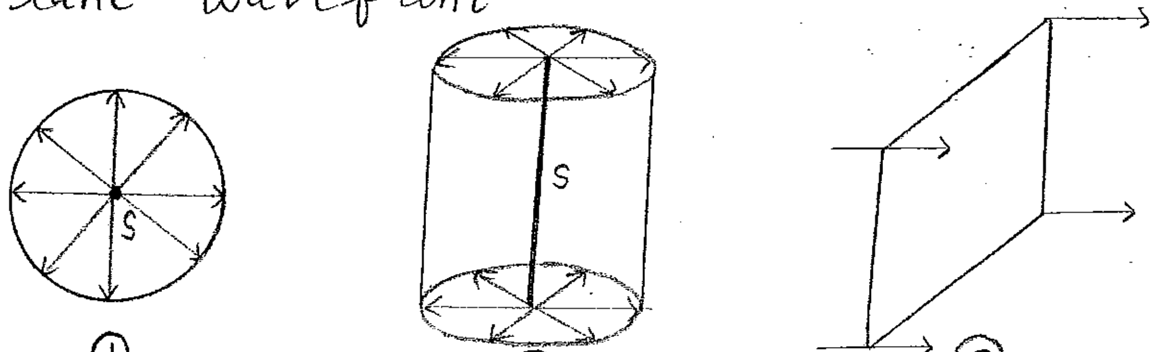
Wave optics

Wavefront → The locus of all the particles of the medium, which at any instant are vibrating in the same phase, is called the wavefront.

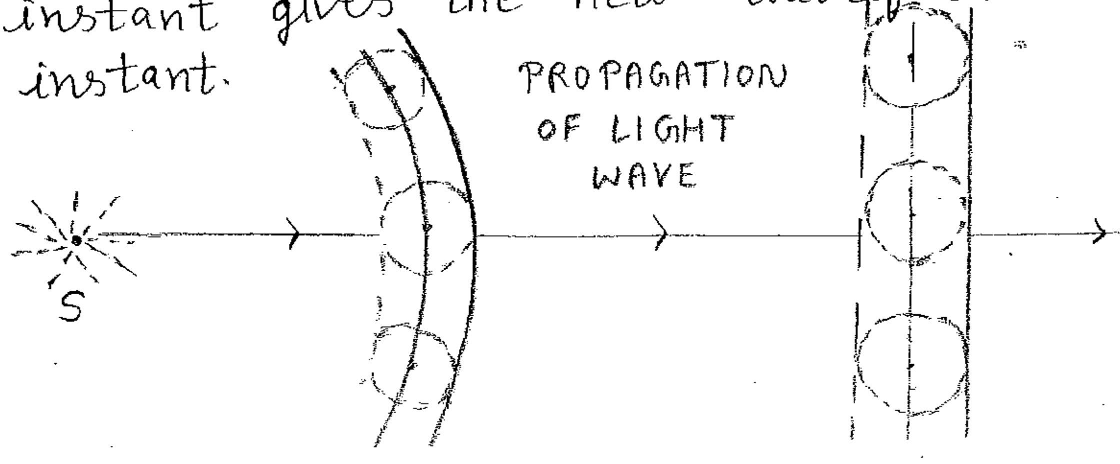
Types of wavefront →

① spherical wavefront
② cylindrical wavefront
③ plane wavefront

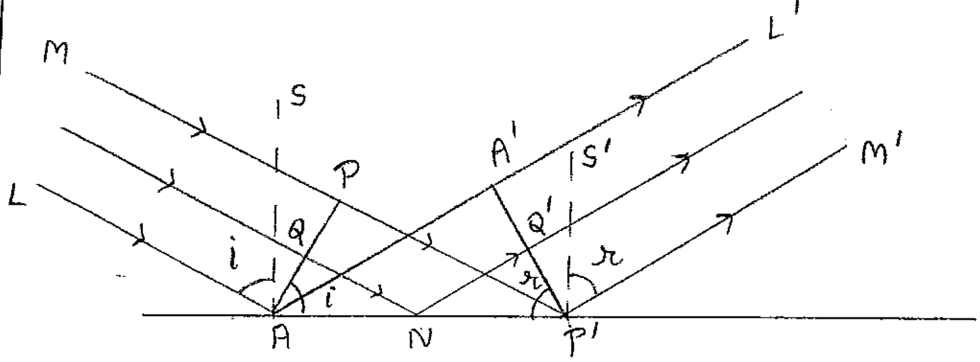


① ② ③

Huygen's Principle → ① Each point on the primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions.
② The disturbance (secondary wavelets) travel in all directions with the velocity of light in the medium.
③ A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wavefront at that instant.



Reflection on the basis of wave theory



To prove the laws of reflection

Let us consider a point Q on the incident wavefront. Suppose that when disturbance from point P on incident wavefront reaches point P', the disturbance from point Q reaches Q' via point N on the reflecting surface. Since P'A' represents the reflected wavefront, the time taken by light to travel from any point on incident wavefront to the corresponding point on the reflected wavefront should always be same.

If c is velocity of light, then time taken by light to go from the point Q to Q' is given by

$$t = \frac{QN}{c} + \frac{NQ'}{c}$$

$$QN = AN \sin i$$

$$NQ' = NP' \sin r$$

$$t = \frac{AN \sin i}{c} + \frac{NP' \sin r}{c}$$

$$= \frac{AN \sin i + (AP' - AN) \sin r}{c}$$

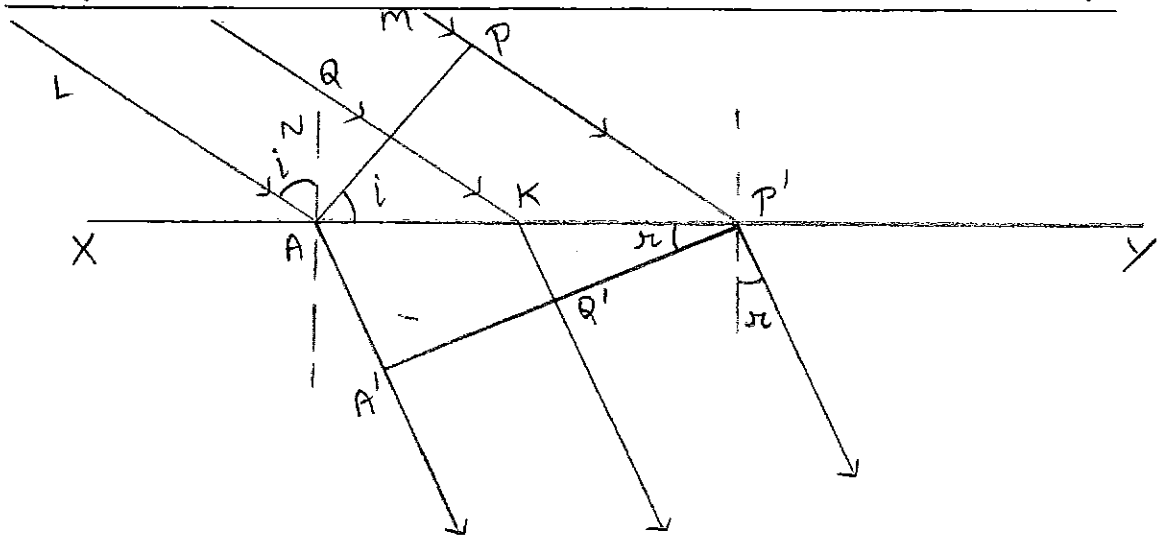
$$t = \frac{AP' \sin r + AN(\sin i - \sin r)}{c}$$

(If t is independent of N)

$\Rightarrow$ $\sin i - \sin r = 0$ or $\sin i = \sin r$ or $i = r$

$\Rightarrow$ Angle of incidence = Angle of reflection

Refraction on the basis of wave theory



To prove the laws of refraction

Let us consider any point Q on the incident wavefront. When disturbance from point P on incident wavefront reaches point P' on the refracted wavefront, the disturbance from point Q reaches point Q' via point K on the refracting surface XY. Since P'A' represents the refracted wavefront, the time taken by light to travel from a point on incident wavefront to the corresponding point on refracted wavefront should always be the same. Now time taken by light to go from Q to Q' will be

$$t = \frac{QK}{c} + \frac{KQ'}{v} \qquad QK = AK\sin i$$

$$= \frac{AK\sin i}{c} + \frac{KP'\sin r}{v} \qquad KQ' = KP'\sin r$$

$$= \frac{AK\sin i}{c} + \frac{(AP' - AK)\sin r}{v}$$

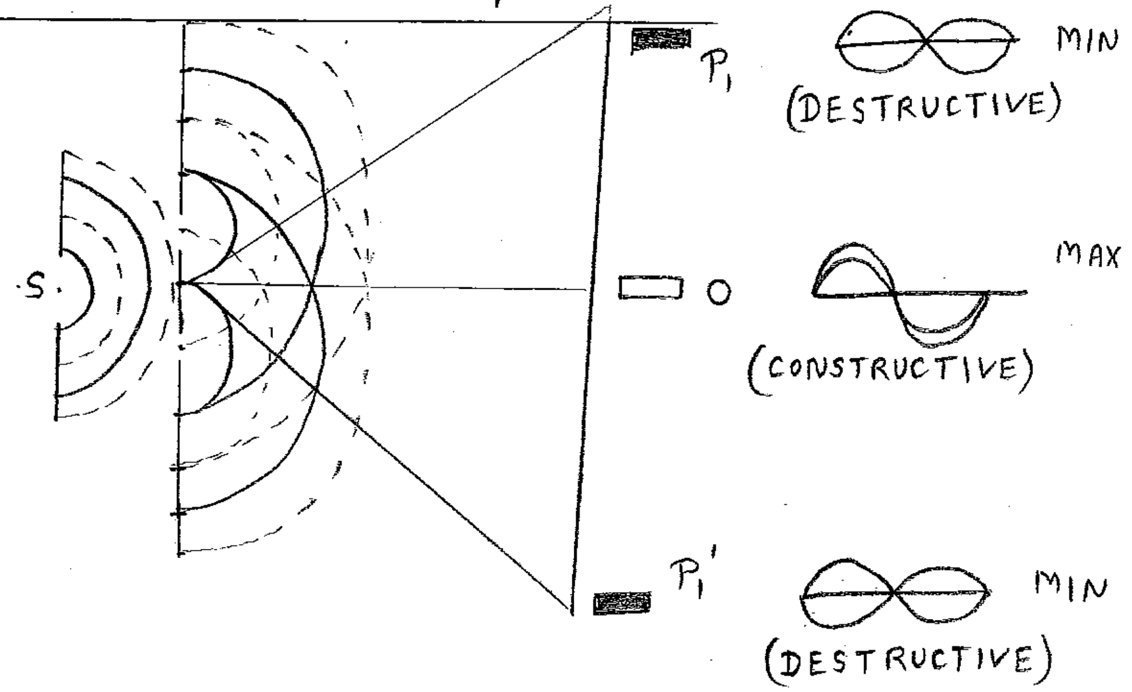
$$= \frac{AP'\sin r}{v} + AK\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right)$$

$$\Rightarrow \quad \frac{\sin i}{c} - \frac{\sin r}{v} = 0 \quad \text{or} \quad \frac{\sin i}{\sin r} = \frac{c}{v} = \mu$$

This is law of refraction.

Interference of light → When two light waves of the same frequency and having zero or constant phase difference travelling in the same direction superimpose on each other, the intensity in the region of superposition gets redistributed becoming maximum at some points and minimum at others. This phenomenon is called interference of light.

Young's double slit experiment



Conditions for constructive & destructive interference

Let us suppose that the displacements of two light waves from two coherent sources S_1 and S_2 at point P on the screen at any time t are given by

$$y_1 = a_1 \sin\omega t$$

$$\& \quad y_2 = a_2 \sin(\omega t + \phi)$$

where a_1 and a_2 are the amplitudes of the two waves, ϕ is constant phase difference between the two waves.

By the superposition principle, the resultant displacement at point P is

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2(\sin \omega t \cos\phi + \cos\omega t \sin\phi)$$

$$\Rightarrow \quad y = (a_1 + a_2 \cos\phi) \sin\omega t + a_2 \sin\phi \cos\omega t$$

Put $a_1 + a_2 \cos\phi = A \cos\theta$

& $a_2 \sin\phi = A \sin\theta$

Then $y = A \cos\theta \sin\omega t + A \sin\theta \cos\omega t$

or $y = A \sin(\omega t + \theta)$

where $A^2(\cos^2\theta + \sin^2\theta) = (a_1 + a_2 \cos\phi)^2 + a_2^2 \sin^2\phi$

or $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$

or $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$

Now Intensity (I) of wave $\propto$ (amplitude)^2

$$\Rightarrow \quad I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

Here $2\sqrt{I_1 \cdot I_2} \cos\phi$ is Interference term

<u>Constructive Interference</u> $\rightarrow$ 'I' will be max. when $\cos\phi = 1$

$\Rightarrow \phi = 0, 2\pi, 4\pi, \text{———}$ $\quad \frac{2\pi n}{\lambda} = 0, 2\pi, 4\pi, \text{—}$

or $n = 0, \lambda, 2\lambda, 3\lambda \text{———}$

<u>Destructive Interference</u> $\rightarrow$ 'I' will be min. when $\cos\phi = -1$

$\Rightarrow \phi = \pi, 3\pi, 5\pi, \text{——}$

or $m = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \text{———}$

Coherent & In Coherent sources

Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called Coherent sources.

Two sources of light which do not emit light waves with a constant phase difference are called incoherent sources.

Need of coherent sources for the production of Interference pattern

We know that, the resultant intensity is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$2\sqrt{I_1 I_2}\cos\phi$ is called Interference term

There are two possibilities

1. If $\cos\phi$ remains constt with time, the total energy at any point will be constt. we get max. intensity $(\sqrt{I_1} + \sqrt{I_2})^2$ at points where $\cos\phi = 1$ and min. intensity at points where $\cos\phi = -1$. The sources in this case are coherent.

2. If $\cos\phi$ varies continuously with time assuming both positive and negative values then the average value of $\cos\phi$ will be zero. At all points intensity $I = I_1 + I_2$ is same at all points. There will be general illumination. The two sources in this case are incoherent.

Conditions for obtaining two coherent sources of light →

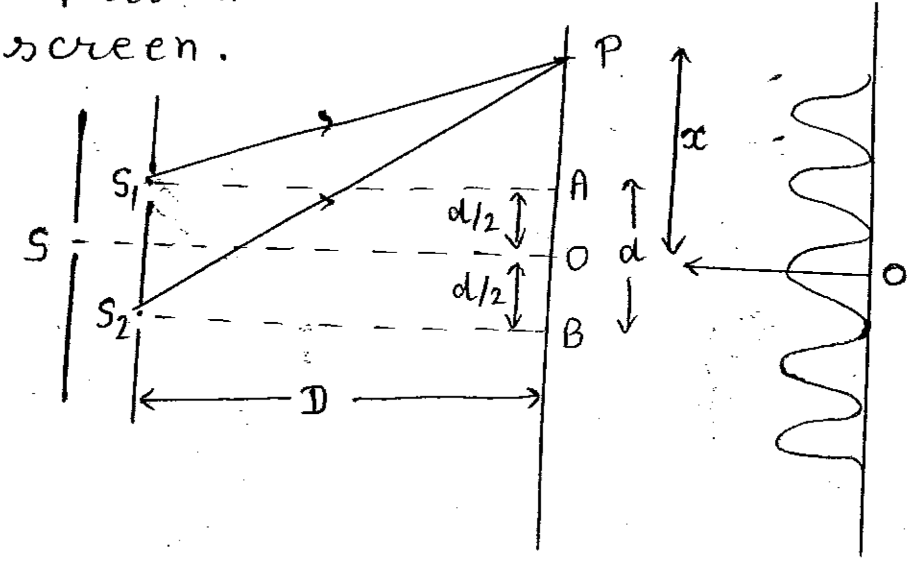
① The two coherent sources of light must be obtained from a single source (As two independent sources can not be coherent because in that case the phase difference will not be constant)

② The two sources must give monochromatic light (As different colours will produce different interference pattern)

③ The path difference between the waves arriving on the screen from the two sources must not be large (It should not exceed 30 cm otherwise phase difference will not be constt)

Theory of Interference fringes : Fringe width

Let us consider that a slit S is illuminated by a monochromatic source of wavelength λ. S_1 and S_2 are two narrow slits at equal & small distance from S such that they act as two coherent sources seperated by distance d. Interference pattern is observed at distance D on the screen.



Let us consider a point P on the screen at a distance x from the centre O

Path difference $\Delta = S_2P - S_1P$

$$S_2P^2 - S_1P^2 = [D^2 + (x + d/2)^2] - [D^2 + (x - d/2)^2]$$

$$\Rightarrow (S_2P - S_1P)(S_2P + S_1P) = 2xd$$

$$\text{or } S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

If Point P lies close to point O then

$$S_2P \approx S_1P \approx D$$

$$\Rightarrow S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D}$$

$$\text{or } \Delta = \frac{xd}{D}$$

<u>Positions of bright fringes</u> → For constructive interference

$$\Delta = \frac{xd}{D} = n\lambda$$

$$\Rightarrow x_n = \frac{n\lambda D}{d} \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$\therefore$ Positions are $x = 0, \frac{\lambda D}{d}, \frac{2\lambda D}{d}, \ldots$

<u>Positions of dark fringes</u> → For destructive interference

$$\Delta = \frac{xd}{D} = (2n+1)\lambda/2 \Rightarrow x_n' = (2n+1)\frac{\lambda D}{2d}$$

$$n = 0, 1, 2, 3, \ldots$$

$\therefore$ Positions are $x = \frac{\lambda D}{2d}, \frac{3\lambda D}{2d}, \ldots$

<u>Fringe width</u> → It is the separation between two successive bright or dark fringes.

$$\beta = x_n - x_{n-1} \quad \text{or} \quad x_n' - x_{n-1}'$$

$$= n\frac{\lambda D}{d} - \frac{(n-1)\lambda D}{d} \quad \text{or} \quad (2n+1)\frac{\lambda D}{2d} - \frac{(2n-1)\lambda D}{2d}$$

In both cases $\beta = \frac{\lambda D}{d}$

{which is same for all fringes.}

Conditions for sustained interference

① Two sources should be monochromatic and coherent and should emit light of same frequency and wavelength nearly along same direction.

② Two sources should be narrow and for better contrast, amplitude of interfering waves should be equal.

③ The distance between two coherent sources should be small and interfering waves should be in same state of polarisation.

Conservation of energy in Interference

We know that $I_{max.} \propto (a_1 + a_2)^2$

$\& \; I_{min.} \propto (a_1 - a_2)^2$

$$\therefore \quad I_{av.} \propto \frac{(a_1 + a_2)^2 + (a_1 - a_2)^2}{2}$$

or $I_{av.} \propto a_1^2 + a_2^2$

$\& \quad I = I_1 + I_2 \Rightarrow I \propto a_1^2 + a_2^2$

which is same as $I_{av.}$ in Interference pattern. So there is no violation of the law of conservation of energy. Whatever energy disappears from a dark fringe, an equal energy appears in a bright fringe.

Comparison of intensities at maxima & minima →

As Intensity $\propto$ (amplitude)^2

$\therefore \quad I_1 \propto a_1^2 \quad \& \quad I_2 \propto a_2^2$

$$\Rightarrow \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

Amplitude at maxima is $(a_1 + a_2)$
& Amplitude at minima is $(a_1 - a_2)$
Therefore ratio of intensities at maxima and minima is

$$\frac{I_{max.}}{I_{min.}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{\left(\frac{a_1}{a_2}+1\right)^2}{\left(\frac{a_1}{a_2}-1\right)^2} = \left[\frac{r+1}{r-1}\right]^2$$

[Here $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio]

Interference pattern with white light

When monochromatic light is replaced by white light, central bright fringe is white. This central bright fringe* is surrounded by a few coloured fringes. [As the wavelength of violet colour is least, so the fringe nearest to either side of the central white fringe is violet and the fringe farthest from the central white fringe is red.]*

Interference in thin films

→ When a thin film is seen with the monochromatic light, we find alternate dark and bright fringes.

In reflected light

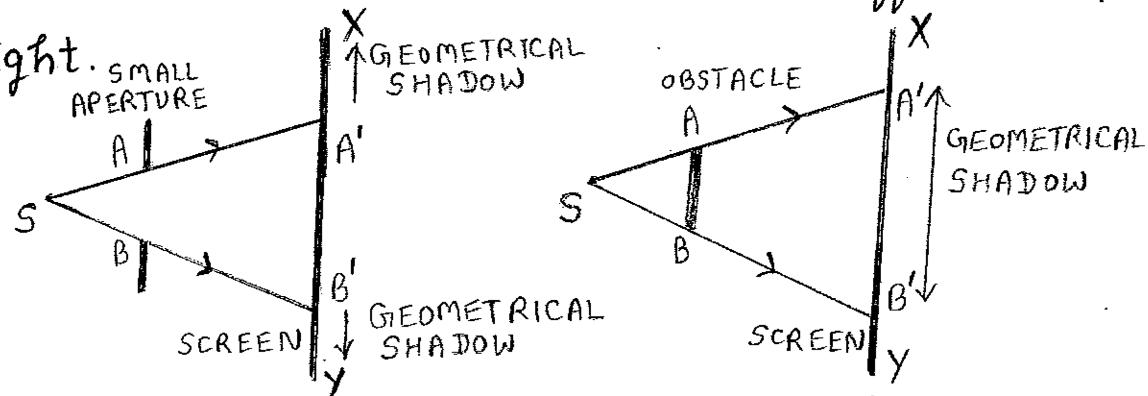
Net path difference $= 2\mu t \cos r + \lambda/2$

For bright fringe, $2\mu t \cos r + \lambda/2 = n\lambda$
or $2\mu t \cos r = (2n-1)\lambda/2$

& For dark fringe $2\mu t \cos r + \lambda/2$ the net path difference is equal to $(2n+1)\lambda/2$
so $2\mu t \cos r = n\lambda$

* In transmitted light, $2\mu t \cos r = n\lambda$ (bright)
$2\mu t \cos r = (2n+1)\lambda/2$ (dark)

Diffraction of light → The phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into the regions of geometrical shadow is called diffraction of light.



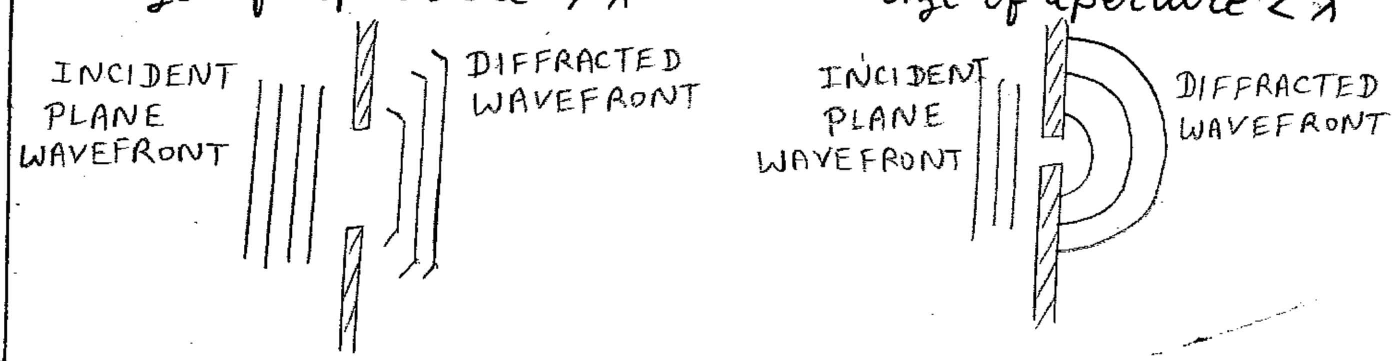
Types of diffraction → ① **Fresnel's diffraction** →

In this diffraction, the source and screen are placed close to the aperture or obstacle and light after diffraction appears converging towards the screen.

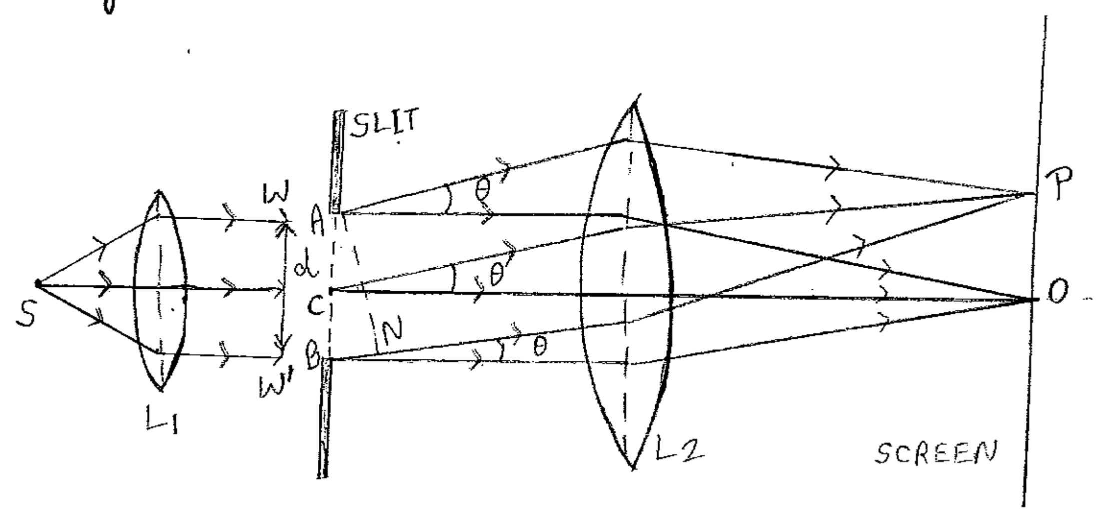
② **Fraunhoffer's diffraction** → In this diffraction, the source and screen are placed at large distances from the aperture so a converging lens is used to observe the diffraction pattern.

