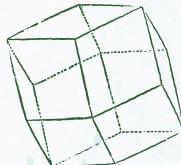
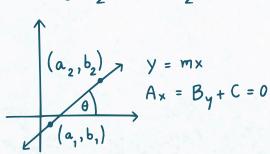




$$\begin{aligned} & \text{Free } \square \quad \leftarrow \frac{\partial y}{\partial t} + \frac{\partial y}{\partial z} = f(t) \\ & \Delta F = x^2 + y^2 - z^2 \quad \leftarrow \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} f(x,y,z) dx dy dz \\ & \int \int \int f(x,y,z) dx dy dz = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial t} + \frac{\partial y}{\partial z} \right) dx dy dz \\ & \int \int \int f(x,y,z) dx dy dz = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \left(\frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial x \partial z} \right) dx dy dz \\ & \int \int \int f(x,y,z) dx dy dz = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \left(\frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial x \partial z} \right) dx dy dz \end{aligned}$$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$



MATHEMATICA

CATHEDRAL MATH SOCIETY

π

1729

e



EDITION III

SEPTEMBER 2025

LETTER FROM THE EDITOR:

Mathematica continues its mission by offering content that challenges, inspires, and broadens the horizons of all who engage with it.

This **Third Edition** brings together a carefully curated collection of problems spanning junior, intermediate, and senior levels. This issue features distinct problems from geometric progressions to percentages, triangles, permutations and combinations, and geometrical figures. Each is designed to sharpen intuition, test logical reasoning, and cultivate resilience in approaching mathematical challenges.

Beyond problem-solving, we turn to the universe and history for perspective. Our Real-Life Math feature explores how mathematics helps determine the age of the cosmos, while the History of Indian Mathematics section reflects on the contributions of Brahmagupta, who laid the groundwork for key mathematical principles still in use today.

Solutions are available through the QR code provided, but we strongly encourage readers to wrestle with the problems themselves first.

As **Carl Friedrich Gauss** once remarked, “Mathematics is the queen of the sciences.” We hope this edition serves as a reminder of both the beauty and the power of mathematics.

Until October,
Editor-In-Chief
Aaditya Agarwal

EDITOR-IN-CHIEF

Aaditya Agarwal 12 D

EDITORS

Aarush Choksi 12 D

Vyom Gupta 12 A

DESIGNER

Vaibhav Sanghai 11 B

CONTRIBUTORS

Vaibhav Sanghai 11 B

Jiana Jhaveri 10 A

Rian Augustine 10 C

Aashi Bubna 10 C

Anvika Nevatia 10 IG

Mahir Goradia 10 A

Advay Gupta 8 D

Tashvi Bhandari 8 A

EQUATION INDEX

A. Problem of the Month (Junior, Intermediate, Senior)

- Geometric Progression (2021 Euclid, Q 6(b)).....
- Percentages (2024 Hypatia, Q1).....
- Triangles (2023 AIME, Q12).....
- Permutations and Combinations (1990 AIME I, Q8).....
- Geometrical Figures (2023 Cayley, Q23).....
- POTW [total of 6 problems].....

B. Real-Life Math

- Math tells us the Age of the Cosmos.....
- 100 Prisoner's Riddle.....

C. History of Indian Mathematics

- Brahmagupta.....
- Shakuntala Devi.....

MATH PROBLEMS

1.0.1 Geometric Progression

Suppose that a and r are real numbers. A geometric sequence with the first term a and the common ratio r has 4 terms. The sum of this geometric sequence is $6 + 6\sqrt{2}$. A second geometric sequence has the same first term a and the same common ratio r , but has 8 terms. The sum of this second geometric sequence is $30 + 30\sqrt{2}$. Determine all possible values for a .

1.0.2 Percentages

At Radford Motors, 4050 trucks were sold. Of the trucks sold, 32% are white, 24% are gray and 44% are black. How many white trucks were sold?

If 1/4th of the gray trucks sold were electric, how many trucks sold were both gray and electric?

In addition to the trucks that were sold, there are k unsold black trucks. In total, 46% of all trucks, sold and unsold, were black. Determine the value of k .

