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THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
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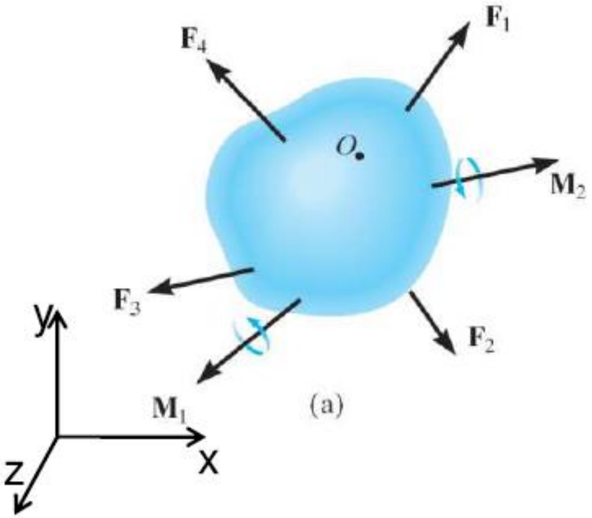
# Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$



(a)

# Rigid Body Equilibrium

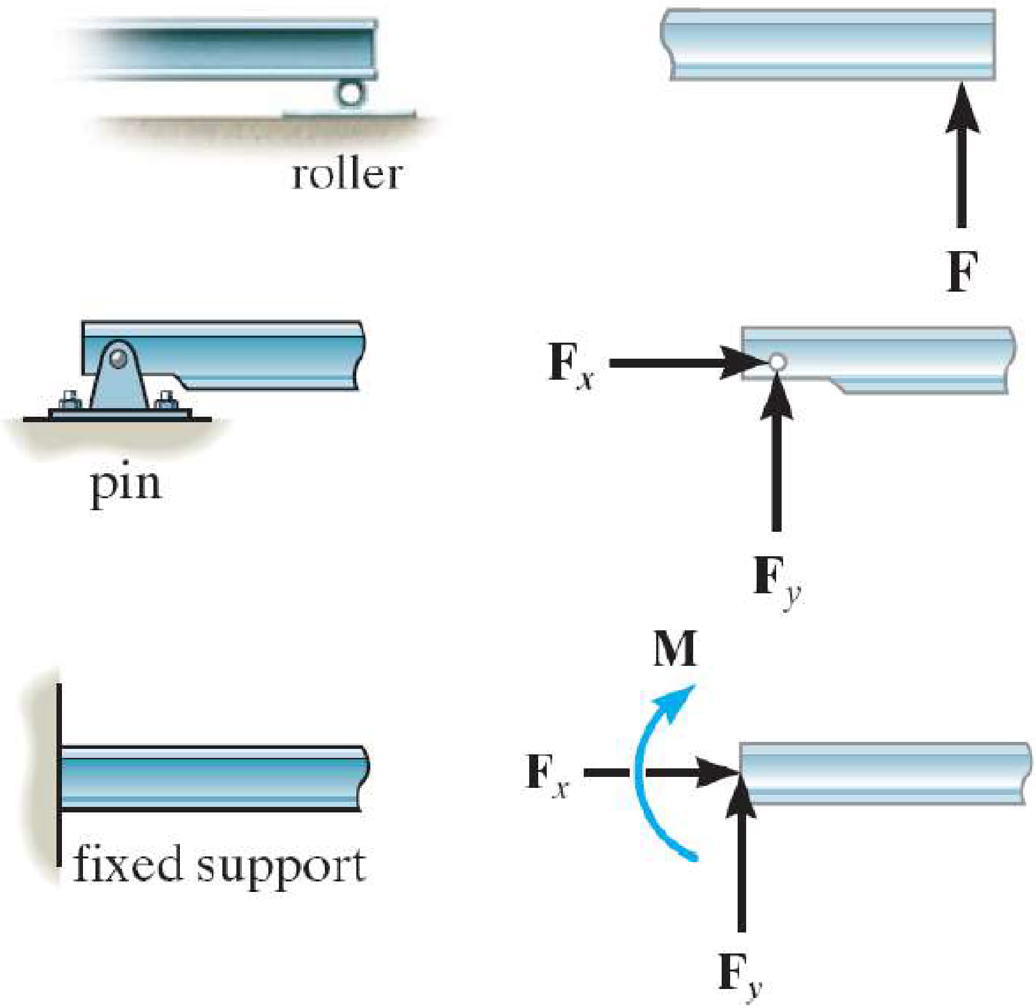
Support Reactions

Prevention of

Translation or

Rotation of a body

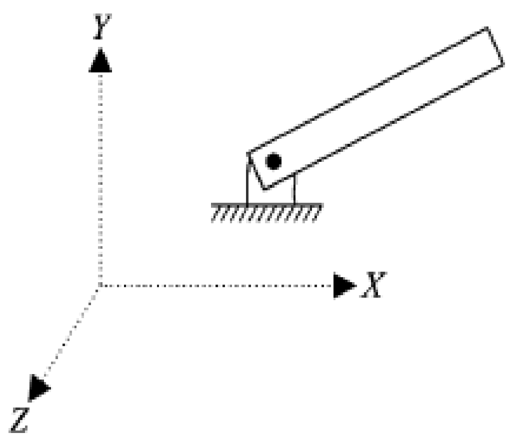
Restraints



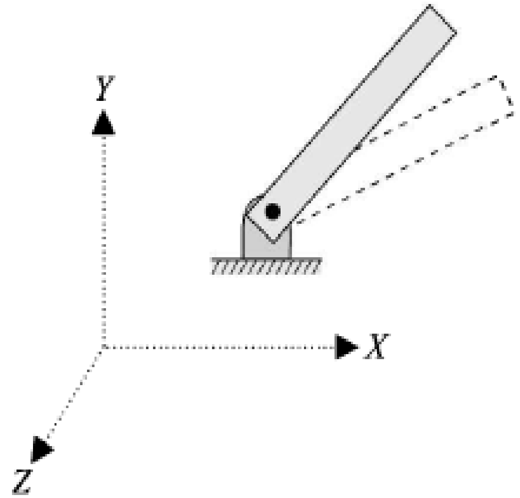
# Supports

In case of rigid bodies, particularly beams, to prevent not only translation motion but also rotational motion, these are normally held by various supports

