Vehicle Dynamics Simulation Entry Programming Assignment

Simulation Usage

A 2D vehicle dynamics simulation was written according to the assignment specifications.

The simulation source code is located in the following git repository: https://github.com/AdiKertesz/2DVehicleSimulation.

Simulation usage:

python3 2d_vehicle_simulation.py -x0 <x0> -y0 <y0> -psi <psi> -v <v> -path <csv file directory>

In the simulation the vehicle localization is not perfect, a white noise of ± 1 [m] is added to each coordinate and a noise of ± 0.5 [deg] is added to the heading angle.

The front wheel servo actuator is modeled as a 2nd order dynamic system with 0.7 damping ratio and 16.5 $[\mathrm{rad/sec}]$ bandwidth, which guarantee 0.2 [sec] rise time.

In the pure pursuit tracking algorithm, the $d_{look\ ahead}$ is set to 10*v, which means the vehicle is aiming for the point on the path it will occupy in 10 seconds.

Theoretical Questions

1. <u>What will be the steady state effect of a bias in mechanical road wheel servo? Show this effect in your simulation.</u>

The dynamic system simulated can be described as follows (assuming small angles):

Kinematics:

$$\dot{y} = v * \sin(\psi) = v * \psi$$
$$\dot{\psi} = \frac{v}{L} * \tan(\delta) = \frac{v}{L} * \delta$$

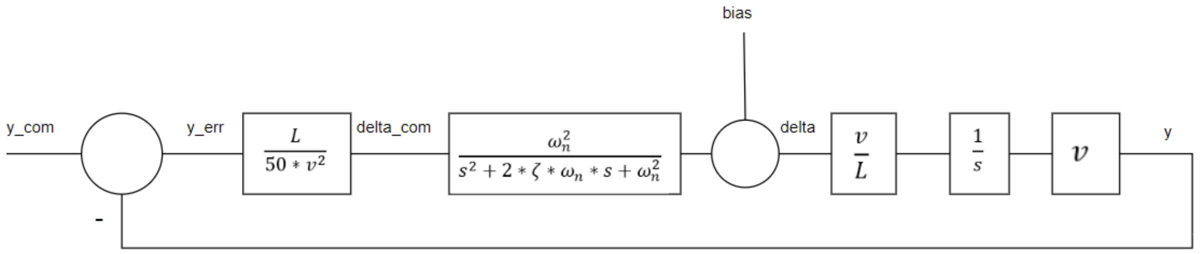
Servo dynamics:

$$\frac{\ddot{\delta}}{\omega_n^2} + 2 * \frac{\zeta}{\omega_n}\dot{\delta} + \delta = \delta_{com}$$

Tacking algorithm (with $d_{look\ ahead} = 10 * v$):

$$\delta_{com} = \tan(\delta_{com}) = \frac{L}{R} = L * \frac{y_{err}}{0.5 * d_{look\ ahead}{}^2} = 2 * L * \frac{y_{err}}{100 * v^2}$$

The system can be described in the following block diagram:



The transfer function between the error in y to the servo bias is:

$$\frac{Y_{err}(s)}{\delta_{bias}(s)} = \frac{\frac{v}{L} * \frac{1}{s} * v}{1 + \frac{v}{L} * \frac{1}{s} * v * \frac{L}{50 * v^2} * \frac{\omega_n^2}{s^2 + 2 * \zeta * \omega_n * s + \omega_n^2}}$$
$$= \frac{\frac{v^2}{L} * (s^2 + 2 * \zeta * \omega_n * s + \omega_n^2)}{s^3 + 2 * \zeta * \omega_n * s^2 + \omega_n^2 * s + \frac{\omega_n^2}{50}}$$

According to the final value law:

$$\lim_{t \to \infty} Y_{err}(t) = \lim_{s \to 0} \frac{Y_{err}(s)}{\delta_{bias}(s)} = 50 * \frac{v^2}{L} * \delta_{bias}$$

The bias is expected to create a constant lateral tracking error relative to the bias size. The following simulation results demonstrate this effect.