1.0.3 Triangles

Let $\triangle ABC$ be an equilateral triangle with side length 55. Points D, E and F lie on BC, CA and AB, respectively, with $BD = 7$, $CE = 30$ and $AF = 40$. Point P inside $\triangle ABC$ having the property that $\angle AEP = \angle BFP = \angle CDP$. Find $\tan^2(\angle AEP)$.

1.0.4 Permutations and Combinations

In a shooting match, eight clay targets are arranged in two hanging columns of three targets each and one column of two targets. A marksman must break all the targets according to the following rules:

1. The marksman first chooses a column from which a target is to be broken.
2. The marksman must then break the lowest remaining target in the chosen column.

If the rules are followed, how many different orders can the eight targets be broken?

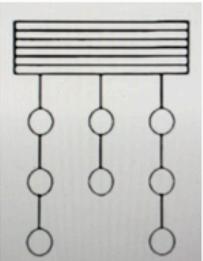


Figure 1: Permutations and Combinations

1.0.5 Geometrical Figures

In the diagram, ABC is a quarter-circle centered at B. Each of square PQRS, square SRTB and square RUVT has side length 10. Points P and S are on AB, T and V are on BC, and Q and U are on the quarter-circle. Line segment AC is drawn. Three triangular regions are shaded, as shown.

What is the integer closest to the total area of the shaded regions?

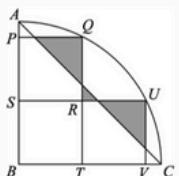


Figure 2: Area of Shaded Region

POTW

JUNIOR

An Average Error

At POTW Farms, Maggie is responsible for collecting eggs each day. She recorded the number of eggs collected each day for one week, and then calculated that the average number of eggs collected in one day that week was 77. When she double-checked the number of eggs she recorded for each day, she noticed that she had recorded 18 eggs on Wednesday. However, she had actually collected 81 eggs. What is the corrected average number of eggs collected in one day for that week?



Five Magnets

Harlow has five magnets, each with a different number from 1 to 5. They arranged these magnets to create a five digit number $ABCDE$ such that:

- the three-digit number ABC is divisible by 4,
- the three-digit number BCD is divisible by 5, and
- the three-digit number CDE is divisible by 3.

Determine the five-digit number that Harlow created.

12345

POTW

INTERMEDIATE

More or Less

Severin works in a factory that produces steel bolts. One type of steel bolt should have a mass of 5 g; however, there is some variation, so each bolt may be heavier or lighter by as much as 1.9%. Severin puts together a box of these bolts and notices that the total mass of the bolts in the box is exactly 1 kg.

Determine the minimum and maximum number of bolts that could be in the box.



Ten Slice Pizza

The DECI-Pizza Company has a special pizza that has 10 slices. Two of the slices are each $\frac{1}{6}$ of the whole pizza, two are each $\frac{1}{8}$, four are each $\frac{1}{12}$, and two are each $\frac{1}{24}$. A group of n friends share the pizza by distributing all of these slices. They do not cut any of the slices. Each of the n friends receives, in total, an equal fraction of the whole pizza. For what values of $n > 1$ is this possible?

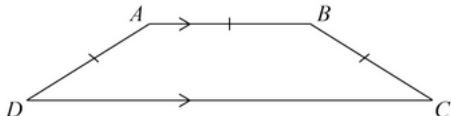


POTW

SENIOR

Trapped

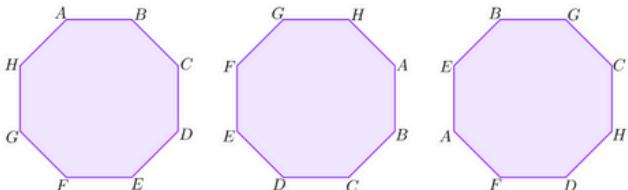
In trapezoid $ABCD$, sides AB and CD are parallel, and the lengths of sides AD , AB , and BC are equal.



If the perpendicular distance between AB and CD is 8 units, and the length of CD is equal to 4 more than half the sum of the other three side lengths, determine the area and perimeter of trapezoid $ABCD$.

Octosquares?

The eight vertices of a regular octagon are randomly labelled A, B, C, D, E, F, G , and H . Each letter is used exactly once. For example, the images below show three different ways to label the vertices of the octagon.



It can be shown that the only way to form a square whose vertices are also vertices of the octagon is by drawing a line segment between every other vertex in the regular octagon.

