MODERN CORPORATE FINANCE

MS6621

END-TERM PROJECT:

MARKET RISK: ANALYSIS OF MARKET INDEX

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INTRODUCTION:

***CHOSEN INDEX*: NSE Oil & Gas**

The NSE Oil and Gas Index is a vital indicator of the performance of India's energy sector, capturing the market movements of major companies in the oil and gas industry. This index includes leading firms engaged in exploration, production, refining, and distribution of oil and gas, reflecting their market significance and operational scale. By analyzing the NSE Oil and Gas Index, investors and analysts can gain insights into the sector's trends, opportunities, and challenges. This report will explore the selection of securities in the index, identify the minimum variance portfolio using market data, and construct a Markowitz Market Index, ultimately comparing the theoretical portfolio weights with those of the actual index.

Selection of Securities Representing the NSE Oil and Gas Index

The NSE Oil and Gas Index comprises top-performing companies in India's oil and gas sector. The selection criteria for these securities include:

1. ***Market Capitalization:*** Companies with the highest market capitalizations are chosen to ensure the index reflects the most influential players, capturing significant market movements and trends within the sector.

2. ***Liquidity:*** Only highly liquid securities with substantial trading volumes are included to ensure ease of trading and accurate representation. High liquidity ensures that these stocks can be traded without causing large price fluctuations.

3. ***Sector Representation:*** The index includes companies from upstream (exploration and production), midstream (transportation and storage), and downstream (refining and marketing) segments to cover the entire value chain of the oil and gas industry. This comprehensive approach provides a holistic view of the sector's performance.

4. ***Financial Health:*** Companies with strong financial performance, including profitability, revenue growth, and robust balance sheets, are selected to ensure the index comprises resilient firms capable of withstanding market volatility.

5. ***Compliance:*** Firms adhering to industry regulations and environmental standards are preferred, promoting sustainable investment practices.

By applying these criteria, the NSE Oil and Gas Index includes a balanced and comprehensive set of securities. This selection process ensures that the index remains relevant and useful for investors seeking exposure to this critical industry, accurately reflecting the sector's overall market performance and economic impact.

DATA COLLECTION:

For this analysis, comprehensive market data was sourced from https://in.investing.com, spanning the period from ***April 1, 2021, to March 31, 2024***, covering three fiscal years: FY21, FY22, and FY23. The dataset for each security included in the NSE Oil and Gas Index comprises 36 months of data, ensuring a thorough and consistent time frame for analysis.

The data collected for each security consists of the following elements:

Adjusted Closing Prices: These prices have been adjusted for corporate actions such as dividends, stock splits, and rights offerings. Adjusted prices provide a more accurate reflection of the security's value over time and are crucial for calculating returns and volatility.

Trading Volume: The number of shares traded each month. Volume data helps assess the liquidity of the securities, which is an important factor in portfolio construction.

Percentage Returns: The monthly percentage change in the adjusted closing price. This metric is essential for understanding the performance and risk profile of each security.

By using this detailed and high-quality dataset, we can perform a robust analysis to identify the minimum variance portfolio. The data provides a solid foundation for applying Markowitz's Modern Portfolio Theory, enabling the construction of an optimal portfolio that balances risk and return. This analysis will also allow us to compare the theoretical portfolio weights with those of the actual NSE Oil and Gas Index, providing insights into the index's composition and performance dynamics.

IDENTIFYING MINIMUM VARIANCE PORTFOLIO:

The concept of the minimum variance portfolio is a cornerstone of Modern Portfolio Theory (MPT) introduced by Harry Markowitz. This theory emphasizes the importance of diversification in investment to minimize risk while maximizing returns. The minimum variance portfolio represents the portfolio with the lowest possible risk, given a set of expected returns.

The minimum variance portfolio is defined as the portfolio that provides the lowest level of volatility or risk for a given level of expected return. It is achieved by allocating assets in such a way that the portfolio's overall variance is minimized.

Harry Markowitz introduced the concept of mean-variance optimization, which involves selecting the optimal combination of assets to achieve the desired level of return with the least amount of risk. The minimum variance portfolio is a key result of this optimization process.

Covariance measures the degree to which the returns of two assets move in relation to each other. Correlation, derived from covariance, provides insights into the strength and direction of the relationship between asset returns. In constructing the minimum variance portfolio, assets with low covariance or correlation are typically selected to enhance diversification.

