B, C, E, K
J	D, E, K
K	E, G, J

Adjacency list for graph G

Breadth-first search and traversal:

The steps involved in breadth first traversal are as follows:

Current Node	QUEUE	Processed Nodes	Status								
			A	B	C	D	E	F	G	J	K
			1	1	1	1	1	1	1	1	1
	A		2	1	1	1	1	1	1	1	1
A	F C B	A	3	2	2	1	1	2	1	1	1
F	C B D	A F	3	2	2	2	1	3	1	1	1
C	B D E G	A F C	3	2	3	2	2	3	2	1	1
B	D E G	A F C B	3	3	3	2	2	3	2	1	1
D	E G J	A F C B D	3	3	3	3	2	3	2	2	1
E	G J K	A F C B D E	3	3	3	3	3	3	2	2	2
G	J K	A F C B D E G	3	3	3	3	3	3	3	2	2
J	K	A F C B D E G J	3	3	3	3	3	3	3	3	2
K	EMPTY	A F C B D E G J K	3	3	3	3	3	3	3	3	3

For the above graph the breadth first traversal sequence is: ***A F C B D E G J K***.

Depth-first search and traversal:

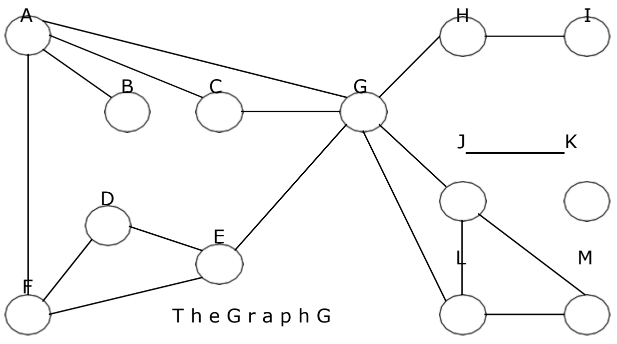
The steps involved in depth first traversal are as follows:

Current Node	Stack	Processed Nodes	Status								
			A	B	C	D	E	F	G	J	K
			1	1	1	1	1	1	1	1	1
	A		2	1	1	1	1	1	1	1	1
A	B C F	A	3	2	2	1	1	2	1	1	1
F	B C D	A F	3	2	2	2	1	3	1	1	1
D	B C E J	A F D	3	2	2	3	2	3	1	2	1
J	B C E K	A F D J	3	2	2	3	2	3	1	3	2
K	B C E G	A F D J K	3	2	2	3	2	3	2	3	3
G	B C E	A F D J K G	3	2	2	3	2	3	3	3	3
E	B C	A F D J K G E	3	2	2	3	3	3	3	3	3
C	B	A F D J K G E C	3	2	3	3	3	3	3	3	3
B	EMPTY	A F D J K G E C B	3	3	3	3	3	3	3	3	3

For the above graph the depth first traversal sequence is: ***A F D J K G E C B***.

Example 2:

Traverse the graph shown below in breadth first order, depth first order and construct the breadth first and depth first spanning trees.

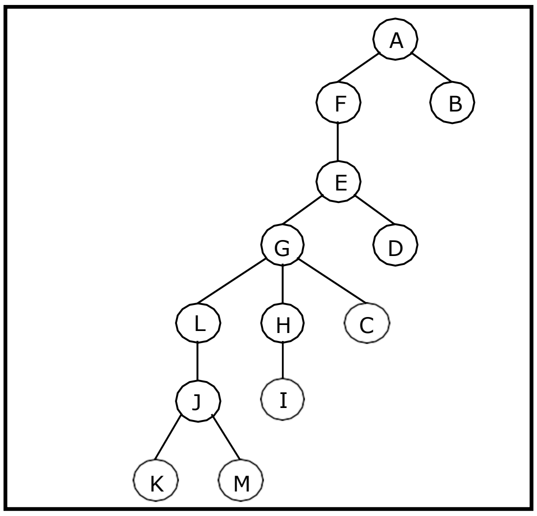


The Graph G

Node	*Adjacency List*
A	F, B, C, G
B	A
C	A, G
D	E, F
E	G, D, F
F	A, E, D
G	A, L, E, H, J, C
H	G, I
I	H
J	G, L, K, M
K	J
L	G, J, M
M	L, J

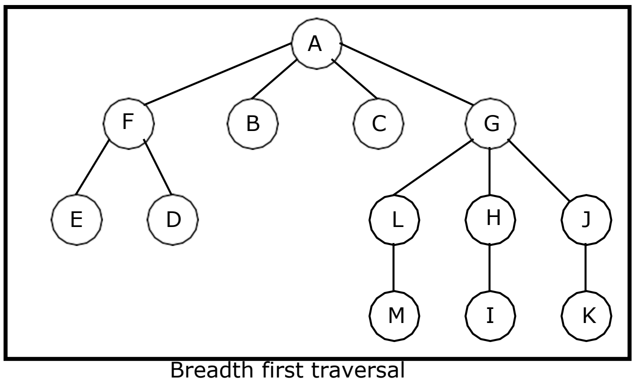
The adjacency list for the graph G

If the depth first traversal is initiated from vertex A, then the vertices of graph G are visited in the order: ***A F E G L J K M H I C D B***. The depth first spanning tree is shown in the figure given below:



