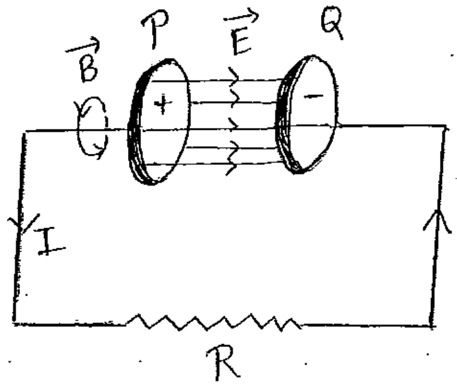
Electromagnetic Waves

6 Maxwell's Displacement current and origin of E.M. waves

According to Ampere's circuital law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$



When this is applied to an electric circuit containing a capacitor as one of circuit elements, the law appeared to be inconsistent. Hence Maxwell introduced the concept of displacement current.

Let us consider that a charged parallel plate capacitor is made to discharge through a resistor R. The current I leaves the left face of plate P of the capacitor, flows through the conducting wire and then enters the right face of plate Q of capacitor. This is called conduction current. As the current can not flow through the space between the two plates, no current exists between the two plates. However, electric field E exists in space between the plates.

To explain the discontinuity of current between the plates, Maxwell assumed that there is displacement current between the plates.

Modified Ampere circuital law is

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (I_C + I_D)$$

Here $I_C = I_D$

$$I_D = \epsilon_0 \frac{d\phi_E}{dt}$$

$$\text{or } \oint \vec{B}\cdot\vec{dl} = \mu_0 \left(I_C + \epsilon_0 \frac{d\phi_E}{dt}\right)$$

The conduction current produces due to charges in motion, whereas the displacement current produces magnetic field due to the time rate of change of electric field. At any instant, the ratio of the amplitudes of electric and magnetic field is always constant and it is equal to speed of E.M. waves. $\frac{E}{B} = C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Thus, the origin of E.M. waves is connected to the concept of displacement current.

Average Energy density of E.M. waves

The average electric energy density of e.m. waves in vacuum is given by

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\epsilon_0 E_0^2$$

& Average magnetic energy of e.m. waves is given by $u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}(B_0/\sqrt{2})^2 = \frac{1}{2}\left(\frac{B_0^2}{2\mu_0}\right)$

Now $E_0 = CB_0 = \frac{B_0}{\sqrt{\mu_0 \epsilon_0}}$

$$\Rightarrow u_E = u_B$$

Now average energy density of e.m. waves

$$u = u_E + u_B = 2u_E = 2u_B$$

$$\text{or } u = \frac{1}{2}\epsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

Thus the energy in electromagnetic waves is divided equally between the constituting electric & magnetic fields.

Maxwell's Equations

① Gauss's law in electrostatics

It states that the total electric flux through any closed surface is always equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

② Gauss's law in magnetism

It states that the net magnetic flux crossing any closed surface is always zero.

$$\oint \vec{B} \cdot \vec{dS} = 0$$

③ Faraday's law of electromagnetic Induction

It states that the induced e.m.f. produced in a circuit is numerically equal to time rate of change of magnetic flux through it.

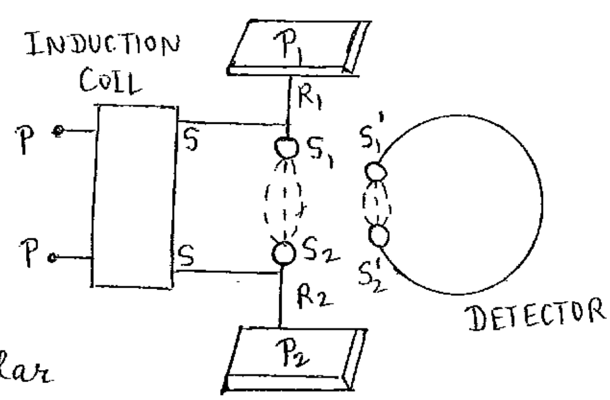
$$e = -\frac{d\phi_B}{dt} = \oint \vec{E} \cdot \vec{dl}$$

④ Maxwell's - Ampere's circuital law

It states that the line integral of magnetic field along a closed path is equal to μ_0 times the total current $(I_C + I_D)$ threading the surface bounded by that closed path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I_C + \epsilon_0 \frac{d\phi_E}{dt}\right)$$

Hertz's experiment



$P_1, P_2 \rightarrow$ Metal Plates

$S_1, S_2 \rightarrow$ Metal Spheres

Distance between P_1 & P_2 was 60 cm & S_1 & S_2 was 2-3 cm. Detector was circular coil with spheres S_1' & S_2'.

Explanation → The high potential difference across the metal plates ionises the air between the plates and spheres S_1 & S_2 and allows a path for discharge of plates which produces spark. Since the two metal plates act as a capacitor of very small capacitance C and the connecting wires offer very low inductance L, the resonant freq. of the arrangement is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \quad [\approx 5 \times 10^7 \, Hz]$$

This results in the oscillations of charges. Due to this, oscillating electric and magnetic fields will be set up which constitute e.m. waves of same frequency (5×10^7 Hz) and these waves are radiated through spark gap.

$$\text{Wavelength} \quad \lambda = \frac{c}{\nu} = 6\,m$$

Properties of e.m. waves

① Electro-magnetic waves are transverse in nature.

② They are produced by accelerated charges.

③ In free space, speed of e.m. waves is given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \, m/s$$

& in material medium, their speed is

$$c' = v = \frac{1}{\sqrt{\mu \epsilon}}$$

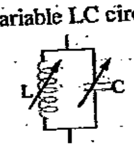
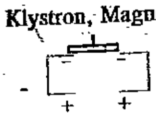
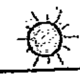
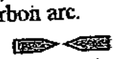
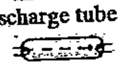
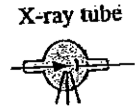
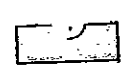
④ The refractive index of material medium

$$n \text{ or } \mu = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\mu_r \epsilon_r}$$

Electromagnetic Spectrum → Main-parts of electro-magnetic spectrum are

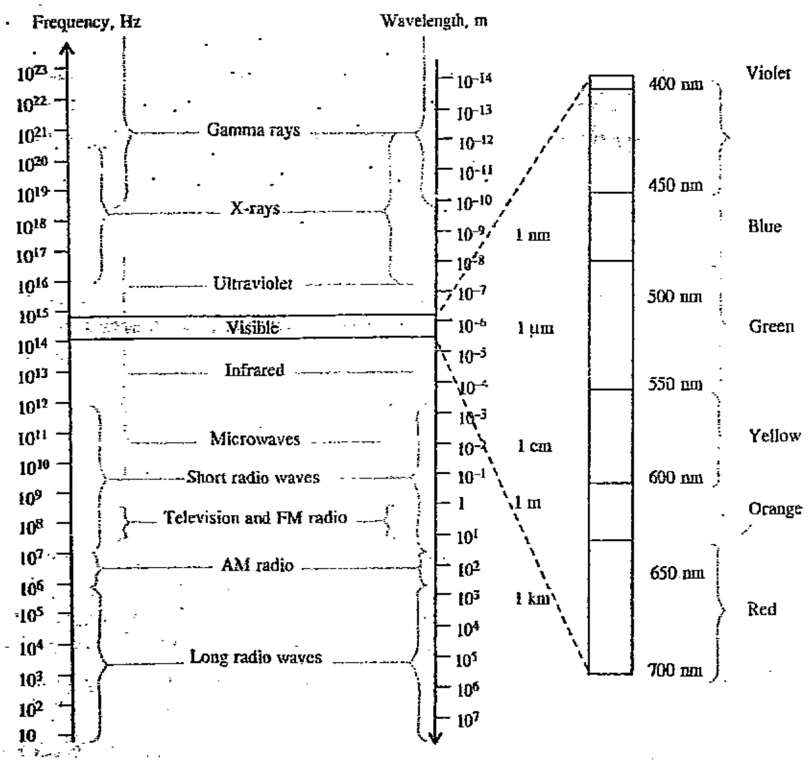
① γ-rays
② X-rays
③ Ultra-violet rays
④ Visible light
⑤ Infra-red rays
⑥ Micro-waves
⑦ Radio-waves

Source, Production and Detection of e.m. waves

Types of e.m. wave	Basic Source	Method of Production	Detection
1. *Radio waves*	Oscillations of electrons	Variable LC circuits 	(i) Telescopic Aerial (ii) Diode
2. *Microwaves*	Inversion/rotation of molecules	(i) Gunn diode (ii) Klystron, Magnetron 	(i) Point contact diodes (ii) Waves guide tubes
3. *Infrared waves*	Vibrations of atoms/molecules	Heaters 	(i) Bolometer (ii) Infrared photofilm (iii) Thermopile
4. *Visible light*	Jumping of electrons in outer orbits	(i) Filament Lamp  (ii) Flames  (iii) Sun 	(i) eye  (ii) Photocell (iii) Photofilm
5. *Ultraviolet rays*	Jumping of electrons in inner shells	(i) Carbon arc.  (ii) Discharge tube  (iii) Sun 	(i) Photocell (ii) Fluorescence (iii) Photofilm
6. *X-rays*	Bombarding targets with very fast electrons	X-ray tube 	(i) Geiger tube (ii) Ionisation chamber (iii) X-ray film
7. *γ-rays*	Radioactive decay of nucleus	(i) Radioactive element  (ii) Cyclotron 	Geiger tube

$C = \nu\lambda$ (58)

S.No.	Name of component	Wavelength Range λ	Frequency Range ν	Source	Discoverer
1.	Radio waves	0·3 m to 600 m	500 kHz to 1000 MHz	Accelerated charged particles in a conducting wire or changing electric current in LC circuit.	Guglielmo Marconi a cavity
2.	Microwaves	0·1 m to 1 mm	10^9 Hz to 10^{12} Hz	Oscillating electrons in a cavity commonly used oscillators to produce micro waves are klystron, Magnetron Gunn diodes.	Guglielmo Marconi
3.	Infra red waves	1 mm to 700 nm	10^{11} Hz to 10^{14} Hz	Hot bodies (Vibrations of atoms/molecules)	William Herschel
4.	Visible light	400 nm to 780 nm	4×10^{14} Hz to 7×10^{14} Hz	Jumping of electron from higher orbit to lower orbit of an atom.	–
5.	Ultra violet waves	380 nm to 0·6 nm	10^{14} Hz to 10^{17} Hz	The sun, welding arc, high voltage gas discharge tube.	Ritter
6.	X-rays	10 nm to 10^{-4} nm	10^{18} Hz to 10^{20} Hz	Fast moving electrons striking a target of high atomic number	Prof. Rontgen
7.	γ-rays	10^{-10} m to 10^{-14} m	10^{18} Hz to 10^{22} Hz	Radioactive decay of nuclei and nuclear reactions	Henri Becqurel



Applications of E.M. waves

① Radio-waves → They are mainly used in radio & television communication system.

② Micro-waves → They are used in RADAR systems, atomic & molecular research, celluler phones, microwave ovens etc.

③ Infra-red waves → They are used in earth satellites, treatment of muscular strains, revealing the secret writing on ancient walls, remote controls of electronic devices.

④ Visible light → It is used in photography, optical microscopy, astronomy, we can't see anything without visible light.

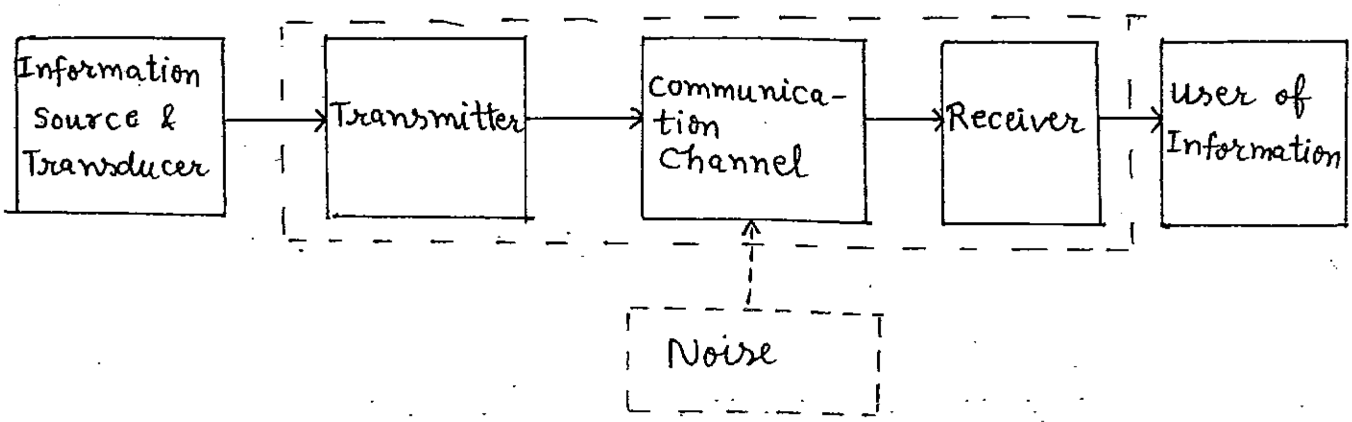
⑤ Ultra-Violet waves → They are used in preserving food stuff, stirlizing the surgical instruments, detecting finger prints, structure of molecules, in LASIK eye surgery.

⑥ X-rays → They are used in locating fracture in bones, curing skin diseases, cancer etc. in detecting gold, silver etc. by detective agencies.

⑦ Gamma-rays → They are used in treatment of Cancer, detecting structural flaws, in food preservation and to get valuable information about structure of atomic nuclei.

Communication Systems

Elements of communiation system.



Transmitter → The electrical signals corresponding to the information are processed, modulated, amplified and fed to the link (communication channel) or radiated through antenna.

Communication channel → The transmitter sends the signal to the receiver through the communication channel. eg. transmission lines, wires, cables, optical fibres or air.

Receiver → A receiver amplifies, filters and demodulates the received signal.

Modes of communication → ① Point to Point communication

There is only one transmitter and one receiver. eg. Telephone communication

② Broadcast (Point to Many point) communication

There is only one transmitter and many receivers. eg Radio broadcast & Television telecast

Signal → Information converted to electromagnetic form is known as signal

① Analog signal ② Digital signal

Few Important terms →

① Encoding → The process of converting an information into analog or digital signal.

② Transducer → A device which converts one form of energy into another form of energy.

③ Attenuation → The loss of strength or power of an electrical signal while travelling through a medium.

④ Noise → A disturbance or unwanted element interfering with the desired information.

⑤ Modulation → The process of placement of low frequency signal over the high freq. wave (carrier wave).

⑥ Demodulation → The process of extracting the low frequency signal from the high frequency carrier wave.

⑦ Amplification → The process of increasing the amplitude (strength) of the signal.

⑧ Range → The maximum distance between the information source and the destination.

⑨ Band-width → The width of the frequency spectrum of a signal is called BW.

$$BW = \nu_{max} - \nu_{min}$$

⑩ Repeaters → To increase the range of transmission of signals, no. of in-between sets of receivers & transmitters are repeaters.

Bandwidth of signals → $BW = \nu_{max.} - \nu_{min.}$

① Speech or voice signals → It is about 3000 Hz eg. in telephony freq. range is 300 Hz – 3100 Hz. BW = 2800 Hz

② Music signals → It is about 20 KHz. Its range is from 20 Hz – 20000 Hz.

③ Radio signals → AM band is from 550 Hz – 1600 KHz & FM band is from 88 MHz – 108 MHz.

④ TV signals → combined BW of picture & voice signals is about 6 MHz. only picture signals have BW of 4 MHz.

⑤ Cellular Mobile phone signals → They have frequency band of 840 MHz to 935 MHz.

⑥ Satellite phone signals → Freq. bands are 3.7 to 4.2 GHz (down link) and 5.9 to 6.4 GHz (uplink).

⑦ Digital signals → Theoretically infinite BW is required for digital signals.

Bandwidth of Transmission medium →

① Co-axial cables → They have BW of about 750 MHz

② optical fibres → Frequency range is 1 THz to 1000 THz. So BW is above 10^{11} Hz

③ Free space → Frequency range is 580 KHz to 6.5 GHz. So BW is nearly 6.5 GHz

Propagation of EM-waves → There are three ways:

① Ground wave ② Sky wave ③ Space wave

Ground wave propagation → In this, radio waves travel along the surface of the Earth. It is limited to a frequency below 1.5 MHz. The minimum length of transmitting antenna is about $\lambda/4$ where λ is the wavelength of signal.

Sky wave propagation → The radio waves which are reflected back to the Earth by ionosphere are known as sky waves. The frequency range is 3 MHz – 40 MHz. The highest frequency that is returned to the earth by the considered layer of the ionosphere after having been sent straight to it, $f_c = 9(N_{max})^{1/2}$, N_{max} is max electron density of ionosphere.

Space wave propagation → High frequency waves (above 40 MHz) called space waves can be transmitted from transmitting to receiving antenna through space is known as space wave propagation.

Range of LOS communication (space wave)

$$AO^2 = AC^2 + OC^2$$

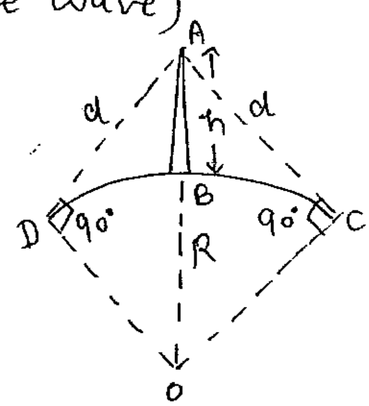
$$\Rightarrow (R+h)^2 = d^2 + R^2$$

$$\text{or } d^2 = h^2 + 2Rh$$

As $R \gg h$ so

$$d^2 \approx 2Rh$$

$$\text{or } d = \sqrt{2Rh} = \text{Radio horizon}$$



Maximum Line of sight distance between the transmitting & receiving antennas

$$d_t = \sqrt{2Rh_t} + \sqrt{2Rh_r}$$

$$d_t = d_1 + d_2$$

Satellite or Extra-terrestrial communication

It is a method in which a beam of microwaves is projected towards a geo-stationary satellite which throws back the same to different parts of Earth. Microwave signals have frequency of about 6 GHz.

