

Section A (20 points)

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|------|------|-------|-------|-------|
| 1. C | 5. D | 9. A | 13. F | 17. B |
| 2. A | 6. B | 10. C | 14. D | 18. D |
| 3. B | 7. A | 11. A | 15. C | 19. B |
| 4. A | 8. C | 12. C | 16. D | 20. B |

Section B (80 points)

21. 2, 9, 4, 3
22. (a) Image 8
(b) Type Ia
(c) No
23. (a) Image 6
(b) Hubble
(c) Asymptotic giant branch
(d) White dwarf/planetary nebula
24. (a) Cygnus
(b) Horizontal branch
(c) Helium
25. (a) Sirius
(b) Chandra
- (c) X-ray
- (d) Large: white dwarf (Sirius B).
Small: main sequence star (Sirius A).
- (e) The Sirius B is much smaller physically, but emits more x-ray radiation than Sirius A since it is much hotter. In fact, the small dot we see for Sirius A may be due to ultraviolet radiation from Sirius A leaking through the filter on the detector.
26. (a) A light echo occurs when light from a bright event reflects off dust clouds at different distances from Earth, creating a visible echo effect as the reflected light reaches us at different times.

- (b) Image 11
- (c) Type II (peculiar)
- (d) Neutron star
27. (a) Cassiopeia A
- (b) JWST
- (c) Emission from hot dust
28. (a) Betelgeuse
- (b) Adaptive optics is a technology that corrects for atmospheric distortion in real-time by using a deformable mirror that changes shape to counteract the wavefront distortions caused by turbulence in Earth's atmosphere. The system measures these distortions using either a natural or laser guide star, then rapidly adjusts the mirror's shape to restore the original wavefront, resulting in much sharper astronomical images.
29. (a) Chandra
- (b) X-ray
- (c) Neither - all electromagnetic radiation travels at exactly the same speed in a vacuum, $c = 299,792,458$ m/s
30. (a) NGC 6543
- (b) Chandra and Hubble
- (c) Hotter
31. (a) A composite image combines data from multiple observations - either from different wavelengths, different times, or different instruments - into a single image. For example, an image might merge X-ray data from Chandra with infrared data from JWST to show different physical processes happening in the same object.
- (b) False
32. (a) Cygnus X-1
- (b) Black hole
- (c) HDE 226868, blue supergiant
- (d) Image 7
33. (a) SS Cygni
- (b) A dwarf nova is a binary star system where a white dwarf accretes matter from a companion star through an accretion disk. Periodically, the disk becomes unstable and rapidly transfers matter onto the white dwarf, causing a sudden, temporary brightening (outburst) of the system.

Section C (55 points)

34. (a) (2 points) Apparent magnitude: the magnitude of an object as it appears from Earth
Absolute magnitude: the magnitude of an object if it were 10 parsecs away
- (b) (3 points) The white dwarf is $12.5 - 5 = 7.5$ magnitudes dimmer than the Sun. The magnitude scale is defined such that a factor of 100 in brightness corresponds to 5 magnitudes. (This is Pogson's ratio.) So, the white dwarf is $100^{7.5/5} = 10^{2 \times 1.5} = \boxed{10^3}$ times dimmer than the Sun.
- (c) (3 points) White dwarfs maintain their size due to electron degeneracy pressure. This quantum mechanical effect prevents electrons from occupying the same energy states (Pauli exclusion principle), creating an outward pressure that balances gravitational collapse regardless of temperature.
35. (a) (4 points) Convection is the bulk transport of material due to temperature differences, where hot material rises and cool material sinks. In the Sun, convection occurs in the outer 30% of its radius, known as the convection zone. (The deep interior is radiative.)
- (b) (6 points) Each dredge-up is worth 2 points:
1. First dredge-up: After core hydrogen fusion stops (i.e., start of RGB).
 2. Second dredge-up: After core helium fusion ceases (i.e., at the end of the RGB)
 3. Third dredge-up: During a helium flash while on the AGB.
- (c) (2 points) The triple-alpha process, where three helium nuclei fuse to form carbon-12.
36. (a) (2 points) The gravitational force is:

$$F = \frac{Gm_1m_2}{a^2}$$

- (b) (4 points) From the definition of center of mass, we know $m_1 a_1 = m_2 a_2$. By definition, $a = a_1 + a_2$.

