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ICT (1)

Q1. (i) all possible combinations = $10 \times 10 \times 10 = 1000$.

(ii) for letters $= 26 \times 26 \times 26 = 17576$.
for numbers $= 10 \times 10 \times 10 = 1000$.

Combining $= 17576 \times 1000$
$= 1,75,76,000$.

(iii) for 10 questions
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^{10} = 9765625$.

Q2

(i) $^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 8 \times 9 \times 10 = 720$.

(ii) $^{20}P_2 = \frac{20!}{(20-2)!} = \frac{20!}{18!} = 20 \times 19 = 380$.

(iii) $^{10}P_{10} = \frac{10!}{0!} = \frac{10!}{1} = 3628800$.

Q3 (i) $^{10}C_3 = \frac{10!}{3!\,7!} = 120$.

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(ii) $^{10}C_6 = \frac{10!}{6!\,4!} = 210.$

Q4.

Let T = student is over 6 feet tall

$P(M) = 2/5 \qquad P(T/M) = 4/1000$

$P(F) = 3/5 \qquad P(T/F) = 1/100$

using Baye's theorem

$$P(F/T) = \frac{1/100 \times 3/5}{(1/100 \times 3/5) + (4/100 \times 2/5)}$$

$$= 3/11.$$

Q5.

$P(A) = 0.25$
$P(B) = 0.35$
$P(C) = 0.40$

Let D = bolt is affected.

$$P(A/D) = \frac{0.05 \times 0.25}{(0.05 \times 0.25) + (0.04 \times 0.35) + (0.02 \times 0.4)}$$

$$= 0.362.$$

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$P(B/D) = \frac{0.04 \times 0.35}{(0.05 \times 0.25) + (0.04 \times 0.35) + (0.02 \times 0.4)}$

$\Rightarrow 0.406.$

$P(C/D) = \frac{0.02 \times 0.4}{(0.05 \times 0.25) + (0.04 \times 0.35) + (0.02 \times 0.4)}$

$= 0.232.$

Q6. R = Reader applies for job.

$P(A/R) = \frac{0.002 \times 1/3}{(0.002 \times 1/3) + (0.001 \times 1/2) + 0.005 \times 1/6)}$

$= 1/3.$

$P(B/R) = \frac{0.001 \times 1/2}{(0.002 \times 1/3) + (0.001 \times 1/2) + (0.005 \times 1/6)}$

$= 1/4.$

$P(C/R) = \frac{0.005 \times 1/6}{(0.002 \times 1/3) + (0.001 \times 1/2) + (0.005 \times 1/6)}$

$= 5/12.$

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P(Both applicants read A) = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Q7. Event H_1 = head on first toss
Event E = head on even numbered toss

$$P(E) = P(E/H_1)P(H_1) + P(E/H_1')P(H_1')$$

→ $P(E/H_1) = 0$

→ $P(E/H_1') = P(E') = 1 - P(E)$

$$P(E) = 0 \times p + (1 - P(E)) \times (1-p) = (1-p)(1-P(E))$$

$$P(E) = \frac{1-p}{2-p}.$$

E_k is event that the first head occurs on toss, Then $E = \bigcup_{k=1}^{\infty} E_k$

$$P(E) = P\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} P(E_k)$$

$$P(E_k) = (1-p)^{2k-1} p$$

$$P(E) = \sum_{k=1}^{\infty} (1-p)^{2k-1} p$$

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$$P(E) = \sum_{k=1}^{\infty} (1-p)^{2k-1} p$$

$$= \frac{p}{1-p} \frac{(1-p)^2}{1-(1-p)^2} = \frac{1-p}{2-p}$$

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ICT(2)