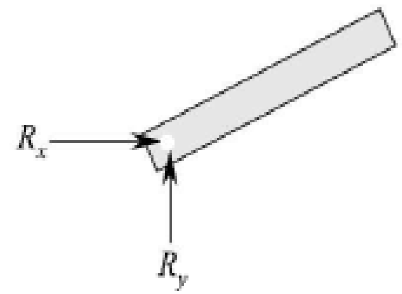
# 1. Hinge *or* pin-support




Hinge support



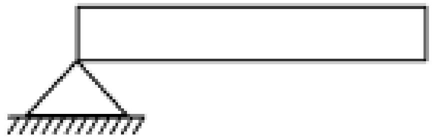

Free to rotate about the *Z*-axis



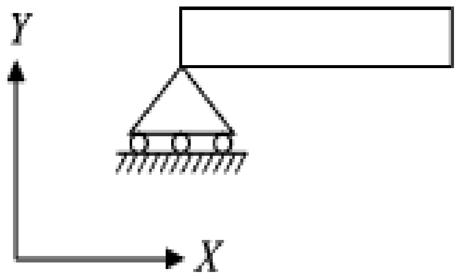

Reactions exerted by hinge support on the body

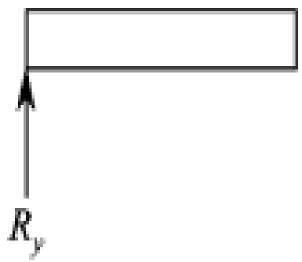
## Alternate representation

This type of support may also be represented as shown below:



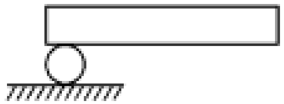
# 2. Roller *or* frictionless support



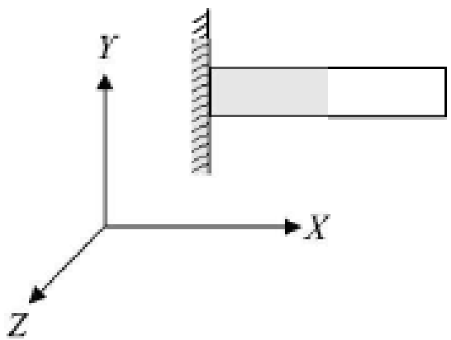



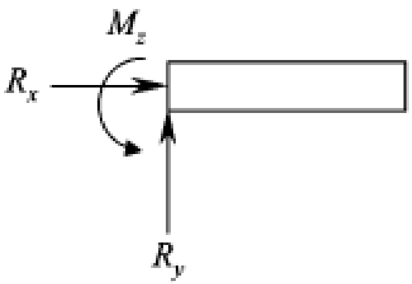

## Alternate representation

This type of support may also be represented as shown below:

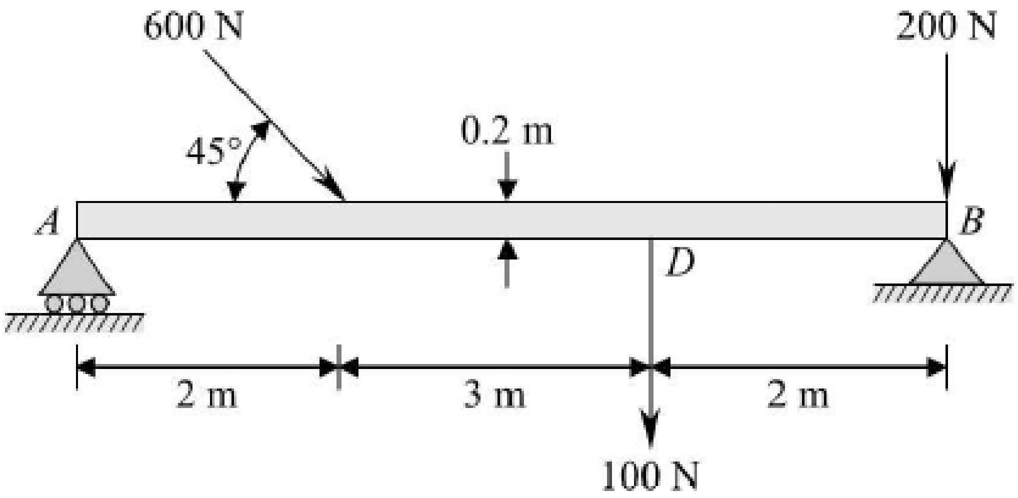


# 3. Fixed *or* built-in support

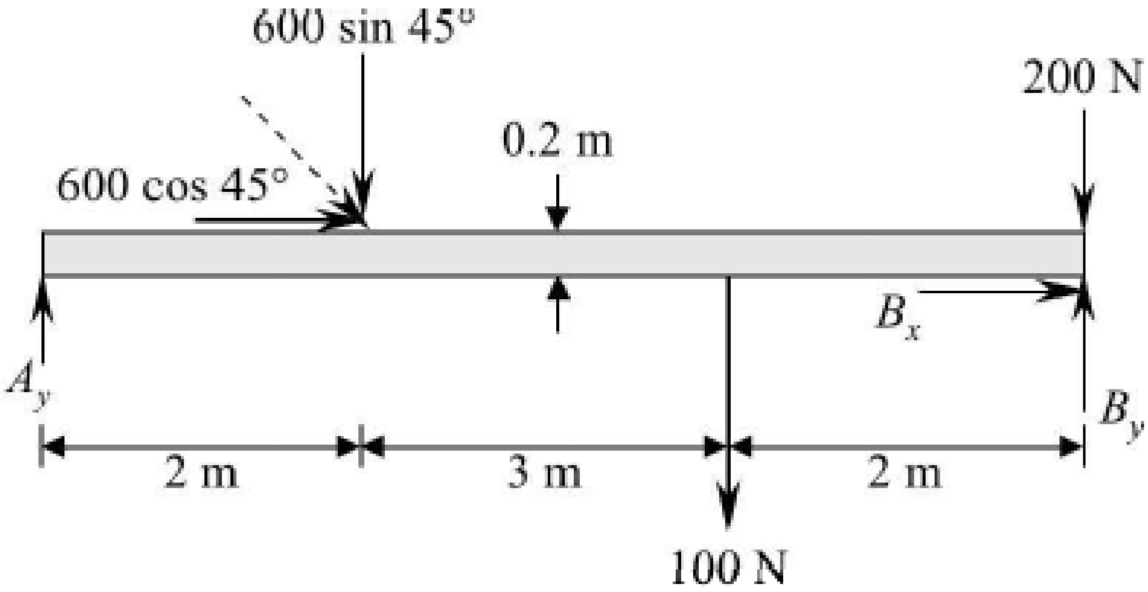


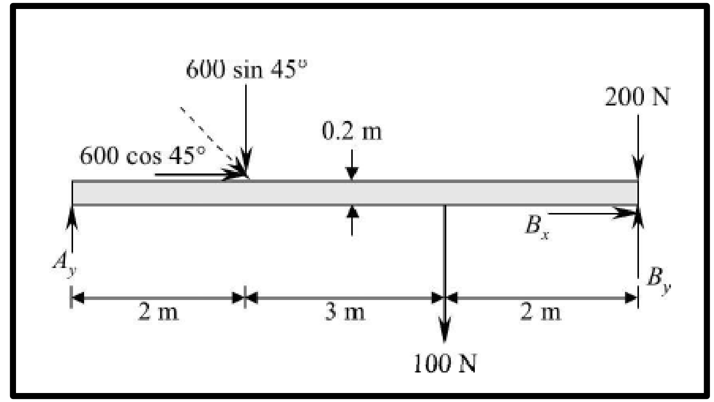
**<u>Problem statement 1:</u> Determine the support reactions for the beam AB loaded as shown. (Neglect the weight of beam)**



# Solution



**Solution** The free-body diagram of the beam is shown in Fig. 5.50(a). Since the end $A$ is a roller support, one vertical reaction $A_y$ is shown; the end $B$ is hinged and hence two reactions $B_x$ and $B_y$ along $X$ and $Y$ axes respectively are shown. The 600 N force is resolved into components as shown.




Applying the conditions of equilibrium,

$\sum F_x = 0 \Rightarrow$

$$B_x + 600\cos 45° = 0$$

$$\therefore \quad B_x = -424.26 \text{ N}$$

$\sum F_y = 0 \Rightarrow$

$$A_y + B_y - 100 - 200 - 600\sin 45° = 0$$

$$\therefore \quad A_y + B_y = 100 + 200 + 600 \sin 45°$$

$$= 300 + 600 \sin 45°$$

$$= 724.26 \text{ N} \qquad \text{(a)}$$

Taking summation of the moments about the point B (as it eliminates more number of unknowns) and equating it to zero,

$\sum M_B = 0 \Rightarrow$

$$-(A_y \times 7) - (600 \cos 45° \times 0.2) + (600 \sin 45° \times 5) + (100 \times 2) = 0$$

$$\Rightarrow \quad (A_y \times 7) = (600 \sin 45° \times 5) + (100 \times 2) - (600 \cos 45° \times 0.2)$$

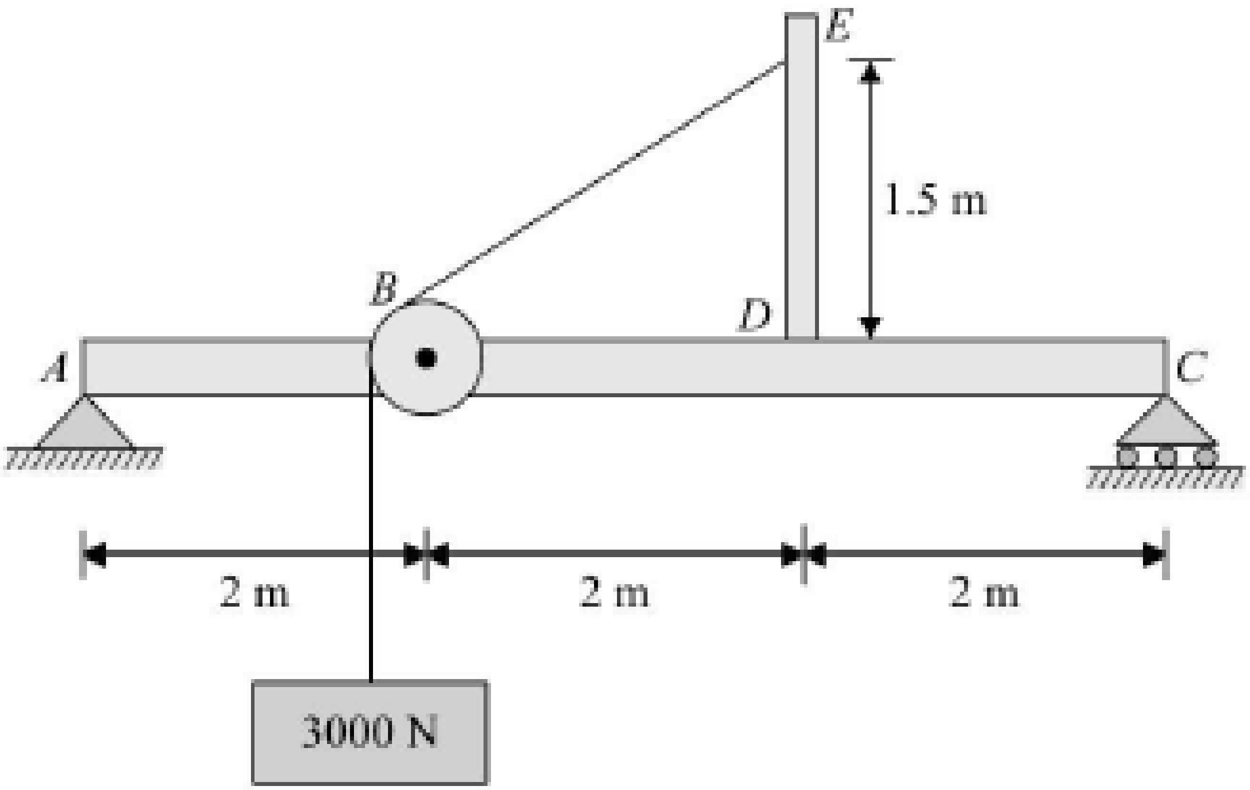
$$\therefore \quad A_y = 319.5 \text{ N} \qquad \text{(b)}$$