For example, in the first labelling example, both $ACEG$ and $BDFH$ are squares. These are the only two squares that can be formed using that specific labelling of the octagon. Similarly, in the second example, $ACEG$ and $BDFH$ are the only two squares that can be formed, and in the third example, $ABCD$ and $EGHF$ are the only two squares that can be formed.

If the vertices of a regular octagon are randomly labelled A, B, C, D, E, F, G , and H and each letter is used exactly once, what is the probability that $ABCD$ is a square?

That is, what is the probability that the shape formed by connecting the vertex labelled A to the vertex labelled B , the vertex B to the vertex labelled C , the vertex labelled C to the vertex labelled D , and the vertex labelled D to the vertex labelled A , is a square?

SOLUTION TO THE POTW



Geometric Progression

By Vaibhav Sanghai - Senior

Since the sum of a geometric series with the first term a , common ratio r and 4 terms is $6 + 6\sqrt{2}$ then,

$$a + ar + ar^2 + ar^3 = 6 + 6\sqrt{2}$$

Similarly for the 8 terms, we get

$$a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 + ar^7 = 30 + 30\sqrt{2}$$

$$a + ar + ar^2 + ar^3 + r^4(a + ar + ar^2 + ar^3) = 30 + 30\sqrt{2}$$

$$(1 + r^4)(a + ar + ar^2 + ar^3) = 30 + 30\sqrt{2}$$

$$(1 + r^4)(6 + 6\sqrt{2}) = 30 + 30\sqrt{2}$$

$$1 + r^4 = \frac{(30 + 30\sqrt{2})}{(6 + 6\sqrt{2})}$$

$$1 + r^4 = 5 \Rightarrow r^4 = 4 \Rightarrow r^2 = \pm 2$$

Since $r^2 > 0$, we reject the answer

$$r^2 = -2$$

$$r = \pm\sqrt{2}$$

If $r = \sqrt{2}$, then,

$$\begin{aligned} a + ar + ar^2 + ar^3 &= a + a\sqrt{2} + a\sqrt{2}^2 + a\sqrt{2}^3 \\ &= a + a\sqrt{2} + a\sqrt{2}^2 + 2a\sqrt{2} = a(3 + 3\sqrt{2}) \end{aligned}$$

Since $a + ar + ar^2 + ar^3 = 6 + 6\sqrt{2}$,

$$a(3 + 3\sqrt{2}) = 6 + 6\sqrt{2} \Rightarrow a = 2$$

Similarly, if we get $r = -\sqrt{2}$,

$$a = \frac{6 + 6\sqrt{2}}{3 - 3\sqrt{2}} = \frac{2 + 2\sqrt{2}}{1 - \sqrt{2}} = \frac{(2 + 2\sqrt{2})(1 + \sqrt{2})}{(1 - \sqrt{2})(1 + \sqrt{2})} = -6 - 4\sqrt{2}$$

The values of a are 2 & $(-6 - 4\sqrt{2})$.

Percentages

By Aashi Bubna - Junior & Intermediate

Total trucks sold = 4050

Percent breakdown of the sold trucks: white = 32%, grey = 24%, black = 44%.

Since, 32% of the 4050 trucks sold were white,

∴ Number of electric trucks

$$= \frac{972}{4} = 243 \text{ trucks}$$

∴ 273 trucks were both grey and electric

There are k unsold black trucks
44% of the sold trucks are black.
46% of all trucks, sold and unsold are black.

We already have the sold black trucks:

$$= 44\% \text{ of } 4050 = \frac{(44 * 4050)}{100} = 1728$$

∴ 1728 of trucks sold were black.

Number of white trucks

$$= 32\% \text{ of } 4050 = \frac{(32 * 4050)}{100} = 1296$$

No of grey trucks

$$= 24\% \text{ of } 4050 = \frac{(24 * 4050)}{100} = 972$$

Now, $1/4^{\text{th}}$ of these are electric

∴ Total black trucks (Sold + Unsold) = $1728 + k$

Total trucks overall = $4050 + k$

$$\frac{(1728 + k)}{(4050 + k)} = \frac{46}{100}$$

$$1728 + k = 0.46(4050 + k)$$

$$1728 + k = 1863 + 0.46k$$

$$k - 0.46k = 1863 - 1728$$

$$0.54k = 81$$

$$k = \frac{8100}{54}$$

$$k = 150$$

Number of unsold black trucks is 150.

