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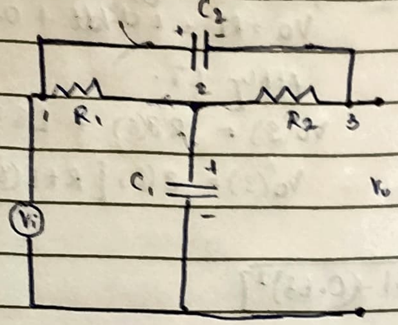
4/4/23

# 20EC530
## Control Systems

5/4/23 Equation for bridge T circuits

Determine the TF for the circuit shown:




Apply KCL

$V_1 = V_i$ ; $\frac{V_2-V_1}{R_1} + \frac{V_2-V_3}{R_2} + C_1\frac{dV_2}{dt}$ → ②

→ ①

at node 3.

③ ← $\frac{V_2-V_3}{R_2} - C_2\frac{d(V_1-V_3)}{dt} = 0$.

Capacitor vtgs : $V_{c1}, V_{c2}$ → $V_1 = V_i$ $V_{c1} = V_2$ ; $V_1 - V_3 = V_{c2}$

$V_3 = V_1 - V_{c2}$

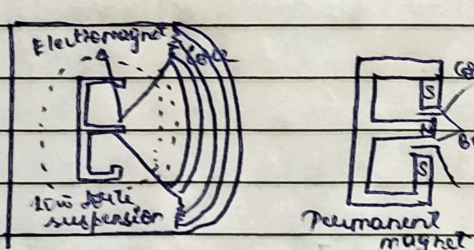
Eqn ② → $\frac{V_{c1} - V_i}{R_1} + \frac{-V_i + V_{c2} + V_{c1}}{R_2} + C_1\frac{d(V_{c1})}{dt}$ :

$$\frac{d(V_{c1})}{dt} = -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_{c1} - \frac{1}{C_1}\left(\frac{1}{R_2}\right)V_{c2} + \frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_i = 0$$

Eqn ③

$$\frac{d}{dt}(V_{c2}) = -\frac{V_{c1}}{C_2R_2} - \frac{V_{c2}}{C_2R_2} + \frac{V_i}{C_2R_2}$$

Loud speaker:




The permanent magnet establishes a radial field in the cyl. gap b/w the poles of the magnet. The force on the conductor wound on the bobbin causes the voice coil to move, producing sound.

The effect of air can be modeled as if the cone had an equivalent mass 'M' & viscous ~~force~~ friction 'v'. The force is given by:

$F = Bil$ Newton

$l = N\pi d$ → diameter of bobbin

No. of turns

Assume $N = 20$, $d = 2cm$, $B = 0.5T$

$F = Bil$ ← $l = 20\times\pi\times2\times10^{-2}$

$= 0.628i$

$F = M\frac{d^2(x)}{dt^2} + b\frac{dx}{dt}$

$0.628i = M\frac{d^2x}{dt^2} + b\frac{dx}{dt}$

$0.628 I(s) = X(s)[Ms^2 + bs]$

$\frac{X(s)}{I(s)} = \frac{0.628}{Ms^2+bs} = \frac{\frac{0.628}{M}}{s^2+\frac{b}{M}s}$

$= \frac{\frac{0.628}{M}}{s(s+\frac{b}{M})}$

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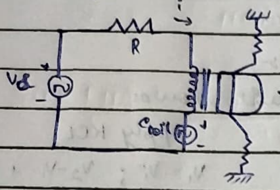
## Loud speaker with Circuit:-




$e_{coil} = Blv = Bl\frac{dx}{dt} = 0.628\frac{dx}{dt}$

Apply KVL:

$V_a = Ri + L\frac{di}{dt} + e_{coil}$

$V_a = Ri + L\,di/dt + 0.63\,dx/dt$

Apply LT.

$V_a(s) = RI(s) + LsI(s) + 0.63s\,X(s)$

$V_a(s) = I(s)[R + L(s)] + 0.63s\,X(s)$

$$\frac{X(s)}{V_a(s)} = \frac{0.63}{s[(Ms+b)(R+Ls) + (0.63)^2]}$$

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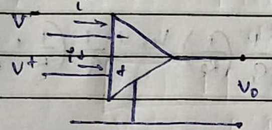
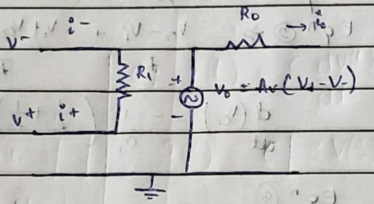
## Modelling of Circuits:

Op - Amp :-

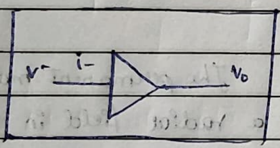



⇒




$V^+ \rightarrow$ Connected to gnd

⇓

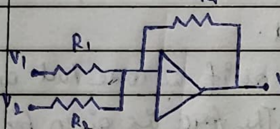



ideal: $R_i = \infty$ | $I = i_+ = 0 \rightarrow (1)$

$R_0 = 0$ | $V_+ = V_- = 0 \rightarrow (2)$

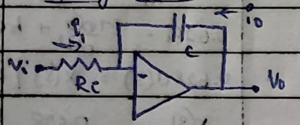
$A_v = \infty$

Summer:




$V_0 = -\left[\frac{R_f V_1}{R_1} + \frac{R_f V_2}{R_2}\right]$

Integrator:-




$V_0 = -\frac{1}{RC}\int V_i\,dt$

$i_1 + i_0 = 0$

$\frac{V_i}{R_i} + C\frac{dV_0}{dt} = 0$

$V_0 = -\frac{1}{R_iC}\int_0^t V_i(\tau)\,d\tau + V_0(0)$

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Replacing $\frac{d}{dt} = s$ ; $\frac{V_0(s)}{V_i(s)} = -\frac{1}{s}\cdot\frac{1}{RiC}$

* Standard form
* Properties of Laplace Transform
* ILT
* Differential form
* Fourier Series
* Properties.

Course Outcomes:-

CO1: ~~Displ~~ Explain the physical systems as

CO2: Analyze various properties of control sys. in time domain & freq. domain using appropriate tools.

CO3: Design & test controllers for transfer fn & state space models.

CO4: Evaluate the state space models of systems using appropriate tools

CO5: Demonstrate the performance of controllers using modern tools.

Textbook:

1. G.F. Franklin, GD Powell, A.E. "Feedback Control of Dynamic systems". Pearson Ed. 5th Edition, 2002
2. M. Gopal, "Control systems: Principles & Design, Tata McGraw Hill

## Laplace Transforms:

$f(t) \longrightarrow F(s)$

$s = \sigma + j\omega$

$$L\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

| | | | |
|---|---|---|---|
| $\delta(t)$ | 1 | • $t^n$ | $\frac{n!}{s^{n+1}}$ |
| • $u(t)$ | $\frac{1}{s}$ | | |
| • $e^{at}$ | $\frac{1}{s-a}$ | • $te^{at}$ | $\frac{1}{(s-a)^2}$ |
| • $e^{-at}$ | $\frac{1}{s+a}$ | • $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ |
| • $t$ | $1/s^2$ | • $t^n f(t)$ | $(-1)^n \frac{d^n}{ds^n}F(s)$ |

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- $\sin\omega(t) = \dfrac{\omega}{s^2+\omega^2}$ ; $\cos\omega t \;\; \dfrac{s}{s^2+\omega^2}$

- $\sin(\omega t+\theta) \;\; \dfrac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$ ; $\cos(\omega t-\theta) \;\; \dfrac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$

- $e^{-at}\sin\omega t = \dfrac{\omega}{(s+a)^2+\omega^2}$ ; $e^{-at}\cos\omega t \;\; \dfrac{s}{(s+a)^2+\omega^2}$

RLC:-

$V(t) = R\,i(t)$

$V(s) = R\,I(s)$

L $\quad V(t) = L\dfrac{di}{dt}$

$\boxed{V(s) = L[sI(s) - i(0^-)]}$

$V(s) = LsI(s) - Li(0^-)$

$\boxed{I(s) = \dfrac{V(s) + i(0^-)}{Ls}}$

C: $\quad i(t) = C\dfrac{dV(t)}{dt}$

$I(s) = C[sV(s) - V(0^-)]$

$V(s) = \dfrac{I(s)}{Cs} + V(0^-)$

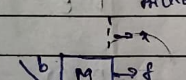
11/9/23 Types of Control systems:

1. Electrical systems $\rightarrow$ R, L, C, active elements
2. Mechanical systems $\rightarrow$ Mass, spring, friction
3. Electromechanical systems. $\rightarrow$ Combo of elec. & mechanical
4. Pneumatic systems.

Objective of mechanical sys $\rightarrow$ Motion

along a straight line $\downarrow$ Translational mech sys — Mass, spring, friction

about an axis $\downarrow$ Rotational mech sys. — Moment of Inertia, spring action, Friction



$x \rightarrow$ displacement

$\dfrac{dx}{dt} \rightarrow$ velocity

$\dfrac{d^2x}{dt^2} \rightarrow$ acceleration





mass : force $\propto$ acceleration

$f = m\dfrac{d^2x}{dt^2}$

Friction: $f \propto$ velocity

$f = b\dfrac{dx}{dt}$ — b: Friction coeff

Spring: $f \propto x$

$f = kx$

Applied force should overcome: Inertia $\rightarrow m\dfrac{d^2x}{dt^2}$ ; Friction $\rightarrow b\dfrac{dx}{dt}$

