Q-IFO: Lab Notebook

1 Bandwidth Tuning via Thermal Expansion in an Effective Two-Mirror Filter Cavity

We model a three-mirror filter cavity as an effective two-mirror system under the assumption that $L_1 \ll L_2$. Mirrors R_1 and R_2 form a short subcavity of length L_1, embedded within a larger cavity defined by length L_2. Thermal expansion of this short segment modulates the effective reflectivity and tunes the total cavity bandwidth.

For the tabletop configurations, we are using Thorlabs BB1-E03Ps (backside polished plano mirrors) with identical reflectivities:

$$R_1 = R_2 = R_{12},$$

the approximate bandwidth of the effective cavity is given by:

$$\gamma = \frac{c}{8\pi L_2}\left(\frac{(1-R_{12})^2}{1-2R_{12}\cos\left(\frac{4\pi L_1}{\lambda}\right)+R_{12}^2}+(1-R_3)\right)$$

Thermal Tunability

The bandwidth tunability with respect to the subcavity length L_1 is:

$$\frac{d\gamma}{dL_1} = \frac{c}{8\pi L_2}\cdot\frac{dT_{\text{eff}}}{d\phi}\cdot\frac{d\phi}{dL_1}, \quad \text{where } \phi = \frac{4\pi L_1}{\lambda}$$

The sensitivity of the effective transmission term is:

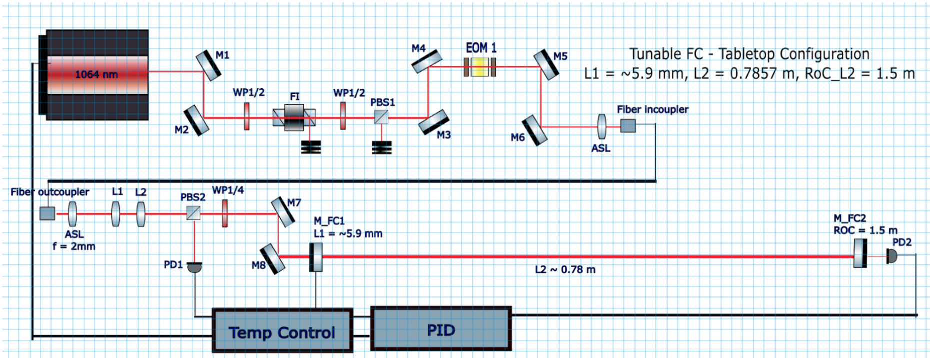
$$\frac{dT_{\text{eff}}}{d\phi} = -\frac{(1-R_1)\cdot(1-R_2)\cdot 2\sqrt{R_1R_2}\sin\left(\frac{4\pi L_1}{\lambda}\right)}{\left[1-2\sqrt{R_1R_2}\cos\left(\frac{4\pi L_1}{\lambda}\right)+R_1R_2\right]^2}$$

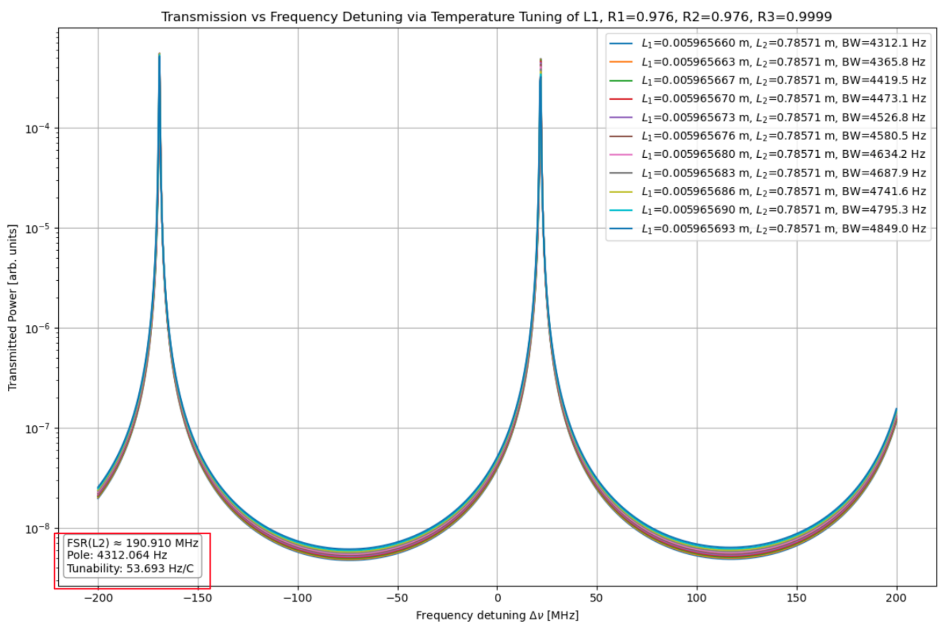
Thermal tunability in Hz/°C is then:

$$\text{Tunability} = \frac{d\gamma}{dL_1}\cdot\frac{dL_1}{dT}, \quad \text{with } \frac{dL_1}{dT} = \alpha\cdot L_1$$

Design Goal

We target a tunability of several hundred Hz over a 10 °C swing, with baseline bandwidths between 10–500 kHz. This method scales favorably to long cavities. For instance, a 300 m cavity incorporating a thin tuning optic (with L_1 on the order of millimeters) can maintain high finesse while achieving controllable bandwidth variation.



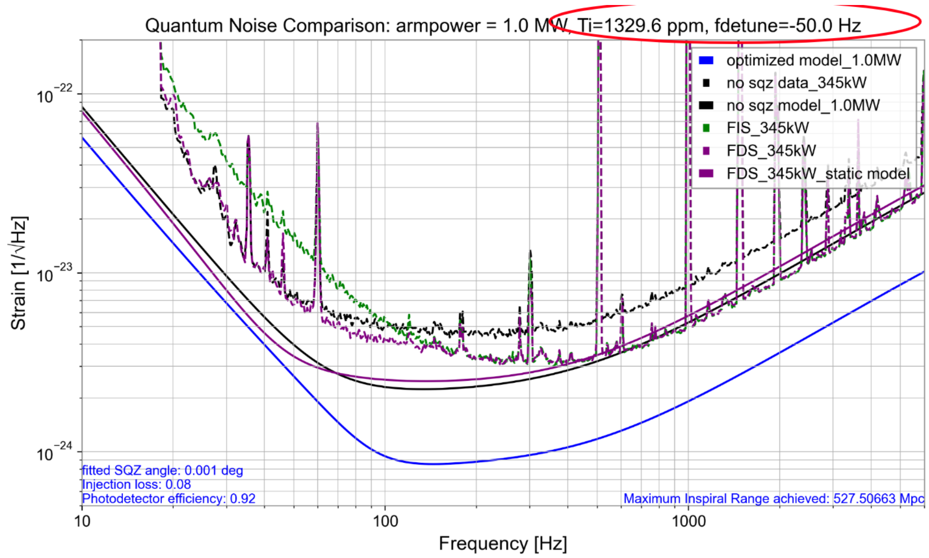


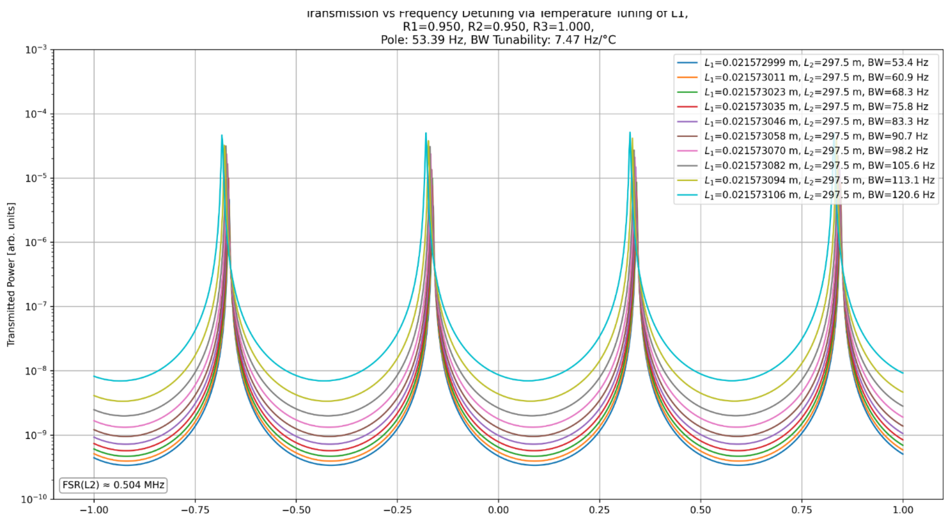
Tabletop Tunable FC Demonstration

We want to demonstrate the feasibility of the tunable filter cavity with a tabletop configuration, in which L2 0.787 m

Filter Cavity Optimization via Noise Budgeting

The tabletop demonstration serves as a proof of concept for implementing a similar tunable configuration in the full scale 300 m cavity. To define optimal filter cavity parameters for the A+ LIGO upgrade—with target circulating arm powers up to 1 MW—we employ quantum noise budgeting tools to simulate the interferometer's sensitivity under various FC configurations. Key