OCTOBER 2025

CATHEDRAL MATH SOCIETY

Mathematica

EDITION IV

Letter From The Editor:

Mathematica's primary goal is to encourage students to think logically and analytically, foster an enthusiasm for problem-solving, and learn how to approach new challenges.

In the spirit of preparing students for the AMC being held in November, this edition of Mathematica focuses on questions ranging from AMC 8 to AMC 12. Students are encouraged to attempt these challenging questions to learn new concepts and hone their skills.

Beyond the problems provided, this edition of Mathematica also focuses on practical applications of Mathematics in the economic and financial fields, and Aryabhatta's revolutionary invention of the concept of zero.

Solutions for all problems are available through the QR codes. However, we encourage students to attempt these questions prior to seeking these solutions.

As Albert Einstein once said, "Pure mathematics is, in its way, the poetry of logical ideas." We hope that this edition of Mathematica inspires students to explore the elegance and creativity inherent in mathematics

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~ Aaditya Agarwal

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EQUATION INDEX

MATH PROBLEMS

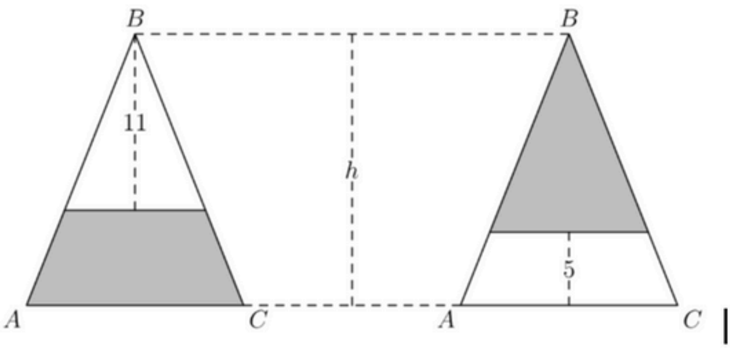
2022 AMC 10A Problem 8

A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 26 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 40$

2023 AMC 8 Problem 24

Isosceles △ABC has equal side lengths AB and BC. In the figure below, segments are drawn parallel to AC so that the shaded portions of △ABC have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height ***h*** of △ABC? (Diagram not drawn to scale.)



A)14.6 B) 14.8 C) 15 D) 15.2 E) 15.4

MATH PROBLEMS

2017 AMC 10A Problem 8

How many integers n satisfy $(n-1)(n-3)(n-5)(n-7) < 9$?

(A) 0 (B) 4 (C) 5 (D) 6 (E) 7

2022 AMC 10A Problem 22

Q. Suppose that 13 cards numbered 1, 2, 3, . . . , 13 are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards 1, 2, 3 are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, 7, 8, 9, 10 on the fourth pass, and 11, 12, 13 on the fifth pass.

7	11	8	6	4	5	9	12	1	13	10	2	3

For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?

(A) 4082
(B) 4095
(C) 4096
(D) 8178
(E) 8191

MATH PROBLEMS

2010 AMC 12A Problem 5

Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot, a bullseye scores 10 points, with other possible scores being 8, 4, 2, and 0 points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bullseyes she will be guaranteed victory. What is the minimum value for n?

A) 38 B) 40 C) 42 D) 44 E) 46

2023 AMC 12A Problem 16

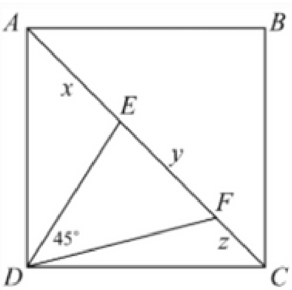
Consider the set of complex numbers z satisfying $|1+z+z^2|=4$.

The maximum value of the imaginary part of z can be written in the form $\frac{\sqrt{m}}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

2016 Euclid Question 9(b)

In the diagram drawn below ABCD is a square



Points E and F are chosen on AC so that $\angle EDF = 45^\circ$. If $AE = x$, $EF = y$, and $FC = z$, prove that $y^2 = x^2 + z^2$.

