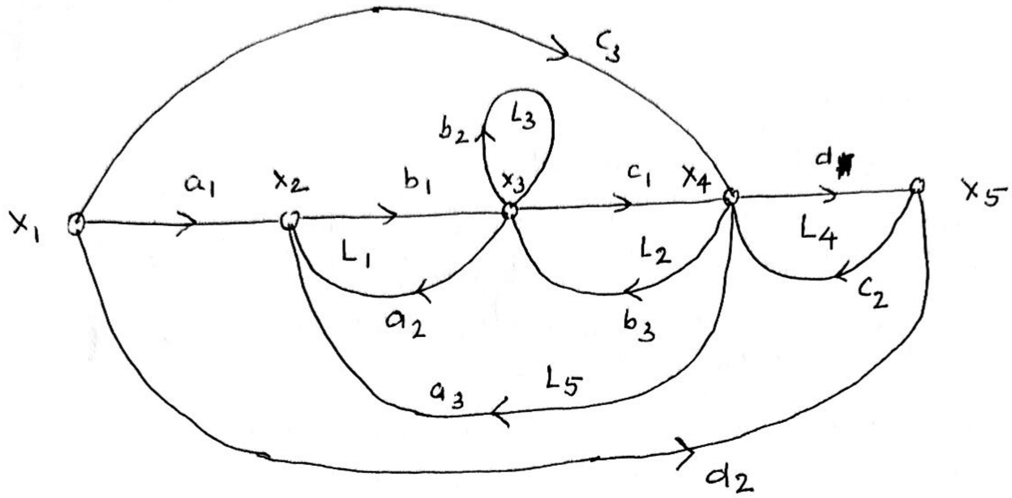



| Forward paths | Gain |
|---|---|
| 1. $x_1 - x_2 - x_3 - x_4 - x_5$ | $M_1 = a_1 b_1 c_1 d_1$ |
| 2. $x_1 - x_4 - x_5$ | $M_2 = c_3 d_1$ |
| 3. $x_1 - x_5$ | $M_3 = d_2$ |

| Loops | Gain |
|---|---|
| 1. $x_2 - x_3 - x_2$ | $L_1 = b_1 a_2$ |
| 2. $x_3 - x_4 - x_3$ | $L_2 = c_1 b_3$ |
| 3. $x_3 - x_3$ | $L_3 = b_2$ |
| 4. $x_4 - x_5 - x_4$ | $L_4 = d_1 c_2$ |
| 5. $x_2 - x_3 - x_4 - x_2$ | $L_5 = b_1 c_1 a_3$. |

| Two Non touch loops | Gain product |
|---|---|
| 1. Loop1 and Loop4 | $L_1 L_4 = a_2 b_1 c_2 d_1$ |
| 2. Loop 3 and Loop4 | $L_3 L_4 = a_3 b_1 b_2 c_1$ |

No Combinations of three non touching loops, four non touching loops etc.

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1 L_4 + L_3 C_4]$$

$$=$$

$$\frac{X_5}{X_1} = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3}{\Delta} \quad \text{— } (1)$$

To find $\Delta_1$

$\Delta_1 = 1 -$ [Sum of loop gains which are non touching with forward path 1] + [Sum of gain product of two non touching loops which are non touching the forward path 1] — + ...

All Loops are in forward path 1. $\Rightarrow \Delta_1 = 1$

To Find $\Delta_2$

Loop 1 and Loop 3 are not in forward path 2 and both are touching loops.

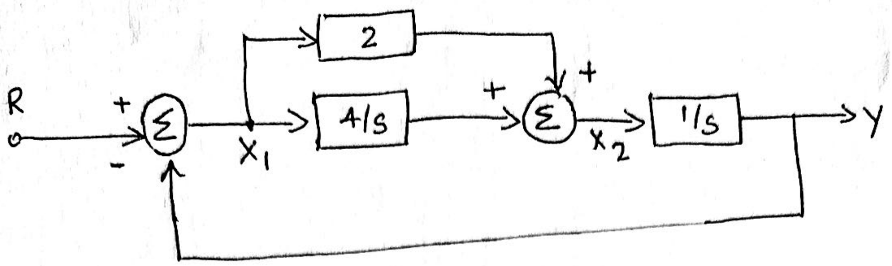
$\Delta_2 = 1 - [L_1 + L_3] =$

To Find $\Delta_3$

Loops 1, 2 and 3, 5 are not in forward path 3 and all four are touching loops.