CODE EXPLANATION:

```
In [3]: return_portfolio = pd.read_csv('securities.csv')
        return_portfolio
```

Loading the dataset into the IDE for further analysis.

```
In [27]: variance_matrix = returns_df.cov() * 12
         means = returns_df.mean()
         mu = (means).values * 12
```

Method .cov( ) returns variance-covariance matrix, method .means( ) return mean return of each security. Factor of 12 is multiplied to get annual mean returns and covariances.

After calculating annual return and covariances, we will calculate return and risk of an equally weighted portfolio. This is done by the code snippet shown below:

```
In [30]: weight = [1/15] * 15
         portfolio_return = mu.dot(weight)
         portfolio_variance = np.transpose(weight) @ variance_matrix @ weight
         portfolio_risk = np.sqrt(portfolio_variance)
```

For an equally weighted portfolio:

Weight of each security = 1/15

Return = 22.49%

Risk = 19.39%

```
In [32]: port_returns = []
         port_volatility = []
         port_weights = []
         num_assets = 15
         num_portfolios = 10000
         for port in range(num_portfolios):
             weights = np.random.random(num_assets)
             weights = weights / sum(weights)
             port_weights.append(weights)
             returns = mu.dot(weights)
             port_returns.append(returns)
             var = variance_matrix.mul(weights, axis=0).mul(weights, axis=1).sum().sum()
             sd = np.sqrt(var)
             port_volatility.append(sd)
```

Above code simulates 10,000 portfolios, by randomly generating weights for each security, such that sum of weights is always equals to 1. Return and risk for each such portfolio is also calculated in the same snippet. Later storing data in the form of a dataframe, which is shown in the below snippet:

```
In [33]: data = {'Returns': port_returns, 'Volatility': port_volatility}
         for counter, symbol in enumerate(returns_df.columns.tolist()):
             data[symbol + ' weight'] = [w[counter] for w in port_weights]
         portfolios_V1 = pd.DataFrame(data)
```

Now out of simulated 10,000 portfolios our aim is to find the one which has minimum risk level or minimum variance portfolio, which is achieved using .idxmin( ) method, show in below snippet of code:

```
In [55]: min_vol_port = portfolios_V1.iloc[portfolios_V1['Volatility'].idxmin()]
         print("Return, Risk and weights of securities for minimum variance portfolio")
         print(min_vol_port)
```

```
Return, Risk and weights of securities for minimum variance portfolio
Returns             0.162929
Volatility          0.166442
ONGC weight         0.055798
IGL weight          0.173424
GUJGASLTD weight    0.106558
MGL weight          0.049115
GSPL weight         0.003759
BPCL weight         0.012855
HINDPETRO weight    0.003953
AEGISCHEM weight    0.040861
OIL weight          0.048975
PETRONET weight     0.165131
RELIANCE weight     0.142890
CASTROLIND weight   0.045540
ATGL weight         0.003934
IOC weight          0.023792
GAIL weight         0.123415
Name: 7448, dtype: float64
```

For minimum variance portfolio:

Risk = 16.57%
Return = 16.77%
Weight of each security is shown in above code's output.

Now we want to plot efficient frontier, for that purpose we use below given code snippet:

```
In [36]: w_min = np.ones(num_assets) / num_assets # Minimum weight portfolio
         w_sharpe = np.linalg.inv(variance_matrix).dot(mu) / np.sum(np.linalg.inv(variance_matrix).dot(mu))
         num_ports = 200
         gap = (np.amax(mu) - ret(mu, w_min)) / num_ports
         all_weights = np.zeros((num_ports, num_assets))
         ret_arr = np.zeros(num_ports)
         vol_arr = np.zeros(num_ports)
         for i in range(num_ports):
             port_ret = ret(mu, w_min) + i * gap
             double_constraint = LinearConstraint([np.ones(num_assets), mu], [1, port_ret], [1, port_ret])
             x0 = w_min
             fun = lambda w: np.sqrt(np.dot(w, np.dot(variance_matrix, w)))
             a = minimize(fun, x0, method='trust-constr', constraints=double_constraint, bounds=[(0, 1)] * num_assets)
             all_weights[i, :] = a.x
             ret_arr[i] = port_ret
             vol_arr[i] = vol(a.x, variance_matrix)
```

```
In [37]: sharpe_arr = ret_arr / vol_arr
```

```
In [38]: plt.figure(figsize=(20, 10))
         plt.scatter(vol_arr, ret_arr, c=sharpe_arr, cmap='viridis')
         plt.colorbar(label='Sharpe Ratio')
         plt.xlabel('Volatility')
         plt.ylabel('Return')
         plt.title('Efficient Frontier')
```

This code constructs the efficient frontier by calculating portfolio weights that offer a range of expected returns and their corresponding volatilities. Initially, it sets up the portfolio with equal weights and calculates the maximum Sharpe ratio portfolio. It determines the number of portfolios to simulate and calculates the increment in returns for each step along the frontier.

