Dynamics and control of a quadrotor with 2 DOF manipulator

Abstract

This paper focusses on the behavior of a quadrotor with a 2 degree of freedom manipulator attached to its bottom. The model is developed using the Lagrangian formulation of the combined system of quadrotor and the manipulator and coupled dynamic equations are derived. This is a MIMO system. To control the system, two control schemes are analyzed. The objective is to control the system's states to follow a specified trajectory.

Introduction

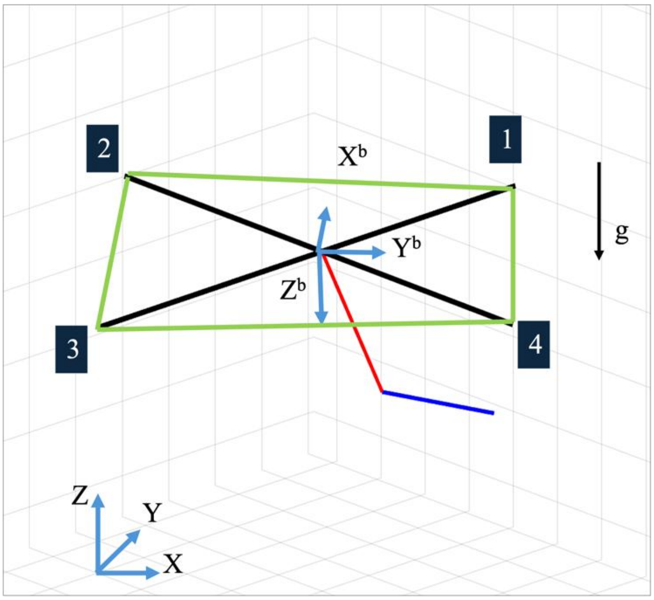
Ariel robots are used for a variety of tasks due robust maneuverability and large workspace. These types robots can operate in hard-to-reach places. Hence these robots are a huge attraction in industry as well as in research labs. The most common type of ariel robots in a quadcopter with a simple 2 DOF manipulator. This is the point of discussion in this paper.

The previous research on aerial manipulation can be divided into 2 categories. The first approach is to install a gripper at the bottom of a UAV to hold the payload. The second approach is to suspend the payload with cables. However, these approaches have limitations for manipulation. In the first category as a gripper is used, payloads get rigidly connected to the body of the UAV, due to which we constrain the dimensions of the payload. As well as the attitude of the payload is restricted to the attitude of the UAV and the accessible range of the gripper is limited because of the UAV body and the rotors. In the second approach it becomes difficult to directly control the movement of the payload, because a cable cannot always control the trajectory of the payload. To overcome the problems faced by the above approaches we can use a robotic manipulator with a gripper. A manipulator has much larger workspace and also actively interact with the environment. Hence the UAV equipped with a robotic manipulator can be used for specialized missions.

The aim of this paper is only the analysis drone and manipulator system and not the gripper. The dynamics of the combined system of drone and manipulator is derived using the Euler-Lagrangian formulation.

The dynamics of the drone and the manipulator is coupled, and thus the control system is designed with high enough complexity. Two different types of controllers are discussed. They are a PD controller with feedback linearization and adaptive sliding mode controller (ASMC). The results are reported. The paper is arranged in the following way. The governing equation of the system is derived using the mathematical model. Two control laws are analyzed. The resulting plots obtained from the two controllers are reported.

Mathematical model of the system



Kinematics

The kinematics of the combined system is obtained in the inertial frame. The moment of inertia matrix of the quadrotor in its principal body frame is assumed to be diagonal. The manipulator is assumed to be sticks having a fixed mass. The manipulator is also taken to be rigid and unbreakable.

The position of the center of mass of the quadrotor in the inertial is taken to be

$$p = [x, y, z]^T$$

The Euler angles of the quadrotor are taken as

$$\Phi = [\phi, \theta, \psi]^T$$

The two joint angles of the manipulator are

$$\eta = [\eta_1, \eta_2]^T$$

The variable containing the generalized coordinates is defined as

$$q = [p^T, \Phi^{\mathrm{T}}, \eta^T]^T$$

The time derivative of p gives the velocity of the center of mass of the quadrotor, and the translational velocity in the body fixed frame is given as $\dot{p^b}$. They are related by the rotation matrix $R_t \in \mathrm{SO}(3)$ as

$$\dot{p} = R_t \, \dot{p^b}$$

The angular velocity of the quadrotor in the inertial and body fixed frame are defined as

$$\omega = \left[\omega_x, \omega_y, \omega_z\right]$$

and

$$\omega_b = [\omega_x^b, \omega_y^b, \omega_z^b]$$

respectively.