$f = M\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt}$

$f(t) \longrightarrow \boxed{\text{Mech sys}} \longrightarrow x(t)$

$F(s) \qquad\qquad\qquad X(s)$

$T\{F(s)\} = X(s)$

Transfer fn: $\dfrac{X(s)}{F(s)}$

- Transfer fn is applicable for LTI (Linear Time invariant) systems.
- Starts from rest.
- No initial condition

$\mathcal{L}\left\{\dfrac{dx(t)}{dt}\right\} = sX(s) - \cancel{x(0)}^{\,0} = sX(s)$

$\mathcal{L}\left\{\dfrac{d^2x(t)}{dt^2}\right\} = s^2X(s)$

$\mathcal{L}\left\{\dfrac{d^nx(t)}{dt^n}\right\} = s^nX(s)$

$f(t) = M\dfrac{d^2x(t)}{dt^2} + b\dfrac{d}{dt}x(t) \rightarrow$ Performance eqn always – Time domain

Integro-differential eqn

$\Downarrow$ LT

$F(s) = Ms^2X(s) + bsX(s) = X(s)[Ms^2+bs]$

$\dfrac{X(s)}{F(s)} = \dfrac{1}{Ms^2+bs} \cdot \dfrac{1/M}{1/M} = \dfrac{1/M}{s^2+\frac{b}{M}s} = \dfrac{1/M}{s[s+\frac{b}{M}]} = \dfrac{A}{s} + \dfrac{B}{s+\frac{b}{M}}$

$f(t) = \delta(t)$

$F(s) = 1$

$x(t) = A + B e^{-(b/M)t}$

$= A + B e^{-t/\tau}$ $\quad \because \tau = M/b$

$V_c(t) = V[1 - e^{-t/RC}]$

$= V[1 - e^{-t/\tau}]$

$\tau = RC =$ time constant [electrical]

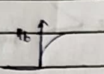
$1/M = A(s + b/M) + B(s)$

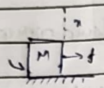
$s = 0 \quad 1/M = Ab/M \rightarrow A = 1/b$

$s = -b/M \quad 1/M = 0 - \frac{bB}{M} \Rightarrow B = -1/b$

$\therefore x(s) = \frac{1}{b}\left[\frac{1}{s} - \frac{1}{s+b/M}\right]$

$x(t) = 1/b\left[1 - e^{-(b/M)t}\right]$

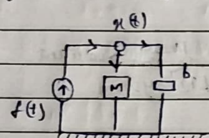




$\delta(t) \longrightarrow \boxed{M, b} \longrightarrow x(t)$

Mechanical equivalent circuit:

No. of nodes = no. of displacements




$f(t) = M\frac{d^2x(t)}{dt^2} + b\frac{dx}{dt}$

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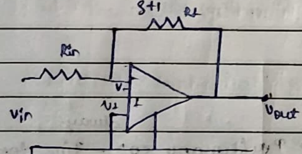
1. First step follower realistic model of op-amp is given by following eqn & it is shown in fig 1.

$V_{out} = \frac{10^7}{s+1}[V_+ - V_-] \qquad i_+ = i_- = 0$

Find the transfer fn of the simple amplification circuit shown using this model.




Apply KCL:

$\frac{V_{in} - V_-}{R_{in}} + \frac{V_{out} - V_-}{R_f} = 0.$

$V_- = \frac{R_f}{R_{in}+R_f} V_{in} + \frac{R_{in}}{R_{in}+R_f} V_{out}$

$V_{out} = \frac{10^7}{s+1}\left[0 - \left(\frac{R_f}{R_{in}+R_f} V_{in} + \frac{R_{in}}{R_{in}+R_f} V_{out}\right)\right]$

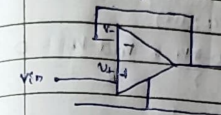




$\frac{10^7 V_{out}}{s+1}\left(1 + \frac{R_{in}}{R_{in}+R_f}\right) = \frac{-10^7}{s+1}\frac{R_f}{R_{in}+R_f} V_{in}.$

$\frac{V_{out}}{V_{in}} = \frac{-10^7 \frac{R_f}{R_{in}+R_f}}{s+1+10^7\frac{R_{in}}{R_{in}+R_f}}$

2. ST op-amp connection shown in fig 2 results in Vout = Vin. If the op amp is ideal. Give the tr. fn if the op-amp has non-ideal tr. fn of problem 1.




ideal

$V_{in} = V_+$

$V_{out} = V_-$

$V_+ = V_- \rightarrow$ ideal

$V_+ \neq V_- \rightarrow$ non-ideal

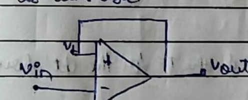
$V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$

