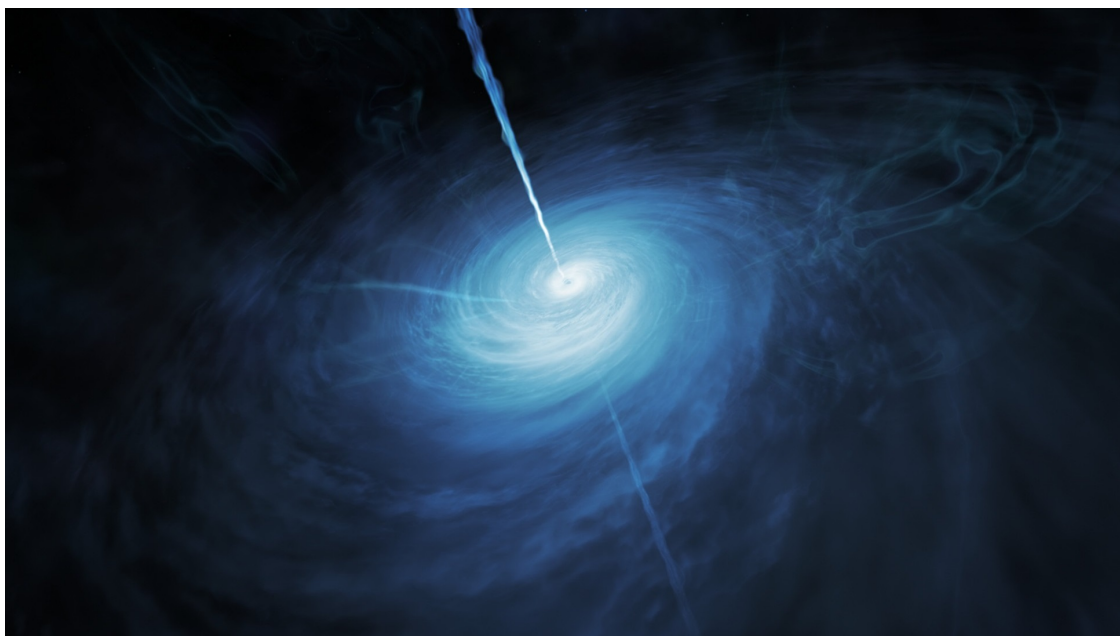


Science Olympiad
Reach for the Stars B
UT Invitational

October 23, 2020



Directions:

- Write **all answers on the lines on the answer pages**. Any marks elsewhere will not be scored.
- Make sure to put your **team name** and **team number** at the top of all answer pages.
- Do not worry about significant figures. Just make sure to **use 3 or more in your answers**.
- Best of luck! And may the odds be ever in your favor.

Written by:

Aditya Shah, *adityashah108@gmail.com*; **Antonio Frigo**, *frigo.antonio.mct@gmail.com*

April Cheng, *aprilcheng0420@yahoo.com*; **Dhruva Karkada**, *dkarkada@gmail.com*

Pranit Mohnot, *pranit_mohnot@berkeley.edu*; **Robert Lee**, *robertyl@ucla.edu*

Sahil Pontula, *sahil31415@gmail.com*

Section A: Multiple Choice

For the following questions, select the correct answer (or correct answers, if specified) for each question. The first several questions involve reading a passage and selecting the right word to complete the blank, which corresponds to a question number. Questions are worth 1 point each, unless otherwise specified, for a total of 69 points.

One of the most remarkable things about astronomy is its scale and its implications on our place in the universe. The average distance between the Sun and the Earth is 1 AU, or 1 __ (1) __. It would take a modern jet about 20 years to travel that distance! Even if we travelled at the speed of light, it would still take over 4 years to reach the closest star to our Solar System, __ (2) __.

There are over 100 million stars in our galaxy, __ (3) __, which is a __ (4) __. At the center of our galaxy, astronomers have discovered a __ (5) __. Our galaxy is a part of a group of galaxies called the __ (6) __, which is in turn a part of the Virgo Supercluster. The Virgo Supercluster is estimated to have a diameter of over 100 million light years!