Size of aperture or obstacle observing diffraction

Size of aperture $> \lambda$ Size of aperture $< \lambda$



Diffraction at a single slit → A source S of monochromatic light is placed at the focus of a convex lens L_1. A parallel beam of light and hence a plane wavefront WW' gets incident on a narrow rectangular slit AB of width d.



The incident wavefront disturbs all parts of the slit AB simultaneously. Diffraction takes place at slit AB and diffraction pattern is focussed by a convex lens L_2 on a screen placed in its focal plane.

Central maximum → All the secondary wavelets going straight across the slit AB are focussed at the central point O on the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a central bright fringe.

Calculation of path difference → Let us suppose that the secondary wavelets diffracted at an angle θ are focussed at point P. The secondary wavelets start from different parts of the slit in same phase but they reach the point P in different phases.

From the diagram,

$$\text{Path difference}, \Delta = BP - AP = BN = AB \sin\theta$$

$$\text{or } \Delta = d \sin\theta$$

Positions of minima → Let point P be located such that $\theta = \theta_1$

$$\& \ d \sin\theta_1 = \lambda$$

We can divide the slit AB into two halves AC & CB. Then the path difference between the wavelets from A and C will be $\lambda/2$. Similarly, corresponding to every point in the upper half AC, there is a point in the lower half CB for which the path difference is $\lambda/2$. Hence they interfere destructively so as to produce minima.

For first dark fringe, $d \sin\theta_1 = \lambda$

& for n^{th} dark fringe,

$$d \sin\theta_n = n\lambda$$

If θ is small, then

$$\sin\theta_n \approx \theta_n = \frac{n\lambda}{d}$$

Positions of secondary maxima → Let us suppose that point P is located such that $\Delta = 3\lambda/2$

when $\theta = \theta_1'$ then $d \sin\theta_1' = \frac{3}{2}\lambda$

We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\lambda/2$. The wavelets from these parts will interfere destructively. However the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum.

For first secondary maximum

$$d \sin\theta_1' = \frac{3}{2}\lambda$$

& for second secondary maximum

$$d \sin\theta_2' = \frac{5}{2}\lambda$$

Hence for nth secondary maximum

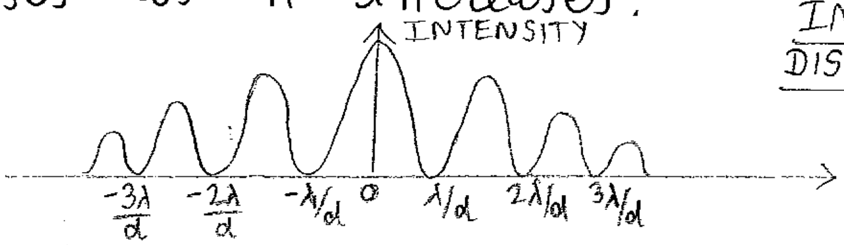
$$d \sin\theta_n' = (2n+1)\frac{\lambda}{2} \qquad [\, n = 1, 2, 3 \ldots \,]$$

If θ is small, then

$$d \sin\theta_n' \simeq d\,\theta_n' \text{ so}$$

$$\theta_n' = (2n+1)\frac{\lambda}{2d}$$

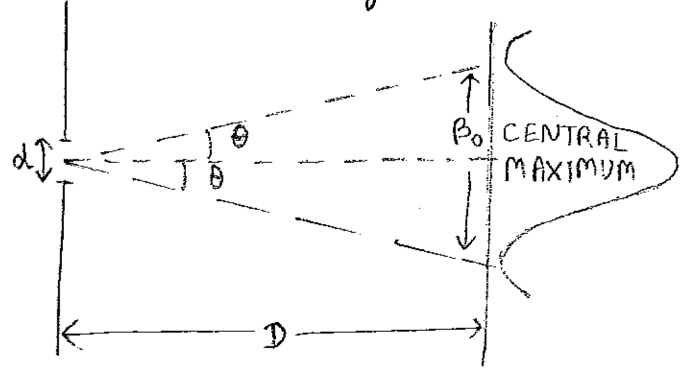
The intensity of secondary maxima decreases as 'n' increases.



INTENSITY DISTRIBUTION CURVE

Widths of central and secondary maxima →

Angular width of central maximum → It is the angular separation between the directions of the first minima on the two sides of the central maximum.



For first minima , $\theta = \lambda / d$

It is half angular width of central maximum.

$\therefore$ Angular width $= 2\theta = 2\lambda/d$

Linear width of central maximum

$$\beta_0 = D \times 2\theta = 2D\lambda/d \quad \left[\because \theta = \frac{Arc}{Radius}\right]$$

Linear width of a secondary maximum

We know that $\theta_n = n\lambda/d$

$$\theta_{n+1} = (n+1)\lambda/d$$

Angular width of n^{th} secondary maximum

$$= \theta_{n+1} - \theta_n = \lambda/d$$

Hence linear width of n^{th} secondary maximum

$$= D \times \lambda/d = \beta$$

Hence $\beta_0 = 2\beta$

The central maximum of a diffraction pattern is twice as wide as any secondary maximum.

Validity of Ray optics : Fresnel's distance

Ray optics as the limiting case of wave optics

Fresnel's distance → The distance at which the diffraction spread of a beam is equal to the size of the aperture is called Fresnel's distance.

We know that $\theta = \lambda / d$

If a screen is placed at distance D, this beam spreads over a linear width, $x = D\lambda/d$

If $x = d$ then $D = D_F$

$$\therefore D_F = \frac{d^2}{\lambda}$$

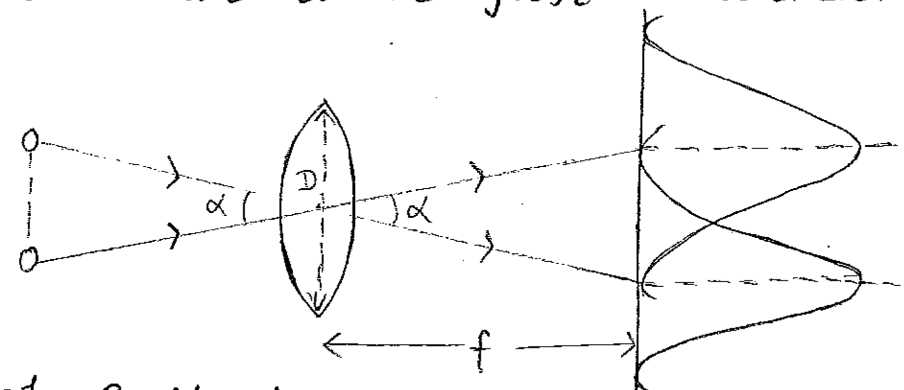
If $D < D_F$, then there will not be too much spreading by diffraction i.e. the light will travel along straight lines and concepts of ray optics will be valid.

Difference between Interference & Diffraction

Interference	Diffraction
① It is result of the superposition of secondary waves from two coherent sources.	① It is result of the superposition of two secondary wavelets coming from different parts of same wavefront.
② All fringes are of equal intensity & width	② Intensities & widths of bright fringes are not equal.
③ Dark fringes are perfectly dark i.e. there is good contrast between dark & bright fringes.	③ Dark fringes are not perfectly dark so there is poor contrast.

Resolving power of optical Instruments

When the two images can not be distinguished, they are said to be unresolved. If the images are well distinguished, they are said to be well resolved. If the images are just distinguished, they are said to be just resolved.



Rayleigh Criterion → Two objects or points are just resolved if the position of the central maximum of the image of one object coincides with the first minimum of the image of the other object.

Limit of resolution → The minimum distance of separation between two points so that they can be seen as separate (or just resolved) by the optical instrument is known as its limit of resolution.

Resolving power → The ability of an optical instrument to form distinctly separate images of the two closely placed points or objects is called its resolving power. It is the reciprocal of limit of resolution.

Resolving Power of Human Eye → Resolving power is reciprocal of limit of resolution. The limit of resolution of human eye is $\alpha = \frac{1.22\lambda}{D}$

D = diameter of pupil of eye

Resolving power $= \frac{1}{\alpha} = \frac{D}{1.22\lambda}$

Diameter of the pupil of human eye is about 2 mm. Eye is most sensitive to the wavelength 555 nm. So $\alpha = \frac{1.22\lambda}{D}$

$\Rightarrow \alpha = 1.2$ minute = limit of resolution

& resolving power is $1/\alpha$

Resolving Power of a telescope → we know that for a telescope $\alpha = \frac{1.22\lambda}{D}$ = Angular resolution

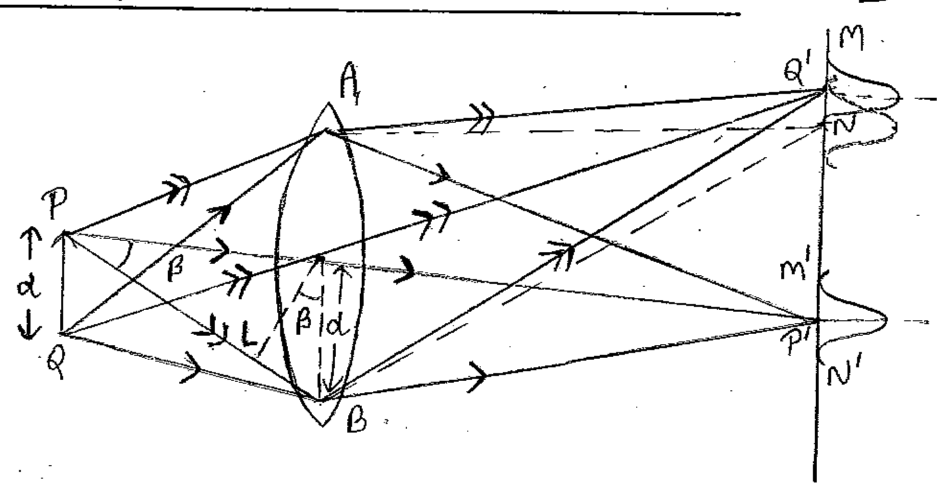
Here λ = wavelength of light

& D = diameter of objective lens

So Resolving Power $= \frac{D}{1.22\lambda}$

When we increase the diameter of objective of telescope, its resolving power increases.

Resolving power of a microscope → [only Result]*



Let PQ be the object of size d placed in front of the objective lens AB of the microscope. The central maxima of the images of P and Q are P′ and Q′ respectively. As the size of object PQ is decreased i.e. d decreases, central maximum P′ moves closer to the central maximum Q′.

According to Rayleigh criterion, two points P and Q are resolved if central max. P′ coincides with the minimum N near to Q′. The corresponding $d = d_{min.}$ gives the resolving limit of microscope.

Since N is the first minimum around Q′,

so $(QA + AN) - (QB + BN) = 1.22\lambda$

or $(QA - QB) + (AN - BN) = 1.22\lambda$ (Min. of Q at N)

Also P′ coincides with N, so

$PA + AN = PB + BN$ (Max. of P at N)

or $PA - PB = BN - AN$ or $PB - PA = AN - BN$

Putting this value in above eqn., we get

$(QA - QB) + (PB - PA) = 1.22\lambda$

If β is the semi-angle subtended by the objective AB at the object PQ, then

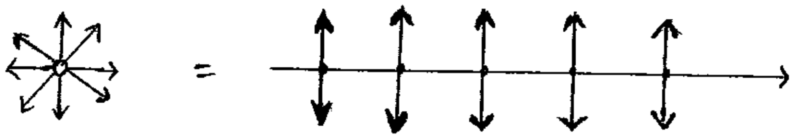
$(QA - QB) + (PB - PA) = 2 d_{min.} \sin\beta$ (From diagram)

$\Rightarrow \quad 2 d_{min.} \sin\beta = 1.22\lambda$

or $\quad d_{min.} = \dfrac{1.22\lambda}{2\sin\beta}$ & Resolving power $= \dfrac{1}{d_{min.}}$

*[If we fill the liquid of refractive index μ between object and objective of microscope then $\quad d_{min.} = \dfrac{1.22\lambda}{2\mu\sin\beta}$]

Polarisation → The phenomenon of restricting the vibration of a light (vector) in a particular direction in a plane perpendicular to the direction of propagation of light is called polarisation of light.

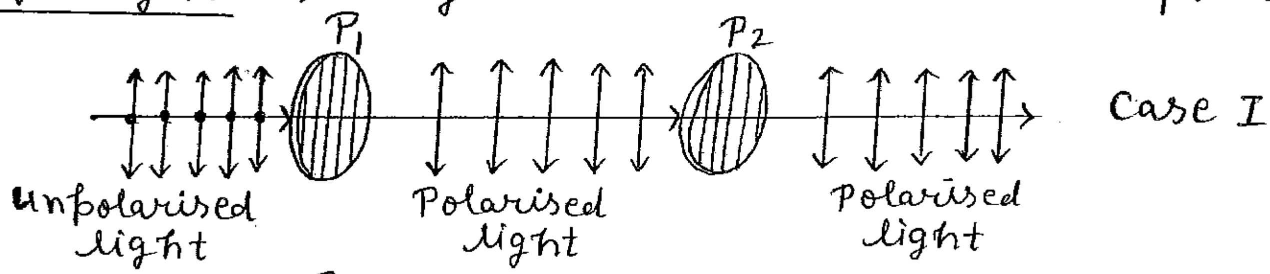


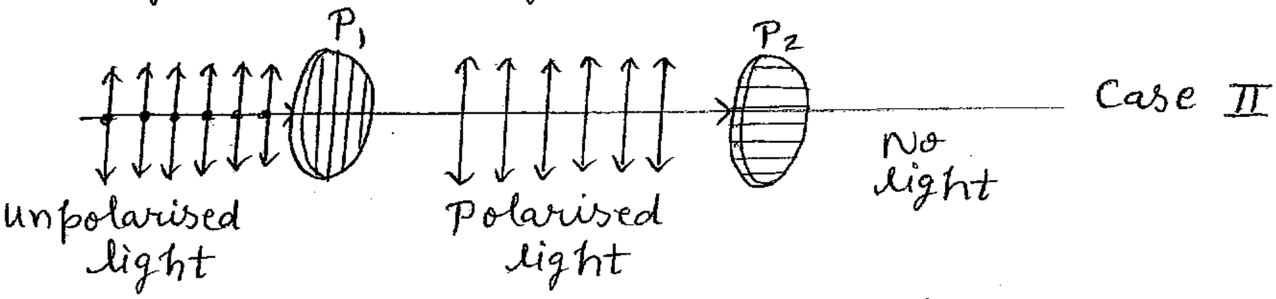
UNPOLARISED LIGHT

PLANE POLARISED LIGHT
(VIBRATIONS IN ONE PLANE)

Polaroids → It is a device used to produce the plane polarised light.
For example Tourmaline crystal

Experimental verification of transverse nature of light → Only transverse waves can be polarised





P_1 → Polariser $\quad$ P_2 → Analyser

In case I P_1 & P_2 have parallel crystallographic axis. i.e. $\theta = 0°$

In case II P_1 & P_2 have crossed crystallographic axis i.e $\theta = 90°$

Malus law → It states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.

We know that Intensity $\propto$ (Amplitude)^2

$\therefore$ Intensity of transmitted light through the analyser is given by $I \propto (E\cos\theta)^2$

or $I = kE^2\cos^2\theta$

Now $kE^2 = I_0$

$\therefore \quad I = I_0 \cos^2\theta$

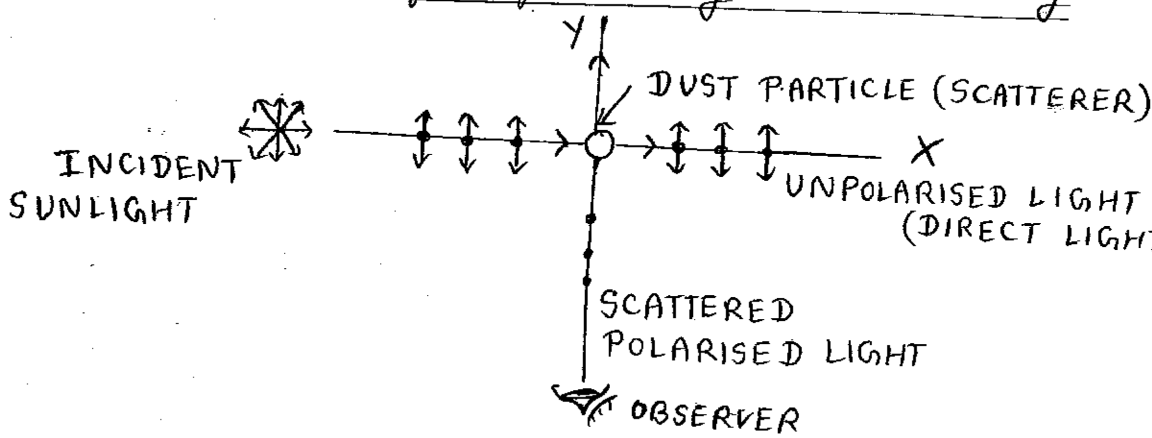
or $I \propto \cos^2\theta$

① If $\theta = 0$ $\quad I = I_0$ (P_1 & P_2 are parallel)

② If $\theta = 90°$ $\quad I = 0$ (P_1 & P_2 are perpendicular)

Methods of producing plane polarised light

① Polarisation of light by scattering



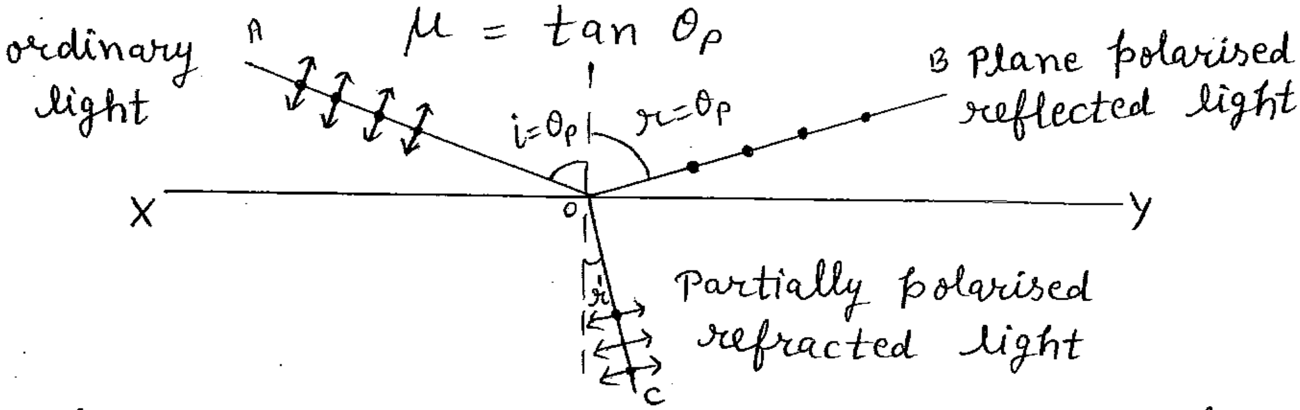
When light is incident on small particles of dust or air molecules etc. (having smaller size as compared to wavelength of light), it is absorbed by the electrons of atoms of these particles and re-radiated in all directions. This phenomenon is known as scattering.

Polarisation by reflection → when ordinary or unpolarised light falls obliquely on a glass slab, it is partly reflected and partly refracted. Both the reflected and refracted beams of light are partly polarised.

At a particular angle of incidence, the reflected light is completely plane polarised. This angle is known as polarising angle (θ_p).

Brewster's law → The refractive index of the medium (μ) is numerically equal to the tangent of the angle of polarisation (θ_p).

$$\mu = \tan \theta_p$$



When $\angle i = \angle \theta_p$ then reflected and refracted components are mutually perpendicular.

From figure $\angle BOY + \angle COY = 90^\circ$

$$(90^\circ - r) + (90^\circ - r') = 90^\circ$$

or $(90^\circ - \theta_p) + (90^\circ - r') = 90^\circ$

or $r' = 90^\circ - \theta_p$

Now $\mu = \dfrac{\sin i}{\sin r'} \Rightarrow \mu = \dfrac{\sin \theta_p}{\sin(90^\circ - \theta_p)}$

or $\mu = \dfrac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p$

Applications of polarised light and Polaroids →

① **In sun-glasses** → They reduce glare produced by direct light.

② **In windows of trains & aeroplanes** → They use a pair of polaroids so that we can adjust the intensity of light.

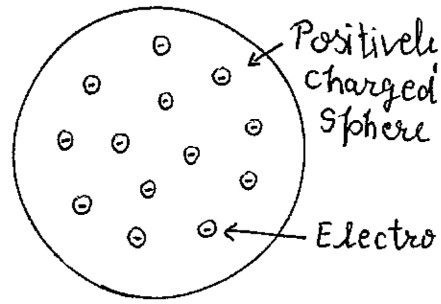
③ **In microscopes** → They reduce glare so that we can see very minute objects clearly.

④ In L.C.D. (i.e liquid crystal display), in calculators, watches, T.V., computers etc. we use principle of polarisation to form numbers and patterns.

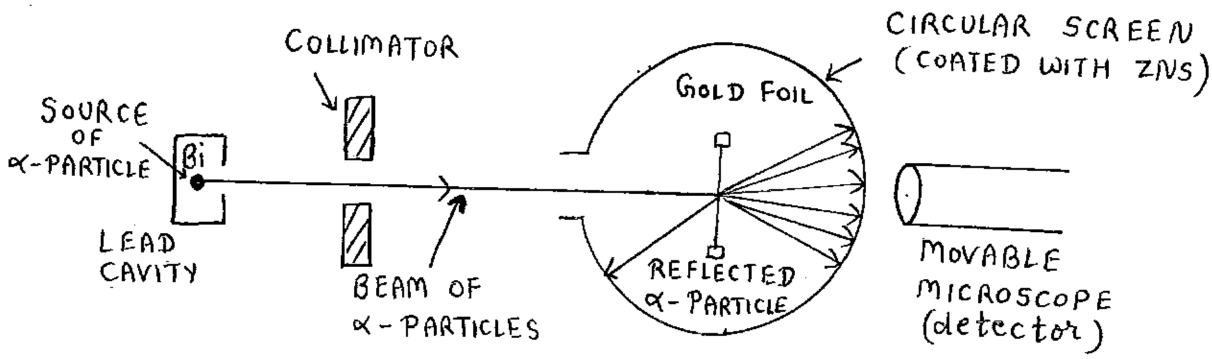
⑤ Polarised laser beams are used as needles for producing sound from compact discs in CD-players.

Atoms and Nuclei

Thomson model → An atom is a sphere of positive charges of uniform density of about 10^{-10}m diameter in which negative charges are embedded like plums in the pudding. This is also called 'Plum pudding model'.



Alpha-Particle scattering experiment and Rutherford's Nuclear model of atom →



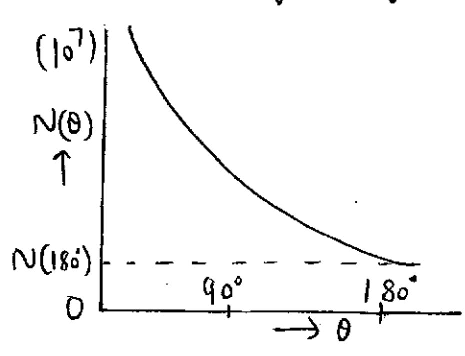
observations → (i) Most of the α-particles passed through the gold foil undeflected.

(ii) Some of the α-particles were deflected through small angles ($>1^\circ$)

(iii) A few α-particles were deflected through large angles ($>90^\circ$) & very few retraced their path.

(iv) The number of α-particles per unit area [$N(\theta)$] that reach the screen at a scattering angle θ were found to vary as

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$



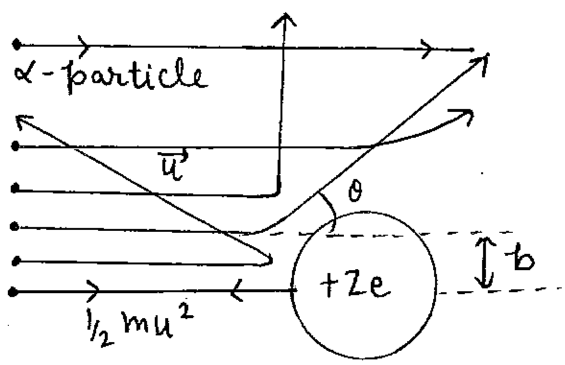
Conclusions → (i) As most of the α-particles pass through the gold foil undeflected, so it indicates that most of the space in an atom is empty.

