DeepLearning.AI

Math for Machine Learning

-

Probability and Statistics - Week 4

W4 Lesson 1

DeepLearning.AI

Confidence Interval

Confidence Interval (Known Standard Deviation)

Confidence Interval - Intuition

Confidence Interval - Intuition

Statistopia

10,000 people

Confidence Interval - Intuition

Statistopia

10,000 people

Estimate μ

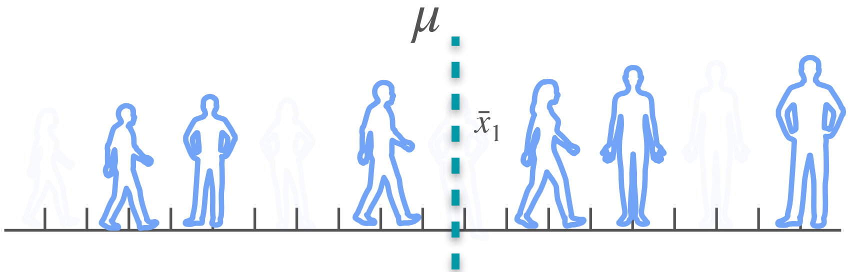
Confidence Interval - Intuition

Statistopia

10,000 people

Estimate μ
(mean height of the population)

Confidence Interval - Intuition

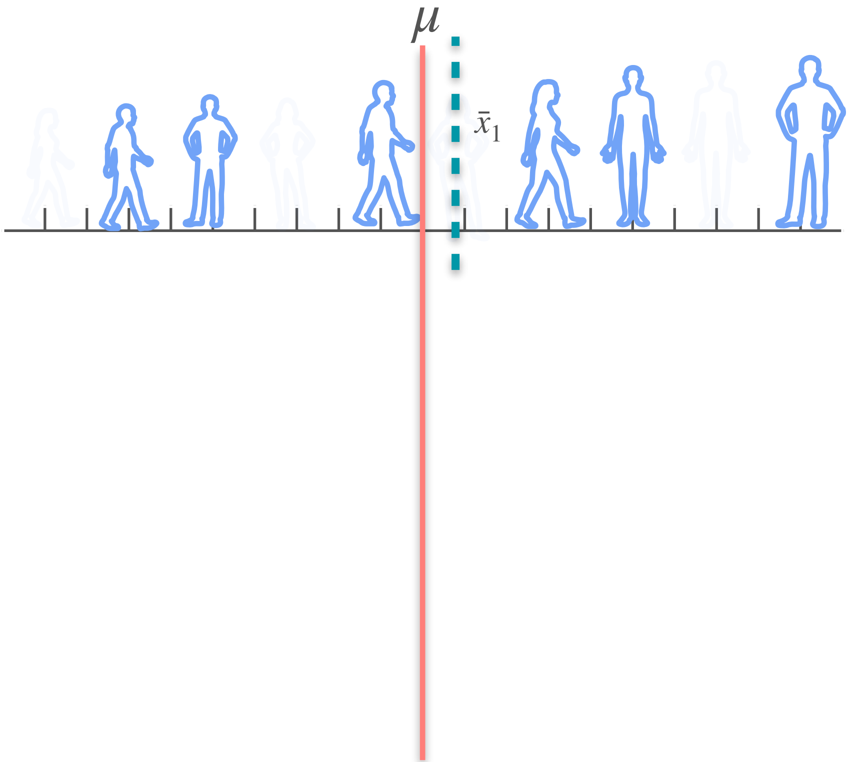


Statistopia

10,000 people

Estimate μ
(mean height of the population)

Confidence Interval - Intuition



Statistopia

10,000 people

Estimate μ
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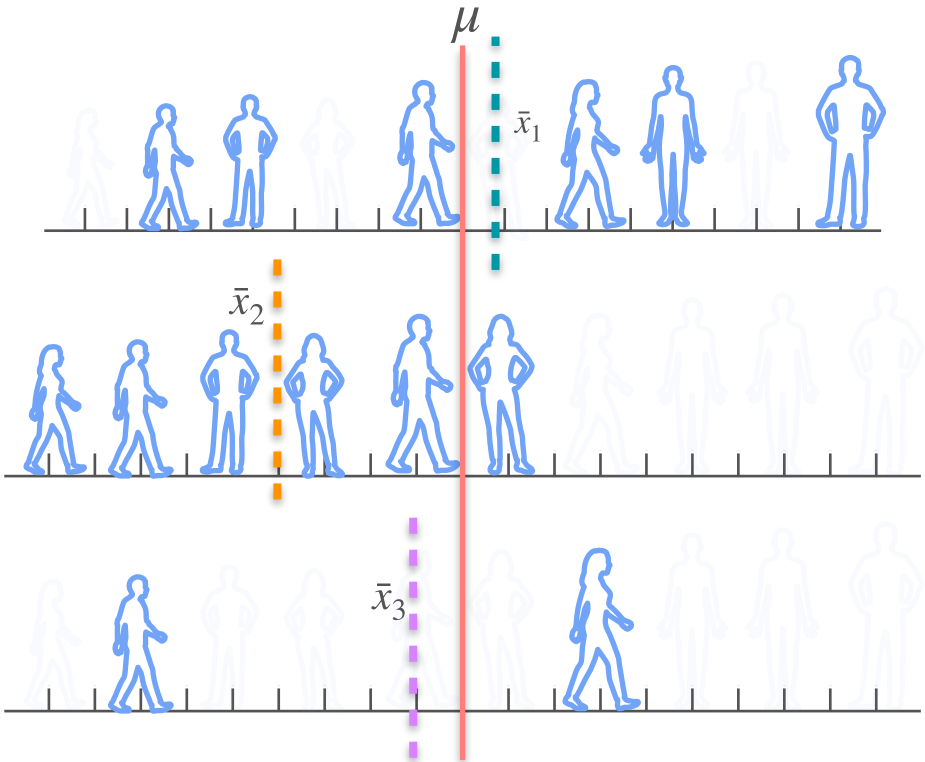
Confidence Interval - Intuition

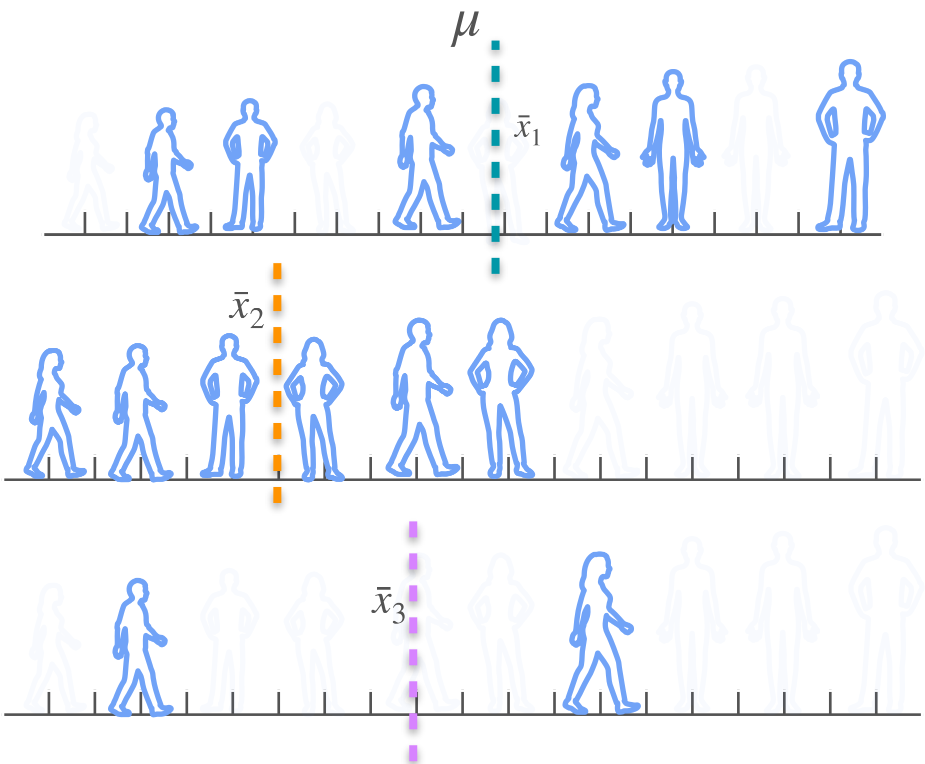


Statistopia

10,000 people

Estimate μ
(mean height of the population)

Confidence Interval - Intuition

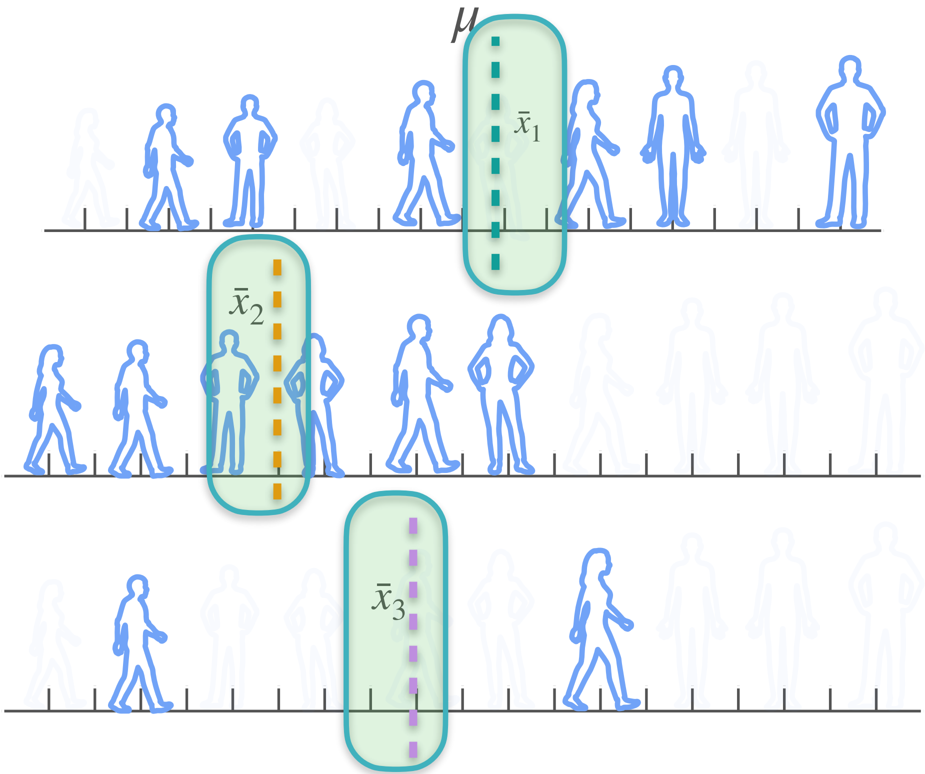


Statistopia

10,000 people

Estimate μ
(mean height of the population)

Confidence Interval - Intuition



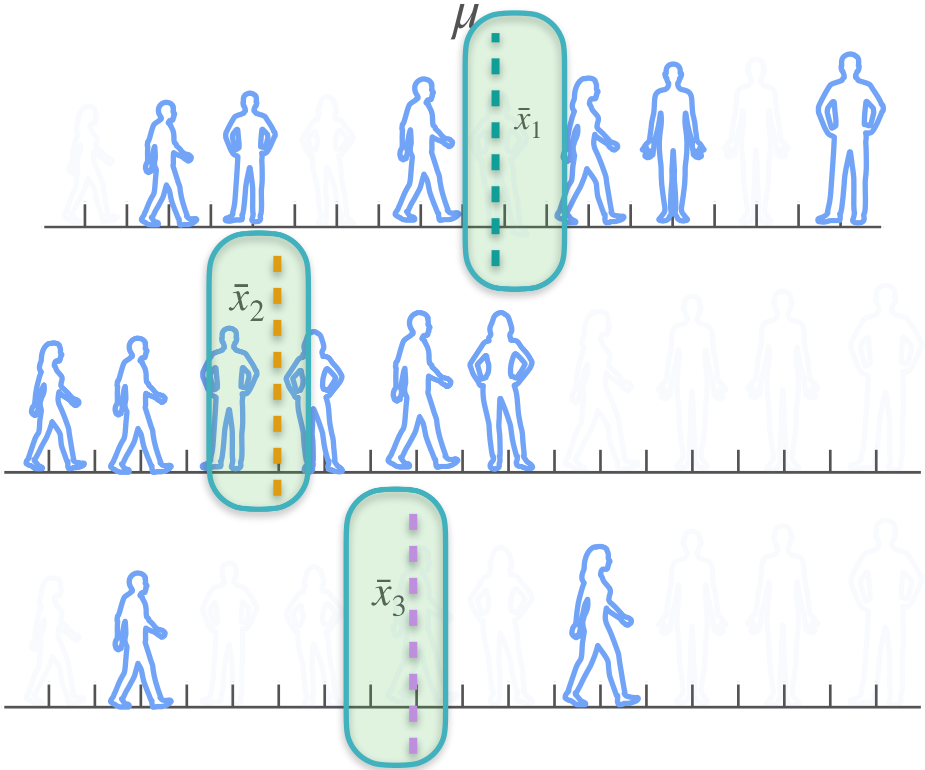
Statistopia

10,000 people

Estimate μ
(mean height of the population)

Can you use these sample means with some degree of certainty?

Confidence Interval - Intuition



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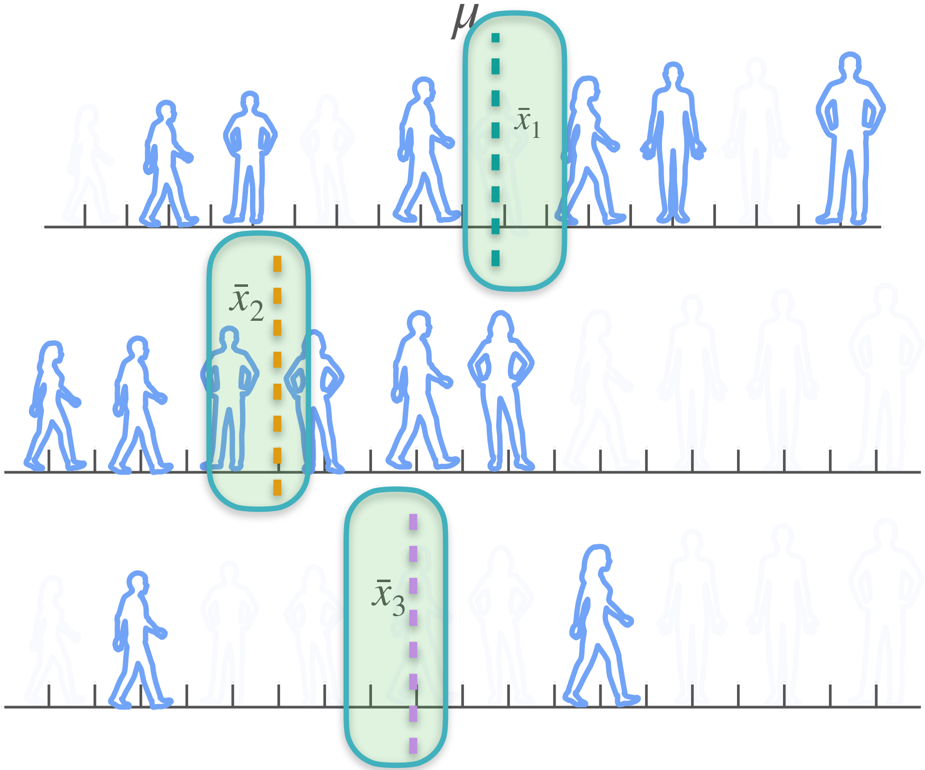

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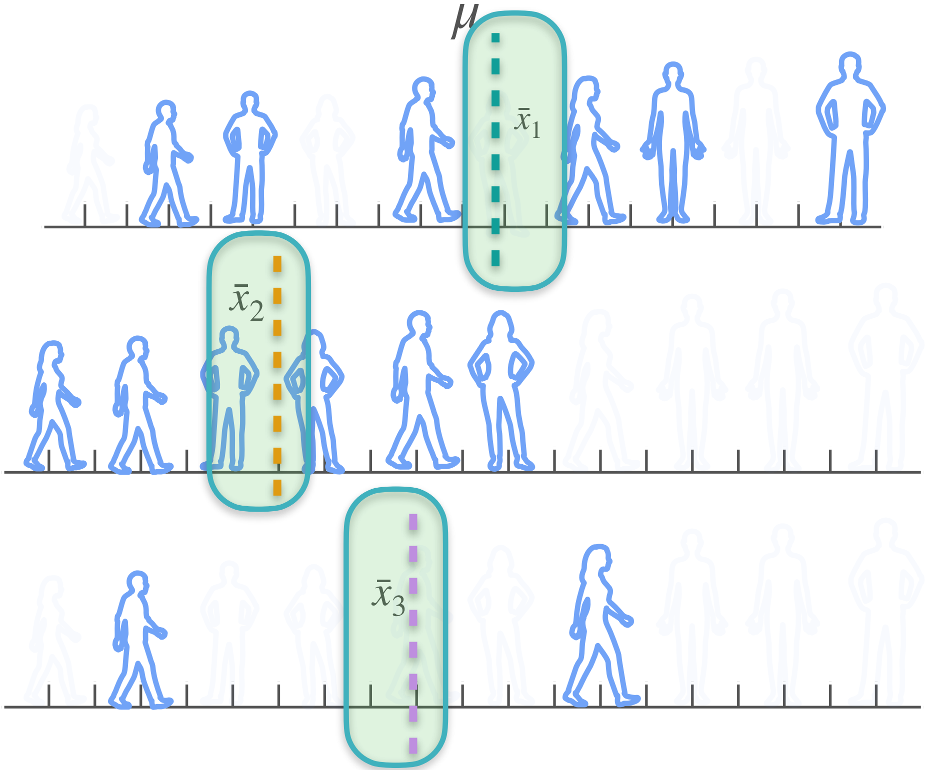
Confidence Interval

Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

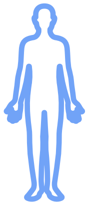
Confidence Interval

lower limit $< \bar{x} <$ upper limit

Confidence Interval - Intuition

Confidence Interval - Intuition

$$n = 1$$



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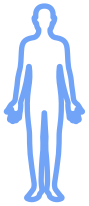
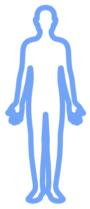


$$\bar{x}$$

Confidence Interval - Intuition

$$n = 1$$

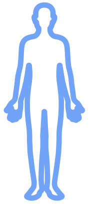


$$\bar{x}$$

Central Limit Theorem

Confidence Interval - Intuition

$$n = 1$$

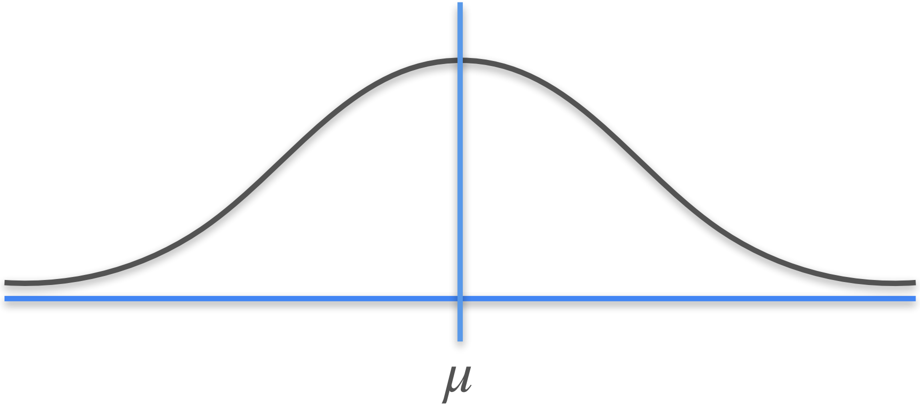


$$\bar{x}$$

Central Limit Theorem

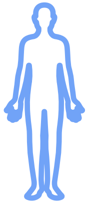
population standard deviation (σ)

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

$n = 1$



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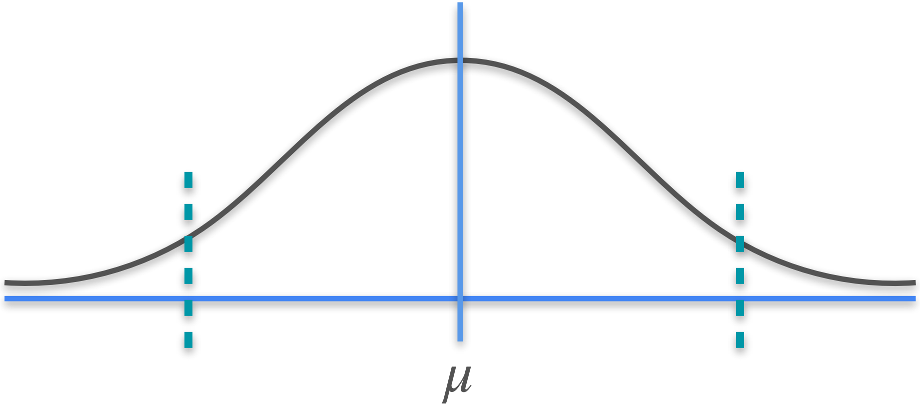

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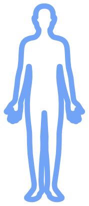

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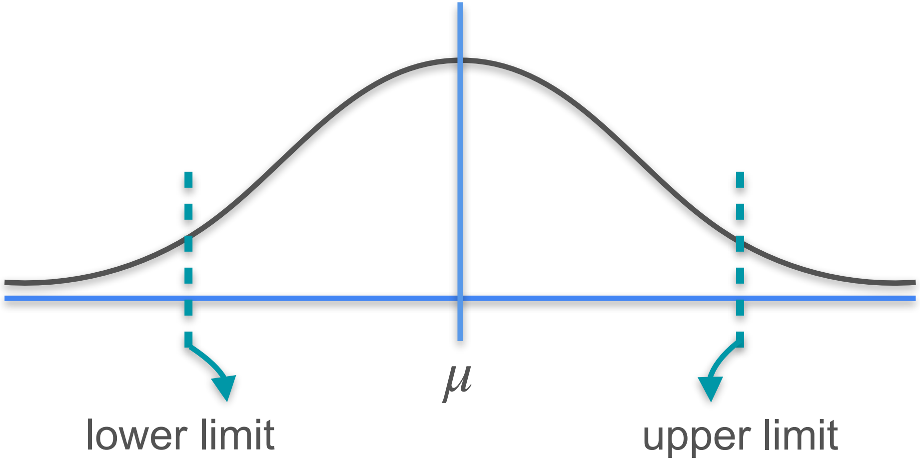

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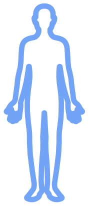

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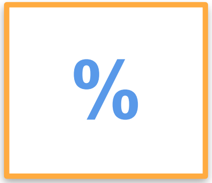

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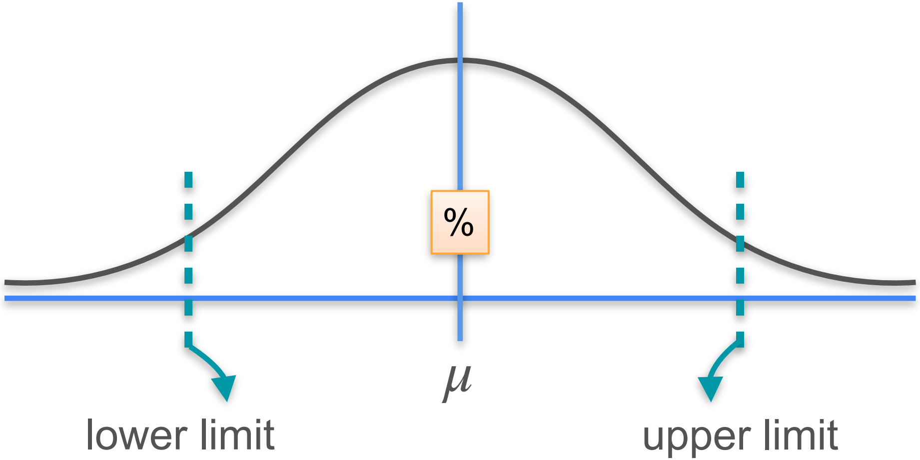

Confidence Interval - Intuition

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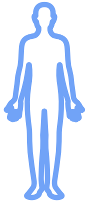

$\bar{x}$

%

α

significance level

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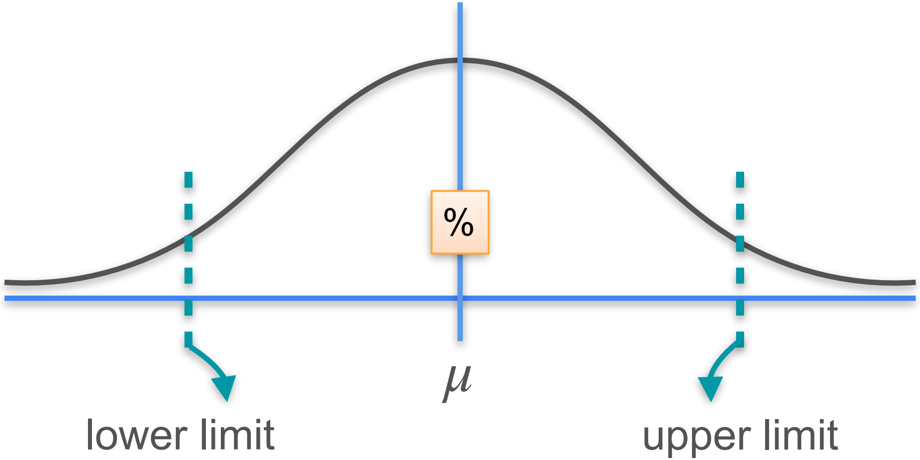

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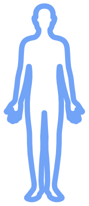

$$\bar{x}$$

$$\boxed{\%}$$

$$1 - \alpha$$

α

significance level

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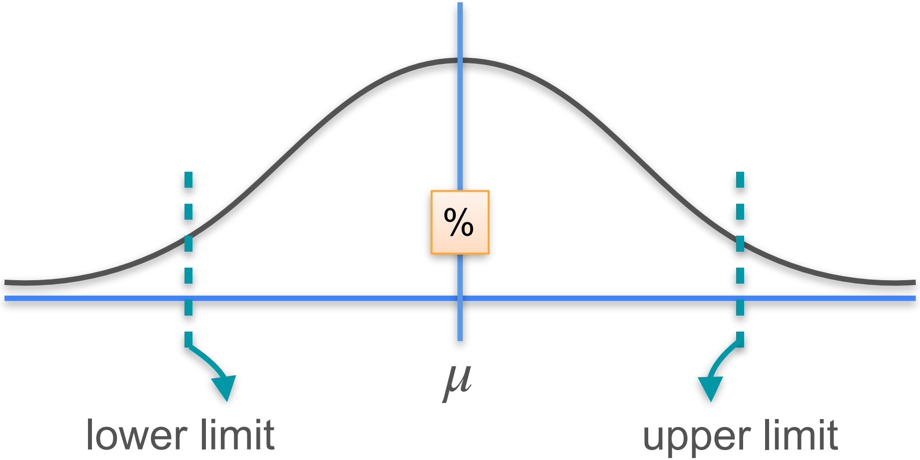

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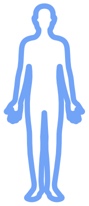

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Confidence level

$\boxed{\%}$

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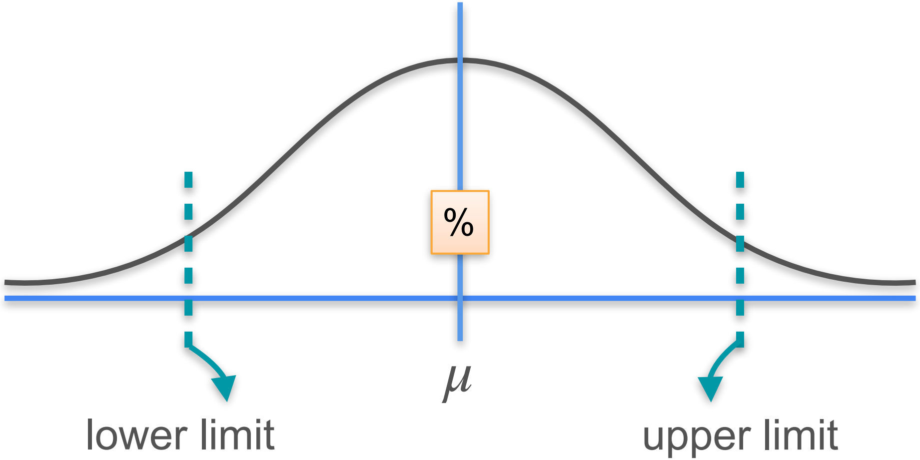

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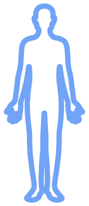

$$\bar{x}$$

Confidence level

%

$$1 - \alpha$$

$$\alpha$$

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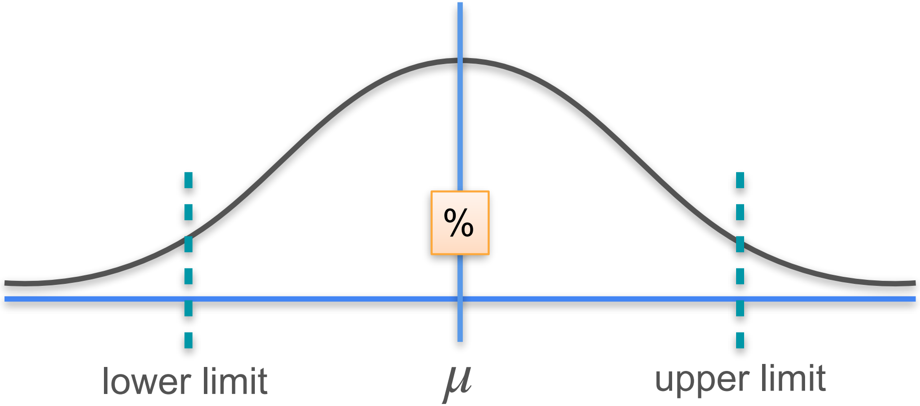

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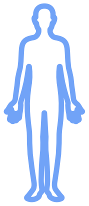

$\bar{x}$

Confidence level

%

$1 - \alpha$

$\alpha = 0.05$

significance level

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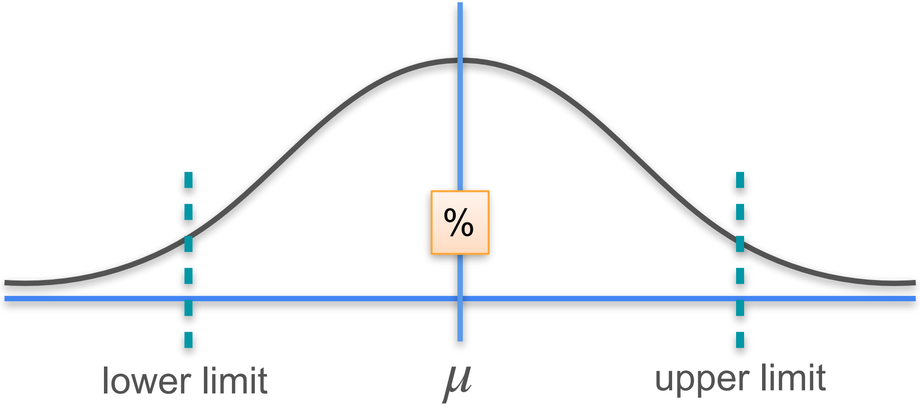

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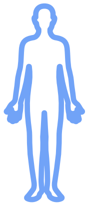

$\bar{x}$

Confidence level

%

$1 - \alpha$

$1 - 0.05 = 0.95$

$\alpha = 0.05$

significance level

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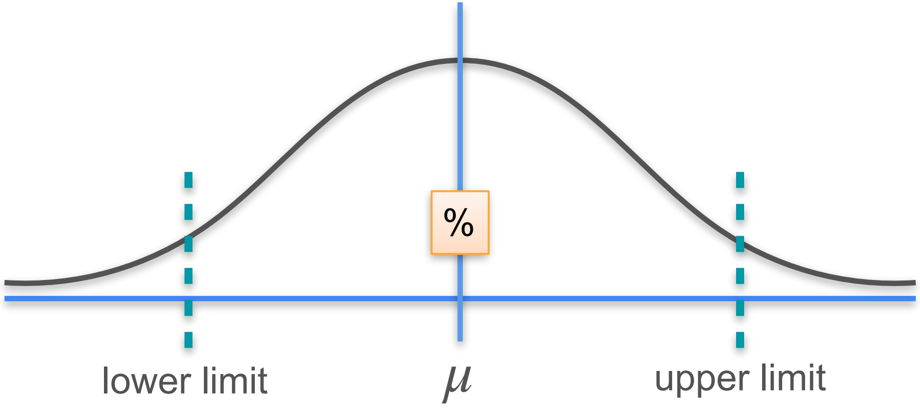

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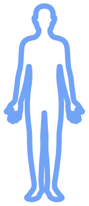

$\bar{x}$

Confidence level

$\boxed{95\%}$

$1 - \alpha$

$1 - 0.05 = 0.95$

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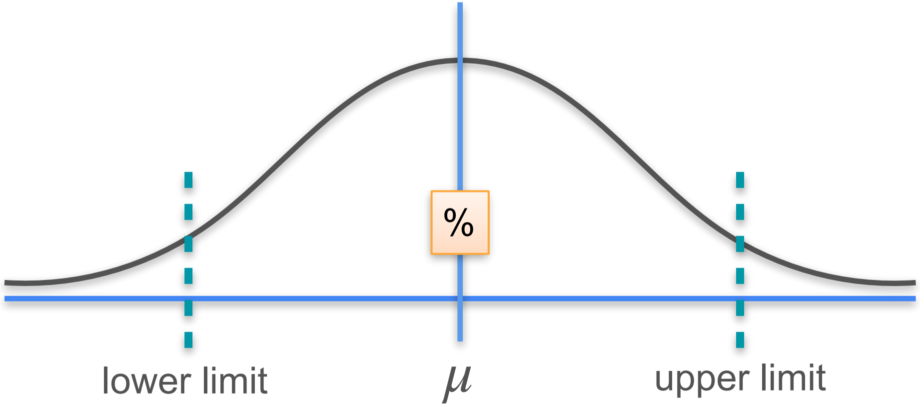

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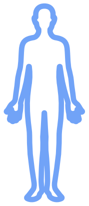

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Confidence level

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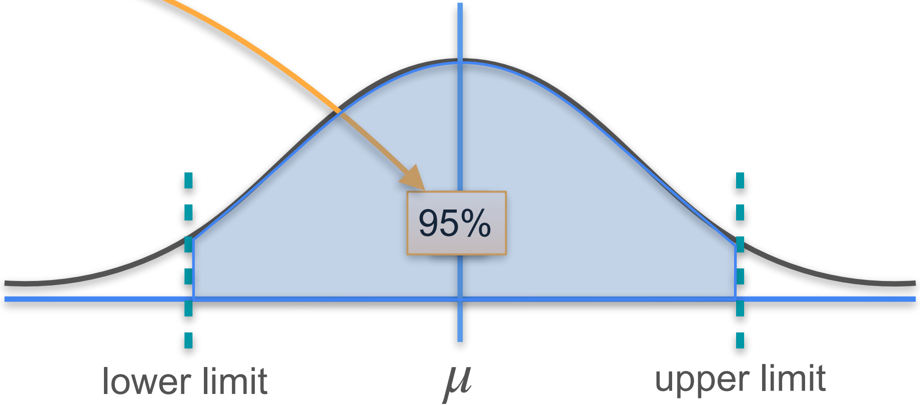

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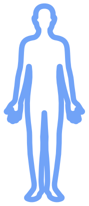

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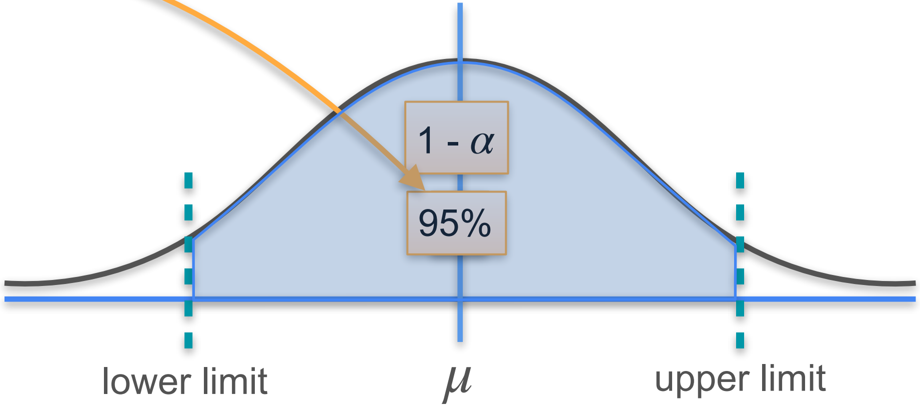

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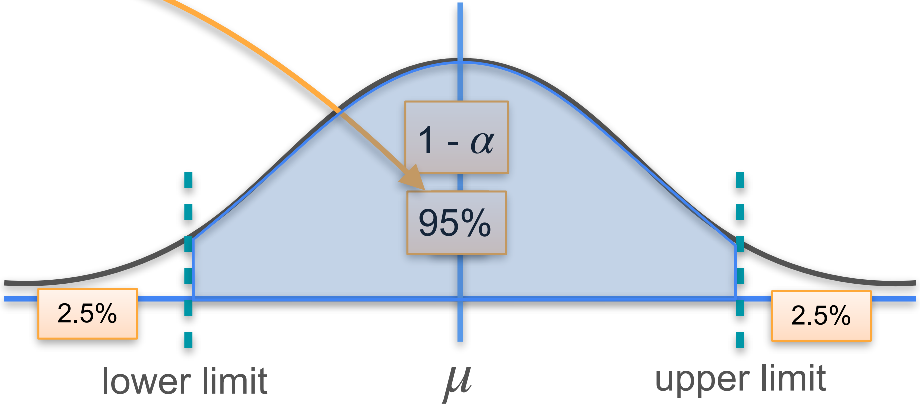

Confidence Interval - Intuition

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$\bar{x}$

Confidence level

$\boxed{95\%}$

$1 - \alpha$

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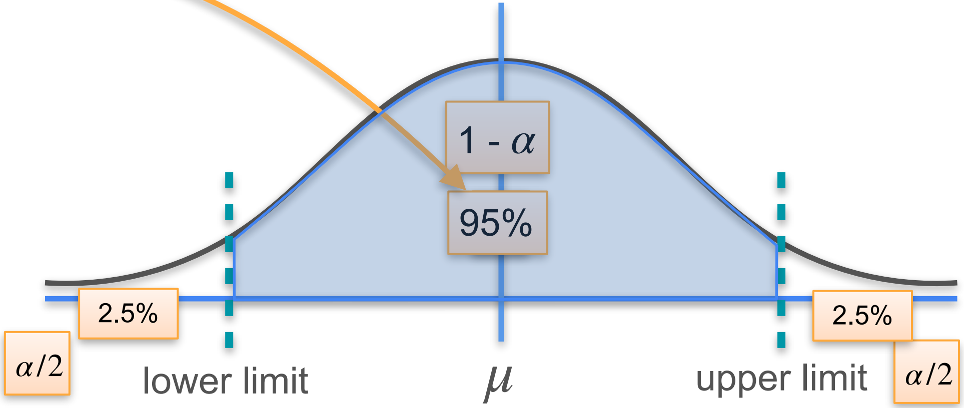

Confidence Interval - Intuition

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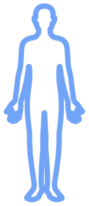

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Confidence level

95%

$1 - \alpha$

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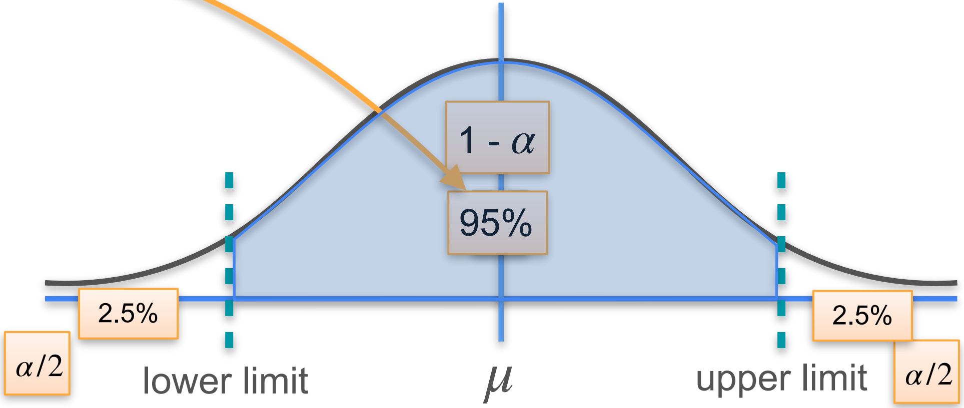

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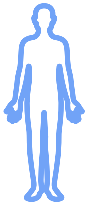

$\bar{x}$

$\alpha = 0.05$

significance level

$1 - 0.05 = 0.95$

Confidence level

95%

$1 - \alpha$

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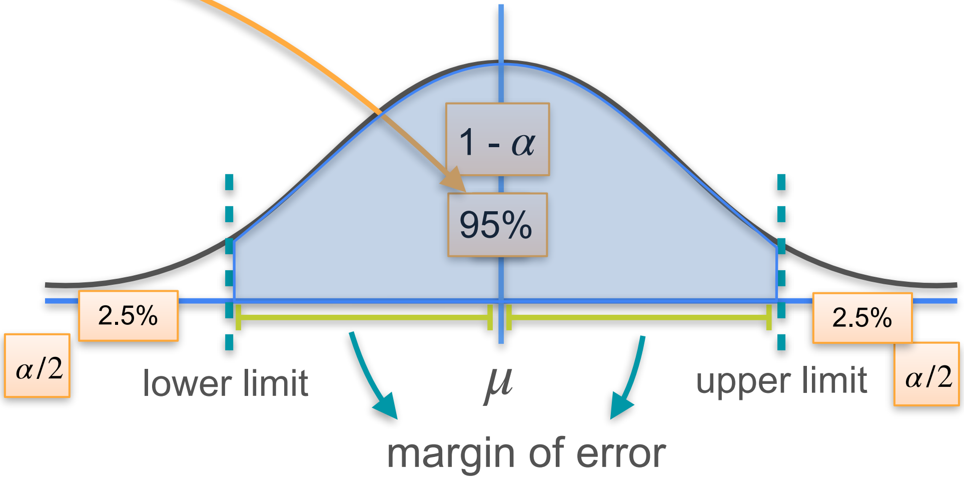

Confidence Interval - Intuition

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Confidence level

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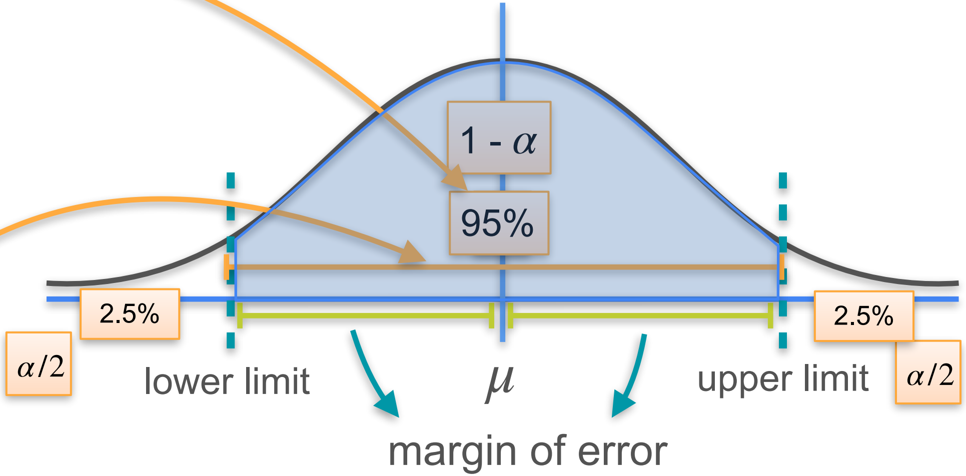


Confidence Interval - Intuition

$$n = 1$$

Known σ

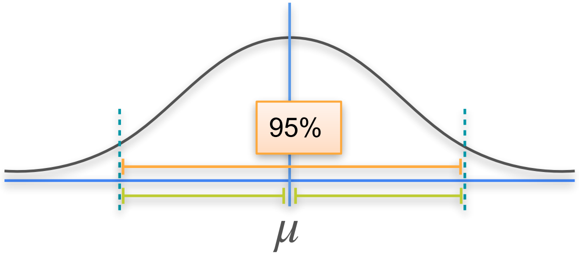
$\boxed{95\%}$

Confidence Interval - Intuition

$$n = 1$$

Known σ

$\boxed{95\%}$



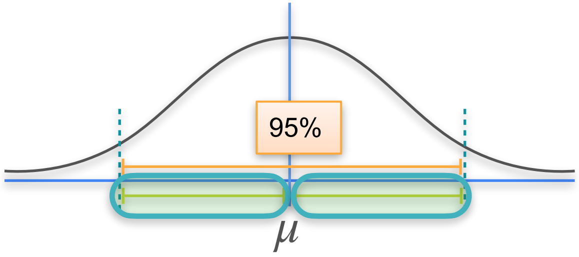
Confidence Interval - Intuition

$$n = 1$$

Known σ

$\boxed{95\%}$

Margin of error



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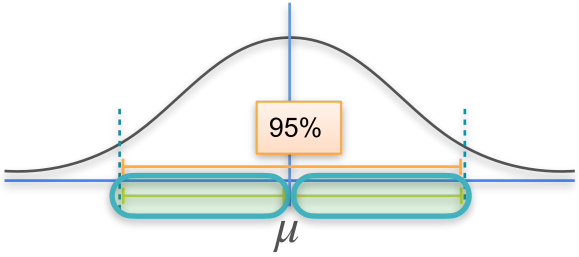

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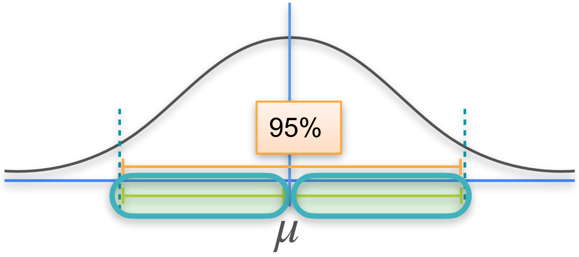

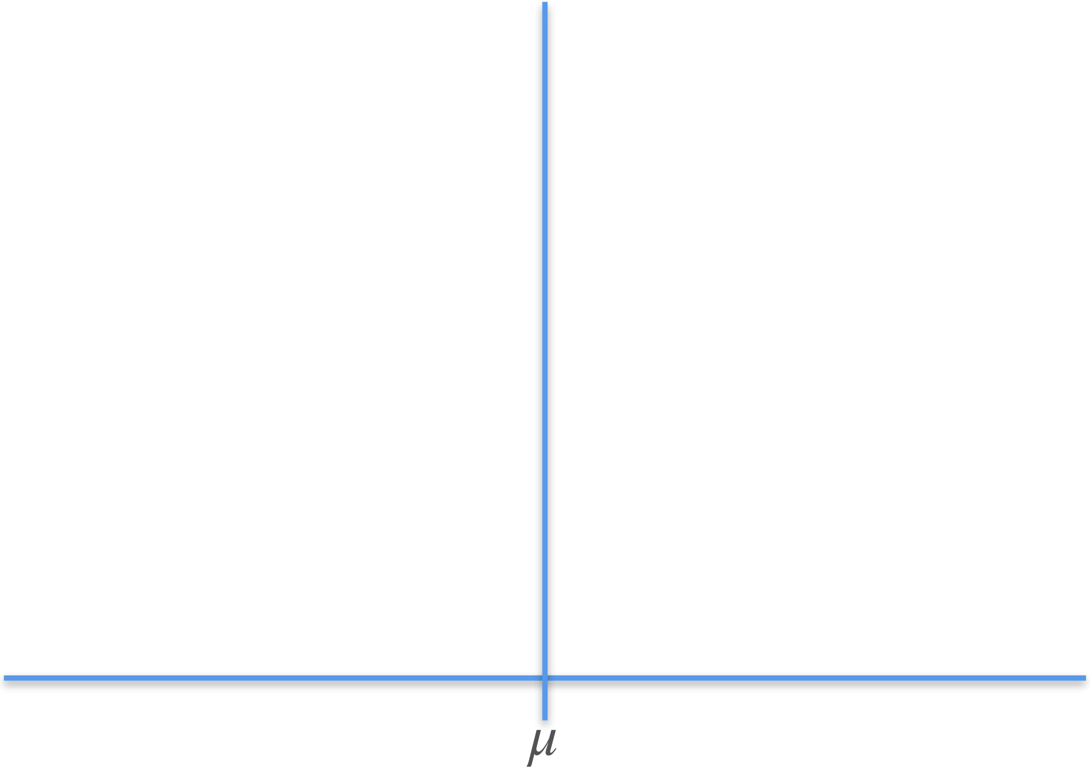

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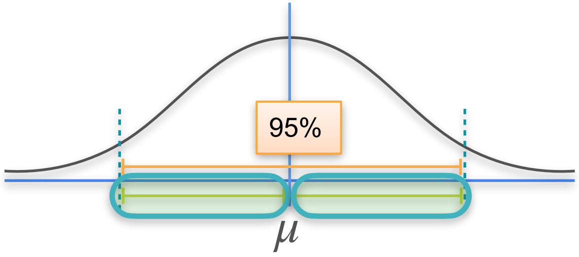

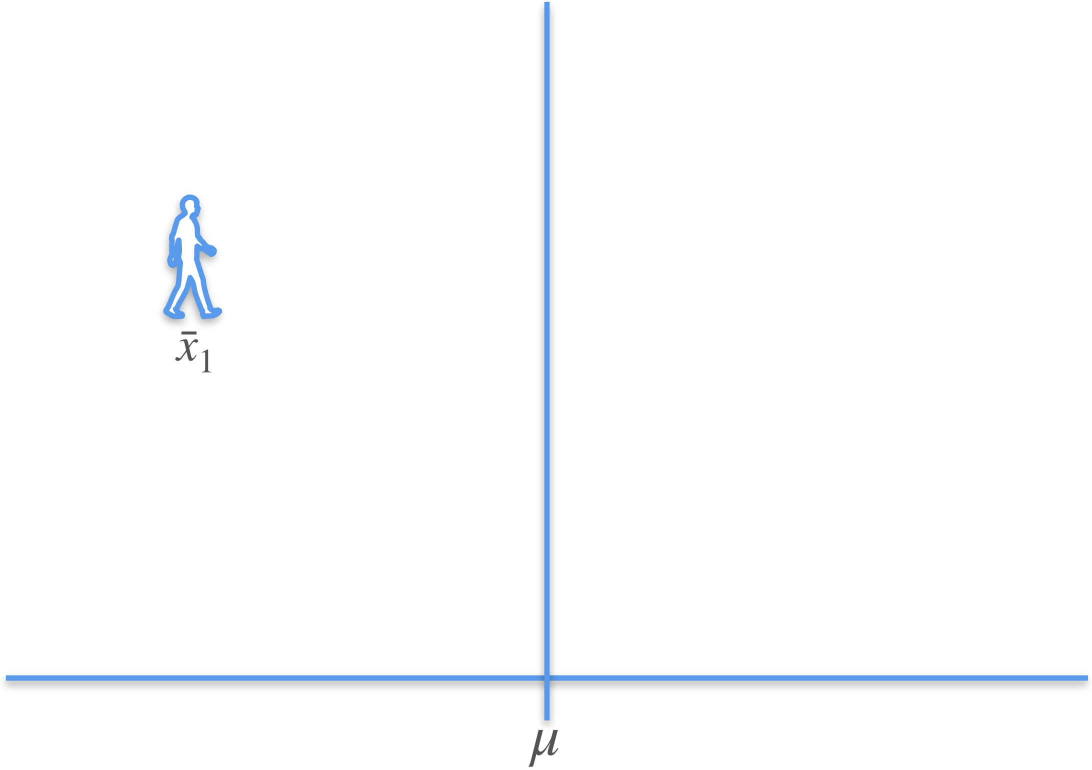

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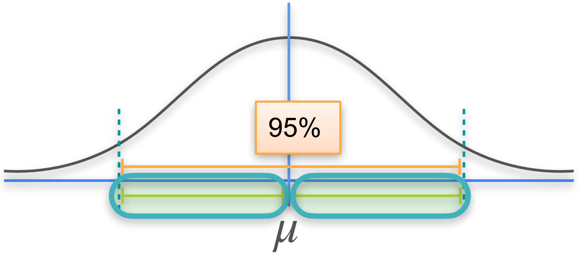

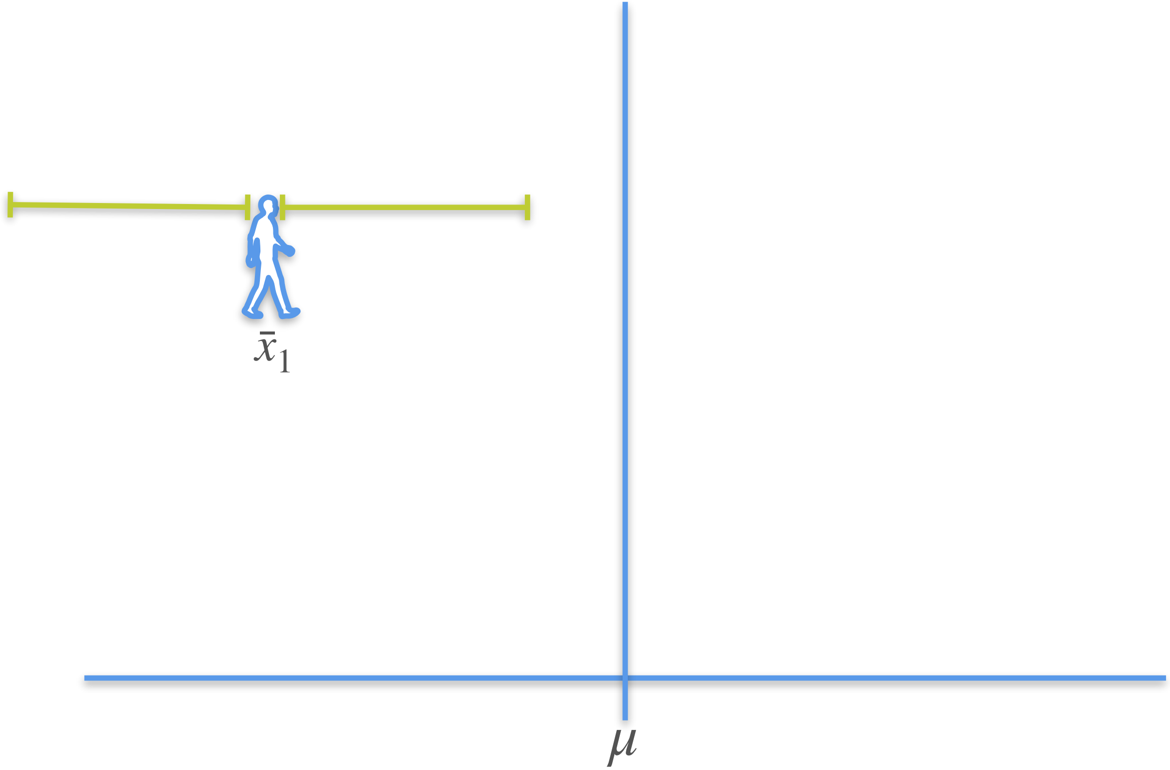

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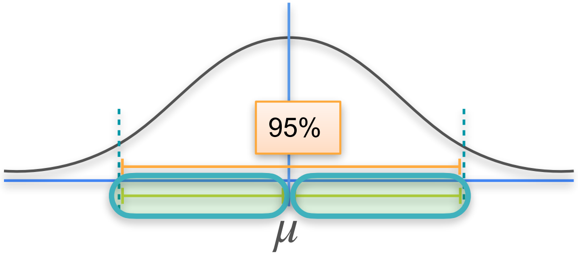
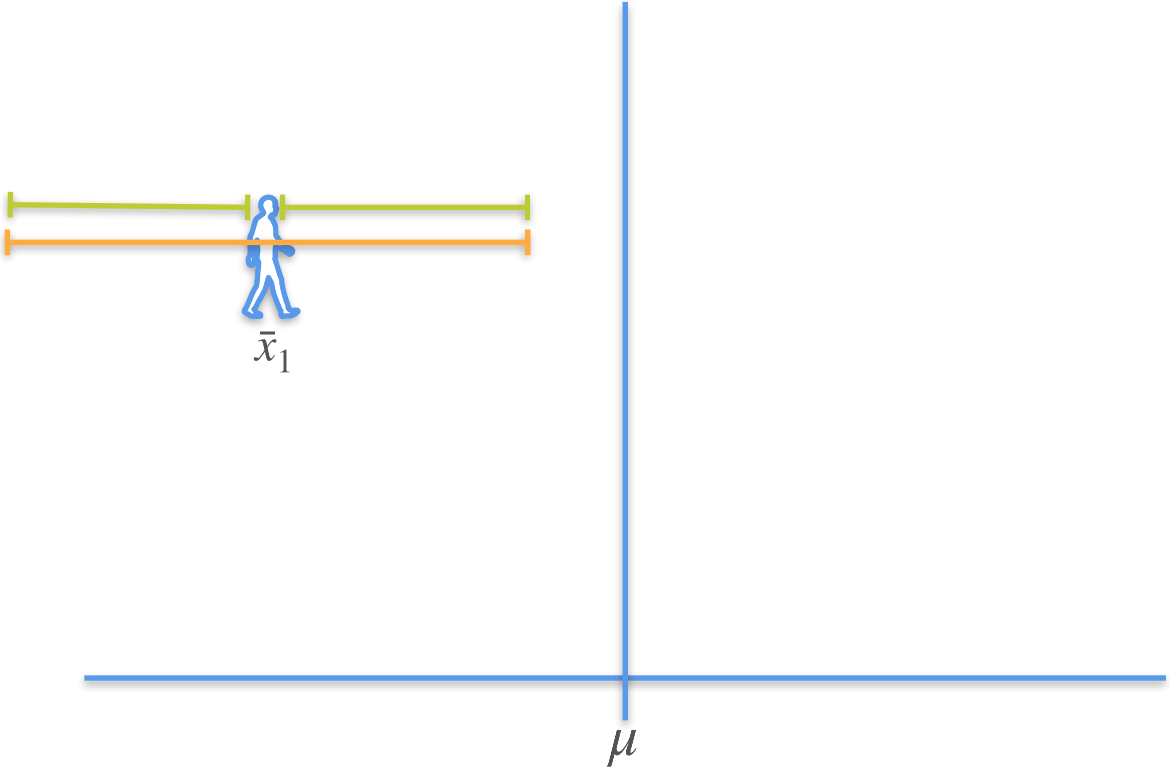




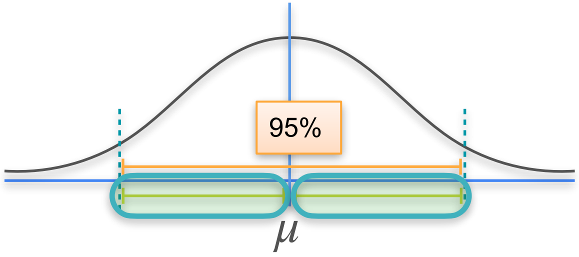
Confidence Interval - Intuition

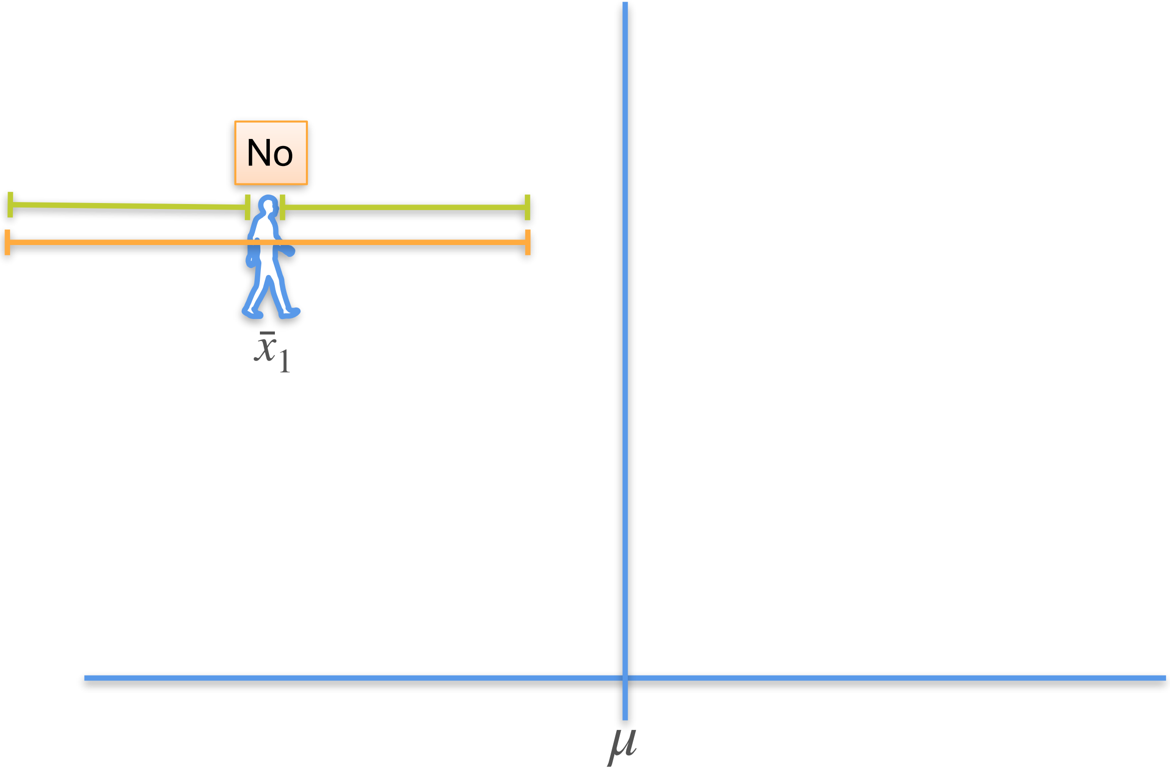
$n = 1$

Known σ

95%

Margin of error





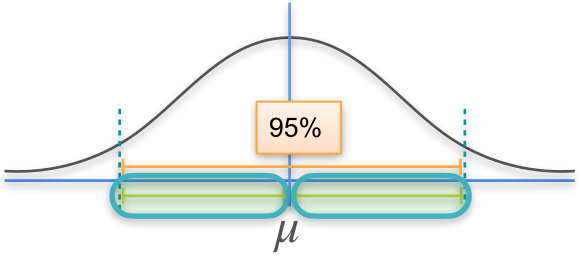
Confidence Interval - Intuition

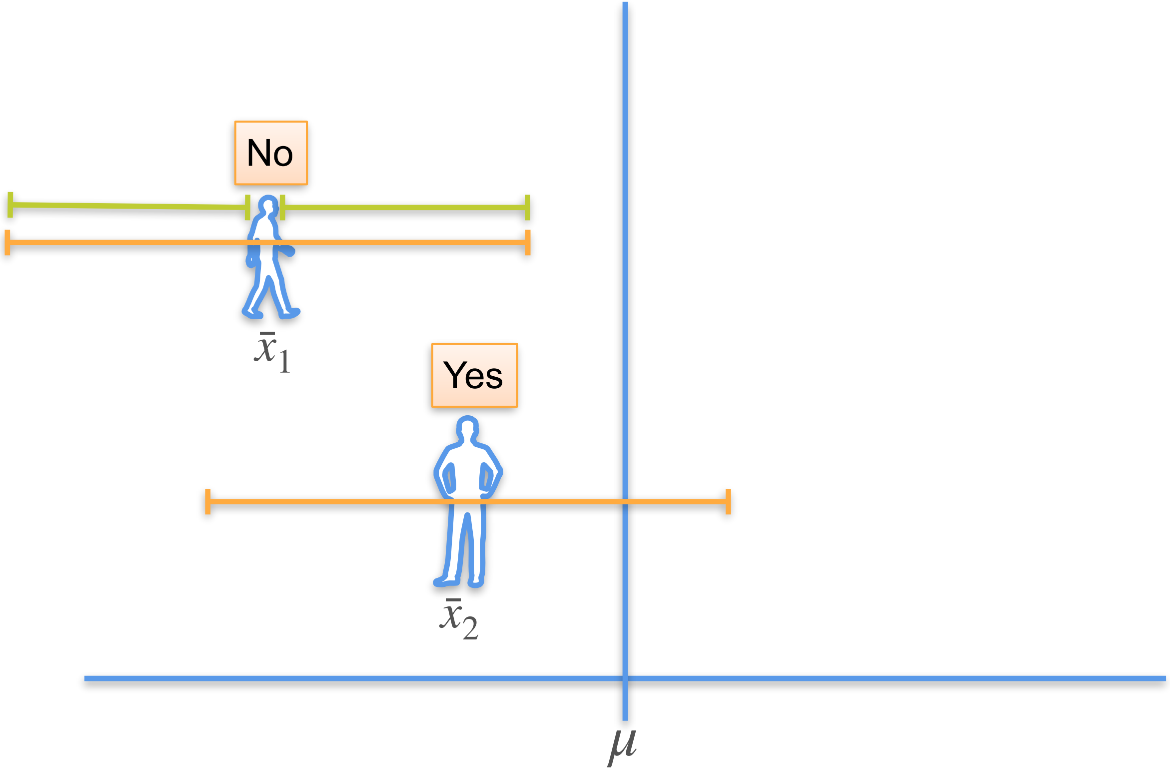
$n = 1$

Known σ

95%

Margin of error





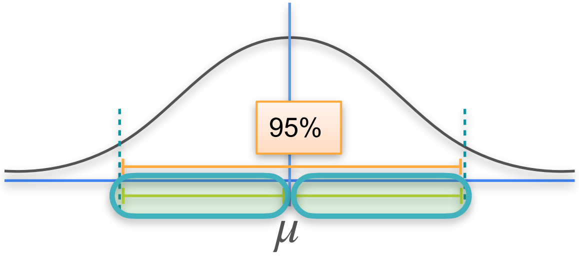
Confidence Interval - Intuition

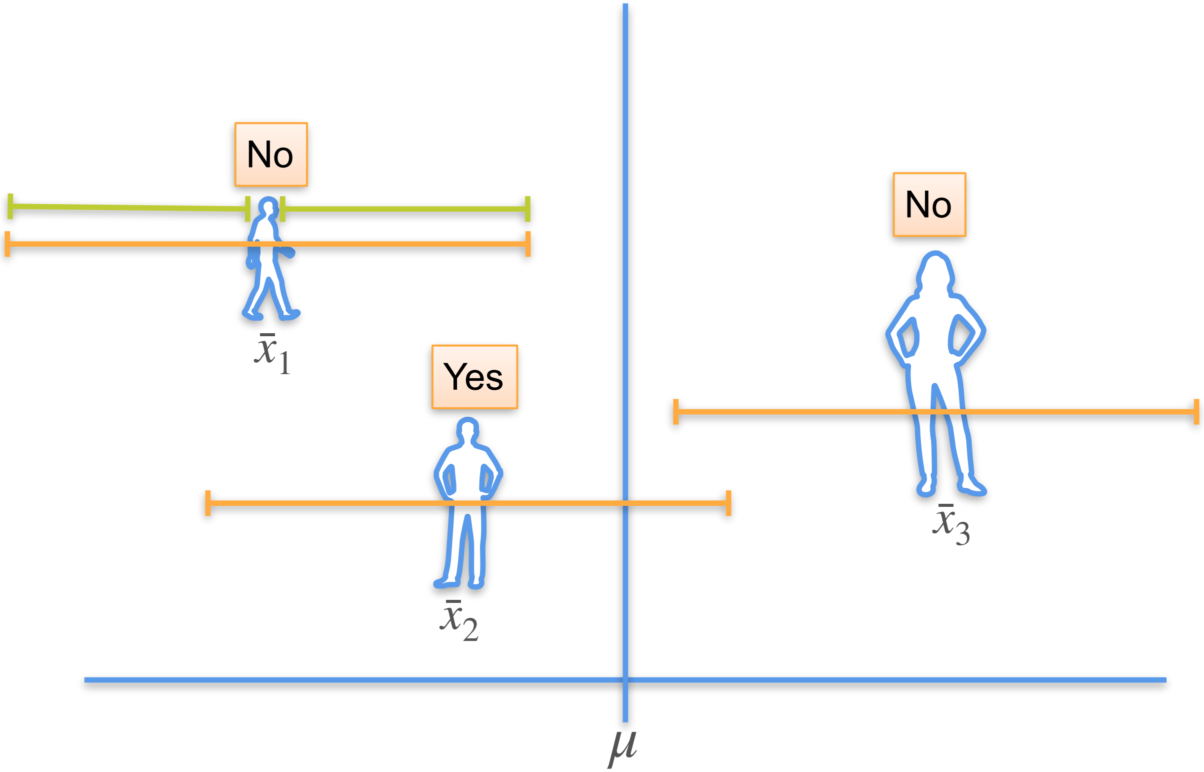
$n = 1$

Known σ

95%

Margin of error





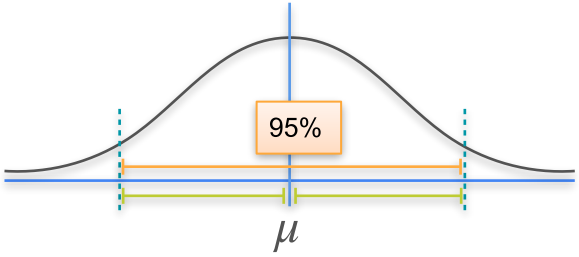
Confidence Interval - Intuition

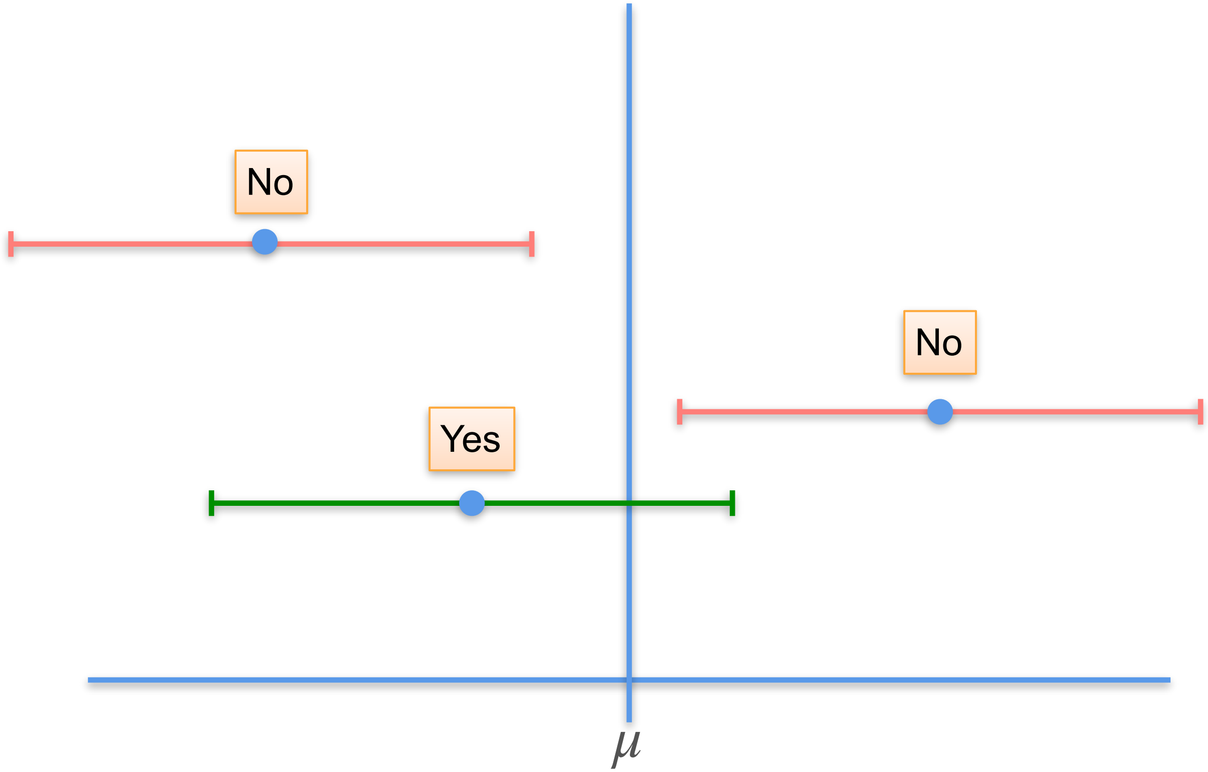
$$n = 1$$

Known σ

95%

Margin of error





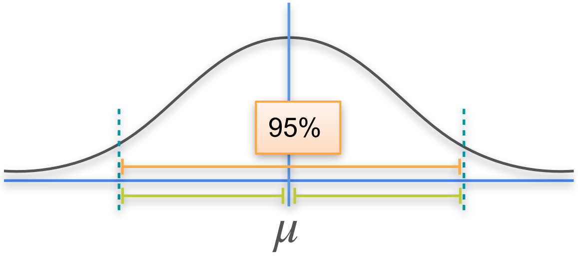
Confidence Interval - Intuition

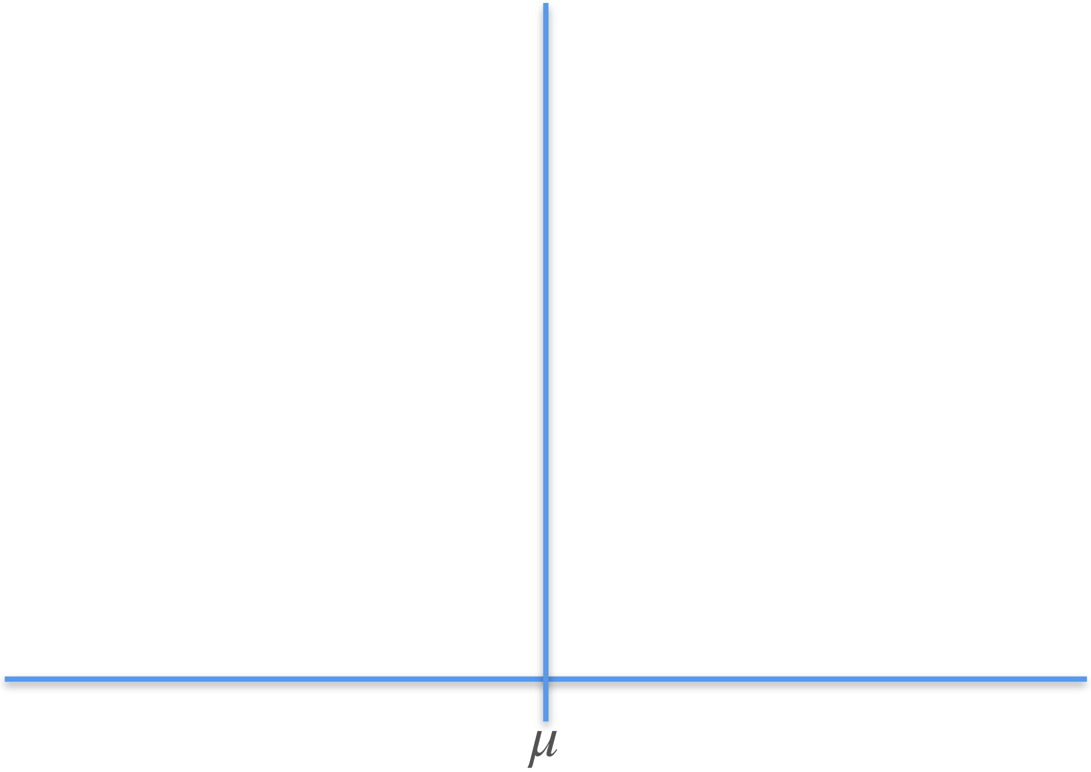
$n = 1$

Known σ

95%

Margin of error





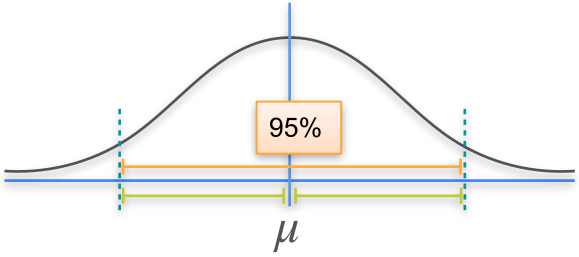
Confidence Interval - Intuition

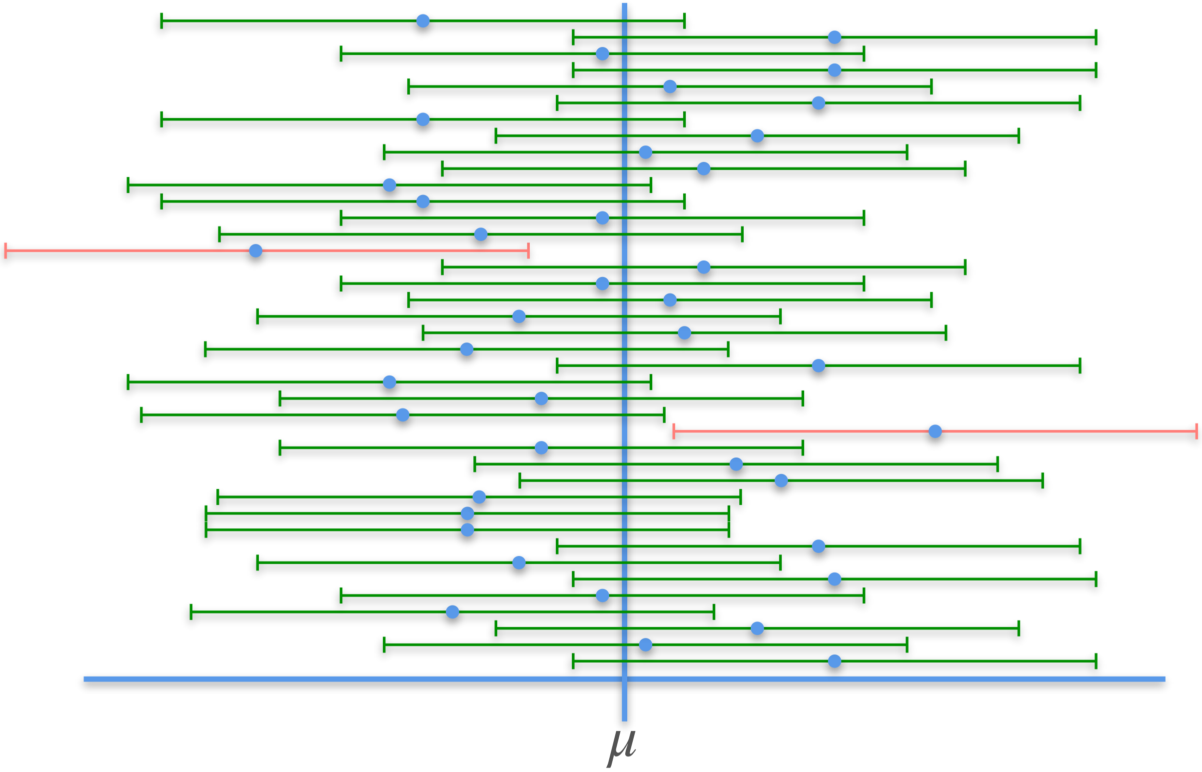
$$n = 1$$

Known σ

95%

Margin of error





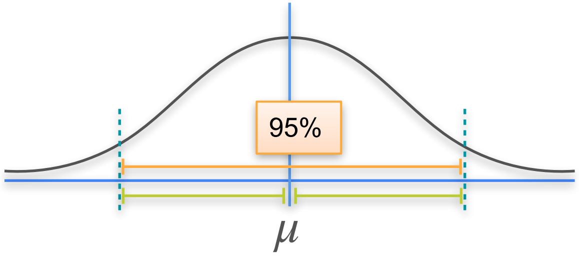
Confidence Interval - Intuition

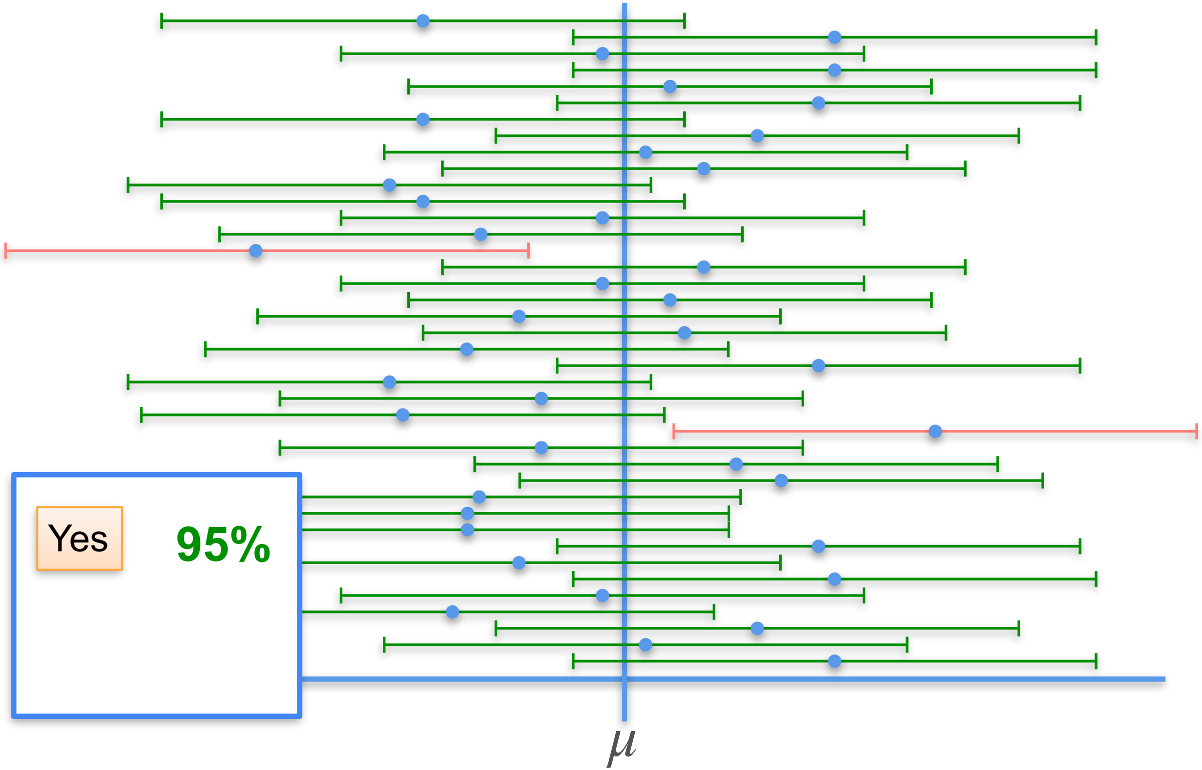
$n = 1$

Known σ

95%

Margin of error





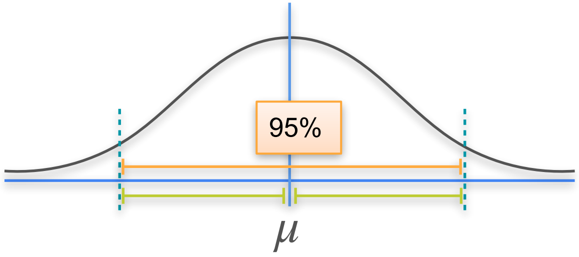
Confidence Interval - Intuition

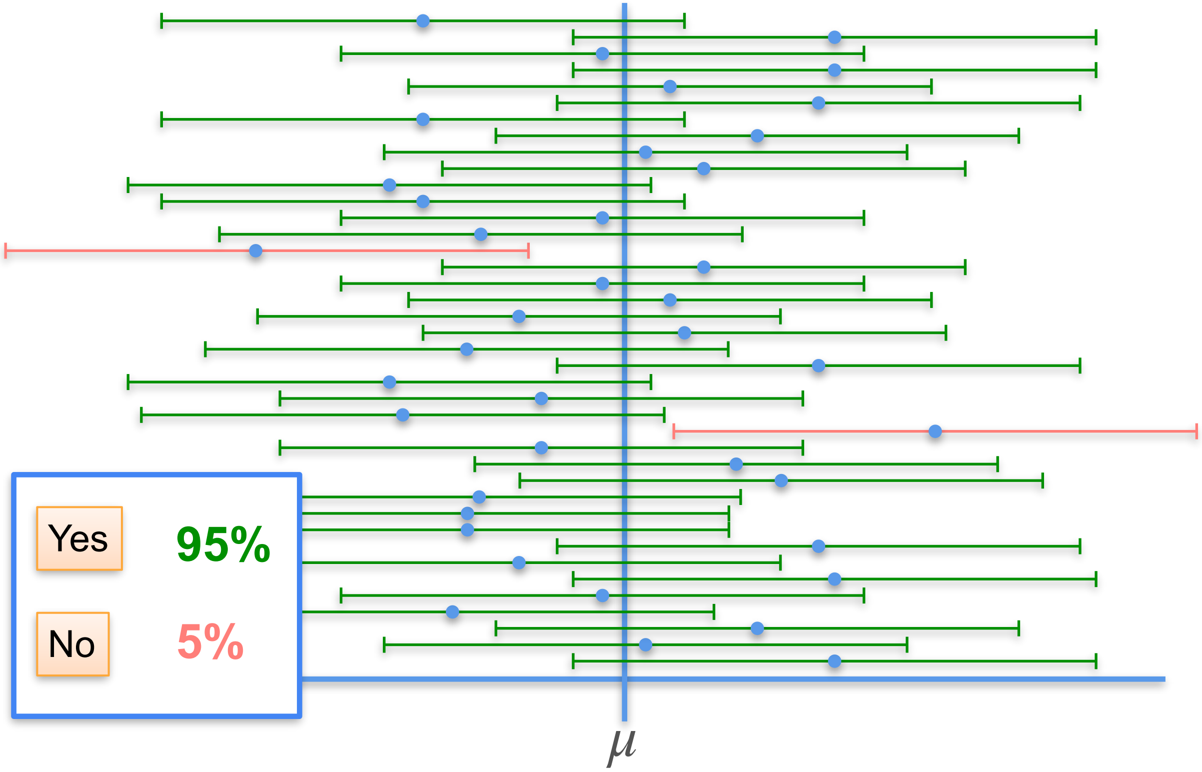
$$n = 1$$

Known σ

95%

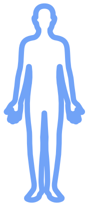
Margin of error





Confidence Interval - Intuition

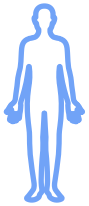
$$n = 1$$



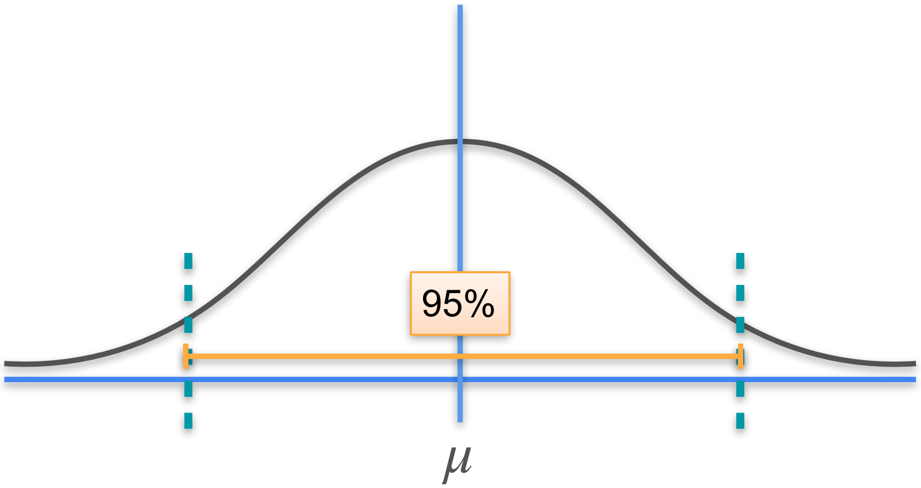
$$\bar{x}$$

Confidence Interval - Intuition

$$n = 1$$

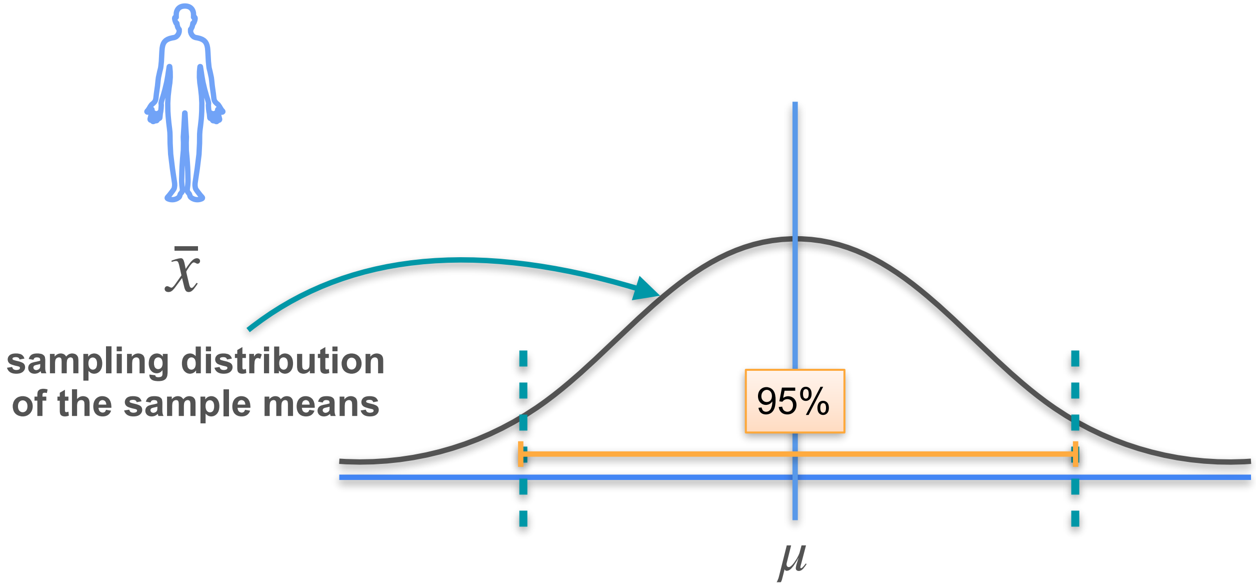


$\bar{x}$



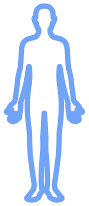
Confidence Interval - Intuition

$n = 1$



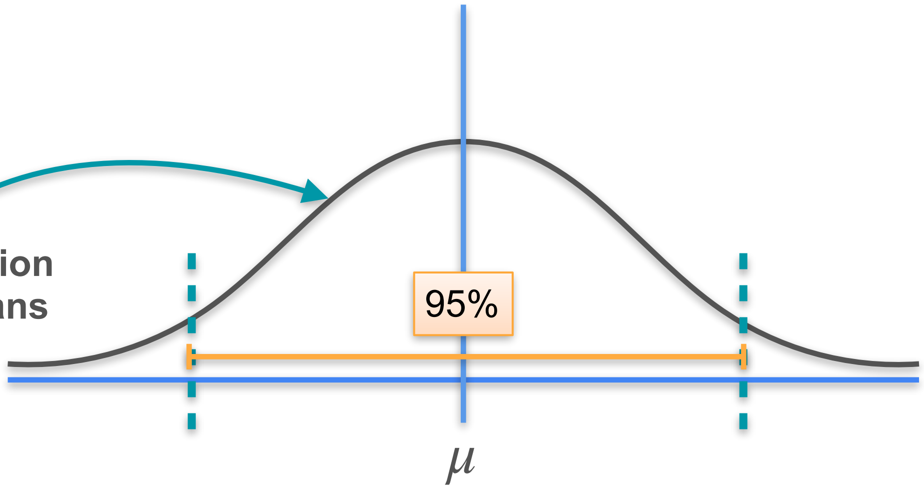
Confidence Interval - Intuition

$n = 1$



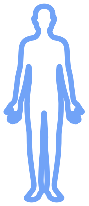
Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

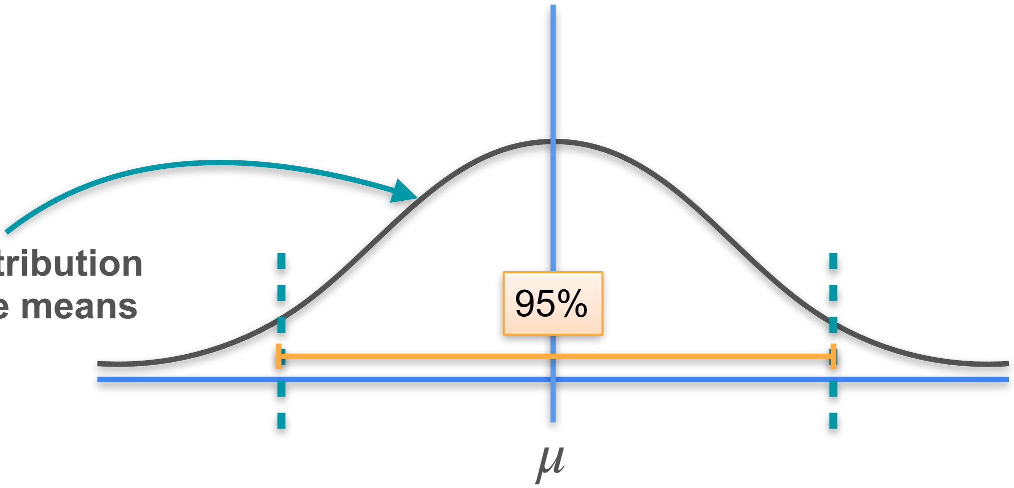
$n = 1$



Central Limit Theorem

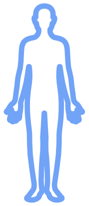
$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$

$$\mu_{\bar{x}} = \mu$$



Confidence Interval - Intuition

$n = 1$



$\bar{x}$

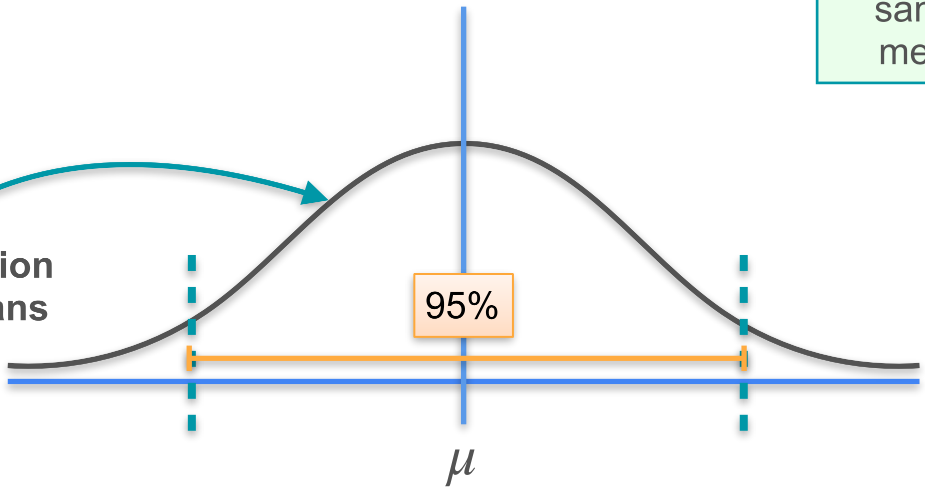
sampling distribution of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$

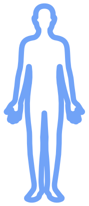
mean of the sample means

$$\mu_{\bar{x}} = \mu$$



Confidence Interval - Intuition

$n = 1$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem

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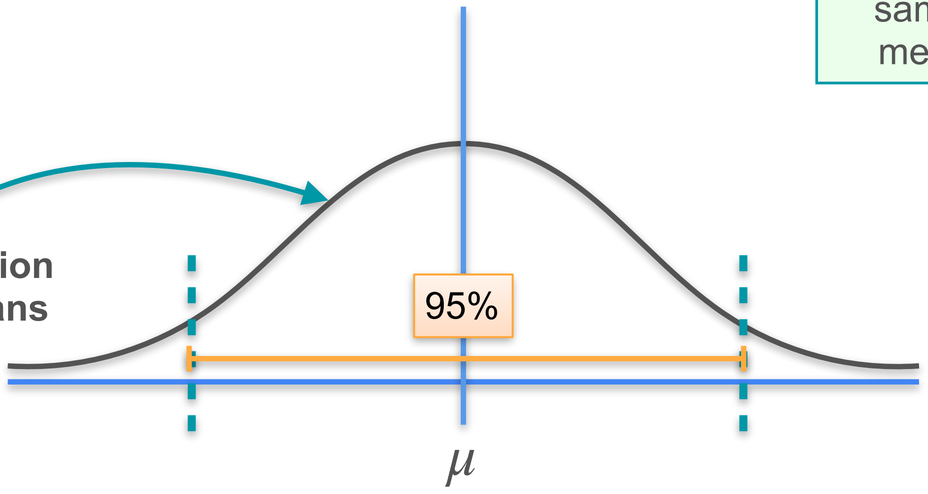


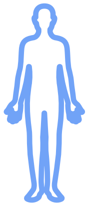
mean of the sample means

$$\mu_{\bar{x}} = \mu$$

population mean

Confidence Interval - Intuition

$$n = 1$$

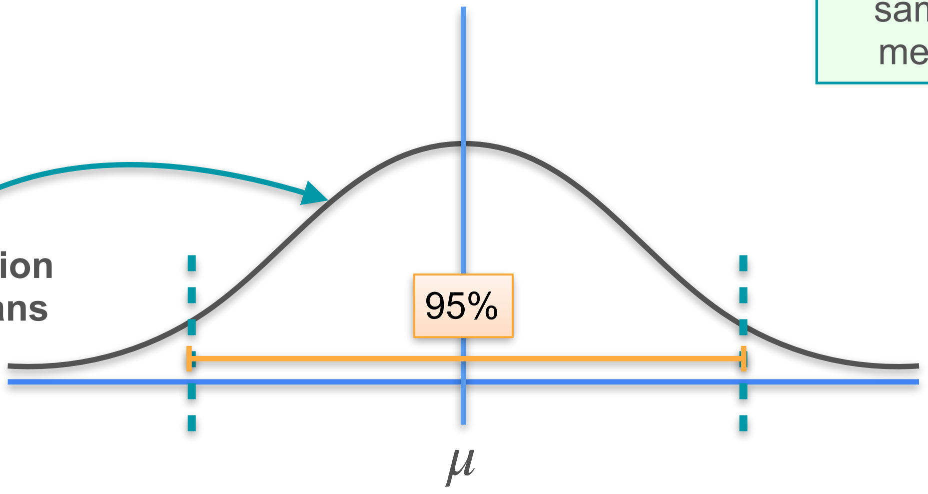


$$\bar{x}$$

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$

sampling distribution of the sample means



mean of the sample means

$$\mu_{\bar{x}} = \mu$$

population mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

standard deviation of the sample means

Confidence Interval - Intuition

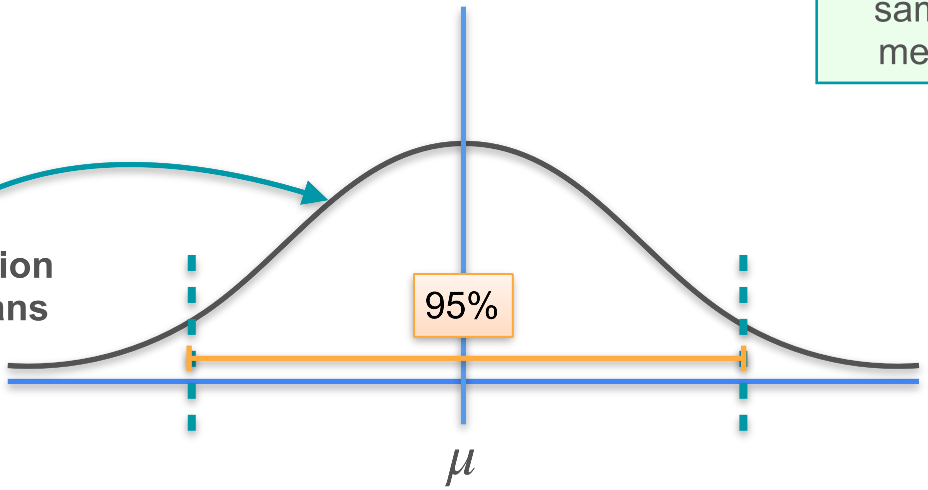
$n = 1$

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$

$\bar{x}$

sampling distribution of the sample means



mean of the sample means

$$\mu_{\bar{x}} = \mu$$

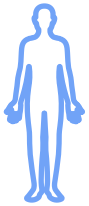
population mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}}$$

standard deviation of the sample means

Confidence Interval - Intuition

$$n = 1$$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem

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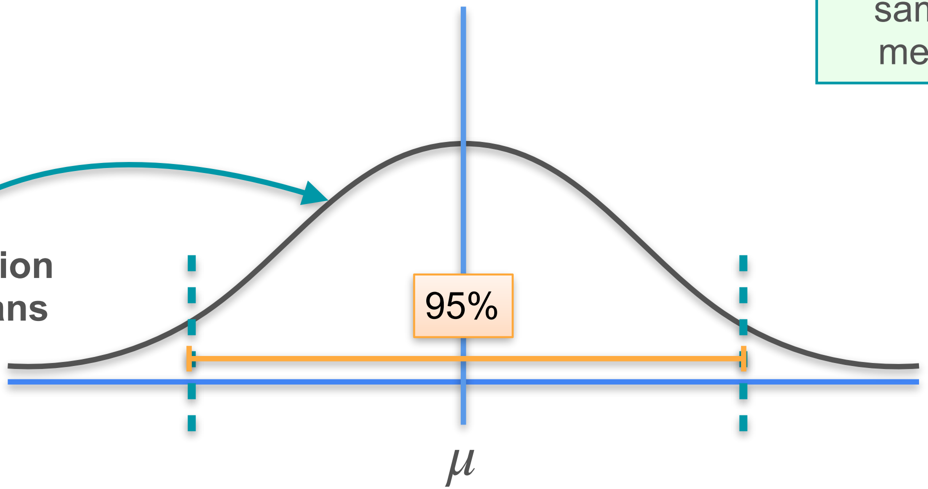


mean of the sample means

$$\mu_{\bar{x}} = \mu$$

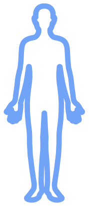
population mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}} = \sigma$$

standard deviation of the sample means

Confidence Interval - Intuition

$n = 1$

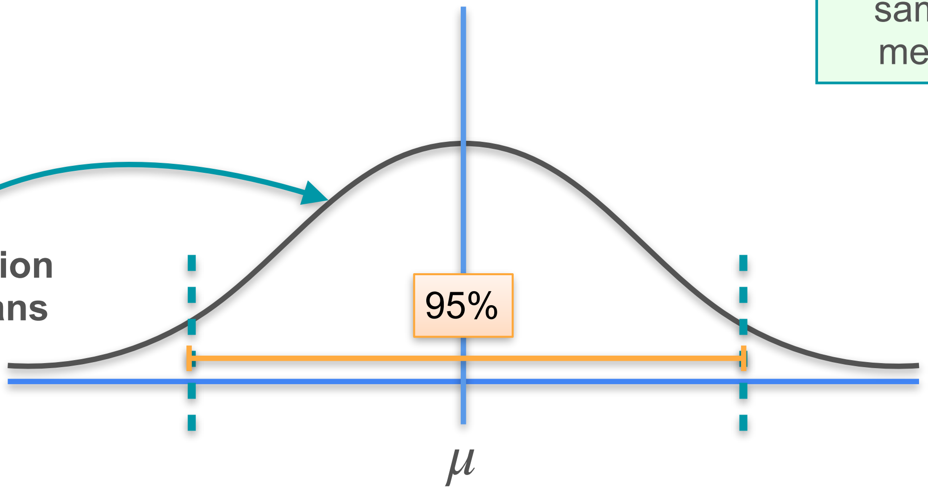


$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Population standard deviation (σ)

mean of the sample means

$$\mu_{\bar{x}} = \mu$$

population mean

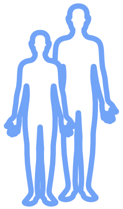
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}} = \sigma$$

standard deviation of the sample means

Confidence Interval - Intuition

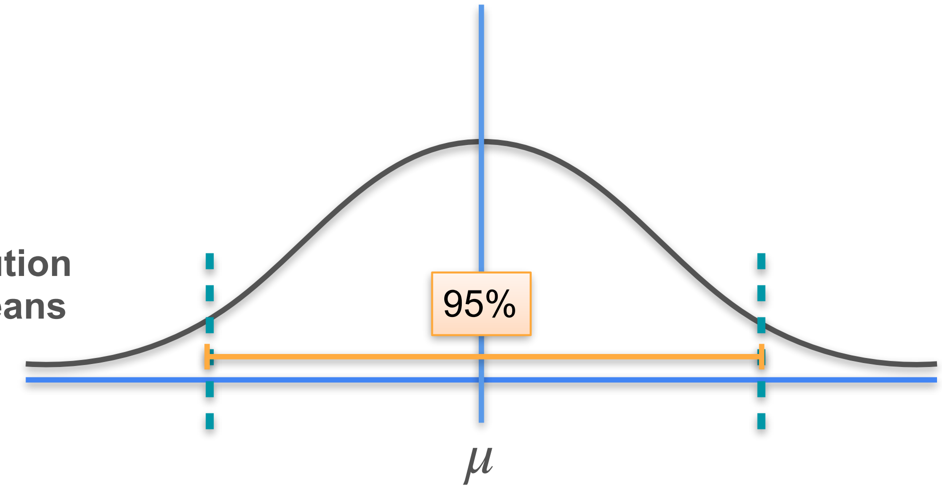
Central Limit Theorem

Population standard deviation (σ)



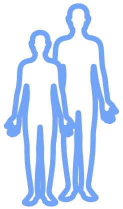
$\bar{x}$

sampling distribution of the sample means



Confidence Interval - Intuition

$n = 2$

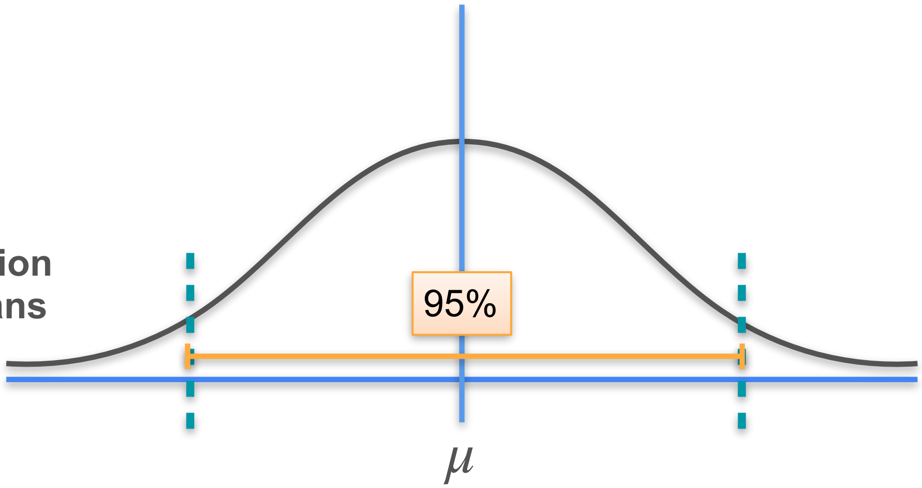


$\bar{x}$

sampling distribution of the sample means

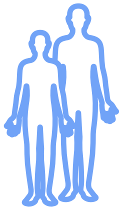
Central Limit Theorem

Population standard deviation (σ)



Confidence Interval - Intuition

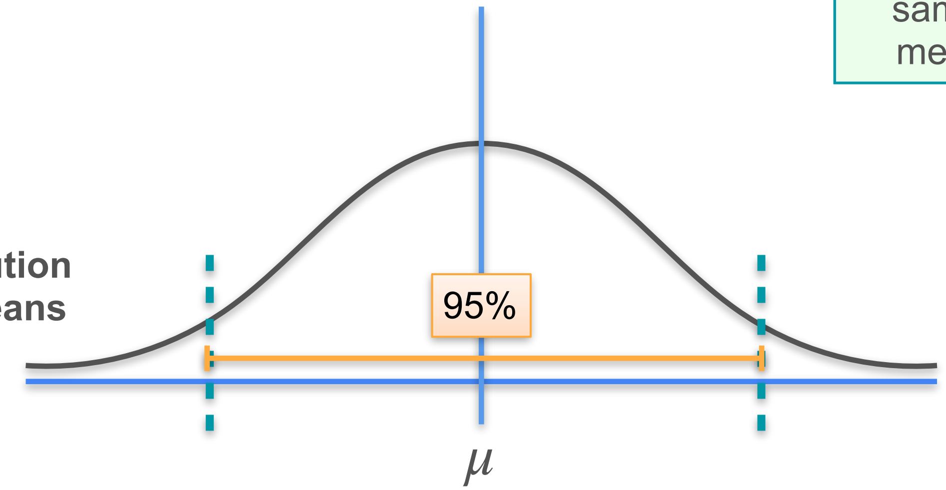
$n = 2$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem



Population standard deviation (σ)

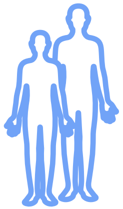
mean of the sample means

$$\mu_{\bar{x}} = \mu$$

population mean

Confidence Interval - Intuition

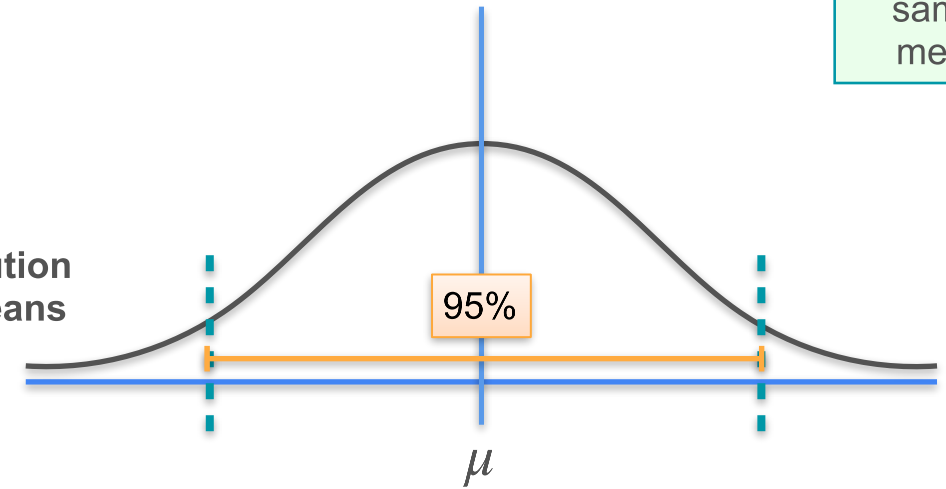
$n = 2$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem



Population standard deviation (σ)

mean of the sample means

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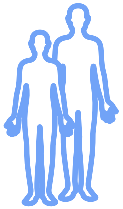
population mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

standard deviation of the sample means

Confidence Interval - Intuition

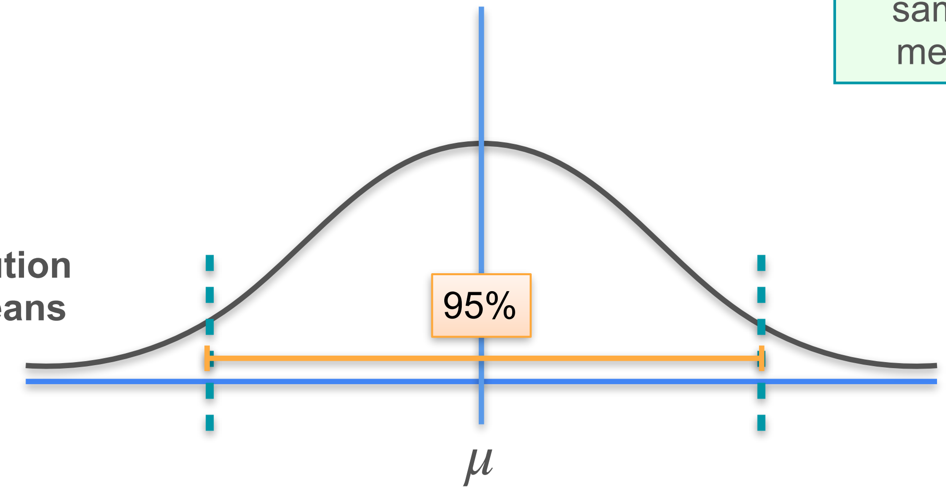
$n = 2$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem



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population mean

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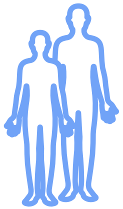
standard deviation of the sample means

Confidence Interval - Intuition

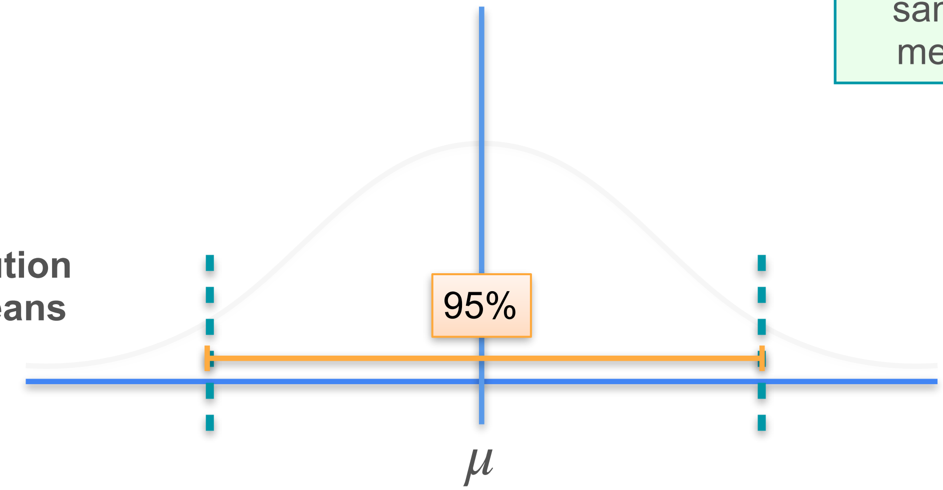
$n = 2$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem



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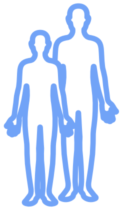
standard deviation of the sample means

Confidence Interval - Intuition

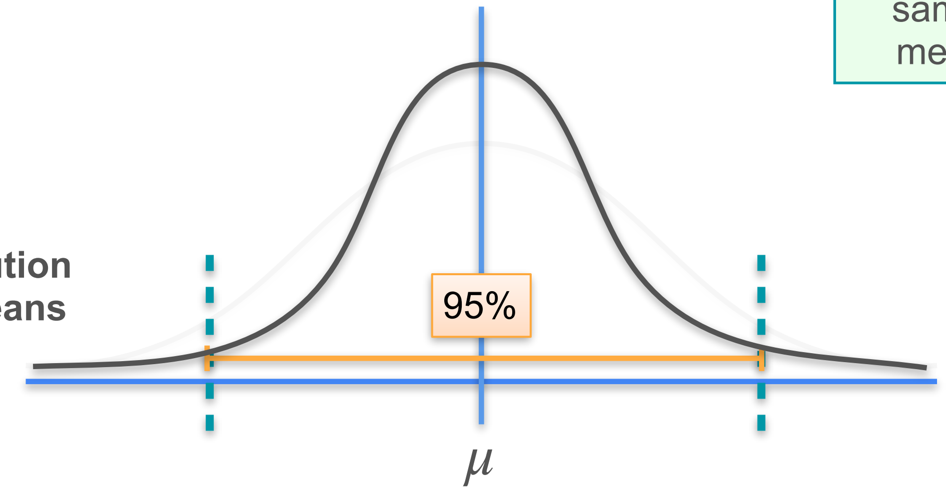
$n = 2$



$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem



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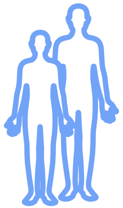
standard deviation of the sample means

Confidence Interval - Intuition

$n = 2$

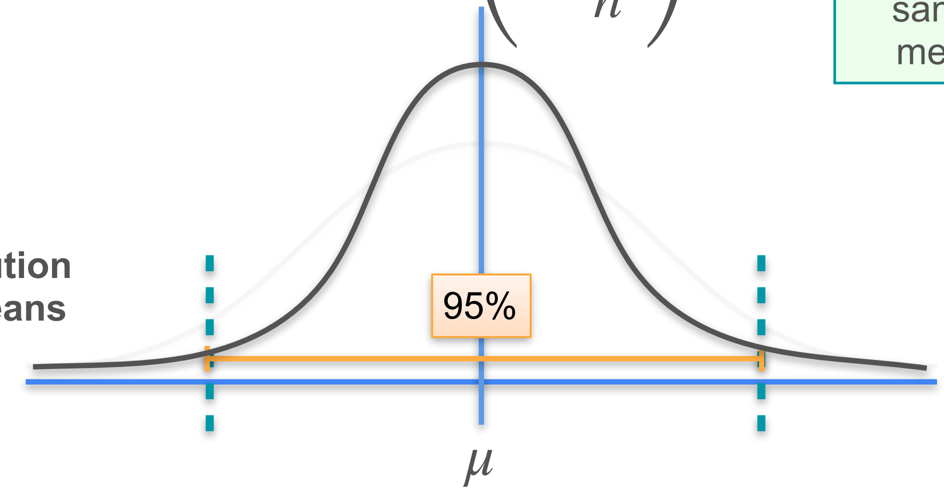


$\bar{x}$

sampling distribution of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$



Population standard deviation (σ)

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population mean

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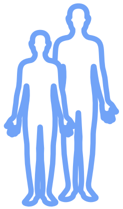
standard deviation of the sample means

Confidence Interval - Intuition

$$n = 2$$

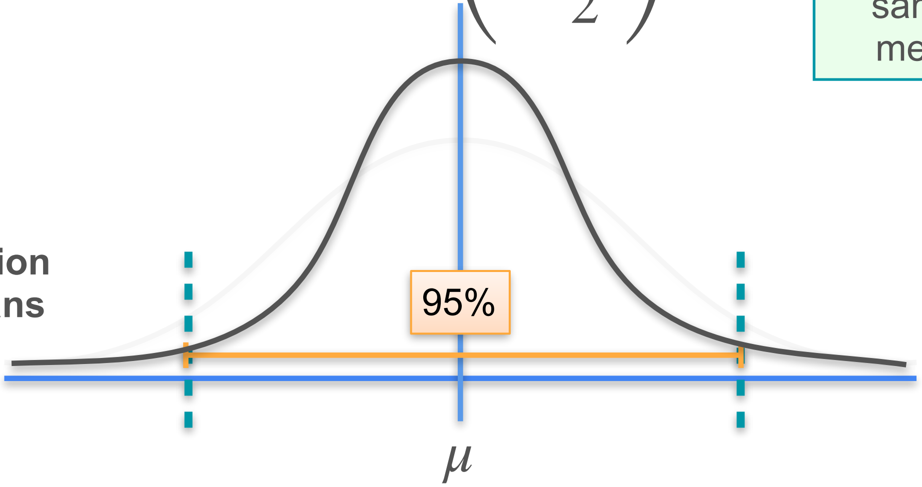


$$\bar{x}$$

sampling distribution of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



Population standard deviation (σ)

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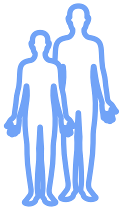
standard deviation of the sample means

Confidence Interval - Intuition

$$n = 2$$



$$\bar{x}$$

sampling distribution of the sample means

Central Limit Theorem

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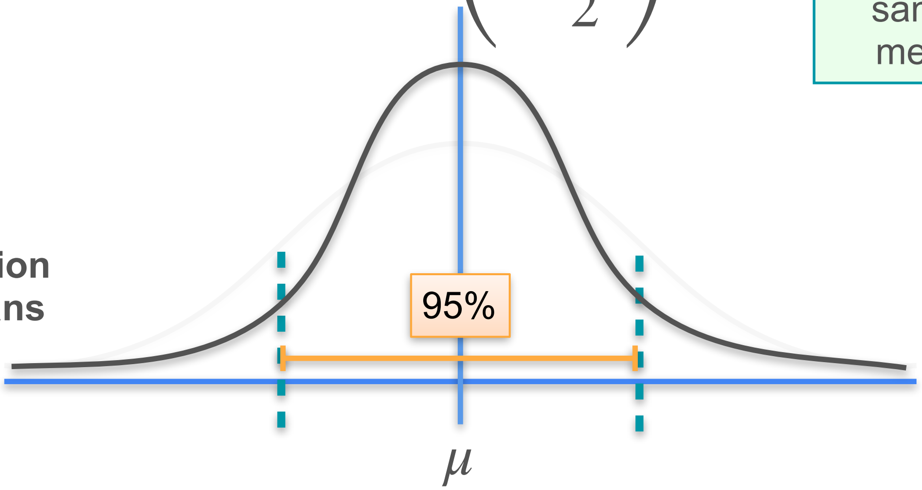