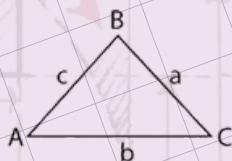
Triangles

By Rian Augustine - Intermediate & Senior

Before attempting this question, we must be familiar with the law of sines and the law of cosines.

Law of Sines: In a triangle, side “a” divided by the sine of $\angle A$ is equal to the side “b” divided by the sine of $\angle B$ is equal to the side “c” divided by the sine of $\angle C$.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Law of Cosines: For a triangle with sides a, b, and c, and angles A, B, and C, the law of cosines states:

$$a^2 = b^2 + c^2 - 2bc * \cos A, b^2 = a^2 + c^2 - 2ac * \cos B, \\ \text{and } c^2 = a^2 + b^2 - 2ab * \cos C.$$

The points D, E, and F lie on BC, CA, and AB with BD = 7, CE = 30, and AF = 40.

From this, DC = 48, EA = 25, and FB = 15.

$$\angle AEP = \angle BFP = \angle CDP$$

This angle will appear in three different triangles.

When this phenomenon occurs, we use a concept called the **Miquel point** to navigate through the problem.

The Miquel point of a triangle is a special point formed when you pick a point on each side of a triangle (or their extensions) and then draw three circles, each passing through a vertex and the two chosen points on the adjacent sides.

The Miquel point of this triangle will be the single point P that lies on the circumcircle of each of the smaller triangles AEP, BFP, and CDP. A circle which passes through all 3 vertices of a triangle is called a circumcircle.

Concyclic points are those points which lie on the same circle, in this case, being A, E, P and D. A key property of concyclic points is that opposite angles in the quadrilateral add to 180° , and angles subtending the same chord are equal.

From these concyclic points, A, E, D and P,
 $\angle AEP = \angle ADP$ (both of which are subtending AP)

Since the points C, D, P, and A are concyclic, $\angle CDP$ and $\angle CAP$ are equal, as both subtend AC. $\therefore \angle AEP = \angle CDP, \angle ADP = \angle CAP$.

$\therefore \triangle ADP$ and $\triangle CAP$ have two equal angles, which proves they are similar.

In an equilateral triangle with a side length of 55, the altitude divides the base into two segments of 27.5 each. Using the Pythagorean theorem:

$$\text{Height} = \sqrt{55^2 - 27.5^2} = \frac{55\sqrt{3}}{2}.$$

D lies on BC, which has $BD = 7$. Making \perp AM from A makes $BM = 27.5$, so $DM = 20.5$. $\triangle AMD$ is a right triangle, where $AM = (55\sqrt{3})/2$ and $MD = 20.5$.

$$\therefore AD = \sqrt{\left(\frac{55\sqrt{3}}{2}\right)^2 + 20.5^2}$$

However, there's a quicker route:

In $\triangle ABD$, side $AB = 55$, $BD = 7$, and $\angle ABD = 60^\circ$ (since ABC is equilateral).

Using the Law of Cosines:

$$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos 60^\circ$$

$$AD^2 = 55^2 + 7^2 - 2(55)(7)(0.5)$$

$$AD^2 = 3025 + 49 - 385 = 2689$$

$$AD = \sqrt{2689}$$

E lies on CA with $CE = 30$, so $AE = 25$.

Again, $\triangle AEC$ is an equilateral triangle with $AC = 55$.

Dropping \perp from E to BC (or using Law of Cosines) lets you verify that $AE = 25$ is already along the side, so no extra calculation needed — it's a straight segment.

We're trying to find $\tan^2(\angle AEP)$.

$\angle AEP$ is in $\triangle AEP$. $\triangle CAP$ shares $\angle CAP = \angle AEP$ based on the Miquel/cyclic argument. This lets us set up one Law of Sines equation for $\triangle AEP$ and another for $\triangle CAP$.

$$\frac{\sin(\angle AEP)}{AE} = \frac{\sin(\angle APE)}{EP} \quad (\text{Equation 1})$$

$$\frac{\sin(\angle CAP)}{AC} = \frac{\sin(\angle APE)}{CP}$$

$\because \angle CAP = \angle AEP$, we can write:

$$\frac{\sin(\angle AEP)}{AC} = \frac{\sin(\angle APE)}{CP} \quad (\text{Equation 2})$$

$\because \angle AEP = \angle CAP$, $\triangle AEP$ and $\triangle CAP$ are similar.