(ii) The positive charges in an atom were concentrated in a very small region at the centre of the atom.

(iii) Electrons being very light do not affect the α-particles.

Alpha-Particle Trajectory and Impact Parameter

The perpendicular distance of the velocity vector ($\vec{u}$) of the incident α-particle from the centre of the nucleus when α-particle is not deflected is known as Impact Parameter (b).



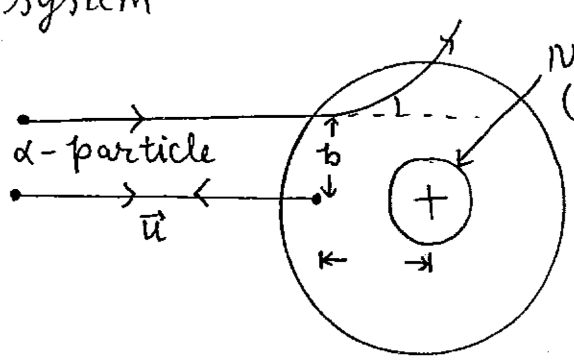
The angle between the direction of approach of the alpha particle and the direction of moving away of α-particle is called scattering angle (θ)

$$b = \frac{Ze^2 \cot \theta/2}{4\pi\epsilon_0 E} \quad \& \quad E = \frac{1}{2}mu^2$$

Distance of closest approach

K.E. of α-particle = P.E. of system

$$\frac{1}{2}mu^2 = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{r_0}$$



$$\Rightarrow r_0 = \frac{2Ze^2}{4\pi\epsilon_0(\frac{1}{2}mu^2)}$$

$$\Rightarrow r_0 = \frac{2Ze^2}{4\pi\epsilon_0 (K.E.)}$$

Drawbacks of Rutherford's model of Atom

(i) It failed to fully explain the stability of the atom.

(ii) It failed to explain the complex spectrum emitted by an atom.

Atomic Spectra → Atomic spectra deals with frequencies (or wavelengths) and intensities of electromagnetic radiations absorbed or emitted by atom.

Types of atomic spectra

(i) Line emission spectrum → It is the series of lines of various wavelengths emitted by an atomic gas or vapour usually excited by passing an electric current through it.

Examples (i) The spectrum of mercury vapours has strong blue & strong green lines

(ii) Spectrum of sodium has strong yellow line ($\lambda = 590 nm$) alongwith a closely spaced dim yellow line ($\lambda = 596 nm$)

(iii) Spectrum of neon has intense red lines responsible for the red colour of neon bulb.

(ii) Line Absorption spectrum → When white light passes through a gas or vapour, then the spectrum of transmitted light observed with a spectrometer shows some dark lines. This is known a absorption spectrum. This is similar to emission spectrum.

Bohr's atomic model

(i) An electron revolves around the nucleus with a definite fixed energy in a fixed path known as stationary state (energy level).

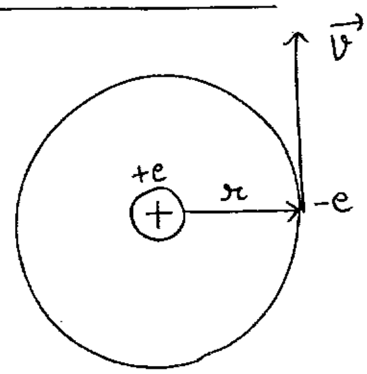
(ii) It states that electrons can revolve only in those energy levels, in which its angular momentum is an integral multiple of $h/2\pi$.

(iii) It states that electron can jump from higher energy level to lower energy level radiating energy in the form of a photon of frequency ν.

$$h\nu = E_i - E_f$$

Bohr's theory of Hydrogen atom (Derivations)

Coulomb's force of attraction between the nucleus and the electron revolving in an orbit of radius r_n is given by



$$F_n = \frac{e^2}{4\pi\epsilon_0 r_n^2}$$

This provides centripetal force for electron

$$\Rightarrow \quad \frac{m v_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2} \quad \text{or} \quad m v_n^2 = \frac{e^2}{4\pi\epsilon_0 r_n}$$

Now $L_n = m v_n r_n = nh/2\pi$

$$\Rightarrow \quad v_n = \frac{nh}{2\pi m r_n}$$

Radius of an orbit $\quad m\left(\frac{nh}{2\pi m r_n}\right)^2 = \frac{e^2}{4\pi\epsilon_0 r_n}$

$$\Rightarrow \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\Rightarrow \quad r_n \propto n^2$$

Speed of an electron in an orbit

$$v_n = \frac{nh}{2\pi m r_n} = \frac{nh}{2\pi m}\left(\frac{\pi m e^2}{n^2 h^2 \epsilon_0}\right)$$

$$\Rightarrow v_n = \frac{e^2}{2h\epsilon_0 n}$$

$$\Rightarrow v_n \propto \frac{1}{n}$$

Total energy of an electron in an orbit

$$E_n = K.E._n + P.E._n$$

$$K.E._n = \tfrac{1}{2} m v_n^2$$

$$= \frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r_n}$$

$$P.E._n = -\frac{e^2}{4\pi\epsilon_0 r_n}$$

$$\Rightarrow E_n = \frac{e^2}{8\pi\epsilon_0 r_n} - \frac{e^2}{4\pi\epsilon_0 r_n}$$

$$= \frac{-e^2}{8\pi\epsilon_0 r_n}$$

or $$E_n = \frac{-me^4}{8h^2\epsilon_0^2 n^2}$$

After putting the values of constants for Hydrogen, we get

$$E_n = \frac{-13.6}{n^2}\ \text{eV}$$

Frequency (ν_n) of an electron in nth orbit

$$\nu_n = \frac{v_n}{2\pi r_n}$$

$$v_n = \frac{e^2}{2h\epsilon_0 n} \quad \& \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\Rightarrow \nu_n = \frac{1}{2\pi} \times \frac{e^2}{2h\epsilon_0 n} \times \frac{\pi m e^2}{n^2 h^2 \epsilon_0} = \frac{me^4}{4h^3\epsilon_0^2 n^3}$$

$$\Rightarrow \nu_n \propto 1/n^3$$

Energy levels of hydrogen atom

$$E_n = \frac{-13.6}{n^2}\ \text{eV}$$

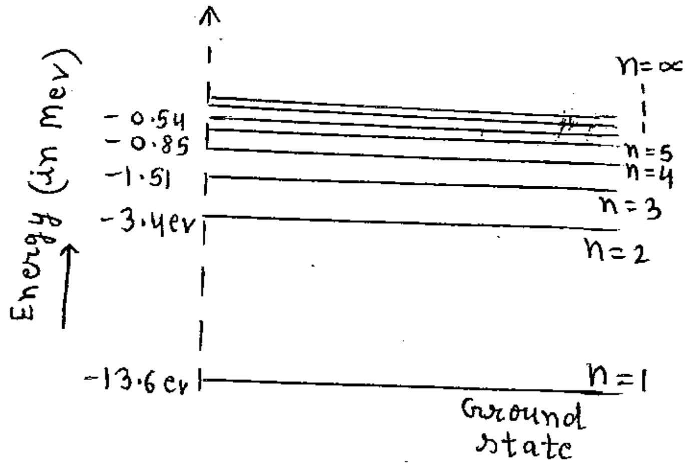
For $n = 1$, $E_1 = -13.6\ \text{eV}$ (ground state)

For $n = 2$, $E_2 = \frac{-13.6}{2^2} = -3.4\ \text{eV}$

For $n = 3$, $E_3 = -\frac{13.6}{3^2} = -1.51\ \text{eV}$

--- For $n = \infty$, $E_\infty = -\frac{13.6}{\infty} = 0$

It is clear from the diagram that when principle quantum number (n) increases, the energy levels come closer & closer to each other.



The line spectra of Hydrogen atom

$$h\nu = E_{n_i} - E_{n_f}$$

Now $$E_n = \frac{-me^4}{8h^2\epsilon_0^2 n^2}$$

$$\Rightarrow h\nu = \frac{-me^4}{8\epsilon_0^2 n_i^2 h^2} - \left(\frac{-me^4}{8\epsilon_0^2 n_f^2 h^2}\right)$$

$$\Rightarrow \nu = \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = \frac{c}{\lambda}$$

or $$\frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 c h^3}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

or $$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \left\{ R = \frac{me^4}{8\epsilon_0^2 c h^3} = 1.097\times 10^7\ m^{-1} \right\}$$

$$\frac{1}{\lambda} = \bar{\nu}\ \text{(wave number)} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

Various spectral series (H-atom)

(i) **Lyman series** → The spectral lines emitted due to the transition of an electron from any outer orbit ($n_i = 2, 3, 4, ---$) to the first orbit ($n_f = 1$) form a spectral series known as Lyman series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\Rightarrow \bar{\nu} = R\left[\frac{1}{1^2} - \frac{1}{n_i^2}\right] \qquad n_i = 2, 3, 4, ---$$

Longest wavelength ($\lambda_{L_{max}}$)

$$\frac{1}{\lambda_{L_{max}}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$$

$$\lambda_{L_{max}} = \frac{4}{3R} = 1.216 \times 10^{-7}\ m$$

Shortest wavelength ($\lambda_{L_{min.}}$)

$$\frac{1}{\lambda_{L_{min.}}} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right] = R$$

$$\lambda_{L_{min.}} = \frac{1}{R} = 0.911 \times 10^{-7}\ m$$

This series lies in ultra-violet region

(ii) **Balmer series** → The spectral lines emitted due to the transition of an electron from any outer orbit ($n_i = 3, 4, 5, 6, --$) to the second orbit ($n_f = 2$) form a spectral series known as Balmer series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right] \qquad n_i = 3, 4, 5, --$$

Longest wavelength $\lambda_{B_{max}}$

$$\frac{1}{\lambda_{B_{max}}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\lambda_{B_{max}} = \frac{36}{5R} = 6560\ Å$$

Shortest wavelength ($\lambda_{B\,min.}$)

$$\frac{1}{\lambda_{B\,min.}} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4}$$

$$\lambda_{B\,min.} = \frac{4}{R} = 3644\ A^\circ$$

This series lies in the visible region.

(iii) Paschen series → The spectral lines emitted due to the transition of an electron from any outer orbit ($n_i = 4, 5, 6$ ---) to the third orbit ($n_f = 3$) form a spectral series known as Paschen series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_i^2}\right] \quad n_i = 4, 5, 6 \ldots$$

Longest wavelength ($\lambda_{Pa\,max}$)

$$\frac{1}{\lambda_{Pa\,max}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{7R}{144}$$

$$\lambda_{Pa\,max} = \frac{144}{7R} = 18741\ A^\circ$$

Shortest wavelength ($\lambda_{Pa\,min.}$)

$$\frac{1}{\lambda_{Pa\,min.}} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = \frac{R}{9}$$

$$\lambda_{Pa\,min.} = \frac{9}{R} = 8199\ A^\circ$$

This series lies in the Infra-red region.

(iv) Brackett Series → The spectral lines emitted due to the transition of an electron from any outer orbit ($n_i = 5, 6, 7$, ---) to the fourth orbit ($n_f = 4$) form a spectral series known as Brackett series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_i^2}\right] \quad n_i = 5, 6, 7, \ldots$$

Longest wavelength ($\lambda_{Br.\ max}$)

$$\frac{1}{\lambda_{Br.\ max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right] = \frac{9R}{400}$$

$$\text{or} \quad \lambda_{Br.\ max} = \frac{400}{9R} = 40589\ A^\circ$$

Shortest wavelength ($\lambda_{Br.\ min.}$)

$$\frac{1}{\lambda_{Br\ min.}} = R\left[\frac{1}{4^2} - \frac{1}{\infty}\right] = \frac{R}{16}$$

$$\text{or} \quad \lambda_{Br.\ min.} = \frac{16}{R} = 14576\ A^\circ$$

This series lies in Infra-red region.

(V) Pfund series → The spectral lines emitted due to the transition of an electron from any outer orbit ($n_i = 6, 7, 8, ---$) to the fifth orbit ($n_f = 5$) form a spectral series known as Pfund series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_i^2}\right] \quad n_i = 6, 7, 8, ---$$

Longest wavelength ($\lambda_{Pf\ max.}$)

$$\frac{1}{\lambda_{Pf\ max.}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right] = \frac{11R}{900}$$

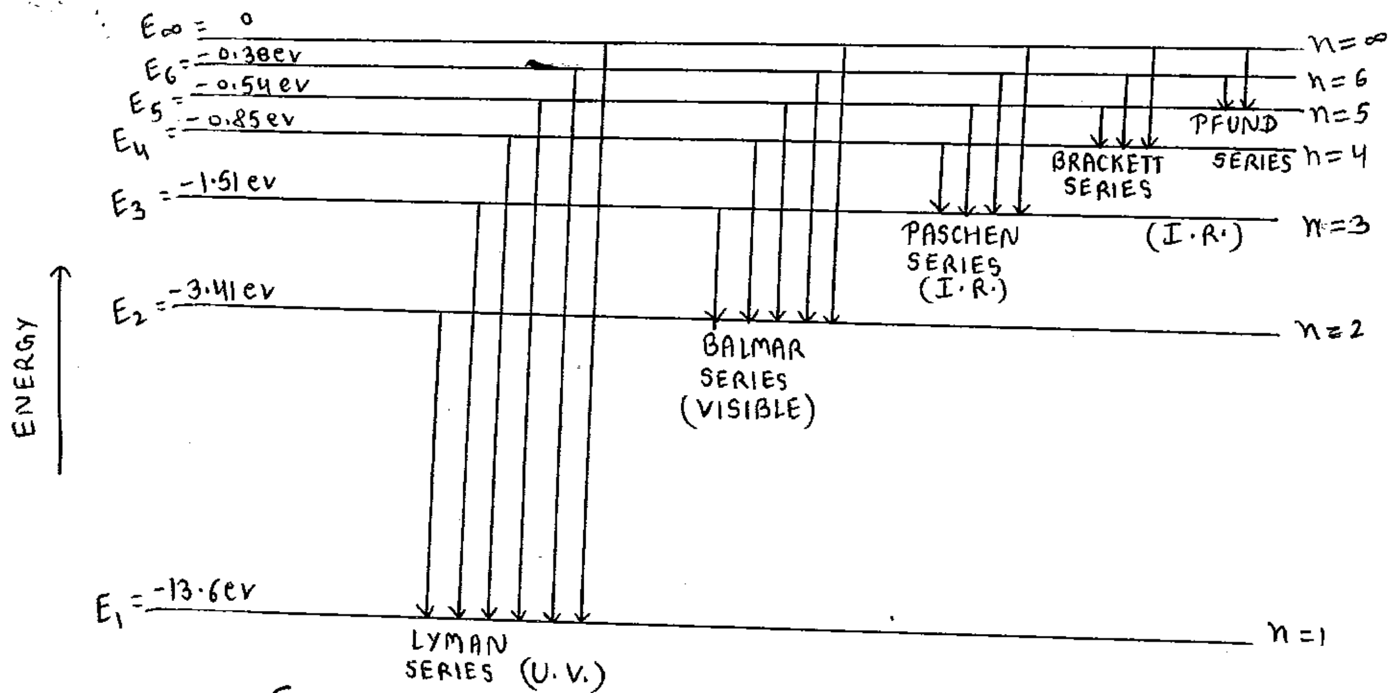
$$\text{or} \quad \lambda_{Pf\ max.} = \frac{900}{11R} = 74536\ A^\circ$$

Shortest wavelength ($\lambda_{Pf\ min.}$)

$$\frac{1}{\lambda_{Pf\ min.}} = R\left[\frac{1}{5^2} - \frac{1}{\infty}\right] = \frac{R}{25}$$

$$\text{or} \quad \lambda_{Pf\ min.} = \frac{25}{R} = 22775\ A^\circ$$

This series lies in Infra-red region.



[Energy level diagram (H-atom)]

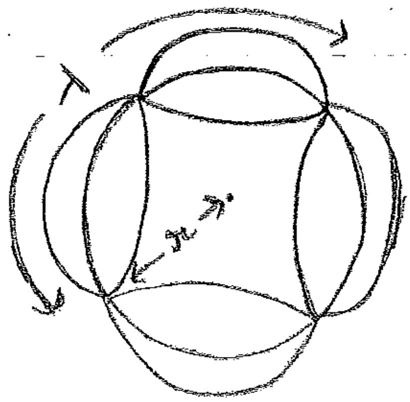
De-Broglie's explanation of Bohr's second postulate of quantization of Angular momentum →

Bohr could not explain as to why only certain energy levels were allowed for orbiting the e^- around the nucleus of atom. De-Broglie assumed that an electron orbit would be stable only if contained an integral multiple of e^- wavelength. According to him

$$2\pi r = n\lambda \quad , \lambda = \frac{h}{mv}$$

$$\Rightarrow 2\pi r = \frac{nh}{mv} \quad \text{or} \quad mvr = \frac{nh}{2\pi}$$

$$\Rightarrow L = n\,(h/2\pi)$$



Drawbacks of Bohr's model

(i) It could not explain spectra of complex atoms.
(ii) It could not explain fine structure of Balmer series spectral lines.
(iii) It could not account for wave nature of electrons
(iv) It could not explain Zeeman and Stark effect.

Composition of a Nucleus

The number of protons in a nucleus is known as atomic number (Z).
The sum of no. of protons (Z) and no. of neutrons (N) is called mass number (A).

$$A = Z + N \quad \text{or} \quad N = A - Z$$

Nucleus is represented by $_ZX^A$ or ^A_ZX

Decay of free neutron

$$\underset{\text{(Neutron)}}{{}_0n^1} \longrightarrow \underset{\text{(Proton)}}{{}_1H^1} + \underset{\text{(electron)}}{{}_{-1}e^0} + \underset{\text{(Antineutrino)}}{\bar{\nu}}$$

The half time of a free neutron is about 15 minutes.

Size of nucleus

$$\text{Volume } V = \frac{4}{3}\pi R^3 \propto A$$

$$\text{or } R \propto A^{1/3} \quad \text{or } R = R_0 A^{1/3}$$

R_0 is empirical constant $R_0 = 1.2$ fermi

Einstein's mass-energy equation

$$E = mc^2 \quad \text{(Acc to Einstein)}$$

Rest mass energy is m_0c^2
If T is K.E. of particle then
Total mass-energy, $E = m_0c^2 + T$

$$\text{or } mc^2 = m_0c^2 + T$$

$$\text{or } T = (m - m_0)c^2 = \Delta mc^2$$

$$T = \Delta mc^2$$

Mass Defect → It is defined as the difference between the mass of constituent nucleons of a nucleus in the free state and the mass of the nucleus. It is denoted by Δm.

$$\Delta m = \{Z m_p + (A-Z) m_n\} - M$$

Nuclear Binding energy → The total energy required to disintegrate the nucleus into its constituent particles (i.e. nucleons) is called nuclear binding energy.

The energy equivalent to mass-defect is the binding energy of the nucleus.

$$E_b = \Delta m . c^2 \quad \text{(Binding energy)}$$

$$\Delta m = \{Z m_p + (A-Z) m_n\} - M$$

$$\Rightarrow E_b = [\{Z m_p + (A-Z) m_n\} - M] \cdot c^2$$

We can also write, $E_b = \Delta m \times 931$ MeV

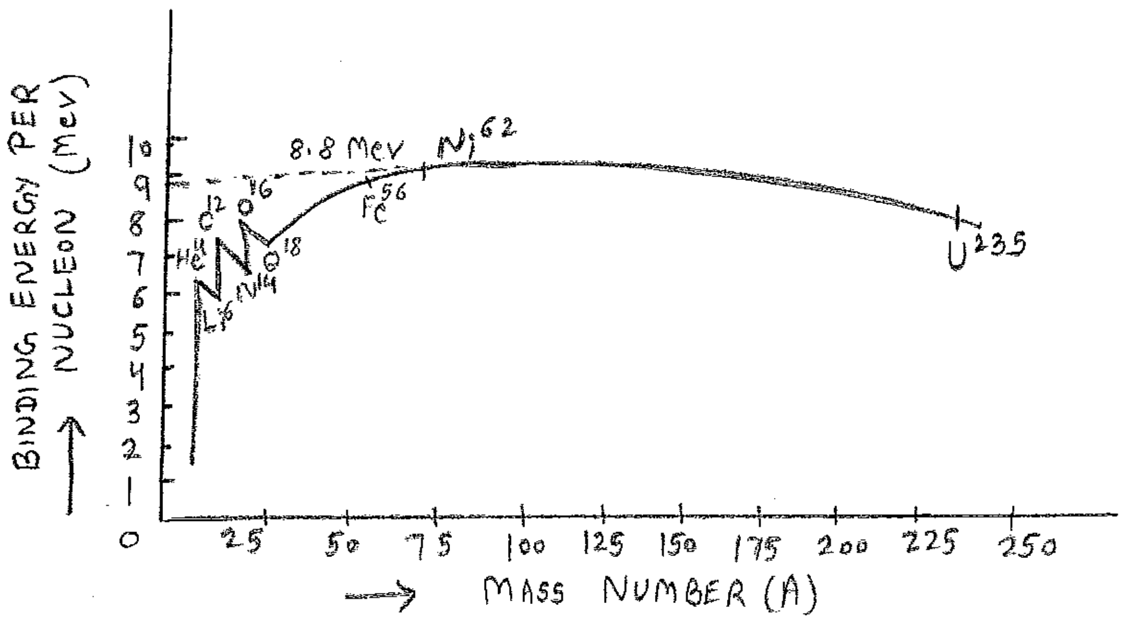
Nuclear Binding Energy Per Nucleon → The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Binding Energy}}{\text{Mass Number}}$$

$$E_{bn} = \frac{\Delta m \times 931}{A} \text{ MeV}$$

B.E./Nucleon determines the stability of a nucleus.

B.E. per Nucleon curve



(i) Average B.E. per nucleon for mass number less than 3 is very small.

(ii) Some nuclei $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$ have greater B.E./N than their neighbouring nuclei.

(iii) For $_{28}Ni^{62}$, B.E. per nucleon is maximum (i.e. 8.8 MeV). Thus Iron, Nickel etc. are stable elements.

(iv) For $A > 62$, B.E./nucleon decreases and for $A = 238$, it drops to 7.5 MeV.

Importance of the curve → (i) When a heavy nucleus splits up into lighter nuclei, the B.E. per nucleon of lighter nuclei is more than that of heavy nucleus. Thus energy is released. (Nuclear fission)

(ii) When two very light nuclei $A \leq 10$ combines to form a relatively heavy nucleus then energy is released (Nuclear fusion).

Packing Fraction → It is defined as the mass difference per nucleon.

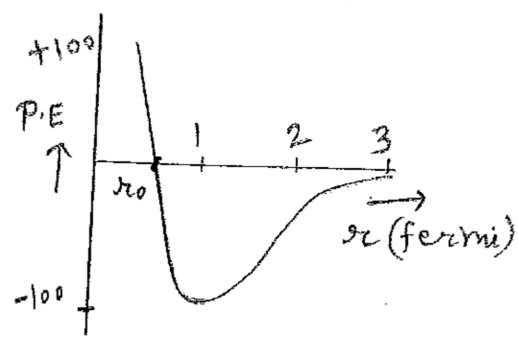
$$P_f = \frac{(M-A)}{A}$$

Here M is actual mass of nucleus.
Packing fraction measures the stability of the nucleus.

Nuclear forces → Nuclear forces are the strongest and short-range forces. Nuclear forces arise due to exchange of particles known as π-mesons between the nucleons. They are attractive forces. They are charge independent.

* π-meson is a fundamental particle whose mass is 270 times the mass of electron. There are three types of π-mesons [π^+, π^-, π^0]

* For distance $r_0 < 0.8$ fermi, nuclear forces become repulsive.



Nuclear Instability

The presence of neutrons in a nucleus plays an important role in nuclear stability. A nucleus becomes stable when no. of neutrons is equal to no. of protons in it. Thus heavy nuclei become stable by converting a neutron into a proton.

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$$

Radio activity → The phenomenon of spontaneous emission of radiation by heavy unstable elements is called radio-activity.

Cause of radioactivity → Instability of heavy nuclei is the cause of radioactivity. Heavy nuclei have large number of protons. Electrostatic repulsion between protons make the heavy nucleus unstable. Heavy nuclei become stable by converting a neutron into a proton by emitting β-particle and antineutrino till the no. of neutrons is equal to no. of protons in the nucleus.

Becqueral rays → There are three types of rays emitted during radioactive decay ; α-rays, β-rays, γ-ray.

α - Decay →

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

β - Decay →

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + \overline{\nu}$$

γ - Decay →

$$\underset{\text{(Excited nucleus)}}{_{Z}X^{A}} \longrightarrow \underset{\text{(Ground state)}}{_{Z}X^{A}} + \underset{\text{(Gamma particle)}}{\gamma}$$

Properties of α, β & γ particles →

(1) α - particles → (i) An α - particle is equivalent to helium nucleus ($_2He^4$).

(ii) They have low penetrating power & high ionising power.

(iii) Their velocities ranges from 1.4×10^7 m/s to 2.2×10^7 m/s.

(iv) They slightly affect the photographic plates.

