

Science Olympiad Astronomy C MIT Invitational

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Cambridge, MA



Directions:

- Each team will be given **50 minutes** to complete the test.
- There are three sections: **§A** (Short Answer) , **§B** (Free Response), and **§C** (JS9). Each section may have a mix of conceptual, quantitative, and DSO-related problems.

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Section A: Short Answer

Use the images in Image Set A to answer the following questions. The total number points for this section is 75 points.

1. [12 pts] Answer true (T) or false (F) for the following questions.
 - (a) [1 pt] What separates protostars from pre-main sequence stars is their energy source.
 - (b) [1 pt] Some stars have no (discernible) pre-main sequence phase.
 - (c) [1 pt] Pre-main sequence stars are less luminous than their main-sequence counterparts.
 - (d) [1 pt] Most T Tauri stars are in binary systems.
 - (e) [1 pt] Most stars are in binary systems.
 - (f) [1 pt] The outer layers of a star are more opaque than the layers closer to the core, which is why many stars have radiative transport in its outer layers.
 - (g) [1 pt] Radiative zones are unstable against the formation of convection cells.
 - (h) [1 pt] When stars are in hydrostatic equilibrium, they do not move on the H-R diagram.
 - (i) [1 pt] Brown dwarfs are more chemically similar to white dwarfs than red dwarfs.
 - (j) [1 pt] Due to the presence of high-energy jets, T Tauri stars do not have protoplanetary disks.
 - (k) [1 pt] In the planetesimal hypothesis of planet formation, planets are built up in hierarchical collisions.
 - (l) [1 pt] The total energy (i.e., the sum of the kinetic energy and potential energy) of a system consisting of one object in an elliptical orbit around another is positive.
2. [8 pts] This question will test your knowledge of a few key terms.
 - (a) [1 pt] Do H II regions evolve from molecular clouds, or do molecular clouds evolve from H II regions?
 - (b) [1 pt] What is causing this ionization or recombination?
 - (c) [2 pts] Sort the following terms from smallest to largest: star, protoplanetary disk, planetesimal, planetary embryo, terrestrial planet, gas giant, Sagittarius A*.
 - (d) [1 pt] The formation of Earth's moon likely resulted from a collision of two of which kind of object? Select from the list in the previous question.
 - (e) [1 pt] Which exoplanet detection technique has found the most exoplanets?
 - (f) [1 pt] Which exoplanet detection technique has found the second most exoplanets?
 - (g) [1 pt] What is the limiting factor for this detection technique, i.e. why has it detected fewer planets than the other method?
3. [3 pts] The following question concerns TW Hya.
 - (a) [1 pt] Identify which image on the image sheet corresponds to this DSO.
 - (b) [1 pt] Which telescope(s) collected data for this spectrum?
 - (c) [1 pt] Is the protoplanetary disk around TW Hya oriented face-on or edge-on to Earth?
4. [2 pts] The following question concerns Image K.
 - (a) [1 pt] Which DSO is depicted?
 - (b) [1 pt] In what portion of the EM spectrum was the data collected to make Image K?

5. [2 pts] The following question concerns Image M.
- (a) [1 pt] Which DSO is depicted?
 - (b) [1 pt] What type of star (e.g., Sun-like star, red giant, white dwarf, etc.) is the central object shown in this image?
6. [5 pts] The following question concerns the Carina Nebula.
- (a) [1 pt] Identify which image on the image sheet corresponds to this DSO.
 - (b) [1 pt] Which telescope(s) collected data for this image?
 - (c) [3 pts] The red, blue, and green in this image correspond to the glow of three different elements. Which elements correspond to which colors?
7. [2 pts] The following question concerns Image Q.
- (a) [1 pt] Which DSO is depicted?
 - (b) [1 pt] Through what exoplanet detection technique (e.g., radial velocity, transit, direct imaging, etc.) were the exoplanets in this system found?
8. [4 pts] The following question concerns Image S.
- (a) [1 pt] Which DSO is depicted?
 - (b) [2 pts] Which telescope collected data for this image? Which range of the EM spectrum did it operate in for this image?
 - (c) [1 pt] What do the green streaks and splotches represent?
9. [9 pts] The following question concerns Image C, which shows $F_{\text{planet}}/F_{\text{star}}$ for one of the DSOs on this year's rules.
- (a) [1 pt] Which DSO does this flux ratio spectrum correspond to?
 - (b) [2 pts] Which telescope collected data for this spectrum? Which range of the EM spectrum did it operate in for this image?
 - (c) [1 pt] Identify the image (on the image sheet) corresponding to the thermal emission spectrum of this DSO.
 - (d) [1 pt] If the DSO was a blackbody with emission spectrum as in the previous part, calculate its temperature.
 - (e) [2 pts] This DSO was notable (among other things) for the identification of a certain molecule in its spectrum in 2017. What molecule was it? How did they find out this molecule existed in the atmosphere?
 - (f) [2 pts] Explain the general shape of the spectrum in Image A (hint: think about blackbody physics).
10. [9 pts] The following question concerns Image J.
- (a) [1 pt] Which DSO is depicted?
 - (b) [2 pts] Which telescope(s) collected data for this spectrum? Which range(s) of the EM spectrum did it (they) operate in for this image?
 - (c) [2 pts] What do the red disk and blue spirals suggest about this system?
 - (d) [1 pt] What type of object is indicated by the green arrow in this image?