This means $\frac{CP}{EP} = \frac{AC}{AE}$

Here $AC = 55$ (side of the equilateral triangle) and $AE = 25$ (found earlier using the perpendicular height), so:

$$\frac{CP}{EP} = \frac{55}{25} = \frac{11}{5}$$

Applying the Law of Sines in both triangles and substituting

$\frac{CP}{EP} = \frac{55}{25} = \frac{11}{5}$ allows us to eliminate EP . This leaves only AE , AC ,

and $\angle AEP$ in the equation.

Placing the figure in coordinates:

$$A = (0, 0)$$

$$B = (55, 0)$$

$$C = (27.5, \frac{55\sqrt{3}}{2})$$

$$E = (12.5, \frac{25\sqrt{3}}{2})$$

$$P = (\frac{1525}{38}, \frac{25\sqrt{3}}{38})$$

The slope of EA is $\sqrt{3}$ and $-\frac{3\sqrt{3}}{7}$

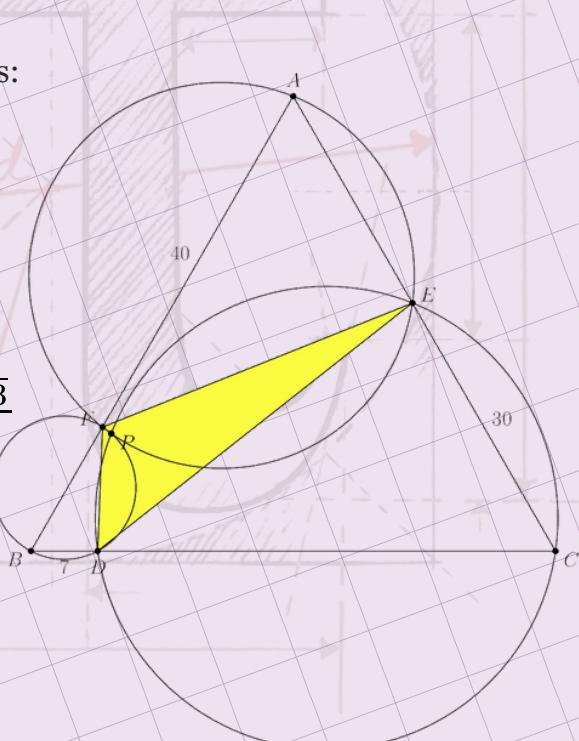
the slope of EP is

Using the angle formula

for two lines:

$$\tan \theta = \left| \frac{m(EP) - m(EA)}{1 + m(EP) * m(EA)} \right|$$

$$\therefore \tan \theta = \left| \frac{-\frac{10\sqrt{3}}{7}}{-\frac{2}{7}} \right| = 5\sqrt{3}$$



Permutations & Combinations

By Mahir Goradia - Intermediate

There are 8 clay targets arranged in two hanging columns of three targets each and one column of two targets.

Rules:

1. A marksman chooses a column.
2. He must break the lowest remaining target in that column.

To find: *The number of different possible orders in which all 8 targets can be broken.*

Step 1 : Total possible shooting orders without restrictions

There are 8 targets in total.
If there were no restrictions at all, you could shoot them in any order, giving:

$$8! = 40320 \text{ possible sequences.}$$

Step 2: Why restrictions matter?

The problem imposes a rule:

In a chosen column, you must shoot from bottom to top.

In Column A (3 targets), the order is fixed → target 1 must come before target 2, and that before target 3.

In Column B (2 targets), the order is also fixed → target 1 before target 2.

In Column C (3 targets), again fixed bottom-to-top.

So although we are permuting 8 objects, within each column the objects can't be freely rearranged.

Step 3. Why divide by factorials?

Think of labeling the targets like this: (bottom to top)

Column A: A1, A2, A3

Column B: B1, B2

Column C: C1, C2, C3

Now, if we just do $8!$, that would count many “fake” sequences where, for example A3 is shot before A1. Those are not allowed.

How do we remove those illegal cases?

