Electrostatics - I
(Electric Charges & field)

Electrostatics is the branch of physics which dea with the study of electric forces, fields, potential due to charges at rest.

Electric charge is a physical quantity which cau matter to experience an electric force when placed near other matter. {S.I. unit → coulomb}

* There are two kinds of electric charges ; positive and negative.
* Like charges repel and unlike charges attrac each other.

Conductors → The materials which easily allow flo of electric charges through them.

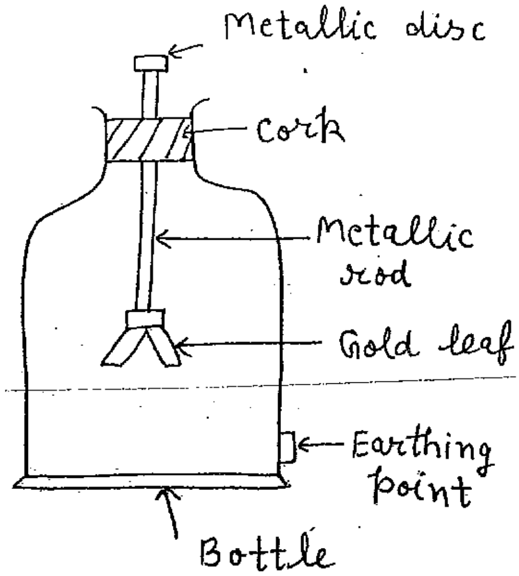
Insulators → The materials which do not allow the flow of electric charge through them.

Electroscope → A device used to detect the presence and nature of electric charge on a bod

eg. Gold Leaf electroscope

(a) Divergence of two halves of the gold leaf of electroscope shows the presence of electric charge on gold leaf.

(b) When a charged body is touched with an already charged electroscope then if charged body has same sign of charge as on gold leaf then divergence of gold leaves increase and vice versa.

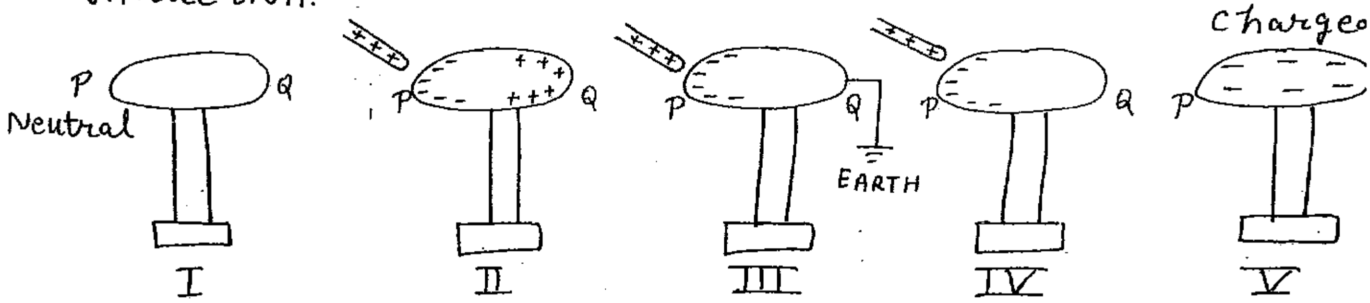


Methods of charging

① **Charging by conduction** → If an uncharged conductor is touched with a charged conductor, the uncharged conductor may acquire charge similar to the charged conductor.

② **Charging by friction** → Suitable materials when rubbed together get electrified (under suitable conditions). This is known a frictional electricity. one material becomes positively charged (loss of e^-s) and other becomes negatively charged (gain of e^-s).
eg. When glass rod is rubbed with silk cloth then glass rod becomes positively charged and silk cloth becomes negatively charged.

③ **Charging by Induction** → The process by which a neutral conductor is made electrically charged when placed near a charged object is known as charging by Induction.



Properties of electric charge

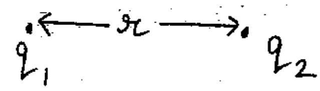
(i) Additivity of electric charge
Total charge on body 'q' is algebraic sum of all the charges ($q_1 + q_2 + q_3 + \text{---}$)

(ii) conservation of electric charge
Charge can neither be created nor be destroyed

(iii) Quantization of charge $q = \pm ne$ ($e = 1.6 \times 10^{-19}$ C)

Coulomb's law → The magnitude of force of attraction or repulsion between any two point charges at rest is directly proportional to the product of the magnitude of charges and inversely proportional to the square of distance between them.

$$F \propto \frac{q_1 q_2}{r^2}$$



$$\text{or } F = \frac{k q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2}$$

$\epsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$ (absolute permittivity)

ϵ_r is relative permittivity ($\epsilon_r = 1$ for free sp

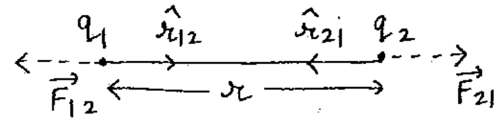
$$F_{air} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}, \quad F_{medium} = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2C^{-2}, \quad \epsilon_r = \frac{F_a}{F_m} = K$$

K = dielectric constant of the medium

Coulomb's law in vector form

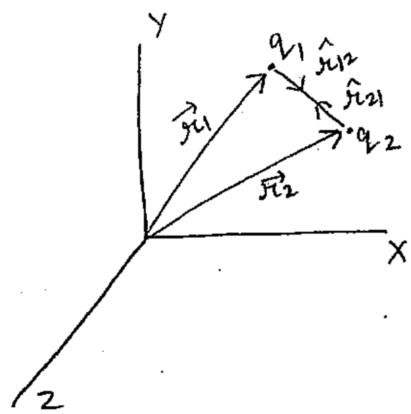
$$\vec{F}_{12} = \frac{k q_1 q_2}{r^2}\hat{r}_{21}$$



$$\& \quad \vec{F}_{21} = \frac{k q_1 q_2}{r^2}\hat{r}_{12} \qquad \hat{r}_{12} = -\hat{r}_{21}$$

$$\Rightarrow \vec{F}_{12} = -\vec{F}_{21}$$ (when q_1 & q_2 are positive

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \qquad \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$



$$\vec{F}_{21} = \frac{k q_1 q_2(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

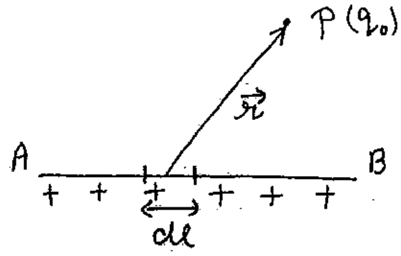
$$\& \quad \vec{F}_{12} = \frac{k q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

* Electrostatic forces are 10^{36} times stronger than gravitational forces.

Continuous charge distribution

A system of closely spaced electric charges forms a continuous charge distribution.

(i) Linear charge distribution



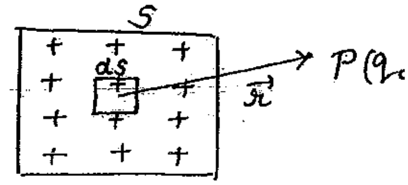
$$dq = \lambda \, dl$$

$\lambda \rightarrow$ linear charge density

$$d\vec{F} = k \frac{dq \cdot q_0}{r^2} \hat{r}$$

$$\Rightarrow \vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_l \frac{\lambda \, dl}{r^2} \hat{r}$$

(ii) surface distribution of charge.



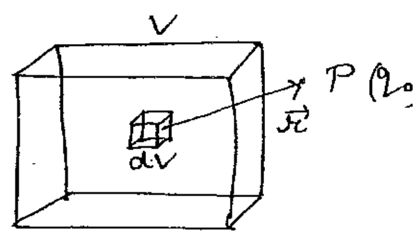
$$dq = \sigma \, ds$$

$\sigma \rightarrow$ surface charge density

$$d\vec{F} = k \frac{dq \cdot q_0}{r^2} \hat{r}$$

$$\Rightarrow \vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{\sigma}{r^2} ds \, \hat{r}$$

(iii) volume distribution of charge



$$dq = \rho \, dV$$

$\rho \rightarrow$ volume charge density

$$d\vec{F} = k \frac{q_0 \cdot dq}{r^2} \hat{r}$$

$$\Rightarrow \vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{\rho}{r^2} dV \, \hat{r}$$

Superposition Principle (For system of discrete charges)

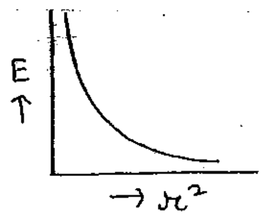
According to this principle, total force acting on a given point charge due to a number of point charges around it is the vector sum of individual forces acting on that point charge due to all other point charges.

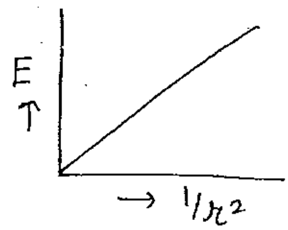
Electric field → The region or space around a charged body within which its influence can be felt by other small charge is called electric field.