- | | |
|----------------------------|-------------------------------|
| 1. A. arbitrary unit | 4. A. spiral |
| B. astronomical union | B. barred-spiral |
| C. astronomical unit | C. elliptical |
| D. none of the above | D. irregular |
| 2. A. Sirius A | 5. A. supermassive black hole |
| B. Proxima Centauri | B. alien civilization |
| C. Betelgeuse | C. globular cluster |
| D. Rigel | D. neutron star |
| 3. A. the Milky Way Galaxy | 6. A. Virgo Group |
| B. the Andromeda Galaxy | B. Solar System |
| C. NGC 17 | C. Andromeda Group |
| D. the LMC | D. Local Group |

The (7) (abbreviated CMB) is blackbody radiation originating from the very early universe. Its spectrum has an effective temperature of about (8), which peaks at a wavelength of (9) according to Wien's displacement law. The radiation in fact used to have an effective temperature of about (10) when it was first emitted at $z=1100$, but the light has been strongly redshifted due to (11). The blackbody radiation we see today originates from (12), but ever since (13), the photons have been freely travelling through the universe. The radiation is pretty much uniform across the sky, except for minor fluctuations which are formally known as (14). These fluctuations are often studied using (15), which studies the magnitude of fluctuations as a function of (16).

- | | |
|-----------------------------------|-------------------------------|
| 7. A. central microwave blackbody | 12. A. primordial black holes |
| B. cosmic microwave background | B. an opaque plasma |
| C. cosmological massive baryon | C. the cosmological constant |
| D. cosmological massive blackbody | D. the big bang |
| 8. A. 3 K | 13. A. the first stars formed |
| B. 300 K | B. dark matter domination |
| C. 3 °C | C. reionization |
| D. 300 °C | D. photon decoupling |
| 9. A. 1.9 nm | 14. A. anharmonicities |
| B. 1.9 mm | B. anisotropies |
| C. 1.9 cm | C. Heisenberg uncertainties |
| D. 1.9 m | D. the inflaton field |
| 10. A. 3000 K | 15. A. Weierstrass functions |
| B. 300 K | B. symplectic geometry |
| C. 30 K | C. multipole expansion |
| D. 3 K | D. diffeomorphic geometry |
| 11. A. time dilation | 16. A. the tangent bundle |
| B. gravitational lensing | B. the Lagrangian |
| C. the doppler effect | C. Lipschitz bounds |
| D. the expansion of the universe | D. angular size |

Gravitational waves are a consequence of (17) , which was developed by Einstein in the early 1900s. It predicts that two orbiting bodies emit (18) . The strength of this emission increases with (19) and decreases with (20) . We have built sensors to detect gravitational waves; the two in the United States are located in the cities (21) . These instruments are light interferometers, much like the interferometer that was famously used to show that (22) . Due to the weakness of gravitational waves, the data collected by these instruments have very low (23) . Nonetheless, scientists have been able to detect many "chirps", which represent (24) .

- | | |
|--|---|
| <p>17. A. the theory of special relativity
 B. the theory of general relativity
 C. the theory of modified Newtonian dynamics (MOND)
 D. the cosmological constant</p> <p>18. A. ripples in spacetime
 B. plane waves in the inflaton field
 C. blackbody radiation in the gravitational field
 D. heavy leptons and pi mesons</p> <p>19. A. quantum coherence
 B. distance from the source
 C. orbital angular velocity
 D. temperature</p> <p>20. A. quantum coherence
 B. distance from the source
 C. orbital angular velocity
 D. temperature</p> | <p>21. A. Hanford and Livingston
 B. San Agustin and Caltech
 C. Phoenix and Socorro
 D. Santo Stefano and Pisa</p> <p>22. A. light is inherently quantum mechanical
 B. there is no ether
 C. Newtonian gravity is wrong
 D. Maxwellian electromagnetism is wrong</p> <p>23. A. multimodality
 B. kurtosis
 C. bias/variance tradeoff
 D. signal-to-noise ratio</p> <p>24. A. the formation of supermassive black holes
 B. starquakes on a neutron star
 C. rotating Kerr black holes
 D. mergers between compact objects</p> |
|--|---|

Stellar evolution describes how stars change during their lifetimes. All stars are “born” from a collapsing cloud of gas and dust, often called a (25) . This process forms a (26) . Eventually, the cloud reaches an equilibrium, and the center becomes hot enough to allow for the fusion of hydrogen, becoming what is known as a (27) .

