DSP

`IDFT:`

$$IDFT[X(K)] = \frac{1}{N}\sum_{k=0}^{N-1} X(K)W_N^{-kn}$$

`DFT:`

$$DFT[x(n)] = \sum_{n=0}^{N-1} x(n)W_N^{kn}$$

`IDFT using DFT:` DFT of conjugate of the input, and the output is the conjugate divided by N.

$$IDFT = \frac{1}{N}(DFT[X^*(K)])^*$$

Properties

`Circular Time Shift:`

- Use IDFT equation to prove.

$$DFT[x(n-m)_N] = w_N^{mk}X(K)$$

`Conjugate Symmetry:`

- Use DFT equation to prove

$$X(K) = X^*(N-K)$$

`Circular Frequency Shift:`

- Use DFT equation to prove.

$$IDFT[X(K-l)_N] = w_N^{-ln}x(n)$$

`Complex Conjugate:`

- Use DFT equation to prove

$$DFT[x^*(n)] = X^*((-K))_N = X^*(N-K)$$

`Parseval Theorem:`

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(K)|^2$$

`Periodicity:`

$$x(n+N)$$

FFT

Number of multiplications = $\frac{N}{2}log_2N$

Number of additions = $Nlog_2N$

`Composite Radix:`

- 3X2 : then 3 summations going from 0 to 1

$$\sum_{n=0}^{1} x(3n)w_6^{3nk} + \sum_{n=0}^{1} x(3n+1)w_6^{(3n+1)k} + \sum_{n=0}^{1} x(3n+2)w_6^{(3n+2)k}$$

- 2X3: then 2 summations going from 0 to 2

$$\sum_{n=0}^{2} x(2n)w_6^{2nk} + \sum_{n=0}^{2} x(2n+1)w_6^{(2n+1)k}$$

Butterworth Filter

1. $LP \rightarrow LP$

$$s \rightarrow \frac{\Omega_p}{\Omega_{LP}} s$$

2. $LP \rightarrow HP$

$$s \rightarrow \frac{\Omega_P \Omega_{HP}}{s}$$

3. $LP \rightarrow BP$

$$s \rightarrow \Omega_p \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$$

4. $LP \rightarrow BS$

$$s \rightarrow \Omega_p \frac{(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u}$$

`NOTE:` Centre frequency $\Omega_o = \sqrt{\Omega_l \Omega_u}$

$$|H(\Omega)|^2 = \frac{1}{1 + (\frac{\Omega}{\Omega_c})^{2N}}$$

$$P_k = \pm\, e^{j(2k+1+N)\pi/2N}$$

$$N \geq \frac{log[\frac{\frac{1}{A_s^2}-1}{\frac{1}{A_p^2}-1}]}{2log(\frac{\Omega_s}{\Omega_p})} or \frac{log[\frac{10^{0.1A_s db}-1}{10^{0.1A_p db}-1}]}{2log(\frac{\Omega_s}{\Omega_p})}$$

`IIT:`

$$\Omega = \frac{\omega}{T}$$

$$\frac{1}{S - P_k} \rightarrow \frac{1}{1 - e^{P_k T} Z^{-1}}$$

$$\frac{b}{(s+a)^2 + b^2} \rightarrow \frac{e^{-aT} sinbT\ Z^{-1}}{1 - e^{-aT} cosbT\ Z^{-1} + e^{-2aT}\ Z^{-2}}$$

`BLT:`

$$\Omega = \frac{2}{T} tan(\frac{\omega}{2})$$

$$s \rightarrow \frac{2}{T}(\frac{1-Z^{-1}}{1+Z^{-1}})$$

Chebysev Filters

$$\Omega'_p = \frac{\Omega_p}{\Omega_p}$$

$$\Omega'_s = \frac{\Omega_s}{\Omega_p}$$

$$\epsilon = \sqrt{\frac{1}{A_p^2} - 1} = \sqrt{10^{0.1 A_p db} - 1}$$

$$|H(\Omega)|_{in\ db} = -20 log(\epsilon) - 6(N-1) - 20\ N\ log\ \Omega'_s$$

$$S_k = \sigma_k + j\Omega_k$$

$$\sigma_k = -sinh[\frac{1}{N} sinh^{-1}(\frac{1}{\epsilon})] sin(\frac{2K-1}{2N})\pi$$

$$\Omega_k = cosh[\frac{1}{N} sinh^{-1}(\frac{1}{\epsilon})] cos(\frac{2K-1}{2N})\pi$$

$$k = b_o\ if\ N\ odd$$
$$\frac{b_o}{\sqrt{1+\epsilon^2}}\ if\ N\ even$$

Windowing techniques

$$h_d(n) = \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{jw(n-\tau)}$$

- `Hanning:` **-44 db**

$$0.5 - 0.5\ cos(\frac{2\pi n}{N-1})$$

- `Hamming;` **-53 db**

$$0.54 - 0.46\ cos(\frac{2\pi n}{N-1})$$

- `Rectangular:` **-21 db**

$$1$$

`Frequency Sampling:`

$$H_d(\omega) = e^{-j\omega(\frac{N-1}{2})};\ |\omega| \leq\ \omega_c$$
$$To\ sample,\ \ \omega = \frac{2\pi k}{N}$$

$$h(n) = \frac{1}{N}[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re(H(k)e^{j\frac{2\pi kn}{N}})]$$

Block Diagrams

$Direct\ form\ I \rightarrow Lattice:$

$$K_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1 - K_m^2}$$

$Lattice\ \rightarrow\ Direct\ form\ I:$

$$a_m(0) = 1$$
$$a_m(m) = K_m$$
$$for\ m = 1, 2, ..., M$$

$$a_m(i) = a_{m-1}(i) + a_m(m)a_{m-1}(m-i)$$