Electric field intensity due to a source charge at any point in its electric field is defined as the force experienced by a unit positive charge placed at that point.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$ S.I. unit → N/C

∴ Electric field intensity due to a point charge Q is given by $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$





In terms of position vectors,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

Electric field intensity due to a system of point charges

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i \vec{r}_i}{r_i^3} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

Electric field intensity due to continuous charge distribution

① For linear charge distribution

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, dl}{r^2} \hat{r}$$

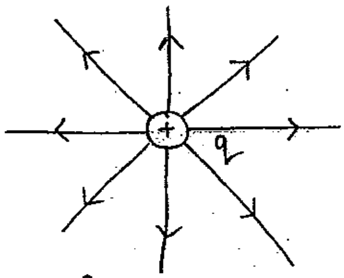
② For surface charge distribution

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \, ds}{r^2} \hat{r}$$

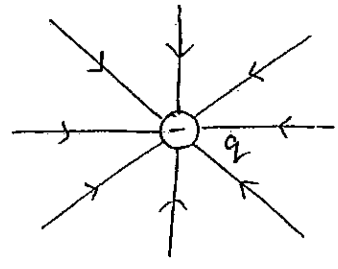
③ For volume charge distribution

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho \, dV}{r^2} \hat{r}$$

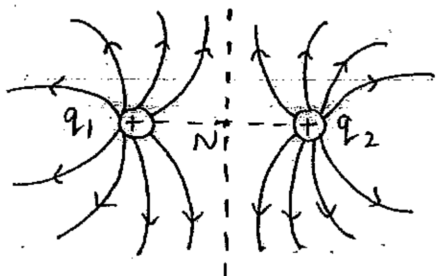
Electric field lines → These are straight or curved imaginary lines in a region such that the tangent at any point on the field line gives the direction of the electric field at that point.



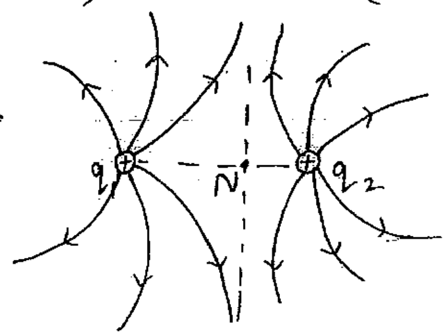
$q > 0$
(Radially outward)



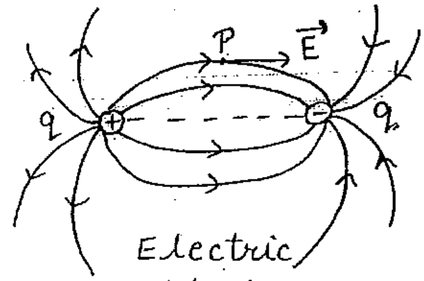
$q < 0$
(Radially inward)



$q_1 = q_2$

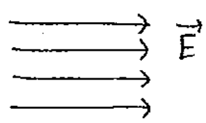


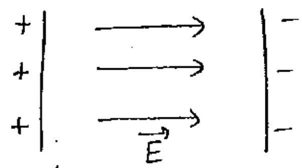
$q_1 > q_2$

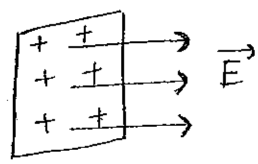


Electric dipole

A uniform electric field is represented by straight, equispaced parallel field lines







Properties of electric field lines

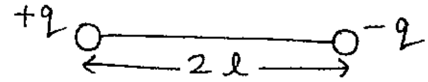
(i) These are imaginary lines which begin from positive charge and terminate on negative charge.

(ii) Two electric field lines do not cross each othe as if they meet at a point then at that point there will be two tangents means two directions of field which is not possible.

(iii) Denser field lines means stronger electric field and vice versa.

(iv) Electric field lines do not form closed loop.

Electric dipole → A pair of two equal and opposite charges separated by certain distance is called an electric dipole.



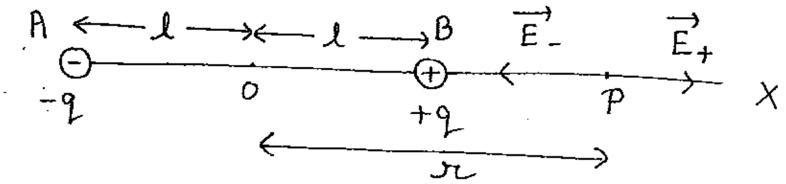
Electric dipole moment ($\vec{p}$) → It is defined as the product of the magnitude of either charge of the electric dipole and the dipole length. $\vec{p} = q\,(2\vec{l})$

Direction of $\vec{p}$ is from −ve to positive charge.

eg. Dipole moment of polar molecule is non-ze (H_2O), (NaCl)

& dipole moment of a non-polar molecule is zero.

Electric field intensity at a point on the axial line of an electric dipole →



Let us consider a point P at a distance 'r' from the centre 'O' of the dipole

$$\vec{E}_+ \text{ (due to } +q) = \frac{1}{4\pi\epsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-l)^2}$$

(along +ve X-axis)

$$\vec{E}_- \text{ (due to } -q) = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+l)^2}$$

(along −ve X-axis)

Net electric field at P, $\vec{E} = \vec{E}_+ + \vec{E}_-$

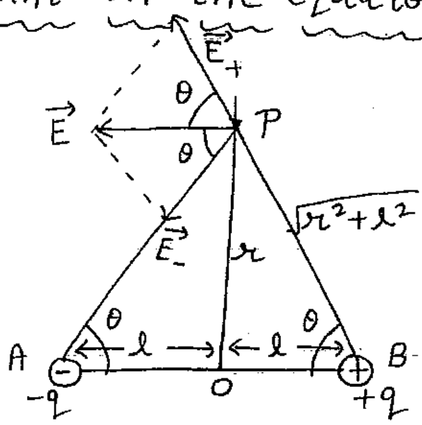
$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]\hat{i} = \frac{q}{4\pi\epsilon_0}\left[\frac{4rl}{(r^2-l^2)^2}\right]\hat{i}$$

$$|\vec{E}| = \frac{(q\cdot 2l)\cdot 2r}{4\pi\epsilon_0 (r^2-l^2)^2} = \frac{2pr}{4\pi\epsilon_0 (r^2-l^2)^2}$$

For short dipole, $E = \dfrac{2p}{4\pi\epsilon_0 r^3}$ $[\because l << r]$

$\Rightarrow E \propto 1/r^3$

Electric field intensity at a point on the equatorial line of an electric dipole



Let us consider a point P at a distance 'r' from O, on equatorial line of dipole.

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{BP^2} \text{ along BP produced}$$

$$= \frac{q}{4\pi\epsilon_0(r^2+l^2)}$$

& $$E_- = \frac{q}{4\pi\epsilon_0(r^2+l^2)} \text{ along PA}$$

Net electric field E is given by vector sum of E_+ and E_-

i.e. $$E = \sqrt{E_+^2 + E_-^2 + 2E_+E_-\cos 2\theta}$$

Now $|\vec{E}_+| = |\vec{E}_-|$

$$\Rightarrow E = \sqrt{2E_+^2 + 2E_+^2\cos 2\theta} = \sqrt{2E_+^2 \cdot 2\cos^2\theta}$$

$$\Rightarrow E = 2E_+\cos\theta$$

$$\cos\theta = \frac{l}{\sqrt{r^2+l^2}}$$

$$\Rightarrow E = \frac{2q}{4\pi\epsilon_0(r^2+l^2)} \cdot \frac{l}{\sqrt{r^2+l^2}} = \frac{p}{4\pi\epsilon_0(r^2+l^2)^{3/2}}$$

or $$\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0(r^2+l^2)^{3/2}}$$

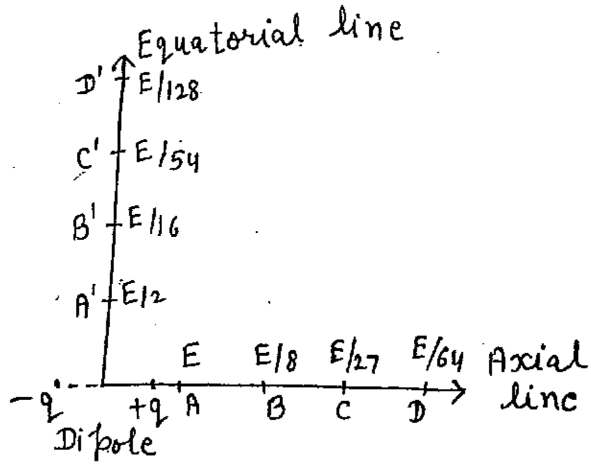
For short dipole

$$E = \frac{p}{4\pi\epsilon_0 r^3} \qquad [\because l << r]$$

or $E \propto 1/r^3$

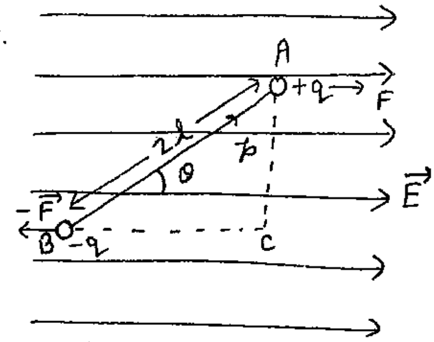
i.e. $E_{axial} = 2E_{equatorial}$

[Comparison of E_{axial} and $E_{equatorial}$]



Torque on a dipole in uniform electric field

Net force on an electric dipole in a uniform electric field is zero. Two equal and opposite forces acting on the dipole constitute a couple.



Torque = moment of couple

$\tau = qE \times AC$ $\quad\quad AC = 2l \sin\theta$ (From fig.)

