Course: Electronic Engineering UEC001

Topic: Boolean Algebra

Presentation by

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Basic Identities of Boolean Algebra

- 1. $X + 0 = X$

OR +

A	0	RESULT
0	0	0
1	0	1

- 2. $X \cdot 1 = X$

AND •

A	1	RESULT
0	1	0
1	1	1

- 3. $X + 1 = 1$
- 4. $X \cdot 0 = 0$
- 5. $X + X = X$
- 6. $X \cdot X = X$

Basic Identities (2)

- 7. $X + X' = 1$

X	X'	RES
0	1	1
1	0	1

- 8. $X \cdot X' = 0$

X	X'	RES
0	1	0
1	0	0

- 9. $(X')' = X$

Basic Properties (Laws)

- Commutative
  - 10. $X + Y = Y + X$
- Associative
  - 12. $X+(Y+Z)=(X+Y)+Z$
- Distributive
  - 14. $X(Y+Z) = XY+XZ$
  - AND distributes over OR

- Commutative
  - 11. $X \cdot Y = Y \cdot X$
- Associative
  - 13. $X(YZ) = (XY)Z$
- Distributive
  - 15. $X+YZ=(X+Y)(X+Z)$
  - OR distributes over AND

Basic Properties (2)

- DeMorgan's Theorem
- Very important in simplifying equations
  - 16. $(X + Y)' = X' \cdot Y'$
  - 17. $(XY)' = X' + Y'$

X	Y	X+Y	$\overline{X+Y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \bullet \overline{Y}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Simplify, simplify

- These properties (Laws and Theorems) can be used to simplify equations to their simplest form.
  - Simplify F=X'YZ+X'YZ'+XZ

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

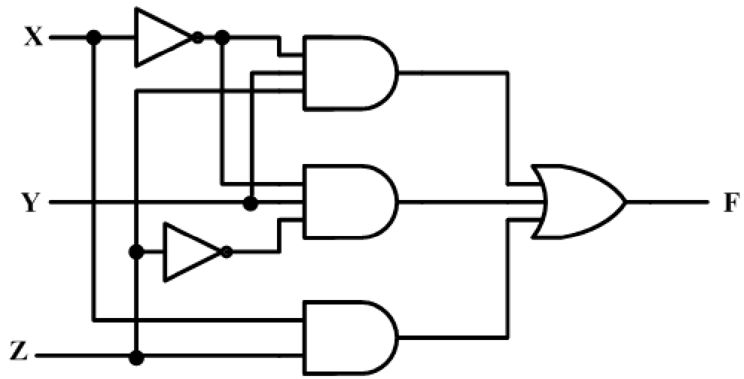
$$= \overline{X}Y(Z + \overline{Z}) + XZ \qquad \text{by identity 14}$$

$$= \overline{X}Y \bullet 1 + XZ \qquad \text{by identity 7}$$

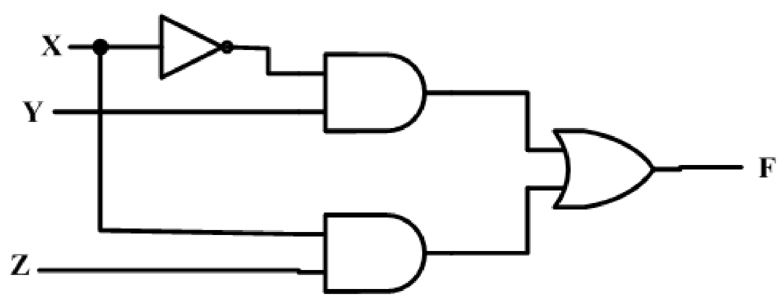
$$= \overline{X}Y + XZ \qquad \text{by identity 2}$$