Arrays are prepared to store the portfolio weights, returns, and volatilities for each portfolio on the efficient frontier. A loop runs through the specified number of portfolios, and for each iteration, the target return for the current portfolio is calculated. Constraints are defined to ensure the portfolio weights sum to one and achieve the target return.

The portfolio weights that minimize volatility are found using the minimize function, and the optimized weights, expected return, and portfolio volatility are stored in the respective arrays.

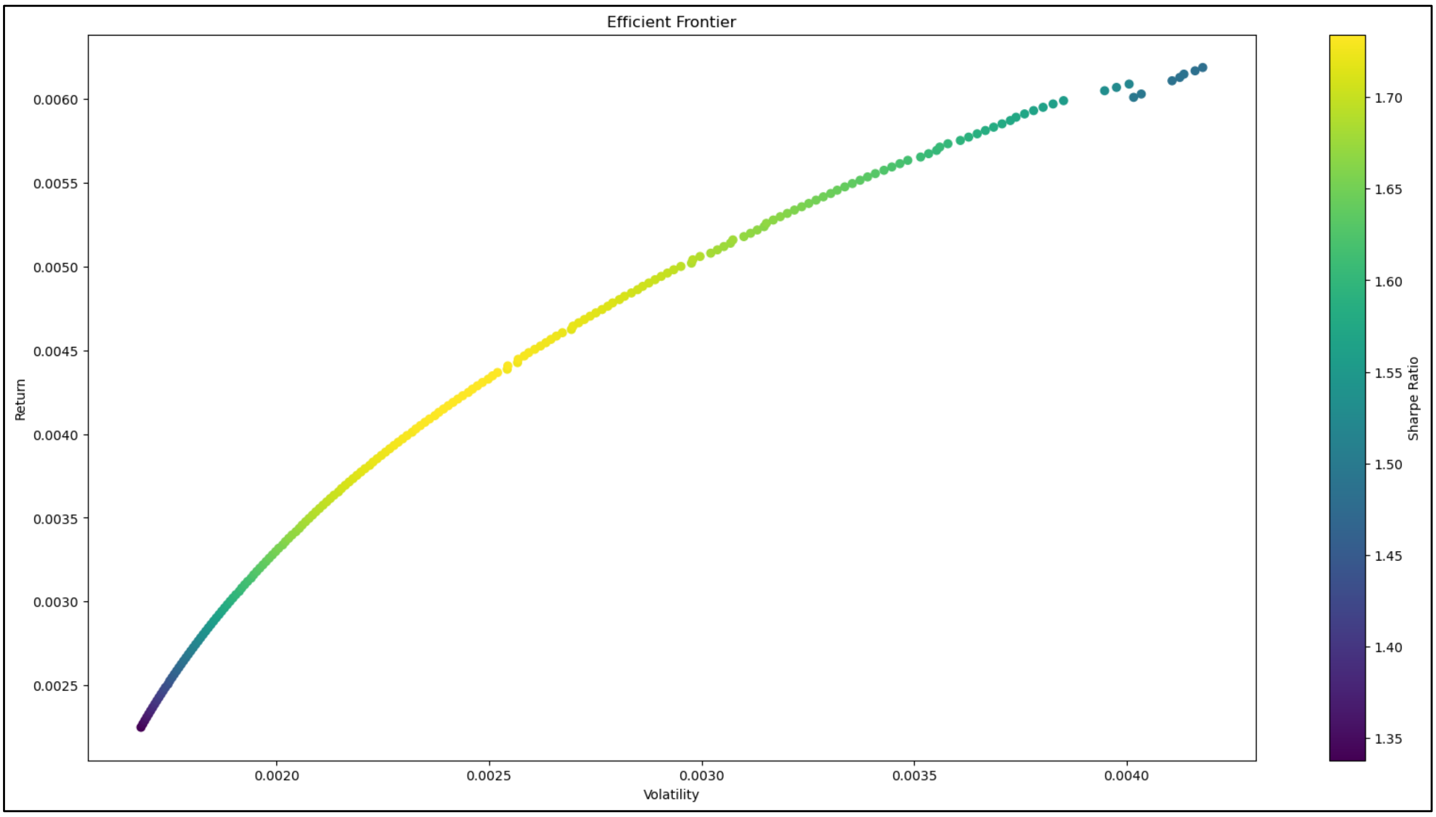

Fig 1. Efficient Frontier

CONSTRUCTING MARKOWITZ MARKET INDEX:

To construct the Markowitz Market Index using an indifference curve, we focus on identifying the optimal portfolio that maximizes the investor's utility. This involves the following steps:

1. *Efficient Frontier:* This represents the set of optimal portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return.

2. *Indifference Curve:* This curve represents the combinations of risk and return that provide the same level of satisfaction or utility to the investor. Each investor has their own indifference curve based on their risk tolerance.

3. *Utility Maximization:* The optimal portfolio is found at the tangency point between the efficient frontier and the highest possible indifference curve. This point maximizes the investor's utility by balancing their return expectations and risk tolerance.

Assumptions for indifference curve:

1. ***Risk Aversion Coefficient (A):*** We assume a specific value for the risk aversion coefficient A. In this case, A=3 is used, which represents a moderately risk-averse investor. The coefficient A affects the curvature and position of the indifference curves.

2. ***Utility Function:*** The utility function used is U = E(R) – $A\sigma^2/2$, where E(R) is the expected return, $\boldsymbol{\sigma^2}$ is the variance, and A is the risk aversion coefficient. This assumes that investors derive utility from returns and disutility from risk (variance).

```
In [93]: A = 3  # Risk aversion coefficient

         for k in range(3):
             utility = ret_arr - A * (vol_arr ** 2) + k * 0.01  # Correct utility function for risk-averse investor
             plt.plot(vol_arr, utility, linestyle='--', label=f'Indifference Curve {k+1}')

         plt.legend()
         plt.grid(True)
         plt.show()
```

Above code plots 3 different indifference curves, as shown below:

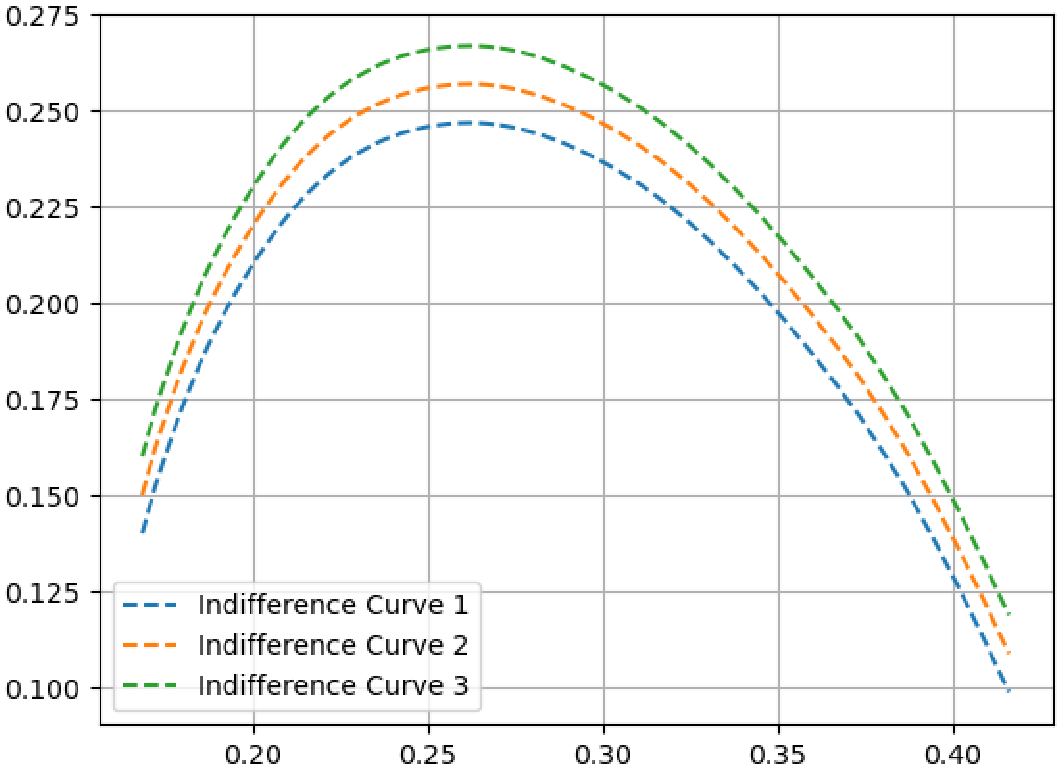

Fig 2. Indifference Curves

Based on the highest utility indifference curve, we'll find the optimal portfolio weights, which is done by solving the indifference curve and efficient frontier at the point of tangency, code snippet for same is provided below:

```
In [92]: # Identify the optimal portfolio based on the highest utility
         optimal_utility = ret_arr - A * (vol_arr ** 2)
         optimal_index = np.argmax(optimal_utility)
         optimal_weights = all_weights[optimal_index]
         print("Optimal Weights for Markowitz Market Index:", optimal_weights)

         Optimal Weights for Markowitz Market Index: [2.04913431e-01 2.31937383e-06 1.21064900e-05 3.02560527e-06
          5.70575098e-04 3.16994830e-06 5.57939578e-06 1.53493107e-01
          4.31739616e-01 4.37019539e-06 5.21156930e-06 4.73834559e-06
          5.93531417e-02 1.49871347e-01 1.82602150e-05]
```

After finding weights for optimal portfolio, we wish to find return and risk level of that portfolio. Below given code snippet calculates risk and return for optimal portfolio, and plots the point of optimal portfolio on efficient frontier.

<u>For optimal portfolio:</u>

Risk = 26.18%
Return = 45.25%

In [90]:
```
# Plot the optimal portfolio
optimal_utility = ret_arr - A * (vol_arr ** 2) + 0.02
optimal_index = np.argmax(optimal_utility)
optimal_return = ret_arr[optimal_index]
optimal_risk = vol_arr[optimal_index]
plt.scatter(vol_arr, ret_arr, c=sharpe_arr, cmap='viridis', label='Portfolios')
plt.plot(optimal_risk, optimal_return, marker='*', color='red', label='Optimal Portfolio')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Return')

plt.legend()

ax.legend(fontsize=12)
plt.tight_layout()
plt.show()

print(f"Risk of Optimal Portfolio: {optimal_risk:.4f}")
print(f"Return of Optimal Portfolio: {optimal_return:.4f}")
```