$\Rightarrow \tau = qE \times 2l \sin\theta$

or $\tau = pE\sin\theta$ $\quad (\because p = q \cdot 2l)$

$\Rightarrow \vec{\tau} = \vec{p} \times \vec{E}$ $\quad\quad$ S.I. unit → Nm

Special cases : (i) If $\theta = 0$ i.e. $\vec{p}$ & $\vec{E}$ are paral

$\tau = 0$ (Stable equilibrium)

(ii) If $\theta = 90^\circ$ i.e. $\vec{p} \perp \vec{E}$

$\tau = pE$ (Maximum)

(iii) If $\theta = 30^\circ$ $\quad \tau = pE\sin 30^\circ = \frac{1}{2}pE$

(iv) If $\theta = 180^\circ$ i.e. $\vec{p}$ & $\vec{E}$ are antiparallel

$\tau = 0$ (unstable equilibrium)

Electric potential energy of an electric dipole in uniform electric field

Let dW be small work done to rotate the dipole through $d\theta$

$\Rightarrow dW = \tau\, d\theta = pE\sin\theta\, d\theta$

$\Rightarrow W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = -pE\left[\cos\theta\right]_{\theta_1}^{\theta_2}$

or $W = -pE\left[\cos\theta_2 - \cos\theta_1\right]$

If $\theta_1 = 90^\circ$ & $\theta_2 = \theta^\circ$ then

$W = -pE\cos\theta = U$

U is min. when $\theta = 0^\circ$ & U is max when $\theta = 180^\circ$

Electric flux linked with any surface is defined as the total number of electric field lines passing through that surface.

OR

Total electric flux through a surface in an electric field may be defined as the surface integral of the electric field over that surfac

$$\phi = \int \vec{E}\cdot\vec{ds} = E\,ds\cos\theta$$

(i) If θ is zero ($\vec{E}$ is $\perp$ to plane of surface) i.e. $\vec{E}$ is parallel to $\vec{ds}$

ϕ is maximum $\quad \phi = ES$

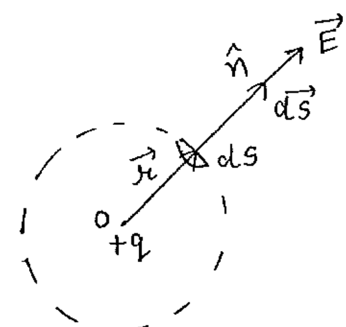
(ii) If $\theta = 90^\circ$ ($\vec{E}$ is $\parallel$ to plane of surface) i.e. $\vec{E}$ is $\perp$ to $\vec{ds}$

ϕ is minimum , $\phi = 0$

Gauss law → According to Gauss theorem, the total electric flux (ϕ) through any close surface (S) in free space is equal to $1/\epsilon_0$ times the total charge (q) enclosed by the surface.

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

Electric field intensity at every point on surface of sphere (gaussian surface) is given by



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$d\phi = \vec{E}\cdot\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}\cdot\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}ds$$

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}\oint_S ds = \frac{q}{4\pi\epsilon_0 r^2}(4\pi r^2) = \frac{q}{\epsilon_0}$$

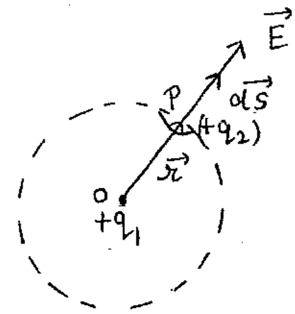
(i) If $q = 0$, $\phi = 0$ (i.e if charge inside the gaussian surface is zero)

(ii) If $q_{net} = 0$ i.e. in case of dipole $\phi = 0$

Deduction of coulomb's law from Gauss theorem

According to Gauss theorem

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{q_1}{\epsilon_0} \quad \text{or} \quad \oint_S E\,ds\cos\theta = \frac{q_1}{\epsilon_0}$$



$$\text{or} \quad E\oint_S ds = \frac{q_1}{\epsilon_0} \quad (\because \cos\theta = 1)$$

$$\text{or} \quad E \times 4\pi r^2 = \frac{q_1}{\epsilon_0} \quad \text{or} \quad E = \frac{q_1}{4\pi\epsilon_0 r^2}$$

Now $F = q_2 E = \dfrac{q_1 q_2}{4\pi\epsilon_0 r^2}$

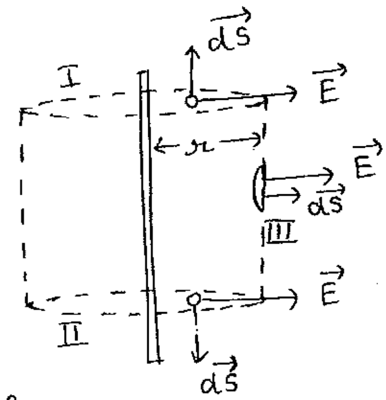
which is mathematical form of coulomb's law.

Applications of Gauss law

① Electric field intensity due to an infinitely lon straight uniformly charged wire

According to Gauss theorem

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$



$$\text{or} \quad \oint_S \vec{E}\cdot\vec{ds} = \int_{I}\vec{E}\cdot\vec{ds} + \int_{II}\vec{E}\cdot\vec{ds} + \int_{III}\vec{E}\cdot\vec{ds}$$

For I & II surface $\vec{E}\cdot\vec{ds} = 0$

as $\vec{E}$ & $\vec{ds}$ are $\perp$ to each other

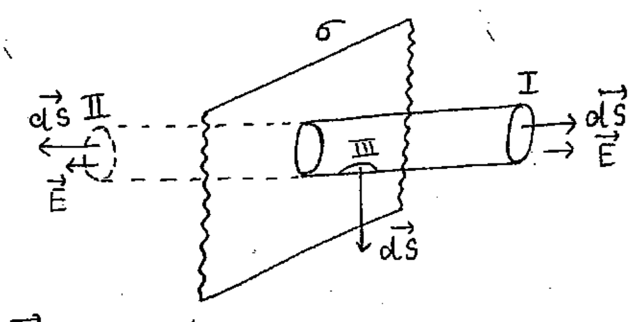
$$\Rightarrow \int_{III}\vec{E}\cdot\vec{ds} = \frac{\lambda l}{\epsilon_0} = \int_{III} E\,ds = E\int ds$$

$$\Rightarrow E\,(2\pi r l) = \frac{\lambda l}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow E \propto \frac{1}{r} \quad \& \quad \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{n}$$

② Electric field intensity due to uniformly charged infinite plane sheet

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{\sigma S}{\epsilon_0} = \frac{q}{\epsilon_0}$$



For surface III

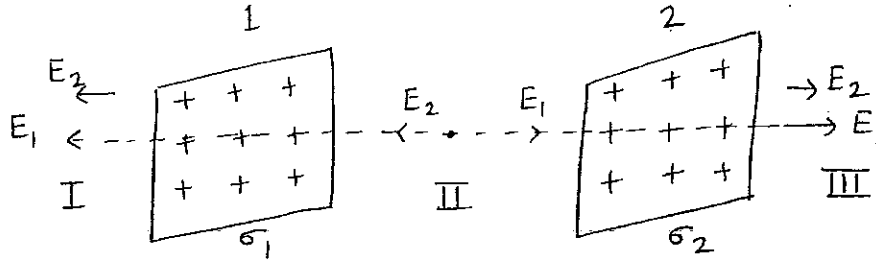
$$\int \vec{E}\cdot\vec{ds} = 0$$

$$\therefore \int_{I}\vec{E}\cdot\vec{ds} + \int_{II}\vec{E}\cdot\vec{ds} = \frac{\sigma S}{\epsilon_0}$$

$$\Rightarrow 2ES = \frac{\sigma S}{\epsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

* For sheet of finite thickness, $\vec{E} = \dfrac{\sigma}{\epsilon_0}\hat{n}$

(3) Electric field intensity due to two infinite plane sheets of charge



We know that

$$E = \frac{\sigma}{2\epsilon_0}$$

& $E_1 = \frac{\sigma_1}{2\epsilon_0}$ & $E_2 = \frac{\sigma_2}{2\epsilon_0}$

$$E_I = -E_1 - E_2 = -\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = -\frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

$$E_{II} = E_1 - E_2 = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$$

$$E_{III} = E_1 + E_2 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

Special case : $\sigma_1 = \sigma$ & $\sigma_2 = -\sigma$

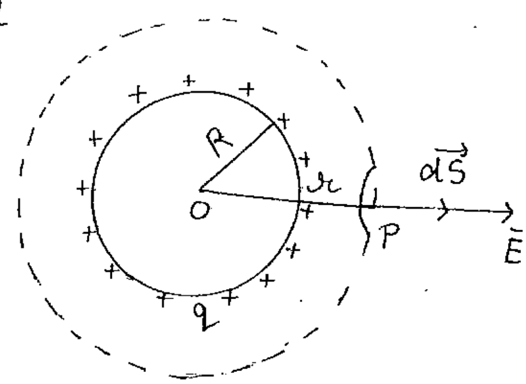
$E_I = 0$, $E_{III} = 0$ & $E_{II} = \frac{\sigma}{\epsilon_0}$

This is the case of capacitor

(4) Electric field intensity due to a uniformly charge thin spherical shell

(a) At a point outside the shell

Draw a gaussian surface in the form of a sphere of radius r ($r > R$) with O as centre.



