

1 Selection Sort

0: [12, 7, 5, 13, 6, 2, 4]
 ↑
 i, min
 ↑
 j
 Search new min

3: [2, 7, 5, 13, 6, 12, 4]
 ↑
 i, min
 ↑
 j
 Search new min

1: [12, 7, 5, 13, 6, 2, 4]
 ↑
 i
 ↑
 min
 ↑
 j
 SWAP

4: [2, 7, 5, 13, 6, 12, 4]
 ↑
 i
 ↑
 min
 ↑
 j
 SWAP

2: [2, 7, 5, 13, 6, 12, 4]
 ↑
 i, min
 ↑
 j

5: [2, 4, 5, 13, 6, 12, 7]
 ↑
 i, min
 ↑
 j

Invariant: Selection sort

For any given value of i in the main loop of selection sort, at the beginning of the iteration, the following invariants hold:

1. $array[1..i-1]$ is sorted
2. For any x in $array[1..i-1]$ and y in $array[i..n]$, $x \leq y$

K-Steps
 ↓
 K: [2, 4, 5, 6, 7, 12, 13]
 ↑
 i
 ↑
 j

LI: $arr[1...0]$ is sorted

for ($i=1$, $i \leq n$; $i++$):

min = i LI: $arr[1...i-1]$ is sorted & $x \in arr[1...i-1]$,
 $x \leq y$ for $y \in arr[i..n]$

for ($j = i + 1$), $j \leq n$, $j++$):

if ($arr[j] < arr[min]$):
 $min = j$

swap($arr[i]$, $arr[min]$)

LI: $arr[1 \dots i]$ is sorted & $x \in arr[1 \dots i]$, $y \in arr[i+1 \dots n]$
where $x \leq y$.

LI: $arr[1 \dots i]$ is sorted where $i = n$ when loop terminates, so
 $arr[1 \dots n]$ is sorted.

$y \in arr[1 \dots n]$
where $x \leq y$

Code:

```
def selectionSort(arr, n):
```

```
    for i in range(n):
```

```
        min = i
```

```
        for j in range(i+1, n):
```

```
            if arr[j] < arr[min]:
```

```
                min = j
```

```
    arr[i], arr[min] = arr[min], arr[i]
```

```
    return arr
```

Time:

$$T(n) = n + (n-1) + (n-2) + \dots + 2 + 1$$

$$T(n) = n \left(\frac{n-1}{2} \right) = \frac{n^2 - n}{2}$$

Best / Worst-Case: $O(n^2)$

Space: $O(1)$

In-Place: Yes, as aux space is $O(1)$.

Stable: No, because selection sort swaps non-adjacent elements which changes the relative order of elements with the same value.

Online: No, because if any new elements are added to the input, the algo. would have to start all over.