ω and ω_b are also related by R_t. Also, the time derivative of the Euler angles $\dot{\Phi}$ can be mapped to ω by the transformation matrix T. The relations among $\dot{\Phi}$, ω and $\omega_b$are

$$\omega = R_t \omega_b$$

$$\omega = T\dot{\Phi}$$

$$\omega_b = R_t^T T \dot{\Phi} = Q\dot{\Phi}$$

Let p_i^b be the position of the center of mass of link i = 1, 2 in the body-fixed frame 0_b. Then, p^i, which is the position of the center of mass of the link i in the inertial frame O_I, is related to p_i^b by

$$p_i = p + R_t p_i^b$$

In addition, the translational and angular velocity of each manipulator link are related by the time derivative of the joint variables $\dot{\eta}$ and the Jacobian matrix $J_t \in \mathcal{R}^{2\times 2}$ and $J_r \in \mathcal{R}^{2\times 2}$ respectively.

$$\dot{p_i^b} = J_t \dot{\eta}$$

$$\omega_i^b = J_r \dot{\eta}$$

With equations (7) and (8), the translational and angular velocity of each link in the inertial frame are computed as

$$\dot{p_i} = \dot{p} + \dot{R}_t p_i^b + R_t \dot{p_i^b}$$

$$\dot{p_\iota} = \dot{p} + \widehat{\omega_b} R_t p_i^b + R_t J_t \dot{\eta}$$

$$\omega_i = \omega + R_t J_r \dot{\eta}$$

For simplicity, equations (2), (4), (9) and (10) are rewritten in the following matrix form.

$$\dot{p} = [I_{3\times 3}\ 0_{3\times 3}\ 0_{3\times 2}]\ \dot{q} = M_{t,b}\dot{q}$$

$$\omega = [0_{3\times 3}\ T\ 0_{3\times 2}]\ \dot{q} = M_{r,b}\ \dot{q}$$

$$\dot{p_\iota} = \left[I_{3\times 3} - \left(R_t p_i^b\right)^T R_t J_{t,i}\right]\ \dot{q} = M_{t,i}\ \dot{q}$$

$$\omega_i = \left[0_{3\times 3}\ T\ R_t J_{r,i}\right]\ \dot{q} = M_{r,i}\ \dot{q}$$

where the subscript ·×· represents the size of I and 0. Also, ∧ is the operator that converts a vector into a skew-symmetric matrix. With the above relations, the time derivative of the generalized coordinate variable vector q˙is easily mapped into the translational and angular velocities of the quadrotor and each link in the inertial coordinate frame.

Dynamics

To derive the dynamics of the combined system, the

Lagrange-D'Alembert equation is used.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = \tau + \tau_{ext}$$

$$\mathcal{L} = \mathcal{K} - \mathcal{U}$$

where $\mathcal{K}$ and $\mathcal{U}$ are the total kinetic and potential energy of the combined system and τ represents the generalized force. τ_{ext} indicates external disturbance applied to the system.

The total kinetic energy is the sum of the kinetic energy of individual components, which are the quadrotor and two links. The total kinetic energy $\mathcal{K}$ and its components are as follows:

$$\mathcal{K} = \mathcal{K}_b + \sum_{i=1}^{2} \mathcal{K}_i$$

$$\mathcal{K} = \frac{1}{2}\dot{p^T} m_b \dot{p} + \frac{1}{2}\dot{\Phi}^T T^T R_t I_b R_t^T T \dot{\Phi}$$

$$\mathcal{K}_i = \frac{1}{2}\dot{p_\iota}^T m_i \dot{p_\iota} + \frac{1}{2}\omega_i^T (R_t R_i) I_i (R_t R_i)^T \omega_i$$

where m is the mass and I is the inertia matrix. The subscripts b and i indicate the corresponding values are with respect to O_b and O_i respectively.