$\Delta_3 = 1 - [L_1 + L_2 + L_3 + L_5] =$

Substitute in eq. (1) and simplify.




- o/p of summer is a new variable
- Signal at pick-off point is a new variable

<u>Equations</u>

$X_1 = R - Y$

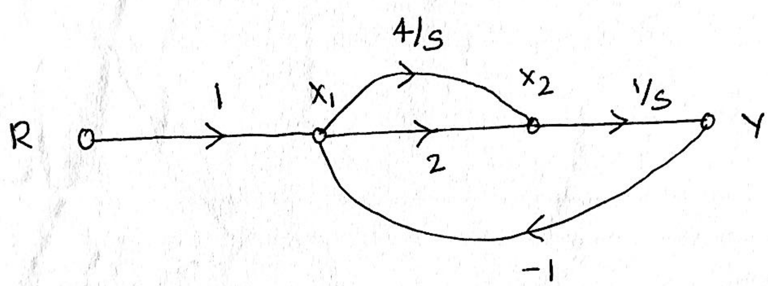
$X_2 = 2X_1 + \frac{4}{s} X_1 = [2 + \frac{4}{s}] X_1$

$Y = 1/s \; X_2$

<u>No of Variables</u>

R, $X_1$, $X_2$ and Y

Four ⇒ 4 nodes




<u>Forward paths</u>

$R - X_1 - X_2 - Y$. Gain $M_1 = (2 + 4/s)(1/s)$

$= \frac{2s+4}{s^2}$

<u>Loops</u>

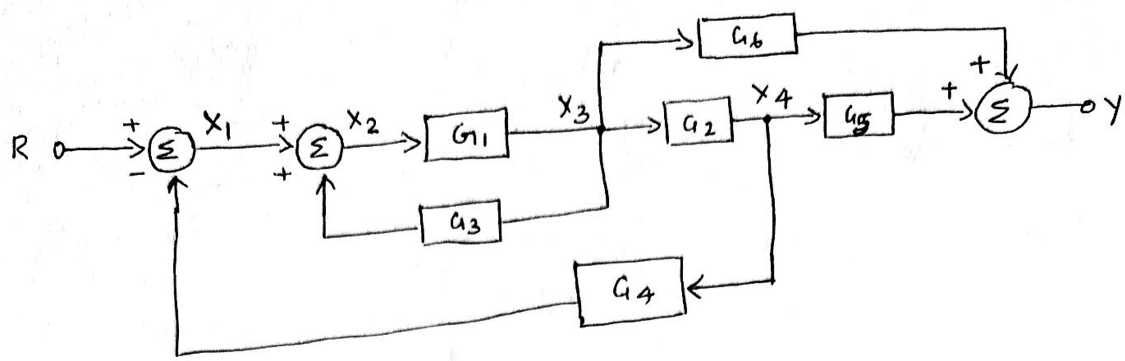
$X_1 - X_2 - Y - X_1$ $L_1 = -[2 + 4/s](1/s)$

$$\frac{Y}{R} = \frac{M_1 \Delta_1}{\Delta}$$

$$\Delta = 1 - L_1 = 1 + (2 + 4/s)(1/s) = \frac{s^2 + 2s + 4}{s^2}$$

$\Delta_1 = 1$ No non touching loops with the forward path

$$\frac{Y}{R} = \frac{2s+4}{s^2} \cdot \frac{s^2}{s^2+2s+4} = \frac{2s+4}{s^2+2s+4}$$