Population standard deviation (σ)

mean of the sample means

$$\mu_{\bar{x}} = \mu$$

population mean

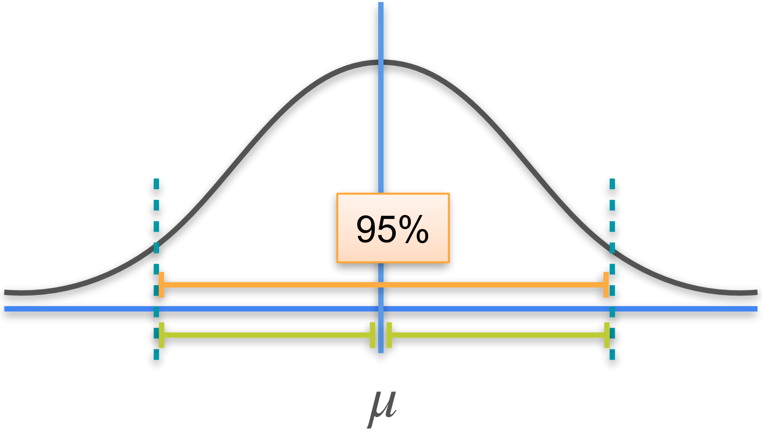
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard deviation of the sample means

Confidence Interval - Intuition

95%

$$n = 1$$

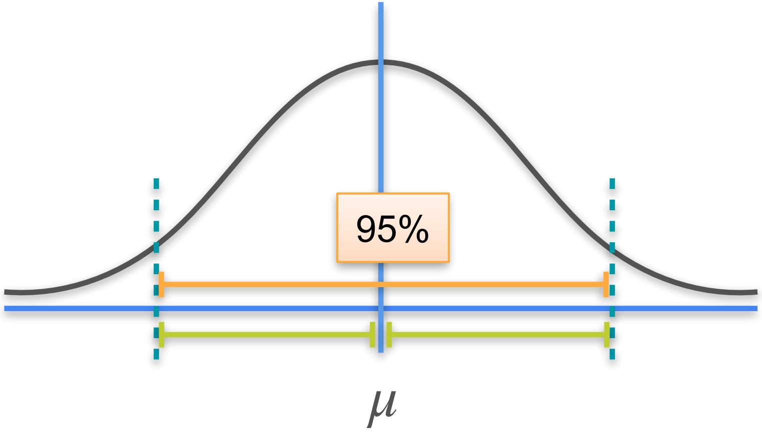


Confidence Interval - Intuition

95%

$$n = 1$$

$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$

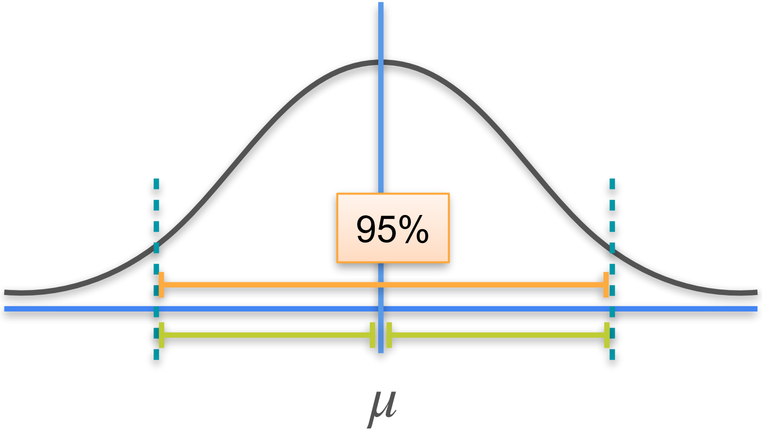


Confidence Interval - Intuition

95%

$$n = 1$$

$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



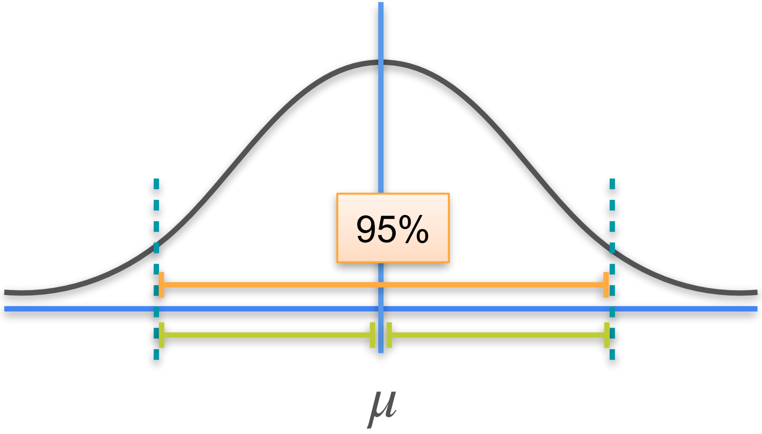
$$n = 2$$

Confidence Interval - Intuition

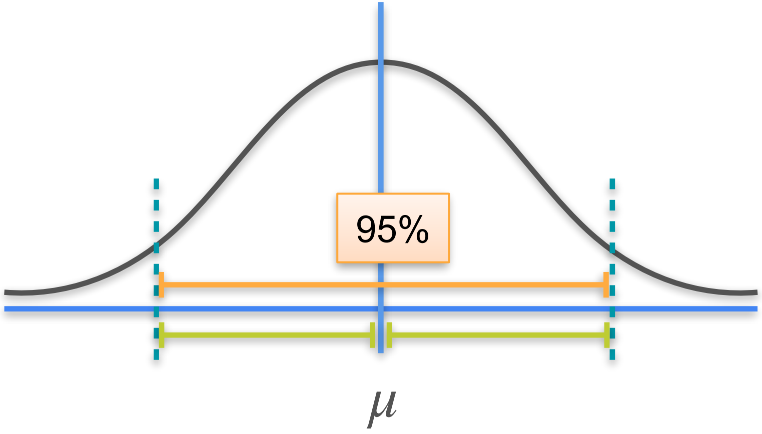
95%

$n = 1$

$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$

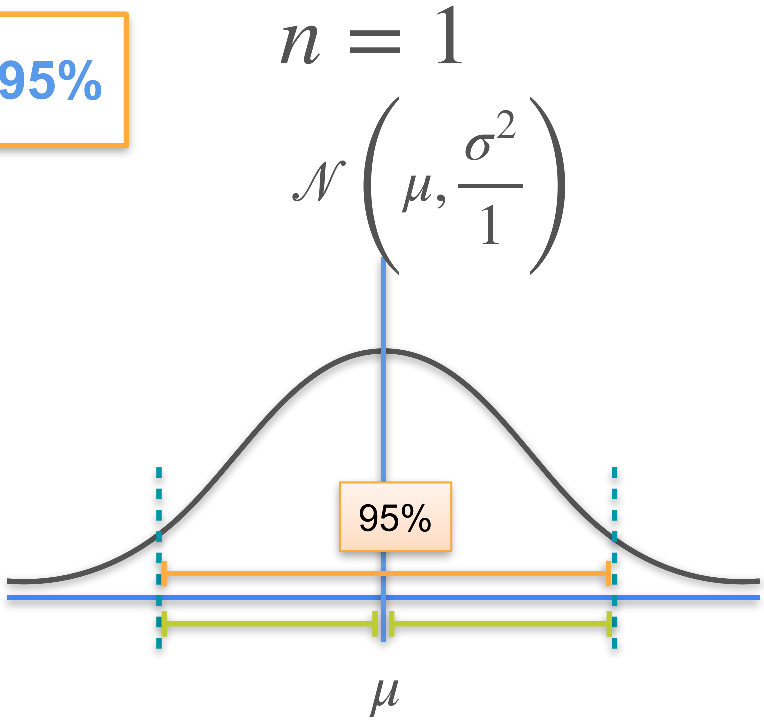


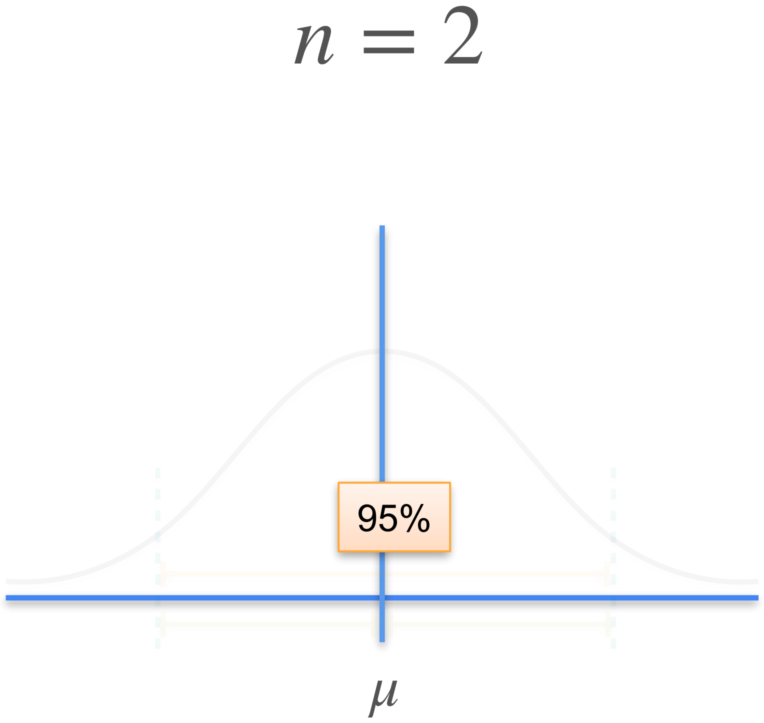
$n = 2$



Confidence Interval - Intuition

95%



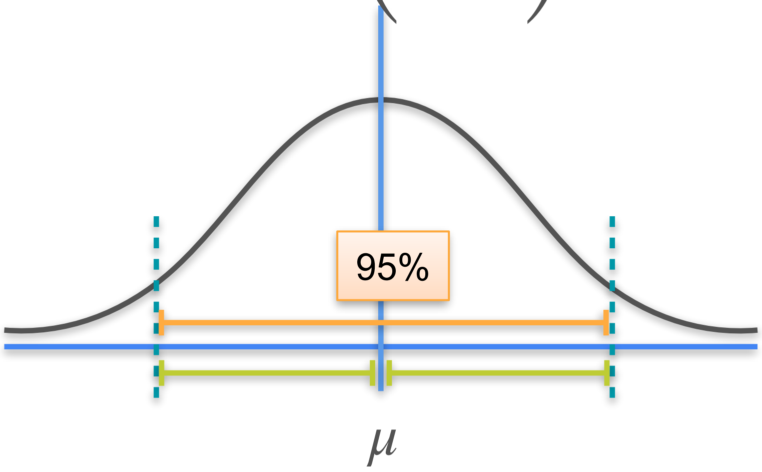


Confidence Interval - Intuition

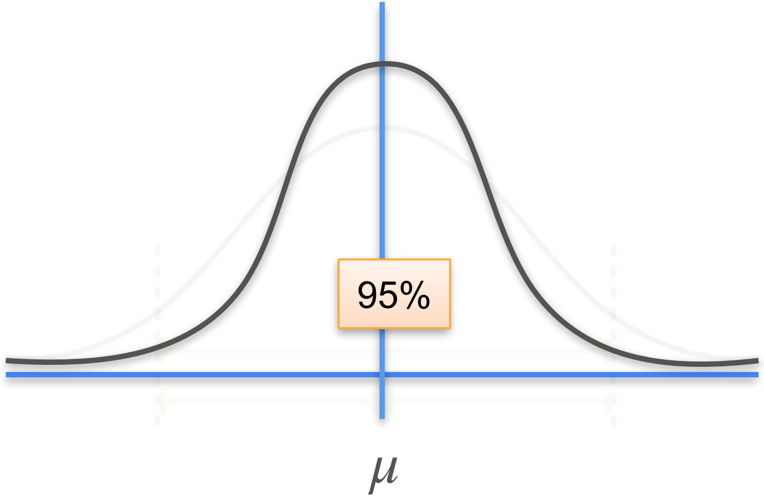
95%

$$n = 1$$

$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



$$n = 2$$



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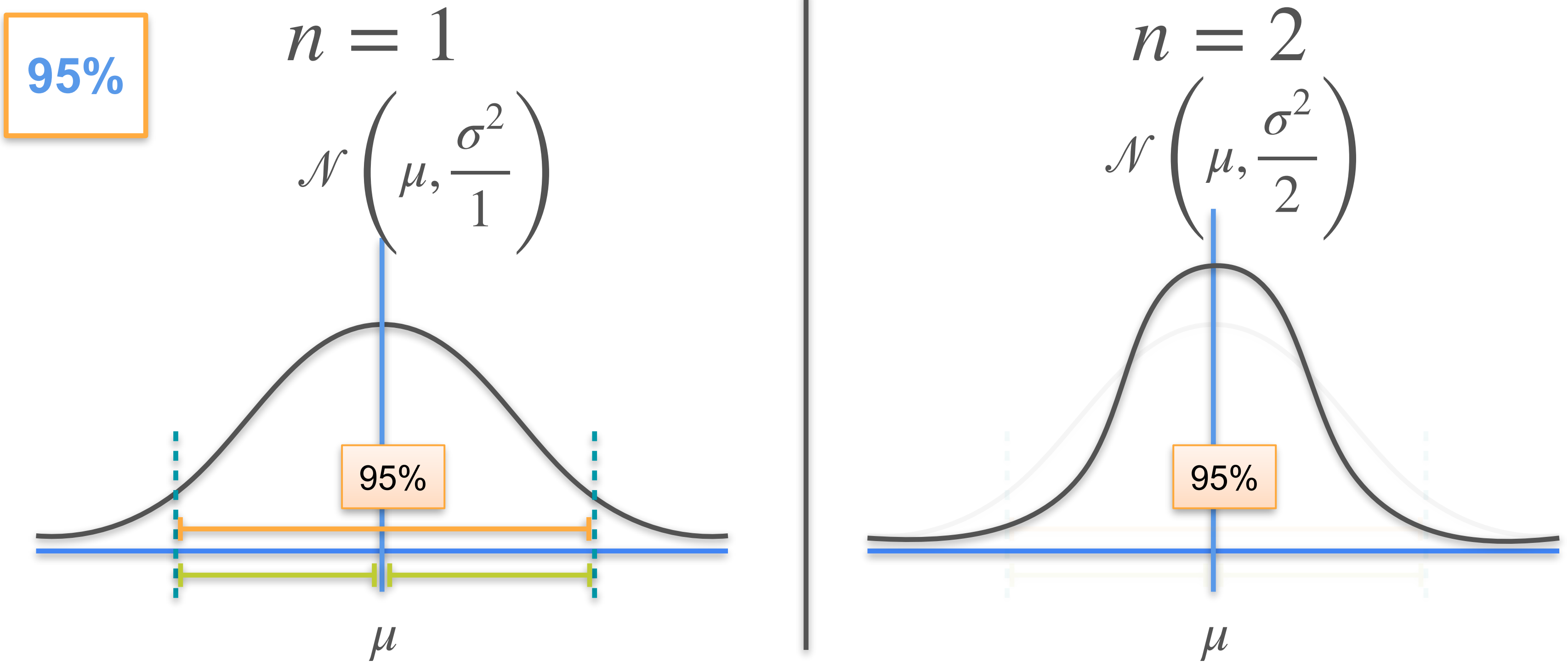

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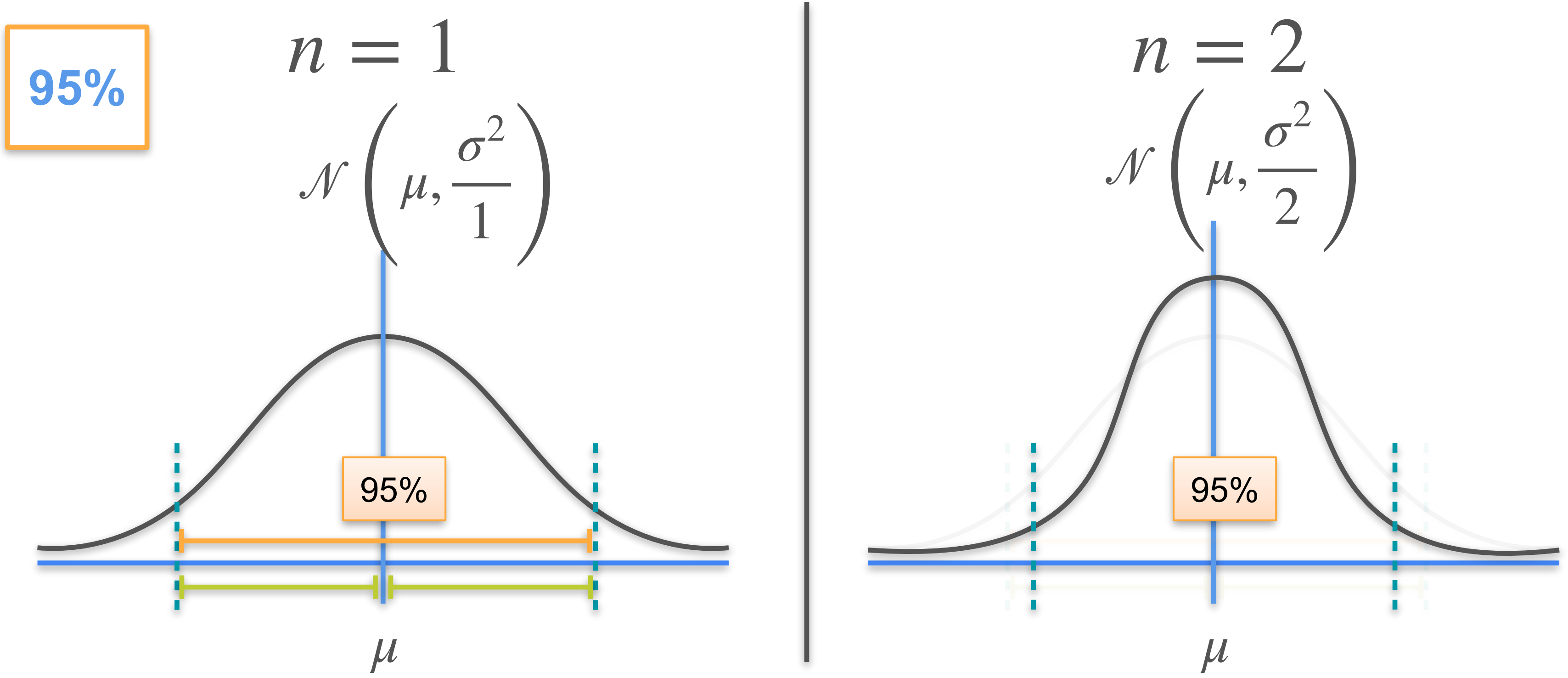
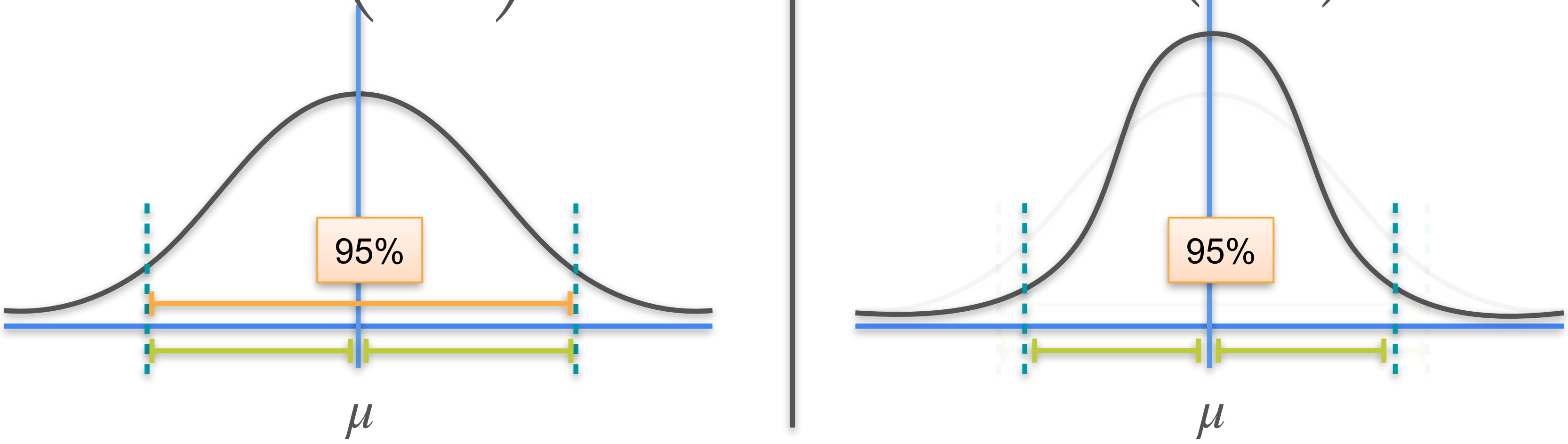


Confidence Interval - Intuition

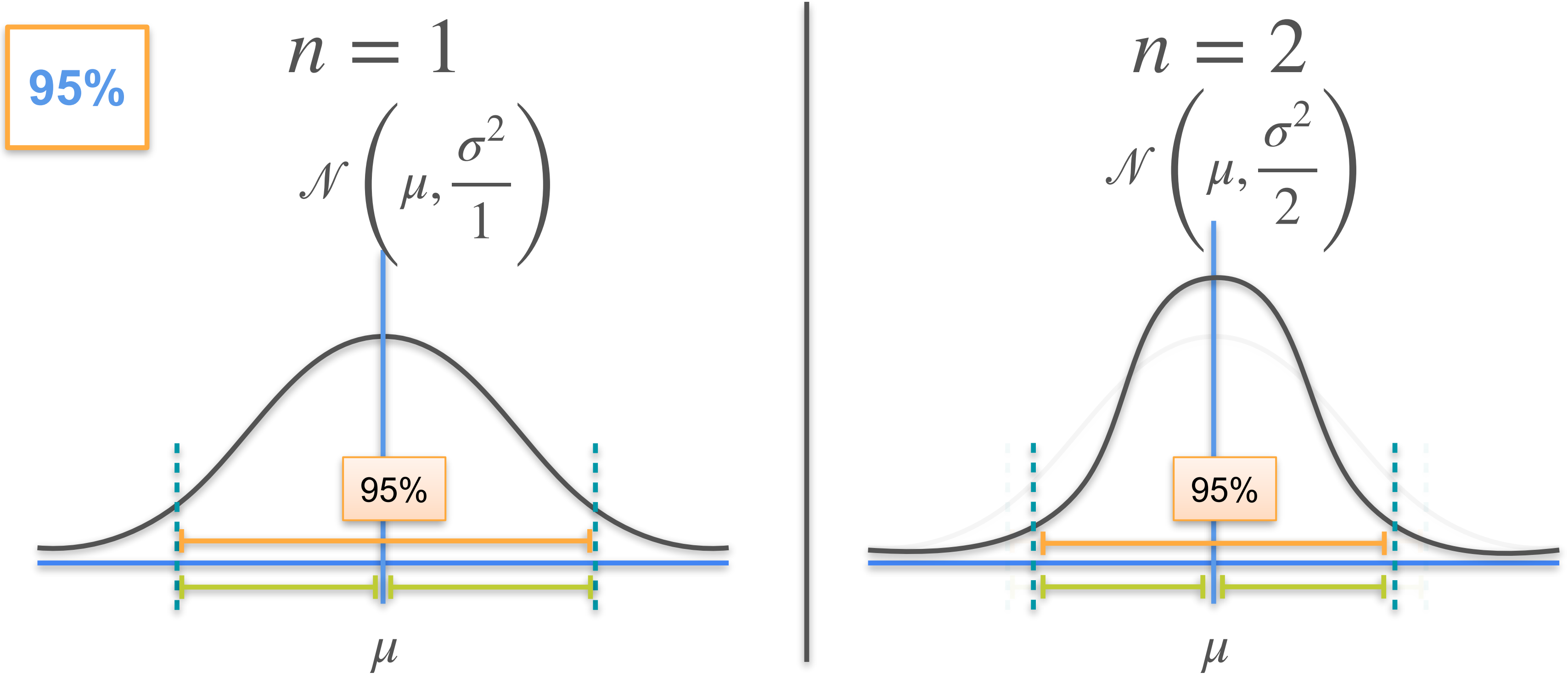
95%

$$n = 1 \qquad \mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$

$$n = 2 \qquad \mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



Confidence Interval - Intuition

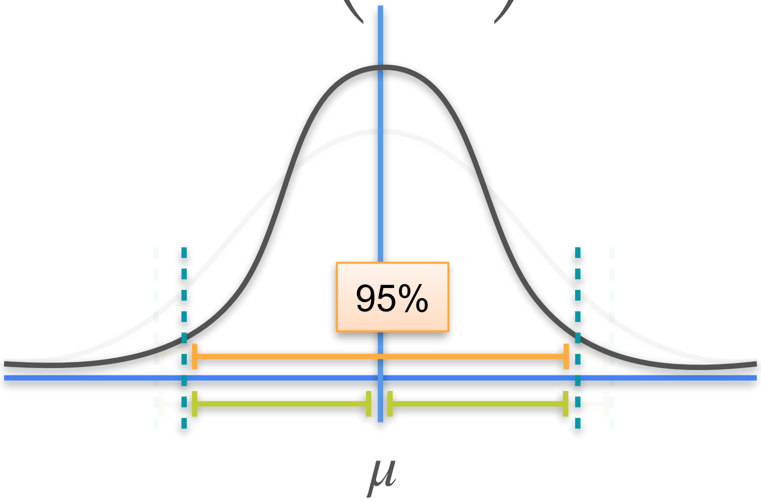


Confidence Interval - Intuition

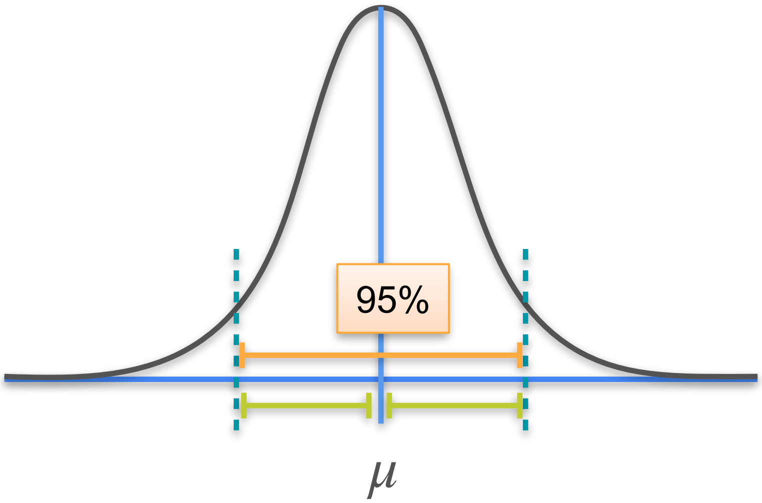
95%

$$n = 2$$

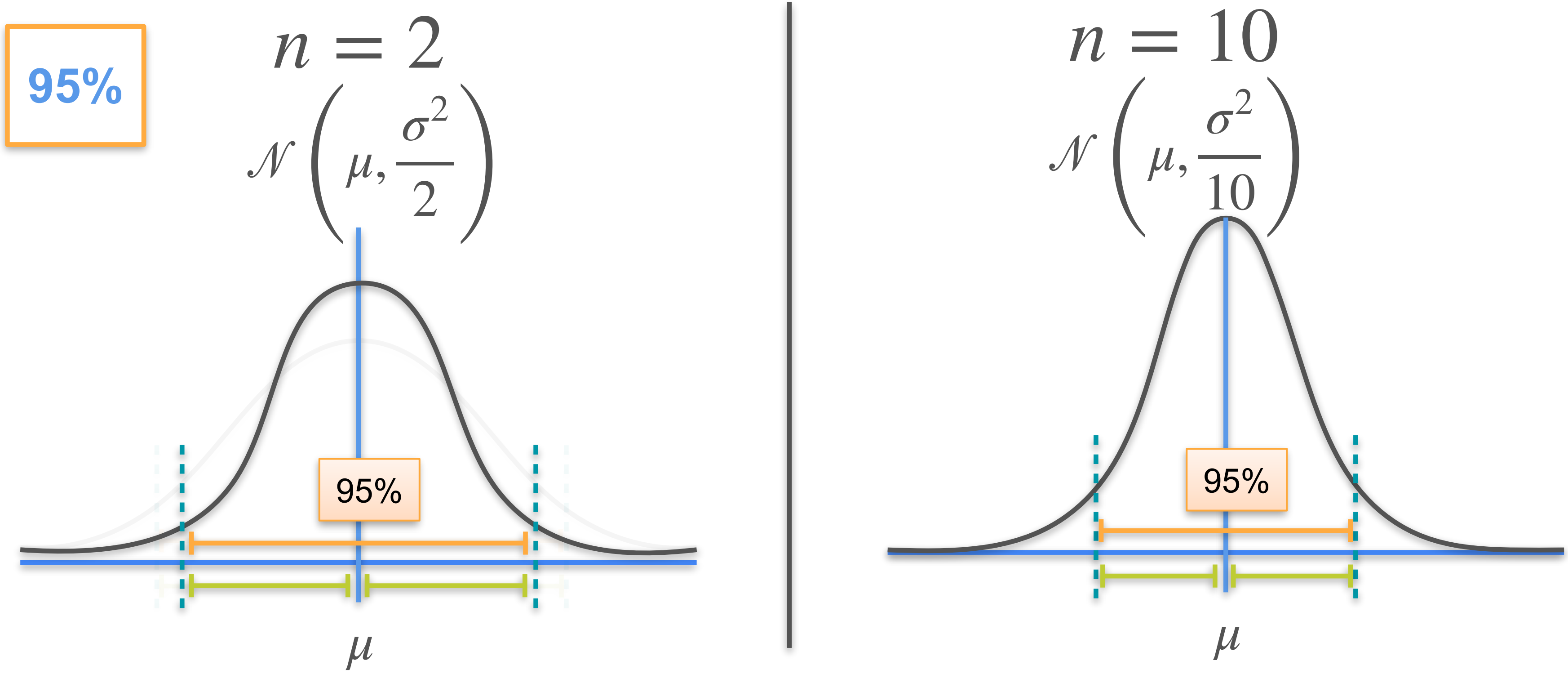
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



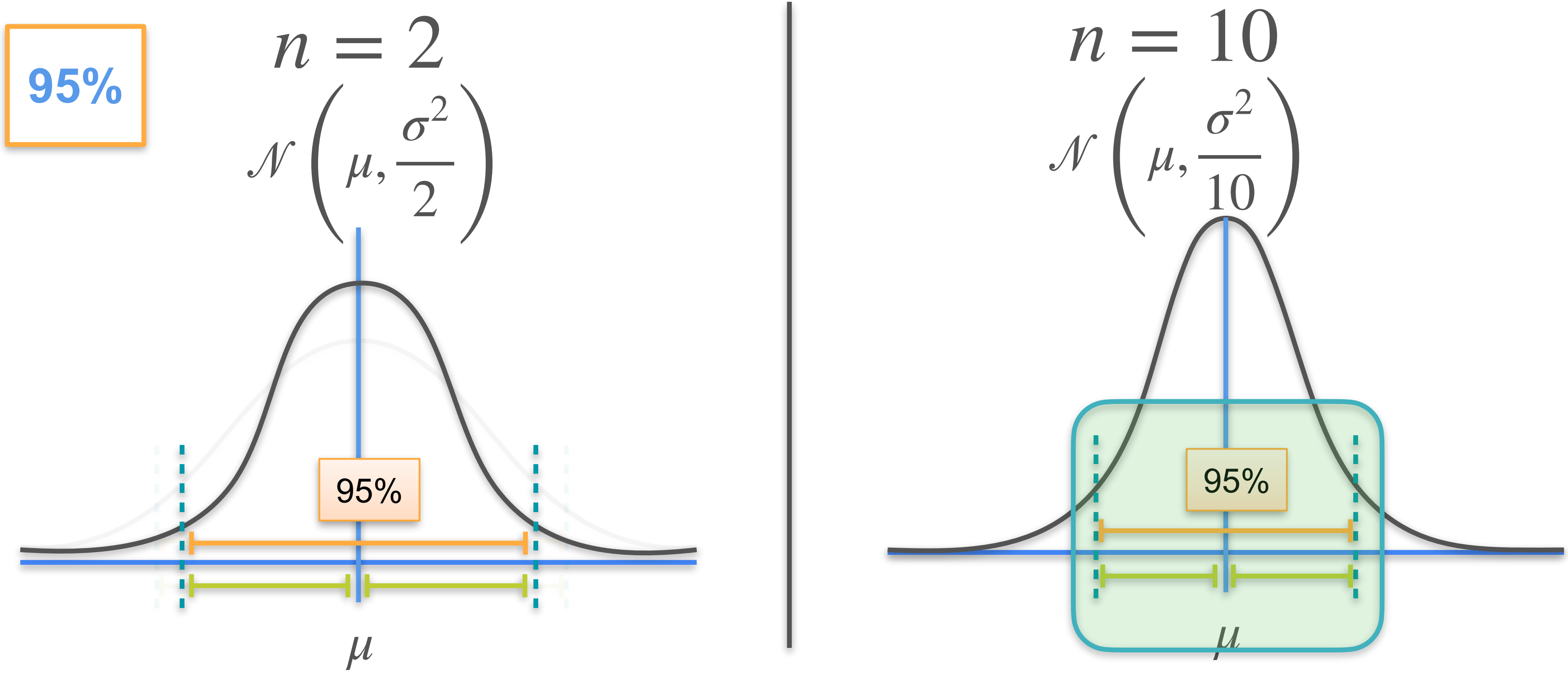
$$n = 10$$



Confidence Interval - Intuition



Confidence Interval - Intuition



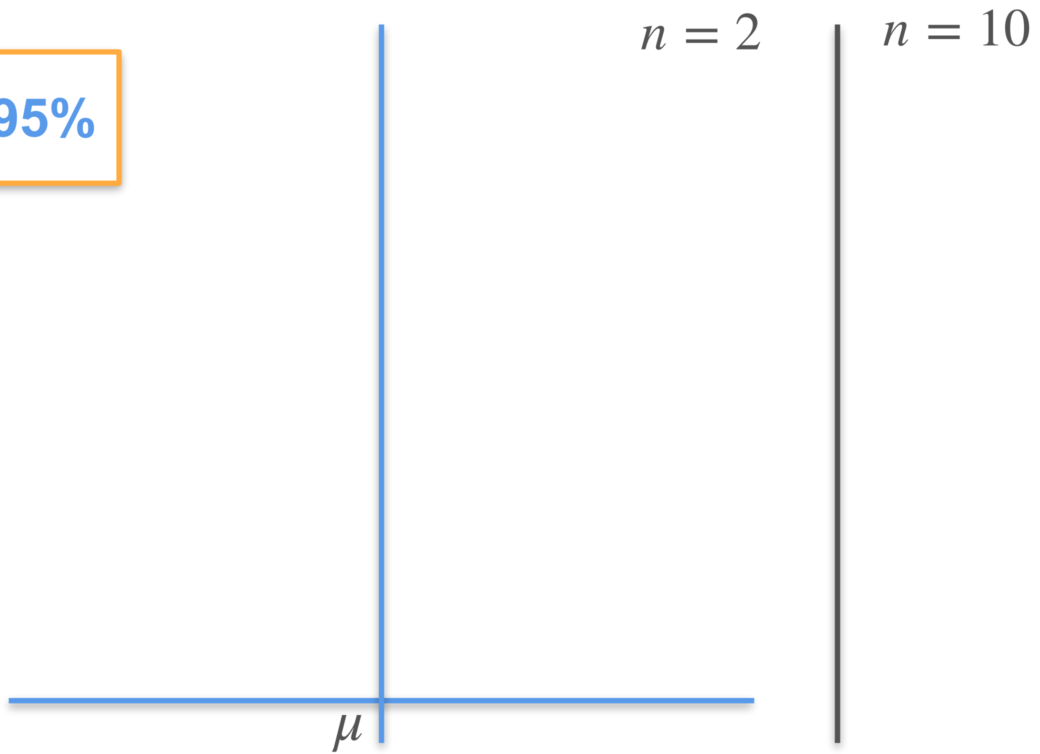
Confidence Interval - Intuition

$n = 2$ $n = 10$

95%

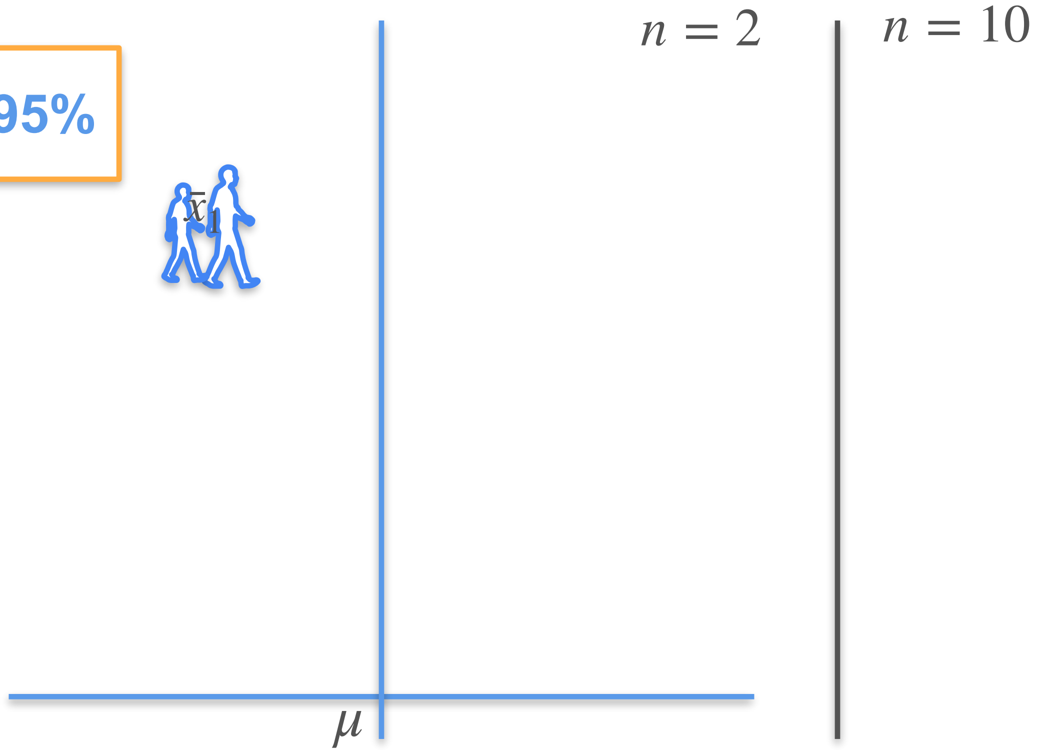
Confidence Interval - Intuition

95%

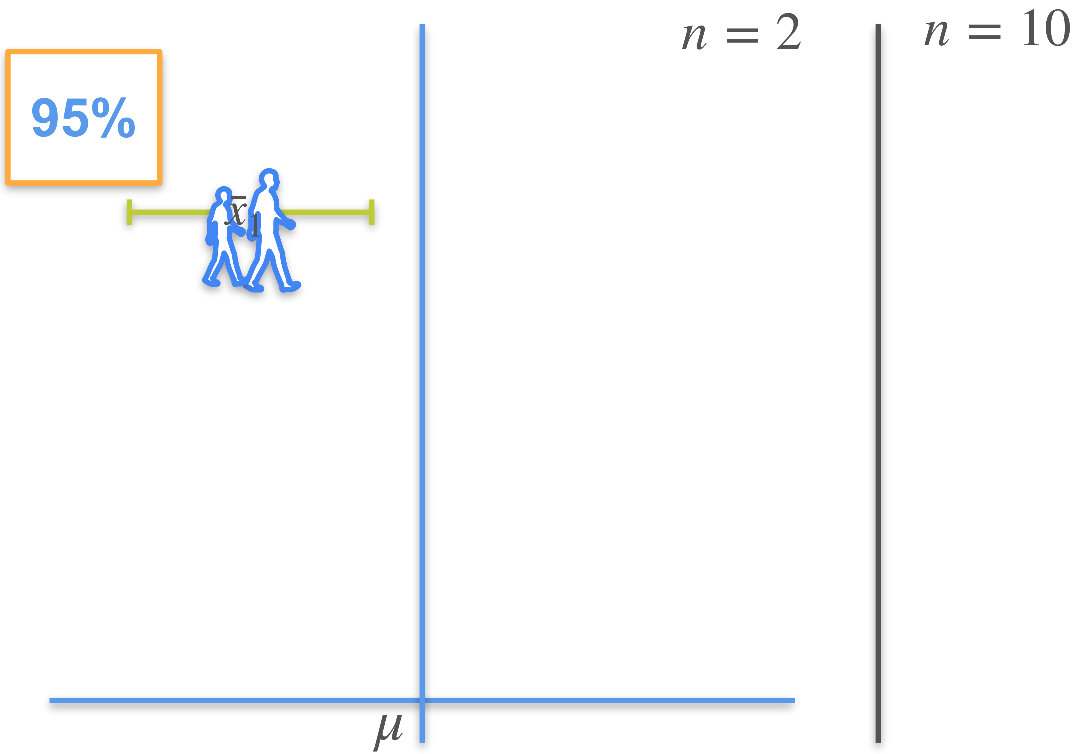


Confidence Interval - Intuition

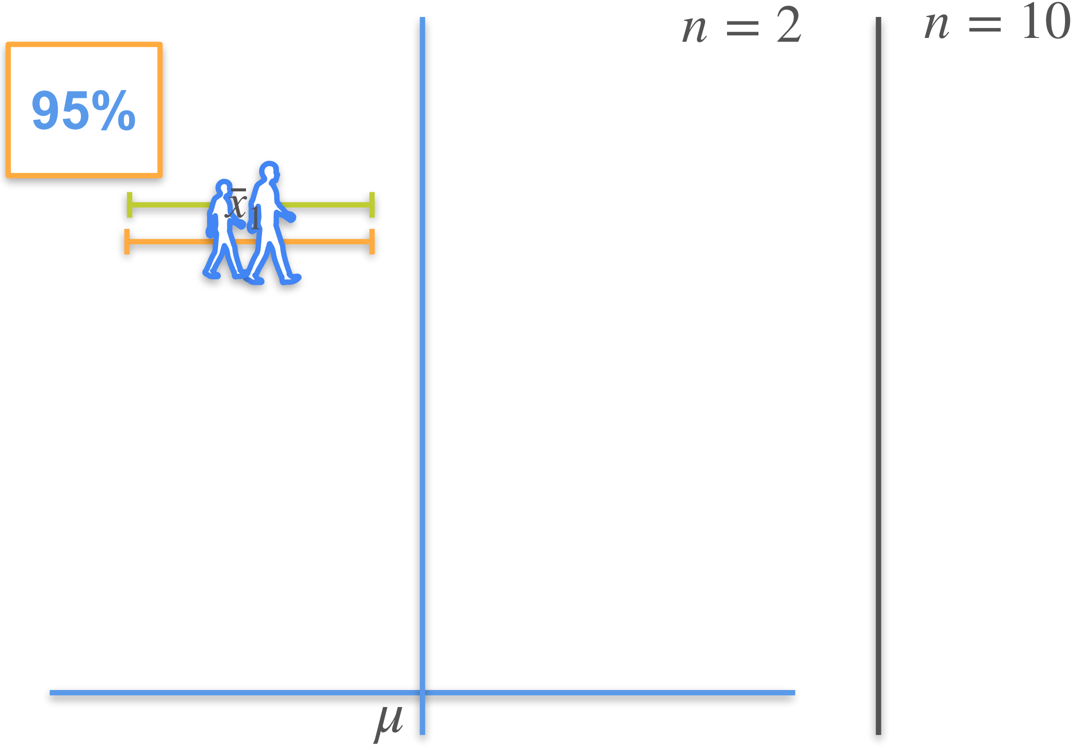
95%



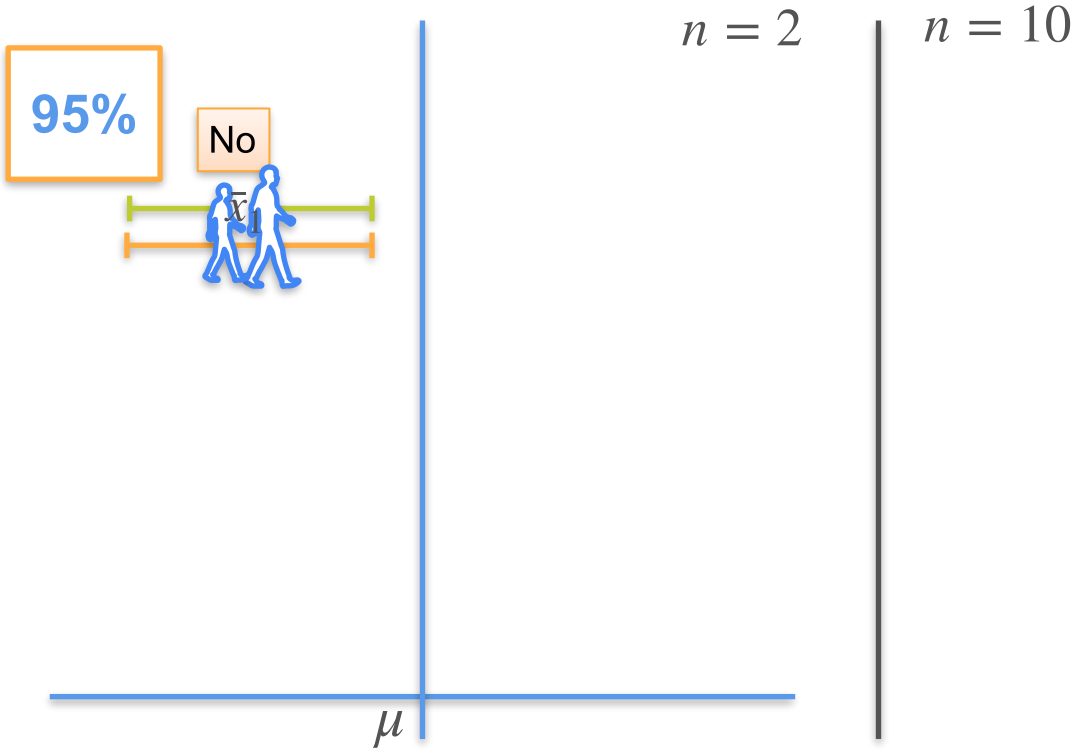
Confidence Interval - Intuition



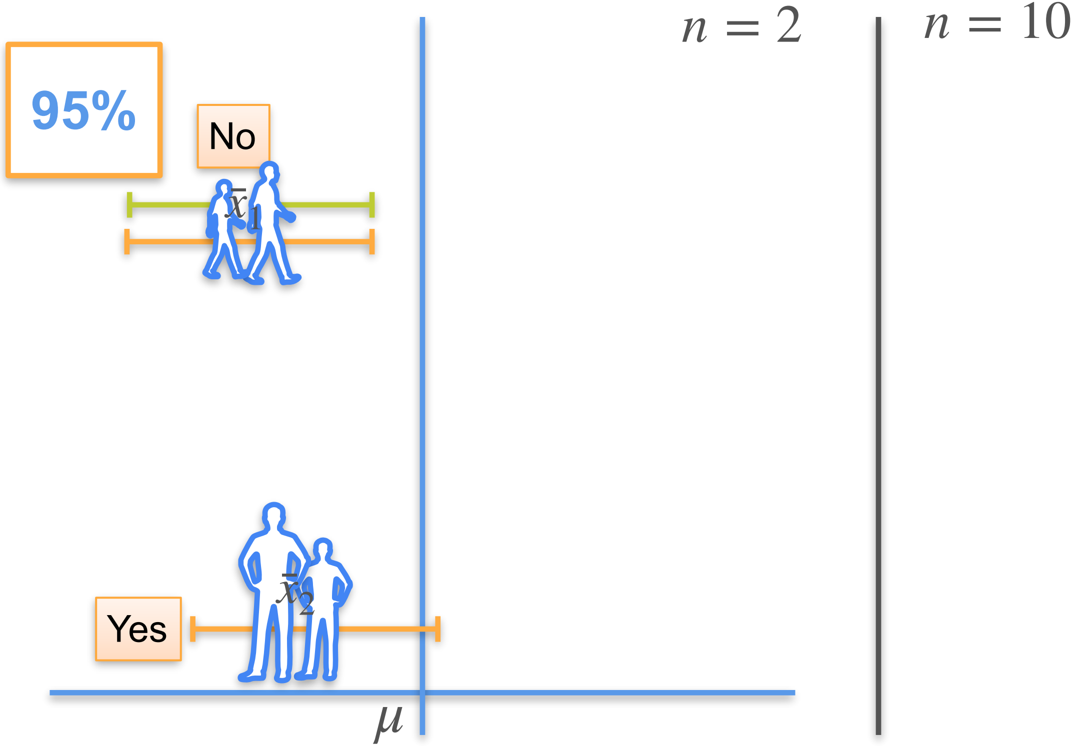
Confidence Interval - Intuition



Confidence Interval - Intuition



Confidence Interval - Intuition



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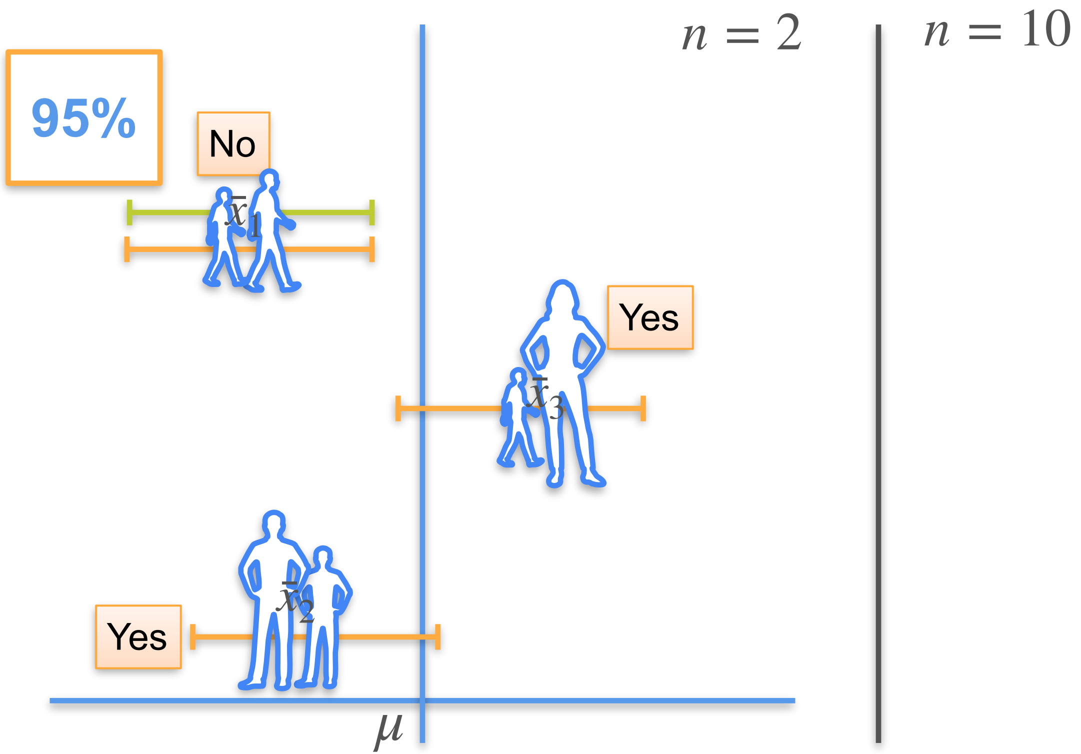

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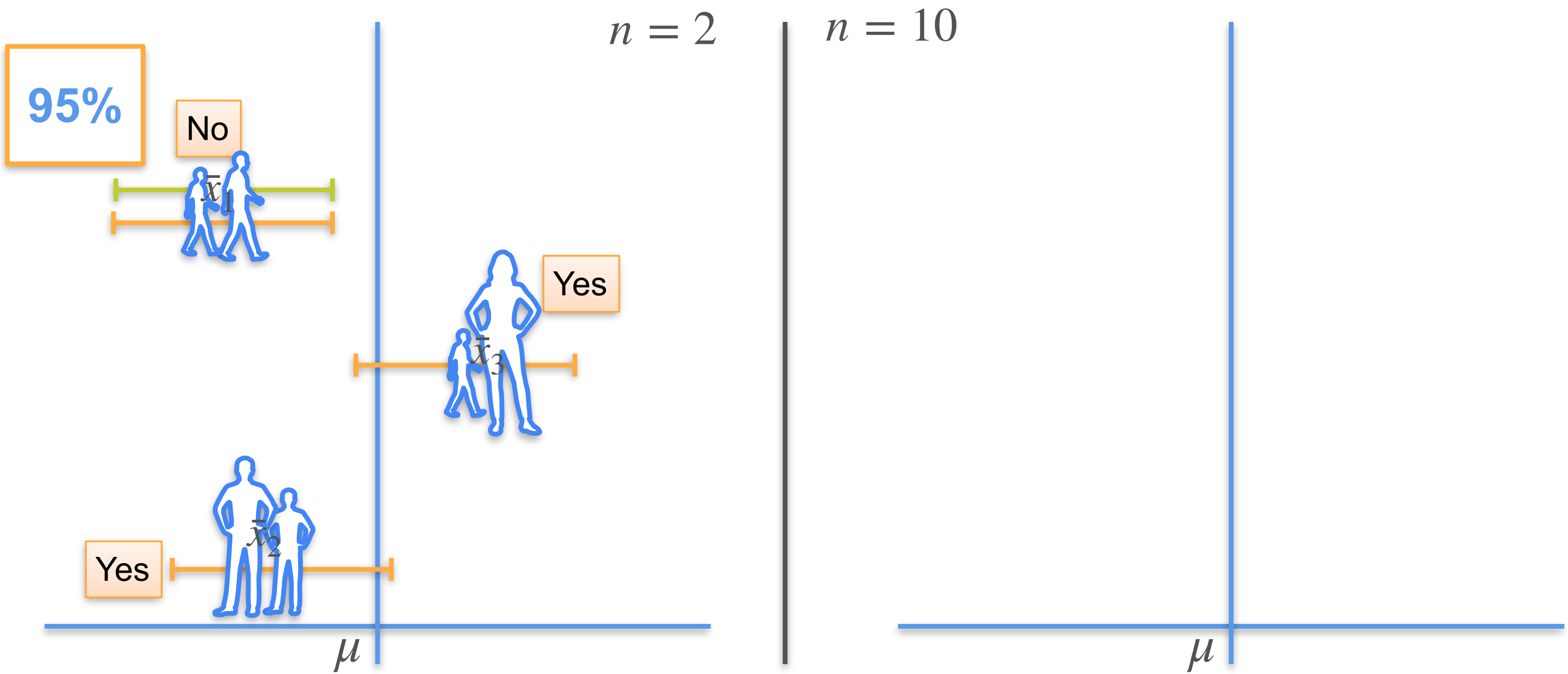
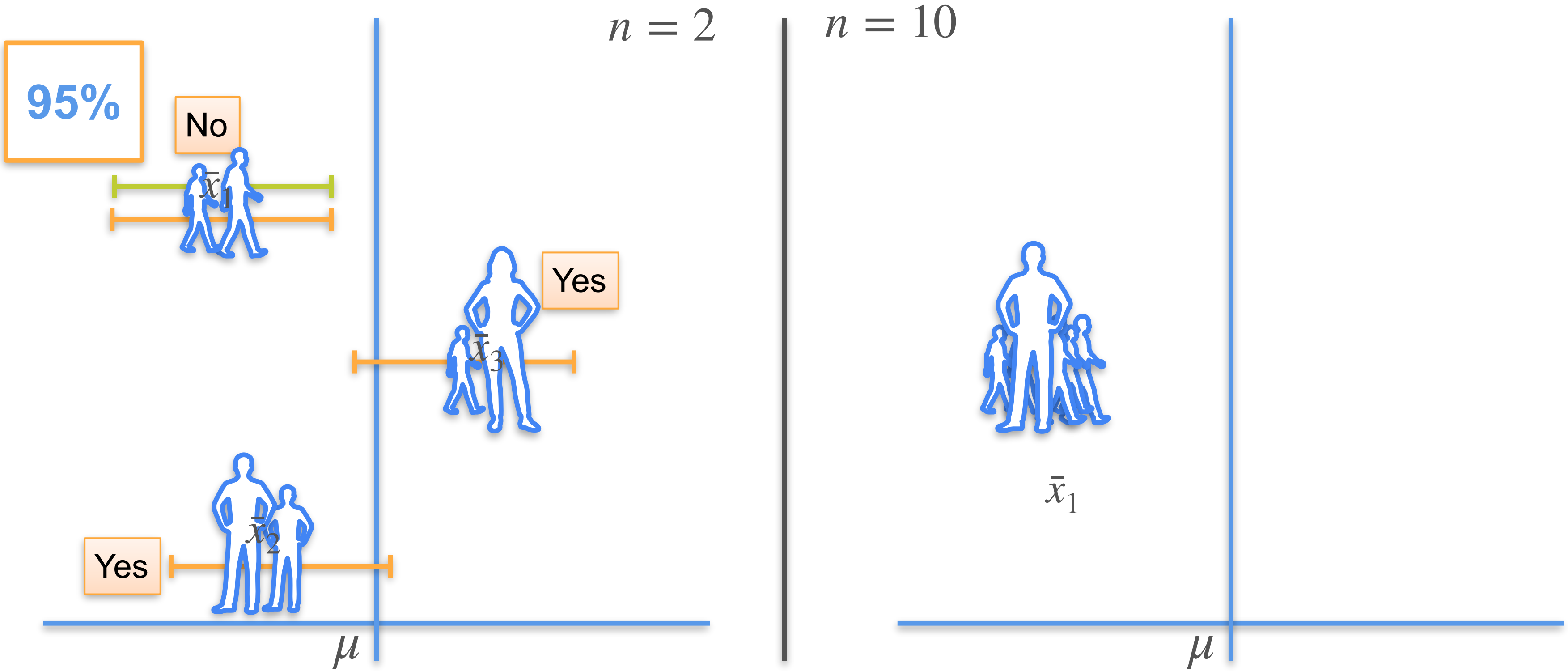
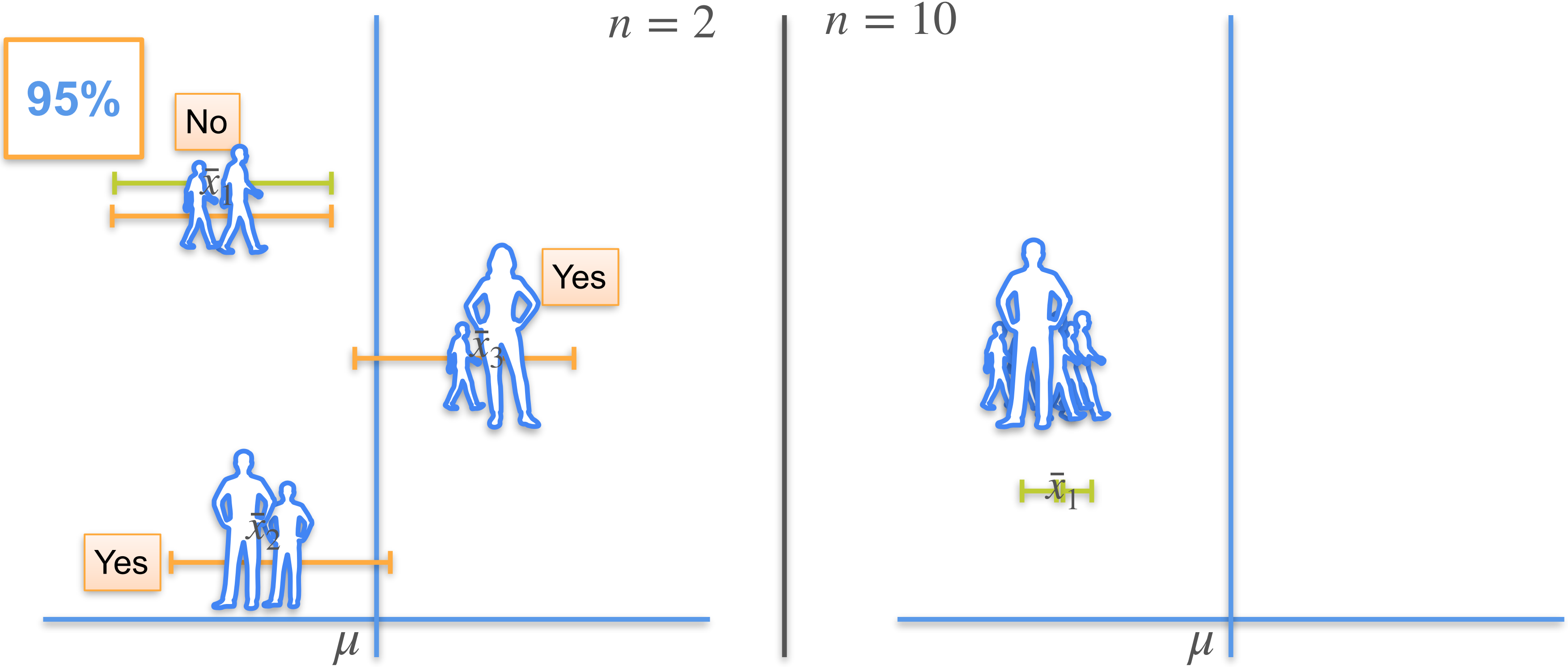


Confidence Interval - Intuition



Confidence Interval - Intuition



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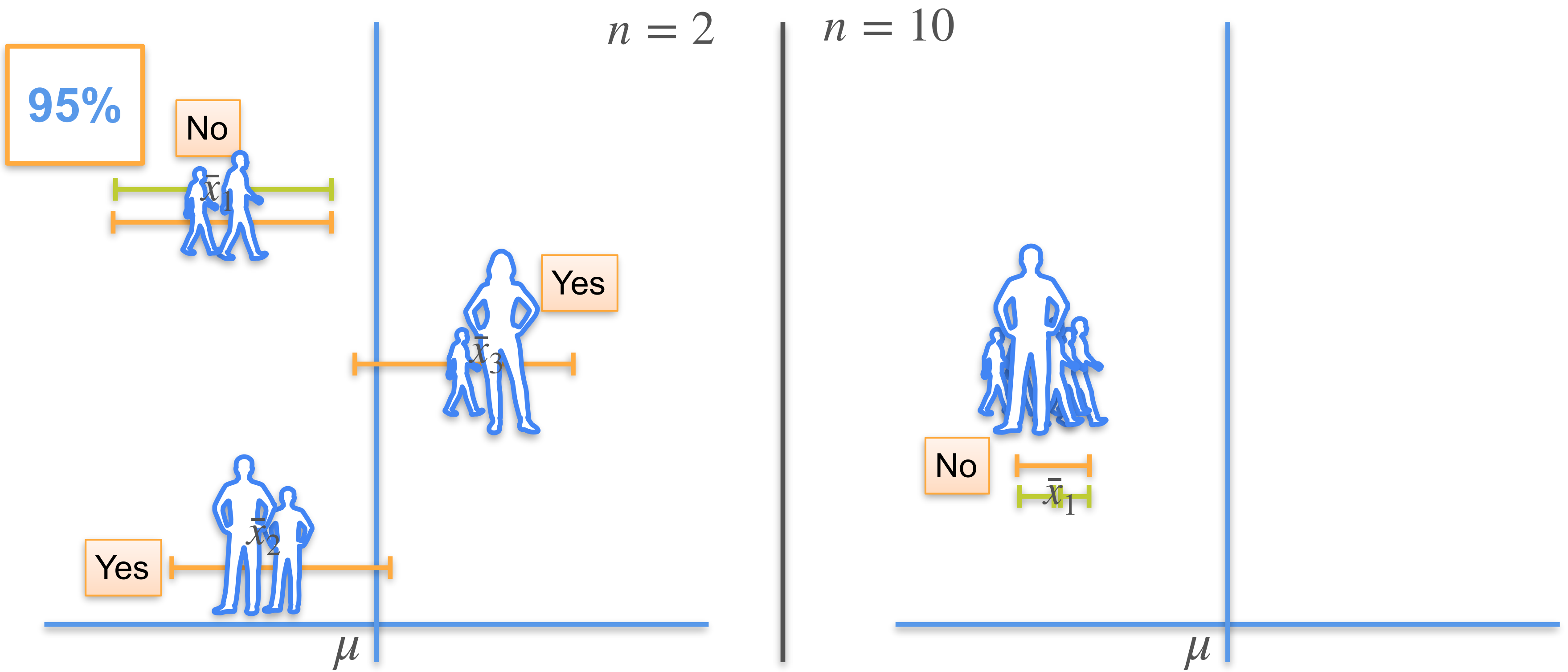
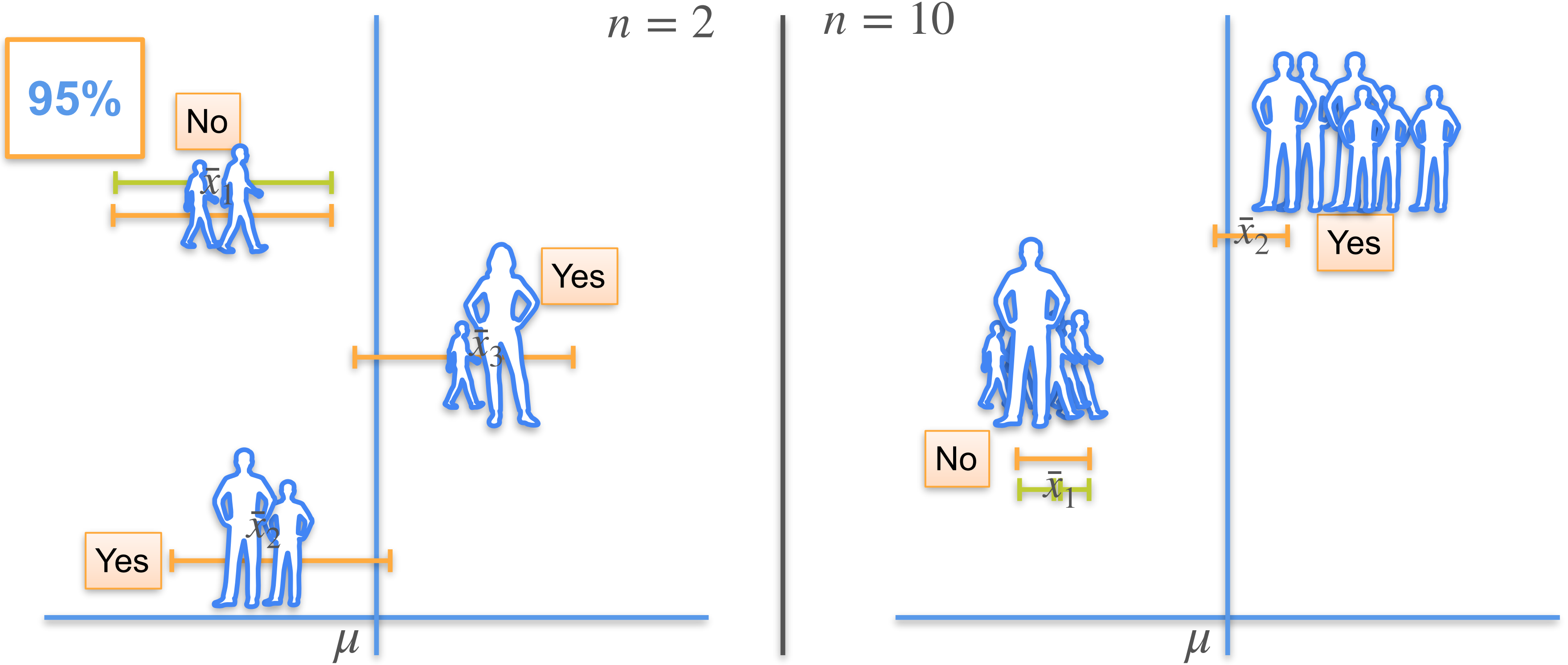
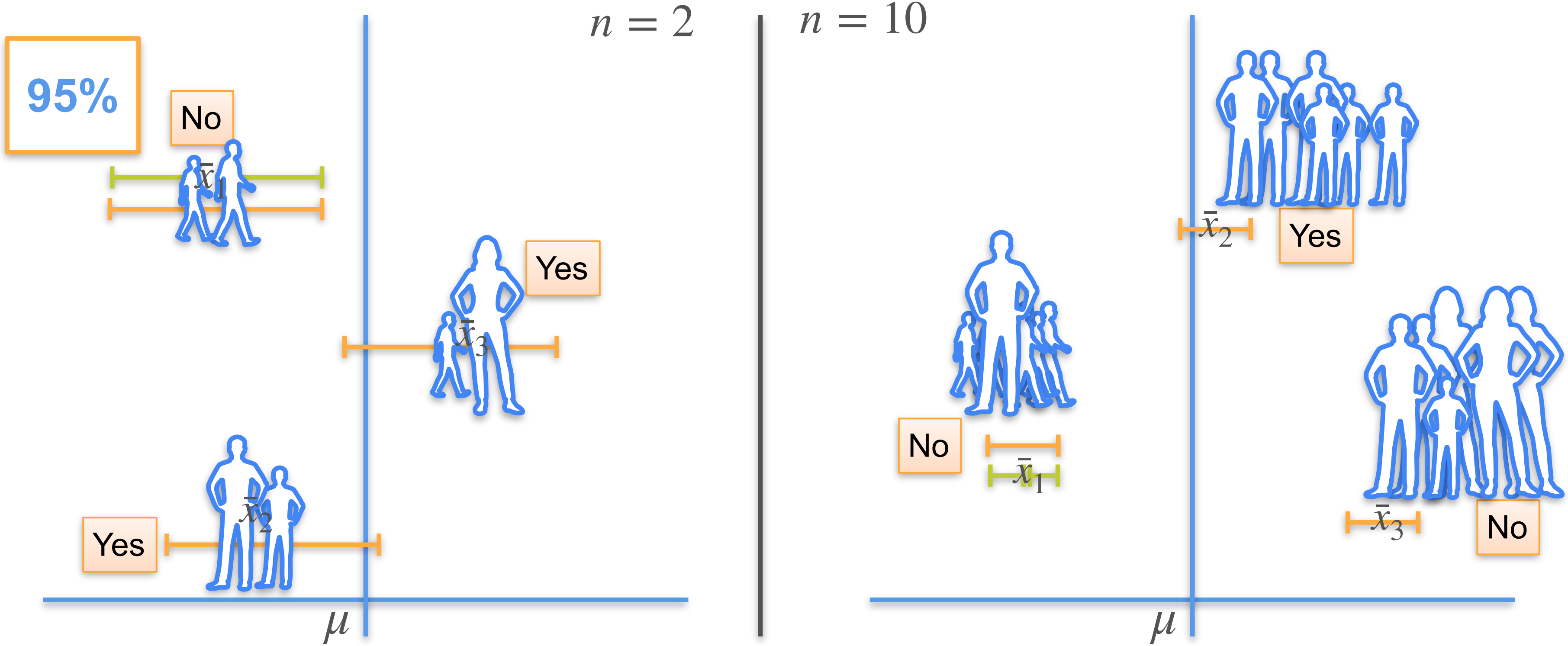


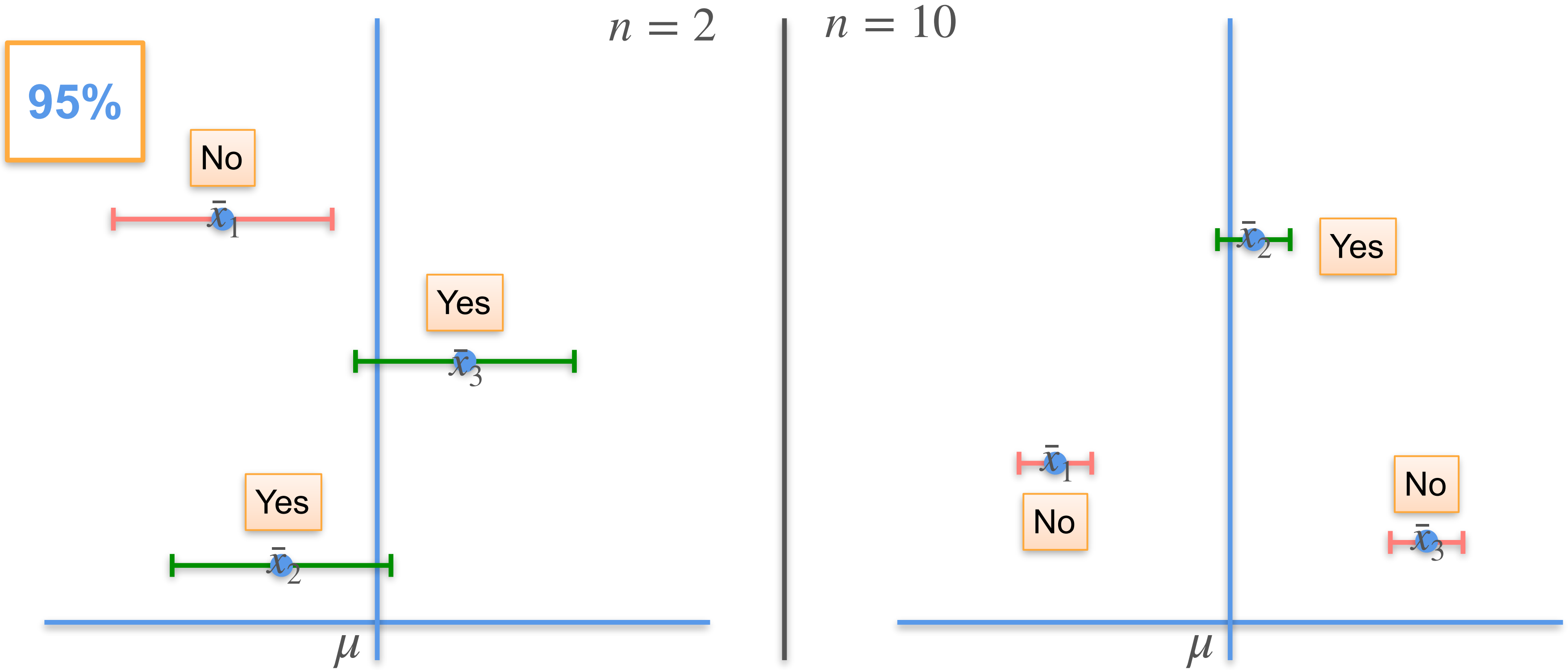
Confidence Interval - Intuition



Confidence Interval - Intuition

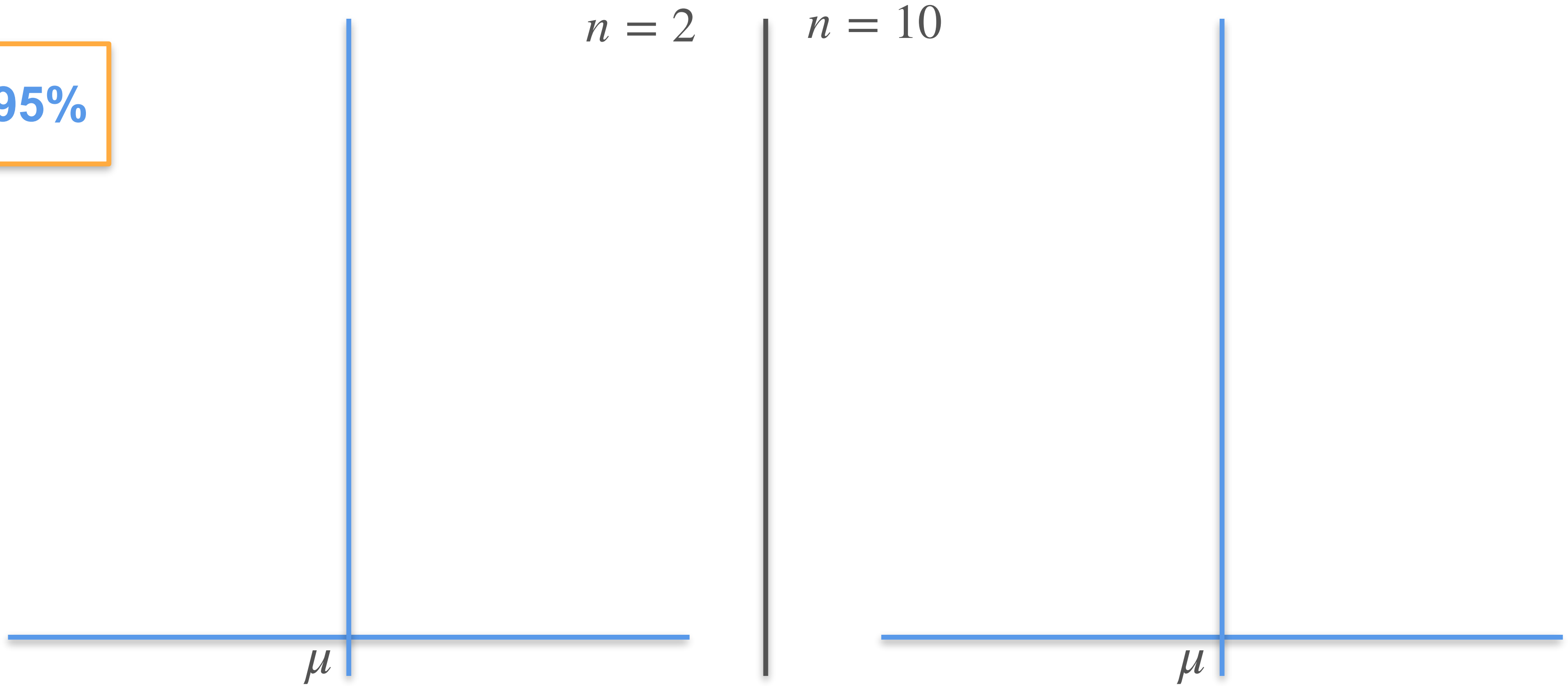


Confidence Interval - Intuition

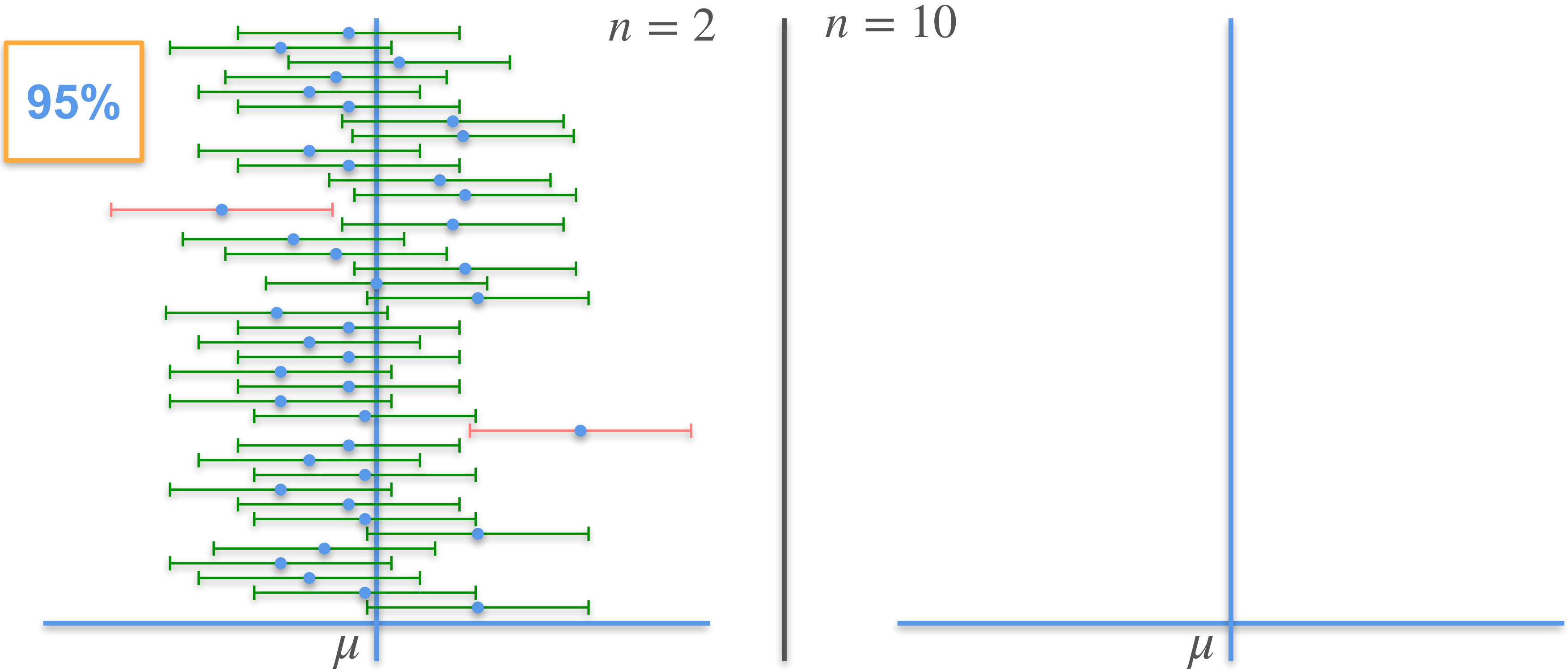


Confidence Interval - Intuition

95%



Confidence Interval - Intuition



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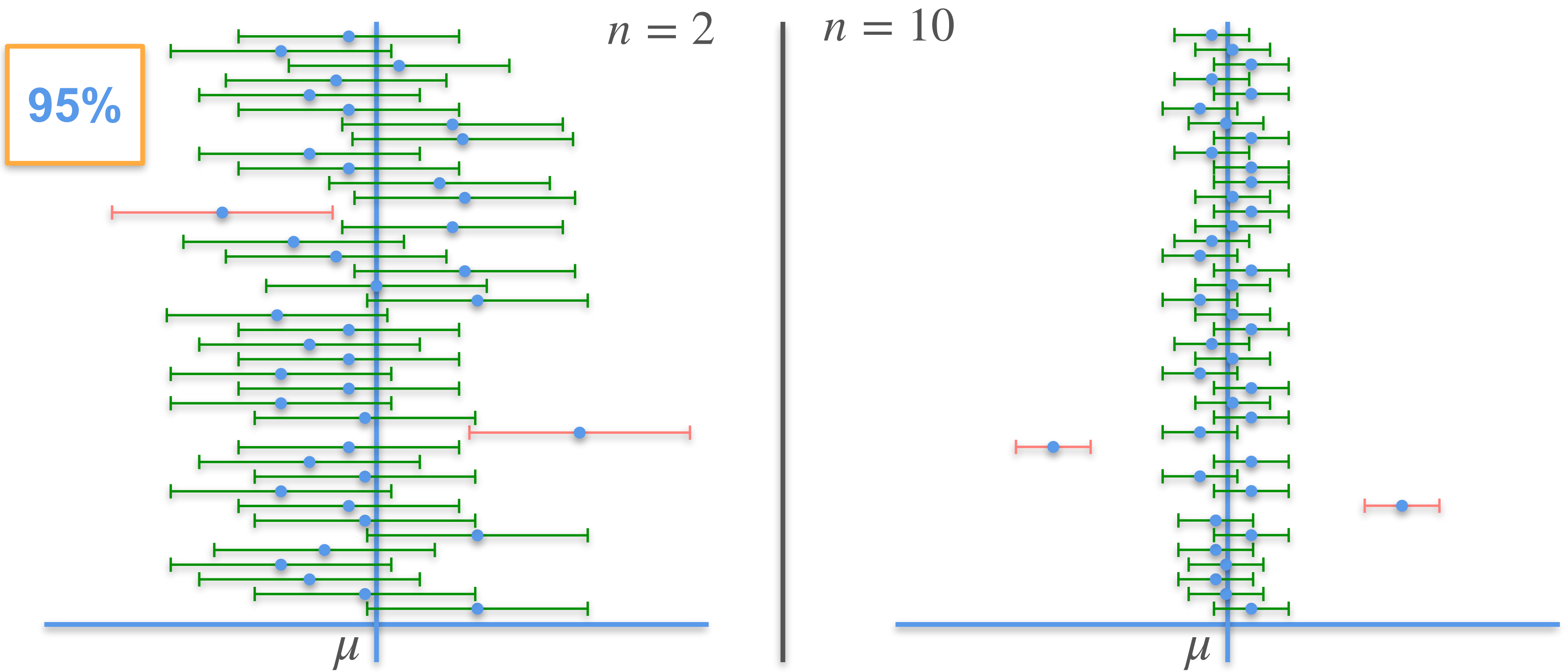

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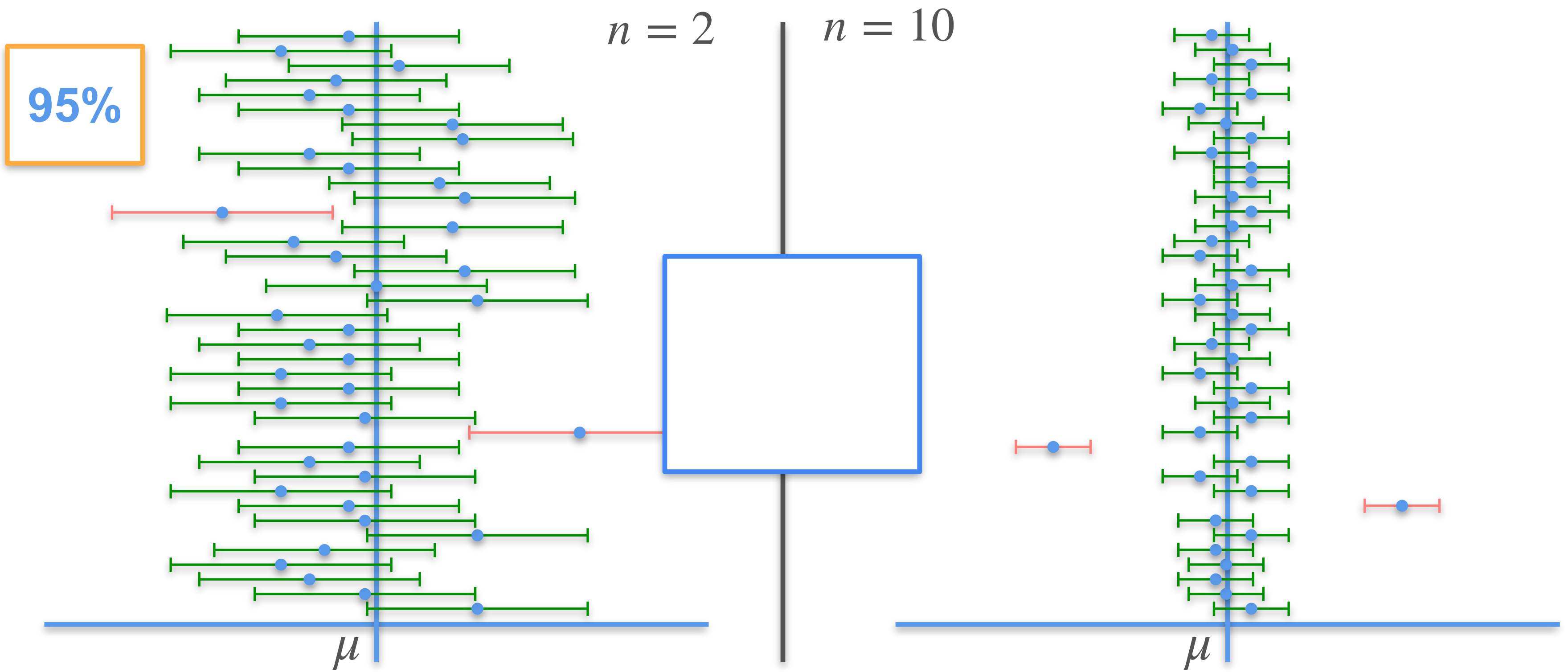
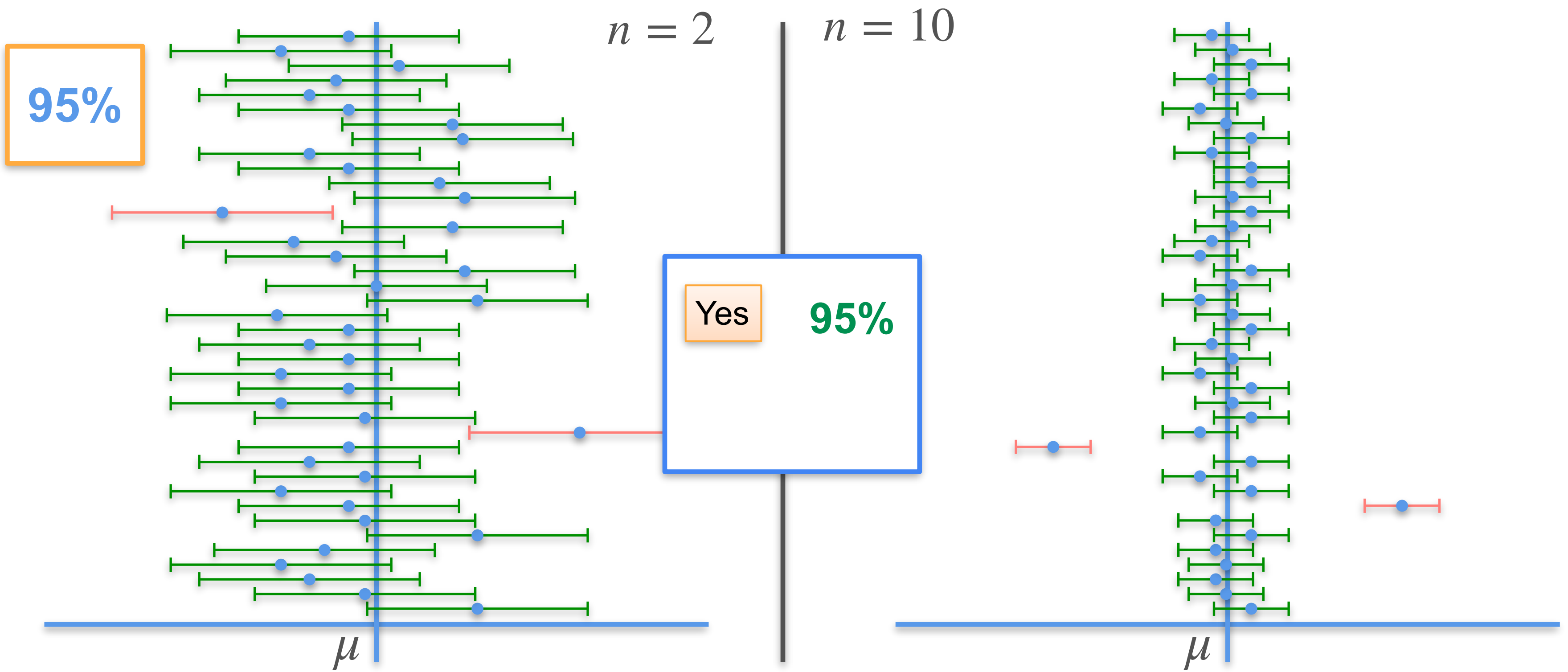


Confidence Interval - Intuition



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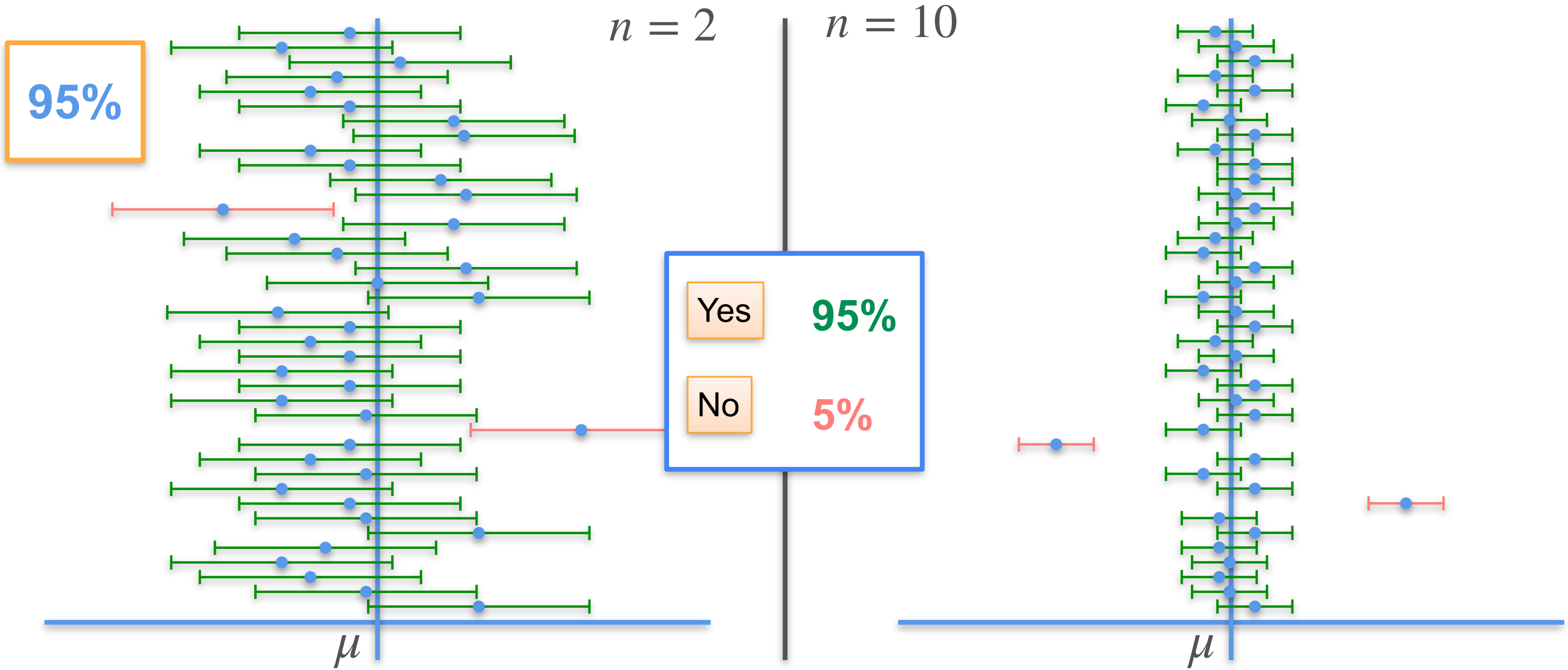
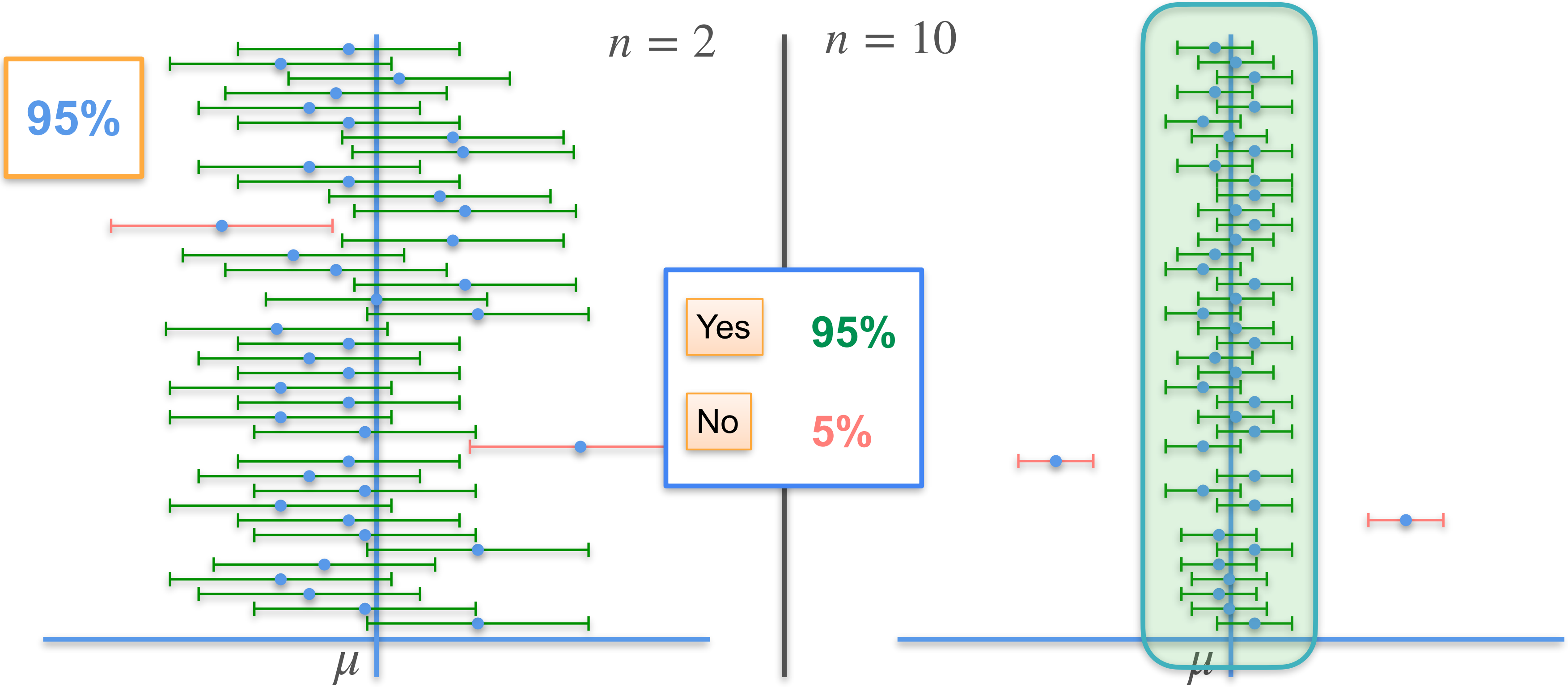


Confidence Interval - Intuition



Effect of the Sample Size

Effect of the Sample Size

sample size

n

1 2 3 4 5 6 7 8 9 10

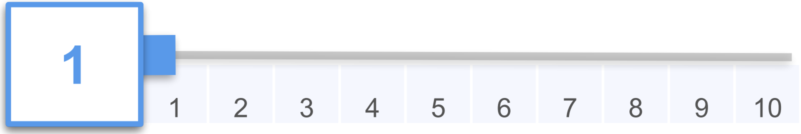
Confidence level

%

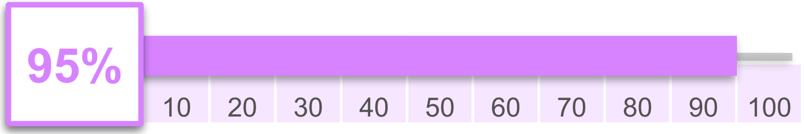
10 20 30 40 50 60 70 80 90 100

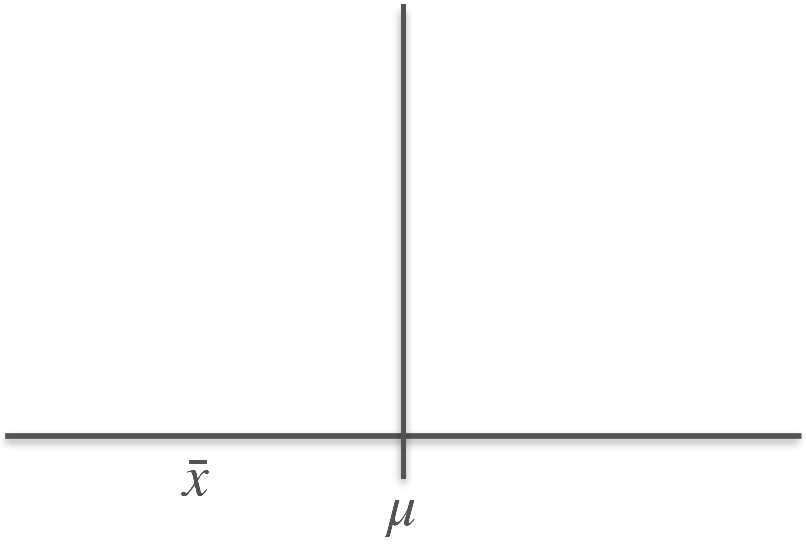
Effect of the Sample Size

sample size

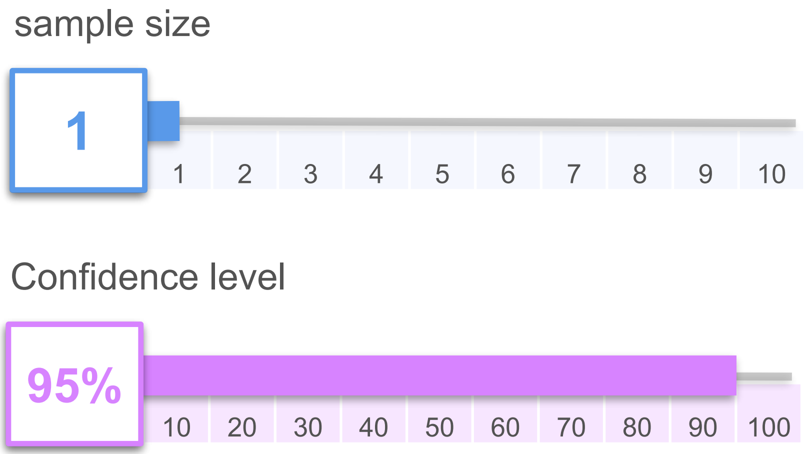


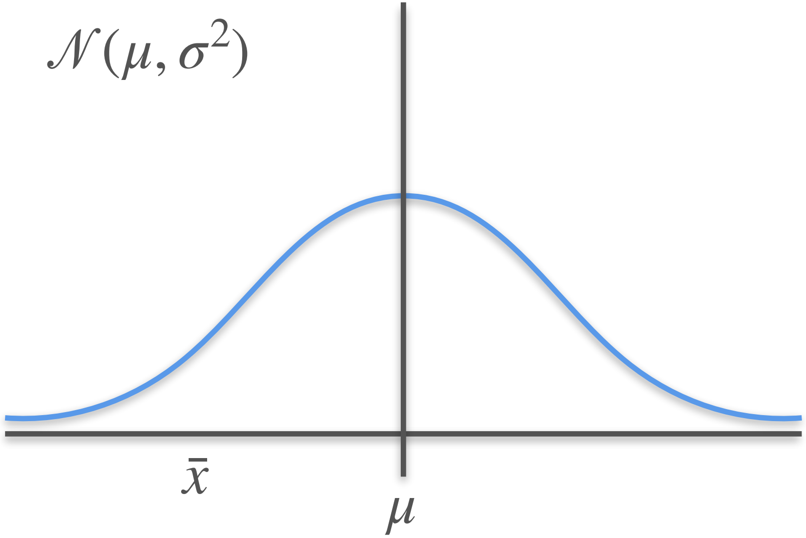
Confidence level



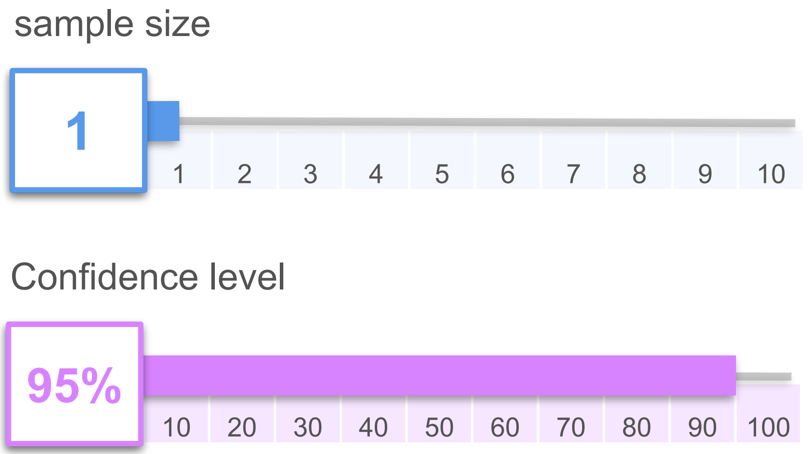


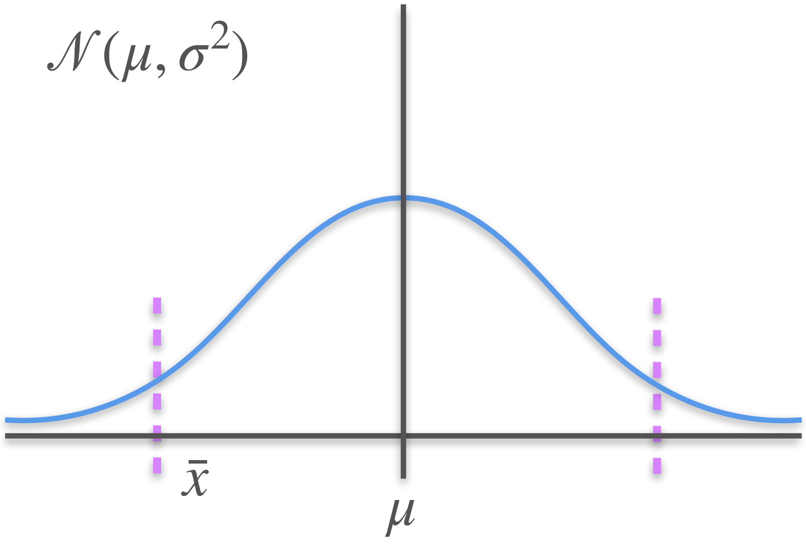
Effect of the Sample Size



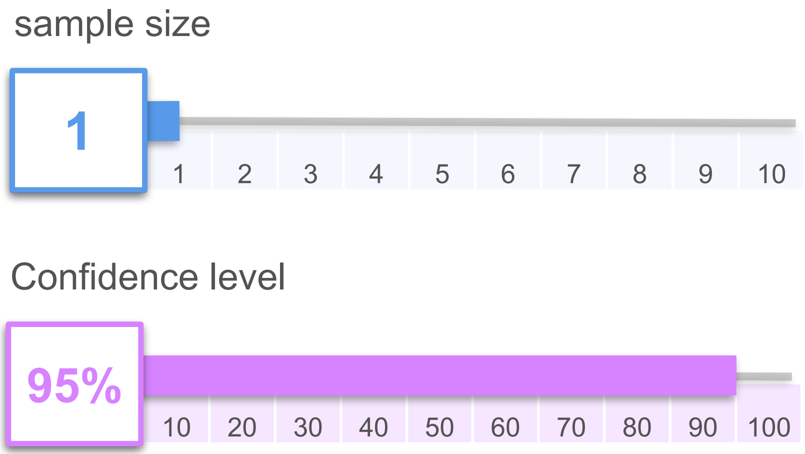


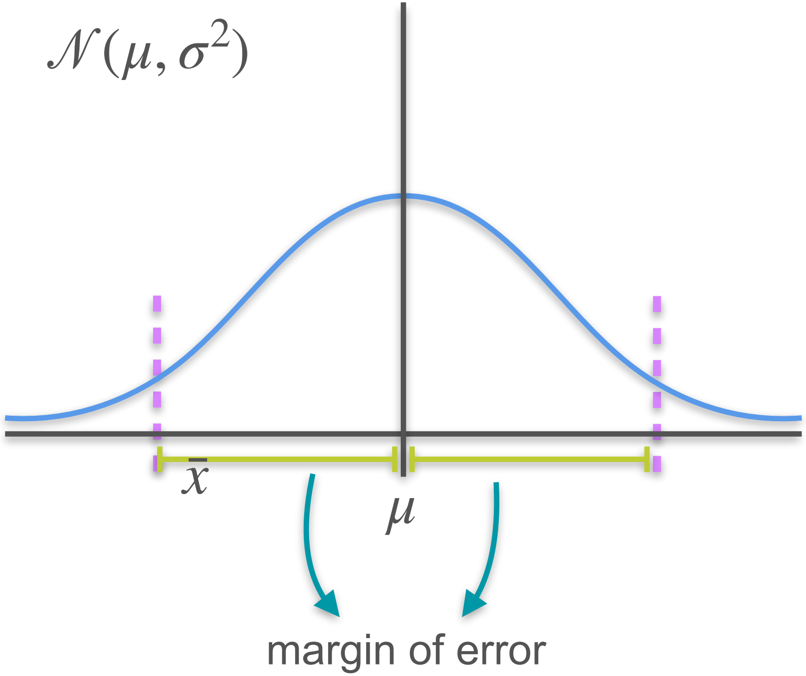
Effect of the Sample Size





Effect of the Sample Size





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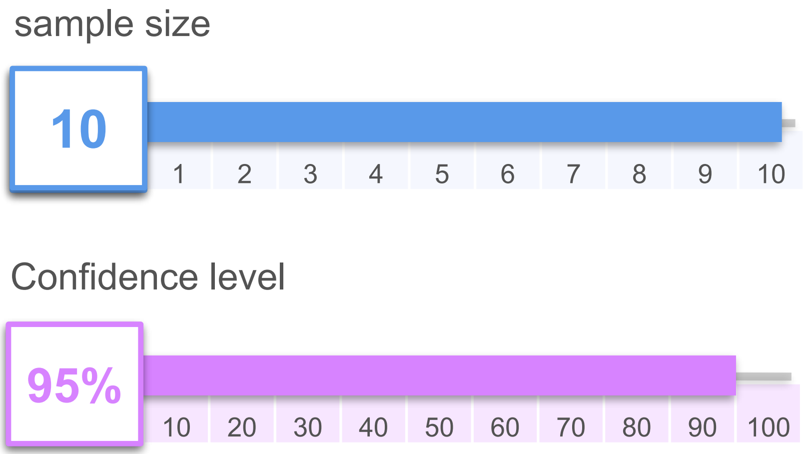
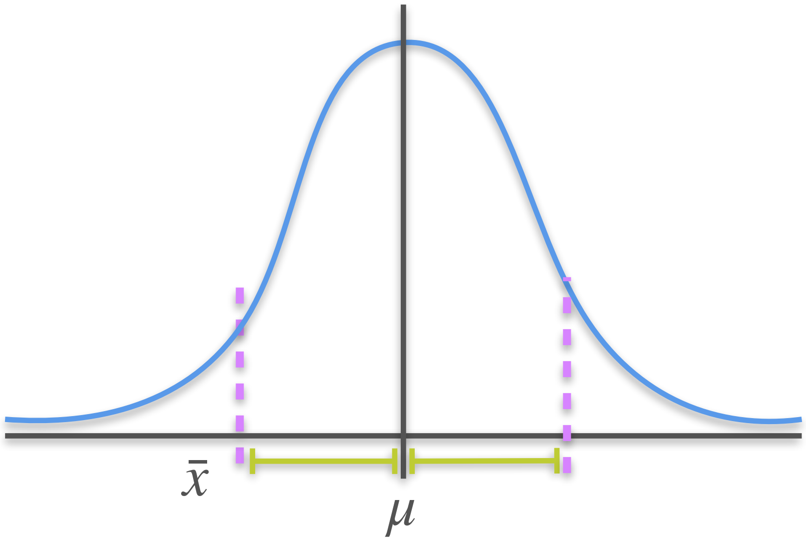