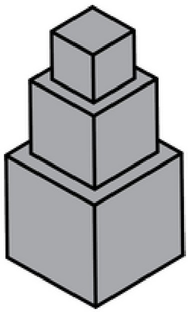
PROBLEM OF THE WEEK

Grades 7 and 8

Imtiaz is an artist. One of the pieces of art that he created is a tower of three cubes. The bottom cube has a side length of 3 m, the middle cube has a side length of 2 m, and the top cube has a side length of 1 m. The top two cubes are each centred on the cube below.

Imtiaz wishes to paint the piece of art. Since the piece will be suspended in the air, the bottom will also be painted.

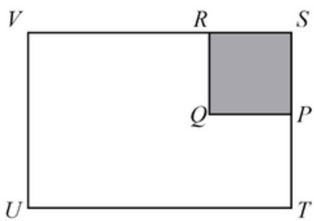
Determine the total surface area of the piece of art, including the bottom.



Grades 9 and 10

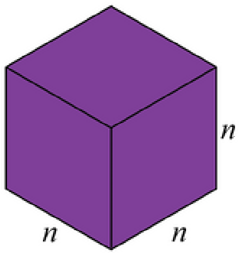
Rectangle $STUV$ has P on ST, R on SV, and Q inside the rectangle such that $PQRS$ is a square. When square $PQRS$ is removed from rectangle $STUV$, the remaining shape has an area of 92 m^2.

If $PT = 4$ m and $RV = 8$ m, what is the area of rectangle $STUV$?



Grades 11 and 12

A cube has side length n, where n is a positive integer. Exactly three faces, which meet at a corner, are painted purple. The cube is then cut into n^3 smaller cubes with side length 1. If exactly 125 of these smaller cubes have no faces painted purple, then determine the value of n.



SOLUTIONS

2022 AMC 10 A Problem 8

Saisha Damodar

We know 5 out of the 6 numbers in the set, and we know that x is a positive integer. Since the formula to find the average of certain numbers is:

= sum of all the integers/ the no. of integers

$$=\frac{1+7+5+2+5+x}{6} \quad = \frac{x+20}{6}$$

Let the average of the 6 numbers be Y

So, $\dfrac{20+x}{6}=Y$

Where Y ∈ {1, 2, 5, 7, x}

Find all the possible values for x, by assuming all the possible values for Y

$$\frac{20+x}{6}=1; x=-14$$

which is not possible since x must be positive

$$\frac{20+x}{6}=2; x=-8$$

which again, is not possible since x must be positive

$$\frac{20+x}{6}=5$$

here, x= 10 which is positive, and hence can be considered as a possible value of x.

$$\frac{20+x}{6}=7$$

here, x = 22 which is positive, and hence can be considered as a possible value for x

$$\frac{20+x}{6}=x$$

here, x = 4 which is positive, and hence can be considered as a possible value for x

So, if we add up all the positive possible values for x, we get

$22+4+10=36$

2023 AMC 8 Problem 24

Reyansh Aggarwal

Main Concept:
When a line is drawn parallel to one side of a triangle, it creates a smaller triangle that is similar to the original one.
For two triangles that are similar, the ratio of their areas is equal to the square of the ratio of their corresponding heights.
If $Area_1$ and $Area_2$ are the areas of two similar triangles, and h_1 and h_2 are their corresponding heights, then:

$$\frac{Area_1}{Area_2} = \left(\frac{h_1}{h_2}\right)^2$$

left side of the figure:
1. The unshaded part is a small triangle at the top with a height of 11. This triangle is similar to the large triangle △ABC
2. Ratio of areas (unshaded△ to whole△) = (Ratio of heights)^2
$= \left(\frac{11}{h}\right)^2 = \frac{121}{h^2}$
3. The shaded area is the whole triangle minus the unshaded part. So, the ratio of area of the shaded△ to the area of whole△ is: $1 - \frac{121}{h^2} = \frac{h^2 - 121}{h^2}$

Right side of the figure:
1. The shaded part itself is a triangle at the top. Its height is (h-5). This shaded△ is similar to the large triangle △ABC.
2. Ratio of areas (shaded△ to whole△) = (Ratio of heights)^2
$= \left(\frac{h-5}{h}\right)^2$

Solving for h:
The problem states the shaded areas are equal. Since they are parts of the same whole triangle (△ABC), their ratios to the whole area must also be equal.