Likewise, the total potential energy is the sum of potential energy of each component. The total potential energy U and its components are described by the equations below:

$$\mathcal{U} = m_b g e_3^T p + \sum_{i=1}^{2} m_i g e_3^T (p + R_t p_i^b)$$

where the first and last terms are the potential energies of the quadrotor and link i, respectively. e_3 is the unit vector $[0\ 0\ 1]^T$ and g is the gravity.

By substituting equations (17), (18) and (21) in equation (16), the dynamics equation which includes all components as one system can be derived as the following:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_{ext}$$

where $M(q) \epsilon\ \mathcal{R}^{8\times 8}$ is the inertia matrix which is positive definite and symmetric. $C(q,\dot{q})$ is the Coriolis matrix and $C(q,\dot{q})\dot{q}$ represents the Coriolis and centrifugal force terms. In addition, $G(q)$ includes gravity effects at each joint.

Control law

1. PD – controller with Feedback Linearization

Our objective of control is to track a trajectory which will be already provided to the quadrotor as a function. Here, we are using a PD-controller with feedback linearization.

As we know that the error caused by any disturbance comes to zero the fastest when our system is critically damped. That is why we choose our control input (τ), such that the system is critically damped even if the M, C, G matrices are not constant with time.

First let us consider a simple 1-DOF mass-spring-damper system. The governing equation for this system is:

$$M\ddot{q} + C\dot{q} + kq = F$$

Choosing a PD-controller:

$$F = -K_P q - K_d \dot{q}$$

Substituting the above equation in the governing equation, we get:

$$M\ddot{q} + (C + K_d)\dot{q} + (k + K_p)q = 0$$

Now, using the relation for critically damped systems ($\zeta = 1$):

$$C' = 2\sqrt{MK'}$$

Where, $C' = C + K_d$ and $K' = K + K_p$

$$C + K_d = 2\sqrt{M(k + K_p)}$$

Therefore,

$$K_d = -C + 2\sqrt{M(k + K_p)}$$

Now, no matter what the values of K and C are, if values of K_p and K_d are chosen such that the above relation is satisfied, we will get a critically damped system.

In a 1-DOF system we predict two values (K_p and K_d) with the help of one relation. In a 2-DOF system, K_p and K_d become 2×2 matrices, so now we need to predict the values of 8 control gains using 2 relations. Similarly for a N-DOF system, we will have to predict $2N^2$ control gains with the help of N equations. This is a very cumbersome method. To overcome this issue, we use Feedback Linearization.

Feedback Linearization:

Let us consider a N-DOF system,

$$M\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

We choose,

$$\tau = M(-K_p q - K_d \dot{q}) + C(q,\dot{q})\dot{q} + G(q)$$

Substituting the above expression of τ in the governing equation of our system,

$$M\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = M(-K_p q - K_d \dot{q}) + C(q,\dot{q})\dot{q} + G(q)$$

$$\Rightarrow M(\ddot{q} + K_d\dot{q} + K_p q) = 0$$

Now as $M \neq 0$,

$$\ddot{q} + K_d\dot{q} + K_p q = 0$$

Here, K_p and K_d are user chosen matrix, so we will specifically choose it to be diagonal. Choosing a diagonal matrix reduces our unknown control gains to $2N$, while the no, of relations among them will still be N. So, now we need to predict only N control gains. The other N will be found out using the relations between these control gains. This makes our task much easier considering the earlier situation where we had to predict N^2 control gains.

Also using a diagonal matrix, our equations get decoupled as following,

$$\ddot{q}_i + K_{d_i}\dot{q}_i + K_{p_i} q_i = 0$$

where $i = 1,2,3 \ldots, N$.

Now we want to make these N 1-DOF systems critically damped. Therefore, we will use our condition for critically damped systems, which is:

$$C' = 2\sqrt{MK'}$$

Here $C' = K_{d_i}$, $K' = K_{p_i}$ and $M = 1$;

This gives us the N relations between our $2N$ control gains as the following,

$$K_{d_i} = 2\sqrt{K_{p_i}}$$

2. Adaptive Sliding Mode Controller

ASMC is a control scheme that adjusts the disturbances and uncertainties to improve performance, stability and reliability. It uses high frequency switching control to drive a system's trajectory to specific constraints.

The main aim of this controller is to make the system linear. This is achieved by defining the following quantity

$$\dot{x} + bx = s$$

where, x is the vector of states of the system at any instant.