Necessity or Need of modulation

(i) **Height of transmitting antenna** → For a signal of 15 KHz, λ is about 20 Km, min. height of antenna required is $\lambda/4 = 5$ Km which is not possible. After modulation, freq. is of the order of $1 MHz = 10^6 Hz$ so height of antenna required is 75 m which is practically possible.

(ii) **Effective power radiated by antenna**

$$P_{\text{dish antenna}} = 6\left(\frac{D}{\lambda}\right)^2 \text{ or } P \propto \frac{1}{\lambda^2}$$

Shorter the wavelength, more power is radiated.

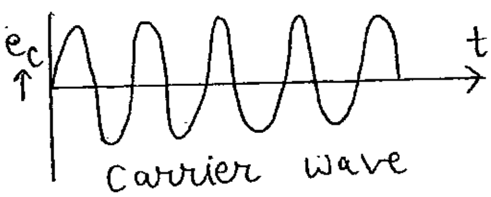
(iii) **Mixing up of signals from different transmitters**

Similar voice signals from different transmitters get mixed up if they are not superimposed with diff. carrier signals.

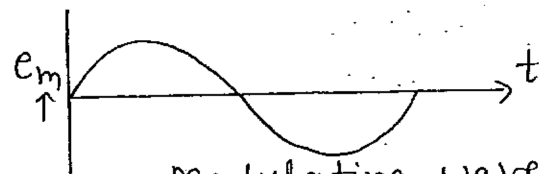
Modulation → The process of mounting a low frequency signal over high frequency signal is known as modulation.

Types of modulation → ① Amplitude modulation
② Frequency modulation

Amplitude modulation → If the amplitude of the carrier wave varies in accordance with the amplitude of very low freq. wave (signal) then it is amplitude modulation.

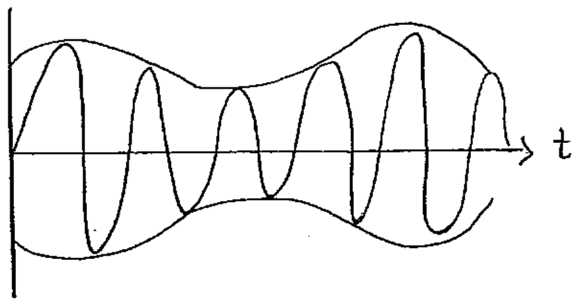


Carrier wave

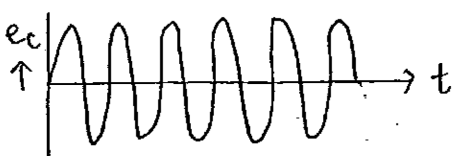


Modulating wave

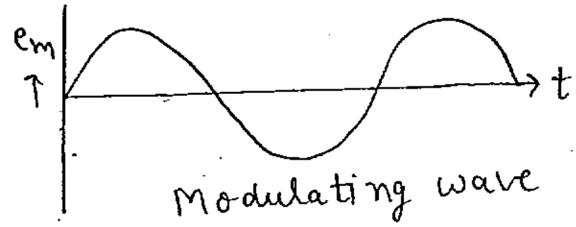
Amplitude Modulated Wave



Frequency Modulation → If the frequency of the carrier wave varies in acc with the freq. of very low freq. wave then it is frequency modulation.

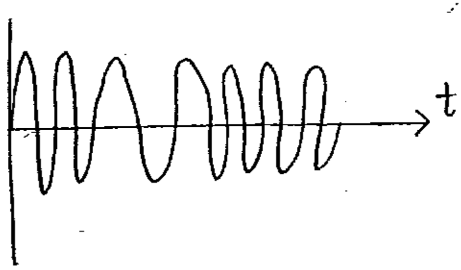


Carrier wave



Modulating wave

Frequency Modulated Wave



(iii) Phase Modulation → If the phase of carrier wave changes in accordance with the phase of the audio frequency wave then it is phase modulation.

Amplitude modulation (Mathematical treatment)

Let carrier wave is represented by

$$e_c = E_c \sin \omega_c t$$

& modulating signal is represented by

$$e_m = E_m \sin \omega_m t$$

Now amplitude modulated wave is given by

$$e = (E_c + e_m)\sin \omega_c t = (E_c + E_m \sin \omega_m t)\sin \omega_c t$$

$$\text{or } e = E_c\left(1 + \frac{E_m}{E_c}\sin \omega_m t\right)\sin \omega_c t$$

Now $\frac{E_m}{E_c} = m_a$ (modulation index)

$$\therefore \quad e = E_c(1 + m_a \sin \omega_m t)\sin \omega_c t$$

$$= E_c \sin \omega_c t + m_a E_c \sin \omega_m t \sin \omega_c t$$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$\text{or } e = E_c \sin \omega_c t + m_a E_c \frac{1}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

$$\Rightarrow \quad e = \underset{\text{(carrier wave)}}{E_c \sin \omega_c t} + \underset{\text{(LSB)}}{\frac{m_a E_c}{2}\cos(\omega_c - \omega_m)t} - \underset{\text{(USB)}}{\frac{m_a E_c}{2}\cos(\omega_c + \omega_m)t}$$

BW of amplitude modulated signal is $2\nu_m$

Significance of amplitude modulation index

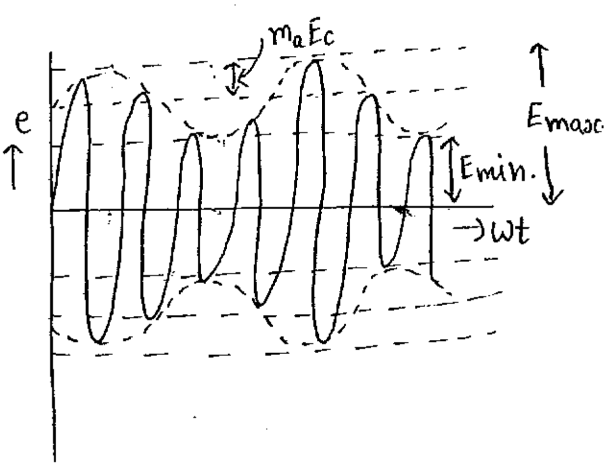
The variation in amplitude of carrier wave is given by

$$m_a E_c = E_{max} - E_c$$

$$\therefore \quad m_a = \frac{E_{max} - E_c}{E_c}$$

$$\& \quad m_a E_c = E_c - E_{min}$$

$$\therefore \quad m_a = \frac{E_c - E_{min}}{E_c}$$



$$\Rightarrow E_c - E_{min.} = E_{max} - E_c$$

$$\Rightarrow E_c = \frac{E_{max} + E_{min.}}{2}$$

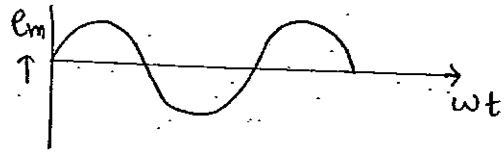
$$\Rightarrow m_a = \frac{E_{max} - E_{min.}}{E_{max} + E_{min.}}$$

If $m_a = 0$, no modulation

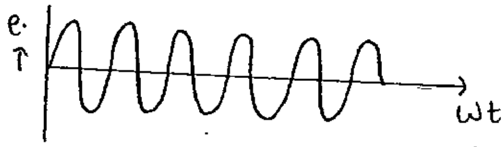
If $m_a = 1/2$, E_{max} is three times $E_{min.}$

If $m_a = 1$, $E_{min.}$ becomes zero

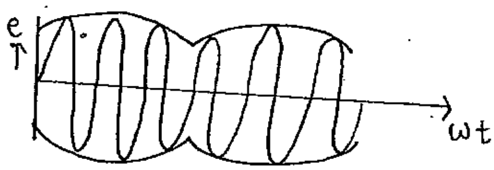
If $m_a > 1$, It is over-modulation



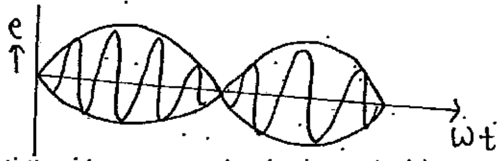
message signal



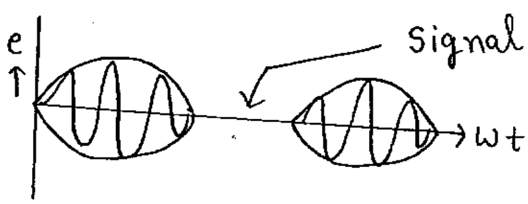
$m_a = 0$ No modulation



$m_a = 1/2$ 50% modulation



$m_a = 1$ 100% modulation



$m_a > 1$ over modulation

$$\% \, m_a = \left(\frac{E_{max} - E_{min.}}{E_{max} + E_{min.}}\right) \times 100$$

Production of Amplitude Modulated wave

Modulated signal m(t) is mounted on carrier signal c(t) to produce x(t). Square law device converts this signal into an output given by

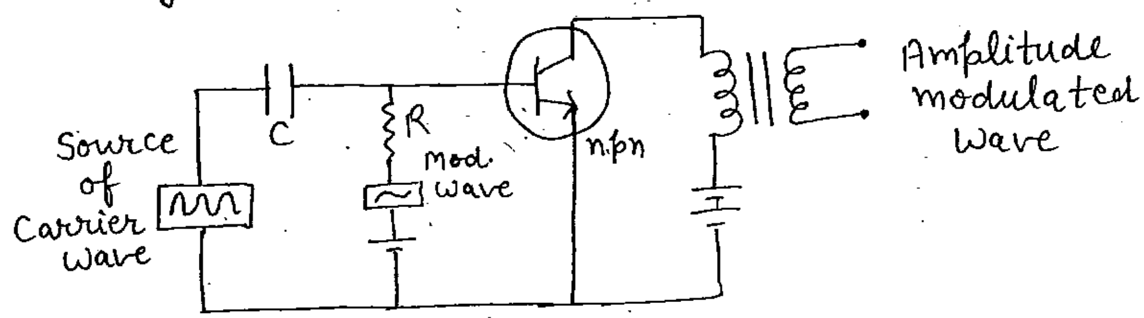
$$Y(t) = A\, x(t) + B\, x^2(t)$$

Now $x(t) = E_m \sin\omega_m t + E_c \sin\omega_c t$

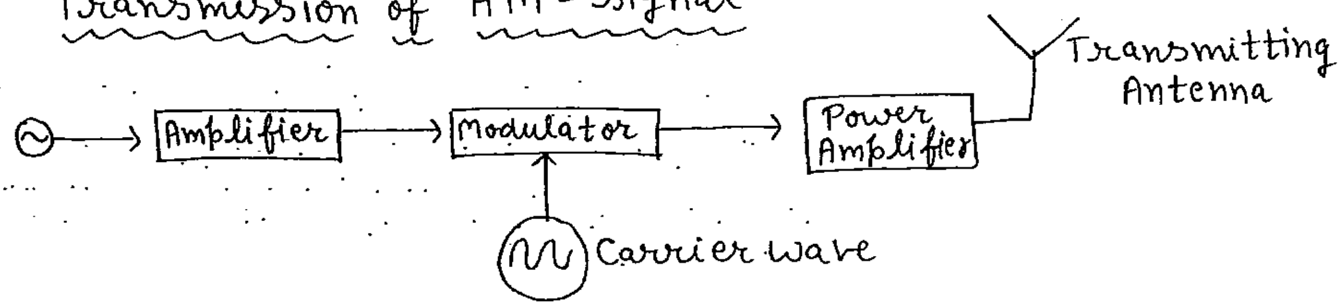
$$Y(t) = A\left[E_m \sin\omega_m t + E_c \sin\omega_c t\right] + B\left[E_m \sin\omega_m t + E_c \sin\omega_c t\right]^2$$

or $$Y(t) = A E_m \sin\omega_m t + A E_c \sin\omega_c t + \frac{B}{2}\left(E_m^2 + E_c^2\right) - \frac{B}{2} E_m^2 \cos 2\omega_m t - \frac{B}{2} E_c^2 \cos^2\omega_c t + B E_m E_c \cos(\omega_c - \omega_m)t - B E_m E_c \cos(\omega_c + \omega_m)t$$

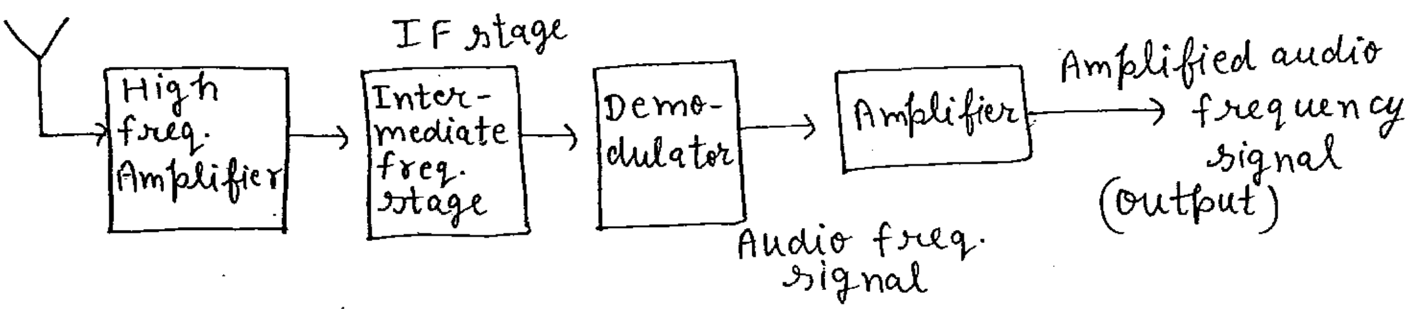
After passing through band pass filter, we get only ω_c, $(\omega_c + \omega_m)$ & $(\omega_c - \omega_m)$ frequencies.



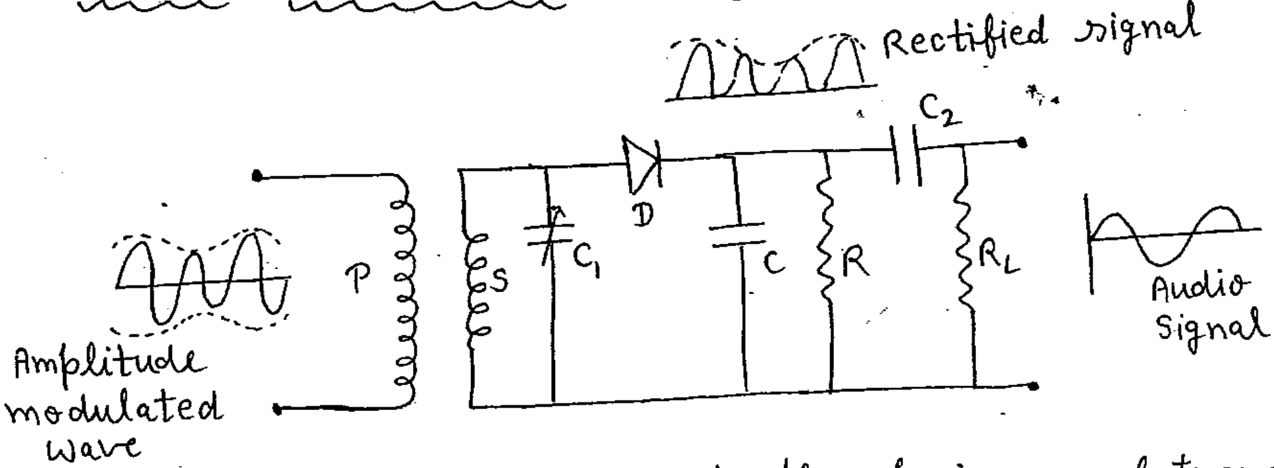
Transmission of AM-signal



Detection of an Amplitude modulated wave

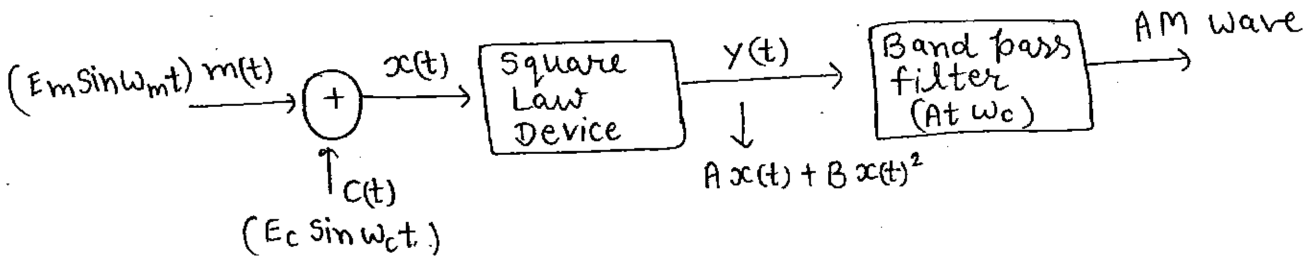


Simple Demodulator circuit



Modulated wave is fed to the primary of transformer. The secondary coil (S) of the transformer and the variable capacitor C_1 constitute LC circuit. This is brought in resonance with input modulated signal. Diode D acts as rectifier. The combination of R & C acts as filter and using these filters we can obtain the desired audio signal.

Imp. **Block diagram of a simple modulator**



Advantages of Amplitude modulation → (i) It is an easy method for the transmission & receiving of speech signals.