A main-sequence star evolves into a (28) when all the hydrogen in its core is consumed. The star’s core contracts and heats up, while its outer layers do the opposite. If the star is massive enough, (29) fusion in the core begins, usually surrounded by a shell of hydrogen fusion. If a star is massive enough, it can go on to fuse even heavier elements. Typically, even the most massive stars can only fuse elements up to (30) , because that’s when the binding energy per nucleon is maximized.

- | | |
|---------------------------|---------------------------|
| 25. A. globular cluster | 28. A. main-sequence star |
| B. molecular cloud | B. red giant |
| C. protoplanetary disk | C. red supergiant |
| D. circumstellar disk | D. red dwarf |
| 26. A. pre-star | 29. A. helium |
| B. star cluster | B. carbon |
| C. protostar | C. oxygen |
| D. open star | D. uranium |
| 27. A. main-sequence star | 30. A. carbon |
| B. red dwarf | B. iron |
| C. white dwarf | C. uranium |
| D. neutron star | D. plutonium |

A high-mass star ends its life with a powerful explosion called a (31) . If the progenitor (original) star is between about 8 and 30 solar masses, it will leave behind a (32) , which is supported by (33) . If the progenitor is even more massive, it can form a (34) .

In contrast, a star of relatively low mass—such as our own Sun—ends its evolution by gently expelling its outer layers into space. These ejected gases form a glowing cloud called a (35) . The burned-out core that remains is called a (36) , which is held up by (37) . White dwarfs can also become a supernova if it accretes enough mass to exceed the (38) .

- | | |
|---------------------------------|-------------------------------------|
| 31. A. quasar | 35. A. stellar halo |
| B. gamma-ray burst | B. galactic halo |
| C. gravitational wave | C. planetary nebula |
| D. supernova | D. circumstellar disk |
| 32. A. black hole | 36. A. white dwarf |
| B. neutron star | B. neutron star |
| C. white dwarf | C. black hole |
| D. red dwarf | D. red dwarf |
| 33. A. radiation pressure | 37. A. electron degeneracy pressure |
| B. electron degeneracy pressure | B. neutron degeneracy pressure |
| C. neutron degeneracy pressure | C. gravitational repulsion |
| D. gravitational repulsion | D. quantum entanglement |
| 34. A. black hole | 38. A. TOV Limit |
| B. quasar | B. Einstein Limit |
| C. active galactic nucleus | C. Chandrasekhar Limit |
| D. blue hypergiant | D. Eddington Limit |

To help visualize different types of stars and stellar evolution, astronomers often plot stars on a (39) (HR) diagram. Usually, HR diagrams have (40) on the x-axis and (41) on the y-axis. As a result, different classes of stars would appear in different locations. For example, white dwarfs appear in the (42), while red supergiants would appear in the (43).

39. A. Heliotropic Radiation
B. Hendricks-Rammer
C. Hayashi-Rhodes
D. Hertzsprung-Russell

40. A. temperature
B. pressure
C. luminosity
D. radius

41. A. temperature
B. pressure
C. luminosity
D. radius

42. A. top left
B. bottom left
C. top right
D. bottom right

43. A. top left
B. bottom left
C. top right
D. bottom right

The remainder of this section will not be “fill-in-the-blank” like the earlier questions.

Use the following information to answer the next five (5) questions.

Two extraordinarily rich friends are comparing their telescopes. One of them has an x-ray telescope in space, while the other has a ground-based radio telescope.

44. Which portion of the electromagnetic spectrum (gamma-ray or radio) has a higher frequency?
- A. Gamma-ray
 - B. Radio
45. Which portion of the electromagnetic spectrum (gamma-ray or radio) has photons of a higher energy?
- A. Gamma-ray
 - B. Radio
46. True or false: the radio telescope must be ground-based since the Earth's atmosphere blocks incoming radio waves
- A. True
 - B. False
47. One of the friends claims that their telescope has a higher angular resolution, so it is better at resolving objects. Are they correct?
- A. Yes
 - B. No
48. It turns out that both of their telescopes have the exact same angular resolution. Which one is bigger?
- A. Gamma-ray
 - B. Radio
-

Use the following information to answer the next four (4) questions.