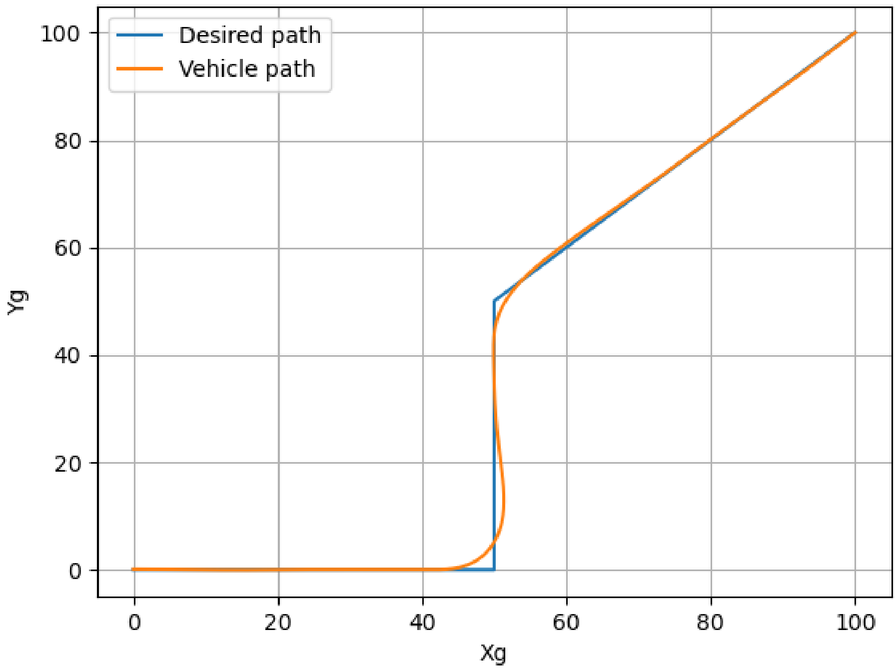


Figure 1 - Simulation results without servo angle bias

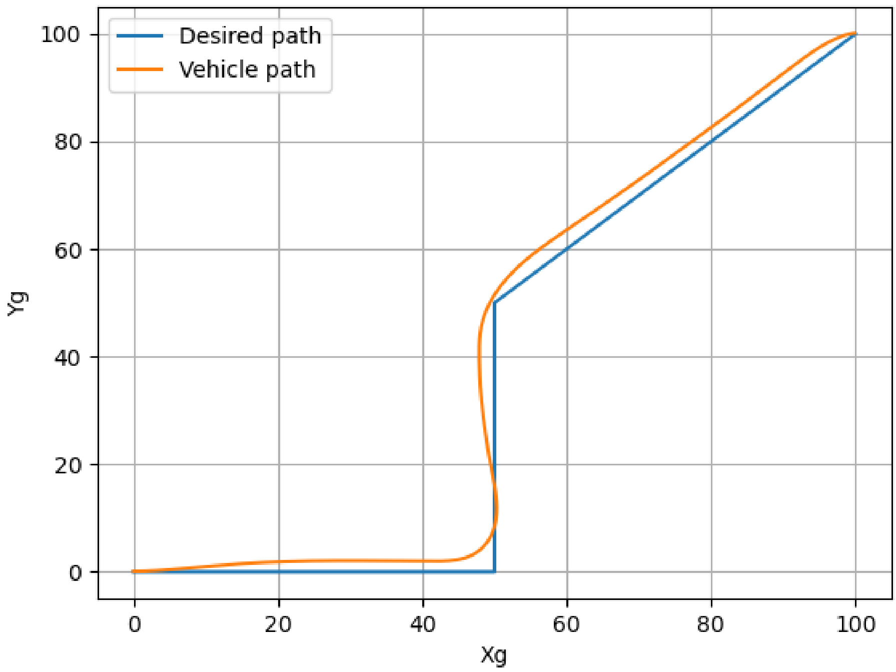


Figure 2 - Simulation results with servo angle bias of 0.1 rad

2. It is known that kinematic bicycle model is only valid in low speed ~5 [m/sec] suggest a model for higher speeds and state what cannot be neglected anymore?

   When the vehicle travels at a higher speed, the lateral forces generated by the tires and the wheels slip angle cannot be neglected anymore.
   Assuming the slip angles of the front and rear wheels are α_f and α_r accordingly, and that the lateral forces on the wheels are:

$$F_f = C * \alpha_f$$
$$F_r = C * \alpha_r$$

   In a constant circular motion, the radial acceleration is:

$$a_R = \frac{v^2}{R}$$

   According to newton's second law of motion, if the vehicle mass is m:

$$\sum F = m * a$$

   Assuming constant angular rate, around the center of gravity:

$$0 = I * \ddot{\psi} = \sum M$$

   If the distances of the front and rear wheels from the center of gravity are l_f and l_r accordingly:

$$\frac{v^2}{R} = a_R = \frac{F_r + F_f}{m} = \frac{C}{m} * (\alpha_f + \alpha_r)$$
$$F_r * l_r = F_f * l_f \rightarrow \alpha_r * l_r = \alpha_f * l_f$$

   After solving for the slip angles, we get:

$$\alpha_r = \frac{m * v^2}{C * R} * \frac{l_f}{l_f + l_r}$$
$$\alpha_f = \frac{m * v^2}{C * R} * \frac{l_r}{l_f + l_r}$$

   The following can be shown (assuming small angles) from the geometry of the problem:

$$\frac{L}{R} = 2 * \sin\left(\frac{\alpha_r - \alpha_f + \delta}{2}\right) = \alpha_r - \alpha_f + \delta = \delta + \frac{m * v^2}{C * R} * \frac{l_f - l_r}{l_f + l_r}$$

   The new model introduces a dynamic term depending on the vehicles mass, velocity, friction coefficient and center of gravity.

3. Explain your consideration for the integration method and step size.

   Because of the relative simplicity of the model, first order Euler method was chosen for the integration.
   Because the fastest element simulated is the servo dynamics, with a rise time of 0.2 [sec], a step size of 0.04 [sec] was chosen, so the dynamic effects won't be lost.