Bode plot

→ It is the plot of $|G(j\omega)H(j\omega)|_{dB}$ and $\angle G(j\omega)H(j\omega)$ as a function of ω

→ The plot of $|G(j\omega)H(j\omega)|_{dB}$ as a function of ω is called the magnitude plot

→ The plot of $\angle G(j\omega)H(j\omega)$ as a function of ω is called the phase plot.

→ Both are plotted on ~~the~~ same semilog sheet

→ The stability of closed loop system is determined by measuring Gain margin (GM) and phase margin (PM) from the plot.

→ The critical point is frequency domain is $1\angle -180°$, which is equivalent to $(-1+j0)$ point or $0\,dB\ \angle -180°$.

→ For example, if the gain of the system is $-40\,dB$ and phase $-180°$ at same 'ω', then there is a margin of $40\,dB$. i.e. If an additional gain of $40\,dB$ is added to the system, then the gain becomes $-40+40=0$ and phase $-180°$ and the system becomes critically stable.

∴ Gain margin is $40\,dB$ in this case.

→ On the other hand, if the gain of the system is $0\,dB$ and phase $-130°$, then the phase margin is $50°$. why?

→ The frequency at which $|G(j\omega)H(j\omega)|_{dB} = 0$ is called 'Gain cross over frequency' denoted by ω_{gc}.

→ The frequency at which $\angle G(j\omega)H(j\omega) = -180^\circ$ is called 'phase cross over frequency' denoted by ω_{pc}.

→ If $\omega_{pc} = \omega_{gc}$ the system is Critically stable.

→ The system is stable for $\omega_{pc} < \omega_{gc}$

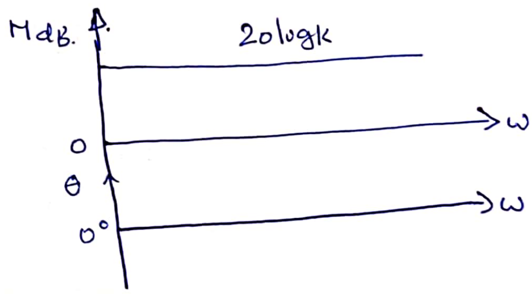
→ The system is unstable for $\omega_{pc} > \omega_{gc}$.

Bode plot for various factors

Constant term K

$$|M(j\omega)|_{dB} = |G(j\omega)H(j\omega)|_{dB} = 20\log K.$$

$$\theta = \angle G(j\omega)H(j\omega) = 0$$



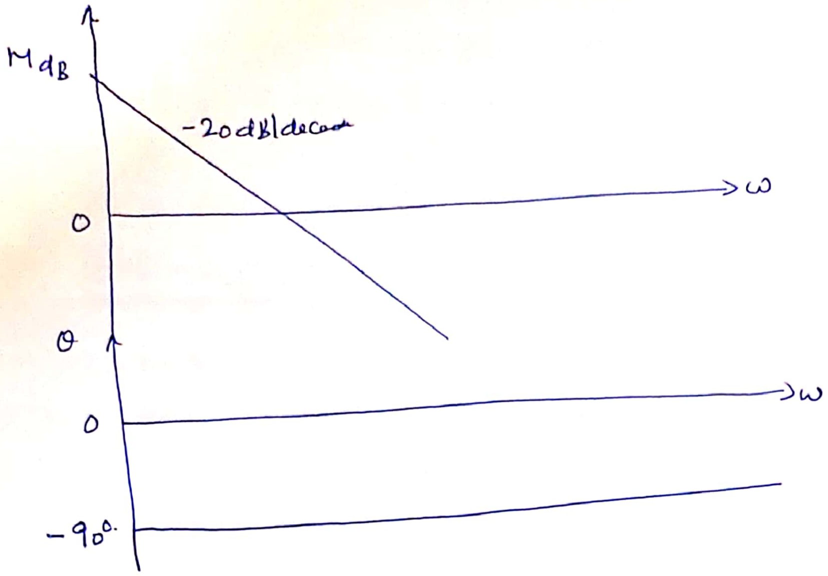
Bode plot of $G(s)H(s) = 1/s$ i.e. pole at $S = 0$

$M(j\omega) = 1/j\omega \Rightarrow |M(j\omega)| = 1/\omega$; $\angle M(j\omega) = \theta = -90^\circ$

$M_{dB} = 20\log(1/\omega) = -20\log\omega$, $\omega > 0$.

ω		MdB	
1	one decade.	0	20dB fall in gain.
10		−20	

Slope of $20\log 1/\omega$ is $-20\,dB/decade$.



