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THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
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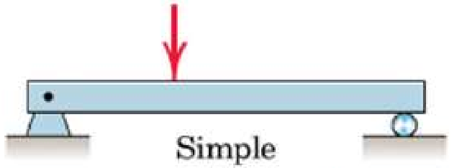
Process of **conversion** **of** **a given distribution of load** to a **Point Load**

Types of Beams

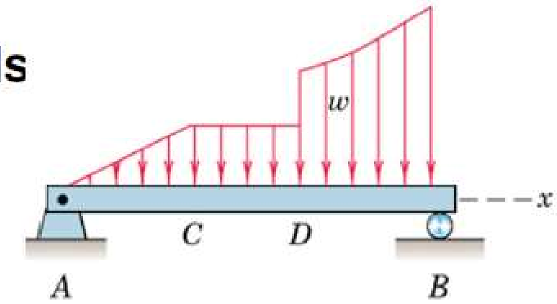
- Based on type of external loading

Beams supporting Concentrated Loads



Beams supporting Distributed Loads

- Intensity of distributed load = *w*
- *w* is expressed as force per unit length of beam (N/m)



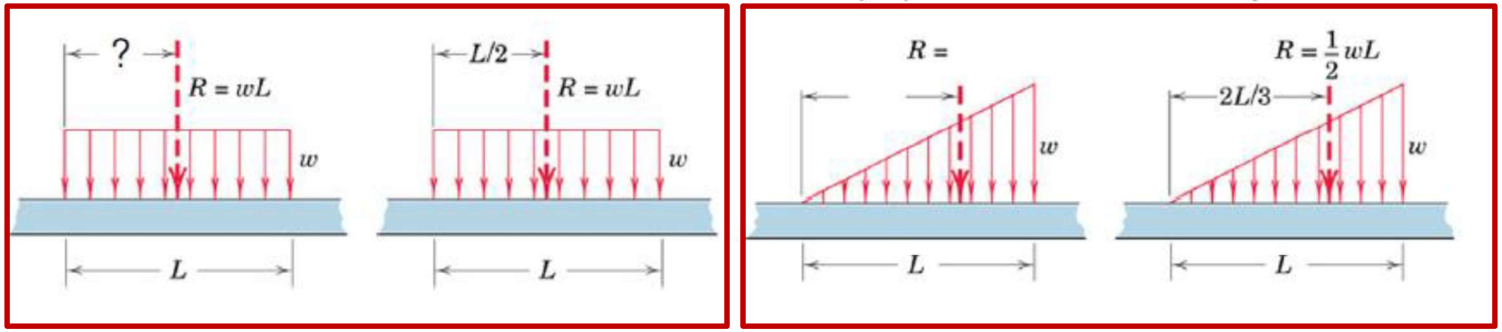
- intensity of loading may be constant or variable, continuous or discontinuous
  - discontinuity in intensity at D (abrupt change)
  - At C, intensity is not discontinuous, but rate of change of intensity (*dw*/*dx*) is discontinuous