(v) They can cause skin burns.

(2) β - particles → (i) A β - particle is compared to a fast moving electron ($_{-1}e^0$).

(ii) They have more penetrating power but less ($\frac{1}{100}$th) ionising power than α-particle

(iii) Their velocity can have value nearly equal to vel of light.

(iv) They affect the photographic plates.

(v) They are deflected by electric & magnetic fields.

(3) γ - particles → (i) They have no charge and their relative rest mass is zero.

(ii) They travel with speed of light in vacuum.

(iii) They have very high penetrating power & low ionising power ($\frac{1}{100}$th of β - particles).

(iv) They affect the photographic plates.

(v) They are not deflected by electric & magnetic field.

Law of Radioactive decay → The rate of disintegration of a radioactive substance at an instant is directly proportional to the number of nuclei in the radioactive substance at that time.

Mathematical form → Let N_0 be the no. of atoms present in radioactive substance initially. When disintegration begins then after time t, no of atoms remained N.

According to decay law

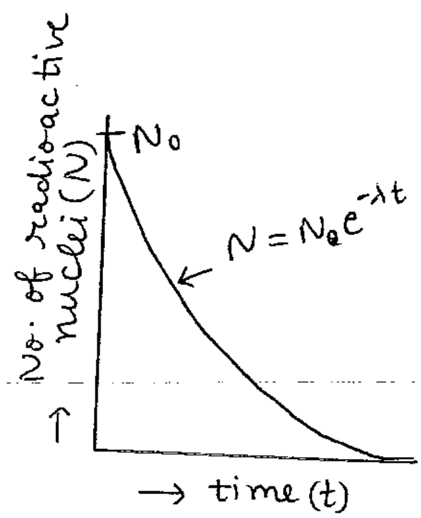
$$-\frac{dN}{dt} \propto N \qquad [\lambda \text{ is disintegration constant}]$$

$$\text{or} \quad R = -\frac{dN}{dt} = \lambda N$$

$$\text{or} \quad \frac{dN}{N} = -\lambda dt$$

$$\Rightarrow \int \frac{dN}{N} = -\int \lambda dt$$

$$\Rightarrow \log_e N = -\lambda t + k$$



at $t = 0$ $N = N_0$

$$\Rightarrow \log_e N_0 = k$$

$$\Rightarrow \log_e N = -\lambda t + \log_e N_0$$

$$\Rightarrow \log_e\left(\frac{N}{N_0}\right) = -\lambda t \qquad \left[\because \log N - \log N_0 = \log \frac{N}{N_0}\right]$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\text{or} \quad N = N_0 e^{-\lambda t}$$

Radioactive decay (disintegration) constant

Radioactive decay constant is the reciprocal of the time during which the number of atoms in the radioactive substance reduces to 36.8% of the original value of atoms in it.

$$N = N_0 e^{-\lambda t}$$

$$\text{At } t = \frac{1}{\lambda} \qquad N = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368 N_0$$

$$\text{or } N = 36.8\% \; N_0$$

Half life of a radioactive substance

The time during which half of the atoms of the radioactive substance disintegrates is called half life of a radioactive substance.

$$N = N_0 e^{-\lambda t}$$

$$N = N_0/2 \qquad \text{At} \quad t = T_{1/2}$$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{-\lambda T_{1/2}} = 2$$

$$\Rightarrow +\lambda T_{1/2} = \log_e 2 = 2.303 \log_{10} 2$$

$$\Rightarrow T_{1/2} = \frac{2.303 \times 0.3010}{\lambda} = \frac{0.6931}{\lambda}$$

$$\Rightarrow T_{1/2} \propto \frac{1}{\lambda}$$

Mean (Average) life of a Radioactive substance

It is given by the total sum of life time of all atoms divided by the total no. of atoms present.

Mean life, $\tau = \dfrac{\text{Sum of lives of all atoms}}{\text{Total no. of atoms present}}$

$$\tau = \frac{T_{total}}{N_0}$$

We know that $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$\Rightarrow dN = -\lambda N_0 e^{-\lambda t}\, dt$$

Total life of dN atoms $= \int_{N=N_0}^{N=0} t\, dN$,

$$T_{total} = \int_{N_0}^{0} t\,(dN)$$

$$= \lambda N_0 \int_0^\infty t\, e^{-\lambda t}\, dt$$

$$= \lambda N_0 \left[\left(\frac{t\, e^{-\lambda t}}{-\lambda}\right)_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda}\, dt\right]$$

$$= -\frac{N_0}{\lambda}\left[e^{-\infty} - e^{0}\right] = \frac{N_0}{\lambda}$$

Now $\tau = \dfrac{N_0}{\lambda N_0} = \dfrac{1}{\lambda}$

Relation between Half life and mean life

$$T_{1/2} = \frac{0.693}{\lambda}, \quad \tau = \frac{1}{\lambda}$$

$$\tau = 1.44\, T_{1/2}$$

Activity of a radioactive substance

The total decay rate of the substance is also known as the activity of a radioactive substance.

$$A = \lambda N$$

S.I. unit is 1 Bq.

Alpha Decay $_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$

$$Q = (m_X - m_Y - m_\alpha)\cdot c^2$$

Examples: $_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$

$_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4}$

K.E. of α-particle $= \left(\frac{A-4}{A}\right) Q$

Escape of α-particle → The motion of an α-particl in the neighbourhood of potential barrier is considered as a wave. It is found that there is a small but definite probability that the α-particle may tunnel through the potential barrier even if the kinetic energy of α-particle is less than the height of potential barrier. This effect is known as tunneling of the nucleus.* (The potential barrier of nucleus is of the order of 26 Mev & no α-particle can have this much amount of energy.)

Beta Decay

(i) β^- decay → The neutron inside the nucleus is converted into a proton & an electron like particle.

$$_{15}P^{32} \longrightarrow {}_{16}S^{32} + {}_{-1}e^{0}\ (\beta^{-1}) + \overline{\nu}$$

$$_{0}n^{1} \longrightarrow {}_{1}H^{1} + {}_{-1}e^{0} + \overline{\nu}$$

(ii) β^+ decay → A proton is converted into a neutron and a positron ($_{+1}e^0$) is emitted if a nucleus has more protons than neutrons.

$$_{11}Na^{22} \longrightarrow {}_{10}Ne^{22} + {}_{+1}e^0\,(\beta^+) + \nu$$

$$_{1}H^{1} \longrightarrow {}_{0}n^{1} + {}_{+1}e^0 + \nu$$

(iii) Electron capture → In this process, the nucleus absorbs one of the inner e^- revolving around it and hence a nuclear proton becomes a neutron and a neutrino (ν) is emitted.

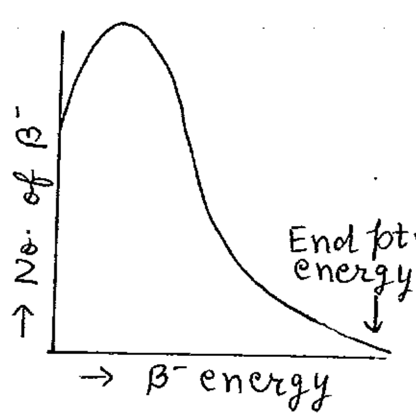
$$_{1}H^{1} + {}_{-1}e^0 \longrightarrow {}_{0}n^{1} + \nu$$

* In heavy elements, e^- capture is more frequent than positron emission.

$$_{54}Xe^{120} + {}_{-1}e^0 \longrightarrow {}_{53}I^{120} + \nu$$

Energy carried by emitted β-particle

(i) only few β-particles carry maximum energy called end point energy.



(ii) The energy spectrum is continuous which indicates that the emitted β-particles have all possible energies from 0 to $(Q_\beta)_{max}$.

(1) The energy calculations showed that the energy of a nucleus emitting β-particles decreases by an amount equal to the end point energy of continuous spectrum but most of the β-particles emitted have energies smaller than $(Q_\beta)_{max}$.

(2) Another contradiction arises in respect of conservation of angular momentum. β-particle has spin $1/2$ so during β-particle emission, spin or angular momentum of nucleus must change by $1/2$ but it never occurs.

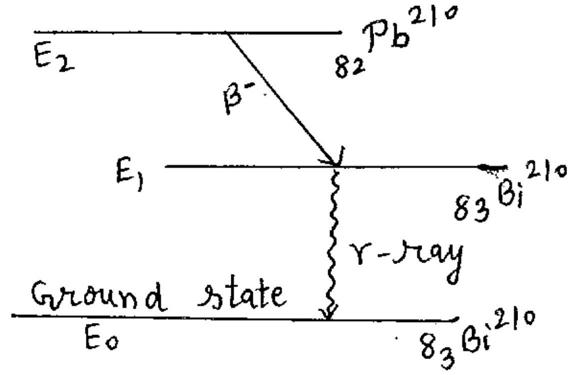
These two problems were resolved by Pauli.

Neutrino Hypothesis → He suggested that the emission of low energy β^- particle by any nucleus is accompanied by an other neutral particle of zero rest mass and spin $1/2$ known as anti-neutrino($\bar{\nu}$). This particle carries energy which solves the first part and due to spin $1/2$ it solves the problem of conservation of momentum.

Gamma-Decay → The spontaneous process of emission of high energy photon from a radioactive nucleus.

$${}_{82}Pb^{210} \longrightarrow {}_{83}^{*}Bi^{210} + {}_{-1}e^{0} + \bar{\nu}$$

$${}_{83}^{*}Bi^{210} \longrightarrow {}_{83}Bi^{210} + \gamma\text{-ray}$$



Nuclear Reactions (Read from NCERT)* for detail

① Fission

$$_{92}U^{235} + {_0}n^1 \rightarrow {_{92}}U^{236} \longrightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3\,{_0}n^1 + Q$$

② Fusion

$$_1H^2 + {_1}H^2 \longrightarrow {_2}He^3 + {_0}n^1 + Q$$

$$4\,{_1}H^1 \longrightarrow {_2}He^4 + 2\,{_{+1}}e^0 + 2\nu + Q$$

Artificial Radioactivity

$$\underset{\text{(Stable)}}{_{13}Al^{27}} + {_2}He^4 \longrightarrow \underset{\text{(unstable)}}{_{15}P^{30}} + {_0}n^1$$

$$\underset{\text{(Stable)}}{_{29}Cu^{66}} \longrightarrow \underset{\text{(unstable)}}{_{30}Zn^{66}} + {_{-1}}e^0$$

The process by which stable nuclei are made unstable by bombarding them with high energy particles and then these unstable nuclei decay to give nuclear radiatio is called artificial radio activity. These unstable nuclei are called radio-isotopes.

Applications of Radio-isotopes

① Tracer technique in field of fluid technology & medicine

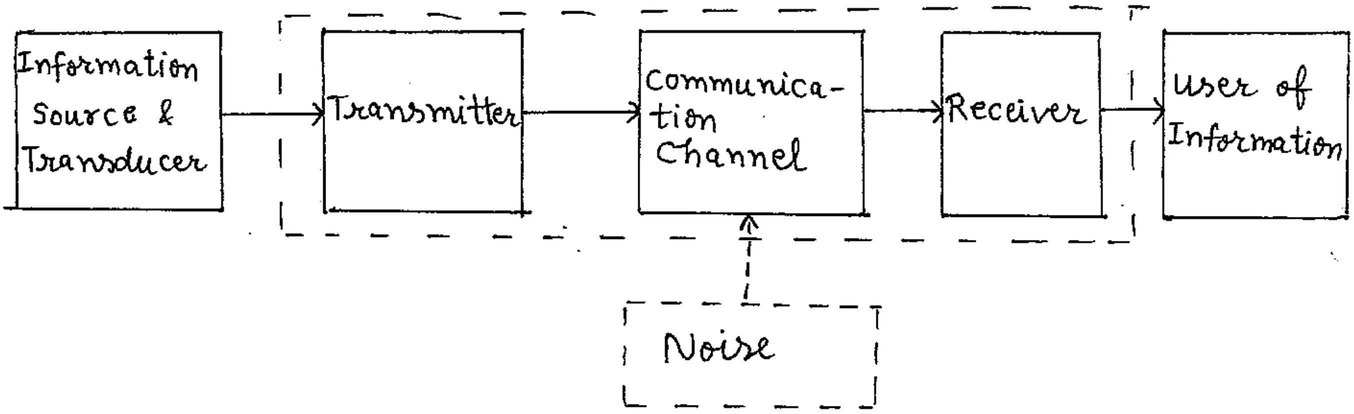
② In treatment of skin cancer

③ To investigate flow of liquids in chemical plants.

④ It can be used in carbon-dating.

Communication Systems

Elements of communiation system



Transmitter → The electrical signals corresponding to the information are processed, modulated, amplified and fed to the link (communication channel) or radiated through antenna.

Communication channel → The transmitter sends the signal to the receiver through the communication channel. eg. transmission lines, wires, cables, optical fibres or air.

Receiver → A receiver amplifies, filters and demodulates the received signal.

Modes of communication → ① Point to Point communication

There is only one transmitter and one receiver.
eg. Telephone communication

② Broadcast (Point to Many point) communication

There is only one transmitter and many receivers.
eg Radio broadcast & Television telecast

Signal → Information converted to electromagnetic form is known as signal

① Analog signal ② Digital signal

Few Important terms →

1. Encoding → The process of converting an information into analog or digital signal.
2. Transducer → A device which converts one form of energy into another form of energy.
3. Attenuation → The loss of strength or power of an electrical signal while travelling through a medium.
4. Noise → A disturbance or unwanted element interfering with the desired information.
5. Modulation → The process of placement of low frequency signal over the high freq. wave (carrier wave).
6. Demodulation → The process of extracting the low frequency signal from the high frequency carrier wave.
7. Amplification → The process of increasing the amplitude (strength) of the signal.
8. Range → The maximum distance between the information source and the destination.
9. Band-width → The width of the frequency spectrum of a signal is called B.W.

   $BW = \nu_{max} - \nu_{min}$.
10. Repeaters → To increase the range of transmission of signals, no. of in-between sets of receivers & transmitters are repeaters.

Bandwidth of signals → $BW = \nu_{max.} - \nu_{min.}$

① Speech or voice signals → It is about 3000 Hz eg. in telephony freq. range is 300 Hz – 3100 Hz, BW = 2800 Hz

② Music signals → It is about 20 KHz. Its range is from 20 Hz – 20000 Hz.

③ Radio signals → AM band is from 550 Hz – 1600 KHz & FM band is from 88 MHz – 108 MHz.

④ TV signals → combined BW of picture & voice signals is about 6 MHz. Only picture signals have BW of 4 MHz.

⑤ Cellular Mobile phone signals → They have frequency band of 840 MHz to 935 MHz.

⑥ Satellite phone signals → Freq. bands are 3.7 to 4.2 GHz (down link) and 5.9 to 6.4 GHz (uplink).

⑦ Digital signals → Theoretically infinite BW is required for digital signals.

Bandwidth of Transmission medium →

① Co-axial cables → They have BW of about 750 MHz

② optical fibres → Frequency range is 1 THz to 1000 THz. So BW is above 10^{11} Hz

③ Free space → Frequency range is 580 KHz to 6.5 GHz. So BW is nearly 6.5 GHz.

Propagation of EM-waves → There are three ways:

① Ground wave ② Sky wave ③ Space wave

Ground wave propagation → In this, radio waves travel along the surface of the Earth. It is limited to a frequency below 1.5 MHz. The minimum length of transmitting antenna is about $\lambda/4$ where λ is the wavelength of signal.

Sky wave propagation → The radio waves which are reflected back to the Earth by ionosphere are known as sky waves. The frequency range is 3 MHz – 40 MHz. The highest frequency that is returned to the earth by the considered layer of the ionosphere after having been sent straight to it, $f_c = 9 (N_{max})^{1/2}$, N_{max} is max. electron density of ionosphere.

Space wave propagation → High frequency waves (above 40 MHz) called space waves can be transmitted from transmitting to receiving antenna through space is known as space wave propagation.

Range of LOS communication (space wave)

$$AO^2 = AC^2 + OC^2$$

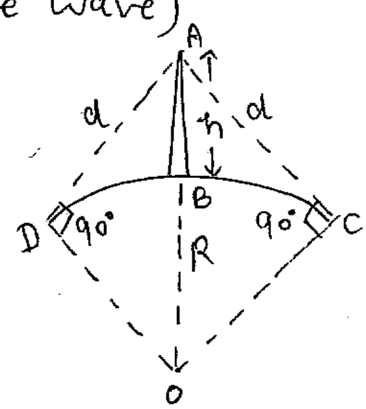
$$\Rightarrow (R+h)^2 = d^2 + R^2$$

$$\text{or } d^2 = h^2 + 2Rh$$

As $R >> h$ so

$$d^2 \approx 2Rh$$

$$\text{or } d = \sqrt{2Rh} = \text{Radio horizon}$$



Maximum Line of sight distance between the transmitting & receiving antennas

$$d_t = \sqrt{2Rh_t} + \sqrt{2Rh_r}$$

$$d_t = d_1 + d_2$$

Satellite or Extra-terrestrial communication

It is a method in which a beam of microwaves is projected towards a geo-stationary satellite which throws back the same to different parts of Earth. Microwave signals have frequency of about 6 GHz.

Necessity or Need of modulation

(i) Height of transmitting antenna → For a signal of 15 KHz, λ is about 20 Km, min. height of antenna required is $\lambda/4 = 5$ Km which is not possible. After modulation, freq. is of the order of 1 MHz $= 10^6$ Hz so height of antenna required is 75 m which is practically possible.

(ii) Effective power radiated by antenna

$$P_{\text{dish antenna}} = 6\left(\frac{D}{\lambda}\right)^2 \text{ or } P \propto \frac{1}{\lambda^2}$$

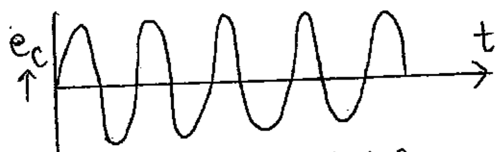
Shorter the wavelength, more power is radiated.

(iii) Mixing up of signals from different transmitters
Similar voice signals from different transmitters get mixed up if they are not superimposed with diff. carrier signals.

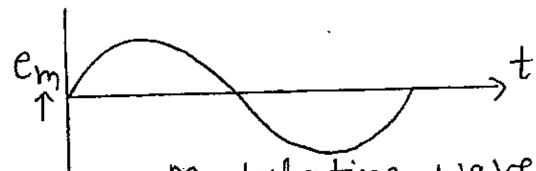
Modulation → The process of mounting a low frequency signal over high frequency signal is known as modulation.

Types of modulation → ① Amplitude modulation
② Frequency modulation

Amplitude modulation → If the amplitude of the carrier wave varies in accordance with the amplitude of very low freq. wave (signal) then it is amplitude modulation.

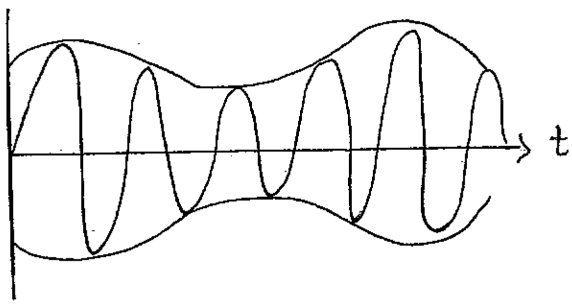


Carrier wave

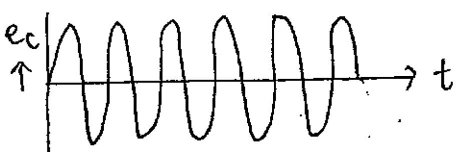


Modulating wave

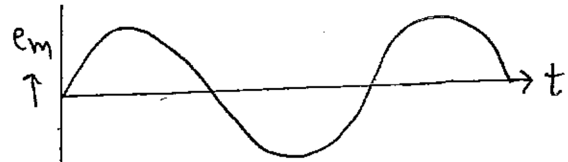
Amplitude Modulated Wave



Frequency Modulation → If the frequency of the carrier wave varies in acc with the freq. of very low freq. wave then it is frequency modulation.

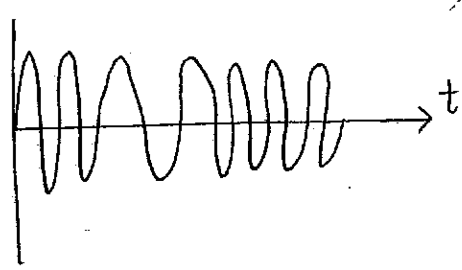


Carrier wave



Modulating wave

Frequency Modulated wave



(iii) Phase Modulation → If the phase of carrier wave changes in accordance with the phase of the audio frequency wave then it is phase modulation.

Amplitude modulation (Mathematical treatment)

Let carrier wave is represented by

$$e_c = E_c \sin \omega_c t$$

& modulating signal is represented by

$$e_m = E_m \sin \omega_m t$$

Now amplitude modulated wave is given by

$$e = (E_c + e_m) \sin \omega_c t = (E_c + E_m \sin \omega_m t) \sin \omega_c t$$

$$\text{or } e = E_c \left(1 + \frac{E_m}{E_c} \sin \omega_m t\right) \sin \omega_c t$$

Now $\frac{E_m}{E_c} = m_a$ (modulation index)

$$\therefore \quad e = E_c (1 + m_a \sin \omega_m t) \sin \omega_c t$$

$$= E_c \sin \omega_c t + m_a E_c \sin \omega_m t \sin \omega_c t$$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

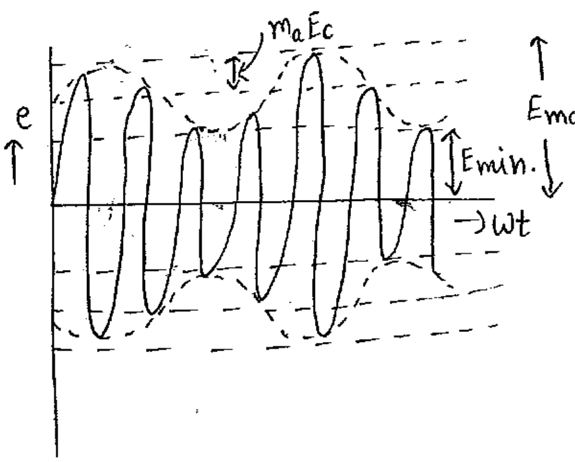
$$\text{or } e = E_c \sin \omega_c t + m_a E_c \frac{1}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$\Rightarrow \quad e = \underset{\text{(Carrier wave)}}{E_c \sin \omega_c t} + \underset{\text{(LSB)}}{\frac{m_a E_c}{2} \cos(\omega_c - \omega_m)t} - \underset{\text{(USB)}}{\frac{m_a E_c}{2} \cos(\omega_c + \omega_m)t}$$

BW of amplitude modulated signal is $2\nu_m$

Significance of amplitude modulation index

The variation in amplitude of carrier wave is given by



$$m_a E_c = E_{max} - E_c$$

$$\therefore \quad m_a = \frac{E_{max.} - E_c}{E_c}$$

$$\& \quad m_a E_c = E_c - E_{min.}$$

$$\therefore \quad m_a = \frac{E_c - E_{min.}}{E_c}$$

$$\Rightarrow E_c - E_{min.} = E_{max} - E_c$$

$$\Rightarrow E_c = \frac{E_{max} + E_{min.}}{2}$$

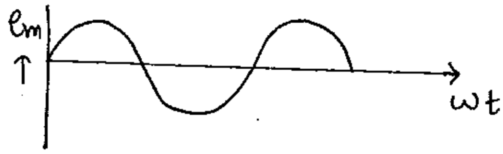
$$\Rightarrow m_a = \frac{E_{max} - E_{min.}}{E_{max} + E_{min.}}$$

If $m_a = 0$, no modulation

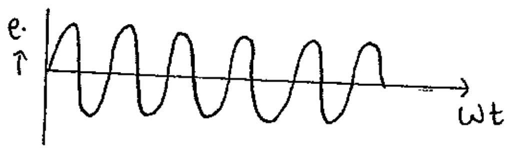
If $m_a = 1/2$, E_{max} is three times $E_{min.}$

If $m_a = 1$, $E_{min.}$ becomes zero

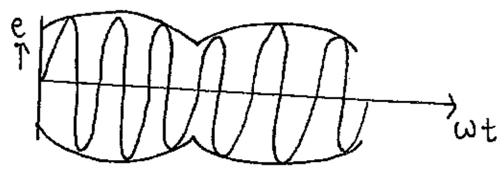
If $m_a > 1$, It is over-modulation



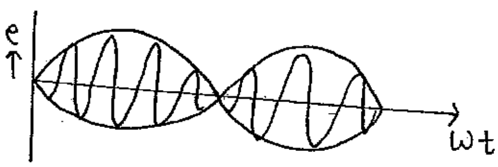
message signal



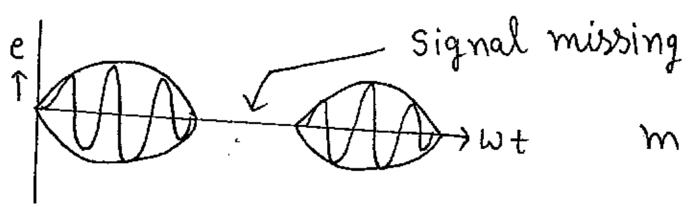
$m_a = 0$ No modulation



$m_a = 1/2$ 50% modulation



$m_a = 1$ 100% modulation



$m_a > 1$ over modulation

$$\% \, m_a = \left(\frac{E_{max} - E_{min.}}{E_{max} + E_{min.}} \right) \times 100$$