*(ii) Its installation cost is very less (For point to many)

(iii) It is fairly efficient system of modulation.

Disadvantages of Amplitude modulation → (i) It is more likely to suffer from noise.

(ii) Energy loss is quite high and it is limited to point to point links.

Some additional topics

① **Internet** → It is a global network of computers linked by high speed data lines and wireless systems. It allows communication and sharing of information between any two or more computers connected through the vast network.

Applications of Internet

(a) **Email** → It is a message sent and received through a computer network. Emailing allows exchange of text/graphic material using email software. Its various advantages are: fast delivery at low cost, easy record maintenance, reduction of the wastage of paper stationery.

(b) **File transfer** → An FTP (File transfer protocol) permits the transfer of files or software from one computer to another connected to the internet.

(c) **WWW (world wide web)** → It is a set of protocols that allows us to access any document on the internet. WWW is based on clients and servers.

A web server is a WWW server that responds to the requests made by web browsers. A web browser is a WWW client that navigates through the world wide web and displays web pages. A location on net server is called web site.

HTTP → Hyper Text Transfer Protocol
HTML → Hyper Text Markup Language
URL → Uniform Resource Locator

(d) E-commerce → E-Commerce is the collection of tools and practices involving internet technologies that allow a company to create, maintain and optimise business relations with consumers & other businesses. It helps in online banking activities, online shopping activities.

(e) Chat → It is the real time conversation among people with common interests through the typed messages on the net.

(2) Mobile Telephony → In mobile telephony, numerous lower power transmitters (base stations) are set up, each covering a fraction of that service area called a cell.

Each base station is connected to a switching office called mobile telephone switching office (MTSO), which co-ordinates communication between all the base stations and the telephone centre office. As a mobile receiver moves from one cell to another, the mobile user is handed over to the new cell's base station. This is called handover. Mobile telephones operate in the UHF range of frequencies around 800-950 MHz

(3) Global Positioning system (GPS) → The global positioning system is a satellite based system that can be used to locate positions anywhere on the earth. This is used for camping, shiping, cell phone location, aircraft navigation, wheather forecasting etc.

Components of GPS

(i) Space segment → It consists of 29 satellites that are continuously orbiting the earth at altitudes of about 19000 Km.

(ii) control segment → It consists of five unmanned monitor stations and one master control station.

(iii) User segment → It consists of the users & their GPS receivers. The number of simultaneous users is limitless.

How GPS works

A part of information sent by a satellite vehicle (SV) is a time stamp. When a GPS unit receives the transmission, it compares the time stamp from the satellite to the time it reached the receiver. The difference between the two, multiplied by the speed of the transmission signal provides the distance that the signal travelled {The receiver finds position by trilateration process that uses distances from atleast these known locations. The intersection of the three spheres having radii equal to these three distances gives the possible position. By using additional SVs, the positional accuracy can be improved.

DUAL NATURE OF MATTER & RADIATION

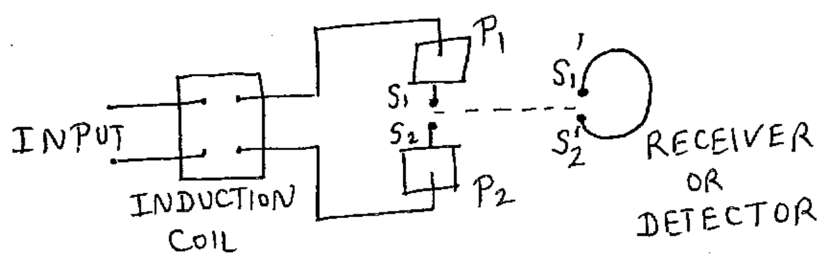
DUAL NATURE → WAVE NATURE
DUAL NATURE → PARTCLE NATURE

Work function → The minimum energy required by the free electrons to just leave the metal surface is known as the work function of the metal.

Definition → one electron-volt → It is the energy acquired by an electron when it is accelerated through a potential difference of 1 V.

Photoelectric effect → The phenomenon of the emission of electrons from preferably metal surfaces (may be non-metals) exposed to the light energy of suitable frequency is known as photo-electric effect.

Hertz observations → He performed an experime to investigate electromagnetic radiations. Electromagnetic radiation falling on the detector induced a potential difference across the gap. He observed spark across the gap.



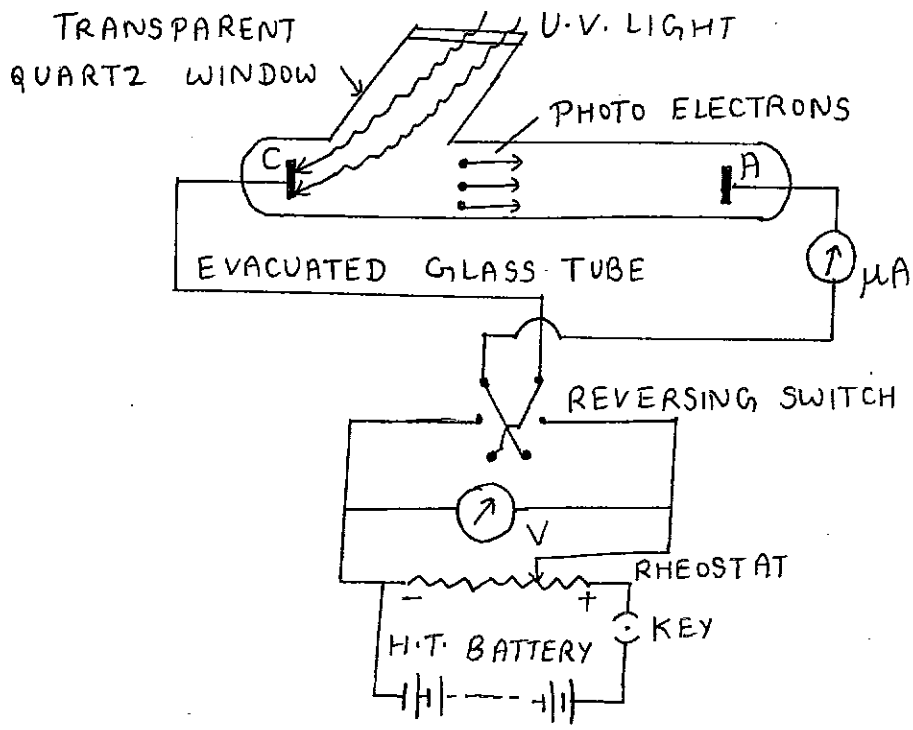
Hertz observed that sparks across the gap $S_1'S_2'$ jumped more readily when the detector was exposed to ultra-violet light. He concluded that light facilitated the emission of some electrons from metallic spheres.

Hallwach's and Lenard's observations →

Hallwach observed that ultraviolet light thrown on a negatively charged zinc plate, connected to a gold leaf electroscope, decreases the divergence of gold leaves. This indicates the decrease in the negative charge on zinc plate. He concluded that a negatively charged zinc plate when exposed to ultra-violet light emits negatively charged particles (photo-electrons).

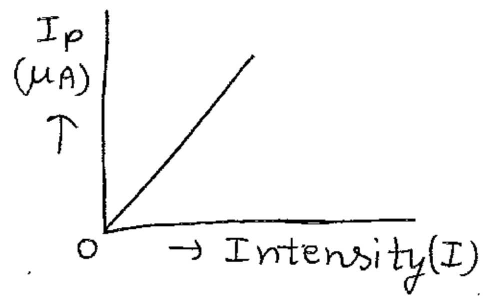
Lenard studied the effect of light on a metallic surface. He observed when ultra-violet light fell on cathod C, electric current appeared in the circuit known as photo-electric current. No effect was observed when ultra-violet light was absent. He also concluded that there should be suitable minimum frequency of ultra-violet light (i.e. threshold frequency) for different materials.

Experimental study of photo-electric effect →



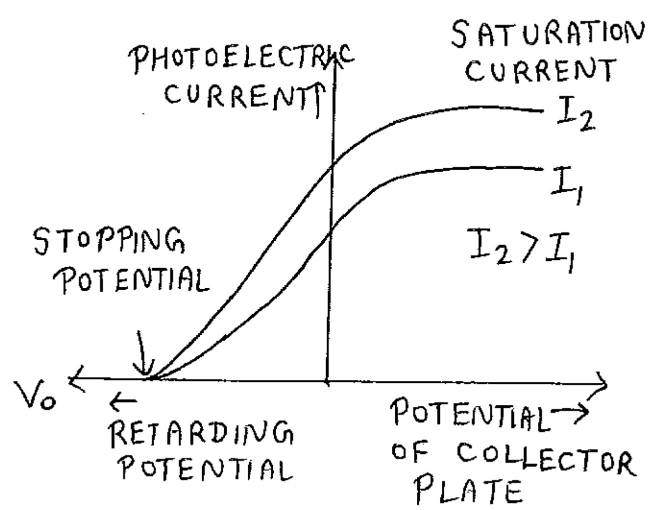
(i) Effect of intensity of incident light on the photo-electric current

Photo-electric current is directly proportional to the intensity of incident radiations, provided the frequency is greater than the threshold frequency



(ii) Effect of potential on photo-electric current

The minimum negative potential (V_0) applied to anode for which the photo-electric current becomes zero is called cut-off or stopping potential.



For fixed frequency and fixed intensity of incident light, the photoelectric current increases with increase in applied positive potential of plate A. When all the photo-electrons emitted by cathode C reach the plate A, the photo-electric current has the maximum value. This maximum current is known as saturation current.

$$\frac{1}{2} m v_{max}^2 = eV_0$$

$$\Rightarrow K.E._{max.} \propto V_0$$

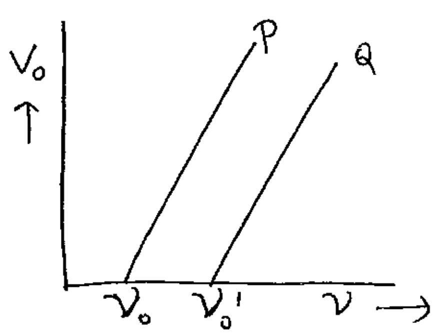
<u>Conclusion</u> → (i) Saturation current depends on the intensity of incident radiation.

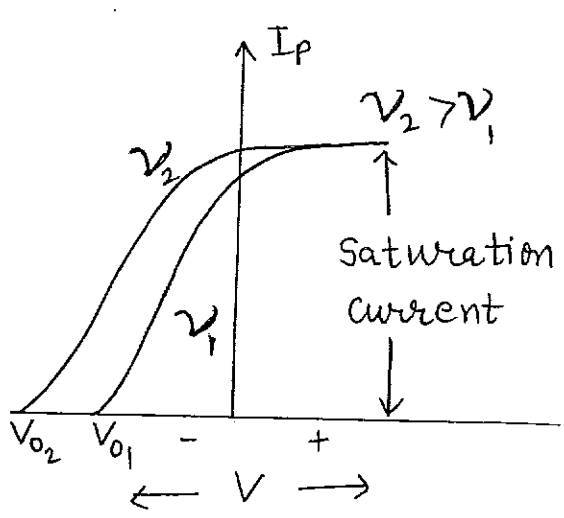
(ii) Intensity of incident radiation does not affect the stopping potential.

(iii) Maximum K.E. of emitted photo-electrons does not depend on intensity.

Effect of frequency of incident light

Stopping potential is directly proportional to the freq. of incident light.





Conclusions → (i) Value of saturation current does not depend on frequency of incident radiation.

(ii) No photo-electric emission is possible below cut-off frequency even if the intensity is very high.

(iii) Kinetic energy varies linearly with frequency.

Laws of photo-electric emission

(i) For a given substance, there is a minimum frequency of incident light called threshold frequency (ν_0) below which no photo-electric emission takes place, whatsoever the intensity of incident light may be.

(ii) Rate of photo-electrons emission (i.e. photo-electric current) is directly proportional to intensity of incident light provided freq. is greater than threshold frequency.

(iii) Max K.E. increases with increase in frequency of incident light ($\nu > \nu_0$)

(IV) The process of photo-electric emission is instantaneous. (As soon as light falls, photo-electrons are emitted).

Photo-electric effect and wave theory of light

Photo-electric effect can not be explained on wave theory of light because

(i) As per wave theory of light, the electric field component of light increases with the increase in the intensity of light. When light falls on a metal surface, the force acting on a free electron in the metal surface increases with the increase in the intensity of incident light. So kinetic energy of emitted electron should increas with intensity but it does not happen.

(ii) As per wave theory of light, electrons are emitted from the surface of metal if it is exposed to a very intense beam of light. In other words, there is no role of minimum or thershold frequency. This is also the contradiction to the observation.

(iii) As per wave theory of light, the energy is uniformly distributed over the wavefronts of incident light. When light falls on the surface of the metal, the energy of incident light is absorbed by a large number of electrons. So each electron will take enough time to have sufficient energy to come out but according to observation this process is an instantaneous process so there is a contradiction.

Einstein's photo-electric equation →

As per Einstein, an incident photon having energy $h\nu$ ejects electron from a metal having work function $\phi_0 = h\nu_0$ and imparts $K.E. = \frac{1}{2} m v^2$ to it.

$$h\nu = \phi_0 + \tfrac{1}{2} m v^2_{max.} \qquad \phi_0 = h\nu_0 = \text{work function}$$

$$\text{or} \quad h\nu = h\nu_0 + \tfrac{1}{2} m v^2_{max}$$

If $\nu = \nu_0$ then $\frac{1}{2} m v^2 = 0$

$$\Rightarrow \quad \tfrac{1}{2} m v^2 = h(\nu - \nu_0)$$

$$\Rightarrow \quad K.E. \propto \nu \quad \text{or} \quad K.E. \propto 1/\lambda$$

verification of the laws of photo-electric emission

(i) If $\nu < \nu_0$ then K.E. is −ve which is not possible.

(ii) one photon can emit only one electron so no of photo-electrons emitted per sec. is directly proportional to intensity.

(iii) $K.E. = h(\nu - \nu_0)$ implies it increases with freq.

(iv) Photo-electric emission is due to elastic collision so process is instantaneous.

Determination of Plank's constant and work function

According to Einstein eqn.

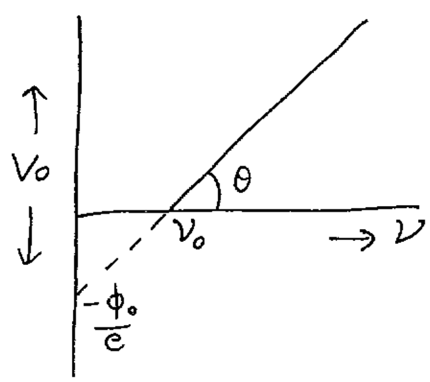
$$h\nu = h\nu_0 + \tfrac{1}{2} m v^2_{max}$$

$$= h\nu_0 + eV_0$$

$$\Rightarrow eV_0 = h(\nu - \nu_0)$$

$$\Rightarrow V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

$$h/e = \tan\theta = \Delta V_0 / \Delta \nu$$



Particle nature of light (PHOTON)

Photon is a packet of energy or quantum of energy ejected at the speed of light by an emitter.

$$E = h\nu$$

Here h is Plank's constant.

$$h = 6.63 \times 10^{-34} \text{ J S}$$

Properties of photons (i) A photon travels with a speed of light in vaccum.

(ii) It has zero rest mass and no charge

(iii) Kinetic energy of photon is $E = mc^2$ or $m = \frac{h\nu}{c^2}$

(iv) Momentum of photon, $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

(V) Energy of photon in terms of wavelength is $E = \frac{hc}{\lambda}$ (measured in electron volt)

Wave nature of matter (Matter waves)

The waves associated with moving material particles are known as de-Broglie waves or matter waves.

Matter waves like electromagnetic waves can travel in vacuum hence they are not the mechanical waves. Matter waves are not the electromagnetic waves because they are not produced by accelerated charged particles. Amplitude of the matter waves is used to give the probability of existence of the particle at a point so matter waves are called probability waves.

De-Broglie relation → Energy of photon is given by $E = h\nu$

and also $E = mc^2$

$\Rightarrow \quad h\nu = mc^2 = \frac{hc}{\lambda}$

Where λ is de-broglie wavelength.

$\therefore \quad mc = \frac{h}{\lambda}$ or $\lambda = \frac{h}{mc} = \frac{h}{p}$

For particle moving with velocity v,

$$\lambda = \frac{h}{mv}$$

De-Broglie wavelength λ of an electron moving through a potential difference V →

$$K.E. = \frac{1}{2} m v^2 = eV \; (P.E.)$$

or $\quad m^2 v^2 = 2meV$

$$mv = \sqrt{2meV}$$

$$\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

For electron, $m = 9.1 \times 10^{-31}$ kg & $e = 1.6 \times 10^{-19}$ C

$$\Rightarrow \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

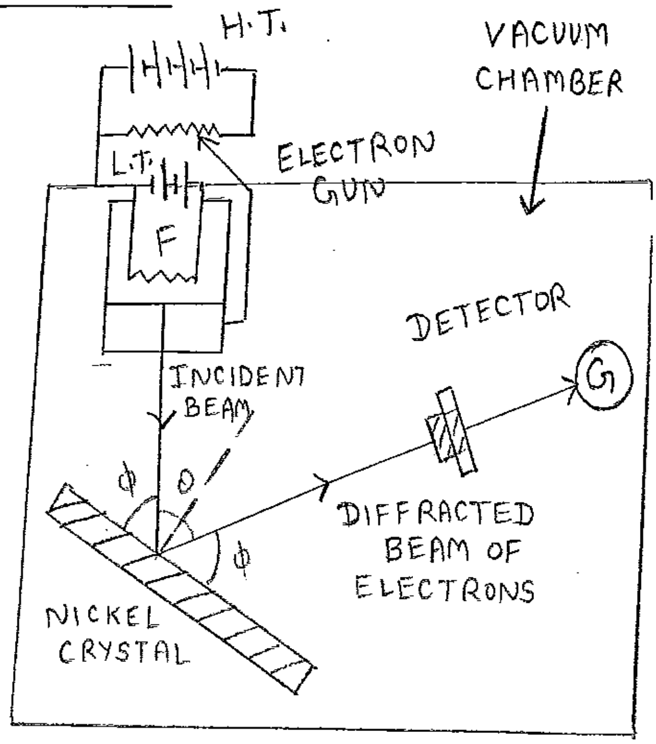
$$\Rightarrow \lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

Heisenberg's uncertainty Principle →

According to this principle, it is impossible to measure simultaneously the position and the momentum of a particle accuratly.