Consider 3 stars, A, B, and C, each with varying temperatures and radii. Star A has a temperature of T and a radius of R . Star B has a temperature of $2T$ and a radius of R . Star C has a temperature of $T/2$ and a radius of $2R$.

49. Which star has the highest luminosity?
- A. A
 - B. B
 - C. C
50. How many times more luminous is Star A than Star C?
- A. 2
 - B. 4
 - C. 8
 - D. 16
 - E. None of the above
51. Each star's spectrum will peak at a certain wavelength, as predicted by Wien's Law. Which star's spectrum will peak at the longest wavelength?
- A. A
 - B. B
 - C. C
52. Suppose that Star B is twice as far as Star A from Earth. What is the ratio of the intensity of light we receive from Star B to that of Star A?
- A. $1/4$
 - B. $1/2$
 - C. 1
 - D. 4
 - E. 16
-

53. Which of the following lists the order of the main sequence spectral types from hottest to coolest?
- A. OBAFGKM
 - B. ABFGKMO
 - C. BOGAFMK
 - D. ABCDEFG
54. How do stars produce energy?
- A. Chemical reactions
 - B. Nuclear reactions
 - C. Release of gravitational potential energy
 - D. Electron degeneracy pressure
55. The most luminous stars are ____ and ____ .
- A. big, hot
 - B. big, cool
 - C. small, hot
 - D. small, cool
56. Lightyears are a measure of ____ .
- A. distance
 - B. energy
 - C. speed
 - D. time
57. Luminosity is a measure of ____ .
- A. energy
 - B. temperature
 - C. distance
 - D. power
58. Stars spend most of their lives on the ____ on an HR diagram.
- A. Main-sequence
 - B. Red giant branch
 - C. Asymptotic branch
 - D. Instability strip
59. A star's temperature
- A. Increases towards the center.
 - B. Decreases towards the center.
 - C. Is fairly constant throughout the star.
 - D. Increases towards the center, until the core, which is relatively mild.
60. The balance between a star's inward pull of gravity and its outwards pressure gradient is called
- A. Stellar equilibrium
 - B. Hydrostatic equilibrium
 - C. Eddington limit
 - D. Stellar buoyancy
61. *The planets in our Solar System orbit the Sun in an ellipse, with the Sun as one of the foci.* This is Kepler's ____ Law.
- A. First
 - B. Second
 - C. Third
 - D. Fourth
62. $P^2 \propto a^3$. This is Kepler's ____ Law.
- A. First
 - B. Second
 - C. Third
 - D. Fourth
63. What is the Sunyaev-Zel'dovich effect?
- A. The distortion of the CMB due to gravitational lensing
 - B. The distortion of spacetime due to the CMB
 - C. The distortion of the CMB due to Compton scattering
 - D. The distortion of spacetime due to gravitational interactions
-

The next three (3) question are worth **two points**.

Each of the following scenarios occurs in a universe in which one physical law, principle, or fact has been changed. Your task is to determine how this difference affects reality. You may assume (highly unrealistically) that all other principles and laws remain unchanged. Select the answer which is most directly linked to the change.

64. The force of gravity is proportional to r^{-3} .

- A. Atoms no longer can exist, as they fly apart due to the lack of internal attraction.
- B. Neutrinos can fly through much more matter on average before getting stopped.
- C. Most planets are rocky and extremely dense.
- D. The speed of light is no longer constant.

65. The ratio of hydrogen to helium in the universe is 1:1. The mass of the universe is remains the same.

- A. Galaxies are substantially larger on average.
- B. Galaxies are substantially redder on average.
- C. Core-collapse supernovae explosions do not occur at all.
- D. The mass of silicon in the universe would remain the same.

66. The binding energy per nucleon peak occurs at neon.

- A. The composition of the ISM does not change.
- B. Classical novae occur more frequently.
- C. The Sun would explode as a supernova.
- D. Supernovae occur more frequently.