Production of Amplitude Modulated wave

Modulated signal $m(t)$ is mounted on carrier signal $c(t)$ to produce $x(t)$. Square law device Converts this signal into an output given by

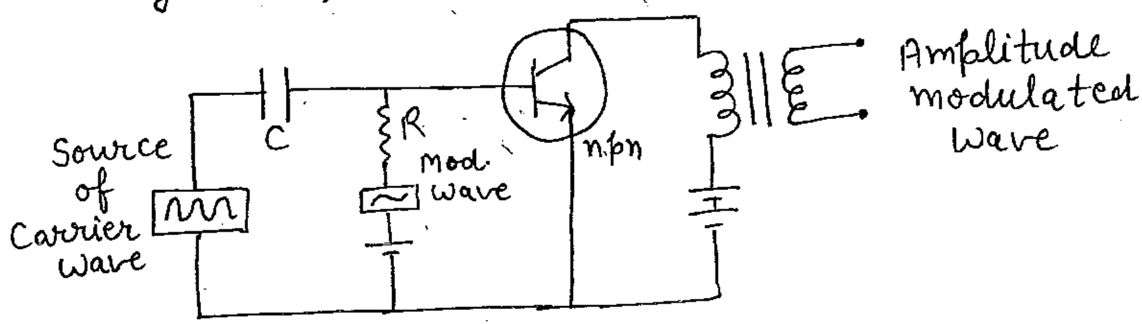
$$Y(t) = A\,x(t) + B\,x^2(t)$$

Now $x(t) = E_m \sin \omega_m t + E_c \sin \omega_c t$

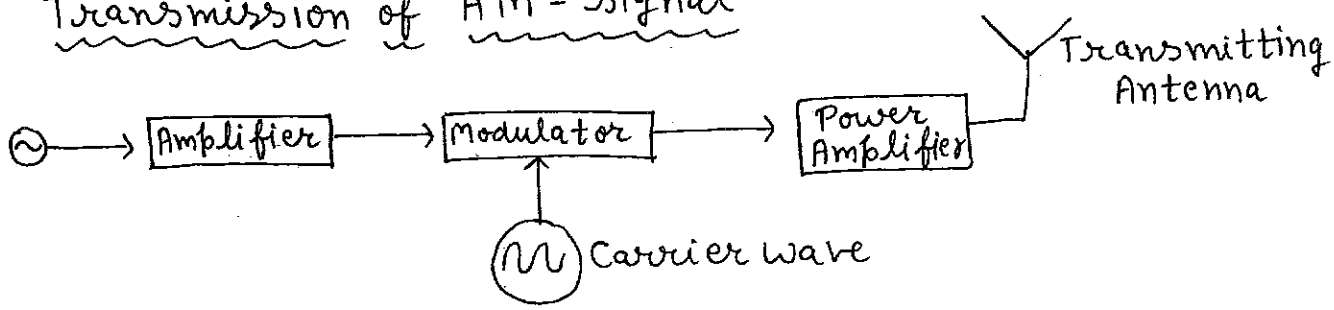
$$Y(t) = A\left[E_m \sin \omega_m t + E_c \sin \omega_c t\right] + B\left[E_m \sin \omega_m t + E_c \sin \omega_c t\right]^2$$

or
$$Y(t) = A E_m \sin \omega_m t + A E_c \sin \omega_c t + \frac{B}{2}\left(E_m^2 + E_c^2\right) - \frac{B}{2} E_m^2 \cos 2\omega_m t - \frac{B}{2} E_c^2 \cos^2 \omega_c t + B E_m E_c \cos(\omega_c - \omega_m)t - B E_m E_c \cos(\omega_c + \omega_m)t$$

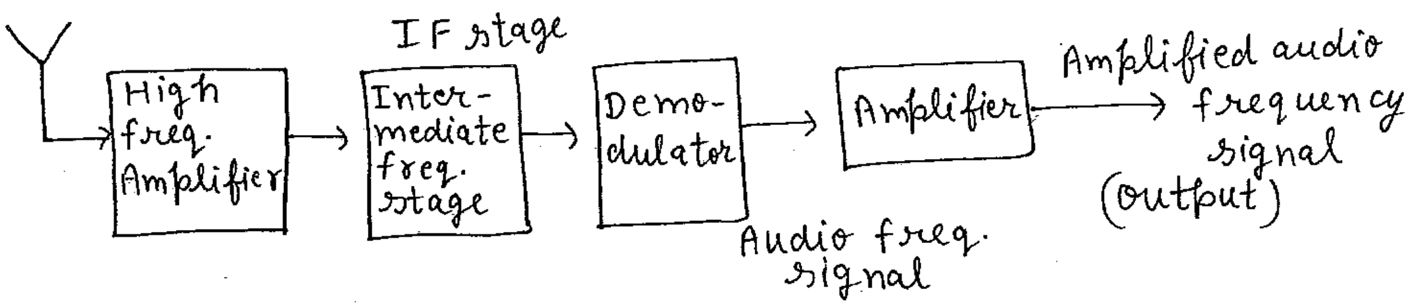
After passing through band pass filter, we get only ω_c, $(\omega_c + \omega_m)$ & $(\omega_c - \omega_m)$ frequencies.



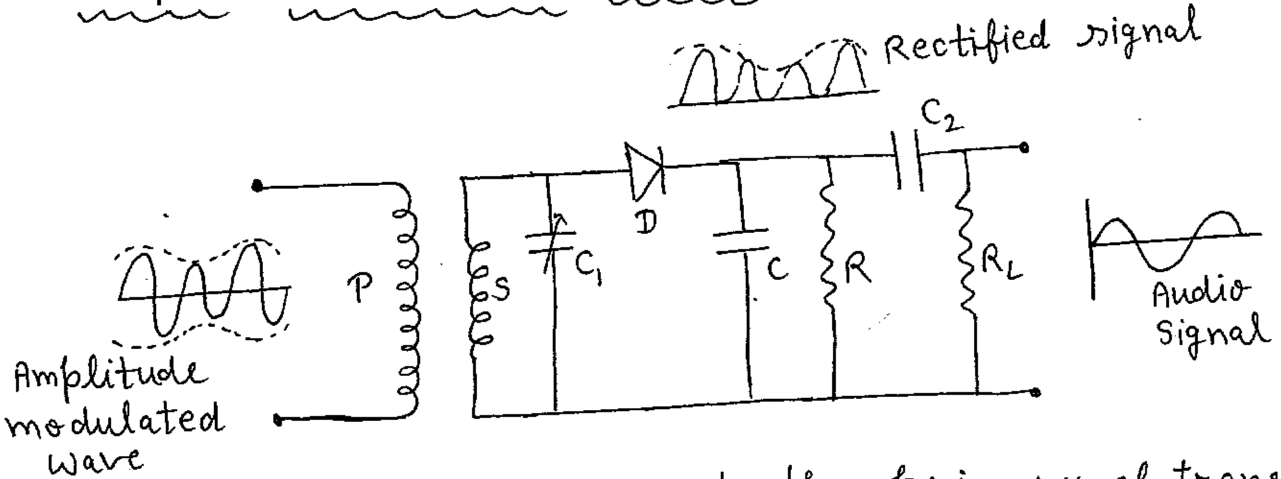
Transmission of AM - signal



Detection of an Amplitude modulated wave

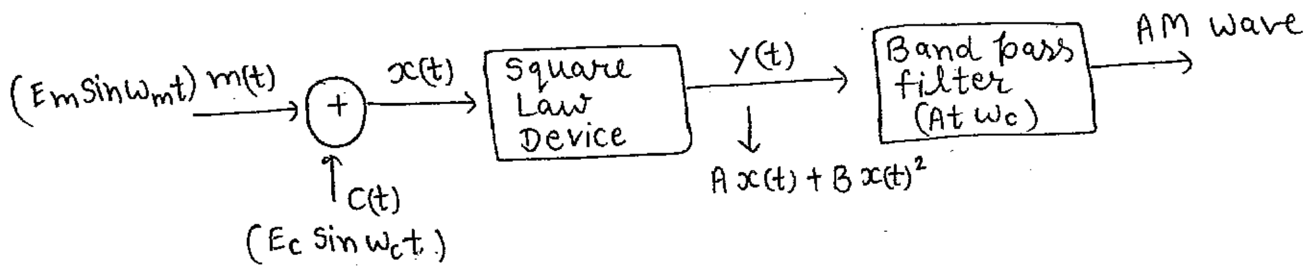


Simple Demodulator circuit



Modulated wave is fed to the primary of transformer. The secondary coil (S) of the transformer and the variable capacitor C_1 constitute LC circuit. This is brought in resonance with input modulated signal. Diode D acts as rectifier. The combination of R & C acts as filter and using these filters we can obtain the desired audio signal.

Imp. Block diagram of a simple modulator



Advantages of Amplitude modulation → (i) It is an easy method for the transmission & receiving of speech signals.

*(ii) Its installation cost is very less (For point to many).

(iii) It is fairly efficient system of modulation.

Disadvantages of Amplitude modulation → (i) It is more likely to suffer from noise.

(ii) Energy loss is quite high and it is limited to point to point links.

Some additional topics

① **Internet** → It is a global network of computers linked by high speed data lines and wireless systems. It allows communication and sharing of information between any two or more computers connected through the vast network.

Applications of Internet

(a) **Email** → It is a message sent and received through a computer network. Emailing allows exchange of text/graphic material using email software. Its various advantages are : fast delivery at low cost, easy record maintenance, reduction of the wastage of paper stationery.

(b) **File transfer** → An FTP (File transfer protocol) permits the transfer of files or software from one computer to another connected to the internet.

c) **WWW (world wide web)** → It is a set of protocols that allows us to access any document on the internet. WWW is based on clients and servers.

A web server is a WWW server that responds to the requests made by web browsers. A web browser is a WWW client that navigates through the worldwide web and displays web pages. A location on net server is called web site.

HTTP → Hyper Text Transfer Protocol
HTML → Hyper Text Markup Language
URL → Uniform Resource Locator

(d) E-commerce → E-Commerce is the collection of tools and practices involving internet technologies that allow a company to create, maintain and optimise business relations with consumers & other businesses. It helps in online banking activities, online shopping activities.

(e) Chat → It is the real time conversation among people with common interests through the typed messages on the net.

(2) Mobile Telephony → In mobile telephony, numerous lower power transmitters (base stations) are set up, each covering a fraction of that service area called a cell.

Each base station is connected to a switching office called mobile telephone switching office (MTSO), which co-ordinates communication between all the base stations and the telephone centre office. As a mobile receiver moves from one cell to another, the mobile user is handed over to the new cell's base station. This is called handover. Mobile telephones operate in the UHF range of frequencies around 800 - 950 MHz

(3) Global Positioning system (GPS) → The global positioning system is a satellite based system that can be used to locate positions anywhere on the earth. This is used for camping, shiping, cell phone location, aircraft navigation, wheather forecasting etc.

Components of GPS

(i) Space segment → It consists of 29 satellites that are continuously orbiting the earth at altitudes of about 19000 Km.

(ii) control segment → It consists of five unmanned monitor stations and one master control station.

(iii) User segment → It consists of the users & their GPS receivers. The number of simultaneous users is limitless.

How GPS works

A part of information sent by a satellite vehicle (SV) is a time stamp. When a GPS unit receives the transmission, it compares the time stamp from the satellite to the time it reached the receiver. The difference between the two, multiplied by the speed of the transmission signal provides the distance that the signal travelled { The receiver finds position by trilateration process that uses distances from atleast these known locations. The intersection of the three spheres having radii equal to these three distances gives the possible position. By using additional SVs, the positional accuracy can be improved.

Energy stored in an inductor →

When current I flows through the inductor, an e.m.f. is induced in it. It is given by

$$\mathcal{E} = -L\frac{dI}{dt}$$

For external battery, e.m.f. $= -\mathcal{E}$

so $\mathcal{E} = L\frac{dI}{dt}$

For small charge dq, small work done by the external supply is given by

$$dW = \mathcal{E}\,dq = L\frac{dI}{dt}dq = LI\,dI$$

$$\Rightarrow \int dW = \int_0^{I_0} LI\,dI = \tfrac{1}{2}LI_0^2$$

$W = U = \frac{1}{2}LI_0^2$ is work done stored in the magnetic field of inductor

Magnetic energy stored in a solenoid →

$$U_m = \tfrac{1}{2}LI_0^2$$

Now $B = \mu_0 n I_0$ or $I_0 = \frac{B}{\mu_0 n}$

& $L = \mu_0 n^2 A l$

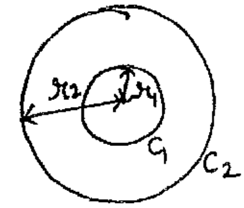
$$\Rightarrow U_m = \tfrac{1}{2}\mu_0 n^2 A l \frac{B^2}{\mu_0^2 n^2} = \frac{1}{2}\left(\frac{B^2 A l}{\mu_0}\right)$$

$(\overline{U_m})$ Magnetic energy density $= \frac{U_m}{V} = \frac{U_m}{Al}$

$$\Rightarrow \overline{U_m} = \frac{B^2}{2\mu_0}$$

Mutual Inductance of two concentric coils

$$\phi_B = BA = \left(\frac{\mu_0 I}{2r_2}\right)\pi r_1^2 = \frac{\mu_0 \pi r_1^2}{2r_2}I$$



Now $\phi = MI$

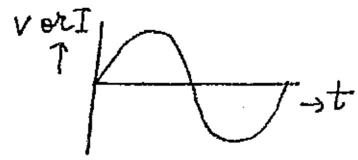
$$\Rightarrow M = \mu_0 \pi r_1^2 / 2r_2$$

$$M_{12} = M_{21} = M = \frac{\mu_0}{4\pi}\left(\frac{2\pi r_1^2}{r_2}\right)$$

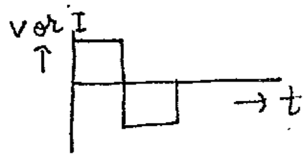
ALTERNATING CURRENTS

Alternating Current → An electric current, magnitude of which changes with time and polarity reverses periodically is called alternating current (A·C).

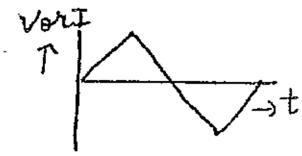
Types of A.C. → (i) Sinusoidal



(ii) Square



(iii) Triangular



(iv) Mixed

* Most common form is sinusoidal

Sinusoidal alternating current (A·C.) is expressed as

$$I = I_0 \sin \omega t$$

Here $\omega = \frac{2\pi}{T} = 2\pi f$ is angular frequency

& Sinusoidal voltage of A·C source is given by

$$V = V_0 \sin \omega t$$

Advantages of A·C →(i) It can be stepped up & stepped down (using transformer)

(ii) A·C can be transmitted to long distances without much loss of heat.

(iii) A·C can be converted into D·C for storage.

(iv) A·C can be better controlled (using choke coil).

Disadvantages of A·C →(i) A·C is more dangerous than D·C due to its peak value which is $\sqrt{2}$ times the r.m.s. value.

(ii) A.C. can not be used for electrolysis.

(iii) Scales of A·C meters are not uniform.

(iv) A·C is not distributed uniformly. It has skin effect.

Mean or Average value of A.C. → It is that value of steady current which sends the same amount of charge through a circuit in a certain time interval as is sent by an A.C. through the same circuit in half cycle.

* Mean value of an a.c over half cycle is 63.7% of its peak value

Let an alternating current be represented by

$$I = I_0 \sin\omega t$$

Now $I = \frac{dq}{dt} \Rightarrow dq = I dt = I_0 \sin\omega t \, dt$

$$\Rightarrow \int dq = \int_0^{T/2} I_0 \sin\omega t \, dt$$

$$\Rightarrow q = I_0 \left(\frac{-\cos\omega t}{\omega}\right)_0^{T/2} = \frac{-I_0 T}{2\pi}(\cos\pi - \cos 0^\circ)$$

$$\Rightarrow q = \frac{I_0 T}{\pi}$$

Now $q = I_{av.} \times T/2 = \frac{I_0 T}{\pi}$

$$\Rightarrow I_{av.} = \frac{2 I_0}{\pi} = 0.637 I_0$$

Root mean square (R.M.S.) or Effective value of A.C →

It is defined as that steady current which produces the same amount of heat in a conductor in a certain time interval as is produced by the A.C in the same conductor during the time period T (i.e full cycle).

Let A.C is given by $I = I_0 \sin\omega t$

Heat produced is given by $dH = I^2 R dt$

or $dH = (I_0^2 \sin^2\omega t) R \, dt$

$$\Rightarrow \int dH = H = \int_0^T I_0^2 R \sin^2\omega t \, dt$$

$$\Rightarrow H = I_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right]$$

$$\Rightarrow H = \frac{I_0^2 R}{2}[T - 0] = \frac{I_0^2 RT}{2}$$

Now $H = I_{r.m.s.}^2 RT = \frac{I_0^2 RT}{2}$

$$\Rightarrow I_{r.m.s.} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

* $I_{r.m.s.}$ is 70.7% of the peak value I_0

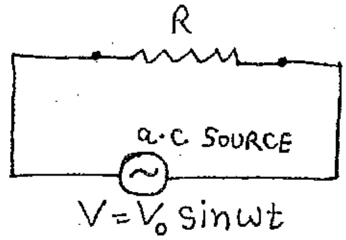
* read only

Phasor and Phasor diagram → Phasor is a rotating vector which represents a quantity varying sinusoidally with time.

In a phasor diagram, two quantities varying sinusoidally with time are represented. The phase differenc between these quantities is represented by the angle between their maximum values.

Alternating voltage applied to a resistor

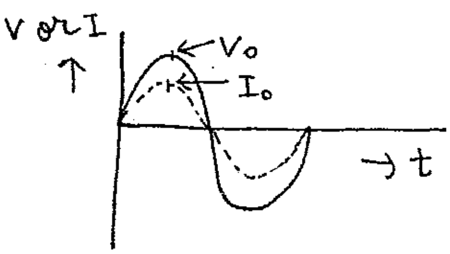
The applied alternating voltage is given by $V = V_0 \sin\omega t$ — I

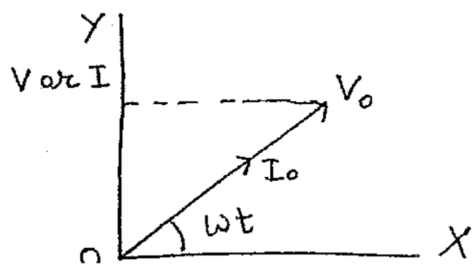


Now $V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t$$

$\Rightarrow I = I_0 \sin\omega t$ — II $\Rightarrow$ * V & I are in same phas





Power supplied to a resistor → Instantaneous power dissipated in resistor is given by

$$P = I^2 R = (I_0^2 \sin^2 \omega t) R$$

$$\text{or } P = I_0^2 R \sin^2 \omega t = I_0^2 R \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$P_{av.} = I_0^2 R \left(\frac{1 - \langle \cos 2\omega t \rangle}{2}\right)$$

$$\langle \cos 2\omega t \rangle = \text{Average value of } \cos 2\omega t$$

$$= \frac{1}{T}\int_0^T \cos 2\omega t \, dt = 0$$

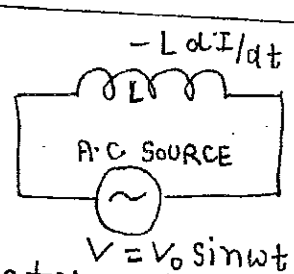
$$\Rightarrow \langle \cos 2\omega t \rangle = 0$$

$$\Rightarrow P_{av.} = \frac{I_0^2 R}{2} = I_{r.m.s.}^2 R$$

Alternating voltage applied to an Inductor

The alternating voltage across the inductor is given by

$$V = V_0 \sin \omega t$$



Induced e.m.f. across the inductor is $-L\frac{dI}{dt}$

$$\Rightarrow V + \left(-L\frac{dI}{dt}\right) = 0 \Rightarrow V = L\frac{dI}{dt}$$

$$\Rightarrow \frac{V_0}{L}\sin \omega t = \frac{dI}{dt} \Rightarrow dI = \frac{V_0}{L}\sin \omega t \, dt$$

$$\Rightarrow I = \frac{V_0}{L}\int \sin \omega t \, dt = -\frac{V_0}{\omega L}(\cos \omega t)$$

$$\Rightarrow I = \frac{V_0}{\omega L}\sin(\omega t - \pi/2)$$

$$\Rightarrow I = I_0 \sin(\omega t - \pi/2) \qquad I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$

$$X_L = \omega L = \text{Inductive reactance}$$

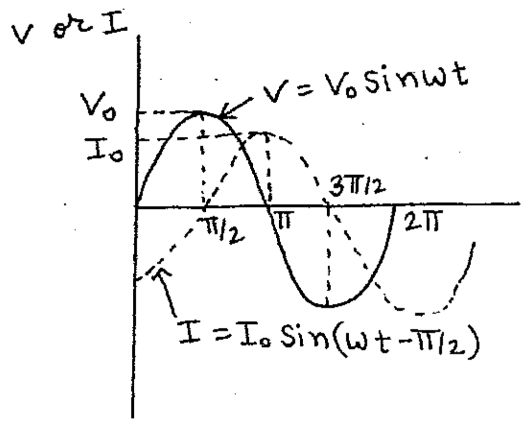
This shows that the current lags behind the voltage by an angle of $\pi/2$.

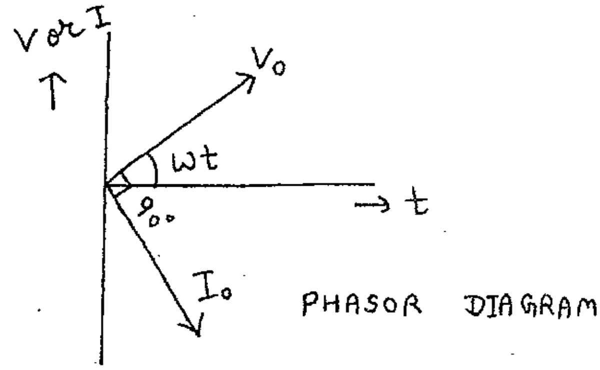
For A.C $\omega = 2\pi\nu$ is finite

So inductor offers opposition to the flow of a.c

For D.C $\omega = 2\pi\nu = 0$

So inductor offers no opposition to the flow of d.c





Power supplied to an Inductor

Instantaneous power supplied to an Inductor

$$P_L = VI = V_0 \sin\omega t \cdot I_0 \sin(\omega t - \pi/2)$$

$$= -\frac{V_0 I_0}{2} \sin 2\omega t$$

$$(P_L)_{av.} = -\frac{V_0 I_0}{2} \langle \sin\omega(2t) \rangle$$

$$\langle \sin 2\omega t \rangle = \frac{1}{T}\int_0^T \sin 2\omega t \, dt = 0$$

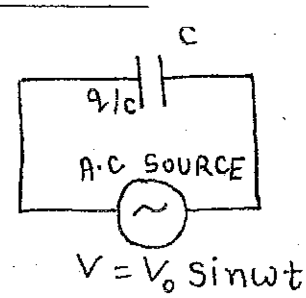
$$\Rightarrow (P_L)_{av.} = 0$$

* Average power for full cycle, in case of Inductor is zero.