Hence,

$$\frac{h^2-121}{h^2} = \left(\frac{h-5}{h}\right)^2$$

Therefore, $\dfrac{h^2-121}{h^2} = \dfrac{(h-5)^2}{h^2}$

Therefore, $h^2 - 121 = (h-5)^2$

Therefore, $h^2 - 121 = h^2 - 10h + 25$

Therefore, $-121 = -10h + 25$

Therefore, $10h = 146$

Therefore, $h = 14.6$ and the correct answer is **(A)**

2017 AMC 10 Problem 8

Manansh Jain

Step 1 : Group the factors (use symmetry)
Group symmetrically:

$$(n-1)(n-3)(n-5)(n-7)$$

$$[(n-1)\times(n-7)]\times[(n-3)\times(n-5)]$$

Each pair multiplies to a quadratic with the same $n^2 - 8n$ part, which later makes a neat substitution possible.

Compute the pairs:
$(n-1)(n-7) = n^2 - 8n + 7$
&
$(n-3)(n-5) = n^2 - 8n + 15$

Step 2 : Substitute to reduce degree

Set $x = n^2 - 8n$

Then the product becomes $(x+7)(x+15)<9$

Both quadratics share the same n^2-8n piece, so the quartic in n becomes a quadratic inequality in x.

Step 3 : Expand and solve the quadratic inequality in x
Expand and move 9 to the left:
$(x+7)(x+15) < 9 \Rightarrow$
$x^2+22x+105 < 9 \Rightarrow$
$x^2+22x+96 < 0$

Factor:
$x^2 + 22x + 96 = (x+6)(x+16)$

Sign analysis of a product shows it is negative exactly between the roots:

$(x+6)(x+16) < 0 \Rightarrow -16 < x < -6$

Step 4 : Translate back to n (complete the square)
Recall
$x = n^2 - 8n$
and complete the square:
$n^2 - 8n = (n-4)^2 - 16$
Substitute into the interval for x:
$-16 < (n-4)^2 - 16 < -6$
Add 16 to everything:
$0 < (n-4)^2 < 10$
Interpretation:
the square $(n-4)^2$ must be strictly positive and strictly less than 10.

Step 5 : Find integer n
Integer squares strictly between 0 and 10 are 1,4,9.
So :
$(n-4)^2 \in \{1,4,9\}$
Solve each:

$(n-4)^2 = 1 \Rightarrow n = 4\pm1 \Rightarrow n = 3,5$
$(n-4)^2 = 4 \Rightarrow n = 4\pm2 \Rightarrow n = 2,6$
$(n-4)^2 = 9 \Rightarrow n = 4\pm3 \Rightarrow n = 1,7$

So the integer solutions are $n=\{1,2,3,5,6,7\}$
Why not $n = 4$?
Because $(n-4)^2 = 0$ which fails the strict inequality
$0 < (n-4)^2$
For $n= 4$ the original product equals 9 (equality), which is not allowed.

Also no other integers work because if
$|n-4| \geq 4$
then
$(n-4)^2 \geq 16$,
which is ≥ 10 and thus outside the allowed range.