Section B: Deep-Sky Objects

Use the Image Sheet to answer the following questions. Questions are worth 2 points each, for a total of 60 points.

Image 5 shows one of the most famous star systems in the night sky: Sirius.

1. In which part of the electromagnetic spectrum was Image 5 taken?
 - A. Radio
 - B. Infrared
 - C. Visible
 - D. X-ray
 2. There are two objects shown in Image 5, one big and one small. Which one is Sirius A?
 - A. The big one
 - B. The small one
 3. What type of object is Sirius B?
 - A. Main-sequence star
 - B. Red giant
 - C. White dwarf
 - D. Red dwarf
 4. Image 10 shows another image of the Sirius system. In which part of the electromagnetic spectrum was it taken?
 - A. Radio
 - B. Infrared
 - C. Visible
 - D. X-ray
 5. Just like Image 5, there are two objects shown in Image 10, one big and one small. Which one is Sirius A?
 - A. The big one
 - B. The small one
 6. Which object is shown in Image 2?
 - A. Galactic formation
 - B. Galactic merger
 - C. Star system merger
 - D. Galactic death
 7. Which of the following images shows star formation happening in the object from the previous question?
 - A. Image 3
 - B. Image 7
 - C. Image 9
 - D. Image 13
 8. Which telescope took the image you chose in the previous question?
 - A. Chandra
 - B. Hubble
 - C. Spitzer
 - D. Fermi
 9. Which image shows the constellation in which this object is in?
 - A. Image 1
 - B. Image 6
 - C. Image 8
 - D. Image 11
 - E. Image 17
 10. Image 2 also has a distinctive reddish color. This color is most likely due to which spectral line?
 - A. H-alpha
 - B. 21 cm line
 - C. [O III]
 - D. None of the above
-

-
11. Which image shows the Baby Boom Galaxy?
- A. Image 3
 - B. Image 7
 - C. Image 12
 - D. Image 13
12. The Baby Boom Galaxy **is** a Starburst Galaxy.
- A. True
 - B. False
13. Image 8 shows the constellation that contains the Baby Boom Galaxy.
- A. True
 - B. False
-
14. There are two images that depict the Rho Ophiuchi cloud complex. Which ones are they?
- A. Image 3
 - B. Image 7
 - C. Image 9
 - D. Image 13
 - E. Image 15
15. The two images you selected in the previous question were taken in which parts of the electromagnetic spectrum? Note: if both of them were taken in the same portion of the electromagnetic spectrum, then you should only select one answer to this question.
- A. Radio
 - B. Infrared
 - C. Visible
 - D. Ultraviolet
 - E. Gamma-ray
16. Which of the following is the closest to the average temperature of the Rho Ophiuchi cloud complex?
- A. 1 Kelvin
 - B. 10 Kelvin
 - C. 100 Kelvin
 - D. 1000 Kelvin
-
17. The light curve of which star is shown in Image 4?
- A. Altair
 - B. Capella
 - C. Betelgeuse
 - D. Polaris
 - E. Spica
18. Which image(s) show(s) the star you chose in the previous question?
- A. Image 14
 - B. Image 16
 - C. Image 18
 - D. Image 19
19. Which of the following best describes this star?
- A. Main-sequence star
 - B. Red giant
 - C. Red supergiant
 - D. White dwarf
20. Which of the following is the brightest star in the constellation the star from the previous question resides in?
- A. Sirius
 - B. Rigel
 - C. Spica
 - D. Deneb
-