Substituting this value in equation (a),

$$A_y + B_y = 724.26 \text{ N}$$

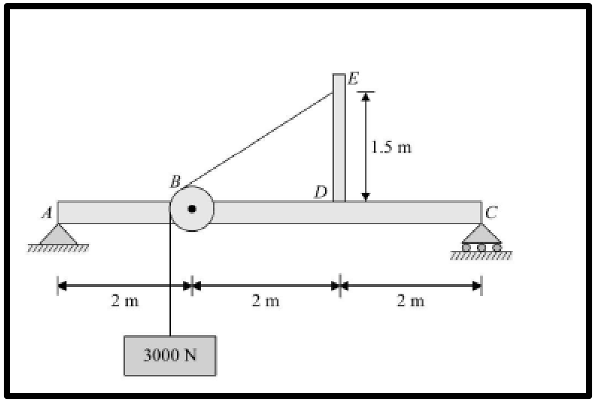
$$\Rightarrow \quad B_y = 724.26 - 319.5 = 404.76 \text{ N}$$

**Problem statement 2:** A smooth pulley supporting a load of 3000 N is mounted at B on a horizontal beam AC as shown in Figure. If the beam weighs 1000 N, find the support reactions at A and C. (Neglect the weight and size of pulley)



# Solution

**Solution** For a clear understanding of the problem, let us isolate the bodies and analyze the free-body diagrams separately. The forces shown in the free-body diagram of the pulley are reactions $B_x$ and $B_y$ as $B$ is a hinge point and equal tension $T$ on both ends of the string as the pulley is frictionless. The forces shown in the free-body diagram of the beam are its weight placed at its centre, reactions $A_x$ and $A_y$ at the support point $A$, reaction $C_y$ at the support point $C$, reactions $B_x$ and $B_y$ shown at the point $B$ in the direction opposite to that shown in the pulley and tension $T$ in the string.

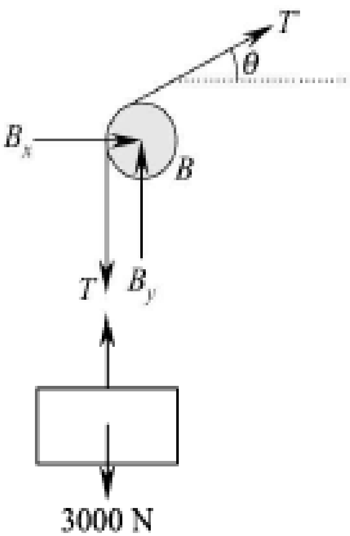


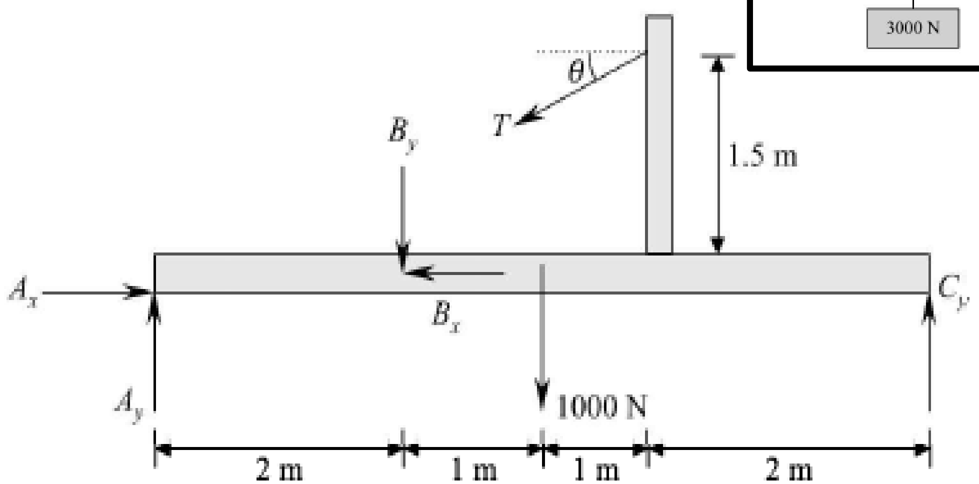

The inclination of the string is given as:

$$\tan\theta = 1.5/2 = 0.75$$

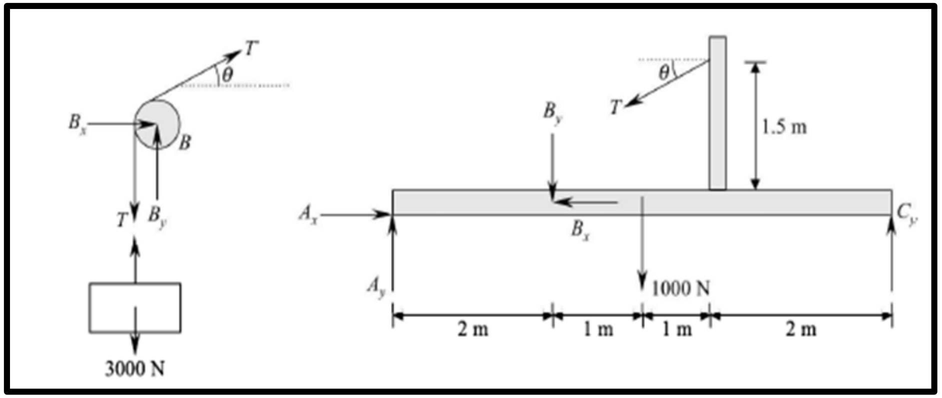
Therefore, $$\sin\theta = 0.6$$

and $$\cos\theta = 0.8$$



# Solution (*cont..*)




*Block*

From the free-body diagram of the load, we see that,

$$\sum F_y = 0 \Rightarrow$$

$$T - 3000 = 0$$

$$\therefore \quad T = 3000 \text{ N}$$

*Pulley*

Applying the conditions of equilibrium to the free-body diagram of the pulley,

$$\sum F_x = 0 \Rightarrow$$

$$B_x + T\cos\theta = 0$$

$$\therefore \quad B_x = -3000 \times 0.8 = -2400 \text{ N}$$

(The negative sign indicates that the force acts in the direction opposite to that of what we have assumed.)

