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TO PASS 80% or higher

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Week 5 - Problem Set

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1. Consider the toy key exchange protocol using an online trusted 3rd party (TTP) discussed in Lecture 9.1. Suppose Alice, Bob, and Carol are three users of this system (among many others) and each have a secret key with the TTP denoted k_a, k_b, k_c respectively. They wish to generate a group session key k_{ABC} that will be known to Alice, Bob, and Carol but unknown to an eavesdropper. How would you modify the protocol in the lecture to accommodate a group key exchange of this type? (note that all these protocols are insecure against active attacks)

0 / 1 point

- ○ Alice contacts the TTP. TTP generates a random k_{ABC} and sends to Alice
  $E(k_a, k_{ABC}), \quad \text{ticket}_1 \leftarrow E(k_c, E(k_b, k_{ABC})), \quad \text{ticket}_2 \leftarrow E(k_b, E(k_c, k_{ABC})).$
  Alice sends k_{ABC} to Bob and k_{ABC} to Carol.
- ○ Bob contacts the TTP. TTP generates a random k_{AB} and a random k_{BC}. It sends to Bob
  $E(k_a, k_{AB}), \quad \text{ticket}_1 \leftarrow E(k_a, k_{AB}), \quad \text{ticket}_2 \leftarrow E(k_c, k_{BC}).$
  Bob sends ticket_1 to Alice and ticket_2 to Carol.
- ○ Bob contacts the TTP. TTP generates random k_{ABC} and sends to Bob
  $E(k_b, k_{ABC}), \quad \text{ticket}_1 \leftarrow E(k_a, k_{ABC}), \quad \text{ticket}_2 \leftarrow E(k_c, k_{ABC}).$
  Bob sends ticket_1 to Alice and ticket_2 to Carol.
- ◉ Alice contacts the TTP. TTP generates a random k_{ABC} and sends to Alice
  $E(k_a, k_{ABC}), \quad \text{ticket}_1 \leftarrow E(k_b, k_{ABC}), \quad \text{ticket}_2 \leftarrow E(k_c, k_{ABC}).$
  Alice sends k_{ABC} to Bob and k_{ABC} to Carol.

! Incorrect
The protocol is insecure because k_{ABC} is sent in the clear and an eavesdropper can easily obtain it.

2. Let G be a finite cyclic group (e.g. $G = \mathbb{Z}_p^*$) with generator g.

0 / 1 point

Suppose the Diffie-Hellman function $\text{DH}_g(g^x, g^y) = g^{xy}$ is difficult to compute in G. Which of the following functions is also difficult to compute?

As usual, identify the f below for which the contra-positive holds: if $f(\cdot,\cdot)$ is easy to compute then so is $\text{DH}_g(\cdot,\cdot)$. If you can show that then it will follow that if DH_g is hard to compute in G then so must be f.

- ☑ $f(g^x, g^y) = g^{x(y+1)}$

✓ Correct
an algorithm for calculating $f(g^x, g^y)$ can easily be converted into an algorithm for calculating $\text{DH}(\cdot,\cdot)$.
Therefore, if f were easy to compute then so would DH, contrading the assumption.

- ☑ $f(g^x, g^y) = g^{x+y}$

! This should not be selected
It is easy to compute f as $f(g^x, g^y) = g^x \cdot g^y$.

- ☑ $f(g^x, g^y) = g^{x-y}$

! This should not be selected
It is easy to compute f as $f(g^x, g^y) = g^x / g^y$.

- ☑ $f(g^x, g^y) = (g^{3xy}, g^{2xy})$ (this function outputs a pair of elements in G)

✓ Correct
an algorithm for calculating $f(\cdot,\cdot)$ can easily be converted into an algorithm for calculating $\text{DH}(\cdot,\cdot)$.
Therefore, if f were easy to compute then so would DH, contrading the assumption.

3. Suppose we modify the Diffie-Hellman protocol so that Alice operates as usual, namely chooses a random a in $\{1,\dots,p-1\}$ and sends to Bob $A \leftarrow g^a$. Bob, however, chooses a random b in $\{1,\dots,p-1\}$ and sends to Alice $B \leftarrow g^{1/b}$. What shared secret can they generate and how would they do it?

1 / 1 point

- ◉ $\text{secret} = g^{a/b}$. Alice computes the secret as B^a and Bob computes $A^{1/b}$.
- ○ $\text{secret} = g^{a/b}$. Alice computes the secret as $B^{1/b}$ and Bob computes A^a.
- ○ $\text{secret} = g^{a/b}$. Alice computes the secret as $B^{1/a}$ and Bob computes A^b.
- ○ $\text{secret} = g^{b/a}$. Alice computes the secret as B^a and Bob computes $A^{1/b}$.

✓ Correct
This is correct since it is not difficult to see that both will obtain $g^{a/b}$

4. Consider the toy key exchange protocol using public key encryption described in Lecture 9.4.

1 / 1 point

Suppose that when sending his reply $c \leftarrow E(pk, x)$ to Alice, Bob appends a MAC $t := S(x, c)$ to the ciphertext so that what is sent to Alice is the pair (c, t). Alice verifies the tag t and rejects the message from Bob if the tag does not verify.

Will this additional step prevent the man in the middle attack described in the lecture?

- ○ it depends on what MAC system is used.
- ◉ no
- ○ yes
- ○ it depends on what public key encryption system is used.

✓ Correct
an active attacker can still decrypt $E(pk', x)$ to recover x and then replace (c, t) by (c', t') where $c' \leftarrow E(pk, x)$ and $t \leftarrow S(x, c')$.

