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Week 2 - Problem Set

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1. **Consider the following five events:** 1 / 1 point

   1. **Correctly guessing a random 128-bit AES key on the first try.**
   2. **Winning a lottery with 1 million contestants (the probability is $1/10^6$).**
   3. **Winning a lottery with 1 million contestants 5 times in a row (the probability is $(1/10^6)^5$).**
   4. **Winning a lottery with 1 million contestants 6 times in a row.**
   5. **Winning a lottery with 1 million contestants 7 times in a row.**

   What is the order of these events from most likely to least likely?

   - ( ) 2, 3, 5, 4, 1
   - (•) 2, 3, 4, 1, 5
   - ( ) 3, 2, 5, 4, 1
   - ( ) 2, 3, 1, 5, 4

   ✓ Correct
   - The probability of event (1) is 1/2^128.
   - The probability of event (5) is 1/(10^6)^7 which is about 1/2^{139}. Therefore, event (5) is the least likely.
   - The probability of event (4) is 1/(10^6)^6 which is about 1/2^{119.5} which is more likely than event (1).
   - The remaining events are all more likely than event (4).

2. **Suppose that using commodity hardware it is possible to build a computer** 1 / 1 point

   for about $200 that can brute force about 1 billion AES keys per second.

   Suppose an organization wants to run an exhaustive search for a single

   128-bit AES key and was willing to spend 4 trillion dollars to buy these

   machines (this is more than the annual US federal budget). How long would

   it take the organization to brute force this single 128-bit AES key with

   these machines? Ignore additional costs such as power and maintenance.

   - (•) More than a billion (10^9) years
   - ( ) More than a year but less than 100 years
   - ( ) More than a week but less than a month
   - ( ) More than a month but less than a year
   - ( ) More than a 100 years but less than a million years

   ✓ Correct
   The answer is about 540 billion years.
   - # machines = 4*10^12/200 = 2*10^10
   - # keys processed per sec = 10^9 * (2*10^10) = 2*10^19
   - # seconds = 2^128 / (2*10^19) = 1.7*10^19

   This many seconds is about 540 billion years.

3. **Let $F: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a secure PRF (i.e. a PRF where the key space, input space, and output space are all $\{0,1\}^n$) and say $n = 128$.** 0 / 1 point

   Which of the following is a secure PRF (there is more than one correct answer):

   - [x] $F'(k,x) = F(k,\ x \bigoplus 1^n)$

   ✓ Correct
   Correct. A distinguisher for F' gives a distinguisher for F.

   - [ ] $F'(k,x) = F(k,\ x)\ \bigoplus\ F(k,\ x \oplus 1^n)$
   - [ ] $F'((k_1,k_2),\ x) = F(k_1,x) \bigoplus F(k_2,x)$
   - [x] $F'((k_1,k_2),\ x) = F(k_1,x)\ \|\ F(k_2,x)$ (here $\|$ denotes concatenation)

   ✓ Correct
   Correct. A distinguisher for F' gives a distinguisher for F.

   - [x] $F'(k,\ x) = k \bigoplus x$

   ! This should not be selected
   Not a PRF. A distinguisher will query at $x = 0^n$ and $x = 1^n$ and

   output not random if the xor of the response is 1^n.

   This is unlikely to hold for a truly random function.

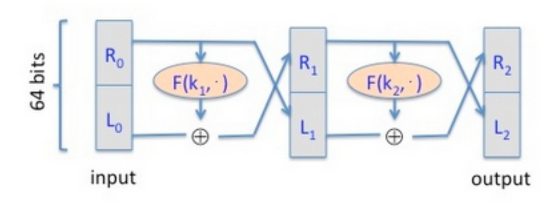
   - [ ] $F'(k,\ x) = \begin{cases} F(k,x) & \text{when } x \neq 0^n \\ 0^n & \text{otherwise} \end{cases}$

4. **Recall that the Luby-Rackoff theorem discussed in The Data Encryption Standard lecture states that applying a three round Feistel network to a secure PRF gives a secure block cipher. Let's see what goes wrong if we only use a two round Feistel.** 1 / 1 point

   Let $F: K \times \{0,1\}^{32} \to \{0,1\}^{32}$ be a secure PRF.

   Recall that a 2-round Feistel defines the following PRP

   $F_2: K^2 \times \{0,1\}^{64} \to \{0,1\}^{64}$:



   Here R_0 is the right 32 bits of the 64-bit input and L_0 is the left 32 bits.

   One of the following lines is the output of this PRP F_2 using a random key, while the other three are the output of a truly random permutation $f: \{0,1\}^{64} \to \{0,1\}^{64}$. All 64-bit outputs are encoded as 16 hex characters.

   Can you say which is the output of the PRP? Note that since you are able to distinguish the output of F_2 from random, F_2 is not a secure block cipher, which is what we wanted to show.

   Hint: First argue that there is a detectable pattern in the xor of $F_2(\cdot,\ 0^{64})$ and $F_2(\cdot,\ 1^{32}0^{32})$. Then try to detect this pattern in the given outputs.

   - (•) On input 0^{64} the output is "e86d2de2 e1387ae9".
     On input $1^{32}0^{32}$ the output is "1792d21d b645c008".
   - ( ) On input 0^{64} the output is "5f67abaf 5210722b".
     On input $1^{32}0^{32}$ the output is "bbe033c0 0bc9330e".
   - ( ) On input 0^{64} the output is "7c2822eb fdc48bfb".
     On input $1^{32}0^{32}$ the output is "325032a9 c5e2364b".
   - ( ) On input 0^{64} the output is "7b50baab 07640c3d".
     On input $1^{32}0^{32}$ the output is "ac343a22 cea46d60".