The quantity, s is related to the linearity of the system in this way. The system tends to be linear when s tends to 0. The phase space is modified in a way to force the system states to lie on a straight line as shown in the fig [2].

Ultimately the tendency of the system is to make $\dot{x} = 0$ and $\dot{x} = 0$. To follow a desired trajectory, it is required that at any time, the error between the current states and the desired states at that instant, is small. Hence the equation [] can be modified as

$$\dot{e} + be = s$$

where, $e(t) = x(t) - x_d(t)$

where, $x_d(t)$ is the desired trajectory.

This definition of s is used for all subsequent analysis.

Now, to make s to vanish, the following is proposed, according to ASMC method,

$$s\dot{s} < 0$$

The control input to the system are forces, which are second derivatives of the states, and hence, the errors. Or,

$$(\dot{e} + \Lambda e)(\ddot{e} + \Lambda \dot{e}) < 0$$

Also,

$$\tau \propto \ddot{e} \propto \frac{ds}{dt}$$

where, τ is the control input.

According to this, the following control inputs are defined,

$$U_1 = K.sgn(s)$$
$$U_2 = A.s$$
$$U_3 = \widehat{\Delta}$$

where, $\widehat{\Delta} = -\int_0^t s\,dt$ up to the current time instant. We append the control inputs to our basic control law to obtain,

$$\tau = M\ddot{e} + C\dot{q} + G - K.sgn(s) - A.s + \widehat{\Delta}$$

where, a negative sign is placed on the terms due direct proportionality of control inputs with the errors.

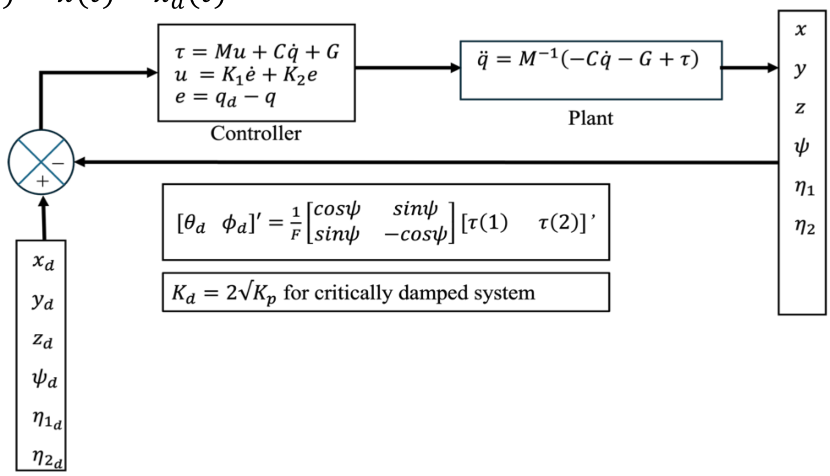


Fig1: A schematic to the control loop indicating how the system states are updated and the effect of input forces on the system.

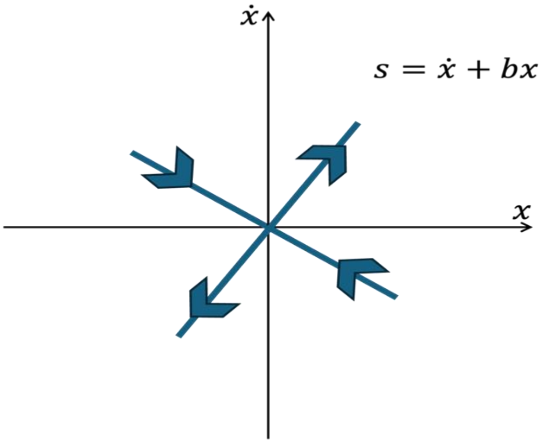


Fig2: A phase diagram representing the condition b>0 required for the system's states to converge.