<u>Equations</u>

$X_1 = R - G_4 X_4$

$X_2 = X_1 + G_3 X_3$

$X_3 = G_1 X_2$

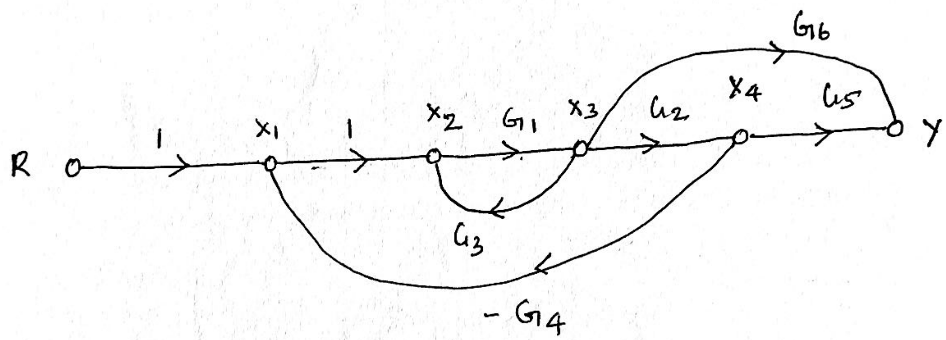
$X_4 = G_2 X_4$

$Y = G_5 X_4 + G_6 X_3$

<u>Variables</u>

$R, X_1, X_2, X_3, X_4, Y$

$\Rightarrow$ Six



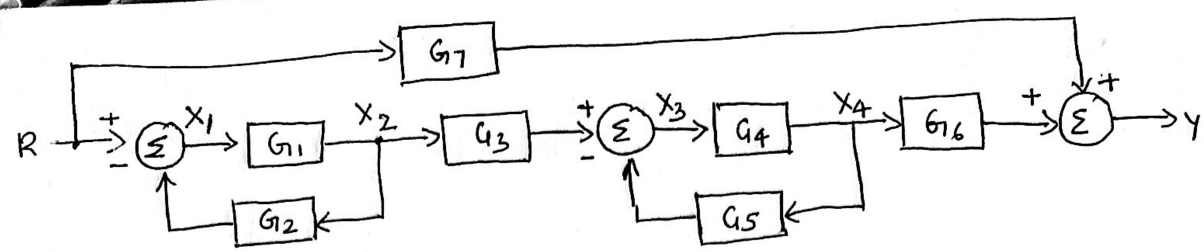

| <u>Forward paths</u> | <u>Gain</u> |
|---|---|
| 1. $R - X_1 - X_2 - X_3 - X_4 - Y$ | $M_1 = G_1 G_2 G_5$ |
| 2. $R - X_1 - X_2 - X_3 - Y$ | $M_2 = G_1 G_6$ |

| <u>Loops</u> | <u>Gain</u> |
|---|---|
| 1. $X_2 - X_3 - X_2$ | $L_1 = G_1 G_3$ |
| 2. $X_1 - X_2 - X_3 - X_4 - X_1$ | $L_2 = -G_1 G_2 G_4$ |

Both loops are touching

$$\frac{Y}{R} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta} \qquad \Delta = 1 - (L_1 + L_2) =$$

$\Delta_1 = 1 \qquad \Delta_2 = 1 \qquad \because$ Both forward paths are touching the loops.




**Equations**

$X_1 = R - G_2 X_2$

$X_2 = G_1 X_1$

$X_3 = G_3 X_2 - G_5 X_4$ ; $X_4 = G_4 X_3$

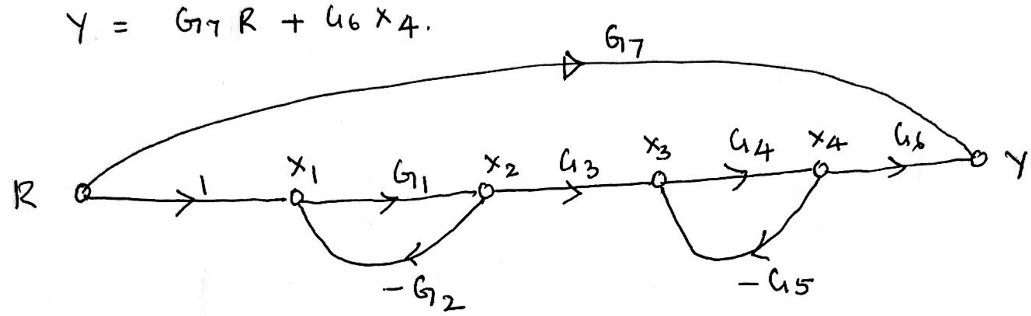
$Y = G_7 R + G_6 X_4$.