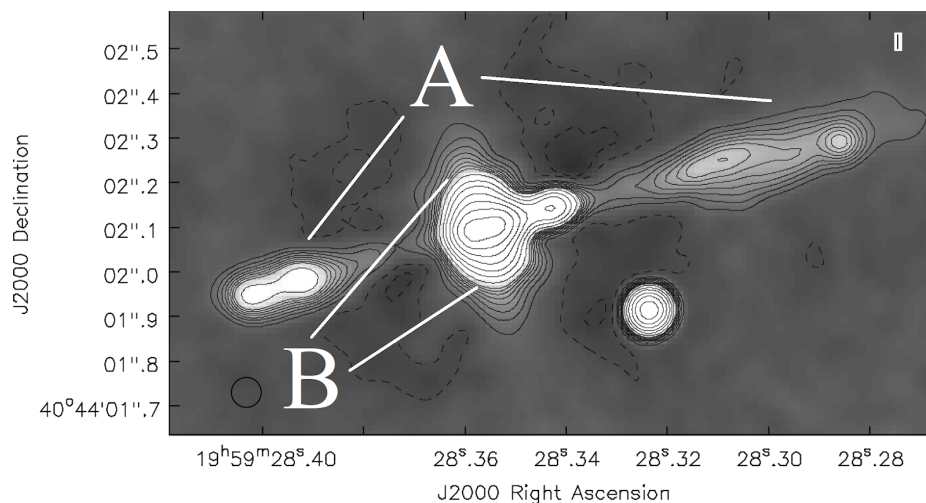
-
21. Image 20 shows Sgr A*. What do astronomers think it is?
- A. A neutron star
 - B. A supermassive black hole
 - C. A galaxy
 - D. A molecular cloud
22. Which telescope collected the data used to make Image 20?
- A. Hubble
 - B. Spitzer
 - C. Fermi
 - D. SOFIA
 - E. WISE
23. Where do astronomers think Sgr A* is located?
- A. At the center of the Milky Way Galaxy
 - B. At the edge of the Oort Cloud
 - C. In the Andromeda Galaxy
 - D. About 10 billion light years away in a distance galaxy
-
24. In which portion of the electromagnetic spectrum was Image 3 taken?
- A. Radio
 - B. Infrared
 - C. Visible
 - D. Ultraviolet
 - E. Gamma-ray
25. Which telescope collected the data used to make Image 3?
- A. Fermi
 - B. Chandra
 - C. Hubble
 - D. Spitzer
 - E. GAIA
26. Which image shows the constellation that contains the object shown in Image 3?
- A. Image 1
 - B. Image 6
 - C. Image 8
 - D. Image 11
 - E. Image 17
-
27. Which image shows T Tauri?
- A. Image 7
 - B. Image 15
 - C. Image 16
 - D. Image 18
 - E. Image 19
28. Which of the following best describes T Tauri?
- A. A molecular cloud
 - B. A pre-main sequence star
 - C. A main sequence star
 - D. A red giant
 - E. A red dwarf
-
29. Which image shows the SMC?
- A. Image 3
 - B. Image 7
 - C. Image 9
 - D. Image 13
 - E. Image 15
30. Which of the following best describes the SMC?
- A. A irregular satellite galaxy
 - B. A starburst galaxy with high rates of star formation
 - C. A distant quasar
 - D. A nebula

Section C: Calculations and Long-Form Questions

Answer the following calculation and long-form questions. For calculations, give a brief summary of your process for credit (and partial credit if the answer isn't correct). Calculations without explanations will be awarded minimal credit. Questions are worth 3-4 points each, for a total of 72 points.

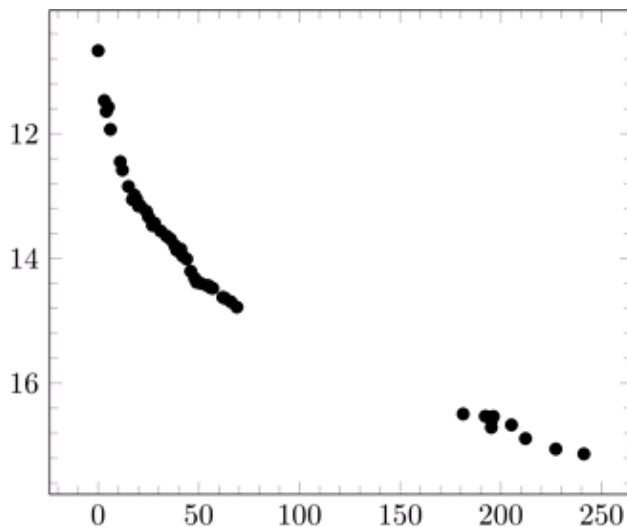
- (9 points) **Orbits.** The supermassive black hole at the center of a galaxy has a mass of 4×10^6 solar masses. A star is travelling around it in a very elliptical orbit. Because the supermassive black hole is far more massive than the star, assume the star's mass is negligible throughout this question.
 - (3 points) Long term observations of the star show that it has an orbital period of 22 years. What is its semimajor axis, in AU? Round to the nearest whole number. *Hint: use Kepler's Laws. The units in this question (Earth years, solar masses, and AU) work out really nicely too!*
 - (3 points) At its periapsis (closest point in the orbit), the star's distance from the supermassive black hole is one-half of its semimajor axis. At this point, how fast is the star moving, in m s^{-1} ? *Hint: use the vis-viva equation*
 - (3 points) Based on the information given in the previous part, what is the eccentricity of the orbit of the binary system around the SMBH? Round to one decimal place.
- (9 points) **AGN / Cygnus A Radio Image Analysis.** Active galactic nuclei (AGNs) are a complicated, ongoing field of research for astronomers. Initially, when astronomers first observed AGNs, they saw a wide collection of objects with disparate properties. However, they ultimately were able to merge all these objects into the class AGN using the unified model.
 - (3 points) Explain how the unified model accounted for all of the AGN classes.
 - (3 points) Name and describe the features present in the unified model.