$\hbar = \Delta x . \Delta p \qquad$ Here $\hbar = \frac{h}{2\pi}$

Davisson and Germer Experiment



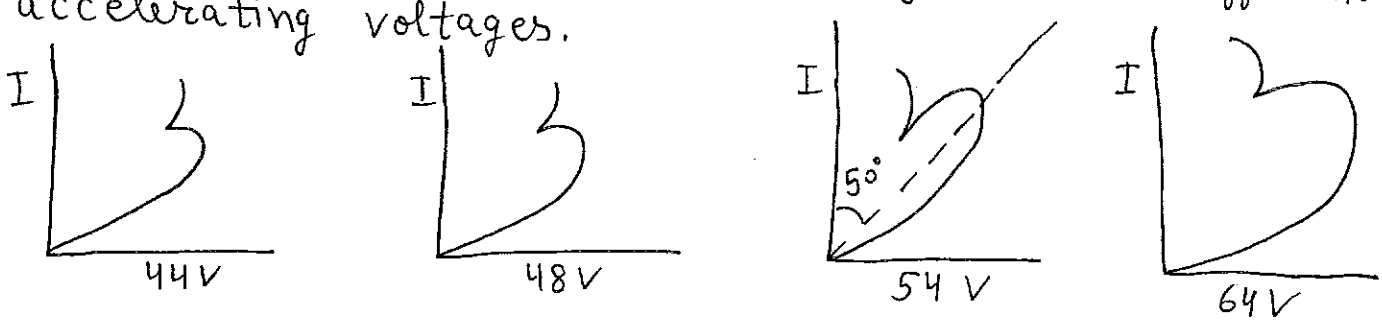
(i) Electron gun : It consists of a tungsten filament F with barium oxide coating.

(ii) Nickel crystal : Atoms of Ni crystal act as scatterer.

(iii) Detector : A movable detector Connected to a sensitive galvanometer to detect the current.

Working → A beam of comparatively slow electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle. The scattered beam of electrons is received by the detector (movable). The energy of incident beam of electrons can also be varied by changing the voltage.

Davisson and Germer plotted the graphs between the intensity of scattered beam of electrons at different angles of scattering and at different accelerating voltages.



When beam of electrons accelerated through a potential of 54 V was made to incident on nickel crystal, the intensity of scattered beam was maximum at scattering angle of 50° and a strong peak was obtained in this case.

$$\phi + \theta + \phi = 180^\circ \quad , \quad \theta = 50^\circ \Rightarrow \phi = 65^\circ$$

ϕ is called glancing angle

According to De-Broglie hypothesis, wavelength of e^- is given by $\lambda = \frac{12.27}{\sqrt{V}} A^\circ$

If $V = 54 V$, $\lambda = \frac{12.27}{\sqrt{54}} = 1.67 A^\circ$

Theoretically, Bragg's eqn. for maxima of the diffraction pattern is $2d \sin\theta = n\lambda$

$d = 0.91 A^\circ$ for Nickel

& $n = 1$

$\Rightarrow \lambda = 2 \times 0.91 \sin 65^\circ = 1.65 A^\circ$

Two results are in close agreement with each other.

So Davisson & Germer experiment provides direct verification of de-broglie hypothesis of wave nature of moving particles.

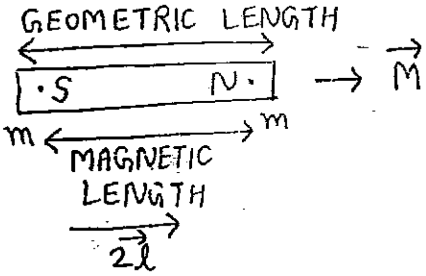
<u>Wave function or probability amplitude</u> → The amplitude of wave associated with moving particle is known as wave function. It is denoted by ψ. $|\psi|^2$ is probability density gives probability per unit volume of finding a particle at a point.

7 8

MAGNETISM and MATTER

Bar Magnet

(OR MAGNETIC DIPOLE)



MAGNETIC DIPOLE MOMENT $\vec{M} = m(2\vec{l})$

S.I. unit – Ampere-metre^2

Properties of bar magnet (i) Attractive property

(ii) Directive property

(iii) Inductive property (Induced magnetism)

(iv) Unlike poles attract & like poles repel each other.

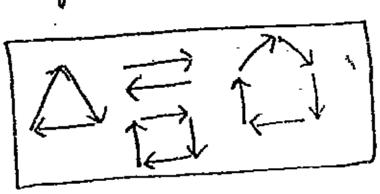
(v) Magnetic poles exist in pairs

(vi) Repulsion is the surest test for distinguishing between a magnet and a piece of iron

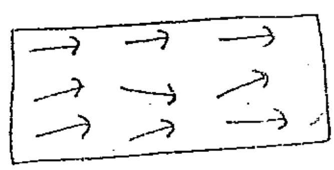
Atomic view of magnetism – Each atom / molecule of a magnetic substance behaves like a magnetic dipole.

In case of un-magnetised piece of iron (or magnetic material), these magnetic dipoles are arranged in such a manner that their net magnetic dipole moment is zero.

In a bar magnet, these molecular magnets are aligned in same direction so as to give net magnetic dipole moment $\vec{M}$.



(In unmagnetised piece of Iron)



(In a bar magnet)

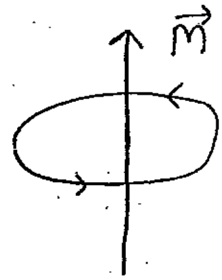
Coulomb's law in magnetism

$$F \propto \frac{m_1 m_2}{r^2}$$

or $F = k\frac{m_1 m_2}{r^2}$ $\quad k = \frac{\mu_0}{4\pi} = 10^{-7}\ Wb/Am$

m – magnetic pole strength
(S.I. unit Ampere-metre)

Magnetic moment of current loop as a magnetic dipole



$\vec{M} = I\vec{A}$

$M \propto I$

$\propto A$

$M = kIA$ (Here $k = 1$)

$\Rightarrow M = IA$

For coil (or loop) having n-turns $M = nIA$

Atom as a magnetic dipole (Magnetic dipole moment of a revolving electron)

Let us calculate the magnetic dipole moment (M) of an atom due to orbital motion of electron.
The angular momentum of electron (of mass m_e) moving with velocity v in a circular orbit of radius r is given by $L = m_e v r$ ----(i)
The orbital motion of electron is equivalent to a current $I = e\left(\frac{1}{T}\right)$, $T = \frac{2\pi r}{v}$

$\Rightarrow I = ev/2\pi r$ Now $A = \pi r^2$ & $M = IA$

$\Rightarrow M = \frac{evr}{2} = \left(\frac{e}{2m_e}\right)L$ using (i)

or $\vec{M} = -\left(\frac{e}{2m_e}\right)\vec{L}$ (–ve sign is due to –ve charge on electron)

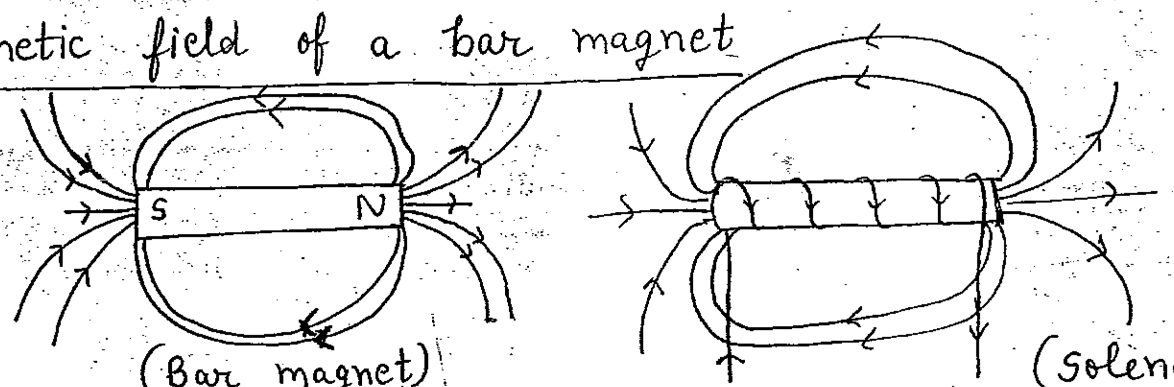
Acc to Bohr's theory

$L = nh/2\pi$

$\Rightarrow M = \left(\frac{e}{2m_e}\right)\frac{nh}{2\pi} = n\left(\frac{eh}{4\pi m_e}\right)$ $\left\{\begin{matrix}\text{For } n=1 \\ M = \frac{eh}{4\pi m_e}\end{matrix}\right\}$

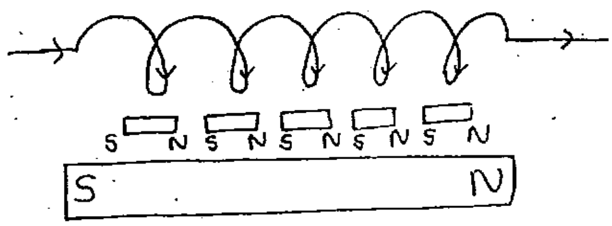
{ Bohr Magneton $= eh/4\pi m_e$ }

Magnetic field of a bar magnet



(Bar magnet) (Solenoid)

(Bar magnet as an equivalent solenoid)



(Each current loop behaves as magnetic dipole)

(Read only)
* **Gauss law in magnetism** – surface integral of magnetic field over a closed surface S is zero.

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

(i.e. An isolated magnetic pole does not exist.)

Properties of magnetic field lines

(i) Magnetic field lines are closed continuous loops.

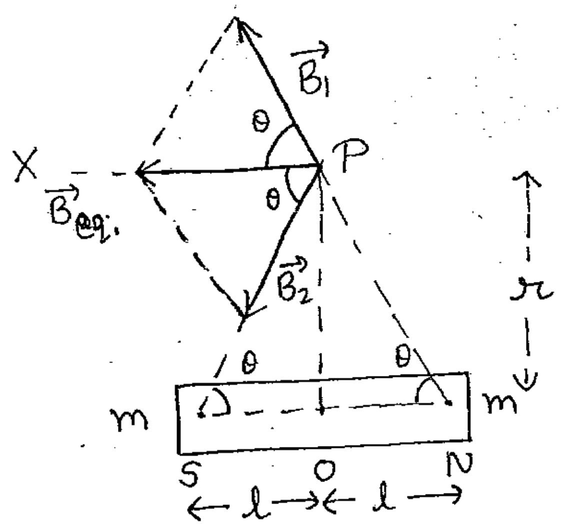
(ii) outside the magnet, field lines are from north to south.

(iii). Tangent to magnetic field lines at any point give the direction of field.

(iv) No two magnetic field lines can intersect each other.

(v) Widely spaced lines represent weak magnetic field and crowded lines represent strong magnetic field.

Magnetic field on equitorial line of a bar magnet



consider a bar magnet of length $2l$ and pole strength m.
consider a point P on equitorial line at a distance r from centre.
At point P

$$\vec{B}_{equitorial} = \vec{B_1} + \vec{B_2}$$

$$|\vec{B_1}| = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2+l^2}\right) \quad \& \quad |\vec{B_2}| = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2+l^2}\right)$$

$\vec{B_1}$ & $\vec{B_2}$ have two components.
$B_1 \sin\theta$ and $B_2 \sin\theta$ are equal & opposite so they will cancel each other

$$\text{so } |\vec{B}_{equitorial}| = |\vec{B_1}|\cos\theta + |\vec{B_2}|\cos\theta$$

$$|\vec{B}_{eq}| = \frac{\mu_0}{4\pi}\left(\frac{2m}{r^2+l^2}\right)\cos\theta$$

$$\text{Now } \cos\theta = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\text{so } |\vec{B}_{eq.}| = \frac{\mu_0}{4\pi}\left(\frac{2m}{r^2+l^2}\right)\frac{l}{(r^2+l^2)^{1/2}}$$

$$|\vec{B}_{eq.}| = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{3/2}} \quad \{\because M = m\times 2l\}$$

$$\vec{B}_{eq.} = -\frac{\mu_0}{4\pi}\cdot\frac{\vec{M}}{(r^2+l^2)^{3/2}}$$

For short bar magnet $|\vec{B}_{eq.}| = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right)$

* Negative sign shows that $\vec{B}_{eq.}$ & $\vec{M}$ are in opposite directions.

* Read only

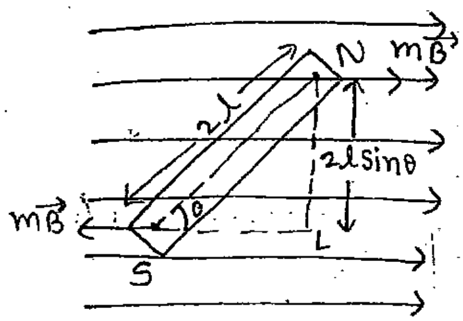
Magnetic field at any point due to a short magnetic dipole

$$B = \frac{\mu_0}{4\pi} \frac{M\sqrt{3\cos^2\theta + 1}}{r^3}$$

For axial line, $\theta = 0° \Rightarrow B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

For equitorial line, $\theta = 90° \Rightarrow B = \frac{\mu_0}{4\pi} \frac{M}{r^3}$

Torque on a bar magnet in a magnetic field



Let us consider a bar magnet of length $2l$ and pole strength m

Force on each pole of magnet $= mB$

These two forces constitute a torque and its magnitude is given by $\tau = mB \times LN$

$$\tau = mB \times 2l\sin\theta = MB\sin\theta$$

$\vec{\tau} = \vec{M} \times \vec{B}$ { $\vec{\tau}$ is in the direction of $\vec{M} \times \vec{B}$ i.e. ⊥ to plane}

Potential energy of bar magnet (Work done in rotating the magnet through an angle θ)

$\tau = MB\sin\theta$

dW = Small work $= \tau\, d\theta = MB\sin\theta\, d\theta$

Total work done $W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$

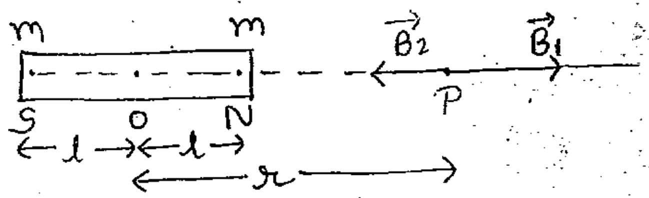
$U = W = MB(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 90°$ & $\theta_2 = \theta$ then $U = MB(-\cos\theta)$

$U = -\vec{M}\cdot\vec{B}$ At $\theta = 0°$ U (is min.) $= -MB$

At $\theta = 180°$ U (is max.) $= MB$

Magnetic field on axial line of a bar magnet



consider a bar magnet of length $2l$ and pole strength m. consider a point P at distance r from centre of magnet.

$$\vec{B}_{axial} = \vec{B_1} + \vec{B_2}$$

$$|\vec{B_1}| = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2} \qquad |\vec{B_2}| = \frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}$$

As $|\vec{B_1}| > |\vec{B_2}|$ and $\vec{B_1}$ & $\vec{B_2}$ are opposite

$$\Rightarrow B_{axial} = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}$$

$$= \frac{\mu_0 \cdot m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$B_{axial} = \frac{\mu_0 m}{4\pi}\cdot\frac{4rl}{(r^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2}$$

$$[\because M = m \times 2l]$$

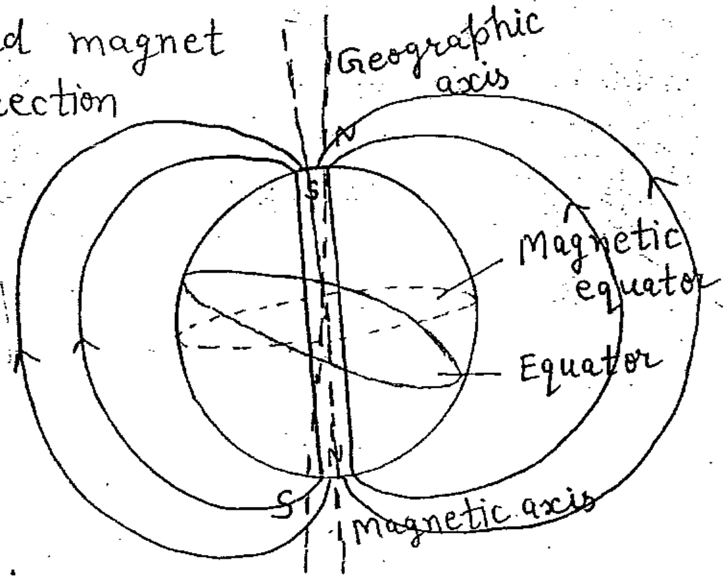
$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\frac{2\vec{M}r}{(r^2-l^2)^2}$$

For very small length of bar magnet
$l << r$

$$\Rightarrow B_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$$

* Magnetic field due to a bar magnet at a point on its axial line has the same direction as that of its magnetic dipole moment vector.