parameters such as input mirror transmissivity (T_i), detuning frequency, and cavity bandwidth are jointly optimized to maximize the binary neutron star (BNS) inspiral range and suppress quantum radiation pressure noise at low frequencies. To efficiently explore this parameter space, we integrate machine learning techniques—particularly Bayesian optimization—to identify favorable configurations for bandwidth and thermal tunability.





Scaling to a 300 m Filter Cavity

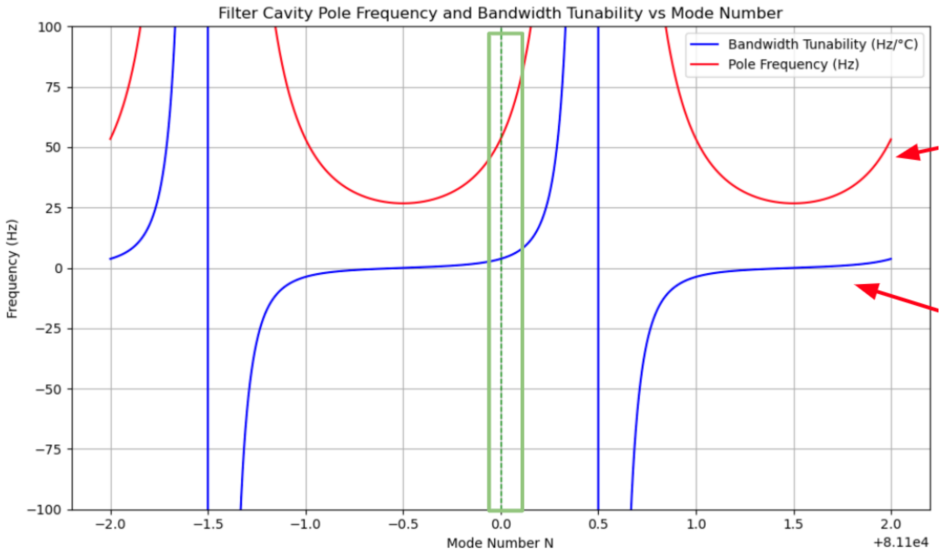
The short subcavity formed by mirrors R_1 and R_2 (with $L_1 \ll L_2$) acts as a compound mirror with an effective transmissivity:

$$T_{\text{eff}} = \frac{(1-R_1)(1-R_2)}{1 - 2\sqrt{R_1 R_2}\cos\left(\frac{4\pi L_1}{\lambda}\right) + R_1 R_2}$$

The total cavity bandwidth is then:

$$\gamma = \frac{c}{8\pi L_2}\left(T_{\text{eff}} + (1 - R_3)\right)$$

This framework allows tunability via thermal expansion of L_1, without compromising the high finesse or stability of the long cavity. The same thermal tuning principles demonstrated in the tabletop setup can thus be extended to a 300 m-scale system.