Homework

Bode of $G(s)H(s) = s$ Zero at $s = 0$

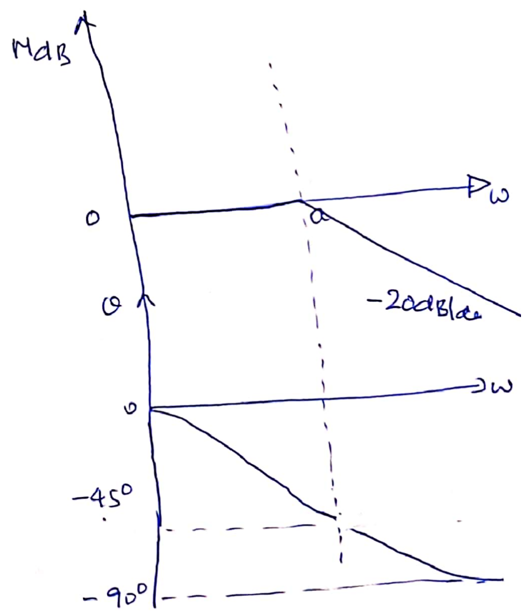
$\theta = +90^\circ$

Slope : $+20$ dB/decade.

Bode plot of $G(s)H(s) = \dfrac{1}{1 + s/a}$ i.e. pole at $s = -a$.

$$M_{dB} = \begin{cases} 0 & \text{for } \omega < a \\ -20\log(\omega/a) & \text{for } \omega > a. \end{cases}$$

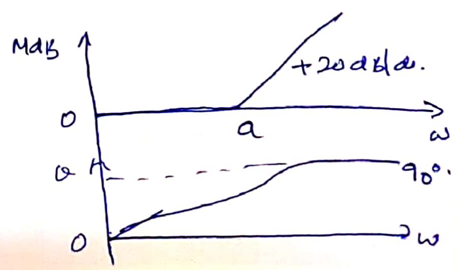
$$\theta = \left\{ -\tan^{-1}(\omega/a) \text{ for all } \omega. \right.$$



Bode plot of $G(s)H(s) = (1 + s/a)$ i.e. zero at $s = -a$.

$$M_{dB} = \begin{cases} 0 & ; \ \omega < a \\ 20\log(\omega/a) & ; \text{ for } \omega > a \end{cases}$$

$\theta = \tan^{-1}(\omega/a)$ for all 'ω'.



Bode plot of Complex poles.

$$G(s)H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{(s/\omega_n)^2 + 2\xi(s/\omega_n) + 1}$$

$$M_{dB} = \begin{cases} 0 & ; \omega < \omega_n \\ -40\log(\omega/\omega_n) & ; \omega > \omega_n. \end{cases}$$

$$\theta = \begin{cases} -\tan^{-1}\left[\dfrac{2\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right] & ; \omega < \omega_n \\ -90^\circ & ; \omega = \omega_n \\ -180 + \tan^{-1}\left[\dfrac{2\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right] & ; \omega > \omega_n. \end{cases}$$

Problem

$$G(s)H(s) = \frac{27(s+2)}{s(s+6)\underbrace{(s^2+4s+9)}_{\text{Complex pole.}}}$$

$$s^2 + 4s + 9 = s^2 + 2\xi\omega_n s + \omega_n^2$$

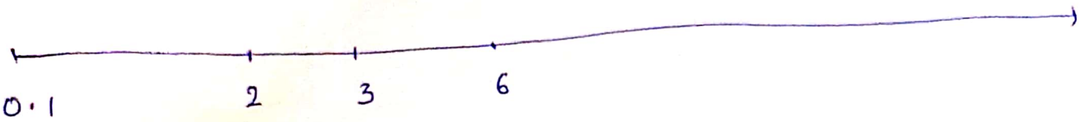
$$\Rightarrow \omega_n = 3\ r/s \qquad \xi = 0.66.$$

$$G(s)H(s) = \frac{27 \cdot 2 \cdot (1 + s/2)}{s \cdot 6 \cdot (1 + s/6) \cdot 9\left((s/3)^2 + \frac{4}{3}(s/3) + 1\right)}.$$

$$G(j\omega)H(j\omega) = M(j\omega) = \frac{1 + j\omega/2}{j\omega(1 + j\omega/6) + \left[(j\omega/3)^2 + 1.33(j\omega/3) + 1\right]}$$

Corner frequencies $\omega = 2\ r/s,\ 3\ r/s\ \&\ 6\ r/s.$

$$M_{dB} = \underbrace{20\log(\omega/2)}_{\omega \geq 2} - \underbrace{20\log(\omega)}_{\omega > 0} - \underbrace{20\log(\omega/6)}_{\omega \geq 6} - \underbrace{40\log(\omega/3)}_{\omega \geq 3.}$$



$0.1 < \omega < 2$

$M_{dB} = -20\log(\omega).$

at $\omega = 0.1$, $M_{dB} =$

at $\omega = 2$ $M_{dB} =$

slope of M_{dB} plot : -20 dB/dec.

$2 < \omega < 3$

$M_{dB} = -20\log\omega - 20\log\omega/2$

$\omega = 3$ $M_{dB} =$

slope : -40 dB/dec

$3 < \omega < 6$

$M_{dB} = -20\log\omega - 20\log\omega/2 - 40\log\omega/3$

$\omega = 6$, $M_{dB} =$

slope : -80 dB/dec.

