

Week 4 - Problem Set

LATEST SUBMISSION GRADE

90%

1. An attacker intercepts the following ciphertext (hex encoded):

20814804c1767293b99f1d8cab3bc3e7 ac1e37bf015599e5f4deef805488281d

He knows that the plaintext is the ASCII encoding of the message "Pay Bob 1000" (excluding the quotes). He also knows that the cipher used is CBC encryption with a random IV using AES as the underlying block cipher.

Show that the attacker can change the ciphertext so that it will decrypt to "Pay Bob 5000". What is the resulting ciphertext (hex encoded)?

This shows that CBC provides no integrity.

20814804c1767293b99f1d8cab3bc3e7 ac1e37bf015599e5f4deef805488281d

✓ Correct  
You got it!

2. Let  $(E, D)$  be an encryption system with key space  $K$ , message

space  $\{0, 1\}^*$  and ciphertext space  $\{0, 1\}^*$ . Suppose  $(E, D)$

provides authenticated encryption. Which of the following systems

provides authenticated encryption: (as usual, we use  $\parallel$  to denote

string concatenation)

☒  $E'(k, m) = (E(k, m), E(k, m))$  and

$D'(k, (c_1, c_2)) = D(k, c_1)$

! This should not be selected  
This system does not provide ciphertext integrity. The attacker can query for  $E'(k, 0^*)$  to obtain  $(c_1, c_2)$ . It then outputs  $(c_1, \emptyset)$  and wins the ciphertext integrity game.

☒  $E'((k_1, k_2), m) = E(k_2, E(k_1, m))$  and

$D'((k_1, k_2), c) = \begin{cases} D(k_1, D(k_2, c)) & \text{if } D(k_2, c) \neq \perp \\ \perp & \text{otherwise} \end{cases}$

✓ Correct  
 $(E', D')$  provides authenticated encryption because an attack on  $(E', D')$  gives an attack on  $(E, D)$ . It's an interesting exercise to work out the ciphertext integrity attack on  $(E, D)$  given a ciphertext integrity attacker on  $(E', D)$ .

☒  $E'(k, m) = (E(k, m), H(m))$  and

$D'(k, (c, h)) = \begin{cases} D(k, c) & \text{if } H(D(k, c)) = h \\ \perp & \text{otherwise} \end{cases}$

(here  $H$  is some collision resistant hash function)

! This should not be selected  
This system is not CPA secure because  $H(m)$  leaks information about the message in the ciphertext.

☒  $E'(k, m) = [c \leftarrow E(k, m), \text{output}(c, c)]$  and

$D'(k, (c_1, c_2)) = \begin{cases} D(k, c_1) & \text{if } c_1 = c_2 \\ \perp & \text{otherwise} \end{cases}$

✓ Correct  
 $(E', D')$  provides authenticated encryption because an attack on  $(E', D')$  directly gives an attack on  $(E, D)$ .

3. If you need to build an application that needs to encrypt multiple

messages using a single key, what encryption

method should you use? (for now, we ignore the question of key generation

and management)

☐ Implement MAC-then-Encrypt yourself

☐ Use a standard implementation of CBC encryption with

a random IV.

☐ Implement Encrypt-and-MAC yourself

☒ Use a standard implementation of one of the authenticated

encryption modes GCM, CCM, EAX or OCB.

✓ Correct

4. Let  $(E, D)$  be a symmetric encryption system with message space  $M$  (think

of  $M$  as only consisting for short messages, say 32 bytes).

Define the following MAC  $(S, V)$  for messages in  $M$ :

$S(k, m) := E(k, m)$  ;  $V(k, m, t) := \begin{cases} 1 & \text{if } D(k, t) = m \\ 0 & \text{otherwise} \end{cases}$

What is the property that the encryption system  $(E, D)$  needs to satisfy

for this MAC system to be secure?

☒ authenticated encryption

☐ semantic security under a deterministic chosen plaintext attack

☐ semantic security

☐ semantic security under a chosen plaintext attack

✓ Correct  
Indeed, authenticated encryption implies ciphertext integrity which prevents existential forgery under a chosen message attack.

5. In Key Derivation we discussed how to derive session keys

from a shared secret. The problem is what to do when the shared

secret is non-uniform. In this question we show that using a PRF with

a non-uniform key may result in non-uniform values. This shows that

session keys cannot be derived by directly using a non-uniform

secret as a key in a PRF. Instead, one has to use a key derivation

function like HKDF.

Suppose  $k$  is a non-uniform secret key sampled from the key space  $\{0, 1\}^{256}$ .

In particular,  $k$  is sampled uniformly from the set of all keys whose most significant

128 bits are all 0. In other words,  $k$  is chosen uniformly from a small subset of the key space. More

precisely,

for all  $c \in \{0, 1\}^{128}$ :  $\Pr[k = c] = \begin{cases} 1/2^{128} & \text{if } \text{MSB}_{128}(c) = 0^{128} \\ 0 & \text{otherwise} \end{cases}$

Let  $F(k, x)$  be a secure PRF with input space  $\{0, 1\}^{256}$ . Which

of the following is a secure PRF when the key  $k$  is uniform in the

key space  $\{0, 1\}^{256}$ , but is insecure when the key is sampled from the non-uniform

distribution described above?