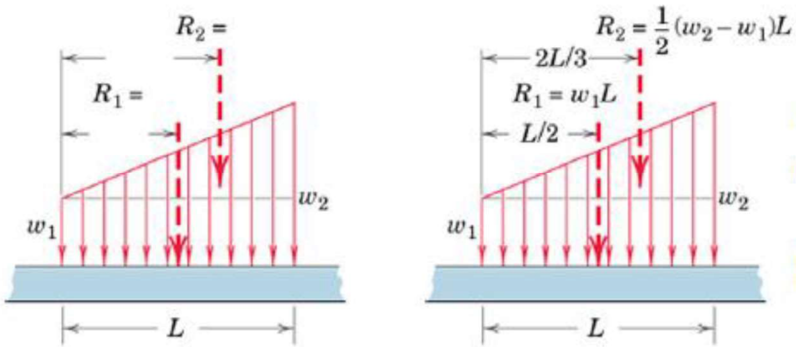
Unit of w is ___kN/m

Beams

Distributed Loads on beams

- Determination of Resultant Force (R) on beam is important



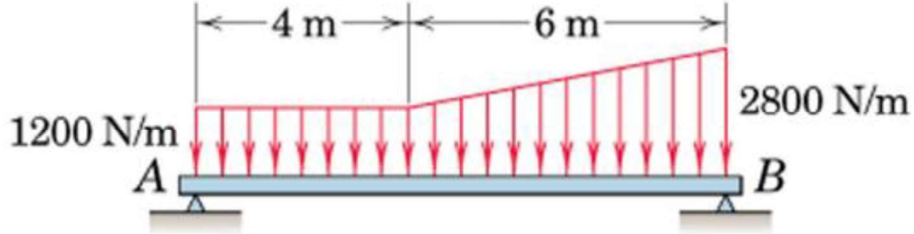


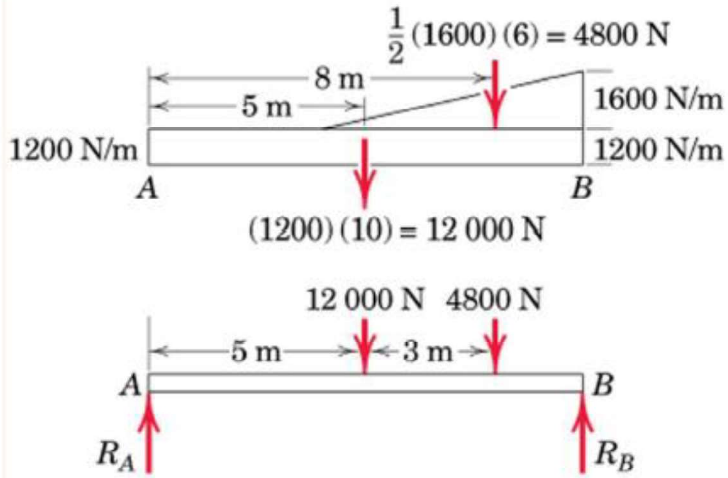
R = area formed by w and length L over which the load is distributed

R passes through centroid of this area

Beams: Example

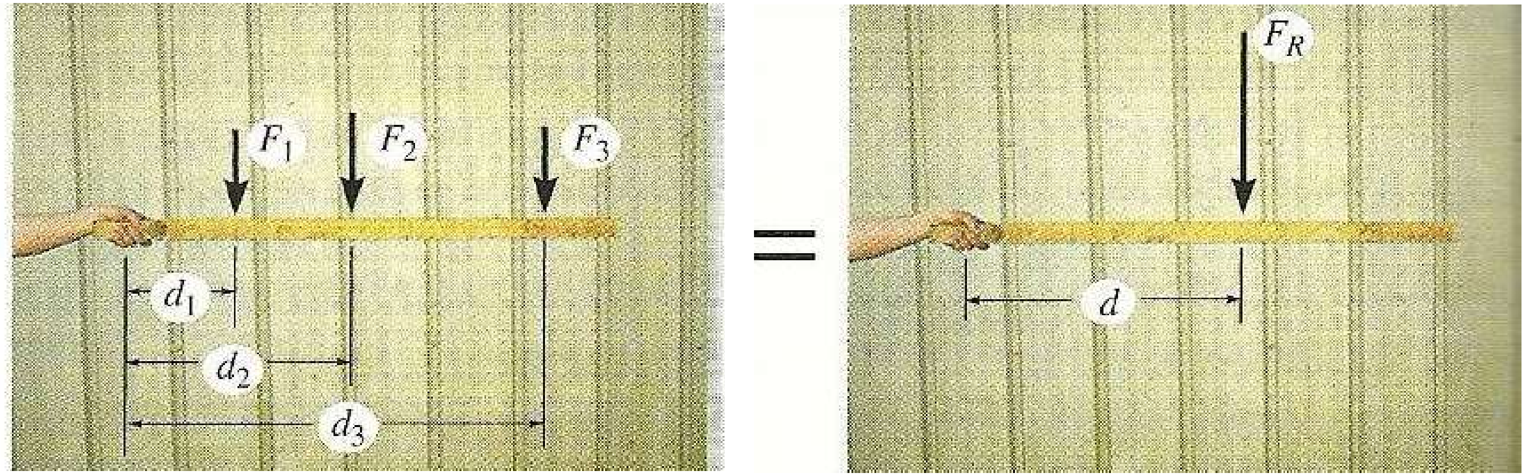
Determine the external reactions for the beam