   ✓ Correct
   Observe that the two round Feistel has the property that

   the left of $F(\cdot,0^{64}) \bigoplus F(\cdot,1^{32}0^{32})$ is 1^{32}.

   The two outputs in this answer are the only ones with this property.

5. **Nonce-based CBC. Recall that in Lecture 4.4 we said that if one wants to use CBC encryption with a non-random unique nonce then the nonce must first be encrypted with an independent PRP key and the result then used as the CBC IV.** 1 / 1 point

   Let's see what goes wrong if one encrypts the nonce with the same PRP key as the key used for CBC encryption.

   Let $F: K \times \{0,1\}^\ell \to \{0,1\}^\ell$ be a secure PRP with, say, $\ell = 128$. Let n be a nonce and suppose one encrypts a message m by first computing $IV = F(k,n)$ and then using this IV in CBC encryption using $F(k,\cdot)$. Note that the same key k is used for computing the IV and for CBC encryption. We show that the resulting system is not nonce-based CPA secure.

   The attacker begins by asking for the encryption of the two block message $m = (0^\ell, 0^\ell)$ with nonce $n = 0^\ell$. It receives back a two block ciphertext (c_0, c_1). Observe that by definition of CBC we know that $c_1 = F(k, c_0)$.

   Next, the attacker asks for the encryption of the one block message $m_1 = c_0 \bigoplus c_1$ with nonce $n = c_0$. It receives back a one block ciphertext c_0'.

   What relation holds between c_0, c_1, c_0'? Note that this relation lets the adversary win the nonce-based CPA game with advantage 1.

   - ( ) $c_0' = c_0 \bigoplus 1^\ell$
   - ( ) $c_1 = c_0 \bigoplus c_0'$
   - ( ) $c_0 = c_1 \bigoplus c_0'$
   - (•) $c_1 = c_0'$

   ✓ Correct
   This follows from the definition of CBC with an encrypted nonce

   as defined in the question.

6. **Let m be a message consisting of ℓ AES blocks** 1 / 1 point

   (say $\ell = 100$). Alice encrypts m using CBC mode and transmits

   the resulting ciphertext to Bob. Due to a network error,

   ciphertext block number $\ell/2$ is corrupted during transmission.

   All other ciphertext blocks are transmitted and received correctly.

   Once Bob decrypts the received ciphertext, how many plaintext blocks

   will be corrupted?

   - (•) 2
   - ( ) ℓ
   - ( ) 3
   - ( ) $\ell/2$
   - ( ) 0

   ✓ Correct
   Take a look at the CBC decryption circuit. Each ciphertext

   blocks affects only the current plaintext block and the next.

7. **Let m be a message consisting of ℓ AES blocks (say $\ell = 100$). Alice encrypts m using randomized counter mode and** 1 / 1 point

   transmits the resulting ciphertext to Bob. Due to a network error,

   ciphertext block number $\ell/2$ is corrupted during transmission.

   All other ciphertext blocks are transmitted and received correctly.

   Once Bob decrypts the received ciphertext, how many plaintext blocks

   will be corrupted?

   - ( ) 2
   - ( ) $1 + \ell/2$
   - (•) 1
   - ( ) 0
   - ( ) $\ell/2$

   ✓ Correct
   Take a look at the counter mode decryption circuit. Each

   ciphertext block affects only the current plaintext block.

8. **Recall that encryption systems do not fully hide the length of** 1 / 1 point

   transmitted messages. Leaking the length of web requests hasbeen used to eavesdrop on encrypted HTTPS traffic to a number of

   web sites, such as tax preparation sites, Google searches, and

   healthcare sites.

   Suppose an attacker intercepts a packet where he knows that the

   packet payload is encrypted using AES in CBC mode with a random IV. The

   encrypted packet payload is 128 bytes. Which of the following

   messages is plausibly the decryption of the payload:

   - ( ) 'If qualified opinions incline to believe in the exponential
     conjecture, then I think we cannot afford not to make use of it.'
   - ( ) 'The most direct computation would be for the enemy to try
     all 2^r possible keys, one by one.'
   - ( ) 'The significance of this general conjecture, assuming its truth, is
     easy to see. It means that it may be feasible to design ciphers that
     are effectively unbreakable.'
   - (•) 'In this letter I make some remarks on a general principle
     relevant to enciphering in general and my machine.'

   ✓ Correct
   The length of the string is 107 bytes, which after padding becomes 112 bytes,

   and after prepending the IV becomes 128 bytes.

9. **Let $R := \{0,1\}^4$ and consider the following PRF $F: R^5 \times R \to R$ defined as follows:** 1 / 1 point

   $$F(k,x) := \begin{cases} t = k[0] \\ \text{for i=1 to 4 do} \\ \qquad \text{if } (x[i-1] == 1) \quad t = t \oplus k[i] \\ \text{output } t \end{cases}$$

   That is, the key is $k = (k[0], k[1], k[2], k[3], k[4])$ in R^5 and the function at, for example, 0101 is defined as $F(k, 0101) = k[0] \oplus k[2] \oplus k[4]$.

   For a random key k unknown to you, you learn that

   $F(k, 0110) = 0011$ **and** $F(k, 0101) = 1010$ **and** $F(k, 1110) = 0110$.

   What is the value of $F(k, 1101)$? Note that since you are able to predict the function at a new point, this PRF is insecure.

   1111

   ✓ Correct