According to Gauss theorem

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

or $\oint_S E\, dS = E \oint_S dS = E(4\pi r^2) = \frac{q}{\epsilon_0}$

or $E = \frac{q}{4\pi\epsilon_0} r^2$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

(b) At a point on the surface of shell

$$r = R$$

$$\therefore \quad E = \frac{q}{4\pi \epsilon_0 R^2}$$

Now $\quad q = \sigma \times 4\pi R^2$

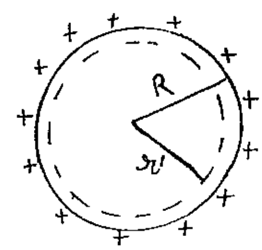
$$\therefore \quad E = \frac{\sigma}{\epsilon_0}$$

(c) At a point inside the shell

Draw a gaussian surface radius r' $(r' < R)$

According to Gauss theorem

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$



Here Gaussian surface does not enclose any charge $\therefore q = 0$

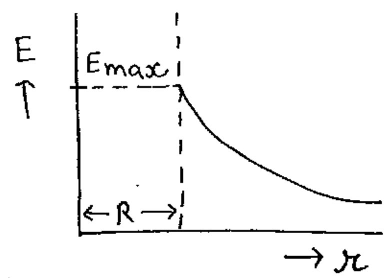
$$\therefore \oint_S \vec{E} \cdot \vec{ds} = 0 \quad \text{or} \quad \vec{E} = 0$$

Variation of electric field intensity with distance (r) from centre O of spherical shell

(i) When $r < R$, $E = 0$

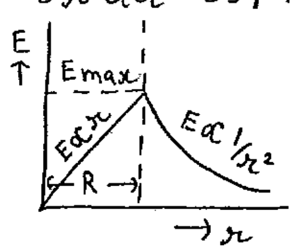
(ii) When $r = R$, $E = \dfrac{q}{4\pi R^2 \epsilon_0} = \dfrac{\sigma}{\epsilon_0}$

(iii) When $r > R$, $E = \dfrac{q}{4\pi \epsilon_0 r^2}$ or $E \propto \dfrac{1}{r^2}$.



For knowledge :

* Variation of electric field intensity due to uniformly charged solid sphere

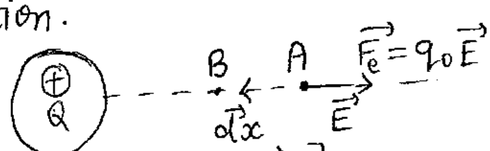


Electrostatics – II

(Electrostatic potential & Capacitance)

Electric potential at any point in the electric field is the work done per unit charge in bringing a unit positive charge from infinity to that point along any path.

Electric potential energy difference between any two points is the work done by some external force in moving a charge without acceleration from one point to another in an electric field due to any charge distribution.



$$W_{AB} = \int_A^B \vec{F}_{ext} \cdot \vec{dx}$$

$$= -q_0 \int \vec{E} \cdot \vec{dx} \quad [\vec{F}_{ext} = -\vec{F_e} = -q_0\vec{E}]$$

$$W_{AB} = U_B - U_A$$

If point A lies at ∞, then $U_\infty = 0$

& $$W_{\infty B} = U_B = -q_0 \int_\infty^B \vec{E} \cdot \vec{dx}$$

Electric potential is defined as the electric potential energy per unit charge. S.I. unit → J/C or Volt

$$V = \frac{U}{q_0} = -\int_\infty^B \vec{E} \cdot \vec{dx}$$

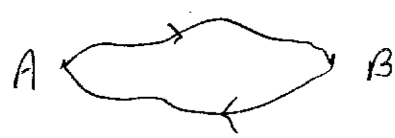
Electric potential difference, $\Delta V = V_B - V_A$

$$\Delta V = \frac{U_B}{q_0} - \frac{U_A}{q_0} = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot \vec{dx}$$

* Electrostatic force is conservative in nature as work done in a closed path is zero.

$$W_{AB}/q_0 = V_B - V_A$$

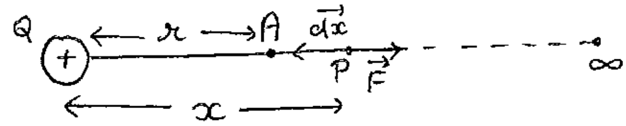
$$W_{BA}/q_0 = V_A - V_B$$



& For closed path $W_{ABA} = V_B - V_A + V_A - V_B = 0$

$$\therefore \oint \vec{E} \cdot \vec{dl} = 0$$

Electric potential due to a point charge



Let us consider a point charge Q. A unit positive charge is placed at a point P in electric field of charge Q.

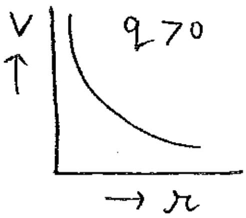
$$F = \frac{1}{4\pi\epsilon_0} \times \frac{Q \times 1}{r^2}$$

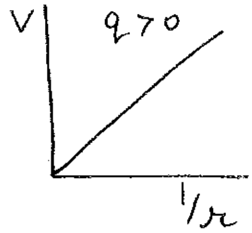
Now $dW = \vec{F} \cdot \vec{dx} = -F\,dx$ (as $\theta = 180^\circ$)

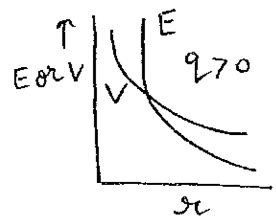
or $dW = -\dfrac{Q}{4\pi\epsilon_0 x^2}\,dx$

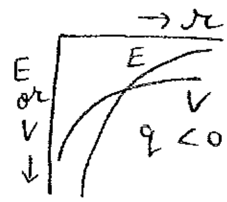
$$W = \int_{\infty}^{r} \frac{-Q}{4\pi\epsilon_0 x^2}\,dx = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = V = \frac{Q}{4\pi\epsilon_0 r}$$

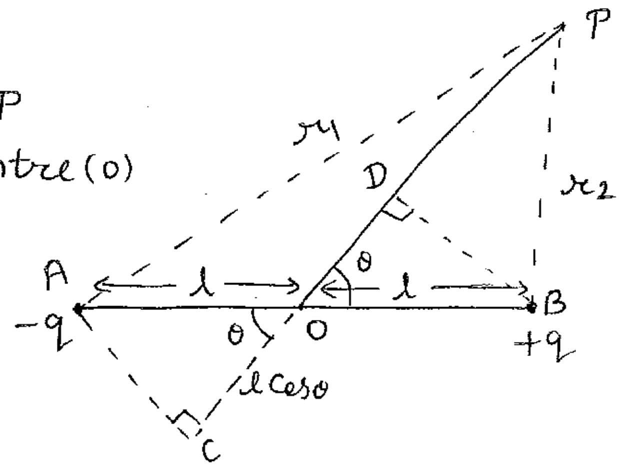








Electric potential at any point due to an electric dipole



Let us consider a point P at a distance r from centre (O) of electric dipole AB.

Potential at P due to charge $-q$

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1}$$

Potential at P due to charge $+q$

$$V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

Potential at P due to dipole

$$V = V_1 + V_2$$

$$\text{or } V = \frac{-q}{4\pi\epsilon_0 r_1} + \frac{q}{4\pi\epsilon_0 r_2}$$

$$\text{or } V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Now $r_1 \approx r + l\cos\theta$ & $r_2 \approx r - l\cos\theta$ (From fig.)

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \times \frac{2l\cos\theta}{(r^2 - l^2\cos^2\theta)} = \frac{p\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}$$

Special cases: ① If P lies on axial line of dipole i.e. $\theta = 0$ & $l << r$ (short dipole)

$$\Rightarrow V = \frac{p}{4\pi\epsilon_0 r^2}$$

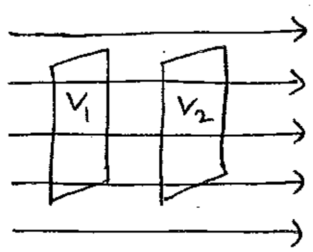
② If P lies on equatorial line of dipole i.e $\theta = 90^\circ$ & $V = 0$

* Electric potential at a point due to system of charges, $V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

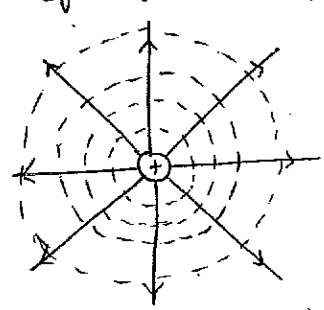
$$V = V_1 + V_2 + \text{— —} V_n$$

* Electric potential is a scalar quantity

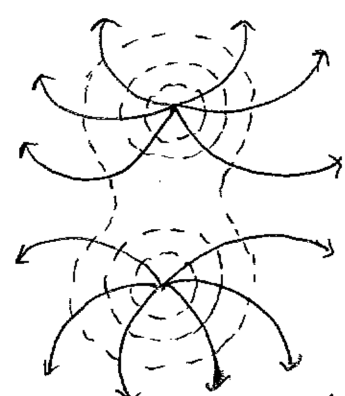
Equipotential surface → A surface at every point of which, the electric potential due to charge distribution is same is called equipotential surface.



uniform field



For isolated point charge



Pair of similar point charges

Properties of equipotential surfaces

① No work is done in moving a test charge from one point to another on an equipotential surface.

$$\frac{W_{AB}}{q_0} = V_B - V_A$$

As $V_B = V_A \quad \therefore \; W_{AB} = 0$

② The electric field is perpendicular to the element $\vec{dl}$ of the equipotential surface.

$$dW = \vec{E} \cdot \vec{dl} = 0$$

$$\Rightarrow E\,dl\cos\theta = 0 \Rightarrow \cos\theta = 0$$

$$\& \quad \theta = 90^\circ$$

$\Rightarrow \vec{E}$ is $\perp$ to $\vec{dl}$

③ $$E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

$$\text{or} \quad dr \propto \frac{1}{E}$$

It means equipotential surfaces are farther apart in the region of weak electric field.