**Variables**

$R, X_1, X_2, X_3, X_4$ and $Y$

$\Rightarrow$ Six Variables

$\Rightarrow$ Six nodes




**Forward paths**

1. $R - X_1 - X_2 - X_3 - X_4 - Y$
2. $R - Y$

**Gain**

$M_1 = G_1 G_3 G_4 G_6$

$M_2 = G_7$

**Loops**

1. $X_1 - X_2 - X_1$
2. $X_3 - X_4 - X_3$

**Gain**

$L_1 = -G_1 G_2$

$L_2 = -G_4 G_5$

Loop 1 and Loop 2 are non touching $\Rightarrow L_1 L_2 = G_1 G_2 G_4 G_5$

$\Delta = 1 - [L_1 + L_2] + L_1 L_2 = 1 + G_1 G_2 + G_4 G_5 + G_1 G_2 G_4 G_5$

Loop 1 and Loop 2 are touching the Ist forward path

$\therefore \Delta_1 = 1$

Loop 1 and Loop 2 are not in the 2nd forward path.

$\Delta_2 = 1 - (L_1 + L_2) + L_1 L_2 = \Delta$.

$$\frac{Y}{R} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

# First order System

$$\frac{Y(s)}{R(s)} = \frac{1}{s+\sigma}$$

pole : $s + \sigma = 0 \Rightarrow \boxed{s = -\sigma}$

$Y(s)[s+\sigma] = R(s)$ Taking inverse LT

$$\boxed{\frac{dy(t)}{dt} + \sigma\, y(t) = r(t)}$$

A first order system has one pole and is described by first order differential equation.

## Impulse response of first order system

$Y(s) = \dfrac{R(s)}{s+\sigma}$. $\quad r(t) = \delta(t)$

$\Rightarrow R(s) = 1$

$Y(s) = \dfrac{1}{s+\sigma}$.

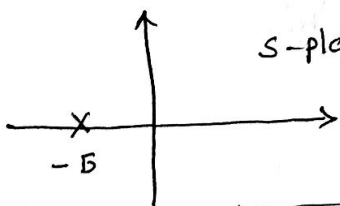
$y(t) = h(t) = e^{-\sigma t}, \; t \geq 0$

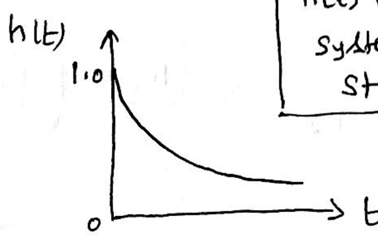
$h(t)$ is impulse response of the system.

**Case I**

$\sigma > 0$

Pole: $s = -\sigma \Rightarrow$ Real & $-$ve

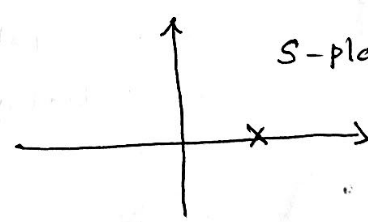


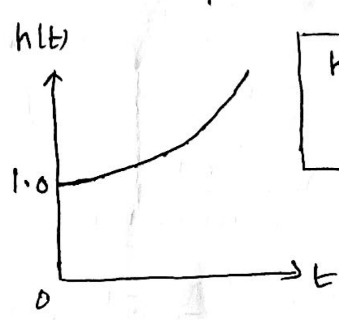




h(t) is bounded
system is Stable

**Case II** $\quad \sigma < 0$

pole: $s = -\sigma \Rightarrow$ Real and +ve

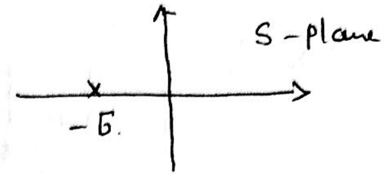



h(t) is unbounded
System is Stable

## Step Response of Stable first order System

$$\frac{Y(s)}{R(s)} = \frac{1}{s+B}$$



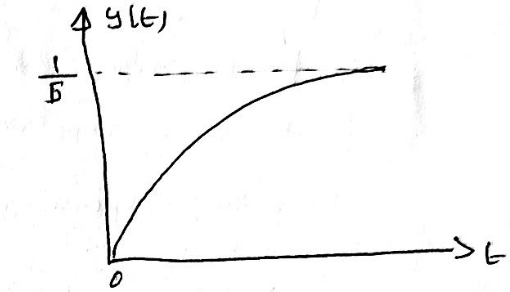

$r(t) = u(t) \Rightarrow R(s) = 1/s.$

$$Y(s) = \frac{1}{s(s+B)} = \frac{A}{s} + \frac{B}{s+B} \qquad A = 1/B \qquad B = -1/B$$