$= \frac{10^7}{s+1}[V_{in} - V_{out}]$

$V_{out}\left(1 + \frac{10^7}{s+1}\right) = \frac{10^7}{s+1} V_{in}$

$\frac{V_{out}}{V_{in}} = \frac{\frac{10^7}{s+1}}{1+\frac{10^7}{s+1}} = \frac{10^7}{s+1+10^7} \approx \frac{10^7}{s+10^7}$

3. ST the non-ideal tr fn of problem ① op amp shown in fig. 3 is unstable.




$V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$

$V_+ = V_{out} \; ; \; V_- = V_{in}$

$V_{out} = \frac{10^7}{s+1}[V_{out} - V_{in}]$

$V_{out}\left(1 - \frac{10^7}{s+1}\right) = -\frac{10^7}{s+1} V_{in}$

$\frac{V_{out}}{V_{in}} = \frac{\frac{-10^7}{s+1}}{1 - \frac{10^7}{s+1}} = \frac{-10^7}{s+1-10^7} \approx \frac{-10^7}{s-10^7}$

NOTE: Tr fn has a dr with $s - 10^7$ & the -ve sign means exp time fn is increasing, which means that it an unstable one.

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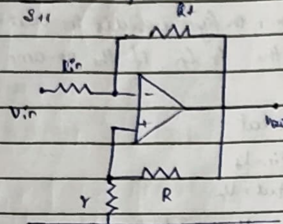
4. An op-amp connection with f/b to both -ve & +ve terminals is shown in fig 4. If the op-amp has non-ideal dc. In ever in problem, give the maximum value possible for the +ve f/b ratio $P = \frac{r}{r+R}$ -ve f/b ratio $N = \frac{R_{in}}{R_{in}+R_f}$ for the ckt to remain stable.

→ $V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$




KCL (-ve)

$$\frac{V_{in} - V_-}{R_{in}} + \frac{V_{out} - V_-}{R_f} = 0 \rightarrow ①$$

KCL : (+ve)

$$\frac{0 - V_+}{r} + \frac{V_{out} - V_+}{R} = 0 \rightarrow ②$$

From ①,

$$\frac{V_-}{R_{in}} + \frac{V_-}{R_f} = \frac{V_{in}}{R_{in}} + \frac{V_{out}}{R_f}$$

$$V_- = \frac{R_f V_{in}}{R_{in}+R_f} + \frac{R_{in} V_{out}}{R_{in}+R_f}$$

From ②

$$V_+ = \frac{r\, V_{out}}{r+R} = P.V_{out}$$

$N = \frac{R_{in}}{R_{in}+R_f}$ $\quad V_- = (1-N)V_{in} + N.V_{out}$

Now,

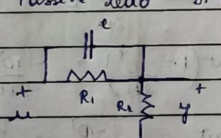
$$V_{out} = \frac{10^7}{s+1}[V_+ - V_-]$$

$$= \frac{10^7}{s+1}[P.V_{out} - (1-N)V_{in} - N.V_{out}]$$

$$\frac{V_{out}}{V_{in}} = \frac{-10^7(1-N)}{s+1-10^7P+10^7N}$$

5. Write the dynamic eqn and find the TF for the ckt shown in fig 5

a. Passive lead b. Active lead c. Passive lag d. Active lag




$$\frac{V_u - V_y}{R_1} + C\frac{d}{dt}(V_u - V_y) - \frac{V_y}{R_2} = 0$$

$$\frac{V_u}{R_1} - \frac{V_y}{R_1} + \frac{Cd(V_u)}{dt} - \frac{Cd(V_y)}{dt} - \frac{V_y}{R_2}$$

$$C.\dot{V}_y + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_y = C.\dot{V}_u + V_u$$

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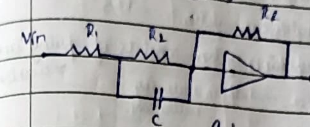
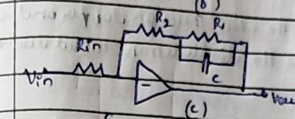
Applying L.T.

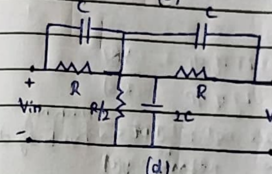
$$\frac{V_y(s)}{V_u(s)} = \frac{SC + \frac{1}{R_1}}{CS + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$




$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{2}\frac{S + \frac{1}{CR_1}}{S + \frac{1}{R_1C} + \frac{1}{R_2C}}$$

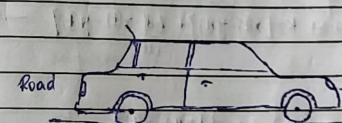



$$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_{in}}\frac{S + \frac{1}{R_2C} + \frac{1}{R_1C}}{S + \frac{1}{R_1C}}$$



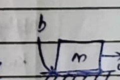

14/9/23 Cruise control system:-

Ex. Car.