Results

1. PD with feedback linearization

A. Trajectory 1: (spiral)

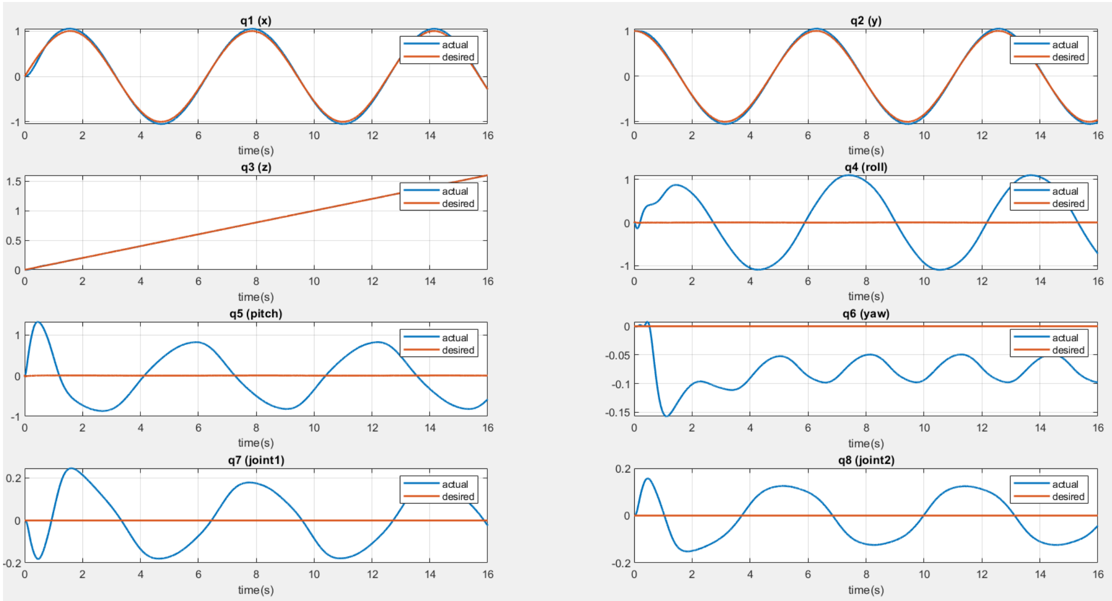


Fig 3: Plot of system states with time.

B. Trajectory 2: (In-plane circular)

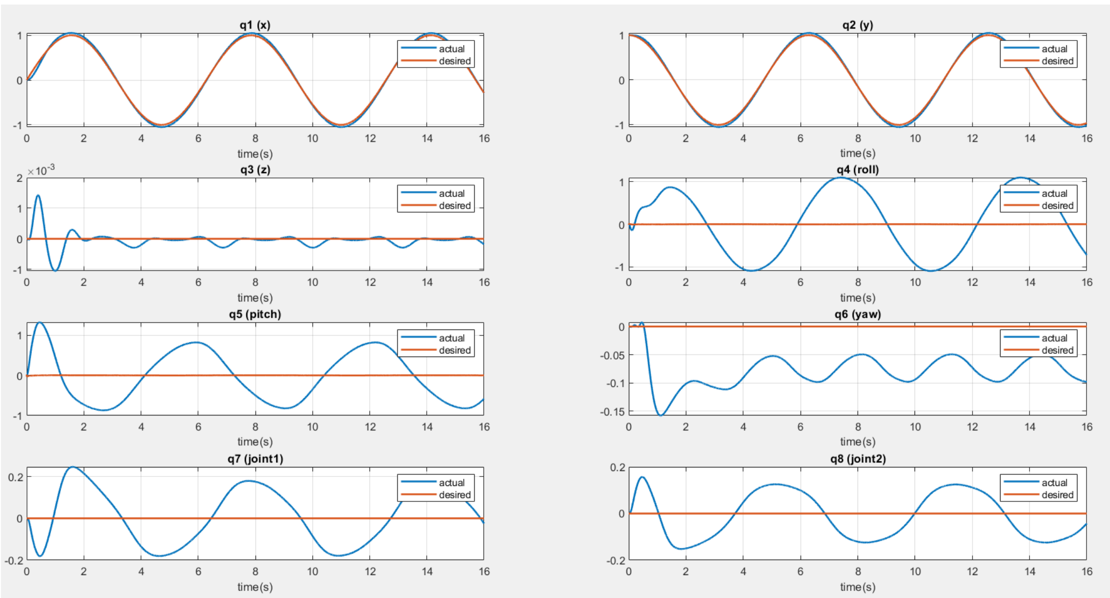


Fig 4: Plot of system states with time.

C. Manipulator movements

Drone is stationary while the manipulator is moving on a certain trajectory.