$\omega > 6$

$M_{dB} = 20\log\omega/2 - 20\log\omega - 20\log(\omega/6) - 40\log(\omega/3).$

$\omega =$ 10 or 100

$M_{dB} =$

slope: -60 dB/decade.

ω	M_{dB}	slope. dB/dec
0.1		-20
2		-40
3		-80
6		-60
10 or 100		

phase angle

$$M(j\omega) = \frac{1 + j\omega/2}{j\omega\,(1 + j\omega/6)\,\left((j\omega/3)^2 + 1.33\left(\frac{j\omega}{3}\right) + 1\right)}$$

$$\theta = \underbrace{\tan^{-1}(\omega/2)}_{\theta_1} \underbrace{- 90}_{\theta_2} - \underbrace{\tan^{-1}(\omega/6)}_{\theta_3} - \underbrace{\tan^{-1}\left[\frac{1.33(\omega/3)}{1 - (\omega/3)^2}\right]}_{\theta_4}$$

$$= \theta_1 + \theta_2 + \theta_3 + \theta_4.$$

$$* \; \theta_4 = \begin{cases} -\tan^{-1}\left[\frac{1.33(\omega/3)}{1-(\omega/3)^2}\right] & ; \; \omega < 3 \\ -90^0 & ; \; \omega = 3 \\ -180 + \tan^{-1}\left[\frac{1.33(\omega/3)}{1-(\omega/3)^2}\right] & ; \; \omega > 3 \end{cases}$$

$\theta_1 = \tan^{-1} \omega/2$

$\theta_2 = -90$

$\theta_3 = -\tan^{-1}(\omega/6)$

$\theta_4 = -\tan^{-1}(\;)$

ω	θ_1	θ_2	θ_3	θ_4	θ
0.1					
2					
3				-90^0	
6					
10 or 100					

problems

Q1: The open Loop transfer function of a closed loop system is given by $G(s)H(s) = \frac{80}{s(s+2)(s+20)}$

Construct the Bode plot and calculate the following

(i) GM (ii) PM (iii) ω_{gc} (iv) ω_{pc}

Is the closed loop system stable.

Soln:

$$G(s)H(s) = \frac{80}{s(s+2)(s+20)} = \frac{80}{s \cdot 2 \cdot (1+s/2) \cdot 20(1+s/20)}$$

$$= \frac{2}{s(1+s/2)(1+s/20)}$$

Corner frequencies $\omega = 2 \cdot 4\ 20$ r/s.

Magnitude plot

$$G(j\omega)H(j\omega) = M(j\omega) = \frac{2}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)} \quad \text{— (1)}$$

$$M_{dB} = \underbrace{20\log 2}_{\text{all } \omega} - \underbrace{20\log(\omega)}_{\omega>0} - \underbrace{20\log(\omega/2)}_{\omega \geqslant 2} - \underbrace{20\log(\omega/20)}_{\omega \geqslant 20}$$

Choose $\omega \approx 0$ as the origin of ω axis. why?

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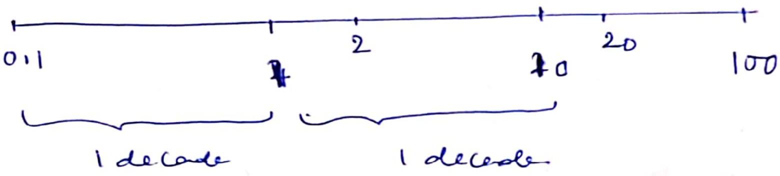
$\omega = 0.1$



For $\underline{0.1 \leqslant \omega \leqslant 2}$ ← First Corner Freq

$M_{dB} = 20\log 2 - 20\log(\omega)$. why?

at $\omega = 0.1$, $M_{dB} =$

at $\omega = 2$, $M_{dB} =$

slope of M_{dB} plot : -20 dB/dec. why?

For $\underline{2 \leq \omega \leq 20}$

$M_{dB} = 20\log 2 - 20\log\omega - 20\log\omega/2$ — why?

at $\omega = 20$, $M_{dB} =$

slope of M_{dB} plot : -40 dB/dec. — why?

For $\underline{\omega > 20}$

$M_{dB} = 20\log 2 - 20\log\omega - 20\log\omega/2 - 20\log\omega/20$ — why?

For $\omega > 20$ say at $\omega = 100$

$M_{dB} =$

slope : -60 dB/decade — why?

Complete the following table

w	MdB	slope
0.1		-20 dB/dec
2		- 40 dB/dec
20		- 60 dB/dec.
100		

phase plot

Consider eqn (1)

$$M(j\omega) = \frac{2}{j\omega\,(1 + j\omega/2)\,(1 + j\omega/20)}$$

$$\theta = \angle 2 - \angle j\omega - \angle 1 + j\omega/2 - \angle 1 + j\omega/20$$

$$\theta = 0 - 90^\circ - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/20) \quad - (2)$$

Complete the following table

ω	θ
0.1	
2	
20	
100	

** Take more points for ω, where θ crosses -180°.