④ Two equipotential surfaces can not intersect.

Relation between Electric field intensity & potential

$$dW = \vec{F} \cdot \vec{dr} = -q_0 E\,dr$$

$$\text{or} \quad \frac{dW}{q_0} = -E\,dr = dV$$

$$\text{or} \quad \vec{E} = -\frac{\vec{dV}}{dr}$$

Electric field intensity is negative of potential gradient (vector quantity)

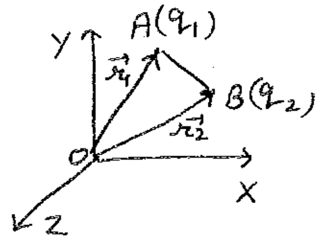
* Negative sign shows that electric field intensity is in the direction of decreasing electric potential.

Electric potential energy of a system of charges

It is the work done on a point charge in bringing it from infinity to a point in the electric field against the electrical force.

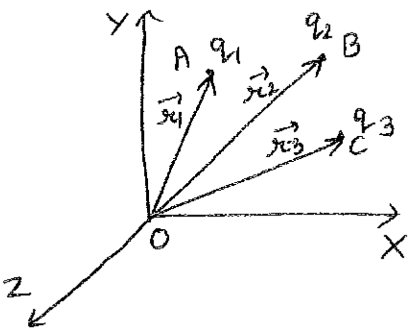
Electric potential energy of a system of point charges is defined as the total work done in bringing these charges to their respective locations from infinity to form the system.

For a system of two point charges



$$W = U = \frac{q_1 q_2}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

For a system of three point charges



$$W_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 |(\vec{r}_2 - \vec{r}_1)|}$$

$$W_3 = W_{13} + W_{23}$$

$$= \frac{q_1 q_3}{4\pi\epsilon_0 |\vec{r}_3 - \vec{r}_1|} + \frac{q_2 q_3}{4\pi\epsilon_0 |\vec{r}_3 - \vec{r}_2|}$$

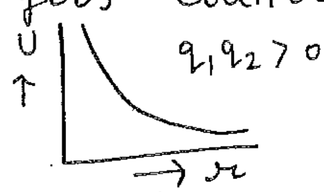
$$W = W_{12} + W_3$$

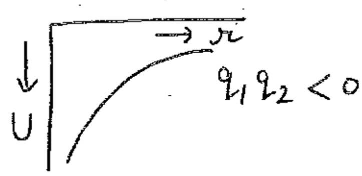
$$W = \frac{q_1 q_2}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|} + \frac{q_1 q_3}{4\pi\epsilon_0 |\vec{r}_3 - \vec{r}_1|} + \frac{q_2 q_3}{4\pi\epsilon_0 |\vec{r}_3 - \vec{r}_2|}$$

In general for system of n-charges

$$U = W = \frac{1}{2}\left[\frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} \frac{q_i q_j}{|\vec{r}_j - \vec{r}_i|}\right] = \frac{1}{2} \sum_{i=1}^{n} q_i V_i$$

The factor $1/2$ is introduced here as each term gets counted twice in the summation





Potential energy of charges in an external field

P.E. of single charge in an external field

$$W = q\,V(r) = U$$

For a system of two charges in external field

$$U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

Read only *

Relation between electric field & potential

In vector form

$$\vec{E} = -\overrightarrow{\nabla V}$$

Here $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ is a vector operator or del operator

So $\vec{E} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

or $E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

Read only *

Electric potential due to uniformly charged thin spherical shell

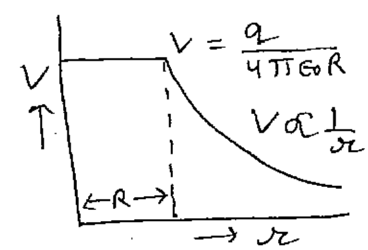
① Potential at a point outside the shell

$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}.$$

② Potential at a point on surface

$r = R$, $\quad V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

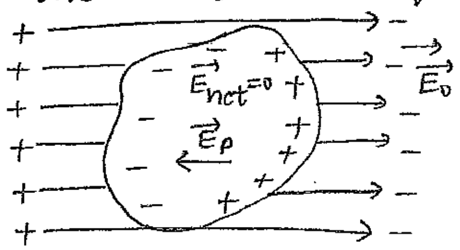
③ Potential at a point inside the shell is same as on the surface. $V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R}$



Capacitance

Behaviours of conductors in the electrostatic field

① Net electric field intensity in the interior of a conductor is zero



Induced electric field ($\vec{E}_p$) is equal & opposite to applied field ($\vec{E}_0$).

There for $\vec{E}_{net} = \vec{E}_0 - \vec{E}_p = 0$

② Electric field just outside the charged conductor is perpendicular to the surface of conductor. If $\vec{E}$ is not perpendicular to the surface then there will be two components $E\cos\theta$ & $E\sin\theta$. component $E\cos\theta$ must be zero as there is no flow of charge along the surface.

$$E\cos\theta = 0 \Rightarrow \cos\theta = 0 \quad \& \quad \theta = 90^\circ$$

③ Net charge in the interior of a conductor is zero

As $\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$ & $\vec{E}$ is zero inside the conductor then $\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0} = 0 \Rightarrow q = 0$

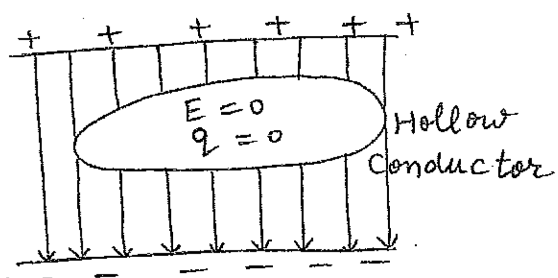
④ Charge resides on the surface of a conductor

⑤ Electric potential is constant for the entire conductor

$$dV = -E\,dr$$

As $E = 0$ for all points in the interior of the conductor $\therefore dV = 0 \Rightarrow V$ is constant

Electrostatic Shielding



The method of protecting a certain region from the effect of electric field is called electrostatic shielding.

eg. The appliances or instruments inside the hollow of a conductor remains shielded from the external field.

Dielectrics and Polarisation behaviour of non-conducting substances in an electric field

Effect of electric field on the polar substance

Effect of electric field on non-polar molecule

Polarizability $\quad \vec{p} \propto \epsilon_0 \vec{E}$.

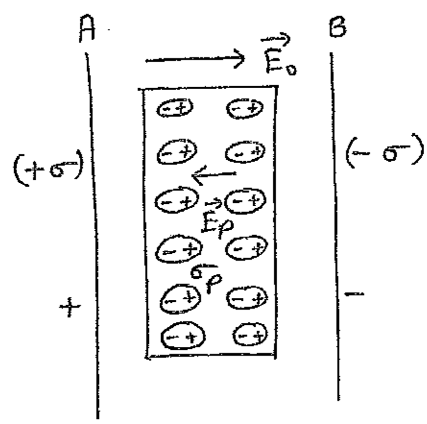
or $\quad \vec{p} = \epsilon_0 \alpha \vec{E}$

or $\quad \alpha = \dfrac{p}{\epsilon_0 E}$ = atomic polarizability

S.I. unit of α is m^3

Dielectrics → The non-conducting materials in which equal and opposite induced charges are produced on their opposite faces on the application of electric field are called dielectrics. eg. Air, glass etc

Electric Polarization → It is the process of inducing equal and opposite charges on the two opposite faces of the dielectric on the application of electric field.



$$E_0 = \frac{\sigma}{\epsilon_0} \quad , \quad E_p = \frac{\sigma_p}{\epsilon_0}$$

$$E = E_0 - E_p = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$$E = \frac{\sigma - \sigma_p}{\epsilon_0}$$

Dielectric constant $\quad K = \dfrac{E_0}{E} = \dfrac{\sigma}{\sigma - \sigma_p}$

As $E_0 > E$ so $K > 1$

Relation between Surface charge density (σ) and Induced surface charge density (σ_p)

$$E = E_0 - E_p$$

$$E = \frac{E_0}{K} = \frac{\sigma}{\epsilon_0 K}$$

$$\Rightarrow \frac{\sigma}{\epsilon_0 K} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$$\Rightarrow \sigma_p = \left(\sigma - \frac{\sigma}{K}\right) = \left(\frac{K-1}{K}\right)\sigma$$

As $K > 1$ $\therefore$ $\sigma_p < \sigma$

In case of conductor $E = 0 \Rightarrow E_0 = E_p$ & $\sigma_p = \sigma$

The Polarization vector → It is defined as the dipole moment per unit volume of the polarized dielectric.