Final answer :
There are 6 integers that satisfy the inequality:
$n=1,2,3,5,6,7$
Hence, the answer is (D) 6

2022 AMC 10A Problem 22

Saniddhya Jain

Splitting into Sequence A and B:
When you pick up cards in two passes, what happens?
In the first pass, you're taking some of the cards in ascending order. In the second pass, you take whatever is left, which also must appear in ascending order (otherwise you would need a third pass).
That means the final permutation is literally a "shuffle" of two already-sorted sequences:
Sequence A = cards picked up in the first pass (in increasing order), and Sequence B = cards picked up in the second pass (in increasing order).
So to build such a permutation, you only need to decide which cards go in A. The ones not in A automatically go into B.
Choosing which cards go in the first pass = choosing a subset.
If you have the numbers {1,2,3,...,13}, then any subset S of those numbers could be your first-pass cards.
The complement of S (the numbers remaining) will be the second-pass cards.
Once you pick S, you fill the chosen positions with the numbers from S in ascending order, and fill the remaining positions with the other numbers in ascending order. This uniquely determines a permutation that can be picked up in at most two passes.
So the entire counting reduces to:
How many subsets S are there of {1,...,13}?"

For each individual card in the 13 element set:

- Either it is in S (chosen for the first pass),
- Or it is not in S (left for the second pass).

That's two choices for each card. Because the choices are independent for each of the 13 cards, the total number of possible subsets is:

$(2 \times 2 \times 2 \times 2.\ldots\ldots \times 2\,(13 times) = 2^{13}$

In other words, each element can be either "in" or "out" of the subset.

Subtracting the single-pass subsets:

Some subsets give you a permutation that can be picked up in one pass (already sorted). That happens when S is just the first smallest cards in order. The possibilities of this occurring are:

- S=∅ (no first-pass cards, everything picked once)
- S={1}
- S={1,2}
- S={1,2,3,...,13}

There are 14 such subsets.

For all these, only one pass would be needed.

So we subtract those 14:

2^13 - 14 = 8192−14 = 8178

This is why the final answer for exactly two passes is 8178 **(D)**.

2010 AMC 12A Problem 5

Agastya Mundhe

Given information:

- We are halfway through the 100 shot tournament. This means 50 shots are over, and 50 shots remain.
- Points system- Bullseye = 10 points, and other possible scores are 8,4,2,0.
- Chelsea leads by 50 points and scores at least 4 points on each shot.
- We also know that if Chelsea's next 'n' shots are bullseyes, she will win.

How to find the value of 'n'?

- To find the value of n let us assume the worst case scenario where Chelsea's opponent scores 500 points , i.e. 50 bullseyes.
- We assume this because, it's written above - "guaranteed" victory, hence we know that even in the worst case scenario Chelsea still wins.
- Let's count Chelsea's possible points in the next 50 shots :

– We know that n shots are bulleseyes therefore, no. of points will be 10n. We also know for the rest of the shots, she will have to score 4 points, as we are assuming the worst case scenario for Chelsea. Therefore, for the rest of the shots: 4(50-n).

Therefore the overall equation is: 10n+4(50-n)

– We know 10n+4(50-n) + 50> 500 to win. We add the 50 point lead as well to the inequality, as that is already in the given information. Let's solve the inequality.

Step 1: 10n +200- 4n +50> 500

Step 2: 6n +250 > 500

Step 3: 6n > 250

Step 4: n>250/6

Step 5: n> 41.6667 ≈ 42

Hence, the answer is C) 42

2023 AMC 12A Problem 16

Raina Kothari

$|1 + z + z^2| = 4$

Completing the square:

$z^2 + z + 1 = (z + \frac{1}{2})^2 + \frac{3}{4}$

$|(z + \frac{1}{2})^2 + \frac{3}{4}| = 4$

Replace with a simpler variable:

Let $w = z + \frac{1}{2}$

Then $|w^2 + \frac{3}{4}| = 4$

Express w in terms of real and imaginary parts:

Let $w = u + iv$

*u = real part of w

*v = imaginary part of w

Then $w^2 = (u + iv)^2 = (u^2 - v^2) + 2iuv$

Substitute into the equation:

$|w^2 + \frac{3}{4}|^2 \rightarrow (u^2 - v^2 + \frac{3}{4})^2 + (2uv)^2 = 16$

Simplify to find v:

$(u^2 - v^2 + \frac{3}{4})^2 + 4u^2v^2 = 16$

For maximum imaginary part, set $u = 0$

Therefore, $(-v^2 + \frac{3}{4})^2 = 16$

$-v^2 + \frac{3}{4} = -4$

$$v^2 = \frac{19}{4}$$

$$v = \frac{\sqrt{19}}{2}$$

6. Therefore,

Maximum imaginary part of z = $\sqrt{19} / 2$

$m = 19$, $n = 2 \rightarrow m + n =$ **21**

Note:

u and v represent the coordinates of w on the complex plane:

u is the horizontal (real) part, and v is the vertical (imaginary) part.

When $u = 0$, the point is directly above the origin, giving the maximum imaginary value.

2016 Euclid Question 9(b)

Aditya Gupta

Since AC is a diagonal of the square, we can say that

$\angle EAD = \angle FCD = 45$

Let $\angle ADE = \theta$

Since the angles in a triangle have a sum of 180,

$\angle AED = 180 - \angle EAD - \angle ADE$
$= 180 - 45 - \theta = 135 - \theta$

Since AEF is a straight angle,

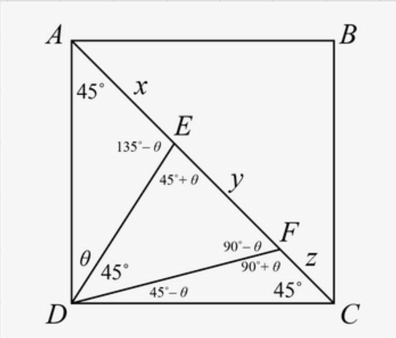
$\angle DEF = 180 - \angle AED$
$180 - (135 - \theta) = 45 + \theta$

Continuing in the same way, we find that $\angle EFD = 90 - \theta$

$\angle DFC = 90 + \theta$

$\angle FDC = 45 - \theta$

We then get this diagram



Using the sine law:

in $\triangle AED : \frac{x}{sin\theta} = \frac{ED}{sin45}$

in $\triangle DEF : \frac{y}{sin45} = \frac{ED}{sin\,(90-\theta)} = \frac{FD}{sin\,(45+\theta)}$

in $\triangle DFC : \frac{z}{sin\,(45-\theta)} = \frac{FD}{sin45}$

Dividing the first equation by the second, we get $\frac{x}{y} = \frac{sin\,(90-\theta) sin\theta}{sin^2 45}$

Dividing the fourth equation by the third, we get $\frac{z}{y} = \frac{sin\,(45-\theta) sin\,(45+\theta)}{sin^2 45}$

$sin\,(90-\theta) = cos\,(\theta)$
$sin\,(45+\theta) = cos\,(45-\theta)$
$\frac{1}{sin^2 45} = 2$

Therefore, $\frac{x}{y} = 2sin\theta cos\theta = sin2\theta$

$\frac{z}{y} = 2cos\,(45-\theta) sin\,(45-\theta) = sin\,(90-2\theta) = cos2\theta$

and, $\left(\frac{x}{y}\right)^2 + \left(\frac{z}{y}\right)^2 = sin^2 2\theta + cos^2 2\theta = 1$

Therefore, $x^2 + z^2 = y^2$

Application of Math in Economics and Finance

Aastha Mehta

One of the certain and more applicable applications of mathematics in economics is calculus, specifically differential calculus. Let's say a firm is attempting to determine how many units of its product it should make so that it maximizes its revenues.

