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Week 2 - Problem Set

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1. Consider the following five events: 1 / 1 point

1. Correctly guessing a random 128-bit AES key on the first try.

2. Winning a lottery with 1 million contestants (the probability is $1/10^6$).

3. Winning a lottery with 1 million contestants 5 times in a row (the probability is $(1/10^6)^5$).

4. Winning a lottery with 1 million contestants 6 times in a row.

5. Winning a lottery with 1 million contestants 7 times in a row.

What is the order of these events from most likely to least likely?

- ( ) 2, 3, 5, 4, 1
- ( ) 3, 2, 5, 4, 1
- ( ) 2, 3, 1, 5, 4
- (•) 2, 3, 4, 1, 5

✓ Correct

- The probability of event (1) is 1/2^128.
- The probability of event (5) is 1/(10^6)^7 which is about 1/2^{139}. Therefore, event (5) is the least likely.
- The probability of event (4) is 1/(10^6)^6 which is about 1/2^{119.5} which is more likely than event (1).
- The remaining events are all more likely than event (4).

2. Suppose that using commodity hardware it is possible to build a computer for about $200 that can brute force about 1 billion AES keys per second. Suppose an organization wants to run an exhaustive search for a single 128-bit AES key and was willing to spend 4 trillion dollars to buy these machines (this is more than the annual US federal budget). How long would it take the organization to brute force this single 128-bit AES key with these machines? Ignore additional costs such as power and maintenance. 1 / 1 point

- ( ) More than a month but less than a year
- ( ) More than a day but less than a week
- (•) More than a billion (10^9) years
- ( ) More than a million years but less than a billion (10^9) years
- ( ) More than a 100 years but less than a million years

✓ Correct

The answer is about 540 billion years.

- # machines = 4*10^12/200 = 2*10^10
- # keys processed per sec = 10^9 * (2*10^10) = 2*10^19
- # seconds = 2^128 / (2*10^19) = 1.7*10^19

This many seconds is about 540 billion years.

3. Let $F:\{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}^n$ be a secure PRF (i.e. a PRF where the key space, input space, and output space are all $\{0,1\}^n$) and say $n=128$. 1 / 1 point

Which of the following is a secure PRF (there is more than one correct answer):

- [ ] $F'(k,x) = F(k,x) \parallel 0$

  (here $\parallel$ denotes concatenation)
- [ ] $F'(k,x) = F(k,\ x) \bigoplus F(k,\ x \oplus 1^n)$
- [x] $F'((k_1,k_2),\ x) = \begin{cases} F(k_1,x) & \text{when } x \neq 0^n \\ k_2 & \text{otherwise} \end{cases}$

✓ Correct

Correct. A distinguisher for F' gives a distinguisher for F.

- [ ] $F'(k,\ x) = k \bigoplus x$
- [x] $F'(k,x) = F(k,x)[0,\dots,n-2]$

  (i.e., $F'(k,x)$ drops the last bit of $F(k,x)$)

✓ Correct

Correct. A distinguisher for F' gives a distinguisher for F.

- [x] $F'((k_1,k_2),\ x) = F(k_1,x) \parallel F(k_2,x)$ (here $\parallel$ denotes concatenation)

✓ Correct

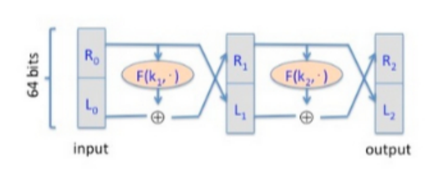
Correct. A distinguisher for F' gives a distinguisher for F.

4. Recall that the Luby-Rackoff theorem discussed in [The Data Encryption Standard lecture] states that applying a three round Feistel network to a secure PRF gives a secure block cipher. Let's see what goes wrong if we only use a two round Feistel. 1 / 1 point

Let $F: K \times \{0,1\}^{32} \rightarrow \{0,1\}^{32}$ be a secure PRF.

Recall that a 2-round Feistel defines the following PRP

$F_2: K^2 \times \{0,1\}^{64} \rightarrow \{0,1\}^{64}$:



Here R_0 is the right 32 bits of the 64-bit input and L_0 is the left 32 bits.

One of the following lines is the output of this PRP F_2 using a random key, while the other three are the output of a truly random permutation $f:\{0,1\}^{64} \rightarrow \{0,1\}^{64}$. All 64-bit outputs are encoded as 16 hex characters.

Can you say which is the output of the PRP? Note that since you are able to distinguish the output of F_2 from random, F_2 is not a secure block cipher, which is what we wanted to show.

Hint: First argue that there is a detectable pattern in the xor of $F_2(\cdot,\ 0^{64})$ and $F_2(\cdot,\ 1^{32}0^{32})$. Then try to detect this pattern in the given outputs.

- (•) On input 0^{64} the output is "e86d2de2 e1387ae9".

  On input $1^{32}0^{32}$ the output is "1792d21d b645c008".
- ( ) On input 0^{64} the output is "5f67abaf 5210722b".

  On input $1^{32}0^{32}$ the output is "bbe033c0 0bc9330e".
- ( ) On input 0^{64} the output is "7c2822eb fdc48bfb".