Quadrature Tuning for Thermal Control

To enable efficient thermal tuning, we park the subcavity length L_1 at a quadrature wavelength, such that the internal phase $\phi = 2kL_1$ satisfies:

$$\phi = 2kL_1 = \frac{4\pi L_1}{\lambda} = (2N+3)\,\frac{\pi}{2}, \quad N \in Z$$

This choice optimizes the slope of the effective transmission function $T_{\text{eff}}(\phi)$, yielding high sensitivity to changes in L_1. For a full-scale cavity with $L_2 = 297.5\,\text{m}$, we target cavity poles in the range of 40–70 Hz, with thermal tunability in the range of 2–10 Hz/°C.

2 Interactive SQZ

Injecting squeezed vacuum reduces quantum shot noise at high frequencies. At low frequencies, quantum radiation pressure noise (QRPN) dominates, which requires a frequency-dependent rotation of the squeezed quadrature. A filter cavity provides this rotation, and an interactive tool helps visualize how detuning, bandwidth, and efficiency affect the quantum noise curve.

Quadrature Variances

The frequency-dependent squeezed quadrature variances after the interferometer or filter cavity are:

$$R_{\pm}(\Omega) = 1 \;\pm\; \eta\,\frac{4x}{(1 \mp x)^2}$$

where

- R_+: anti-squeezed quadrature variance,
- R_-: squeezed quadrature variance,

- $x = 1 - \frac{1}{\sqrt{\mathrm{NLG}}}$, with NLG $\approx 17 \;\Rightarrow\; x \approx 0.76$,
- η: overall efficiency (includes propagation loss, mode-mismatch, and detection inefficiency).

Inclusion of Angular Fluctuations (Rotations)

To account for angular fluctuations in the squeezing ellipse, we average over a Gaussian distribution of angle jitter θ with width $\tilde{\theta}$:

$$R'_{\pm}(\tilde{\theta}) = \int \frac{1}{\sqrt{2\pi}\,\tilde{\theta}} \exp\left(-\frac{\theta^2}{2\tilde{\theta}^2}\right) \left(R_{\pm}\cos^2\theta + R_{\mp}\sin^2\theta\right) d\theta.$$

For small angular spreads $\tilde{\theta}$, this expression can be approximated as

$$R'_{\pm}(\tilde{\theta}) \;\approx\; R_{\pm}\cos^2\tilde{\theta} \;+\; R_{\mp}\sin^2\tilde{\theta}.$$

This shows that angular jitter mixes the squeezed and anti-squeezed quadratures, degrading the observed squeezing.