The firm considers two significant figures: the expense of making an extra unit and the amount of money it makes by selling an extra unit. These are referred to as marginal revenue and marginal cost. Through the use of calculus, the company measures at what rate total cost and total revenue will change if production is altered. The most magical moment comes when the two rates are the same. When marginal revenue is higher than marginal cost, the firm can increase profit by raising production. If it is reversed, cutting down production would be preferable. This seemingly easy concept actually entails calculus of derivatives to comprehend such rates of change and determine the most profitable production level. It informs firms on what is happening and makes output and pricing decisions based on facts. Apart from profit, calculus is employed to comprehend how consumers react to price changes and the rate of changes in costs, thereby being an effective tool for policymakers and firms.Calculus also works in the reverse direction with integration. If a firm, say, knows how cost or revenue varies with every marginal product but desires to learn total cost or revenue for some interval of production, integrals add together all these small increments across this interval and deliver the whole picture. This method allows economists and business managers to make forecasts of total revenues and costs on the basis of marginal information, thus allowing them to plan and forecast better. Changing to finance, the most famous example of mathematics in use is the Black-Scholes option pricing model. Options are agreements whereby investors are granted the ability to purchase or sell a certain asset at a given price within a set timeframe. Black-Scholes applies high-level mathematics, such as probability theory and differential equations, in order to calculate an option's fair value. It estimates variables such as the current price of the asset, the negotiated contract price, time left until the option expiration date, expected asset price volatility, and the risk-free rate. The economists and mathematicians developed this in the 1970s, and it changed the way options are priced. Its principal feature is the use of statistical modeling to simulate the random movement of asset prices such that they arrive at a price at which both buyer and seller are not in a favorable position. The equilibrium is responsible for keeping financial markets in working condition and enables investors to deal with risk more effectively. Moreover, it is the cornerstone of contemporary quantitative finance and shapes an array of other models for risk and prices. Although there are some simplifying assumptions in the original Black-Scholes model, actual financial markets are more complex. Scholars have extended the model by incorporating volatile volatility and interest rates and pricing more complex contracts by computer simulation.

Methods such as Monte Carlo simulations iterate through thousands of random potential asset price scenarios to gain a better understanding of risk in uncertain environments. Other computer software handles challenging-to-specify equations that are beyond the capability of straightforward formulas. All these developments preserve the model's essence but refashion it to suit reality, illustrating the way mathematics adapts similarly to the financial markets that it supports. These pictures are used to highlight that mathematics in finance and economics is never theoretical. It is real, meaningful, and always steering the making of decisions that influence businesses, governments, and individuals. Calculus makes businesses run efficiently by determining the best points of production and price, and mathematical finance safeguards investors and maintains fair and stable markets.

The Invention of Zero

Vaibhav Sanghai

The invention of zero is considered one of the most revolutionary transformations in mathematics of the millennium. Aryabhatta, the great Indian mathematician and astronomer of the 5th century CE, is often credited with laying the foundation for the use of zero as a number. The idea of “nothingness” existed before Aryabhatta; however, no one had given it a numerical value. Aryabhatta was the first to do so. He placed zero at the start of the natural numbers on the number line. This important placement allowed mathematicians to denote large numbers with ease. In his book Aryabhatiya (499 CE), he used the Sanskrit word “kha” to denote zero. Zero later even found its way into the decimal system, which is still accepted globally today.In ancient India, zero was the most important number. It allowed astronomers to accurately calculate distances between planets, track planetary movements, and predict eclipses. Merchants and scholars used zero for basic arithmetic in trade, engineering, and architecture. Zero is even used in sports scores and bank balances. Representing extremely large quantities without zero had been a tedious task until the invention of zero. In modern times, zero has only become more significant. All software is dependent on the binary code, which consists of only zeroes and ones. Computer Science and Artificial Intelligence measure all data in terms of numbers, and zero is used most frequently. Zero is used to represent debt, credit, and balances in financial accounting. It is also critical for concepts such as Absolute Zero Temperature in science and the Null Hypothesis in statistics. Thus, Aryabhatta’s introduction of zero to mathematics was the turning point for mathematics, and the doors it opened for mathematicians to manipulate numbers grew vastly.