5. The numbers 7 and 23 are relatively prime and therefore there must exist integers a and b such that $7a + 23b = 1$.

1 / 1 point

Find such a pair of integers (a, b) with the smallest possible $a > 0$.

Given this pair, can you determine the inverse of 7 in $\mathbb{Z}_{23}$?

Enter below comma separated values for a, b, and for 7^{-1} in $\mathbb{Z}_{23}$.

10,-3,10

✓ Correct
$7 \times 10 + 23 \times (-3) = 1$.
Therefore $7 \times 10 = 1$ in $\mathbb{Z}_{23}$ implying that $7^{-1} = 10$ in $\mathbb{Z}_{23}$.

6. Solve the equation $3x + 2 = 7$ in $\mathbb{Z}_{19}$.

1 / 1 point

8

✓ Correct
$x = (7-2) \times 3^{-1} \in \mathbb{Z}_{19}$

7. How many elements are there in $\mathbb{Z}_{35}^*$?

1 / 1 point

24

✓ Correct
$|\mathbb{Z}_{35}^*| = \varphi(7 \times 5) = (7-1) \times (5-1)$.

8. How much is $2^{10001} \bmod 11$?

1 / 1 point

Please do not use a calculator for this. Hint: use Fermat's theorem.

2

✓ Correct
By Fermat $2^{10} = 1$ in $\mathbb{Z}_{11}$ and therefore
$1 = 2^{10} = 2^{20} = 2^{30} = 2^{40}$ in $\mathbb{Z}_{11}$.
Then $2^{10001} = 2^{10001 \bmod 10} = 2^1 = 2$ in $\mathbb{Z}_{11}$.

9. While we are at it, how much is $2^{245} \bmod 35$?

1 / 1 point

Hint: use Euler's theorem (you should not need a calculator)

32

✓ Correct
By Euler $2^{24} = 1$ in $\mathbb{Z}_{35}$ and therefore
$1 = 2^{24} = 2^{48} = 2^{72}$ in $\mathbb{Z}_{35}$.
Then $2^{245} = 2^{245 \bmod 24} = 2^5 = 32$ in $\mathbb{Z}_{35}$.

10. What is the order of 2 in $\mathbb{Z}_{35}^*$?

1 / 1 point

12

✓ Correct
$2^{12} = 4096 = 1$ in $\mathbb{Z}_{35}$ and 12 is the smallest such positive integer.

11. Which of the following numbers is a generator of $\mathbb{Z}_{13}^*$?

0 / 1 point

- ☑ $7, \quad \langle 7 \rangle = \{1, 7, 10, 5, 9, 11, 12, 6, 3, 8, 4, 2\}$

✓ Correct
correct, 7 generates the entire group $\mathbb{Z}_{13}^*$

- ☑ $4, \quad \langle 4 \rangle = \{1, 4, 3, 12, 9, 10\}$

! This should not be selected
No, 4 only generates six elements in $\mathbb{Z}_{13}^*$.

- ☑ $6, \quad \langle 6 \rangle = \{1, 6, 10, 8, 9, 2, 12, 7, 3, 5, 4, 11\}$

✓ Correct
correct, 6 generates the entire group $\mathbb{Z}_{13}^*$

- ☑ $3, \quad \langle 3 \rangle = \{1, 3, 9\}$

! This should not be selected
No, 3 only generates three elements in $\mathbb{Z}_{13}^*$.

- ☑ $8, \quad \langle 8 \rangle = \{1, 8, 12, 5\}$

! This should not be selected
No, 8 only generates four elements in $\mathbb{Z}_{13}^*$.

12. Solve the equation $x^2 + 4x + 1 = 0$ in $\mathbb{Z}_{23}$.

1 / 1 point

Use the method described in Lecture 10.3 using the quadratic formula.

14,5

✓ Correct
The quadratic formula gives the two roots in $\mathbb{Z}_{23}$.

13. What is the 11th root of 2 in $\mathbb{Z}_{19}$?

1 / 1 point

(i.e. what is $2^{1/11}$ in $\mathbb{Z}_{19}$)

Hint: observe that $11^{-1} = 5$ in $\mathbb{Z}_{18}$.

13

✓ Correct
$2^{1/11} = 2^5 = 32 = 13$ in $\mathbb{Z}_{19}$.

14. What is the discete log of 5 base 2 in $\mathbb{Z}_{13}$?

1 / 1 point

(i.e. what is $\text{Dlog}_2(5)$)

Recall that the powers of 2 in $\mathbb{Z}_{13}$ are $\langle 2 \rangle = \{1, 2, 4, 8, 3, 6, 12, 11, 9, 5, 10, 7\}$

9

✓ Correct
$2^9 = 5$ in $\mathbb{Z}_{13}$.

15. If p is a prime, how many generators are there in $\mathbb{Z}_p^*$?

1 / 1 point

- ○ $(p+1)/2$
- ○ $(p-1)/2$
- ○ $\varphi(p)$
- ◉ $\varphi(p-1)$

✓ Correct
The answer is $\varphi(p-1)$. Here is why. Let g be some generator of $\mathbb{Z}_p^*$ and let $h = g^x$ for some x.
It is not difficult to see that h is a generator exactly when we can write g as $g = h^y$ for some integer y (h is a generator because if $g = h^y$ then any power of g can also be written as a power of h
Since $y = x^{-1} \bmod p-1$ this y exists exactly when x is relatively prime to $p-1$. The number of such x is the size of $\mathbb{Z}_{p-1}^*$ which is precisely $\varphi(p-1)$.