Model Parameters

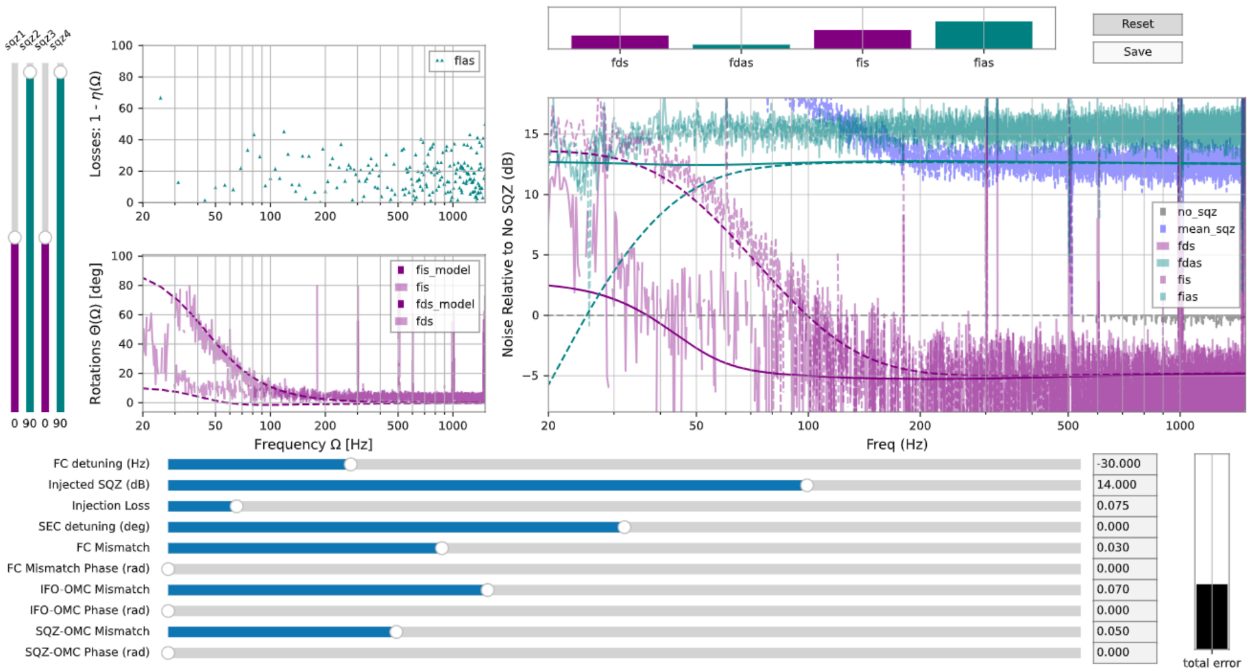
The interactive squeezing model is controlled by:

- **Detuning** (Δ): sets the frequency of quadrature rotation,
- **Bandwidth** (γ): determines how rapidly the rotation occurs,
- **Homodyne angle**: selects the readout quadrature,
- **Efficiency** (η): single parameter capturing all optical losses.

Interactive Visualization

The GUI implements sliders for Δ, γ, homodyne angle, and η. The output consists of dynamic plots of the strain spectrum showing:

- No squeezing (baseline),
- Frequency-independent squeezing (fixed ellipse),
- Frequency-dependent squeezing (rotation via filter cavity).



Insights

- Detuning shifts the rotation frequency, aligning the squeezing ellipse with the interferometer noise spectrum,
- Bandwidth sets the slope of quadrature rotation with frequency,
- Homodyne angle tunes the readout axis, balancing shot noise and QRPN suppression,
- Efficiency η directly reduces observable squeezing and enhances anti-squeezing leakage,
- Angular jitter $\tilde{\theta}$ couples anti-squeezing into the measurement, limiting achievable noise reduction.

Loss Extraction and Angle Jitter Fitting

In practice, the overall efficiency η can be directly solved by comparing the measured anti-squeezed quadrature variance R_+ with the frequency-independent anti-squeezing (FIAS) trace. Since the anti-squeezed quadrature dominates, we can isolate η as

$$R_+^{\text{meas}}(\Omega) \;\approx\; 1 + \eta\,\frac{4x}{(1-x)^2}.$$

This provides a direct calibration of the effective losses in the system.

Once η is fixed from the anti-squeezed quadrature, the residual differences between the measured squeezed quadrature and the model can be attributed to angular fluctuations of the squeezing ellipse. These are described by the jitter-averaged variance formula

$$R'_\pm(\tilde{\theta}) = \int \frac{1}{\sqrt{2\pi}\,\tilde{\theta}} \exp\left(-\frac{\theta^2}{2\tilde{\theta}^2}\right) \left(R_\pm \cos^2\theta + R_\mp \sin^2\theta\right) d\theta \quad \approx \quad R_\pm \cos^2\tilde{\theta} + R_\mp \sin^2\tilde{\theta}.$$

Fitting this expression to the squeezed quadrature data allows extraction of the jitter width $\tilde{\theta}$, which quantifies the amount of quadrature mixing induced by angle noise. In this way, the anti-squeezed quadrature variance determines the loss η, while the squeezed quadrature variance constrains $\tilde{\theta}$.