- (e) [2 pts] Assuming the object in the previous part orbits around the central star in this DSO with semimajor axis 93 AU, estimate its orbital period in (Earth) years.
 - (f) [1 pt] Which other image on the image sheet characterizes the central star of this system?
11. [11 pts] The following question concerns Luhman 16.
- (a) [2 pts] Identify which image on the Image Sheet corresponds to this DSO and the wavelength range of this image.
 - (b) [2 pts] Is this DSO moving towards or away from the Solar System? What parameter of the system would you measure to determine this?
 - (c) [2 pts] Which element's spectral lines did researchers use to estimate the age of this system?
 - (d) [1 pt] In which part (thin disk, thick disk, core, etc.) of the Milky Way does this system reside?
 - (e) [2 pts] Why did researchers originally hypothesize a potential exoplanet existed in this system?
 - (f) [2 pts] Identify the image depicting a periodogram of one of the components in this DSO and identify a reason for the multiple peaks in this graph.
12. [14 pts] The following question concerns Image R.
- (a) [1 pt] Examine the evolutionary track of the $3 M_{\odot}$ star. What kind of evolutionary track is this?
 - (b) [1 pt] What kind of pre-main sequence star is this?
 - (c) [1 pt] Examine the evolutionary track of the $0.2 M_{\odot}$ star. What kind of evolutionary track is this?
 - (d) [1 pt] What kind of pre-main sequence star is this?
 - (e) [2 pts] Now, examine the evolutionary track of the sun-like pre-main sequence star. What causes the "bend" in the evolutionary track?
 - (f) [2 pts] What force(s) are acting to stop the collapse of this star when it reaches the main sequence?
 - (g) [3 pts] What force(s) are acting to stop the collapse of this star at a later evolutionary stage (i.e. red giant)?
 - (h) [2 pts] Now, imagine a large protostar, say of mass $9 M_{\odot}$. Why do you think this star is not depicted on this H-R diagram?
 - (i) [1 pt] Finally, imagine a very small protostar, say of mass $0.05 M_{\odot}$. What kind of object will this protostar become?

Section B: Free Response

Points are shown for each question or sub-question, for a total of 80 points. If showing work, please box your final answer.

1. [14 pts] **Cloud collapse.** Stars are thought to be formed from the collapse of large clouds of gas and dust. But what conditions lead to cloud collapse? Let's take a look...
 - (a) [2 pts] Consider a simpler system: are two isolated protons gravitationally bound if, at time $t = 0$, one is stationary, the other is moving away at 1 m/s, and both are initially separated by 1 m?
 - (b) [2 pts] Now, consider the entire cloud, which consists of N uniformly distributed, spherically symmetric molecules of mass μ at a temperature T . The kinetic energy of the cloud is $K = \frac{3}{2}Nk_B T$. Write an expression for K in terms of its radius R and density ρ .
 - (c) [2 pts] The gravitational potential energy of our cloud is approximately $U = -\frac{3}{5}\frac{GM^2}{R}$, where M is the total mass of the cloud. Write an expression for U in terms of its radius R and density ρ .
 - (d) [4 pts] Using the virial theorem, solve for the *Jeans' length*, the radius at which the cloud is in equilibrium. Will the cloud collapse if its radius is larger or smaller than this value?
 - (e) [2 pts] Real clouds aren't uniformly dense, however. What area of the cloud is likely to be the densest? Would this non-uniformity in the density increase or decrease the Jeans' length?
 - (f) [2 pts] These clouds contain lots of H_2 , and as a homonuclear diatomic molecule, one would think that its energy is better described by $K = \frac{5}{2}Nk_B T$. It turns out that using $\frac{3}{2}$ is actually better due to how cold these clouds are. Why is that the case? Is the difference observationally significant?
2. [20 pts] **Disk instability.** As a star forms, the material around it forms a rotating disk of gas and dust in which multiple forces are at play, such as self-gravity, shear, and pressure.
 - (a) [2 pts] Many objects in the universe are spinning disks. In your own words, explain why.
 - (b) [2 pts] What causes shear forces in a protoplanetary disk? *Hint: think about Kepler's third law*
 - (c) [1 pt] Within the disk, which will dominate: the force with the shortest or longest timescale?