The applied force has to overcome:

1. Inertia of the car $- m\frac{d^2x}{dt^2}$
2. Friction between Road surface and tires $- b\frac{dx}{dt}$




$$u = m\frac{d^2x}{dt^2} + b\frac{dx}{dt}$$

i/p ← $u = m\frac{dv}{dt} + bv \rightarrow$ o/p

$U(s) = msV(s) + bV(s)$

$U(s) = V(s).(ms+b)$

$$\Rightarrow \frac{V(s)}{U(s)} = \frac{1}{ms+b} = \frac{1/m}{s + b/m}$$

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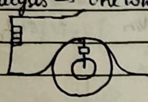
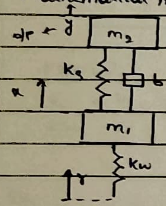
Automobile suspension system:-

* To reduce the impact of road irregularities on the rider / passenger
→ Car → Four wheels. Fig: Suspension is shown for two wheels.
* For analysis → One wheel → Quarter car system



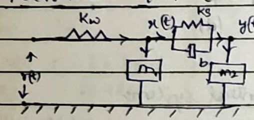
* Suspension system consists of spring and damper. → helps in reducing oscillations. → cushion effect
* Body of the car is connected to the wheel through damper and spring
* Input to the suspension system is the displacement caused due to irregularity of the road

Mathematical model:




Transfer fn = $\dfrac{\text{LT of o/p}}{\text{LT of i/p}} = \dfrac{Y(S)}{R(S)}$

Performance eq$^n$ :- ODE ⇉ Mech ntw, Free body diagram




* Displacement for mass is always independent.

Node x(t):

$$k_w[r(t)-x(t)] = m_1\frac{d^2x(t)}{dt^2} + k_s[x(t)-y(t)] + b\frac{d}{dt}[x(t)-y(t)] \rightarrow ①$$

Node y(t):

$$m_2\frac{d^2y(t)}{dt^2} = k_s[x(t)-y(t)] + b\frac{d}{dt}[x(t)-y(t)] \rightarrow ②$$

Apply LT:

$$k_w[R(S)-X(S)] = m_1S^2X(S) + k_s[X(S)-Y(S)] + bS[X(S)-Y(S)] \rightarrow ③$$

$$m_2S^2Y(S) = k_s[X(S)-Y(S)] + bS[X(S)-Y(S)]$$

Required: $\dfrac{Y(S)}{R(S)}$ → eliminate X(S)

$$\therefore \frac{Y(S)}{R(S)} = \frac{\frac{k_wb}{m_1m_2}\left(S+\frac{k_s}{b}\right)}{S^4+\left(\frac{b}{m_1}+\frac{b}{m_2}\right)S^3+\left(\frac{k_s}{m_1}+\frac{k_s}{m_2}+\frac{k_w}{m_1}\right)S^2+\left(\frac{k_wb}{m_1m_2}\right)S+\frac{k_wk_s}{m_1m_2}}$$



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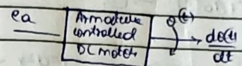
Electro mechanical system

I/P → Electric vtg / current    O/P → Mech. displacement < Translational, Rotational

Ex: DC motor → i/p → vtg ; o/p → rotational displacement

Application: Need to control speed of DC motor → Armature vtg control, Field current control

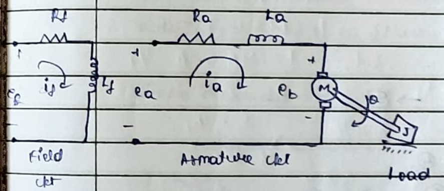



Motor speed ∝ $\dfrac{\text{Armature vtg}}{\text{Field flux}}$ ⇒ Arm vtg ↑ speed ↑, field flux ↓ speed ↑

In armature control, we keep field flux constant ⇒ speed ∝ arm. vtg.

∴ Tr. fn = $\dfrac{\text{LT of o/p}}{\text{LT of i/p}} = \dfrac{\theta(S)}{E_a(S)}$ (or) $\dfrac{S\theta(S)}{E_a(S)}$

Armature controlled DC motor:-




In armature controlled DC motor, the field flux is kept constant and the armature voltage is varied to vary the speed.