Effect of the Sample Size

sample size

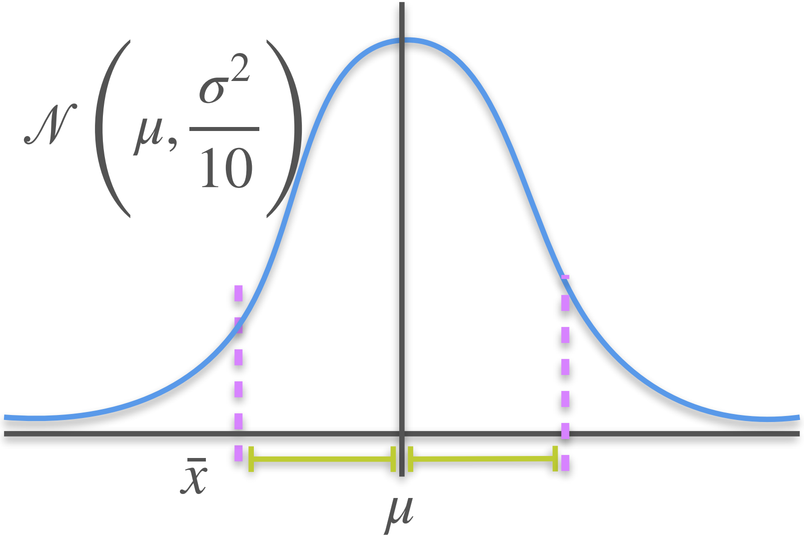
10

1 2 3 4 5 6 7 8 9 10

Confidence level

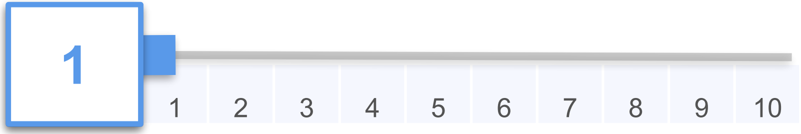
95%

10 20 30 40 50 60 70 80 90 100

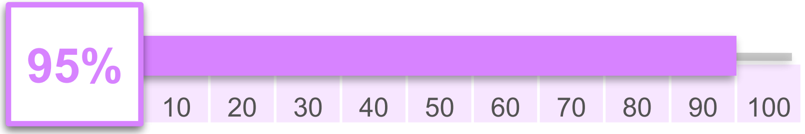


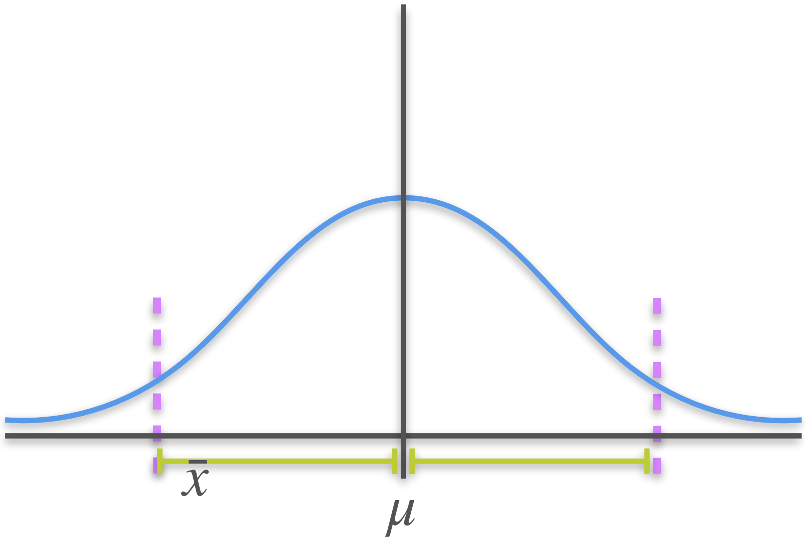
Effect of the Sample Size

sample size

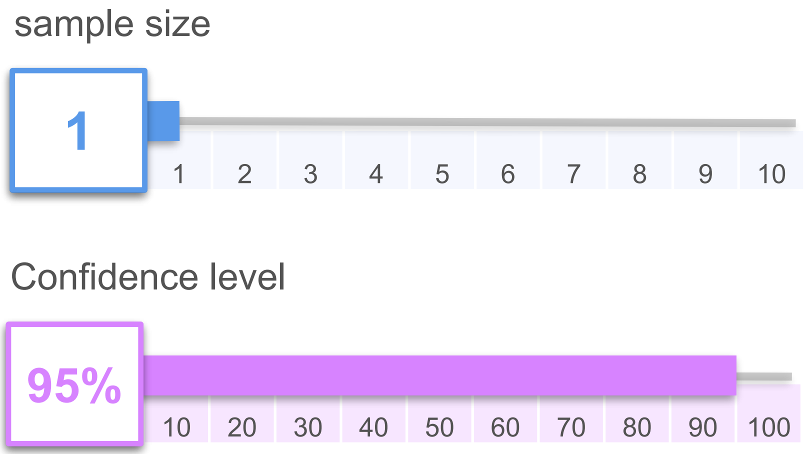


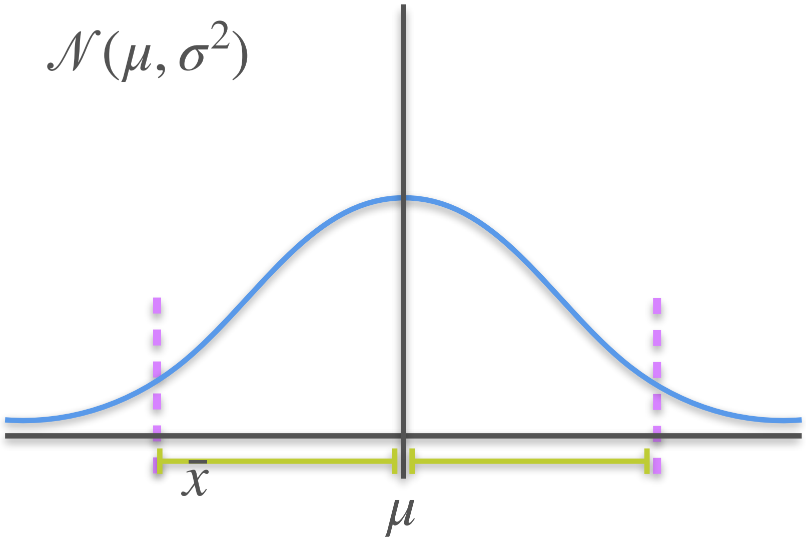
Confidence level





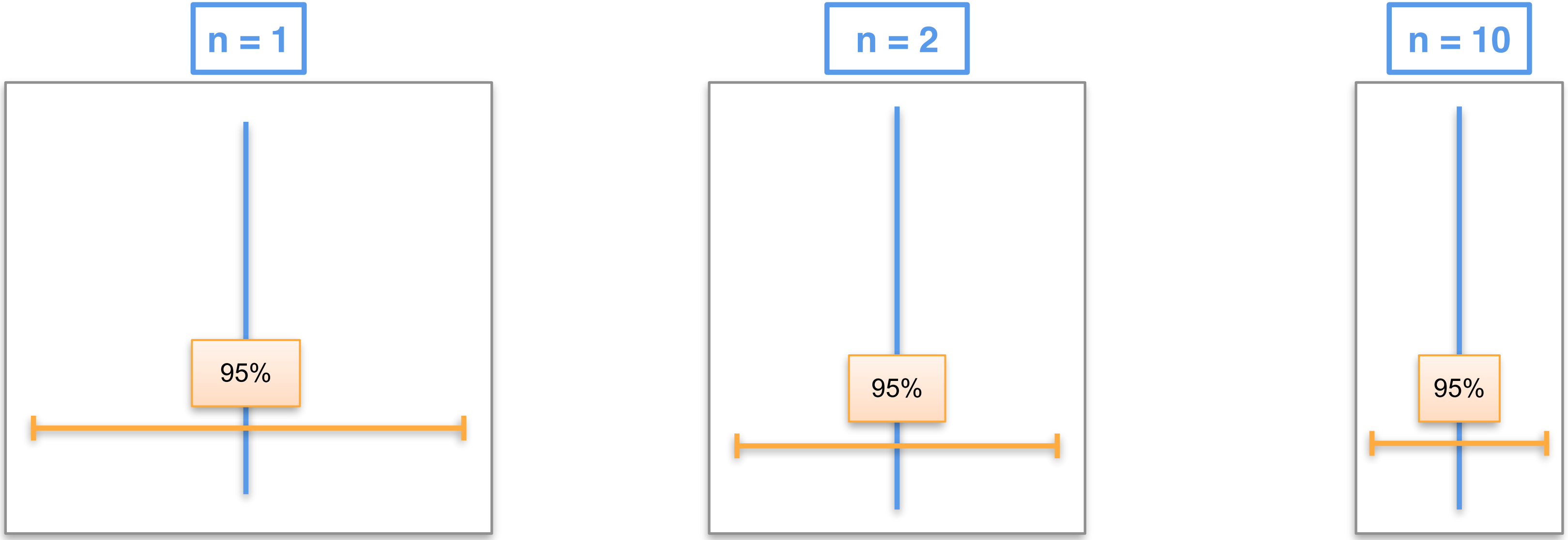
Effect of the Sample Size



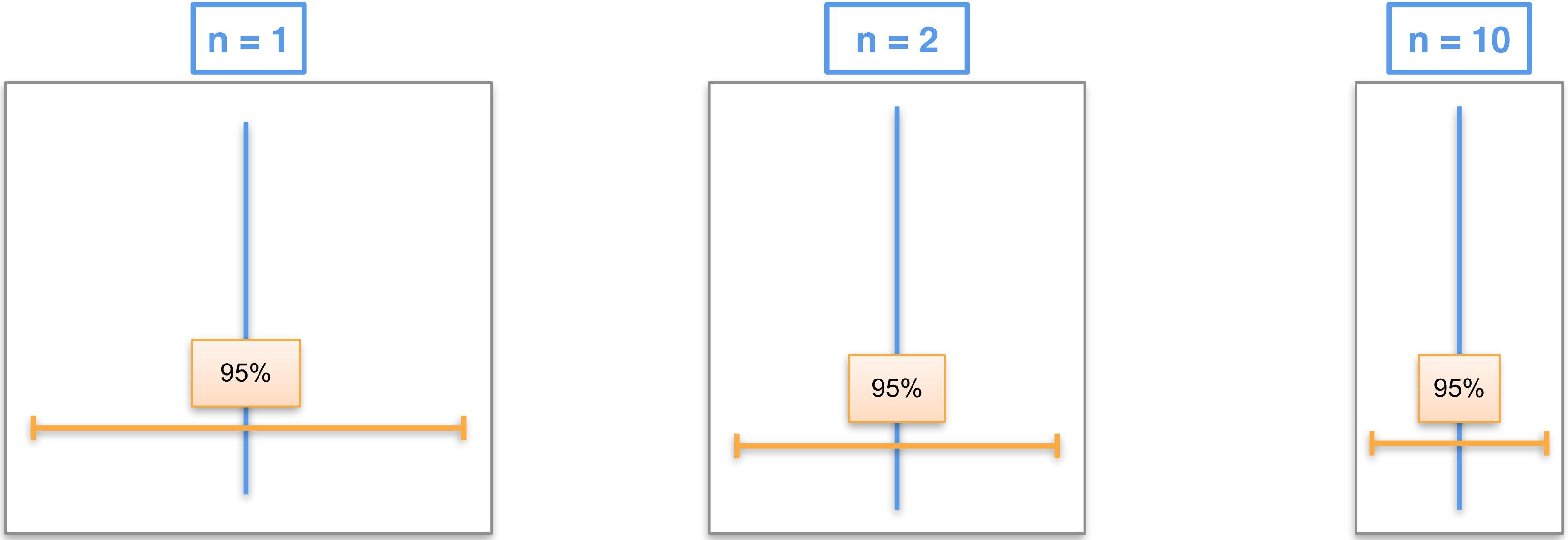


Effect of the Sample Size

Effect of the Sample Size



Effect of the Sample Size



As n increases, the confidence interval shrinks

Effect of the Confidence Level

Effect of the Confidence Level

$n = 1$

Effect of the Confidence Level

$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

Effect of the Confidence Level

$$n = 1$$

$\boxed{95\%}$

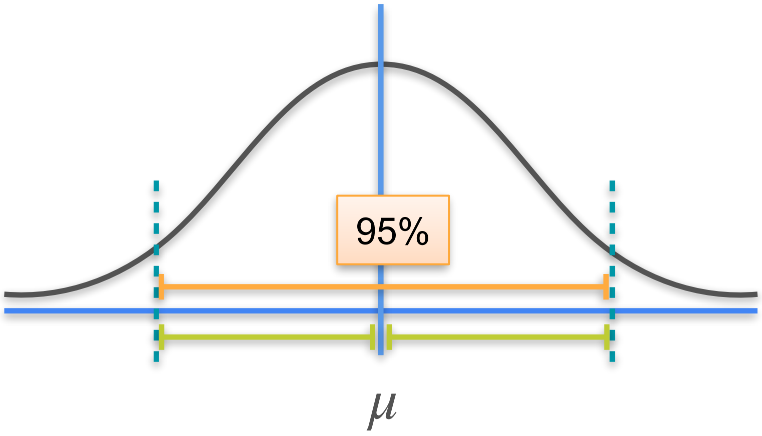
$$\mathcal{N}(\mu, \sigma^2)$$

Effect of the Confidence Level

$$n = 1$$

95%

$$\mathcal{N}(\mu, \sigma^2)$$

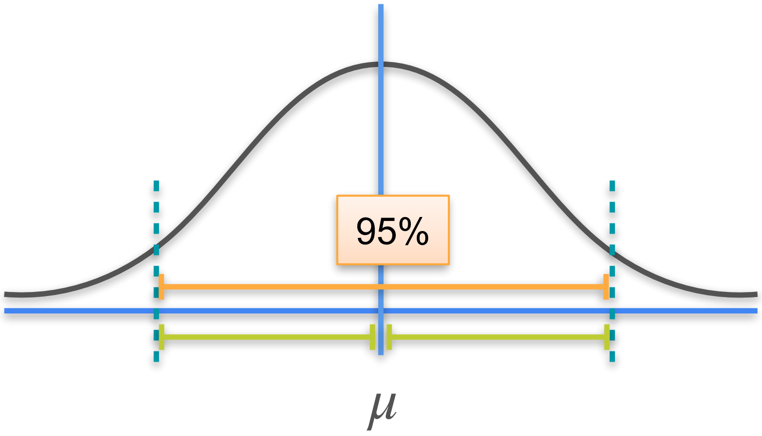


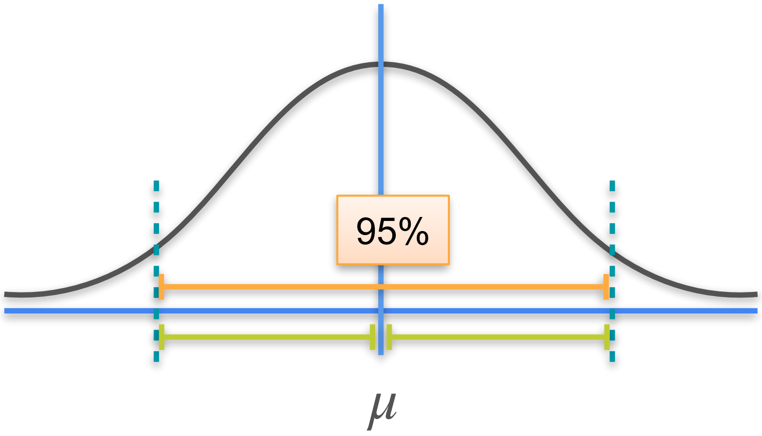
Effect of the Confidence Level

$n = 1$

95%

$\mathcal{N}(\mu, \sigma^2)$



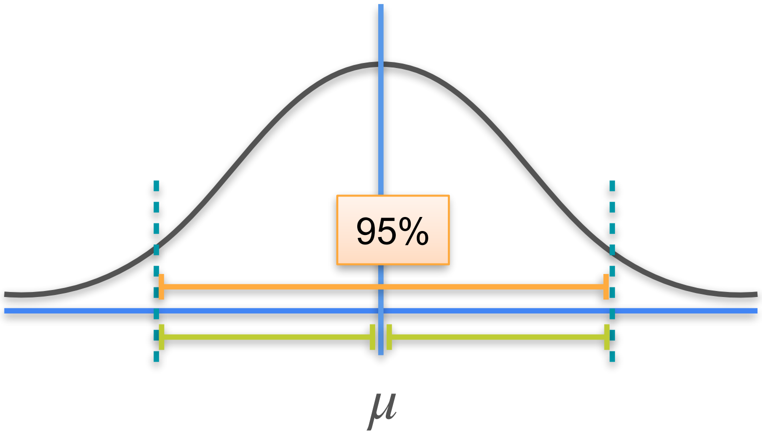


Effect of the Confidence Level

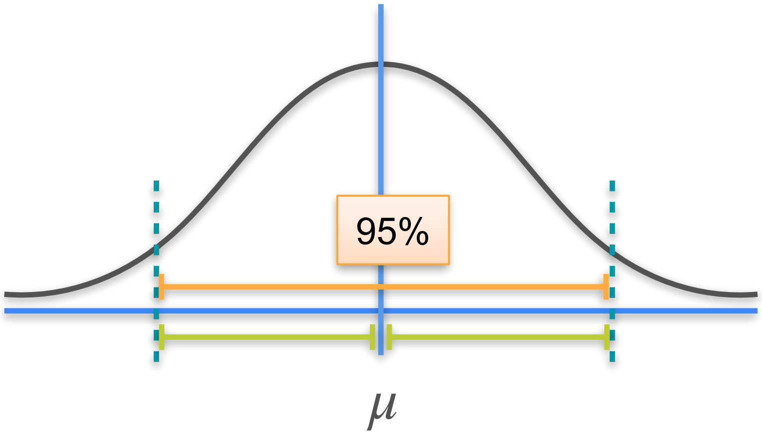
$n = 1$

$\mathcal{N}(\mu, \sigma^2)$

95%



70%

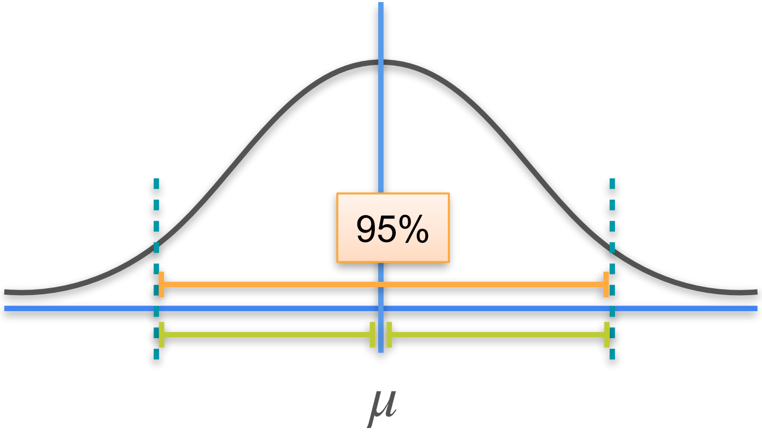


Effect of the Confidence Level

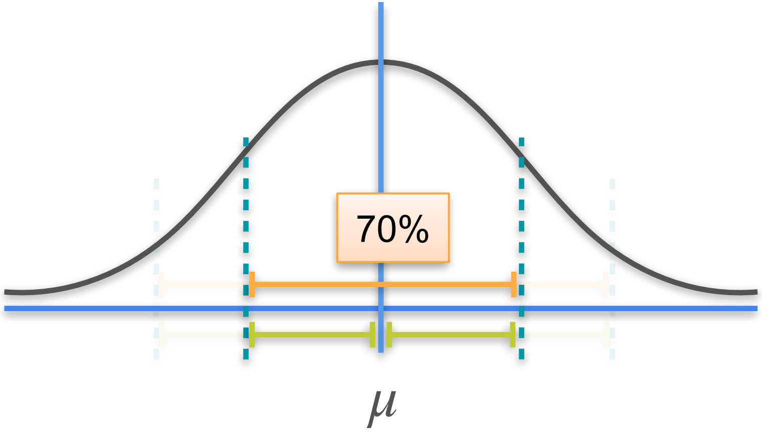
$n = 1$

$\mathcal{N}(\mu, \sigma^2)$

95%



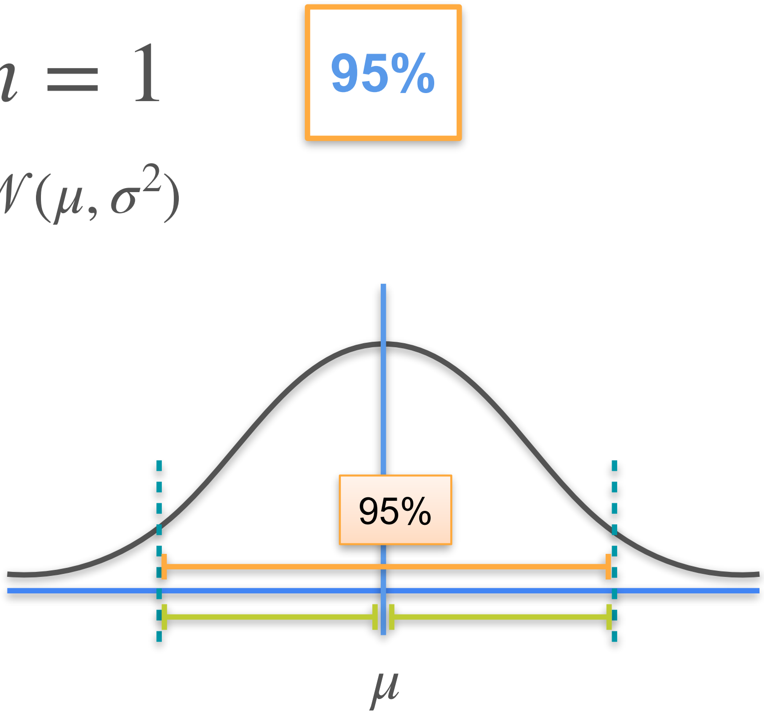
70%

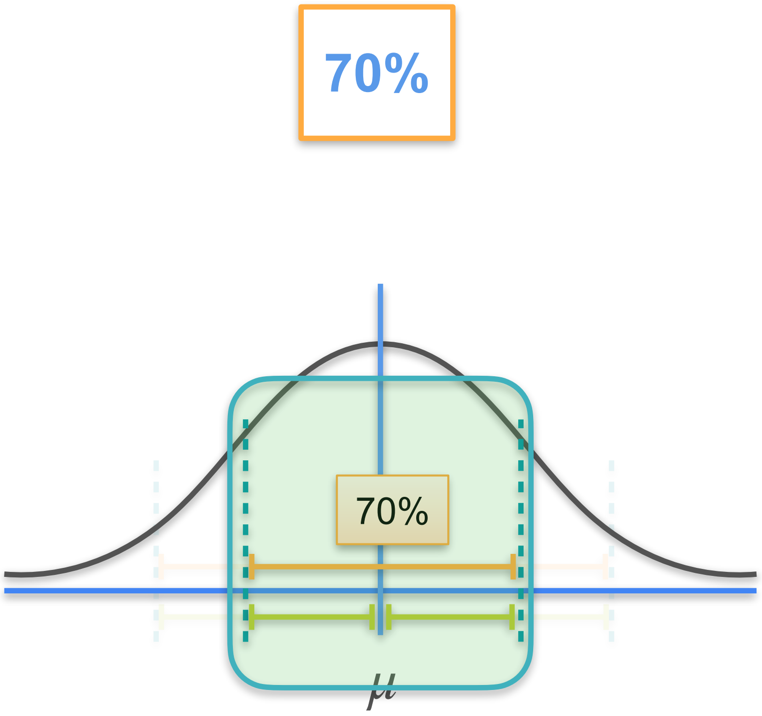


Effect of the Confidence Level

$n = 1$

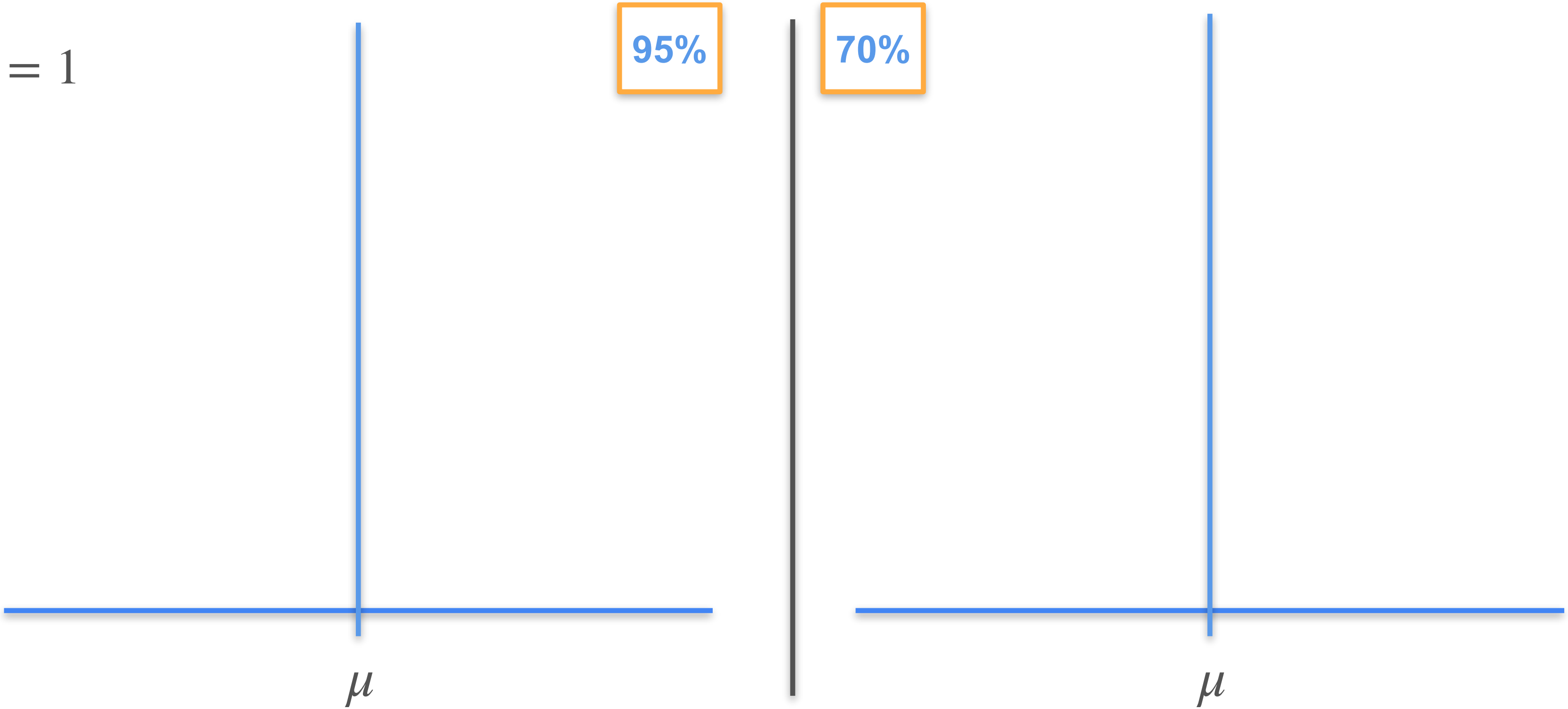
$\mathcal{N}(\mu, \sigma^2)$



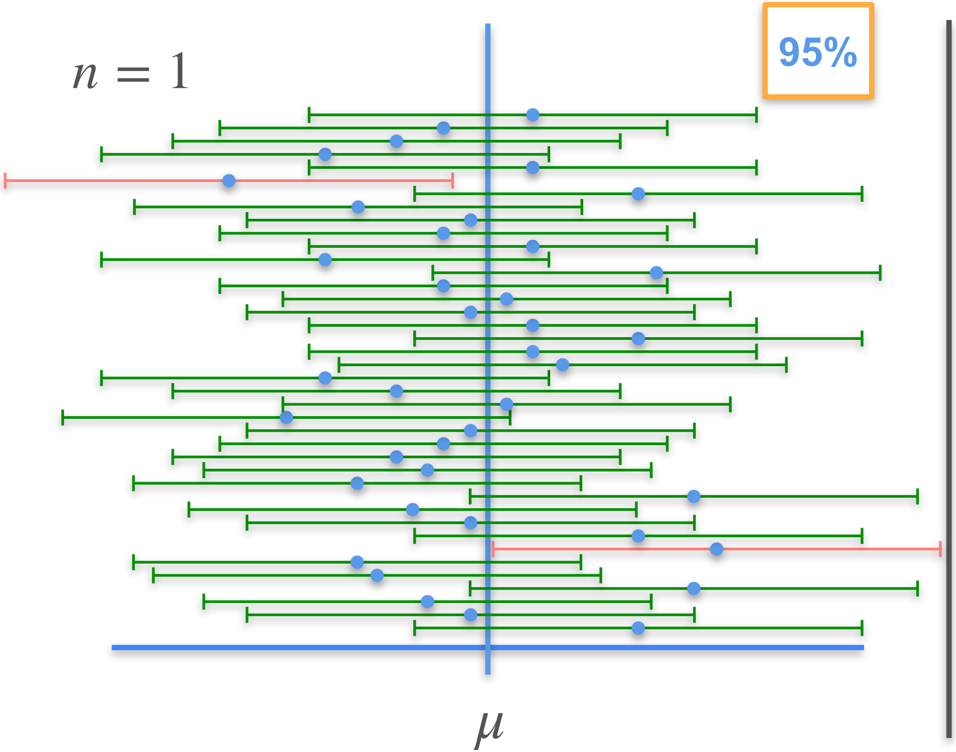


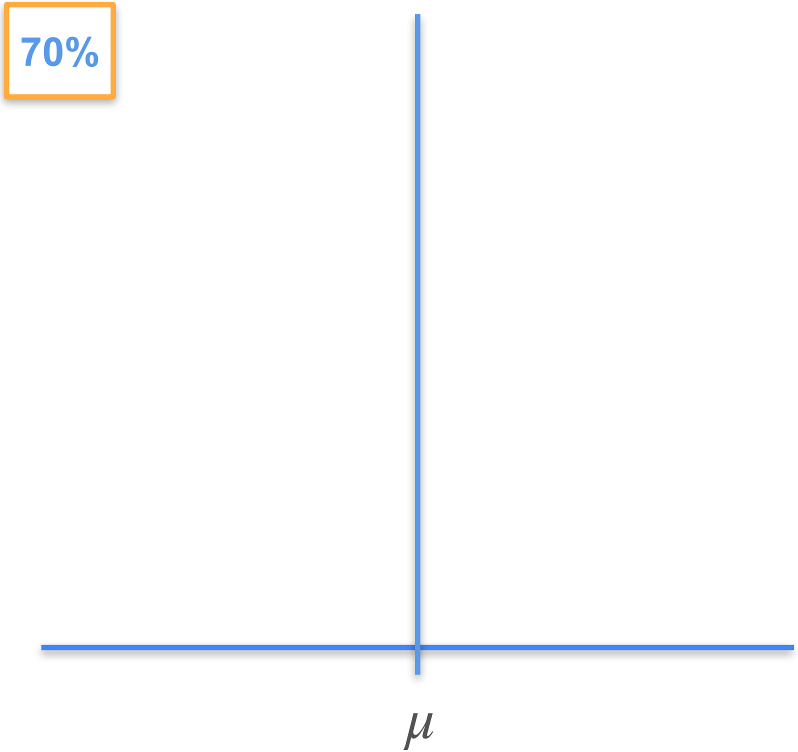
Effect of the Confidence Level

$n = 1$

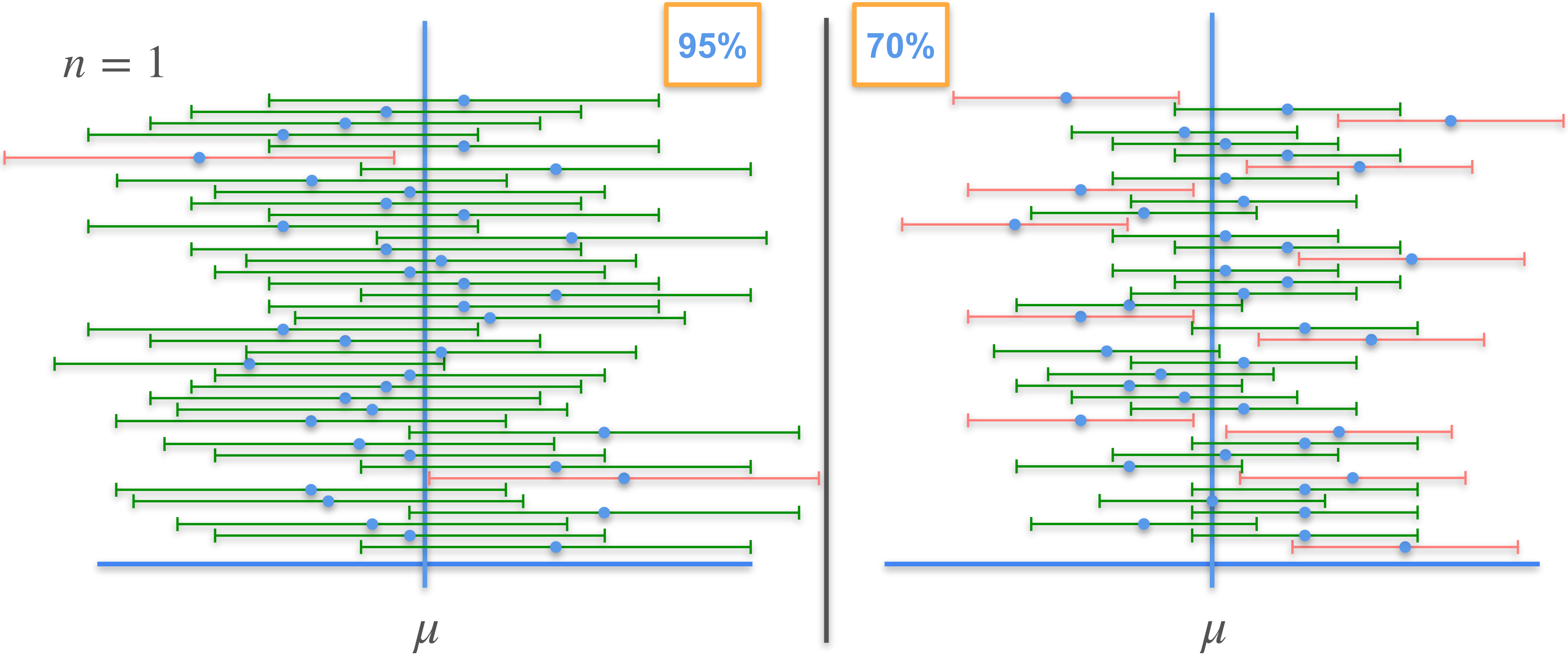


Effect of the Confidence Level

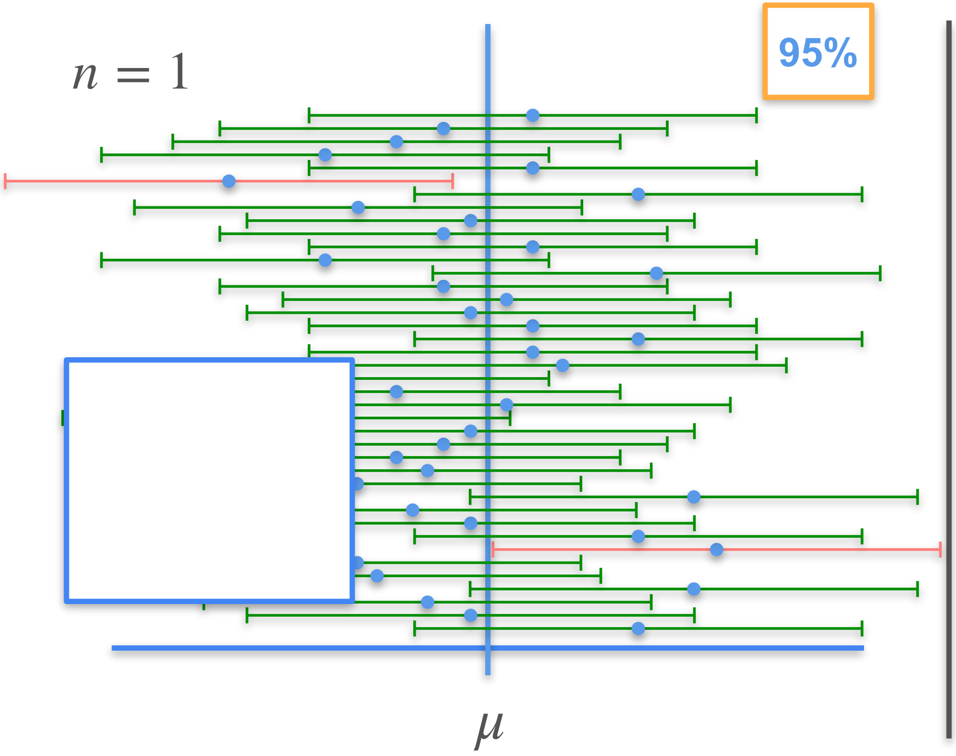


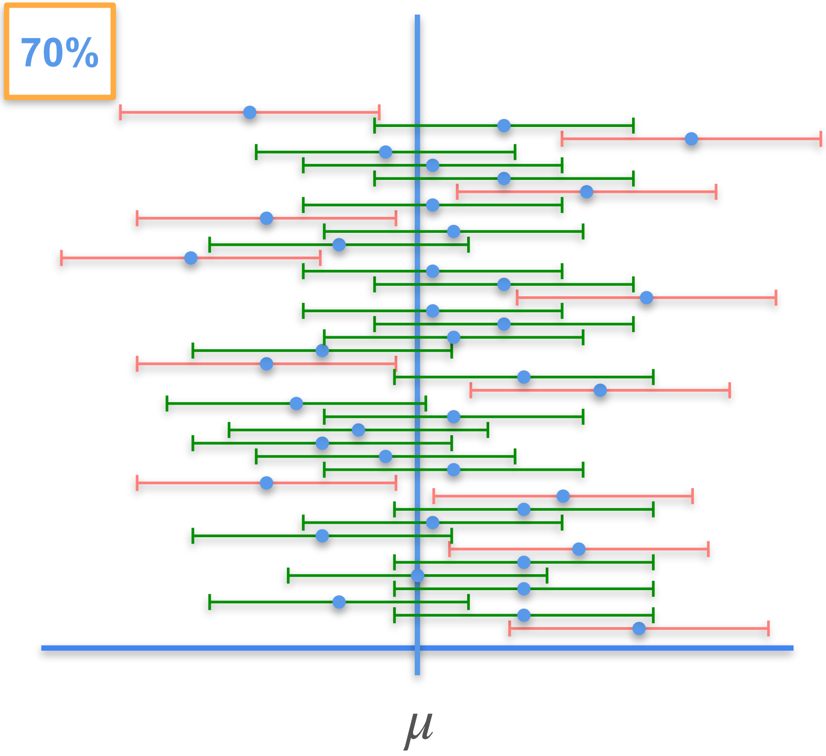


Effect of the Confidence Level

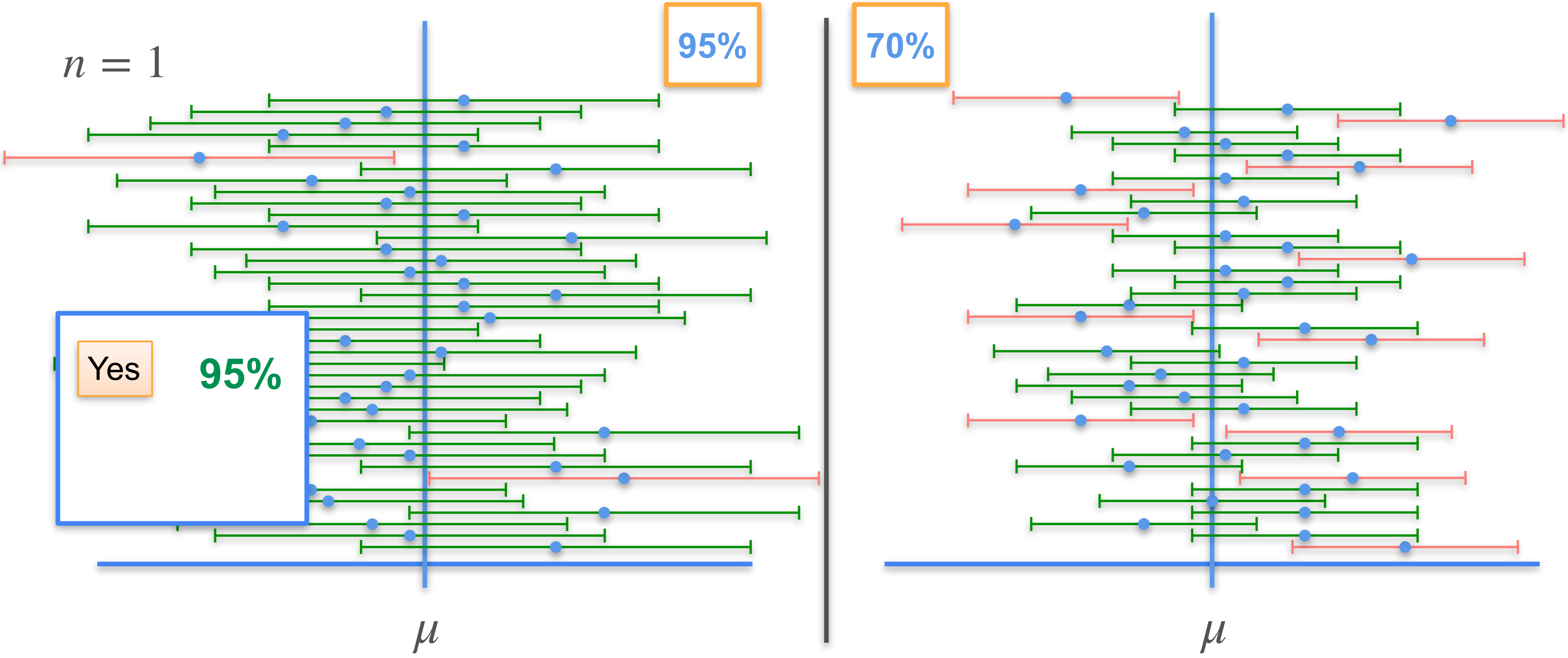


Effect of the Confidence Level





Effect of the Confidence Level



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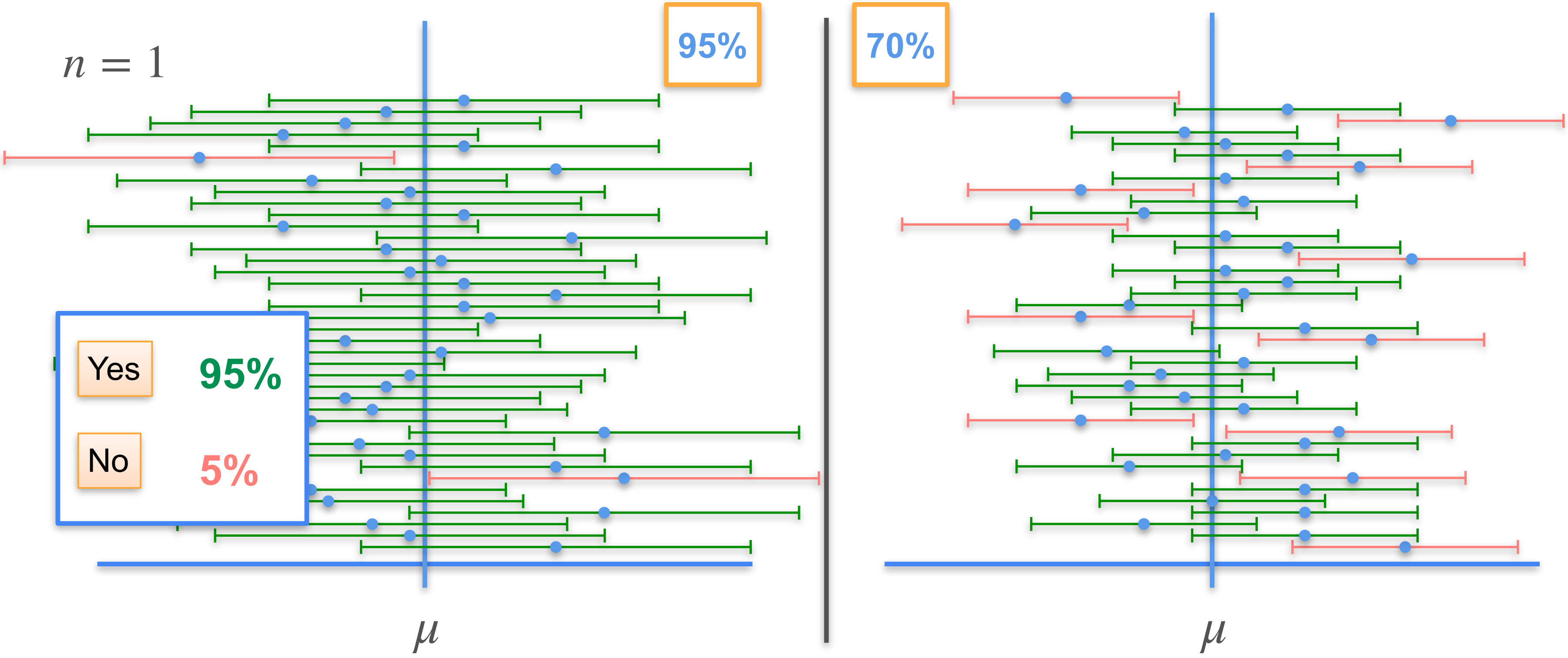

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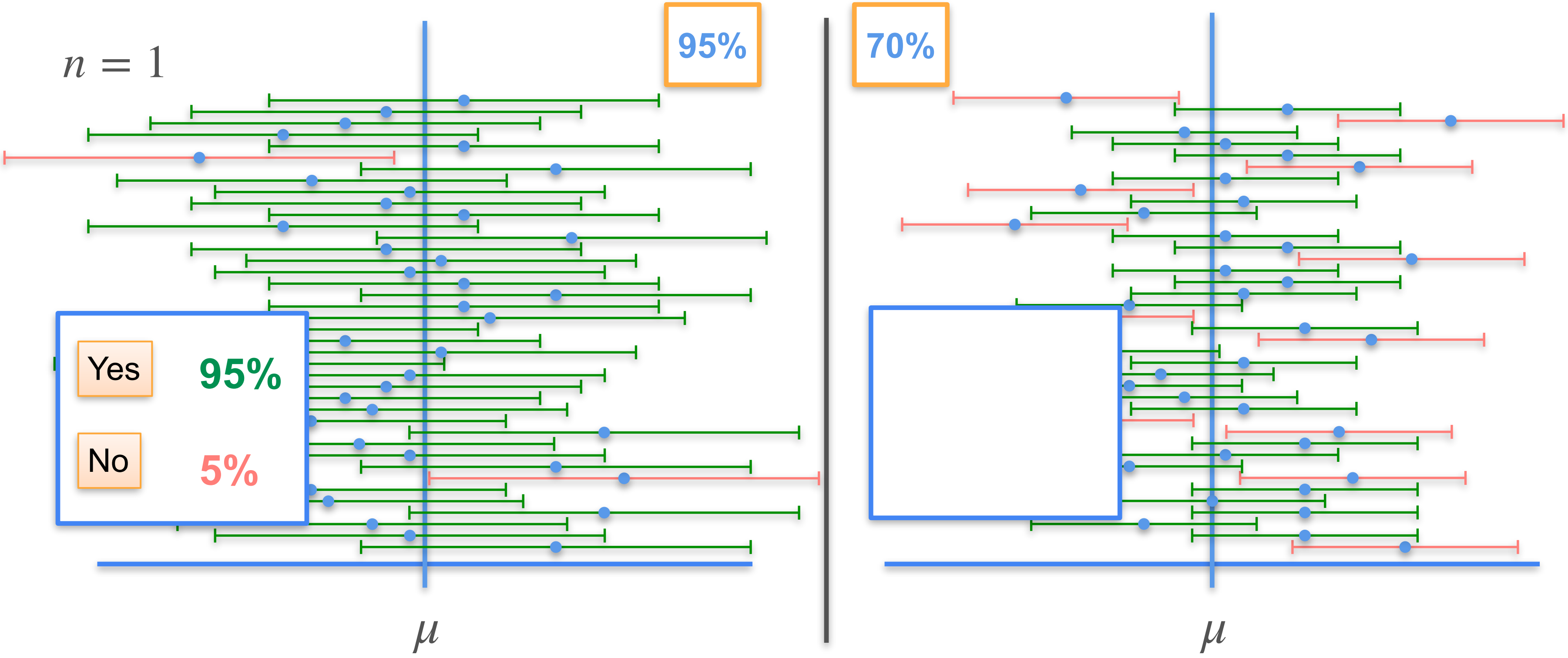

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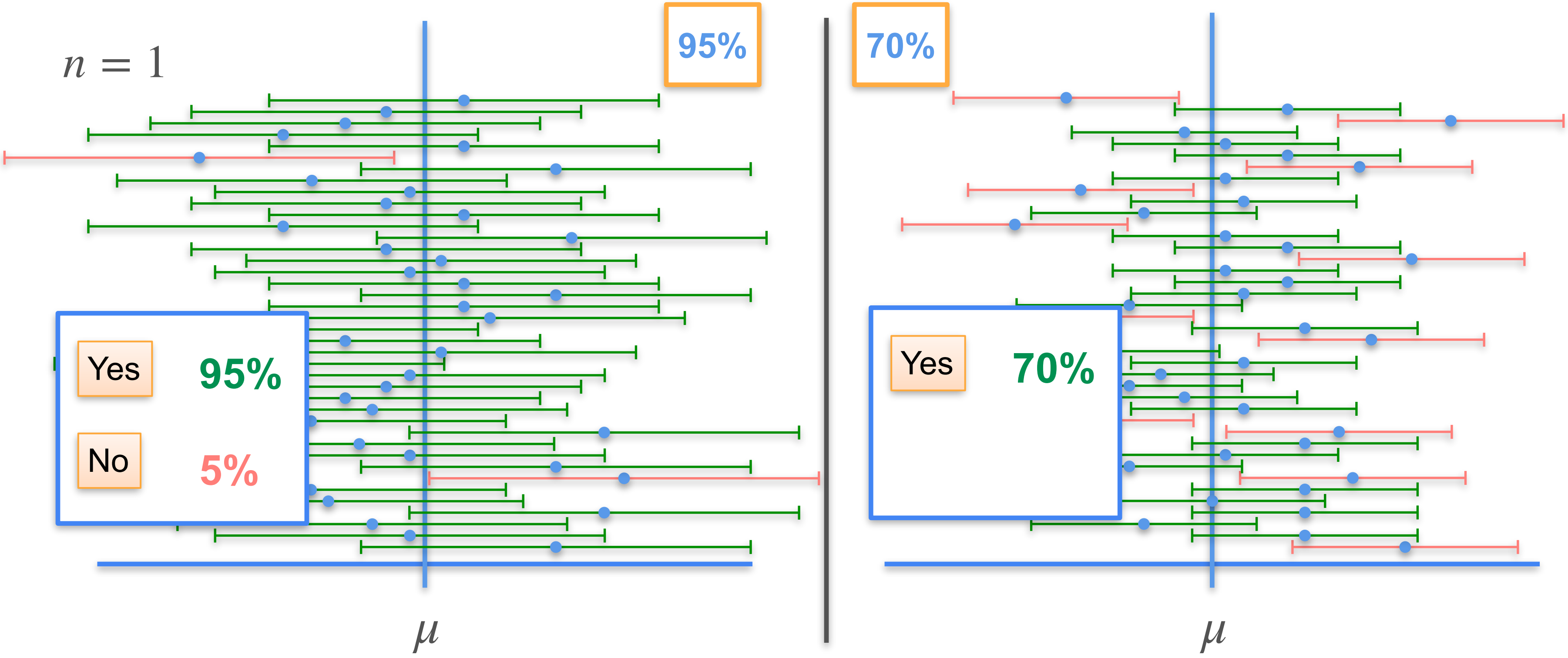

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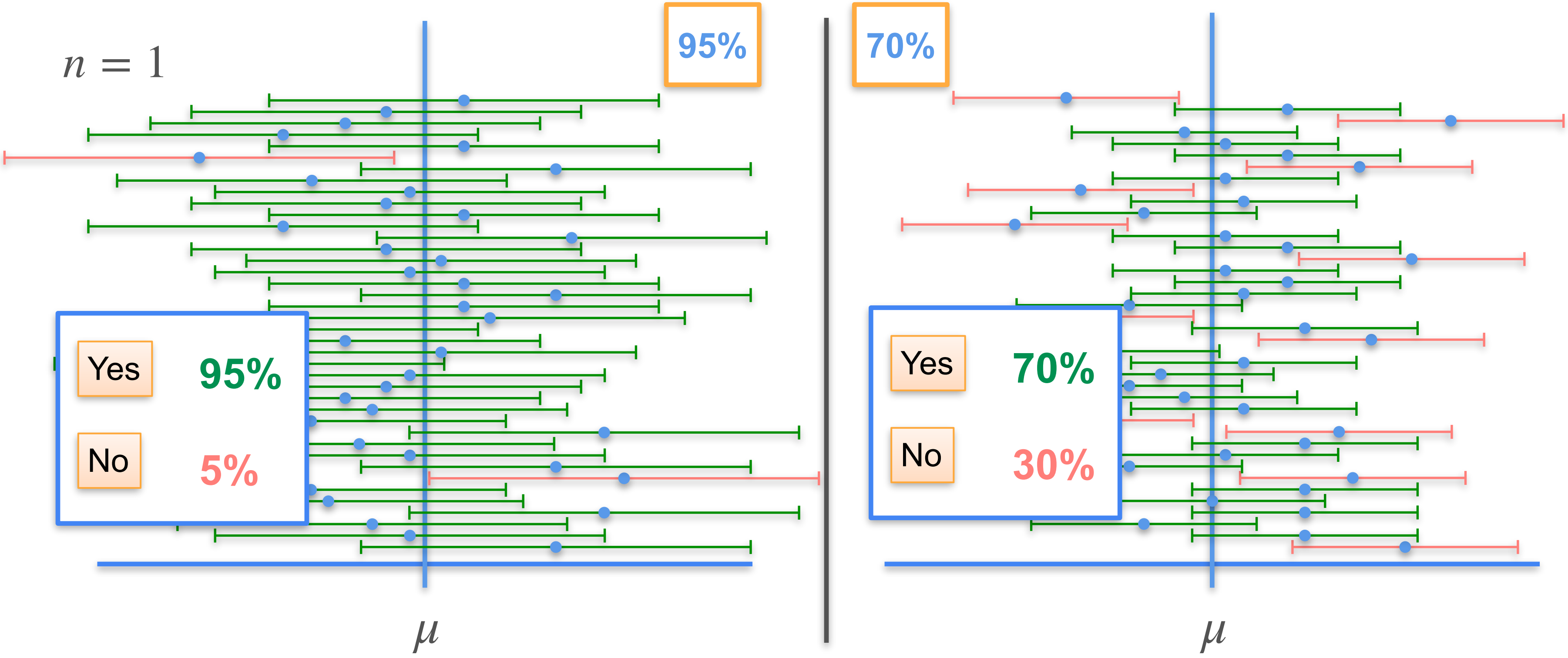

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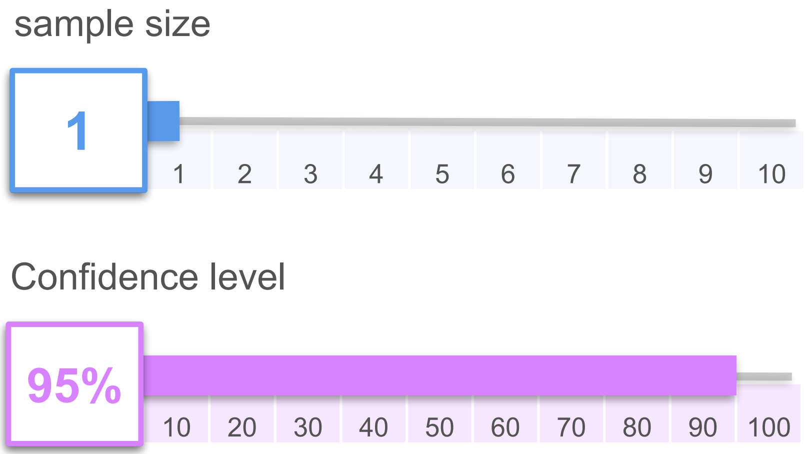

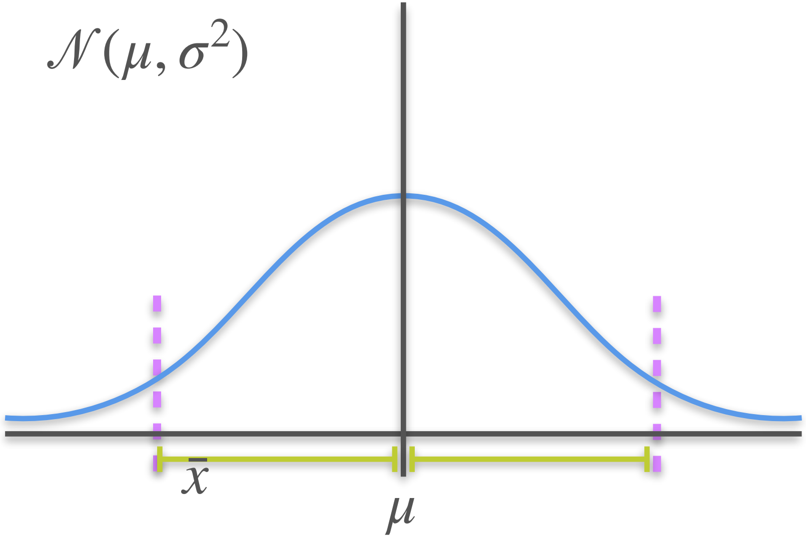

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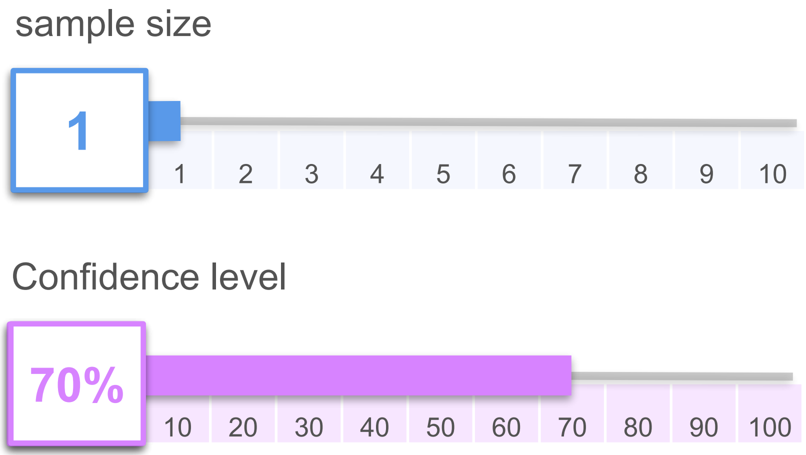

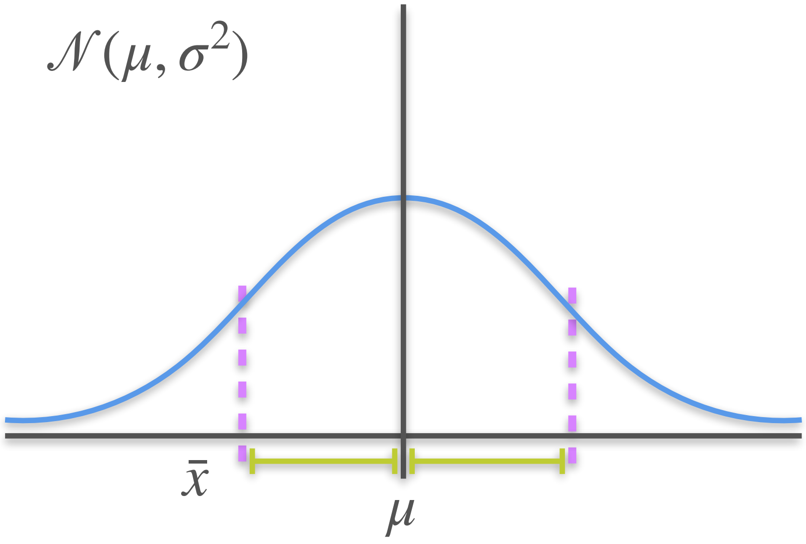

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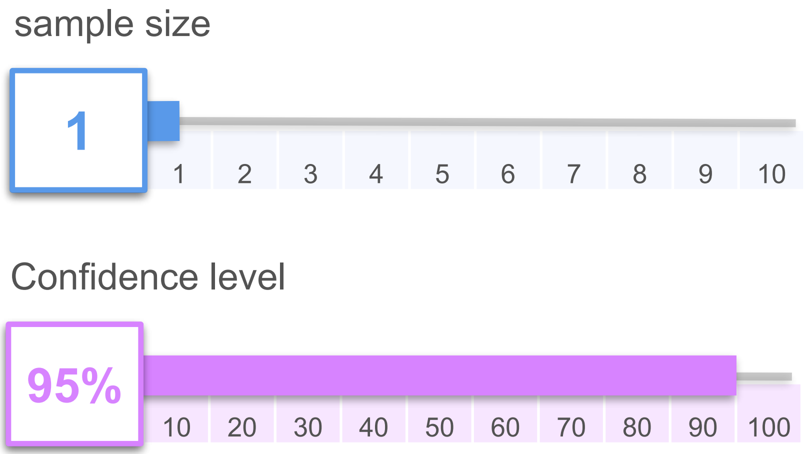

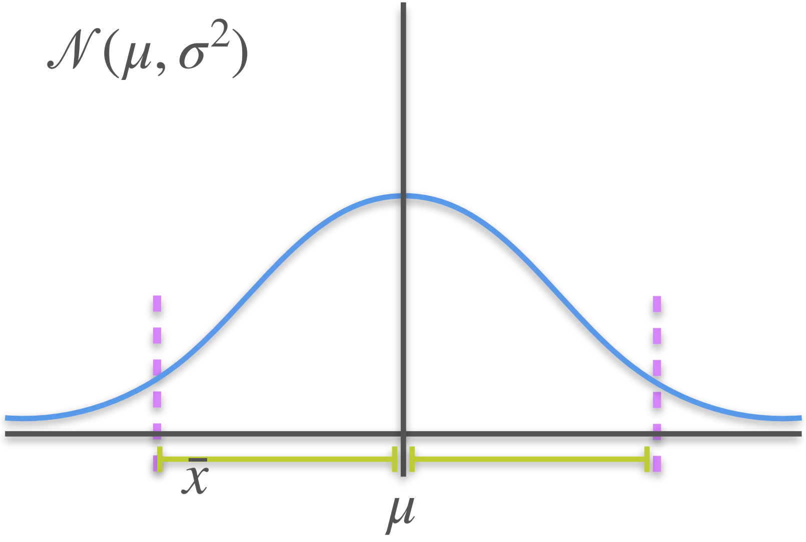

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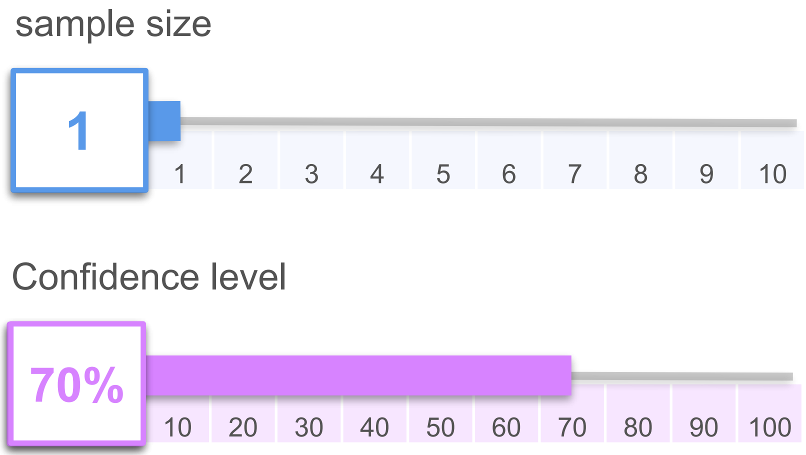
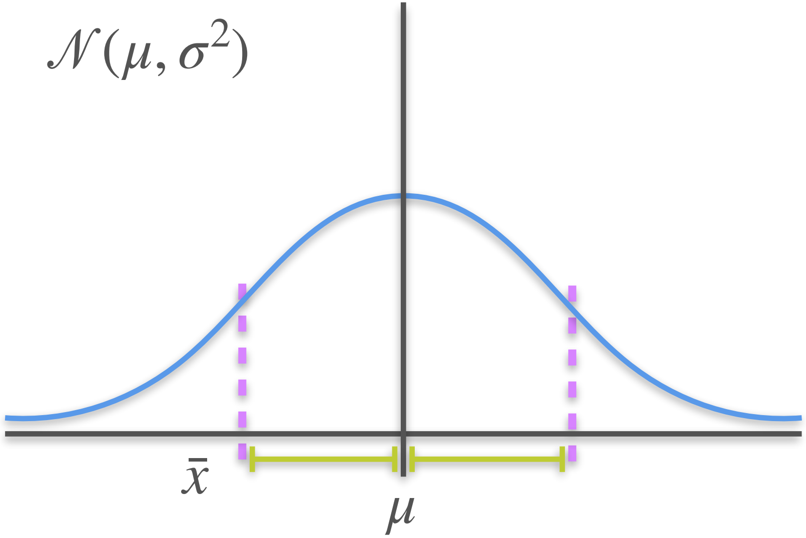


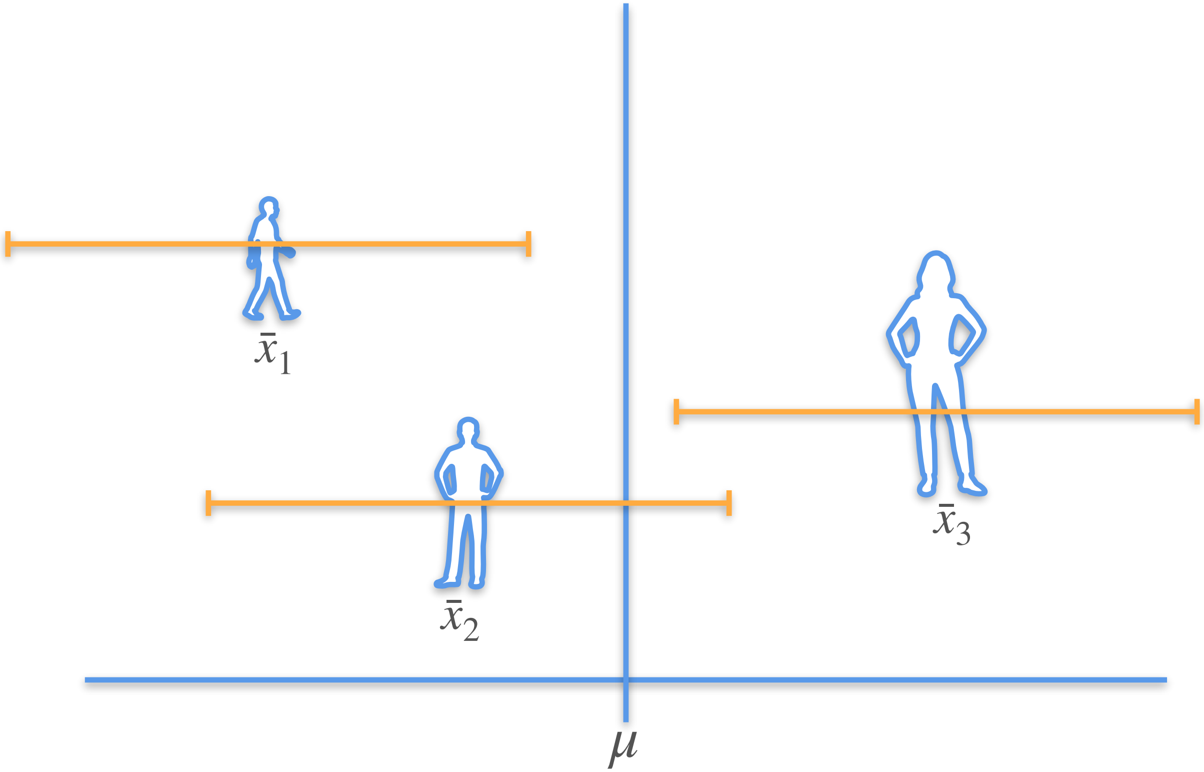


DeepLearning.AI

Confidence Interval

Margin of Error

Margin of Error - Introduction



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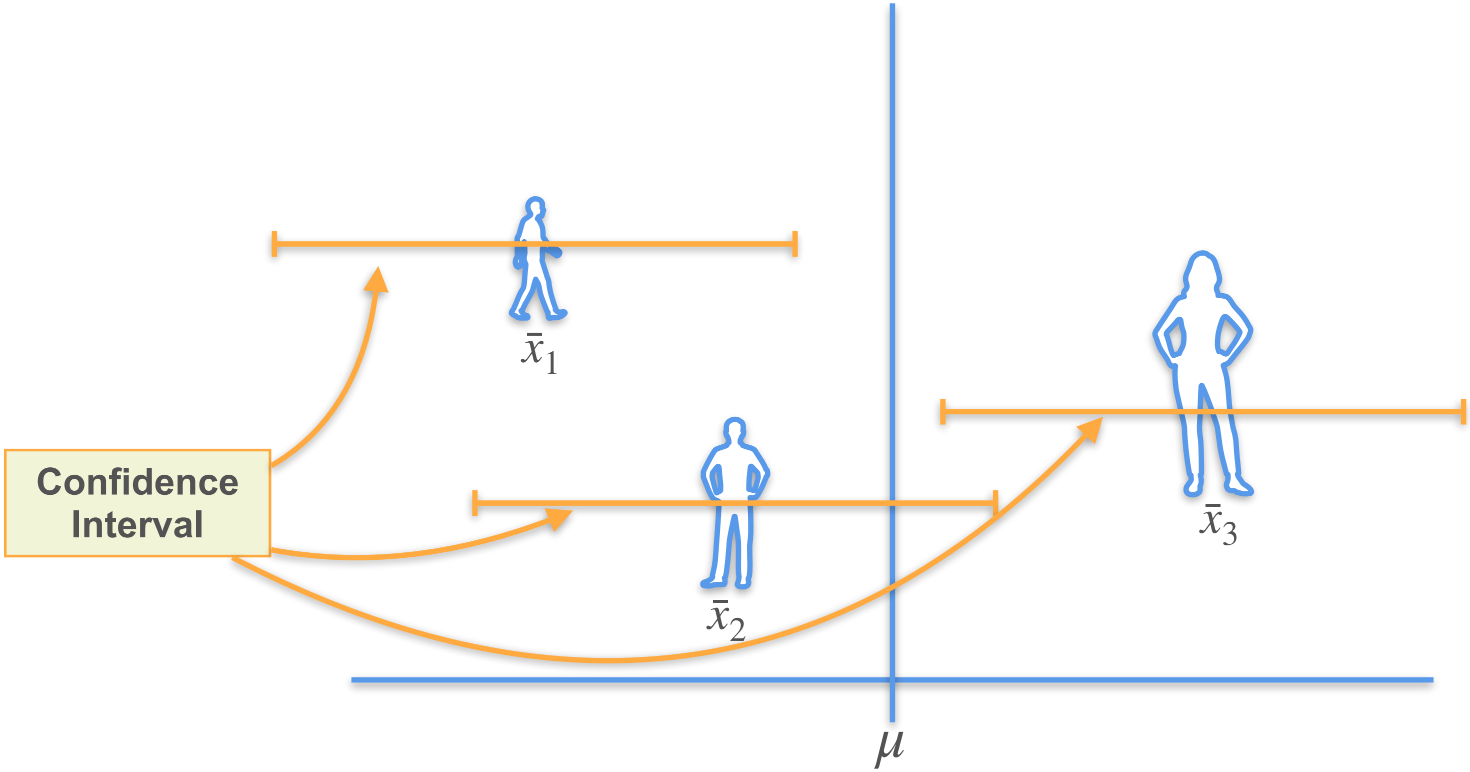

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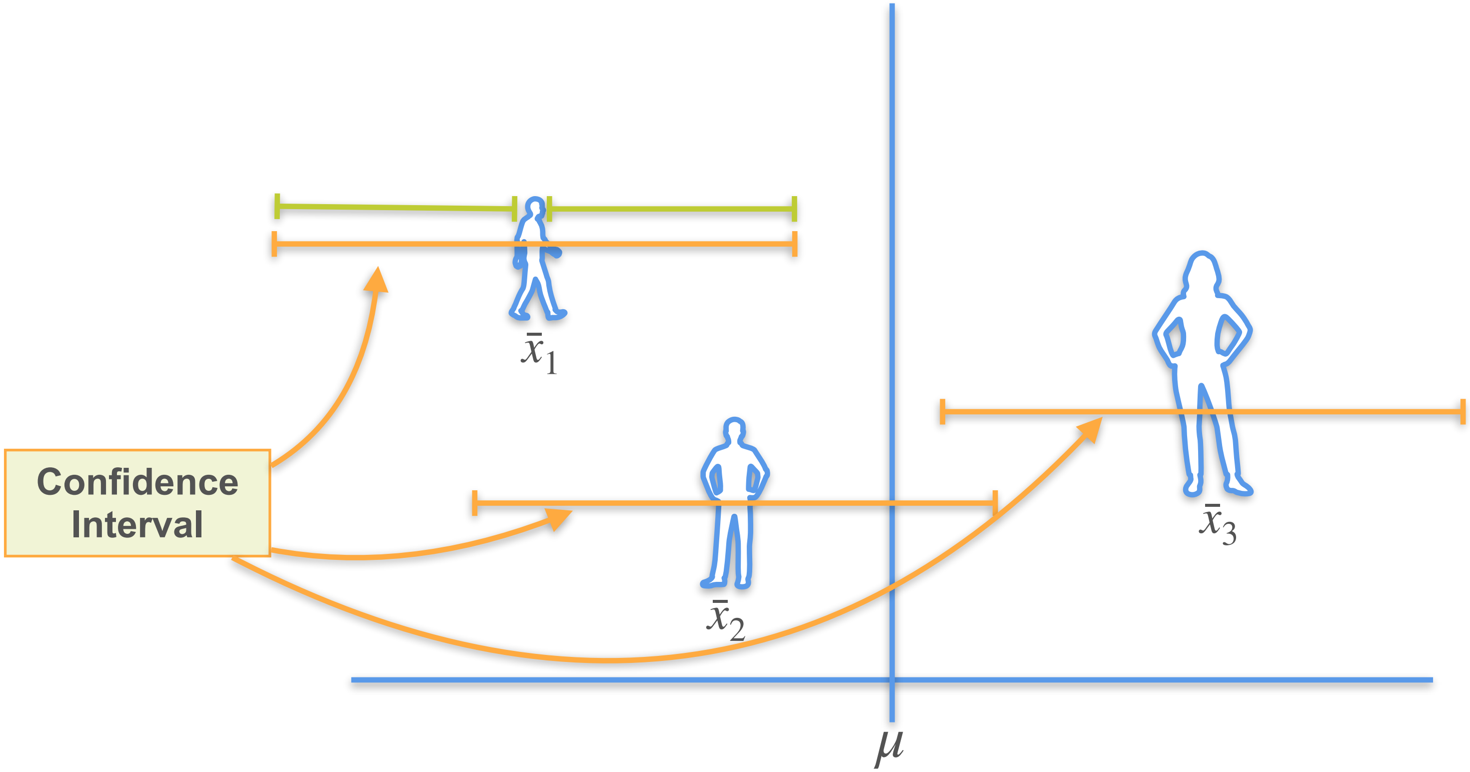

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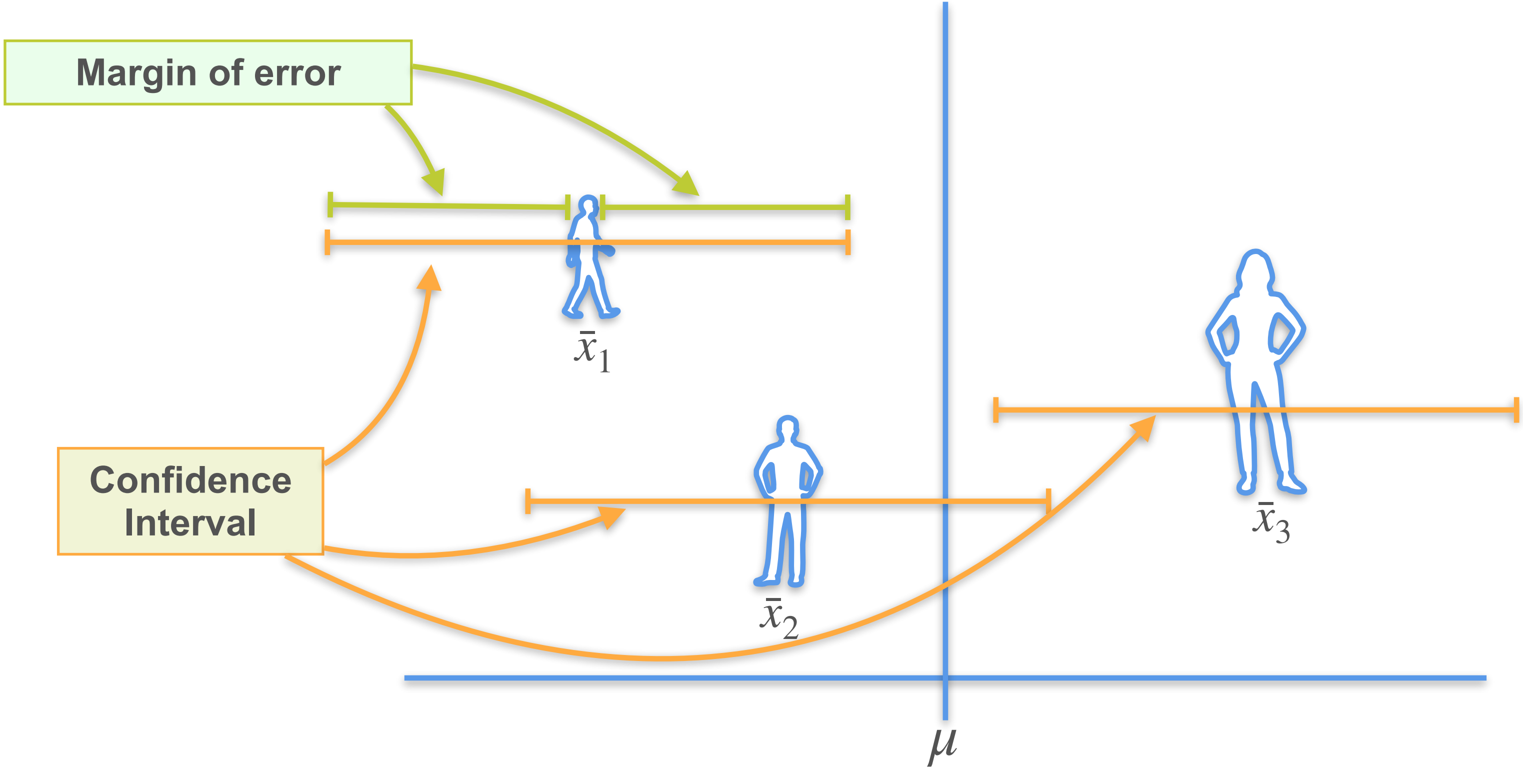

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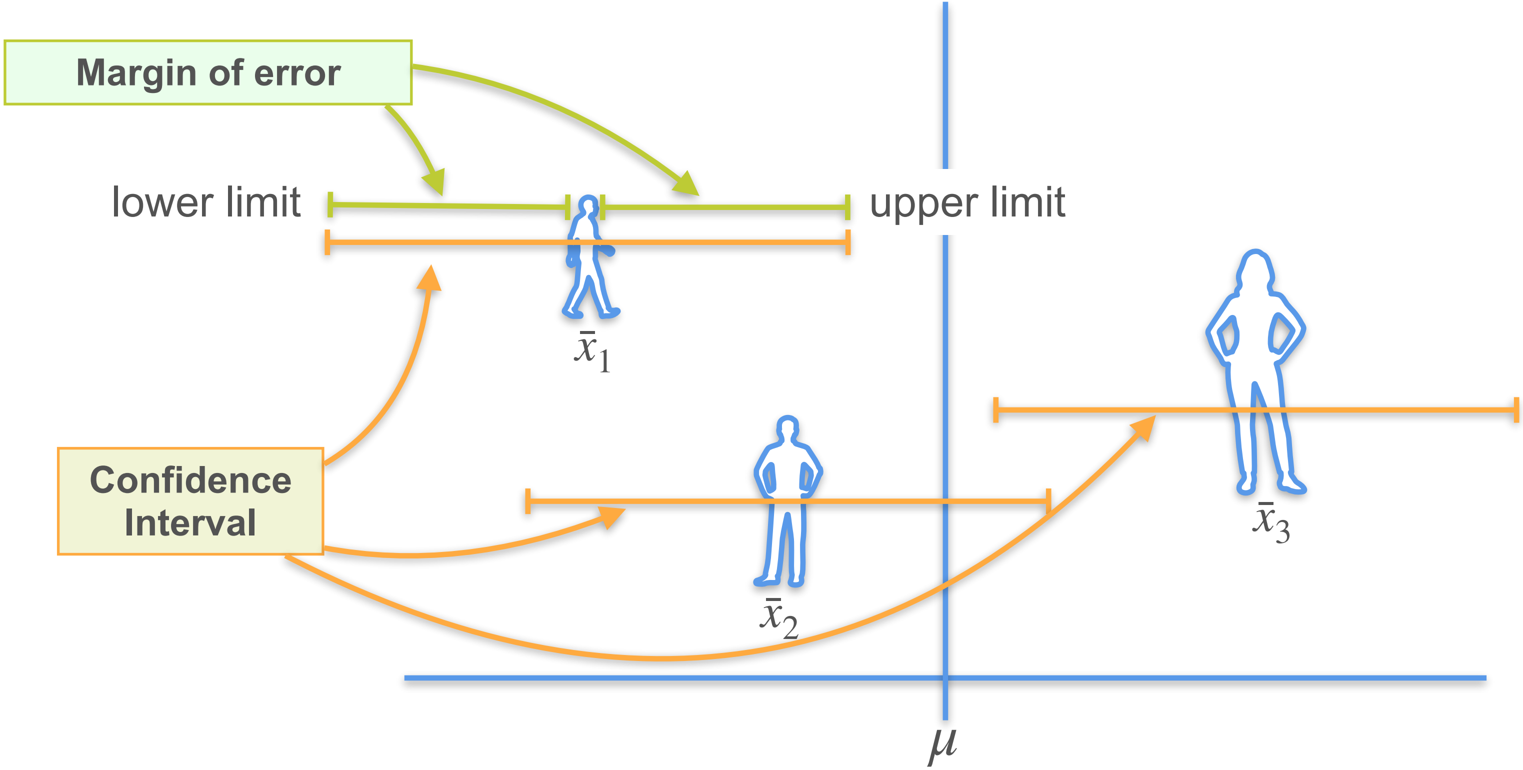

Margin of Error

Margin of Error

Taking multiple samples of size n

sampling distribution of the sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

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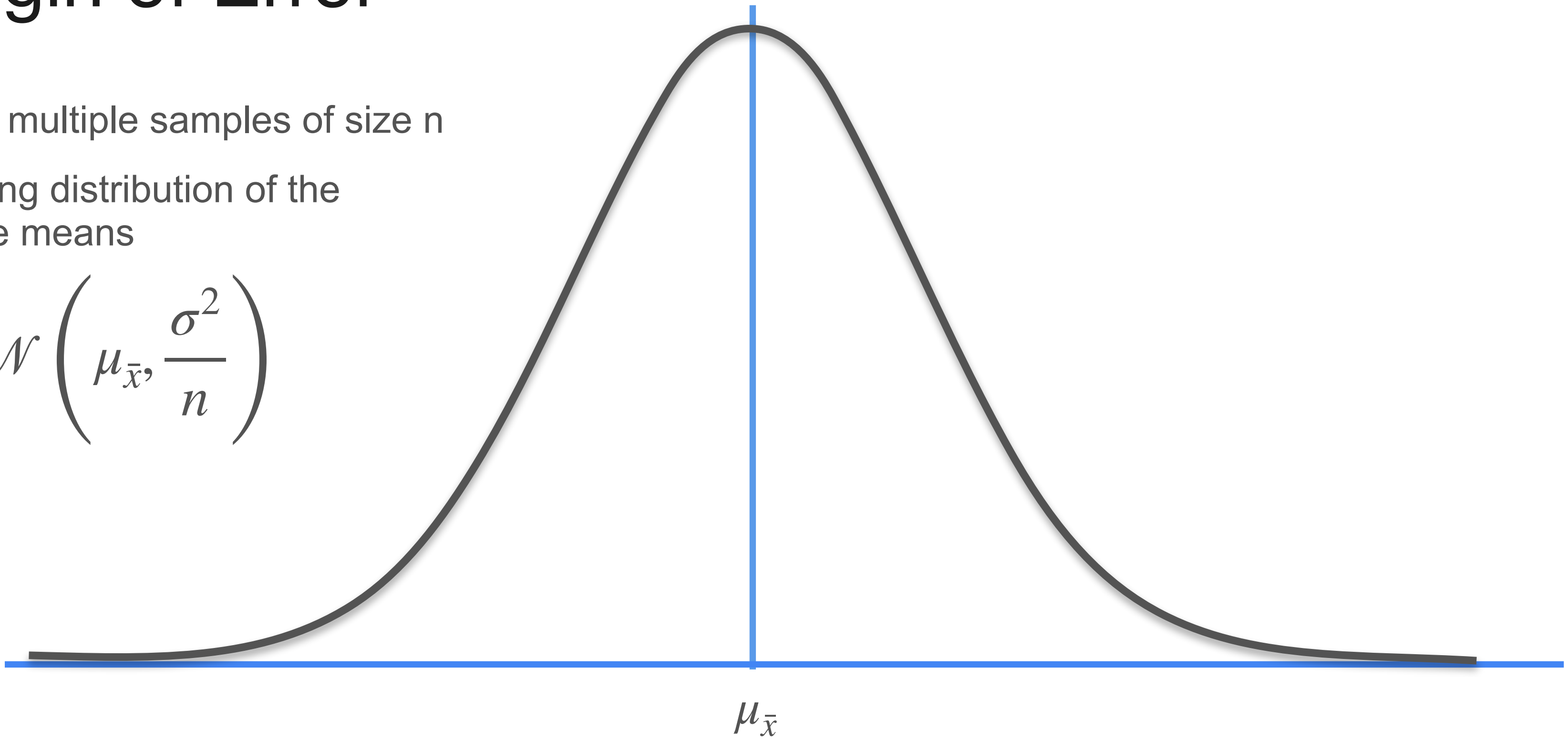

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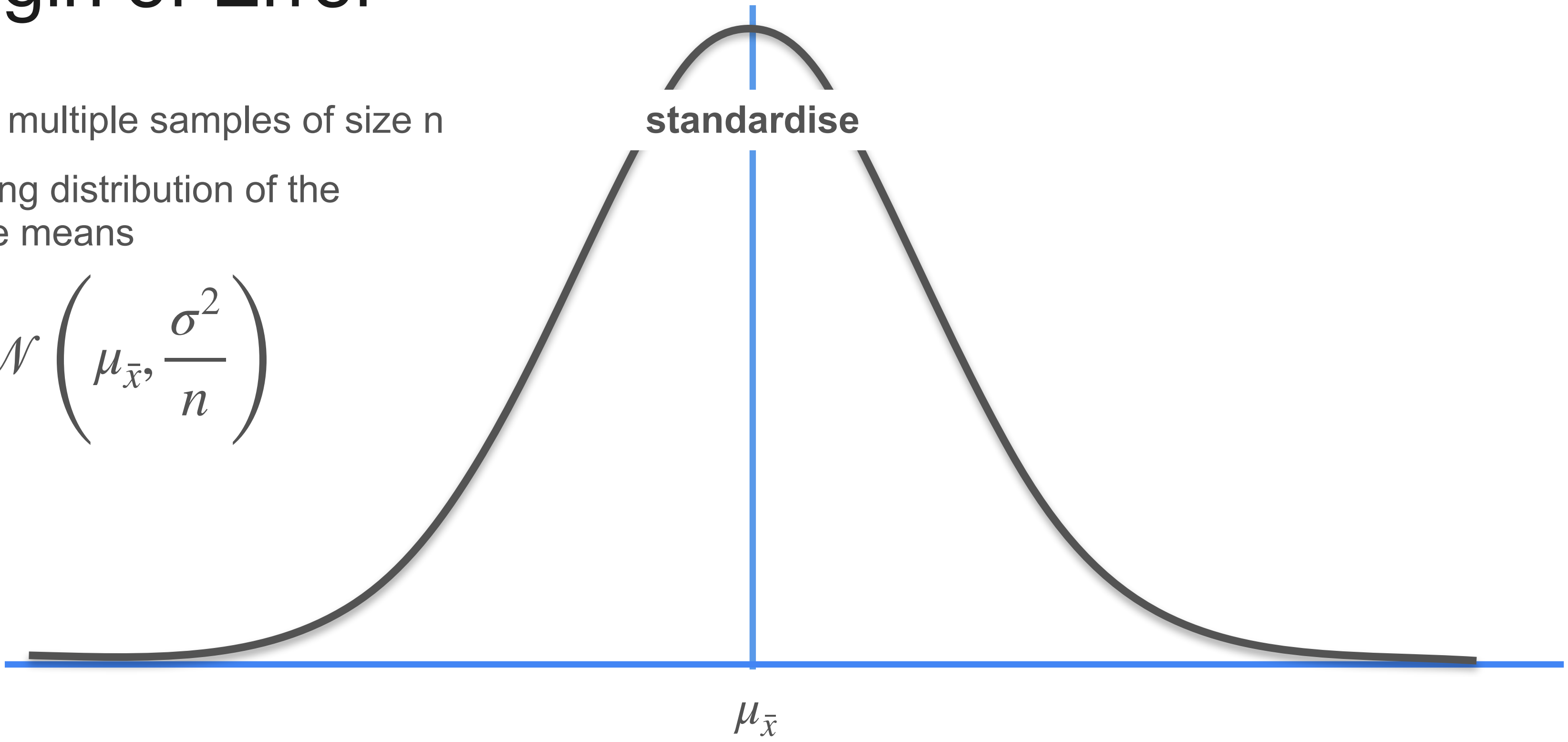

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Taking multiple samples of size n

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$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

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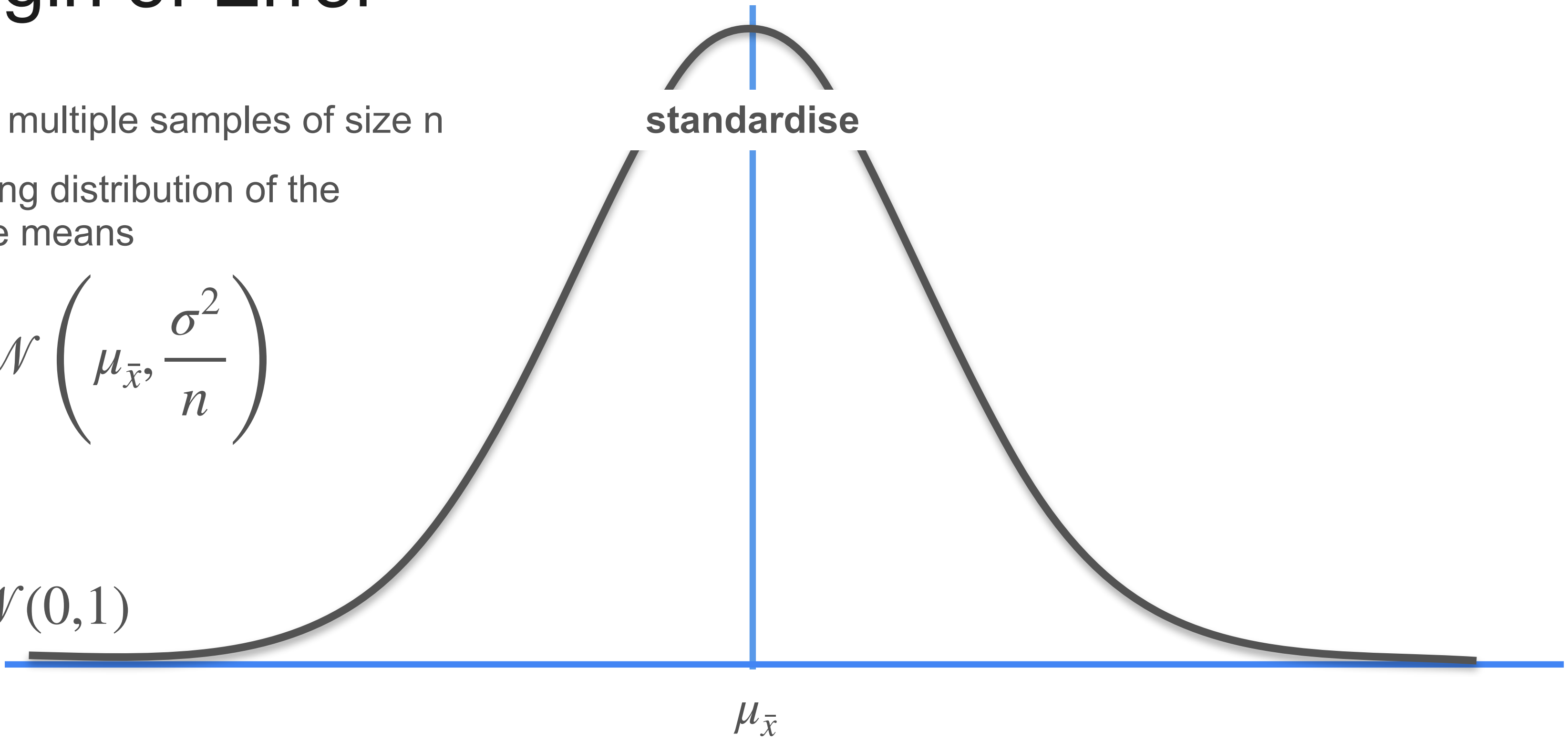
$$\bar{X} \sim \mathcal{N}(0,1)$$



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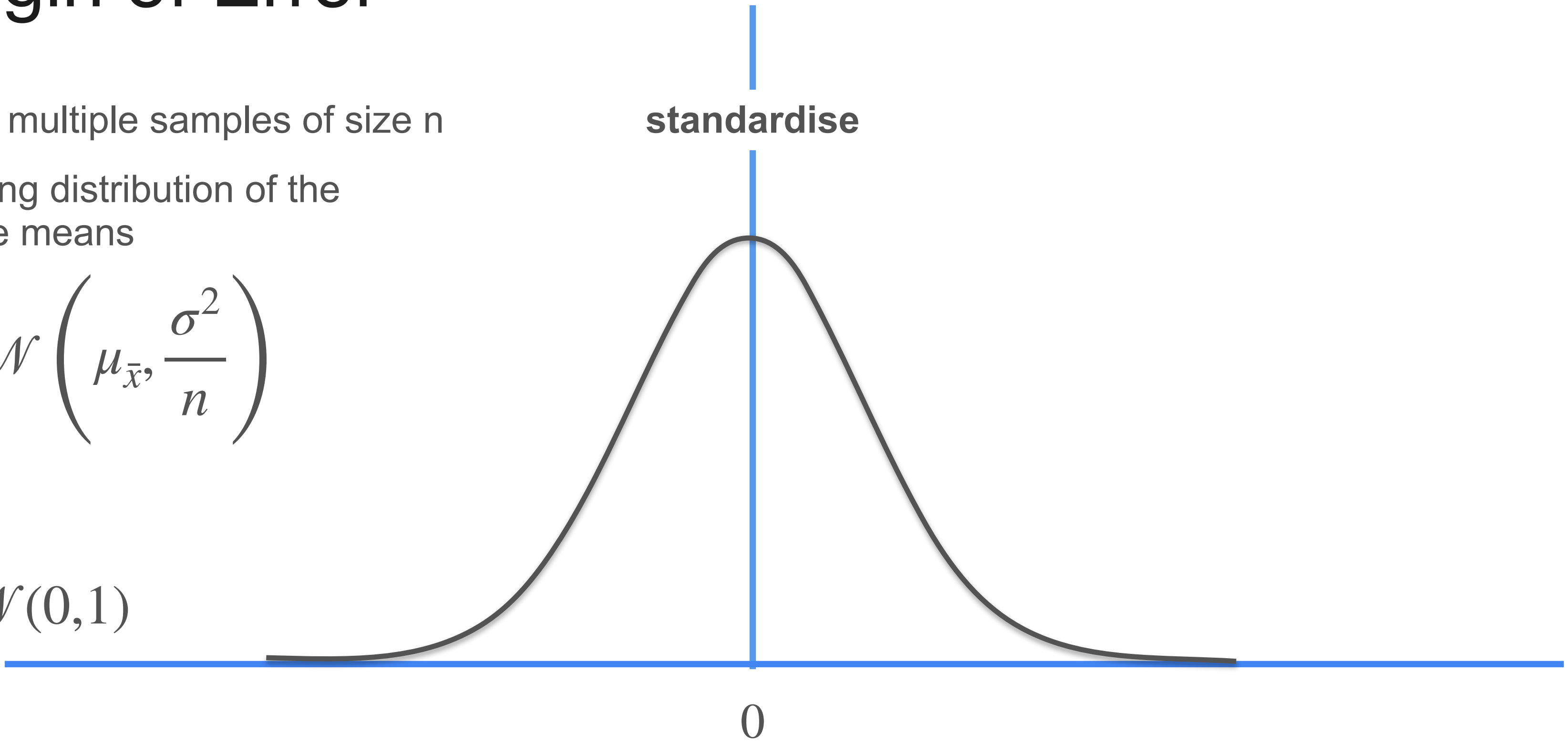

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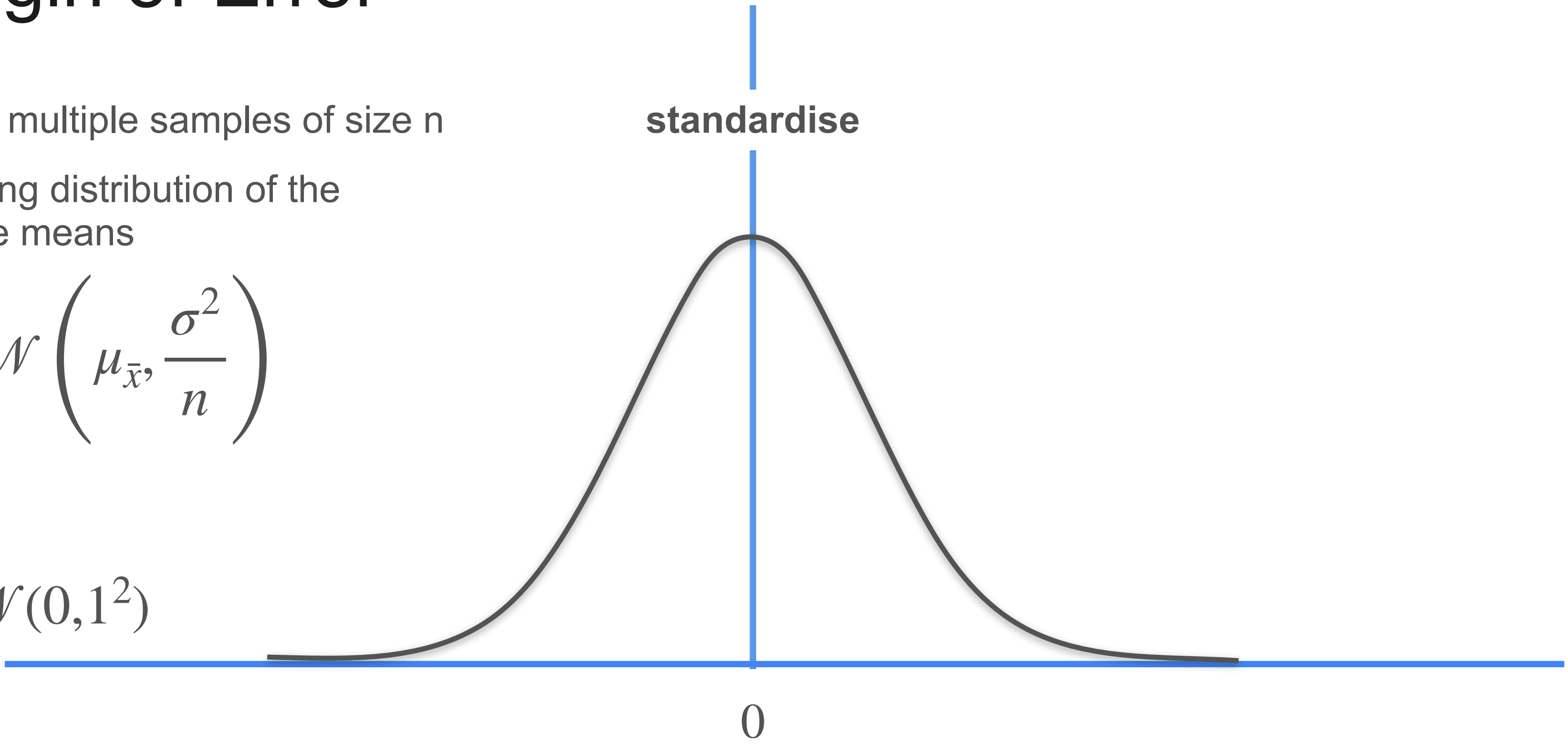

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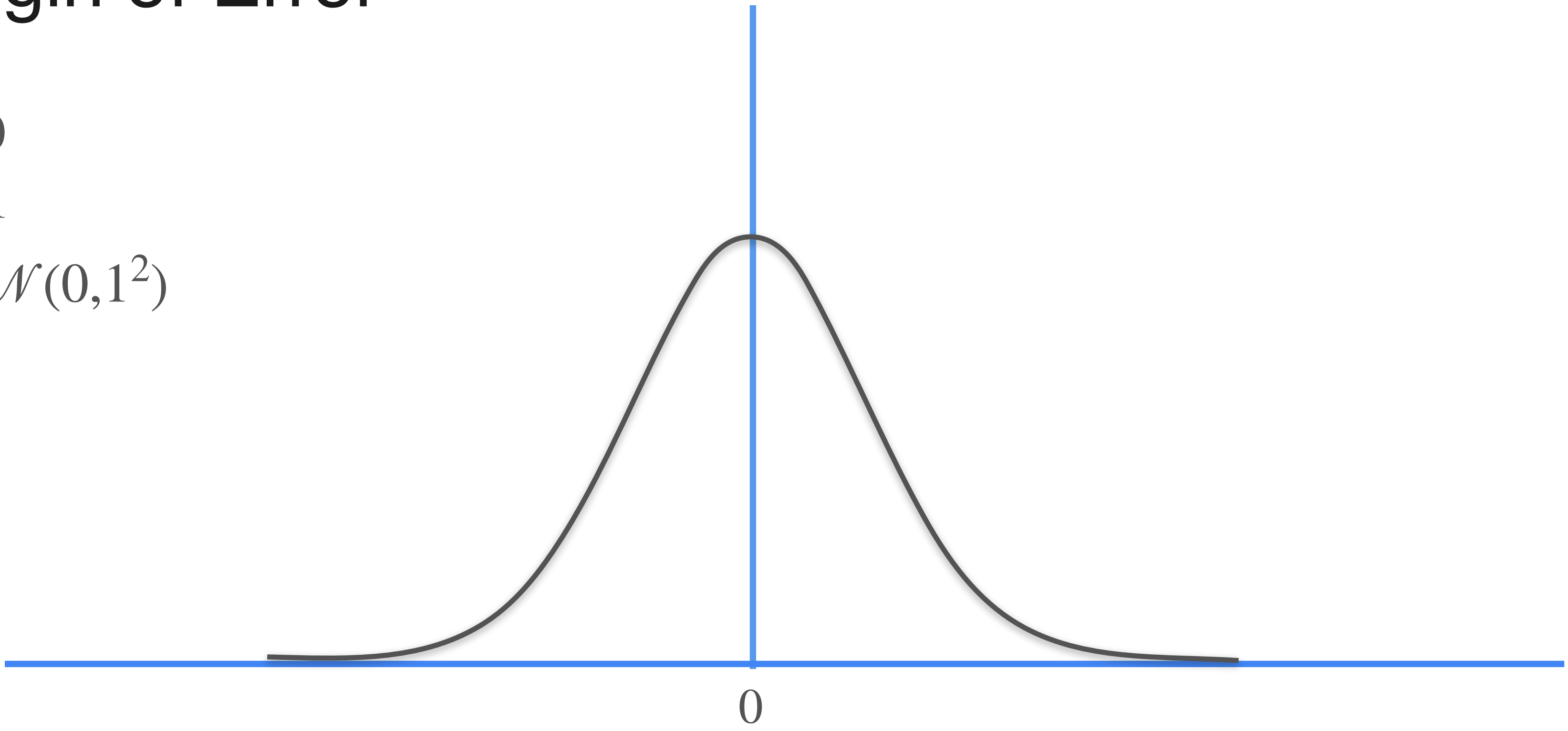

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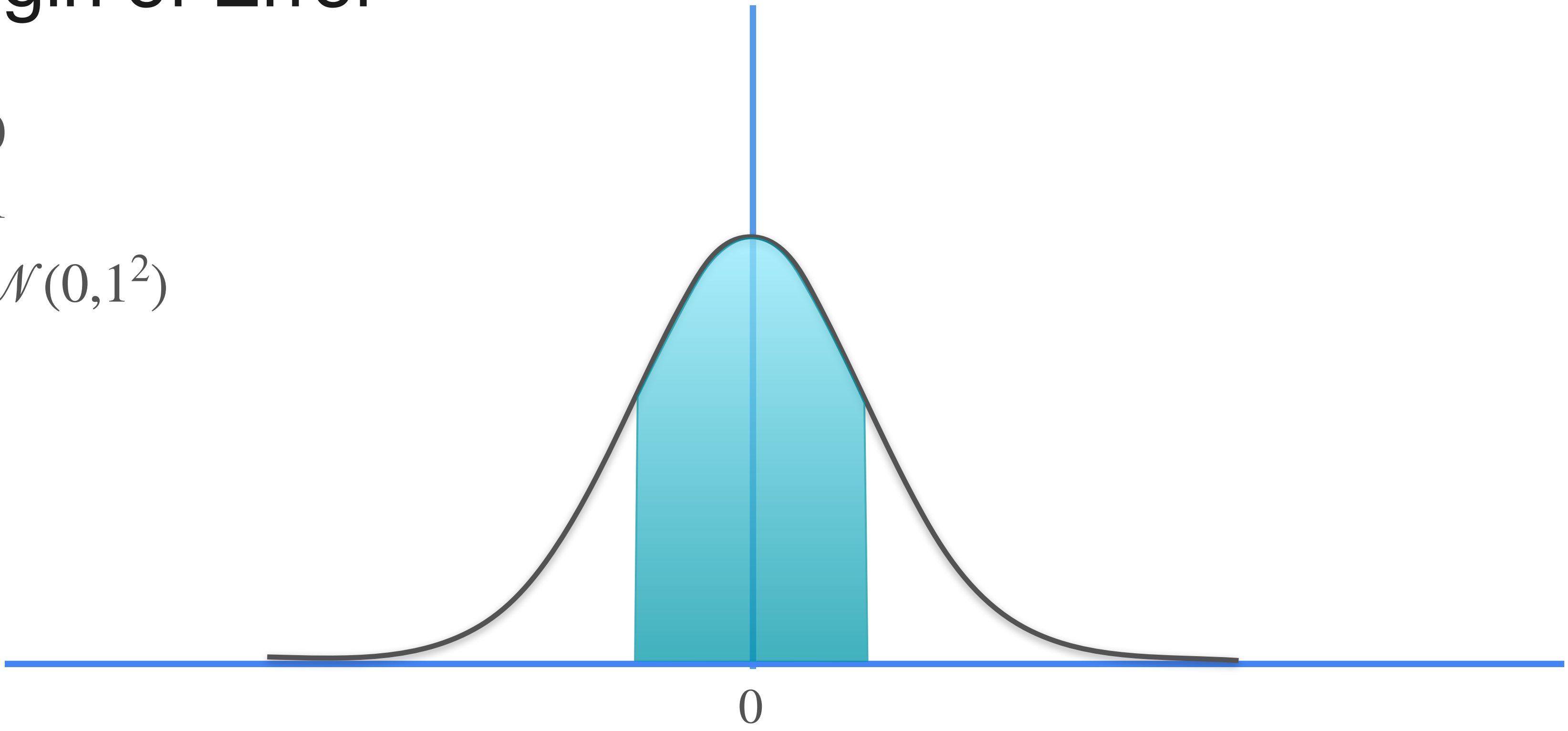

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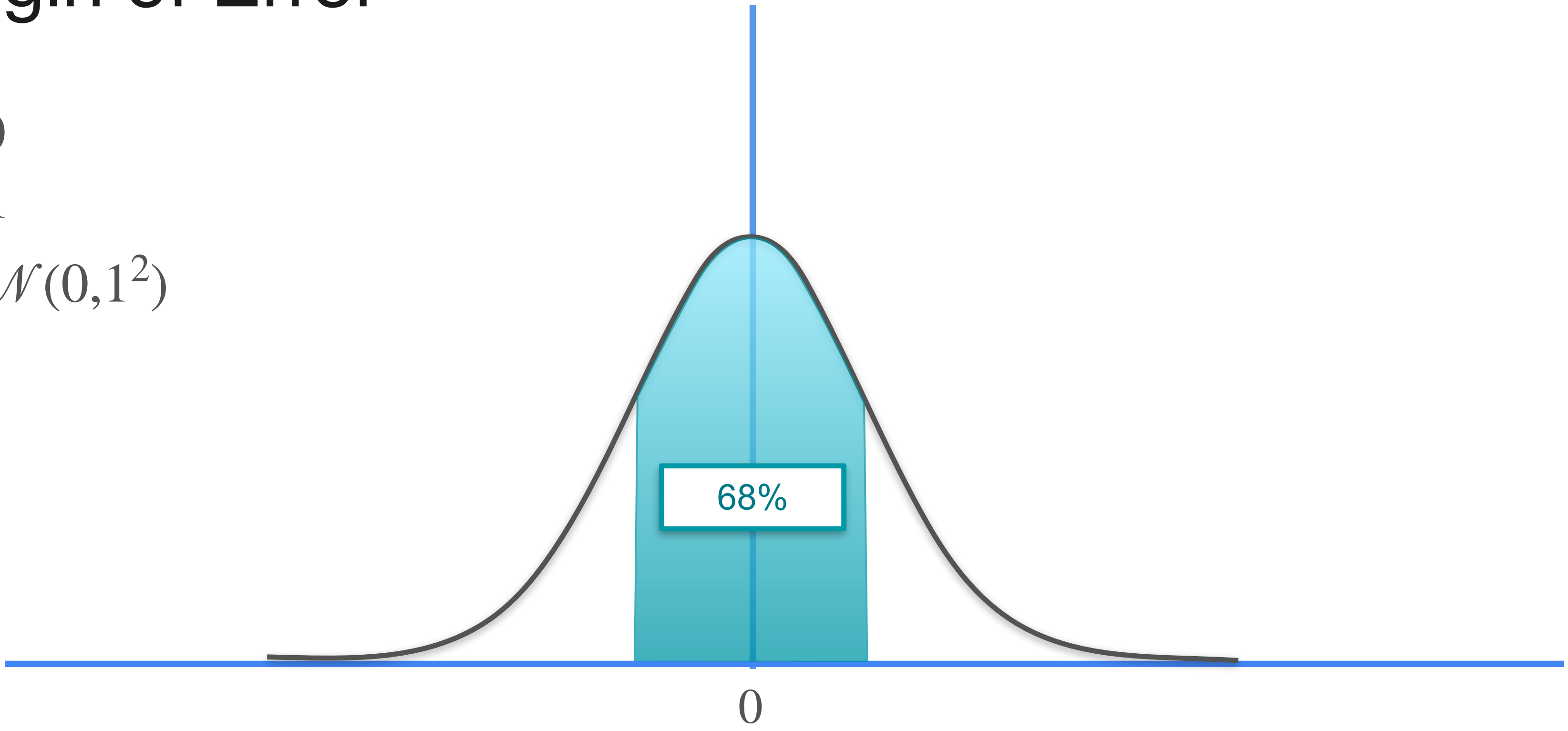

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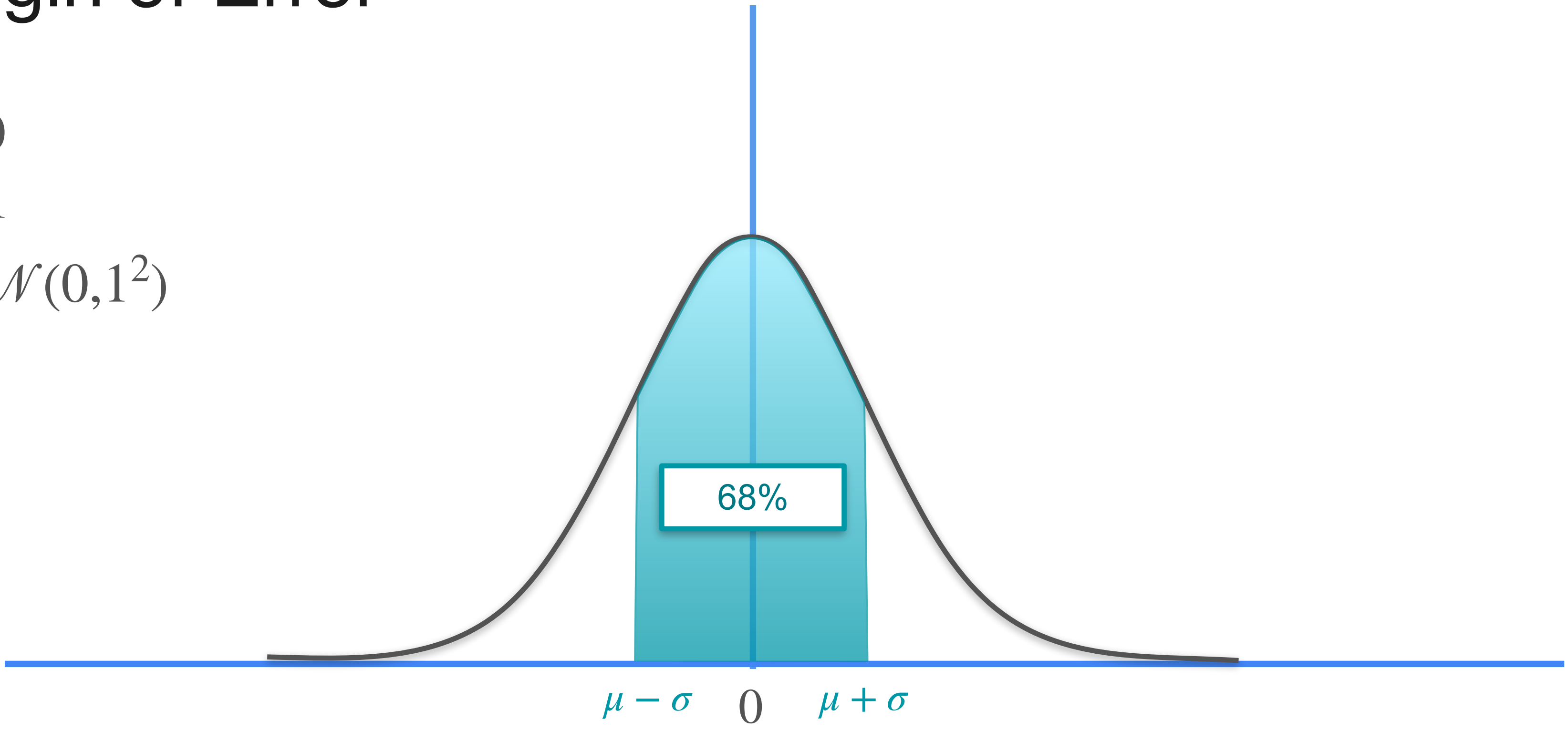

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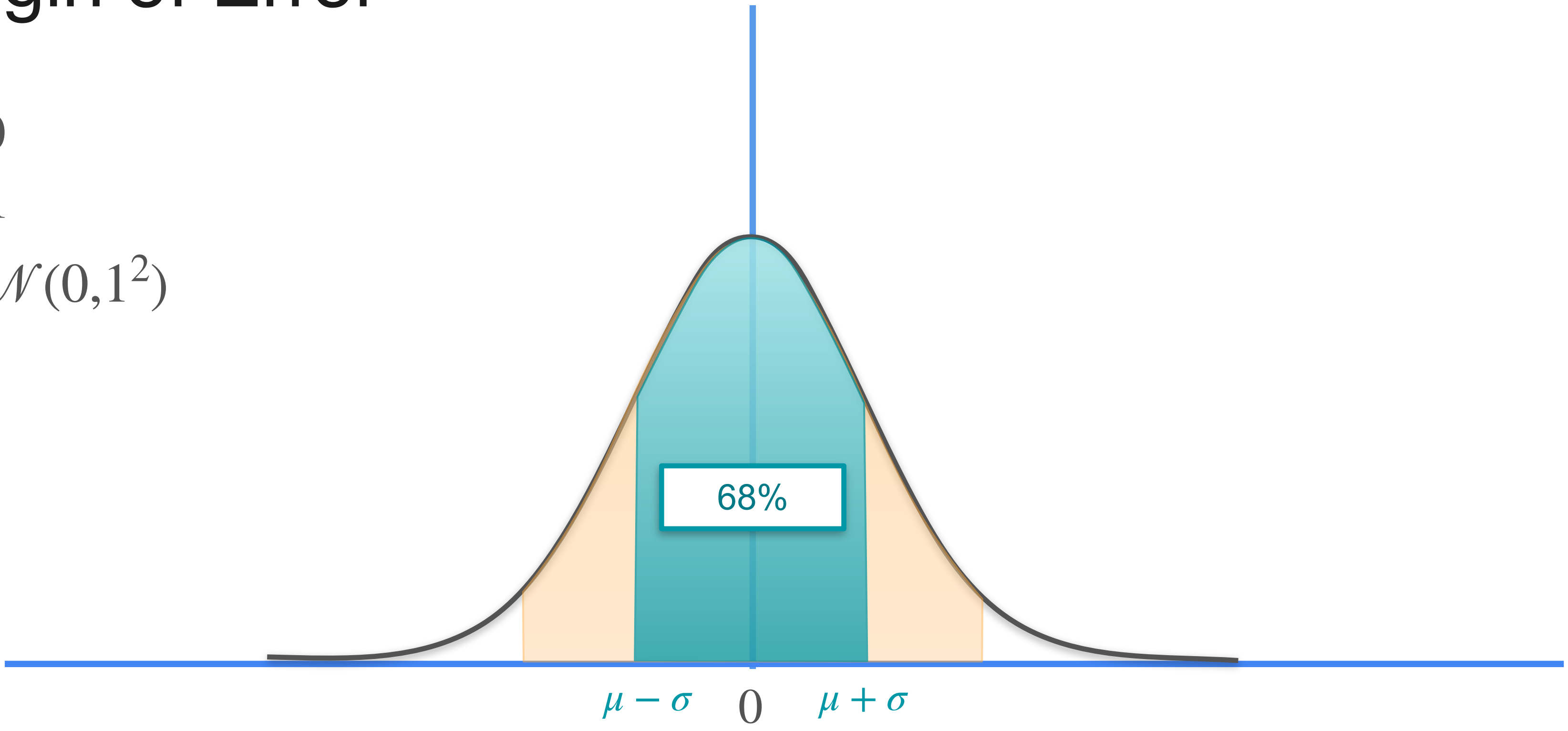

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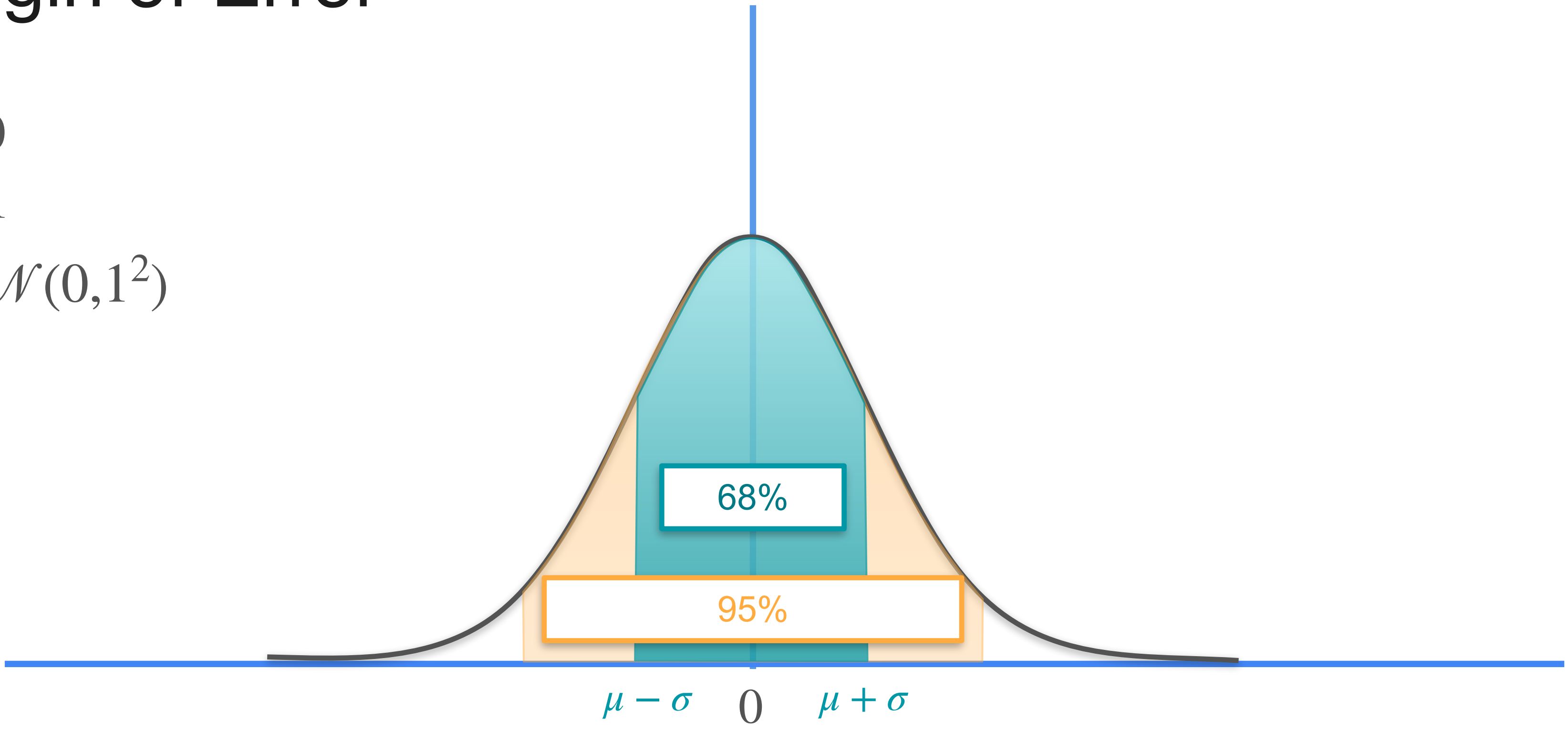

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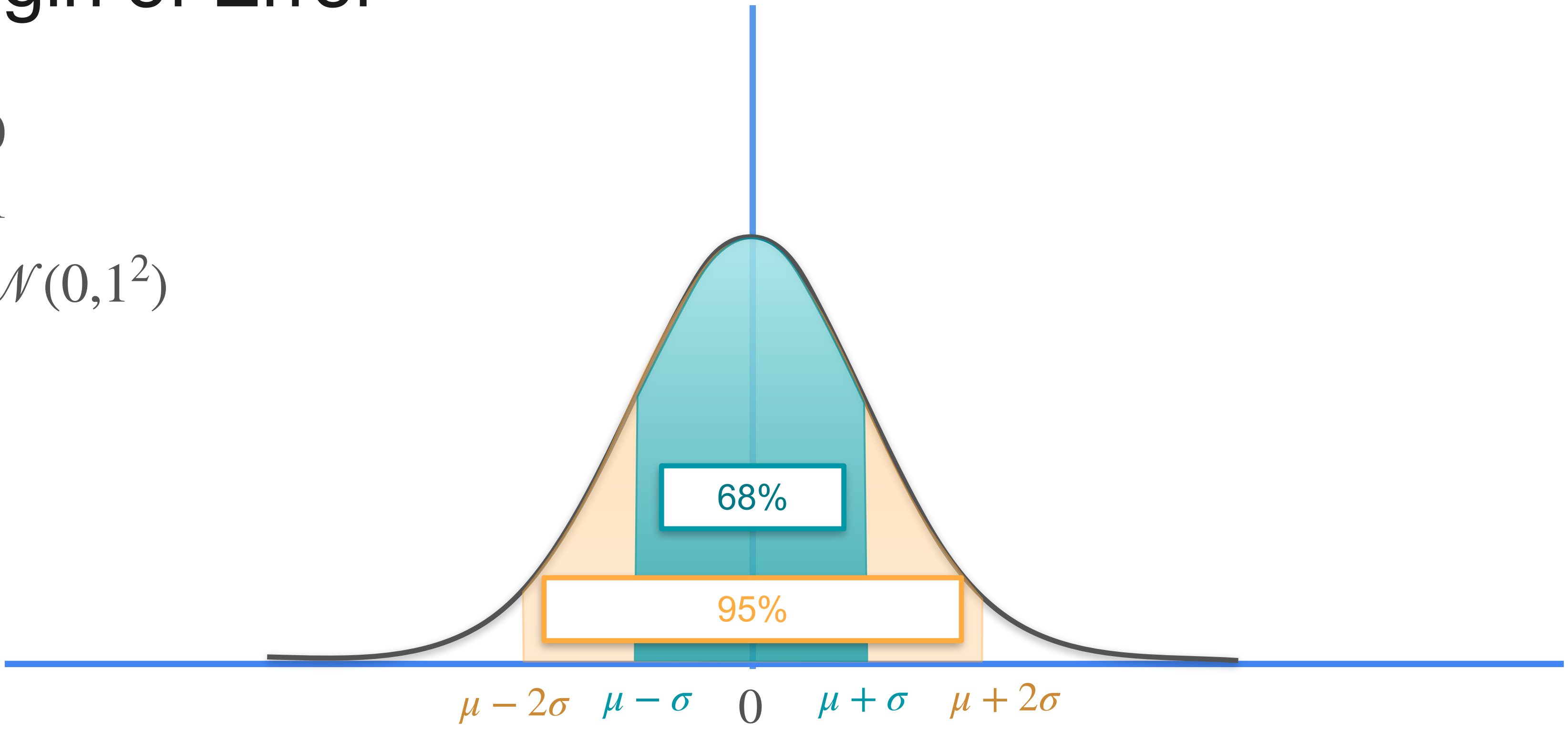

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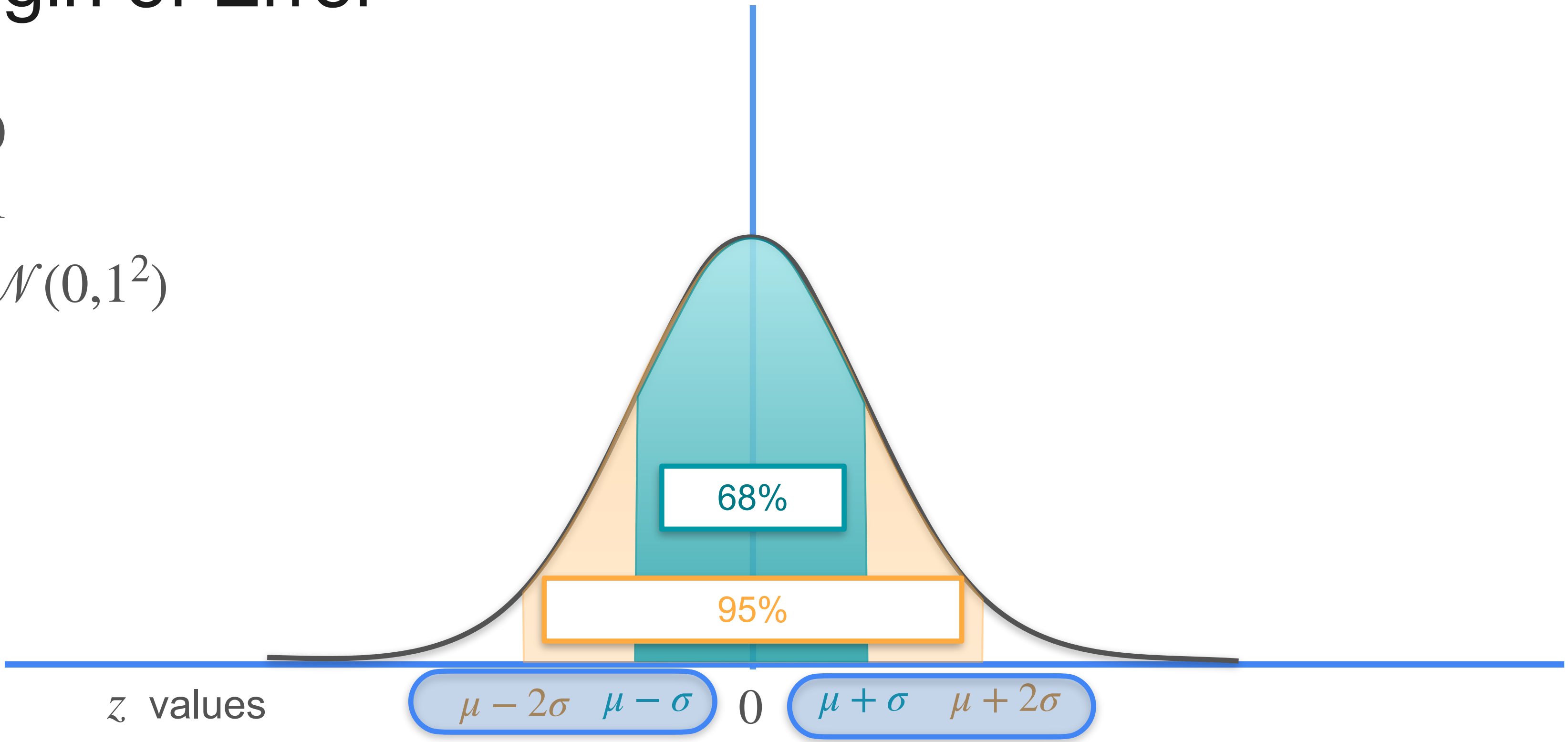