Cygnus A is one of the first radio galaxies discovered by astronomers. The image below (Carilli et al. 2019) depicts a high resolution, radio observation of Cygnus A:



- (3 points) Given that the distance to Cygnus A is 750 million light years and using the largest contour, estimate the length (longest side) of feature B, in pc.

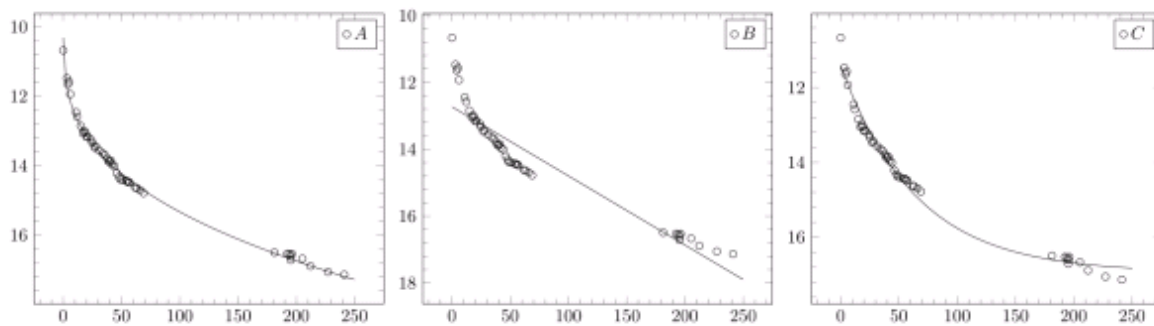
3. (9 points) **Fitting Data.** Consider the following V-band photometry of V1187 Sco (Walter et al. 2012):



You fit the data with the following three functional forms:

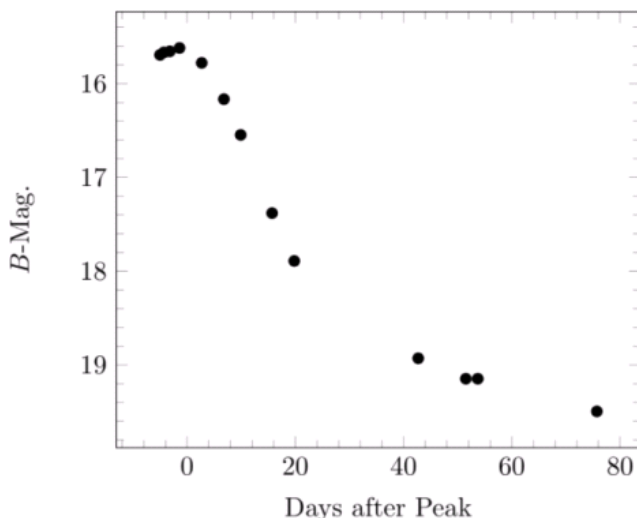
1. A linear regression in the form $M = at + b$
2. A power regression in the form $M = at^b + c$
3. An exponential regression in the form $M = ae^{bt} + c$

The fits are shown (in an arbitrary order) in the image below:



- (a) (3 points) Which regression fits the data best (A-C)?
- (b) (3 points) Identify the power regression and explain how you found it.
- (c) (3 points) Why might this comparison between fits be “unfair”? In other words, what disadvantage does one of these regression methods have over the others?