$R_A = 6.96$ kN, $R_B = 9.84$ kN

Equivalent Systems: Resultants



$$F_R = F_1 + F_2 + F_3$$

What is the value of d?

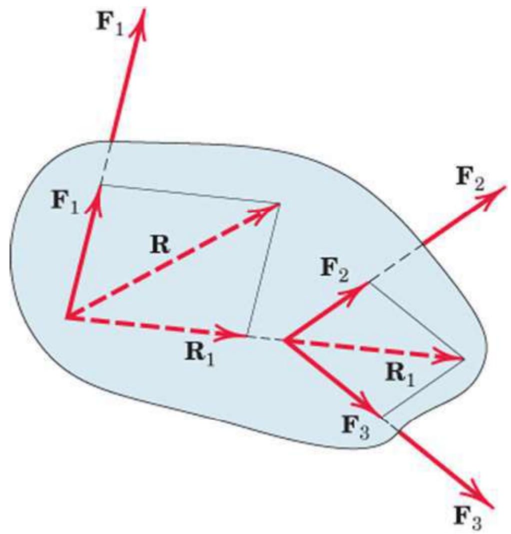
Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

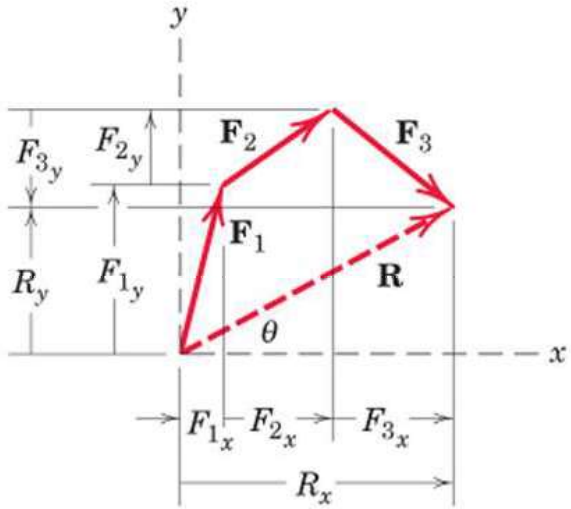
$$\boxed{F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3}$$

Equilibrium Conditions

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





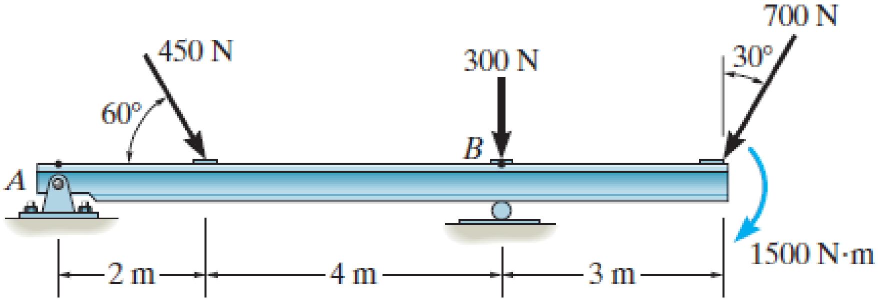
Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Problem Statement 1: Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end *A*



Solution:

Supports : A = (Simple support / Hinge support) [R_{ah} , R_{av}]

B = (Roller support) [R_{bv}]

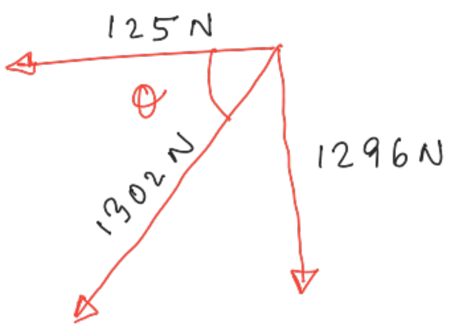
Action : 450 N , 300 N , 700 N , 1500 N-m.