$$Y(s) = \frac{1}{B}\left[1/s - \frac{1}{s+B}\right] \Rightarrow y(t) = \frac{1}{B}\left[1 - e^{-Bt}\right];\ t \geq 0$$

$$y(t)\big|_{t=0} = 1/B\,[1-1] = 0$$

$$y(t)\big|_{t=\infty} = 1/B\,[1-e^{-\infty}] = 1/B.$$




## Time Constant

$$\frac{1}{s+B} \xrightarrow{LT^{-1}} e^{-Bt} = e^{-t/\tau} \Rightarrow \tau = 1/B = \text{Time Constant of the system.}$$

| |
|---|
| Smaller time constant (pole far away from origin) $\Rightarrow$ Faster decay. |

## Second order System

$$\frac{C(s)}{R(s)} = \frac{2s+1}{s^2+3s+2} = \frac{2s+1}{(s+1)(s+2)}$$

| |
|---|
| A second order system has two poles and is described by 2nd order differential equation. |

## Impulse response

$R(s) = 1$

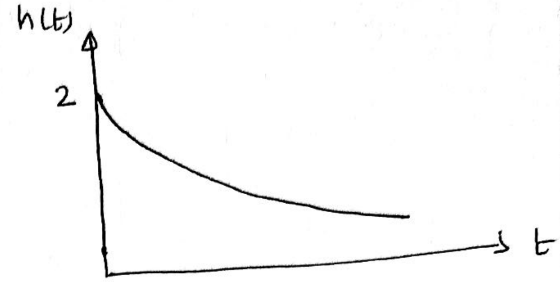
$$C(s) = \frac{2s+1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \qquad \boxed{A = -1} \quad \boxed{B = 3}$$

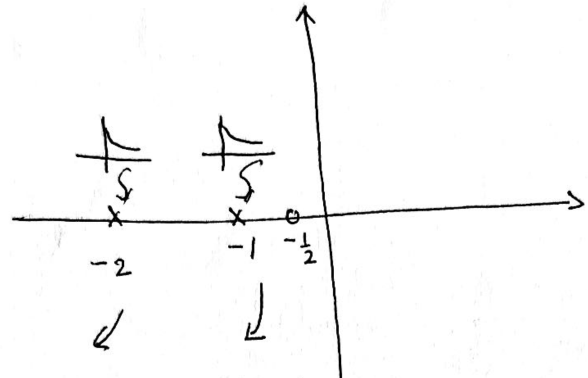
$$c(t) = -e^{-t} + 3e^{-2t};\quad t \geq 0$$

$$h(t) = -e^{-t} + 3e^{-2t} \;;\; t \geq 0$$

$$h(t)\big|_{t=0} = -1 + 3 = 2$$

$$h(t)\big|_{t=\infty} = 0$$




$\tau_2 = 1/2$ $\quad$ $\tau_1 = 1$

$= 0.5$

$\tau_2 < \tau_1$

$e^{-2t}$ decays faster than $e^{-t}$.

## Step response

$$C(s) = \frac{2s+1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$\boxed{A = 1/2}$ $\quad$ $\boxed{B = 1}$ $\quad$ $\boxed{C = 3/2}$

$$C(s) = \frac{1/2}{s} + \frac{1}{s+1} + \frac{3/2}{s+2}$$

$$c(t) = \frac{1}{2} + e^{-t} + \frac{3}{2}e^{-2t} \;;\; t \geq 0$$

$$c(t)\big|_{t=0} = 1/2 + 1 + 3/2 = 2$$

$$c(t)\big|_{t=\infty} = 1/2$$