  On input $1^{32}0^{32}$ the output is "325032a9 c5e2364b".
- ( ) On input 0^{64} the output is "7b50baab 07640c3d".

  On input $1^{32}0^{32}$ the output is "ac343a22 cea46d60".

✓ Correct

Observe that the two round Feistel has the property that

the left of $F(\cdot,0^{64}) \bigoplus F(\cdot,1^{32}0^{32})$ is 1^{32}.

The two outputs in this answer are the only ones with this property.

5. Nonce-based CBC. Recall that in [Lecture 4.4] we said that if one wants to use CBC encryption with a non-random unique nonce then the nonce must first be encrypted with an independent PRP key and the result then used as the CBC IV. 1 / 1 point

Let's see what goes wrong if one encrypts the nonce with the same PRP key as the key used for CBC encryption.

Let $F: K \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$ be a secure PRP with, say, $\ell = 128$. Let n be a nonce and suppose one encrypts a message m by first computing $IV = F(k,n)$ and then using this IV in CBC encryption using $F(k,\cdot)$. Note that the same key k is used for computing the IV and for CBC encryption. We show that the resulting system is not nonce-based CPA secure.

The attacker begins by asking for the encryption of the two block message $m = (0^\ell, 0^\ell)$ with nonce $n = 0^\ell$. It receives back a two block ciphertext (c_0, c_1). Observe that by definition of CBC we know that $c_1 = F(k, c_0)$.

Next, the attacker asks for the encryption of the one block message $m_1 = c_0 \bigoplus c_1$ with nonce $n = c_0$. It receives back a one block ciphertext c_0'.

What relation holds between c_0, c_1, c_0'? Note that this relation lets the adversary win the nonce-based CPA game with advantage 1.

- (•) $c_1 = c_0'$
- ( ) $c_1 = c_0 \bigoplus c_0'$
- ( ) $c_0' = c_0 \bigoplus 1^\ell$
- ( ) $c_0 = c_1 \bigoplus c_0'$

✓ Correct

This follows from the definition of CBC with an encrypted nonce

as defined in the question.

6. Let m be a message consisting of ℓ AES blocks (say $\ell = 100$). Alice encrypts m using CBC mode and transmits the resulting ciphertext to Bob. Due to a network error, ciphertext block number $\ell/2$ is corrupted during transmission. All other ciphertext blocks are transmitted and received correctly. Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted? 1 / 1 point

- (•) 2
- ( ) 0
- ( ) ℓ
- ( ) 3
- ( ) $\ell/2$

✓ Correct

Take a look at the CBC decryption circuit. Each ciphertext

blocks affects only the current plaintext block and the next.

7. Let m be a message consisting of ℓ AES blocks (say $\ell = 100$). Alice encrypts m using randomized counter mode and transmits the resulting ciphertext to Bob. Due to a network error, ciphertext block number $\ell/2$ is corrupted during transmission. All other ciphertext blocks are transmitted and received correctly. Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted? 1 / 1 point

- ( ) $\ell/2$
- (•) 1
- ( ) 0
- ( ) 2
- ( ) $1 + \ell/2$

✓ Correct

Take a look at the counter mode decryption circuit. Each

ciphertext block affects only the current plaintext block.

8. Recall that encryption systems do not fully hide the length of transmitted messages. Leaking the length of web requests [hasbeen used] to eavesdrop on encrypted HTTPS traffic to a number of web sites, such as tax preparation sites, Google searches, and healthcare sites. Suppose an attacker intercepts a packet where he knows that the packet payload is encrypted using AES in CBC mode with a random IV. The encrypted packet payload is 128 bytes. Which of the following messages is plausibly the decryption of the payload: 1 / 1 point

- (•) 'In this letter I make some remarks on a general principle relevant to enciphering in general and my machine.'
- ( ) 'The most direct computation would be for the enemy to try all 2^r possible keys, one by one.'
- ( ) 'If qualified opinions incline to believe in the exponential conjecture, then I think we cannot afford not to make use of it.'
- ( ) 'The significance of this general conjecture, assuming its truth, is easy to see. It means that it may be feasible to design ciphers that are effectively unbreakable.'

✓ Correct

The length of the string is 107 bytes, which after padding becomes 112 bytes,

and after prepending the IV becomes 128 bytes.

9. Let $R := \{0,1\}^4$ and consider the following PRF $F: R^5 \times R \rightarrow R$ defined as follows: 1 / 1 point

$$F(k,x) := \begin{cases} t = k[0] \\ \text{for i=1 to 4 do} \\ \quad \text{if } (x[i-1] == 1) \quad t = t \oplus k[i] \\ \text{output } t \end{cases}$$

That is, the key is $k = (k[0], k[1], k[2], k[3], k[4])$ in R^5 and the function at, for example, 0101 is defined as $F(k, 0101) = k[0] \oplus k[2] \oplus k[4]$.

For a random key k unknown to you, you learn that

$F(k, 0110) = 0011$ **and** $F(k, 0101) = 1010$ **and** $F(k, 1110) = 0110$.

What is the value of $F(k, 1101)$? Note that since you are able to predict the function at a new point, this PRF is insecure.

1111

✓ Correct