If n is no. of atoms or molecules / volume

then $\vec{P} = n\vec{p}$ S.I. unit → C/m^2

$$|\vec{P}| = \sigma_p$$

Susceptibility & Dielectric constant

$$\vec{P} \propto \epsilon_0 \vec{E}$$

or $\vec{P} = \epsilon_0 \chi_e \vec{E}$

or $\chi_e = \frac{P}{\epsilon_0 E}$ (No unit)

Relation between χ_e & K

$E = E_0 - E_p$ or $E_0 = E + E_p = E + \frac{\sigma_p}{\epsilon_0} = E + \frac{P}{\epsilon_0}$

or $E_0 = E + \frac{\epsilon_0 \chi_e E}{\epsilon_0} = (1 + \chi_e)E$

$$\Rightarrow K = \frac{E_0}{E} = 1 + \chi_e$$

Dielectric strength → The maximum value of electric field intensity that can be applied to dielectric without electric breakdown. $E_{break} = V_{break}/d$

Electric Displacement vector $E = E_0 - E_p$

$E = \frac{\sigma - \sigma_p}{\epsilon_0}$ $\therefore$ $\vec{E}\cdot\hat{n} = \frac{\sigma - \vec{P}\cdot\hat{n}}{\epsilon_0}$ $(\because \sigma_p = \vec{P}\cdot\hat{n})$

or $(\epsilon_0 \vec{E} + \vec{P})\cdot\hat{n} = \sigma = \vec{D}\cdot\hat{n}$

$\Rightarrow$ $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ is electric displacement vector.

Electrical capacitance

Electrical capacitance is the measure of the ability of a conductor to store electric charge or energy is known as electrical capacitance.

If q be the charge on a conductor & V be its potential then $q \propto V$ or $q = CV$

or $C = \frac{q}{V}$ If $V = 1$, $C = q$

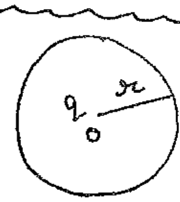
S.I. unit Farad (F) or C/V (very big unit)

other practical units μF, pF, nF

Capacitor → A capacitor consists of two conducting bodies separated by a non-conducting medium such that it can store large amount of charge in small space.

Capacitance of isolated spherical conductor

Potential at any point on the surface of sphere is given by



$$V = \frac{q}{4\pi\epsilon_0 r}$$

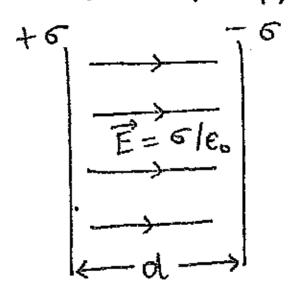
$$\text{Now} \quad C = \frac{q}{V} = 4\pi\epsilon_0 r \Rightarrow C \propto r$$

*Capacitance of Earth ≈ 711 μF = 711×10^{-6} F

Parallel plate capacitor

Principle → Capacitance of a charged conductor is appreciably increased by bringing another uncharged or low potential conductor connected to Earth near it provided there is some non-conducting medium between them.

Capacitance $E = \frac{\sigma}{\epsilon_0}$ = Electric field



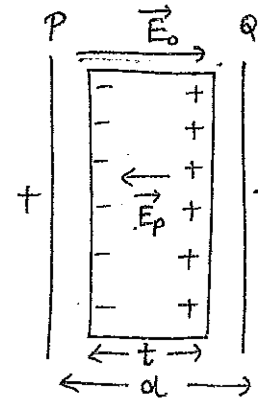
$$E = \frac{V}{d} = \frac{dV}{dr} \quad \text{or} \quad V = Ed = \frac{\sigma}{\epsilon_0} d$$

$$\sigma = \frac{q}{A} \quad \therefore \quad V = \frac{q}{A\epsilon_0} d$$

$$\text{Now} \quad C = q/V = \epsilon_0 A/d$$

Capacitance of parallel plate capacitor with dielectric slab

→ Capacitance of parallel plate capacitor having air or vacuum between the plates



$$C_0 = \frac{\epsilon_0 A}{d}$$

Potential difference between the two plates of the capacitor is

$$V = E_0(d-t) + Et$$

$$\because \frac{E_0}{E} = K \quad \text{or} \quad E = \frac{E_0}{K}$$

$$V = E_0(d-t) + \frac{E_0}{K}t = E_0[d-t+t/K]$$

$$\text{or } V = \frac{\sigma}{\epsilon_0}[d-t+t/K] = \frac{q}{A\epsilon_0}[d-t+t/K]$$

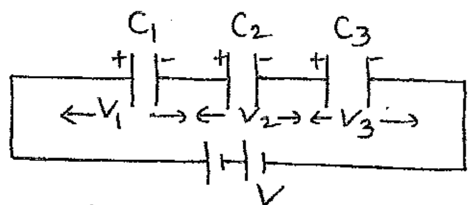
$$C = \frac{q}{V} = \frac{\epsilon_0 A}{(d-t+t/K)} = \frac{\epsilon_0 A}{d\left[1-\frac{t}{d}+\frac{t}{dK}\right]} = \frac{C_0}{\left[1-\frac{t}{d}+\frac{t}{dK}\right]}$$

$$\Rightarrow \quad C > C_0$$

* If $t = d$ then $C = KC_0$ or $K = C/C_0$

Combination of capacitors

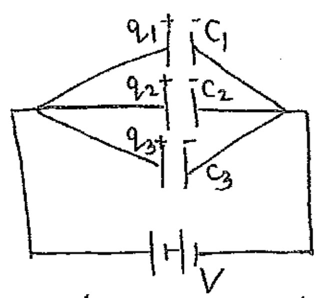
① In series



$$V = V_1 + V_2 + V_3$$

$$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \quad \text{or} \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

② In parallel



$$q = q_1 + q_2 + q_3$$

$$q = C_1V_1 + C_2V_2 + C_3V_3 = (C_1 + C_2 + C_3)V$$

$$C_p = C_1 + C_2 + C_3$$

Energy stored in a charged capacitor

Let at any instant q be the charge on capacitor

$$\therefore \quad V = \frac{q}{C}$$

Now $dW = dU = V\,dq = \frac{q}{C}dq$ (small work done)

$$U = \int_0^Q \frac{q}{C}dq = \left(\frac{q^2}{2C}\right)_0^Q = Q^2/2C$$

$$\text{or } U = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Effect of dielectric introduced between the plates of the capacitor on energy stored in capacitor

(i) When battery is disconnected

$$U_0 = \frac{Q_0^2}{2C_0} \quad \text{Now } C = KC_0 \Rightarrow U = \frac{U_0}{K}$$

As $K > 1$ $\therefore U < U_0$ so energy stored decreases

(ii) When battery remains connected

$$U_0 = \frac{Q_0^2}{2C_0} \quad \text{Now } Q = KQ_0 \ \& \ C = KC_0$$

$\Rightarrow U = KU_0$ $\therefore U > U_0$ (energy increases)

Energy stored in combination of capacitors

① Series combination

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \text{——} \frac{1}{C_n}$$

$$U = \frac{Q^2}{2C_s} = \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \text{——} \frac{1}{C_n}\right]$$

$$\text{or } U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \text{——} \frac{Q^2}{2C_n} = U_1 + U_2 + \text{——} U_n$$

② Parallel combination

$$C_P = C_1 + C_2 + \text{——} C_n$$

$$U = \frac{1}{2}C_pV^2 = \frac{V^2}{2}(C_1 + C_2 + \text{——} C_n)$$

$$\text{or } U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \text{——} \frac{1}{2}C_nV^2 = U_1 + U_2 + \text{——} U_n$$

Energy density in parallel plate capacitor

$$U = \frac{1}{2}CV^2 \quad \& \quad C = \frac{\epsilon_0 A}{d} \quad \& \quad V = Ed$$

$$\Rightarrow U = \frac{1}{2}\frac{\epsilon_0 A}{d}E^2d^2 = \frac{1}{2}\epsilon_0 A d E^2$$

$$\text{Energy density} = \frac{U}{V} = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2 \ (J/m^3)$$

Common Potential when two capacitors having capacitances C_1 & C_2 are connected in parallel then $q = C_1V_1 + C_2V_2$

After connection, on sharing $q' = C_1V + C_2V$

$\Rightarrow C_1V_1 + C_2V_2 = (C_1 + C_2)V$ (charge conservation)

$$\Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

* ✓ There is loss of energy on sharing charges

Current Electricity – I

Electric Current → The rate of flow of electric charges flowing through any cross-section of a conductor.

$$I = \frac{dQ}{dt} \quad , \quad Q = \int I\,dt$$

Direction of conventional current is the direction of flow of positive charges.

S.I. unit → 1 Ampere = 1 Coulomb/sec.

Types of current (i) Steady Direct current (D.C)
(ii) Variable Direct current
(iii) Alternating current (A.C)

Drift velocity → It is the average velocity with which free electrons in a conductor get drifted in a direction opposite to the direction of applied electric field.