Consider a small chunk of the disk that has radius Δr at a distance R from the star. Ignoring dimensionless factors of order unity, the three timescales in the disk at R are

$$t_{\text{grav}} \sim \sqrt{\frac{\Delta r}{G\Sigma}} \quad t_{\text{shear}} \sim \frac{1}{\Omega} \quad t_{\text{pr}} \sim \frac{\Delta r}{c_s}$$

where G is the gravitational constant, Σ is the surface mass density of the disk, c_s is the speed of sound in the disk, and Ω is the angular velocity of the disk at R .

- (d) [3 pts] Physically, what is happening when t_{shear} and t_{pr} are both much less than t_{grav} ? Do these conditions make it easier or harder for a chunk of the disk to collapse?
- (e) [4 pts] One model for planet formation involves a local overdensity in this disk collapsing to form the planet. Using the timescales above, derive Toomre's instability criterion: $Q \sim \frac{c_s \Omega}{G\Sigma} < 1$.
- (f) [4 pts] Having $Q < 1$ at the start of the collapse is a necessary, but not sufficient condition for disk fragmentation. What else do we need, and why?
- (g) [2 pts] The characteristic length for gravitational instability can be written in the form $\lambda \sim c_s^\alpha G^\beta \Sigma^\gamma$. Using dimensional analysis, find the values of α , β , and γ . *Hint: set up a system of linear equations*
- (h) [2 pts] Suppose a chunk of the disk actually collapses. Write an expression (omitting dimensionless constants of order unity) for the mass of the object it initially forms using your answer to part (g) in terms of c_s , G , and Σ . *Hint: if needed, you can use dimensional analysis for this too.*

3. [14 pts] **Exoplanet warming.** In this question, we look at how the radiation laws shape a planet's temperature. For concreteness, we consider a system with a single solar-mass star (Solaris) and a single Earth-mass planet (Terra). Furthermore, Terra's radius is the same as that of Earth and Solaris' radius and temperature are the same as that of the Sun. Throughout this problem, assume that Terra emits predominantly in the infrared with emissivity $\varepsilon = 1$.
- (a) [2 pts] Let's begin by assuming both the Earth and star are blackbodies with zero albedo. Find the "goldilocks" zone, i.e. the minimum and maximum distance (in AU) that Terra can be from Solaris in order to be "habitable." Assume that "habitable" means that Terra's surface temperature is between the freezing point and boiling point of water.
 - (b) [2 pts] Refine your estimate assuming Solaris is still a black body, but now Terra has albedo α . Express your answer as the ratio between the maximum "habitable" distance with Terra having nonzero albedo to your upper bound from part (a). Your answer should be a function of α .

Now, assume Terra has an atmosphere with albedo α_a and a surface with albedo α_s .

- (c) [3 pts] Find the surface temperature of Terra if it is a distance d from Solaris.

Finally, just for fun, let's introduce humans (or a similar species also with low levels of intelligence) to Terra in the hopes that they learn from the outcome of this problem what not to do on Earth. Suppose Terra's atmosphere has a "natural" albedo (without human intervention) of $\alpha_{a0} = 0.3$. Terra's surface is 100% ice (albedo 0.9). Water's albedo is 0.1. Terra's average temperature is -1°C .

- (d) [1 pt] Find Terra's effective albedo from the surface and atmosphere.
- (e) [2 pts] Does the introduction of particulate matter in the atmosphere (soot, for example) increase or decrease Terra's temperature? For full credit, you should use a rigorous mathematical argument.
- (f) [4 pts] Terra's ice starts warming (rip). Is it possible for humans to be boiled alive if enough ice melts (purely on the grounds of the effective equilibrium temperature calculations we are assuming)? If so, calculate how much ice can melt until this boiling point is reached.