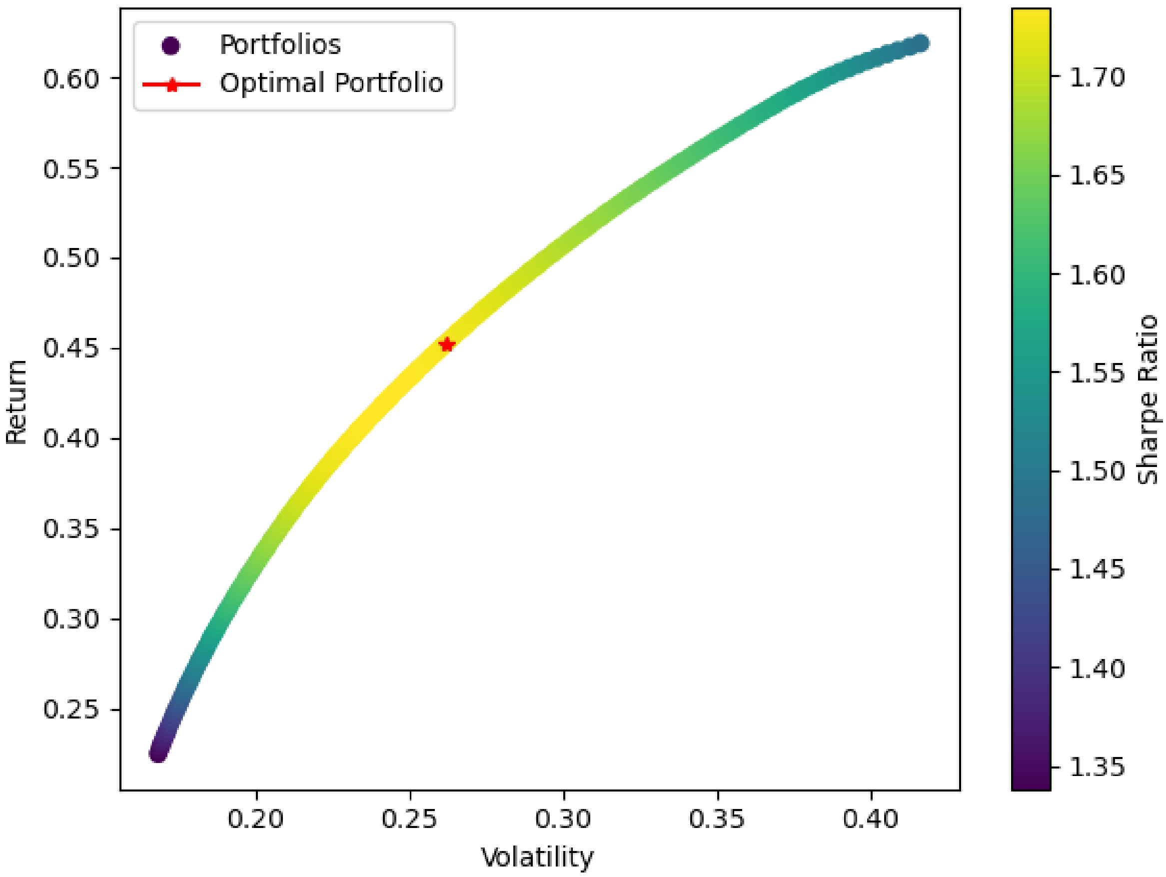

Fig 3. Efficient Frontier and optimal portfolio

THEORETICAL vs ACTUAL WEIGHTS:

In the previous section we calculated weight of each stock as per Markowitz optimal portfolio, actual weight of each stock is obtained from https://www.smart-investing.in/indices-bse-nse.php?index=NOILGAS (based on current market capitalization). Let's compare both of them:

Company Name	Theoretical	Actual
Adani Total Gas Ltd.	5.94E-02	3.11%
Aegis Logistics Ltd.	1.53E-01	0.69%
Bharat Petroleum Corporation Ltd.	3.17E-06	4.23%
Castrol India Ltd.	4.74E-06	0.60%
GAIL (India) Ltd.	1.83E-05	4.25%
Gujarat Gas Ltd.	1.21E-05	1.18%
Gujarat State Petronet Ltd.	5.71E-04	0.52%
Hindustan Petroleum Corporation Ltd.	5.58E-06	2.22%
Indian Oil Corporation Ltd.	1.50E-01	7.21%
Indraprastha Gas Ltd.	2.32E-06	0.96%
Mahanagar Gas Ltd.	3.03E-06	0.40%
Oil & Natural Gas Corporation Ltd.	2.05E-01	10.88%
Oil India Ltd.	4.32E-01	2.17%
Petronet LNG Ltd.	4.37E-06	1.46%
Reliance Industries Ltd.	5.21E-06	60.13%