Force experienced by a free electron is given by $\vec{F} = q\vec{E} = -e\vec{E}$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$$

If $\vec{u}_{av.}$ is initial velocity due to thermal energy & $\tau_{av.}$ is relaxation time then

$$\vec{v}_d = \vec{u}_{av.} + \vec{a}\,\tau_{av.}$$

$$= \frac{\vec{u}_1 + \vec{u}_2 + \text{---} \vec{u}_n}{n} + \vec{a}\,\frac{(\tau_1 + \tau_2 + \text{---} \tau_n)}{n}$$

$$= 0 + a\,\tau_{av.}$$

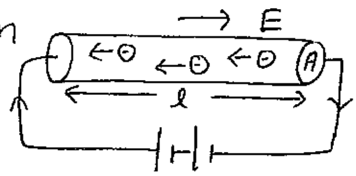
$$\Rightarrow \vec{v}_d = -\frac{e\vec{E}}{m}\tau_{av.}$$

$$|\vec{v}_d| = \frac{eE\tau}{m} \quad [\because \tau_{av.} = \tau]$$

Mobility (μ) → Drift velocity per unit electric field ; $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$

Relation between current & drift velocity

Let A be area of cross-section
l is length of conductor



Now $E = \frac{V}{l}$

If n is number density of electrons then total no. of electrons is nAl

& Total charge on conductor $= (nAl)e$

Now $t = \frac{l}{v_d}$

$$\Rightarrow I = \frac{Q}{t} = \frac{nAle\,v_d}{l} = neAv_d$$

For the derivation of resistivity *

Now $v_d = \frac{eE\tau}{m}$

$$\Rightarrow I = \frac{ne^2A\tau E}{m} = \frac{ne^2A\tau V}{ml}$$

Now $I = \frac{V}{R} \Rightarrow R = \frac{ml}{ne^2\tau A}$

& $R = \rho\frac{l}{A} \Rightarrow \rho = \frac{m}{ne^2\tau}$

Ohm's law → The current flowing through a conductor is directly proportional to potential difference (V) across the ends of conductor provided physical conditions remain same.

$I \propto V$ or $V \propto I$

$$\Rightarrow V = IR \quad \text{or} \quad \frac{V}{I} = R$$

From above $R = \frac{ml}{ne^2\tau A}$

or $R \propto l$ & $R \propto \frac{1}{A}$

S.I. unit → ohm = Volt / Ampere

Factors affecting resistivity or specific resistance

$$\rho = \frac{m}{ne^2\tau}$$

$$\Rightarrow \rho \propto \frac{1}{n} \quad , \rho \propto \frac{1}{\tau}$$

(i) Resistivity is different for different materials (for different values of n). So it depends on nature of material.

(ii) Relaxation time τ decreases with increase in temperature in conductors so increase in temperature results in increase in resistivity in conductors.

* In semiconductors, $\rho = \rho_0 e^{-E_g/2KT}$ so as temperature increases, resistivity decreases.

* In insulators, at absolute zero, resistivity is infinitely large & $\rho = \rho_0 e^{E_g/KT}$

Current Density → It is defined as the amount of current passing per unit area of conductor held perpendicular to the flow of charges.

$$J = \frac{I}{A}$$

It is a vector quantity whose direction is same as the direction of conventional current $\quad I = \int_S \vec{J} \cdot d\vec{A}$

conductance, $G = 1/R$ ohm^{-1} or mho or siemen

conductivity, $\sigma = 1/\rho \quad \Omega^{-1} m^{-1}$

Relation between J, σ & E
(Microscopic form of ohm's law)

We know that $I = neAv_d$

Now $v_d = \frac{eE\tau}{m}$

$\Rightarrow \quad I = \frac{ne^2 E\tau A}{m}$

$\Rightarrow \quad \frac{I}{A} = \frac{ne^2\tau}{m} E = J$.

$\rho = \frac{m}{ne^2\tau}$ or $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

$\Rightarrow \quad \boxed{J = \sigma E}$

Effect of temperature on resistance of a conductor

$$R = R_0(1 + \alpha\Delta\theta)$$

or $$\alpha = \frac{R - R_0}{R_0\Delta\theta}$$

* α is positive for metallic conductors
* α is −ve for insulators and semiconductors
* α is very small for alloys (eg. constantan and manganin)

Limitations of ohm's law

(i) $V \propto I$ is not valid for high voltages & currents as temperature does not remain constant in that case.

(ii) ohm's law is not valid for few cases

(a) Semiconductor diodes (b) Transistors

(c) Thyristors (d) For materials like GaAs, I decreases with increase in V after certain values.

Thermistors → It is highly temperature dependent resistor.

Thermistors can be of two types:

(i) Thermistor with negative temperature co-efficient of resistivity

(ii) Thermistor with positive temperature co-efficient of resistivity.

* They are used in electronic circuits for controlling the variations in applied voltages. (eg. Voltage regulators)

Read only

Super-conductivity → It is the property by virtue of which a metal, alloy, oxide or a poor conductor shows almost zero resistance at a very low temperature is called superconductivity.

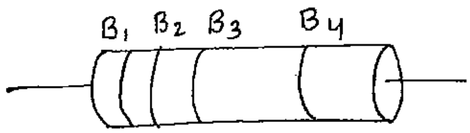
eg. The resistance of mercury (Hg) drops to zero below 4.2 K

Carbon colour code resistors

B	B	R	O	Y	G	B	V	G	W
0	1	2	3	4	5	6	7	8	9

Tolerance

Gold	Silver	No colour
5%	10%	20%



Black, Brown, Red, orange, Yellow, Green, Blue, violet, Grey, White

eg. $B_1 \rightarrow$ Green $\quad 5\ 1 \times 10^4 \pm 5\%$

$B_2 \rightarrow$ Brown $\quad B_1 B_2 \times 10^{B_3} \pm B_4\%$

$B_3 \rightarrow$ Yellow

$B_4 \rightarrow$ Gold

Electric Energy → work done (W) is given by

$$W = Vq = VIt$$

Now $V = IR$

$$\Rightarrow \quad W = \text{Energy} = I^2Rt$$

or $\frac{V^2}{R}t$ S.I. unit → Joule

Electric Power → $P = \frac{dW}{dt} = \frac{W}{t} = \frac{I^2Rt}{t} = I^2R$

or $P = \frac{V^2}{R} = VI$ S.I. unit → Watt

1 h.p. = 746 Watt (other units)

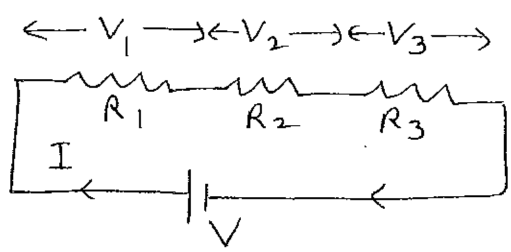
1 K Watt = 1000 Watt

* Commercial unit of electrical energy is KWhr.

$$1\,KWhr = 3.6 \times 10^6\,J$$

Combination of resistors

(i) In Series

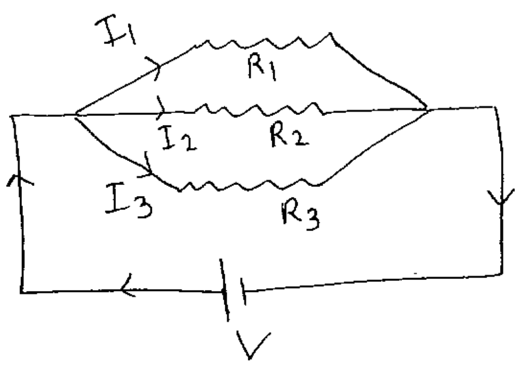


$$V = V_1 + V_2 + V_3$$

$$IR_S = IR_1 + IR_2 + IR_3$$

$$\Rightarrow \quad R_S = R_1 + R_2 + R_3$$

(ii) In parallel



$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

* If there are two resistors then $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

or $$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

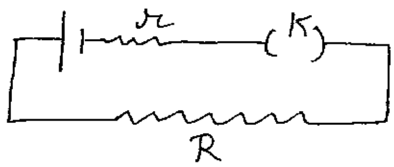
E.M.F. of a cell → It is potential difference between the terminals of the cell when no current flows (i.e. open circuit)

Terminal potential difference → It is potential difference between the terminals of the cell in closed circuit.

Internal resistance of a cell → It is opposition offered by the electrolyte or electrodes of a cell to the flow of current. It is denoted by 'r'.

Relation between E.M.F. & Terminal P.D. of cell

If E be the e.m.f. of cell & V be the P.D. then



$$E = I(R+r)$$

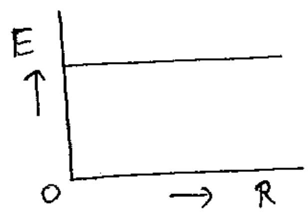
or $$I = \frac{E}{R+r}$$

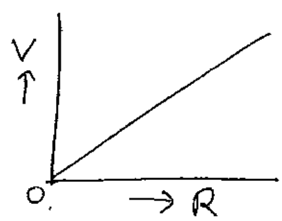
Now $V = IR \Rightarrow E = V + Ir$

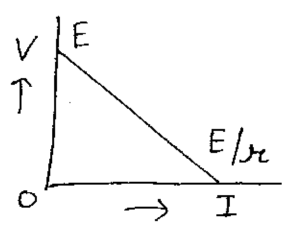
or $$r = \frac{E-V}{I} = \frac{(E-V)R}{V} = \left(\frac{E}{V} - 1\right)R$$

$$\boxed{r = \left[\frac{E}{V} - 1\right]R}$$

Variations of E and V with R







* When the cell / battery is charged then charger voltage $V >$ e.m.f. (E) of the cell

$$\Rightarrow V = E + Ir$$

Combination of cells

(i) In Series

(a) Identical cells in series

$$E_{eff.} = nE \quad , r_{eff.} = nr$$

$$I = \frac{nE}{R+nr}$$

* If $R >> nr$ then $I \approx \frac{nE}{R}$

* If $R << nr$ then $I = \frac{nE}{nr} = \frac{E}{r}$

[i.e current due to single cell.]