Alternating voltage applied to a capacitor

The alternating voltage applied across a capacitor is given by

$$V = V_0 \sin\omega t$$



P.D. across capacitor, $V_c = q/c$

Now $V = \frac{q}{C} = V_0 \sin\omega t \Rightarrow q = CV_0 \sin\omega t$

$$I = dq/dt = \frac{d}{dt}(CV_0 \sin\omega t) = \omega C V_0 \cos\omega t$$

or $I = \dfrac{V_0}{1/\omega C} \cos\omega t = I_0 \sin(\omega t + \pi/2)$

$I_0 = \dfrac{V_0}{X_c}$ $\quad X_c = 1/\omega C$ = Capacitive reactance

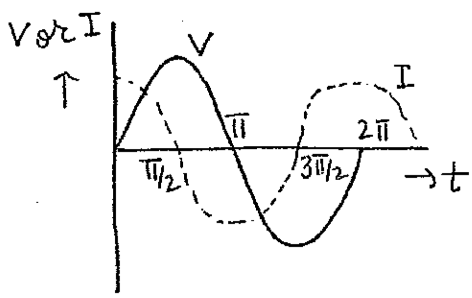
This means that current leads the voltage by an angle of $\pi/2$

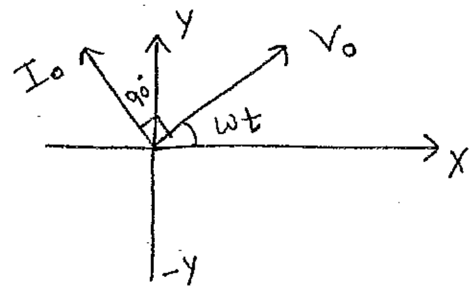
For A.C. $\omega = 2\pi\nu$ is finite so X_c is small

$\Rightarrow$ Capacitor offers small opposition to flow of A.C

For D.C. $\omega = 2\pi\nu = 0$ so X_c is infinite

$\Rightarrow$ Capacitor blocks d.c as X_c is infinite





<u>Power supplied to a capacitor →</u>

Instantaneous power supplied to capacitor is given by

$$P_c = VI = V_0 \sin\omega t \; I_0 \sin(\omega t + \pi/2)$$

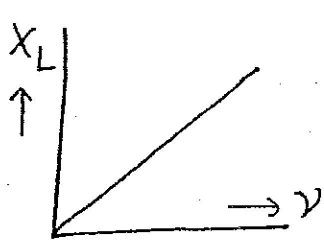
$$\Rightarrow P_c = \frac{V_0 I_0}{2} \sin 2\omega t$$

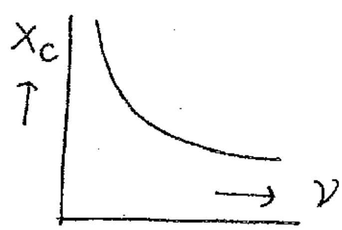
$$(P_c)_{av.} = \frac{V_0 I_0}{2} \langle \sin 2\omega t \rangle$$

$$\langle \sin 2\omega t \rangle = \frac{1}{T}\int_0^T \sin 2\omega t \, dt = 0$$

$$\Rightarrow (P_c)_{av.} = 0$$

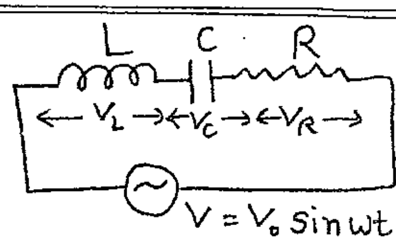
$\Rightarrow$ Average power, for full cycle in capacitor is zero





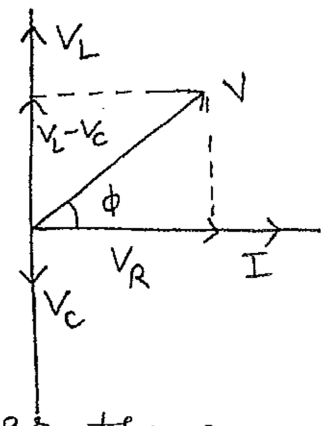
L, C and R in series across an alternating supply

Let I be the r.m.s. value of current flowing through all the circuit elements.



The potential difference across inductor

$$V_L = I X_L$$



P.D. across capacitor, $V_C = I X_C$

P.D. across resistor, $V_R = IR$

In case of inductor, voltage leads the current by $\pi/2$.

In case of capacitor, voltage lags the current by $\pi/2$.

According to above phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad V = \sqrt{I^2 R^2 + (I X_L - I X_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

or $Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$ = Impedance of LCR circuit

(i) If $X_L = X_C$ then $Z = R$

It behaves as pure resistive circuit

(ii) If $X_L = 0$ then $Z = \sqrt{R^2 + X_C^2}$

It is series RC circuit

(iii) If $X_C = 0$ then $Z = \sqrt{R^2 + X_L^2}$

It is series LR circuit

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I X_L - I X_C}{IR} = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega}{R}$$

$$\& \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

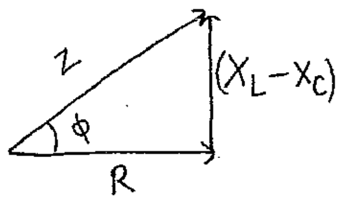
Impedance Triangle → (i) In series LCR circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

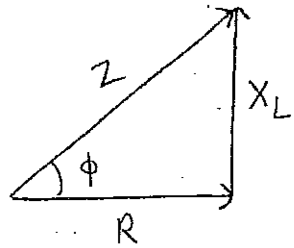
(ii) In LR circuit $Z = \sqrt{R^2 + X_L^2}$

(iii) In CR circuit $Z = \sqrt{R^2 + X_C^2}$

They are represented by Impedance triangle

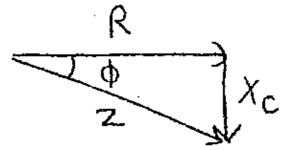


$$\tan\phi = \frac{X_L - X_C}{R}$$



$$\tan\phi = \frac{X_L}{R}$$

$$\tan\phi = \frac{X_C}{R}$$



S.I. unit of Impedance is Ohm.

Admittance → Reciprocal of Impedance of circuit is called admittance of circuit

It is given by $A = \frac{1}{Z}$

S.I. unit of admittance is ohm^{-1}, mho or siemen.

LR - series circuit across alternating electric supply

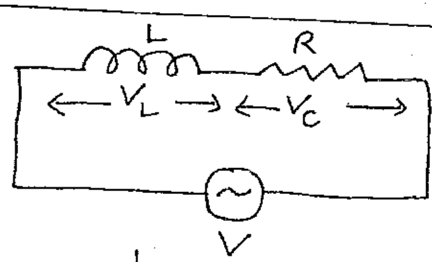
Here $V = \sqrt{V_R^2 + V_L^2}$

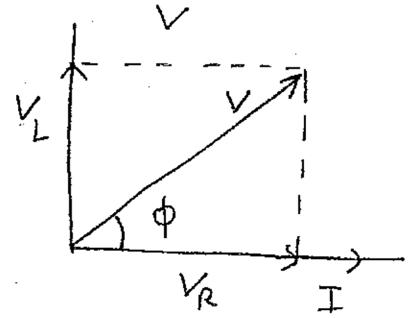
$$= I\sqrt{R^2 + X_L^2}$$

$$Z_{LR} = \frac{V}{I} = \sqrt{R^2 + X_L^2}$$

$$\tan\phi = \frac{I X_L}{I R}$$

$$\Rightarrow \tan\phi = \frac{X_L}{R}$$





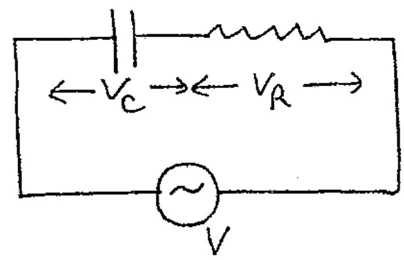
Voltage leads the current by an angle ϕ.

CR-series circuit across alternating electric supply

Here $V = \sqrt{V_R^2 + V_C^2}$

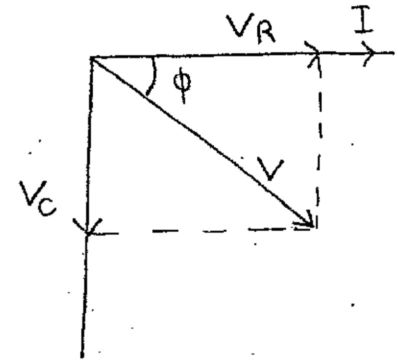
$= I\sqrt{R^2 + X_C^2}$

or $Z_{CR} = \frac{V}{I} = \sqrt{R^2 + X_C^2}$



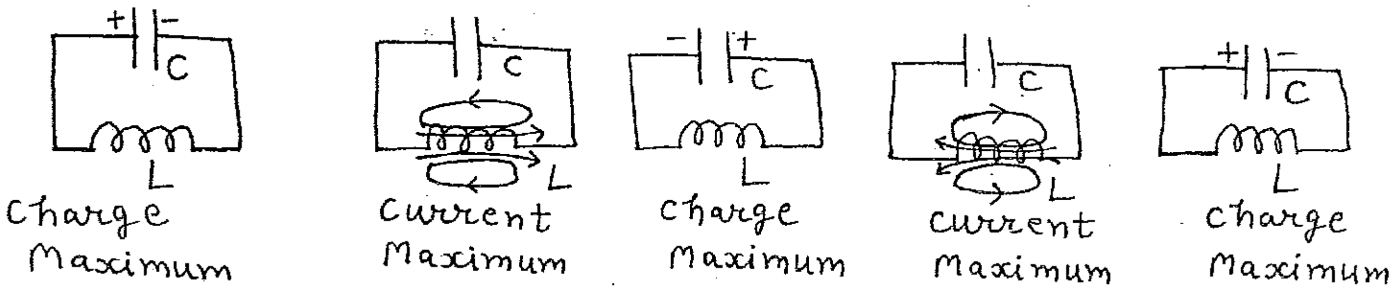
$\tan\phi = \frac{V_C}{V_R} = \frac{I X_C}{I R}$

or $\tan\phi = \frac{X_C}{R}$



Voltage lags behind the current by an angle ϕ.

LC-oscillations → Electrical oscillations produced by the exchange of energy between a capacitor (which stores electrical energy) and an inductor (which stores magnetic energy) are called LC-oscillations.

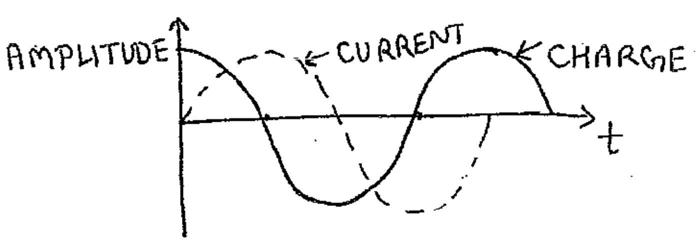


Let us consider that the capacitor of LC-circuit is fully charged. A potential difference exists between the plates of the capacitor and the energy is stored in the electric field of capacitor. ($U_E = Q_0^2/2C$). Now the capacitor begins to discharge through the inductor and hence current starts flowing through the inductor. As a result of this, magnetic field is set up around the inductor. When the capacitor is discharged completely, the energy is stored in the magneti field around the inductor ($U_m = \frac{1}{2} L I_0^2$). Now electrical energy is completely converted into magnetic energy.

When the magnetic field energy becomes maximum, the capacitor begins to recharge itself in the opposite direction. Now the energy stored in the magnetic field is converted into the energy stored in the electric field of capaci

When the capacitor is fully recharged, whole of the energy is stored in the electric field of the capacitor. At that instant, the capacitor is again discharged through the inducto The current flows through the inductor and energ is stored in the magnetic field around it.

Now again the capacitor is re-charged in the opposite direction and the energy is stored in the electric field of capacitor. This process continues and results in LC-oscillations.



$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \nu = \frac{1}{2\pi\sqrt{LC}}$$

ν = freq. of LC-oscillations

Electrical Resonance → Resonance in a series LCR circuit takes place when the circuit allows maximum current for a given frequency of A.C. source for which the Capacitive reactance becomes equal to inductive reactance.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

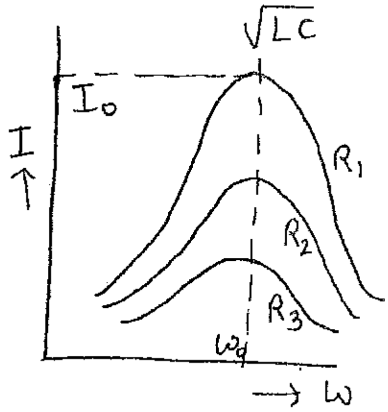
If $X_L = X_c$ or $\omega_o L = \omega_o C$ or $\omega_o = \frac{1}{\sqrt{LC}}$

then $I = \frac{V}{R}$ is maximum

$$\omega_o = \frac{1}{\sqrt{LC}} = 2\pi \nu_o$$

$$\Rightarrow \nu_o = \frac{1}{2\pi\sqrt{LC}}$$

[Here $R_3 > R_2 > R_1$



This is used in acceptor circuit.

Q- factor → It is defined as the ratio of the voltage developed across the inductor (or capacitor) at the resonance to the voltage applied (i.e. voltage across resistance) to the circuit.

$$Q = \frac{V_L \text{ or } V_C}{IR} = \frac{X_L}{R} \text{ or } \frac{X_C}{R}$$

$$\text{or } Q = \frac{\omega_o L}{R} \text{ or } \frac{1}{\omega_o C R}$$

Now $\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

In terms of bandwidth $Q = \frac{\omega_o}{2\Delta\omega} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$

Let us consider that

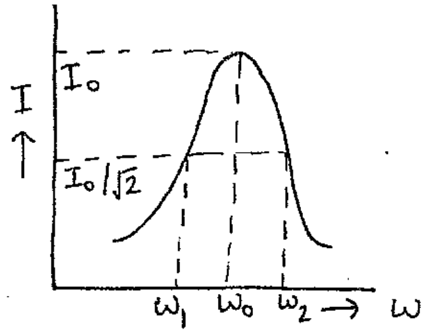
$\omega_1 = \omega_0 - \Delta\omega$

$\omega_2 = \omega_0 + \Delta\omega$

$\omega_2 - \omega_1 = 2\Delta\omega$ = Bandwidth

$$Q = \frac{\omega_0}{2\Delta\omega} = Q \text{ factor}$$

or sharpness of resonance



Power consumed in a series LCR circuit

Power dissipated in an a.c circuit is the product of r.m.s. value of voltage and component of current in phase with r.m.s. voltage.

For LCR circuit

$$V = V_0 \sin\omega t \quad \& \quad I = I_0 \sin(\omega t + \phi)$$

$$P_i = VI = V_0 I_0 \sin\omega t \sin(\omega t + \phi)$$

$$= V_0 I_0 \sin\omega t (\sin\omega t \cos\phi + \cos\omega t \sin\phi)$$

$$\Rightarrow P_i = V_0 I_0 \left[\sin^2\omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi\right]$$

$$P_{av.} = \frac{1}{T}\int_0^T P_i \, dt$$

$$= \frac{V_0 I_0}{T}\left[\int_0^T \sin^2\omega t \cos\phi \, dt + \int_0^T \frac{\sin 2\omega t \sin\phi}{2} dt\right]$$

$$= \frac{V_0 I_0}{T}\left[\int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right)\cos\phi \, dt + \int_0^T \frac{\sin 2\omega t \sin\phi}{2} dt\right]$$

Now $\int_0^T \sin 2\omega t \, dt = \int_0^T \cos 2\omega t \, dt = 0$

$$\Rightarrow P_{av.} = \frac{V_0 I_0}{2T}(\cos\phi \, T) = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$\Rightarrow P_{av.} = V_{r.m.s.} I_{r.m.s.} \cos\phi$$

Here $\cos\phi$ is called power factor.

<u>Special Cases</u> : (i) In pure resistor circuit

$\phi = 0$, $\cos\phi = 1$

$\Rightarrow P_{av.} = V_{r.m.s.} I_{r.m.s.}$

(ii) For pure inductor circuit

$\phi = 90^\circ$, $\cos\phi = 0$

$\Rightarrow P_{av.} = 0 \Rightarrow$ No power loss

(iii) For pure capacitor circuit

$\phi = 90^\circ$, $\cos\phi = 0$

$\Rightarrow P_{av.} = 0 \Rightarrow$ No power loss

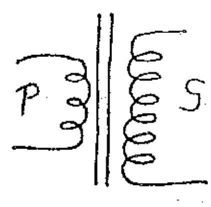
<u>Wattless Current or Idle current</u> → It is that curren due to which the power consumed in the circuit is zero.

eg. Average power consumed in pure inductor or pure capacitor circuit is zero so in these cases, current is wattless current.

<u>Transformers</u> → It is a device used to convert low alternating voltage at higher current into high alternating voltage at lower current and vice-versa.

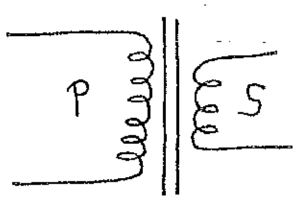
<u>Types of Transformers</u> →

(i) <u>Step-up Transformer</u>



It converts low voltage at high current to high voltage at lower current.

(ii) <u>Step-down Transformer</u>



It converts high voltage at low current to low voltage at higher current

Principle → It is based on the principle of mutual induction. An e.m.f. is induced in a coil, when a changing current flows through its nearby coil.

Acc to Faraday's law, the induced e.m.f. in the primary coil, $E_P = -N_P \frac{d\phi}{dt}$

The induced e.m.f. in the secondary coil,

$$E_S = -N_S \frac{d\phi}{dt}$$

Now $\frac{E_S}{E_P} = \frac{N_S}{N_P} = K$ = Transformation ratio

$K < 1$ for step-down transformer

$K > 1$ for step-up transformer

For an ideal transformer

output power = Input power

$$E_S I_S = E_P I_P \quad \text{or} \quad \frac{E_S}{E_P} = \frac{I_P}{I_S}$$

$$\Rightarrow E \propto \frac{1}{I}$$

$$\text{Efficiency}, \eta = \frac{E_S I_S}{E_P I_P} \leqslant 1$$

Energy losses in a Transformer →

(i) Copper losses → Energy lost in windings of the transformer is known as copper loss

(ii) Flux leakage losses → Magnetic flux linked with primary is not equal to that of secondary.

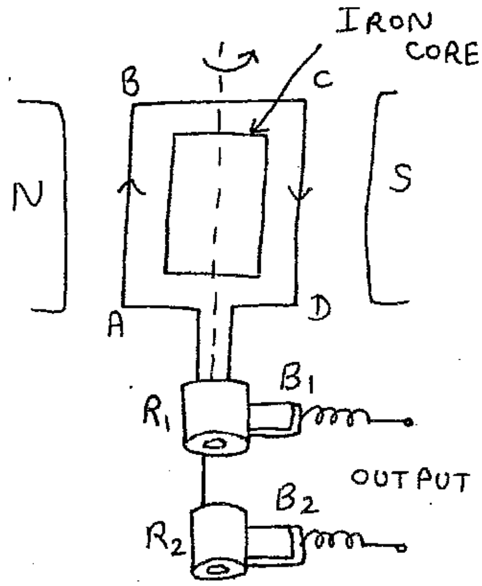
(iii) Iron losses → It is of two types

(a.) Eddy current losses (b) Hysteresis losses

A.C generator → A device used to convert mechanical energy into electrical energy.

Principle → It is based on the principle of electromagnetic induction.

Construction →



(i) Armature → Armature coil ABCD having large no. of turns

(ii) Strong field magnets → Two pole pieces of permanent magnet cylindrical in shape.

(iii) Slip Rings → Two brass slip rings R_1 & R_2 connected to coil.

(iv) Brushes → Two carbon brushes B_1 & B_2 attached with R_1 & R_2

Working → When the armature coil ABCD rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. The magnetic flux linked with the coil changes and hence e.m.f. is induced. The direction of induced current is given by Flaming's Right hand rule. For half revolution, current through B_1 comes out & for next half, current through B_2 comes out. Hence e.m.f. induced is of alternating nature.

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t) = NBA\omega\sin\omega t$$

$$\mathcal{E} = E_0 \sin\omega t \qquad \therefore E_0 = NBA\omega$$

$$I = \frac{\mathcal{E}}{R} = \frac{E_0}{R}\sin\omega t = I_0 \sin\omega t$$

Variation of induced e.m.f. with different positions of coil

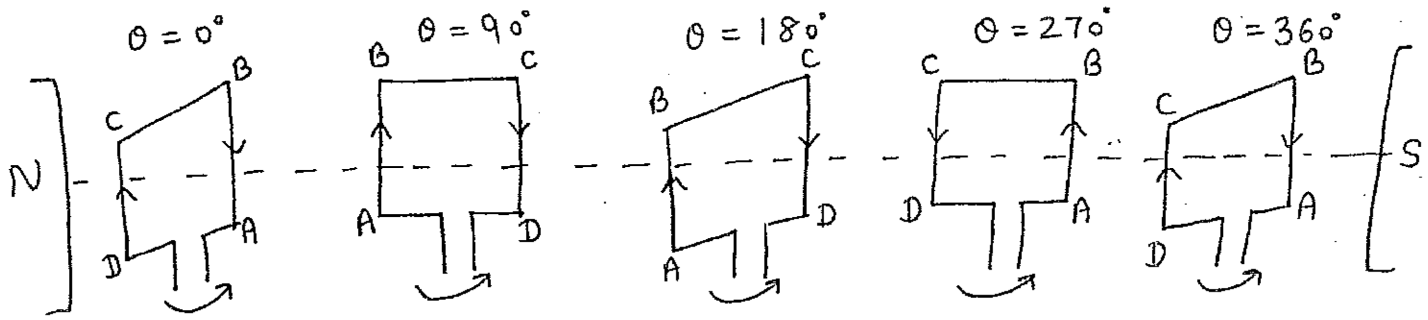
(i) When $\theta = 0°$, $\epsilon = \epsilon_0 \sin \omega t = \epsilon_0 \sin 0° = 0$

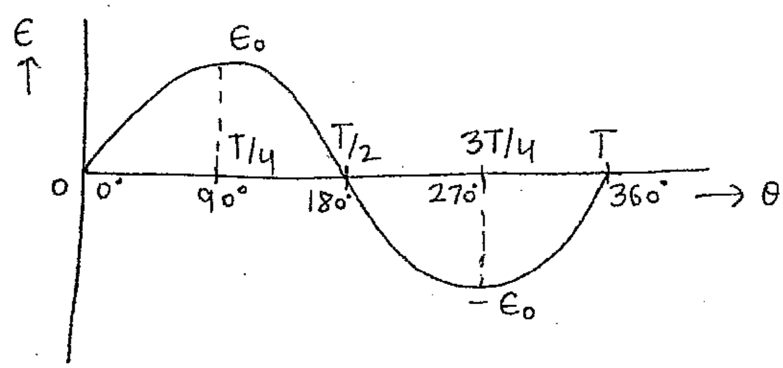
(ii) When $\theta = 90°$, $\epsilon = \epsilon_0 \sin 90° = \epsilon_0$

(iii) When $\theta = 180°$, $\epsilon = \epsilon_0 \sin 180° = 0$

(iv) When $\theta = 270°$, $\epsilon = \epsilon_0 \sin 270° = -\epsilon_0$

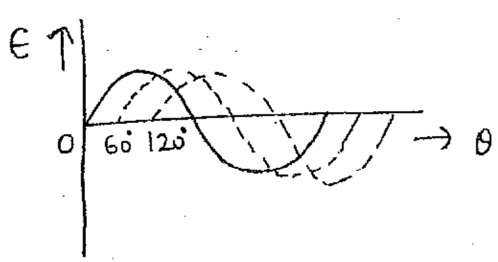
(v) When $\theta = 360°$, $\epsilon = \epsilon_0 \sin 360° = 0$





This is single phase A.C

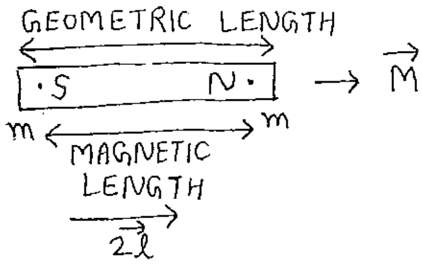
In Poly-phase generators (generally 3-phase) there are three separate coils inclined to one another at equal angles. They produce a phase difference of 120°.



MAGNETISM and MATTER

Bar Magnet

(OR MAGNETIC DIPOLE)



MAGNETIC DIPOLE MOMENT $\vec{M} = m(2\vec{l})$

S.I. unit – Ampere-metre^2

Properties of bar magnet (i) Attractive property

(ii) Directive property

(iii) Inductive property (Induced magnetism)

(iv) Unlike poles attract & like poles repel each other.

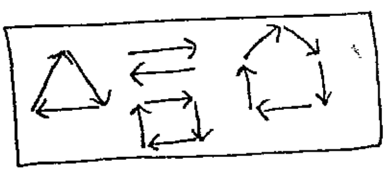
(v) Magnetic poles exist in pairs

(vi) Repulsion is the surest test for distinguishin between a magnet and a piece of iron

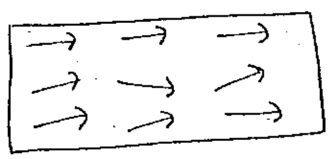
Atomic view of magnetism – Each atom/molecule of a magnetic substance behaves like a magnetic dipole.

In case of un-magnetised piece of iron (or magnetic material), these magnetic dipoles are arranged in such a manner that their net magnetic dipole moment is zero.

In a bar magnet, these molecular magnets are aligned in same direction so as to give net magnetic dipole moment $\vec{M}$.



(In unmagnetised piece of Iron)



(In a bar magnet)

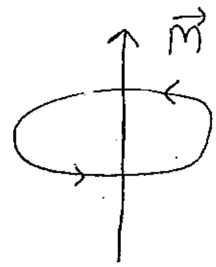
Coulomb's law in magnetism

$$F \propto \frac{m_1 m_2}{r^2}$$

or $F = k\frac{m_1 m_2}{r^2}$ $\quad k = \frac{\mu_0}{4\pi} = 10^{-7}\ Wb/Am$

m – magnetic pole strength
(S.I. unit Ampere-metre)

Magnetic moment of current loop as a magnetic dipole



$\vec{M} = I\vec{A}$

$M \propto I$
$\quad \propto A$

$M = kIA$ (Here $k = 1$)

$\Rightarrow M = IA$

For coil (or loop) having n-turns $M = nIA$

Atom as a magnetic dipole (Magnetic dipole moment of a revolving electron)

Let us calculate the magnetic dipole moment (M) of an atom due to orbital motion of electron.