$$\sum F_y = 0 \Rightarrow$$

$$B_y + T\sin\theta - T = 0$$

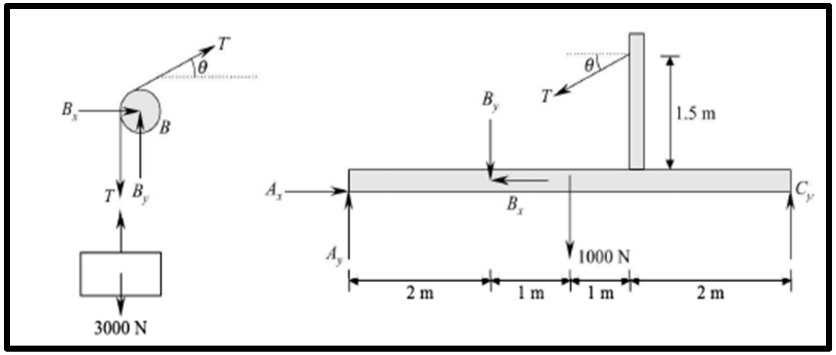
$$\therefore \quad B_y = 3000 - (3000 \times 0.6)$$

$$= 1200 \text{ N}$$

# Solution (*cont..*)

*Beam*

Applying the conditions of equilibrium to the free-body diagram of the beam,




$$\sum F_x = 0 \Rightarrow$$

$$A_x - B_x - T\cos\theta = 0$$

$$\therefore \quad A_x = B_x + T\cos\theta$$

$$= -2400 + (3000 \times 0.8) = 0$$

$$\sum F_y = 0 \Rightarrow$$

$$A_y + C_y - B_y - T\sin\theta - 1000 = 0$$

$$\Rightarrow \quad A_y + C_y = B_y + T\sin\theta + 1000$$

$$= 1200 + 3000(0.6) + 1000$$

$$= 4000 \text{ N} \qquad \text{(b)}$$

As the forces are non-concurrent, in addition we also take summation of the moments about $A$ and equate it to zero,

$$\sum M_A = 0 \Rightarrow$$

$$[C_y \times 6] + [T\cos\theta \times 1.5] - [T\sin\theta \times 4] - [B_y \times 2] - [1000 \times 3] = 0$$

$$C_y \times 6 = -[3000 \times 0.8 \times 1.5] + [3000 \times 0.6 \times 4] + [1200 \times 2] + [1000 \times 3]$$