KVL - Armature ckt:

$$e_a = i_aR_a + L_a\frac{di_a}{dt} + e_b \rightarrow ①$$

$e_a$ → Applied armature vtg; $i_a$ → Armature current; $R_a$ → Armature winding resistance; $L_a$ → Armature winding inductance; $e_b$ → back emf

Taking LT, we have

$$E_a(S) = I_a(S)R_a + L_aSI_a(S) + E_b(S) \rightarrow ②$$

→ Back emf ∝ speed of the armature

$$e_b \propto \frac{d\theta}{dt}$$

$$e_b = K_e\frac{d\theta}{dt} \rightarrow ③$$

$$E_b(S) = K_eS\theta(S) \rightarrow ④$$

Torque developed by the motor is proportional to the product of field flux and armature current

$T_a \propto \phi\, i_a \rightarrow T_a \propto i_a$

$\phi$: constant $\qquad T_a = K_t\, i_a$

$T_a(s) = K_t I_a(s) \rightarrow (5)$

$T_a <$ load torque, Friction

$M\frac{d^2x}{dt^2}$ - Mass

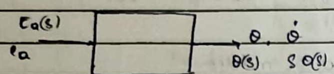
$T_a = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} \rightarrow (6)$

$J$: Moment of Inertia

$b\frac{dx}{dt}$ → Friction

$Kx$ = Spring

$T_a(s) = J s^2 \theta(s) + bs\theta(s) \rightarrow (7)$




T. Fn → $\frac{\theta(s)}{E_a(s)}$

$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + L_a s} \rightarrow (3)$

$E_b(s) = K_e s\,\theta(s) \rightarrow (4)$

$T_a(s) = K_t I_a(s) \rightarrow (5)$

$T_a(s) = \theta(s)[Js^2 + bs] \rightarrow (7)$

$K_t I_a(s) = \theta(s).[Js^2 + bs]$

$K_t\left[\frac{E_a(s) - E_b(s)}{R_a + sL_a}\right] = \theta(s)[Js^2+bs]$

$K_t\left[\frac{E_a(s) - K_e s\,\theta(s)}{R_a + L_a s}\right] = \theta(s)[Js^2+bs]$

$\frac{K_t E_a(s)}{R_a + sL_a} - \frac{K_t K_e s\,\theta(s)}{R_a + L_a s} = \theta(s)[Js^2+bs]$

$\frac{K_t E_a(s)}{R_a + L_a s} = \theta(s)\left[\frac{Js^2+bs + K_t K_e s}{R_a + L_a s}\right]$

$\frac{K_t E_a(s)}{\cancel{R_a + L_a s}} = \theta(s)\left[\frac{Js^2+bs\,(R_a + L_a s) + K_t K_e s}{\cancel{R_a + L_a s}}\right]$



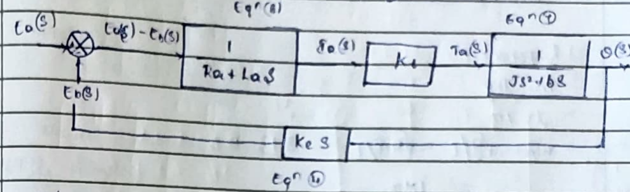
$\frac{\theta(s)}{E_a(s)} = \frac{K_t}{(Js^2+bs)(R_a+L_a s) + K_t K_e s}$

or

$\frac{s\,\theta(s)}{E_a(s)} = \frac{K_t}{(Js+b)(R_a+L_a s) + K_t K_e}$

$K_t$ not present in class eqⁿ

Block diagram: Interconnection of several blocks

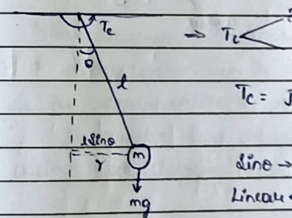



Observation:-

Armature controlled DC motor is a closed loop system with f/b.

Stability: Stable, as it uses f/b.

Simple Pendulum:-
↓
harmonic oscillations




→ $T_c <$ Inertia of the mass (J/I), Opposing torque

$T_c = J\frac{d^2\theta}{dt^2} + (mg)(l\sin\theta) \rightarrow (1)$

Force distance

$\sin\theta$ → Non linear, term

Linear — for small $\theta$, $\sin\theta \approx \theta$.

$\rightarrow T_c = J\frac{d^2\theta}{dt^2} + mg(l\theta)$

$J \rightarrow (m)l^2$

$T_c = ml^2\frac{d^2\theta}{dt^2} + mgl\theta$

$\frac{T_c}{ml^2} = \frac{d^2\theta}{dt^2} + \frac{g}{l}\theta$

But $\omega_n = \sqrt{\frac{g}{l}}$

DATE

→ $\boxed{\frac{T_c}{ml^2} = \frac{d^2\theta}{dt^2} + \omega_n^2\theta}$ → Performance eqn.