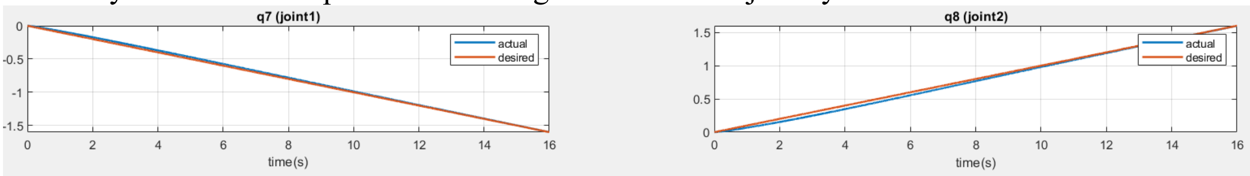


Fig 5: Plot of system states with time.

**Here the other states are very close to zero (error of order 1e-3).

2. Adaptive Sliding Mode Controller (ASMC)

A. Trajectory 1: (In-plane circular)

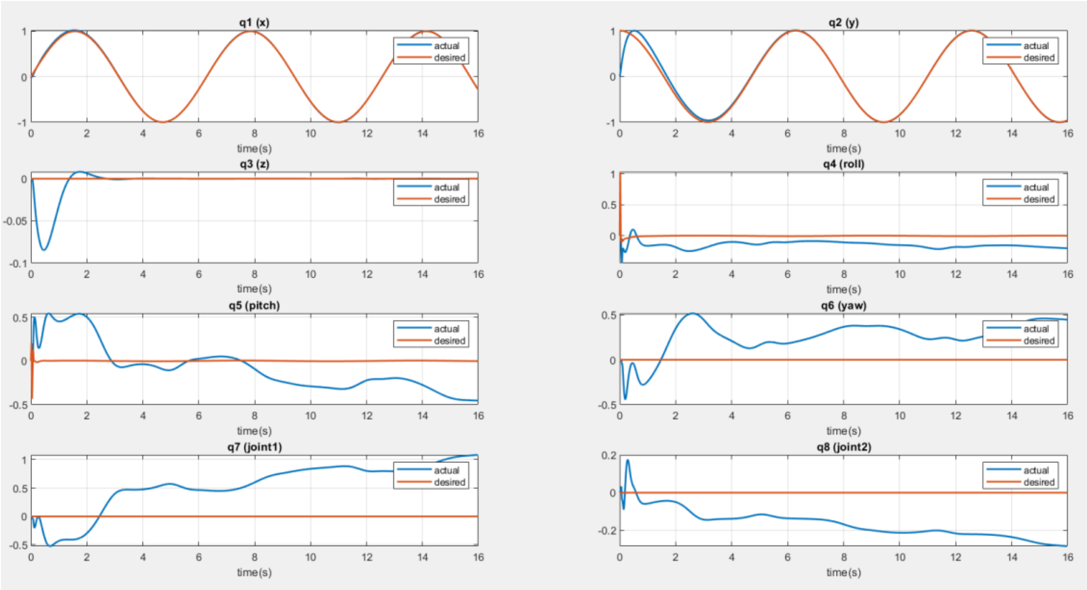


Fig 6: Plot of system states with time.