Table 1: Optimal vs Actual weights

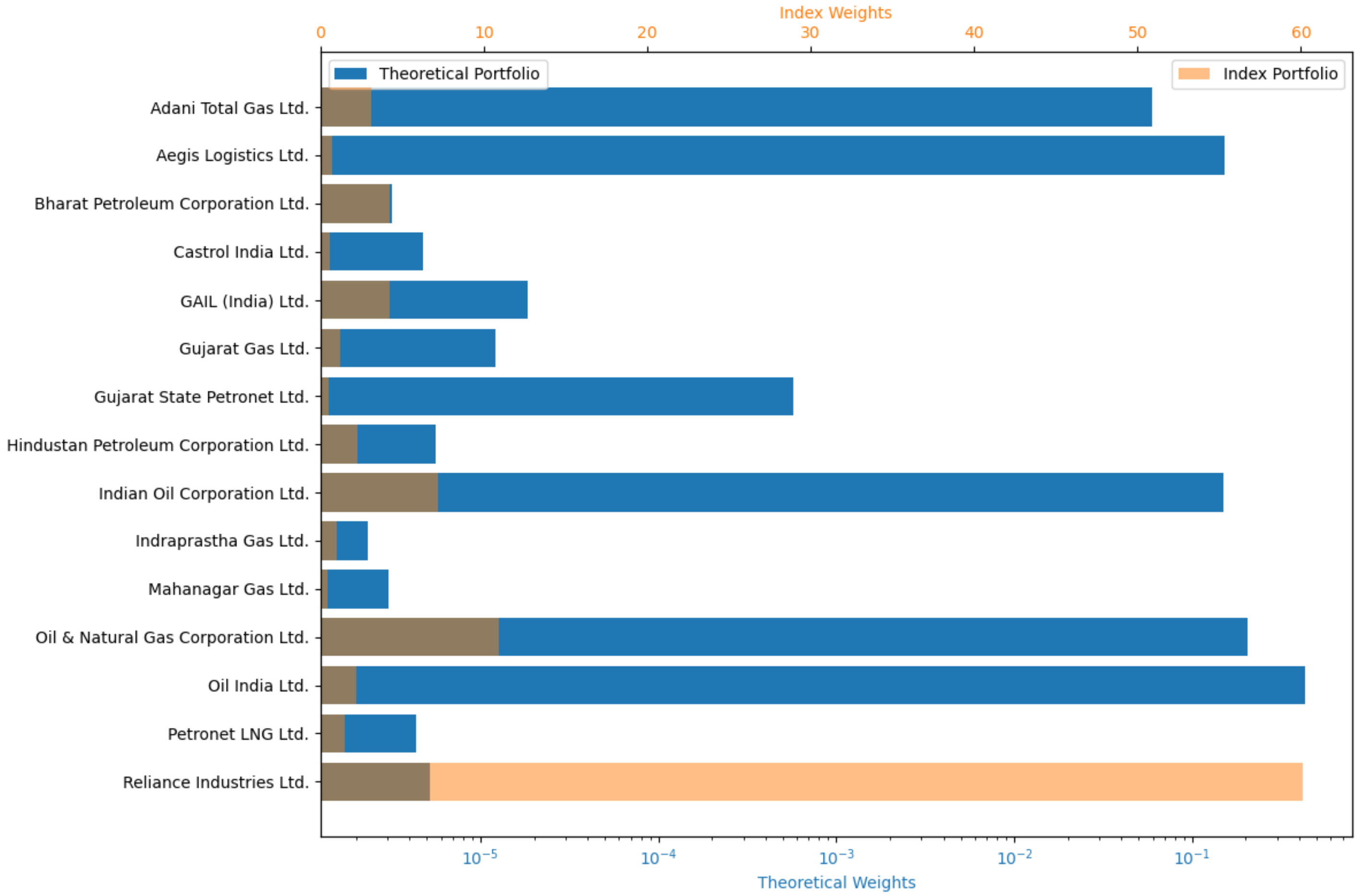

Fig 4. Theoretical vs Actual Weights (Theoretical weight scale is on logarithmic scale)

1. ***<u>Significant Differences in Weights:</u>*** There are notable differences between the weights assigned to various companies in the theoretical portfolio and the index. For instance, Oil India Ltd. has a weight of 43.2% in the theoretical portfolio, but only 2.17% in the index, while Reliance Industries Ltd. has a weight of 0.0005% in the theoretical portfolio but a whopping 60.13% in the index.

2. ***<u>Concentration vs. Diversification</u>***: The theoretical portfolio appears to be more concentrated, with a few companies having relatively high weights (e.g., Oil India Ltd. at 43.2%, Oil & Natural Gas Corporation Ltd. at 20.5%, and Indian Oil Corporation Ltd. at 15%). In contrast, the index seems to be more diversified, with weights distributed across a broader range of companies.

3. ***<u>Sector Exposure:</u>*** Both the theoretical portfolio and the index are likely focused on the oil and gas sector, given the companies included. However, the theoretical portfolio may have a different sector exposure or risk-return profile compared to the index, based on the varying weights assigned to individual companies.

4. ***<u>Optimization Objective:</u>*** The weights in the theoretical portfolio are likely derived from an optimization process that aims to achieve a specific objective, such as maximizing returns, minimizing risk, or optimizing the risk-return trade-off. The index, on the other hand, may be constructed based on different criteria, such as market capitalization or equal weighting.

5. ***<u>Rebalancing Considerations:</u>*** If the theoretical portfolio is intended for active management, the weights may need to be periodically rebalanced to maintain the desired risk-return characteristics. Indices, on the other hand, typically follow a predetermined rebalancing schedule (e.g., quarterly or annually) to maintain their intended composition.