Earth's Magnetism



* A freely suspended magnet aligns along N-S direction
* Angle between magnetic & geographic axis is ≈ 20°
* A vertical plane passing through the geographic axis is Geographic meridian.
* A vertical plane passing through magnetic axis is magnetic meridian.

Cause of Earth's magnetism

* Read only ✓

① It is guessed that earth's magnetism is due to molten charged metallic fluid

② According to some other theory, since every substance is made up of charged particles hence a substance rotating about an axis will become magnetised.

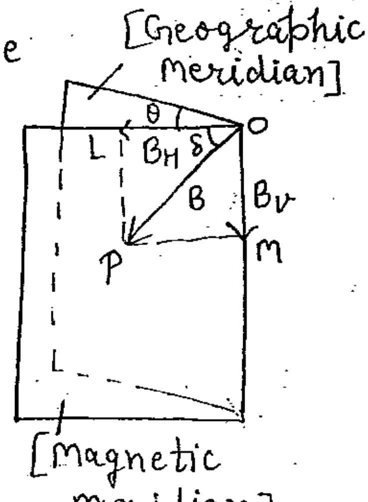
③ The earth's magnetism is also due to the rotation of earth about its own axis.

④ The gases in the atmosphere are in the ionised states. The high energy radiation coming from sun ionises the atoms in the upper part of atmosphere. Hence due to rotation of earth, strong current flows & earth is magnetised.

⑤ Earth's magnetism is also due to presence of magnetic materials (eg. iron, nickel) in the core of earth. Due to rotation, they get magnetised.

Magnetic Elements – The physical quantities which determine the intensity of earth's total magnetic field completely are called magnetic elements.

① **Magnetic Declination** – Declination at a place is the angle between the magnetic-meridian and geographic-meridian. It is denoted by θ. (As shown in figure)



② **Magnetic Inclination or dip** – Dip at a place is defined as the angle made by the direction of the total earth's magnetic field with the horizontal direction. It is denoted by δ. (As shown in figure)

③ **Horizontal component of Earth's magnetic field**

It is the component of earth's magnetic field in horizontal direction.

$$B_H = B\cos\delta \quad , \quad B_V = B\sin\delta$$

$$B = \sqrt{B_H^2 + B_V^2}$$

$$\tan\delta = \frac{\sin\delta}{\cos\delta} = \frac{B_V}{B_H}$$

* The value of angle of dip at any place on the magnetic equator of earth is zero
* The value of angle of dip at the magnetic poles of earth is 90°.

Few Definitions

1) **Magnetic Intensity** → It is also known as magnetic field strength. It is denoted by H & $H = \frac{B_0}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
S.I. unit of H is Ampere/metre.

2) **Intensity of magnetisation** → It is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to magnetising field.
It is denoted by I. $I = \frac{M}{V}$ in (A/m)
M → Magnetic dipole moment, V is volume

3) **Magnetic flux** → Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through the surface.
$$\phi = \int \vec{B} \cdot \vec{ds}$$

4) **Magnetic Induction** → It is defined as the number of magnetic lines of induction crossing per unit area normally through the magnetic substance. It is denoted by B.
$$B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I = \mu_0 (H + I)$$
S.I. unit is Tesla or weber/metre^2

5) **Magnetic Susceptibility** → It is defined as the ratio of the intensity of magnetisation to the magnetic intensity. It is denoted by χ_m
& $\chi_m = \frac{I}{H}$ (⇒ no units)

6) **Magnetic Permeability** → It is defined as the ratio of the magnetic induction to the magnetic intensity. It is denoted by μ.
$$\mu = \frac{B}{H} \quad \text{(S.I. unit is Tm/A)}$$
$$\mu_r = 1 + \chi_m$$

Classification of magnetic materials

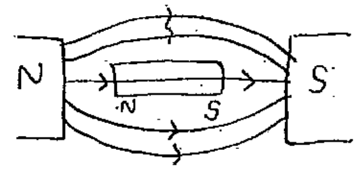
① **Diamagnetic** → Those substances which when placed in a magnetic field are feebly magnetised in a direction opposite to that of the magnetising field, are called diamagnetic substances. eg. Cu, Zn, Bi, Au, Ag, Pb, glass, marble, water, helium, argon, NaCl etc

Properties → ① A diamagnetic substance is feebly repelled by a magnet.

② If a diamagnetic liquid contained in a watch glass is placed on two closely spaced pole pieces of a magnet, it suffers a slight depression in the middle. The liquid shows rise in the middle when the pole pieces are moved apart.

③ The intensity of magnetisation (I) has a small negative value.

④ When diamagnetic material is placed in a magnetic field, the magnetic field lines becomes less dense inside the material.



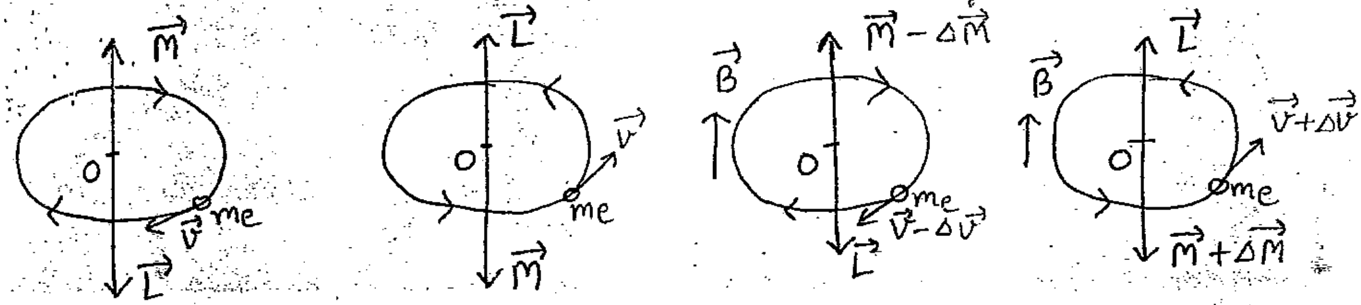
⑤ The magnetic susceptibility (χ_m) has a small negative value.

⑥ Relative permeability μ_r is slightly less than 1.

⑦ Diamagnetic substances do not obey Curie's law.

Electron theory of diamagnetism → We know that there is magnetic dipole moment $\vec{M}$ due to each revolving electron.

In a diamagnetic material, there are only paired electrons. The paired electrons possess equal & opposite dipole moments due to opposite orbital motion. There is no net magnetic moment present in the absence of magnetic field (external)



When external magnetic field $\vec{B}$ is applied then magnetic Lorentz force($\vec{F}$) acts on the electron.

$$\vec{F} = -e(\vec{v} \times \vec{B})$$

In case of e^- revolving in clockwise direction force $\vec{F}$ tends to decrease the centripetal force and velocity decreases to $\vec{v} - \Delta\vec{v}$. Similarly the e^- revolving in anticlockwise direction experience a force such that it tends to increase the centripetal force and velocity increases to $\vec{v}+\Delta\vec{v}$. Now pair of electrons possesses net magnetic dipole moment $2\Delta\vec{M}$ in a direction opposite to $\vec{B}$. This explains the behaviour of diamagnetic substance in presence of external magnetic field.

② Paramagnetic → Those substances which when placed in a magnetic field are feebly magnetised in the direction of the magnetising field are called ~~para~~magnetic substances. eg. Al, Na, Sb, Pt, Mn, Cr, copper chloride, liquid oxygen etc.

Properties → ① A paramagnetic substance is feebly attracted by the magnet.

② A freely suspended rod of paramagnetic material aligns itself parallel to the direction of magnetic field.

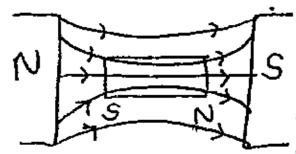
③ If a paramagnetic liquid contained in a watch glass is placed on two closely spaced pole pieces of a magnet, it shows a slight rise in the middle. when the poles are moved apart, the paramagnetic liquid gets depressed in the middle.

④ The intensity of magnetisation (I) has a small positive value.

⑤ The magnetic susceptibility (χ_m) has a small positive value.

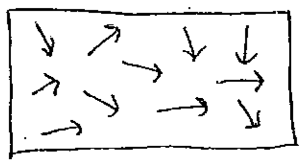
⑥ When a paramagnetic material is placed inside a magnetic field, the field lines becomes slightly more dense in the paramagnetic material.

⑦ The relative permeability is slightly greater than 1.

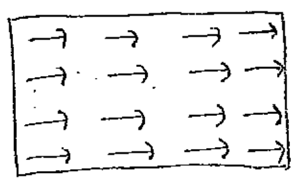


⑧ Paramagnetic substances obey curie's law.

Electron theory of Paramagnetism → In case of paramagnetic substances, the interaction between the atomic magnetic dipoles is very weak and they are almost independent of each other. Due to thermal agitation, the atomic magnetic dipoles are randomly oriented.



When external magnetic field is applied, the field aligns the magnetic dipoles along its own direction and temperature increases the thermal agitation. So at low temperature alignment is more



Curie law → $I \propto H$, $I \propto \frac{1}{T}$ $\quad I \propto \frac{H}{T}$

$$I = C\frac{H}{T} \Rightarrow \frac{I}{H} = \chi_m = \frac{C}{T} \quad [C \to \text{Curie's Constt}]$$

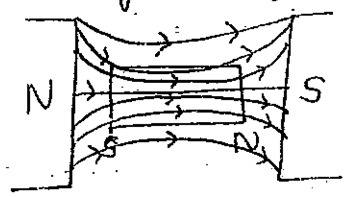
③ Ferromagnetic → Those substances which when placed in a magnetic field are strongly magnetised in the direction of the magnetising field are called ferromagnetic substances. eg. Iron, cobalt, Nickel, Alnico etc

Properties → ① A ferromagnetic substance is strongly attracted by a magnet.

② When a rod of ferromagnetic substance is suspended in a magnetic field, it quickly aligns itself along the direction of the magnetic field.

③ The ferromagnetic materials move from weaker part to the stronger part of the magnetic field.

④ When a ferromagnetic material is placed in a magnetic field, the magnetic field lines become highly dense inside ferromagnetic substance



⑤ The intensity of magnetisation (I) has a large positive value.

⑥ The magnetic susceptibility (χ_m) has a large positive value.

⑦ Relative permeability is very large as compared to one.

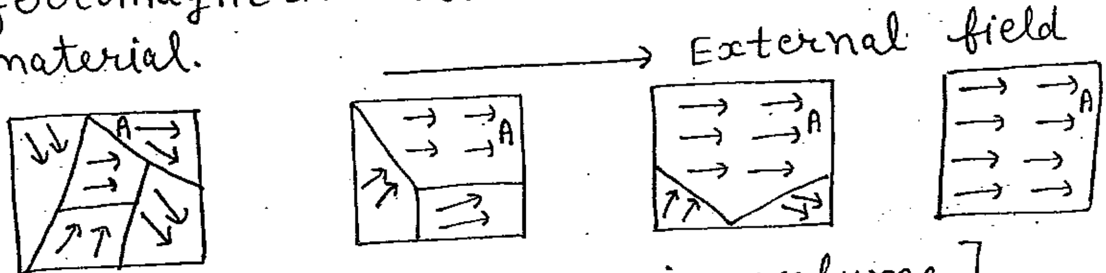
⑧ Ferromagnetic substances do not obey curie's law.

Electron theory of ferromagnetism (Domain theory)

In case of ferromagnetic substances, an unpaired electron in one atom interacts strongly with the unpaired electron of the neighbouring atom in such a way that they align themselves in a common direction over a small volume of material. These small volumes are called Domains.

Due to their random orientations, the net magnetic moment of the material is zero. When external magnetic field is applied, the magnetic moments of domains align along the direction of external field ; one due to alignment of atomic magnetic dipoles within the domain and second due to alignment of whole domain along the magnetic field.

* At curie temperature or curie point, the ferromagnetic material reduces to paramagnetic material.

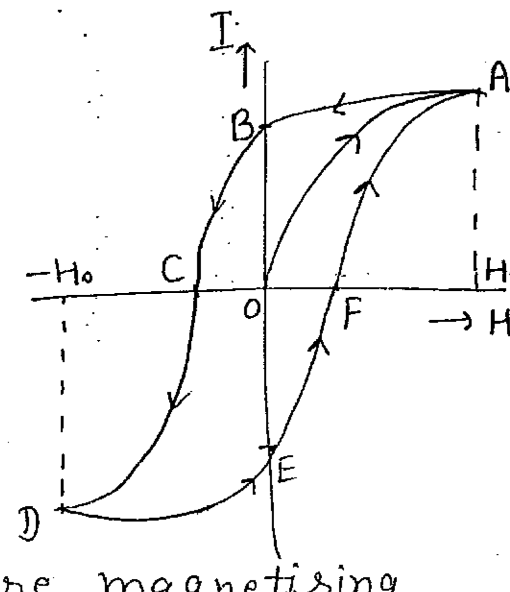


[Domain A increases in volume]

Hysteresis → The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation & demagnetisation of the ferromagnetic material is called Hysteresis.

OB → Retentivity

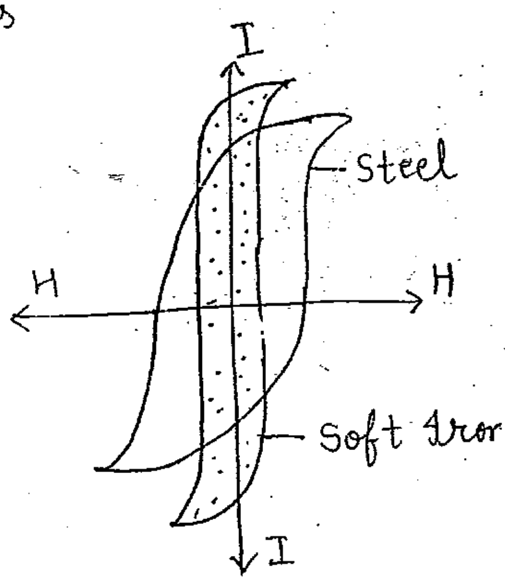
OC → Coercivity



Retentivity → The value of the intensity of magnetisation of material, when the magnetising field is reduced to zero, is called retentivity or residual magnetism

Coercivity → The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.

Comparison of Hysteresis loop for soft iron & steel



① Due to small area of hysteresis loop, the loss of energy per unit volume in case of soft iron is relatively small.

② Soft iron is much strongly magnetised than steel.

③ The retentivity of soft iron is comparatively more. On removing magnetising field, a large amount of magnetisation is retained by soft iron.

④ Coercivity of steel is comparatively much larger. The residual magnetism in steel can not be destroyed easily as compared to soft iron.

Applications of ferromagnetic substances

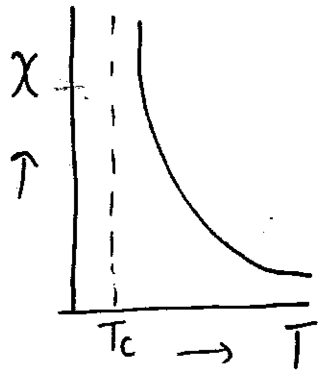
① Permanent magnets (Steel is used generally)

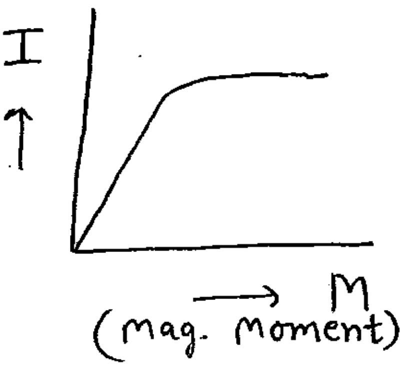
② Electromagnets (Generally soft iron cores are used)

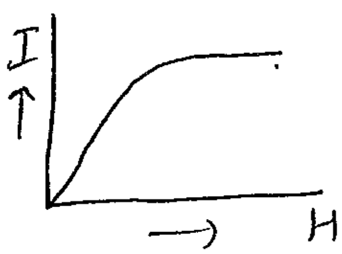
③ Transformer cores (Soft iron core is generally used. Mumetal (76% nickel, 17% iron 5% copper & 2% chromium) is also used for it.

Few Important graphs

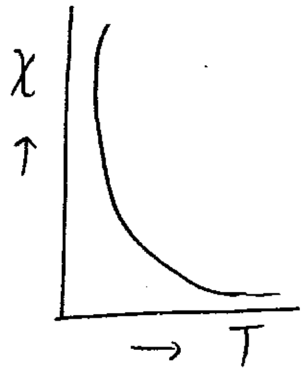
Ferromagnetic

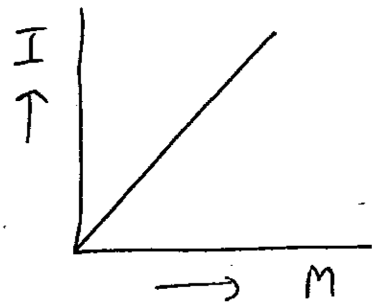


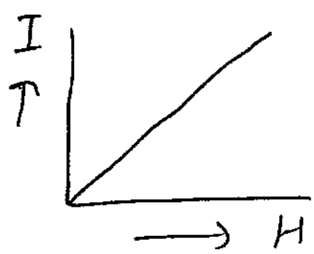




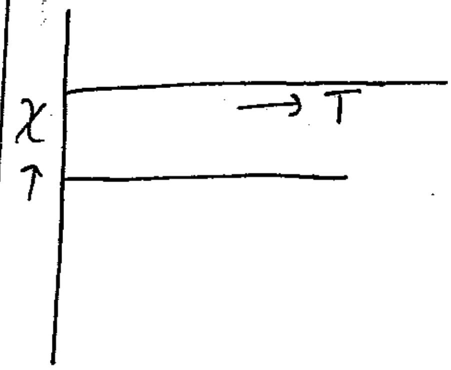
Paramagnetic

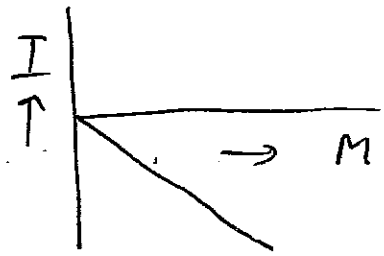


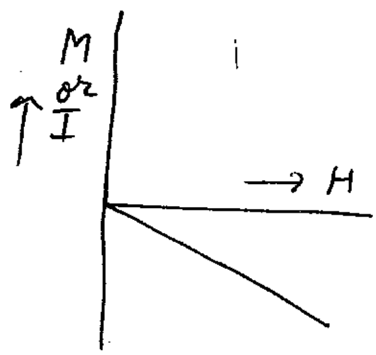




Diamagnetic







ELECTROMAGNETIC INDUCTION

Magnetic Flux → The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

Also $\phi = \int_A \vec{B} \cdot d\vec{A}$ $\quad B = \frac{F}{qv\sin\theta}$

$$\phi = \frac{F}{qv\sin\theta} \cdot A$$

S.I. unit of flux is 1 weber $= 1$ Tesla$\cdot m^2$

C.G.S. unit is 1 maxwell $= 1$ Gauss$\cdot cm^2$

1 weber $= 10^8$ maxwell

If $\theta = 0^\circ$ $\quad \phi = BA$ (positive)

& if $\theta = 180^\circ$, $\phi = -BA$ (negative)

Electromagnetic Induction → The phenomenon of production of induced e.m.f. (and hence induced current) due to a change in magnetic flux linked with a closed circuit is called electromagnetic induction.