B. Trajectory 2: (Spiral)

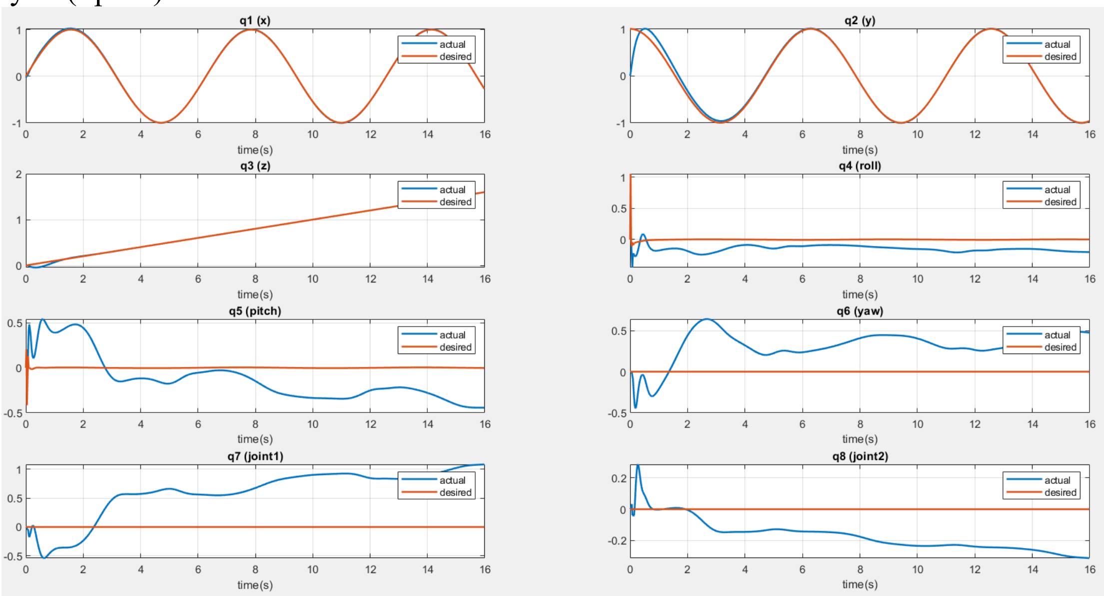


Fig 7: Plot of system states with time.

C. Manipulator movements

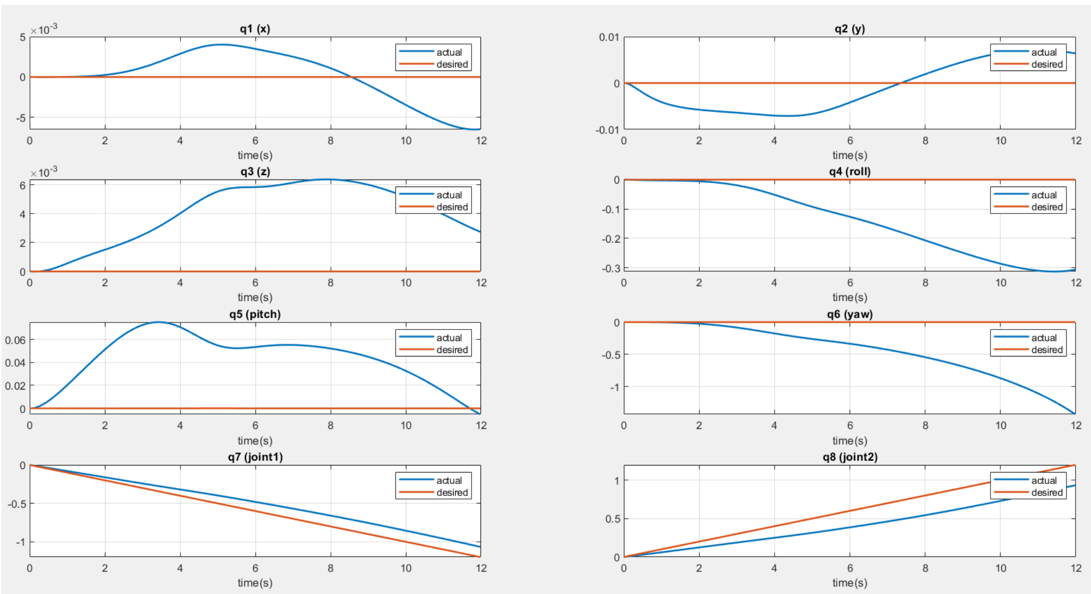


Fig 8: Plot of system states with time.

Discussion

1. For the PD with feedback linearization controller it can be seen that UAV is able to follow the given position quiet well but, the joint angles of the manipulator are still not able to follow the trajectory.
2. Also the roll, pitch and yaw of the UAV also deviate from the desired trajectory.
3. The manipulator was only controlled when the UAV was hovering at a certain location.
4. ASMC was also able to follow the given trajectories accurately but, the joint angles of the manipulator are still not able to follow the trajectory.
5. Also the roll, pitch and yaw of the UAV also deviated from the desired trajectory.
6. This controller was not able to control the manipulator even when the UAV was hovering, and also other states were affected.
7. PD with feedback linearization controller was found to be better as compared to ASMC in this scenario which can be inferred from the results.
8. It was observed that with both controllers, the manipulator was unable to follow the desired trajectory while the UAV was in motion.
9. For different trajectories the parameters for both the controllers needed to be tuned to obtain the above results.
10. PD with feedback linearization controller had less number parameters to tune than as compared to ASMC but still gave good results as compared to ASMC.