Take LT

$$\frac{T_c(s)}{ml^2} = s^2\theta(s) + \omega_n^2\theta(s) = \theta(s)[s^2+\omega_n^2]$$

$$\boxed{\text{T. fn} \rightarrow \frac{\theta(s)}{T_c(s)} = \frac{1}{ml^2[s^2+\omega_n^2]}} \rightarrow \frac{1/ml^2}{[s^2+\omega_n^2]}$$

Response:

m = 1kg

l = 1m

$\omega_n = \sqrt{g/l} = \sqrt{9.8/1} = 3.13 \approx 3$

$T_c = u(t)$ 1Nm

$T_c(s) = 1/s$

$$\theta(s) = \frac{1}{s(s^2+3^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+3^2}$$

$1 = A(s^2+3^2) + (Bs+C)s$

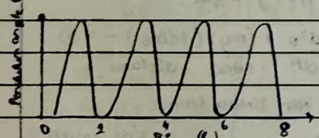
s = 0, $1 = 9A \Rightarrow A = 1/9$

Equate coeff of $s^2$: $A+B=0 \Rightarrow B = -1/9$

coeff of s: $C = 0$

$$\theta(s) = \frac{1}{9}\left[\frac{1}{s} - \frac{s}{s^2+3^2}\right]$$

$$\theta(t) = \frac{1}{9}[1-\cos 3\omega t]$$




22/9/23

Modelling of Floppy disk drive:

→ Group → Picture -x-

- Motor develops the torque ($T_c$ / $M_c$)
- along with this there will be some disturbing torque ($T_D$/$M_D$)
- $J_1$ is the inertia of the motor.
- Motor torque is coupled to head through a flexible shaft which has K, b.
- $J_2$ is the inertia of the head.



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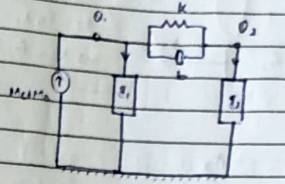
- Motor shaft displacement $\theta_1$ and head displacement is $\theta_2$




Node $\theta_1$:

$$M_c + M_D = J_1\frac{d^2\theta_1}{dt^2} + K[\theta_1-\theta_2] + b\frac{d}{dt}(\theta_1-\theta_2) \rightarrow (1)$$

Node $\theta_2$:-

$$K[\theta_1-\theta_2] + b\frac{d}{dt}(\theta_1-\theta_2) = J_2\left[\frac{d^2\theta_2}{dt^2}\right]$$

Neglect disturbing torque → $M_c = J_1\frac{d^2\theta_1}{dt^2} + K[\theta_1-\theta_2] + b\frac{d}{dt}(\theta_1-\theta_2) \rightarrow (2)$

LT & Simplify

$$\frac{\theta_2(s)}{M_c(s)} = \frac{K}{J_1J_2s^2\left(s^2+\frac{K}{J_1}+\frac{K}{J_2}\right)}$$

$$\frac{\theta_1(s)}{M_c(s)} = \frac{J_2s^2+K}{J_1J_2s^2\left(s^2+\frac{K}{J_1}+\frac{K}{J_2}\right)}$$

Block diagram reduction techniques:-

Every system can be represented by a block diagram which is the interconnection of several blocks.

The complex block diagram can be reduced to single block using block diagram reduction techniques.

Rules for Block diagram reduction:

$G(s) = G$
$H(s) = H$ } all in s domain
$X(s) = X$

$x(t)$, $X(s)$ → $g(t)$, $G(s)$ → $y(t)$, $Y(s)$

$y(t) = g(t) * x(t)$

Impulse response

$Y(s) = G(s) \cdot X(s)$

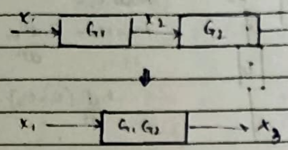
- Convolution in time domain is equal to multiplication in freq. domain