Depth first Traversal

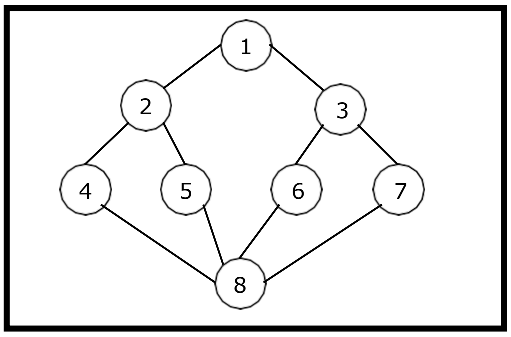
If the breadth first traversal is initiated from vertex A, then the vertices of graph G are visited in the order: **A *F B C G E D L H J M I K***. The breadth first spanning tree is shown in the figure given below:



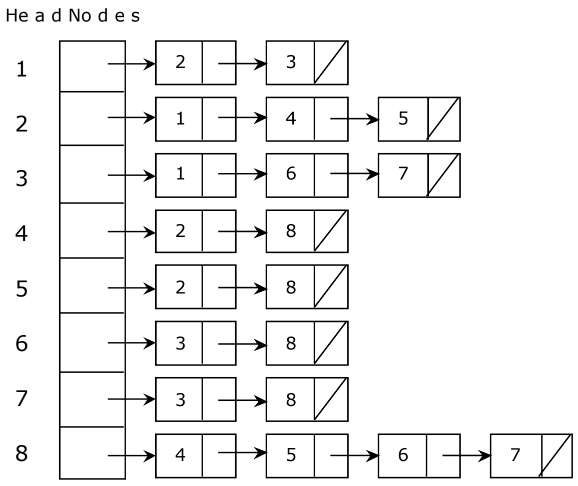
Breadth first traversal

Example 3:

Traverse the graph shown below in breadth first order, depth first order and construct the breadth first and depth first spanning trees.



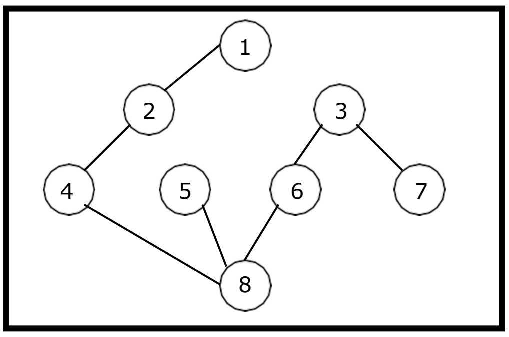
Graph G



Adjacency list for graph G

Depth first search and traversal:

If the depth first is initiated from vertex 1, then the vertices of graph G are visited in the order: 1, 2, 4, 8, 5, 6, 3, 7. The depth first spanning tree is as follows:

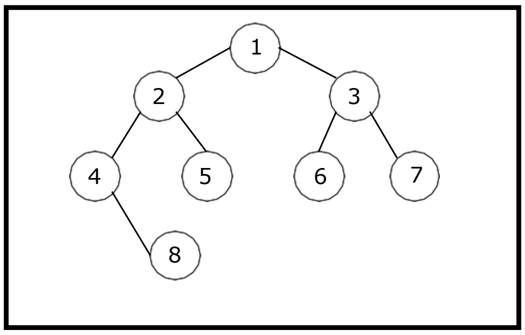


Depth First Spanning Tree

Breadth first search and traversal:

If the breadth first search is initiated from vertex 1, then the vertices of G are visited in the order: 1, 2, 3, 4, 5, 6, 7, 8. The breadth first spanning tree is as follows:

Breadth First Spanning Tree



Dijkstra's Algorithm

Dijkstra's Algorithm solves the single source shortest path (SSSP) problem. The **sssp** is to find the shortest distance from the source vertex to all other vertices in the graph. We can store this in a simple array.

Algorithm basically starts at the node that you choose (the source node) and it analyzes the graph to find the shortest path between that node and all the other nodes in the graph.

The algorithm keeps track of the currently known shortest distance from each node to the source node and it updates these values if it finds a shorter path.

Once the algorithm has found the shortest path between the source node and another node, that node is marked as "visited" and added to the path.

The process continues until all the nodes in the graph have been added to the path. This way, we have a path that connects the source node to all other nodes following the shortest path possible to reach each node.

Psuedo-code

1. Set the distance to the source to 0 and the distance to the remaining vertices to infinity.
2. Set the **current** vertex to the source.
3. Flag the **current** vertex as visited.
4. For all vertices adjacent to the **current** vertex, set the distance from the source to the **adjacent** vertex equal to the minimum of its present distance and the **sum** of the **weight of the edge** from the current vertex to the adjacent vertex and the distance from the source to the **current** vertex.