$$\therefore \quad C_y = 1500 \text{ N} \qquad \text{(c)}$$

Substituting the value of $C_y$ in equation (b),

$$A_y + C_y = 4000$$

$$\Rightarrow \quad A_y = 2500 \text{ N}$$

# Problems on “Combination of Members”

# Example 1

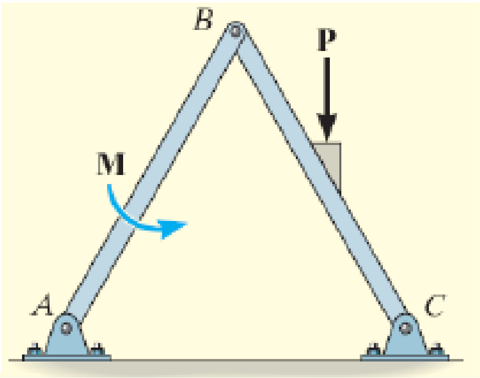
## Example: Free Body Diagrams

Draw FBD of

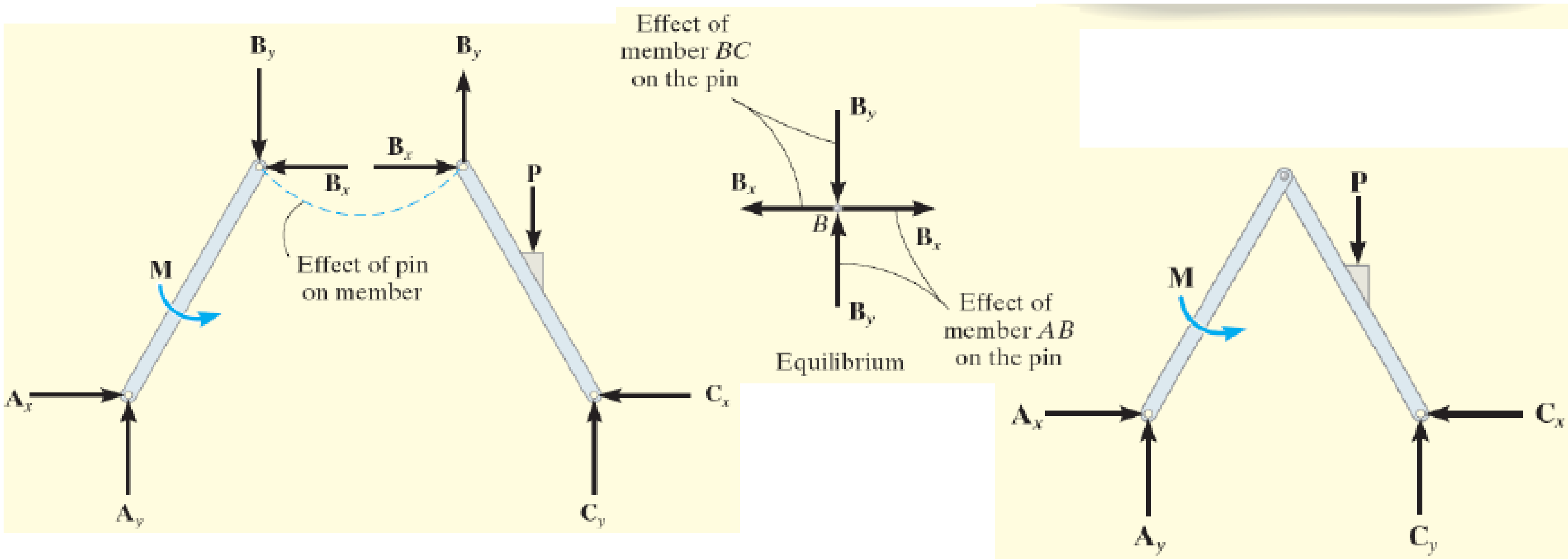
(a) Each member

(b) Pin at B, and

(c) Whole system

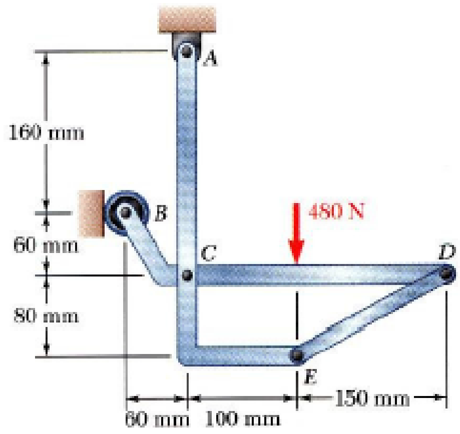


# Solution



B_y
B_y
B_x
B_x
P
M
Effect of pin on member
A_x
A_y
C_x
C_y
Effect of member BC on the pin
B_y
B_x
B
B_x
B_y
Effect of member AB on the pin
Equilibrium
M
P
A_x
A_y
C_x
C_y

# Example 1

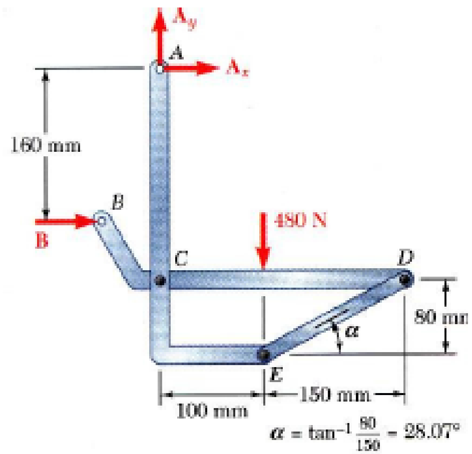



Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.
- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at *C*.
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

# Solution




SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\text{ N} \qquad \boxed{A_y = 480\text{ N} \uparrow}$$

$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$\boxed{B = 300\text{ N} \rightarrow}$$

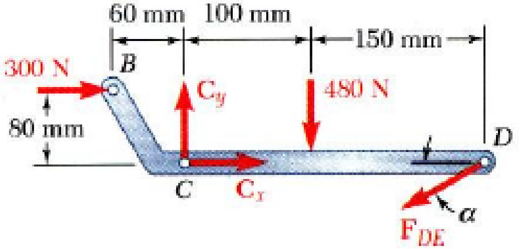
$$\sum F_x = 0 = B + A_x \qquad \boxed{A_x = -300\text{ N} \leftarrow}$$

Note:

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

# Solution (*cont..*)

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.




$$\sum M_C = 0 = (F_{DE}\sin\alpha)(250\text{ mm}) + (300\text{ N})(60\text{ mm}) + (480\text{ N})(100\text{ mm})$$

$$F_{DE} = -561\text{ N} \qquad \boxed{F_{DE} = 561\text{ N}\ C}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

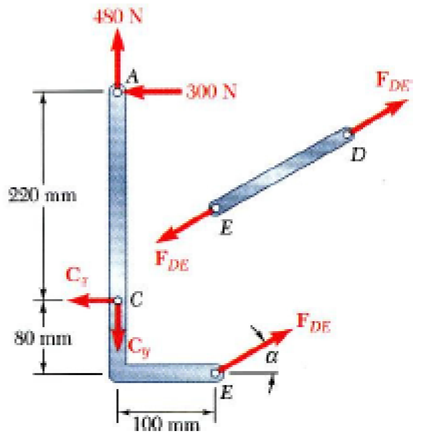
$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$

$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N} \qquad \boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$

$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N} \qquad \boxed{C_y = 216\text{ N}}$$

# Solution (*cont..*)




- With member *ACE* as a free-body, check the solution by summing moments about *A*.

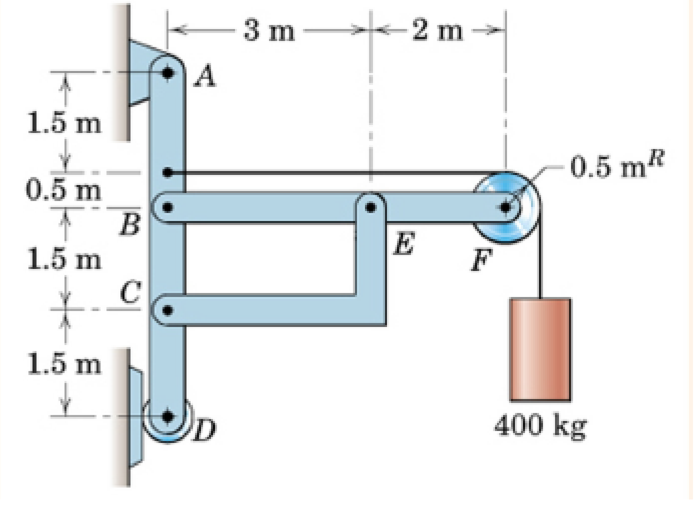
$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)

# Example 3 (Home work)

## Frames and Machines

**Example:** Compute the horizontal and vertical components of all forces acting on each of the members (neglect self weight)



# Frames and Machines

Example Solution:

3 supporting members form a
rigid non-collapsible assembly

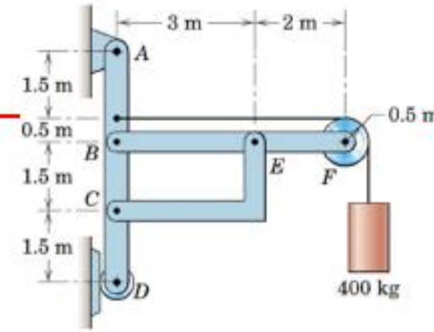
Frame Statically Determinate Externally

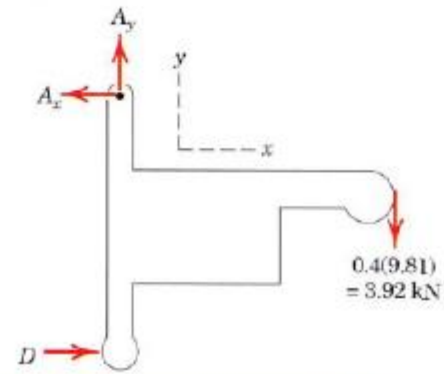
Draw FBD of the entire frame

3 Equilibrium equations are available

Pay attention to sense of Reactions

Reactions can be found out



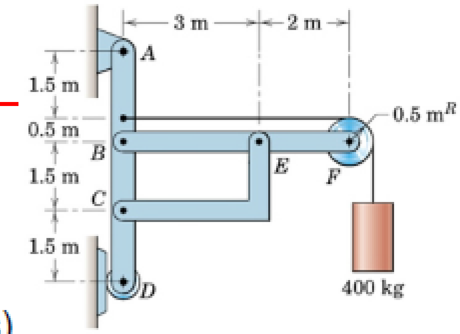




$$[\Sigma M_A = 0] \quad 5.5(0.4)(9.81) - 5D = 0 \quad D = 4.32 \text{ kN}$$

$$[\Sigma F_x = 0] \quad A_x - 4.32 = 0 \quad A_x = 4.32 \text{ kN}$$

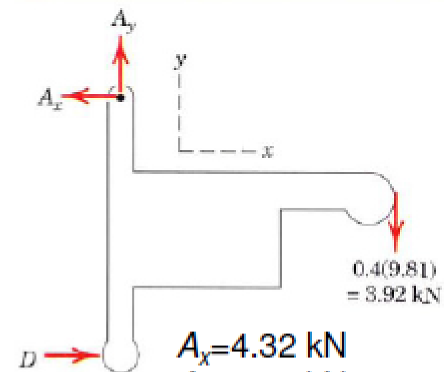
$$[\Sigma F_y = 0] \quad A_y - 3.92 = 0 \quad A_y = 3.92 \text{ kN}$$

# Frames and Machines

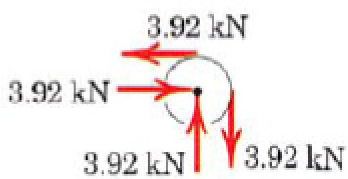



Example Solution: Dismember the frame and draw separate FBDs of each member

- show loads and reactions on each member due to connecting members (interaction forces)



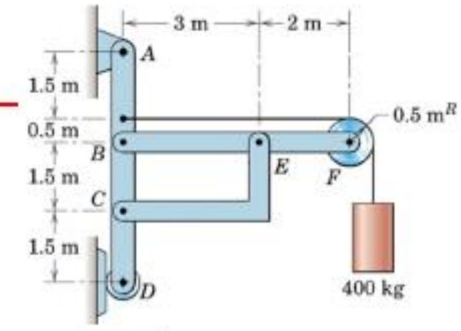

Begin with FBD of Pulley




Then draw FBD of Members BF, CE, and AD

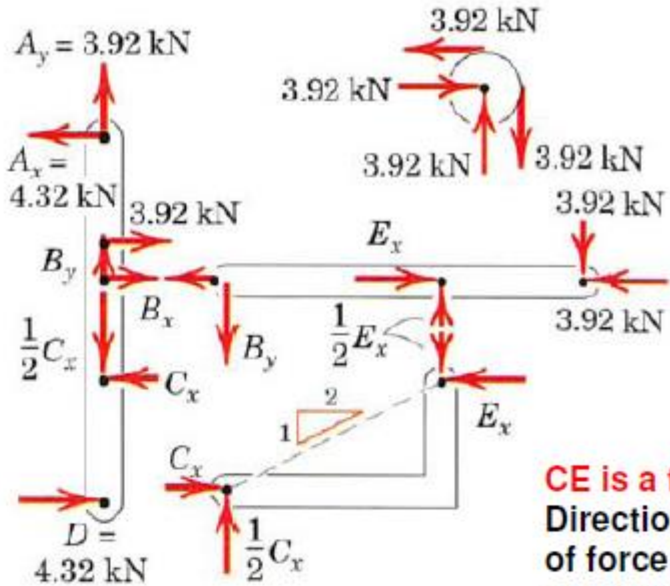
$A_x$=4.32 kN
$A_y$=3.92 kN
$D$=4.32 kN

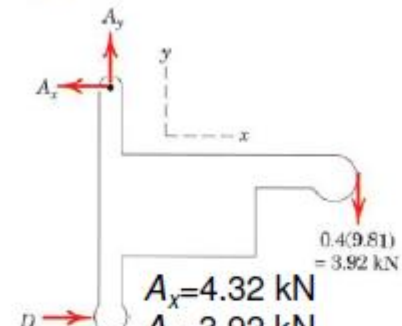
# Frames and Machines




Example Solution:

FBDs




$A_x$=4.32 kN

$A_y$=3.92 kN

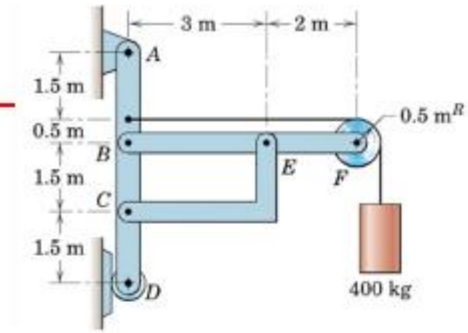
D=4.32 kN

**CE is a two-force member**

**Direction of the line joining the two points of force application determines the direction of the forces acting on a two-force member.**

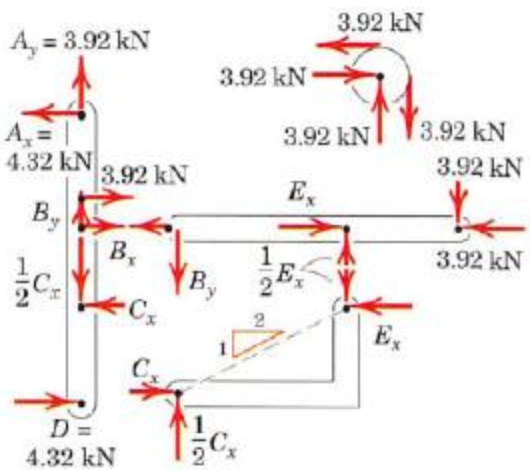
**Shape of the member is not important.**