The three A's behave like identical “slots” in terms of ordering, because only the order is valid.

Same for the two B's (must appear as $B1 < B2$) & the 3 C's.

So for each column, we divide by the number of “fake” rearrangements of its shots:

Column A: divide by $3!$

Column B: divide by $2!$

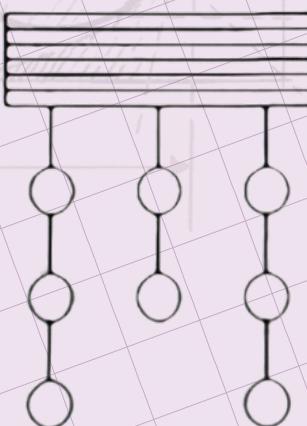
Column C: divide by $3!$

Step 4. Final count

Thus, the number of valid shooting orders is:

$$= \frac{8!}{3!*2!*3!} = \frac{40320}{6*2*6} = \frac{40320}{72} = 560$$

Answer: 560 different orders



Geometrical Figures

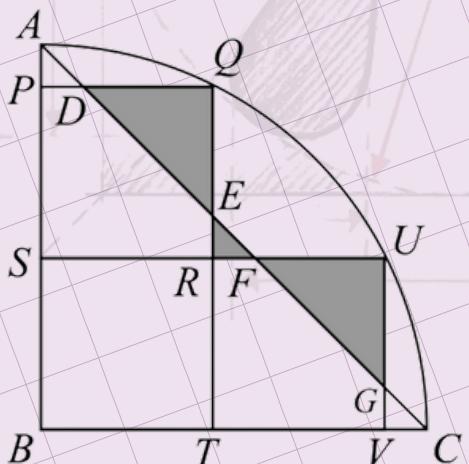
By Anvika Nevatia - Intermediate

At first look, we can see that AB and BC are both radii of the same quarter-circle, $AB = BC$.

Because ABC is a quarter-circle centred at B, $\angle ABC = 90^\circ$
(SRTB is a square)

That makes $\triangle ABC$ isosceles and right-angled, so $\angle BAC$ and $\angle BCA$ are each 45° .

To make it easier, we can add some additional variables:



The lines at P, Q, R, and U meet at right angles.

Since $\angle PAD = \angle BAC = 45^\circ$, we also have:

$$\begin{aligned}\angle PAD &= \angle ADP = \angle QDE = \\ \angle DEQ &= \angle REF = \angle EFR = \\ \angle UFG &= \angle UGF = 45^\circ\end{aligned}$$

This means $\triangle APD$, $\triangle DQE$, $\triangle ERF$, and $\triangle FUG$ are all right-angled and isosceles triangles.

Side length of each square = 10.

$$\begin{aligned}BT &= 10, \text{ and } TQ = TR + RQ \\ &= 10 + 10 = 20\end{aligned}$$

In $\triangle BTQ$, by applying pythagoras:

$$\begin{aligned}BQ &= \sqrt{BT^2 + TQ^2} = \sqrt{10^2 + 20^2} \\ &= \sqrt{500} = 10\sqrt{5}\end{aligned}$$

BQ is the radius of the quarter-circle, so

$$BA = BC = BQ = 10\sqrt{5}$$

Using this value we can calculate the length of AP and so on.

$$BP = TQ = 20, \text{ so } AP = BA - BP = 10\sqrt{5} - 20$$

$$\text{Thus, } PD = AP = 10\sqrt{5} - 20$$

$$PQ = 10, \text{ so } DQ = PQ - PD = 10 - (10\sqrt{5} - 20) = 30 - 10\sqrt{5}.$$

$$QE = DQ = 30 - 10 * \sqrt{5}$$

$$\text{Since } PQ = QR \text{ and } DQ = QE, \text{ we have } PD = ER = 10\sqrt{5} - 20$$

$$\text{Similarly, } ER = RF = 10\sqrt{5} - 20 \text{ and } UF = UG = 30 - 10\sqrt{5}$$

To calculate the total area we must calculate each area of the 3 triangles and add them.