Reaction : R_{ah} , R_{av} , R_{bv}

$$\xrightarrow{+ve} \Sigma F_x \Rightarrow 450\cos 60^\circ - 700\sin 60^\circ = -125\,N \leftarrow$$

$$+ve\uparrow \Sigma F_y \Rightarrow -450\sin 60^\circ - 700\cos 30^\circ - 300 = -1296\,N \downarrow$$

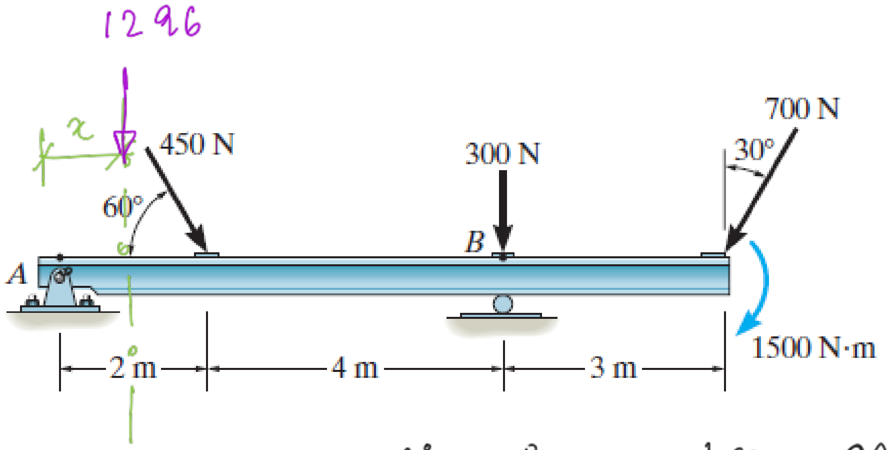
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(125)^2 + (1296)^2} = 1302\,N$$



$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{1296}{125}\right)$$

$$= 84.5^\circ$$



Note: since in the question it as been asked to replace the loading acting on a beam by single resultant, we will calculate the distance of equivalent resultant force for the loading system and its distance.

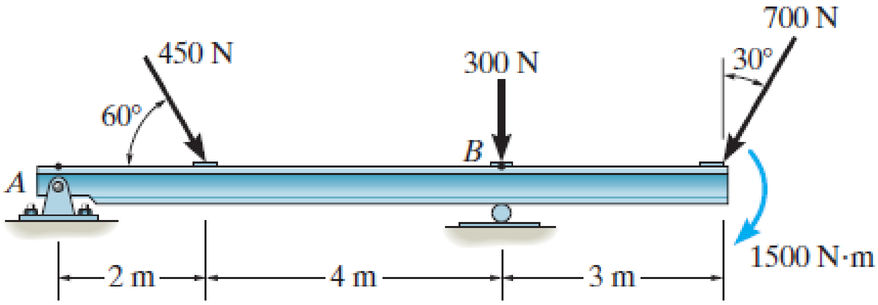
Let us assume that the sum total of all the forces in y-direction (ΣF_y) is acting at a distance x from point "A".