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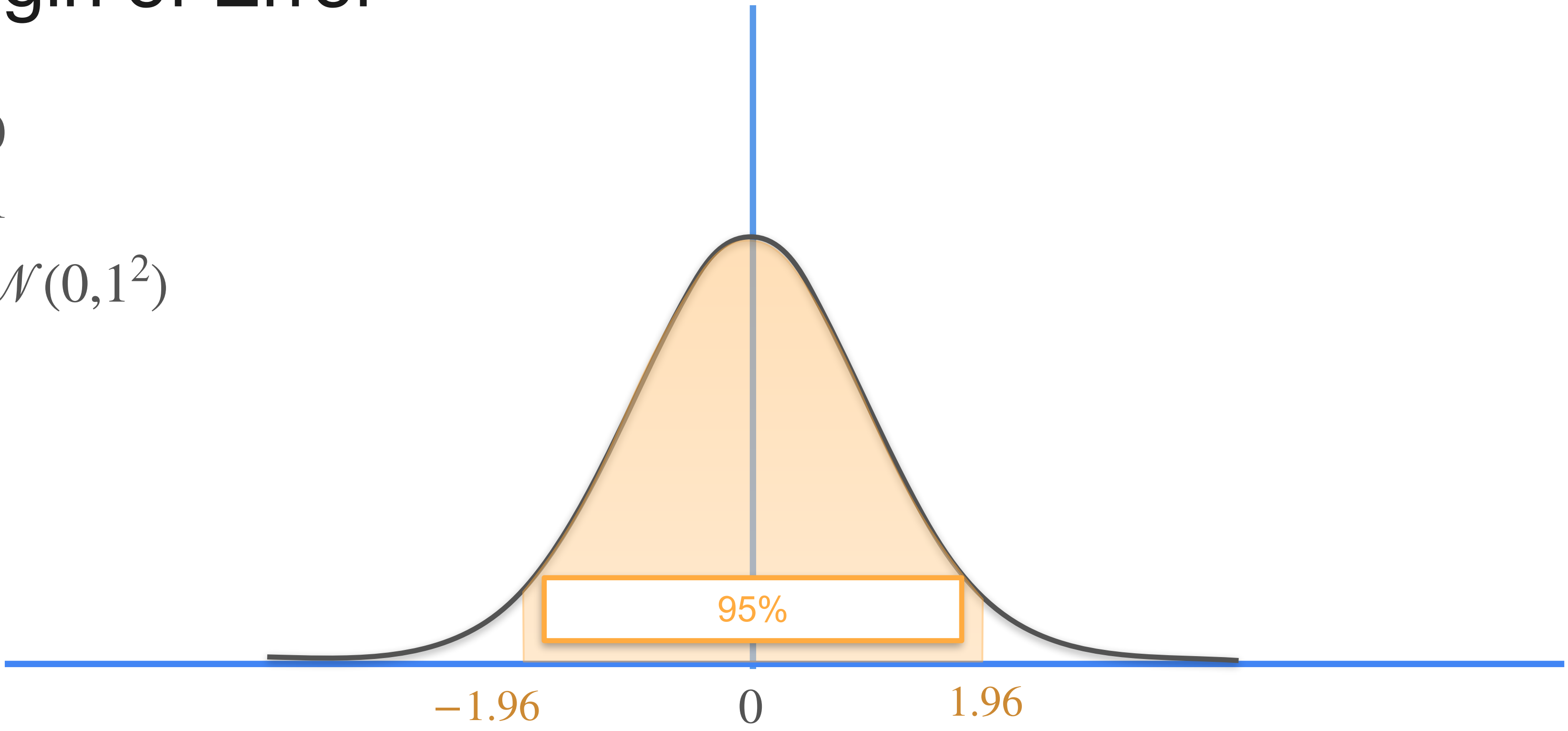

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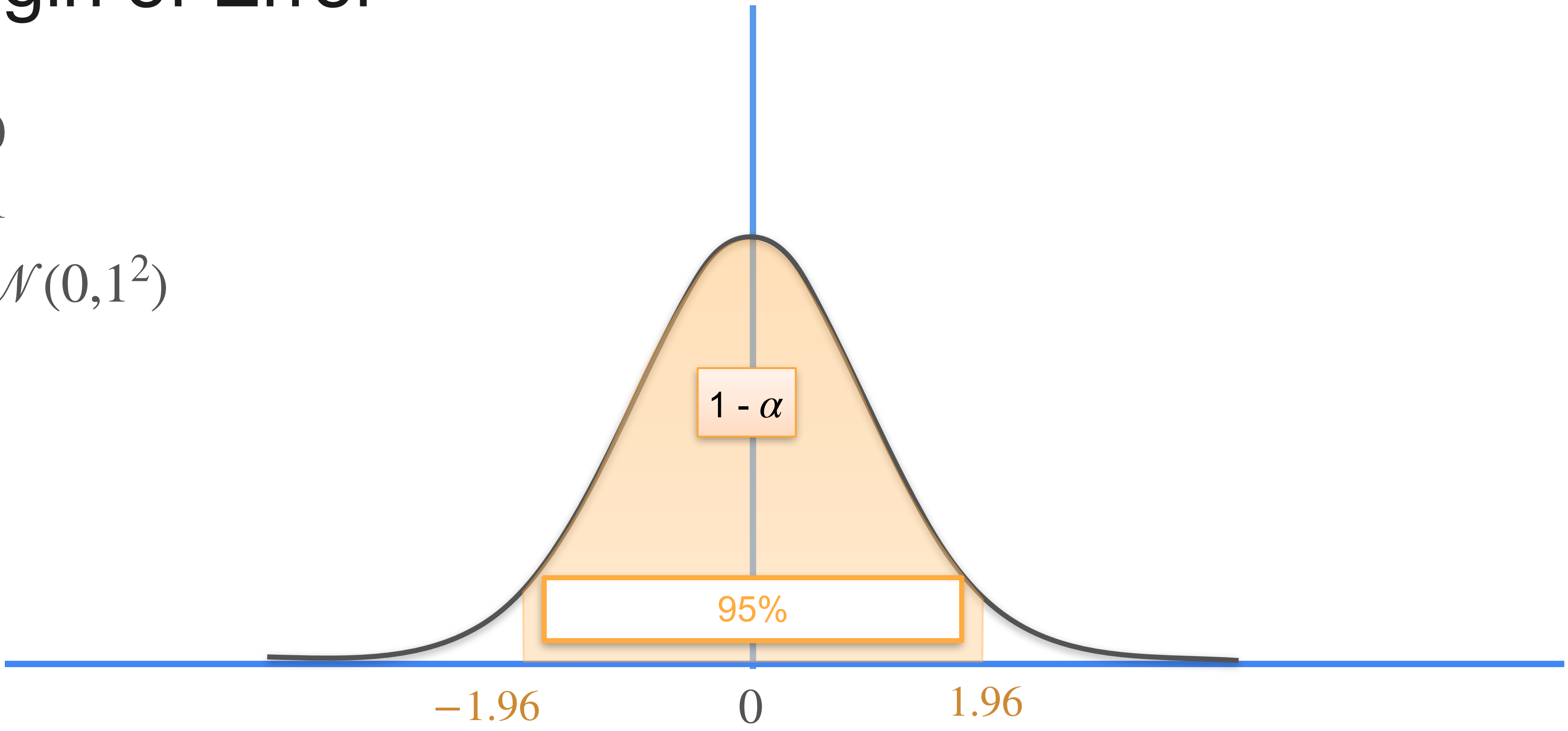

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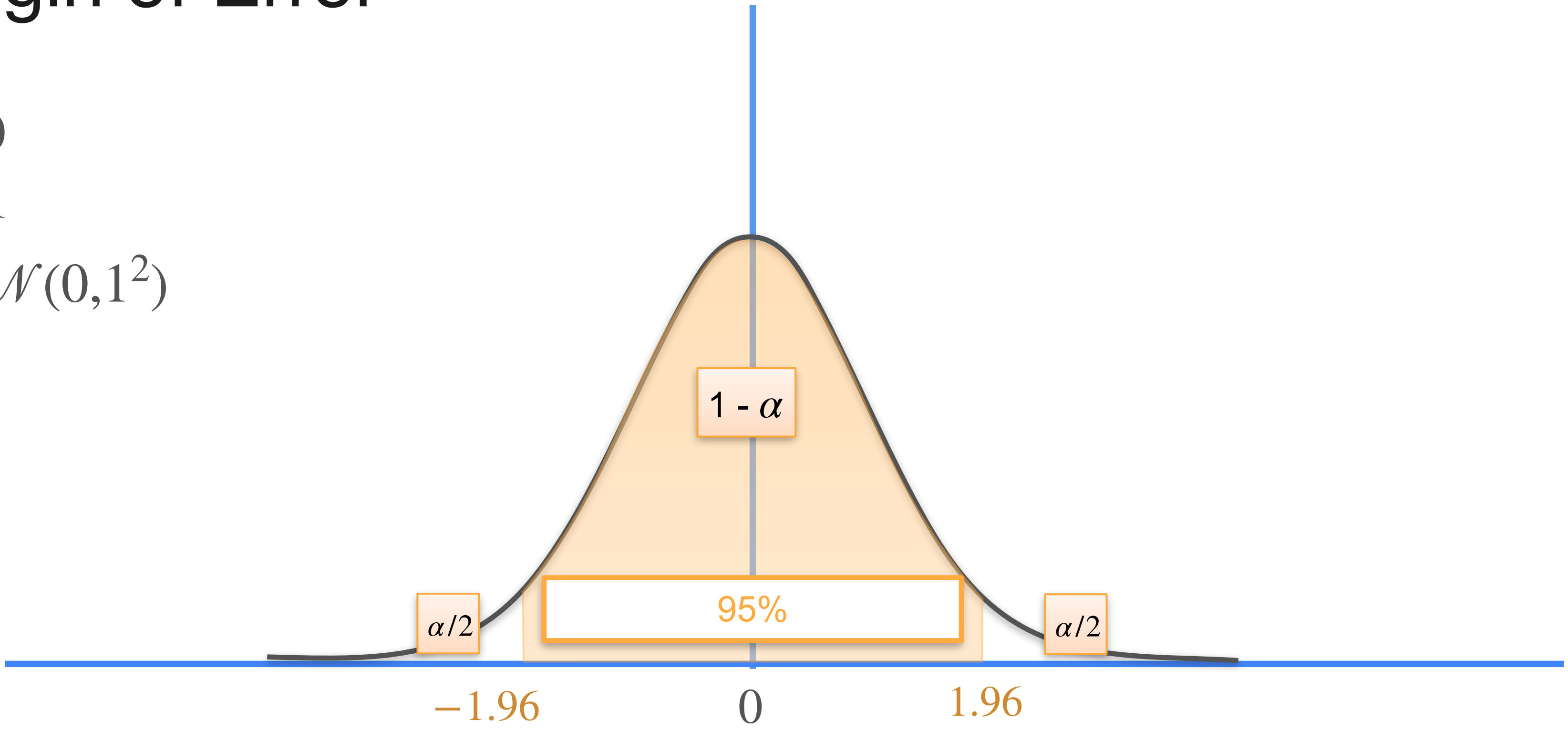

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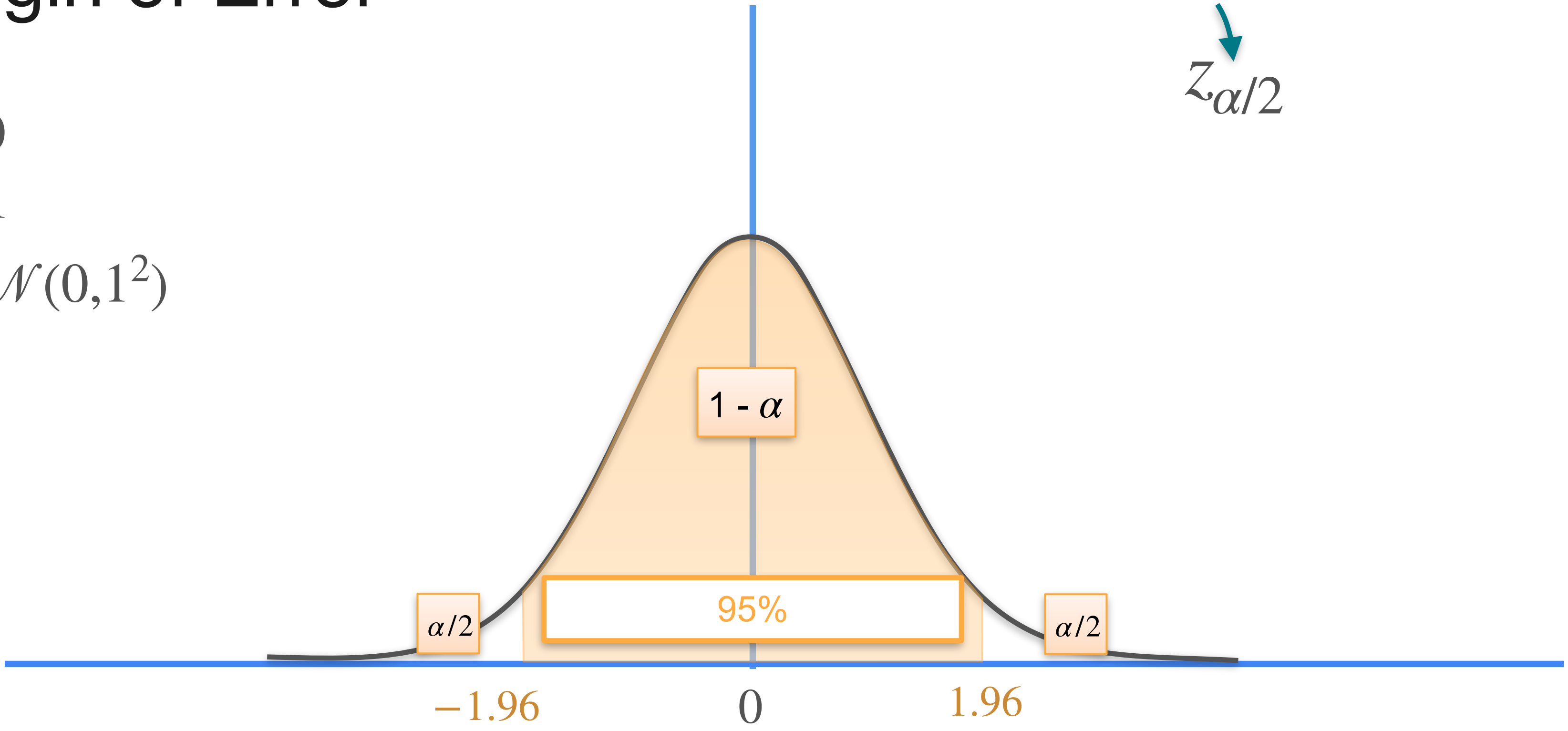
Critical Value

$z_{\alpha/2}$



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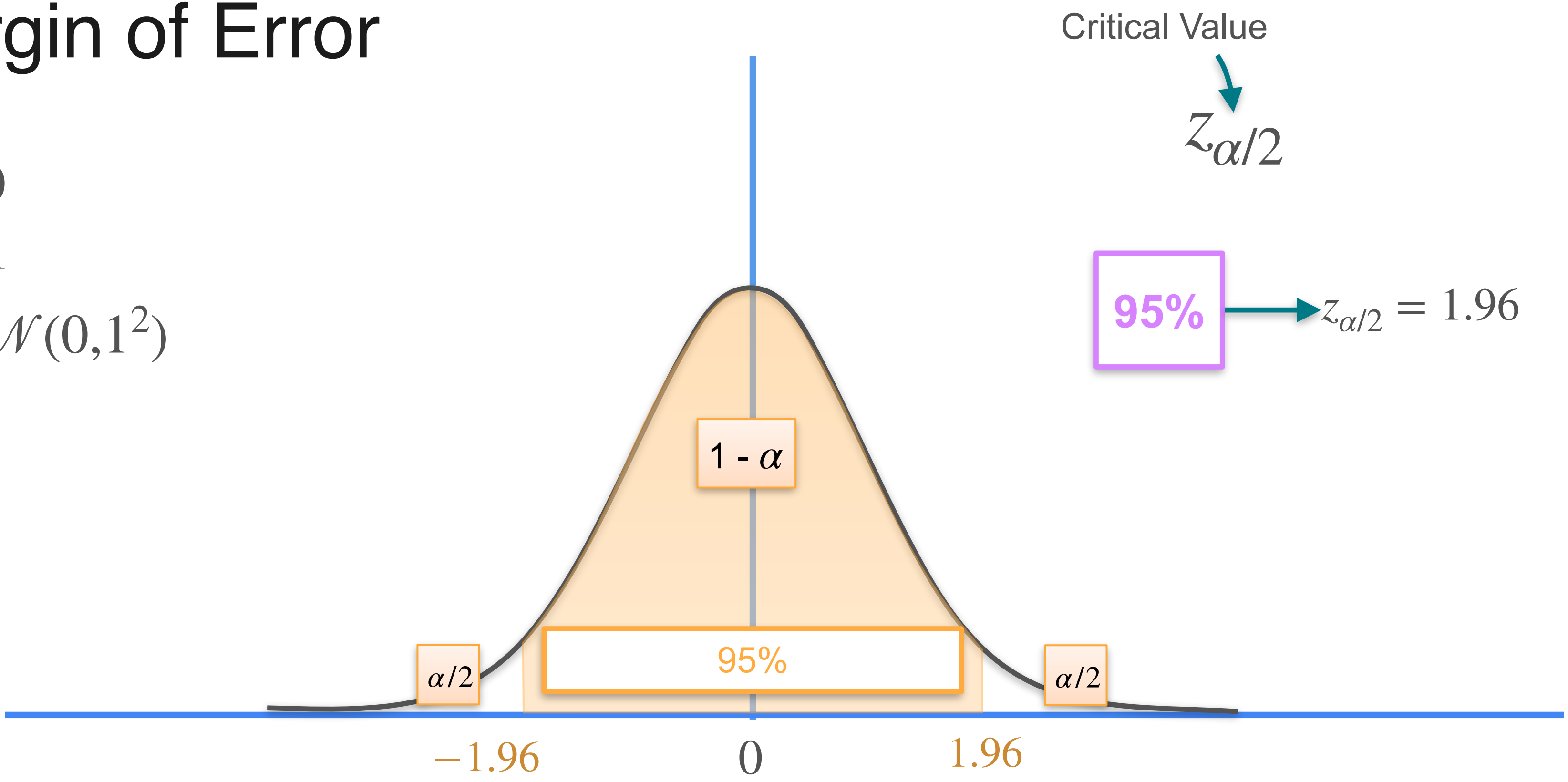

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$\mu = 0$

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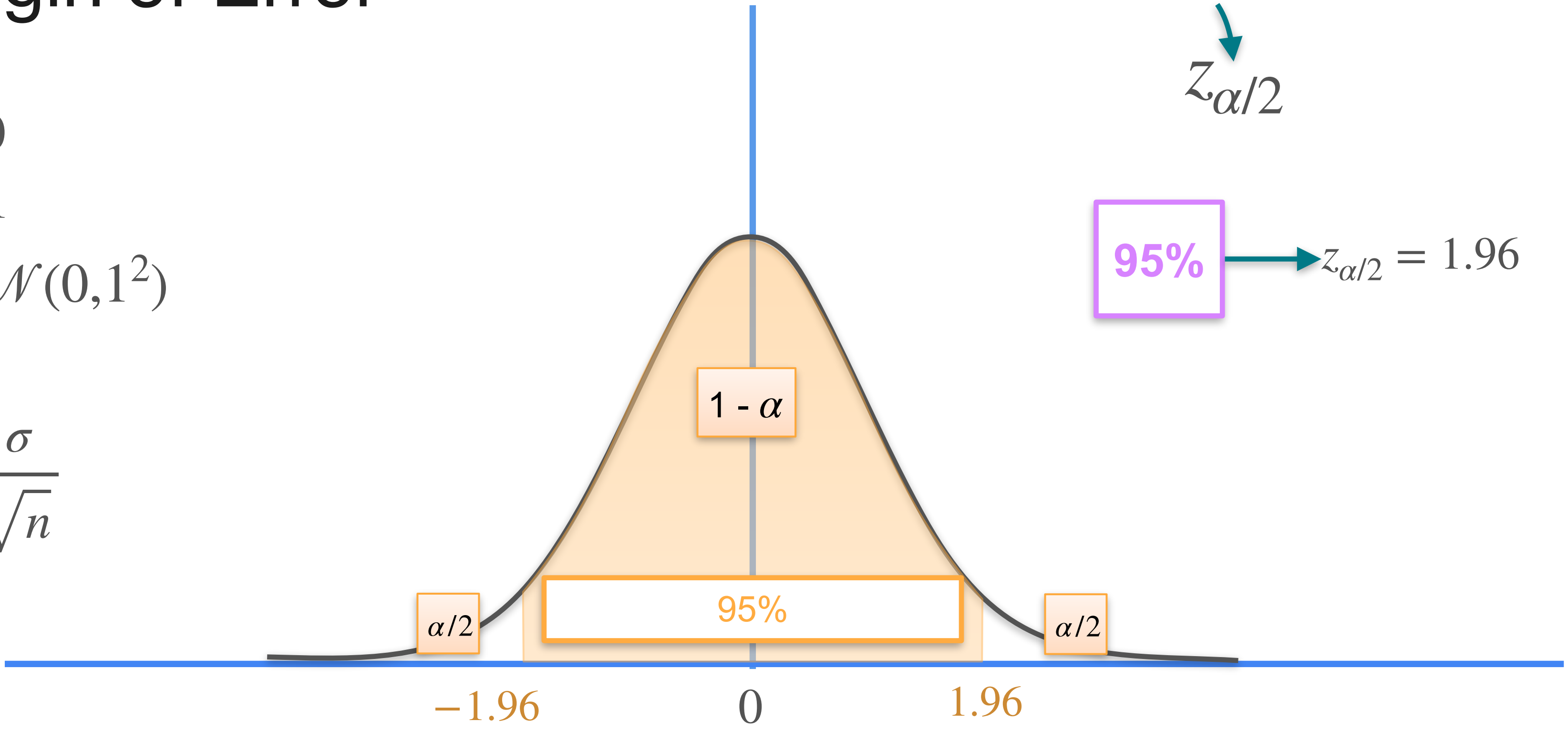
$\bar{X} \sim \mathcal{N}(0, 1^2)$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Critical Value

$z_{\alpha/2}$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$



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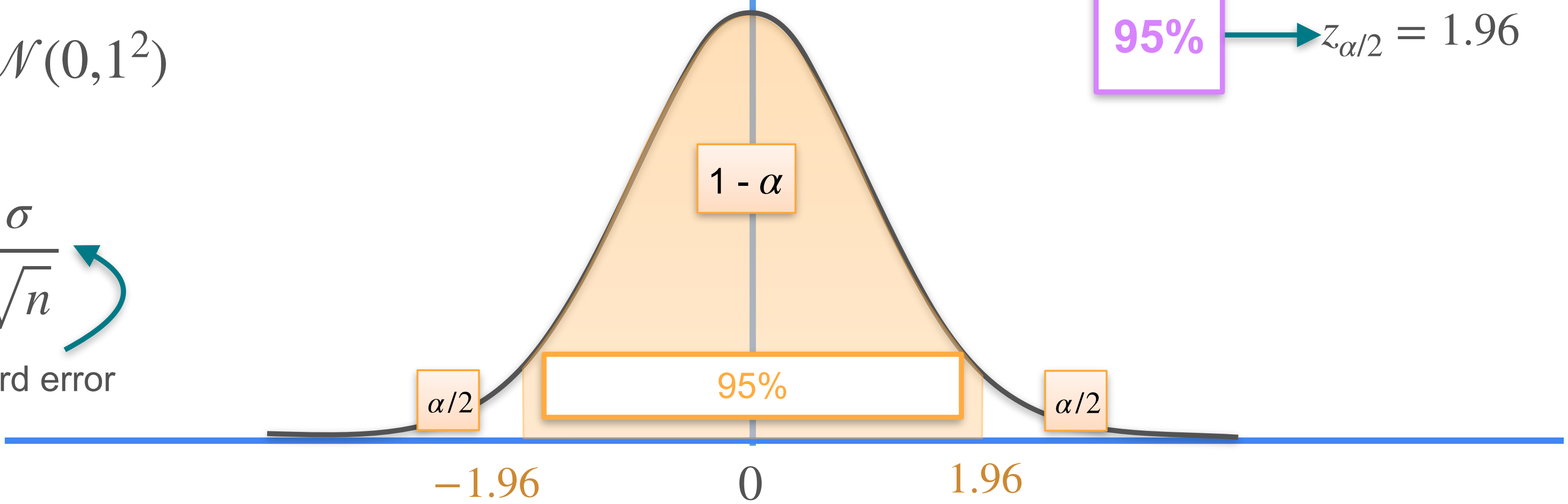

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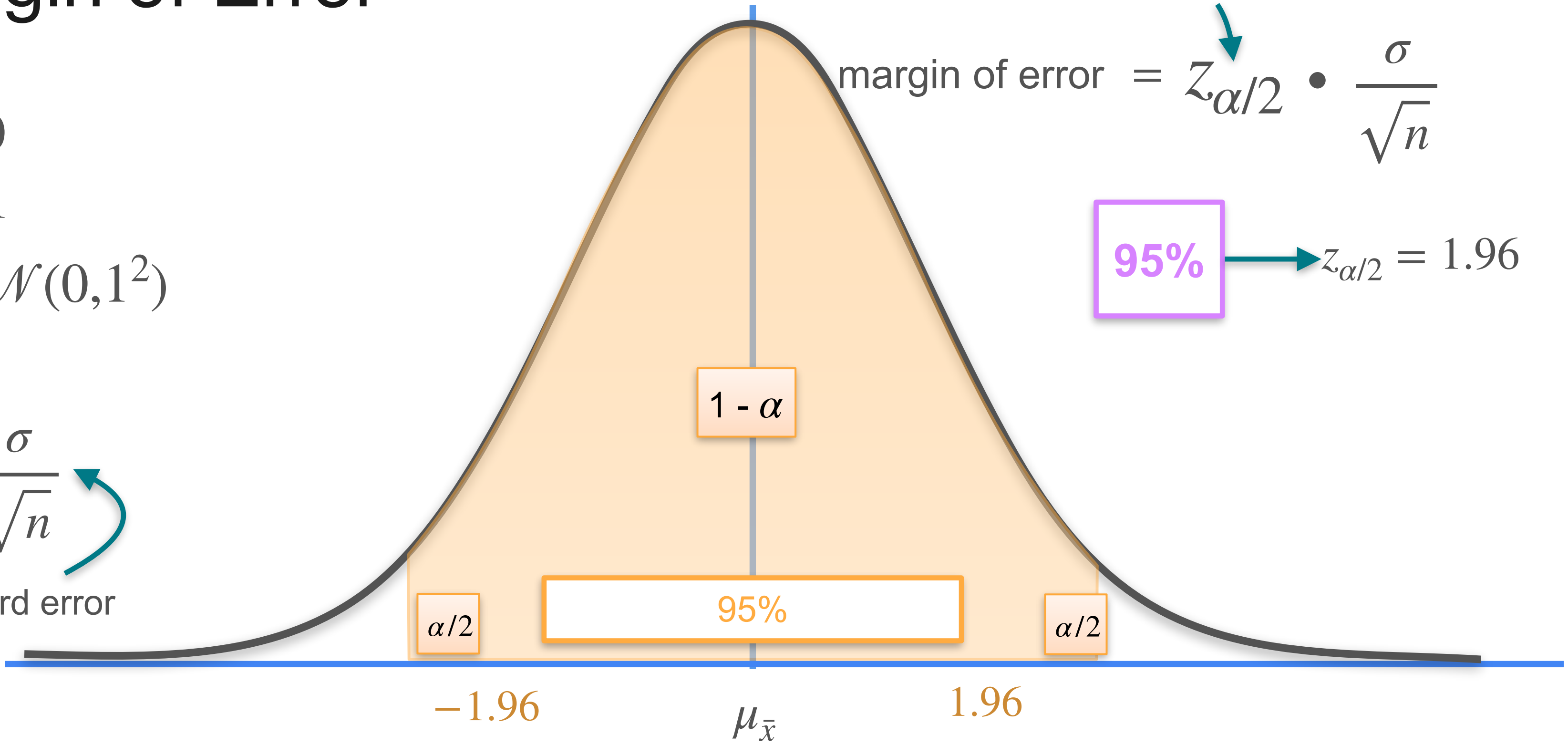

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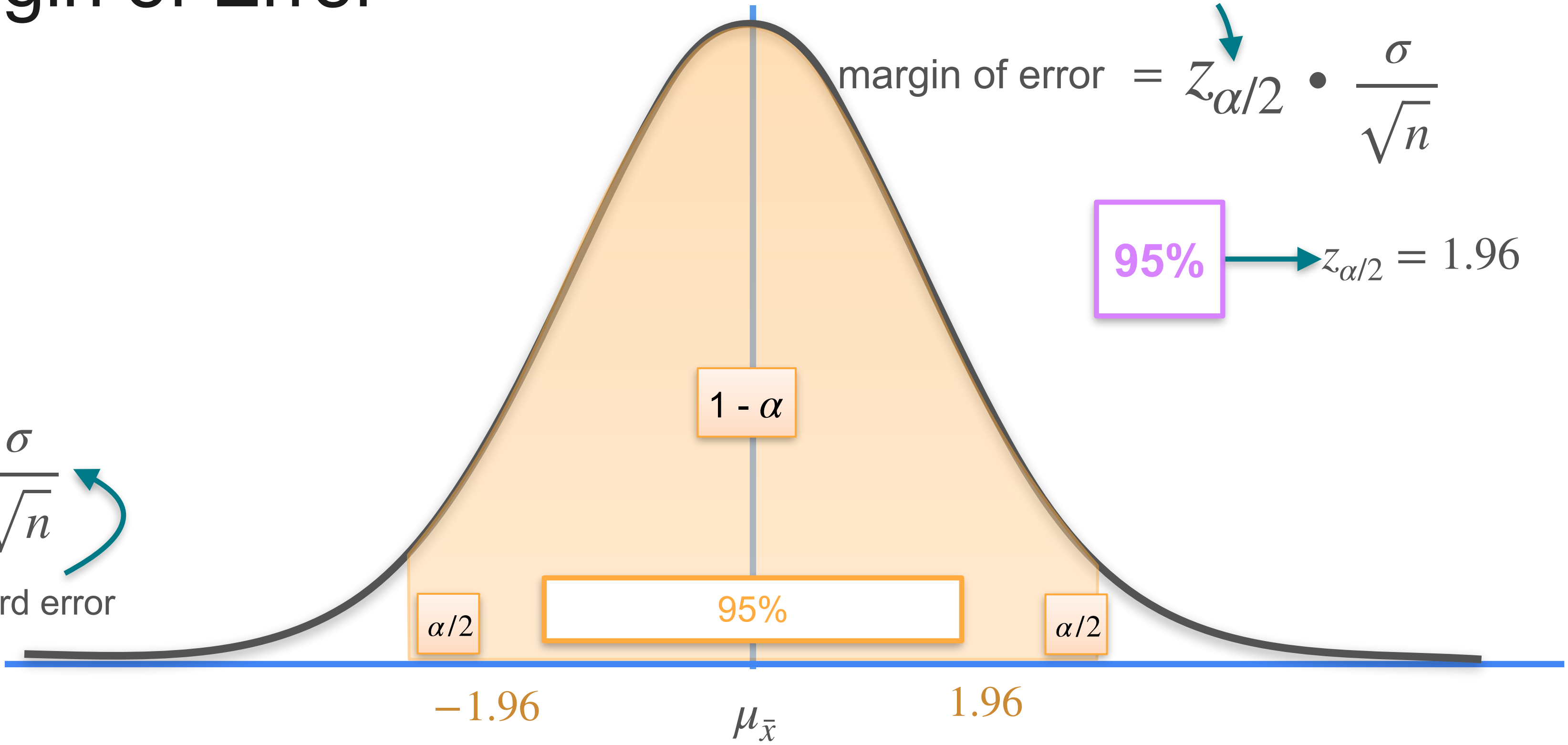

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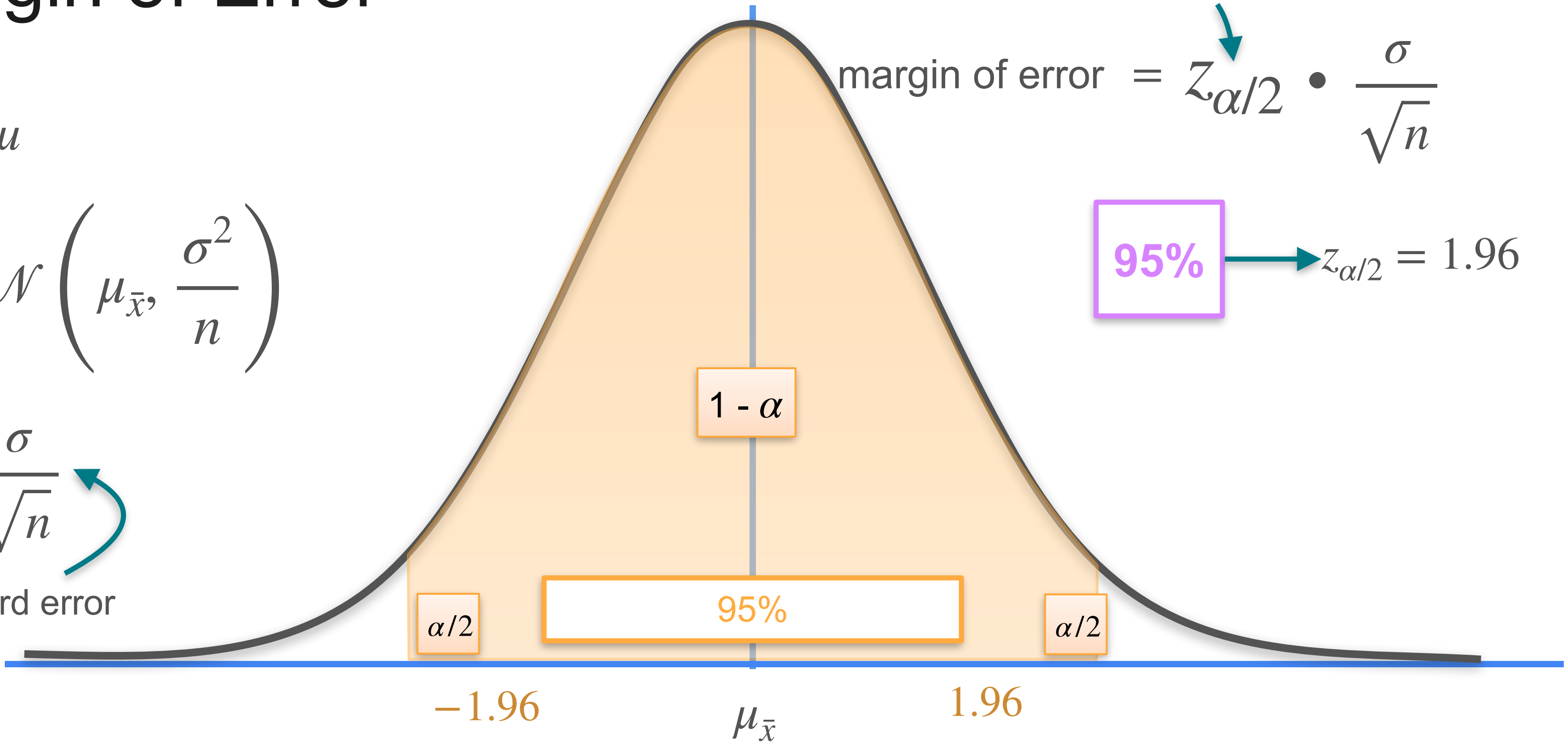

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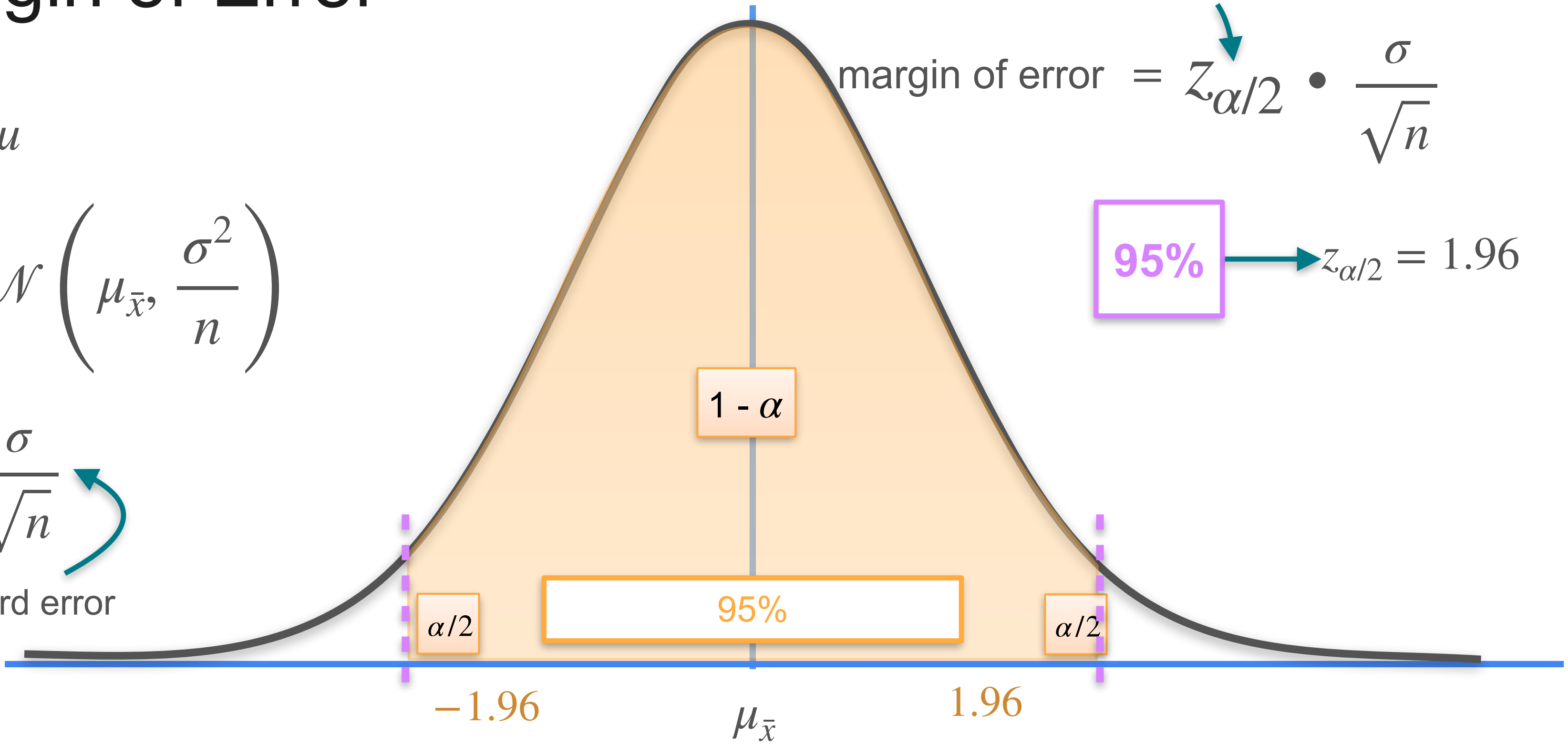

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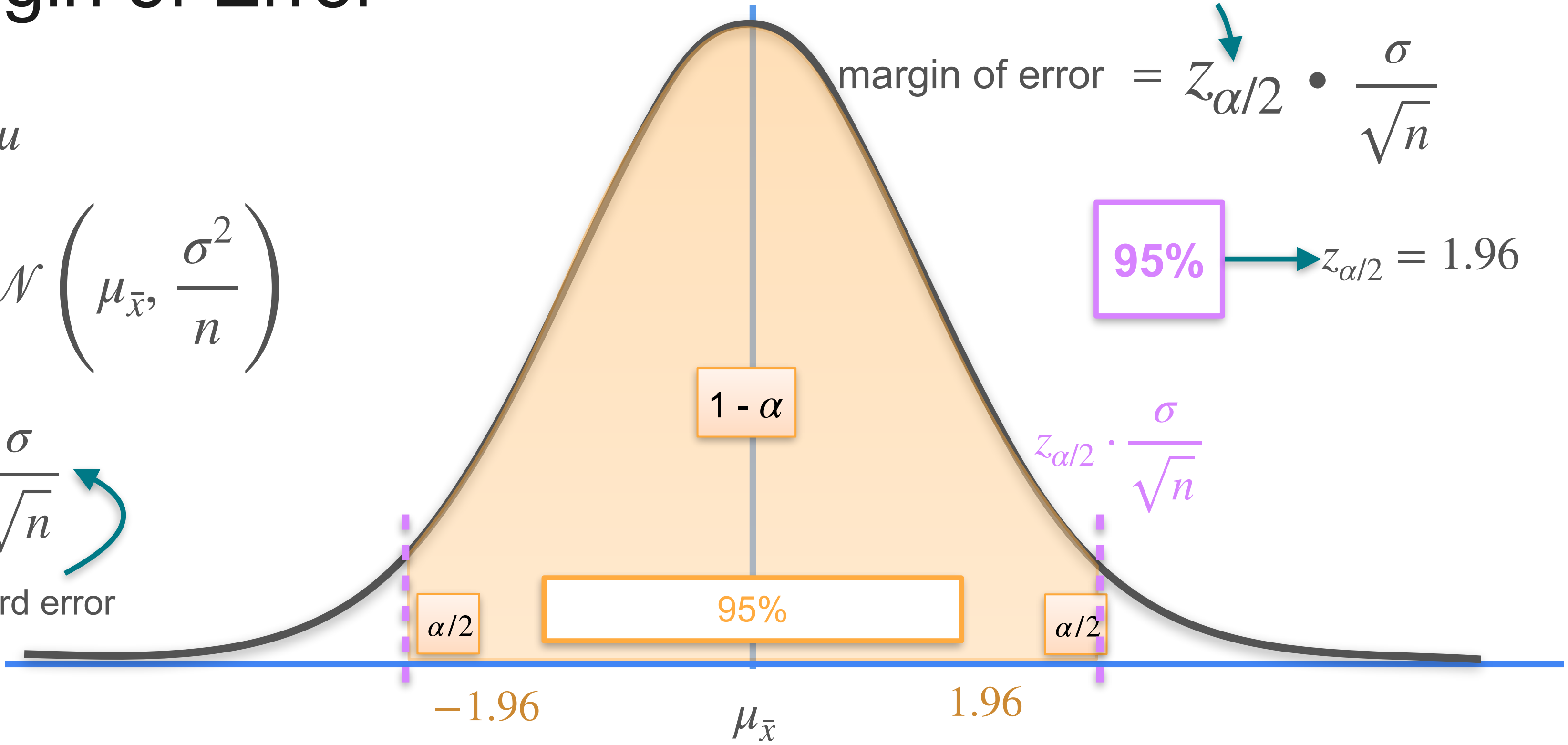

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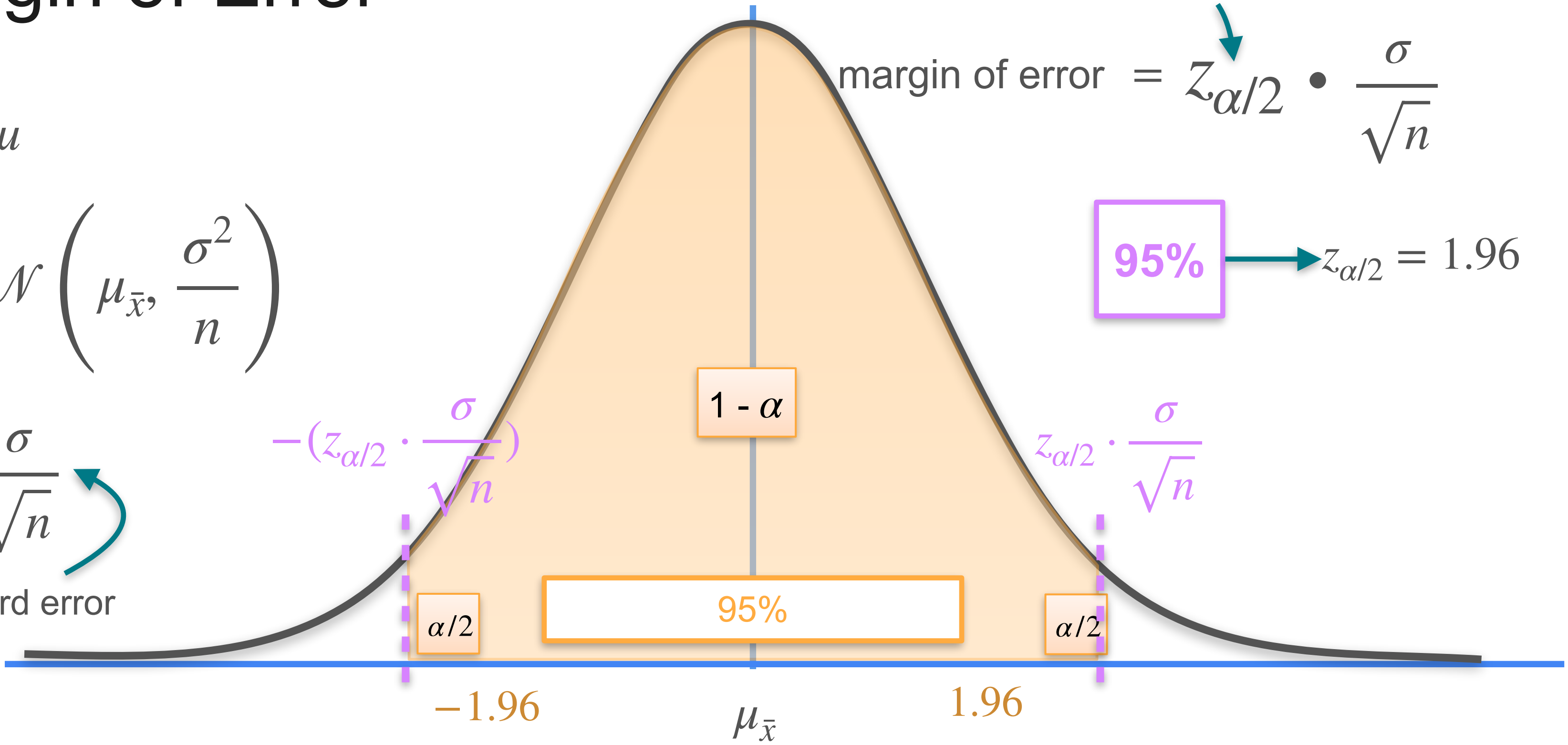

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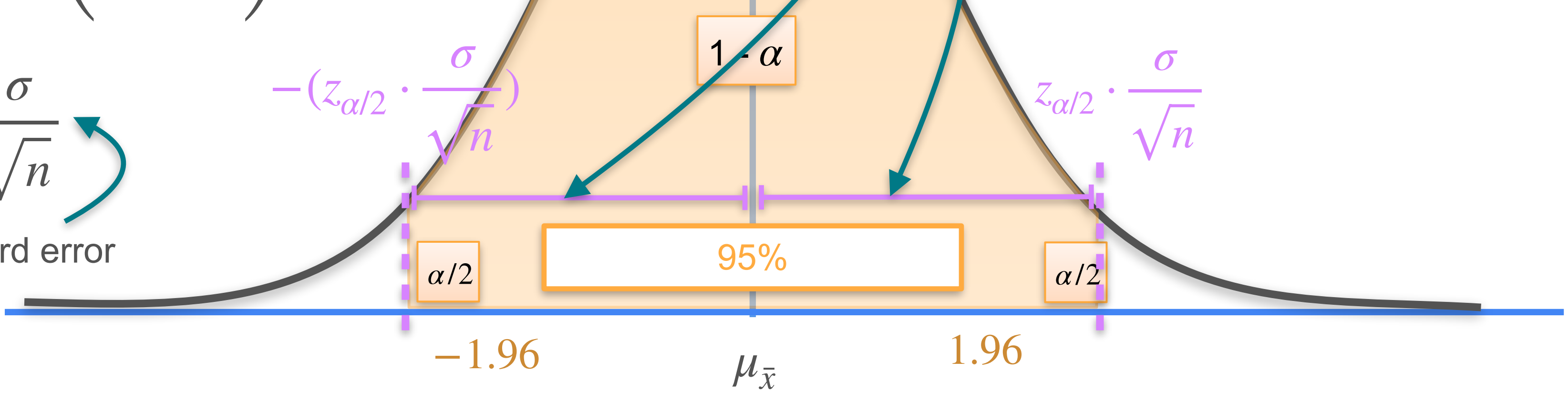

DeepLearning.AI

Confidence Interval

Confidence Interval - Calculation Steps

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STEPS:

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean

$$\bar{x}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)

$\bar{x}$

DeepLearning.AI

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
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$\bar{x}$

95%

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{\alpha/2}$)

$\bar{x}$

95%

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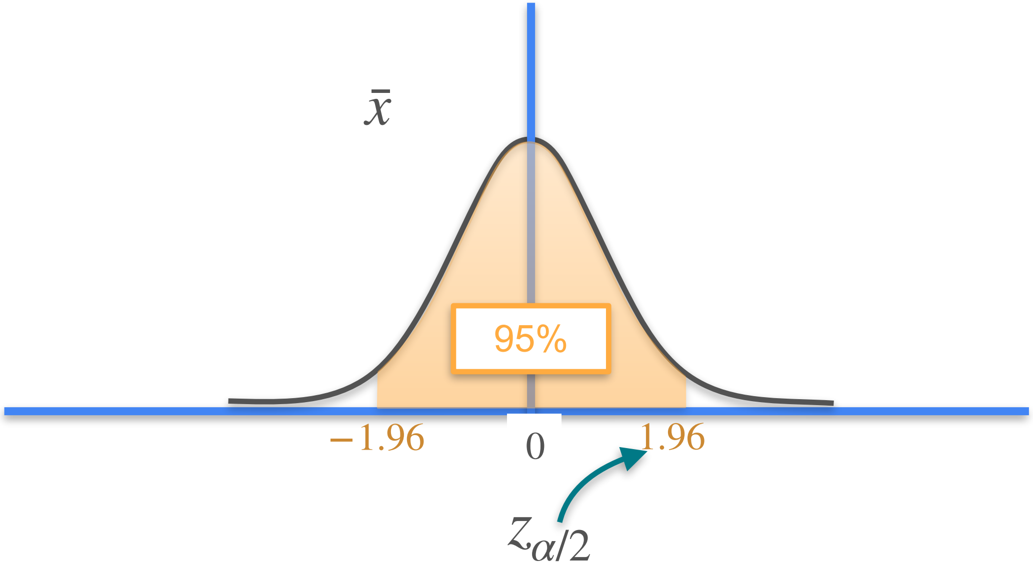

Confidence Interval - Calculation Steps

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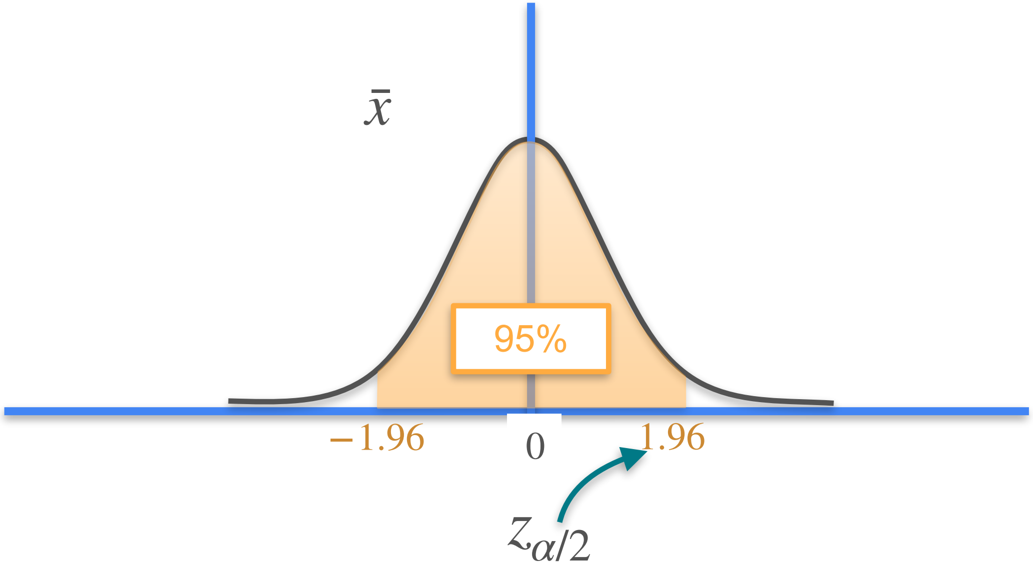

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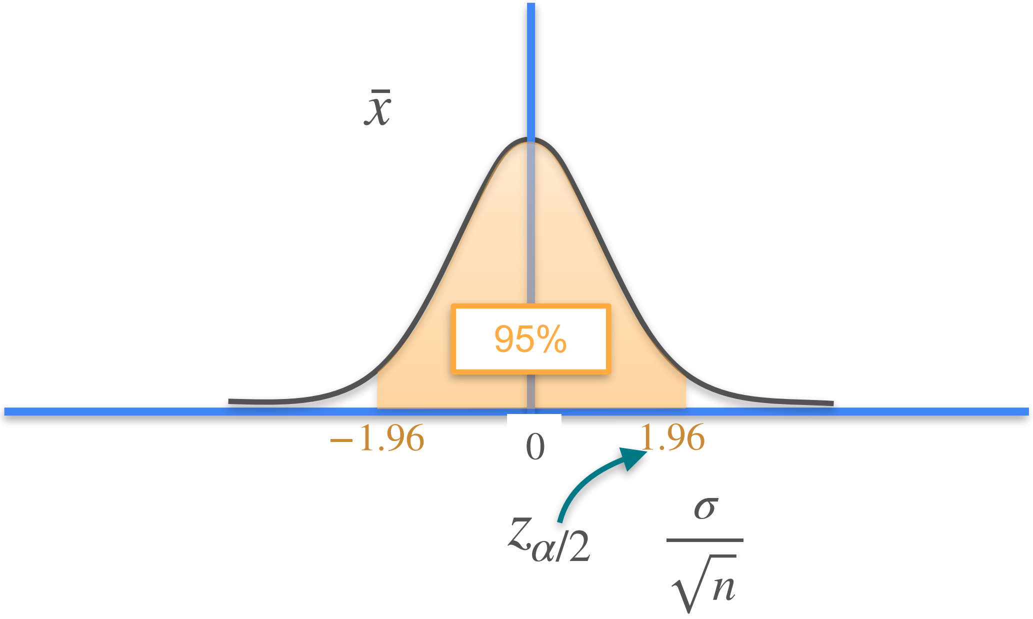

Confidence Interval - Calculation Steps

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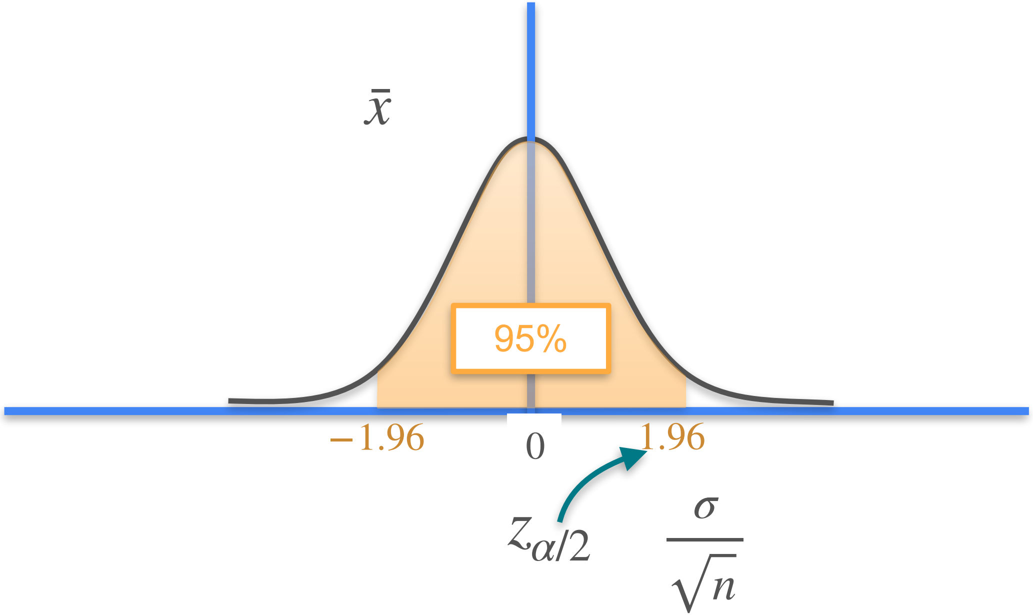
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error



Confidence Interval - Calculation Steps

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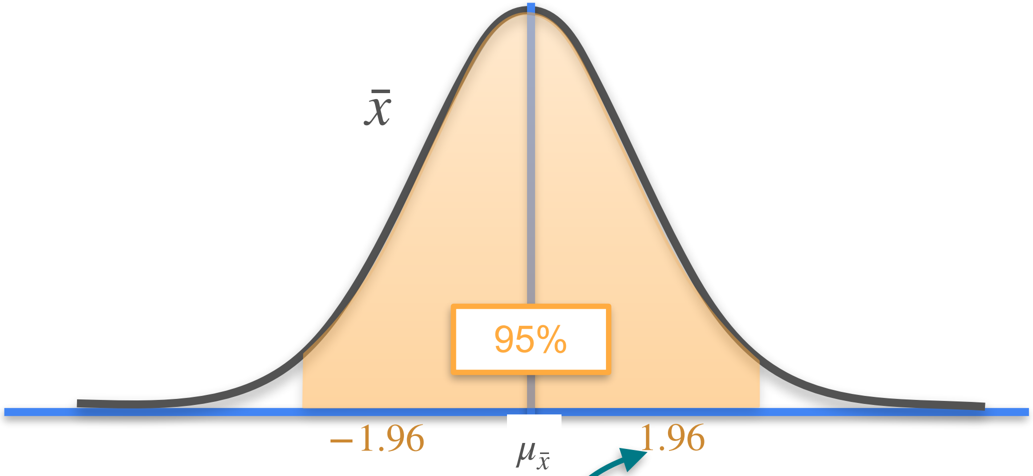

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval - Calculation Steps

STEPS:

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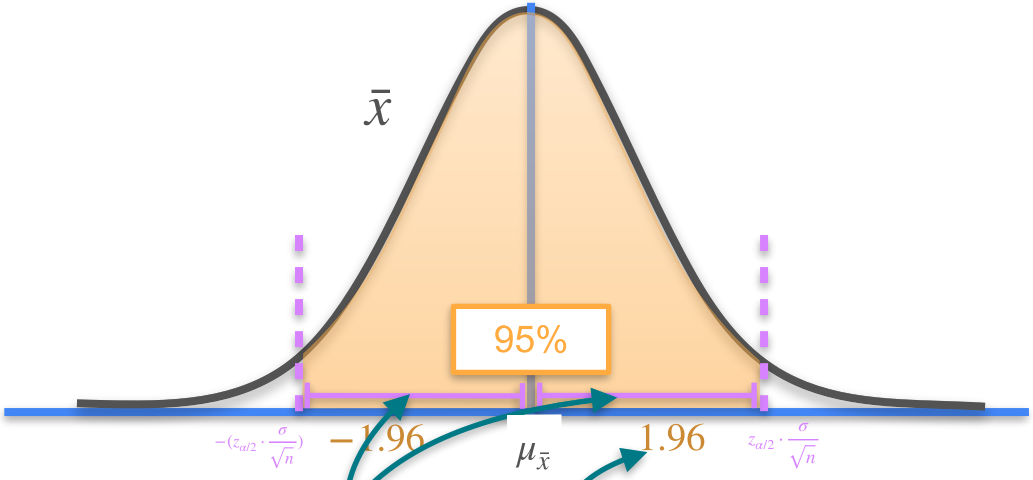
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error

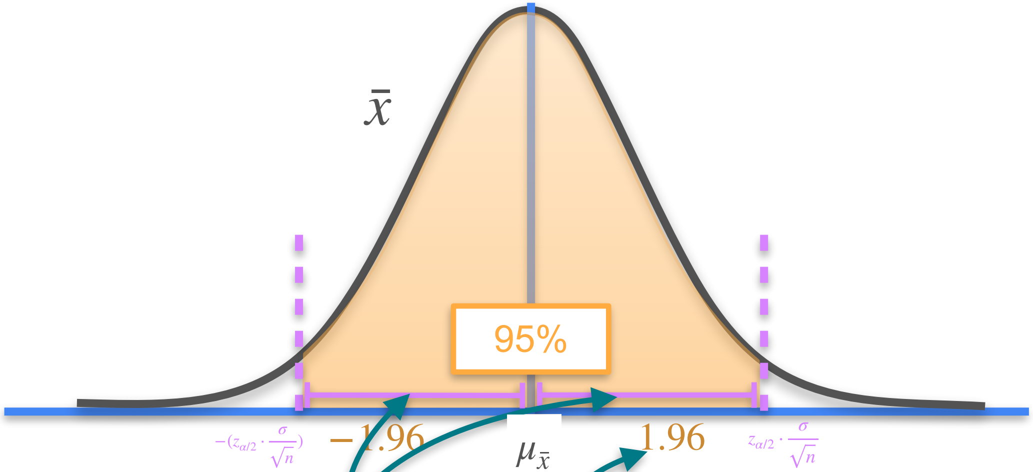


$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

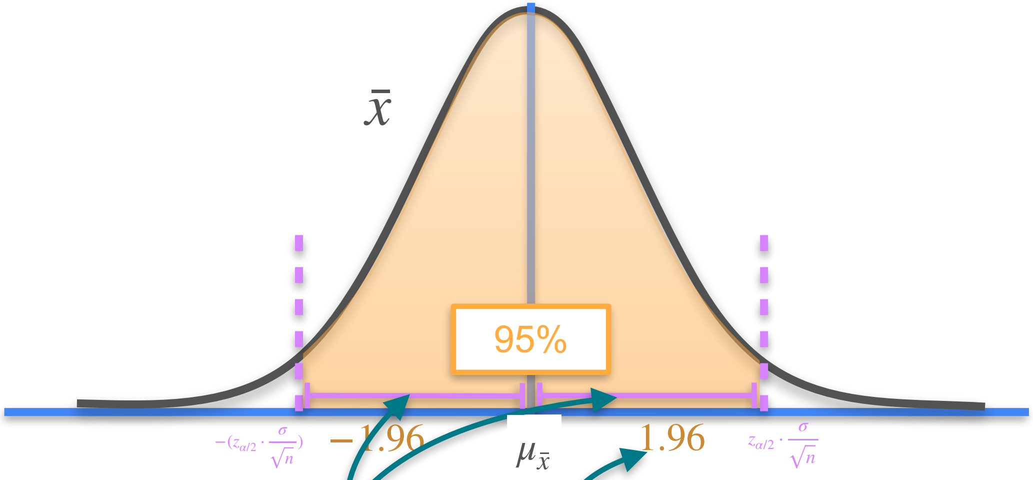


margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level $(1 - \alpha)$
- Get the critical value $(z_{\alpha/2})$
- Find the standard error $(\frac{\sigma}{\sqrt{n}})$
- Find the margin of error
- Add/subtract the margin of error to the sample mean



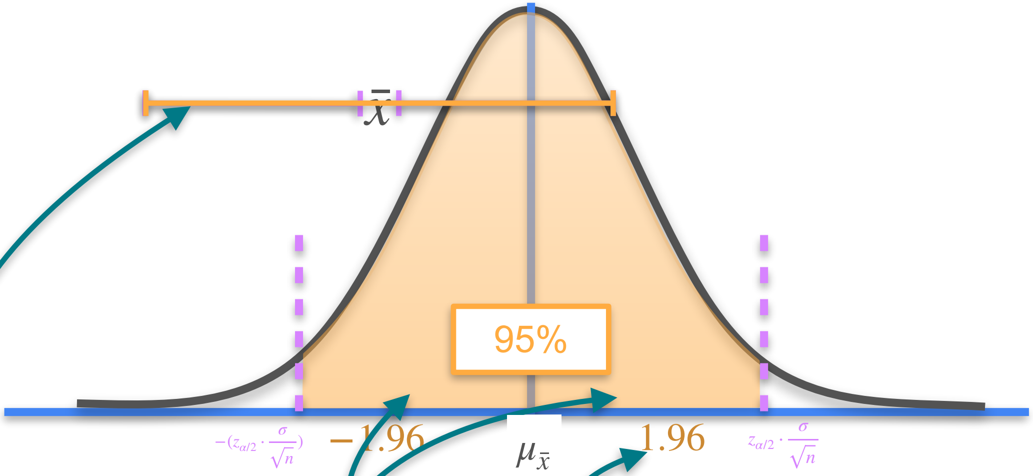
$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level $(1 - \alpha)$
- Get the critical value $(z_{\alpha/2})$
- Find the standard error $(\frac{\sigma}{\sqrt{n}})$
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Assumptions

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample
- Sample size > 30 or population is approximately normal

DeepLearning.AI

Confidence Interval

Confidence Interval - Example

Confidence Interval - Example

Confidence Interval - Example

Statistopia

6,000 adults

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

$\sigma = 25cm$

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

$\sigma = 25cm$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$$\bar{x} = 170cm$$

$$\sigma = 25cm$$

95% → $z_{\alpha/2} = 1.96$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

Confidence Interval - Example

Random Selection

49

$$\sigma = 25cm$$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% → $z_{\alpha/2} = 1.96$

margin of error $=$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% → $z_{\alpha/2} = 1.96$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \bullet \frac{25}{\sqrt{49}}$$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \bullet \frac{25}{\sqrt{49}}$$

$$= 1.96 \bullet \frac{25}{7}$$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \bullet \frac{25}{\sqrt{49}}$$

$$= 1.96 \bullet \frac{25}{7}$$

$$= 7$$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% → $z_{\alpha/2} = 1.96$

Confidence Interval

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \cdot \frac{25}{\sqrt{49}}$$

$$= 1.96 \cdot \frac{25}{7}$$

$$= 7$$

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \bullet \frac{25}{\sqrt{49}}$$

$$= 1.96 \bullet \frac{25}{7}$$

$$= 7$$

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

95% → $z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

$170cm - 7 = 163cm$

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

$170cm - 7 = 163cm$

$170cm + 7 = 177cm$

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

95% → $z_{\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

$170cm - 7 = 163cm$

$170cm + 7 = 177cm$

$163cm < \mu < 177cm$

Confidence Interval

Calculating Sample Size

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Calculating Sample Size

6,000 adults

95% → $z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Random Selection

49

Calculating Sample Size

6,000 adults

Random Selection

Margin of error: 7cm

49

95% → $z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Random Selection

49

Margin of error: 7cm

$\bar{x} \pm 7cm$

Calculating Sample Size

6,000 adults

95% → $z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Random Selection

49

Margin of error: 7cm

$\bar{x} \pm 7cm$

$163cm < \mu < 177cm$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Random Selection

49

Margin of error: 7cm

$\bar{x} \pm 7cm$

$163cm < \mu < 177cm$

Calculating Sample Size

6,000 adults

95% → $z_{\alpha/2} = 1.96$

$\bar{x} = 170cm$ $\sigma = 25cm$

Random Selection

49

Margin of error: 7cm

$\bar{x} \pm 7cm$

$163cm < \mu < 177cm$

Margin of error: 3 cm

$\bar{x} \pm 3cm$

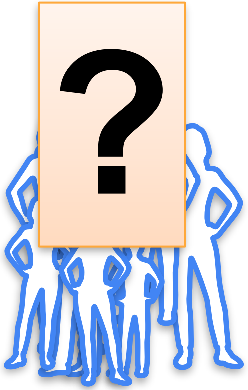
Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Random Selection



Margin of error: 7cm

$\bar{x} \pm 7cm$

$163cm < \mu < 177cm$

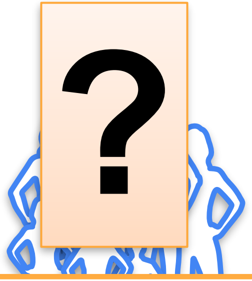
Margin of error: 3 cm

$\bar{x} \pm 3cm$

Calculating Sample Size

6,000 adults

Random Selection



Margin of error: 7cm

$\bar{x} \pm 7cm$

$163cm < \mu < 177cm$

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm$ σ

What is the smallest sample size to obtain the desired margin of error?

Margin of error: 3 cm

$\bar{x} \pm 3cm$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$3$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$3 =$$

$\bar{x} = 170cm$ $\sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$3 = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = 170cm$ $\sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3$$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq$$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

95% → $z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

$$3 \geq 1.96 \times \frac{25}{\sqrt{n}}$$

Margin of error: 3 cm

Calculating Sample Size

$\boxed{\text{6,000 adults}}$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

$$\frac{3}{1.96} \geq \frac{25}{\sqrt{n}}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{3}{1.96 \times 25} \geq \frac{1}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

Calculating Sample Size

$\boxed{\text{6,000 adults}}$

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

$$\left(\frac{1.96 \times 25}{3}\right)^2$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

$$\left(\frac{1.96 \times 25}{3}\right)^2 \leq n$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$$

Calculating Sample Size

6,000 adults

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

95% $\rightarrow z_{\alpha/2} = 1.96$

$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% $\rightarrow z_{\alpha/2} = 1.96$

$$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$$

$\bar{x} = 170cm \qquad \sigma = 25cm$

$$n \geq 266.78 \approx 267$$

Margin of error: 3 cm

Calculating Sample Size

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$$

Calculating Sample Size

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$$

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{MOE}\right)^2$$

Confidence Interval

Difference Between Confidence and Probability

Difference Between Confidence and Probability

Difference Between Confidence and Probability

$\bar{x}$

Difference Between Confidence and Probability

95% Confidence Level

Difference Between Confidence and Probability

95% Confidence Level

The confidence interval contains the true population parameter approximately 95% of the time.

Difference Between Confidence and Probability

95% Confidence Level

The confidence interval contains the true population parameter approximately 95% of the time.

Difference Between Confidence and Probability

95% Confidence Level

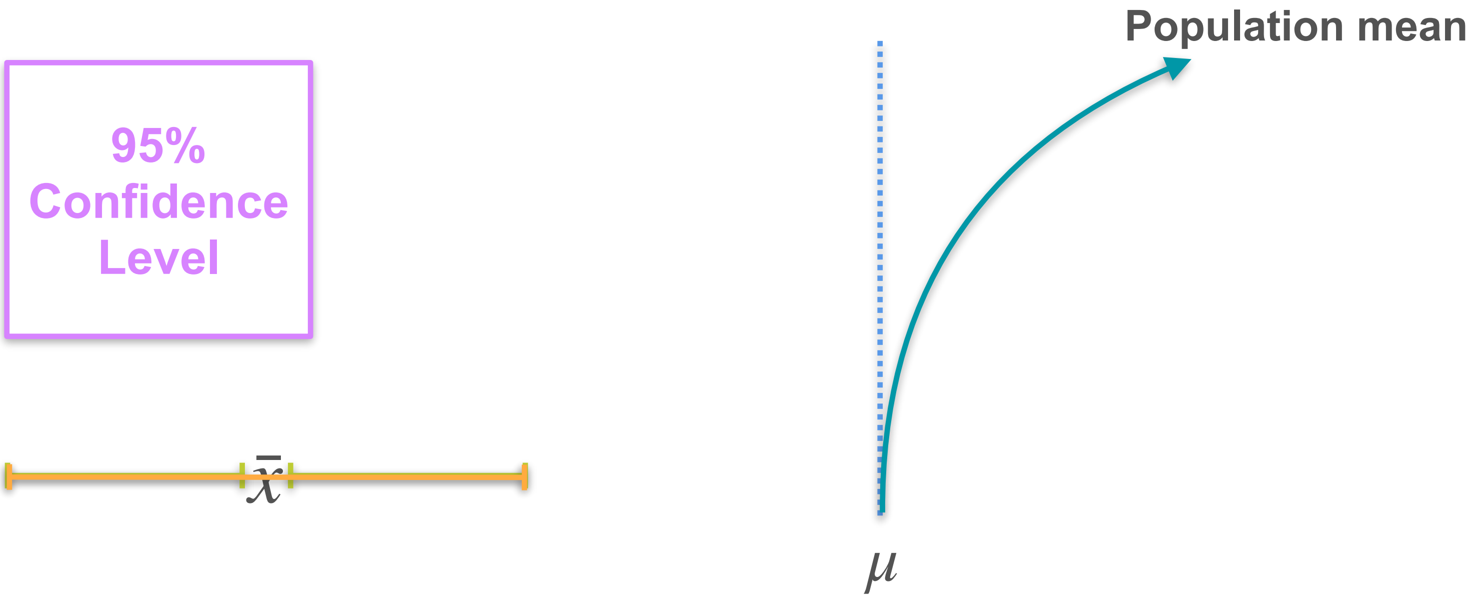
The confidence interval contains the true population parameter approximately 95% of the time.

There's a 95% probability that the population parameter falls within the confidence interval.

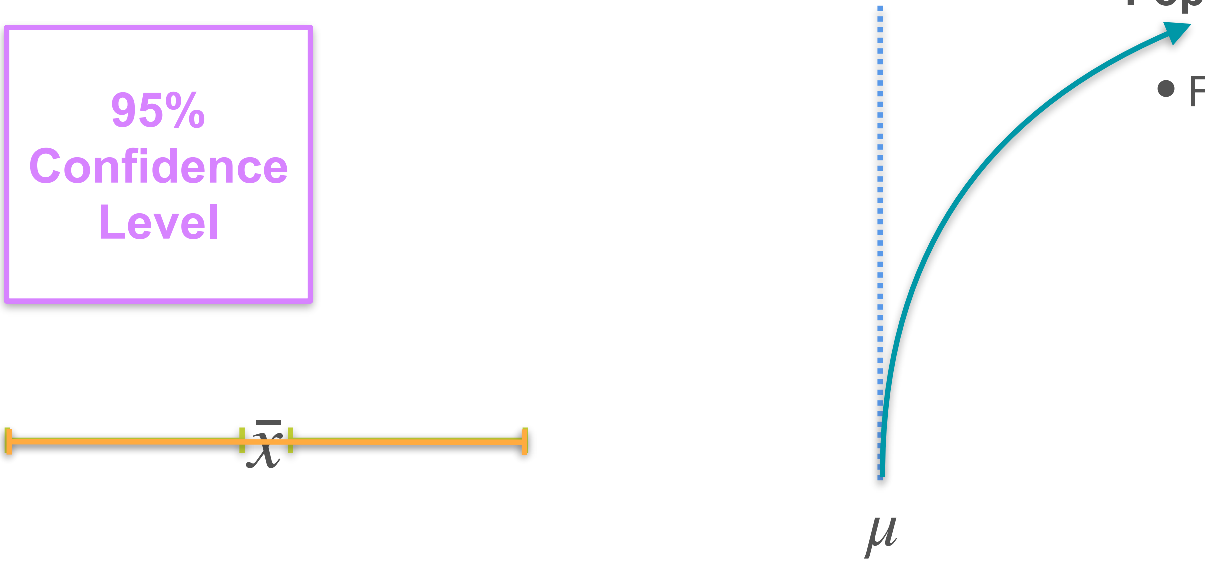
Difference Between Confidence and Probability

95% Confidence Level

Difference Between Confidence and Probability



Difference Between Confidence and Probability

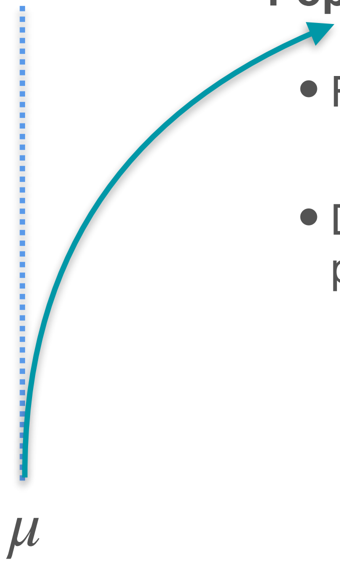


Population mean

- Fixed but unknown

Difference Between Confidence and Probability

95% Confidence Level

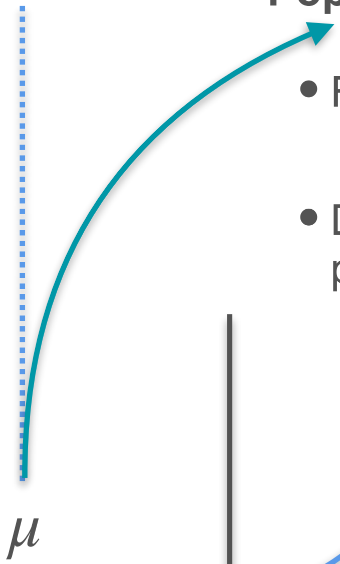


Population mean

- Fixed but unknown
- Does not have a probability distribution

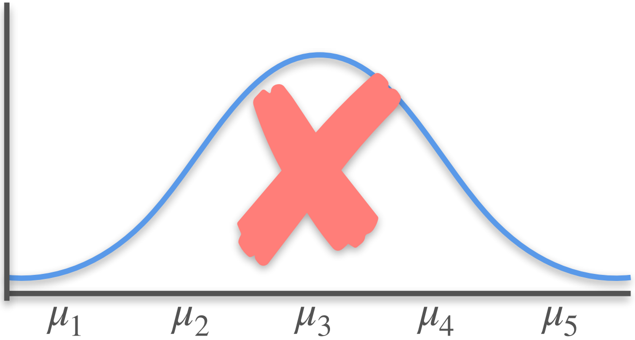
Difference Between Confidence and Probability

95% Confidence Level



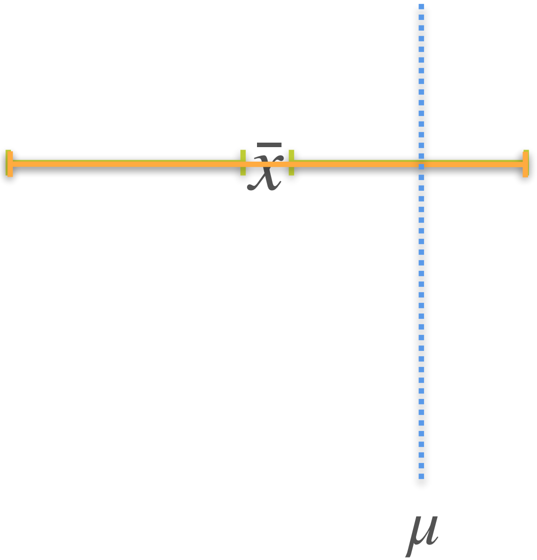
Population mean

- Fixed but unknown
- Does not have a probability distribution



Difference Between Confidence and Probability

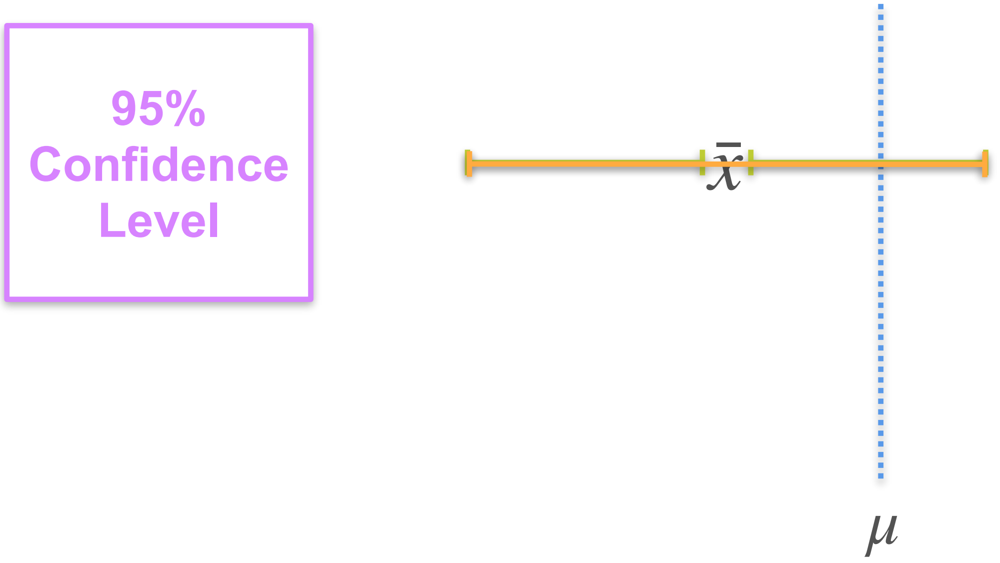
95% Confidence Level



Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval

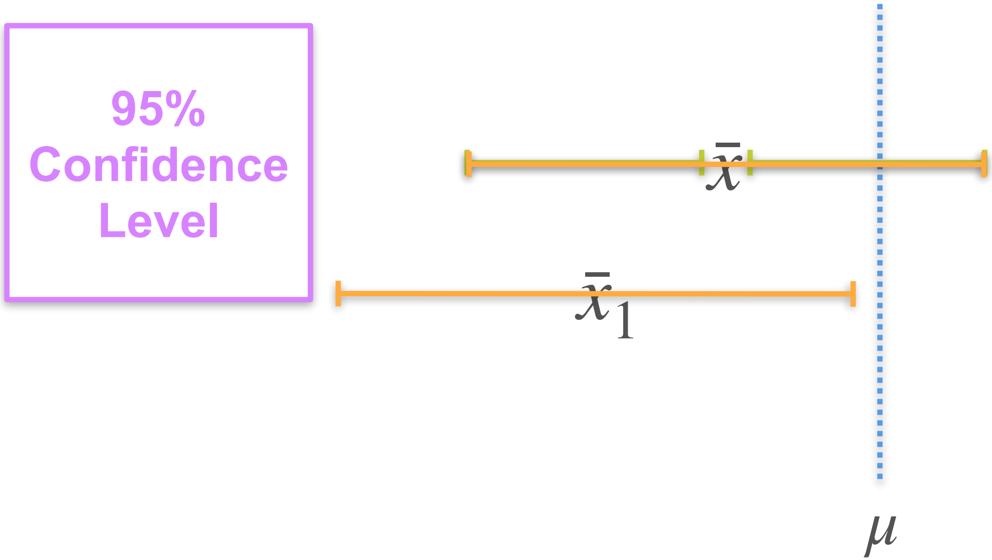
Difference Between Confidence and Probability



Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

Difference Between Confidence and Probability

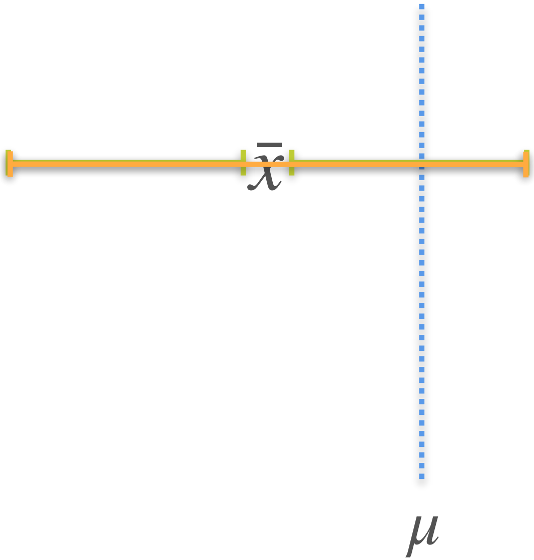


Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

Difference Between Confidence and Probability

95% Confidence Level

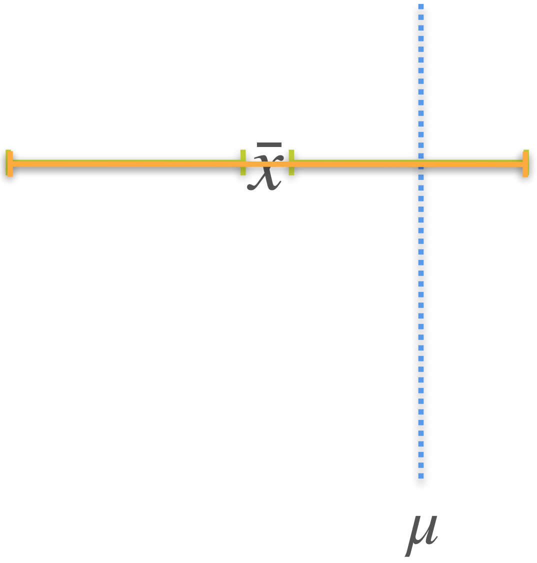


Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

Difference Between Confidence and Probability

95% Confidence Level



Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

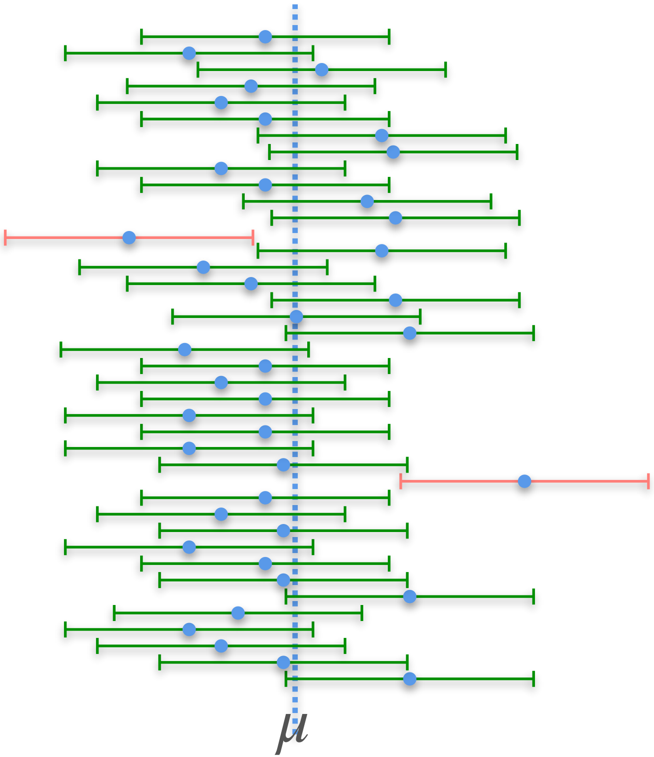
Difference Between Confidence and Probability

95% Confidence Level

μ

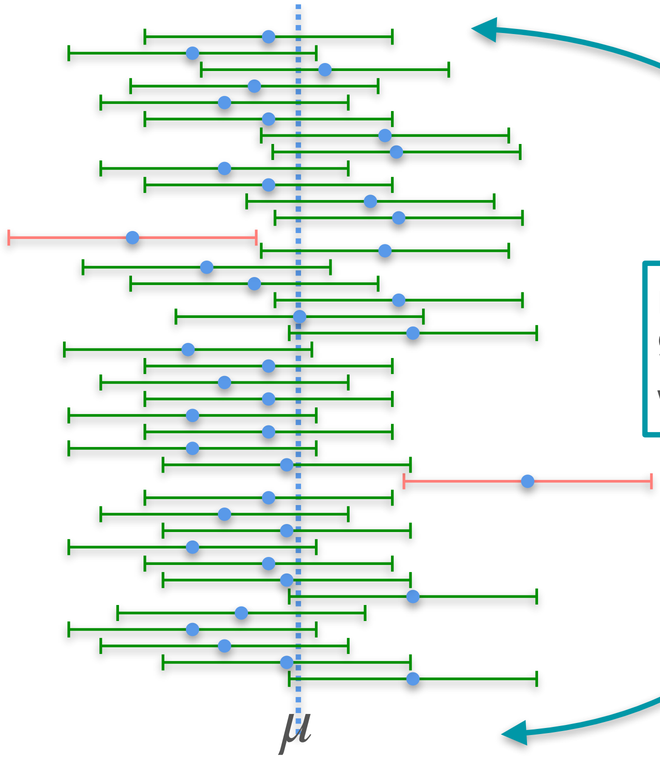
Difference Between Confidence and Probability

95% Confidence Level



Difference Between Confidence and Probability

95% Confidence Level

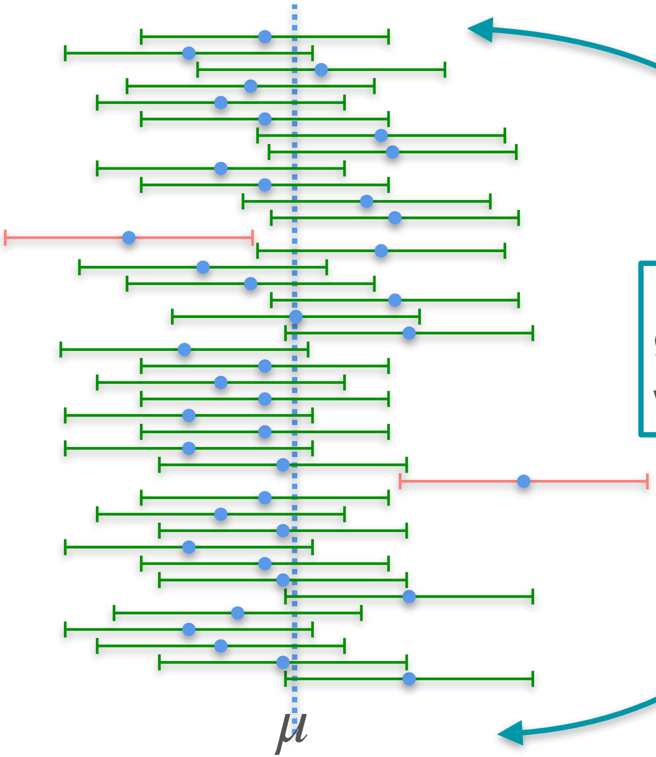


repeated many times, 95% of these intervals will contain μ

Difference Between Confidence and Probability

95% Confidence Level

success rate for constructing the confidence interval

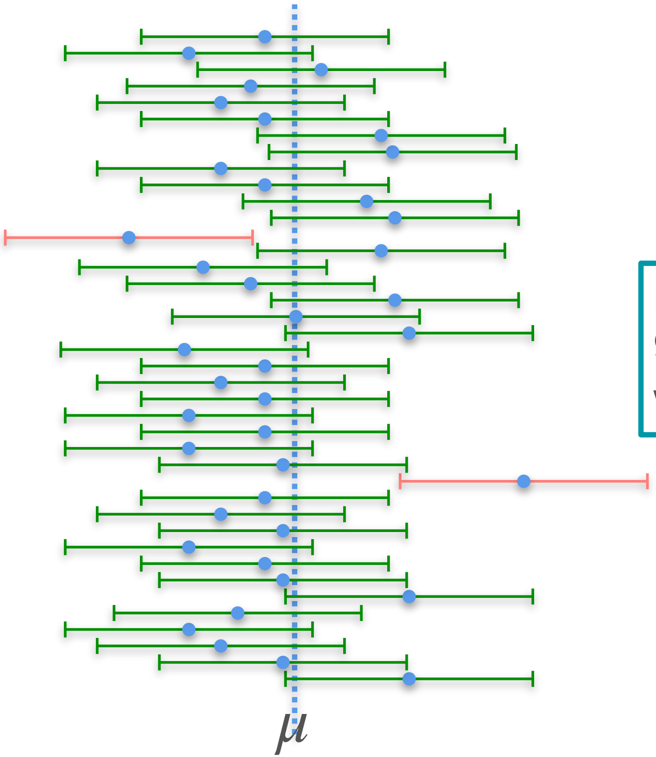


repeated many times, 95% of these intervals will contain μ

Difference Between Confidence and Probability

95% Confidence Level

success rate for constructing the confidence interval

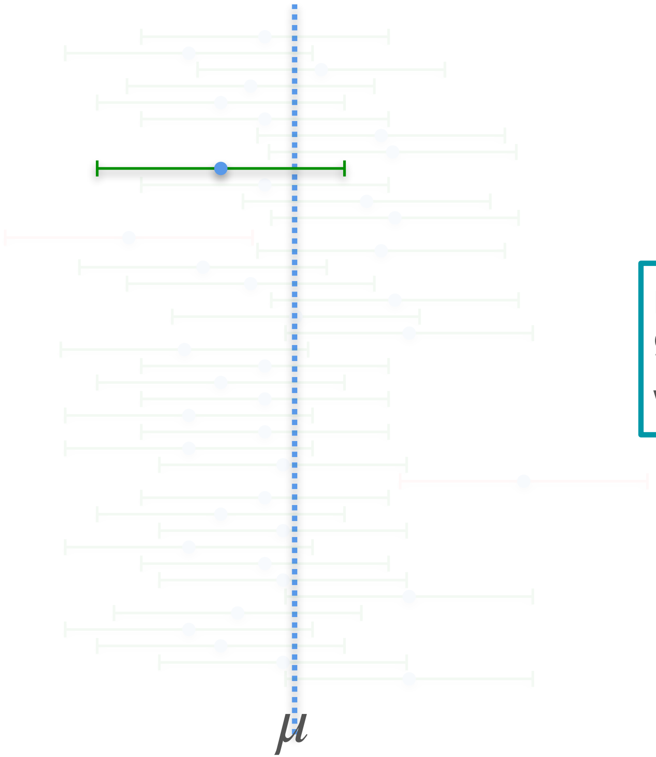


repeated many times, 95% of these intervals will contain μ

Difference Between Confidence and Probability

95% Confidence Level

success rate for constructing the confidence interval

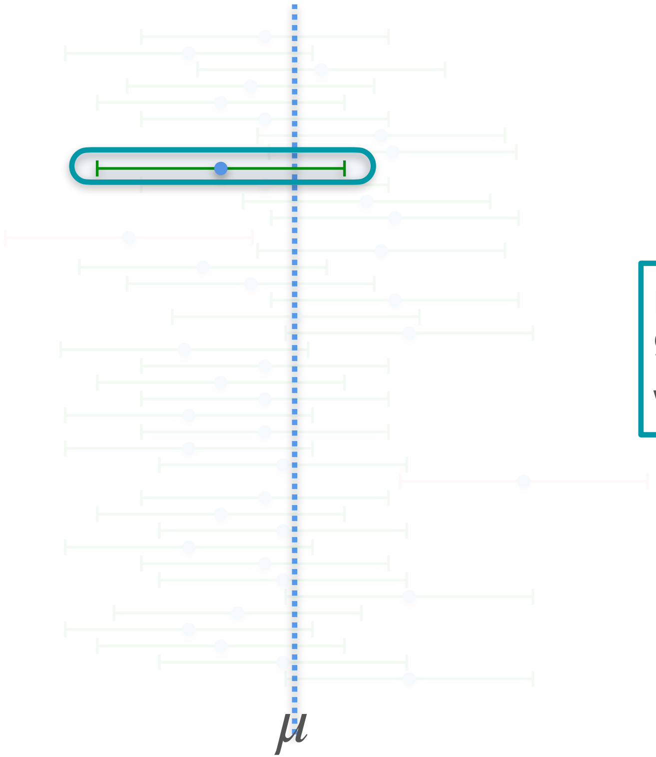


repeated many times, 95% of these intervals will contain μ

Difference Between Confidence and Probability

95% Confidence Level

success rate for constructing the confidence interval



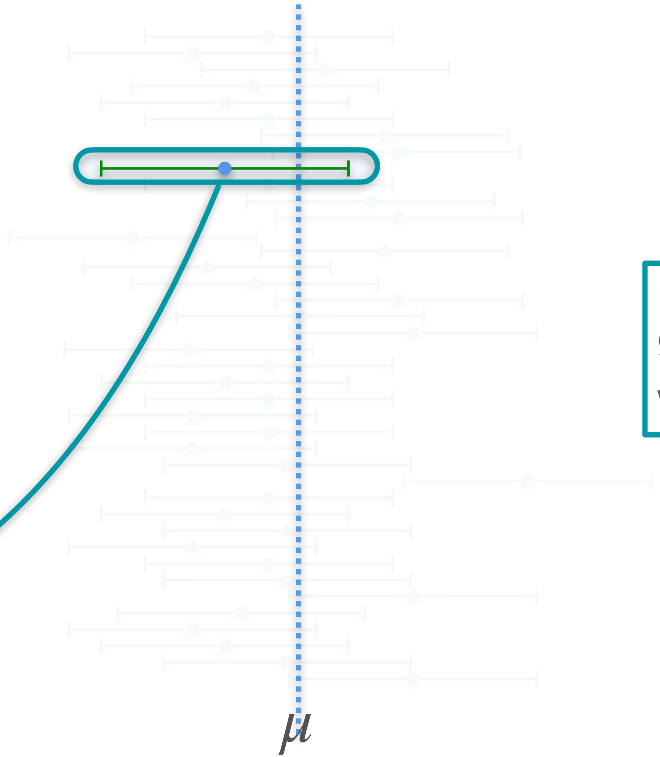
repeated many times, 95% of these intervals will contain μ

Difference Between Confidence and Probability

95% Confidence Level

success rate for constructing the confidence interval

not the probability that one specific intervals contains the population mean



repeated many times, 95% of these intervals will contain μ

DeepLearning.AI

Confidence Interval

Confidence Interval (Unknown Standard Deviation)

Confidence Interval - t Distribution

Confidence Interval - t Distribution

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \quad \pm \quad z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

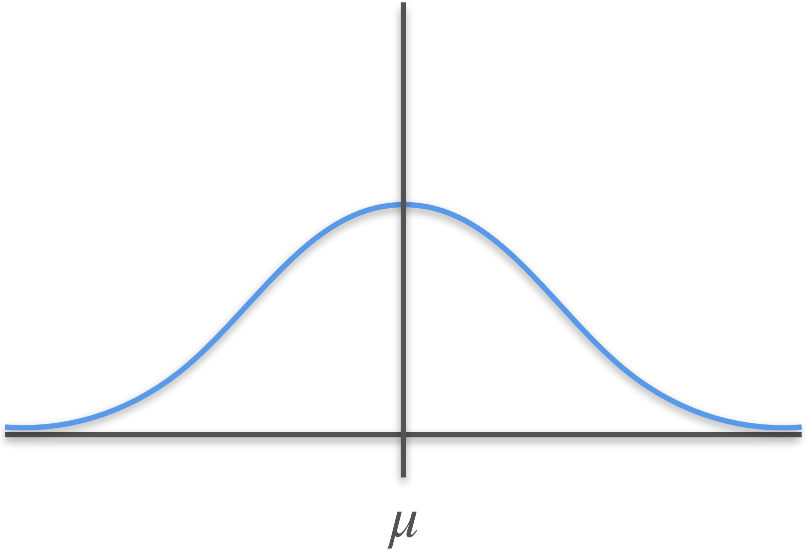
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution



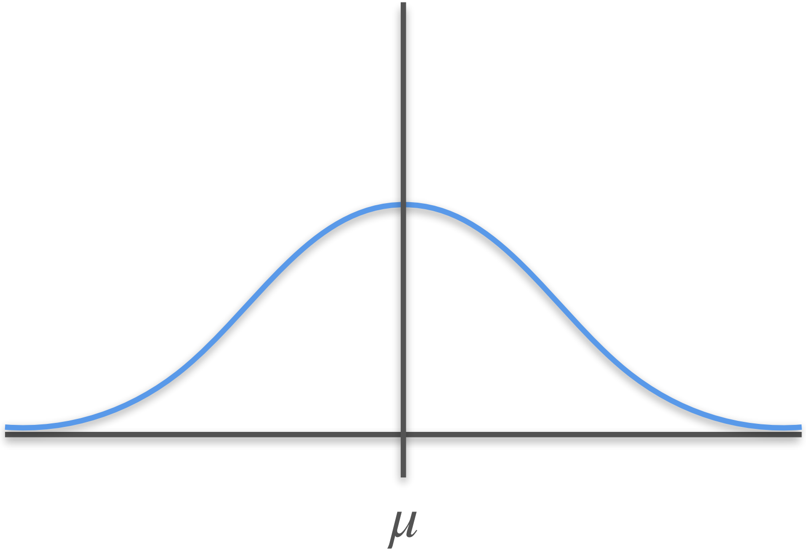
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution



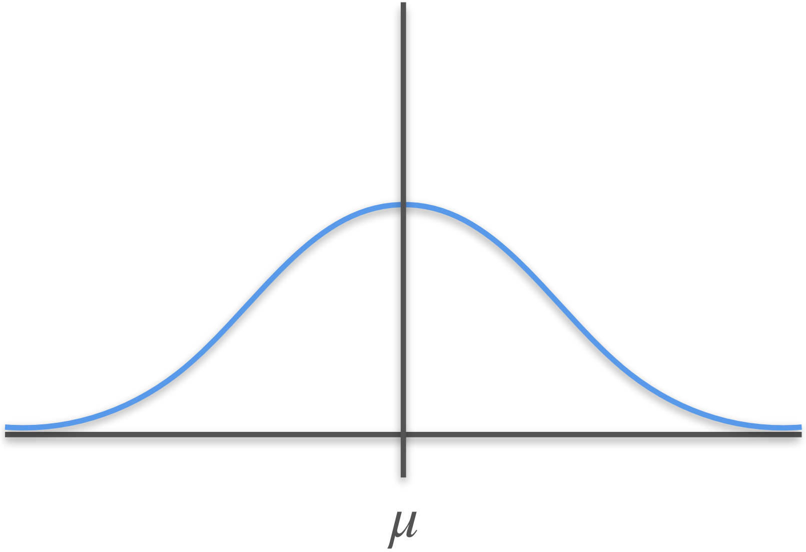
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution



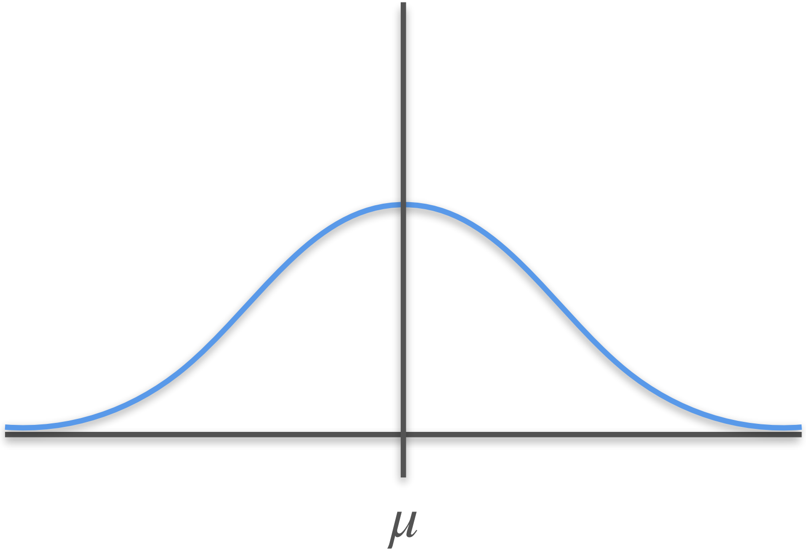
Confidence Interval - t Distribution

known $\boxed{\sigma}$?

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution



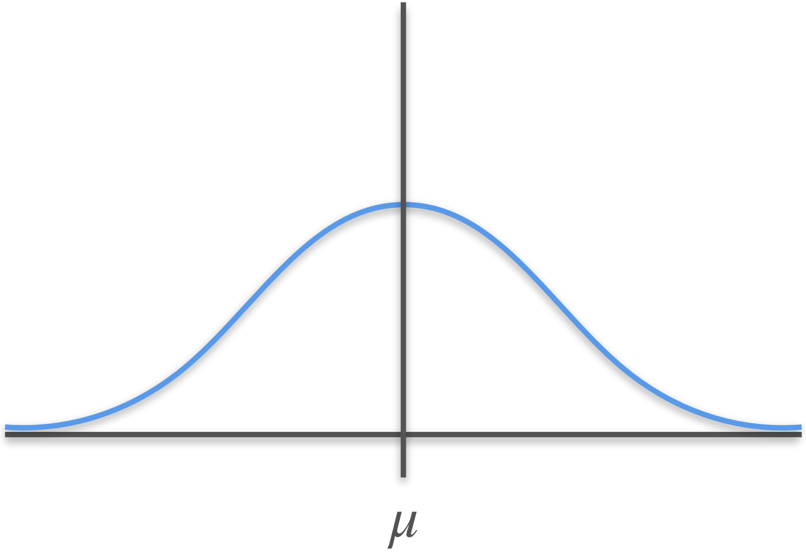
Confidence Interval - t Distribution

known σ ? $\rightarrow$ s

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution



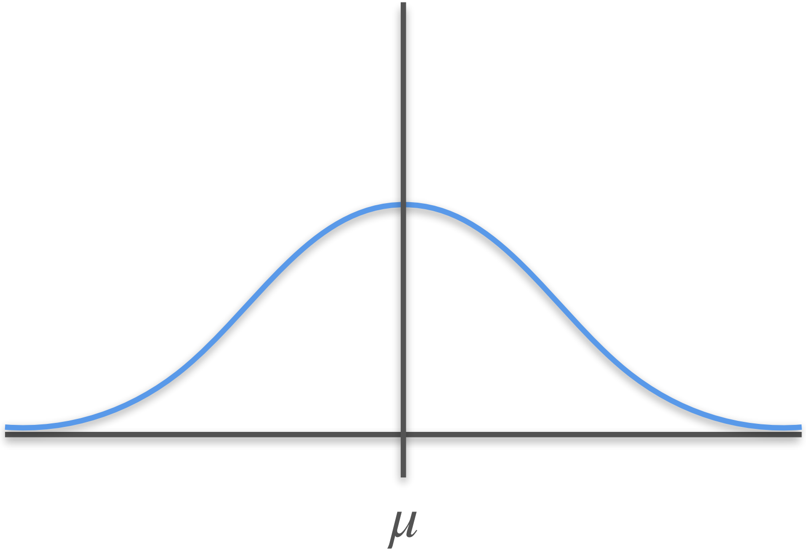
Confidence Interval - t Distribution

known $\boxed{\sigma}$? $\longrightarrow$ s

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \dashrightarrow \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution



Confidence Interval - t Distribution

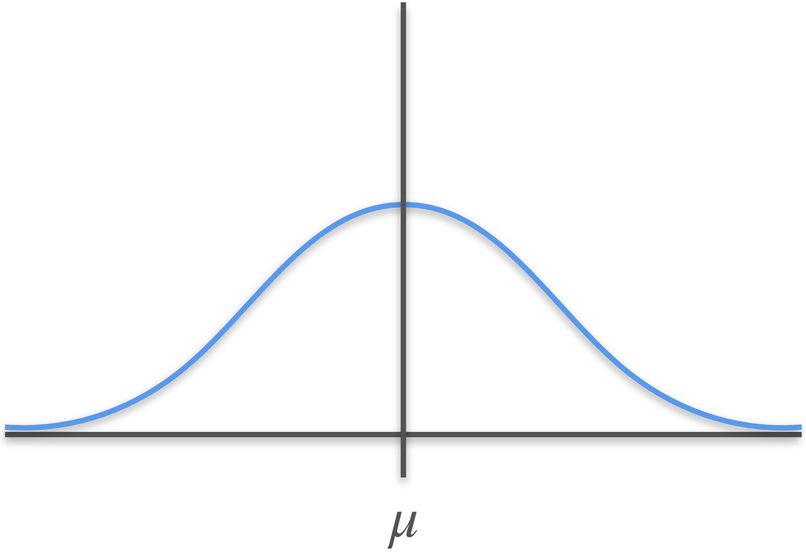
known $\boxed{\sigma}$? $\longrightarrow$ s

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \longrightarrow \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution



Confidence Interval - t Distribution

known $\boxed{\sigma}$? $\longrightarrow$ s

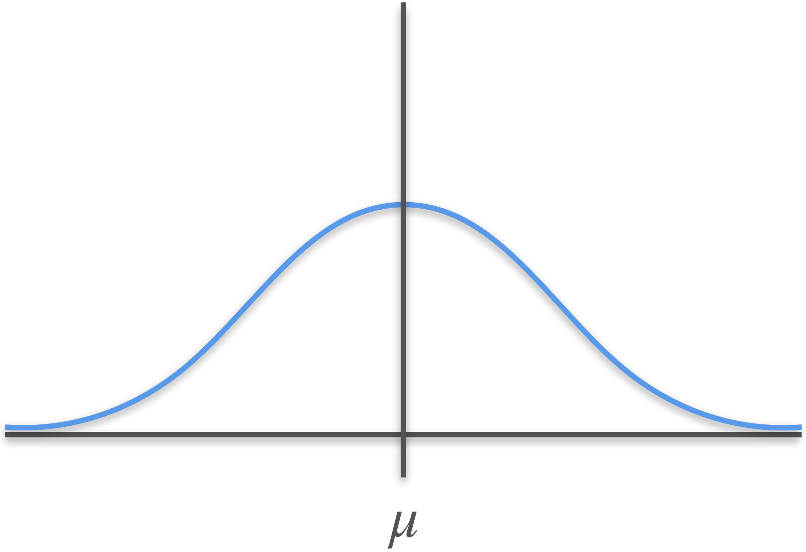
$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \longrightarrow \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution

student's t distribution



Confidence Interval - t Distribution

known $\boxed{\sigma}$? $\longrightarrow s$

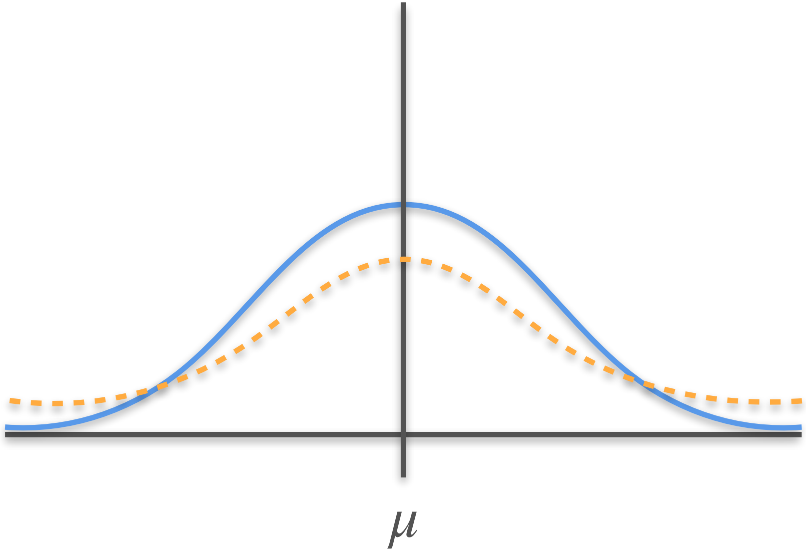
$$\bar{x} \quad \pm \boxed{z_{\alpha/2}} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \dashrightarrow \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution

student's t distribution



Confidence Interval - t Distribution

known $\boxed{\sigma}$? $\longrightarrow s$

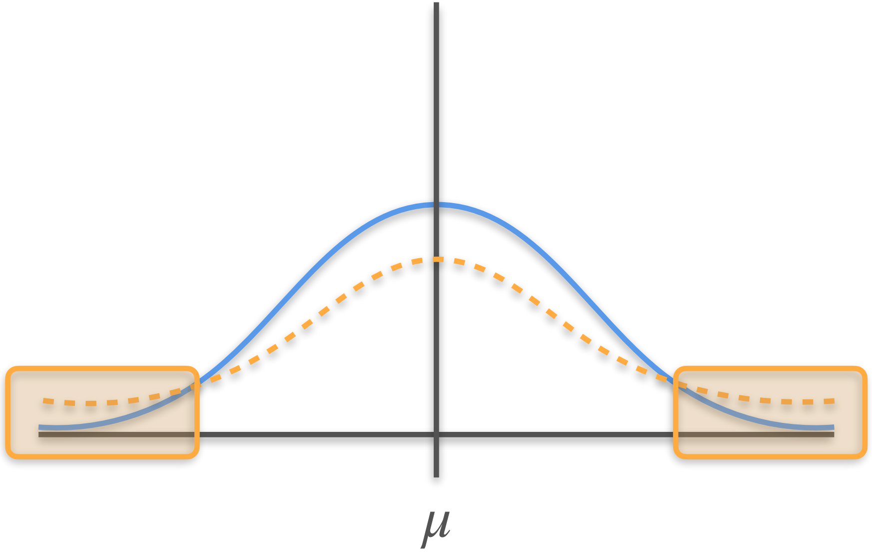
$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \quad \longrightarrow \quad \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution

student's t distribution



Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

$$\bar{x} \quad \pm \quad z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

$$\bar{x} \quad \pm \quad z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

$$\bar{x} \quad \pm \quad z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \quad \pm \quad z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ → replace with s

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ → replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ $\rightarrow$ replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \bullet \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ replace with s

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \quad \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ → replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ → replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ replace with s

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \qquad \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \bullet \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm \boxed{z_{\alpha/2}} \cdot \frac{\boxed{\sigma}}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{\boxed{s}}{\sqrt{n}}$$

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \bullet \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$

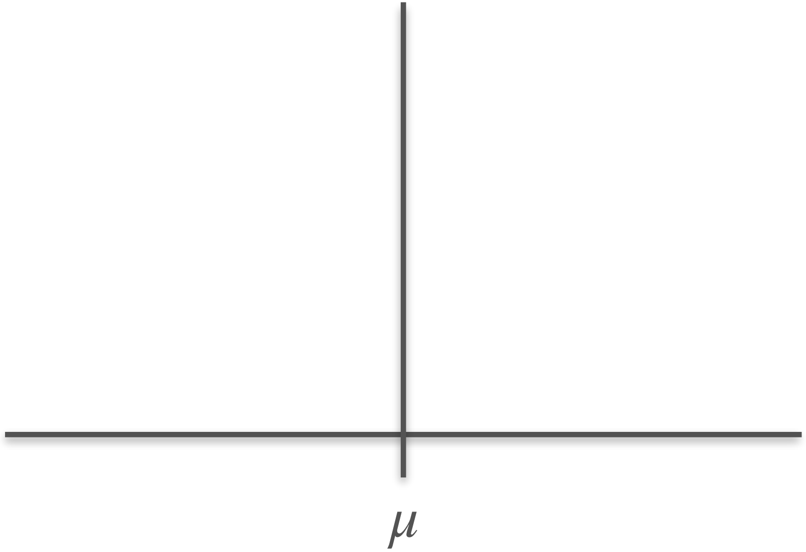
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \bullet \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



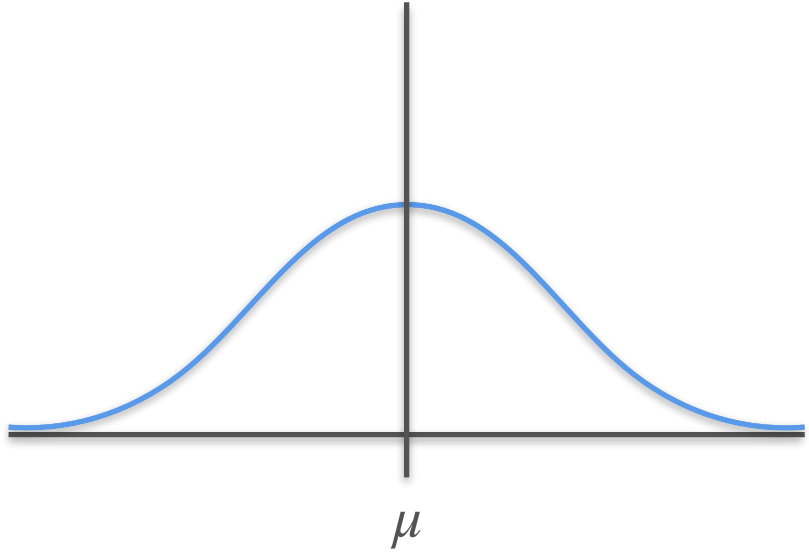
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \bullet \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



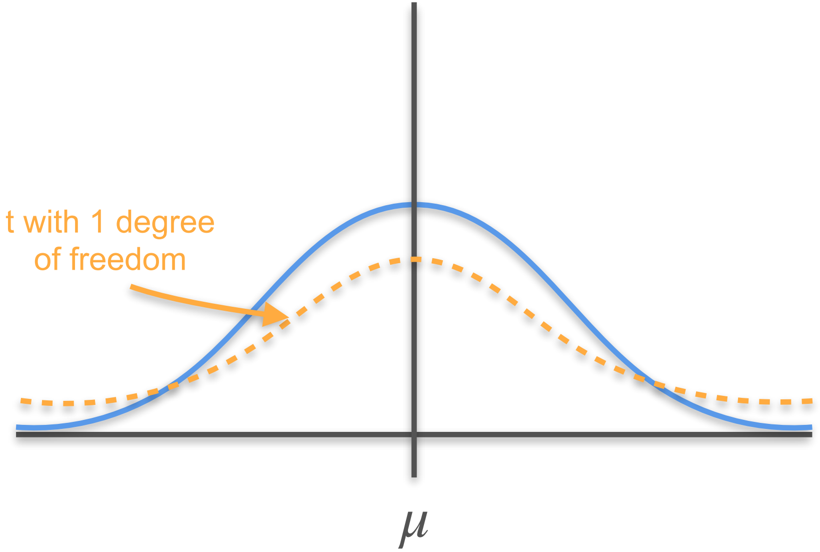
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



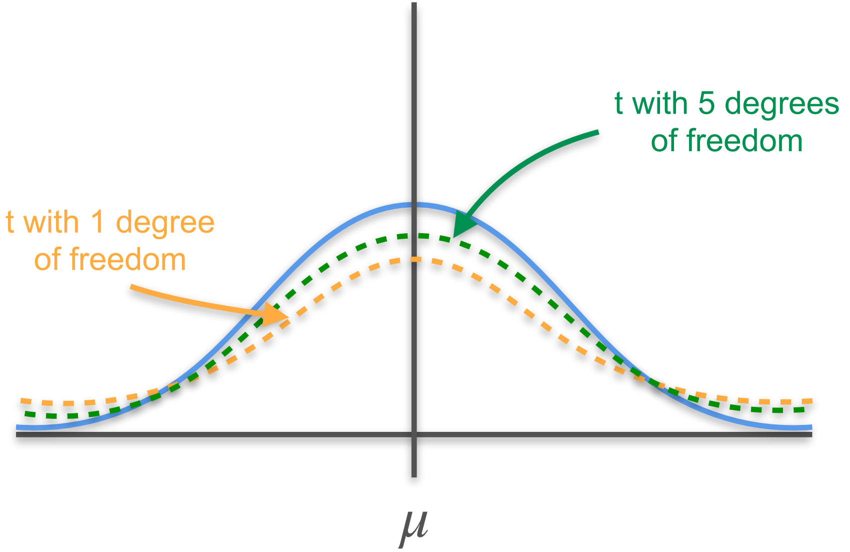
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



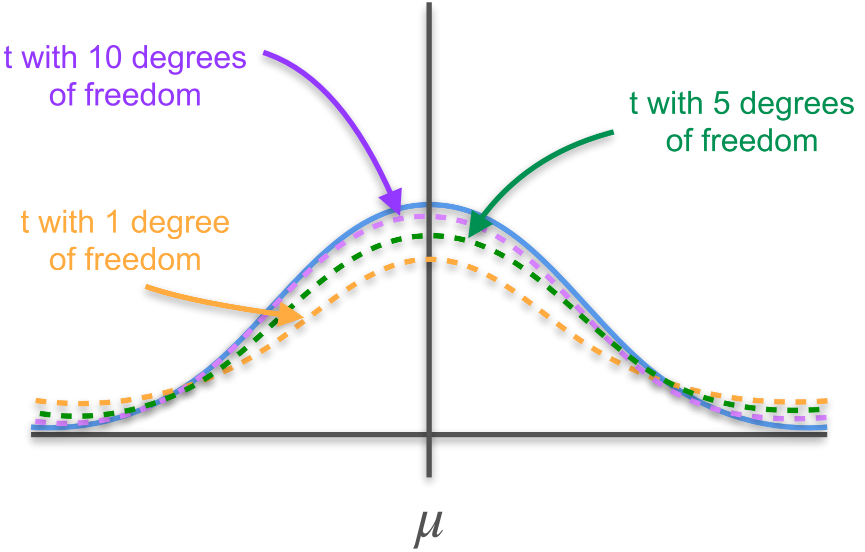
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm \boxed{t_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



DeepLearning.AI

Confidence Interval

Confidence Intervals for Proportion

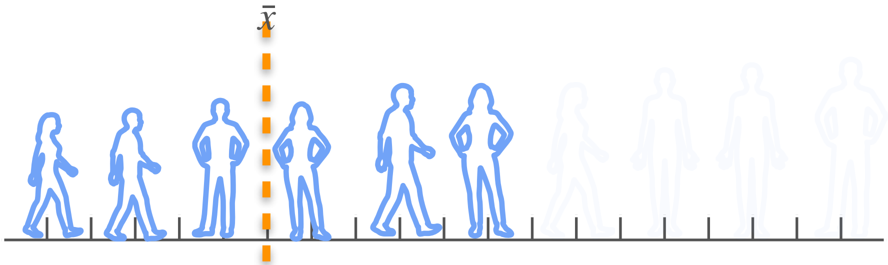
Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Means

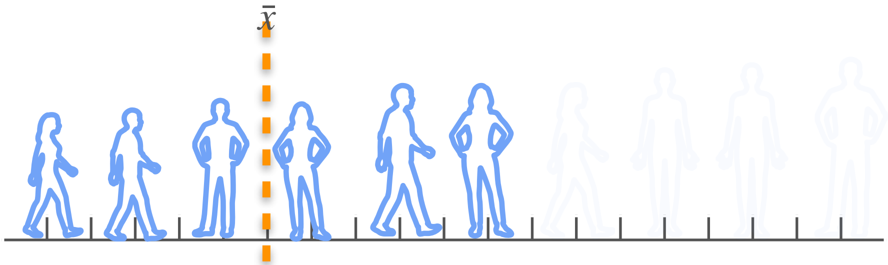
Confidence Interval for Proportions

Confidence Interval for Means



Confidence Interval for Proportions

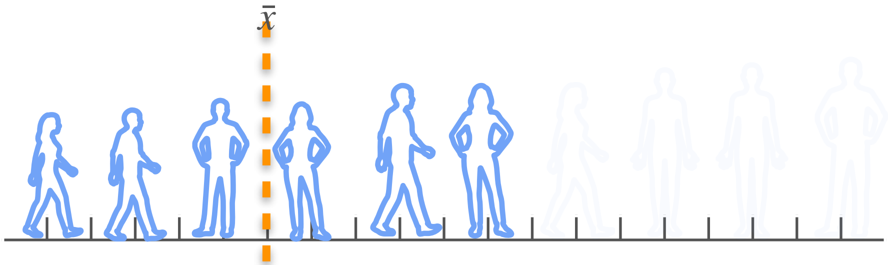
Confidence Interval for Means



confidence interval $=$ $\bar{x} \pm$ margin of error

Confidence Interval for Proportions

Confidence Interval for Means

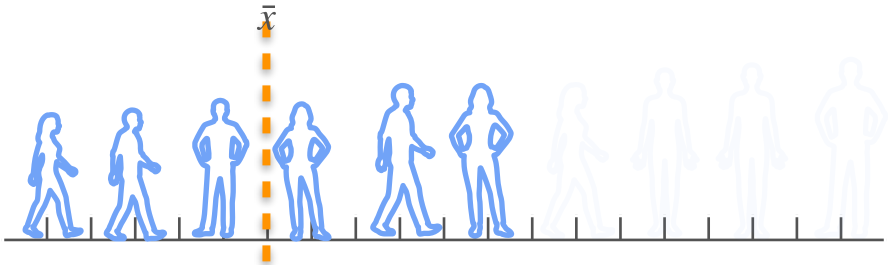


confidence interval $= \bar{x} \pm$ margin of error

margin of error $=$

Confidence Interval for Proportions

Confidence Interval for Means

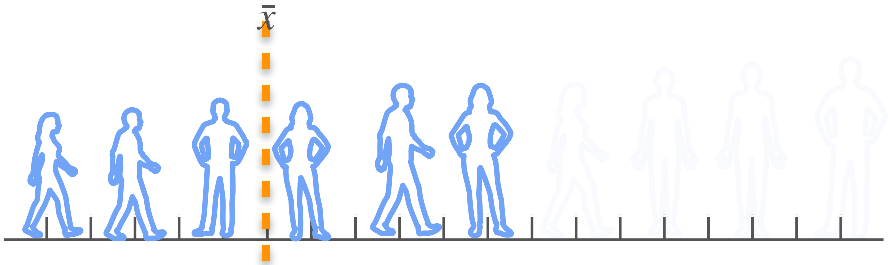


confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

Confidence Interval for Means

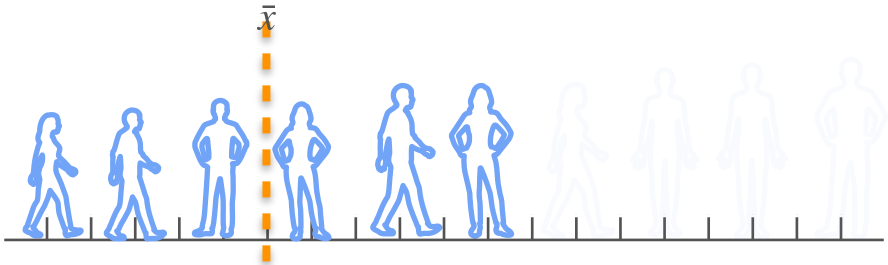


confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

Confidence Interval for Means



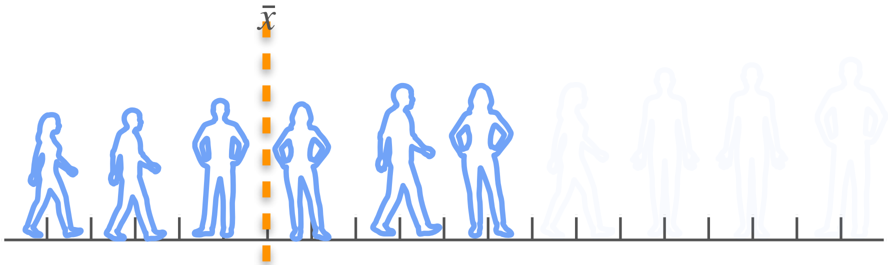
confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

Confidence Interval for Proportions

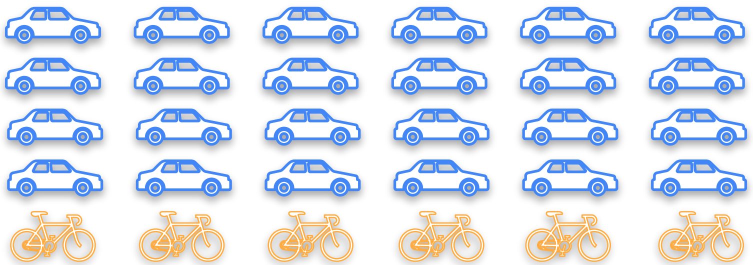
Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

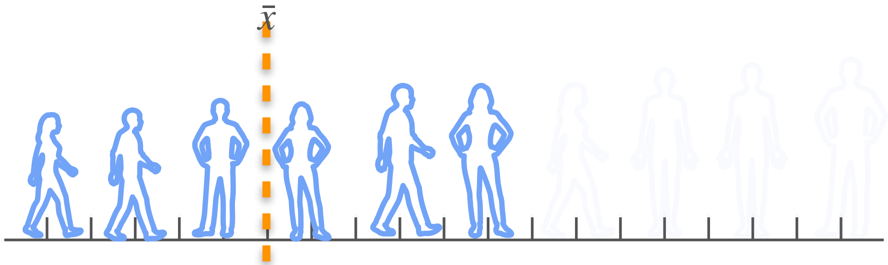
margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions



Confidence Interval for Proportions

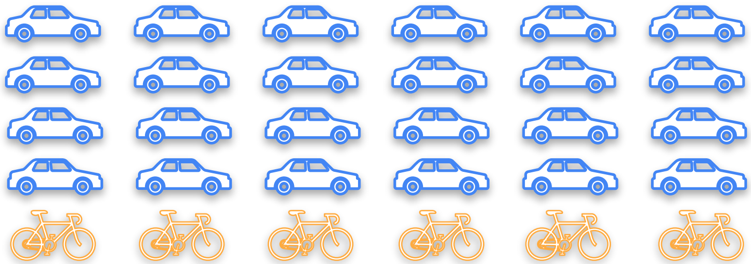
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

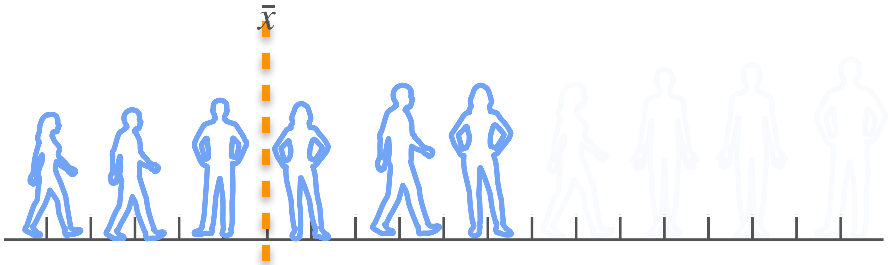
Confidence Interval for Proportions



$$n = 30$$

Confidence Interval for Proportions

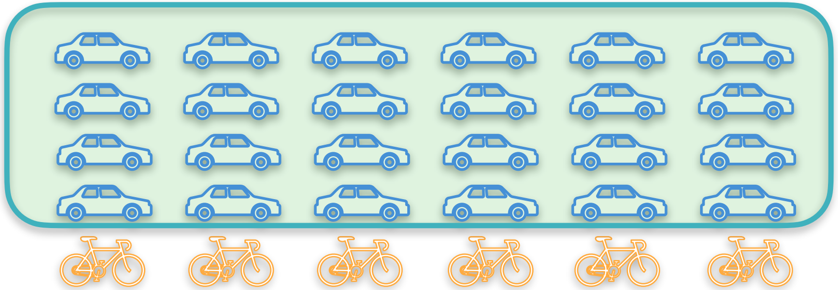
Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$

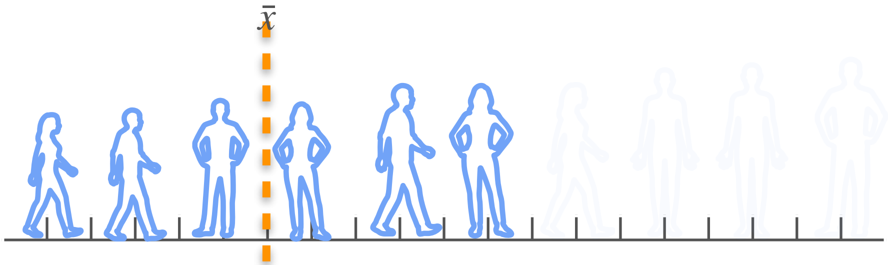
Confidence Interval for Proportions



$n = 30$

Confidence Interval for Proportions

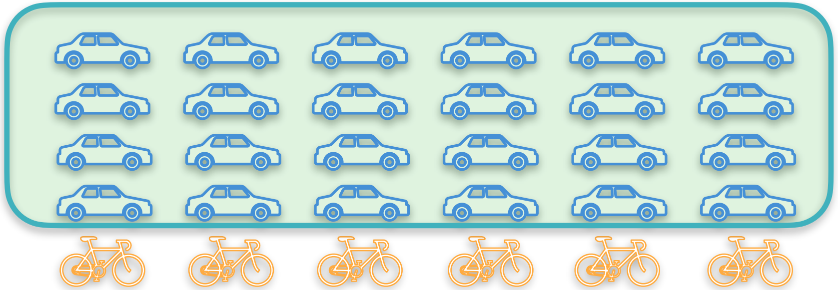
Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

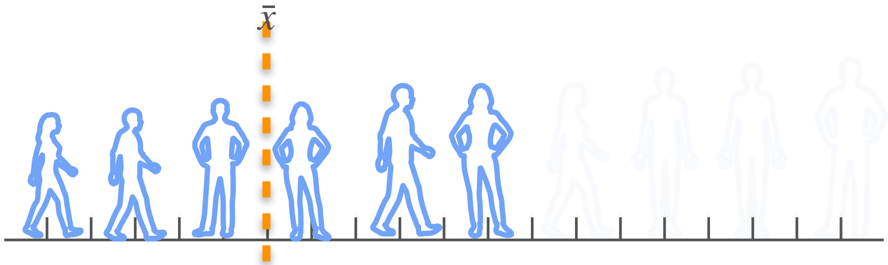


$x = 24$

$n = 30$

Confidence Interval for Proportions

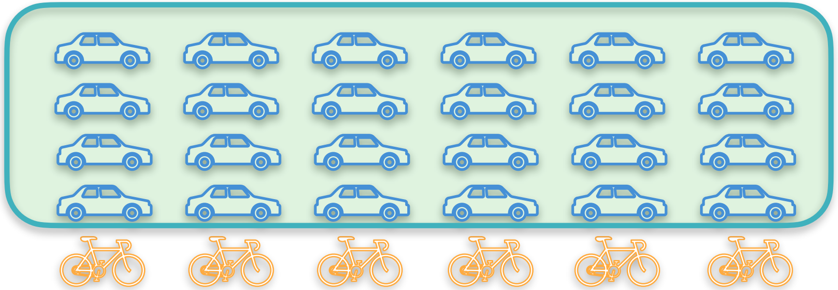
Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions



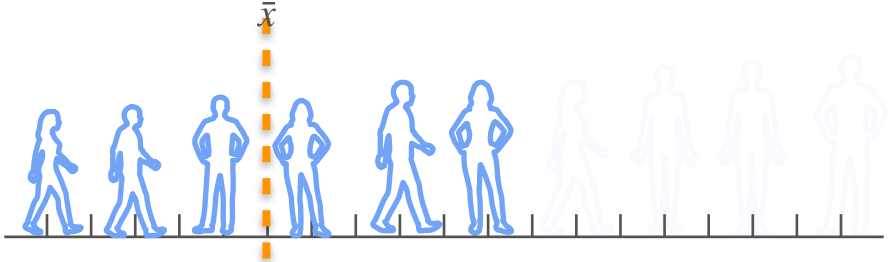
$x = 24$

$n = 30$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

Confidence Interval for Proportions

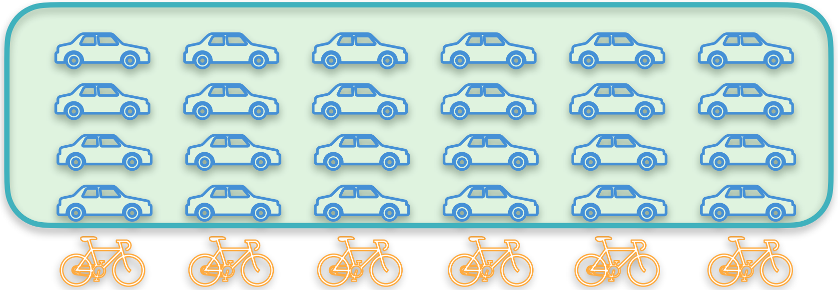
Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions



$x = 24$

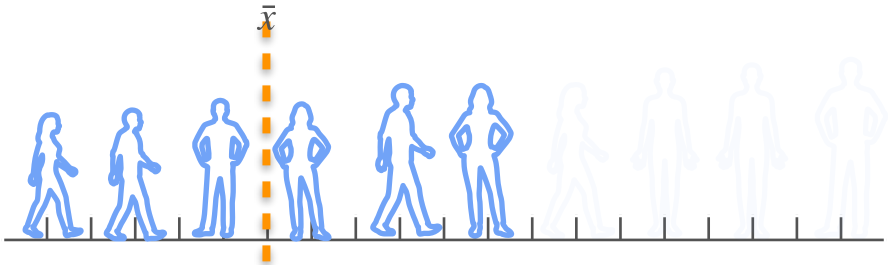
$n = 30$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

How do you calculate a 95% confidence interval for this sample proportion?