4. [32 pts] **Selection effect mayhem.** In this question, we will infer exoplanet properties from the transit method, as well as explore some of the selection effects in this detection method. Suppose we are aliens looking at the solar system through a telescope. You may assume circular orbits for all parts.

- (a) [4 pts] What is the probability that aliens are able to observe Earth in transit around the sun? That is, what is the probability that Earth's orbital inclination is close enough to edge-on to observe a transit?

For the rest of the problem, assume we take the perspective of an alien located in the ecliptic plane (i.e. Earth's orbital plane is edge-on to the alien).

- (b) [3 pts] How long is Earth's transit duration?
- (c) [4 pts] Consider the transit duration of other solar system bodies. We can parameterize the transit duration as $T_{trans} = T_0 \left(\frac{a}{1 \text{ AU}}\right)^\gamma$, where T_0 is a quantity with units of time and a is the orbital radius of the planet. Compute T_0 and γ .
- (d) [2 pts] Suppose the aliens invest one month (30 days) of continuous observing time on the sun. What is the probability that they observe a (partial or full) transit from Jupiter? You may assume Jupiter has a circular orbit with a radius of 5.20 AU.
- (e) [2 pts] Suppose the aliens invest one month (30 days) of continuous observing time on the sun. What is the probability that they observe a (partial or full) transit from the Earth?
- (f) [2 pts] Calculate the percent decrease in flux from the sun from Earth's transit.
- (g) [1 pt] Assuming the probability of transit detection is proportional to the fractional decrease in flux, how many times (more or less) likely is it to observe Jupiter transiting than Earth?
- (h) [4 pts] Describe qualitatively another way aliens would be able to infer the planet radius from the transit light curve, besides transit depth. List an advantage of this method.

Now, consider a hypothetical hot-Jupiter planet in our solar system: a planet the size and composition of Jupiter orbiting in Earth's orbital path.

- (i) [1 pt] What is the probability that a (partial or full) transit is observed?
- (j) [1 pt] Now also taking into account the selection effect from transit depth, how many times more likely is it to observe a hot-Jupiter like planet than an Earth-like planet?

For the remaining parts, refer to the exoplanet mass-period distribution diagram in Figure AA.

- (k) [2 pts] Explain the skew of planets discovered with the transit method towards high masses and low periods.
- (l) [2 pts] Why might the radial velocity method also be skewed towards higher-mass planets?
- (m) [4 pts] Does the radial velocity method have a selection bias in period? If so, is it towards lower or higher periods, and is this selection bias stronger or weaker than the bias from the transit method? Explain your reasoning.

Section C: JS9

Points are shown for each question or sub-question, for a total of 10 points.

1. [11 pts] GRS 1915+105 is certainly one of the most spectacular objects in the observable universe. Let's examine some observations of this fascinating object, to see why. Go to <https://chandra.harvard.edu/js9/index.html>
 - (a) [1 pt] Using the Unofficial Chandra Archive, how many times the satellite has observed this object?
 - (b) [1 pt] Load the ObsID 7485 observation into JS9. This object is a point source. Why does the data consist of an elongated "x" shape?
 - (c) [1 pt] Create a box region, rotate it and change its size to encompass most of the emission. Create an energy spectrum. Describe the result in one or two sentences.
 - (d) [1 pt] Assuming you can approximate this spectrum as a black-body, what physical "law" can you use if you want to estimate the temperature of this object?
 - (e) [1 pt] What is the approximate temperature of this object? (show 2 significant figures.)
 - (f) [1 pt] Create a light curve of this observation and describe the result in one or two sentences. (NOTE: do a "Server-side Analysis" Light Curve, NOT a "NSO Analysis" Light Curve.)
 - (g) [1 pt] In a new JS9 tab, load the ObsID 12462 observation. Create an appropriate region, do a server-side light curve and describe what you see, in one or two sentences.
 - (h) [1 pt] In a new JS9 tab, load the ObsID 16710 observation. Create an appropriate region, do a server-side light curve and describe what you see, in one or two sentences.
 - (i) [1 pt] In a new JS9 tab, load the ObsID 22213 observation. Create an appropriate region, but this time, do a "NSO Analysis" Light Curve. Describe the result.
 - (j) [1 pt] Search for periodic emission by doing a power spectrum, and zooming in on the peak. What period (in seconds) does this peak correspond to?