Faraday's experiments → (i) Induced e.m.f. with a stationary coil and moving magnet.

(ii) Induced e.m.f. with a stationary magnet & moving coil

(iii) Induced e.m.f. by varying current in the neighbouring coil.

Laws of electromagnetic Induction.

First law → Whenever the magnetic flux linked with a closed circuit changes, an e.m.f. (and hence a current) is induced in it which lasts only so long as the change in flux is taking place.

Second law → The magnitude of the induced e.m.f. is equal to the rate of change of magnetic flux linked with the closed circuit.

$$|e| = \frac{d\phi}{dt}$$

Lenz's law → This states that the direction of induced e.m.f. is such that it opposes the cause which produces it i.e it opposes the change in magnetic flux.

Expression for induced e.m.f. → Acc to Faraday law

$$|e| = \frac{d\phi}{dt}$$

Acc to Lenz's law, direction of induced e.m.f. is opposite to $\frac{d\phi}{dt}$

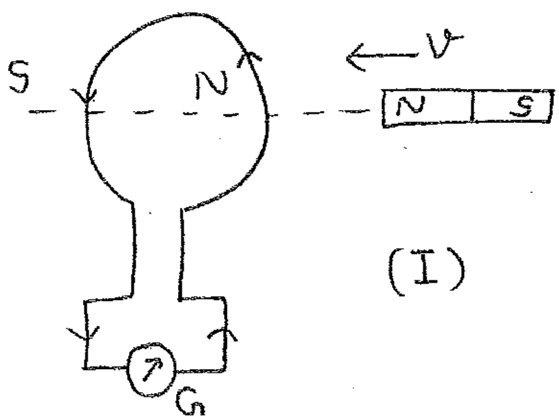
$$\Rightarrow \quad e = \frac{-d\phi}{dt}$$

For N turns $\quad e = -N\frac{d\phi}{dt}$

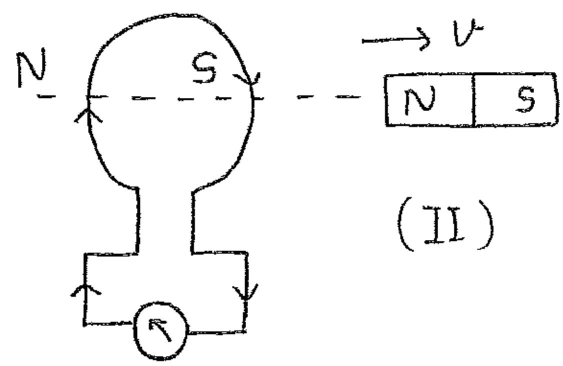
or $e = -N\left(\frac{\phi_2 - \phi_1}{t}\right)$

Explanation of Lenz's law → When the north pole of a bar magnet is moved towards a closed coil, the induced current in the coil flows in anticlockwise direction, as seen from the magnet side. Thus, that face develops north polarity to oppose the north pole of the magnet. (Fig I)

When north pole of magnet is taken away, the induced current in the coil flows in clockwise direction, as seen from the magnet side. Thus, that face develops south polarity to attract the north pole of the magnet. (Fig. II)



(I)

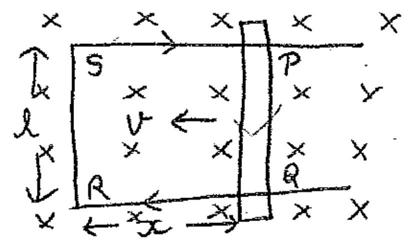


(II)

Lenz's law and law of conservation of energy

Whether a magnet is moved towards or away from a closed coil, the induced e.m.f. always opposes the motion of the magnet. It means work has to be done in moving the magnet closer to the coil against the force of repulsion. Similar work will be done in moving the magnet away from coil against the force of attraction. This work done appears as electric energy in the circuit.

Motional e.m.f. from Faraday's laws



$$\phi = BA$$
$$= Blx$$
$$e = -\frac{d\phi}{dt} = -\frac{d(Blx)}{dt}$$
$$e = -Bl\frac{dx}{dt} = -Blv$$

As v increases, e increases. When v is zero e is zero (as this is motional e.m.f.).

Fleming's Right hand rule

If we stretch the thumb and the first two fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of conductor; then the central finger points in the direction of current induced in the conductor.

Motional e.m.f. from Lorentz force and conservation of energy

→ When a conductor moves through a magnetic field, a Lorentz force acting on the free electrons can set up a current.

In the above figure, as the arm PQ is moved towards left with a speed v, the free electrons of PQ also move with same speed towards left. The electrons experience a magnetic Lorentz force $F_m = qvB$ in the direction QP. So free e^- will move towards P so electric field E is set up in the conductor from Q to P. This field exerts a force $F_e = qE$ on free electrons.

The accumulation of charges at the two ends continues till these two forces balance each other, i.e., $F_m = F_e$

or $qvB = qE$ or $vB = E$

Now $V = El = vBl$

It means it is the magnetic force on the moving electrons that maintains the potential difference and produces the e.m.f.

$$e = Blv$$

Current induced in the loop → If R is the resistance of arm PQ then $I = \frac{e}{R} = \frac{Blv}{R}$

Force on movable arm, $F = BIl \sin 90°$

or $F = \left(\frac{Blv}{R}\right) Bl = \frac{B^2 l^2 v}{R}$

*Power delivered by external force

$$P = Fv = \frac{B^2 l^2 v^2}{R}$$

*Power dissipated as heat loss

$$P_H = I^2 R = \frac{B^2 l^2 v^2}{R^2} \times R = \frac{B^2 l^2 v^2}{R}$$

Here $P_H = P$. So the mechanical energy expended to maintain the motion of the movable arm is first converted in to electrical energy (i.e. induced e.m.f.) and then to thermal energy. This is consistent with law of conservation of energy.

Methods of generating induced e.m.f.

(i) By changing the magnetic field B

(a) moving magnet towards the stationary coil

(b) moving coil towards the stationary magnet

(c) Changing the relative orientation θ of B & A

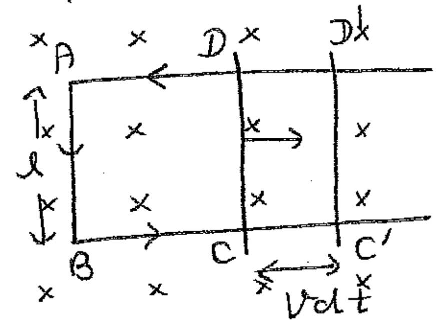
(ii) By changing the area of the coil

$$\phi = B \times \text{Area}$$

$$d\phi = B \times \text{change in area}$$

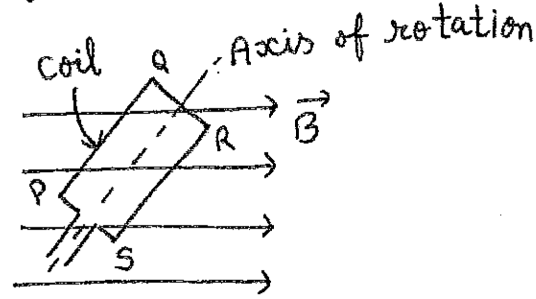
$$= B \times l v dt$$

$$|e| = \frac{d\phi}{dt} = Blv$$



Acc to Fleming's right hand rule, the induced current is in anticlockwise direction.

(iii) By changing the relative orientation of the coil and the magnetic field
(Theory of A.C. generator)



$$\phi = BA\cos\theta$$

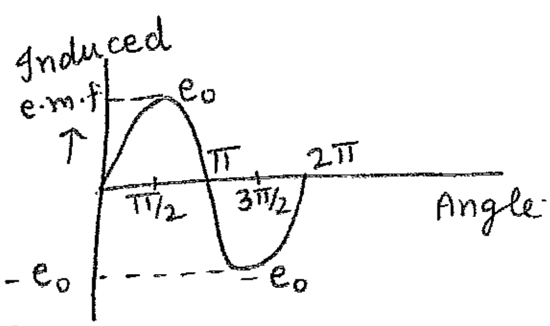
$$\theta = \omega t \quad \therefore \phi = BA\cos\omega t$$

Now $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(BA\cos\omega t)$

or $e = \omega BA \sin\omega t$

For coil of N turns

$$e = NBA\omega \sin\omega t$$



At $\theta = \omega t = 0 \quad e = 0$

At $\theta = \omega t = 90^\circ \quad e = NBA\omega = e_{max.} = e_0$

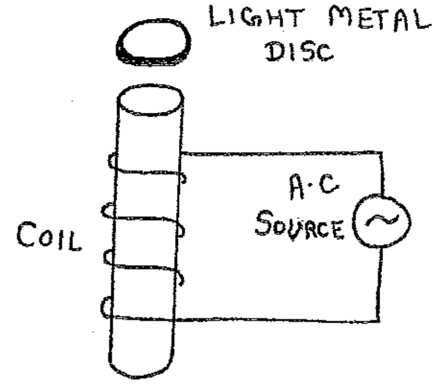
At $\theta = \omega t = 180^\circ \quad e = 0$

At $\theta = \omega t = 270^\circ \quad e = -NBA\omega = -e_{max.} = -e_0$

At $\theta = \omega t = 360^\circ \quad e = 0$

EDDY CURRENTS → Eddy currents are the current induced in solid metallic masses when the magnetic flux threading through them changes.

Experiment to demonstrate Eddy currents



Take a cylindrical electromagnet Connected by an A.C source and place a small metal disc on its top. As the current is switched on, the magnetic field at the disc rises from zero to a finite value. Eddy currents set up & it converts in to a small magnet. If initially the top end of electromagnet acquires N-polarity then by Lenz's law, the lower face of the small magnetic disc will also have N-polarity, resulting in a repulsive force. Thus, the disc is seen to be thrown up.

Undesirable effects of eddy currents

Eddy currents cause unnecessary heating and wastage of power. The heat produced by eddy currents may even demage insulation of coils.

Applications of eddy currents (concept)

(i) It is used in Induction furnace due to its heating effect.

(ii) It is used in electromagnetic damping.

(iii) It is used in electric brakes.

(iv) It is used in speedometer.

Self Induction → Self induction is the phenomenon of production of induced e.m.f. in a coil when a changing current passes through it

Co-efficient of self-induction At any instant

Magnetic flux, $\phi \propto I$

or $\phi = LI$

Now $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

Here L is co-eff of self induction or Self Inductance. It is numerically equal to the magnetic flux linked with the coil when a unit current flows through it.

Also, it is numerically equal to induced e.m.f. set up in the coil when rate of change of current is unity. Its S.I. unit is Henry. 1 henry (H) = 1 weber per Ampere

Self Inductance of a long solenoid

Let us consider a long solenoid of length 'l' and radius 'r' ($r \ll l$) having n turns per unit length. If a current I flows through the coil then magnetic field inside the solenoid is given by

$B = \mu_0 n I$

Magnetic flux linked with each turn is

$\phi = BA = \mu_0 n I A$

For $N = nl$ turns, $\phi = \mu_0 n^2 l I A$

$\phi = LI = \mu_0 n^2 l I A$

$\Rightarrow L = \mu_0 n^2 l A = \mu_0 \frac{N^2}{l} A$

If coil is wound on core of $\mu = \mu_r \mu_0$

$L = \mu_0 \mu_r N^2 A / l$

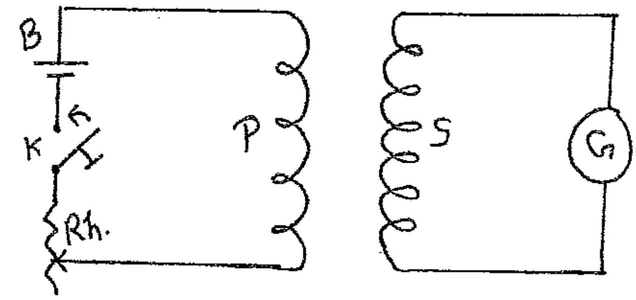
Mutual Induction → Mutual Induction is the phenomenon of production of induced e.m.f. in one coil due to a change in current in the neighbouring coil.

Co-efficient of mutual induction

At any instant

$\phi \propto I$

or $\phi = MI$



Now $e = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$

Here M is co-eff. of mutual induction or Mutual Inductance which is numerically equal to magnetic flux linked with one coil when a unit current passes through the other coil.

& $e = -M$ if $\frac{dI}{dt} = 1$

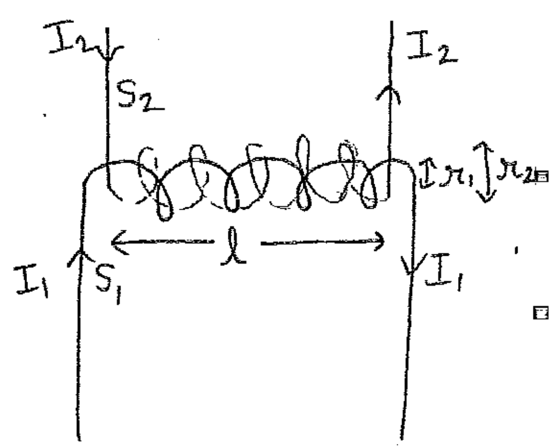
S.I. unit of M is Henry $1H = 1 VsA^{-1}$

Mutual Inductance of two coils is said to be 1 Henry if an induced e.m.f. of one Volt is set up in one coil when the current in the neighbouring coil changes at the rate of 1 ampere per second.

Mutual Inductance of two long solenoids

Magnetic field B_2 inside S_2, due to current I_2 is

$$B_2 = \mu_0 n_2 I_2 = \mu_0 N_2 I_2 / l$$



Total magnetic flux linked with inner solenoid S_1 is

$$\phi_1 = B_2 A N_1 = \frac{\mu_0 N_1 N_2 I_2 A}{l}$$

Mutual Inductance of coil 1 w.r.t. coil 2 is

$$M_{12} = \frac{\phi_1}{I_2} = \frac{\mu_0 N_1 N_2 A}{l}$$

Now magnetic field B_1 due to I_1 is

$$B_1 = \mu_0 n_1 I_1 = \mu_0 N_1 I_1 / l$$

& $\phi_2 = B_1 A N_2 = \frac{\mu_0 N_1 N_2 A I_1}{l}$.

& Mutual inductance of coil 2 w.r.t. 1 is

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l$$

$$\Rightarrow \quad M = M_{12} = M_{21} = \mu_0 n_1 n_2 A l = \mu_0 n_1 n_2 \pi r_1^2 l$$

Factors on which mutual inductance depends

(i) Number of turns

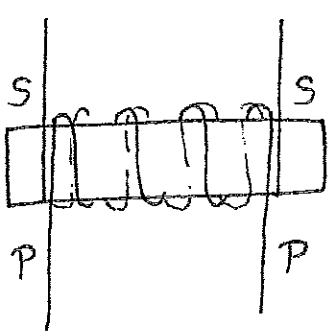
$$M \propto N_1 N_2 \text{ (no of turns)}$$

(ii) Common cross-sectional area

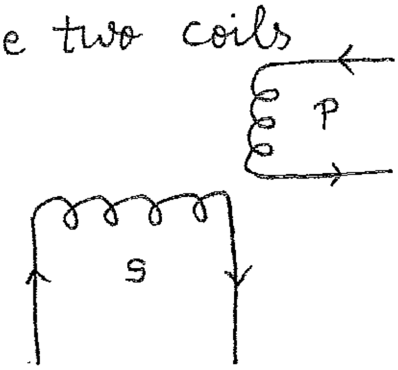
$$M \propto A \text{ (Area)}$$

(iii) Relative separation
Larger the distance between two solenoid, smaller will be the magnetic flux linked with the secondary coil.