$$\text{Area}(DQE) = 0.5 \times DQ \times QE = 0.5 \times (30 - 10\sqrt{5})^2$$

$$\text{Area}(ERF) = 0.5 \times ER \times RF = 0.5 \times (10\sqrt{5} - 20)^2$$

$$\text{Area}(FUG) = 0.5 \times UF \times UG = 0.5 \times (30 - 10\sqrt{5})^2$$

Total Area

$$= 0.5 \times (30 - 10\sqrt{5})^2 + 0.5 \times (30 - 10\sqrt{5})^2 + 0.5 \times (10\sqrt{5} - 20)^2$$

$$= 0.5 \times [(30 - 10\sqrt{5})^2 + (30 - 10\sqrt{5})^2 + (10\sqrt{5} - 20)^2]$$

$$= 0.5 \times (3700 - 1600\sqrt{5})$$

$$= 1850 - 800\sqrt{5}$$

$$\approx 61.14$$

Answer : 61

Math tells us the Age of the Cosmos

By Jiana Jhaveri

How old is the universe? We yet can't go back in time, but mathematics gives us the answers. Astronomers use **Hubble's Law**, it tells us that, the farther away a galaxy is, the faster it's moving away from us.

$$v = H_0 * d$$

- v : the speed a galaxy is moving away (in km/s)
- H_0 : the Hubble constant (how fast the universe is expanding per unit distance)
- d : the distance of the galaxy from us (in Mpc - 1 Mpc = 3.26 million light years)

If galaxies are moving apart now, in the past they had to be closer. Around 13.8 billion years ago, all the matter and energy in the universe was packed into an extremely hot and dense state; this is what is called the Big Bang.

Hubble's Law:

$$v = H_0 * d$$

Age:

$$t \approx \frac{1}{H_0}$$

Given:

$$H_0 = 70 \text{ km/s/Mpc},$$

$$1 \text{ Mpc} = 3.086 * 10^{19} \text{ km}$$

Convert:

$$H_0 \approx \frac{70}{3.086 * 10^{19}} \text{ s}^{-1} \approx 2.27 * 10^{-18} \text{ s}^{-1}$$

Invert:

$$t \approx \frac{1}{2.27 * 10^{-18}} = 4.4 * 10^{17} \text{ s}$$

Years:

$$t \approx \frac{4.4 * 10^{17}}{3.154 * 10^7} \approx 1.4 * 10^{10} \text{ yr}$$

$$\approx 13.8 \text{ billion yr}$$

That's how scientists have used Hubble's law to theorise how old the cosmos is.

This is how Hubble's law was found

1. He used Cepheid variable stars in other galaxies. Their brightness period tells their true luminosity. Comparing true brightness to how dim they look gives distance (d).
2. Vesto Slipher studied light from galaxies and noticed their spectral lines were shifted toward the red end. This "redshift" happens because, if a galaxy is moving away, the light waves it emits get stretched, just like a siren's pitch drops when it passes you (the Doppler effect). This gave the speed of many galaxies (v)

When Hubble plotted v versus d , the points formed a straight line whose slope was the Hubble's constant.

100 Prisoners Riddle

By Advay Gupta

- There are 100 prisoners and 100 boxes.
- Each box contains the name of a prisoner, randomly shuffled.
- Each prisoner may open up to 50 boxes to find their own name.
- If all prisoners find their names, they all go free. If even one fails, they all remain locked up.

At first glance, random guessing seems hopeless. The chance of success is:

$$\frac{1}{2}^{100} = 7.9 * 10^{-31}$$

This is essentially zero.

The Mathematical Insight

The breakthrough comes from thinking mathematically about permutations:

Label the prisoners 1 through 100.

- Label the boxes 1 through 100 and view the contents as a permutation of the numbers 1-100.
- Every permutation can be broken into cycles.

Example of a cycle:

If prisoner 1 opens box 1 and finds name 42, then opens box 42 and finds 77, then opens box 77 and finds 1, they've completed a cycle of length 3.

The Strategy

1. Each prisoner starts at their own numbered box.
2. They open the box, see the name inside, then go to the box with that number.
3. They repeat this process, following the cycle in the permutation, until they find their own name or reach 50 boxes.

Why this works:

- If all cycles in the permutation are ≤ 50 , every prisoner will find their name within 50 boxes.