Solving:

$$a_1 = \frac{m_2}{m_1} a_2$$

$$a = a_2 \left(1 + \frac{m_2}{m_1} \right) = a_2 \left(\frac{m_1 + m_2}{m_1} \right)$$

Therefore,

$$a_2 = \frac{m_1}{m_1 + m_2} a$$

When $m_1 = m_2$, the quantity $m_1/(m_1 + m_2) = 1/2$. So, the objects will be the same distance from the center of mass and $a_2 = \frac{a}{2}$.

- (c) (6 points) Start by equating the gravitational force and the centripetal force:

$$\frac{Gm_1 m_2}{a^2} = \frac{m_2 v_2^2}{a_2}$$

Then, plug in $a = a_2(m_1 + m_2)/m_1$:

$$Gm_1 m_2 \times \frac{m_1^2}{a_2^2 (m_1 + m_2)^2} = \frac{m_2 v_2^2}{a_2}$$

Solving for a_2 gives:

$$a_2 = \frac{Gm_1}{v_2^2} \left(\frac{m_1}{m_1 + m_2} \right)^2$$

- (d) (2 points) The period is the same for both objects. So, it will be the distance that an object travels divided by that object's speed. For a circular orbit, the distance the object travels is the circumference of the orbit. In the case of Object 2, it will be:

$$P = \frac{2\pi a_2}{v_2}$$

- (e) (4 points) Gravitational waves, which are propagating gravitational fields produced by the motion of massive objects. They are often called ripples of space-time curvature. The orbit of the Earth around the Sun is decaying in the same way, but the effect is much weaker since the Earth is not that massive and they are separated by such a large distance.
- (f) (10 points) From our knowledge of stellar evolution, we know that the mass of a neutron star should be between the Chandrasekhar limit and the Tolman-Oppenheimer-Volkoff limit. So, $1.4 M_{\odot} \lesssim m_{NS} \lesssim 3 M_{\odot}$. (People think the Tolman-Oppenheimer-Volkoff limit is between $1.5 M_{\odot}$ and $3 M_{\odot}$, probably around $2.1 M_{\odot}$.) To make the math easier, we can let $m_{NS} \approx 2 M_{\odot} = 4 \times 10^{30}$ kg. (Any value between 1.4 and $3 M_{\odot}$ would be accepted.) This means $m_{BH} \approx 4 M_{\odot} = 8 \times 10^{30}$ kg. After this, there are several ways to estimate a and P . Plugging in numbers to estimate a_2 :

$$\begin{aligned}
 a_2 &= \frac{Gm_1}{v_2^2} \left(\frac{m_1}{m_1 + m_2} \right)^2 \\
 a_2 &\approx \frac{7 \times 10^{-11} \times 4 \times 10^{30}}{(0.2 \times 3 \times 10^8)^2} \left(\frac{4 \times 10^{30}}{4 \times 10^{30} + 8 \times 10^{30}} \right)^2 \\
 a_2 &\approx \frac{7 \times 10^{-11} \times 4 \times 10^{30}}{(0.2 \times 3 \times 10^8)^2} \left(\frac{1}{3} \right)^2 \\
 a_2 &\approx \frac{28 \times 10^{19}}{0.36 \times 10^{16}} \left(\frac{1}{3} \right)^2 \\
 a_2 &\approx \frac{30 \times 10^{19}}{0.3 \times 10^{16}} \left(\frac{1}{3} \right)^2 \\
 a_2 &\approx \frac{100 \times 10^{19}}{10^{16}} \times 10^{-1} \\
 a_2 &\approx 10^4 \text{ m}
 \end{aligned}$$

However, this was for a_2 . To find a :

$$a = \frac{m_1 + m_2}{m_1} a_2 = 3a_2 \approx 3 \times 10^4 \approx 10^4 \text{ m}$$

The masses of the objects in our system were pretty similar, so it makes sense that a is the same order of magnitude as a_2 . Even if you assume a different value for the mass of the neutron star,

you should still get the same order of magnitude. Now, to estimate the period:

$$\begin{aligned}P &= \frac{2\pi a_2}{v_2} \\P &\approx \frac{2\pi \times 10^4}{0.2 \times 3 \times 10^8} \\P &\approx \frac{6 \times 10^4}{0.6 \times 10^8} \\P &\approx \frac{10 \times 10^4}{10^8} \\P &\approx 10^{-3} \text{ s}\end{aligned}$$

(g) (2 points) Newtonian mechanics assumes weak gravitational fields and low velocities ($v \ll c$) Here, we have strong fields and velocities of $0.2c$, requiring relativity for accurate results. Generally people say relativity becomes important when you are at speeds of $\gtrsim 0.1c$.

37. (5 points) Tiebreaker — answers will vary.