The angular momentum of electron (of mass m_e) moving with velocity v in a circular orbit of radius r is given by $L = m_e v r$ ----(i)

The orbital motion of electron is equivalent to a current $I = e\left(\frac{1}{T}\right)$, $T = \frac{2\pi r}{v}$

$\Rightarrow I = ev/2\pi r$ $\quad$ Now $A = \pi r^2$ & $M = IA$

$\Rightarrow M = \frac{evr}{2} = \left(\frac{e}{2m_e}\right)L$ $\quad$ using (i)

or $\vec{M} = -\left(\frac{e}{2m_e}\right)\vec{L}$ (-ve sign is due to -ve charge on electron)

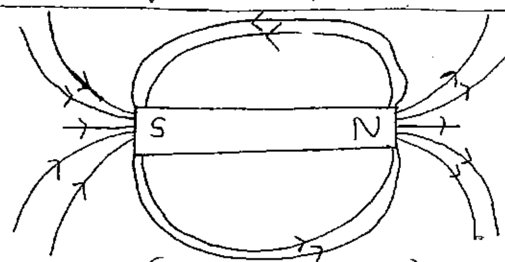
Acc to Bohr's theory

$L = nh/2\pi$

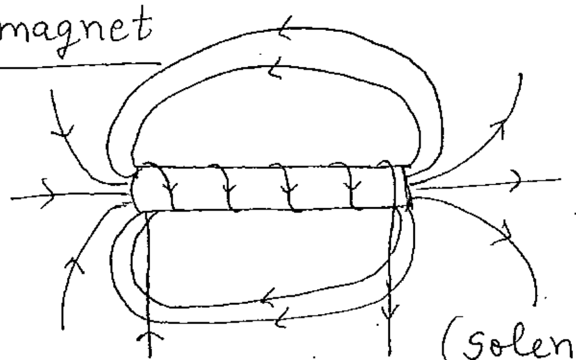
$\Rightarrow M = \left(\frac{e}{2m_e}\right)\frac{nh}{2\pi} = n\left(\frac{eh}{4\pi m_e}\right)$ $\left\{ \begin{array}{l} \text{For } n=1 \\ M = \frac{eh}{4\pi m_e} \end{array} \right\}$

{ Bohr Magneton $= eh/4\pi m_e$ }

Magnetic field of a bar magnet

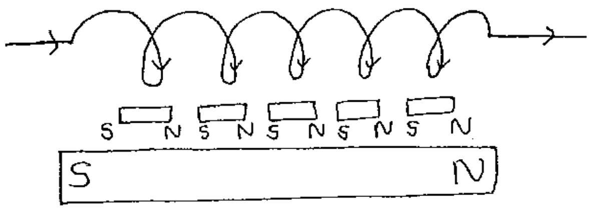


(Bar magnet)



(Solenoid)

(Bar magnet as an equivalent solenoid)



(Each current loop behaves as magnetic dipole)

(Read only)
* <u>Gauss law in magnetism</u> – surface integral of magnetic field over a closed surface S is zero.

$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

(i.e. An isolated magnetic pole does not exist.)

Properties of magnetic field lines

(i) Magnetic field lines are closed continuous loops.

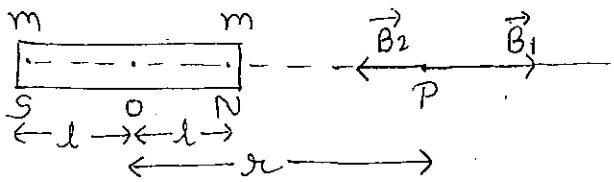
(ii) outside the magnet, field lines are from north to south.

(iii) Tangent to magnetic field lines at any point give the direction of field.

(iv) No two magnetic field lines can intersect each other.

(v) widely spaced lines represent weak magnetic field and crowded lines represent strong magnetic field.

Magnetic field on axial line of a bar magnet



consider a bar magnet of length $2l$ and pole strength m. consider a point P at distance r from centre of magnet.

$$\vec{B}_{axial} = \vec{B_1} + \vec{B_2}$$

$$|\vec{B_1}| = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2} \qquad |\vec{B_2}| = \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}$$

As $|\vec{B_1}| > |\vec{B_2}|$ and $\vec{B_1}$ & $\vec{B_2}$ are opposite.

$$\Rightarrow B_{axial} = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}$$

$$= \frac{\mu_0 m}{4\pi} \left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$B_{axial} = \frac{\mu_0 m}{4\pi} \cdot \frac{4rl}{(r^2-l^2)^2} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2-l^2)^2}$$

$$[\because M = m \times 2l]$$

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi} \frac{2\vec{M} r}{(r^2-l^2)^2}$$

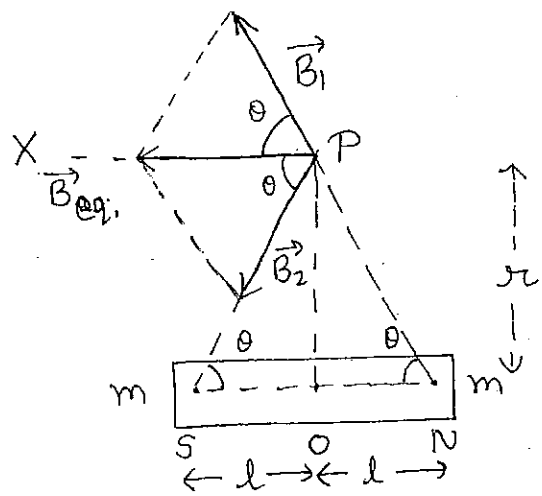
For very small length of bar magnet
$l << r$

$$\Rightarrow B_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$$

* Magnetic field due to a bar magnet at a point on its axial line has the same direction as that of its magnetic dipole moment vector.

Magnetic field on equitorial line of a bar magnet

Consider a bar magnet of length $2l$ and pole strength m.
Consider a point P on equitorial line at a distance r from centre.



At point P

$$\vec{B}_{equitorial} = \vec{B_1} + \vec{B_2}$$

$$|\vec{B_1}| = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2+l^2}\right) \quad \& \quad |\vec{B_2}| = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2+l^2}\right)$$

$\vec{B_1}$ & $\vec{B_2}$ have two components.
$B_1 \sin\theta$ and $B_2 \sin\theta$ are equal & opposite so they will cancel each other

so $|\vec{B}_{equitorial}| = |\vec{B_1}| \cos\theta + |\vec{B_2}| \cos\theta$

$$|\vec{B}_{eq}| = \frac{\mu_0}{4\pi}\left(\frac{2m}{r^2+l^2}\right) \cos\theta$$

Now $\cos\theta = \dfrac{l}{(r^2+l^2)^{1/2}}$

so $|\vec{B}_{eq.}| = \dfrac{\mu_0}{4\pi}\left(\dfrac{2m}{r^2+l^2}\right)\dfrac{l}{(r^2+l^2)^{1/2}}$

$$|\vec{B}_{eq.}| = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{3/2}} \quad \{\because M = m \times 2l\}$$

$$\vec{B}_{eq.} = \frac{-\mu_0}{4\pi}\frac{\vec{M}}{(r^2+l^2)^{3/2}}$$

For short bar magnet $|\vec{B}_{eq.}| = \dfrac{\mu_0}{4\pi}\left(\dfrac{M}{r^3}\right)$

* Negative sign shows that $\vec{B}_{eq.}$ & $\vec{M}$ are in opposite directions.

* Read only

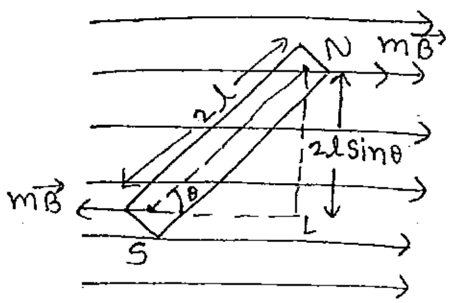
Magnetic field at any point due to a short magnetic dipole

$$B = \frac{\mu_0}{4\pi} \frac{M\sqrt{3\cos^2\theta + 1}}{r^3}$$

For axial line, $\theta = 0° \Rightarrow B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

For equitorial line, $\theta = 90° \Rightarrow B = \frac{\mu_0}{4\pi} \frac{M}{r^3}$

Torque on a bar magnet in a magnetic field



Let us consider a bar magnet of length $2l$ and pole strength m

Force on each pole of magnet $= mB$

These two forces constitute a torque and its magnitude is given by $\tau = mB \times LN$

$$\tau = mB \times 2l\sin\theta = MB\sin\theta$$

$$\vec{\tau} = \vec{M} \times \vec{B}$$ $\{\vec{\tau}$ is in the direction of $\vec{M} \times \vec{B}$ i.e. $\perp$ to plane$\}$

Potential energy of bar magnet (work done in rotating the magnet through an angle θ)

$\tau = MB\sin\theta$

dW = Small work $= \tau\, d\theta = MB\sin\theta\, d\theta$

Total work done $W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$

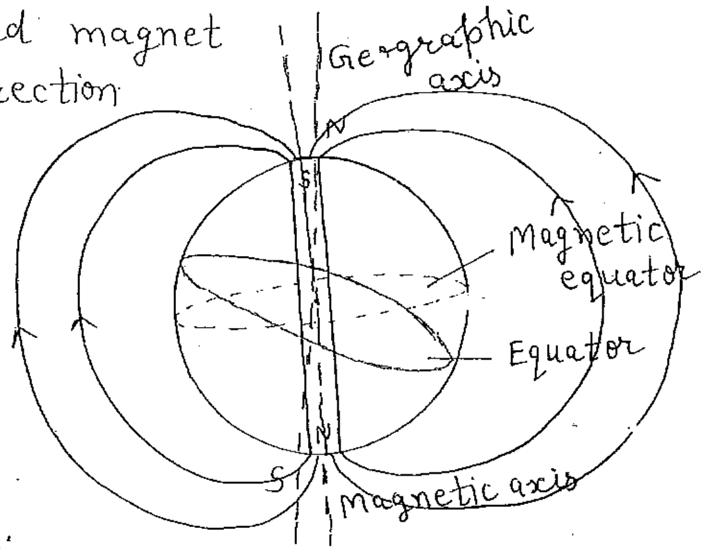
$U = W = MB(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 90°$ & $\theta_2 = \theta$ then $U = MB(-\cos\theta)$

$U = -\vec{M}\cdot\vec{B}$ At $\theta = 0°$ U (is min.) $= -MB$

At $\theta = 180°$ U (is max.) $= MB$

Earth's Magnetism



* A freely suspended magnet aligns along N-S direction
* Angle between magnetic & geographic axis is $\approx 20°$
* A vertical plane passing through the geographic axis is Geographic meridian.
* A vertical plane passing through magnetic axis is magnetic meridian.

Cause of Earth's magnetism

* Read only

① It is guessed that earth's magnetism is due to molten charged metallic fluid

② According to some other theory, since every substance is made up of charged particles hence a substance rotating about an axis will become magnetised.

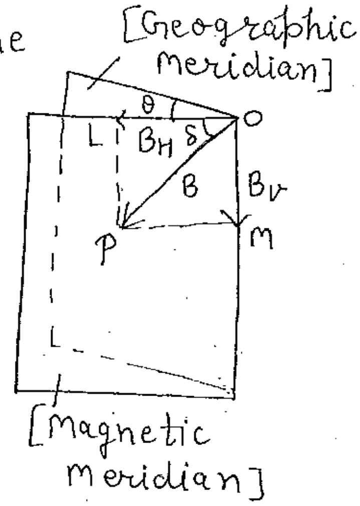
③ The earth's magnetism is also due to the rotation of earth about its own axis.

④ The gases in the atmosphere are in the ionised states. The high energy radiation coming from sun ionises the atoms in the upper part of atmosphere. Hence due to rotation of earth, strong current flows & earth is magnetised.

⑤ Earth's magnetism is also due to presence of magnetic materials (eg. iron, nickel) in the core of earth. Due to rotation, they get magnetised.

Magnetic Elements - The physical quantities which determine the intensity of earth's total magnetic field completely are called magnetic elements.

① Magnetic Declination - Declination at a place is the angle between the magnetic-meridian and geographic-meridian. It is denoted by θ. (As shown in figure)



② Magnetic Inclination or dip - Dip at a place is defined as the angle made by the direction of the total earth's magnetic field with the horizontal direction. It is denoted by δ. (As shown in figure)

③ Horizontal component of Earth's magnetic field

It is the component of earth's magnetic field in horizontal direction.

$$B_H = B\cos\delta \quad , \quad B_V = B\sin\delta$$

$$B = \sqrt{B_H^2 + B_V^2}$$

$$\tan\delta = \frac{\sin\delta}{\cos\delta} = \frac{B_V}{B_H}$$

* The value of angle of dip at any place on the magnetic equator of earth is zero
* The value of angle of dip at the magnetic poles of earth is 90°.

Few Definitions

① **Magnetic Intensity** → It is also known as magnetic field strength. It is denoted by H & $H = \frac{B_0}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ Tm/A

S.I. unit of H is Ampere/metre.

② **Intensity of magnetisation** → It is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to magnetising field.

It is denoted by I. $I = \frac{M}{V}$ in (A/m)

M → Magnetic dipole moment, V is volume

③ **Magnetic flux** → Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through the surface.

$$\phi = \int \vec{B} \cdot d\vec{s}$$

④ **Magnetic Induction** → It is defined as the number of magnetic lines of induction crossing per unit area normally through the magnetic substance. It is denoted by B.

$$B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I = \mu_0 (H + I)$$

S.I. unit is Tesla or weber/metre^2

⑤ **Magnetic Susceptibility** → It is defined as the ratio of the intensity of magnetisation to the magnetic intensity. It is denoted by χ_m

& $\chi_m = \frac{I}{H}$ ($\Rightarrow$ no units)

⑥ **Magnetic Permeability** → It is defined as the ratio of the magnetic induction to the magnetic intensity. It is denoted by μ.

$\mu = \frac{B}{H}$ (S.I. unit is Tm/A)

$\mu_r = 1 + \chi_m$

Classification of magnetic materials

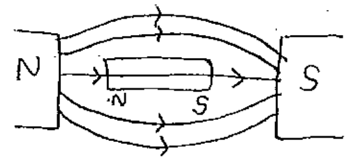
① **Diamagnetic** → Those substances which when placed in a magnetic field are feebly magnetised in a direction opposite to that of the magnetising field, are called diamagnetic substances. eg. Cu, Zn, Bi, Au, Ag, Pb, glass, marble, water, helium, argon, NaCl etc

Properties → ① A diamagnetic substance is feebly repelled by a magnet.

② If a diamagnetic liquid contained in a watch glass is placed on two closely spaced pole pieces of a magnet, it suffers a slight depression in the middle. The liquid shows rise in the middle when the pole pieces are moved apart.

③ The intensity of magnetisation (I) has a small negative value.

④ When diamagnetic material is placed in a magnetic field, the magnetic field lines becomes less dense inside the material.



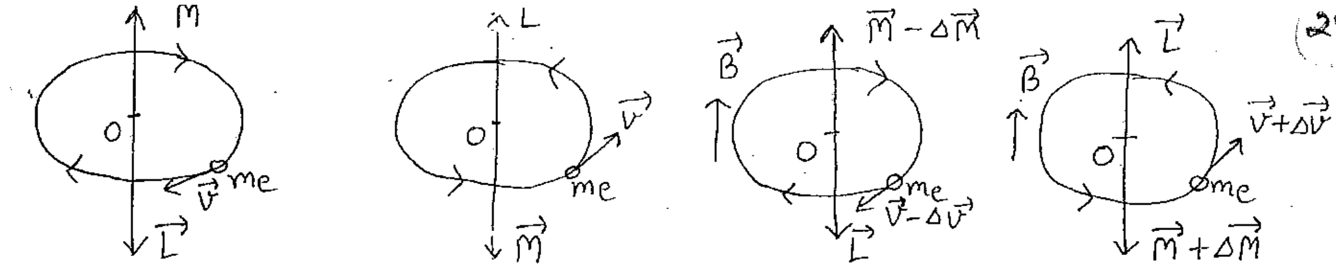
⑤ The magnetic susceptibility (χ_m) has a small negative value.

⑥ Relative permeability μ_r is slightly less than 1.

⑦ Diamagnetic substances do not obey curie's law.

Electron theory of diamagnetism → We know that there is magnetic dipole moment $\vec{M}$ due to each revolving electron.

In a diamagnetic material, there are only paired electrons. The paired electrons possess equal & opposite dipole moments due to opposite orbital motion. There is no net magnetic moment present in the absence of magnetic field (external)



When external magnetic field $\vec{B}$ is applied then magnetic Lorentz force($\vec{F}$) acts on the electron.

$$\vec{F} = -e(\vec{v} \times \vec{B})$$

In case of e^- revolving in clockwise direction force $\vec{F}$ tends to decrease the centripetal force and velocity decreases to $\vec{v} - \Delta\vec{v}$. Similarly the e^- revolving in anticlockwise direction experience a force such that it tends to increase the centripetal force and velocity increases to $\vec{v} + \Delta\vec{v}$. Now pair of electrons possesses net magnetic dipole moment $2\Delta\vec{M}$ in a direction opposite to $\vec{B}$. This explains the behaviour of diamagnetic substance in presence of external magnetic field.

② Paramagnetic → Those substances which when placed in a magnetic field are feebly magnetised in the direction of the magnetising field are called paramagnetic substances. eg. Al, Na, Sb, Pt, Mn, Cr, copper chloride, liquid oxygen etc

Properties → ① A paramagnetic substance is feebly attracted by the magnet.

② A freely suspended rod of paramagnetic material aligns itself parallel to the direction of magnetic field.

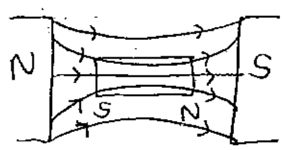
③ If a paramagnetic liquid contained in a watch glass is placed on two closely spaced pole pieces of a magnet, it shows a slight rise in the middle. When the poles are moved apart, the paramagnetic liquid gets depressed in the middle.

④ The intensity of magnetisation (I) has a small positive value.

⑤ The magnetic susceptibility (χ_m) has a small positive value.

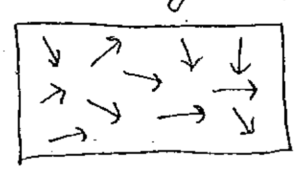
⑥ When a paramagnetic material is placed inside a magnetic field, the field lines becomes slightly more dense in the paramagnetic material.

⑦ The relative permeability is slightly greater than 1.

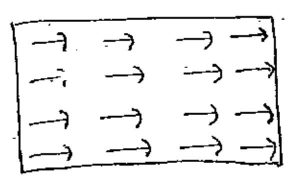


⑧ Paramagnetic substances obey. curie's law.

<u>Electron theory of Paramagnetism</u> → In case of paramagnetic substances, the interaction between the atomic magnetic dipoles is very weak and they are almost independent of each other. Due to thermal agitation, the atomic magnetic dipoles are randomly oriented.



When external magnetic field is applied, the field aligns the magnetic dipoles along its own direction and temperature increases the thermal agitation. So at low temperature alignment is more



<u>Curie law</u> → $I \propto H$, $I \propto \frac{1}{T}$ $\quad I \propto \frac{H}{T}$

$$I = C\frac{H}{T} \Rightarrow \frac{I}{H} = \chi_m = \frac{C}{T} \quad [C \rightarrow \text{curie's constt}]$$

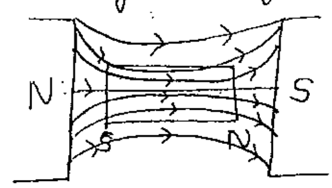
③ Ferromagnetic → Those substances which when placed in a magnetic field are strongly magnetised in the direction of the magnetising field are called ferromagnetic substances. eg. Iron, cobalt, Nickel, Alnico etc.

Properties → ① A ferromagnetic substance is strongly attracted by a magnet.

② When a rod of ferromagnetic substance is suspended in a magnetic field, it quickly aligns itself along the direction of the magnetic field.

③ The ferromagnetic materials move from weaker part to the stronger part of the magnetic field.

④ When a ferromagnetic material is placed in a magnetic field, the magnetic field lines become highly dense inside ferromagnetic substance



⑤ The intensity of magnetisation (I) has a large positive value.

⑥ The magnetic susceptibility (χ_m) has a large positive value.

⑦ Relative permeability is very large as compared to one.

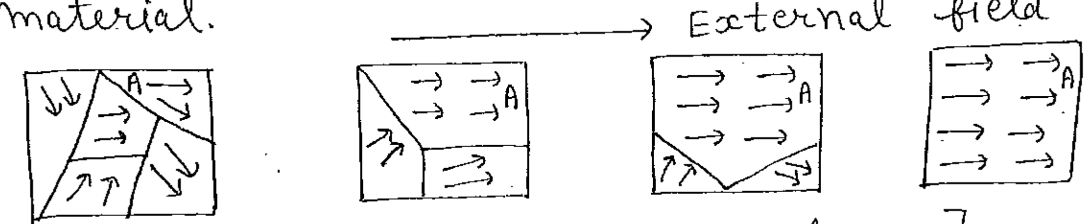
⑧ Ferromagnetic substances do not obey curie's law.

Electron theory of ferromagnetism (Domain theory)

In case of ferromagnetic substances, an unpaired electron in one atom interacts strongly with the unpaired electron of the neighbouring atom in such a way that they align themselves in a common direction over a small volume of material. These small volumes are called Domains.

Due to their random orientations, the net magnetic moment of the material is zero. When external magnetic field is applied, the magnetic moments of domains align along the direction of external field; one due to alignment of atomic magnetic dipoles within the domain and second due to alignment of whole domain along the magnetic field.

* At curie temperature or curie point, the ferromagnetic material reduces to paramagnetic material.

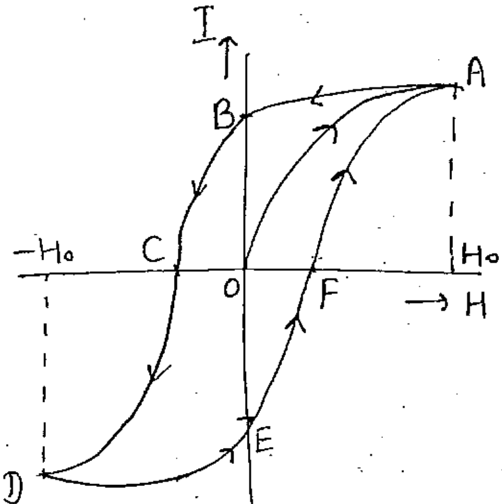


[Domain A increases in volume]

Hysteresis → The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation & demagnetisation of the ferromagnetic material is called Hysteresis.

OB → Retentivity

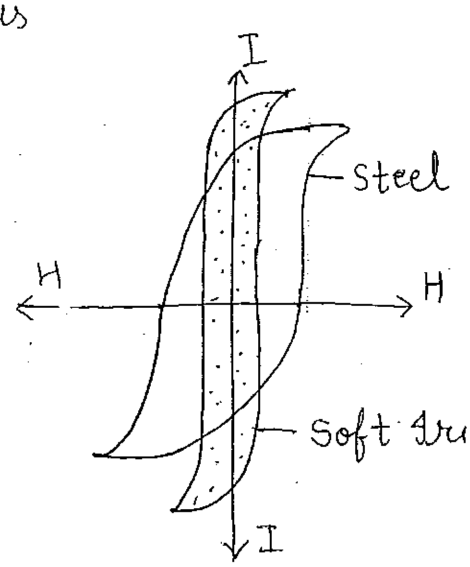
OC → Coercivity



Retentivity → The value of the intensity of magnetisation of material, when the magnetising field is reduced to zero, is called retentivity or residual magnetism

Coercivity → The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.

Comparison of Hysteresis loop for soft iron & steel



① Due to small area of hysteresis loop, the loss of energy per unit volume in case of soft iron is relatively small.

② Soft iron is much strongly magnetised than steel.

③ The retentivity of soft iron is comparatively more. On removing magnetising field, a large amount of magnetisation is retained by soft iron.

④ Coercivity of steel is comparatively much larger The residual magnetism in steel can not be destroyed easily as compared to soft iron.

Applications of ferromagnetic substances

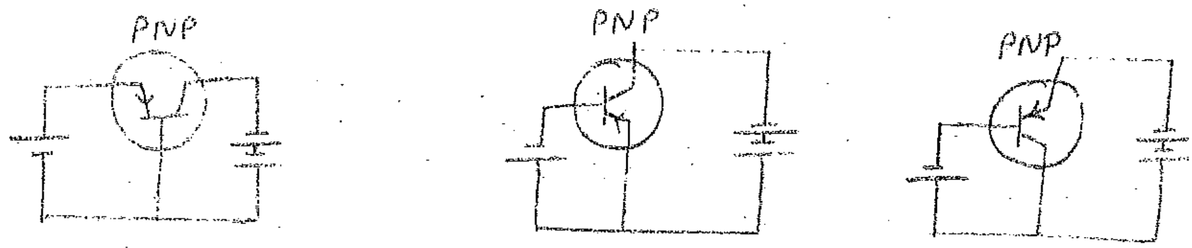
① Permanent magnets (Steel is used generally)

② Electromagnets (Generally soft iron cores are used).