Probability Analysis

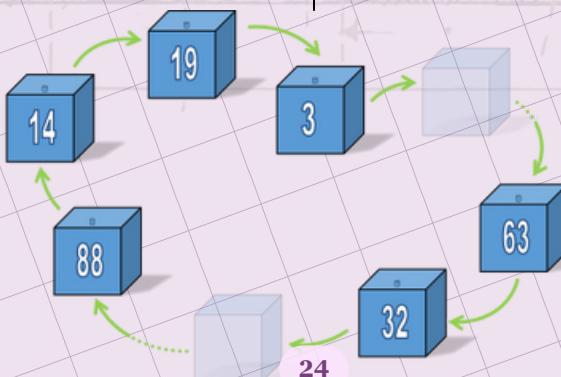
- Let $n = 100$ (total prisoners) and $k = 50$ (boxes allowed per prisoner).
- Statistics prove that in a random shuffle, the chance the longest loop is too long is about 0.693.
- The chance that all loops are short enough $= 1 - 0.693 = 0.307 \approx 31\%$.
- The probability that a random permutation has all cycles $\leq k$ is roughly 31%.

So, using this cycle-following strategy, the prisoners' chance of success jumps from almost zero to about 1 in 3.

Extra Insight:

- On average, the longest cycle in a random permutation of 100 elements is about 62.
- Many permutations fail, but a significant fraction has all cycles ≤ 50 , which makes success

"The probability of success is difficult to estimate; but if we never search the chance of success is zero". - Philip Morrison



Brahmagupta

By Aashi Bubna

Brahmagupta, born in 598 CE in Bhillamal, emerged during a vibrant era of Indian mathematics and astronomy. Despite limited records about his early childhood, it is known that he was drawn to intellectual life in an environment renowned for scholarly pursuits. By age 30, Brahmagupta had composed his most celebrated work, the *Brahmasphutasiddhanta*, a treatise blending innovative mathematics and astronomy.

Brahmagupta's most influential contribution was the formal acceptance and operationalization of zero. He defined zero as a valid number, not just a placeholder, and established rules for arithmetic involving zero and negative numbers.

These are principles that underpin modern mathematics. His rules made calculations with debts (negatives) and fortunes (positives) organized and logical, laying groundwork for generations to come.

He also solved quadratic equations, presented algorithms for finding square roots, and tackled solutions for linear and quadratic indeterminate equations.

Brahmagupta introduced what is now called "Brahmagupta's formula," a technique to compute the area of any cyclic quadrilateral (where all vertices lie on a circle), a landmark result in geometry.

Beyond pure math, his treatises included planetary motions and eclipse calculations, influencing both Indian and later Islamic mathematics. Through creativity and logic, Brahmagupta's work elevated Indian science and left a legacy still vital to mathematics worldwide.



Shakuntala Devi

By Tashvi Bhandari

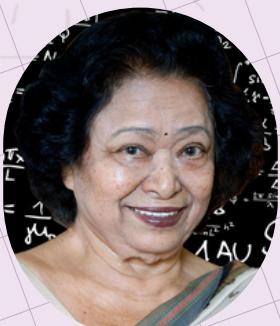
'Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers' -Shakuntala Devi

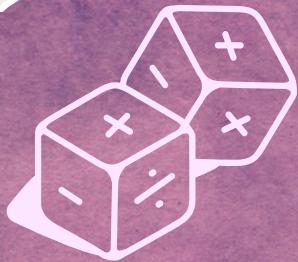
Shakuntala Devi, born on 4th November 1929, was one of the most brilliant mathematicians of her time. Also known as the 'human computer', she won the Guinness World Record in 1980 for being the fastest human calculator. She first displayed her talents as a mathematician at the University of Mysore at the age of six.

She is most famous for her displays of unparalleled mental skills. In 1977 she calculated the 23rd root of a 201-digit number in 50 seconds, while a UNIVAC computer took 62.

She was also a skilled author, and wrote 'The Joy of Numbers' and 'More Puzzles to Puzzle You' on mathematics, along with 'Astrology for You' as a guide to Vedic astrology. She was also famous for her academic study on homosexuality in India, 'The World of Homosexuals'.

She not only shattered records and redefined the limit of the human brain but she was also a philanthropist and used her success to support education for underprivileged children. She broke gender stereotypes and served as a role model for girls and women.





EDITOR IN-CHIEF
AADITYA AGARWAL

CATHEDRAL MATH SOCIETY



SEPTEMBER
EDITION III



CATHEDRAL