**Blocks in cascade:-**

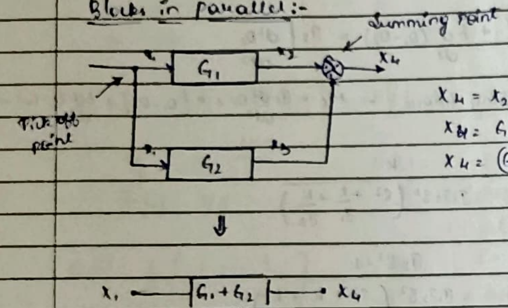
$x_3 = G_2 x_2$

$x_2 = G_1 x_1$

$\Rightarrow x_3 = G_2 G_1 x_1$



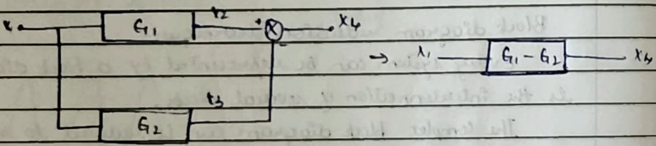

**Blocks in parallel:-**



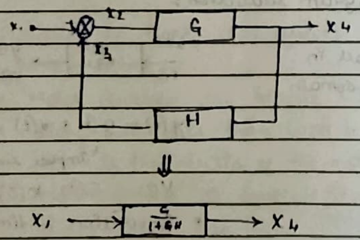

$x_4 = x_2 + x_3$

$x_4 = G_1 x_1 + G_2 x_1$

$x_4 = (G_1 + G_2)\, x_1$




**Feedback loop:-**




$x_4 = G x_2$

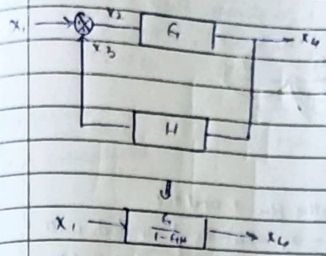
$x_4 = G(x_1 - x_3)$

$x_4 = G(x_1 - H x_4)$

$x_4 [1 + GH] = G x_1$

$\frac{x_4}{x_1} = \frac{G}{1+GH}$

$x_4 = \frac{G}{1+GH} \cdot x_1$




$x_4 = G x_2$

$x_4 = G(x_1 + x_3)$

$x_4 = G(x_1 + H x_4)$

$x_4 (1 - GH) = G x_1$

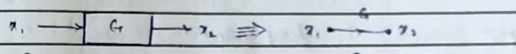
$x_4 = \frac{G}{1 - GH} \cdot x_1$

* Absent

27/9/23 Signal Flow Graph [SFG]

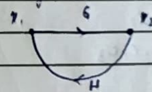
Alternative representation of block diagram.

Terminologies:




$G \rightarrow$ Block gain $\qquad$ $G \rightarrow$ Branch gain

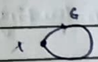
Loop: originates and terminates on the same node.




$x_1 - x_2 - x_1$

loop gain $L = GH$

Self loop:



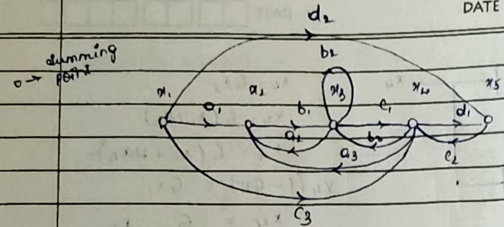

Example: A control system is described by the following set of eqn's. Construct the signal flow graph

$x_2 = a_1 x_1 + a_2 x_3 + a_3 x_4$

$x_3 = b_1 x_2 + b_2 x_3 + b_3 x_4$

$x_4 = c_1 x_3 + c_2 x_5 + c_3 x_1$

$x_5 = d_1 x_4 + d_2 x_1$




$x_1$ = i/p
$x_5$ = o/p

Forward paths b/w $x_1$ and $x_5$:

FP1: $x_1 - x_2 - x_3 - x_4 - x_5$
FP2: $x_1 - x_4 - x_5$
FP3: $x_1 - x_5$

No node will repeat in FP

Gain of forward paths :-

$M_1 = a_1 b_1 c_1 d_1$

$M_2 = c_3 d_1$

$M_3 = d_2$

| loops:- | loop gains |
|---|---|
| 1. $x_2 - x_3 - x_2$ | $L_1 = b_1 a_2$ |
| 2. $x_3 - x_3$ | $L_2 = b_2$ |
| 3. $x_3 - x_4 - x_3$ | $L_3 = c_1 b_3$ |
| 4. $x_4 - x_5 - x_4$ | $L_4 = d_1 c_2$ |
| 5. $x_2 - x_3 - x_4 - x_2$ | $L_5 = b_1 c_1 a_3$ |

Two non touching loops:-

Two loops are said to be non touching if they do not share a common node

1. $L_1$ & $L_4$
2. $L_2$ & $L_4$

Gain product of non-touching loops:

$L_1 L_4 = b_1 a_2 d_1 c_2$

$L_2 L_4 = b_2 d_1 c_2$

Mason's gain formula:-

$$\frac{\text{LT of o/p}}{\text{LT of i/p}} = \frac{C(s)}{R(s)} = \frac{\sum_{k=1}^{n} M_k \Delta_k}{\Delta}$$

$n \to$ No. of forward paths b/w $C(s)$ & $R(s)$

$m \to$ gain of fwd paths

$\Delta \to$ Determinant of SFG

$$= \frac{m_1\Delta_1 + m_2\Delta_2 + \dots + m_n\Delta_n}{\Delta}$$

$\Delta = 1 - \{$Sum of individual loop gains$\}$ + $\{$Sum of gain products of two non-touching groups$\}$ + $\{$Sum of " — three non touching groups$\}$ + ...

$\Delta_1$ = Det of FP1 , $\Delta_2$ = Det of FP2

$$\frac{x_5}{x_1} = \frac{\sum_{k=1}^{3} m_k \Delta_k}{\Delta} = \frac{m_1\Delta_1 + m_2\Delta_2 + m_3\Delta_3}{\Delta}$$

$$\Delta = 1 - \{(L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_4 + L_2 L_4)\}$$

$$= 1 - \{(b_1 a_2 + b_2 + c_1 b_3 + d_1 c_2 + b_1 c_1 a_3) + (b_1 a_2 d_1 c_2 + b_2 d_1 c_2)\}$$