# Frames and Machines




Example Solution:

Find unknown forces from equilibrium




## Member BF

| | | |
|---|---|---|
| $[\Sigma M_B = 0]$ | $3.92(5) - \frac{1}{2}E_x(3) = 0$ | $E_x = 13.08$ kN |
| $[\Sigma F_y = 0]$ | $B_y + 3.92 - 13.08/2 = 0$ | $B_y = 2.62$ kN |
| $[\Sigma F_x = 0]$ | $B_x + 3.92 - 13.08 = 0$ | $B_x = 9.15$ kN |

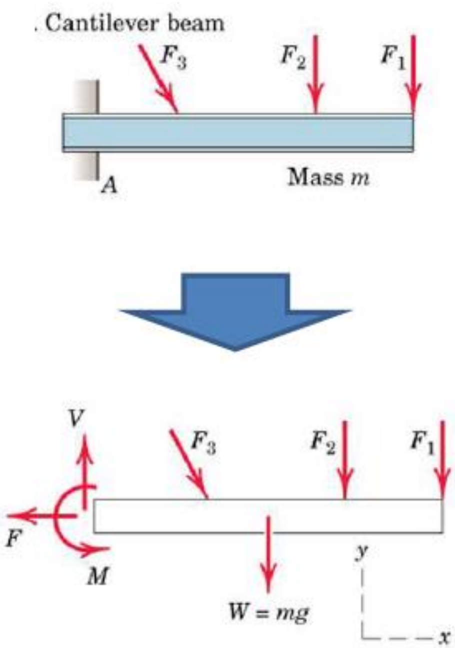
## Member CE

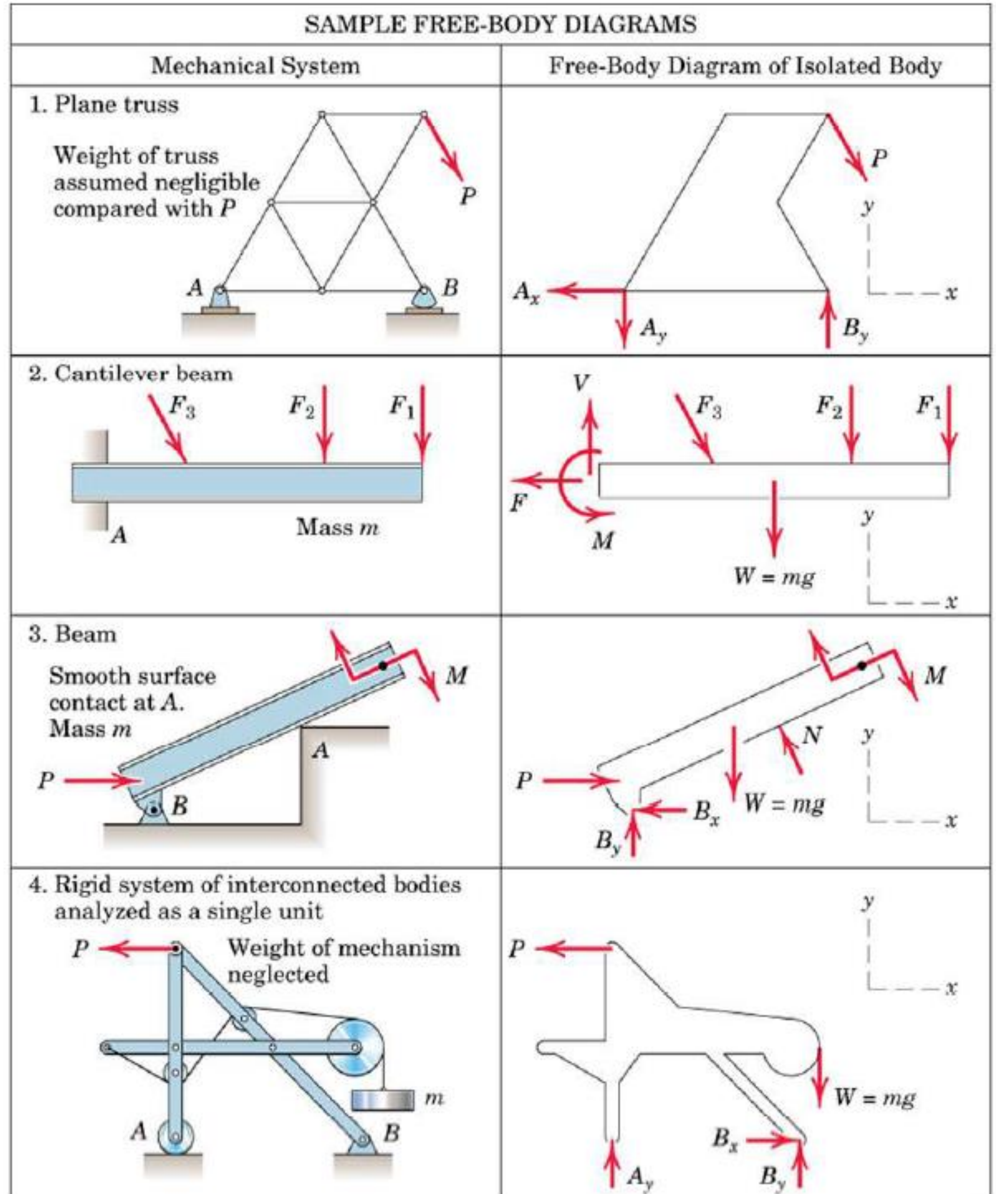
$[\Sigma Fx = 0]$ $C_x = E_x = 13.08$ kN

## Checks:

| | |
|---|---|
| $[\Sigma M_C = 0]$ | $4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$ |
| $[\Sigma F_x = 0]$ | $4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$ |
| $[\Sigma F_y = 0]$ | $-13.08/2 + 2.62 + 3.92 = 0$ |

# Free body diagram



**Thank you**