(iv) Relative orientation of the two coils



[M is maximum]



[M is minimum]

(v) Permeability of core material

Imp. Co-eff. of coupling $K = \frac{M}{\sqrt{L_1 L_2}}$

Energy stored in an inductor →

When current I flows through the inductor, an e.m.f. is induced in it. It is given by

$$\epsilon = -L\frac{dI}{dt}$$

For external battery, e.m.f. = $-\epsilon$

so $\epsilon = L\frac{dI}{dt}$

For small charge dq, small work done by the external supply is given by

$$dW = \epsilon\, dq = L\frac{dI}{dt}dq = LI\,dI$$

$$\Rightarrow \int dW = \int_0^{I_0} LI\,dI = \tfrac{1}{2}LI_0^2$$

$W = U = \tfrac{1}{2}LI_0^2$ is work done stored in the magnetic field of inductor

Magnetic energy stored in a solenoid →

$$U_m = \tfrac{1}{2}LI_0^2$$

Now $B = \mu_0 n I_0$ or $I_0 = \frac{B}{\mu_0 n}$

& $L = \mu_0 n^2 A l$

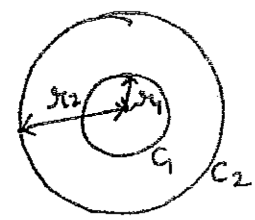
$$\Rightarrow U_m = \tfrac{1}{2}\mu_0 n^2 A l \frac{B^2}{\mu_0^2 n^2} = \frac{1}{2}\left(\frac{B^2 A l}{\mu_0}\right)$$

($\overline{U_m}$) Magnetic energy density $= \frac{U_m}{V} = \frac{U_m}{Al}$

$$\Rightarrow \overline{U_m} = \frac{B^2}{2\mu_0}$$

Mutual Inductance of two concentric coils

$$\phi_B = BA = \left(\frac{\mu_0 I}{2r_2}\right)\pi r_1^2 = \frac{\mu_0 \pi r_1^2}{2r_2}I$$



Now $\phi = MI$

$$\Rightarrow M = \mu_0 \pi r_1^2/2r_2$$

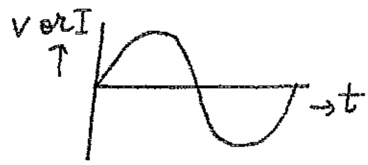
$$M_{12} = M_{21} = M = \frac{\mu_0}{4\pi}\left(\frac{2\pi r_1^2}{r_2}\right)$$

(4)

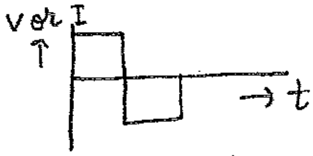
ALTERNATING CURRENTS

Alternating current → An electric current, magnitude of which changes with time and polarity reverses periodically is called alternating current (A.C).

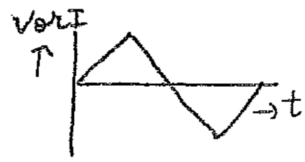
Types of A.C. → (i) Sinusoidal



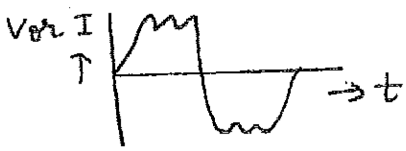
(ii) Square



(iii) Triangular



(iv) Mixed



* Most common form is Sinusoidal

Sinusoidal alternating current (A.C.) is expressed as

$$I = I_0 \sin \omega t$$

Here $\omega = \frac{2\pi}{T} = 2\pi f$ is angular frequency

& Sinusoidal voltage of A.C source is given by

$$V = V_0 \sin \omega t$$

Advantages of A.C → (i) It can be stepped up & stepped down (using transformer)

(ii) A.C can be transmitted to long distances without much loss of heat.

(iii) A.C can be converted into D.C for storage.

(iv) A.C can be better controlled (using choke coil).

Disadvantages of A.C → (i) A.C is more dangerous than D.C due to its peak value which is $\sqrt{2}$ times the r.m.s. value.

(ii) A.C. can not be used for electrolysis.

(iii) Scales of A.C meters are not uniform.

(iv) A.C. is not distributed uniformly. It has skin effect.

Mean or Average value of A.C. → It is that value of steady current which sends the same amount of charge through a circuit in a certain time interval as is sent by an A.C. through the same circuit in half cycle.

* Mean value of an a.c over half cycle is 63.7% of its peak value

Let an alternating current be represented by

$$I = I_0 \sin\omega t$$

Now $\quad I = \frac{dq}{dt} \quad \Rightarrow \quad dq = I\,dt = I_0 \sin\omega t\, dt$

$$\Rightarrow \int dq = \int_0^{T/2} I_0 \sin\omega t\, dt$$

$$\Rightarrow q = I_0 \left(-\frac{\cos\omega t}{\omega}\right)_0^{T/2} = \frac{-I_0 T}{2\pi}(\cos\pi - \cos 0^\circ)$$

$$\Rightarrow q = \frac{I_0 T}{\pi}$$

Now $\quad q = I_{av.} \times T/2 = \frac{I_0 T}{\pi}$

$$\Rightarrow I_{av.} = \frac{2 I_0}{\pi} = 0.637\, I_0$$

Root mean square (R.M.S.) or Effective value of A.C →

It is defined as that steady current which produces the same amount of heat in a conductor in a certain time interval as is produced by the A.C in the same conductor during the time period T (i.e full cycle).

Let A.C is given by $I = I_0 \sin\omega t$

Heat produced is given by $dH = I^2 R\,dt$

or $dH = (I_0^2 \sin^2\omega t) R\, dt$

$$\Rightarrow \int dH = H = \int_0^T I_0^2 R \sin^2\omega t\, dt$$

$$\Rightarrow H = I_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right]$$

$$\Rightarrow H = \frac{I_0^2 R}{2}\left[T - 0\right] = \frac{I_0^2 R T}{2}$$

Now $H = I_{r.m.s.}^2 R T = \frac{I_0^2 R T}{2}$

$$\Rightarrow I_{r.m.s.} = \frac{I_0}{\sqrt{2}} = 0.707 \, I_0$$

* $I_{r.m.s.}$ is 70.7% of the peak value I_0

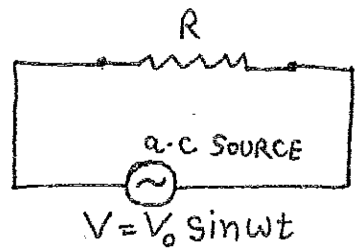
* Read only

Phasor and Phasor diagram → Phasor is a rotating vector which represents a quantity varying sinusoidally with time.

In a phasor diagram, two quantities varying sinusoidally with time are represented. The phase difference between these quantities is represented by the angle between their maximum values.

Alternating voltage applied to a resistor

The applied alternating voltage is given by $V = V_0 \sin\omega t$ — I

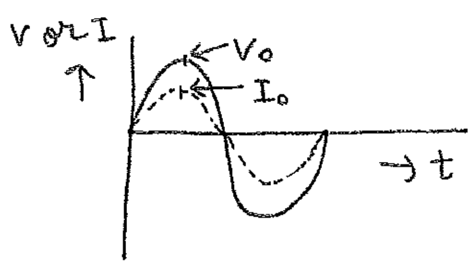


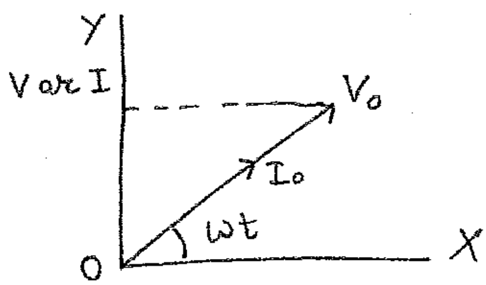
Now $V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{V_0 \sin\omega t}{R}$$

$$\Rightarrow I = I_0 \sin\omega t \quad \text{— II}$$

$\Rightarrow$ * V & I are in same phase





Power supplied to a resistor → Instantaneous power dissipated in resistor is given by

$$P = I^2 R = (I_0^2 \sin^2 \omega t) R$$

$$\text{or } P = I_0^2 R \sin^2 \omega t = I_0^2 R \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$P_{av.} = I_0^2 R \left(\frac{1 - \langle \cos 2\omega t \rangle}{2}\right)$$

$$\langle \cos 2\omega t \rangle = \text{Average value of } \cos 2\omega t$$

$$= \frac{1}{T}\int_0^T \cos 2\omega t \, dt = 0$$

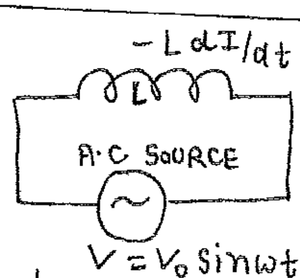
$$\Rightarrow \langle \cos 2\omega t \rangle = 0$$

$$\Rightarrow P_{av.} = \frac{I_0^2 R}{2} = I_{r.m.s.}^2 R$$

Alternating voltage applied to an Inductor

The alternating voltage across the inductor is given by

$$V = V_0 \sin \omega t$$



Induced e.m.f. across the inductor is $-L\frac{dI}{dt}$

$$\Rightarrow V + \left(-L\frac{dI}{dt}\right) = 0 \quad \Rightarrow \quad V = L\frac{dI}{dt}$$

$$\Rightarrow \frac{V_0}{L}\sin \omega t = \frac{dI}{dt} \quad \Rightarrow \quad dI = \frac{V_0}{L}\sin \omega t \, dt$$

$$\Rightarrow I = \frac{V_0}{L}\int \sin \omega t \, dt = -\frac{V_0}{\omega L}(\cos \omega t)$$

$$\Rightarrow I = \frac{V_0}{\omega L}\sin(\omega t - \pi/2)$$

$$\Rightarrow I = I_0 \sin(\omega t - \pi/2) \qquad I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$

$$X_L = \omega L = \text{Inductive reactance}$$

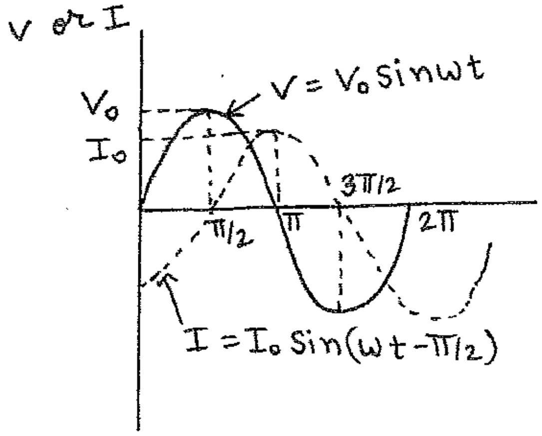
This shows that the current lags behind the voltage by an angle of $\pi/2$.

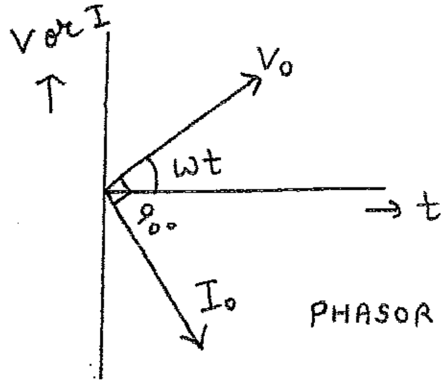
For A.C $\omega = 2\pi\nu$ is finite

So inductor offers opposition to the flow of a.c

For D.C $\omega = 2\pi\nu = 0$

So inductor offers no opposition to the flow of d.c





PHASOR DIAGRAM

Power supplied to an Inductor

Instantaneous power supplied to an Inductor

$$P_L = VI = V_0 \sin\omega t \cdot I_0 \sin(\omega t - \pi/2)$$

$$= -\frac{V_0 I_0}{2} \sin 2\omega t$$

$$(P_L)_{av.} = -\frac{V_0 I_0}{2} \langle \sin\omega(2t) \rangle$$

$$\langle \sin 2\omega t \rangle = \frac{1}{T}\int_0^T \sin 2\omega t \, dt = 0$$

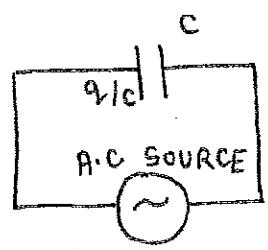
$$\Rightarrow (P_L)_{av.} = 0$$

* Average power for full cycle, in case of Inductor is zero.

Alternating voltage applied to a capacitor

The alternating voltage applied across a capacitor is given by

$$V = V_0 \sin\omega t$$



P.D. across capacitor, $V_c = q/c$

Now $V = \frac{q}{C} = V_0 \sin\omega t \Rightarrow q = CV_0 \sin\omega t$

$$I = dq/dt = \frac{d}{dt}(CV_0 \sin\omega t) = \omega C V_0 \cos\omega t$$

or $I = \dfrac{V_0}{1/\omega C} \cos\omega t = I_0 \sin(\omega t + \pi/2)$

$I_0 = \dfrac{V_0}{X_C}$ $X_C = 1/\omega C$ = Capacitive reactance

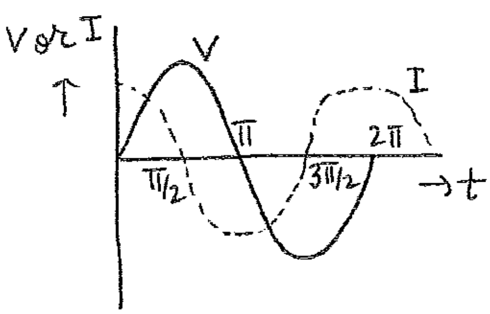
This means that current leads the voltage by an angle of $\pi/2$

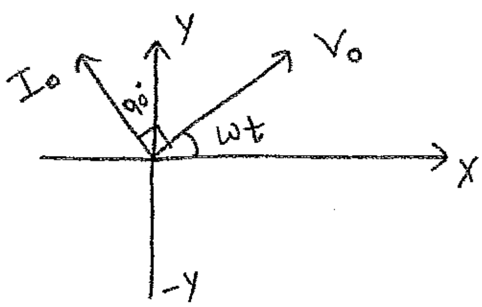
For A.C. $\omega = 2\pi\nu$ is finite so X_C is small

$\Rightarrow$ Capacitor offers small opposition to flow of A.C

For D.C. $\omega = 2\pi\nu = 0$ so X_C is infinite

$\Rightarrow$ Capacitor blocks d.c as X_C is infinite





Power supplied to a capacitor →

Instantaneous power supplied to capacitor is given by

$$P_C = VI = V_0 \sin\omega t \; I_0 \sin(\omega t + \pi/2)$$

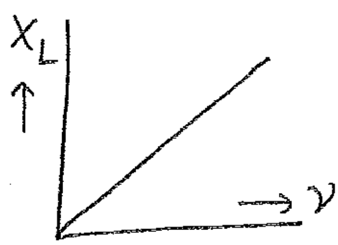
$$\Rightarrow P_C = \frac{V_0 I_0}{2} \sin 2\omega t$$

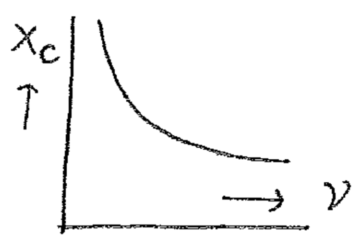
$$(P_C)_{av.} = \frac{V_0 I_0}{2} \langle \sin 2\omega t \rangle$$

$$\langle \sin 2\omega t \rangle = \frac{1}{T}\int_0^T \sin 2\omega t \, dt = 0$$

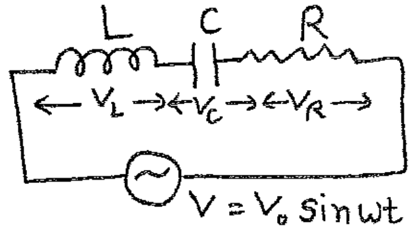
$$\Rightarrow (P_C)_{av.} = 0$$

$\Rightarrow$ Average power, for full cycle in capacitor is zero





L, C and R in series across an alternating supply



Let I be the r.m.s. value of current flowing through all the circuit elements.

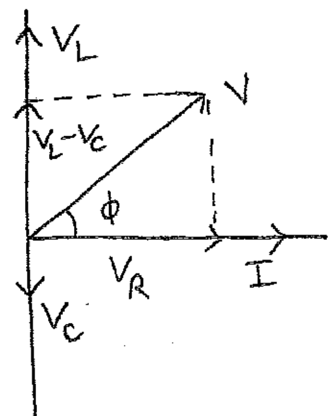
The potential difference across inductor

$$V_L = I X_L$$

P.D. across capacitor, $V_C = I X_C$

P.D. across resistor, $V_R = IR$

In case of inductor, voltage leads the current by $\pi/2$.



In case of capacitor, voltage lags the current by $\pi/2$.

According to above phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad V = \sqrt{I^2R^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

or $Z = \dfrac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$ = Impedance of LCR circuit

(i) If $X_L = X_C$ then $Z = R$

It behaves as pure resistive circuit

(ii) If $X_L = 0$ then $Z = \sqrt{R^2 + X_C^2}$

It is series RC circuit

(iii) If $X_C = 0$. then $Z = \sqrt{R^2 + X_L^2}$

It is series LR circuit

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega}{R}$$

$$\& \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

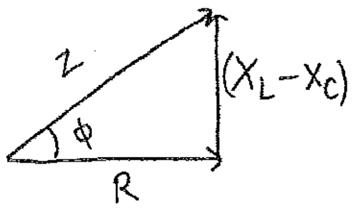
Impedance Triangle → (i) In series LCR circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

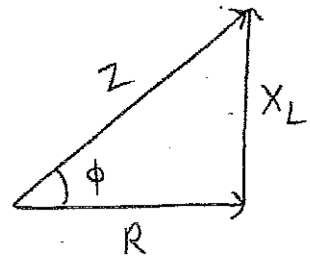
(ii) In LR circuit $Z = \sqrt{R^2 + X_L^2}$

(iii) In CR circuit $Z = \sqrt{R^2 + X_C^2}$

They are represented by Impedance triangle

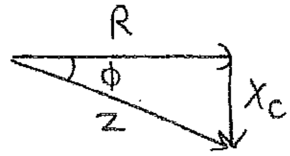


$\tan\phi = \dfrac{X_L - X_C}{R}$



$\tan\phi = \dfrac{X_L}{R}$

$\tan\phi = \dfrac{X_C}{R}$



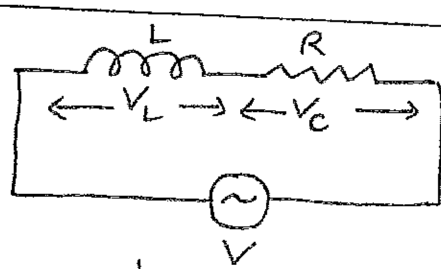
S.I. unit of Impedance is Ohm.