5. From the set of **unvisited vertices**, arbitrarily set one as the new **current** vertex, provided that there exists an edge to it such that it is the minimum of all edges from a vertex in the set of **visited vertices** to a vertex in the set of **unvisited vertices**. To reiterate: The new current vertex must be unvisited and have a minimum weight edges from a visited vertex to it. This can be done trivially by looping through all visited vertices and all adjacent unvisited vertices to those visited vertices, keeping the vertex with the minimum weight edge connecting it.
6. Repeat steps 3-5 until all vertices are flagged as visited.

Example-

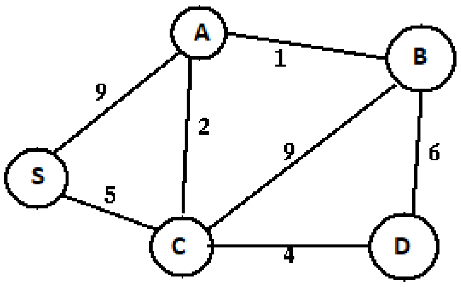


Fig 1: Input Graph (Weighted and Connected)

Given the above weighted and connected graph and source vertex s, following steps are used for finding the tree representing shortest path between s and all other vertices-

Step A- Initialize the distance array (dist) using the following steps of algorithm –

Step 1- Set dist[s]=0, S=ϕ // s is the source vertex and S is a 1-D array having all the visited vertices

Set of visited vertices (S)	S	A	B	C	D
	0	∞	∞	∞	∞

Step 2- For all nodes v except s, set dist[v]= ∞

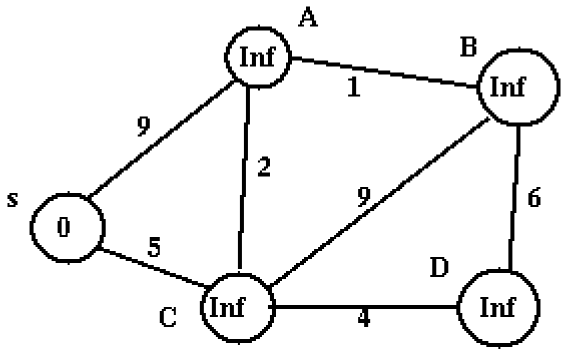


Fig 2: Graph after initializing dist[]

Step B- a)Choose the source vertex s as dist[s] is minimum and s is not in S.

Step 3- find q not in S such that dist[q] is minimum // vertex should not be visited

Visit s by adding it to S

Step 4- add q to S // add vertex q to S since it has now been visited

Step c) For all adjacent vertices of s which have not been visited yet (are not in S) i.e A and C, update the distance array using the following steps of algorithm -

Step 5- update dist[r] for all r adjacent to q such that r is not in S //vertex r should not be visited

Thus dist[] gets updated as follows-

Set of visited vertices (S)	S	A	B	C	D
[s]	0	9	∞	5	∞

Step C- Repeat Step B by

1. Choosing and visiting vertex C since it has not been visited (not in S) and dist[C] is minimum
2. Updating the distance array for adjacent vertices of C i.e. A, B and D

Set of visited vertices (S)	S	A	B	C	D
[s]	0	9	∞	5	∞
[s,C]	0	7	14	5	9

Step 6- Repeat Steps 3 to 5 until all the nodes are in S

This updates dist[] as follows-

Continuing on similar lines, Step B gets repeated till all the vertices are visited (added to S). dist[] also gets updated in every iteration, resulting in the following –

Set of visited vertices (S)	S	A	B	C	D
[s]	0	9	∞	5	∞
[s,C]	0	7	14	5	9
[s, C, A]	0	7	8	5	9

[s, C, A, B]	0	7	8	5	9
[s, C, A, B, D]	0	7	8	5	9

The last updation of dist[] gives the shortest path values from s to all other vertices

The resultant shortest path spanning tree for the given graph is as follows-

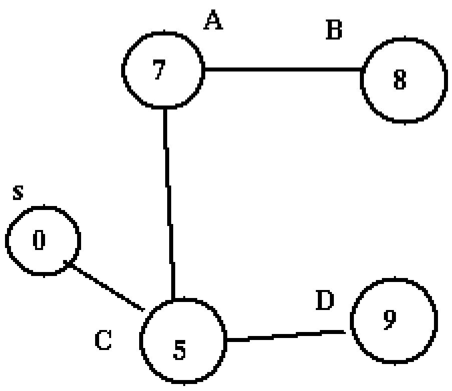
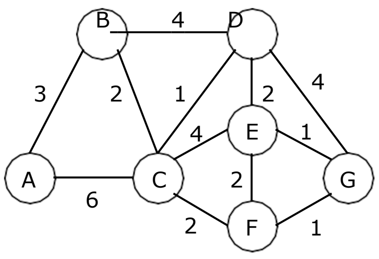


Fig 3: Shortest path spanning tree

Note-

- There can be multiple shortest path spanning trees for the same graph depending on the source vertex

- Example 2:


B
4
D
3
2
1
2
4
4
E
1
A
C
2
G
6
2
F
1