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

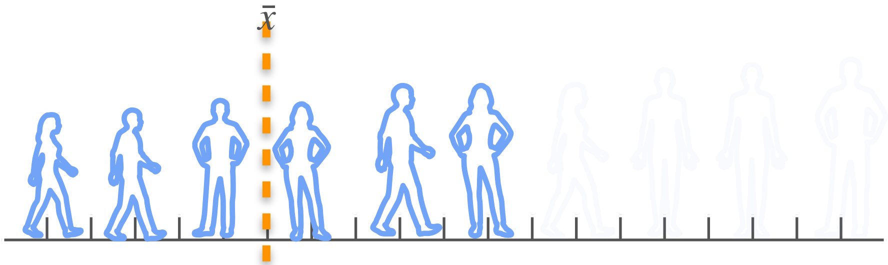
margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $=$ $\bar{x} \pm$ margin of error

margin of error $=$ $z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

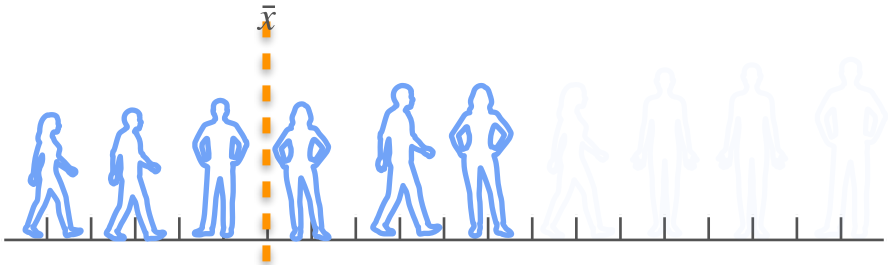
Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $=$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

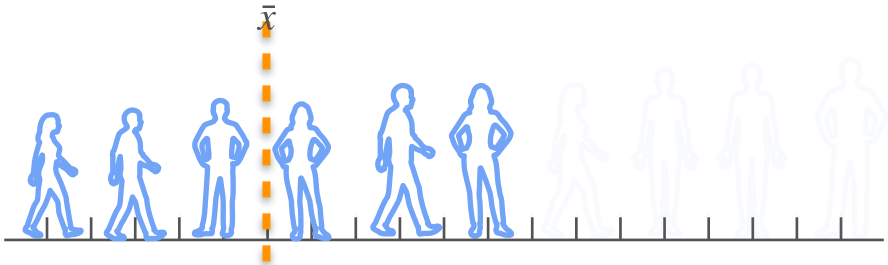
Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p}$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

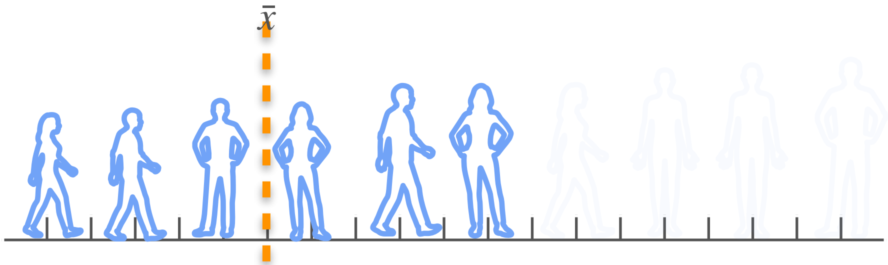
Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

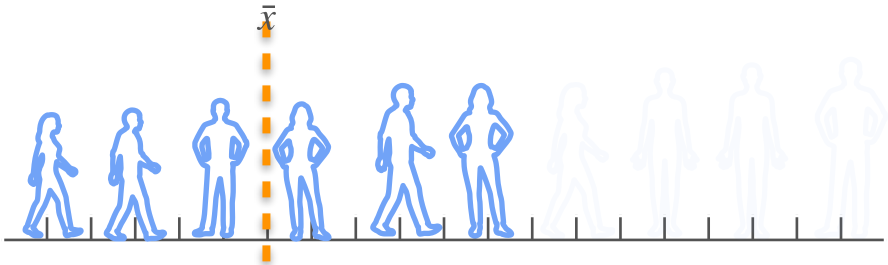
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $=$

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

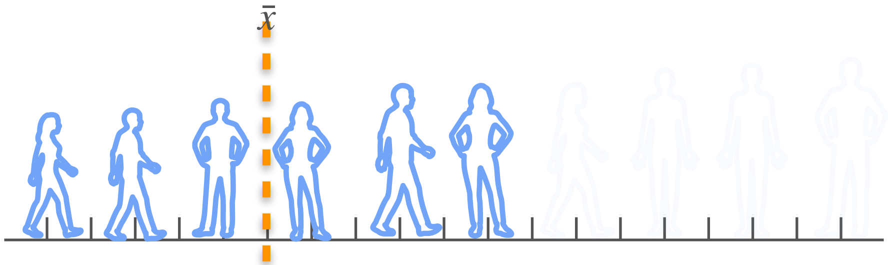
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet$$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

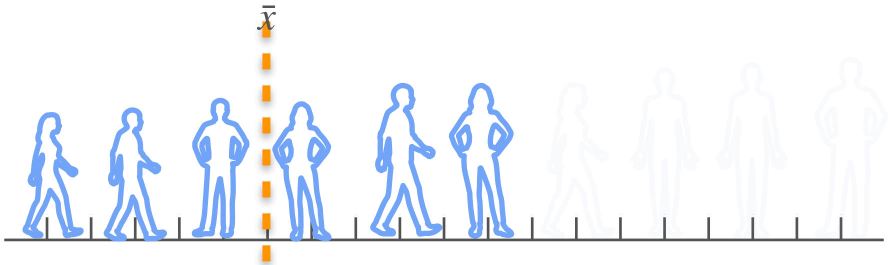
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= \boxed{z_{\alpha/2}} \cdot \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

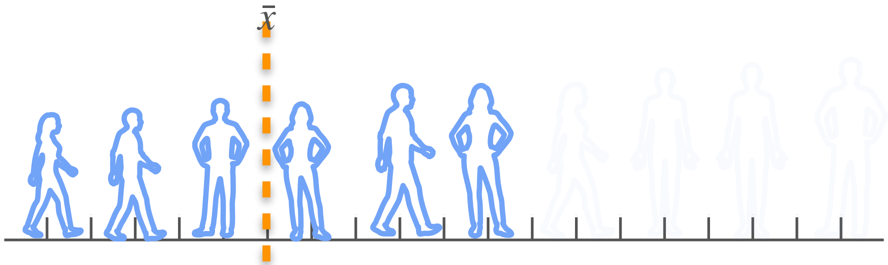
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= \boxed{z_{\alpha/2}} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \dfrac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

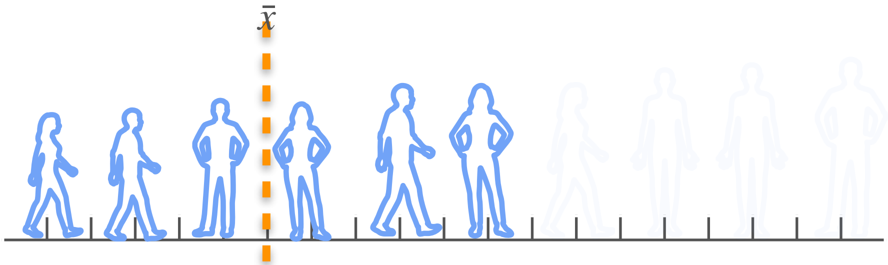
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \boxed{\dfrac{\sigma}{\sqrt{n}}}$

Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot \boxed{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}}$

standard error

Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $=$

Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p}$$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $=$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \cdot$

Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

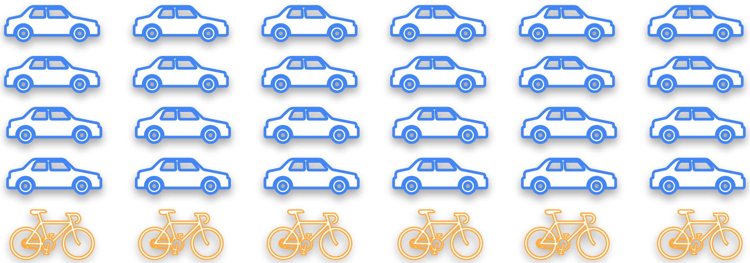
$$\text{margin of error} = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

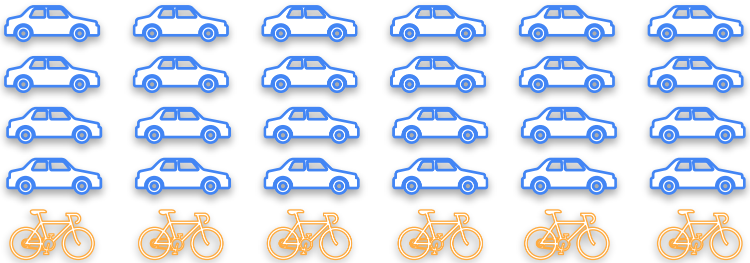


Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$



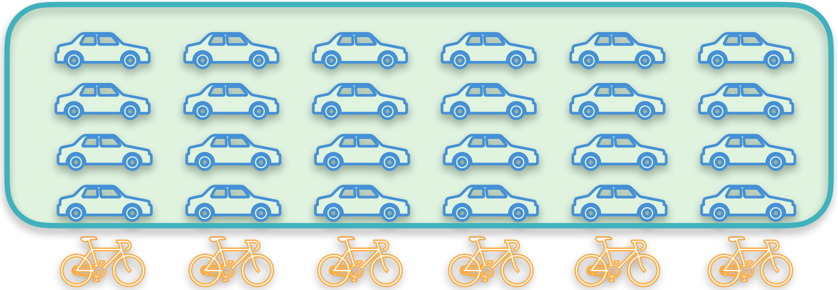
$n = 30$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$



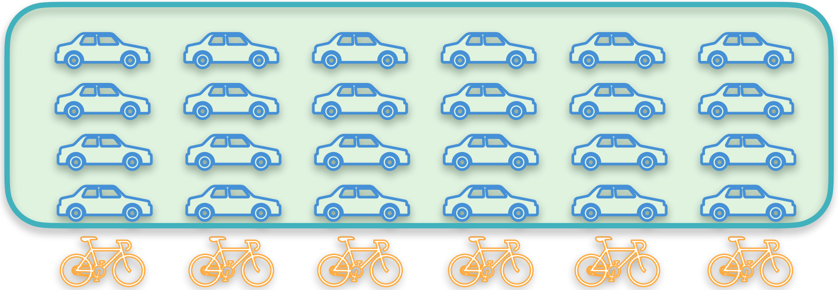
$n = 30$

Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$



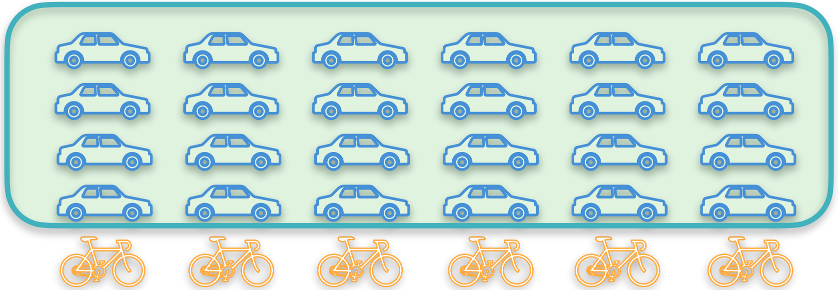
$n = 30$ $\hat{p} = 80\,\% = 0.8$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

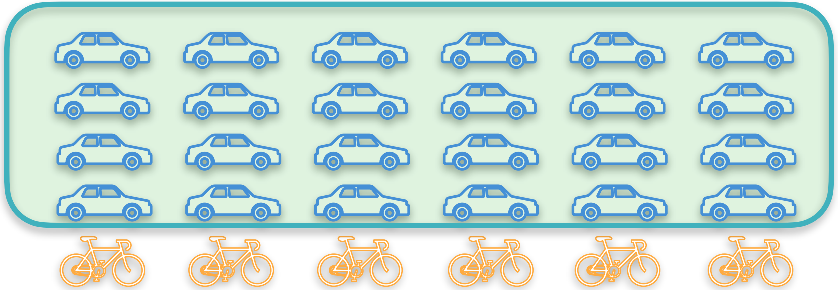
Calculate a 95% confidence interval for this sample proportion

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$



$n = 30 \qquad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

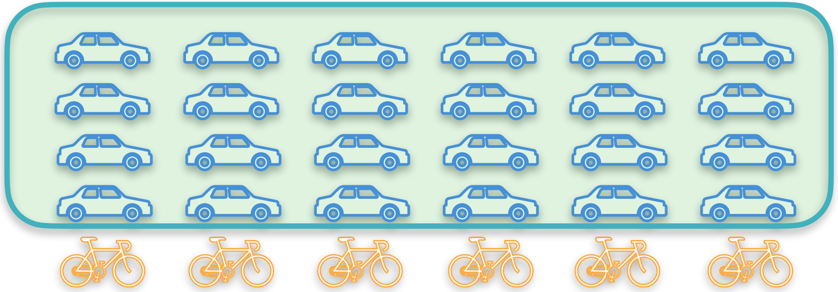
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

margin of error $=$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\rightarrow z_{\alpha/2} = 1.96$

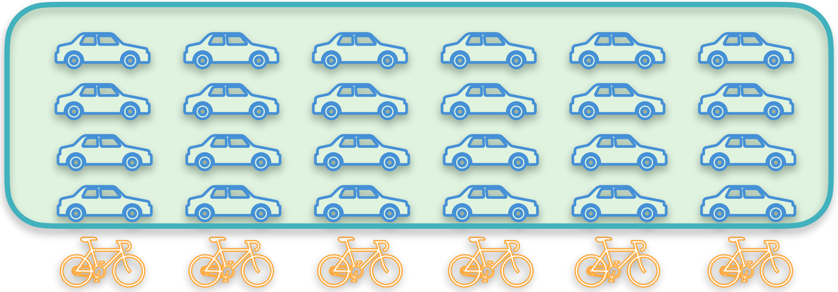
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

margin of error $= 1.96$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\rightarrow z_{\alpha/2} = 1.96$

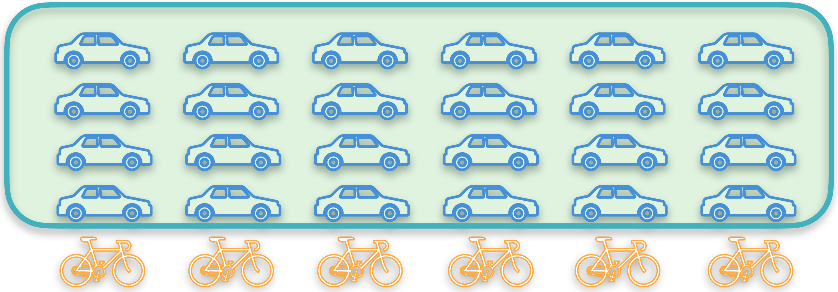
Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \bullet \sqrt{\frac{0.8(1-0.8)}{30}}$$



$n = 30$ $\quad$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

Confidence Interval for Proportions

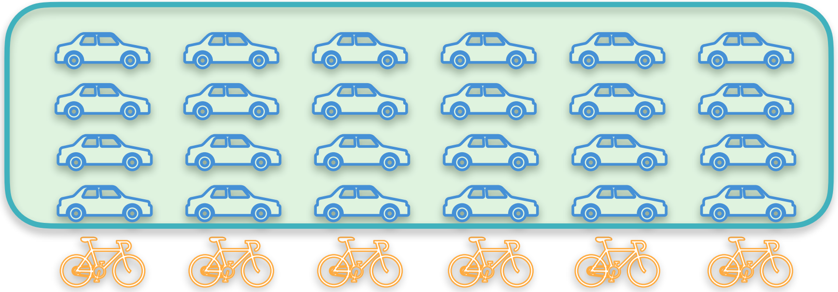
Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \bullet \sqrt{\frac{0.8(1-0.8)}{30}}$$

$$\text{margin of error} = 0.14$$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

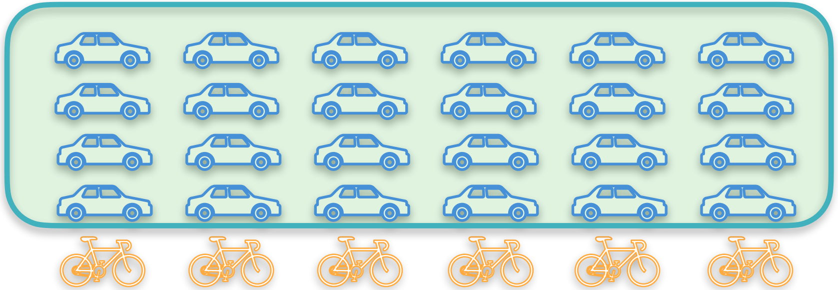
95% $\rightarrow z_{\alpha/2} = 1.96$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\longrightarrow z_{\alpha/2} = 1.96$

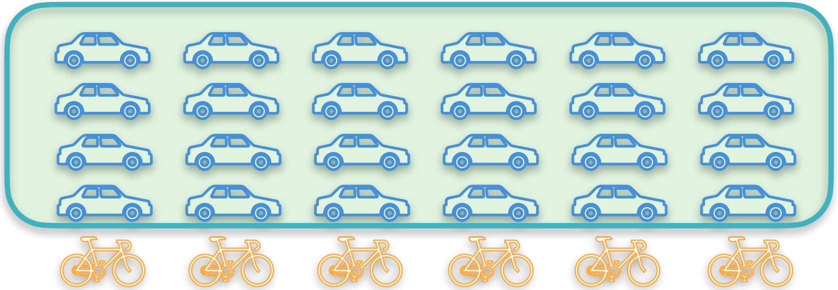
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$

confidence interval $=$



$n = 30$ $\hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\rightarrow z_{\alpha/2} = 1.96$

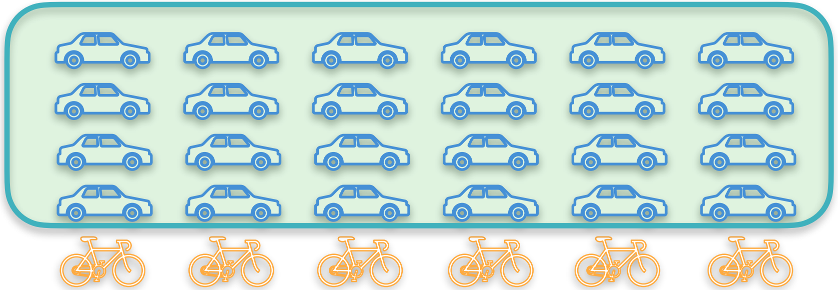
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$

confidence interval $= 0.8 \pm 0.14$



$n = 30$ $\quad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\rightarrow z_{\alpha/2} = 1.96$

Confidence Interval for Proportions

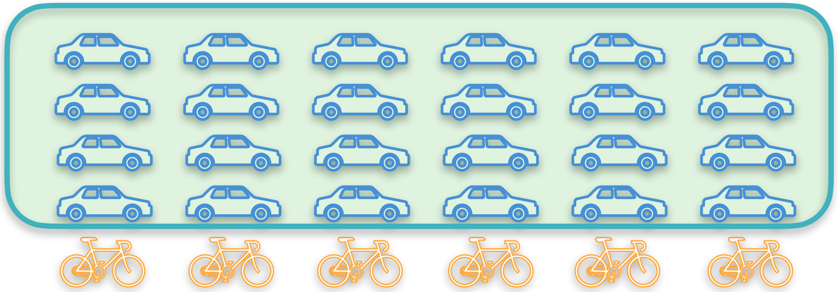
Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$

confidence interval $= 0.8 \pm 0.14$

$$0.66 < p < 0.94$$



$n = 30 \qquad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

$\boxed{95\%} \longrightarrow z_{\alpha/2} = 1.96$

Confidence Interval for Proportions

Confidence Interval for Proportions

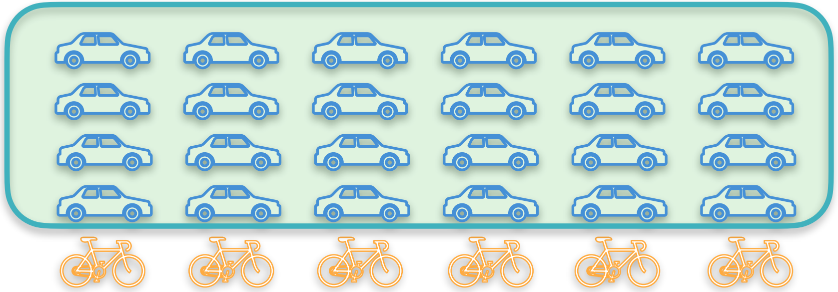
confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$

confidence interval $= 0.8 \pm 0.14$

$$0.66 < p < 0.94$$

$$66\,\% < p < 94\,\%$$



$n = 30$ $\quad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

95% $\rightarrow z_{\alpha/2} = 1.96$

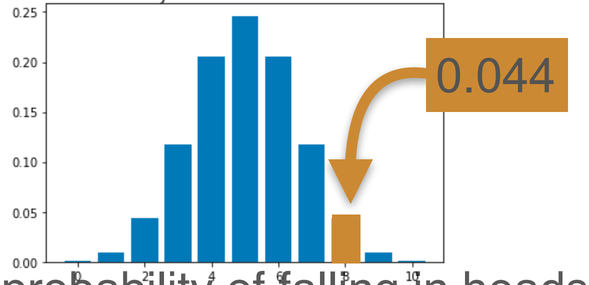
W4 Lesson 2

Hypothesis Testing

- Data Science interview
  - I throw a coin 10 times. I get heads 8 times.
  - Quiz 1: Do you think the coin is biased, or not?
    - Keep both answers correct. Specify that we don’t have enough information.
  - More information: We agree to say that something is *unlikely* if the probability of it happening is less than 5%.

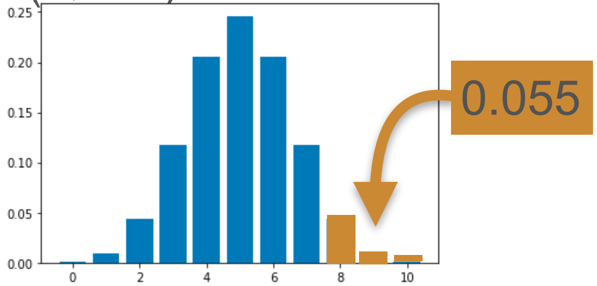
Hypothesis Testing

- Quiz: If we were to assume that the coin is unbiased, what is the probability that it falls in heads 8 times?
  - Answer: (10 choose 8) * 0.5^10



this is for unbiased

  - However, should we instead look at the probability of falling in heads 8 or more times? How much is this one? (Quiz?)
  - Answer:
    ((10 choose 8) +
    (10 choose 9) +
    (10 choose 10)) * 0.5^10

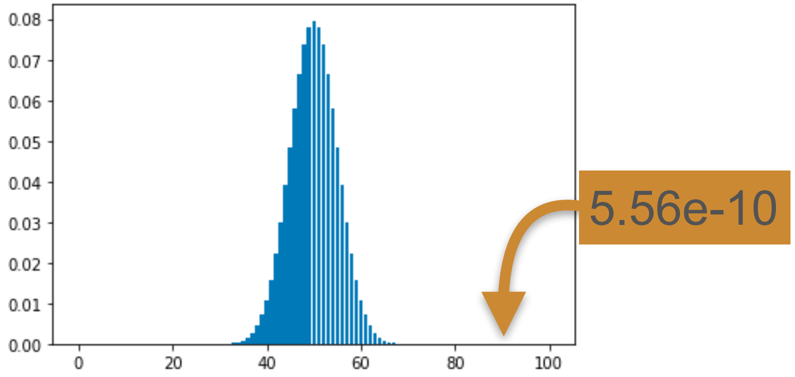


Hypothesis Testing

- If we assume that the coin is fair (**null hypothesis**)
- Then the probability that it lands in heads 8 times (**outcome**) is 0.055
- That probability is higher than 5%
    - Therefore the coin *could* potentially be fair (i.e., **we can't reject the null hypothesis**)

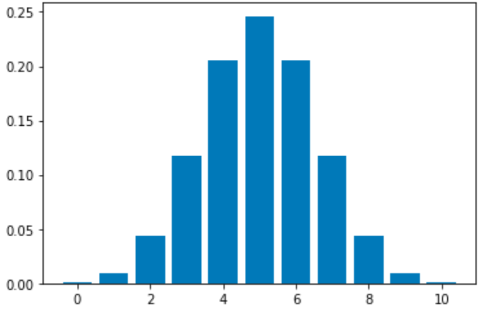
Hypothesis Testing

- Now, same problem except that we toss the coin 100 times and get heads 80 times.
- Probability of 80 or more heads:
  - Sum from i=80 to 100 of (100 choose i) * 0.5^100
  - The probability is TINY
- Therefore, we reject the null hypothesis
  - We conclude the coin is not biased
- Problem: A huge sum of tiny numbers to calculate
- Solution: Approximate with a Gaussian!

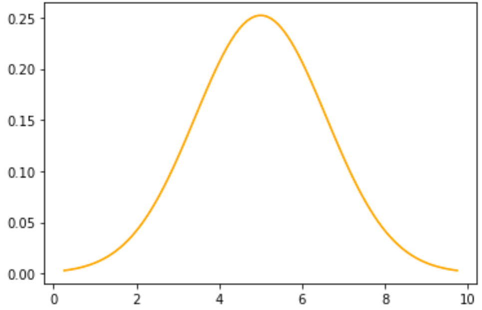


this is covered on W3

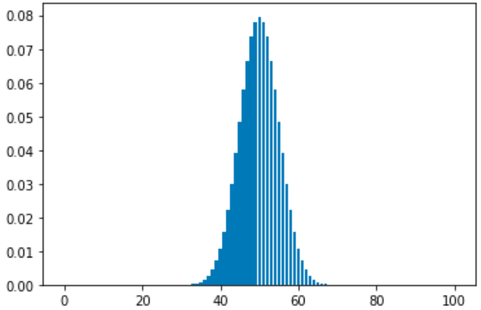
Central Limits Theorem (-Ish)



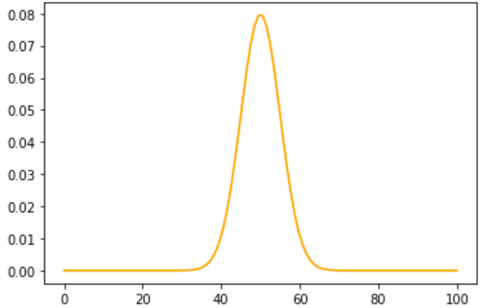
Binomial(10, 0.5)



Gaussian(5, $\sqrt{2.5}$)



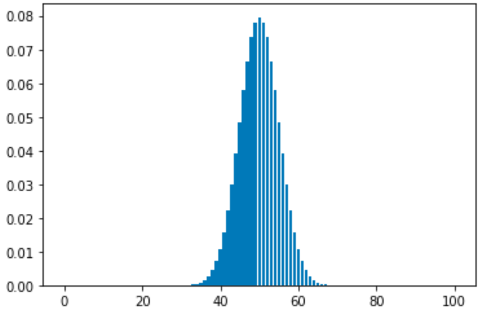
Binomial(100, 0.5)



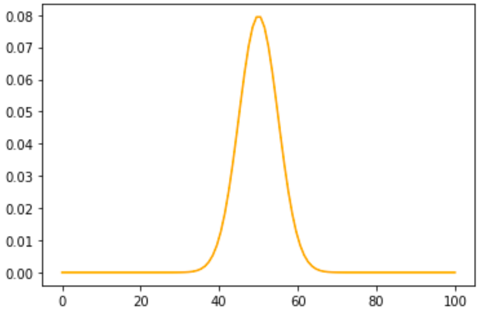
Gaussian(50, 5)

DeepLearning.AI

Central Limits Theorem (-Ish)



Binomial(n, p)



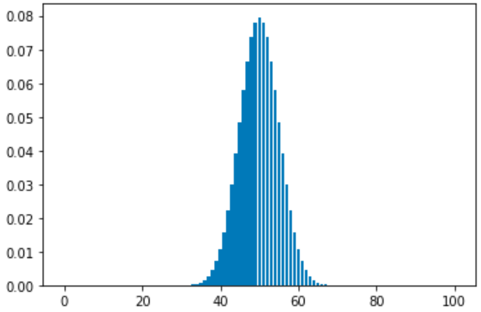
Gaussian(?, ?)

Gaussian$(np, \sqrt{np(1-p)})$

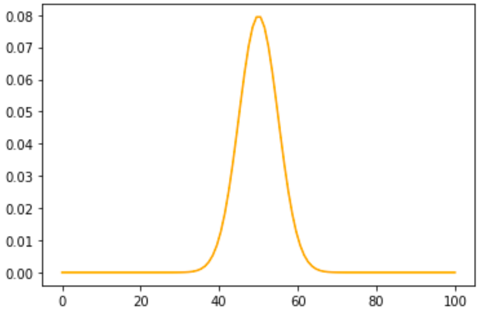
Mean = np
Variance = np(1-p)

Gaussian with:
mean=np
Variance = np(1-p)

Approximating the Binomial With a Gaussian



Binomial(100, 0.5)



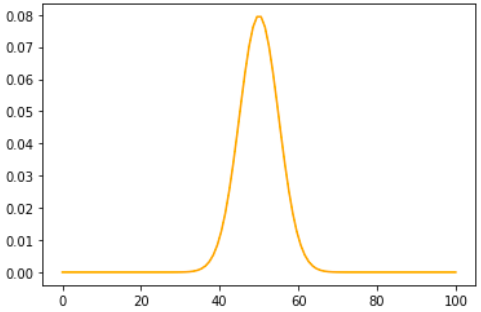
Gaussian(50, 5)

Sum of binomials from 80 to 100

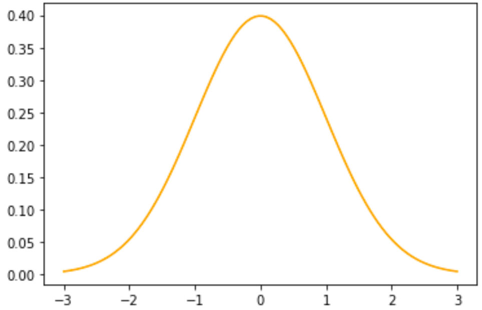
Area under the curve from 80 to 100

Can be calculated using CDF!

How To Do It? Normalize



Binomial(50, 5)



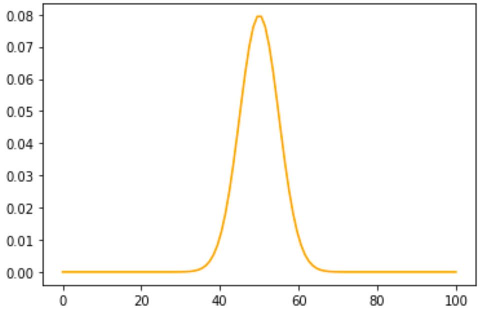
Gaussian(0, 1)

$$\frac{X - \text{mean}}{\text{std}}$$

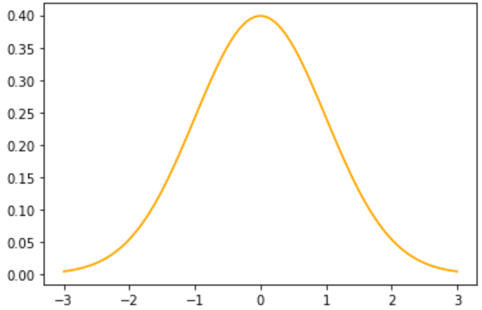
X

$$\frac{X - 50}{5}$$

How To Do It? Normalize



Binomial(n, p)



Gaussian(0, 1)

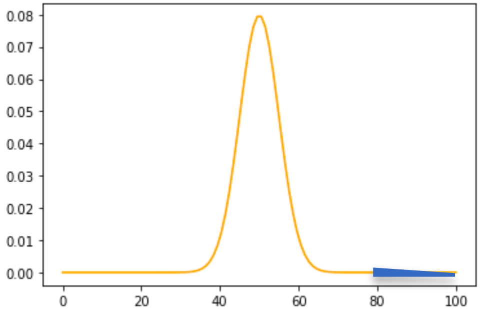
$$\frac{X - \text{mean}}{\text{std}}$$

X

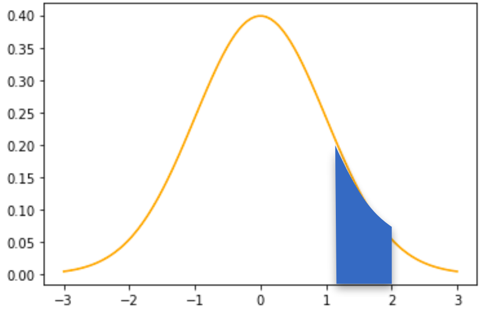
$$\mu = np$$
$$\sigma = np(1-p)$$

$$\frac{X - \mu}{\sigma}$$

How To Do It? Normalize



Binomial(n, p)



Gaussian(0, 1)

P(80 < X < 100) →

$$P\left(\frac{80-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{100-\mu}{\sigma}\right)$$

$$P\left(\frac{80-50}{5} < \frac{X-50}{5} < \frac{100-50}{5}\right)$$

$$= P(12 < Z < 20)$$

How To Do It? Normalize

$$P\left(\frac{80-50}{25} < \frac{X-50}{25} < \frac{100-50}{25}\right)$$

$$= P(12 < Z < 20)$$

$$= \Phi(12) - \Phi(20)$$

= almost 0

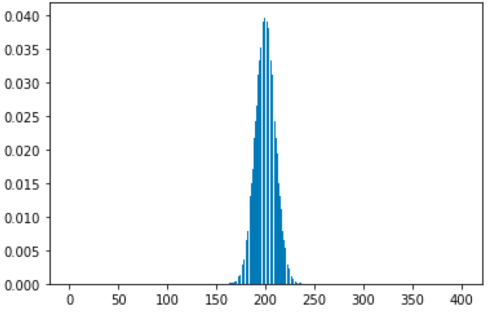
Conclusion: The coin must be biased

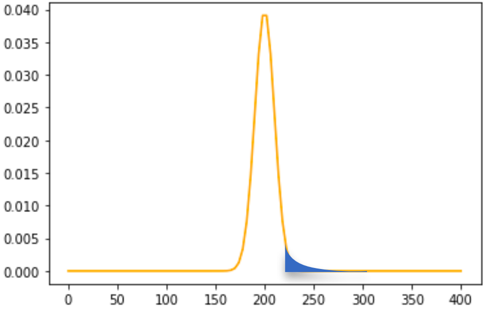
Why?

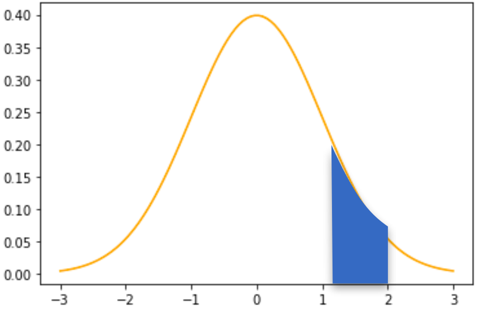
Because we reject the **null hypothesis** which says the coin is not biased.

Another Example

I toss a fair coin 400 times. What is the probability that the number of heads is between 250 and 350?







We can use this
on [illegible]

$$P\left(\frac{250-200}{100} < \frac{X-200}{100} < \frac{350-200}{100}\right)$$

$$P\left(0.5 < \frac{X-200}{100} < 1.5\right)$$

$$= P(0.5 < Z < 1.5)$$

$$= 0.2417$$

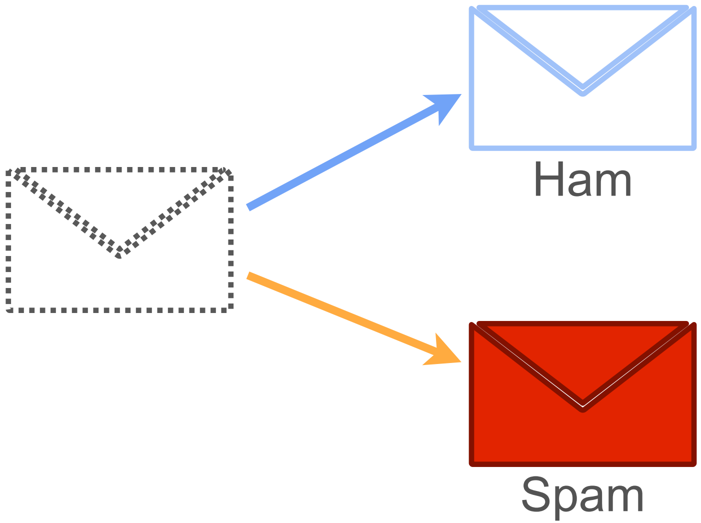
0.9332 - 0.6915

Hypothesis Testing

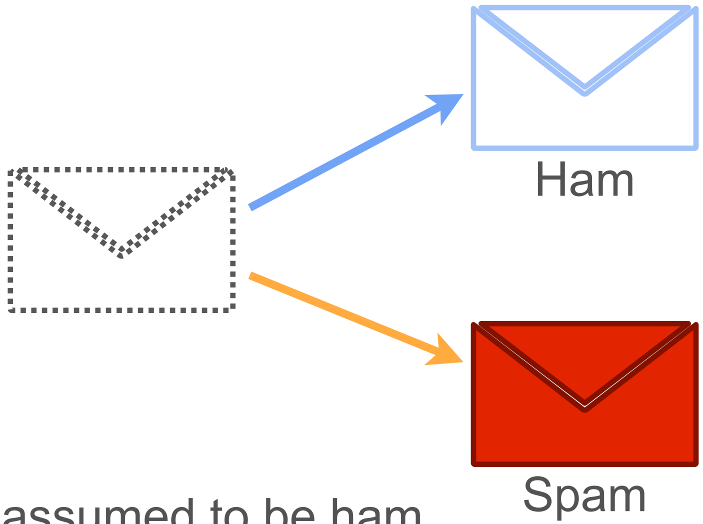
Defining hypothesis

Motivation

Motivation

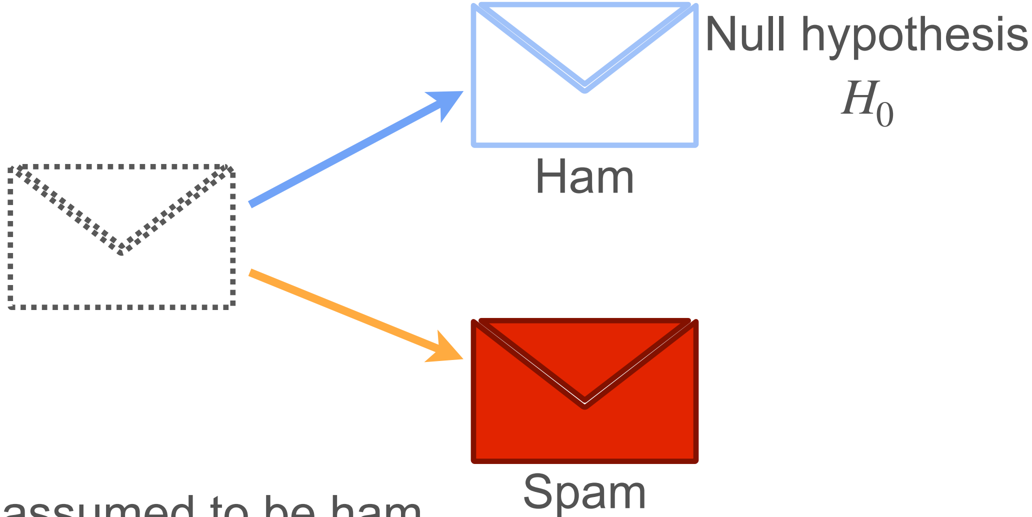


Motivation



Email is assumed to be ham

Motivation



Email is assumed to be ham

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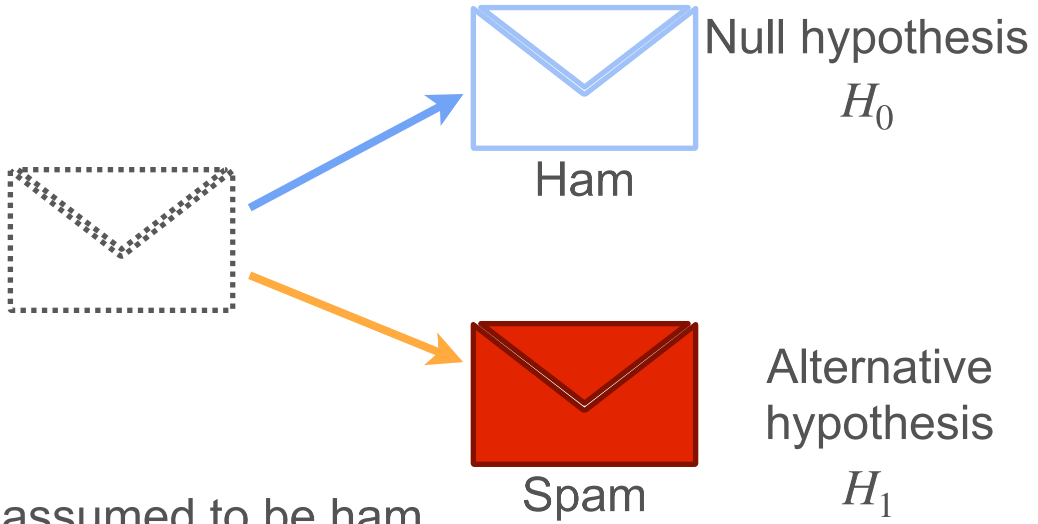

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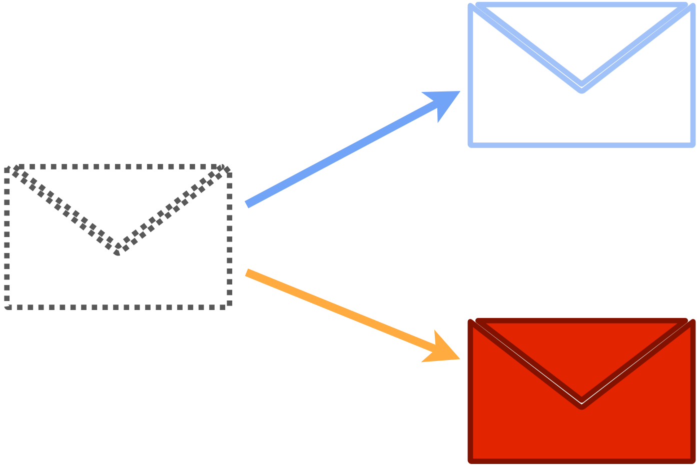
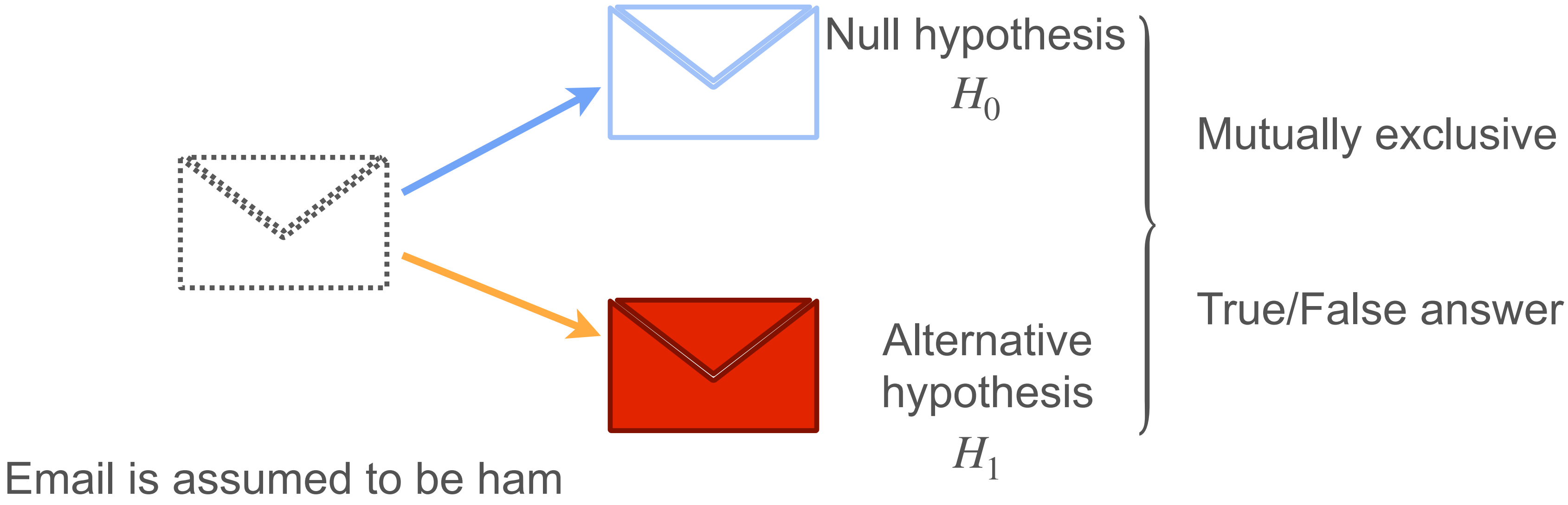


Null hypothesis
H_0

Alternative hypothesis
H_1

Email is assumed to be ham

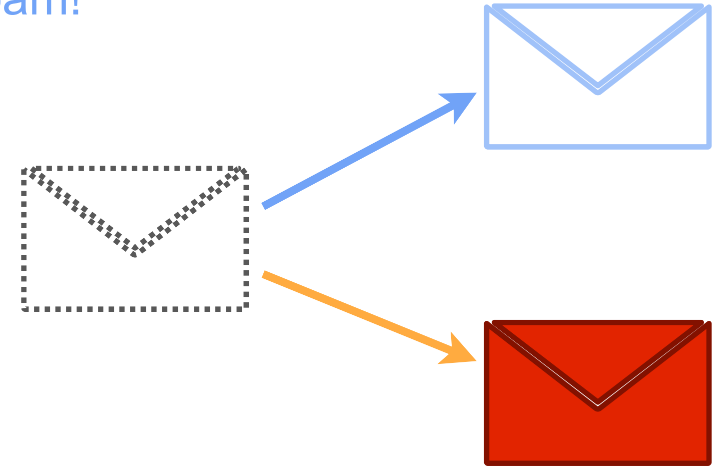
Motivation



Null hypothesis
H_0
Alternative hypothesis
H_1
Mutually exclusive
True/False answer
Email is assumed to be ham

Motivation

When rejecting that the email is not spam, you are accepting that the email is spam!



Null hypothesis

H_0

Alternative hypothesis

H_1

Mutually exclusive

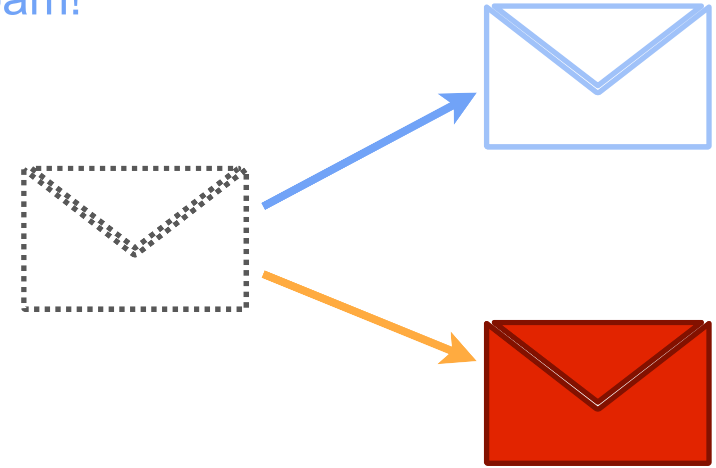
True/False answer

Email is assumed to be ham

Motivation

When rejecting that the email is not spam, you are accepting that the email is spam!

By failing to reject that the email IS spam, you are **not** accepting that it's ham



Null hypothesis

H_0

Alternative hypothesis

H_1

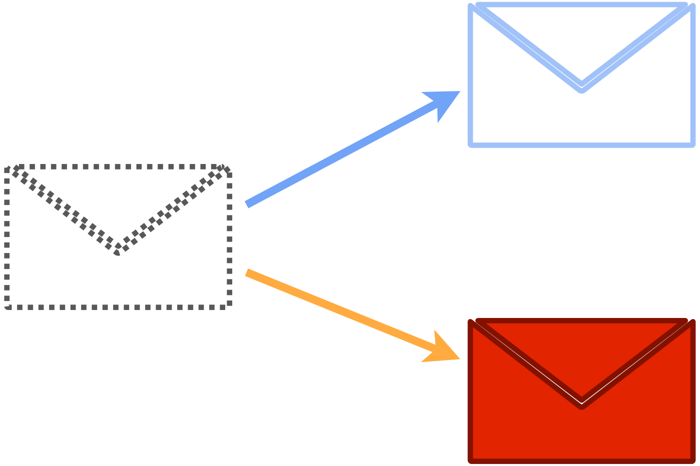
Mutually exclusive

True/False answer

Email is assumed to be ham

Motivation

Not labeling the email spam, doesn’t mean the email is ham!



Null hypothesis

H_0

Alternative hypothesis

H_1

Email is assumed to be ham

Motivation

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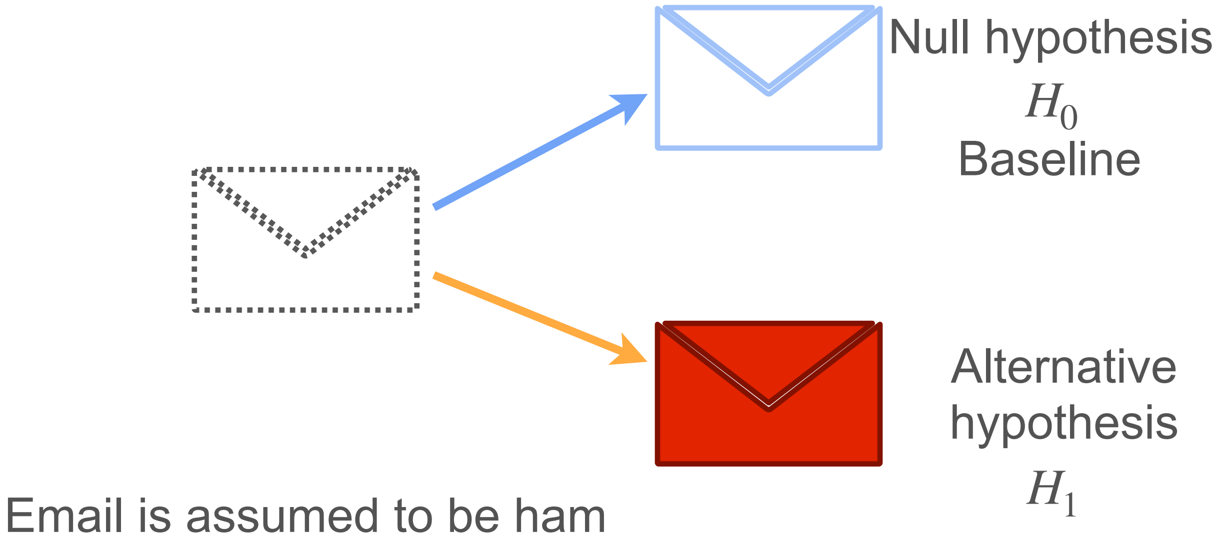

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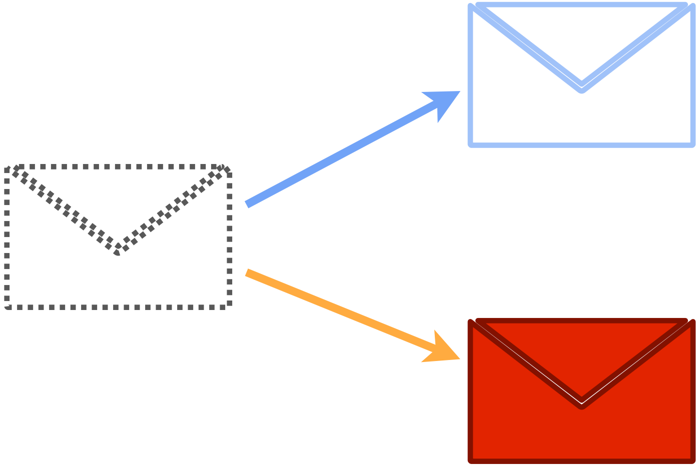


Null hypothesis

H_0

Baseline

Alternative hypothesis

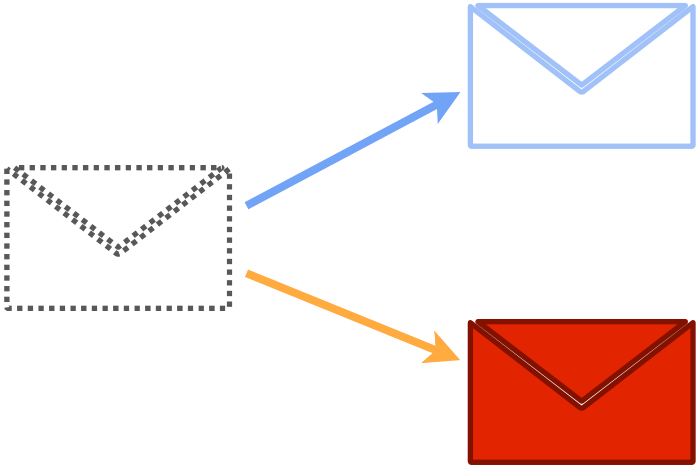
H_1

What you want to prove

Email is assumed to be ham

Motivation

Not labeling the email spam, doesn't mean the email is ham!



Null hypothesis

H_0

Baseline

Data

Evidence

Alternative hypothesis

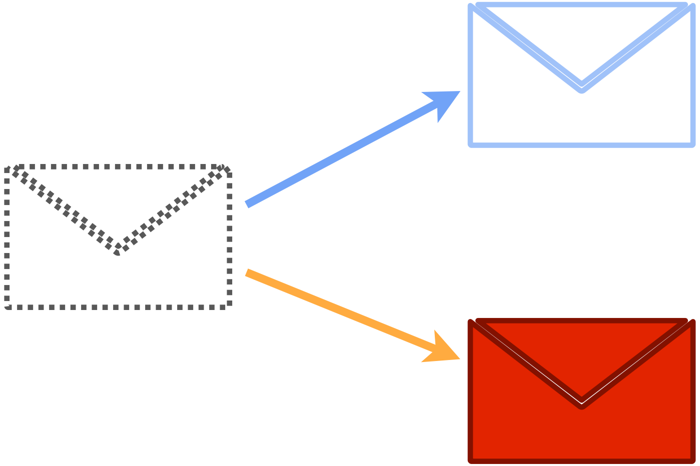
H_1

What you want to prove

Email is assumed to be ham

Motivation

Not labeling the email spam, doesn't mean the email is ham!



Null hypothesis

H_0

Baseline

Data

Evidence

Alternative hypothesis

H_1

What you want to prove

Sender

Attachments

Size

Keywords

Email is assumed to be ham

How To Determine the Result of the Test

How To Determine the Result of the Test

Plenty of evidence against H_0 → Reject H_0 (and accept H_1)

How To Determine the Result of the Test

Plenty of evidence against H_0 → Reject H_0 (and accept H_1)

"Earn extra cash"

"Risk free"

"Dear friend"

"Act immediately"

"Apply now"

"Winner"

How To Determine the Result of the Test

Plenty of evidence against H_0 → Reject H_0 (and accept H_1)

"Earn extra cash"

"Risk free"

"Dear friend"

"Act immediately"

"Apply now"

"Winner"

Very unlikely under H_0 (regular email) → Reject H_0 and send email to spam

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Hypothesis Testing

Type I and Type II errors

Sometimes Things Go Wrong…

Sometimes Things Go Wrong…

What if I make the wrong decision?

Sometimes Things Go Wrong…

What if I make the wrong decision?

Type I error
(False positive)

Sometimes Things Go Wrong…

What if I make the wrong decision?

Type I error
(False positive)

Type II error
(False negative)

Sometimes Things Go Wrong…

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Type I error
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Sometimes Things Go Wrong…

What if I make the wrong decision?

Type I error
(False positive)

Type II error
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Type I and Type II Errors

Type I and Type II Errors

Decision	Reality	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	
Reject H_0 (Decide Guilty)	Type I error	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	
Reject H_0 (Decide Guilty)	Type I error	
Don't reject H_0 (Decide not guilty)	Correct	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	H_0 False (Guilty)
Reject H_0 (Decide Guilty)	Type I error	Correct
Don't reject H_0 (Decide not guilty)	Correct	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	H_0 False (Guilty)
Reject H_0 (Decide Guilty)	Type I error	Correct
Don't reject H_0 (Decide not guilty)	Correct	Type II error

Significance Level

Significance Level

Type I error

Type II error

Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error

Type II error

Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error

Type II error

What is the greatest probability of type I error you are willing to tolerate?

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error

Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Significance level

Type I error

Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Significance level (α)

Type I error

Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Significance level (α)

Type I error

Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Significance level (α)

Type I error

Innocent person determined guilty

0 1 α

Defendants are always considered 'not guilty' → No Type I error

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error

Innocent person determined guilty

Significance level (α)

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Significance level (α)

Type I error

Every defendant is considered guilty

Every time the defendant is innocent you make a Type I error

Innocent person determined guilty

Significance Level

Significance level (α)

0 — 1 — α

Defendants are always considered 'not guilty'

No Type I error

Every defendant is considered guilty

Every time the defendant is innocent you make a Type I error

Significance Level

Significance level (α)

Both options are awful decision makers

Defendants are always considered ‘not guilty’

No Type I error

Every defendant is considered guilty

Every time the defendant is innocent you make a Type I error

Significance Level

Significance level (α)

Defendants are always considered 'not guilty'

No Type I error

Both options are awful decision makers

You want a small α, but not zero

Every defendant is considered guilty

Every time the defendant is innocent you make a Type I error

Significance Level

Significance level (α)

Defendants are always considered 'not guilty'

No Type I error

Both options are awful decision makers

You want a small α, but not zero

Catch:

↓Type I error ↑Type II error

Every defendant is considered guilty

Every time the defendant is innocent you make a Type I error

Significance Level

Significance Level

Type I error

Innocent person determined guilty

Significance Level

Type I error

Innocent person determined guilty

$$\alpha = \max \mathbf{P}(\text{Type I error})$$

Significance Level

Type I error

Innocent person determined guilty

$$\alpha = \max \mathbf{P}(\text{Type I error})$$
$$= \max \mathbf{P}(\text{Reject } H_0 \,|\, H_0)$$

Significance Level

Type I error

Innocent person determined guilty

$$\alpha = \max \mathbf{P}(\text{Type I error})$$
$$= \max \mathbf{P}(\text{Reject } H_0 \,|\, H_0)$$

The value of α is your criteria for designing your test

Significance Level

Type I error

Innocent person determined guilty

$$\alpha = \max \mathbf{P}(\text{Type I error})$$
$$= \max \mathbf{P}(\text{Reject } H_0 \,|\, H_0)$$

The value of α is your criteria for designing your test

For a given sample, α will determine if you reject H_0 or not

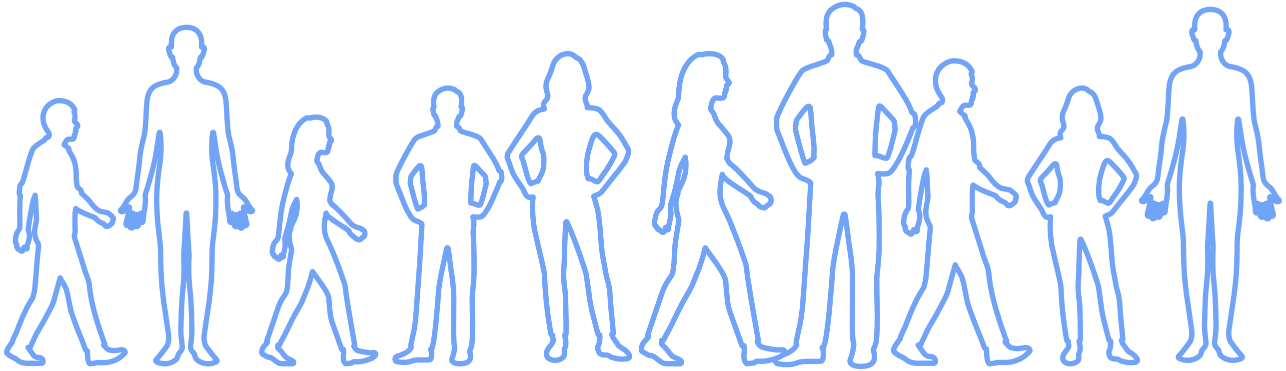
DeepLearning.AI

Hypothesis Testing

Right-Tailed, Left-Tailed and Two-Tailed tests

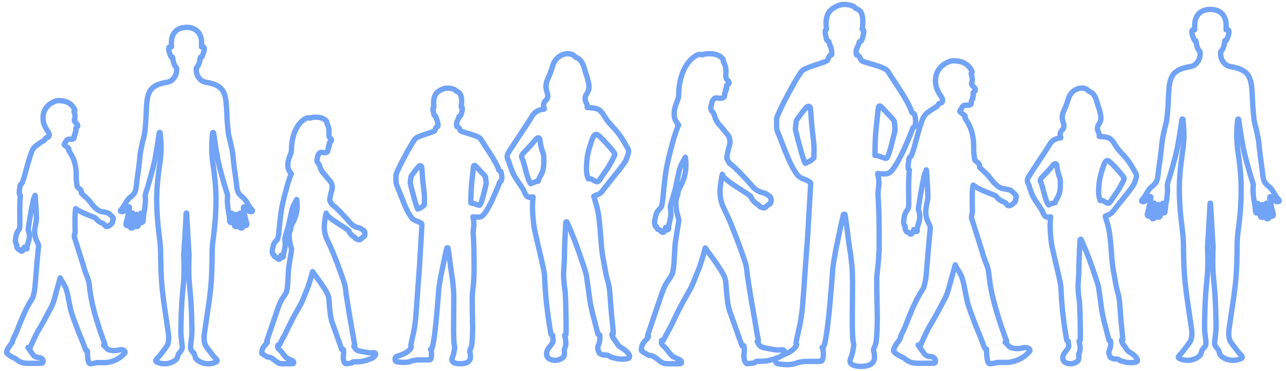
Example: Heights

Example: Heights



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Example: Heights



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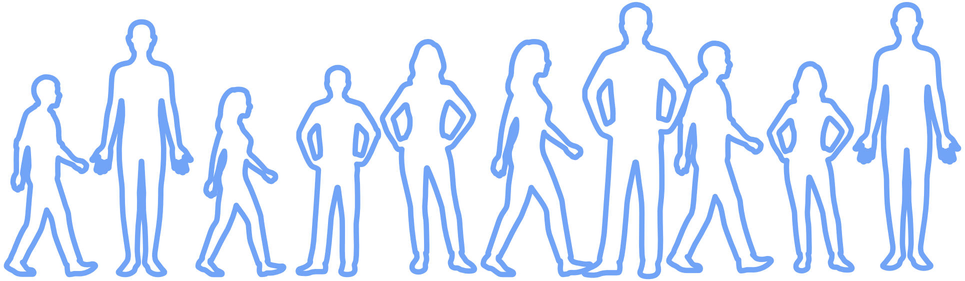
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$$\bar{x} = 68.442$$

Data Quality



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$$\bar{x} = 68.442$$

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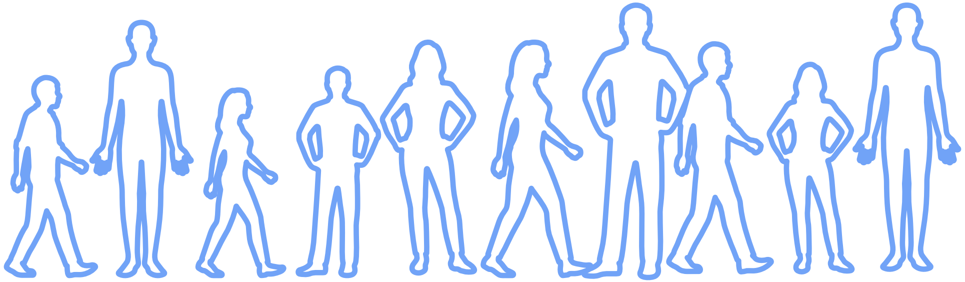

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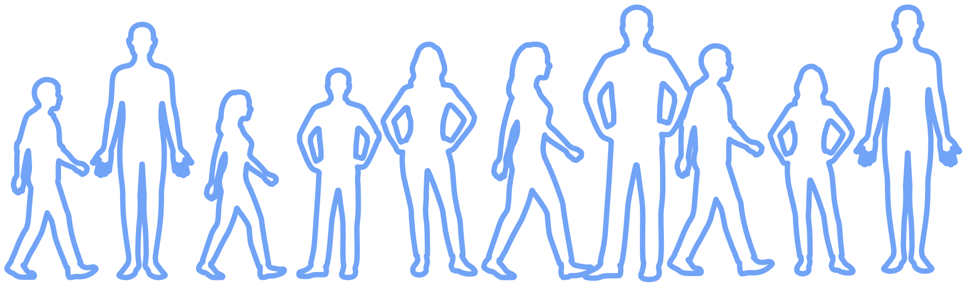
$\bar{x} = 68.442$

Reliable

Data Quality



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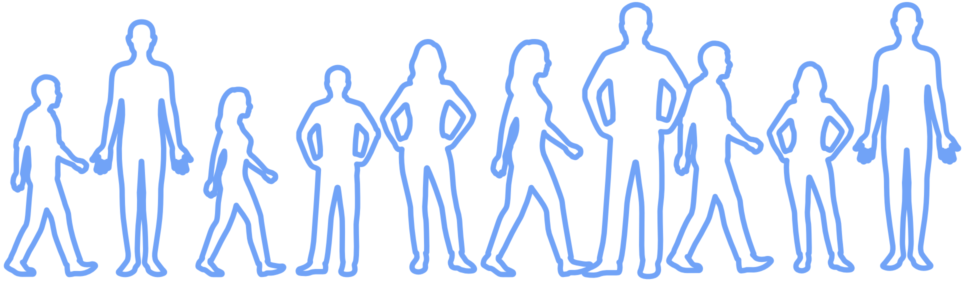
$$\bar{x} = 68.442$$

Reliable

Representative

Data Quality



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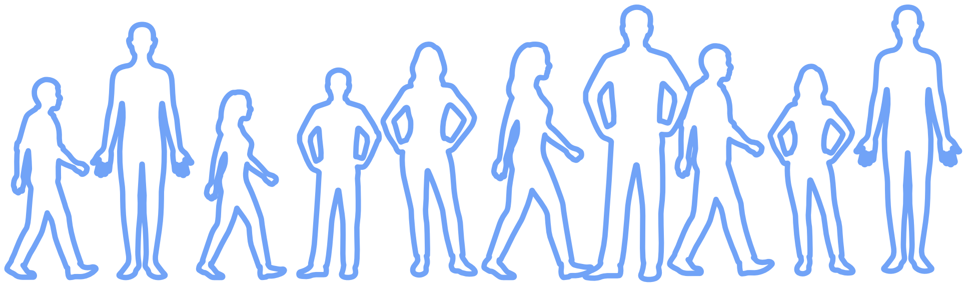
$$\bar{x} = 68.442$$

Reliable

Representative

Randomized

Data Quality



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$$\bar{x} = 68.442$$

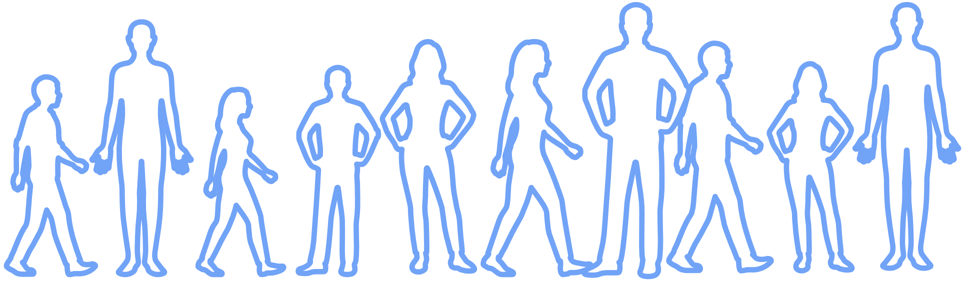
Reliable

Representative

Randomized

Sample size

Determining the Hypothesis



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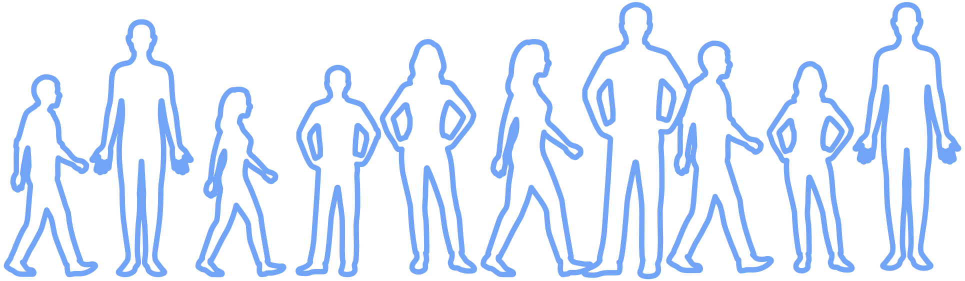
$\bar{x} = 68.442$

vs. $H_1 : \mu > 66.7$

Population

Determining the Hypothesis



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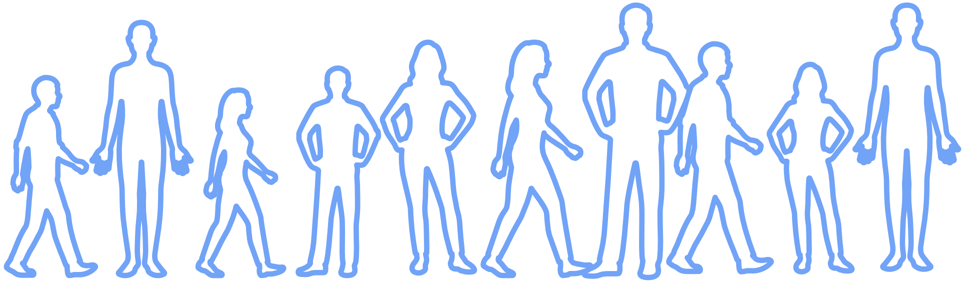
$\bar{x} = 68.442$

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Population

Test Statistic



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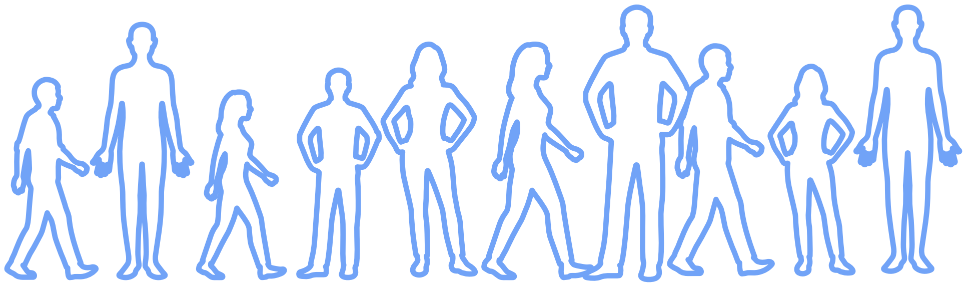
$\bar{x} = 68.442$

Observed statistic

vs. $H_1 : \mu > 66.7$

Test statistic $\longrightarrow \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$

Test Statistic



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$\bar{x} = 68.442$

Observed statistic

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

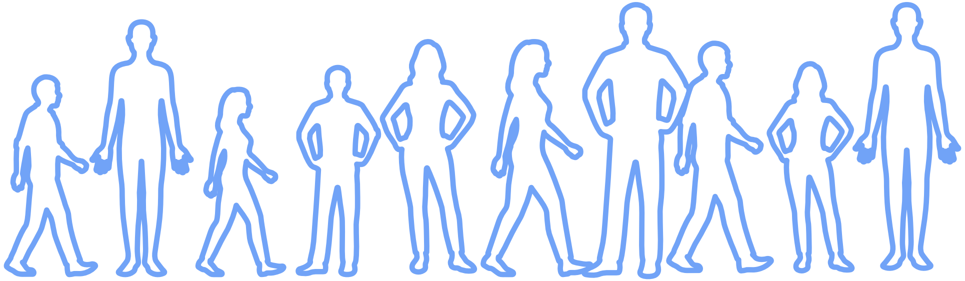
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Test statistic $\longrightarrow \bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$

Test Statistic

Test statistic

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



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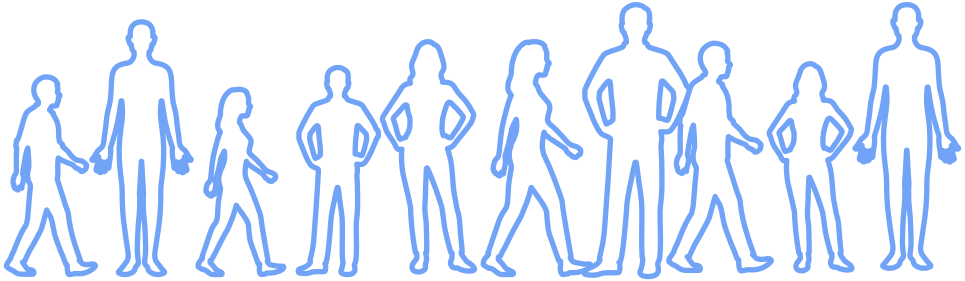
Test Statistic

Test statistic

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

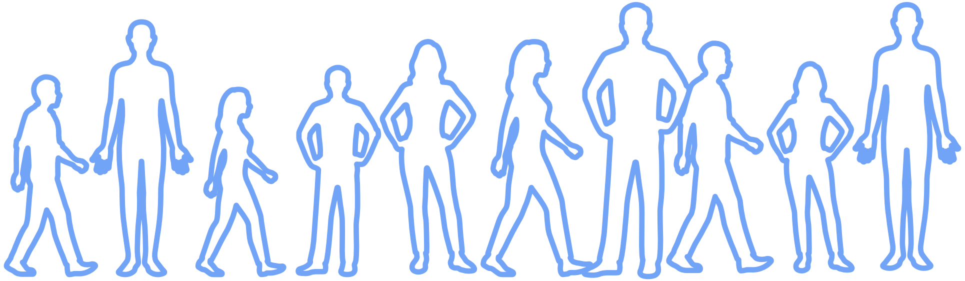
$\bar{x} = 68.442$

Test statistic: $T(\boldsymbol{X})$ $\quad \boldsymbol{X} = (X_1, \ldots, X_n)$

Test Statistic

Test statistic

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



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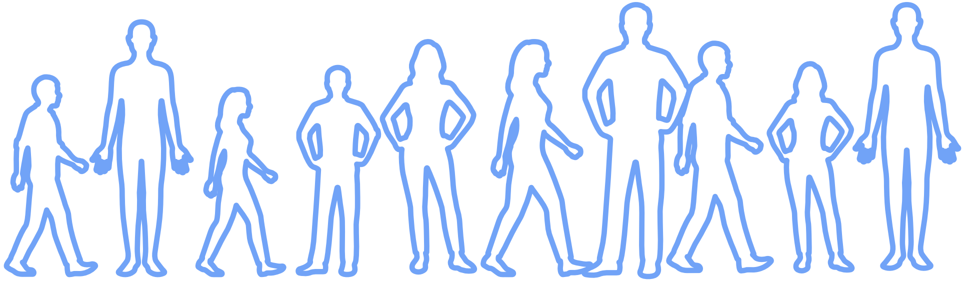
Information about the population parameter under study

Test Statistic

Test statistic

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



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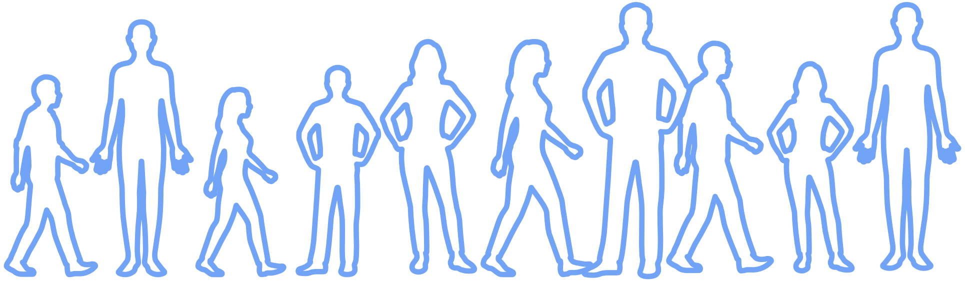
Information about the population parameter under study

$\mu \rightarrow \overline{X}$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



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$\bar{x} = 68.442$

Test statistic: $T(\boldsymbol{X})$ $\quad \boldsymbol{X} = (X_1, \ldots, X_n)$

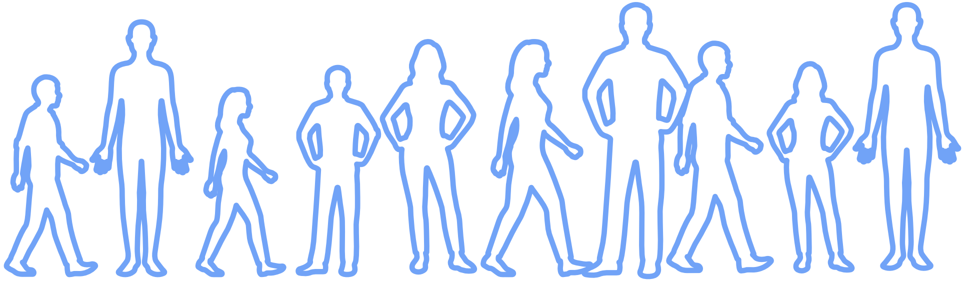
Information about the population parameter under study

$\mu \to \bar{X}$ $\qquad p \to \bar{X}$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
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$\bar{x} = 68.442$

Test statistic: $T(\boldsymbol{X})$ $\quad \boldsymbol{X} = (X_1, \ldots, X_n)$

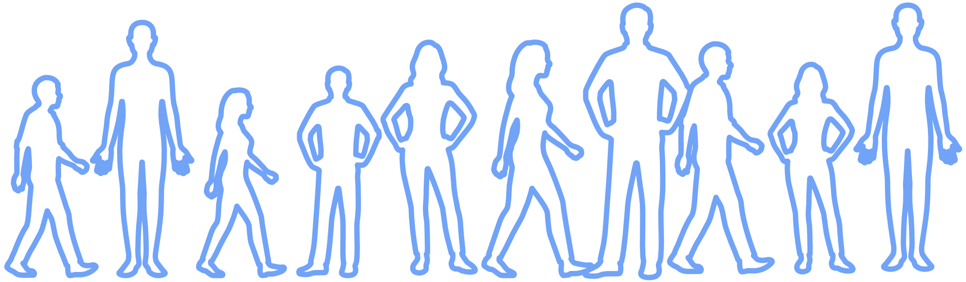
Information about the population parameter under study

$\mu \to \bar{X}$ $\qquad p \to \bar{X}$ $\qquad \sigma^2 \to S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

Test Statistic

Test statistic

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
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$$\bar{x} = 68.442$$

Test statistic: $T(\boldsymbol{X})$ $\quad \boldsymbol{X} = (X_1, \ldots, X_n)$

Information about the population parameter under study

$\mu \rightarrow \overline{X}$ $\qquad p \rightarrow \overline{X}$ $\qquad \sigma^2 \rightarrow S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$

Not unique!

Example: Heights

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions

3 sets of hypothesis

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions

3 sets of hypothesis

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions

3 sets of hypothesis

Right-Tailed Test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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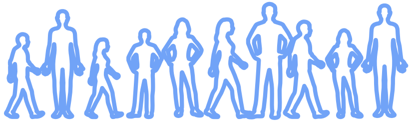
Right-Tailed Test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

 → $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions

3 sets of hypothesis

Right-Tailed Test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Left Tailed Test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Example: Heights

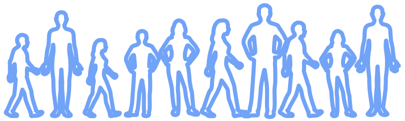
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions

3 sets of hypothesis

Right-Tailed Test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Left Tailed Test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



$\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

3 questions	**3 sets of hypothesis**
Right-Tailed Test	$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$
Left Tailed Test	$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$
Two-Tailed Test	$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Example: Heights

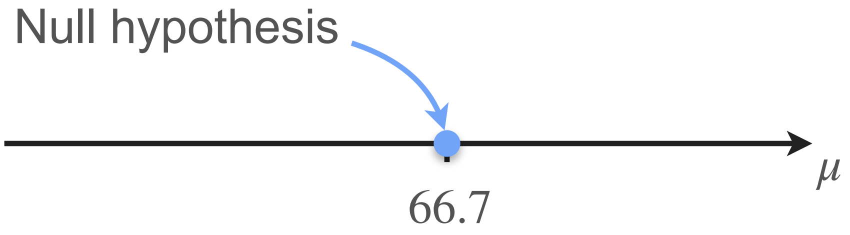
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Right-tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Right-tailed test $\rightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



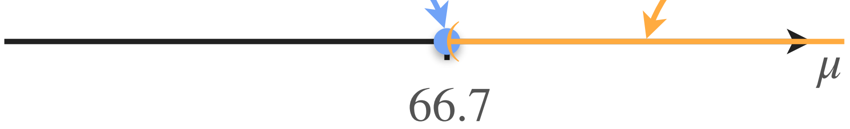
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Right-tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis

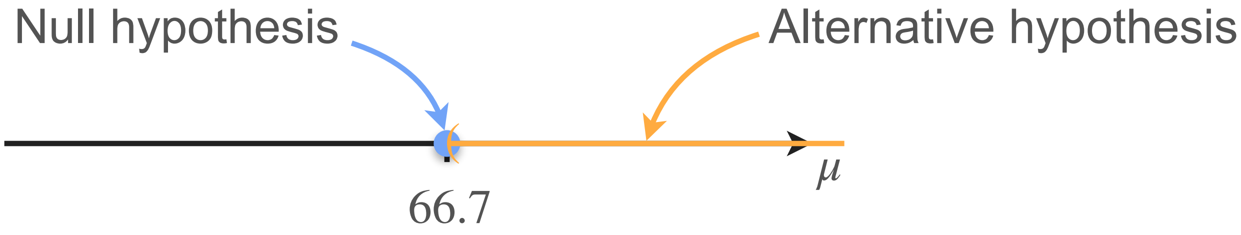
Alternative hypothesis



Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Right-tailed test $\rightarrow$ $H_0 : \mu \leq 66.7$ vs. $H_1 : \mu > 66.7$



Example: Heights

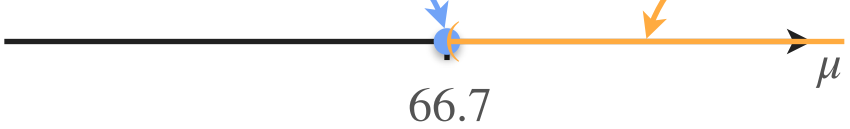
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\bar{X}$ Test statistic

Right-tailed test $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis

Alternative hypothesis



Example: Heights

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Right-tailed test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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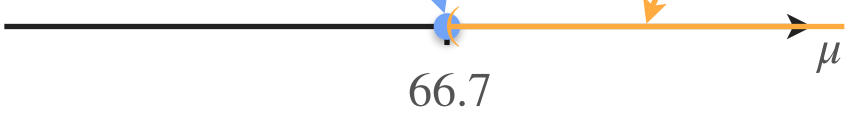


If $\overline{x} \gg 66.7 \Rightarrow$ Reject H_0

Example: Heights

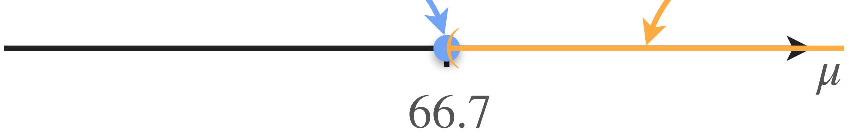
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$ Test statistic

Right-tailed test $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis

Alternative hypothesis



Type I error: Determine $\mu > 66.7$, when population mean did not change

If $\overline{x} \gg 66.7 \Rightarrow$ Reject H_0

Example: Heights

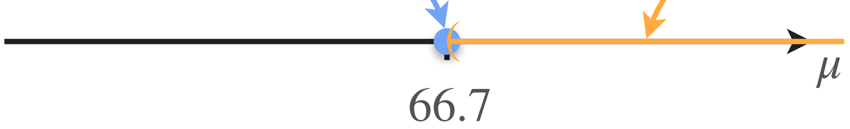
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$ Test statistic

Right-tailed test $\rightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis

Alternative hypothesis



If $\overline{x} \gg 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu > 66.7$, when population mean did not change

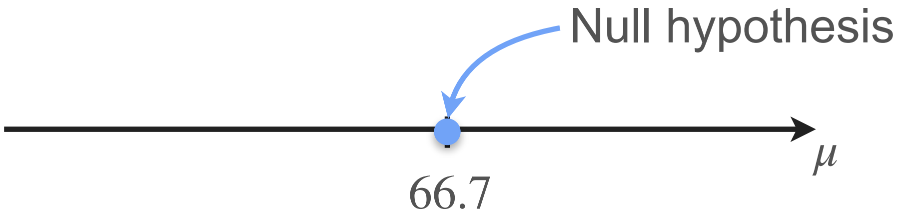
Type II error: Do not reject that $\mu = 66.7$ when in true $\mu > 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



Type I error:

Type II error:

Example: Heights

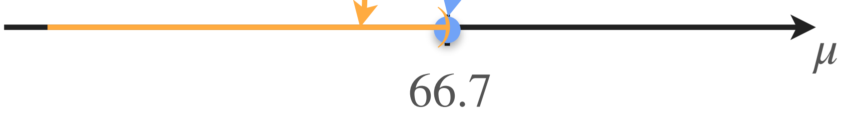
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\bar{X}$

Left tailed test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



Type I error:

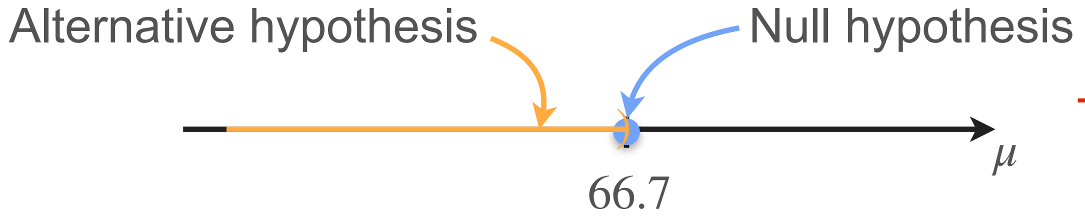
Type II error:

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Left tailed test $\longrightarrow$ $H_0 : \mu \geq 66.7$ vs. $H_1 : \mu < 66.7$



Type I error:

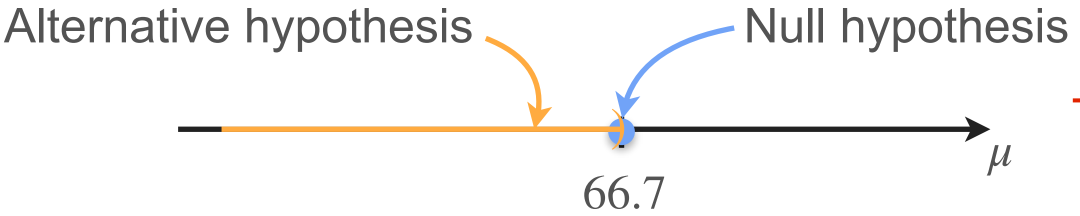
Type II error:

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Left tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



Type I error:

If $\overline{x} \ll 66.7 \Rightarrow$ Reject H_0

Type II error:

Example: Heights

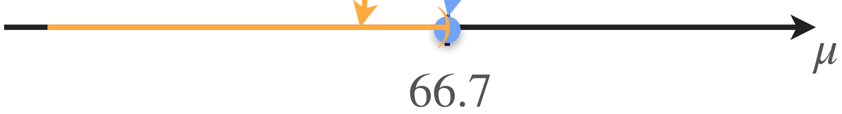
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Left tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu < 66.7$, when population mean did not change

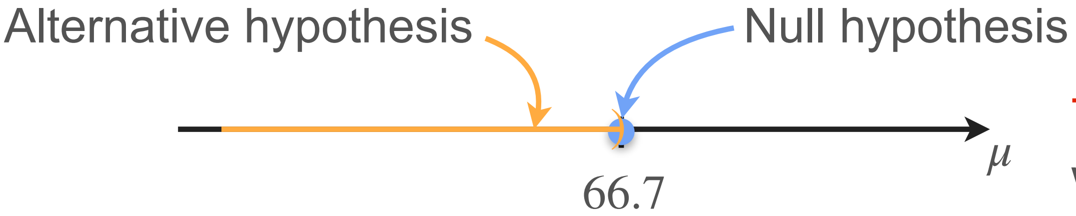
Type II error:

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Left tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu < 66.7$, when population mean did not change

Type II error: Don't reject that $\mu = 66.7$ when true $\mu < 66.7$

Example: Heights

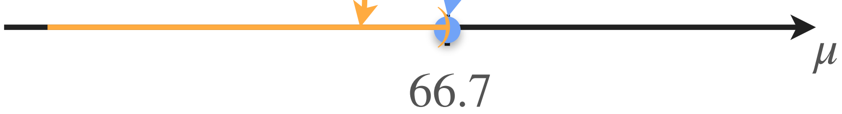
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$ Test statistic

Left tailed test $H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu < 66.7$, when population mean did not change

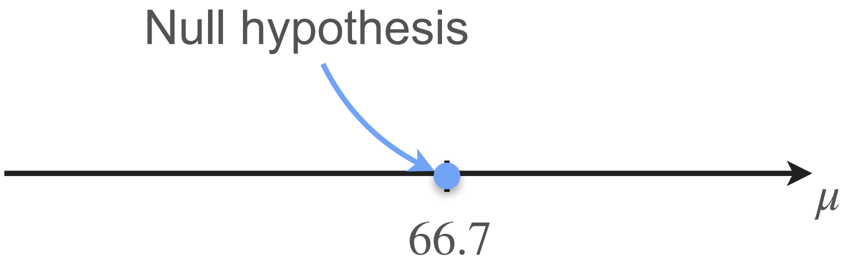
Type II error: Don't reject that $\mu = 66.7$ when true $\mu < 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



Type I error:

Type II error:

Example: Heights

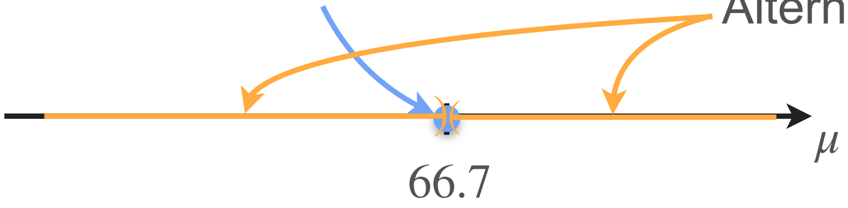
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\bar{X}$

Two tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



Type I error:

Type II error:

Example: Heights

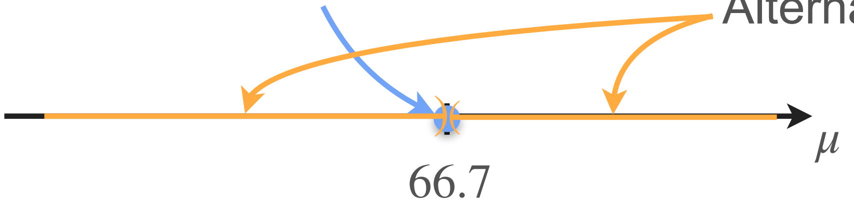
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Two tailed test → $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



Type I error:

Type II error:

$$\text{If} \quad \begin{matrix} \bar{x} \gg 66.7 \\ \text{or} \\ \bar{x} \ll 66.7 \end{matrix} \quad \Rightarrow \quad \text{Reject } H_0$$

Example: Heights

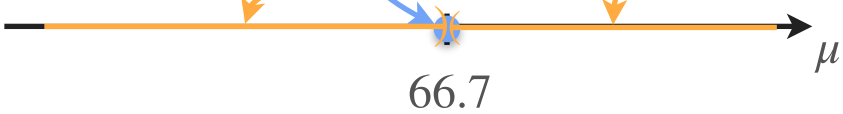
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Two tailed test $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



If $\bar{x} \gg 66.7$ or $\bar{x} \ll 66.7$ $\Rightarrow$ Reject H_0

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

Type II error:

Example: Heights

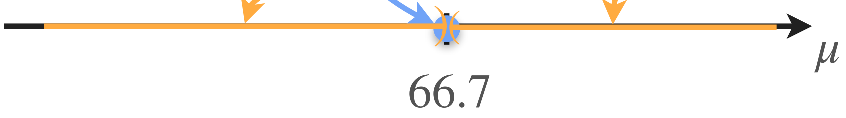
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$

Two tailed test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



$$\text{If} \quad \begin{matrix} \bar{x} \gg 66.7 \\ \text{or} \\ \bar{x} \ll 66.7 \end{matrix} \quad \Rightarrow \quad \text{Reject } H_0$$

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

Type II error: Don't reject that $\mu = 66.7$ when true $\mu \neq 66.7$

Example: Heights

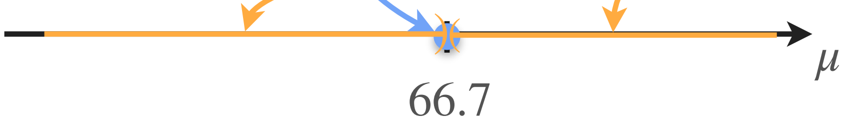
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\overline{X}$ Test statistic

Two tailed test $\longrightarrow$ $H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



If $\bar{x} \gg 66.7$ or $\bar{x} \ll 66.7$ $\Rightarrow$ Reject H_0

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

Type II error: Don't reject that $\mu = 66.7$ when true $\mu \neq 66.7$

DeepLearning.AI

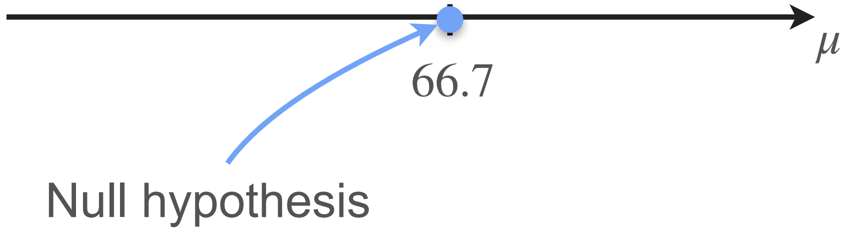
Hypothesis Testing

p-Value

Example: Heights

Example: Heights

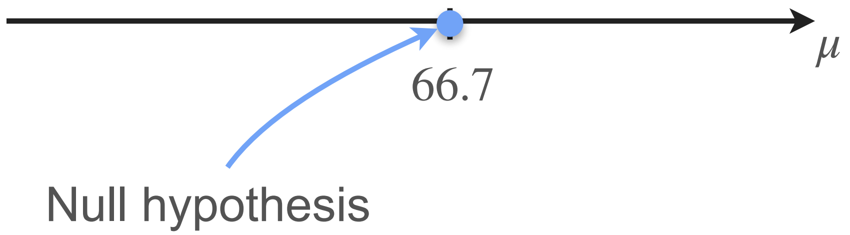
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.



Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$$\sigma = 3$$
$$n = 10$$



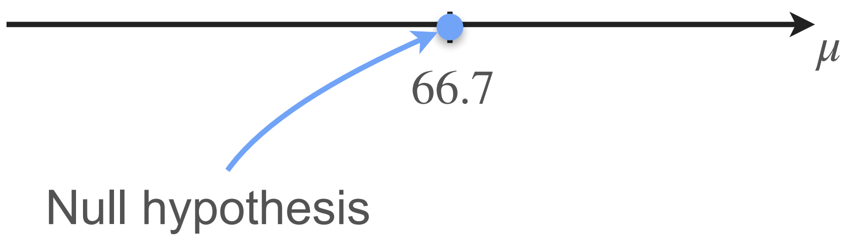
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

If H_0 is true:

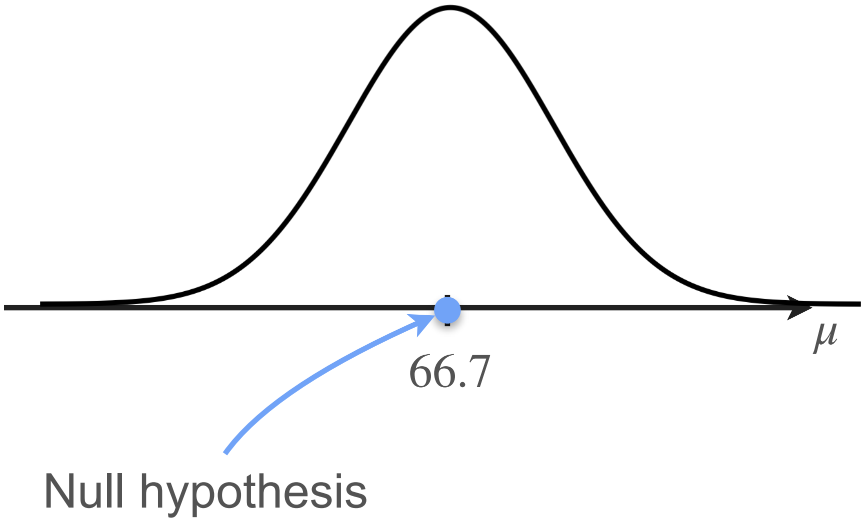


Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$
$n = 10$

If H_0 is true: $\overline{X} \sim \mathcal{N}\left(\quad , \quad \right)$



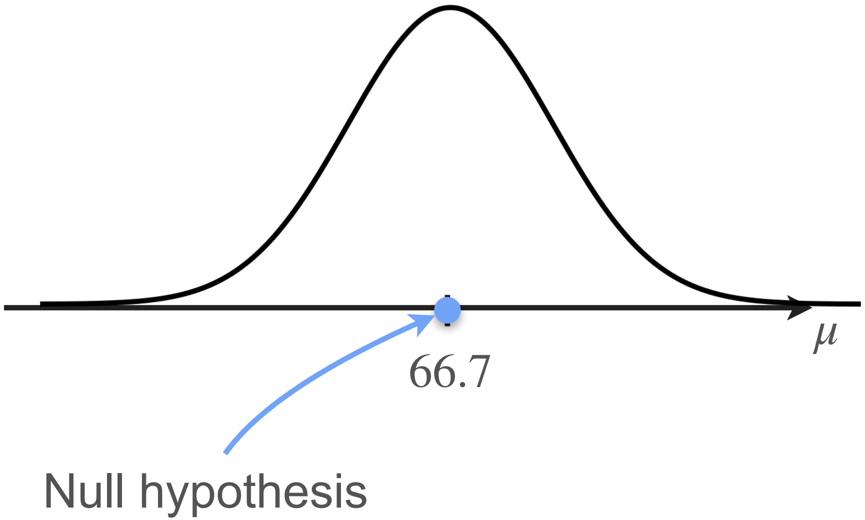
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

If H_0 is true: $\overline{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

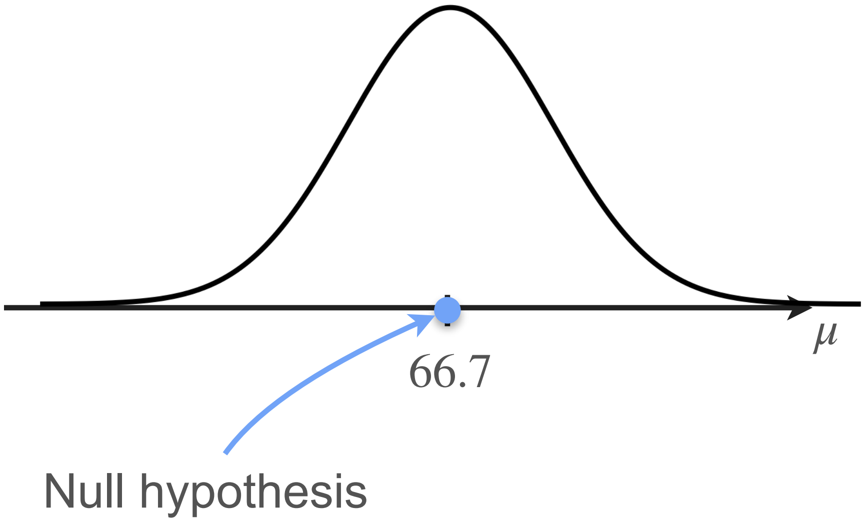


Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$
$n = 10$

If H_0 is true: $\overline{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$



How likely was your sample if H_0 is true?