+ve ↻

$$1296(x) = 450 \sin 60 * 2 + 300 * 6 + 700 \cos 30^\circ * 9 + 1500$$

$$\therefore \quad x = 7.36 \text{ m}$$

Problem Statement 2: Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from *B*

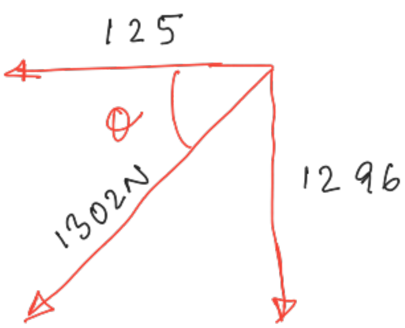


Solution:

$\xrightarrow{+ve}$

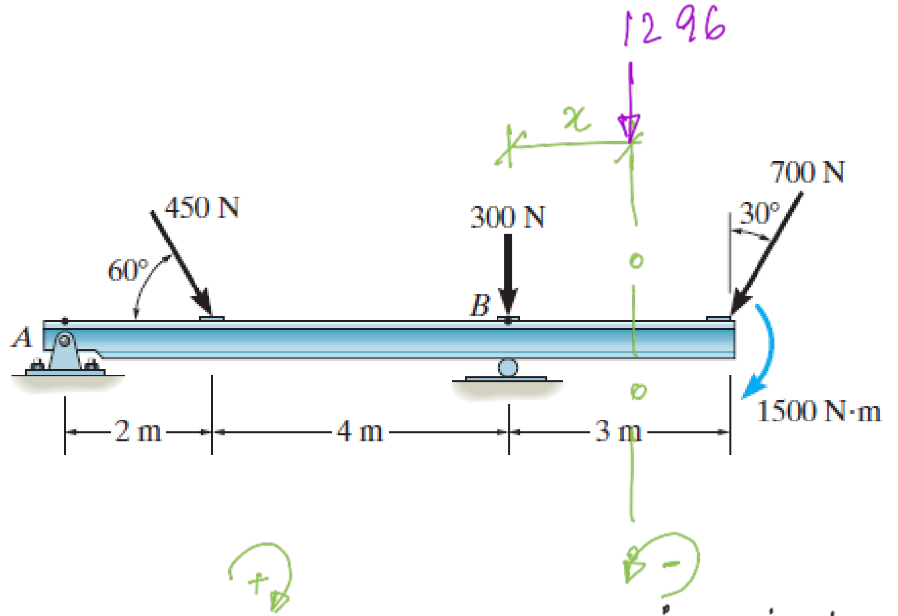
$\sum F_x \Rightarrow 450\cos 60^\circ - 700\sin 30^\circ = -125 \leftarrow$

$+ve\uparrow \quad \sum F_y \Rightarrow -450\sin 60^\circ - 300 - 700\cos 30^\circ = -1296 \downarrow$



$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$

$= 1302\ N$

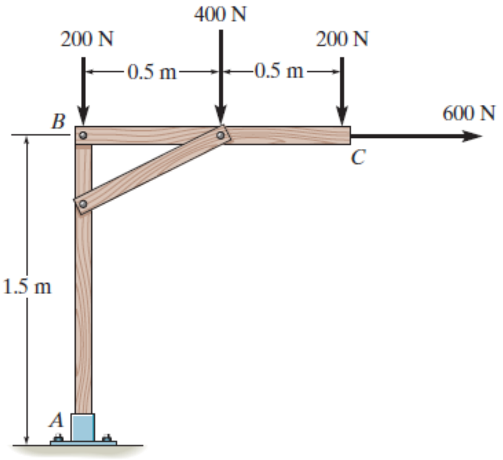


+ve

$$1296(x) = -450 \sin 60^\circ * 4 + 700 \cos 30^\circ * 3 + 1500$$

$\therefore$ $x = 1.36$ m   Ans.