③ Transformer cores (Soft iron core is generally used. Mumetal (76% nickel, 17% iron 5% copper & 2% chromium) is also used for it.

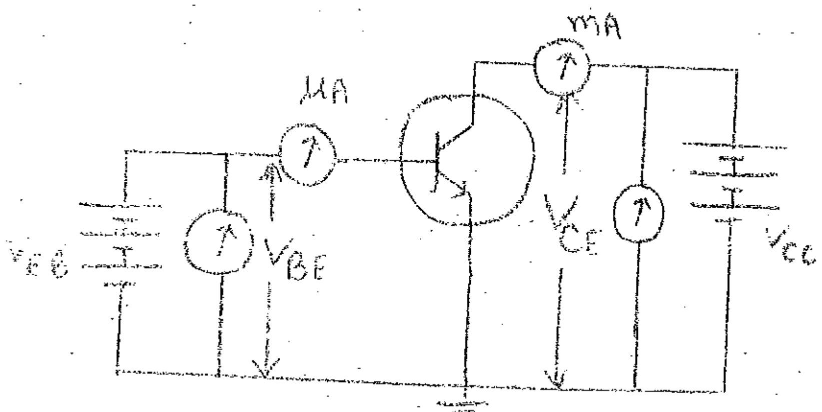
Transistor circuit configurations & characteristics

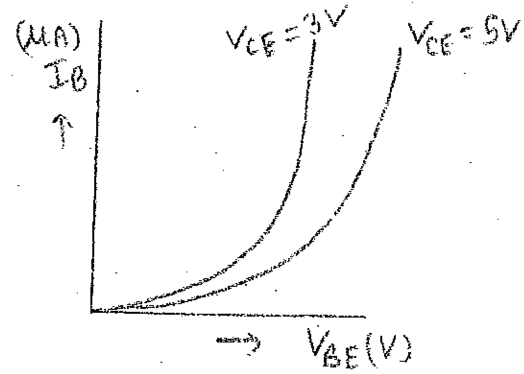
Common - base   common-emitter   common-collector



Common - emitter transistor characteristics

(i) Input characteristics





First keep the collector - emitter voltage (V_{CE}) constant & change V_{BE} in steps and note down the corresponding change in the values of V_{BE} Vs I_B, plot the graph. The curve obtained is input characteristic.

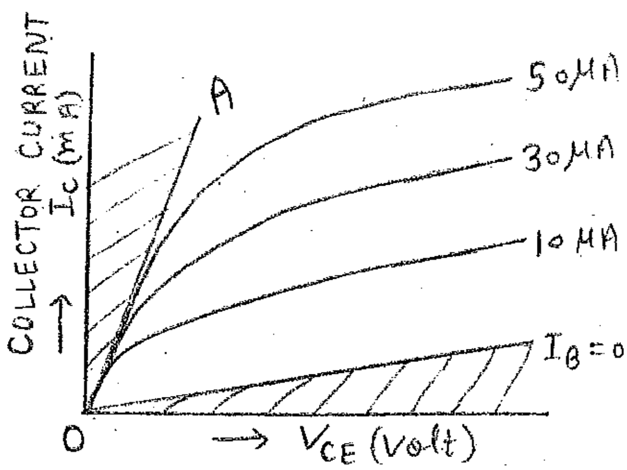
Input characteristics are similar to forward biased characteristics of a junction diode.

For a given value of V_{BE}, the base current decreases with increase of V_{CE}.

Input dynamic resistance $r_i = \left(\dfrac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}\ \text{constt}}$

output characteristics →

First keep the base current I_B constant and change V_{CE} in steps and note down the corresponding values of collector current I_c.



The graph of V_{CE} Vs I_c gives output charac. When the voltage V_{CE} increases from 0 to about 0.5 V, I_c increases rapidly. This value of V_{CE} is called knee voltage. once V_{CE} exceeds V_{BE}, I_c varies slowly & linearly with V_{CE} for given value of I_B. Larger the value of I_B, larger is the value of I_c for a given value of V_{CE}.

Three regions of output characteristic

(i) Saturation region → when $V_{CE} < V_{BE}$, both the junctions are forward biased. I_c does not depend on input current I_B. Shaded region towards the left of line OA is Called saturation region.

(ii) Cut-off region → The shaded region lying below the curve $I_B = 0$ is Called cut-off region. In this region, both the junctions are reverse biased & $I_c = 0$

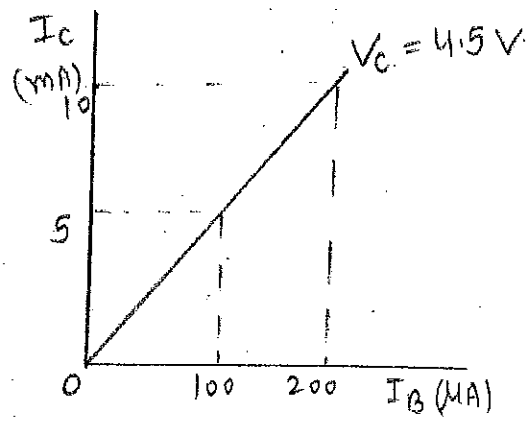
* In the shaded regions, the transistor acts as a switch cut-off (OFF) & saturation (ON)

<u>Active region</u> → The non-shaded central region of output characteristic is called active region. In this region, the emitter-base junction is forward biased and collector-base junction is reverse biased. A transistor works as an audio amplifier in this region.

output resistance, $r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B \text{ Constt.}}$

<u>Transfer characteristic</u>

It is a graph showing the variation of collector current I_C with base current I_B at constant Collector emitter voltage.



<u>Current amplification factor (β)</u> → It is defined as the ratio of Change in collector current to small change in base current at constant V_{CE} when the transistor is in active state.

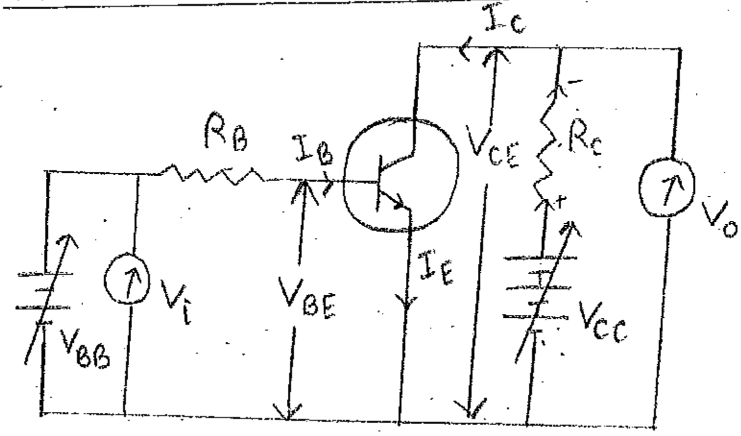
$$\beta_{a.c} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE} \text{ Constt.}}$$

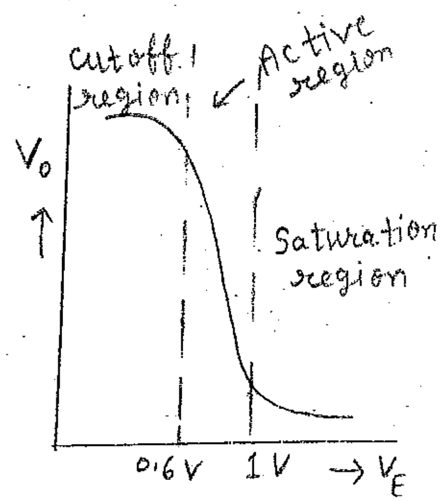
= Signal current gain

$$\& \quad \beta_{d.c} = \frac{I_C}{I_B}$$

Here values of $\beta_{a.c.}$ and $\beta_{d.c.}$ are nearly equal.

Transistor as a switch





Applying Kirchhoff's rule to the input & output circuits separately, we get

$$V_{BB} = I_B R_B + V_{BE} \quad \text{or} \quad V_i = I_B R_B + V_{BE}$$

$$\& \quad V_{CC} = I_C R_C + V_{CE} \quad \text{or} \quad V_{CE} = V_o = V_{CC} - I_C R_C$$

Switching action of a transistor

When base input voltage V_{BB} is very low so that transistor is not forward biased then no current flows through R_C i.e. $I_C = 0$. Hence output voltage is V_{CC}. Here transistor is in OFF state & acts as open circuit.

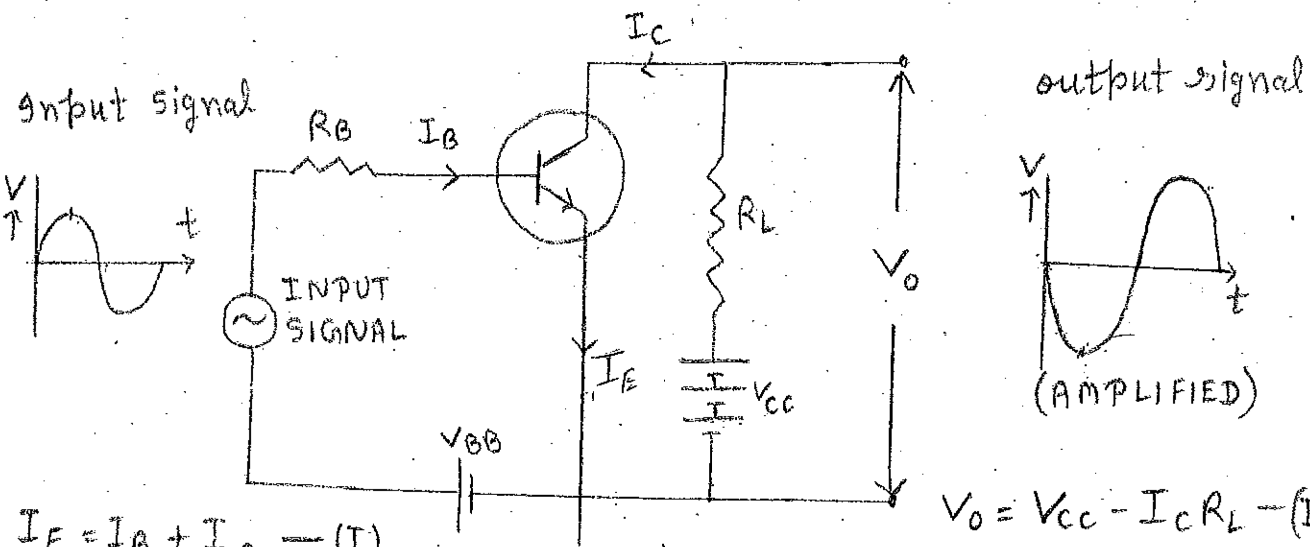
When base input voltage is made positive sufficiently, the transistor is forward biased. Now current I_C flows through R_C. Voltage drop across R_C is V_{CC} and $V_{CE} = 0$. Transistor is in saturation state and transistor acts as closed switch i.e. ON state.

* Transistor switching circuit is so designed that it never remains in active state.

Transistor as an amplifier (CE Configuration)

A device which increases the amplitude of the input signal is called amplifier.

Amplifier circuit using n-p-n transistor



$I_E = I_B + I_C$ — (I)

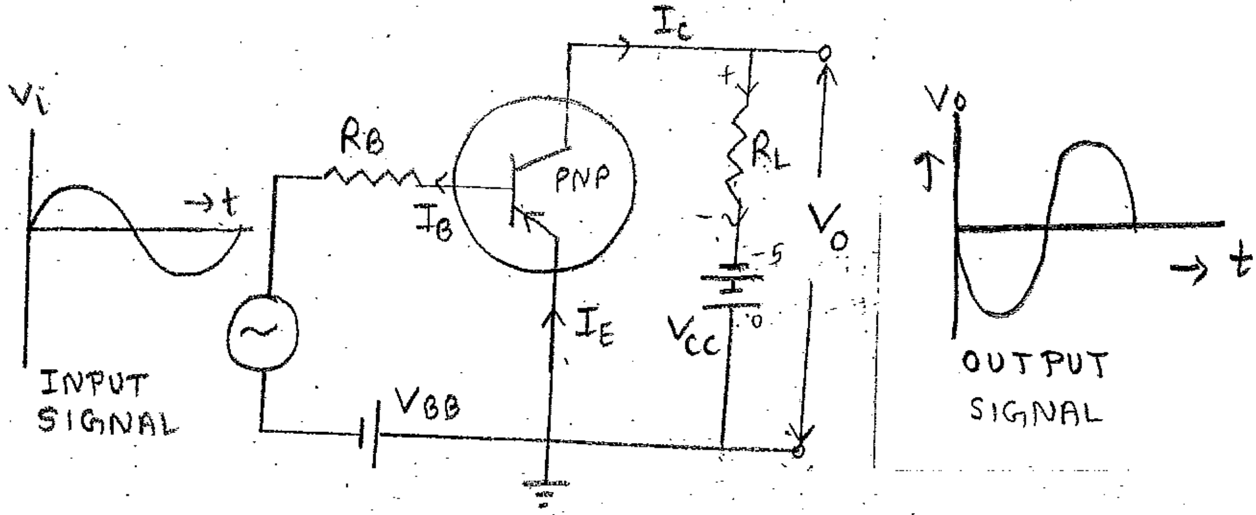
$V_o = V_{CC} - I_C R_L$ — (II)

During positive half cycle of input signal, the forward bias of emitter-base junction increases. Due to increased forward bias, I_E increases & hence I_C also increases. Therefore voltage drop across R_L (i.e. $I_C R_L$) increases. & hence V_o decreases according to eqn. II.

Similarly, during negative half cycle of input signal, the forward bias of emitter base junction decreases. As a result of this, I_E decreases and hence I_C decreases. Therefore voltage drop across R_L (i.e. $I_C R_L$) decreases & hence V_o increases according to eqn. II

Hence there is a phase difference of π between input & output signal.

Amplifier circuit using p-n-p transistor



$I_E = I_B + I_C$ — I $\qquad$ $V_o = V_{CC} - I_C R_L$ — II

During positive half cycle of input signal, the forward bias of emitter-base junction decreases. As a result of this, I_E and hence I_C decreases and according to eqn II V_o increases. Since collector is connected to the negative terminal of the battery so V_o becomes more negative.

During negative half cycle of input signal, the forward bias of emitter-base junction increases. As a result of this, I_E and hence I_C increases and according to eqn. II V_o decreases. Since collector is connected to the negative terminal of the battery so V_o becomes less negative.

In common-emitter amplifier, input & output signal are out of phase i.e. there is a phase difference of π between the input & output signals.

Transistor gains in C.E. amplifiers

Current gain (β)

A.C current gain, $\beta_{a.c.} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE} = \text{Constt.}}$

D.C. current gain, $\beta = \frac{I_C}{I_B}$

Voltage gain (A_V)

$$A_V = \frac{\Delta V_o}{\Delta V_i}$$

$$V_o = V_{CC} - I_C R_L$$

$$V_i = V_{BE} + I_B R_B$$

$$|A_V| = \left(\frac{\Delta I_C}{\Delta I_B}\right)\left(\frac{R_L}{R_B}\right) = \beta_{a.c.} \times \text{resistance gain}$$

Resistance gain $= \frac{R_o}{R_i} \simeq \frac{R_L}{R_B}$

Power gain

$$\text{Power gain} = \frac{\Delta P_o}{\Delta P_i}$$

$$P_o = V_o I_C = I_C^2 R_o$$

$$P_i = V_i I_B = I_B^2 R_i$$

$$\Rightarrow \text{Power gain} = \frac{\Delta I_C^2 R_o}{\Delta I_B^2 R_i}$$

$$= \left(\frac{\Delta I_C}{\Delta I_B}\right)^2 \left(\frac{R_o}{R_i}\right) = \beta_{a.c}^2 \times \text{resistance gain}$$

Trans conductance

$$g_m = \left(\frac{\Delta I_C}{\Delta I_B}\right)\left(\frac{\Delta I_B}{\Delta V_B}\right) = \left(\frac{\Delta I_C}{\Delta V_B}\right)$$

or $g_m = \beta_{a.c} / R_i$

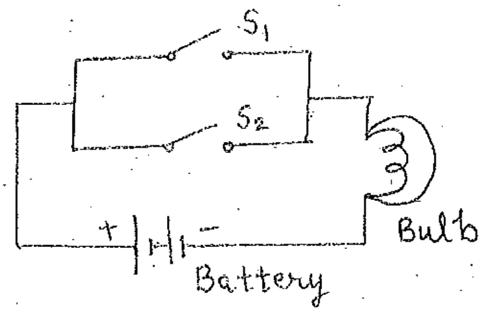
Logic gates → A gate is a digital circuit that is designed for performing a particular logical operation. As it works according to some logical relationship between input and output voltages, so it is generally known as a logic gate. There are three basic gates:

① OR gate ② AND gate ③ NOT gate

The 'OR' gate

$A + B = Y$

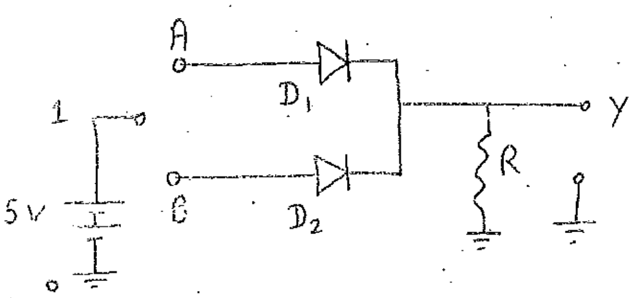
Analog circuit



Truth Table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Realisation of OR gate



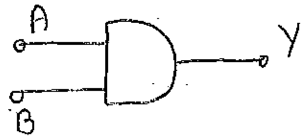
(i) When $A = 0$, $B = 0$, both D_1 & D_2 will not conduct & output across R i.e. Y is zero.

(ii) When $A = 0$, $B = 1$, D_1 will not conduct & D_2 will conduct and we get full voltage across R, $Y = 1$

(iii) When $A = 1$, $B = 0$, D_1 will conduct & D_2 will not conduct & we get full voltage across R, $Y = 1$

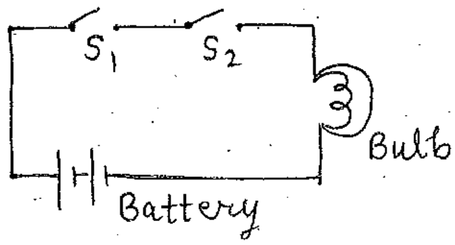
(iv) When $A = 1$, $B = 1$, both D_1 & D_2 will conduct & they are in parallel & output is high $Y = 1$

The 'AND' gate



$A \cdot B = Y$

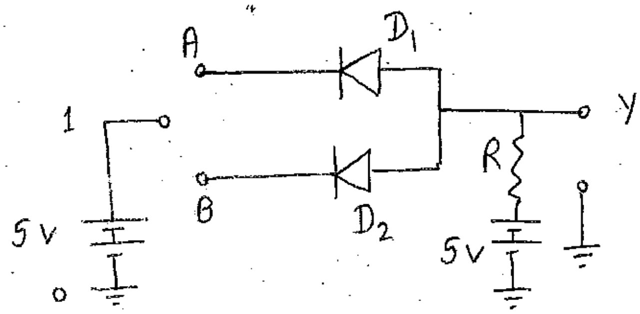
Analog circuit



Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Realisation of AND gate



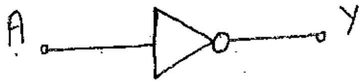
(i) When $A=0$ & $B=0$, both D_1 & D_2 are forward biased and they conduct to give voltage drop across R and we get output $Y=0$

(ii) When $A=0$ & $B=1$, diode D_1 will conduct & D_2 will not conduct. Due to D_1, current flows through R & there is voltage drop across R & output $Y=0$

(iii) When $A=1$ & $B=0$, diode D_2 will conduct & D_1 will not conduct. Due to D_2, current flows & there is voltage drop across R. We get output $Y=0$

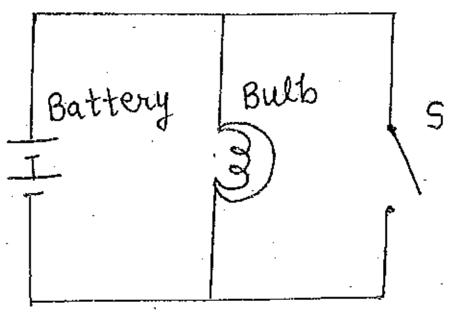
(iv) When $A=1$ & $B=1$, both D_1 & D_2 will not conduct and we get output high & $Y=1$

The 'NOT' gate (or NOT circuit)



$\overline{A} = Y$

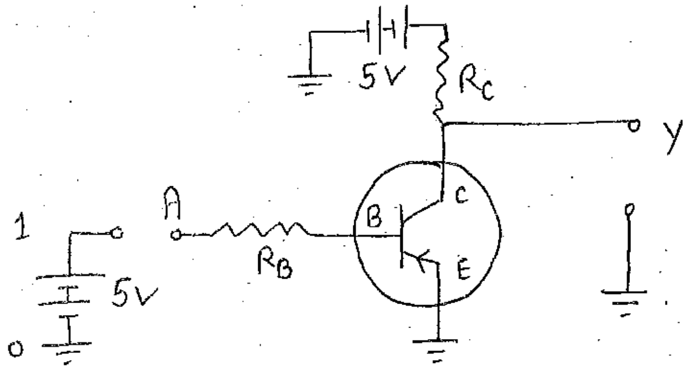
Analog circuit



Truth Table

A	Y
0	1
1	0

Realisation of NOT gate

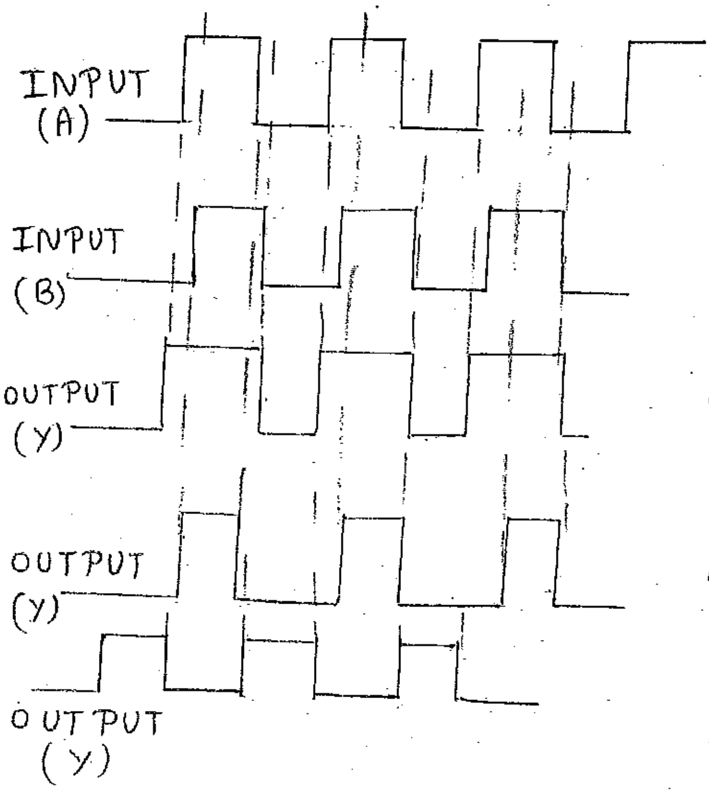


(i) When input $A=0$, emitter-base junction is not forward biased and the collector-base junction is reverse biased. Both I_B & I_C are zero and transistor is in cut off mode. There is no voltage drop across R_c and output $Y=1$

(ii) When input $A=1$, both emitter and collector are forward biased. A large collector current flows. The transistor is in saturation mode. The voltage drop across R_c is almost 5V.
Hence output $Y=0$

Sketch the output waveform obtained from (i) an OR gate (ii) an AND gate (iii) a NOT gate for the given square wave input A.

* Square wave Input (Digital input)



OR gate $Y = A + B$

AND gate $Y = A \cdot B$.

NOT gate $Y = \bar{A}$

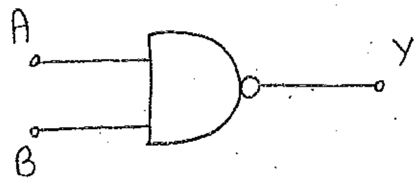
Key points

OR gate → output is high, when either A or B is high.

AND gate → output is high, when both A & B are high.

NOT gate → output is high, when input is low and output is low, when input is high.

The 'NAND' gate

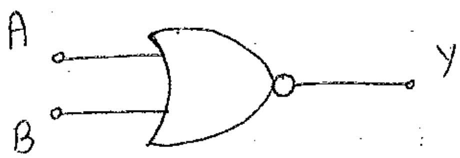


$$Y = \overline{A \cdot B}$$

Truth Table

A	B	$Y' = AB$	$Y = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

The 'NOR' gate



$$Y = \overline{A + B}$$

Truth Table

A	B	$Y' = A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

* NAND gate → output will be high if anyone of inputs is low.

* NOR gate → output will be low if anyone of inputs is high.