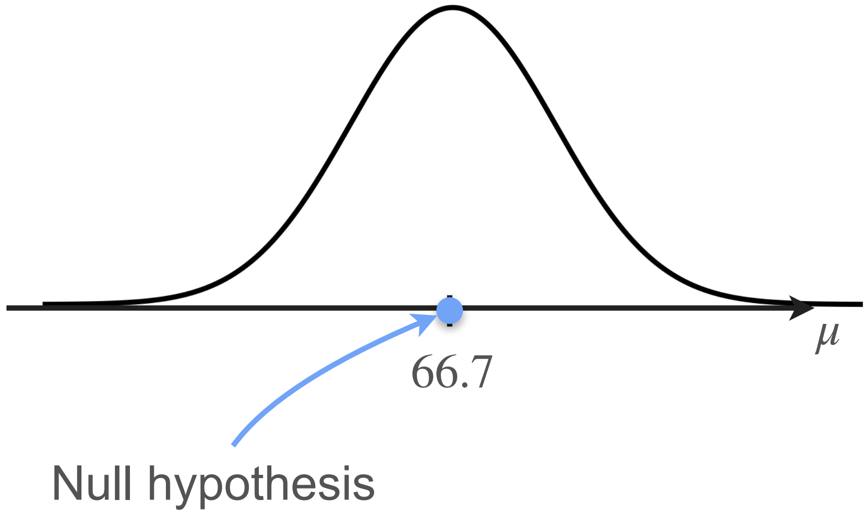
If the answer is very unlikely, then reject H_0

Right-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$



Right-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

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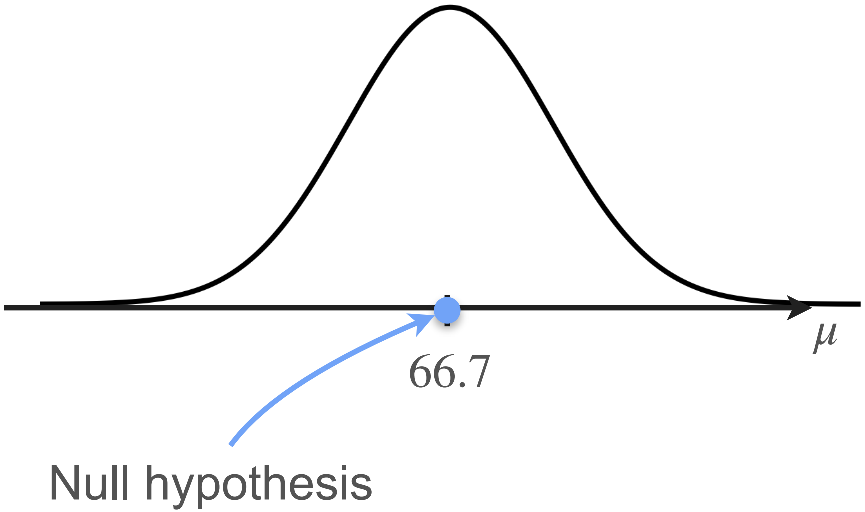
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



Right-Tailed Test for Gaussian Data (Known σ)

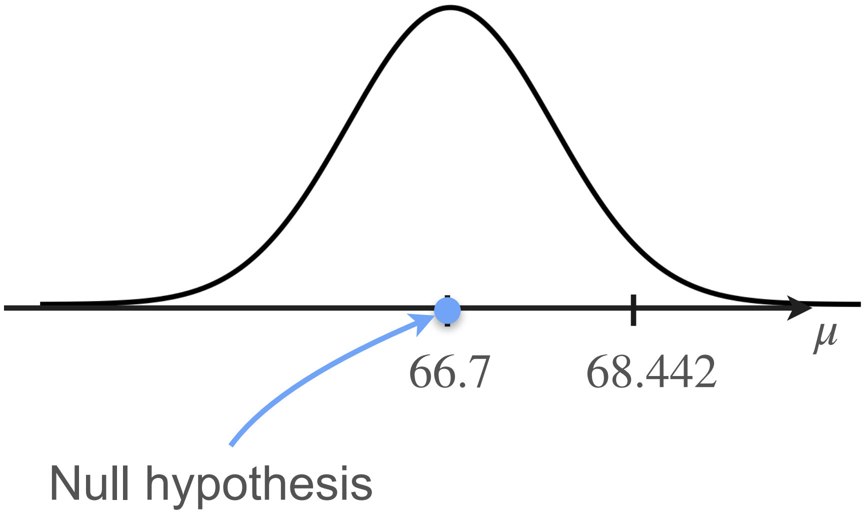
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

$\bar{x} = 68.442$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



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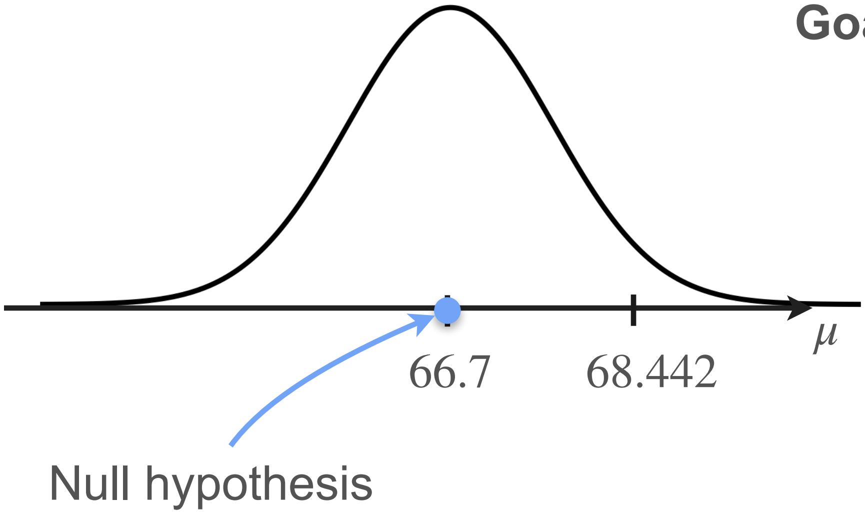
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Goal: Type I error probability < $\alpha = 0.05$



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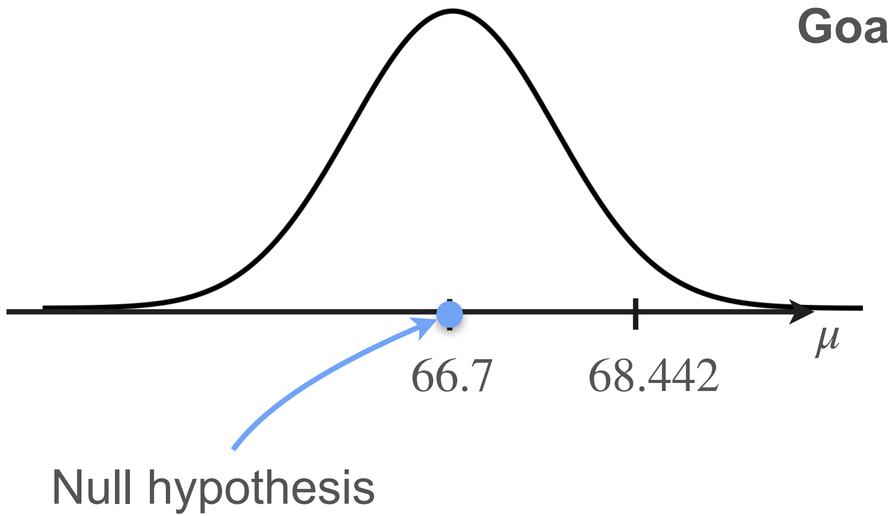
Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change



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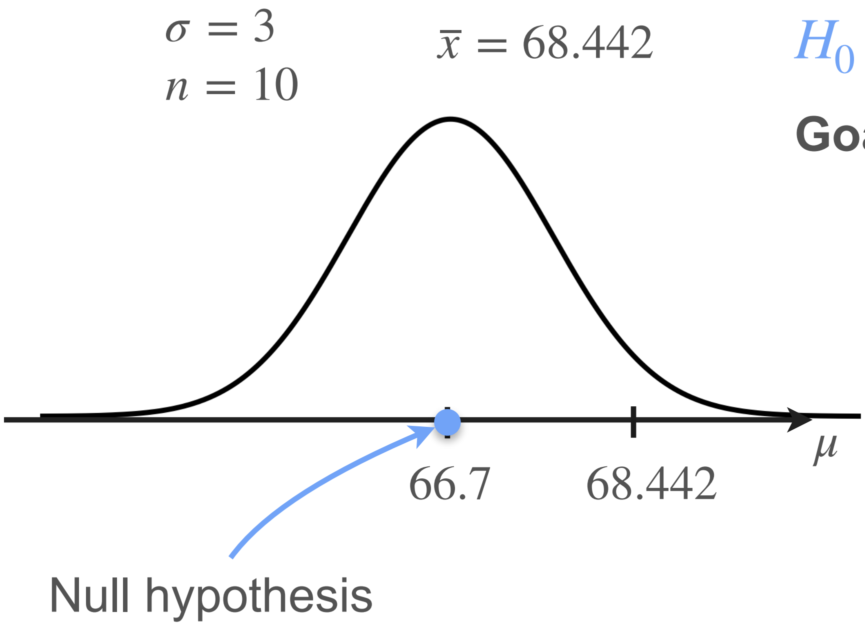

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} > 68.442 \,\middle|\, \mu = 66.7\right)?$$

Right-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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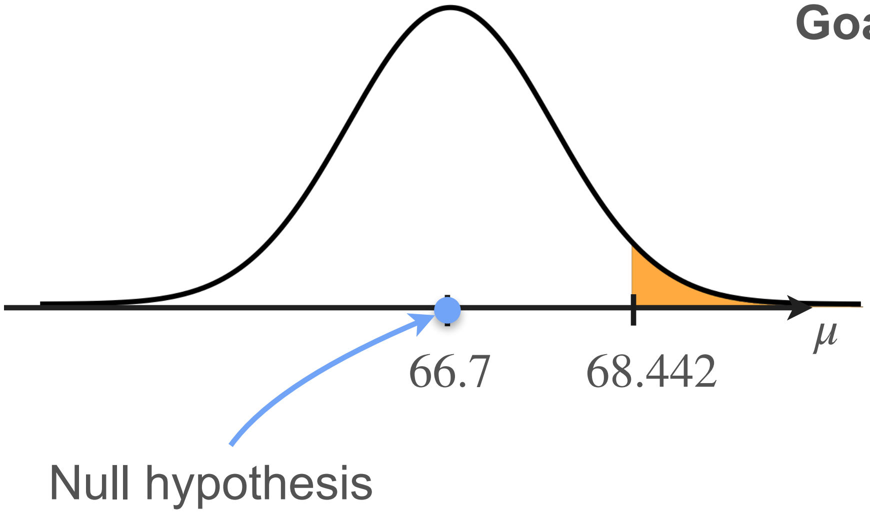

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} > 68.442 \,\middle|\, \mu = 66.7\right) = 0.0407$$

Right-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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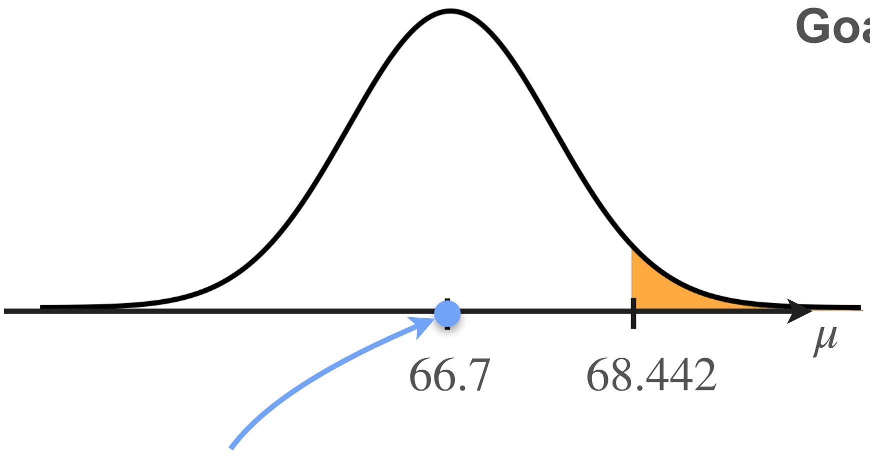
$\bar{x} = 68.442$



$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} > 68.442 \,\middle|\, \mu = 66.7\right) = 0.0407 < \alpha$$

Conclusion: reject H_0
(with a 5% significance level)

Right-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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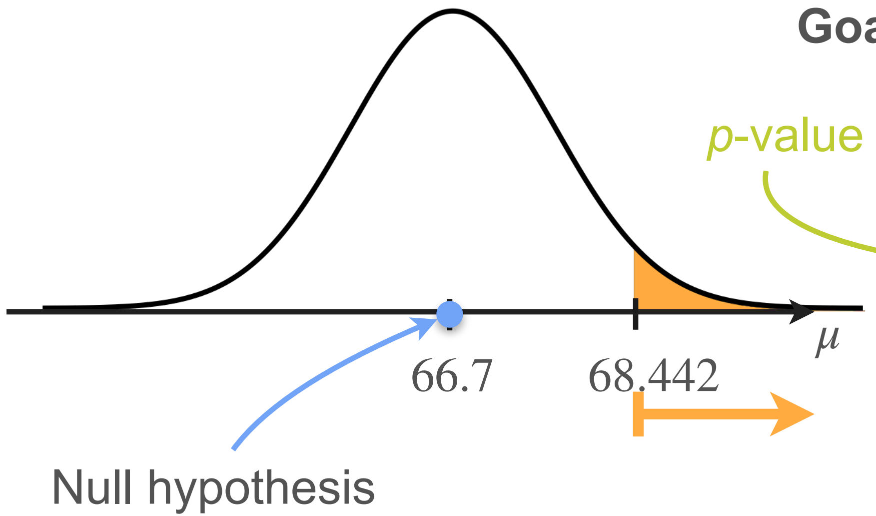

$$\mathbf{P}\left(\bar{X} > 68.442 \,\middle|\, \mu = 66.7\right) = 0.0407 < \alpha$$

Conclusion: reject H_0
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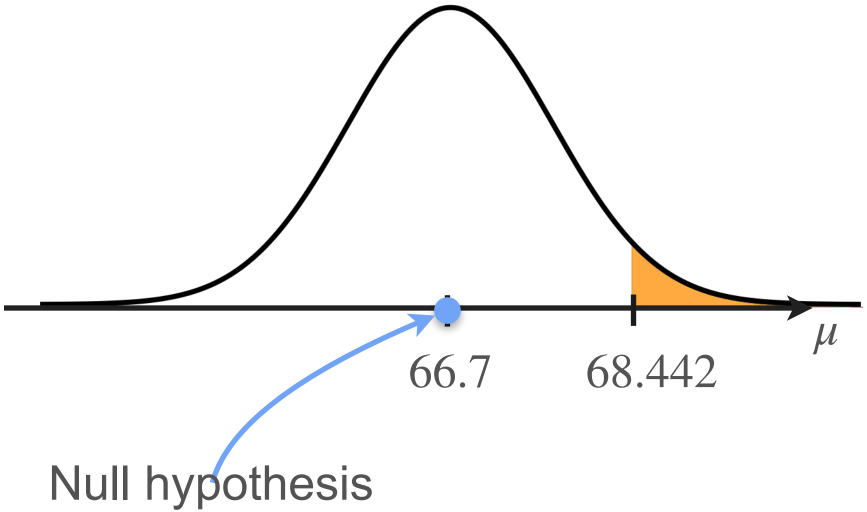
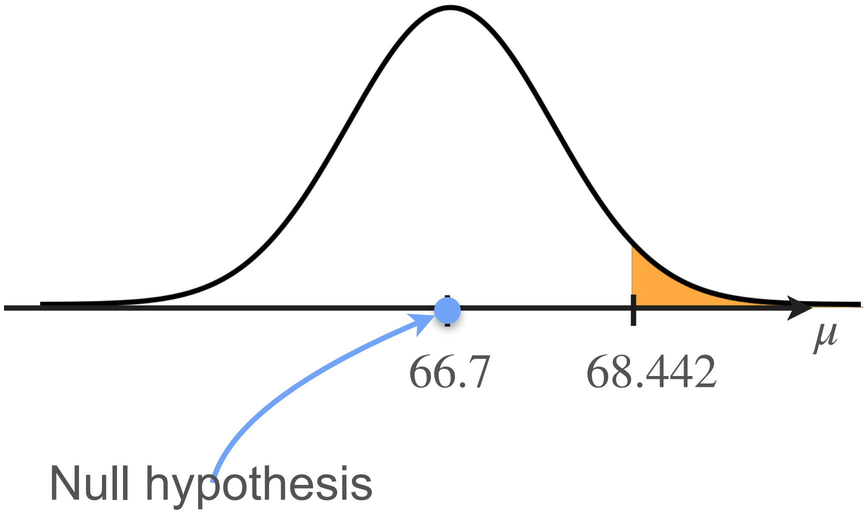
P-Values



$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

P-Values

A ***p*-value** is the probability, assuming H_0 is true, that the test statistic takes on a value as extreme as or more extreme than the value observed



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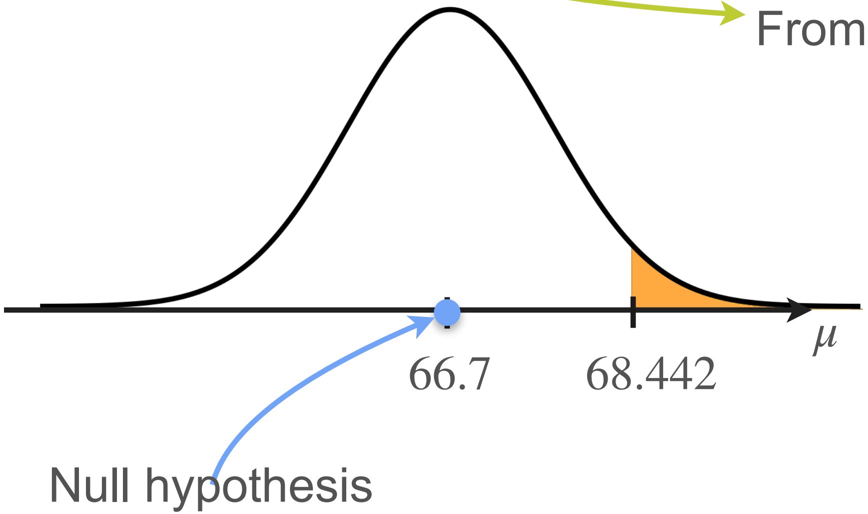
From the observed value to the direction of H_1

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



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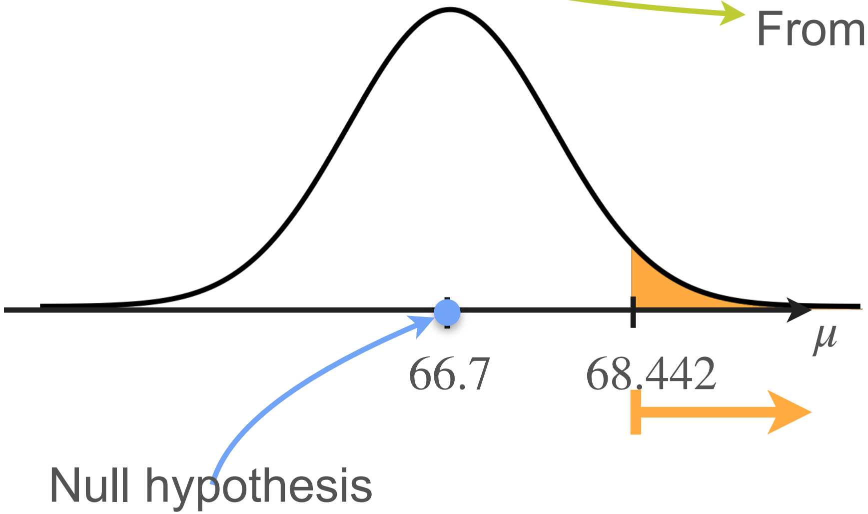

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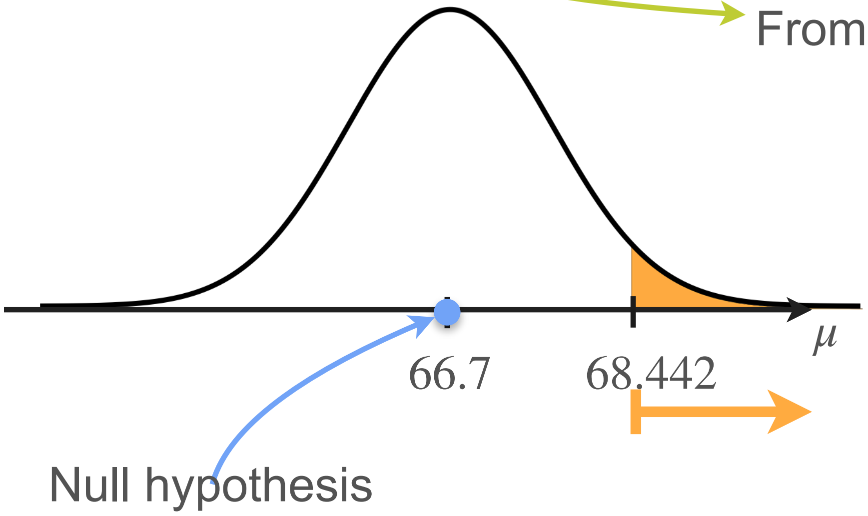
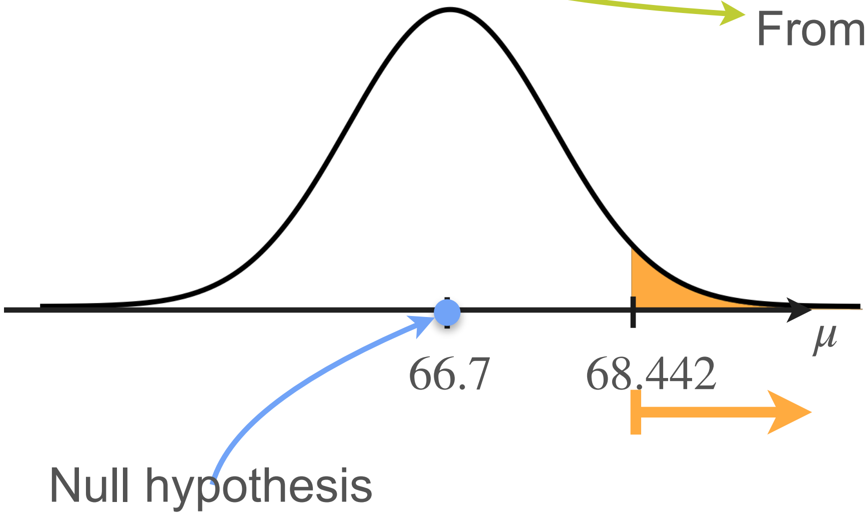


$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Decision rule:

P-Values

A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value as extreme as or more extreme than the value observed



From the observed value to the direction of H_1

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Decision rule:

If *p*-value $< \alpha$ reject H_0 (and accept H_1 as true)

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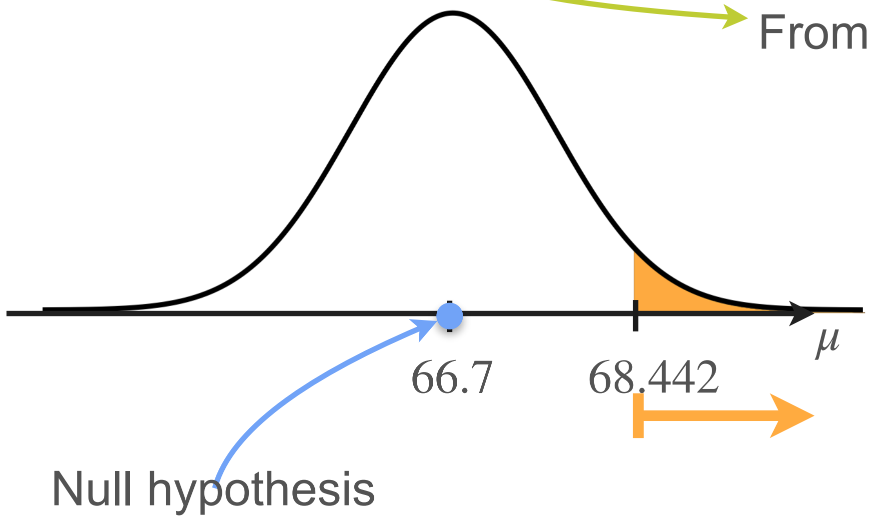

From the observed value to the direction of H_1

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Decision rule:

If p-value $< \alpha$ reject H_0 (and accept H_1 as true)

If p-value $> \alpha$ don't reject H_0

p-values

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$T(X)$: test statistic t: observed statistic H_0: $\mu = \mu_0$

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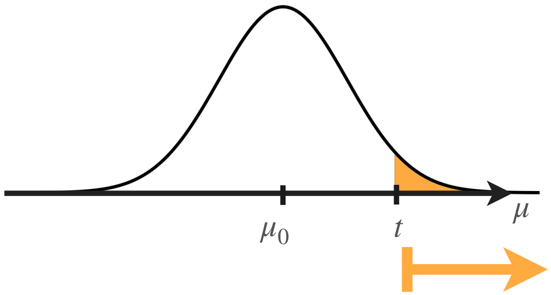
Right-tailed test



$$\mathbf{P}(T(X) > t \mid H_0)$$

p-values

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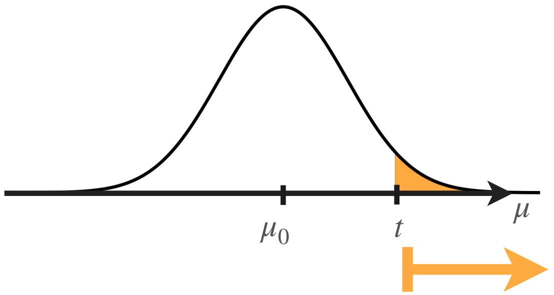
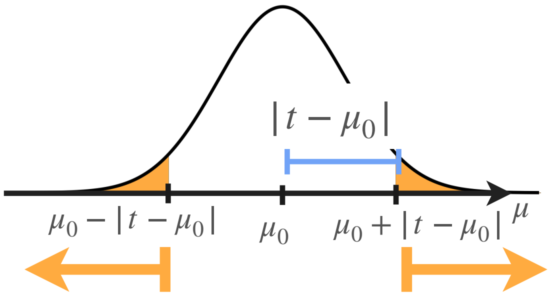
$T(X)$: test statistic t: observed statistic H_0: $\mu = \mu_0$

Right-tailed test



$\mathbf{P}(T(X) > t \mid H_0)$

Two-tailed test



$\mathbf{P}(|T(X) - \mu_0| > |t - \mu_0| \mid H_0)$

p-values

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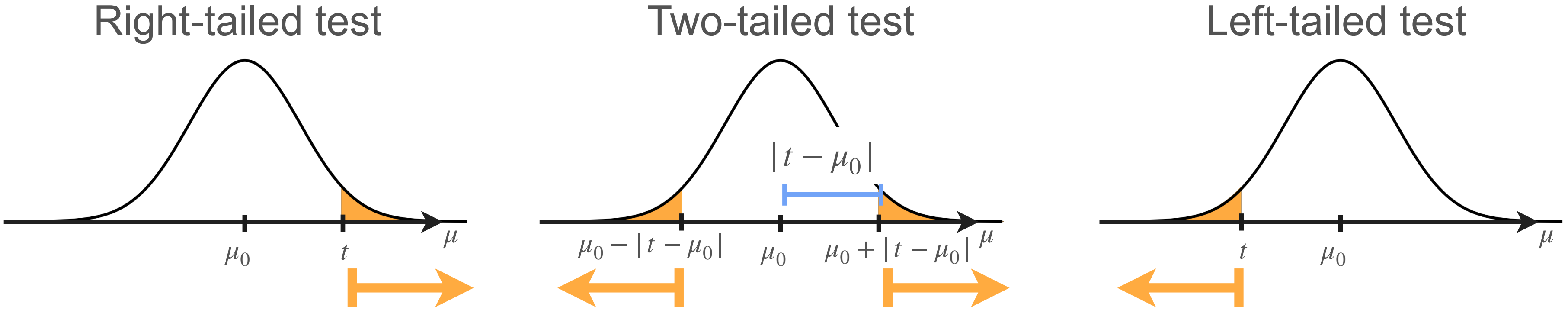
$T(X)$: test statistic t: observed statistic H_0: $\mu = \mu_0$



$\mathbf{P}(T(X) > t \mid H_0)$

$\mathbf{P}(|T(X) - \mu_0| > |t - \mu_0| \mid H_0)$

$\mathbf{P}(T(X) < t \mid H_0)$

Two-Tailed Test for Gaussian Data (Known σ)

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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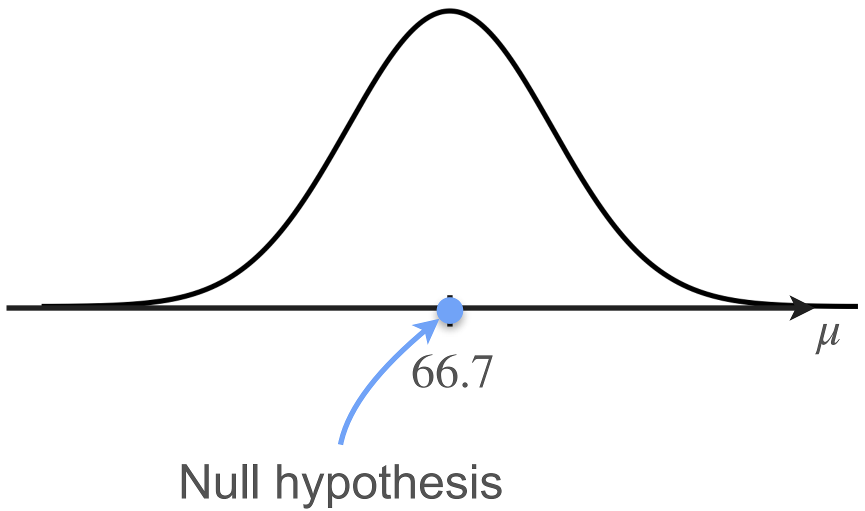
$$n = 10$$



Two-Tailed Test for Gaussian Data (Known σ)

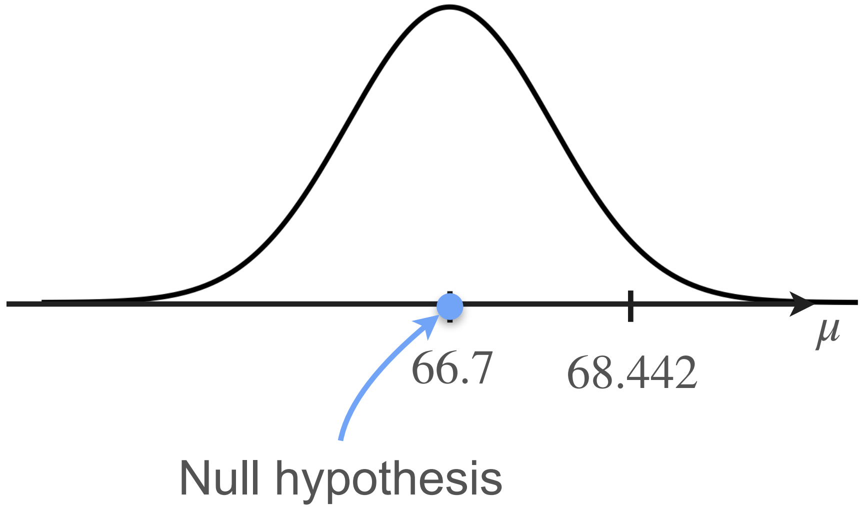
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

$\bar{x} = 68.442$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



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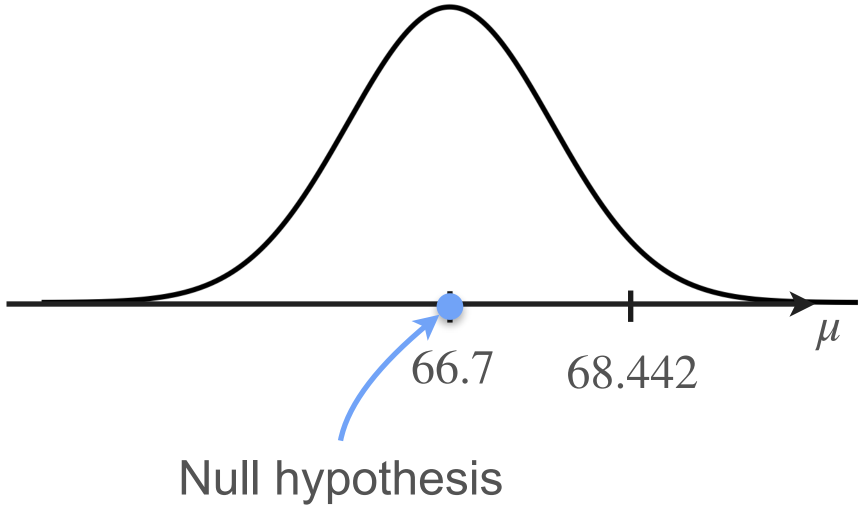

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

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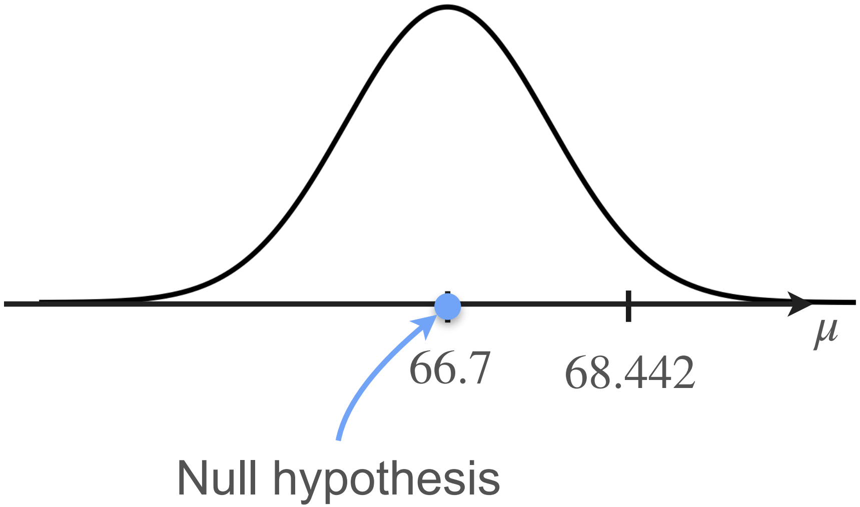

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

$$\mathbf{P}\left(\left| \bar{X} - 66.7 \right| > \left| 68.442 - 66.7 \right| \,\middle|\, \mu = 66.7 \right)?$$

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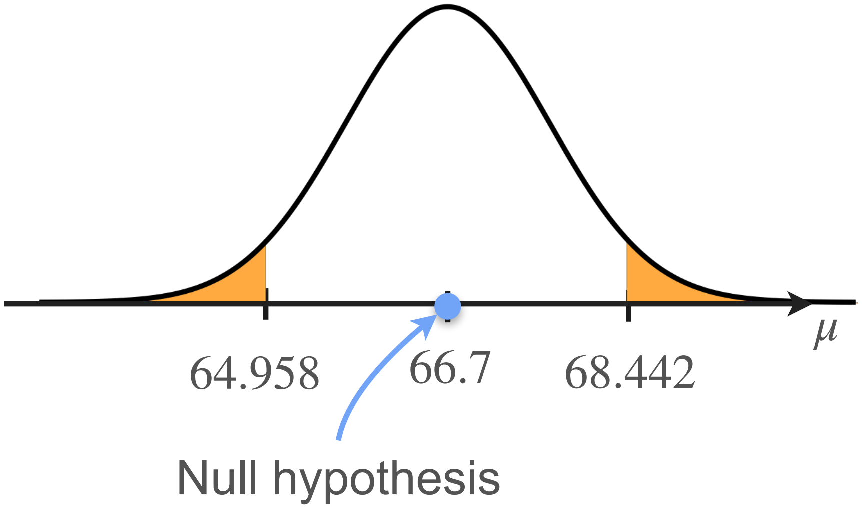


Type I error: Determine $\mu \neq 66.7$, when population mean did not change

$$\mathbf{P}\left(\left|\bar{X} - 66.7\right| > \left|68.442 - 66.7\right| \,\middle|\, \mu = 66.7\right)$$

$$= 0.082$$

Two-Tailed Test for Gaussian Data (Known σ)

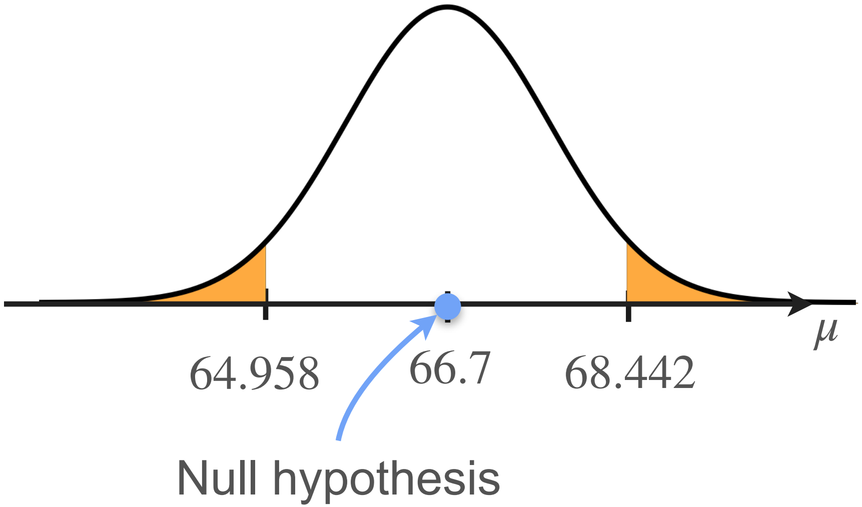
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

$\bar{x} = 68.442$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



Type I error: Determine $\mu \neq 66.7$, when population mean did not change

$$\mathbf{P}\left(\left|\bar{X} - 66.7\right| > \left|68.442 - 66.7\right| \,\middle|\, \mu = 66.7\right)$$

$$= 0.082 > \alpha$$

Conclusion: Do not reject H_0

(with a 5% significance level)

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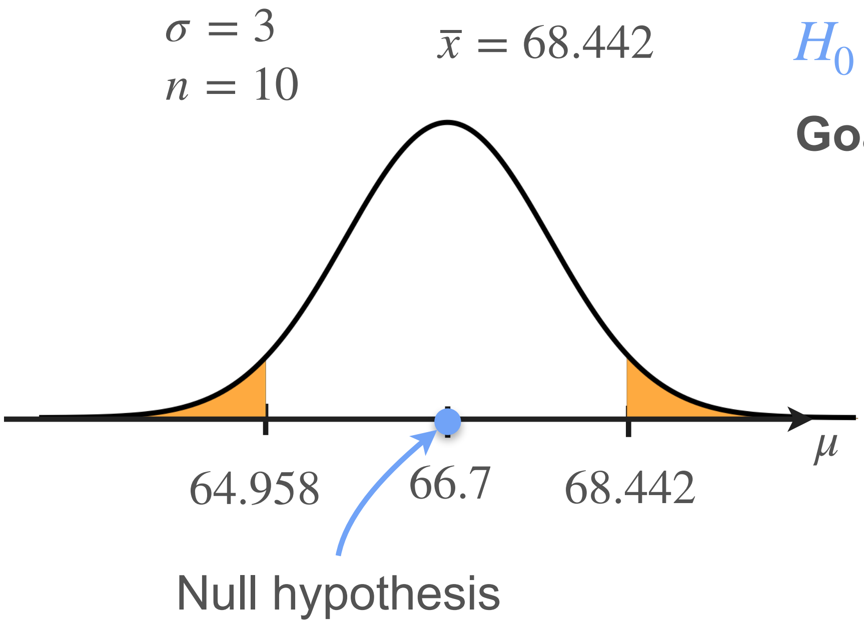
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.



$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

$$\mathbf{P}\left(\left|\bar{X} - 66.7\right| > \left|68.442 - 66.7\right| \,\middle|\, \mu = 66.7\right)$$

$$= 0.082 > \alpha$$

Conclusion: Do not reject H_0

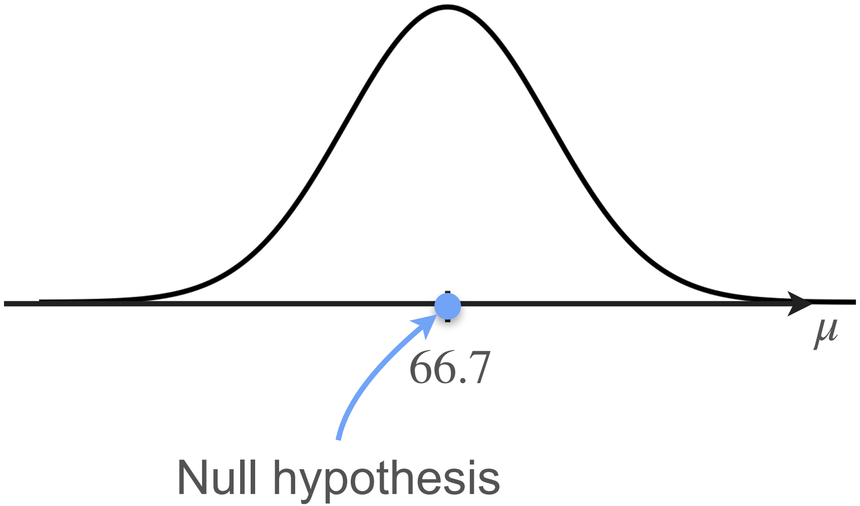
(with a 5% significance level)

Left-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$



Left-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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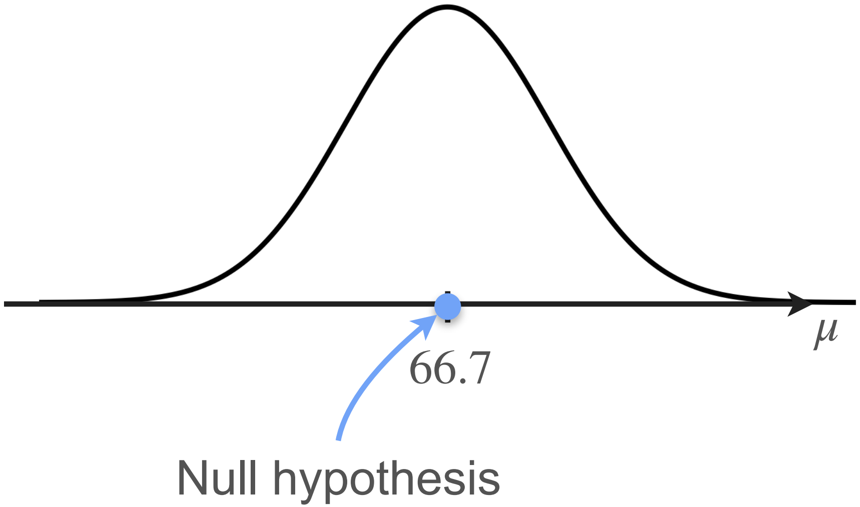
$n = 10$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



Left-Tailed Test for Gaussian Data (Known σ)

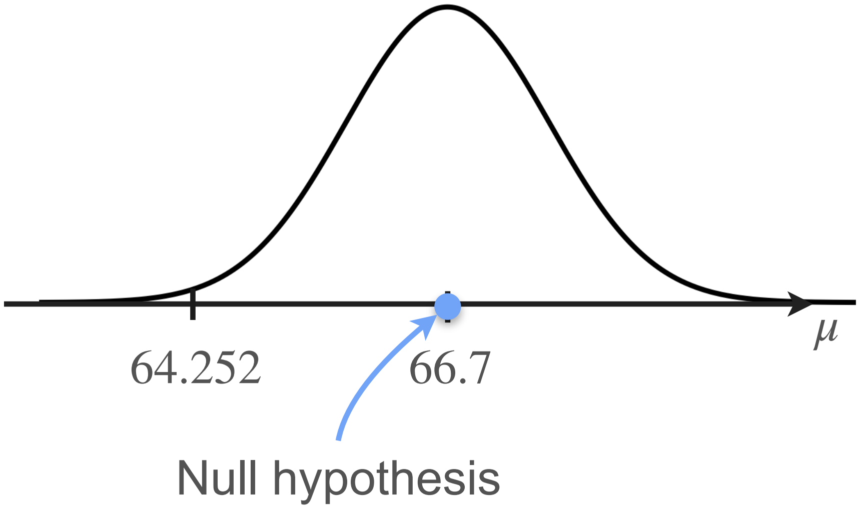
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\sigma = 3$

$n = 10$

$\bar{x} = 64.252$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



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The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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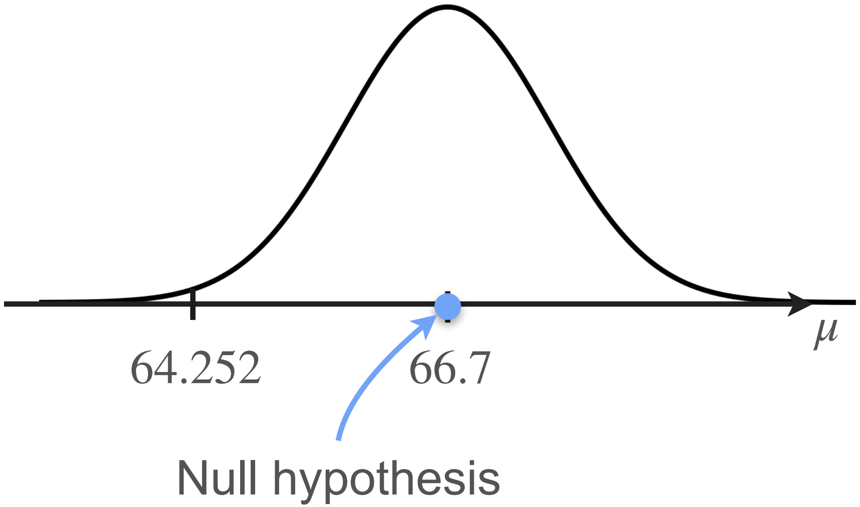

Type I error: Determine $\mu < 66.7$, when population mean did not change

Left-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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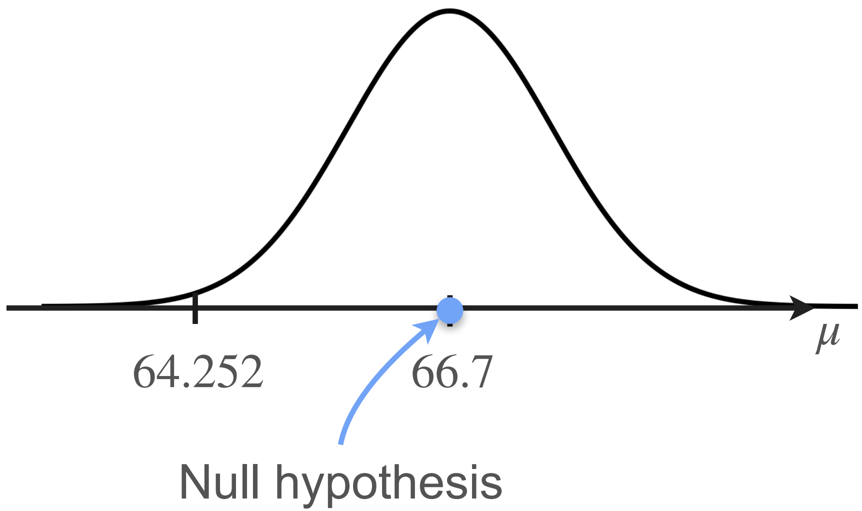

Type I error: Determine $\mu < 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} < 64.252 \,\middle|\, \mu = 66.7\right)?$$

Left-Tailed Test for Gaussian Data (Known σ)

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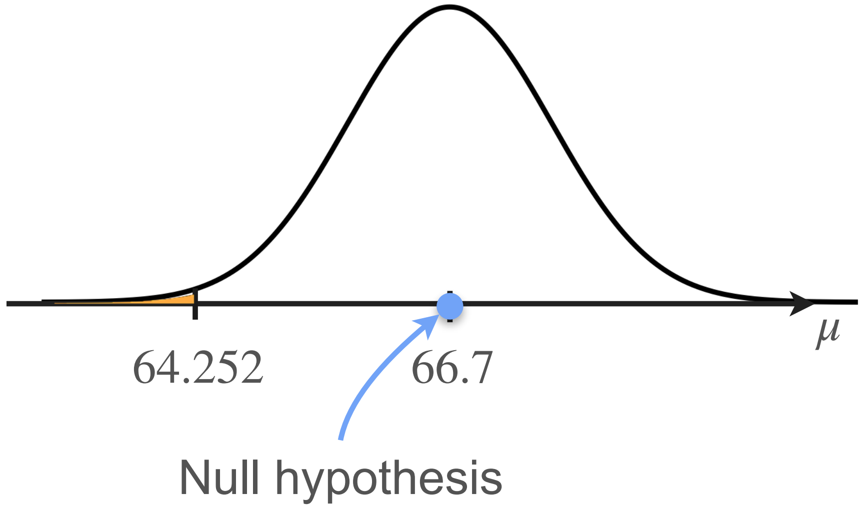
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



Type I error: Determine $\mu < 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} < 64.252 \,\middle|\, \mu = 66.7\right) = 0.0094$$

Left-Tailed Test for Gaussian Data (Known σ)

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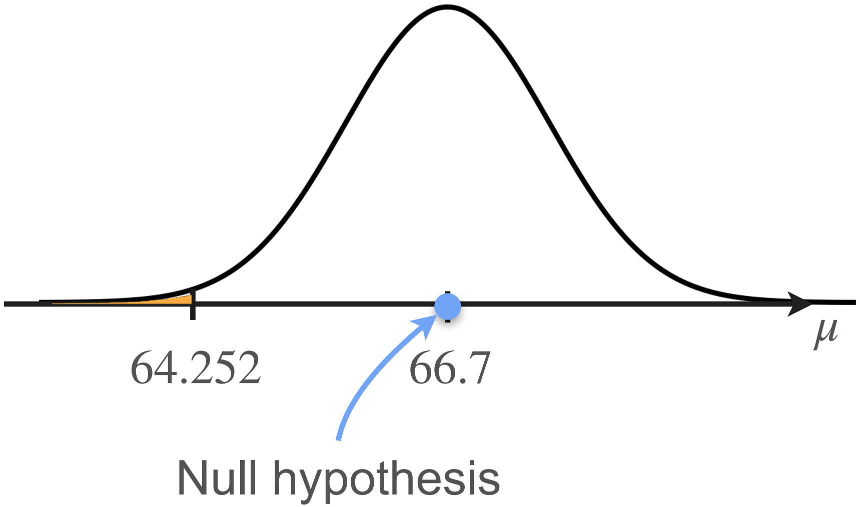
$\sigma = 3$

$n = 10$

$\bar{x} = 64.252$

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



Type I error: Determine $\mu < 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} < 64.252 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0094 < \alpha$$

Conclusion: reject H_0
(with a 5% significance level)

Left-Tailed Test for Gaussian Data (Known σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

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$n = 10$

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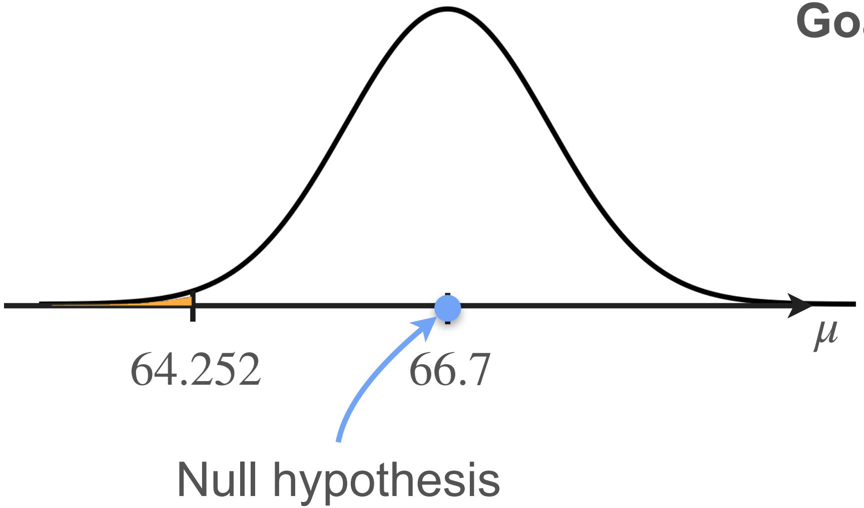

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu < 66.7$, when population mean did not change

$$\mathbf{P}\left(\bar{X} < 64.252 \,\middle|\, \mu = 66.7\right) = 0.0094 < \alpha$$

Conclusion: reject H_0

(with a 5% significance level)

Tests Using the Z-Statistic

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So far, you used the statistic $\overline{X}$

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$$\text{If } H_0 \text{ is true: } \overline{X} \sim \mathcal{N}\left(\mu_0, \frac{3^2}{10}\right)$$

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So far, you used the statistic $\overline{X}$

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Right-Tailed Test Using the Z Statistic

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Goal: Type I error probability < $\alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

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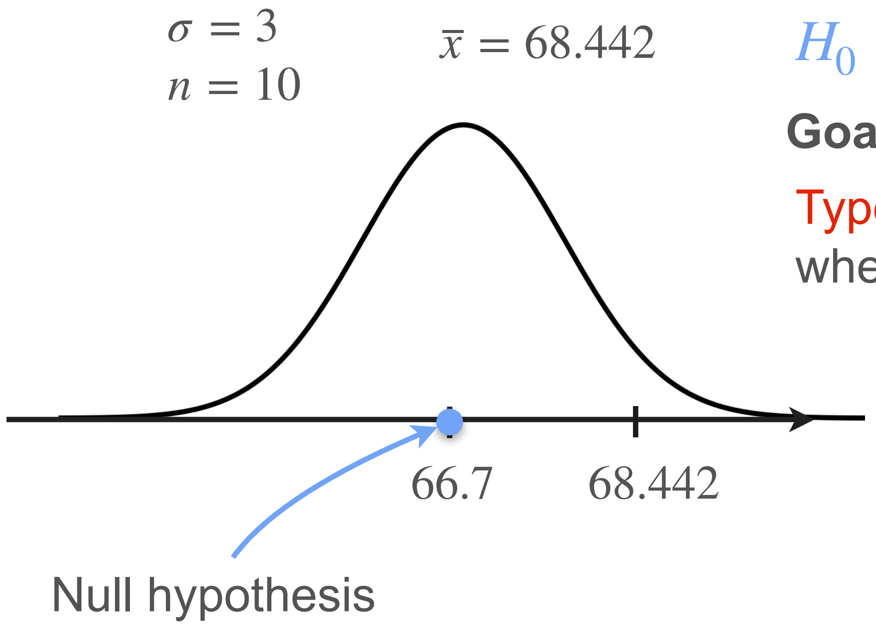

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Right-Tailed Test Using the Z Statistic

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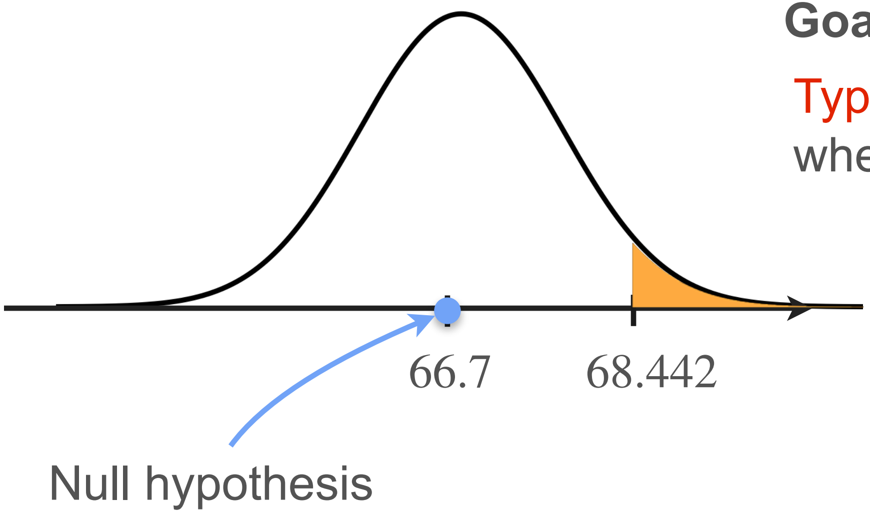
$\bar{x} = 68.442$



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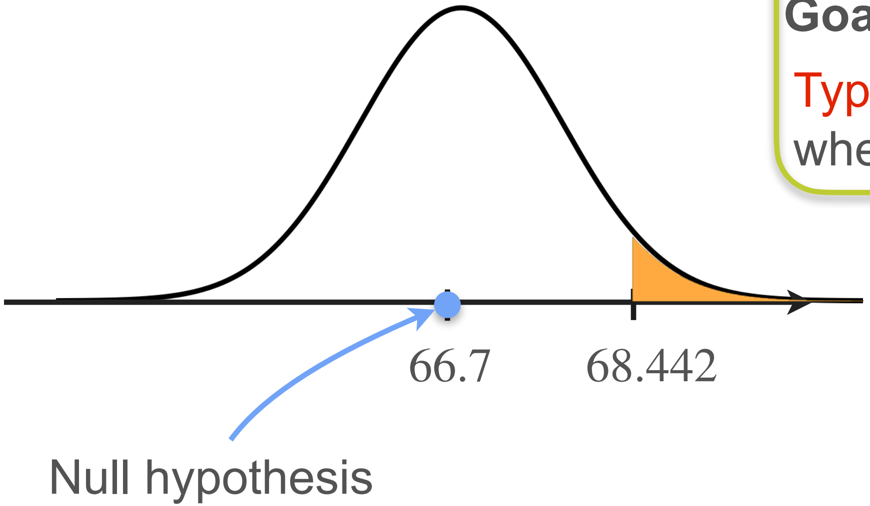

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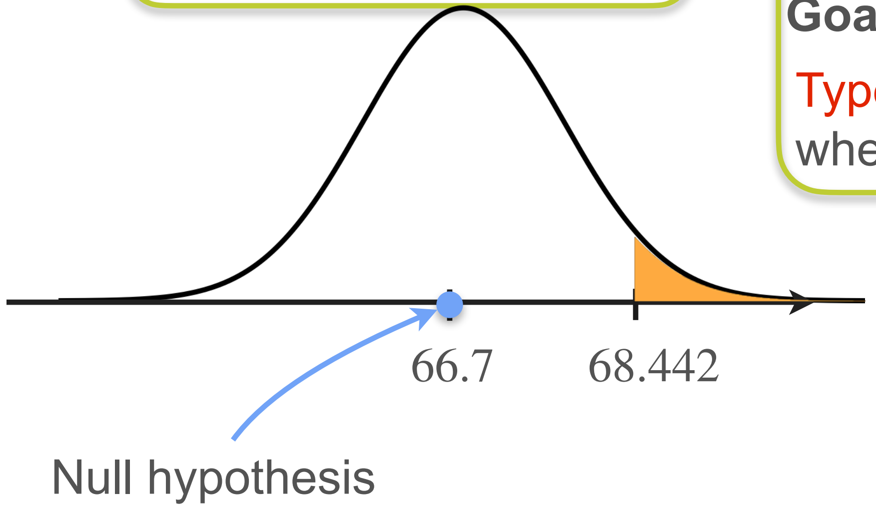

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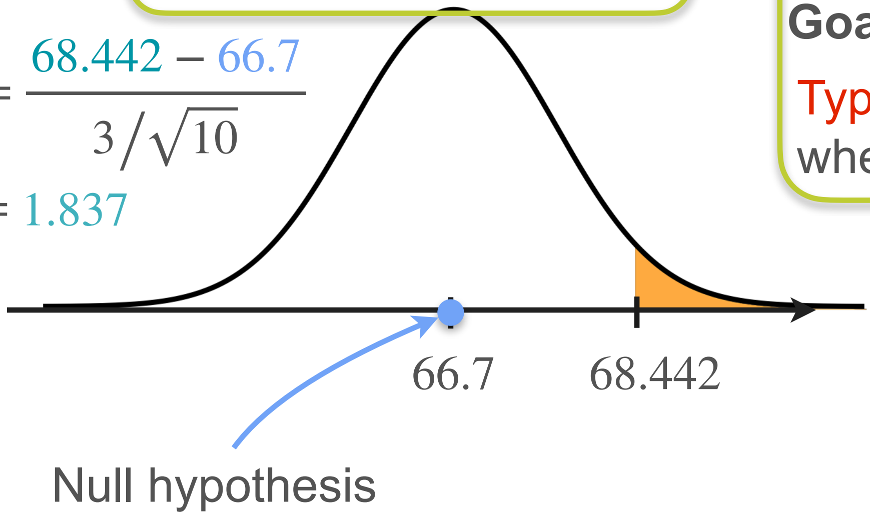

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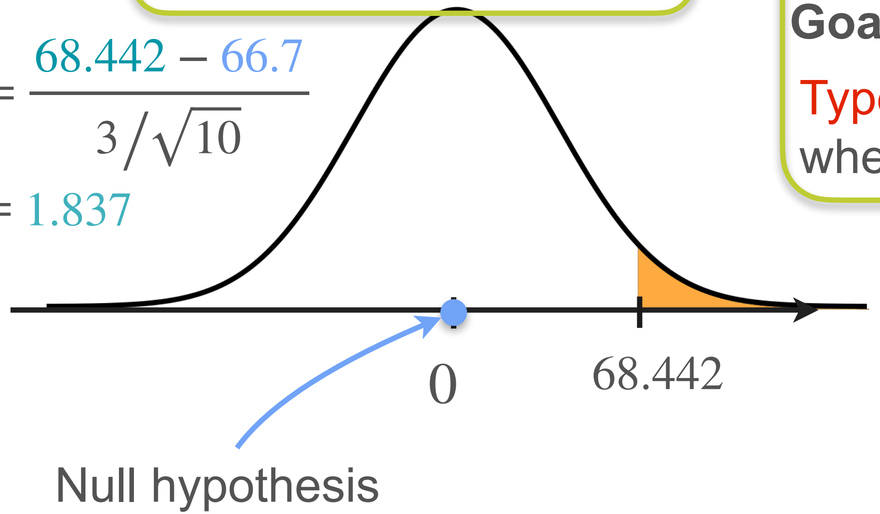

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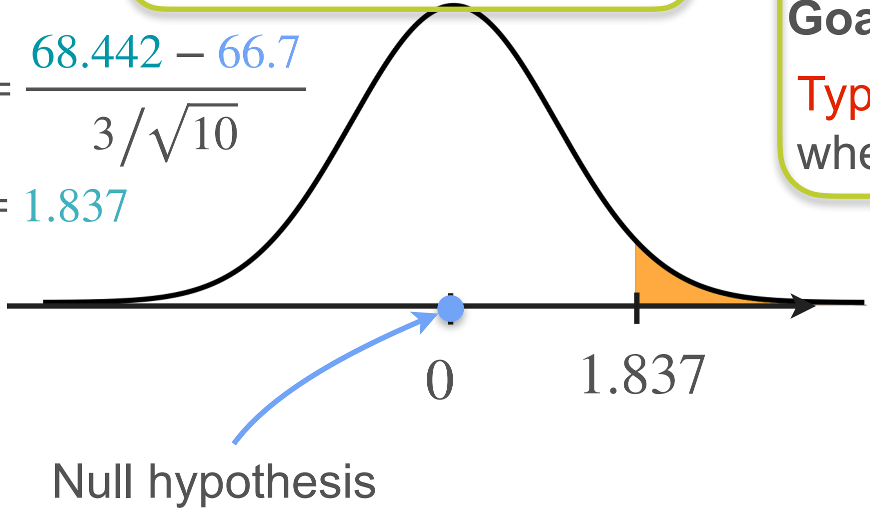

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Hypothesis Testing

Critical Values

P-Values and Critical Values

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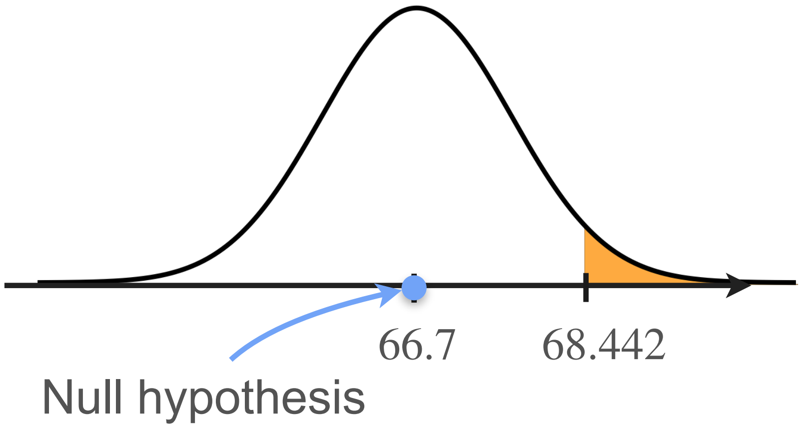
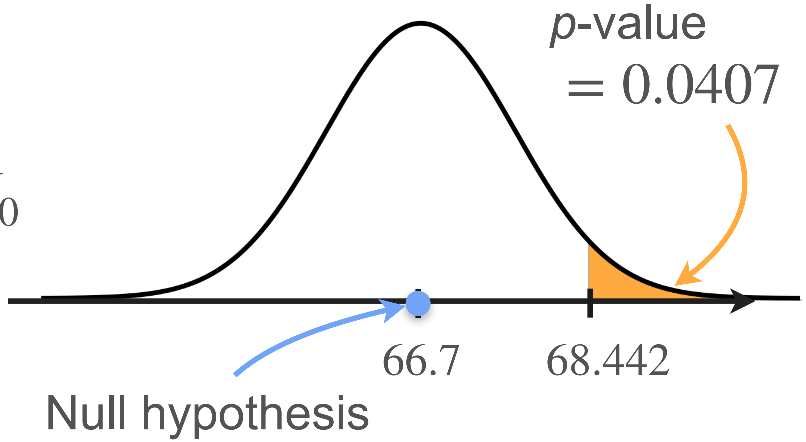


P-Values and Critical Values

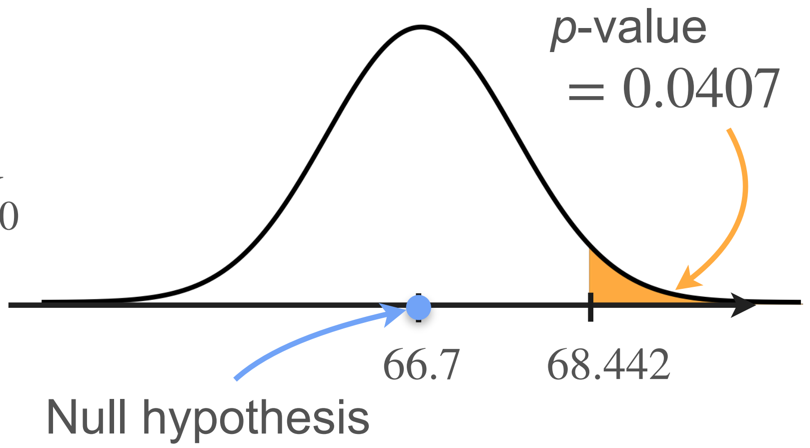
If p-value $< \alpha$ $\Rightarrow$ Reject H_0



P-Values and Critical Values

If p-value $< \alpha$ ⇨ Reject H_0

What is the least extreme sample you could get that you would still reject H0?



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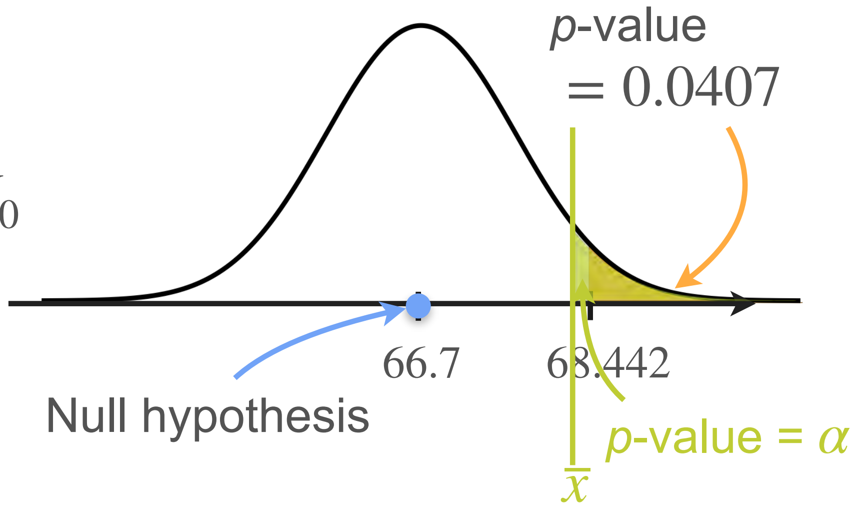

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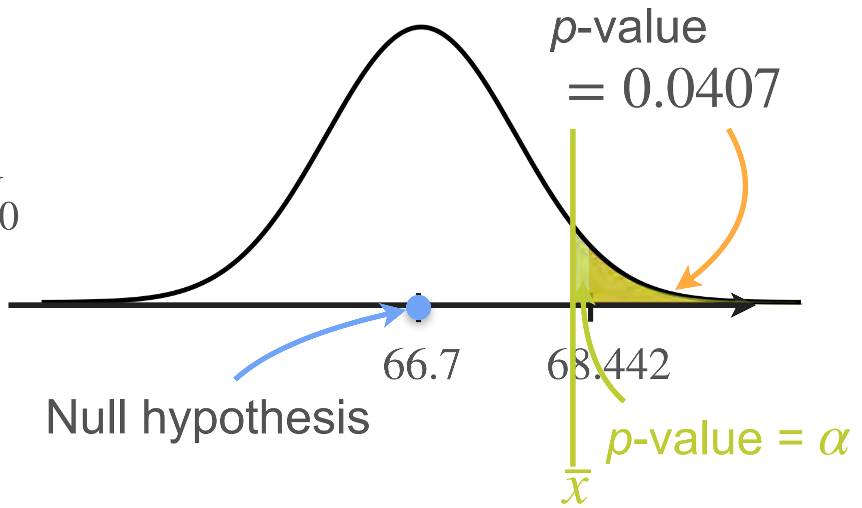

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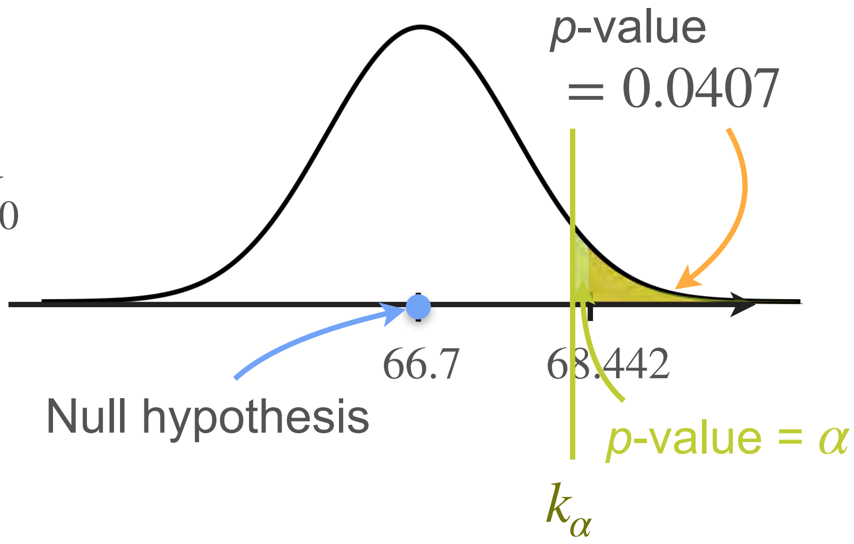

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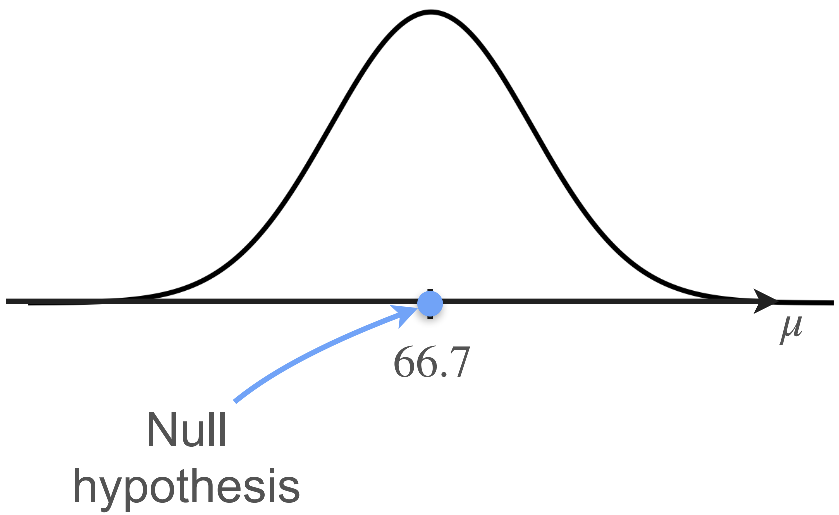

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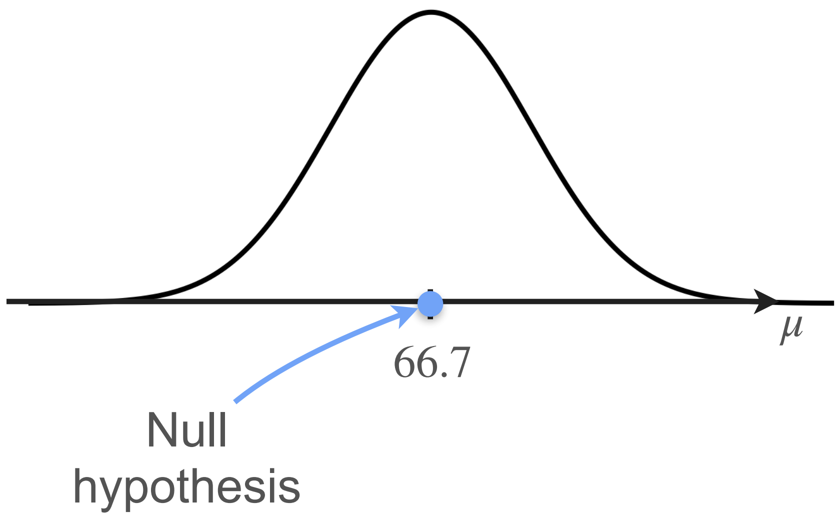

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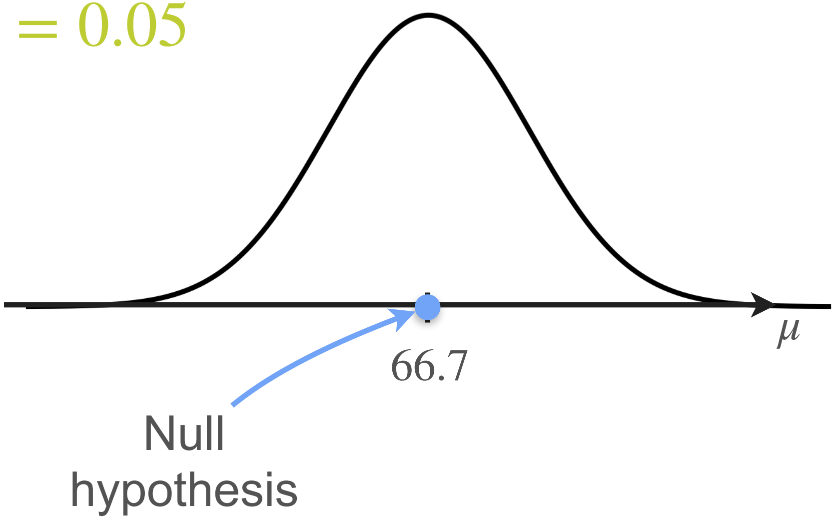

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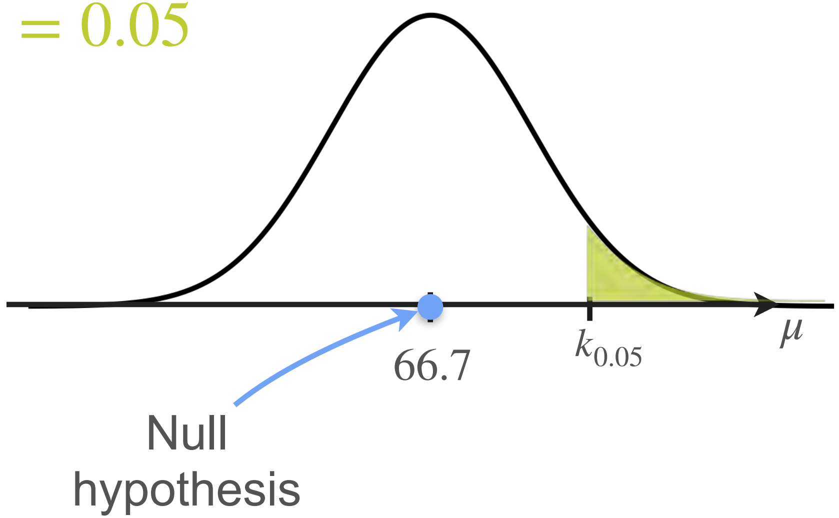

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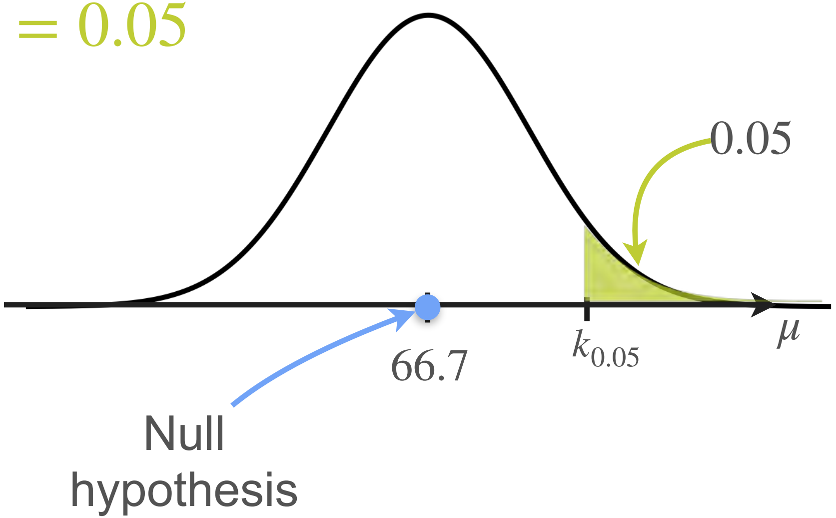

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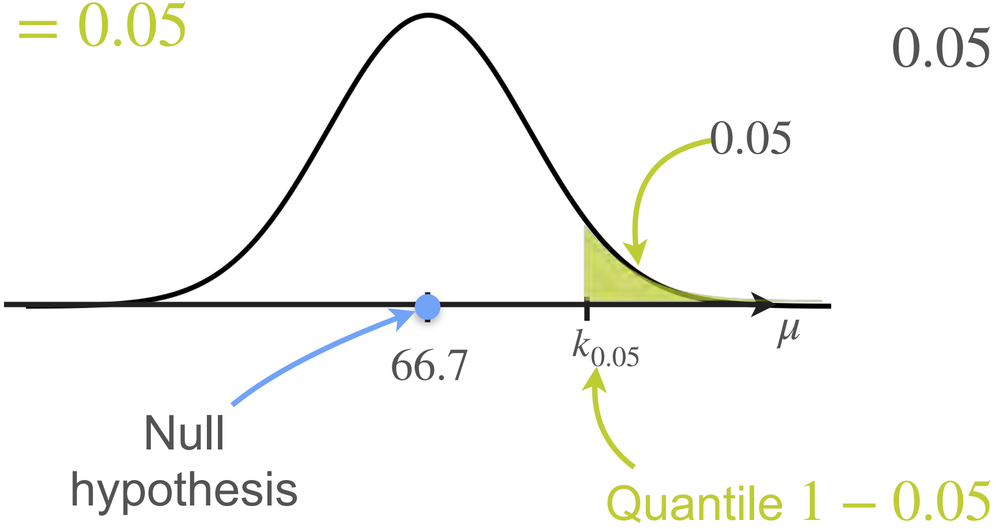

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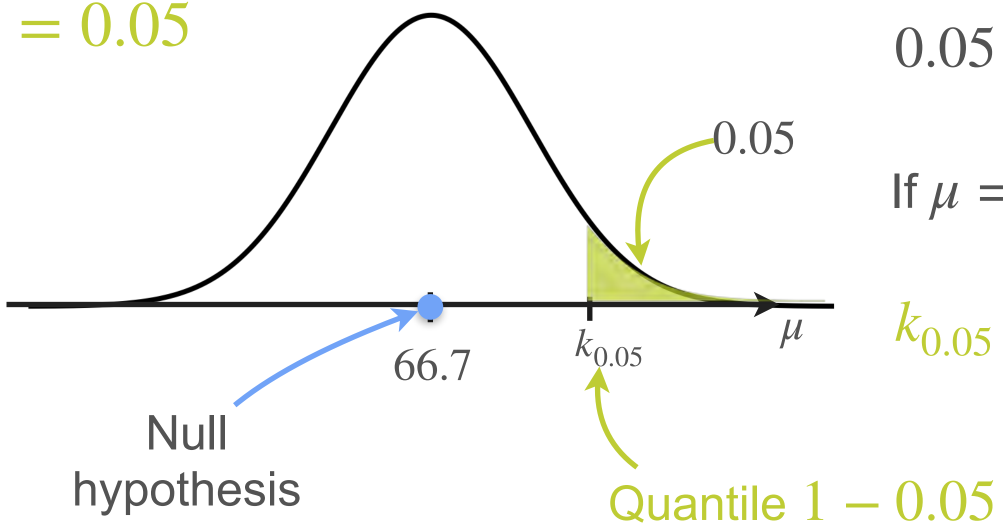

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If $\mu = 66.7 \;\; \bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$k_{0.05} = 68.26$

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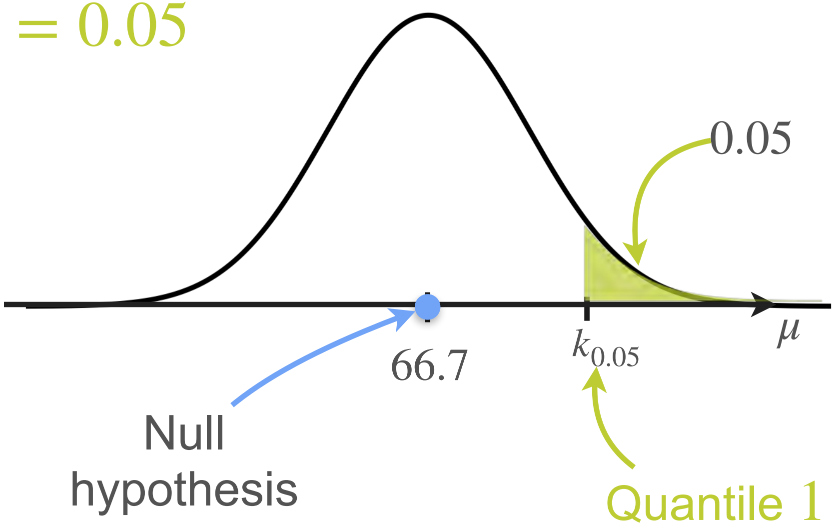

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Decision rule: Reject H_0 if $\bar{x} > 68.26$

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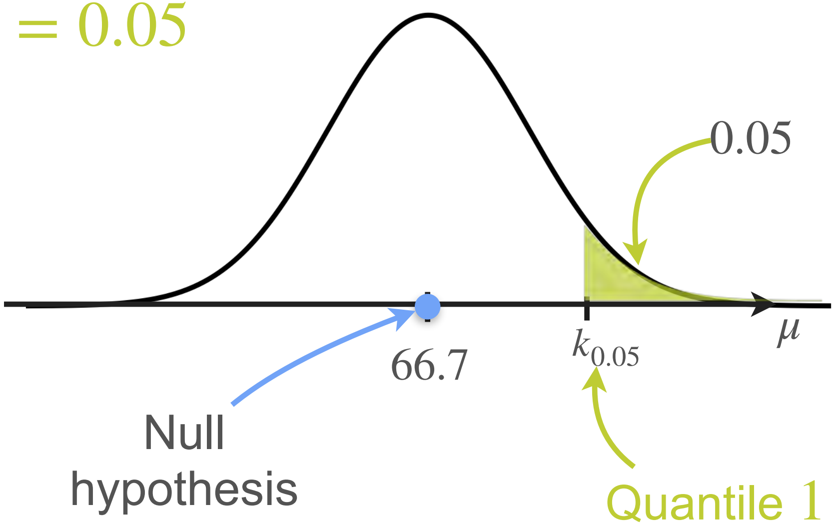

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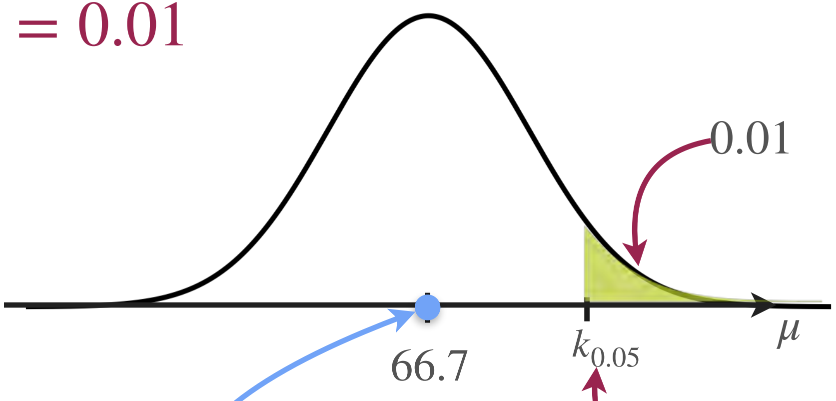

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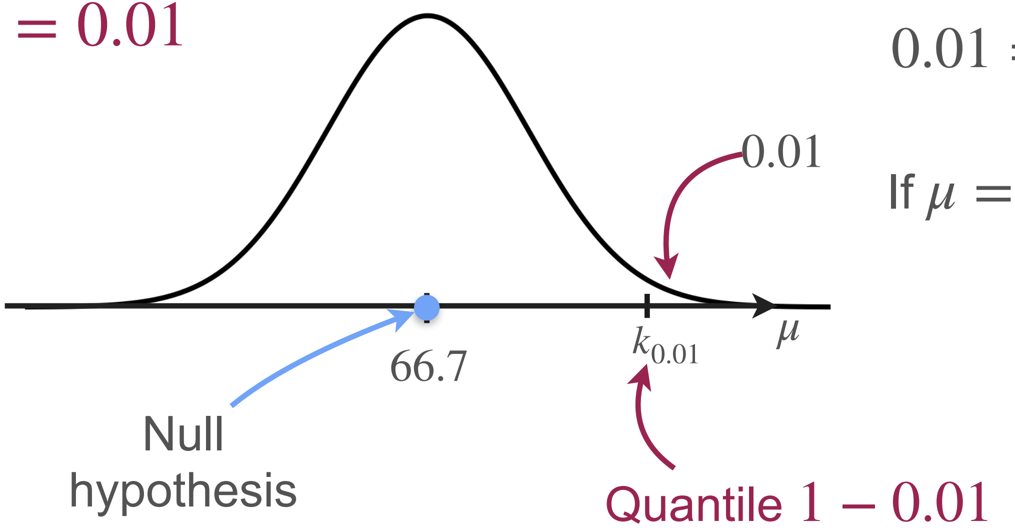

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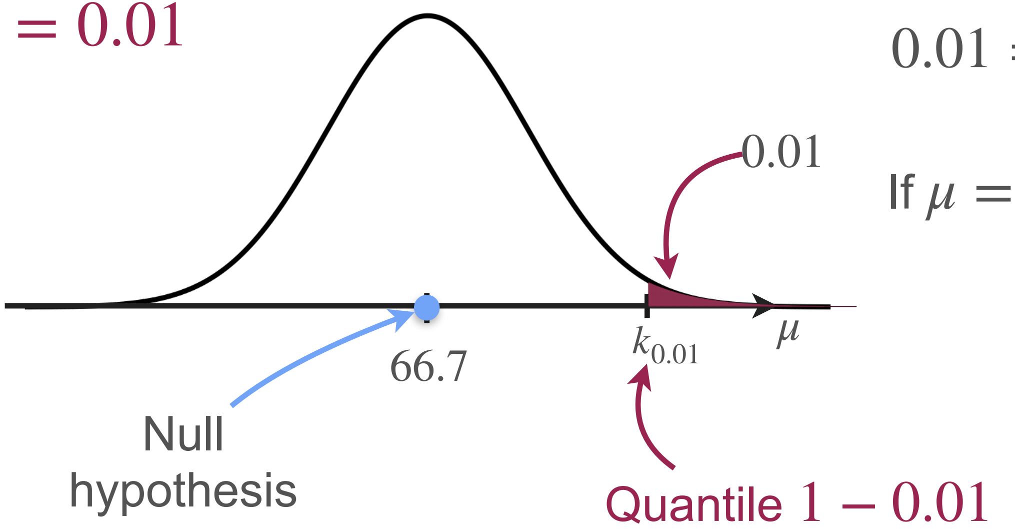


Computing Critical Values

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

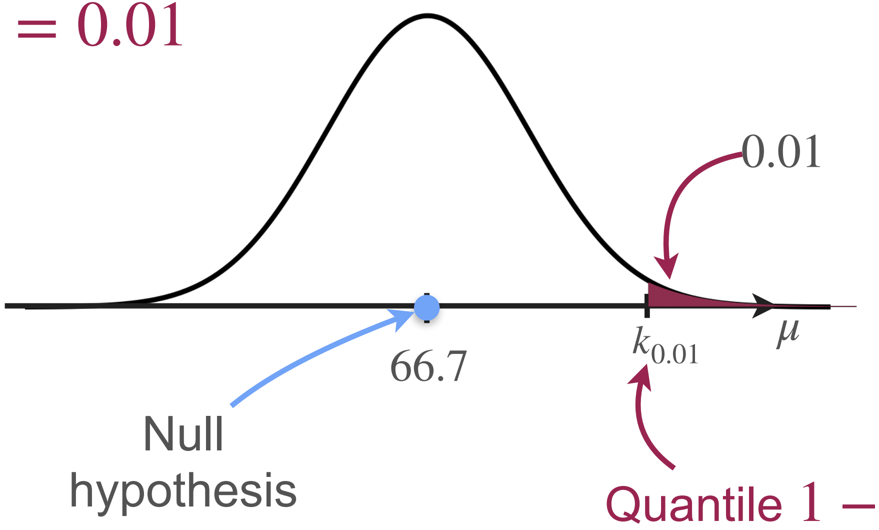
$\alpha = 0.01$

$n = 10 \qquad \sigma = 3 \qquad \bar{x} = 68.442$

$$0.01 = \mathbf{P}\left(\bar{X} > k_{0.01} \,\middle|\, \mu = 66.7\right)$$

If $\mu = 66.7$ $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$k_{0.01} = 68.91$



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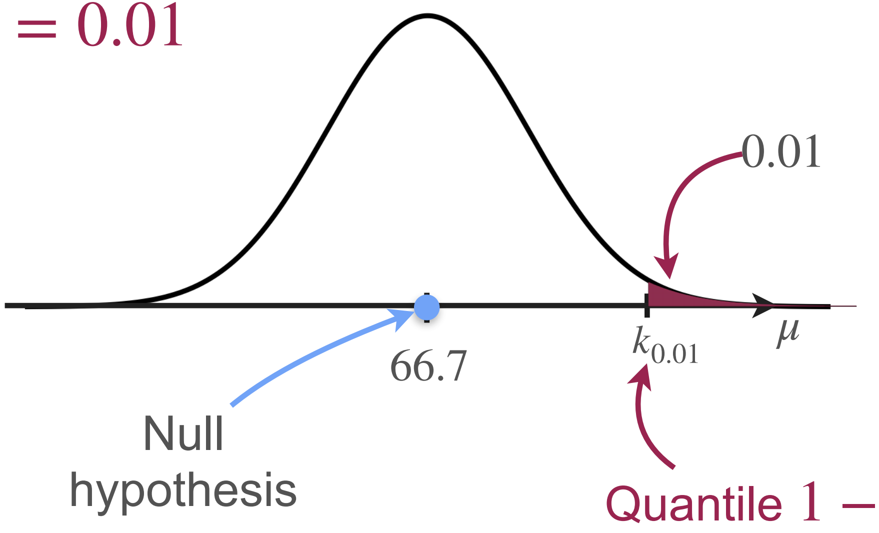

$n = 10 \qquad \sigma = 3 \qquad \bar{x} = 68.442$

$$0.01 = \mathbf{P}\left(\bar{X} > k_{0.01} \,\middle|\, \mu = 66.7\right)$$

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Decision rule: Reject H_0 if $\bar{x} > 68.91$

Computing Critical Values

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

Do not reject H_0

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

$\alpha = 0.01$

$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$

$$0.01 = \mathbf{P}\left(\bar{X} > k_{0.01} \,\middle|\, \mu = 66.7\right)$$

If $\mu = 66.7$ $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

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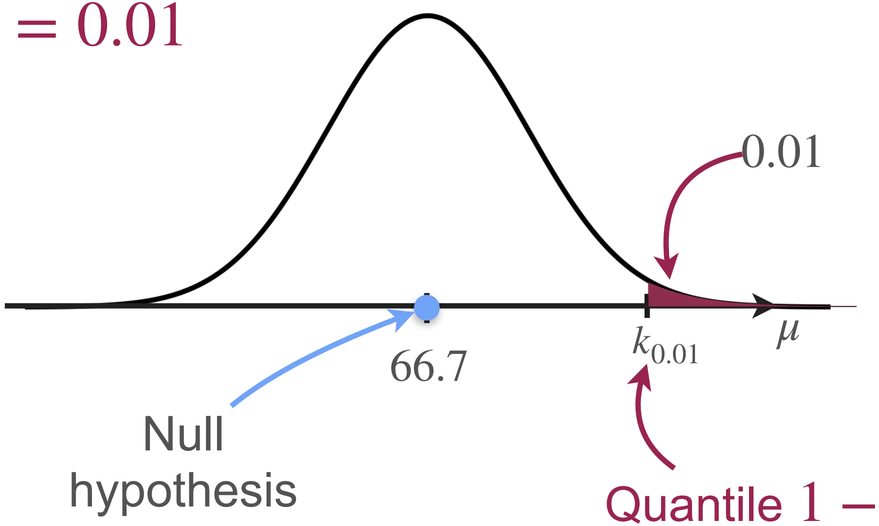
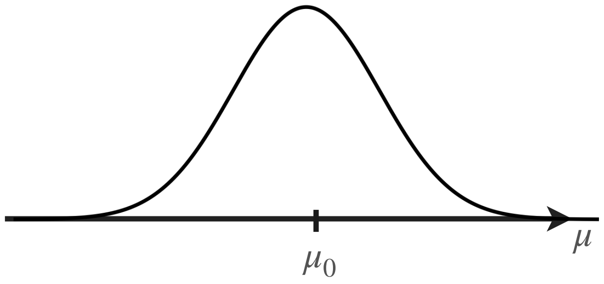


Decision rule: Reject H_0 if $\bar{x} > 68.91$

Critical Values

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$



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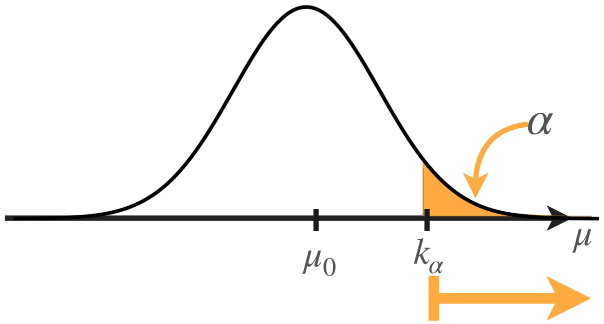

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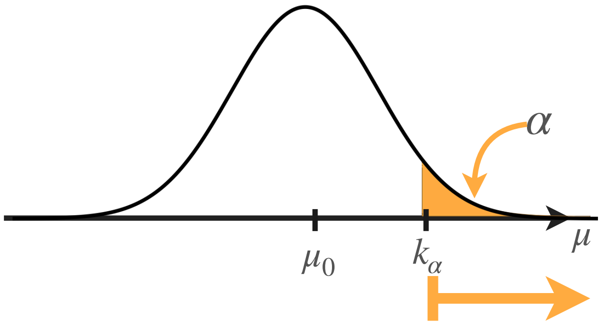
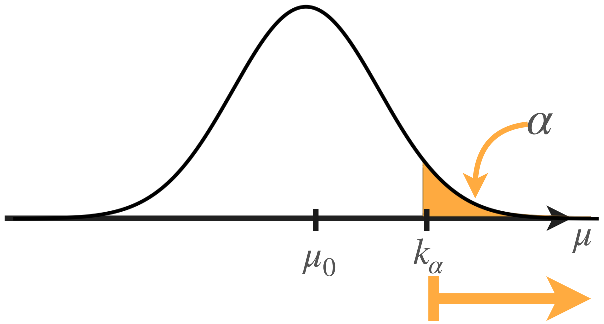


k_α is the quantile $1 - \alpha$

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

Decision rule: Reject H_0 if $t > k_\alpha$



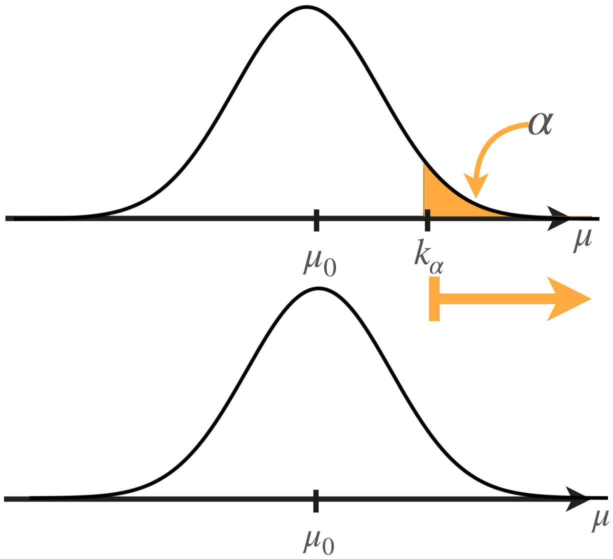
k_α is the quantile $1 - \alpha$

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

Decision rule: Reject H_0 if $t > k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu < \mu_0$



k_α is the quantile $1 - \alpha$

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

Decision rule: Reject H_0 if $t > k_\alpha$

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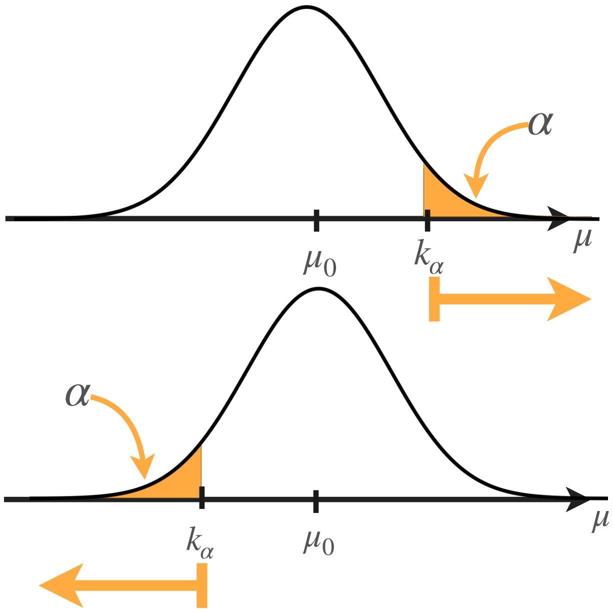

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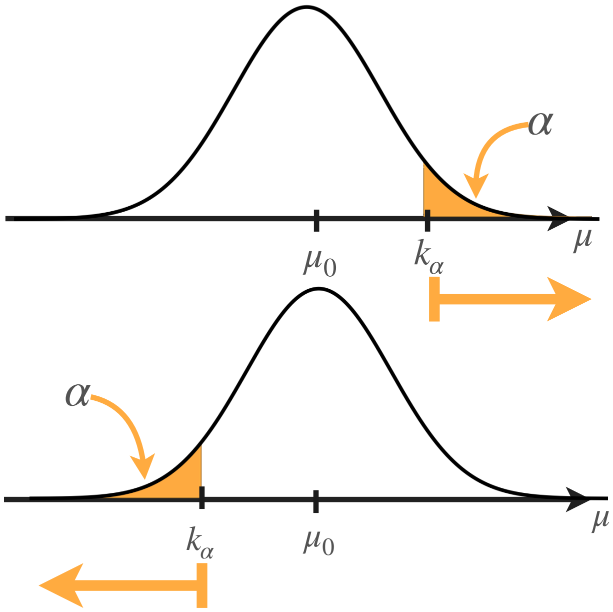

k_α is the quantile $1 - \alpha$

k_α is the quantile α

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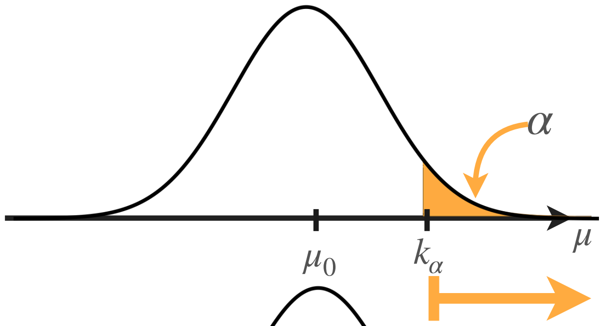
Decision rule: Reject H_0 if $t > k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu < \mu_0$

Decision rule: Reject H_0 if $t < k_\alpha$



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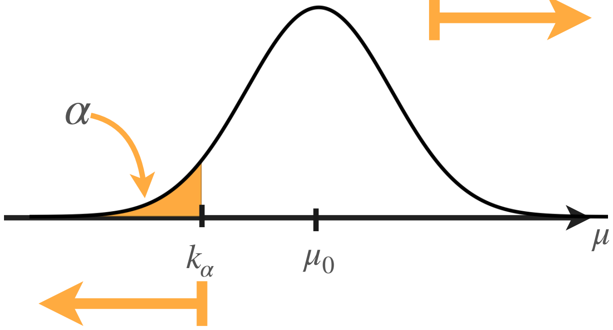

k_α is the quantile α

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

Decision rule: Reject H_0 if $t > k_\alpha$

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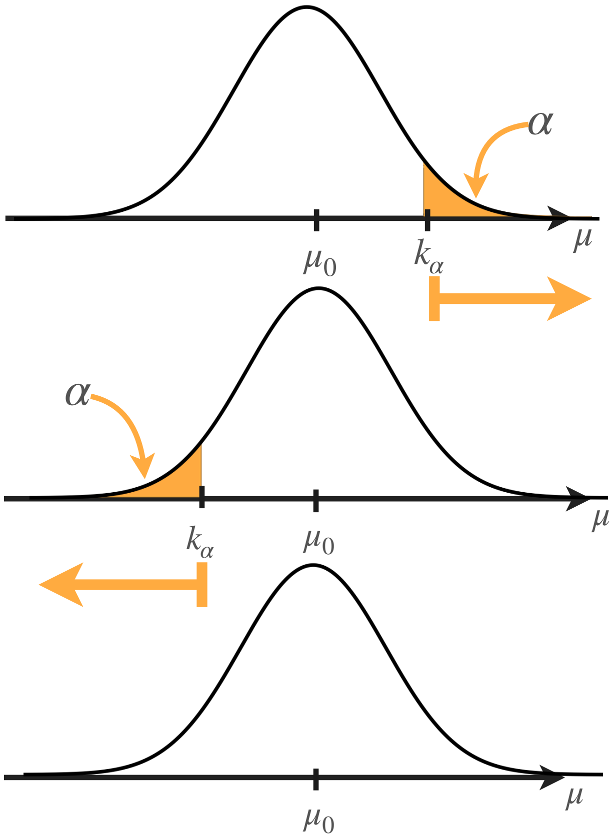
Decision rule: Reject H_0 if $t < k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$



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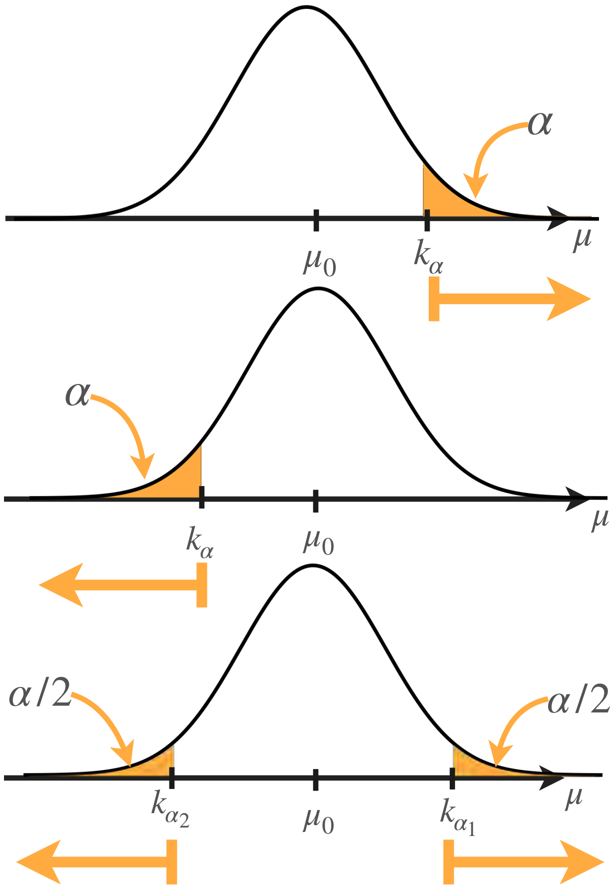

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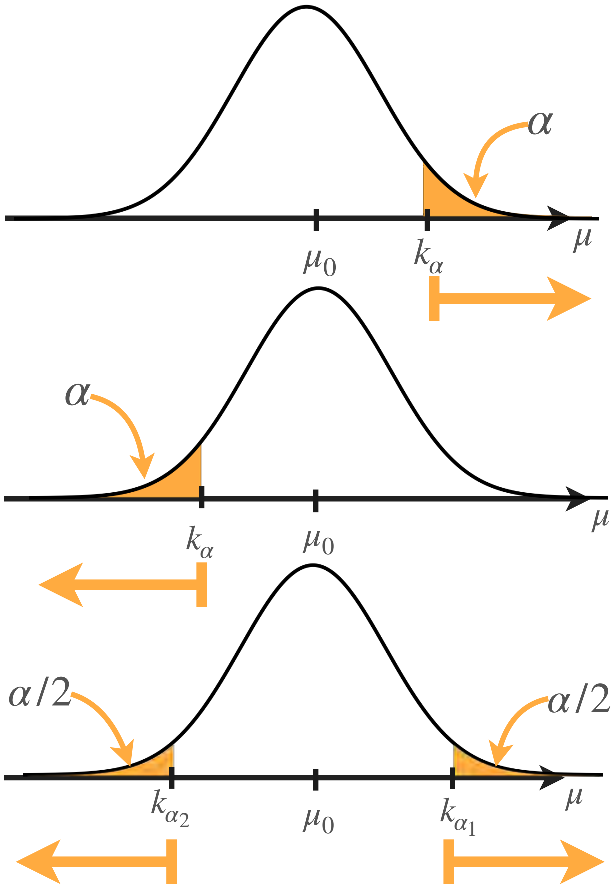

k_α is the quantile $1-\alpha$

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k_{α_1} is the quantile $1-\dfrac{\alpha}{2}$, and k_{α_2} is the quantile $\dfrac{\alpha}{2}$

Critical Values

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

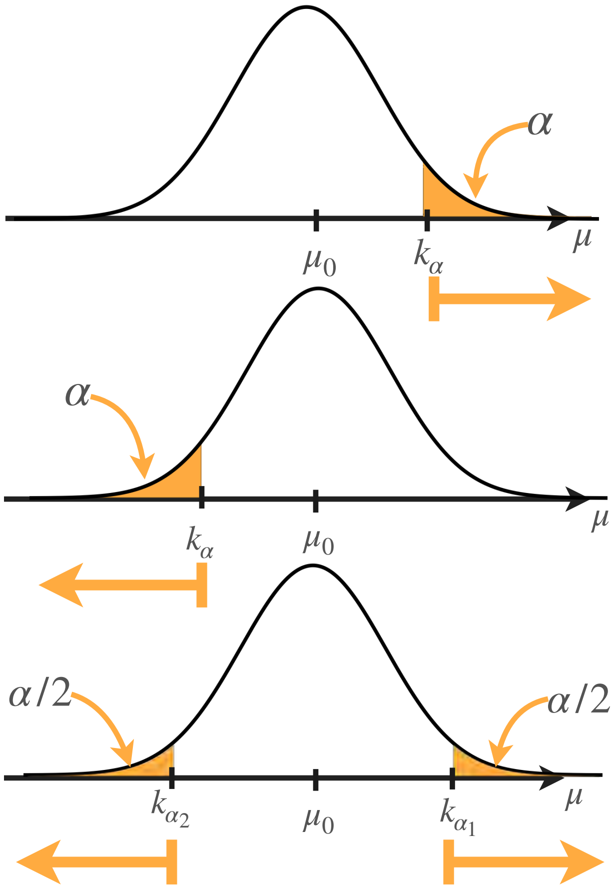
Decision rule: Reject H_0 if $t > k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu < \mu_0$

Decision rule: Reject H_0 if $t < k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$

Decision rule: Reject H_0 if $t > k_{\alpha_1}$ or $t < k_{\alpha_2}$



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- For a given sample, using *p*-value and critical value will lead to the same conclusion
- Defining a test in terms of critical values makes determining Type II error probabilities for the decision rule.

Hypothesis Testing

Power of a test

Type I and Type II Errors

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Decision	**Reality**	
	H_0 True ($\mu = 66.7$)	H_0 False ($\mu > 66.7$)
Reject H_0 (Decide $\mu > 66.7$)	Type I error	Correct
Don't reject H_0 (Decide $\mu = 66.7$)	Correct	Type II error

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What is the **Type II error probability** if the true value is $\mu = 70$?

$\mathbf{P}\left(\text{Do not reject } H_0 \,|\, \mu = 70\right) \longrightarrow \mathbf{P}(\bar{X} < 68.26 \,|\, \mu = 70)$

Finding the Type II Error Probabilities

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10 \quad \sigma = 3$

For $\alpha = 0.05$: $k_\alpha = 68.2604$ $\quad$ Decision rule: Reject H_0 if $\overline{X} > 68.26$

What is the **Type II error probability** if the true value is $\mu = 70$?

$$\mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70)$$

Finding the Type II Error Probabilities

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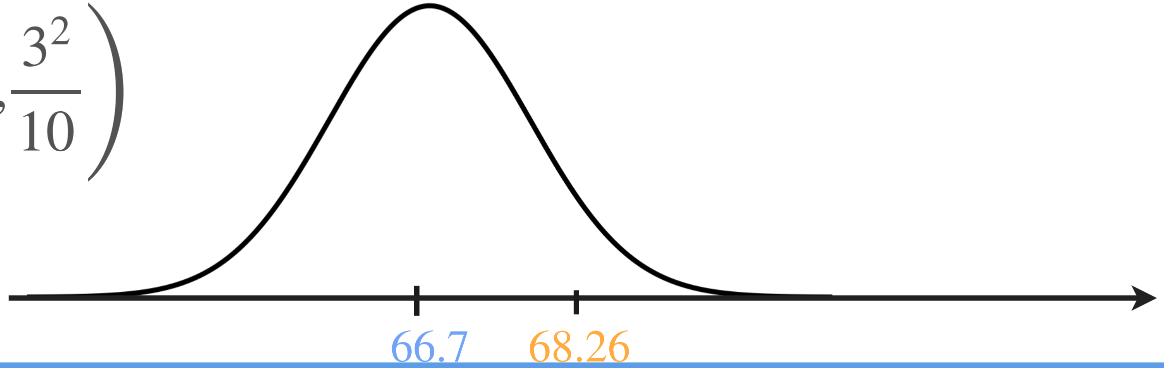
If $\mu = 66.7$ $\overline{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$\mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70)$



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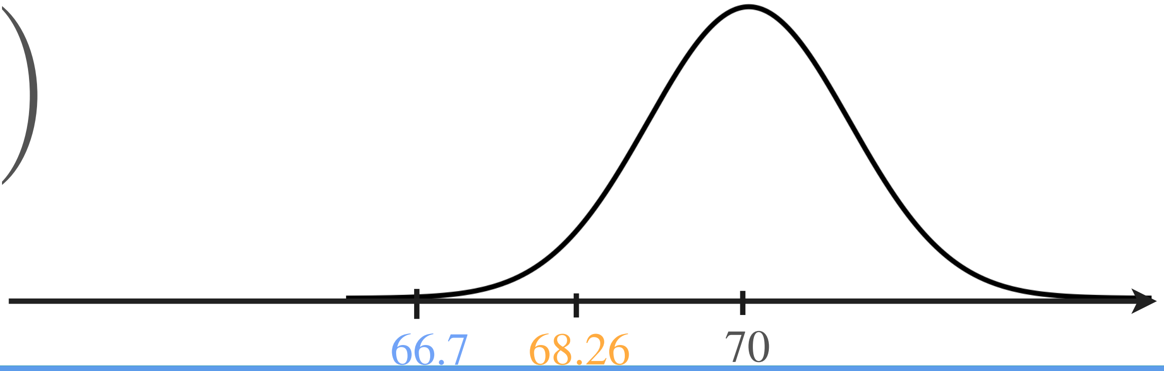
If $\mu = 70$ $\overline{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$

$$\mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70)$$



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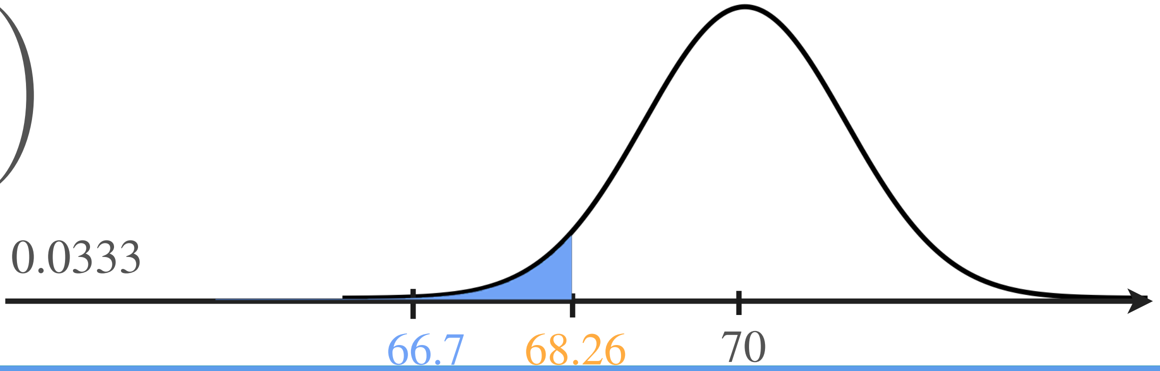
What is the **Type II error probability** if the true value is $\mu = 70$?

If $\mu = 70$ $\quad \overline{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$

$$\mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70) = 0.0333$$



Finding the Type II Error Probabilities

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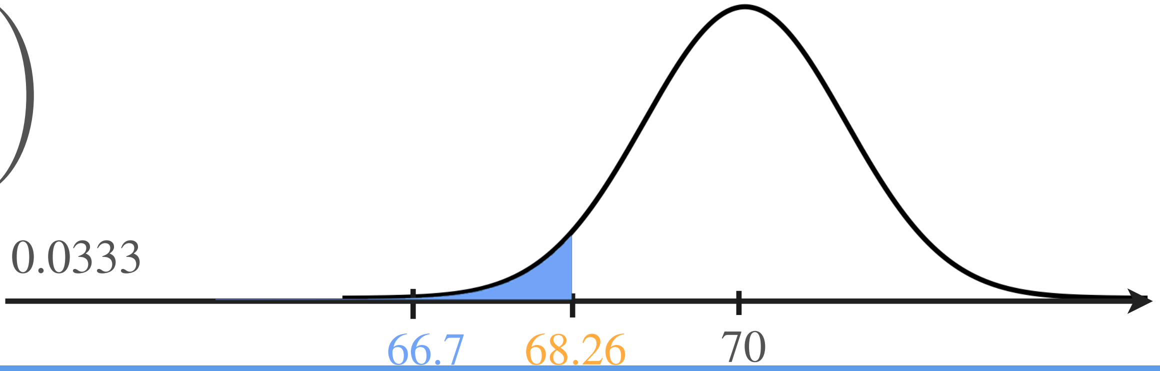
What is the **Type II error probability** if the true value is $\mu = 70$?

If $\mu = 70$ $\overline{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$

$\beta = \mathbf{P}(\overline{X} < 68.26 \,|\, \mu = 70)$ $= 0.0333$



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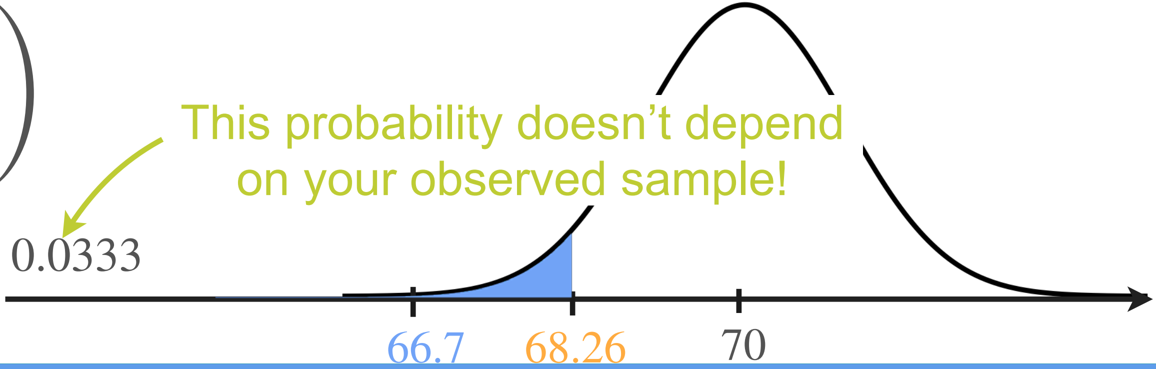
$\beta = \mathbf{P}(\overline{X} < 68.26 \mid \mu = 70) = 0.0333$

This probability doesn't depend on your observed sample!