4. (9 points) **Type Ia Supernovae.** Type Ia supernovae decays are partially powered by radioactive decay. Consider the following light curve of SN 97e:



- (a) (3 points) The Phillips relation in the B, V and I bands are

$$M_B = -21.726 + 2.698 \times \Delta m_{15}(B)$$

$$M_V = -20.883 + 1.949 \times \Delta m_{15}(B)$$

$$M_I = -19.591 + 1.076 \times \Delta m_{15}(B)$$

where $\Delta m_{15}(B)$ is the decline in the B-magnitude light curve from maximum light to the magnitude 15 days after B-maximum.

If extinction **is** a factor, which of the above relations (if any) would yield the most accurate calculation for the distance?

- A. B band
 - B. V band
 - C. I band
 - D. None of the above/other
- (b) (3 points) Use the appropriate relation to calculate the distance to this event, in Mpc.
- (c) (3 points) Suppose we instead assumed the absolute magnitude of the event was -19.5. Would this affect our distance measurement?
- A. We'd overestimate the distance
 - B. We'd underestimate the distance
 - C. This would not impact our measurement of the distance

5. (12 points) **Neutron Stars.** Neutron stars are curious objects that are formed from the collapse of a massive star at the end of its life. Let's examine some of their properties.
- (a) (4 points) If a 1 kilogram mass weighs 7.5×10^{12} N at the surface of a neutron star, find the approximate radius of the neutron star in R_{\odot} . You may assume that its density is uniform and of order $10^{17} \text{ kg m}^{-3}$.
 - (b) (4 points) For all neutron stars, there is a maximum mass above which the neutron star is not stable. Briefly explain why this is.
 - (c) (4 points) Many neutron stars, once born, are rapidly spinning. Explain why rotation increases the upper mass bound for neutron star stability.
6. (8 points) **Black Holes.** Black holes are perhaps even more curious objects. Different types of black holes exist, but stellar-mass black holes are formed from the collapse of an extremely massive star, usually over $8 - 10 M_{\odot}$. Much research has been devoted to understanding the mysterious properties of these objects.
- (a) (4 points) Two commonly used conjectures about black holes are the cosmic censorship hypothesis and the “no hair” theorem. Describe what each of these conjectures say about black holes.
 - (b) (4 points) Singularities are the innermost parts of black holes where the laws of physics are thought to break down. Furthermore, some astrophysicists believe that it is possible for singularities not enclosed by event horizons to occur, resulting in so-called “naked” singularities. Describe the shape a singularity would take for a rotating black hole, and propose an explanation for how “naked singularities” might emerge.
7. (16 points) **Expansion of the Universe.** In this problem, you will explore a uniformly expanding universe. There are two types of distances that astronomers use in this context: **comoving distance** and **proper distance**. The comoving distance between two objects is the distance between these two objects **right now**, and **doesn't change** as time goes on. On the other hand, the proper distance between the two (i.e. “actual” distance) increases with time as the universe expands. At $t = 0$ (in other words, present-day), the proper distance is equal to the comoving distance, by definition.
- Astronomers often use a number called a “scale factor” to describe the ratio of the proper distance to the comoving distance as the universe expands: $a(t) = l_p/l_c$. So, if $a(t)$ is 1.5, then the proper distance is 1.5 times the comoving distance. The comoving distance will not have changed!
- Suppose that Galaxy A is currently 500 Mpc from us. Using this information, answer the next four (4) questions.
- (a) (4 points) What is the comoving distance to Galaxy A right now, in Mpc?
 - (b) (4 points) What is the proper distance to Galaxy A right now, in Mpc?
 - (c) (4 points) What is the comoving distance to Galaxy A when $a(t) = 2$, in Mpc?
 - (d) (4 points) What is the proper distance to Galaxy A when $a(t) = 2$, in Mpc?