Q1 (i) Let $p \rightarrow$ probability of head

$p = 0.48$

$1-p = 0.52$

By Law of Large Numbers

$$P\left[|\bar{X}-0.48| > 0.02\right] \leq \frac{0.2496}{n(0.02)^2}$$

So for 95% Confidence

$$\frac{0.2496}{n(0.02)^2} = 0.05$$

Thus 'n' should be

$$0.2496 \times 2500 \times 20 = 12480$$

(ii) $P(X_1 = 1) = 0.27$ $\qquad n = 1500$

$\mu = 0.27$ $\qquad \sigma^2 = 0.27 \times 0.73 = 0.197$

By LLN,

$$P\left(\left|\frac{X_1+X_2+\dots+X_{1500}}{1500} - 0.27\right| \geq \epsilon\right) \leq \frac{0.1971}{1500\,\epsilon^2}$$

So if we set $\frac{1}{10} = \frac{0.1971}{1500\,\epsilon^2}$

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$$E = \sqrt{\frac{0.1971 \times 10}{1500}} = 0.036$$

Thus margin of error is less than 4% with 90% confidence.

Q2. (i) $f(x) = \begin{cases} \frac{1}{4}x & ; \quad 1 \le x \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$

$$F(x) = \int_1^x \frac{1}{4} x \, dx$$

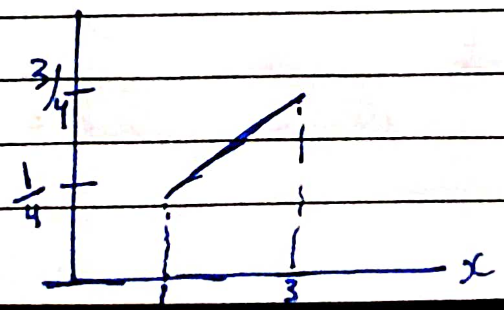
$$= \frac{1}{4}\int_1^x x \, dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_1^x = \frac{1}{4}\left[\frac{x^2}{2} - \frac{1}{2}\right]$$

x is an unknown point that gives area

$$F(x) = \frac{x^2 - 1}{8}$$

When $x = 3$; $F(3) = \frac{9-1}{8} = 1.$

$$F(x) = \begin{cases} 0 & ; \quad x < 1 \\ \frac{x^2-1}{8} & ; \quad 1 \le x \le 3 \\ 1 & ; \quad x > 3 \end{cases}$$



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(ii) $f(x) = \begin{cases} 1/3 & ; \ 0 \le x < 1 \\ 2x^2/7 & ; \ 1 \le x \le 2 \\ 0 & ; \ \text{otherwise} \end{cases}$

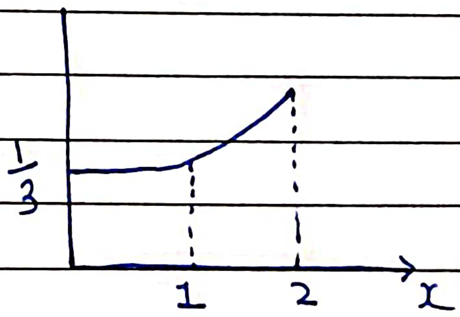
$$\int_0^x \frac{1}{3}\,dx = \left[\frac{x}{3}\right]_0^x = \frac{x}{3}$$

$$\int_1^x \frac{2}{7}x^2\,dx + \frac{1}{3} = \frac{2}{7}\left[\frac{x^3}{3}\right]_1^x + \frac{1}{3}$$

$$= \frac{2x^2}{21} + \frac{5}{21}$$

Put $x = 2$

$$F(2) = \frac{2.8 + 5}{21} = 1$$



$\therefore F(x) = \begin{cases} 0 & ; \ x < 0 \\ x/3 & ; \ 0 \le x \le 1 \\ \frac{2x^3+5}{21} & ; \ 1 \le x \le 2 \\ 1 & ; \ x > 2 \end{cases}$

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Q3. $E = \bigcup_{k=1}^{\infty} E_k$

$$P(E) = P\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} P(E_k)$$

$$P(E_k) = (1-p)^{2k-1} p$$

$$P(E) = \sum_{k=1}^{\infty} (1-p)^{2k-1} p$$

$$= \frac{p}{1-p} \sum_{k=1}^{\infty} (1-p)^{2k}$$

$$= \frac{p}{1-p} \cdot \frac{(1-p)^2}{1-(1-p)^2}$$

$$= \frac{1-p}{2-p}.$$