Date
Page No.

ICT TUT-3

Q1 $E(x) = 7/2$

Value (x)	1	3	5
$P(x)$	$1/4$	$1/4$	$1/2$
$(\mu - 7/2)^2$	$25/4$	$1/4$	$9/4$

$$Var(x) = \frac{25}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4} + \frac{9}{4} \cdot \frac{1}{2} = \frac{11}{4}.$$

Q2

Value (x)	1	2	3	4	5
Prob $P(x)$	$1/5$	$1/5$	$1/5$	$1/5$	$1/5$
$(x-\mu)^2$	4	1	0	1	4

$$Var(x) = E\left[(x-\mu)^2\right] = 4/5 + 1/5 + 0 + 1/5 + 4/5$$

$$= 2.$$

Q3

Value (y)	1	2	3	4	5
$P(y)$	$1/10$	$2/10$	$4/10$	$2/10$	$1/10$
$(y-\mu)^2$	4	1	0	1	4

Date
Page No.

$var(y) = E[(y-\mu)^2] = 4/10 + 2/10 + 0 + 2/10 + 4/10$

$= \cancel{}\ 12/10 = \underline{\underline{1.2}}$ Ans.

(iii)

Value (z)	1	2	3	4	5
P(z)	5/10	0	0	0	5/10
$(z-\mu)^2$	4	1	0	1	4

$Var(z) = E[(z-\mu)^2] = \frac{20}{10} + 0 + 0 + 0 + \frac{20}{10}$

$= \underline{\underline{4}}$ Ans.

(iv)

Value (w)	1	2	3	4	5
P(W)	0	0	1	0	0
$(w-\mu)^2$	μ	1	0	1	4

$Var(w) = E[(w-\mu)^2] = \underline{\underline{0}}$ Ans.

Q3. $E(X) = p$

value(x)	0	1
P(X)	1-p	p
$(x-\mu)^2$	$(0-p)^2$	$(1-p)^2$

Date
Page No.

$$\text{var}(x) = (1-p)p^2 + p(1-p)^2$$
$$= (1-p)p(1-\cancel{p}+\cancel{p}) = (1-p)p.$$

Q4.

$\text{var}(X) = 3$, $\text{var}(y) = 5$

Also, x and y are independent.

(i) $\text{var}(X+y) = \text{var}(X) + \text{var}(y) = 8.$

(ii) $\text{var}(aX+b) = a^2.(\text{var}(X)) = 9.3 = 27.$

(iii) $\text{var}(X+X) = \text{var}(2X) = 4\,\text{var}(X) = 12.$

(iv) $\text{var}(X+3y) = \text{var}(X) + \text{var}(3y)$
$= 3 + 9(5) = 48.$

~~Q5.~~ $\cancel{E(X) = \sum_{k=0}^{n} k\, p_X(k) = \sum_{k=0}^{n} k \frac{n!}{(n-k)!\,k!} p^k q^{n-k}}$

$\cancel{= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!\,(k-1)!} p^{k-1} q^{n-k}}$

Page No.

Q5 For calculating mean, μ $(E(X)) \rightarrow$

$$E(X) = \sum_{x=0}^{n} x . \binom{n}{x} p^{x} (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!\,(n-x)!} p^{x} (1-p)^{n-x}$$

Since term $(x=0)$ can be ignored at we can Sub. $x = y+1$ & $n = m+1$,

$$E(X) = \sum_{y=0}^{m} \frac{(m+1)!}{y!\,(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)\, p \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}$$

Binomial theorem states that

$$(a+b)^{m} = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^{y} b^{m-y}$$

Let $a = p$, $b = 1-p$

$$\Rightarrow \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y} = 1.$$

$\Rightarrow E(X) = (m+1)p$

$\Rightarrow E(X) = n.p$

Date

Page No.

Let $y = x-2, \quad m = n-2$

$$E\left(X(X-1)\right) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^{x} (1-p)^{x}$$

$$\Rightarrow \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x}$$

$$\Rightarrow \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^{x} (1-p)^{n-x}$$

$$\Rightarrow n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$\Rightarrow n(n-1)p^2.$$

$$Var(X) = E(X^2) - E(X)^2 = E\left(X(X-1)\right) + E(X) - E(X)^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$\Rightarrow Var(X) = np(1-p)$$

= Hence Proved

Date

Page No.

Q6 X has range [0, 1] & density $f(x) = 1$

Therefore,

$$E(X) = \int_0^1 x\,dx = \left.\frac{x^2}{2}\right]_0^1 = 1/2.$$

Q7 We found $\mu = 1/2$

Next we compute

$$Var(X) = E\left[(x-\mu)^2\right] = \int_0^1 (x - 1/2)^2\,dx = 1/12.$$

Q8 This is an exercise in change of variables.

Let $Z = \frac{(x-\mu)}{\sigma}$

We have $var(x) = E\left[(x-\mu)^2\right] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} (x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2}\,dx$

$$\Rightarrow \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z^2 e^{-z^2/2}\,dz = \sigma^2$$

The integral in the last line is the same one computed for Var(Z).