Admittance → Reciprocal of Impedance of circuit is called admittance of circuit

It is given by $A = \dfrac{1}{Z}$

S.I. unit of admittance is ohm^{-1}, mho or siemen.

LR - series circuit across alternating electric supply



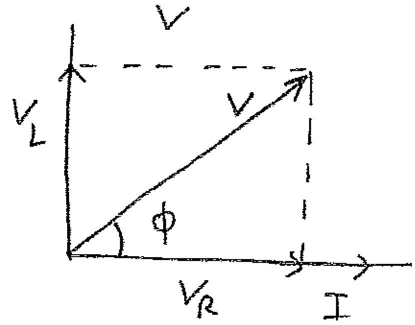
Here $V = \sqrt{V_R^2 + V_L^2}$

$= I\sqrt{R^2 + X_L^2}$

$Z_{LR} = \dfrac{V}{I} = \sqrt{R^2 + X_L^2}$

$\tan\phi = \dfrac{I X_L}{I R}$

$\Rightarrow \tan\phi = \dfrac{X_L}{R}$



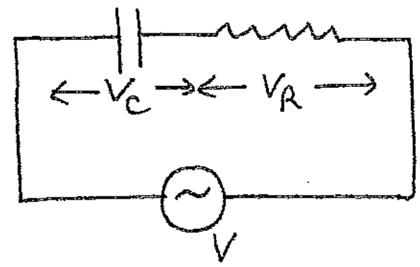
voltage leads the current by an angle ϕ.

CR-series circuit across alternating electric supply

Here $V = \sqrt{V_R^2 + V_C^2}$

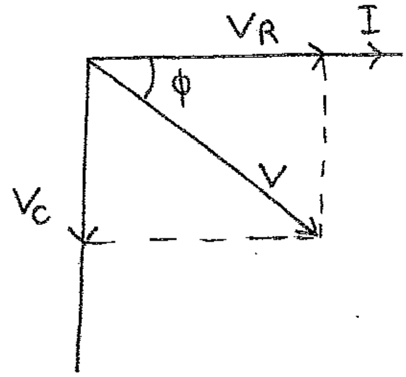
$= I\sqrt{R^2 + X_C^2}$

or $Z_{CR} = \frac{V}{I} = \sqrt{R^2 + X_C^2}$



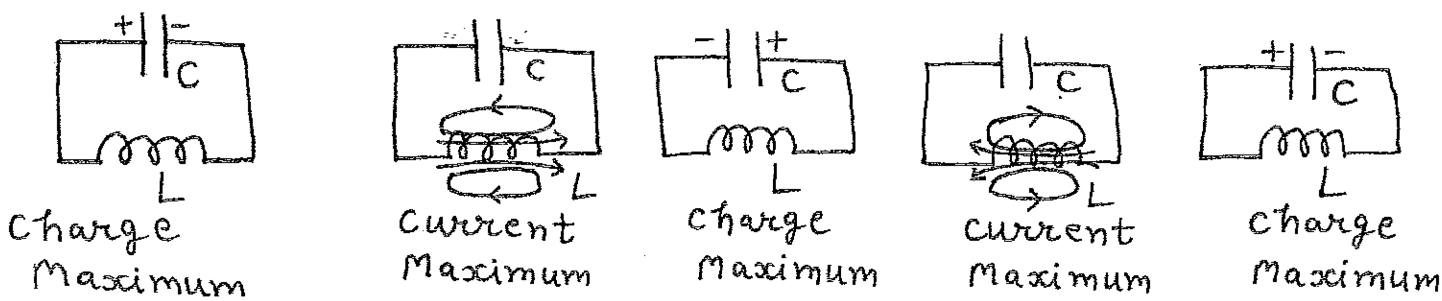
$\tan\phi = \frac{V_C}{V_R} = \frac{I X_C}{I R}$

or $\tan\phi = \frac{X_C}{R}$



Voltage lags behind the current by an angle ϕ.

LC-Oscillations → Electrical oscillations produced by the exchange of energy between a capacitor (which stores electrical energy) and an inductor (which stores magnetic energy) are called LC-oscillations.

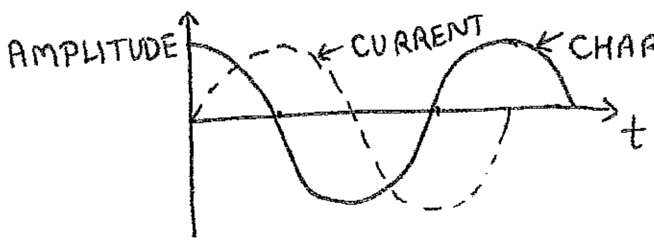


Let us consider that the capacitor of LC-circuit is fully charged. A potential difference exists between the plates of the capacitor and the energy is stored in the electric field of capacitor. ($U_e = Q_0^2/2C$). Now the capacitor begins to discharge through the inductor and hence current starts flowing through the inductor. As a result of this, magnetic field is set up around the inductor. When the capacitor is discharged completely, the energy is stored in the magnetic field around the inductor ($U_m = \frac{1}{2} L I_0^2$). Now electrical energy is completely converted into magnetic energy.

When the magnetic field energy becomes maximum, the capacitor begins to recharge itself in the opposite direction. Now the energy stored in the magnetic field is converted into the energy stored in the electric field of capacitor.

When the capacitor is fully recharged, whole of the energy is stored in the electric field of the capacitor. At that instant, the capacitor is again discharged through the inductor. The current flows through the inductor and energy is stored in the magnetic field around it.

Now again the capacitor is re-charged in the opposite direction and the energy is stored in the electric field of capacitor. This process continues and results in LC-oscillations.



$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \nu = \frac{1}{2\pi\sqrt{LC}}$$

ν = freq. of LC-oscillations

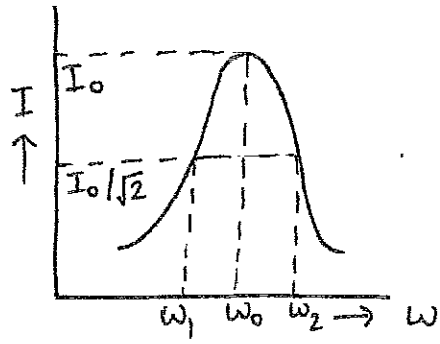
Let us consider that
$\omega_1 = \omega_0 - \Delta\omega$

$\omega_2 = \omega_0 + \Delta\omega$

$\omega_2 - \omega_1 = 2\Delta\omega$ = Bandwidth

$$Q = \frac{\omega_0}{2\Delta\omega} = Q \text{ factor}$$

or sharpness of resonance



Power consumed in a series LCR circuit

Power dissipated in an a.c circuit is the product of r.m.s. value of voltage and component of current in phase with r.m.s. voltage.

For LCR circuit

$$V = V_0 \sin\omega t \quad \& \quad I = I_0 \sin(\omega t + \phi)$$

$$P_i = VI = V_0 I_0 \sin\omega t \sin(\omega t + \phi)$$

$$= V_0 I_0 \sin\omega t (\sin\omega t \cos\phi + \cos\omega t \sin\phi)$$

$$\Rightarrow P_i = V_0 I_0 \left[\sin^2\omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi\right]$$

$$P_{av.} = \frac{1}{T} \int_0^T P_i \, dt$$

$$= \frac{V_0 I_0}{T} \left[\int_0^T \sin^2\omega t \cos\phi \, dt + \int_0^T \frac{\sin 2\omega t \sin\phi}{2} dt\right]$$

$$= \frac{V_0 I_0}{T} \left[\int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right)\cos\phi \, dt + \int_0^T \frac{\sin 2\omega t \sin\phi}{2} dt\right]$$

Now $\int_0^T \sin 2\omega t \, dt = \int_0^T \cos 2\omega t \, dt = 0$

$$\Rightarrow P_{av.} = \frac{V_0 I_0}{2T} (\cos\phi \, T) = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$\Rightarrow P_{av.} = V_{r.m.s.} I_{r.m.s.} \cos\phi$$

Here $\cos\phi$ is called power factor.

Electrical Resonance → Resonance in a series LCR circuit takes place when the circuit allows maximum current for a given frequency of A.C. source for which the capacitive reactance becomes equal to inductive reactance.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

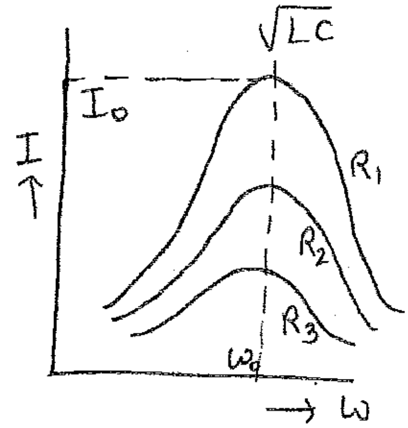
If $X_L = X_C$ or $\omega_0 L = 1/\omega_0 C$ or $\omega_0 = \frac{1}{\sqrt{LC}}$

then $I = \frac{V}{R}$ is maximum

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi \nu_0$$

$$\Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

[Here $R_3 > R_2 > R_1$



This is used in acceptor circuit.

Q-factor → It is defined as the ratio of the voltage developed across the inductor (or capacitor) at the resonance to the voltage applied (i.e. voltage across resistance) to the circuit.

$$Q = \frac{V_L \text{ or } V_C}{IR} = \frac{X_L}{R} \text{ or } \frac{X_C}{R}$$

$$\text{or } Q = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 C R}$$

Now $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

In terms of bandwidth $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$

Special cases : (i) In pure resistor circuit

$\phi = 0$, $\cos\phi = 1$

$\Rightarrow P_{av.} = V_{r.m.s.} I_{r.m.s.}$

(ii) For pure inductor circuit

$\phi = 90°$, $\cos\phi = 0$

$\Rightarrow P_{av.} = 0 \Rightarrow$ No power loss

(iii) For pure capacitor circuit

$\phi = 90°$, $\cos\phi = 0$

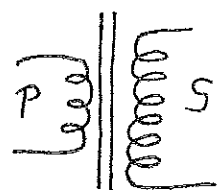
$\Rightarrow P_{av.} = 0 \Rightarrow$ No power loss

Wattless Current or Idle current → It is that curren due to which the power consumed in the circuit is zero.

eg. Average power consumed in pure inductor or pure capacitor circuit is zero so in these cases, current is wattless current.

Transformers → It is a device used to convert low alternating voltage at higher current into high alternating voltage at lower current and vice-versa.

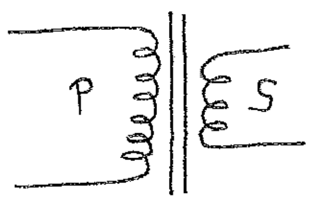
Types of Transformers →



(i) Step-up Transformer

It converts low voltage at high current to high voltage at lower current.

(ii) Step-down Transformer



It converts high voltage at low current to low voltage at higher current

Principle → It is based on the principle of mutual induction. An e.m.f. is induced in a coil, when a changing current flows through its nearby coil.

Acc to Faraday's law, the induced e.m.f. in the primary coil, $E_P = -N_P \frac{d\phi}{dt}$

The induced e.m.f. in the secondary coil,

$$E_S = -N_S \frac{d\phi}{dt}$$

Now $\frac{E_S}{E_P} = \frac{N_S}{N_P} = K$ = Transformation ratio

$K < 1$ for step-down transformer

$K > 1$ for step-up transformer

For an ideal transformer

output power = Input power

$$E_S I_S = E_P I_P \quad \text{or} \quad \frac{E_S}{E_P} = \frac{I_P}{I_S}$$

$$\Rightarrow E \propto \frac{1}{I}$$

$$\text{Efficiency}, \eta = \frac{E_S I_S}{E_P I_P} \leqslant 1$$

Energy losses in a Transformer →

(i) Copper losses → Energy lost in windings of the transformer is known as copper loss

(ii) Flux leakage losses → Magnetic flux linked with primary is not equal to that of secondary.

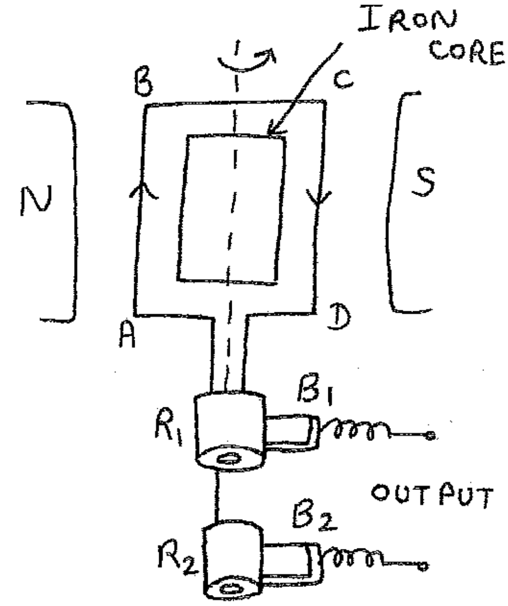
(iii) Iron losses → It is of two types

(a.) Eddy current losses (b) Hysteresis losses

A.C. generator → A device used to convert mechanical energy into electrical energy.

Principle → It is based on the principle of electromagnetic induction.

Construction →



(i) **Armature** → Armature coil ABCD having large no. of turns

(ii) **Strong field magnets** → Two pole pieces of permanent magnet cylindrical in shape.

(iii) **Slip Rings** → Two brass slip rings R_1 & R_2 Connected to coil.

(iv) **Brushes** → Two carbon brushes B_1 & B_2 attached with R_1 & R_2

Working → When the armature coil ABCD rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. The magnetic flux linked with the coil changes and hence e.m.f. is induced. The direction of induced current is given by Flaming's Right hand rule.

For half revolution, current through B_1 comes out & for next half, current through B_2 comes out.

Hence e.m.f. induced is of alternating nature.

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t) = NBA\omega\sin\omega t$$

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \qquad \therefore \mathcal{E}_0 = NBA\omega$$

$$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_0}{R}\sin\omega t = I_0 \sin\omega t$$

Variation of induced e.m.f. with different positions of coil

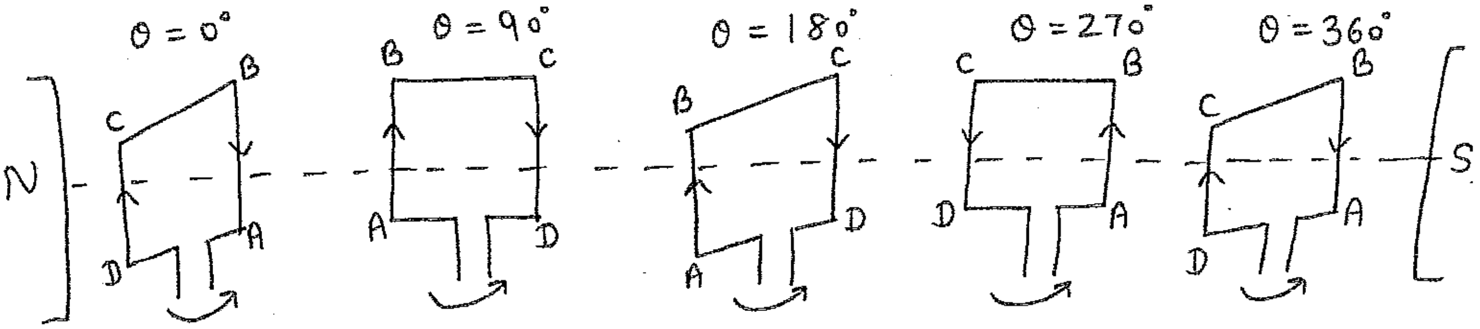
(i) When $\theta = 0°$, $\epsilon = \epsilon_0 \sin \omega t = \epsilon_0 \sin 0° = 0$

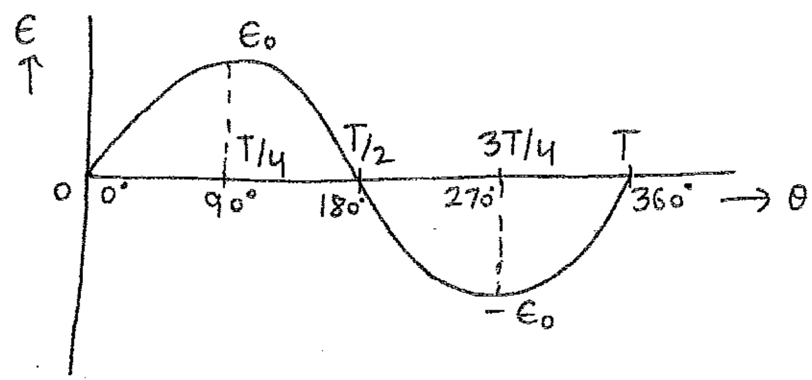
(ii) When $\theta = 90°$, $\epsilon = \epsilon_0 \sin 90° = \epsilon_0$

(iii) When $\theta = 180°$, $\epsilon = \epsilon_0 \sin 180° = 0$

(iv) When $\theta = 270°$, $\epsilon = \epsilon_0 \sin 270° = -\epsilon_0$

(v) When $\theta = 360°$, $\epsilon = \epsilon_0 \sin 360° = 0$





This is single phase A.C

In Poly-phase generators (generally 3-phase) there are three separate coils inclined to one another at equal angles. They produce a phase difference of 120°.