(b) Different cells in series

$$E_{eq.} = E_1 + E_2 + \text{——} E_n$$

$$\& \ r_{eq.} = r_1 + r_2 + \text{——} r_n$$

(ii) In Parallel

(a) Identical cells in parallel

$$E_{eff.} = E$$

$$\frac{1}{r_{eff}} = \frac{1}{r_1} + \frac{1}{r_2} + \text{———} \text{ upto } m \text{ terms}$$

$$r_1 = r_2 = \text{——} r_m$$

$$r_{eff} = \frac{r}{m}$$

$$I = \frac{E}{\frac{r}{m}+R} = \frac{mE}{r+mR}$$

* If $R >> r$ then $I = \frac{E}{R}$

* If $R << r$ then $I = mE/r$

(b) Different cells in parallel (Two cells)

$$E_{eq.} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

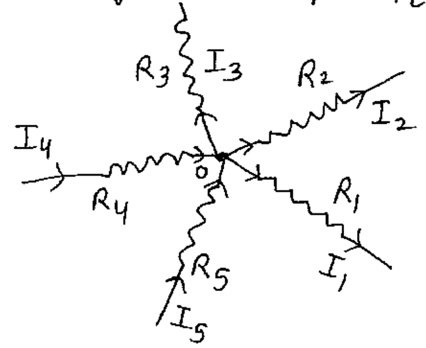
$$r_{eff.} = \frac{r_1 r_2}{r_1 + r_2} \quad , E_{eq} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) r_{eq.}$$

Current Electricity - II

Kirchhoff's Laws

① **Kirchoff's current law** → It states that the sum of all the currents entering any point (or junction) must be equal to the sum of all currents leaving that point.

$$\Sigma I = 0$$



$$I_1 + I_2 + I_3 = I_4 + I_5$$

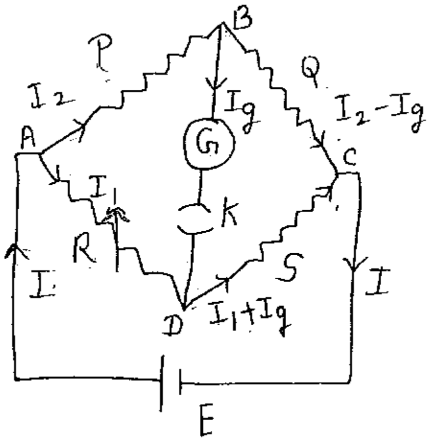
or $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

or $\Sigma I = 0$

② **Kirchhoff's voltage law** → It states that the algebraic sum of all voltages i.e. the potential differences across all elements and e.m.f.s of all sources in any closed electrical circuit is zero.

$$\Sigma E + \Sigma \Delta V = 0$$

Wheatstone Bridge → It is an arrangement of four resistors in the form of a bridge.



Principle → When key K is closed and R is adjusted such that galvanometer shows no deflection then

$$\frac{P}{Q} = \frac{R}{S}$$

or $S = \left(\frac{Q}{P}\right) R$

Knowing P, Q & R, the value of S can be calculated.

<u>Proof</u> → Applying Kirchhoff's laws, loop ABDA

$$-I_2 P - I_g G + I_1 R = 0$$

In loop BCDB

$$-(I_2 - I_g) Q + (I_1 + I_g) S + I_g G = 0$$

If Wheatstone bridge is balanced then

$$-I_2 P + I_1 R = 0 \qquad \{\because I_g = 0\}$$

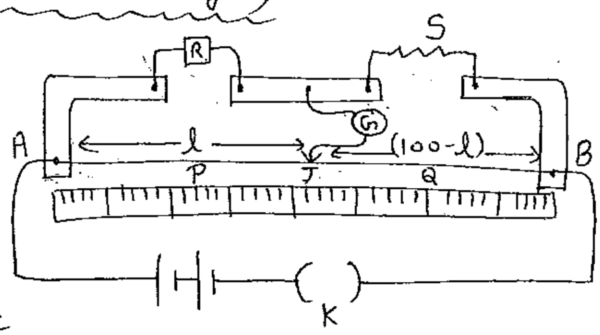
& $-I_2 Q + I_1 S = 0$

$$\Rightarrow \quad I_2 P = I_1 R \quad \& \quad I_2 Q = I_1 S$$

$$\Rightarrow \quad \frac{P}{Q} = \frac{R}{S} \quad \left\{ \text{or} \quad \frac{P}{R} = \frac{Q}{S} = \frac{I_1}{I_2} \right\}$$

(Hence Proved)

Meter Bridge (Slide Wire bridge)



P → Resistance of wire between A & J

Q → Resistance of wire between B & J

If r is resistance per unit length of wire then

$$P = rl \quad \& \quad Q = r(100 - l)$$

Now $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{R}{S} = \frac{l}{100-l}$ or $S = \left(\frac{100-l}{l}\right) R$

Knowing the values of l & R, the value of unknown resistance S can be calculated.

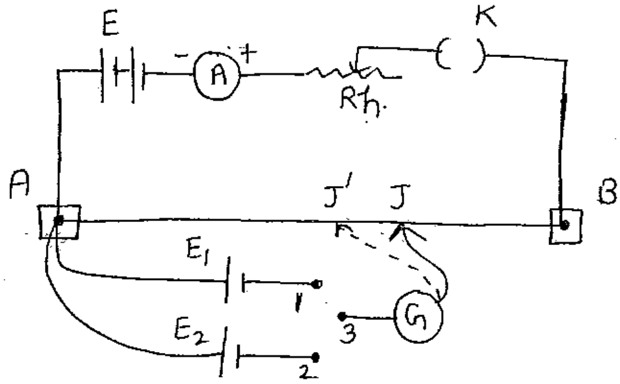
<u>Potentiometer</u> → A potentiometer is a device that can be used to measure the e.m.f. of a source (cell) without drawing any current from the source.

<u>Principle</u> → $V \propto l$ or $V = kl$ is principle of potentiometer where $k = I\rho/A$

Applications

① Comparison of e.m.f.s of two cells using potentiometer

When circuit is closed and key is inserted between 1 & 3 then cell E_1 is brought in the circuit & null point (zero deflection) is obtained at J i.e.



$$E_1 = V_{AJ} \qquad \text{Now} \quad V_{AJ} = k l_1$$

$$\Rightarrow \quad E_1 = k l_1 \quad \text{— I}$$

Now cell E_2 is brought into the circuit by putting the key between 2 & 3. Now null point is obtained at J' i.e.

$$E_2 = V_{AJ'} \qquad \text{Now} \quad V_{AJ'} = k l_2$$

$$\Rightarrow \quad E_2 = k l_2 \quad \text{— II}$$

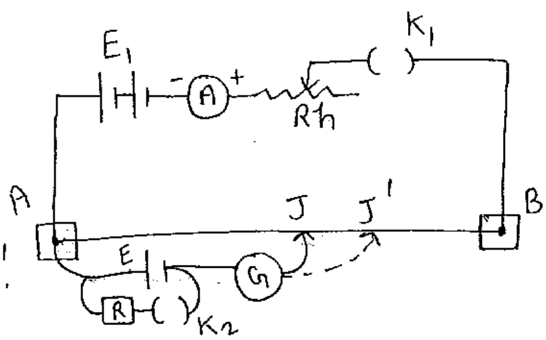
From I & II, we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

By knowing the values of l_1 & l_2, we can calculate E_1 / E_2

② Determination of internal resistance of cell using potentiometer

Close the key K_1 keeping K_2 open, find the null point on wire at J and now put key k_2 and find the null point J'.



Let $AJ = l_1$ & $AJ' = l_2$

$\Rightarrow E = V_{AJ} = k l_1$

When key is closed, potential difference

$V = k l_2 = V_{AJ'}$

$$\Rightarrow \frac{E}{V} = \frac{l_1}{l_2}$$

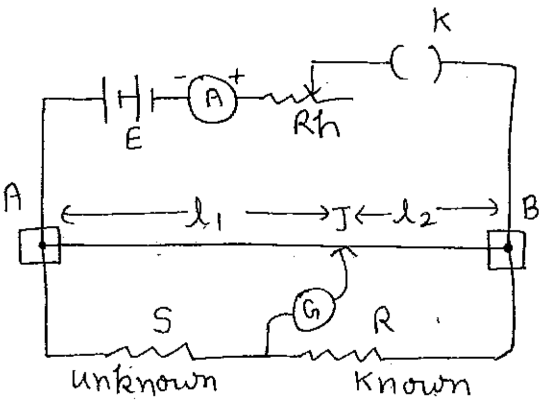
Now $r = \left(\frac{E}{V} - 1\right) R$

or $r = \left(\frac{l_1}{l_2} - 1\right) R = \left(\frac{l_1 - l_2}{l_2}\right) R$

By knowing the values of l_1, l_2 & R, internal resistance 'r', can be calculated.

(3) Measurement of unknown small resistance using potentiometer

To find the unknown resistance S, find the null point on AB i.e. J



Now according to balanced Wheatstone bridge condition

$$\frac{l_1}{S} = \frac{l_2}{R}$$

or $$S = \frac{l_1}{l_2} R$$

By knowing the value of l_1, l_2 & R, the value of S can be calculated.

* Potentiometer can be considered as an ideal voltmeter of infinite resistance. It is used to measure small potential difference.