Power of the Test

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Power of the test

$\mathbf{P}\left(\text{Reject } H_0 \,|\, \mu \in H_1\right)$

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Reject H_0 (Decide $\mu > 66.7$)	Type I error	Correct
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Type II error: $\mathbf{P}\left(\overbrace{\text{Do not reject } H_0 \,|\, \mu \in H_1}^{\beta}\right)$

Power of the Test

Type II error: $\overbrace{\mathbf{P}\left(\text{Do not reject } H_0 \mid \mu \in H_1\right)}^{\beta}$

Power of the test: $\underbrace{\mathbf{P}\left(\text{Reject } H_0 \mid \mu \in H_1\right)}$

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Type II error: $\mathbf{P}\overbrace{\left(\text{Do not reject } H_0 \,|\, \mu \in H_1\right)}^{\beta}$

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Complementary probabilities

Power of the Test

$$\text{Type II error: } \overbrace{\mathbf{P}\left(\text{Do not reject } H_0 \,|\, \mu \in H_1\right)}^{\beta}$$

$$\text{Power of the test: } \underbrace{\mathbf{P}\left(\text{Reject } H_0 \,|\, \mu \in H_1\right)}_{1-\beta}$$

Complementary probabilities

Power of the Test

$$\text{Type II error: } \overbrace{\mathbf{P}\left(\text{Do not reject } H_0 \,|\, \mu \in H_1\right)}^{\beta}$$

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Complementary probabilities

$$\text{Power of the test } = 1 - \text{Type II error probability}$$

$$= 1 - \mathbf{P}\left(\text{Do not reject } H_0 \,|\, \mu \in H_1\right)$$

Power of the Test

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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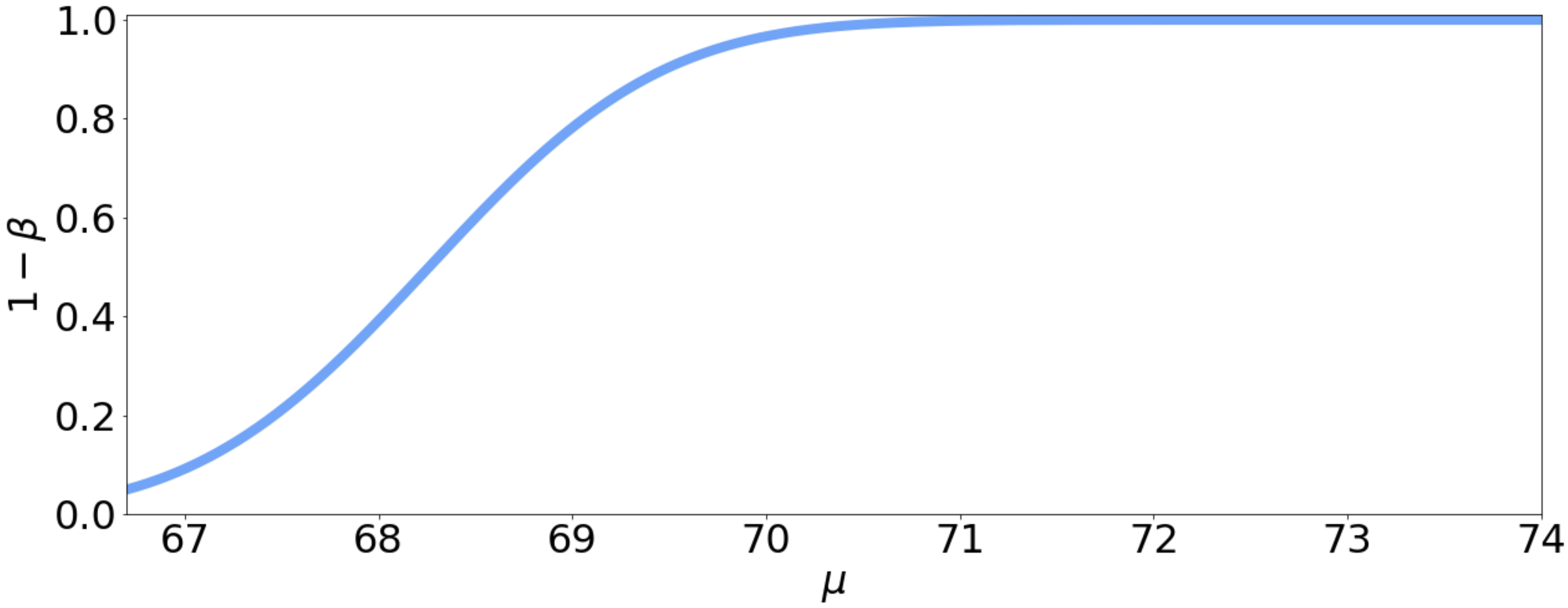

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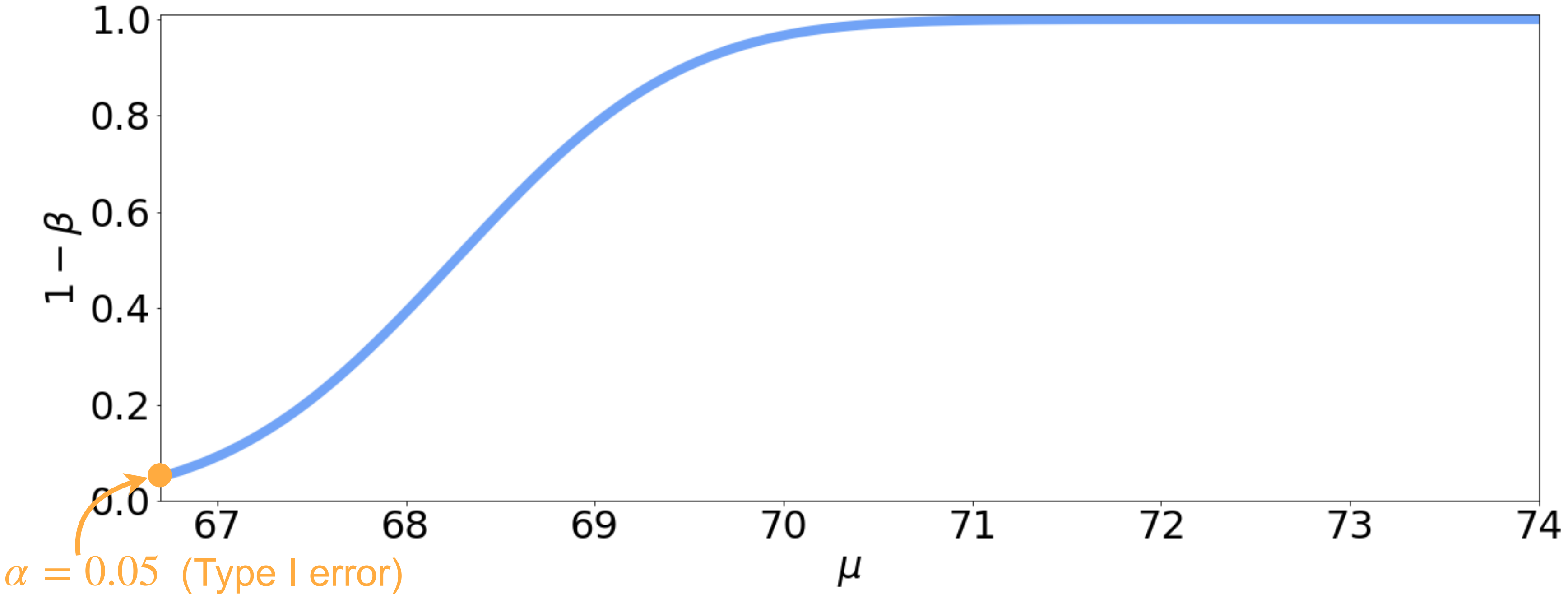

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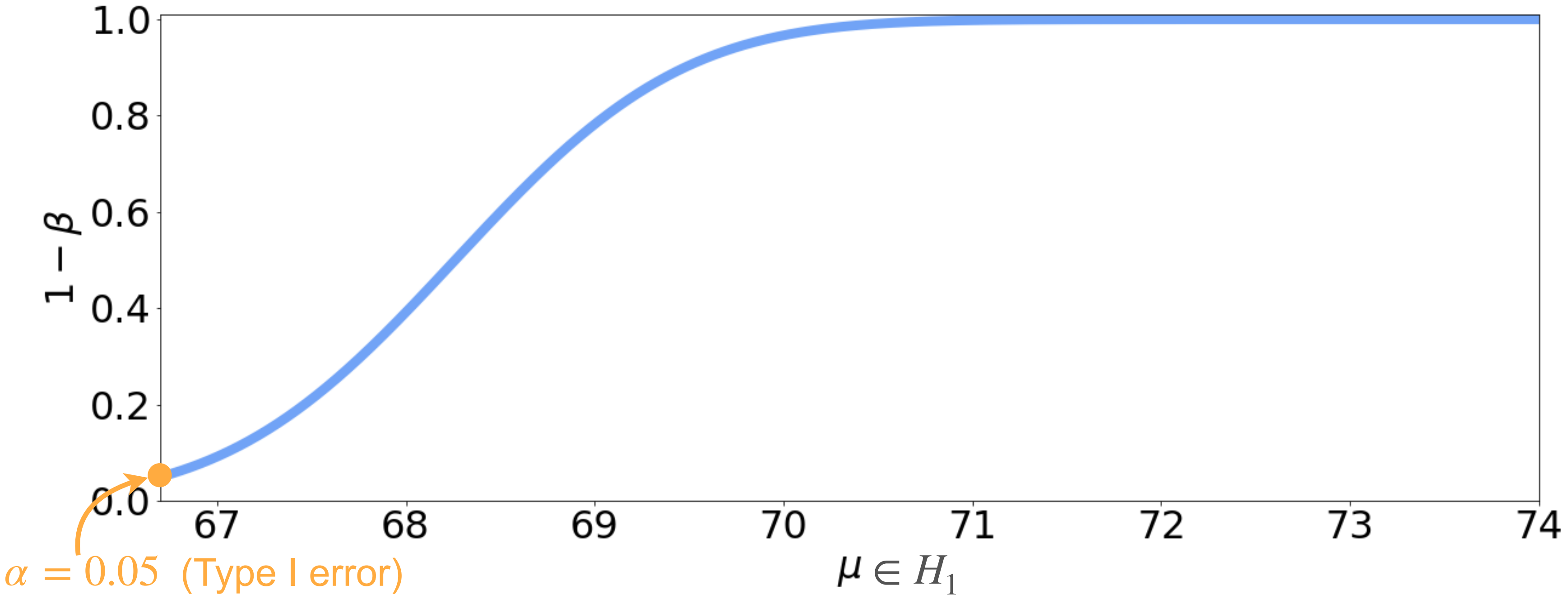

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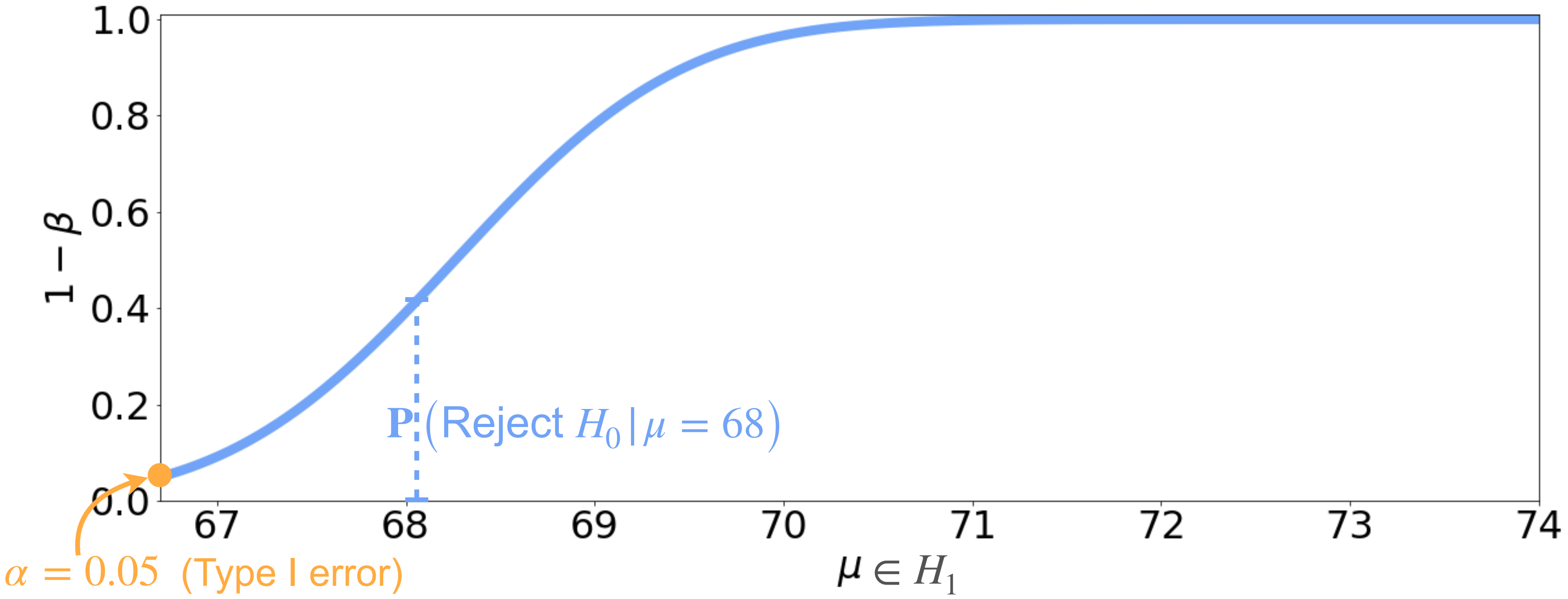

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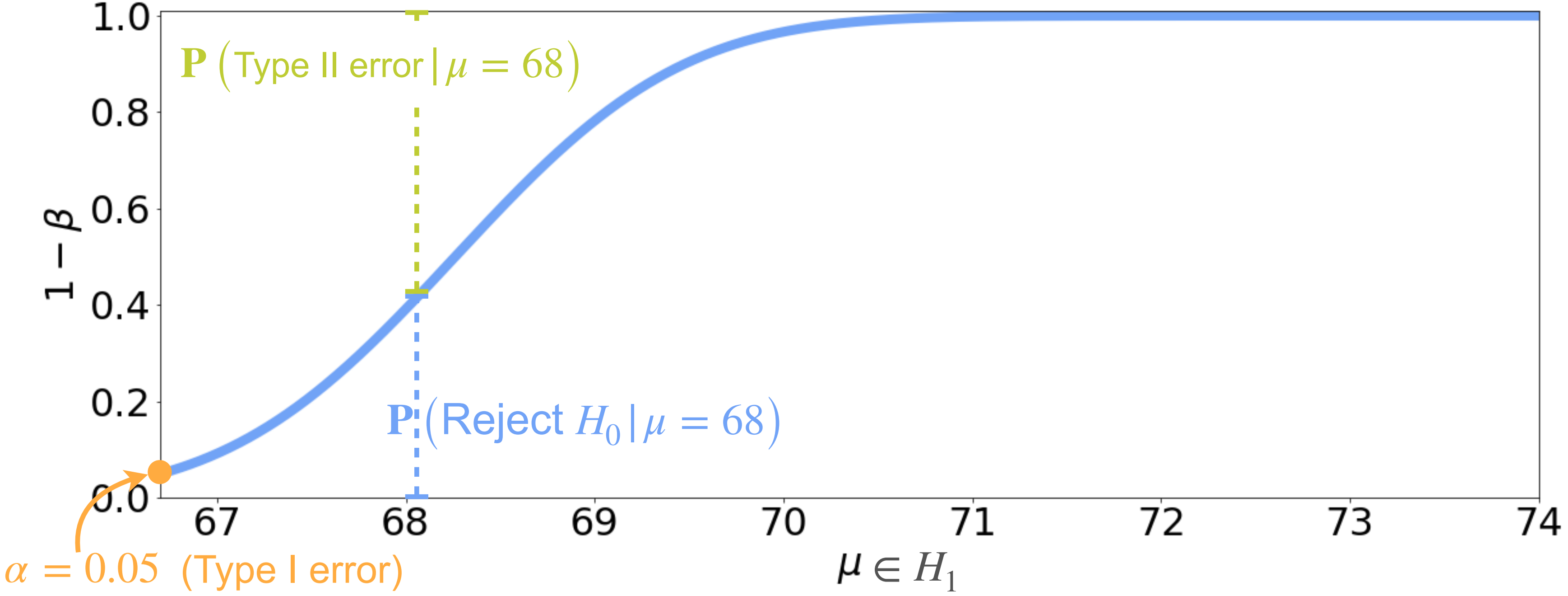

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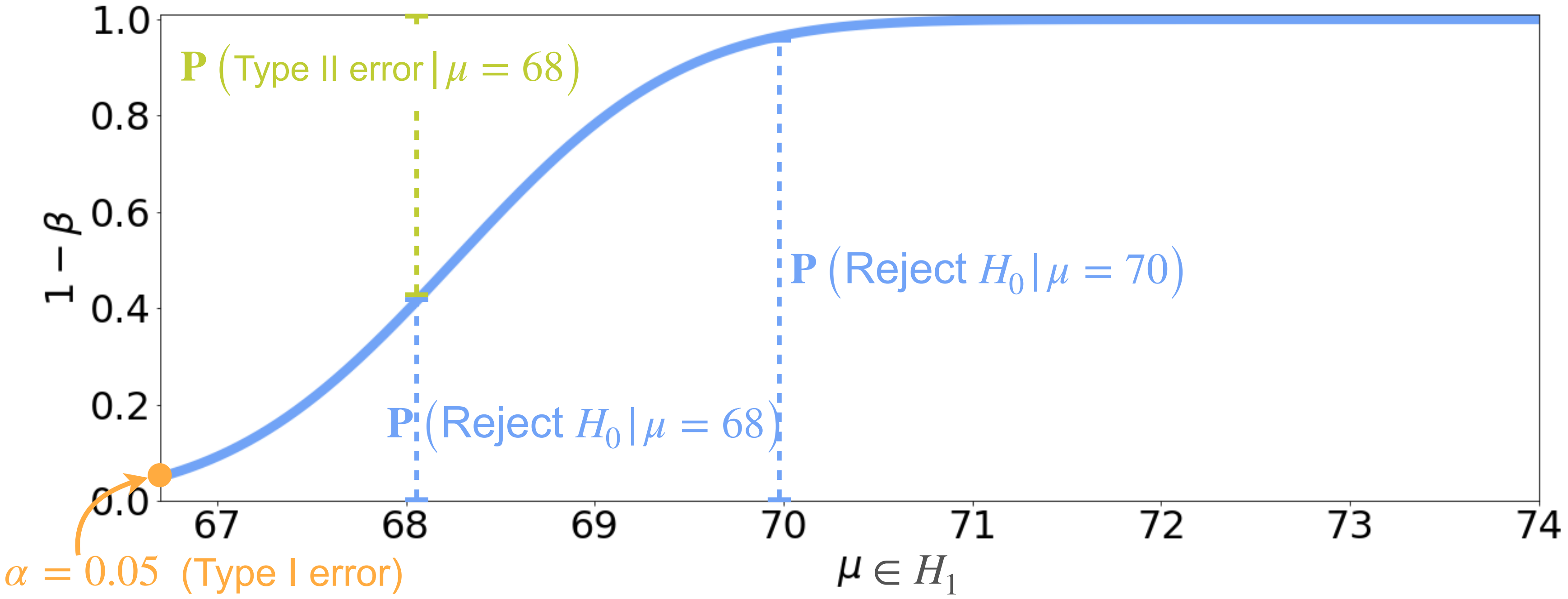

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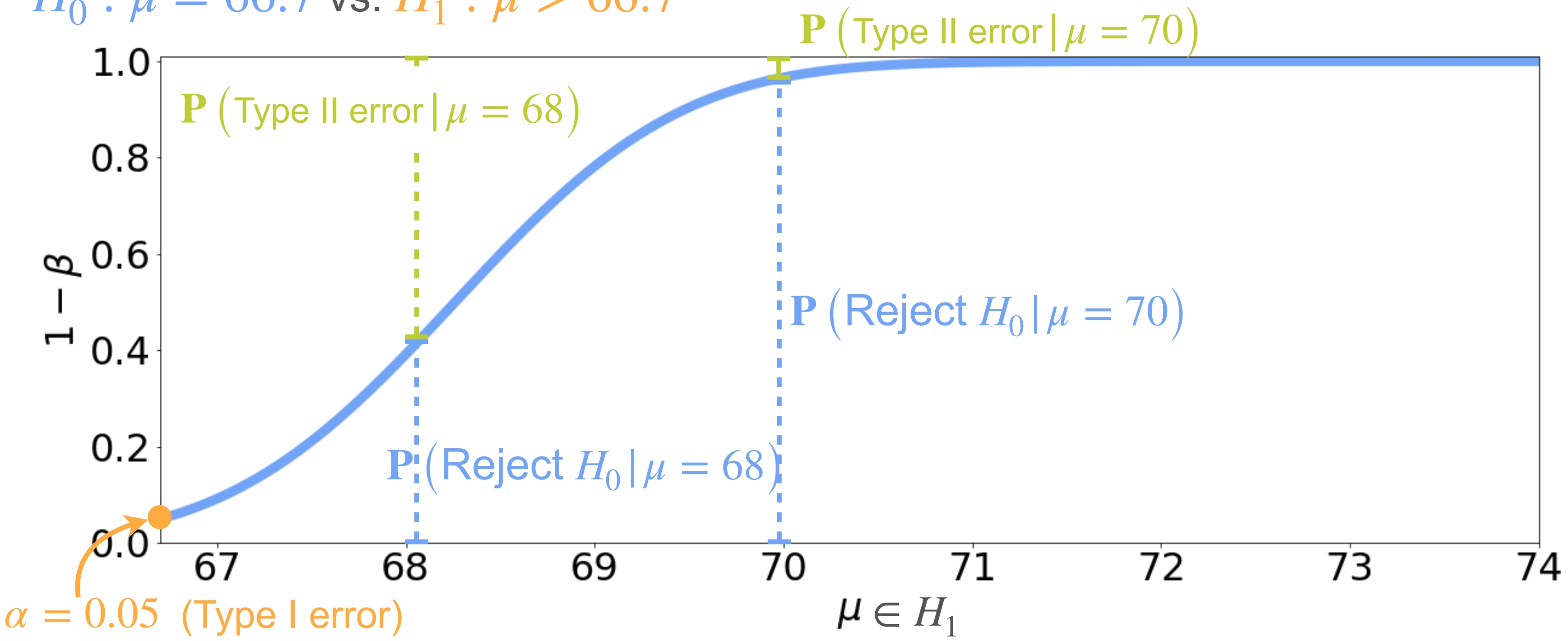

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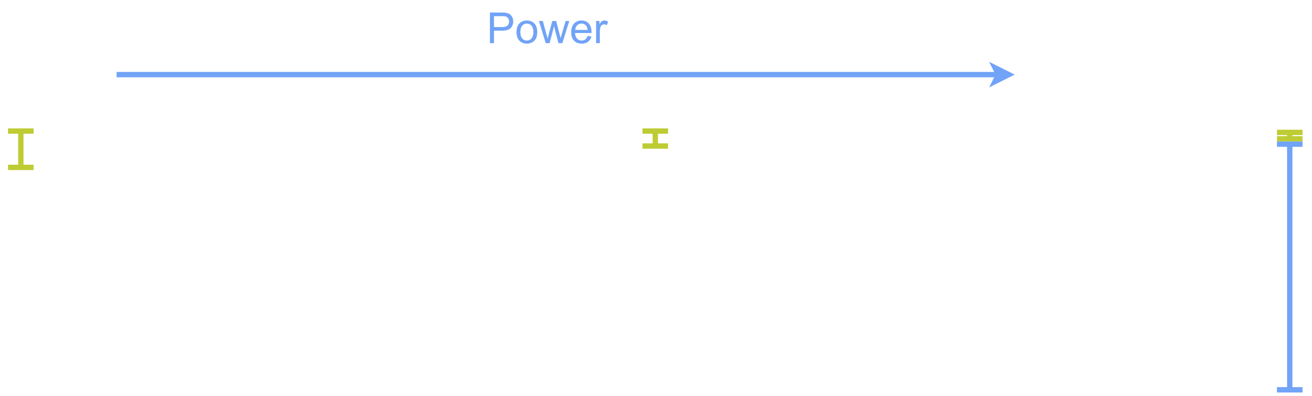

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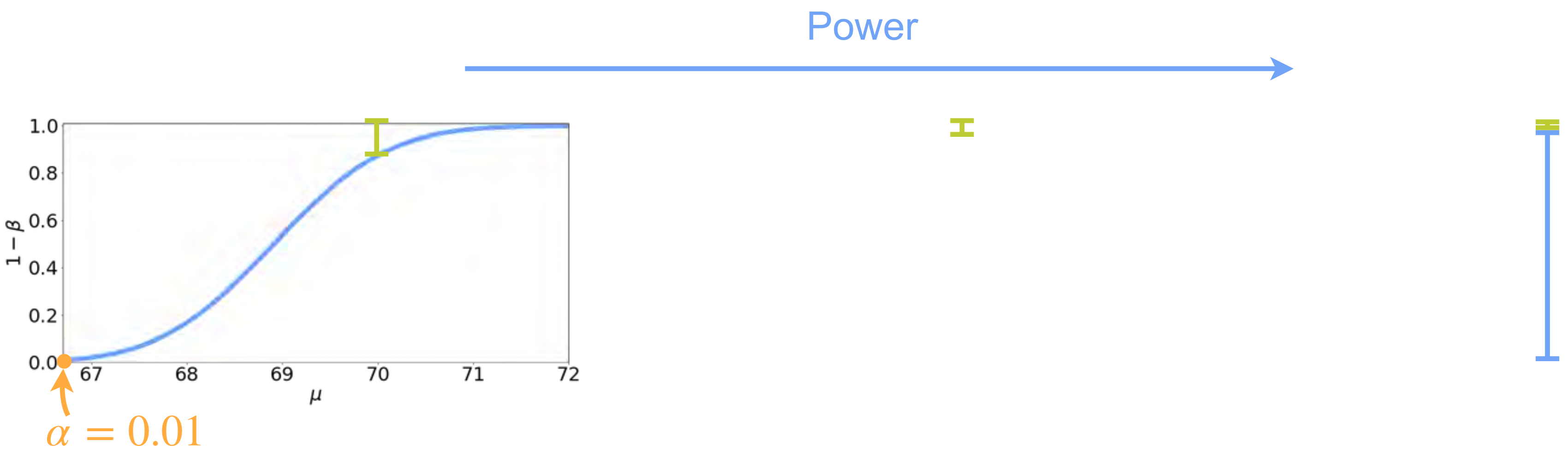

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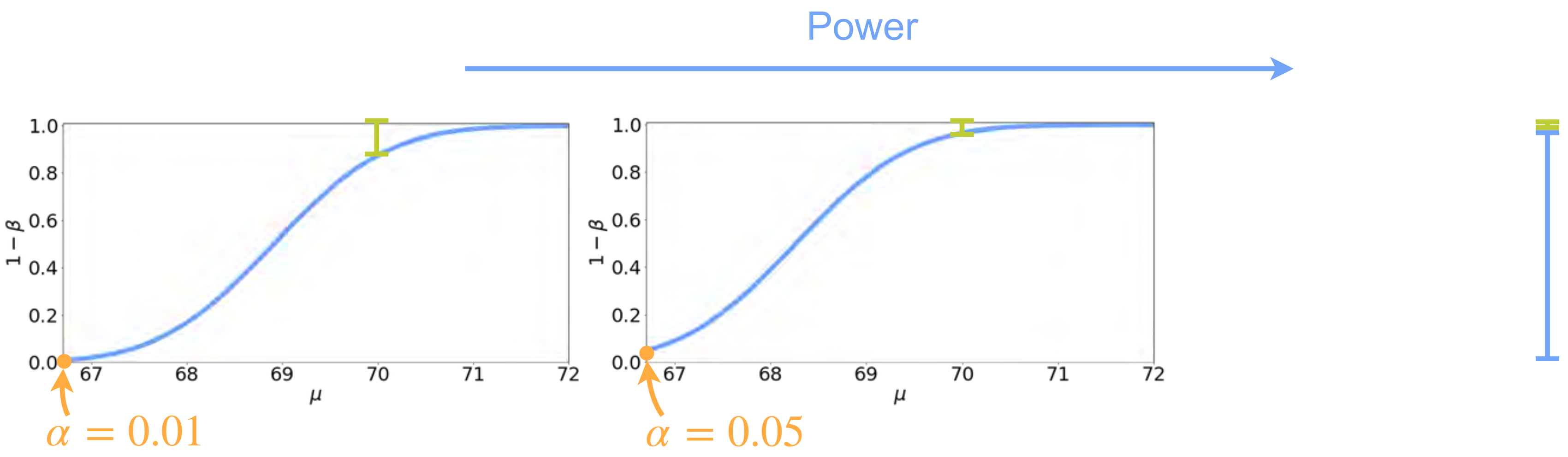

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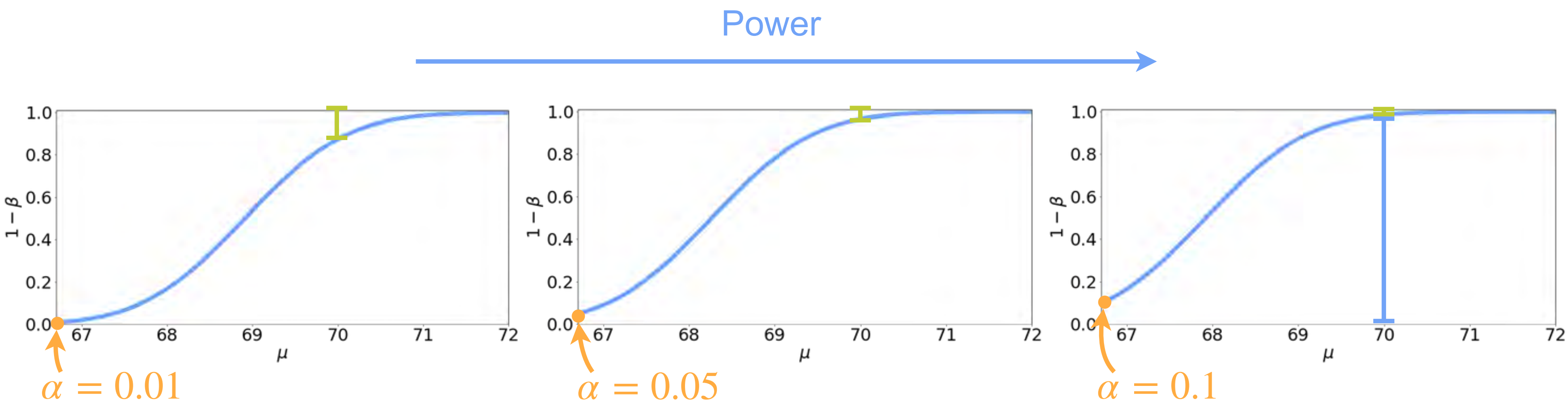

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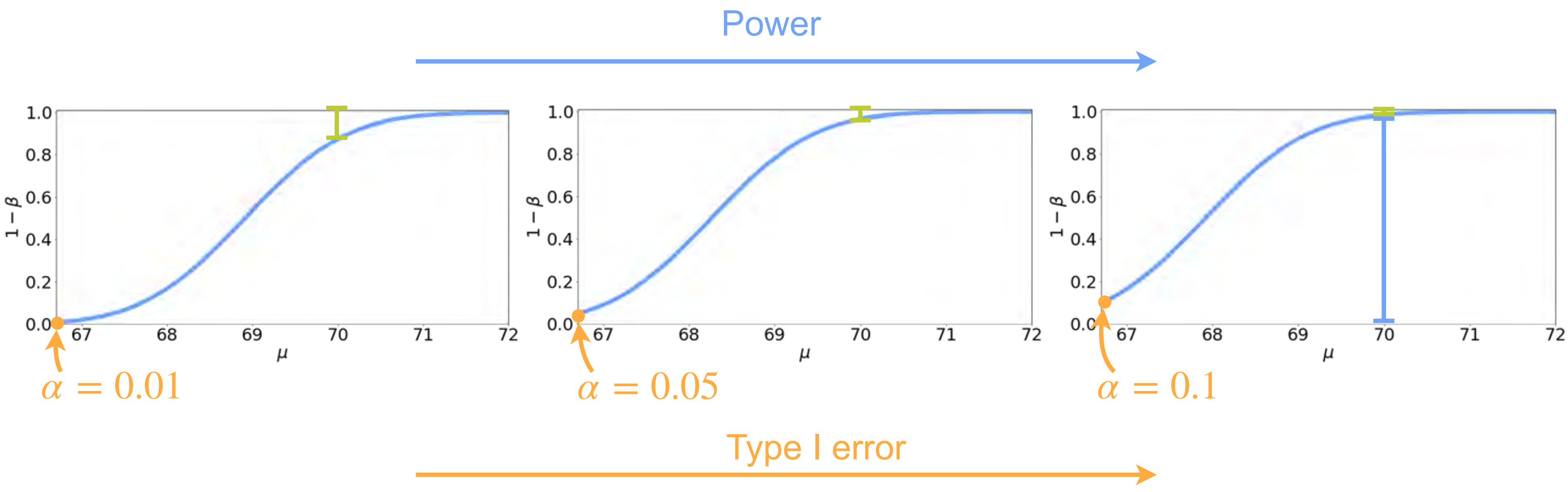

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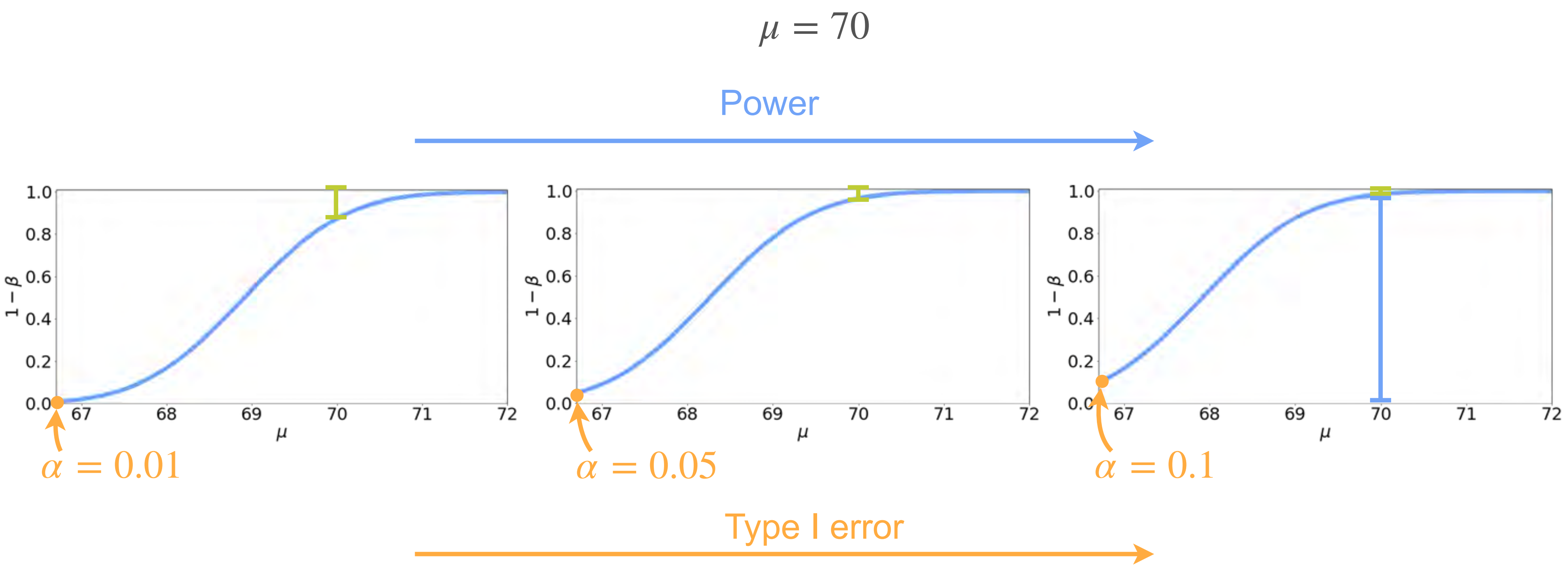
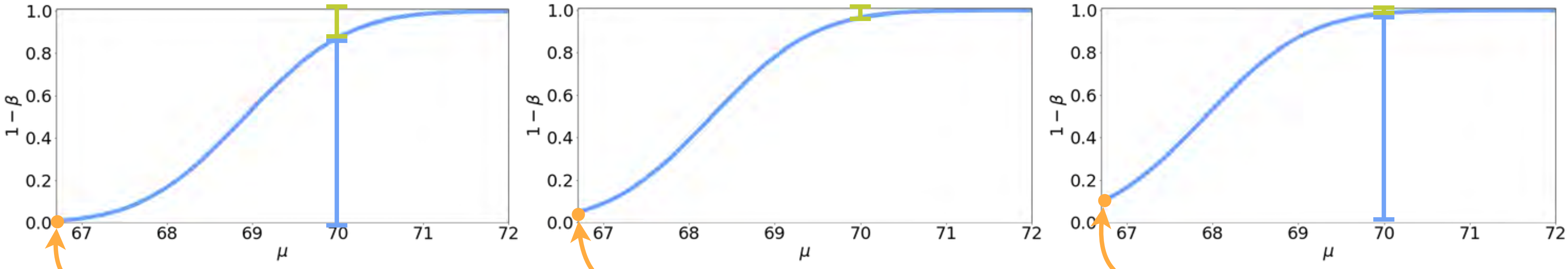


Power of the Test

$\mu = 70$

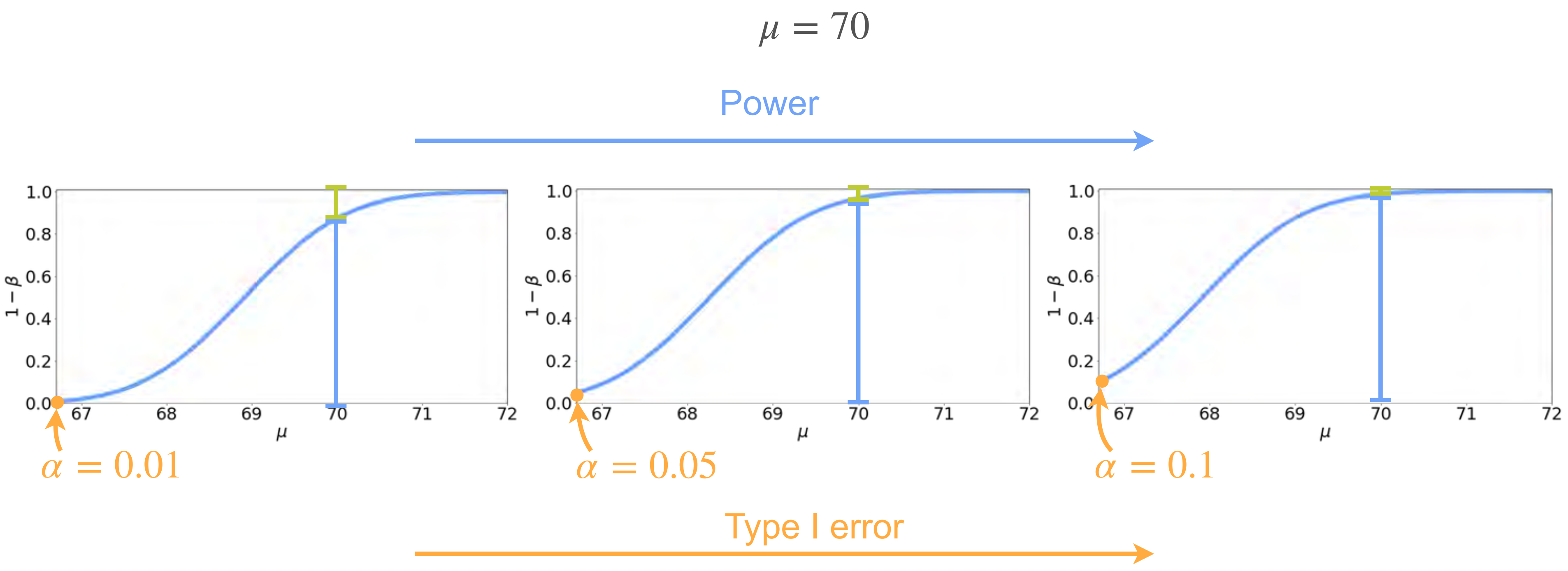
Power



$\alpha = 0.01$ $\alpha = 0.05$ $\alpha = 0.1$

Type I error

Power of the Test



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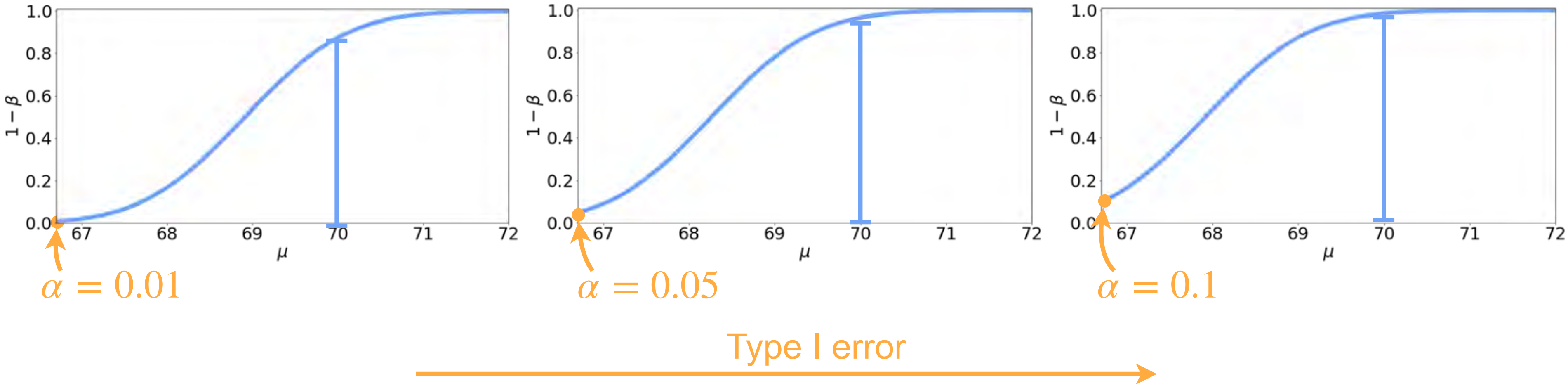

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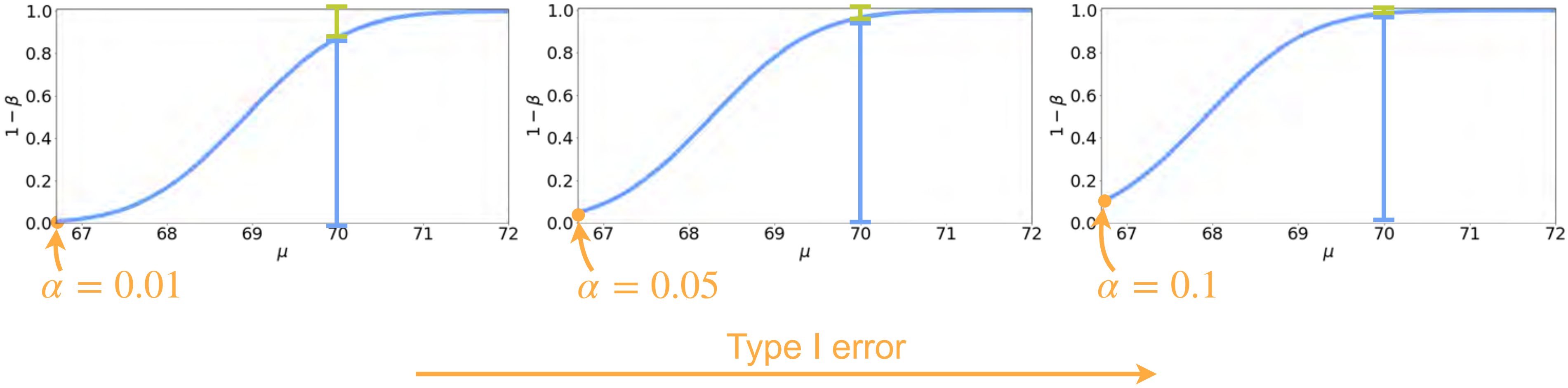

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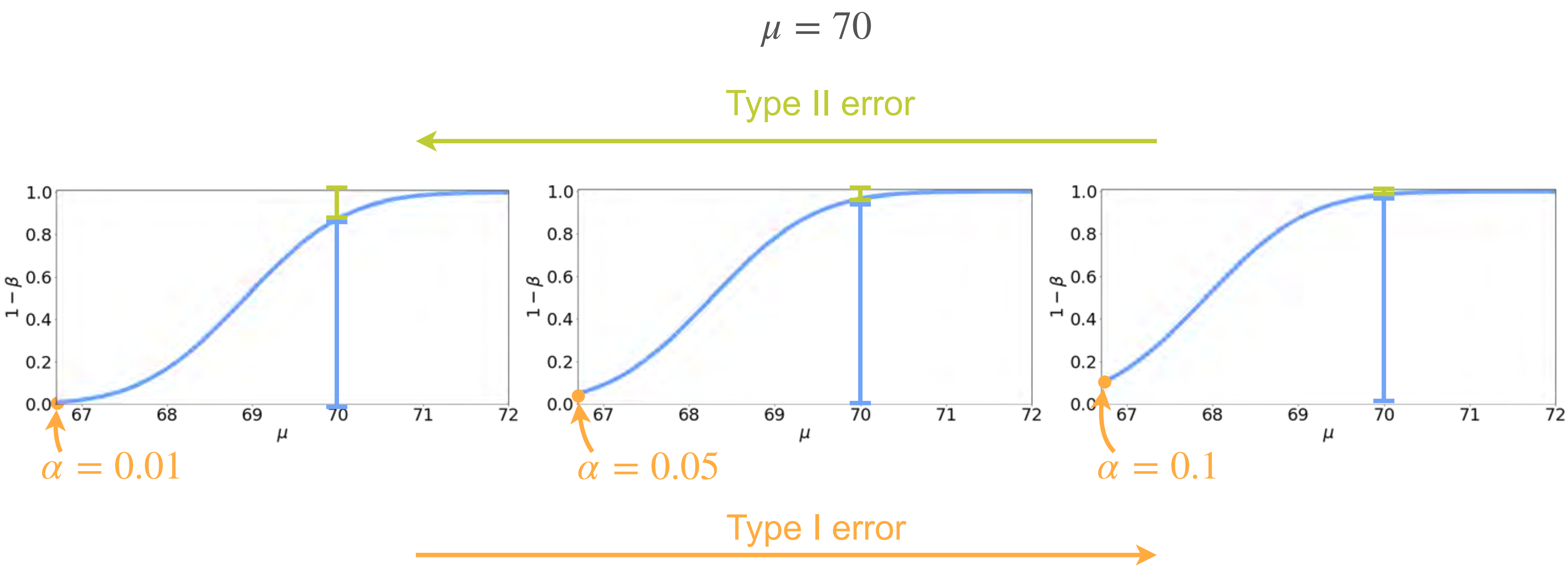

DeepLearning.AI

Hypothesis Testing

Interpreting results

Steps for Performing Hypothesis Testing

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1. **State your hypotheses.**

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3. **Compute the observed statistic** (based on your sample) $\longrightarrow \bar{x} = 68.442$
4. **Reach a conclusion:**
   - If the *p*-value is less than the significance level reject H_0

$$\longrightarrow \mathbf{P}\left(\bar{X} > 68.442 \,|\, \mu = 66.7\right) \overset{?}{>} 0.05$$

Important Remarks - Interpreting Tests

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- Type I error:
  - Reject null hypothesis when it as true $\longrightarrow$ Reject H_0 when $\mu = 66.7$

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- Errors:
  - $\downarrow \alpha \uparrow \beta$

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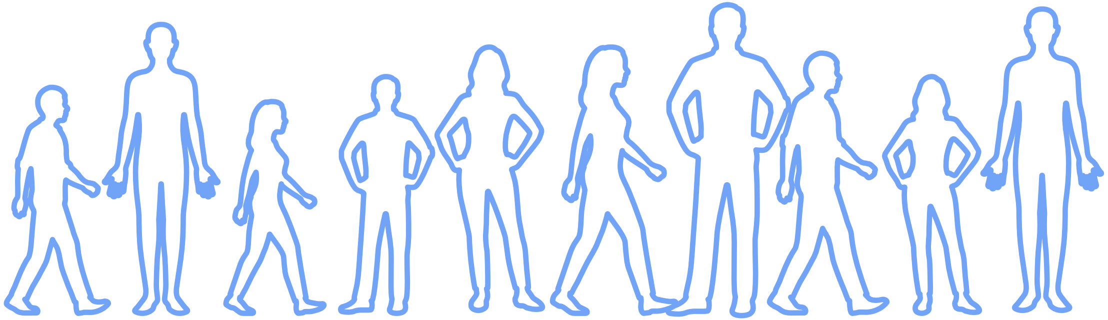
You can only say that there is not enough evidence

Hypothesis Testing

t-Distribution

t-Distribution: Motivation

t-Distribution: Motivation



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

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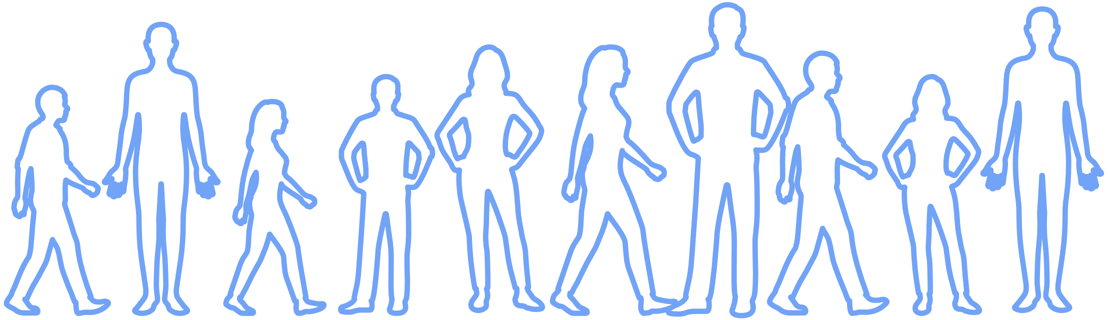

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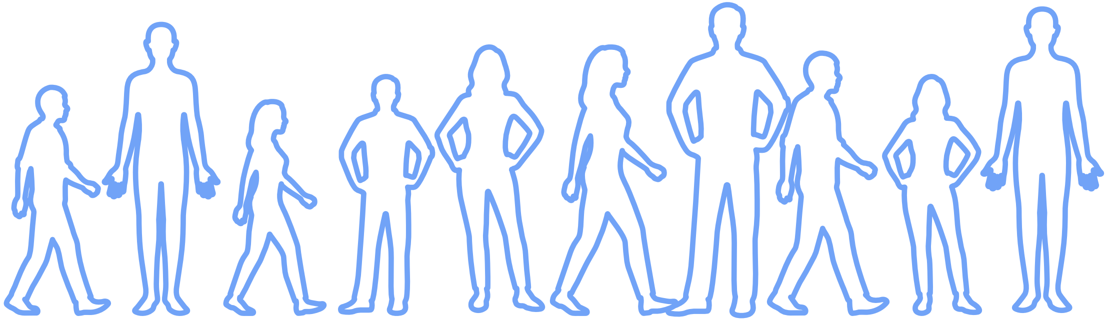

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$$X_i \overset{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$

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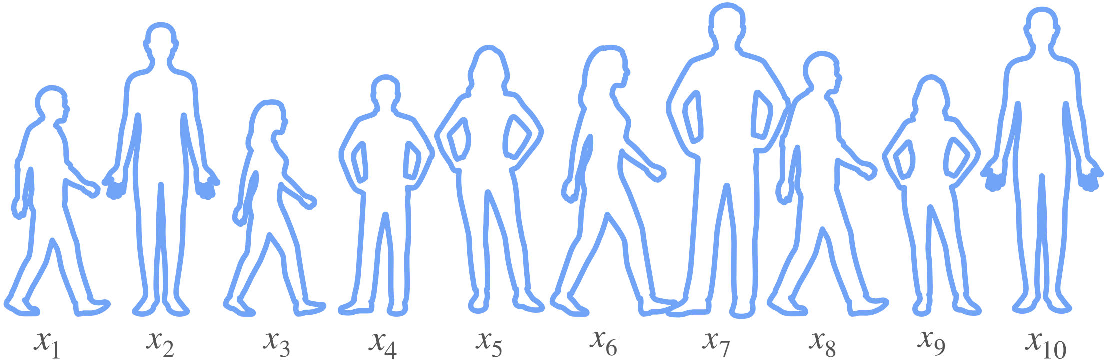

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This is fine if you know μ and σ

What if σ is unknown?

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If μ, σ are known

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

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Z statistic

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Replace σ with its estimate $\dfrac{\bar{X} - \mu}{S/\sqrt{10}}$

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$$S = \sqrt{\frac{1}{10-1}\sum_{i=1}^{10}(X_i - \bar{X})^2}$$

If μ, σ are known

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t-Distribution

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$\dfrac{\overline{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

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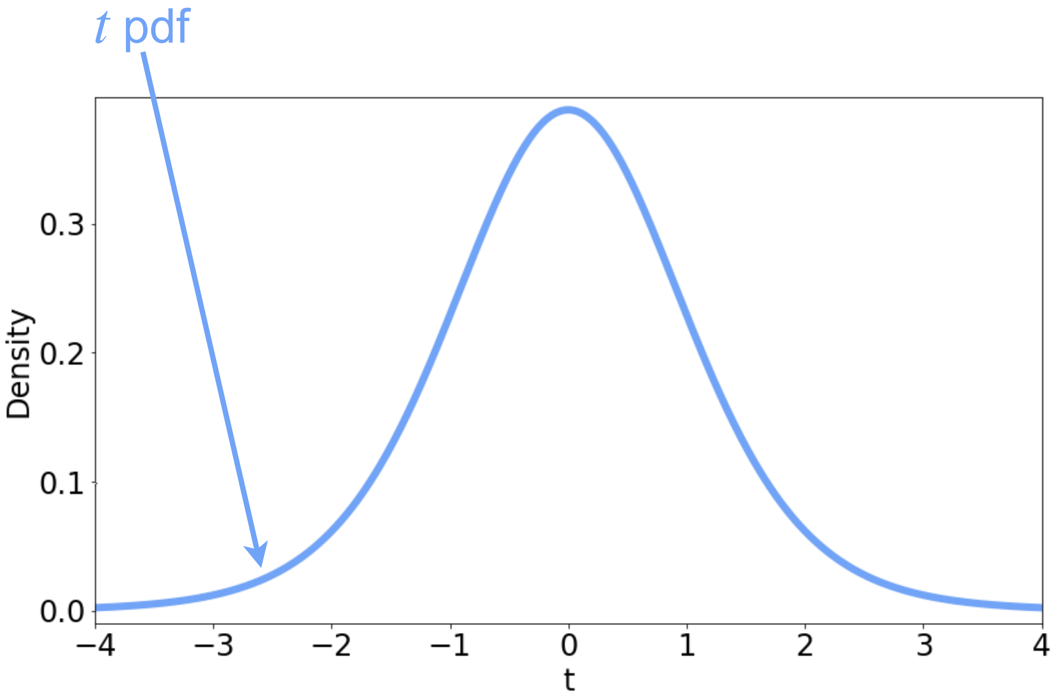
$$\frac{\overline{X} - \mu}{S/\sqrt{10}}$$ follows a t distribution

What does it look like?

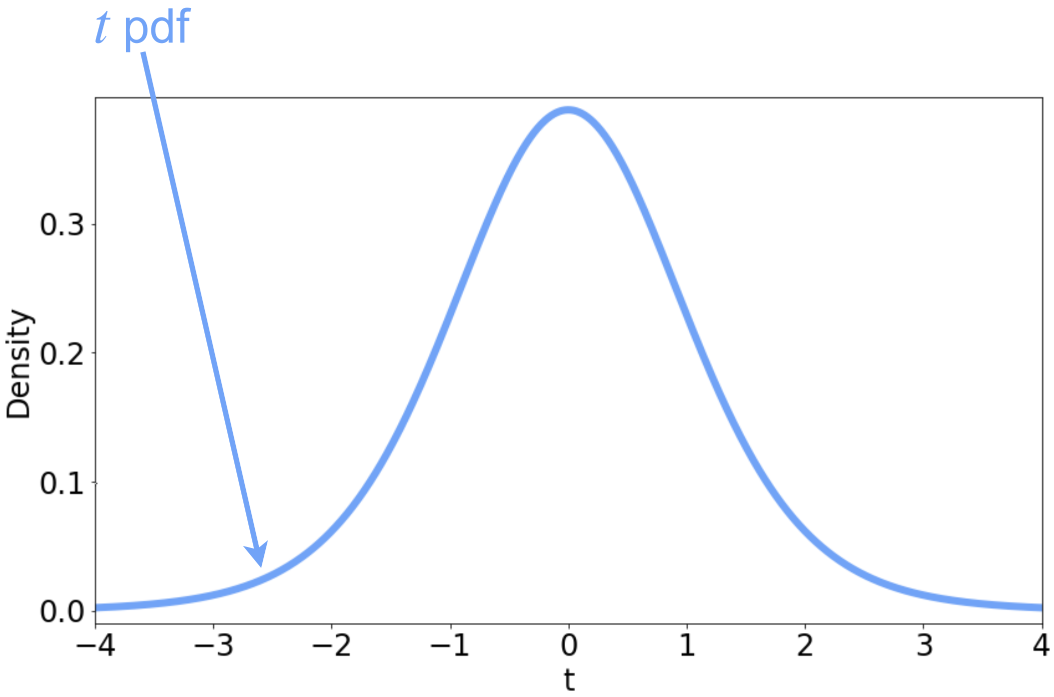


t-Distribution

$\dfrac{\overline{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

What does it look like?

Still bell-shaped



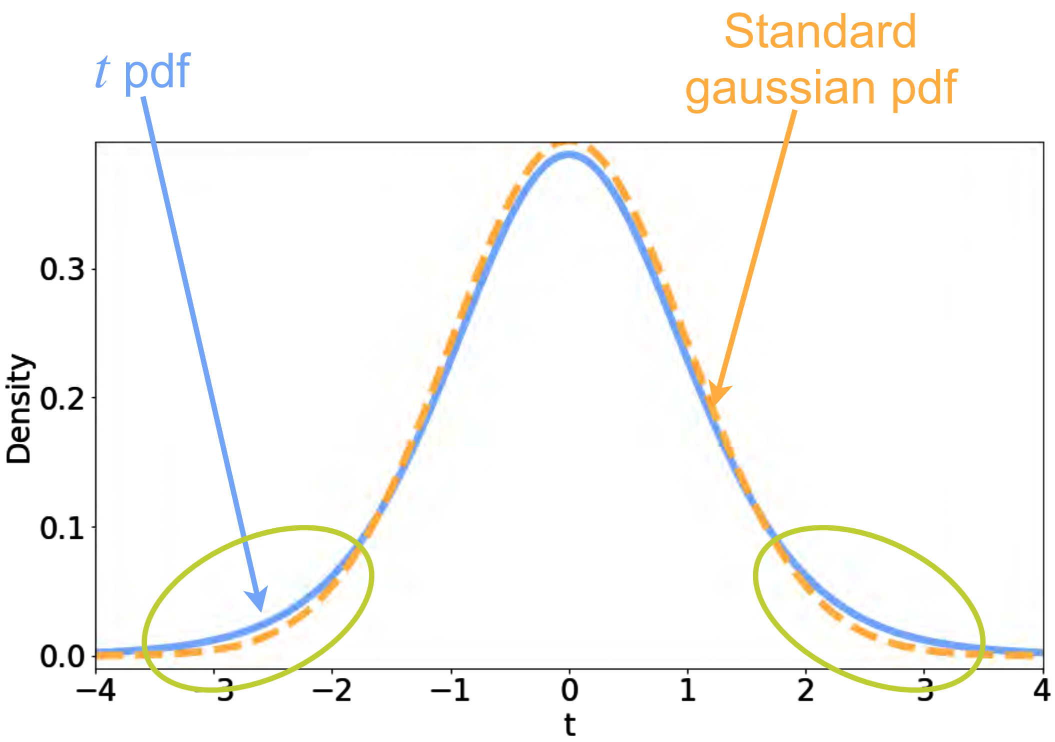
t-Distribution

$$\frac{\overline{X} - \mu}{S/\sqrt{10}}$$ follows a t distribution

What does it look like?

Still bell-shaped

It has heavier tails that account for the uncertainty introduced with the std estimation



t-Distribution

t-Distribution

Parameters:

- Degrees of freedom

t-Distribution

Parameters:

- Degrees of freedom (ν)

t-Distribution

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Controls how heavy the tails are

t-Distribution

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$$X \sim t_{\nu}$$

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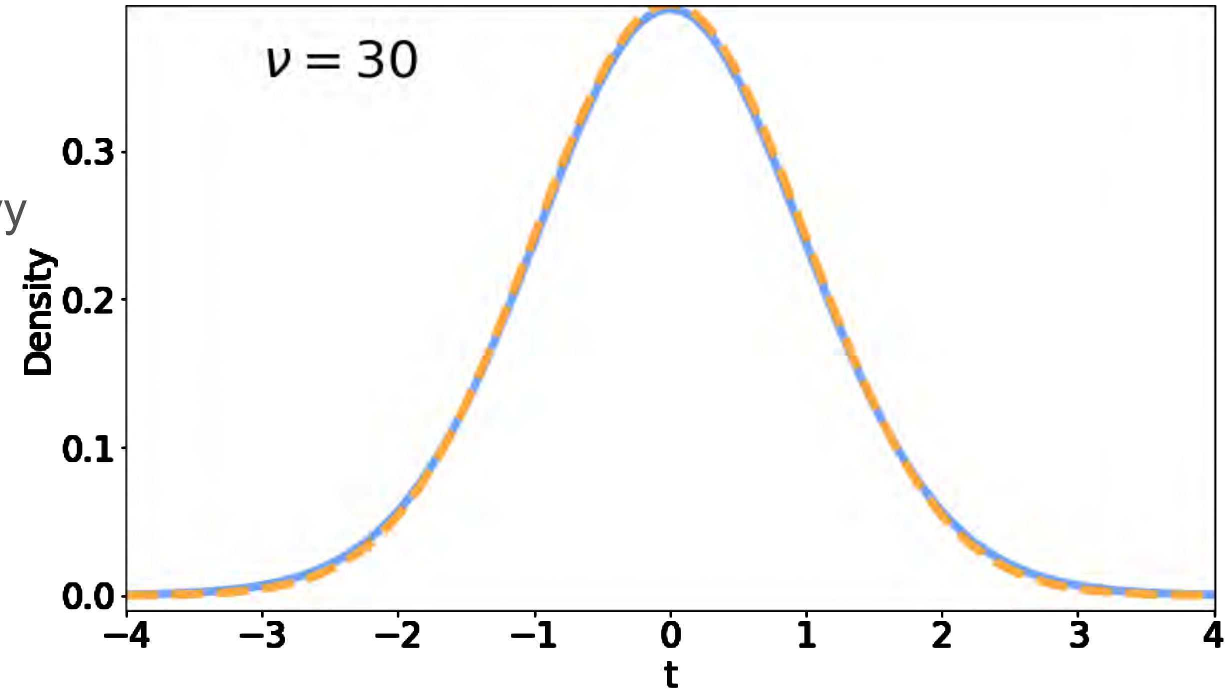

t-Distribution and T-Statistic

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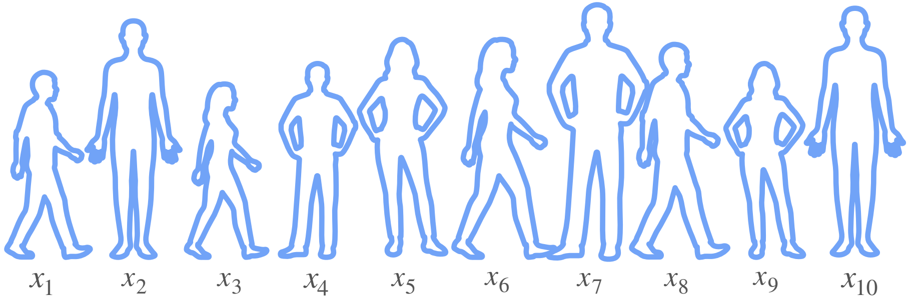
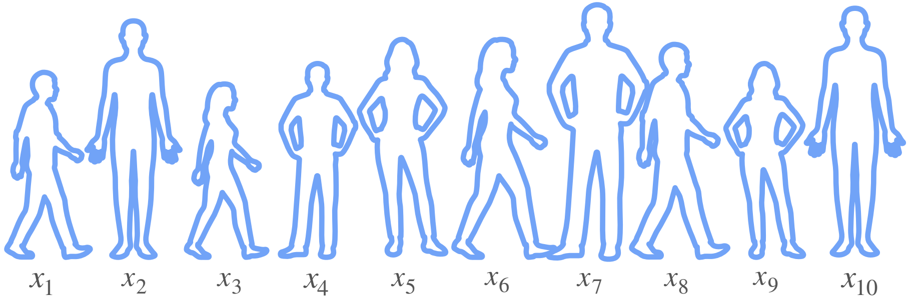


$n = 10$

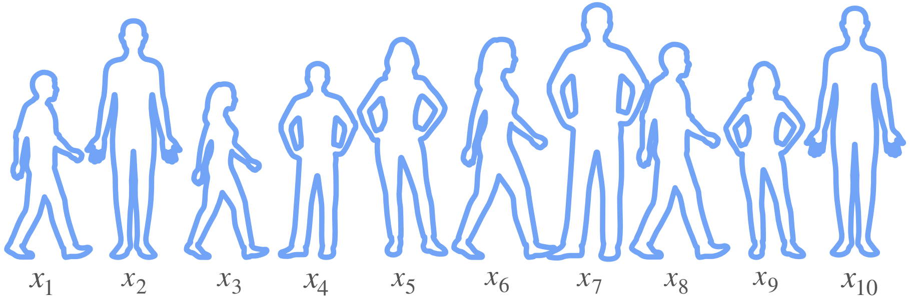
t-Distribution and T-Statistic



$n = 10$

$$T = \frac{\overline{X} - \mu}{S/\sqrt{10}} \sim t_\nu$$

t-Distribution and T-Statistic

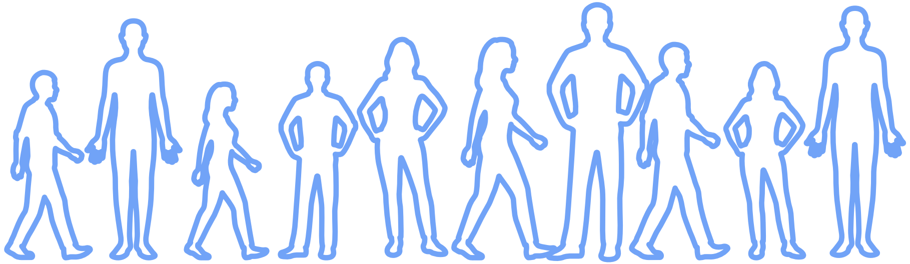


$n = 10$

$\nu = 10 - 1$

$$T = \frac{\overline{X} - \mu}{S/\sqrt{10}} \sim t_9$$

t-Distribution and T-Statistic



x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}

$n = 10$

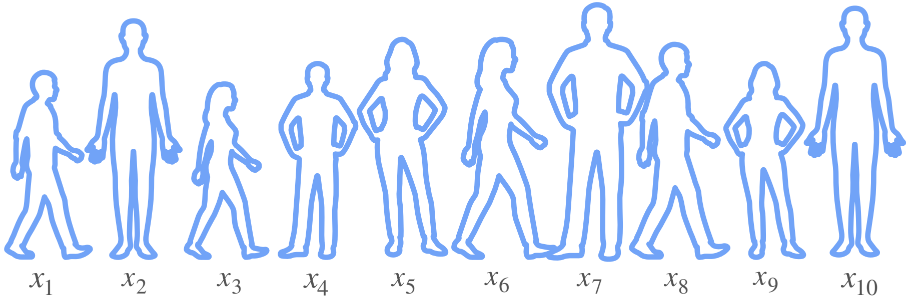
$\nu = 10 - 1$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

Degrees of freedom (ν) = sample size -1

= $(n - 1)$

t-Distribution and T-Statistic



$n = 10$

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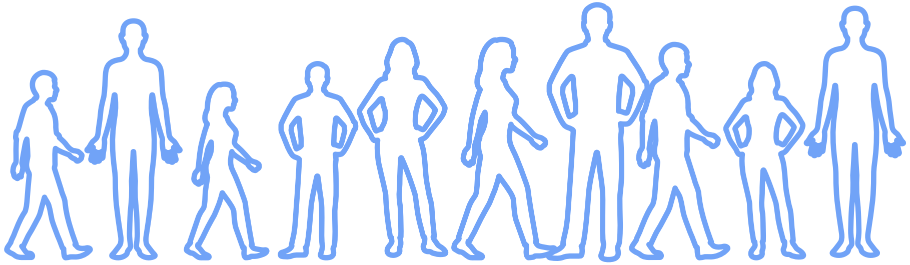
Degrees of freedom (ν) = sample size -1

= $(n - 1)$

As n increases, this looks more like a $\mathcal{N}(0,1^2)$

t-Distribution and T-Statistic



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$\nu = 10 - 1$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

Degrees of freedom (ν) = sample size -1

= $(n - 1)$

As n increases, this looks more like a $\mathcal{N}(0,1^2)$

T-statistic is used when

- The population has a Gaussian distribution
- But you don't know the variance

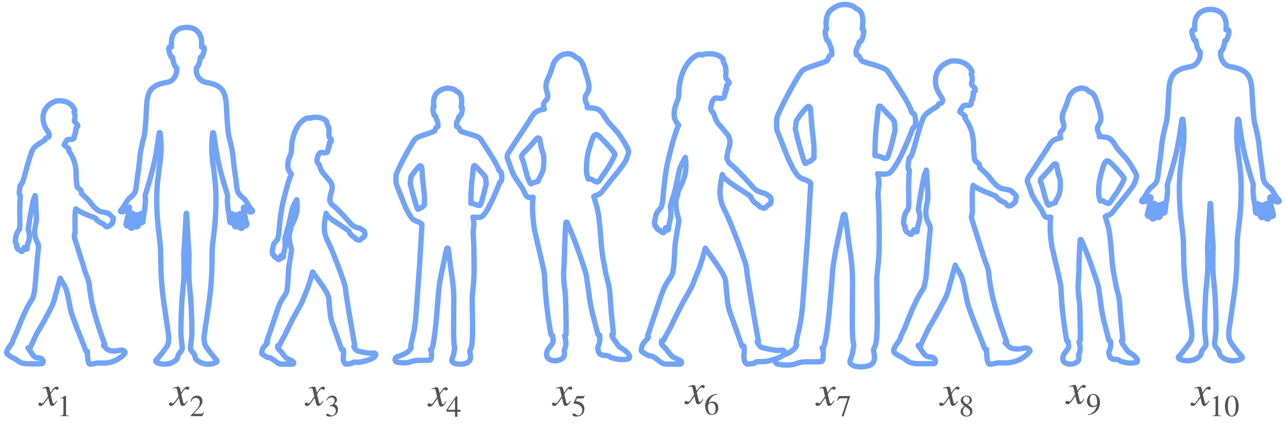
DeepLearning.AI

Hypothesis Testing

t-Tests

Example: Heights

Example: Heights



66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$$\bar{x} = 68.442$$

Example: Heights

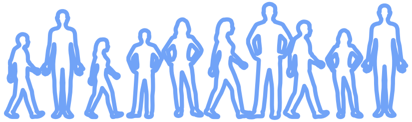
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

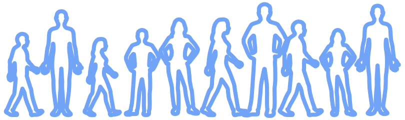
3 questions

3 sets of hypothesis

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

.

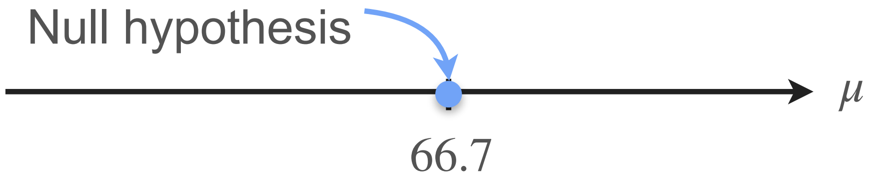
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$$\sigma = 3$$
$$n = 10$$

$$H_0 : \mu = 66.7$$

If H_0 is true: $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$



Example: Heights

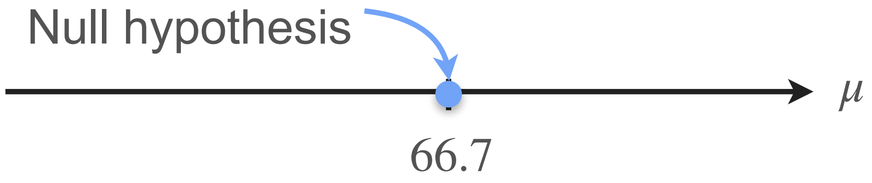
The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\cancel{\sigma = 3}$

$n = 10$

$H_0 : \mu = 66.7$

If H_0 is true: $\overline{X} \sim$



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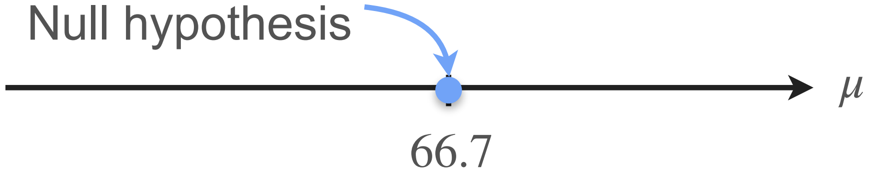

If H_0 is true: $T = \dfrac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$

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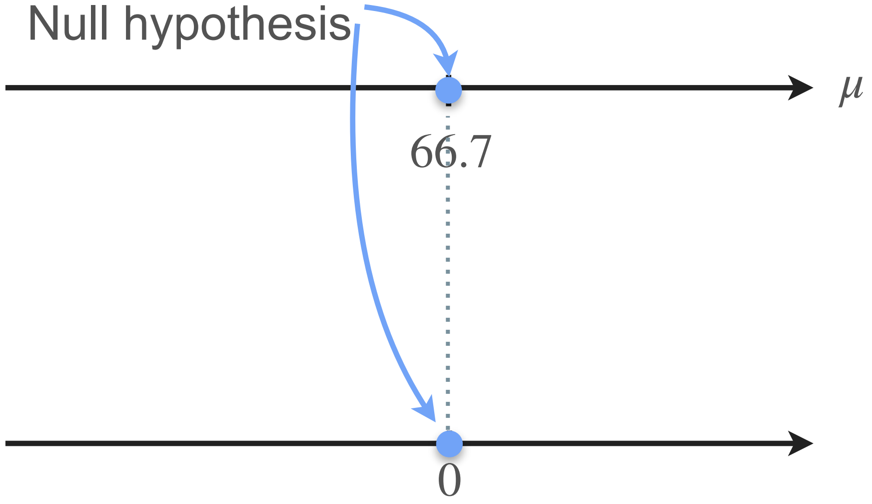

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Example: Heights

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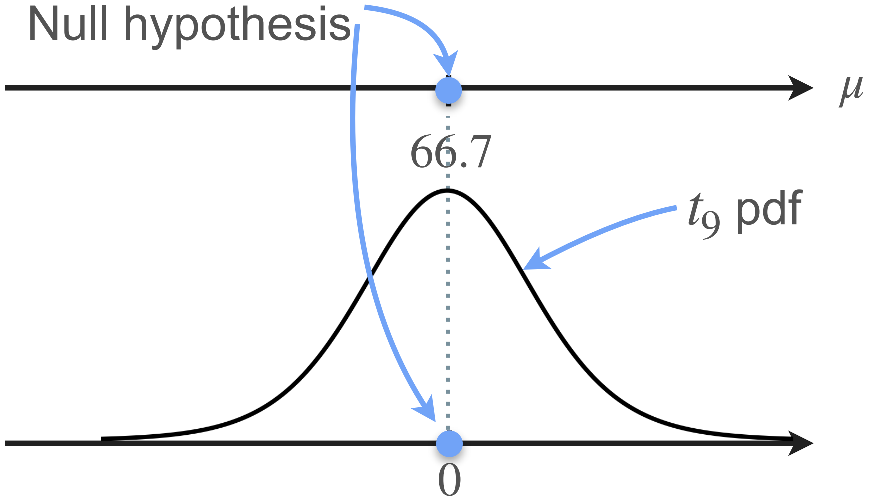
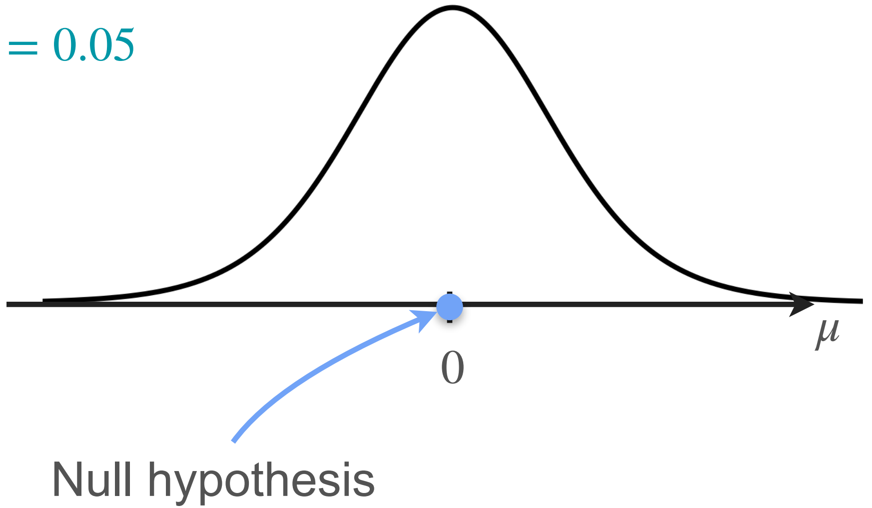


If H_0 is true: $T = \dfrac{\overline{X} - 66.7}{S/\sqrt{10}} \sim t_9$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$\alpha = 0.05$

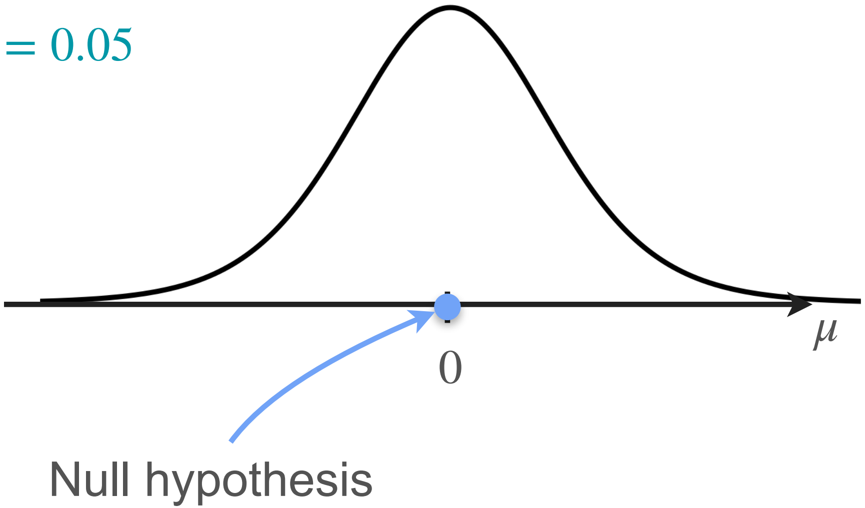


Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

$\alpha = 0.05$



Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

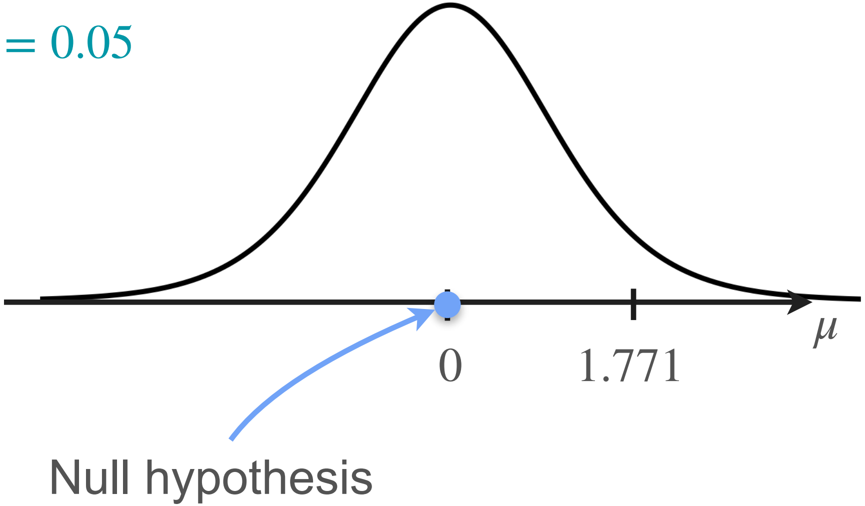
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



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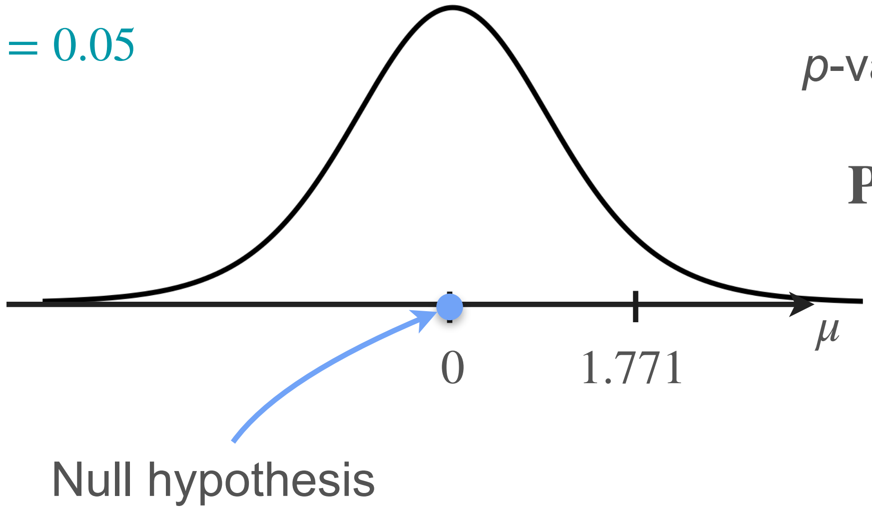

p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7 \right)?$$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

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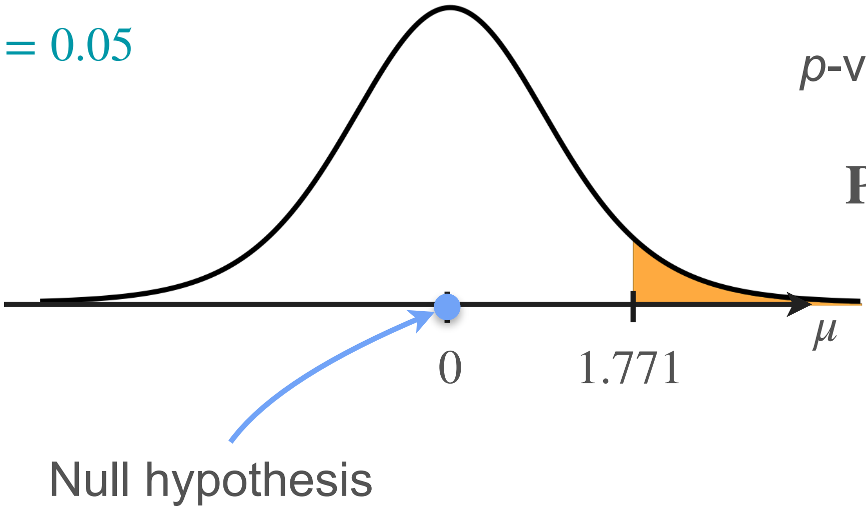


p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0552$$

Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

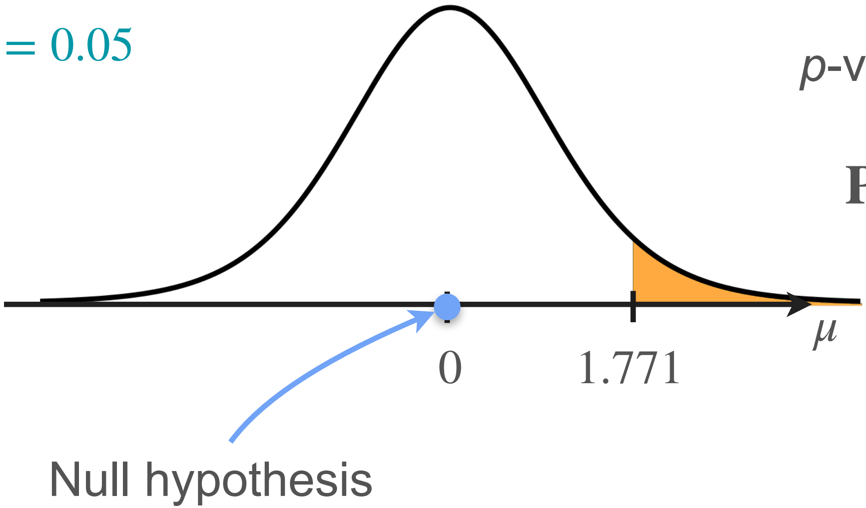
$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0552 > \alpha$$

Conclusion: do not reject H_0

(with a 5% significance level)

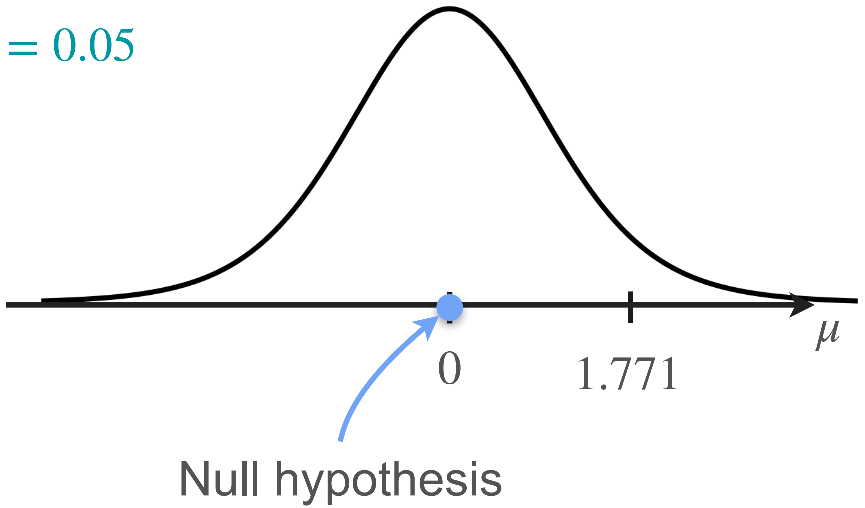
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$n = 10$ $\quad \bar{x} = 68.442 \quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

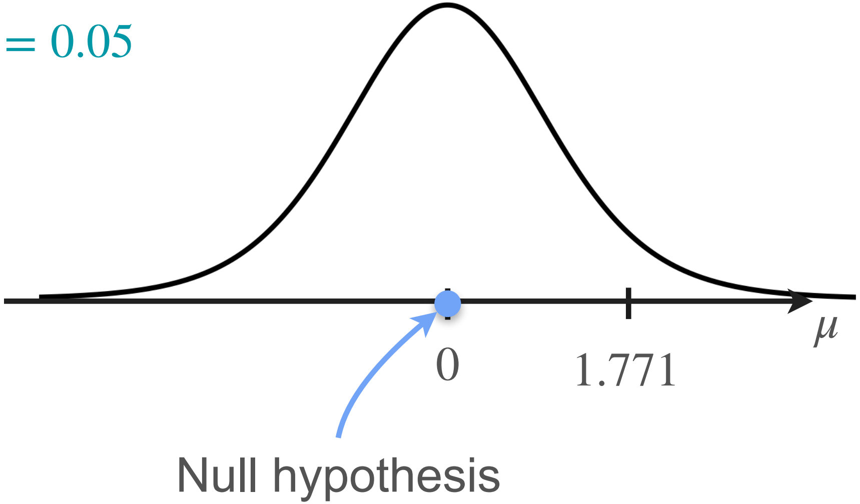
$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$

$\bar{x} = 68.442$

$s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

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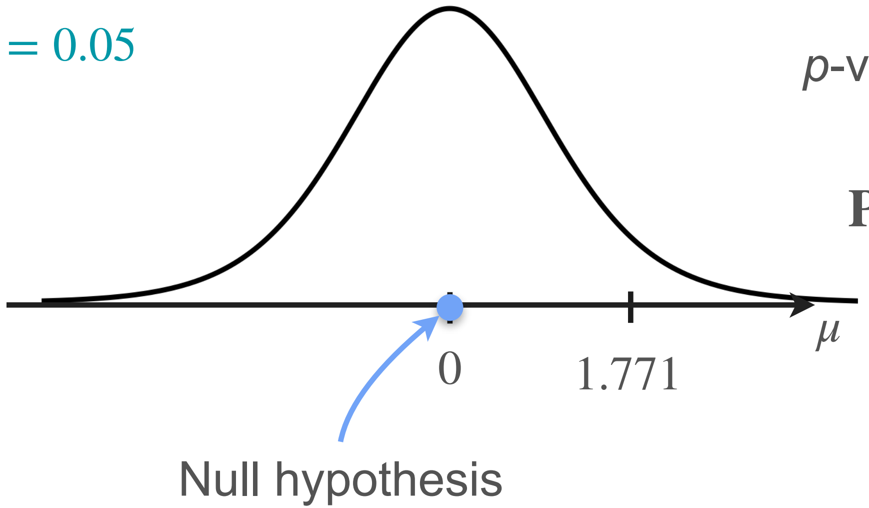

p-value:

$$\mathbf{P}\left(\left| \frac{\bar{X} - 66.7}{S/\sqrt{10}} \right| > 1.771 \,\middle|\, \mu = 66.7 \right)?$$

Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

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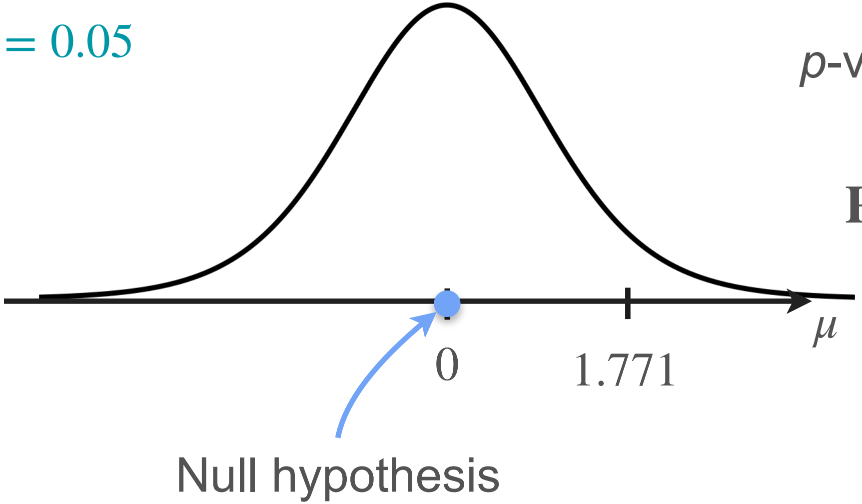


p-value:

$$\mathbf{P}\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \,\middle|\, \mu = 66.7\right)?$$

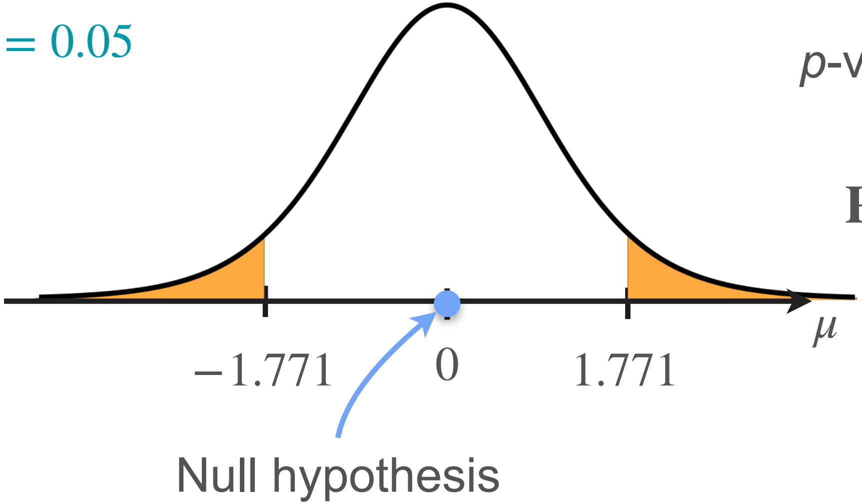
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10 \quad \bar{x} = 68.442 \quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \,\middle|\, \mu = 66.7\right) = 0.1103$$

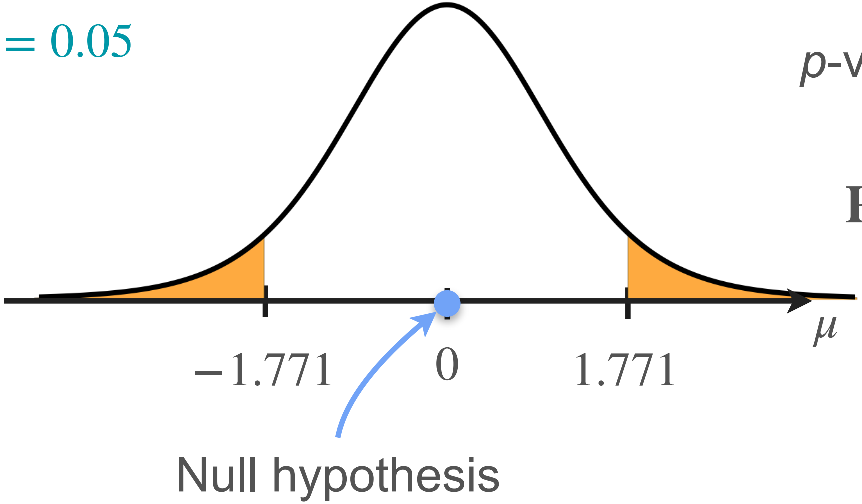
Two-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$ $\quad n = 10$ $\quad \bar{x} = 68.442$ $\quad s = 3.113$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\left| \frac{\bar{X} - 66.7}{S/\sqrt{10}} \right| > |1.771| \,\middle|\, \mu = 66.7 \right) = 0.1103 > \alpha$$

Conclusion: do not reject H_0
(with a 5% significance level)

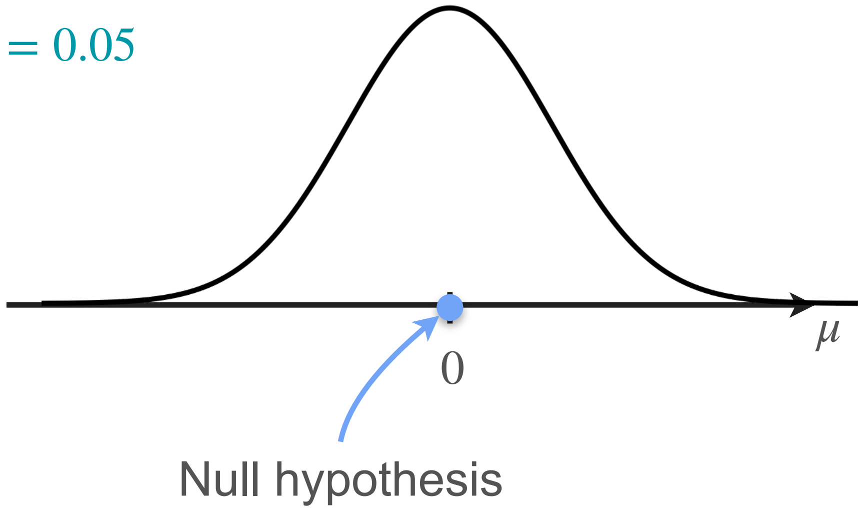
Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$n = 10$ $\quad \bar{x} = 64.252$ $\quad s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$\alpha = 0.05$



Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

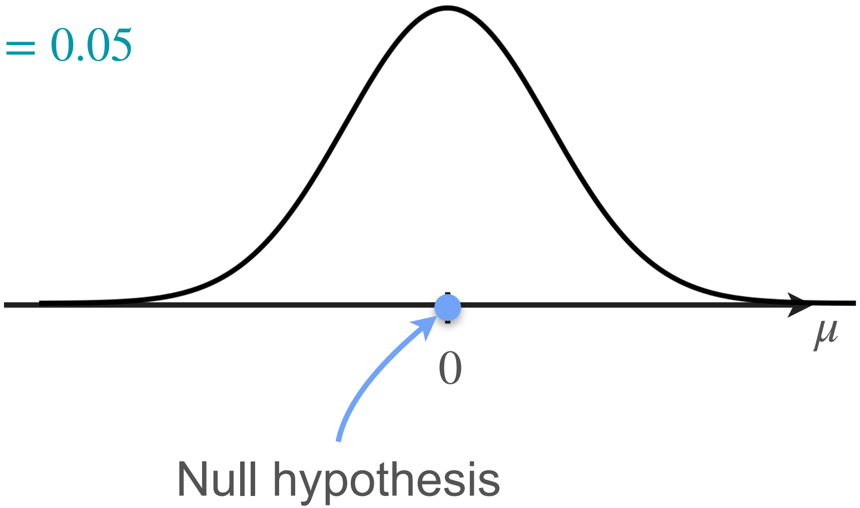
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$\alpha = 0.05$



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The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

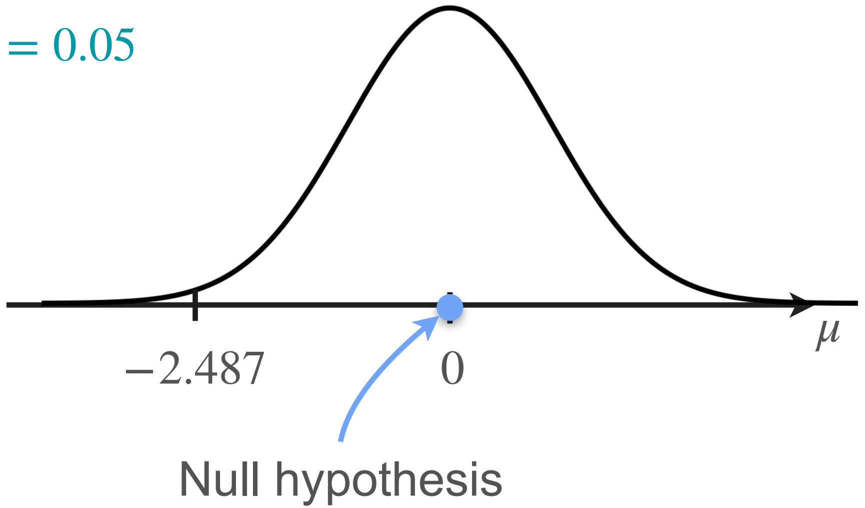
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



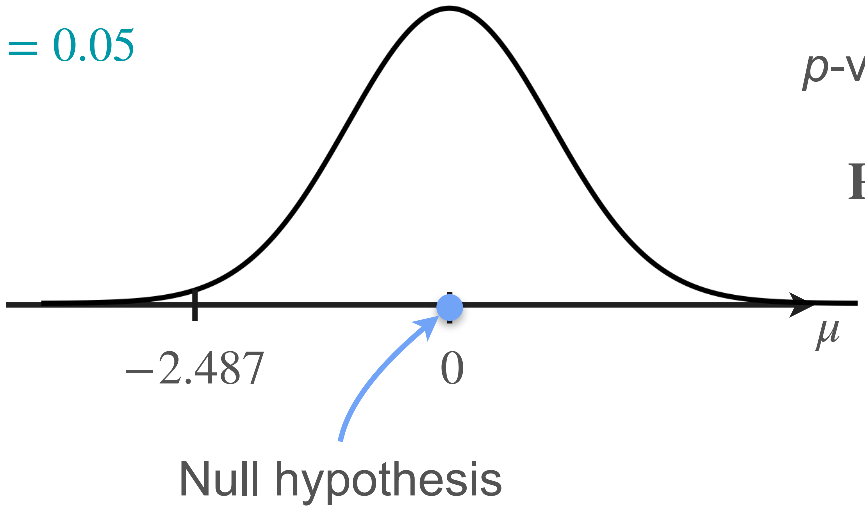
Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$ $\quad \bar{x} = 64.252$ $\quad s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)?$$

Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

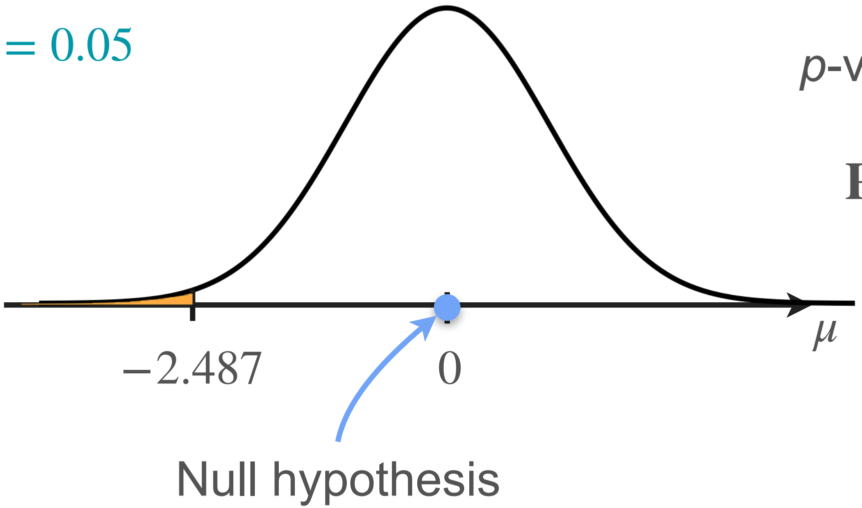
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0173$$

Left-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in**.

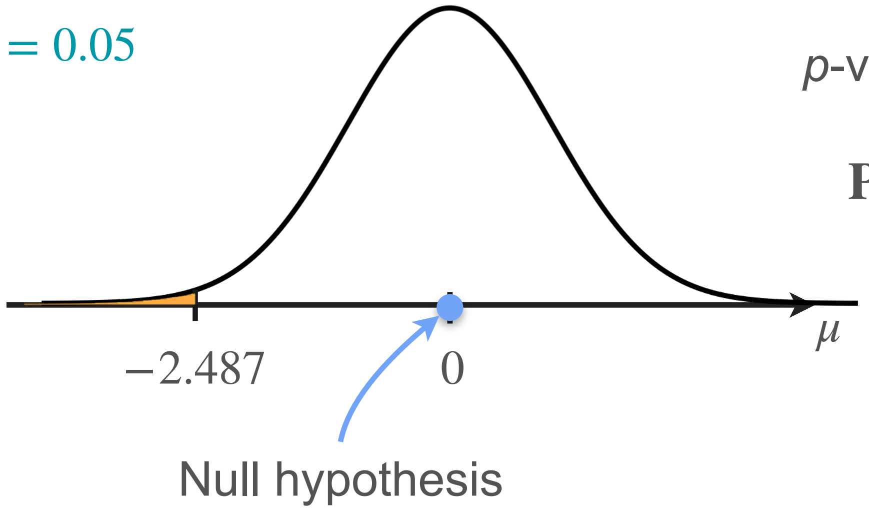
$H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$ $\quad n = 10$

$\bar{x} = 64.252$

$s = 3.113$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$\alpha = 0.05$



p-value:

$$\mathbf{P}\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \,\middle|\, \mu = 66.7\right)$$

$$= 0.0173 < \alpha$$

Conclusion: reject H_0

(with a 5% significance level)

DeepLearning.AI

Hypothesis Testing

Two sample t-test

Independent Two-Sample t-Test

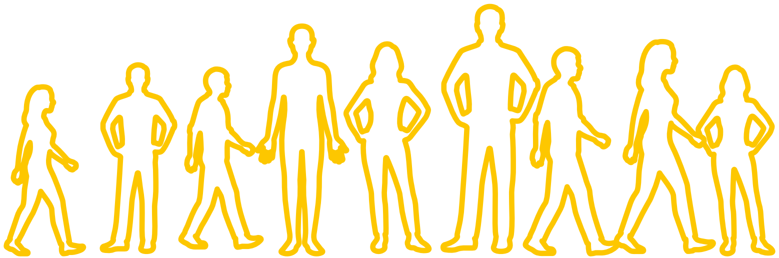
Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

Height of 18 y/o in the US

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74



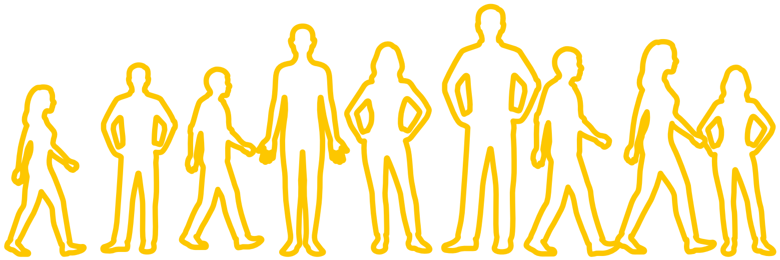
y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

Height of 18 y/o in the US

Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74



y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

$$n_X = 10 \qquad \bar{x} = 68.442 \qquad s_X = 3.113$$

Height of 18 y/o in the US

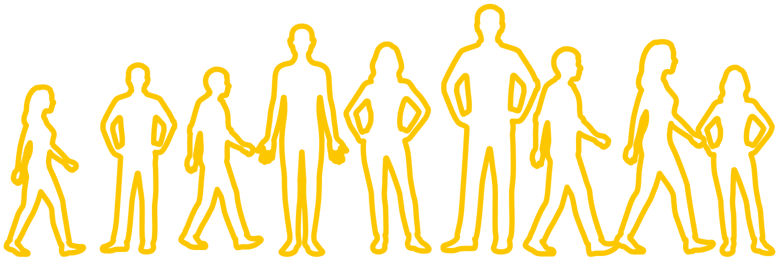
Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$n_X = 10$ $\bar{x} = 68.442$ $s_X = 3.113$

Height of 18 y/o in the US



y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

$n_Y = 9$ $\bar{y} = 65.949$ $s_Y = 3.106$

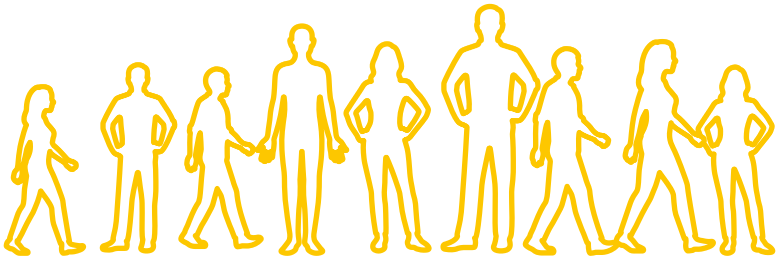
Height of 18 y/o in Argentina

Independent Two-Sample t-Test

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74

$n_X = 10$ $\quad \bar{x} = 68.442$ $\quad s_X = 3.113$

Height of 18 y/o in the US



y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
71.16	64.45	66.00	67.17	63.45	65.91	65.90	60.44	69.06

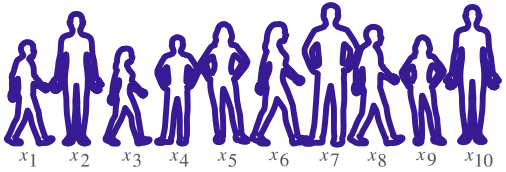
$n_Y = 9$ $\quad \bar{y} = 65.949$ $\quad s_Y = 3.106$

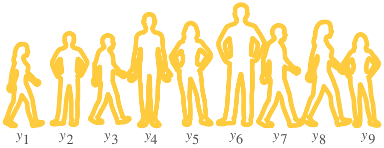
Height of 18 y/o in Argentina

$$\mu_{US} \neq \mu_{Ar}$$

Independent Two-Sample t-Test: Hypothesis

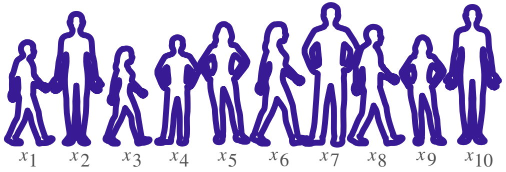
Independent Two-Sample t-Test: Hypothesis

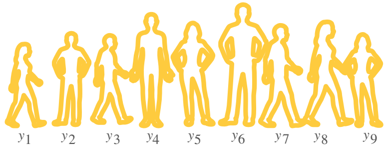




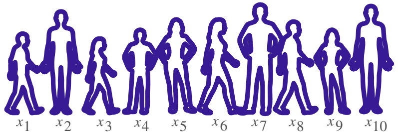
$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$

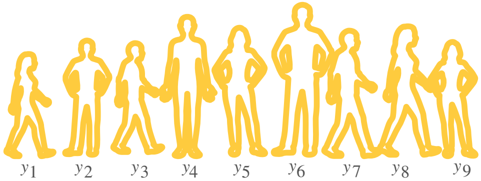
Independent Two-Sample t-Test: Hypothesis





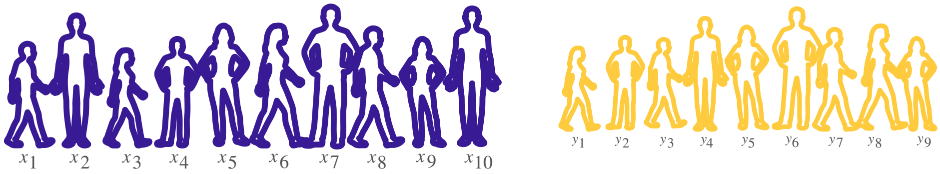
$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$



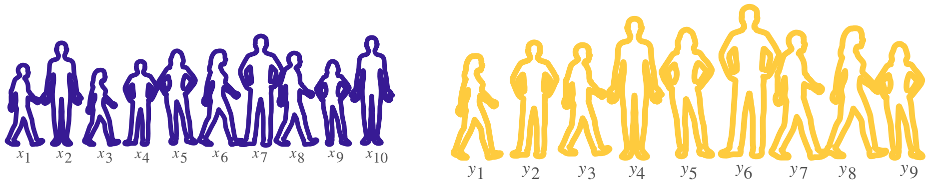


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$

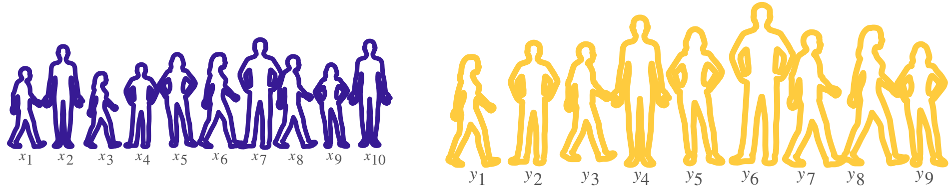
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} > \mu_{Ar}$

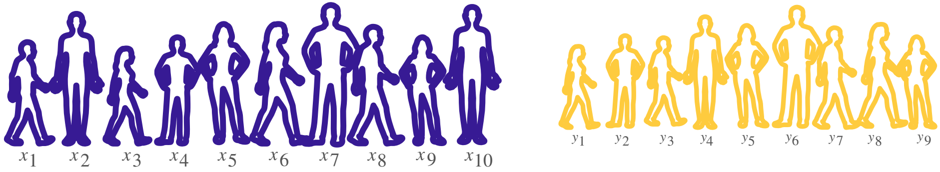


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$

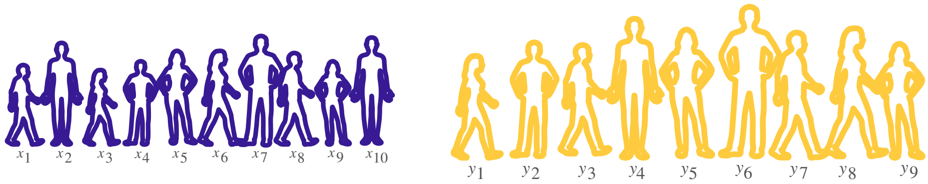


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} \neq \mu_{Ar}$

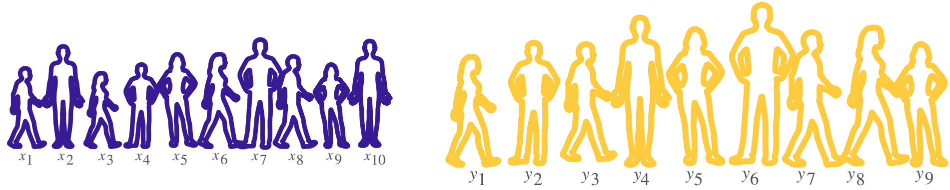
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

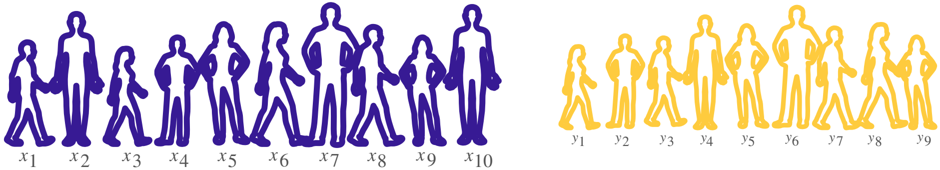


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} < \mu_{Ar}$

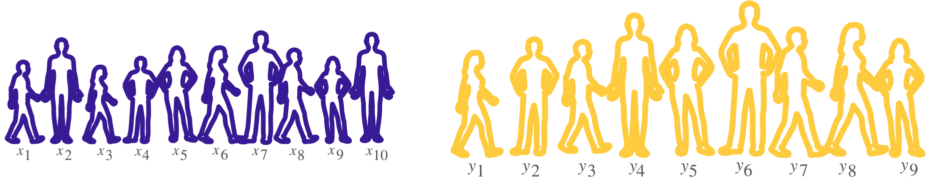


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} \neq \mu_{Ar}$

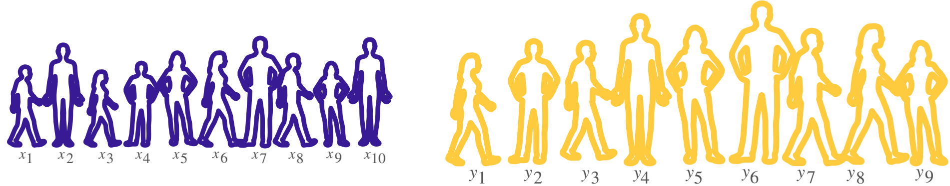
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

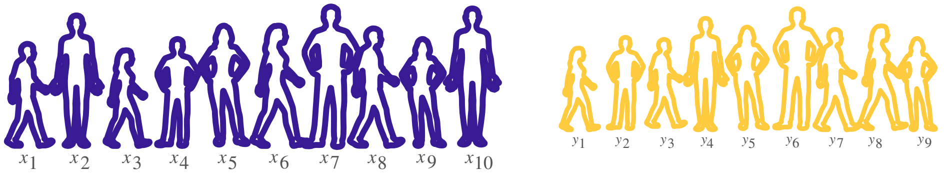


$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} < 0$

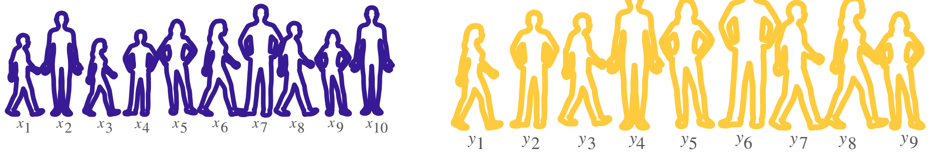


$H_0 : \mu_{US} = \mu_{Ar}$ vs. $H_1 : \mu_{US} \neq \mu_{Ar}$

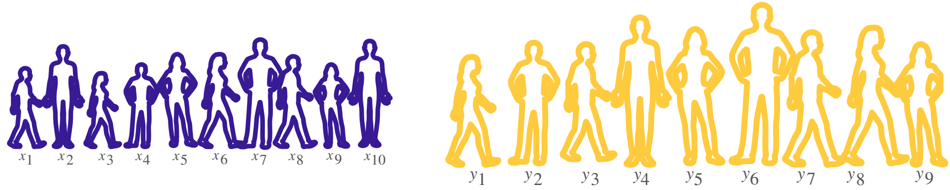
Independent Two-Sample t-Test: Hypothesis



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} < 0$



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

Independent Two-Sample t-Test: Assumptions

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- All people in the sample from the two groups are different

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- All people in the sample from the two groups are different
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$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}^2\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}^2\right)$$

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\overline{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample *t*-Test: Assumptions

- All people in the sample from the two groups are different
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$$\overline{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\overline{X} - \overline{Y} \sim ?$$

$$\overline{Y} = \frac{1}{9} \sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
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$$X \sim \mathcal{N}\left(\mu_{US}, \sigma^2_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma^2_{Arg}\right)$$

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\qquad , \qquad \right)$$

$$\bar{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}^2\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}^2\right)$$

$$\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg},\qquad\qquad\right)$$

$$\overline{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

Independent Two-Sample t-Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}^2\right)$$

$$Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}^2\right)$$

$$\overline{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\overline{Y} = \frac{1}{9} \sum_{i=1}^{9} Y_i$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

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$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

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$$\bar{Y} = \frac{1}{9}\sum_{i=1}^{9} Y_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right)$$

Independent Two-Sample t-Test: Statistic

$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right)$ $\quad$ $Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right)$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don’t know σ_{US}, σ_{Ar} 😱

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$\frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}}$$

Independent Two-Sample t-Test: Statistic

$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right)$ $\quad$ $Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \rightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know $\sigma_{US}, \ \sigma_{Ar}$ 😱 → Replace it with the sample standard deviation

$$T = \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{\nu}$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

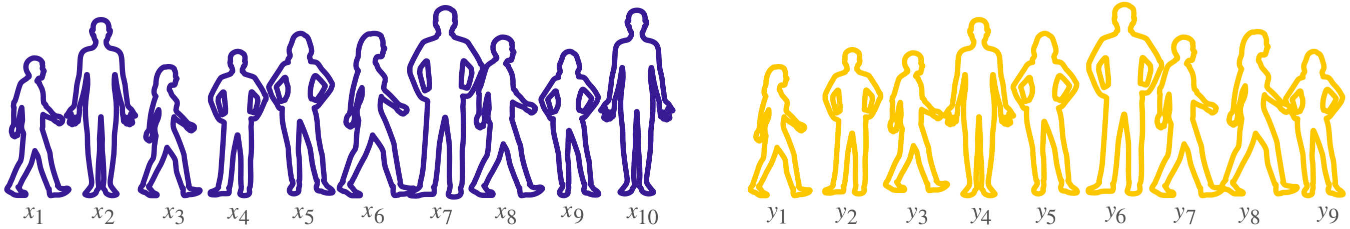
$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}\right) \longrightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1^2)$$

You don't know σ_{US}, σ_{Ar} → Replace it with the sample standard deviation

$$T = \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_\nu$$

Degrees of freedom $= \dfrac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t-Test



$n_X = 10$

$\bar{x} = 68.442$

$s_X = 3.113$

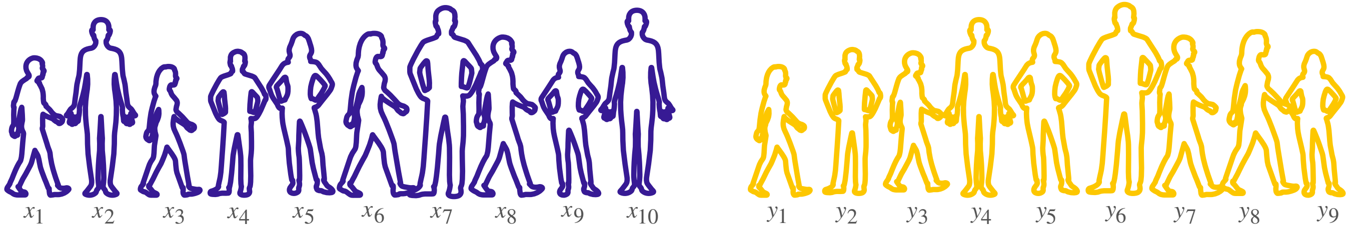
$n_Y = 9$

$\bar{y} = 65.949$

$s_Y = 3.106$

$$\text{Degrees of freedom} = \frac{\left(\frac{^2}{} + \frac{^2}{}\right)^2}{\frac{\left(\frac{^2}{}\right)^2}{-1} + \frac{\left(\frac{^2}{}\right)^2}{-1}}$$

Independent Two-Sample t-Test



$n_X = 10$

$\bar{x} = 68.442$

$s_X = 3.113$

$n_Y = 9$

$\bar{y} = 65.949$

$s_Y = 3.106$

$$\text{Degrees of freedom} = \frac{\overbrace{\left(\frac{3.113^2}{10} + \frac{3.106^2}{9}\right)^2}^{16.8}}{\frac{\left(\frac{3.113^2}{10}\right)^2}{10-1} + \frac{\left(\frac{3.106^2}{9}\right)^2}{9-1}}$$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right) \rightarrow \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{\left(\overline{X} - \overline{Y}\right) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom $= \dfrac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t-Test: Statistic

$$X \sim \mathcal{N}\left(\mu_{US}, \sigma_{US}\right) \qquad Y \sim \mathcal{N}\left(\mu_{Arg}, \sigma_{Arg}\right)$$

$$\overline{X} - \overline{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}\right) \longrightarrow \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Ar}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know σ_{US}, σ_{Ar} 😱 → Replace it with the sample standard deviation

$$T = \frac{(\overline{X} - \overline{Y}) - \left(\mu_{US} - \mu_{Arg}\right)}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom $= \dfrac{\left(\dfrac{s_X^2}{n_X} + \dfrac{s_Y^2}{n_y}\right)^2}{\dfrac{\left(\dfrac{s_X^2}{n_X}\right)^2}{n_X - 1} + \dfrac{\left(\dfrac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t-Test: Right Tailed Test

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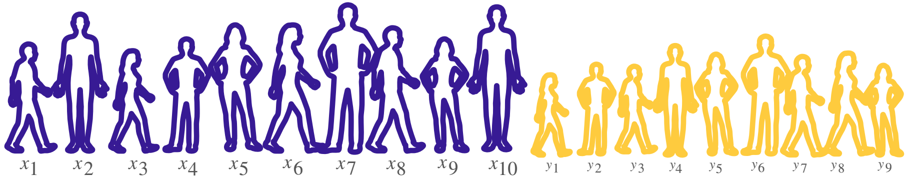
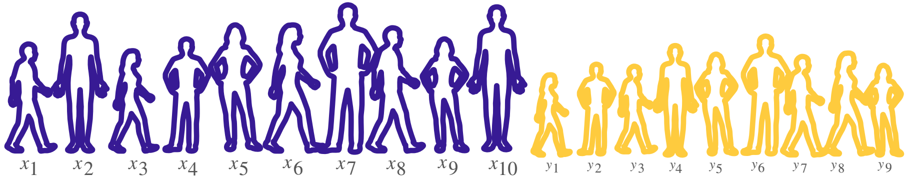


$n_X = 10$ $n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test



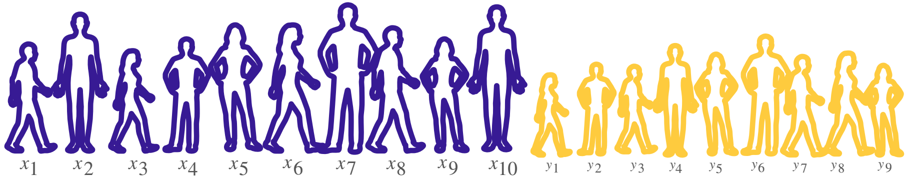
$\bar{x} = 68.442$

$s_X = 3.113$

$n_X = 10$

$n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$

$s_X = 3.113$

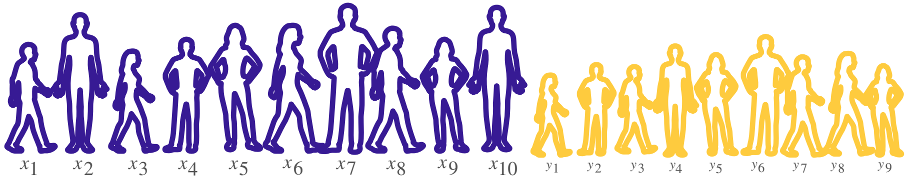
$n_X = 10$

$\bar{y} = 65.949$

$s_Y = 3.106$

$n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test

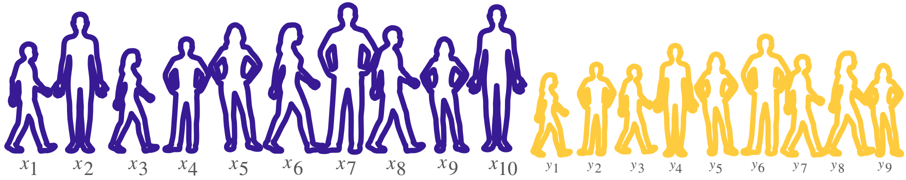


$\bar{x} = 68.442$ $\quad s_X = 3.113$ $\quad n_X = 10$

$\bar{y} = 65.949$ $\quad s_Y = 3.106$ $\quad n_Y = 9$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

Independent Two-Sample t-Test: Right Tailed Test



$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

$\bar{x} = 68.442$

$s_X = 3.113$

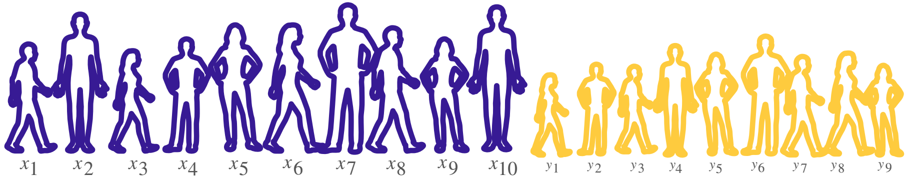
$n_X = 10$

$\bar{y} = 65.949$

$s_Y = 3.106$

$n_Y = 9$

Independent Two-Sample t-Test: Right Tailed Test



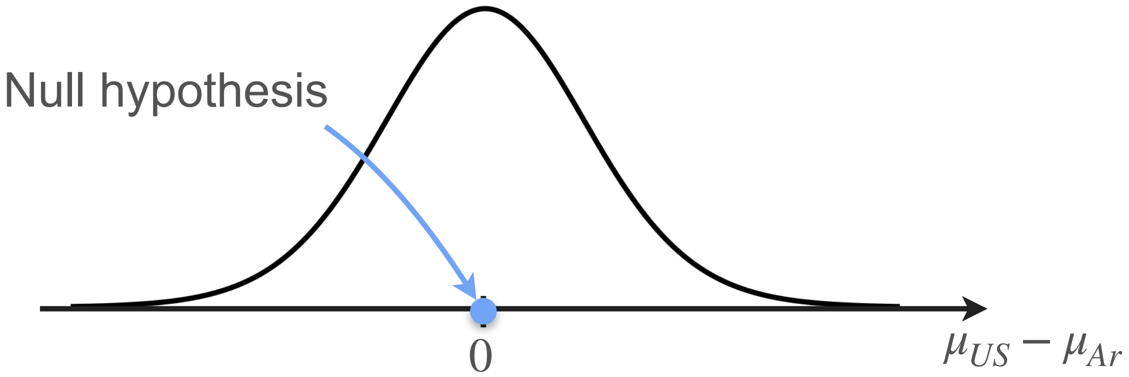
$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

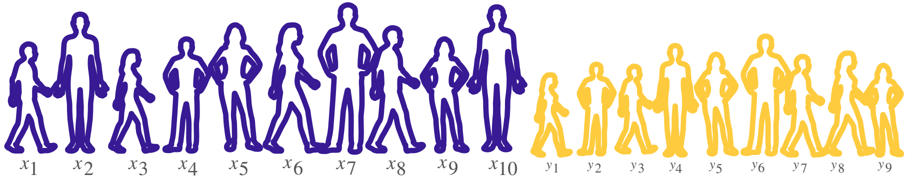
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



Independent Two-Sample t-Test: Right Tailed Test

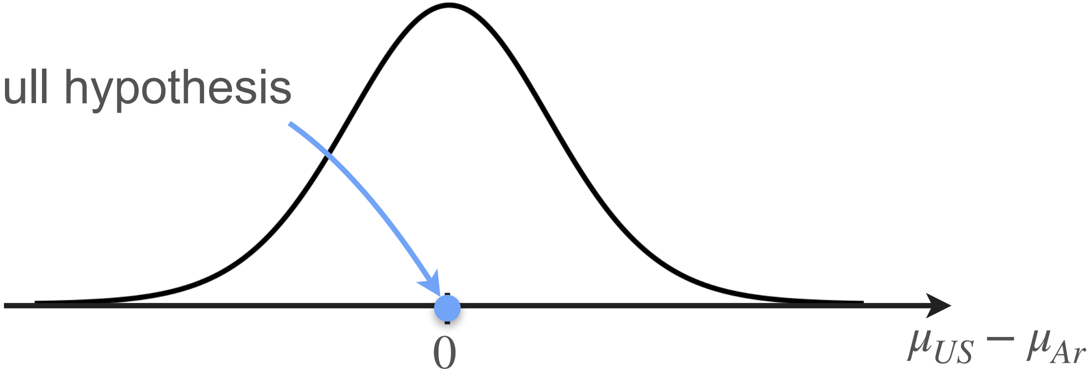


$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t = 1.7459}$$

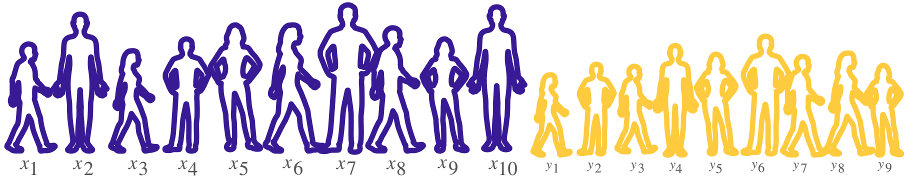
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



Independent Two-Sample t-Test: Right Tailed Test

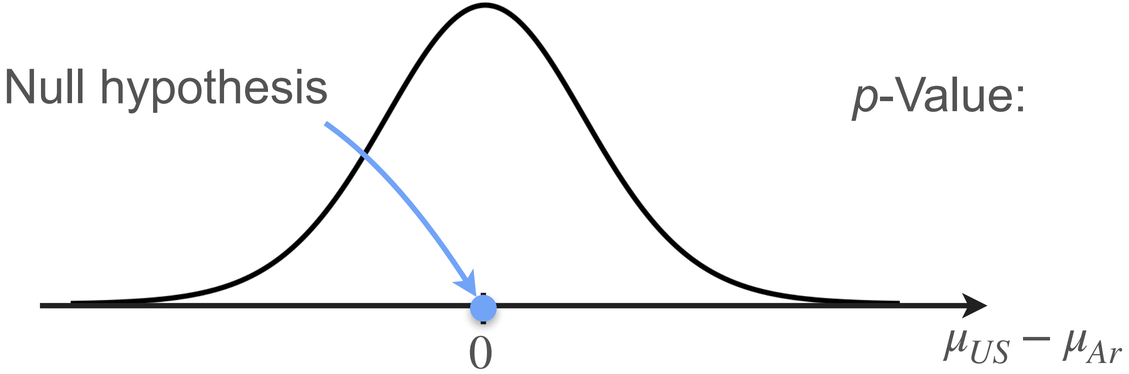


$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t\, =\, 1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