☒  $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) \neq 0^{128} \\ 0^{256} & \text{otherwise} \end{cases}$

☐  $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) = 0^{128} \\ 0^{256} & \text{otherwise} \end{cases}$

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☐  $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) = 0^{128} \\ 1^{256} & \text{otherwise} \end{cases}$

✓ Correct  
 $F'(k, x)$  is a secure PRF because for a uniform key  $k$  the probability that  $\text{MSB}_{128}(k) = 0^{128}$  is negligible. However, for the "non-uniform" key  $k$  this PRF always outputs 0 and is therefore completely insecure. This PRF cannot be used as a key derivation function for the distribution of keys described in the problem.

6. In what settings is it acceptable to use deterministic authenticated

encryption (DAE) like SIV?

☒ when messages are chosen at random from a large enough space so that

messages are unlikely to repeat.

☐ when a fixed message is repeatedly encrypted using a single key.

☐ to individually encrypt many packets in a voice conversation with a single key.

☐ to encrypt many records in a database with a single key when the same record may repeat multiple

times.

✓ Correct  
Deterministic encryption is safe to use when the message/key pair is never used more than once.

7. Let  $E(k, x)$  be a secure block cipher. Consider the following

tweakable block cipher:

$E'((k_1, k_2), t, x) = E(k_1, x) \oplus E(k_2, t)$

Is this tweakable block cipher secure?

☒ no because for  $t \neq t'$  we have

$E'((k_1, k_2), t, 0) \oplus E'((k_1, k_2), t', 1) = E'((k_1, k_2), t', 0) \oplus E'((k_1, k_2), t', 1)$

☐ no because for  $t \neq t'$  we have

$E'((k_1, k_2), t, 0) \oplus E'((k_1, k_2), t', 1) = E'((k_1, k_2), t', 1) \oplus E'((k_1, k_2), t', 0)$

☐ no because for  $x \neq x'$  and  $t \neq t'$  we have

$E'((k_1, k_2), t, x) \oplus E'((k_1, k_2), t', x) = E'((k_1, k_2), t, x') \oplus E'((k_1, k_2), t', x')$

☐ no because for  $x \neq x'$  we have

$E'((k_1, k_2), 0, x) \oplus E'((k_1, k_2), 0, x) = E'((k_1, k_2), 0, x') \oplus E'((k_1, k_2), 0, x')$

☐ yes. It is secure assuming  $E$  is a secure block cipher.

✓ Correct  
since this relation holds, an attacker can make 4 queries to  $E'$  and distinguish  $E'$  from a random collection of one-to-one functions.

8. In Format Preserving Encryption we discussed format preserving encryption

which is a PRP on a domain  $\{0, \dots, s - 1\}$  for some pre-specified

value of  $s$ .

Recall that the construction we presented worked in two steps,

where the second step worked by iterating the PRP until the output

fell into the set  $\{0, \dots, s - 1\}$ .

Suppose we try to build a format preserving credit card encryption

system from AES using "only" the second step. That is, we start with

a PRP with domain  $\{0, 1\}^{128}$  from which we want to build a PRP

with domain  $10^{16}$ . If we only used step (2), how many iterations of

AES would be needed in expectation for each evaluation of the PRP

with domain  $10^{16}$ ?

☐ 2

☒  $2^{16}/10^{16} \approx 3.4 \times 10^{12}$

☐  $10^{16}/2^{16}$

☐  $2^{16}$

✓ Correct  
On every iteration we have a probability of  $10^{16}/2^{16}$  of falling into the set  $\{0, \dots, 10^{16}\}$  and therefore in expectation we will need  $2^{16}/10^{16}$  iterations. This should explain why step (1) is needed.

9. Let  $(E, D)$  be a secure tweakable block cipher.

Define the following MAC  $(S, V)$ :

$S(k, m) := E(k, m, 0)$  ;  $V(k, m, \text{tag}) := \begin{cases} 1 & \text{if } E(k, m, 0) = \text{tag} \\ 0 & \text{otherwise} \end{cases}$

In other words, the message  $m$  is used as the tweak and the plaintext given to  $E$  is always set to 0.

Is this MAC secure?

☒ yes

☐ It depends on the tweakable block cipher.

☐ no

✓ Correct  
A tweakable block cipher is indistinguishable from a collection of random permutations. The chosen message attack on the MAC gives the attacker the image of 0 under a number of the permutations in the family. But that tells the attacker nothing about the image of 0 under some other member of the family.

10. In CBC Padding Attacks we discussed padding oracle attacks. These chosen-ciphertext attacks can

break poor implementations of MAC-then-encrypt.

Consider a system that implements MAC-then-encrypt where encryption is done using CBC with a random IV using AES as the block cipher. Suppose the system is vulnerable to a padding oracle attack. An attacker intercepts a 64-byte ciphertext  $c$  (the first 16 bytes of  $c$  are the IV and the remaining 48 bytes are the encrypted payload). How many chosen ciphertext queries would the attacker need in the worst case in order to decrypt the entire 48 byte payload? Recall that padding oracle attacks decrypt the payload one byte at a time.

☒ 12288

☐ 48

☐ 1024

☐ 16384

☐ 256

✓ Correct

Correct. Padding oracle attacks decrypt the payload one byte at a time. For each byte the attacker needs no more than 256 guesses in the worst case. Since there are 48 bytes total, the number queries needed is  $256 \times 48 = 12288$ .