Problem Statement 3: Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member *AB*, measured from *A*



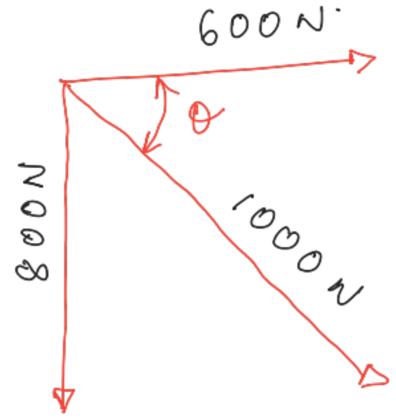
Solution:

$\xrightarrow{+ve}$ $\Sigma F_x \Rightarrow +600\,N \rightarrow$

$+ve \uparrow$ $\Sigma F_y \Rightarrow -200-400-200 = -800\,N \downarrow$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(600)^2 + (-800)^2}$$

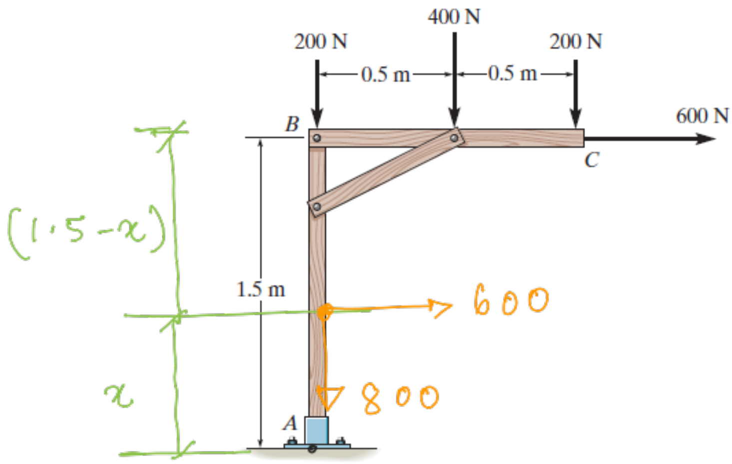
$$= 1000\,N.$$



$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{800}{600}\right)$$

$$\therefore \theta = 53.1^\circ$$



Now, in the question it has been asked to calculate the line of action intersects the vertical line.

Which means we are interested in the calculation of y-intercept.

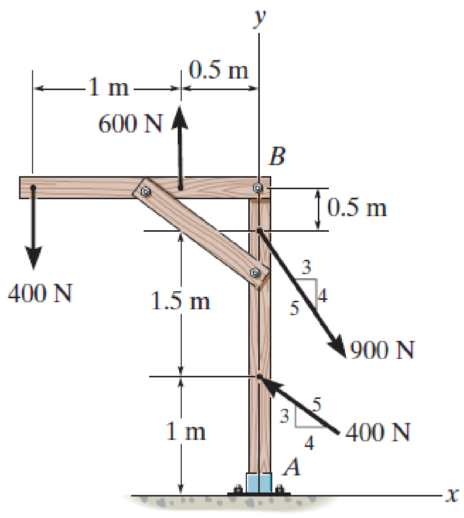
- Let us take y-intercept distance as x from point 'A'
- Now moment about point 'B' is zero.

+ve ↻

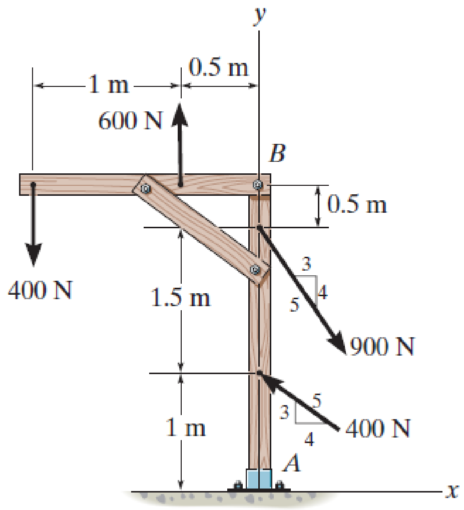
$$+400 * 0.5 + 200 * 1.0 = -600 * (1.5 - x)$$

$$\therefore \quad x = 2.17 \text{ m} \qquad \text{Ans.}$$

Problem Statement 4: Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member *CB*, measured from end *C*.



Solution:



y
1 m
0.5 m
600 N
B
0.5 m
400 N
1.5 m
3
4
5
900 N
5
3
4
400 N
1 m
A
x