Affect on implementation

- F = X’YZ + X’YZ’ + XZ



- Reduces to F = X’Y + XZ



More examples

– 1. $X + XY$
$= X(1+Y)$
$= X \cdot 1 = X$

– 2. $XY+XY'$
$= X(Y + Y')$
$= X \cdot 1 = X$

– 3. $X+X'Y$
– $= (X+X')(X+Y)$
– $= 1 \cdot (X+Y) = X+Y$

– 4. $X \cdot (X+Y)$
$=X \cdot X+X \cdot Y$
$= X+XY=X(1+Y)$
$=X \cdot 1=X$

5. $(X+Y) \cdot (X+Y')$
$=XX+XY'+XY+YY'$
$=X+XY'+XY+0$
$=X(1+Y'+Y)$
$=X \cdot 1=X$

Consensus Theorem

- The Theorem gives us the relationship
  - $XY + X'Z + YZ = XY + X'Z$

$$
\begin{aligned}
XY + X'Z + YZ &\rightarrow XY + X'Z + (X + X')YZ \\
&\rightarrow XY + X'Z + XYZ + X'YZ \\
&\rightarrow (XY + XYZ) + (X'Z + X'YZ) \\
&\rightarrow XY(1 + Z) + X'Z(1 + Y) \\
&\rightarrow XY + X'Z
\end{aligned}
$$

Application of Consensus Theorem

- Consider
  - $(A+B)(A'+C) = AA' + AC + A'B + BC$
  - $= AC + A'B + BC$
  - $= AC + A'B$

Canonical and standard forms

Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

Similarly, it may be easily verified that

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

$$f_1' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

If we take the complement of f_1', we obtain the function f_1:

$$f_1 = (x + y + z)(x + y' + z)(x + y' + z')(x' + y + z')(x' + y' + z)$$
$$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

Similarly, it is possible to read the expression for f_2 from the table:

$$f_2 = (x + y + z)(x + y + z')(x + y' + z)(x' + y + z)$$
$$= M_0 M_1 M_2 M_4$$

Express the Boolean function $F = A + B'C$ in a sum of minterms. The function has three variables, A, B, and C. The first term A is missing two variables; therefore:

$$A = A(B + B') = AB + AB'$$

This is still missing one variable:

$$\begin{aligned} A &= AB(C + C') + AB'(C + C') \\ &= ABC + ABC' + AB'C + AB'C' \end{aligned}$$

The second term $B'C$ is missing one variable:

$$B'C = B'C(A + A') = AB'C + A'B'C$$

Combining all terms, we have

$$\begin{aligned} F &= A + B'C \\ &= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C \end{aligned}$$

But $AB'C$ appears twice, and according to theorem 1 $(x + x = x)$, it is possible to remove one of them. Rearranging the minterms in ascending order, we finally obtain

$$\begin{aligned} F &= A'B'C + AB'C' + AB'C + ABC' + ABC \\ &= m_1 + m_4 + m_5 + m_6 + m_7 \end{aligned}$$

■

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

$$F = A + B'C$$

Truth Table for $F = A + B'C$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Express the Boolean function $F = xy + x'z$ in a product of maxterm form. First, convert the function into OR terms using the distributive law:

$$
\begin{aligned}
F = xy + x'z &= (xy + x')(xy + z) \\
&= (x + x')(y + x')(x + z)(y + z) \\
&= (x' + y)(x + z)(y + z)
\end{aligned}
$$

The function has three variables: x, y, and z. Each OR term is missing one variable; therefore:

$$
\begin{aligned}
x' + y &= x' + y + zz' = (x' + y + z)(x' + y + z') \\
x + z &= x + z + yy' = (x + y + z)(x + y' + z) \\
y + z &= y + z + xx' = (x + y + z)(x' + y + z)
\end{aligned}
$$

Combining all the terms and removing those that appear more than once, we finally obtain:

$$
\begin{aligned}
F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z') \\
&= M_0 M_2 M_4 M_5
\end{aligned}
$$

■

A convenient way to express this function is as follows:

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

The product symbol, Π, denotes the ANDing of maxterms; the numbers are the maxterms of the function.

We can't multiply here

$$(x+\bar{x})(y+\bar{x}) \quad (x+z)(y+z)$$

$$(xy+\cancel{x\bar{x}}+\bar{x}y+\bar{x}\bar{x})\,(xy+xz+yz+\underbrace{zz}_{z})$$

$$(xy+\bar{x}y+\bar{x})\,(xy+xz+yz+z)$$

$$(xy+(\overbrace{1+y}^{1})\bar{x}) \quad (xy+z(\overbrace{1+y}^{1})+xz)$$

$$(xy+\bar{x}) \quad (xy+z+xz)$$

$$(xy+\bar{x}) \quad (xy+(\overbrace{1+x}^{1})z)$$

$$(xy+\bar{x}) \quad (xy+z)$$

Binary Arithmetic Operations: Subtraction

- Learn new borrow rules
  - 0-0 = 0b0 (result 0 with borrow 0)
  - 1-0 = 1b0
  - 0-1 = 1b1
  - 1-1 = 0b0

			0	0	1	1	1	1	1	0	0	**Borrow**
X	229			1	1	1	0	0	1	0	1	
Y	- 46	-		0	0	1	0	1	1	1	0	
	183			1	0	1	1	0	1	1	1	

Binary Codes

Decimal digit	(BCD) 8421	Excess-3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

- To code a number with *n* decimal digits, we need *4n* bits in BCD
  e.g. $(365)_{10} = (0011\ 0110\ 0101)_{BCD}$

- This is different to converting to binary, which is $(365)_{10} = (101101101)_2$

BCD Addition

- Example: Add 448 and 489 in BCD.

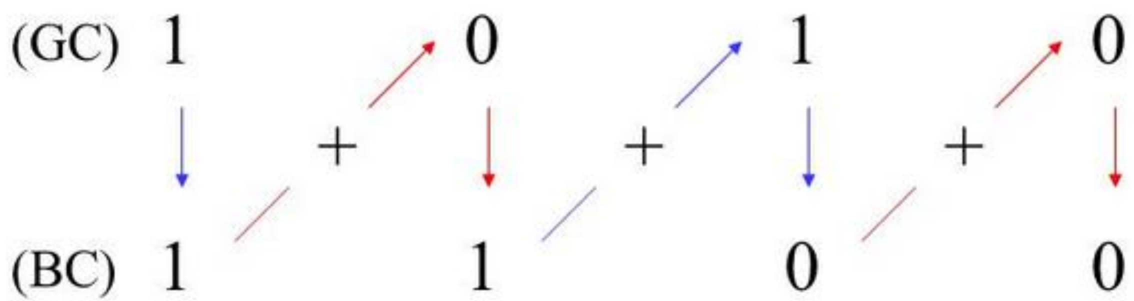
0100	0100	1000	(448 in BCD)
0100	1000	1001	(489 in BCD)
		10001	(greater than 9, add 6)
		1 0111	(carry 1 into middle digit)
	1101		(greater than 9, add 6)
1001	1 0011		(carry 1 into leftmost digit)
1001	0011	0111	(BCD coding of 937_{10})

Binary to Gray Code Conversion

$$
\begin{array}{lccccccc}
\text{(BC)} & 1 & \rightarrow + \rightarrow & 1 & \rightarrow + \rightarrow & 0 & \rightarrow + \rightarrow & 0 \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{(GC)} & 1 & & 0 & & 1 & & 0
\end{array}
$$

- MSB does not change as a result of conversion
- Start with MSB of binary number and add it to neighboring binary bit to get the next Gray code bit
- Repeat for subsequent Gray coded bits

Gray To Binary



- MSB does not change as a result of conversion
- Start with MSB of binary number and add it to the second MSB of the Gray code to get the next binary bit
- Repeat for subsequent binary coded bits