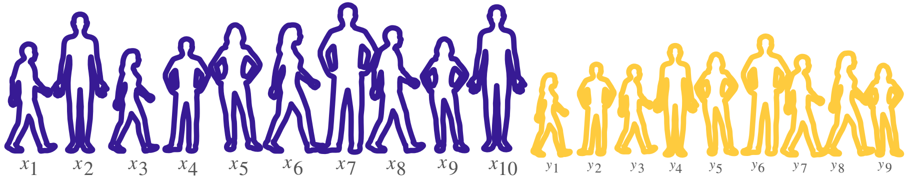
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value:

Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

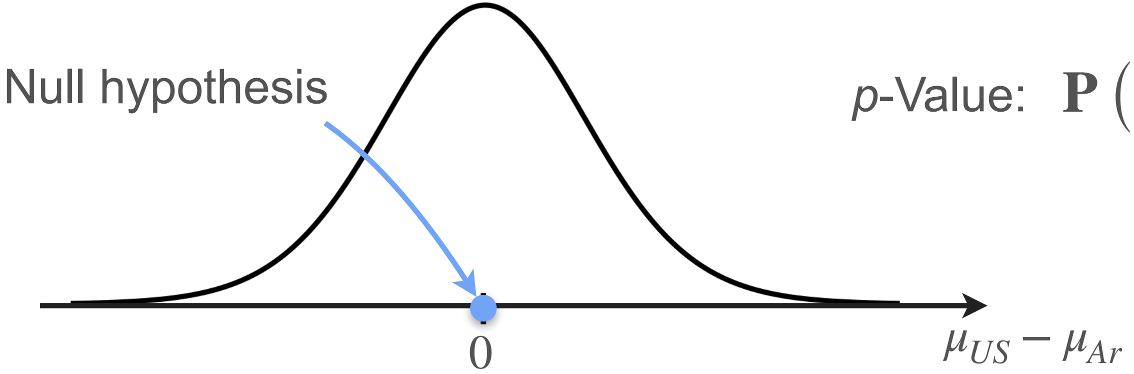
$$\underbrace{\hphantom{s_X = 3.113 \quad s_Y = 3.106}}_{t\,=\,1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

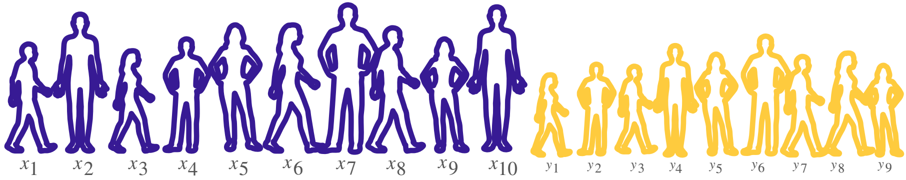
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$



Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad s_X = 3.113$ $\quad n_X = 10$

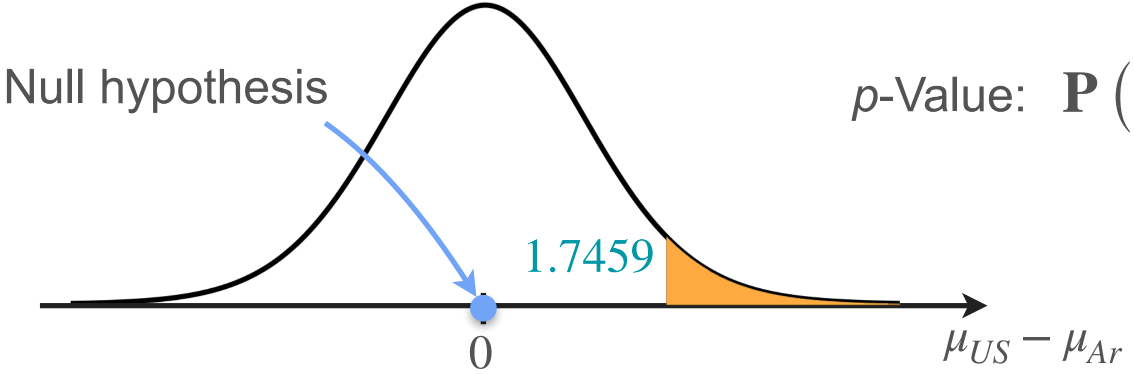
$\bar{y} = 65.949$ $\quad s_Y = 3.106$ $\quad n_Y = 9$

$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

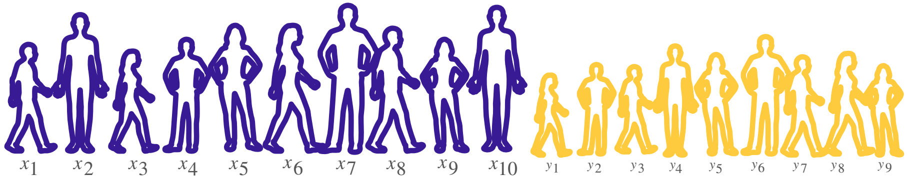
$\alpha = 0.05$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$



p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

Independent Two-Sample *t*-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

$t = 1.7459$

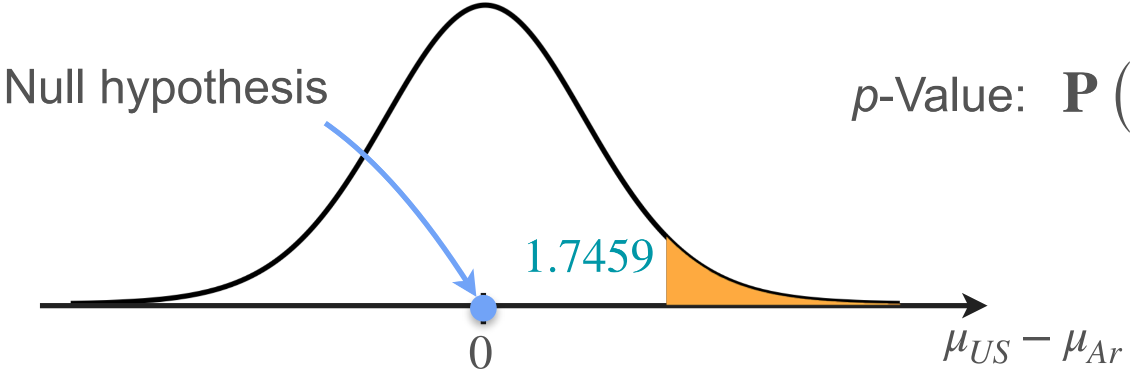
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

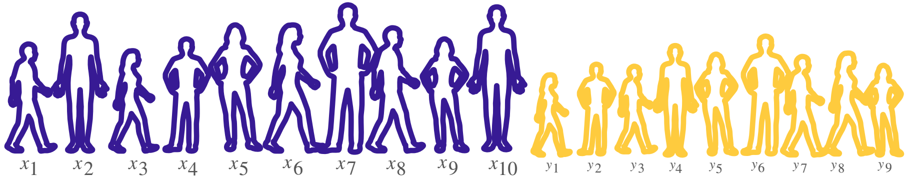
If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

p-Value: $\mathbf{P}\left(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0495$



Independent Two-Sample t-Test: Right Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10 \quad$ $\bar{y} = 65.949$ $\quad n_Y = 9$

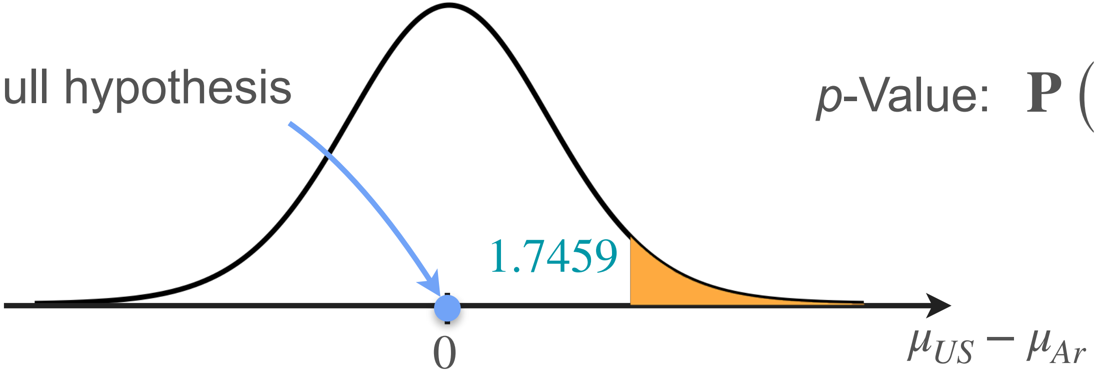
$s_X = 3.113$ $\quad s_Y = 3.106$

$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} > 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(T > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0495 < 0.05$

$\Rightarrow$ Reject H_0 (and accept H_1)

(with a 5% significance level)

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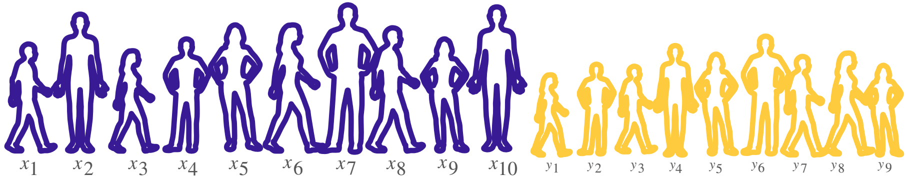
Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

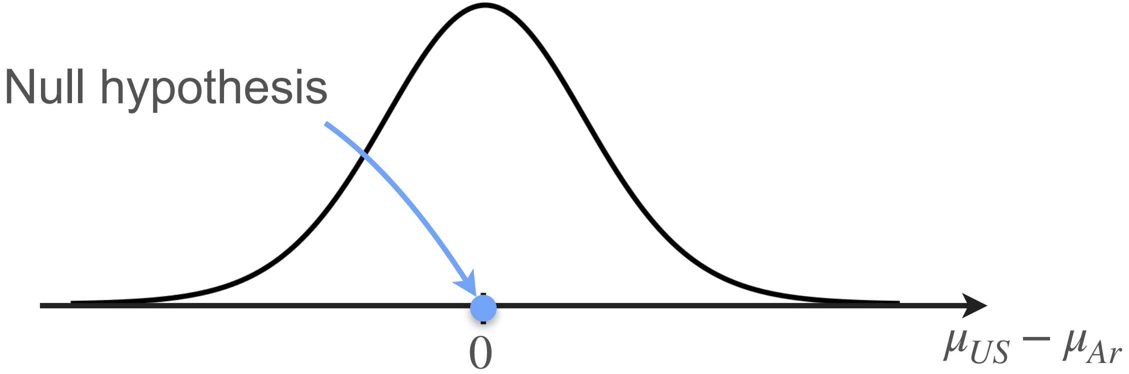
$s_X = 3.113$ $\quad s_Y = 3.106$

$t = 1.7459$

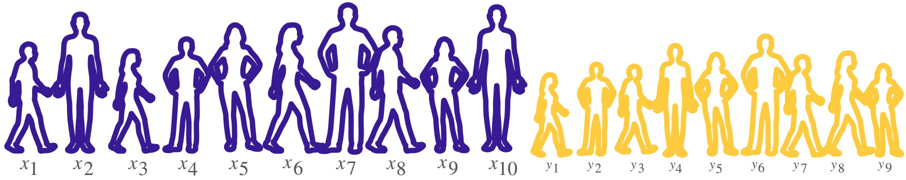
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

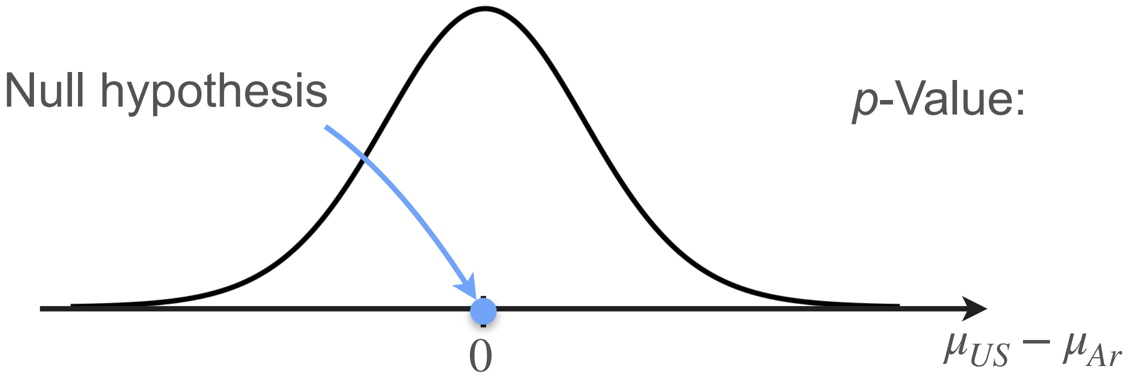
$s_X = 3.113$ $\quad s_Y = 3.106$

$$\underbrace{\qquad\qquad\qquad}_{t\,=\,1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

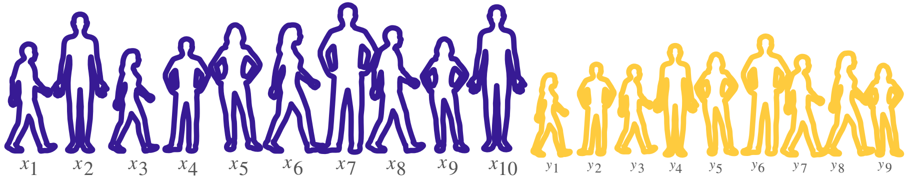
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value:

Independent Two-Sample t-Test: Two Tailed Test



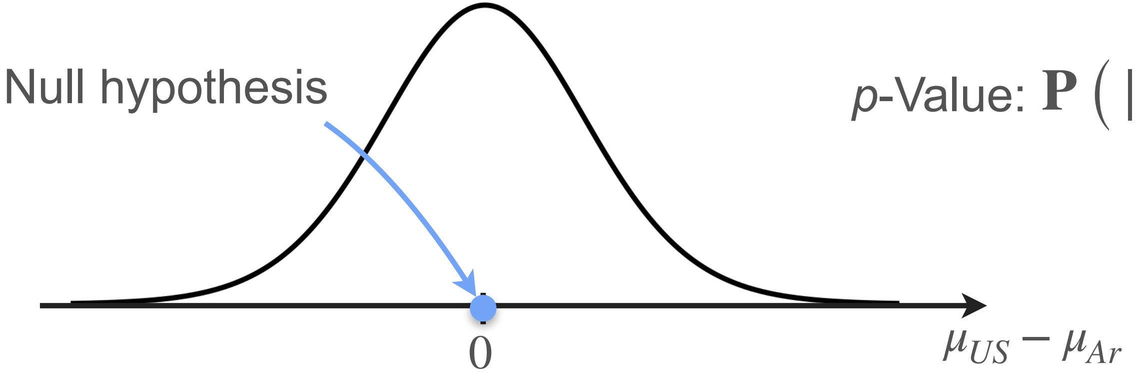
$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t\,=\,1.7459}$$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

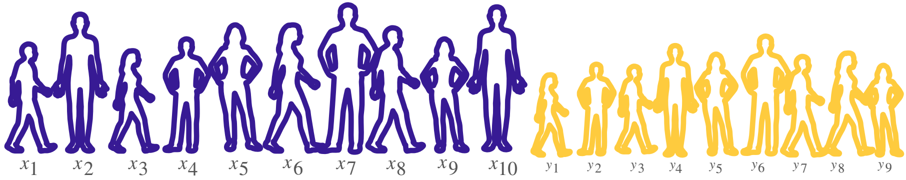
p-Value: $\mathbf{P}\left(\,|T| > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$



Independent Two-Sample t-Test: Two Tailed Test

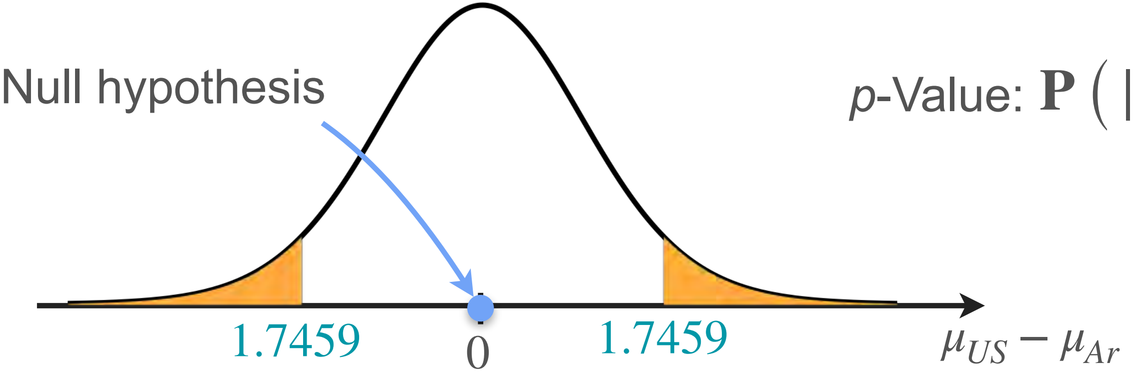
$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$



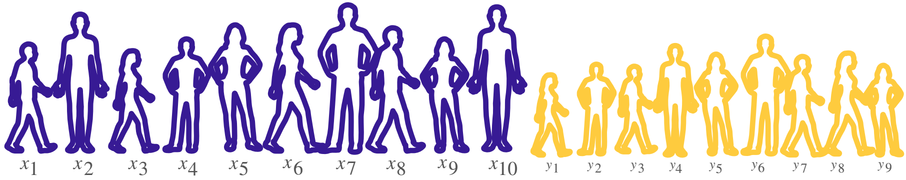
$$\underbrace{\bar{x} = 68.442 \quad s_X = 3.113 \quad n_X = 10 \quad \bar{y} = 65.949 \quad s_Y = 3.106 \quad n_Y = 9}_{t = 1.7459}$$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(|T| > 1.7459 \;\middle|\; \mu_{US} - \mu_{Ar} = 0\right)$

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

$s_X = 3.113$ $\quad s_Y = 3.106$

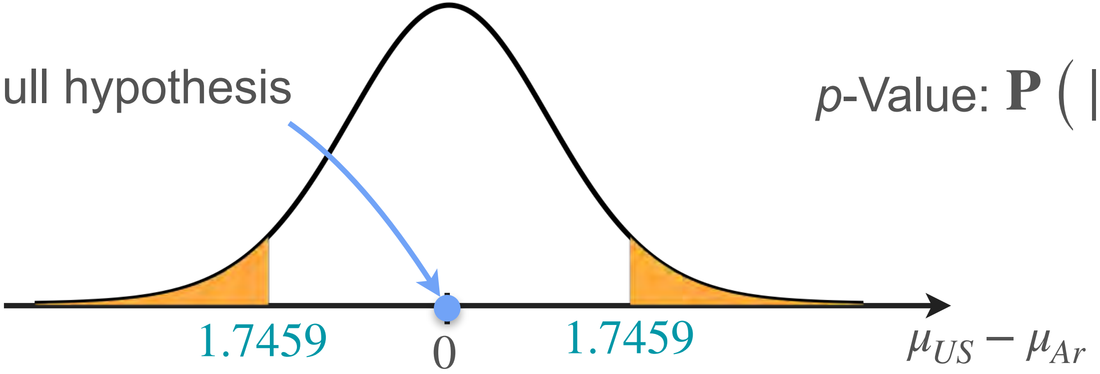
$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$

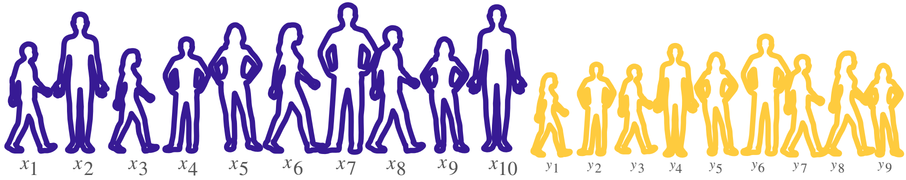
Null hypothesis



p-Value: $\mathbf{P}\left(|T| > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0\right)$

$= 0.0991$

Independent Two-Sample t-Test: Two Tailed Test



$\bar{x} = 68.442$ $\quad n_X = 10$ $\quad \bar{y} = 65.949$ $\quad n_Y = 9$

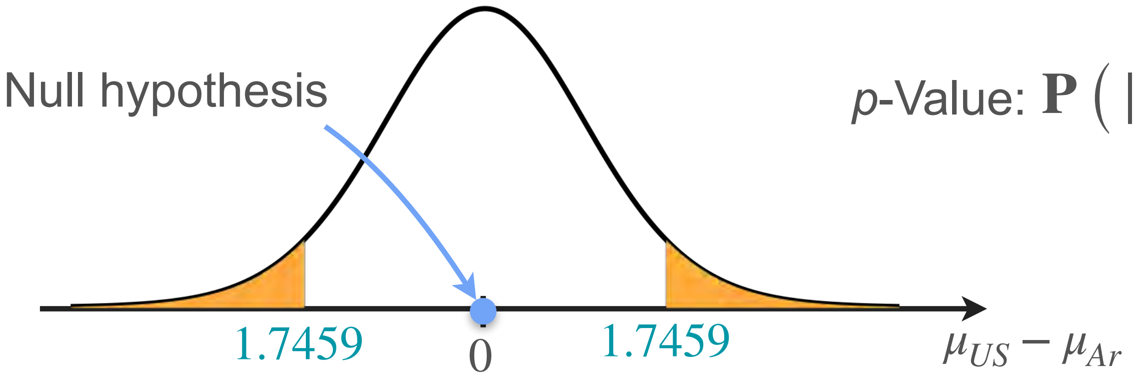
$s_X = 3.113$ $\quad s_Y = 3.106$

$t = 1.7459$

$H_0 : \mu_{US} - \mu_{Ar} = 0$ vs. $H_1 : \mu_{US} - \mu_{Ar} \neq 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{(\overline{X} - \overline{Y}) - 0}{\sqrt{\dfrac{S_X^2}{10} + \dfrac{S_Y^2}{9}}} \sim t_{16.8}$



p-Value: $\mathbf{P}\left(|T| > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0 \right)$

$= 0.0991 > 0.05$

$\Rightarrow$ Do not reject H_0

(with a 5% significance level)

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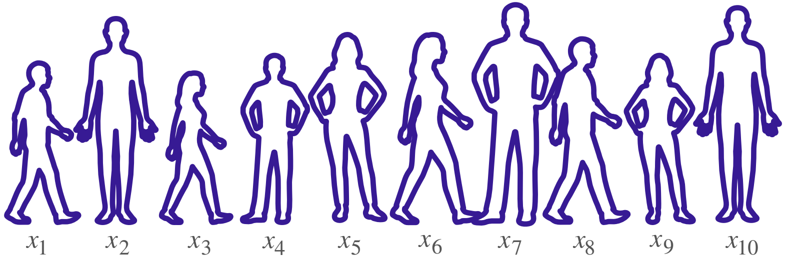
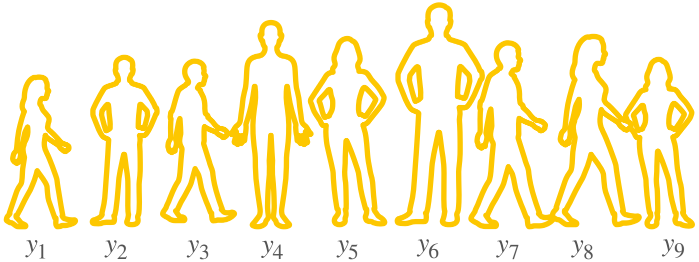
Hypothesis Testing

Paired t-test

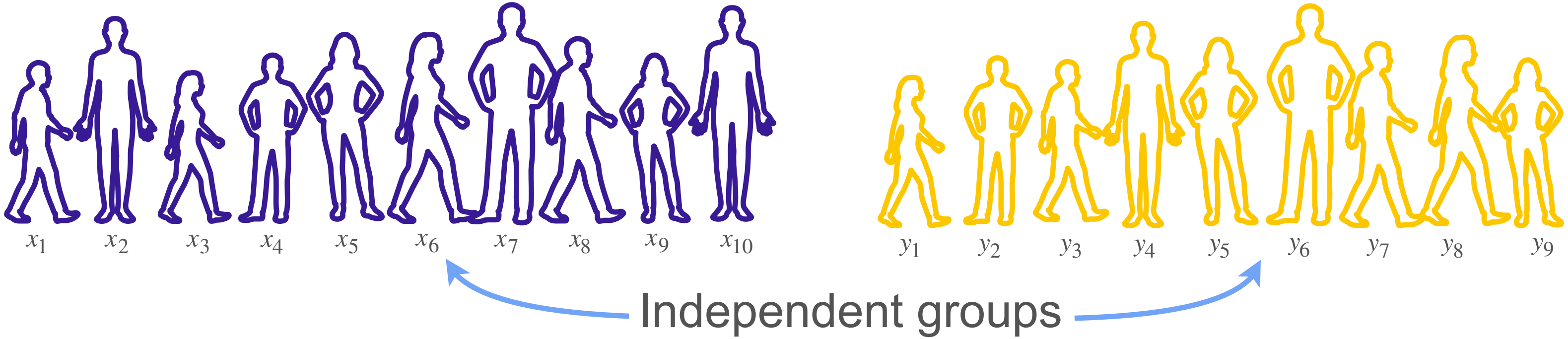
Paired t-Test and Two-Sample t-Test

Paired t-Test and Two-Sample t-Test

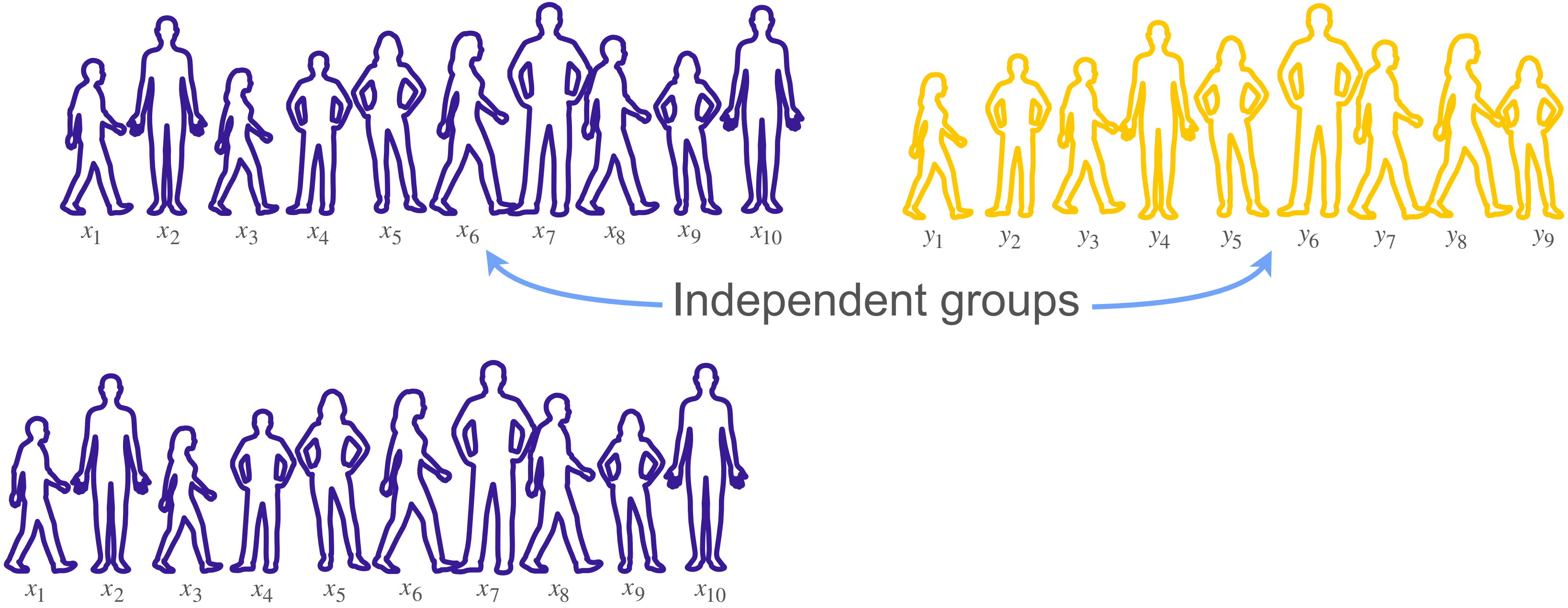




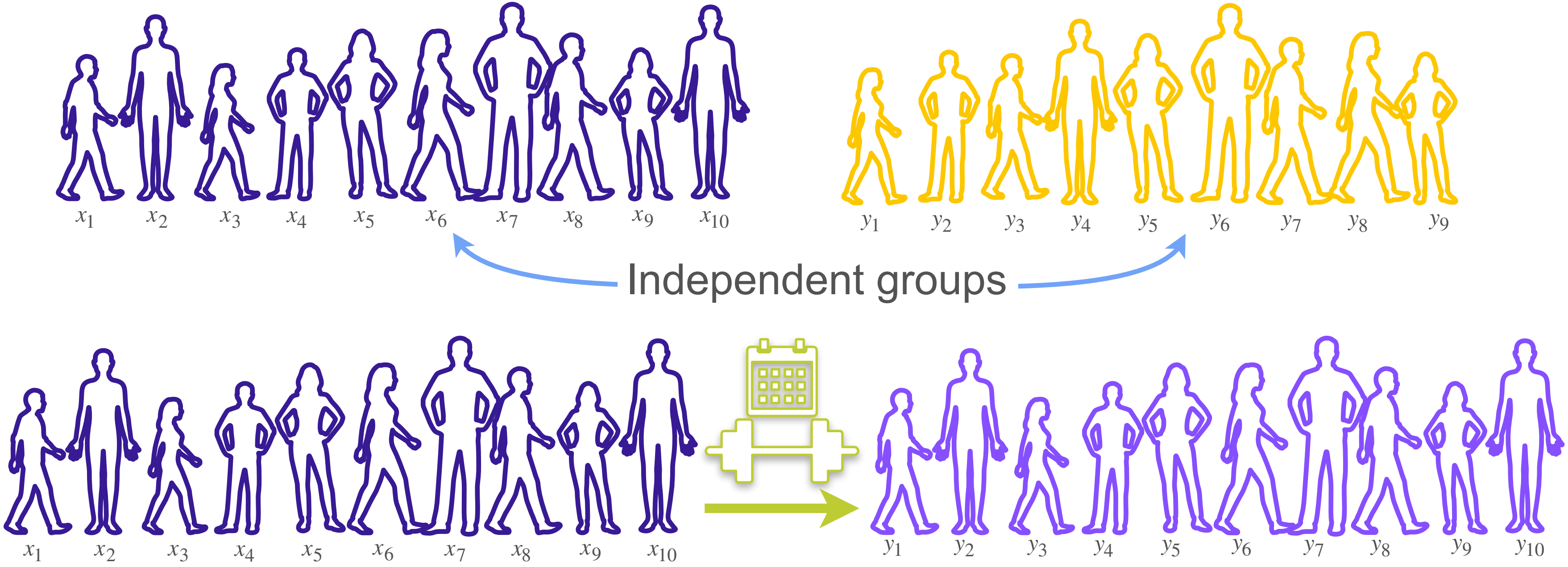
Paired t-Test and Two-Sample t-Test



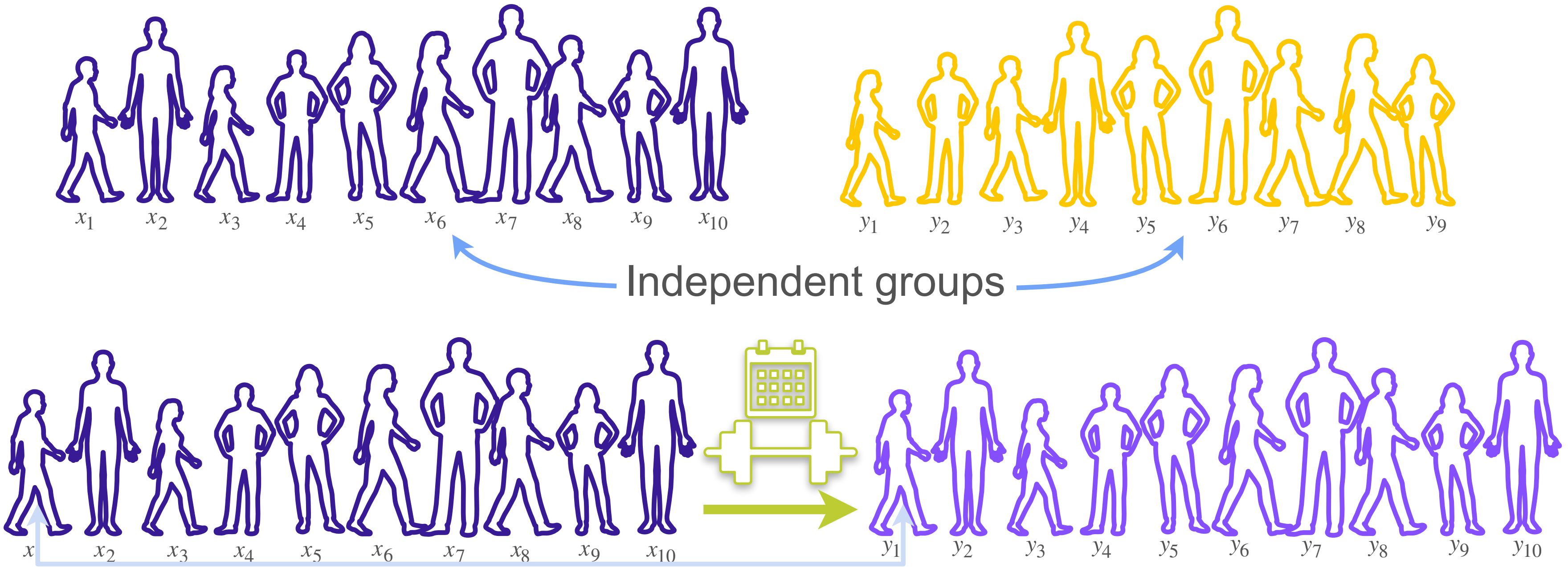
Paired t-Test and Two-Sample t-Test



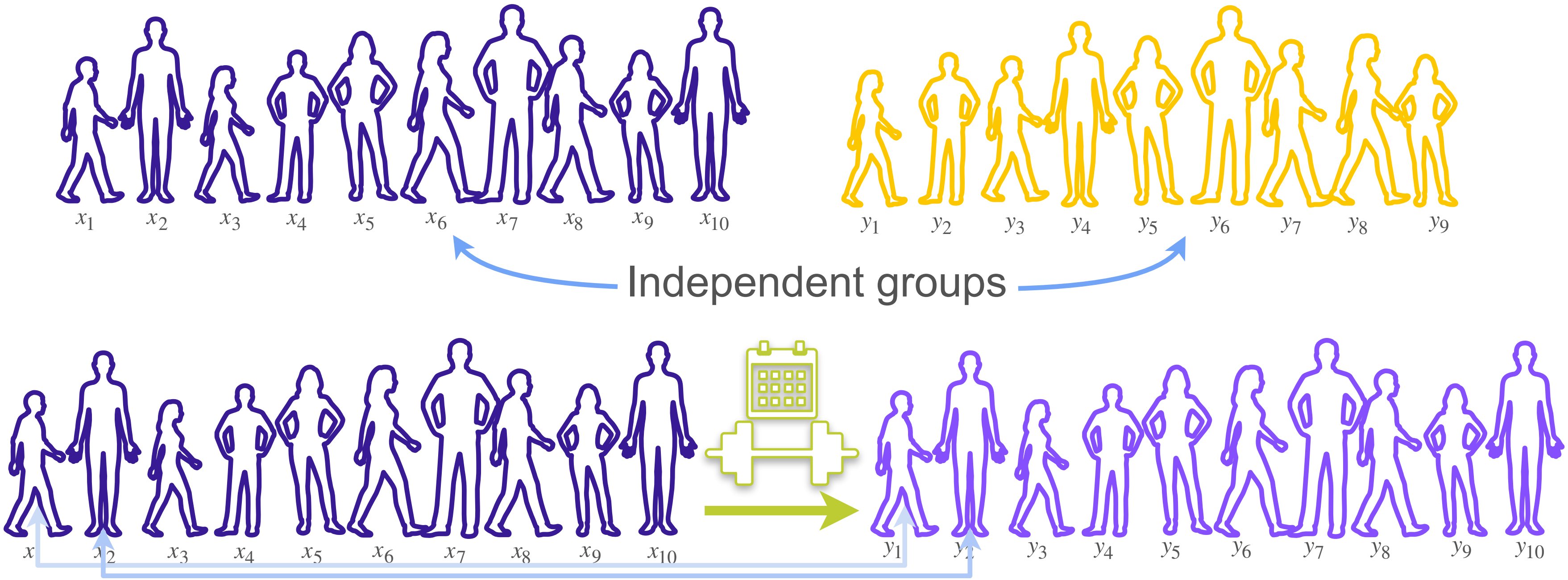
Paired t-Test and Two-Sample t-Test



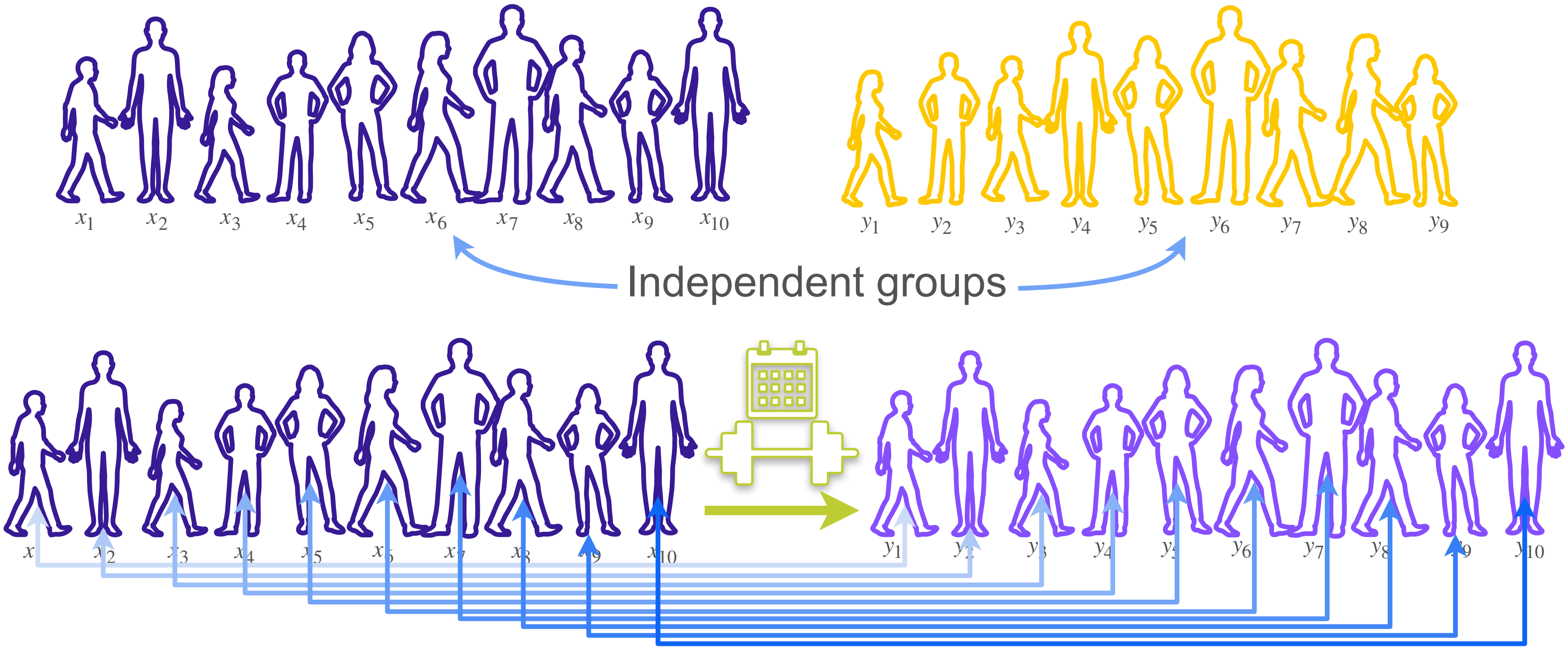
Paired t-Test and Two-Sample t-Test



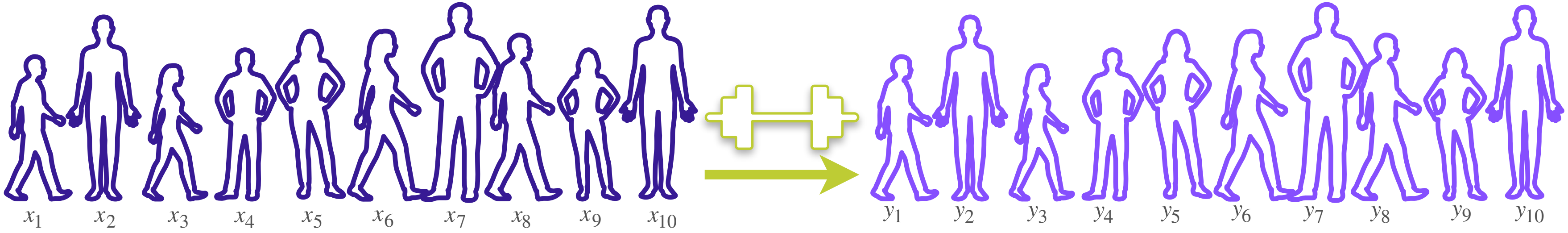
Paired t-Test and Two-Sample t-Test



Paired t-Test and Two-Sample t-Test

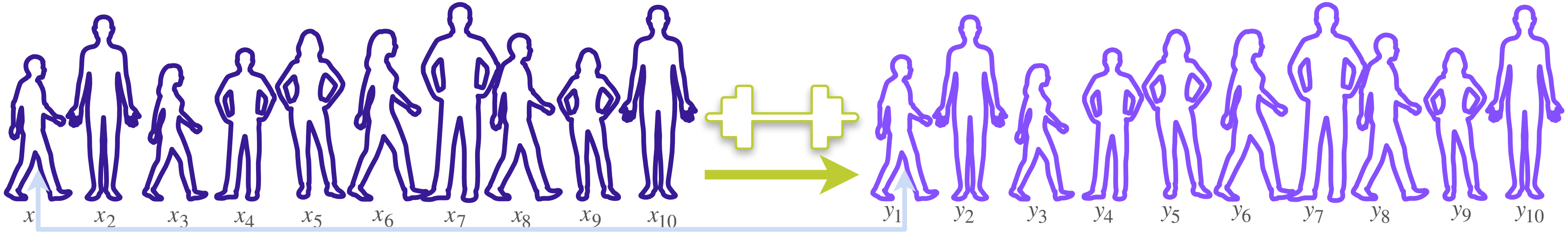


Paired *t*-Test: Statistic



Now you're interested in the difference between pair of samples

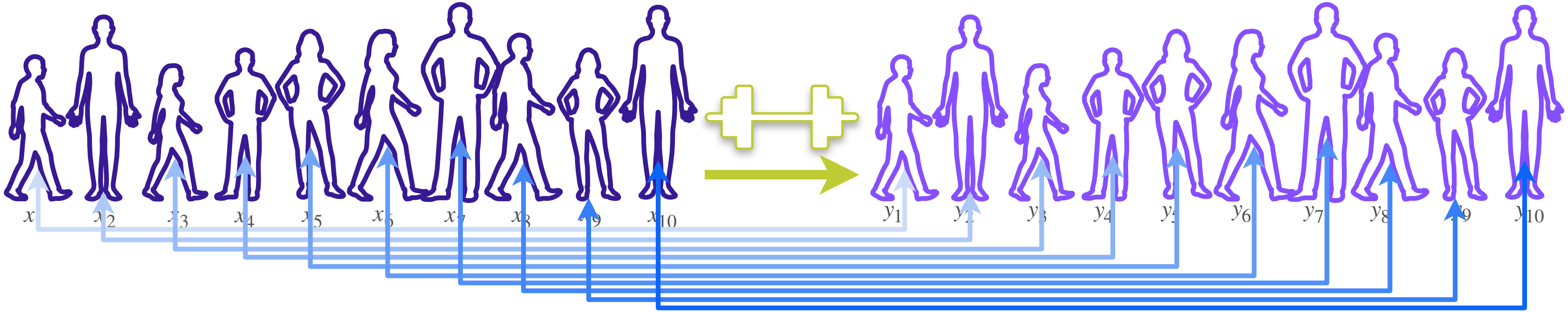
Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$(X_1 - Y_1)$

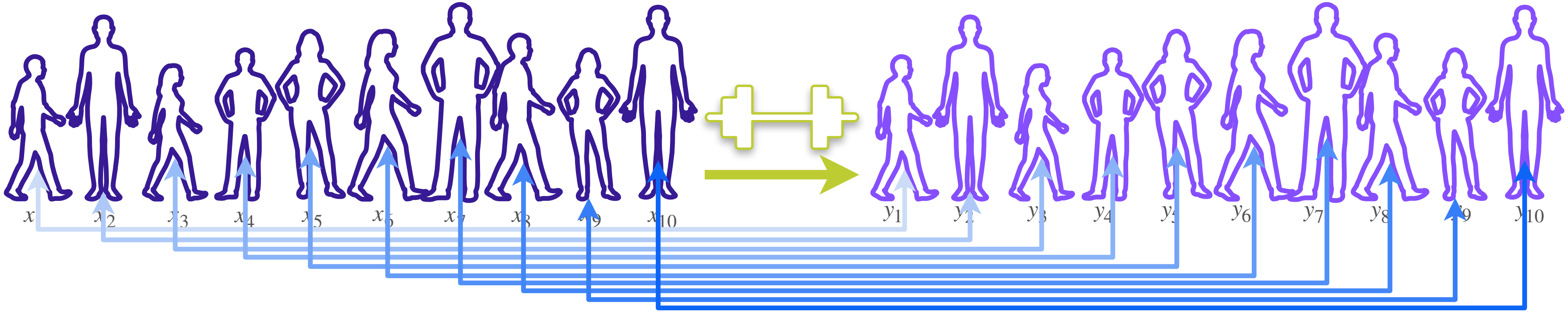
Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$$(X_1 - Y_1) \quad (X_2 - Y_2) \quad (X_3 - Y_3) \quad (X_4 - Y_4) \quad (X_5 - Y_5) \quad (X_6 - Y_6) \quad (X_7 - Y_7) \quad (X_8 - Y_8) \quad (X_9 - Y_9) \quad (X_{10} - Y_{10})$$

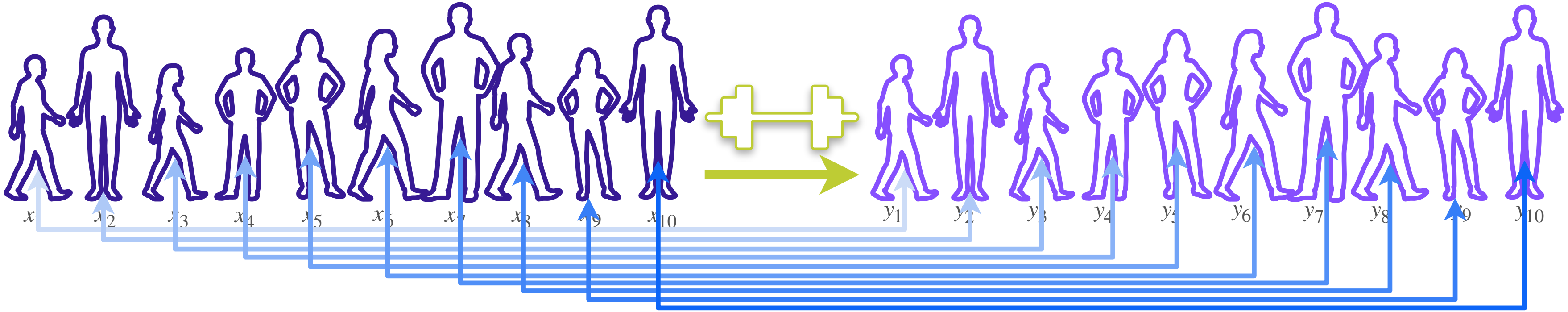
Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

Paired t-Test: Statistic

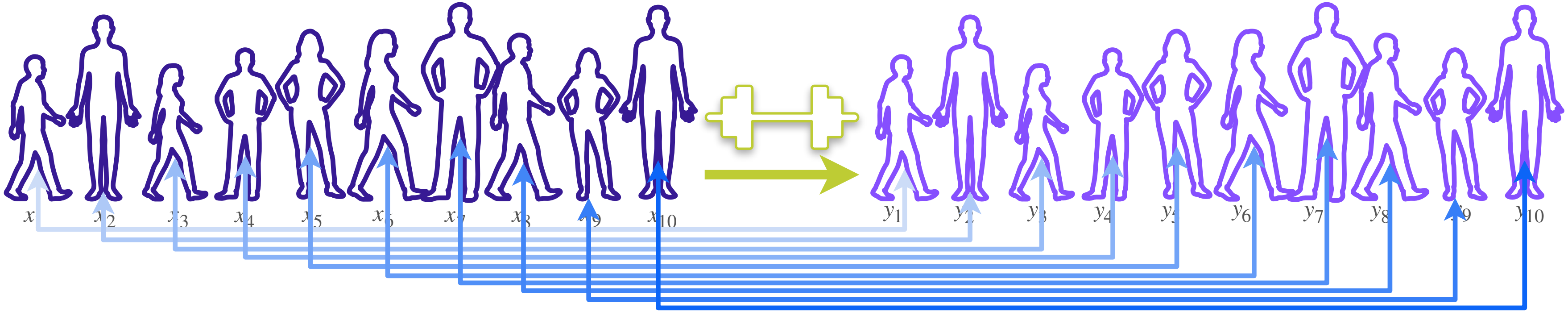


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

D_1

Paired t-Test: Statistic

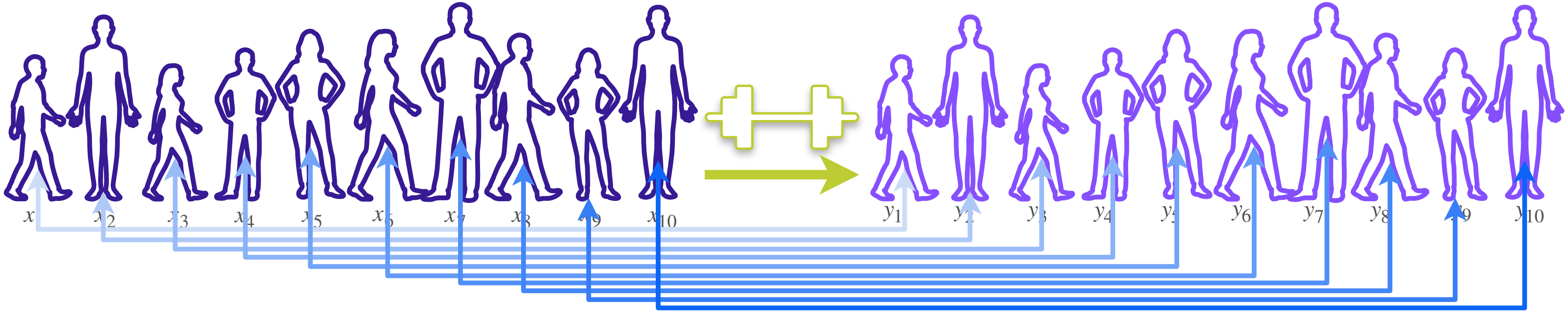


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

D_1 D_2

Paired t-Test: Statistic

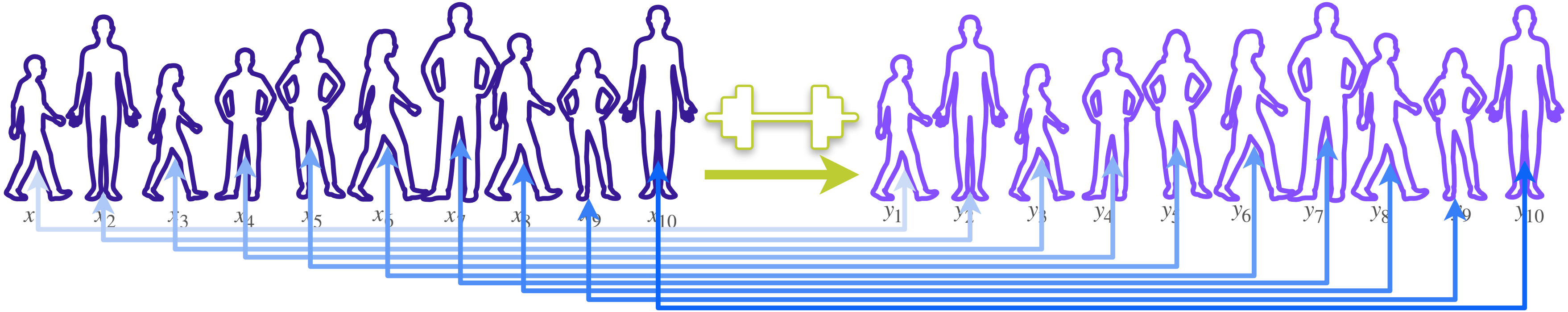


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t-Test: Statistic

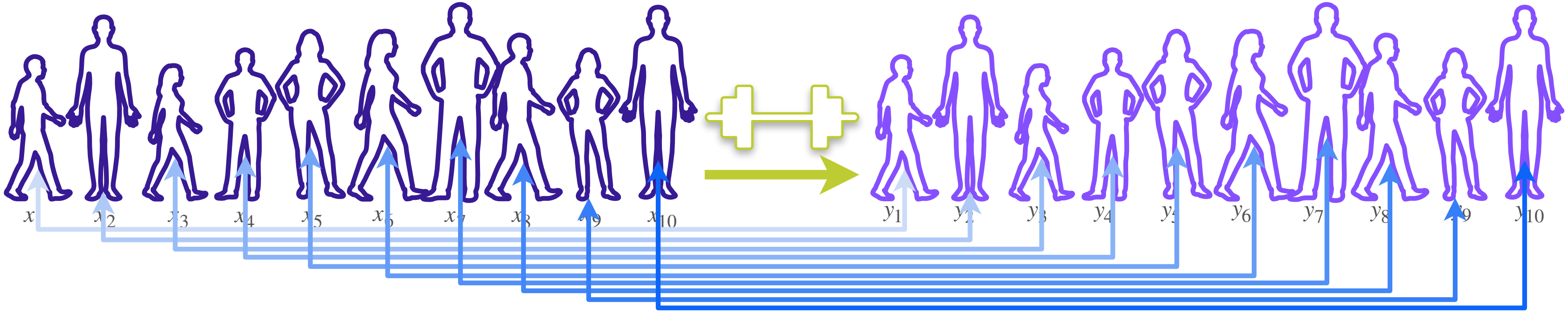


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t-Test: Statistic

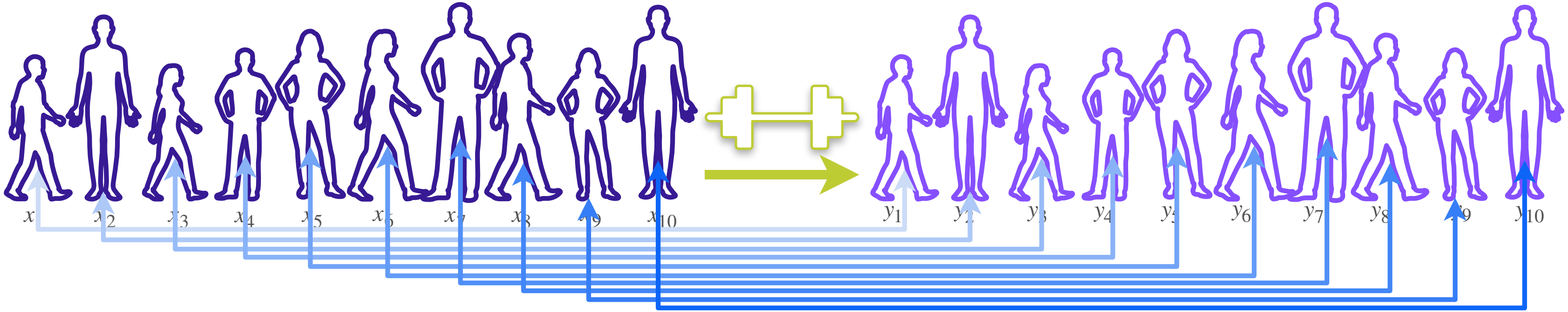


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1)+(X_2 - Y_2)+(X_3 - Y_3)+(X_4 - Y_4)+(X_5 - Y_5)+(X_6 - Y_6)+(X_7 - Y_7)+(X_8 - Y_8)+(X_9 - Y_9)+(X_{10} - Y_{10})}{10}$$

$$\overline{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

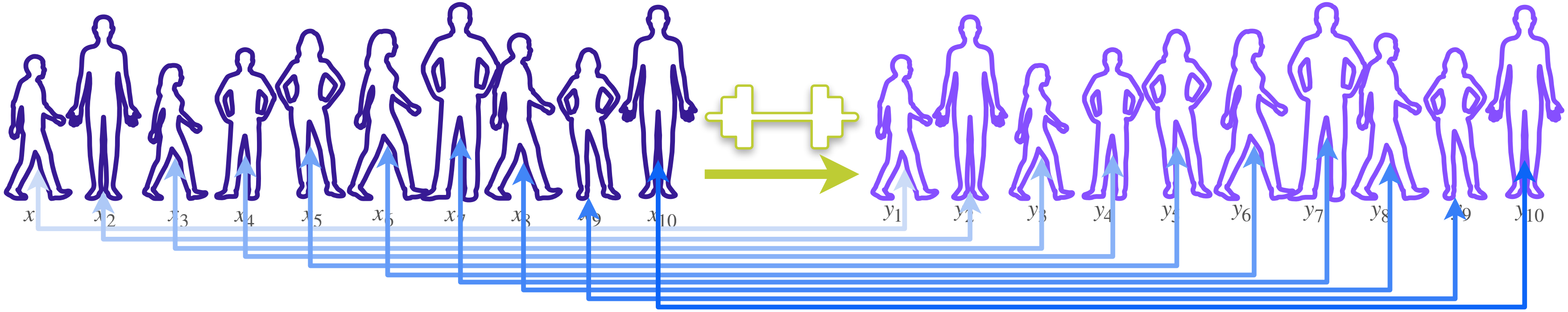
Paired t-Test: Statistic



Now you're interested in the difference between pair of samples

$$\overline{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t-Test: Statistic



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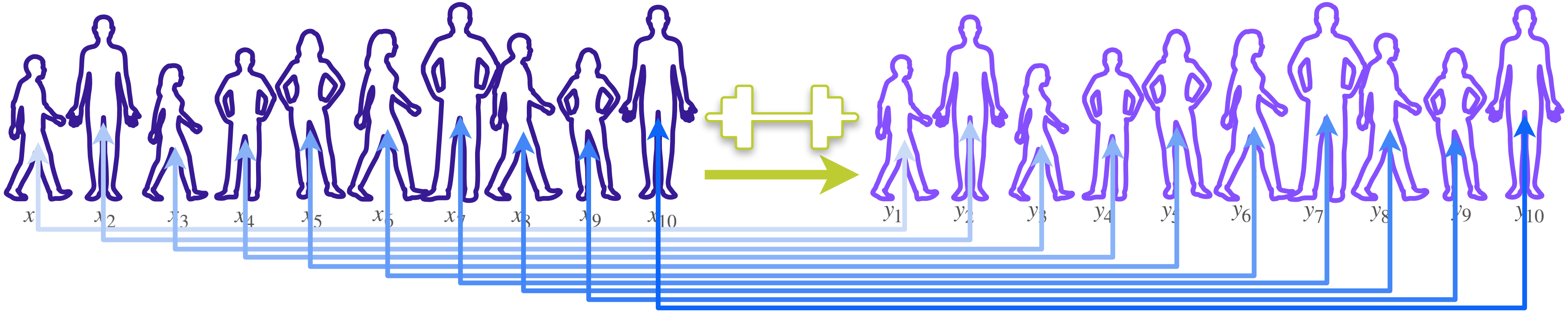
If X_i, Y_i are gaussian $\Rightarrow D_i = X_i - Y_i$ is gaussian.

$$D_i \overset{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

Paired t-Test: Statistic



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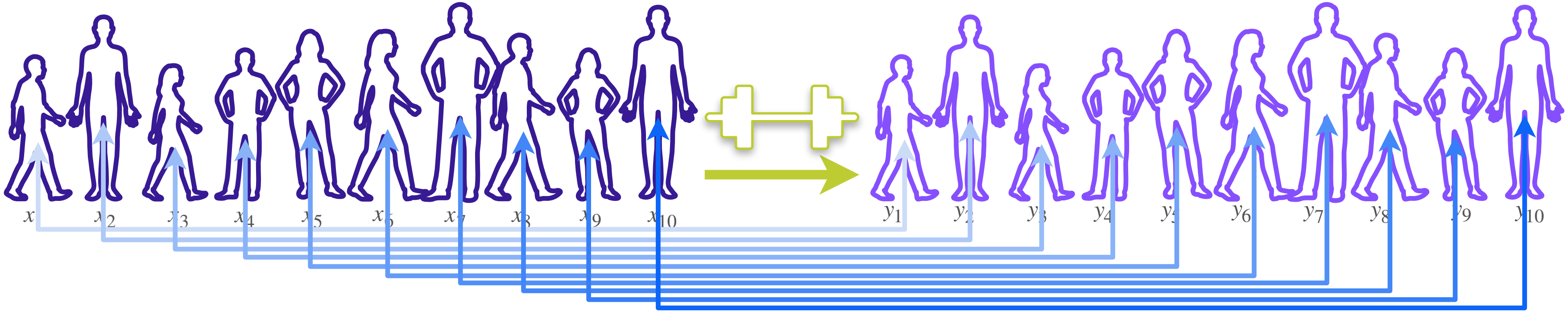

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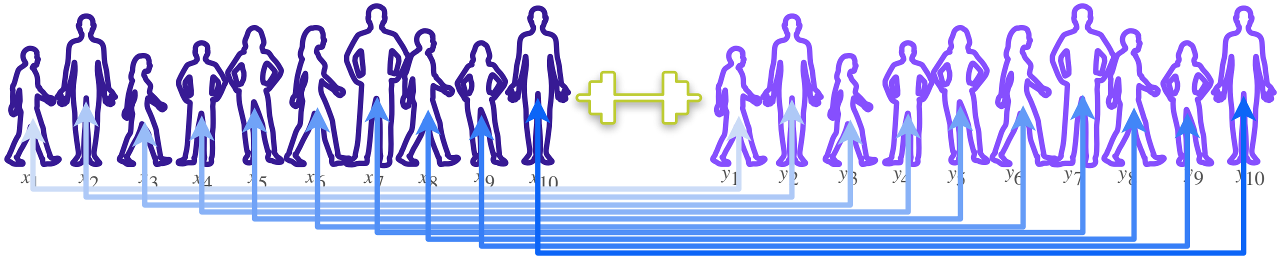

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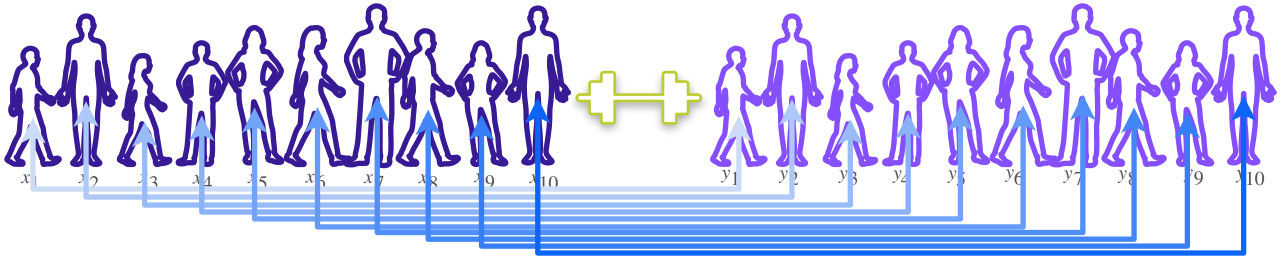

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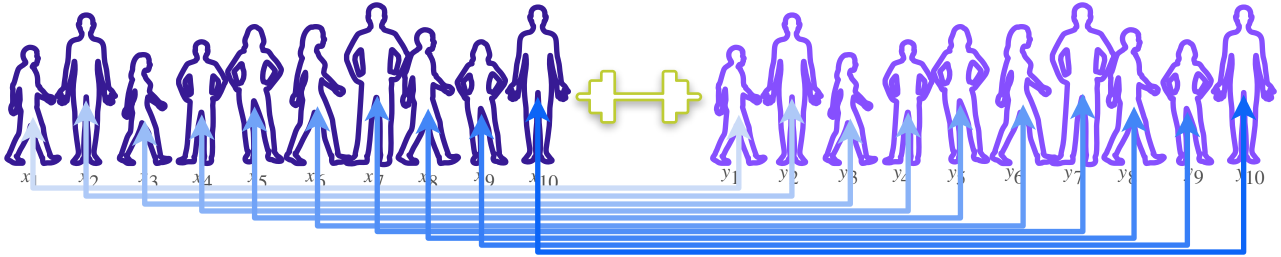
Paired t-Test: Statistic



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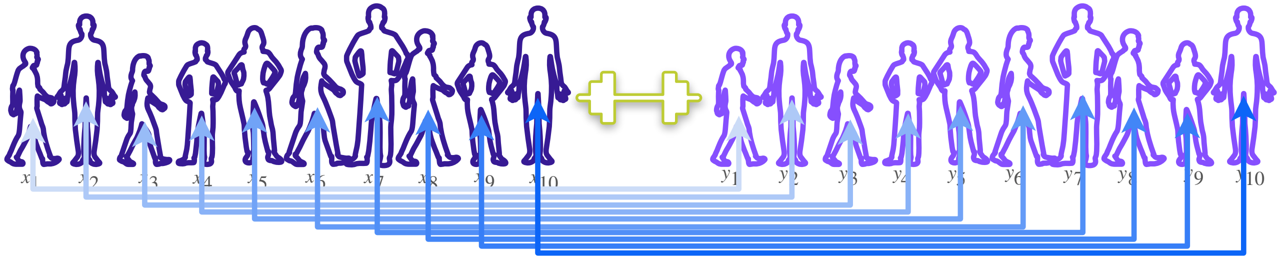
But σ_D is unknown

Paired t-Test: Statistic



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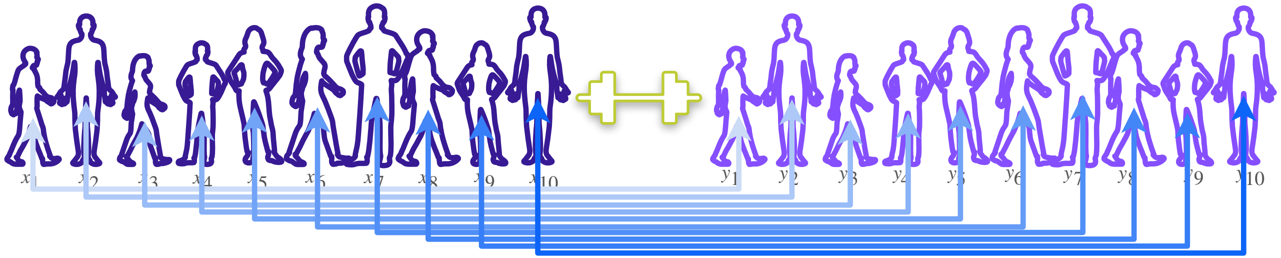
Paired t-Test: Statistic



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$$\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \overline{D})^2}{10-1}} \Rightarrow T = \frac{\overline{D} - \mu_D}{S_D/\sqrt{10}}$$

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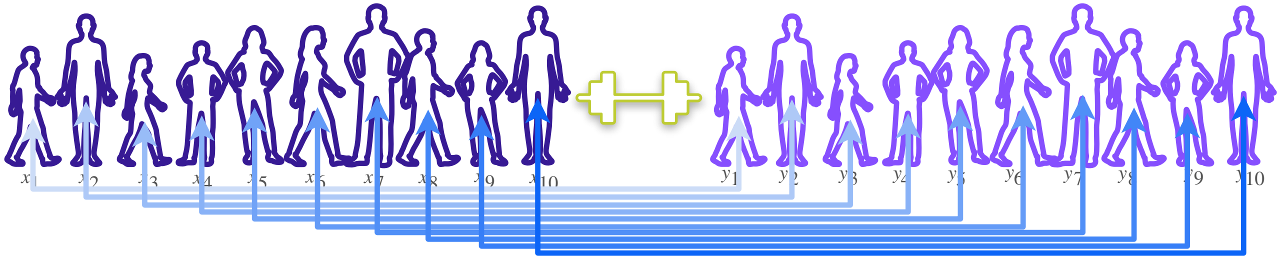

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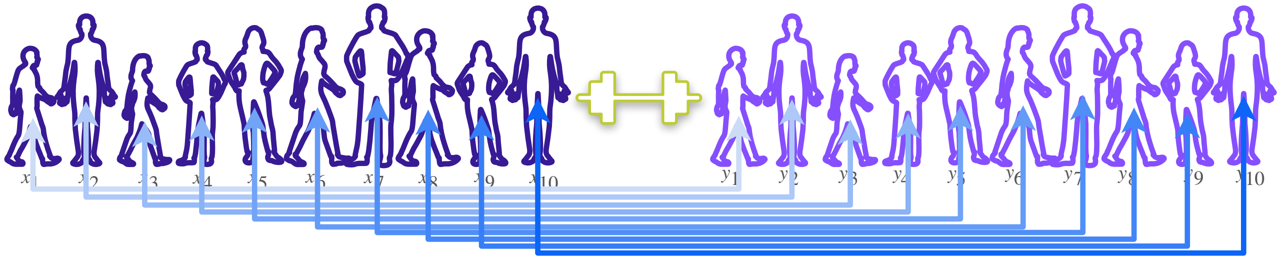
Paired t-Test: Statistic



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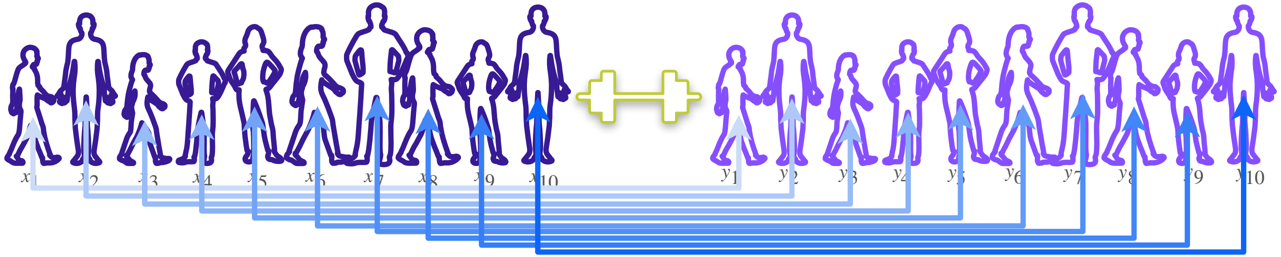
$H_0 : \mu_D = 0$

Paired t-Test: Statistic



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$H_0 : \mu_D = 0$

Test statistic

Paired t-Test: Observations

$=$

d_i

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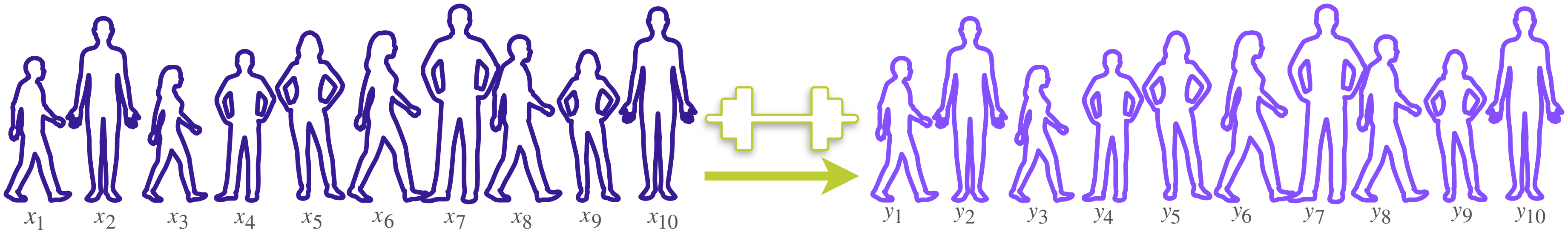
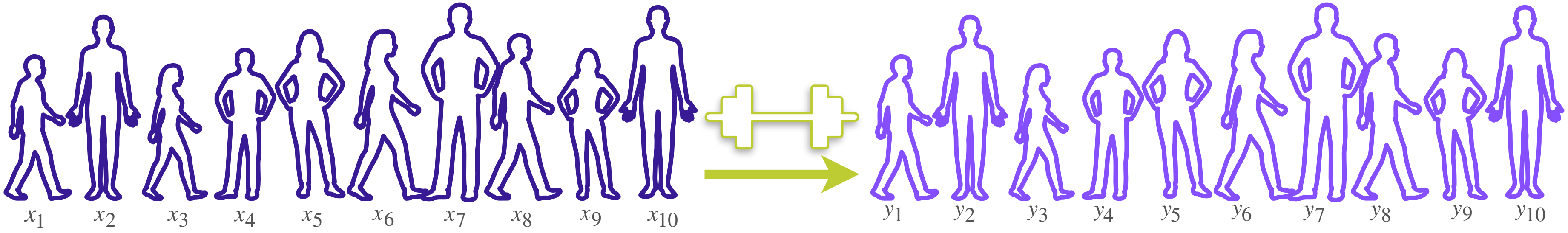
Paired t-Test: Observations



$=$

d_i

Paired t-Test: Observations



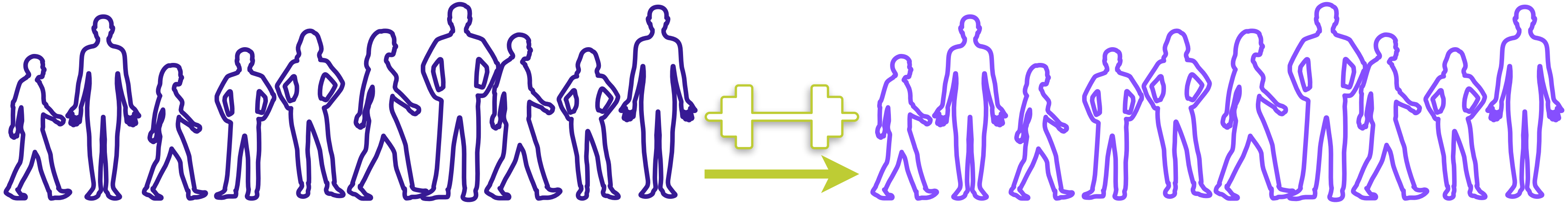
251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

$=$

d_i

Paired t-Test: Observations



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

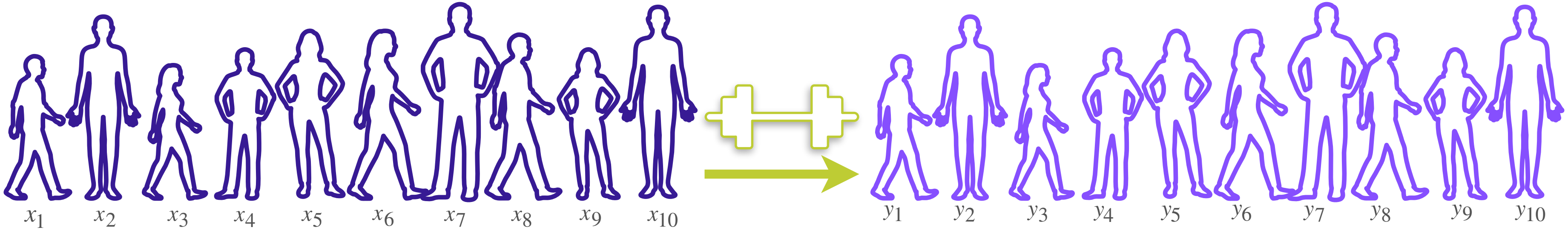
y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

$=$

d_i

Paired t-Test: Observations



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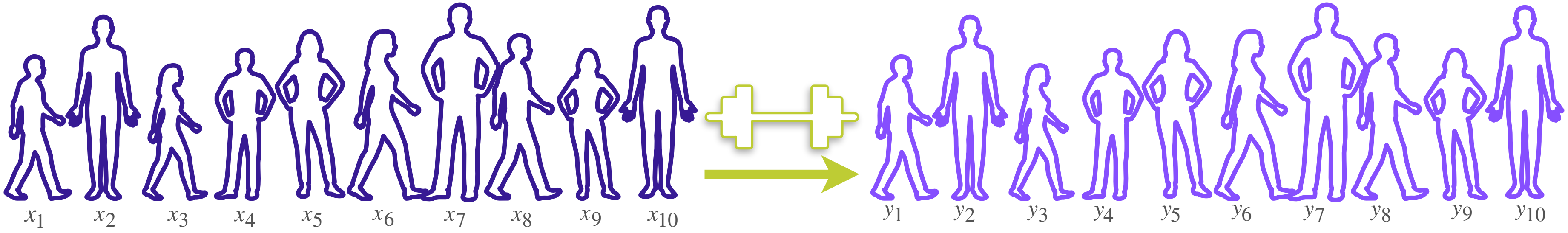
x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0
y_i	251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

$=$

d_i

Paired t-Test: Observations



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x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0
					$-$					
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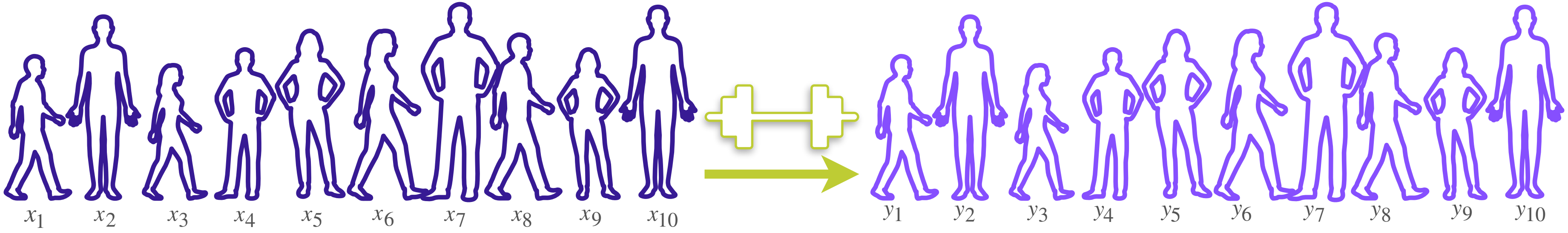
					$=$					
d_i										

Paired t-Test: Observations



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251.5	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

x_i	251.9	166.9	186.6	201.4	154.2	185.4	185.3	116.2	225.4	209.0

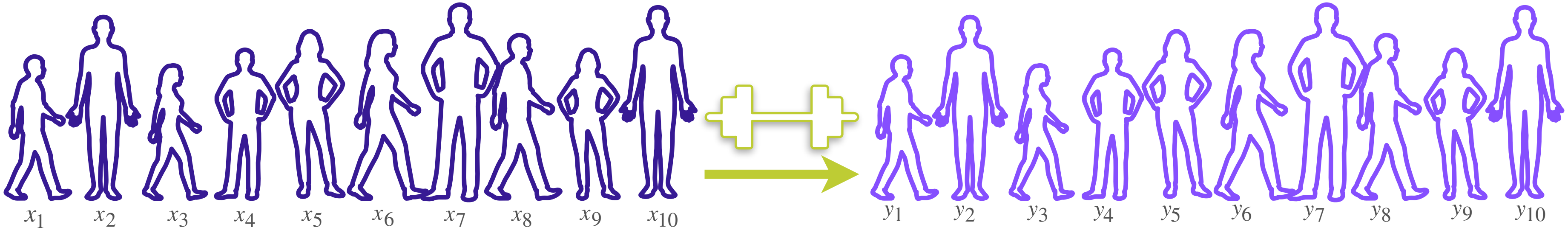
$-$

y_i	251.9	165.6	183.9	200.3	153.0	186.7	182.5	114.3	223.2	210.4

$=$

d_i	0.4

Paired t-Test: Observations



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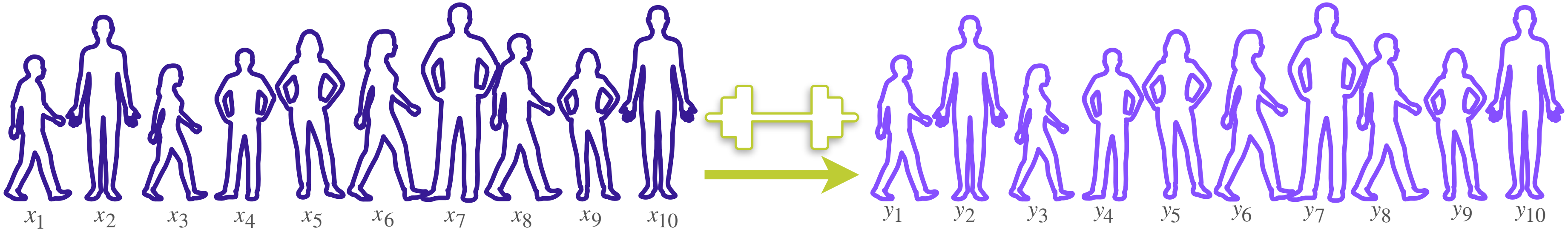
$=$										
d_i	0.4	1.3	2.7	1.1	1.2	-1.3	2.8	1.9	2.2	-1.4

Paired t-Test: Observations



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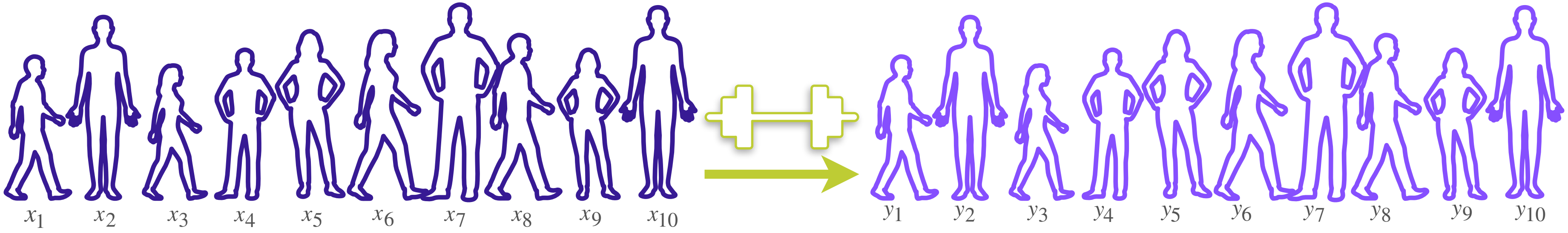
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

Paired t-Test: Observations



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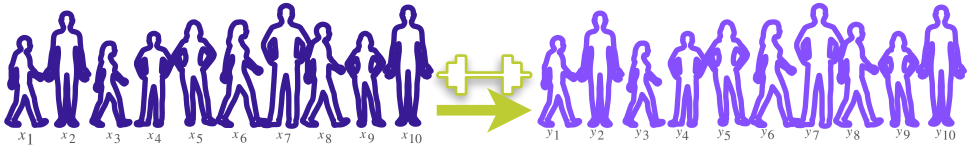
d_i	0.4	1.3	2.7	1.1	1.2	-1.3	2.8	1.9	2.2	-1.4

$$\bar{d} = 1.09$$

$$s_D = 1.485$$

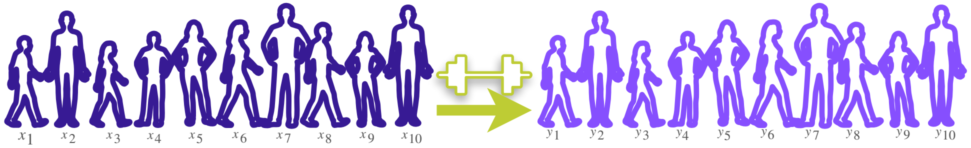
$$t = \frac{1.09 - 0}{1.485/\sqrt{10}} = 2.321$$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{d} = 1.09$ $\quad s_D = 1.485$ $\quad n = 10$

Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\,=\,2.321}$$

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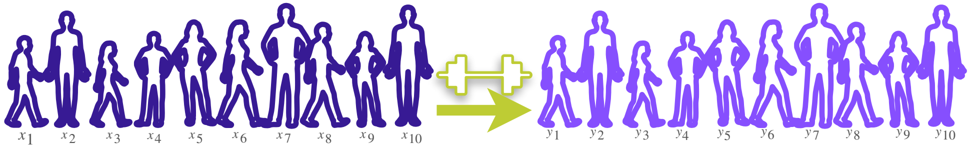

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

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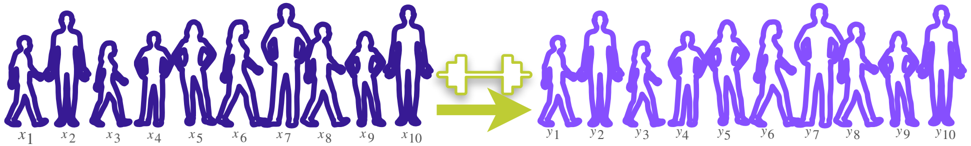
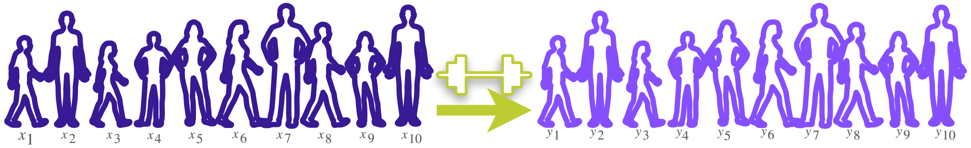


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

Independent Two-Sample t-Test: Right Tailed Test



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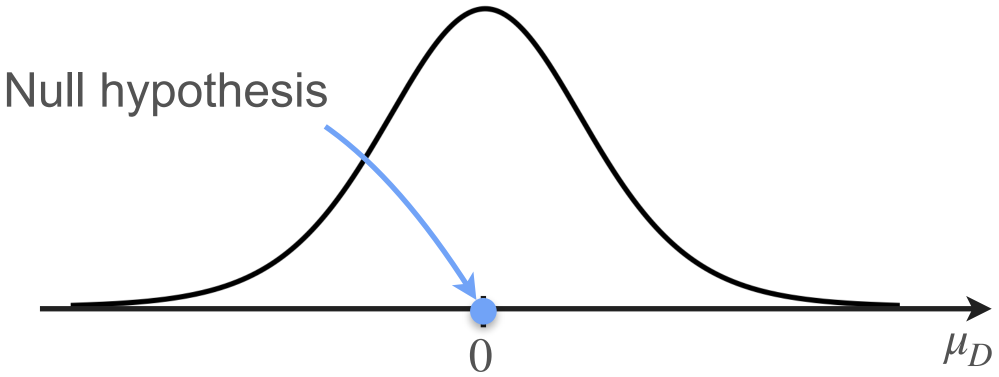
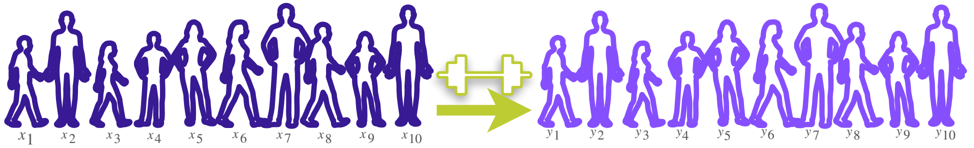
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D/\sqrt{10}} \sim t_{10-1}$



Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

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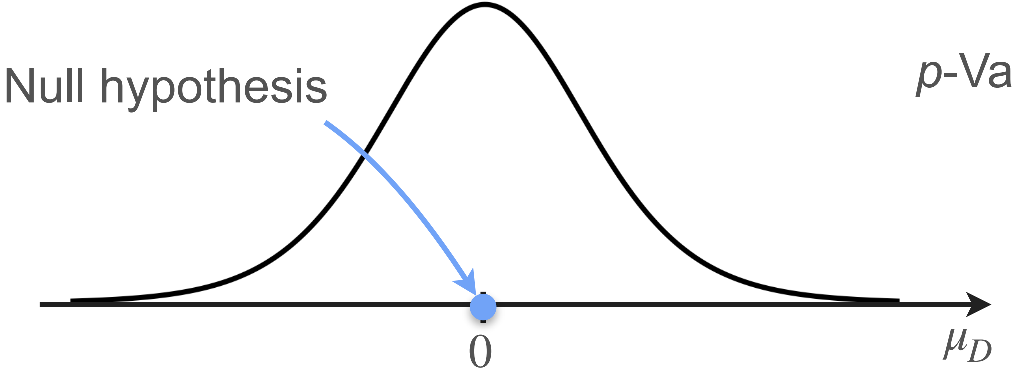
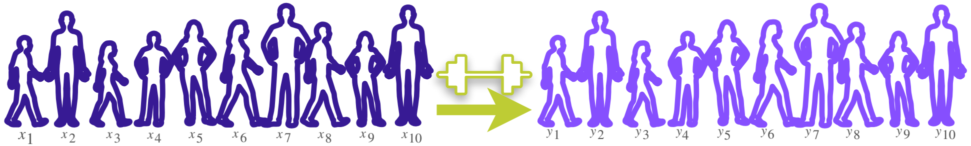


p-Value:

Independent Two-Sample t-Test: Right Tailed Test



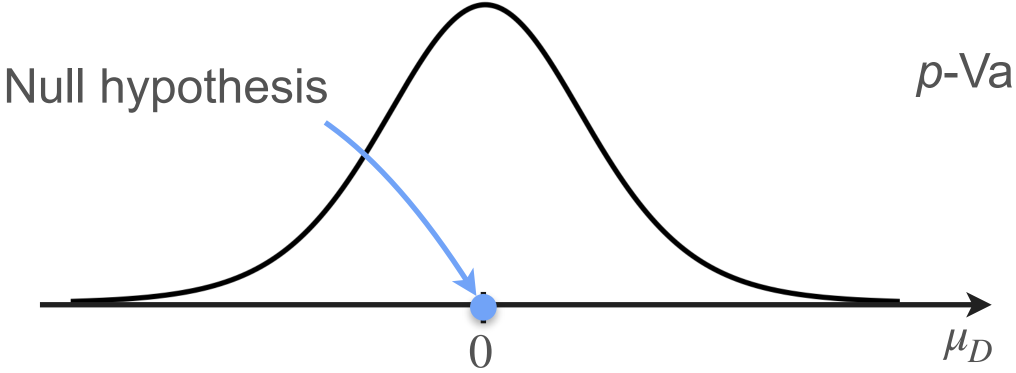
$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\,=\,2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

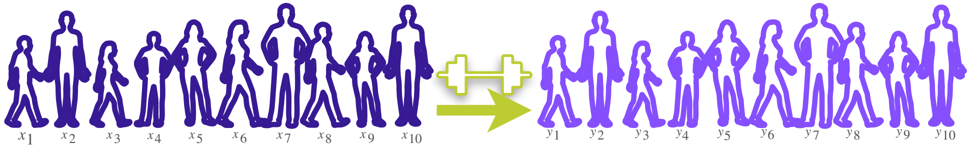
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$

p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$



Independent Two-Sample t-Test: Right Tailed Test



$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t \,=\, 2.321}$$

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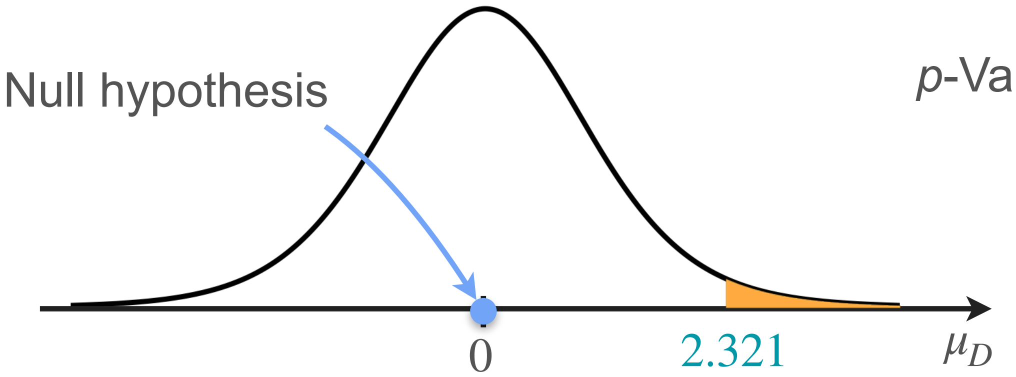
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D/\sqrt{10}} \sim t_{10-1}$



p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$

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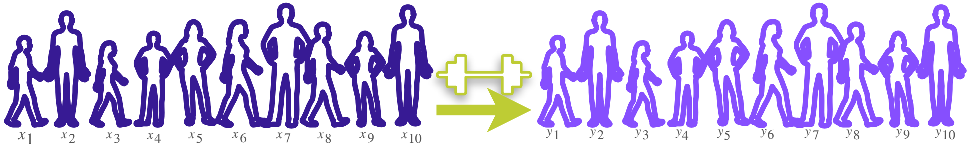
Independent Two-Sample t-Test: Right Tailed Test

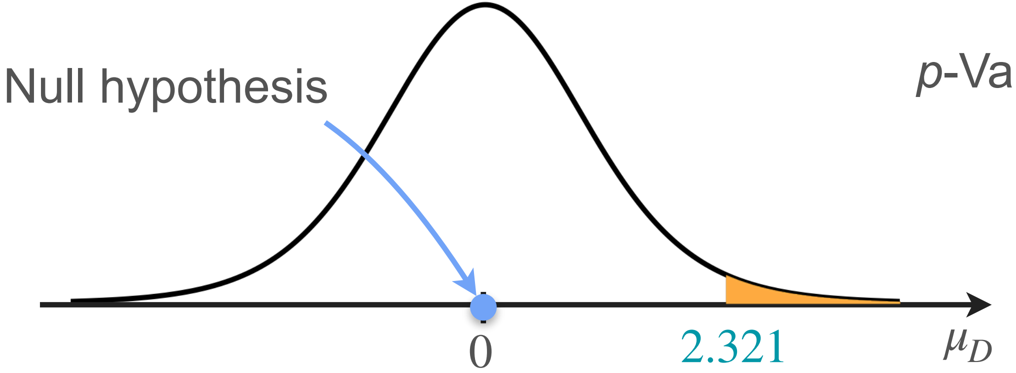


$$\underbrace{\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10}_{t\ =\ 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

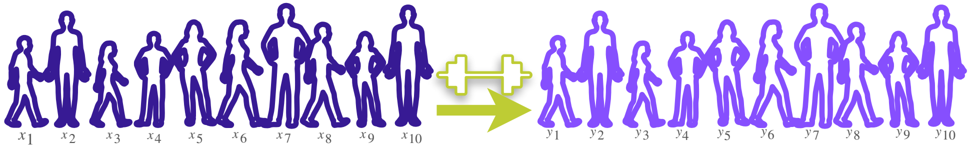
If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D/\sqrt{10}} \sim t_{10-1}$



p-Value: $\mathbf{P}\left(T > 2.321 \mid \mu_D = 0\right)$

$= 0.0227$

Independent Two-Sample t-Test: Right Tailed Test



$\bar{d} = 1.09 \qquad s_D = 1.485 \qquad n = 10$

$$\underbrace{\qquad\qquad\qquad\qquad}_{t \,=\, 2.321}$$

$H_0 : \mu_D = 0$ vs. $H_1 : \mu_D > 0$

$\alpha = 0.05$

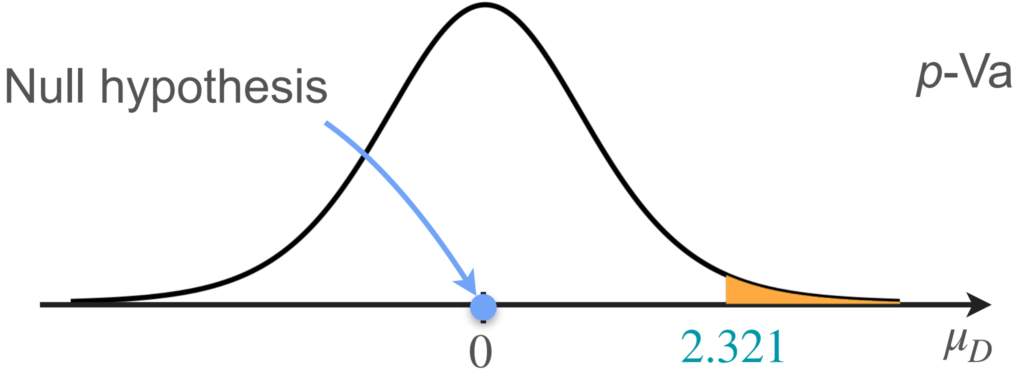
If H_0 is true: $T = \dfrac{\overline{D} - 0}{S_D/\sqrt{10}} \sim t_{10-1}$

p-Value: $\mathbf{P}\left(T > 2.321 \;\middle|\; \mu_D = 0\right)$

$= 0.0227 < 0.05$

$\Rightarrow$ Reject H_0 (and accept H_1)

(with a 5% significance level)



Hypothesis Testing

ML Application: A/B testing

A/B Testing: Purchase Amount

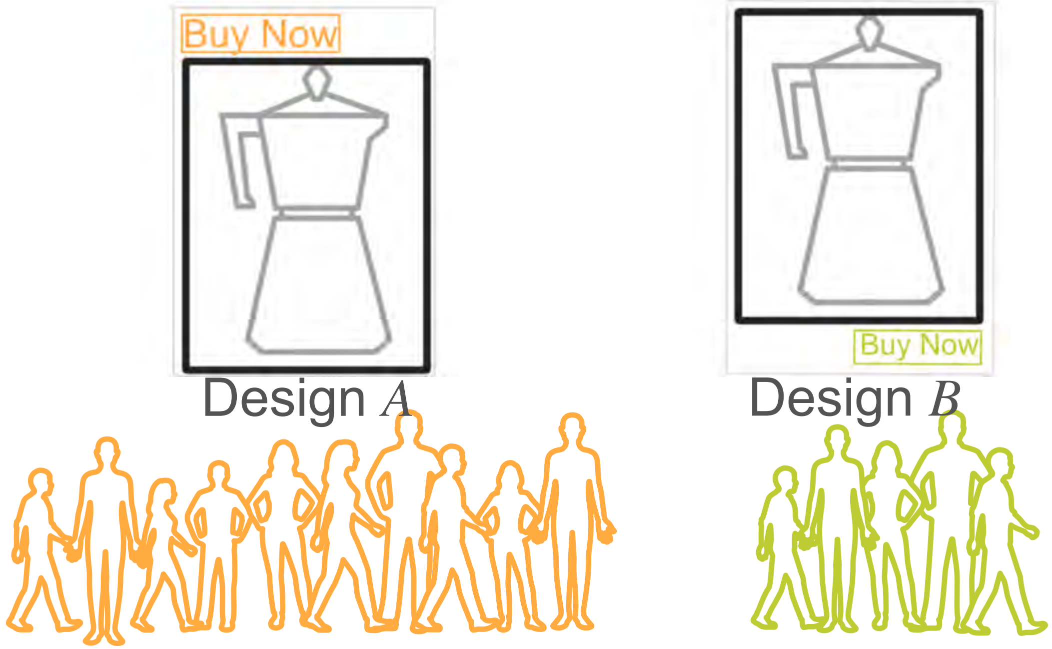
A/B Testing: Purchase Amount

Design A

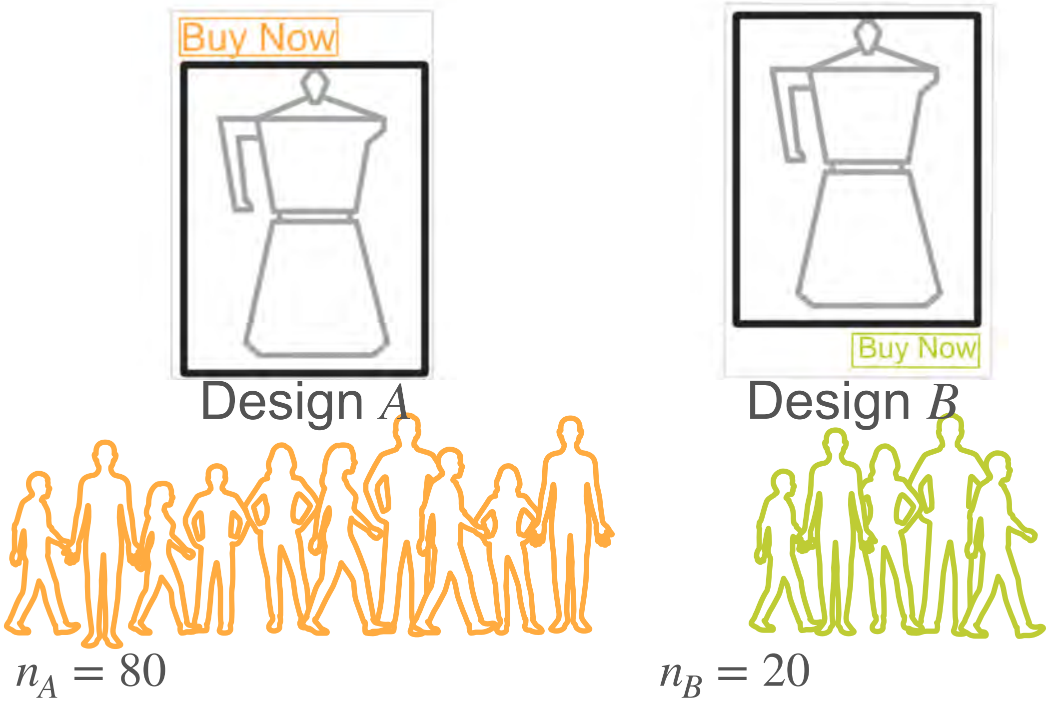
Design B

A/B Testing: Purchase Amount

A/B Testing: Purchase Amount



A/B Testing: Purchase Amount



A/B Testing: Purchase Amount

$n_A = 80$ $\bar{x} = 50$ $n_B = 20$

$s_A = 10$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\bar{x} = 50$

$s_A = 10$

$n_B = 20$ $\bar{y} = 55$

$s_B = 15$

A/B Testing: Purchase Amount

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\;^2}{\quad} + \frac{\;^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$H_0 : \mu_A = \mu_B$ vs. $H_1 : \mu_A < \mu_B$

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

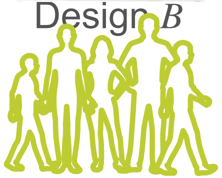
A/B Testing: Purchase Amount

Design A

$n_A = 80$ $\bar{x} = 50$

$s_A = 10$

Design B



$n_B = 20$ $\bar{y} = 55$

$s_B = 15$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\quad^2}{\quad} + \frac{\quad^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

$n_A = 80$

$\bar{x} = 50$

$s_A = 10$

Design B

$n_B = 20$

$\bar{y} = 55$

$s_B = 15$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

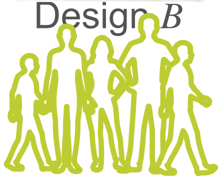
$\alpha = 0.05$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\ ^2}{\quad} + \frac{\ ^2}{\quad}}}$$

A/B Testing: Purchase Amount

Design A

Design B



$n_A = 80$ $\quad \bar{x} = 50$ $\quad n_B = 20$ $\quad \bar{y} = 55$

$s_A = 10$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\quad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(\quad - \quad) - 0}{\sqrt{\frac{\;^2}{\;} + \frac{\;^2}{\;}}}$$

A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\qquad \bar{x} = 50$ $\qquad n_B = 20$ $\qquad \bar{y} = 55$

$s_A = 10$ $\qquad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

A/B Testing: Purchase Amount

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

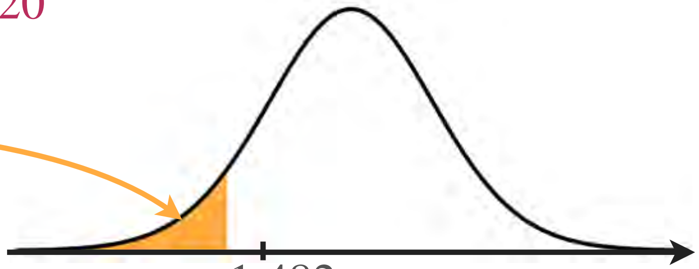
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

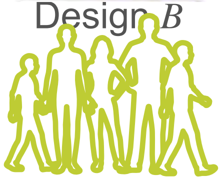
p-Value:



A/B Testing: Purchase Amount

Design A

Design B



$n_A = 80$ $\quad \bar{x} = 50$ $\quad n_B = 20$ $\quad \bar{y} = 55$

$s_A = 10$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\quad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

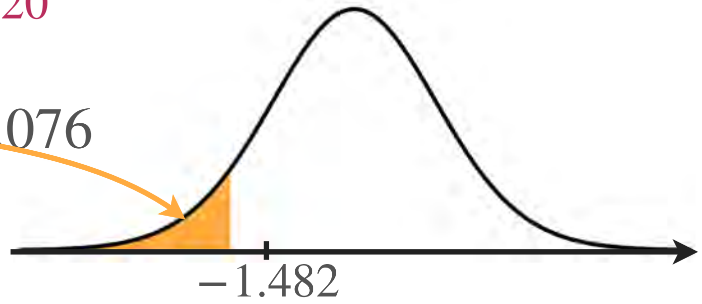
$\alpha = 0.05$

If H_0 is true: $T = \dfrac{\left(\overline{X} - \overline{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\dfrac{10^2}{80} + \dfrac{15^2}{20}}}$$

-1.482

p-Value: 0.076



A/B Testing: Purchase Amount

Design A

Design B

$n_A = 80$ $\quad \bar{x} = 50$ $\quad s_A = 10$

$n_B = 20$ $\quad \bar{y} = 55$ $\quad s_B = 15$

$X \sim \mathcal{N}\left(\mu_A, \sigma_A^2\right)$ $\qquad Y \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$

$H_0 : \mu_A - \mu_B = 0$ vs. $H_1 : \mu_A - \mu_B < 0$

$\alpha = 0.05$

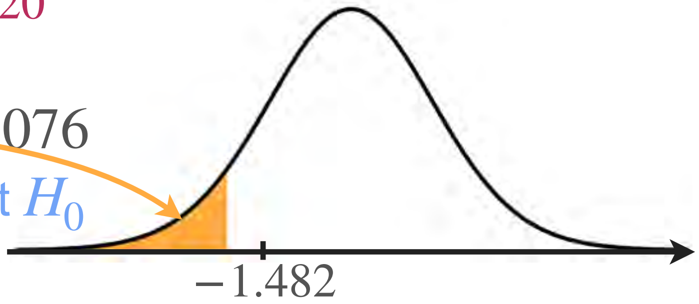
If H_0 is true: $T = \dfrac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\dfrac{S_A^2}{10} + \dfrac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}}$$

-1.482

p-Value: 0.076

Don't reject H_0



A/B Testing and t-Tests

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

Propose variations (A/B)

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

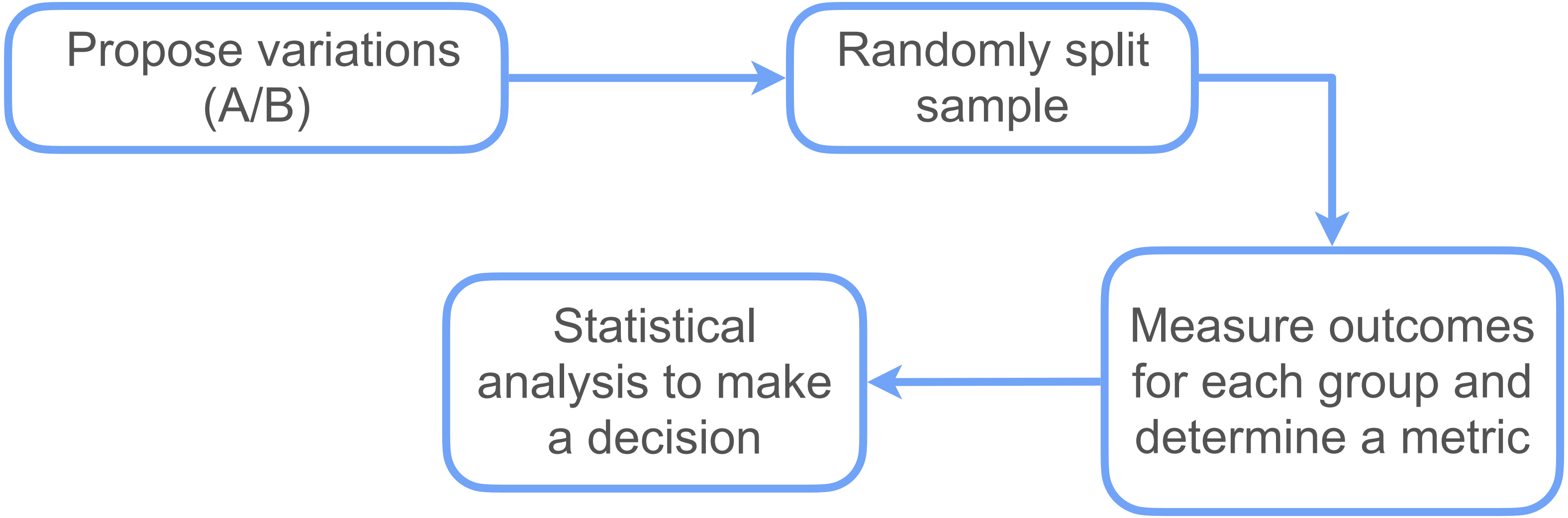
Propose variations (A/B) → Randomly split sample

A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)

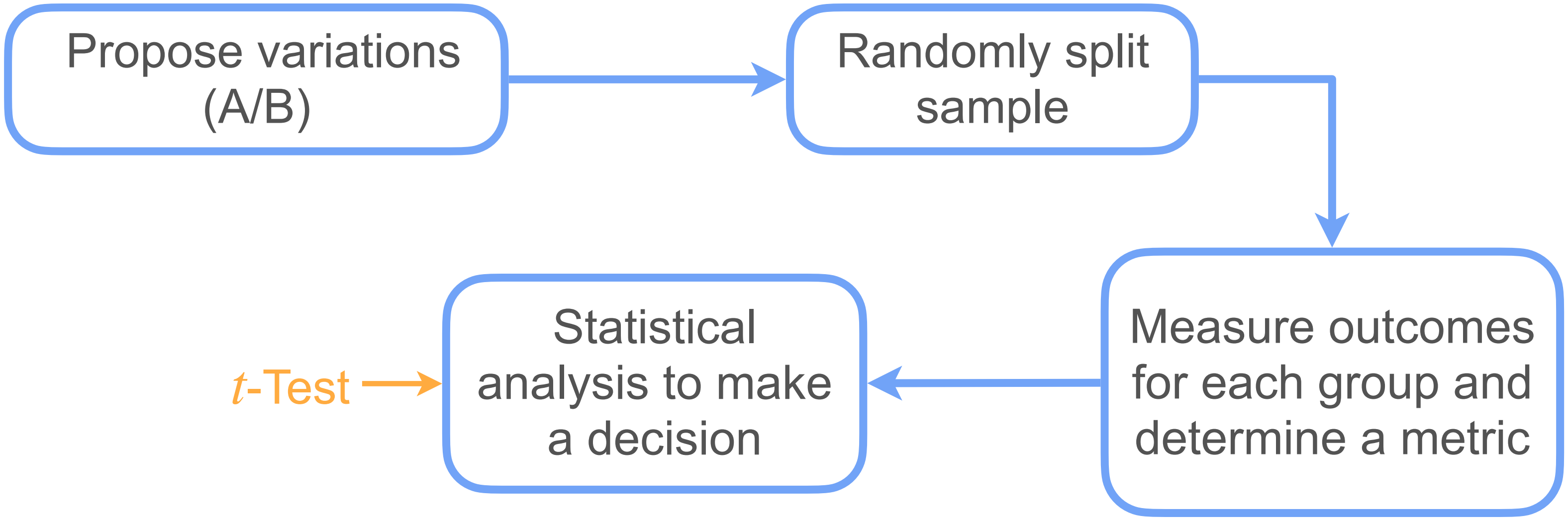
A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing and t-Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Design A

Design B

A/B Testing: Conversion Rates

Design A

Design B

Higher conversion rates?

A/B Testing: Conversion Rates

Higher conversion rates?

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Design A

$n_A = 80$

Design B

$n_B = 20$

A/B Testing: Conversion Rates

Design A

$n_A = 80$ $\quad x = 20$

Design B

$n_B = 20$

A/B Testing: Conversion Rates

Design A

Design B

$n_A = 80$ $x = 20$

$n_B = 20$ $y = 8$

A/B Testing: Conversion Rates

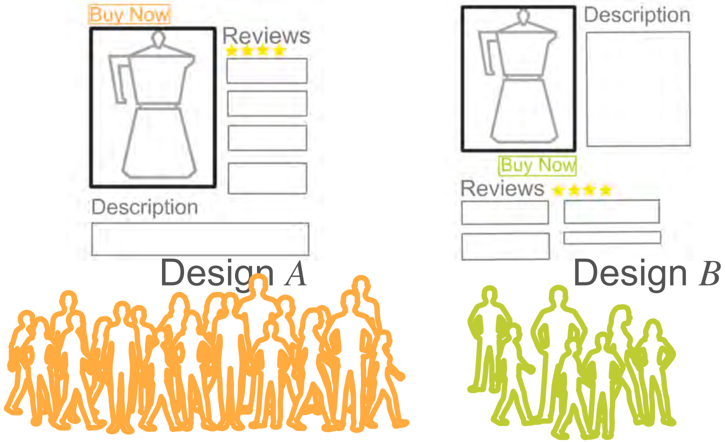
$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

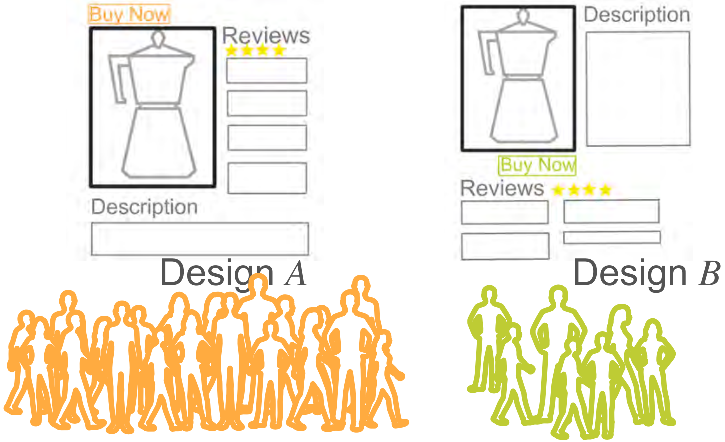
$y = 8$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

p_A = Conversion rate from Design A

p_B = Conversion rate from Design B

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

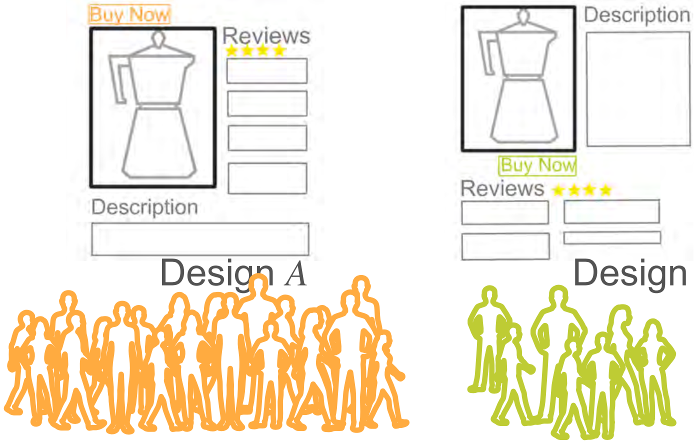
$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$p_A =$ Conversion rate from Design A

$p_B =$ Conversion rate from Design B

$\alpha = 0.05$

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

p_A = Conversion rate from Design A

p_B = Conversion rate from Design B

$\alpha = 0.05$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Statistic?

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \to p_A$$

$$\frac{Y}{n_B} \to p_B$$

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \to p_A \quad \xrightarrow{\hspace{3cm}} \quad \frac{X}{n_A} \sim \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

C.L.T.

$$\frac{Y}{n_B} \to p_B \quad \xrightarrow{\hspace{3cm}} \quad \frac{Y}{n_B} \sim \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{aligned}\frac{X}{n_A} &\overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right) \\[2em] \frac{Y}{n_B} &\overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)\end{aligned}\right\}$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{array}{l} \dfrac{X}{n_A} \overset{a}{\sim} \mathcal{N}\left(p_A, \dfrac{p_A(1-p_A)}{n_A}\right) \\[3em] \dfrac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_B, \dfrac{p_B(1-p_B)}{n_B}\right) \end{array}\right\} \dfrac{X}{n_A} - \dfrac{Y}{n_B} \rightarrow p_A - p_B$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{aligned}\frac{X}{n_A} &\overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)\\ \frac{Y}{n_B} &\overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)\end{aligned}\right\}$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}\right)$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \to p_A - p_B$$

A/B Testing: Conversion Rates

Statistic?

$$\left.\begin{aligned} \frac{X}{n_A} &\overset{a}{\sim} \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right) \\ \frac{Y}{n_B} &\overset{a}{\sim} \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right) \end{aligned}\right\} \frac{X}{n_A} - \frac{Y}{n_B} \to p_A - p_B$$

$$\frac{X}{n_A} - \frac{Y}{n_B} \overset{a}{\sim} \mathcal{N}\left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}\right)$$

$$\Updownarrow$$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - \left(p \quad - p \quad\right)}{\sqrt{\frac{p \; (1 - p \;)}{n_A} + \frac{p \; (1 - p \;)}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0, 1^2\right)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\boxed{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}}} \overset{a}{\sim} \mathcal{N}(0, 1^2)$$

$$= p(1-p)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\boxed{\frac{p \ (1-p \ )}{n_A} + \frac{p \ (1-p \ )}{n_B}}}} \overset{a}{\sim} \mathcal{N}(0, 1^2)$$

$$= p(1-p)\left(\frac{1}{n_A} + \frac{1}{n_B}\right) = p(1-p)(n_A + n_B)\frac{1}{n_A n_B}$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \quad (1 - p \quad)}{n_A} + \frac{p \quad (1 - p \quad)}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0, 1^2\right)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \ (1 - p \ )}{n_A} + \frac{p \ (1 - p \ )}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0, 1^2\right) \longrightarrow$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \ (1-p \ )}{n_A} + \frac{p \ (1-p \ )}{n_B}}} \overset{a}{\sim} \mathcal{N}\left(0,1^2\right) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}}\sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}\left(0,1^2\right)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \;(1-p\;)}{n_A} + \frac{p \;(1-p\;)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}(0,1^2)$$

But you don't know p

$\downarrow$

Replace it by estimation! $\hat{p} = \dfrac{X+Y}{n_A + n_B}$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p \quad - p \quad)}{\sqrt{\frac{p \;(1-p\;)}{n_A} + \frac{p \;(1-p\;)}{n_B}}} \overset{a}{\sim} \mathcal{N}(0,1^2)$$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}}\sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}(0,1^2)$$

Test statistic

But you don't know p

Replace it by estimation! $\hat{p} = \dfrac{X+Y}{n_A + n_B}$

$$\boxed{\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}}\sqrt{n_A n_B} \overset{a}{\sim} \mathcal{N}(0,1^2)}$$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Design A

Design B

A/B Testing: Conversion Rates

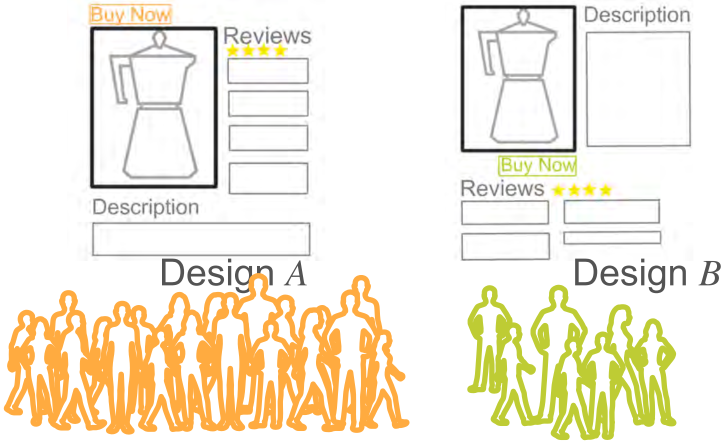
Design A

Design B

$$n_A = 80$$

$$x = 20$$

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

A/B Testing: Conversion Rates

Design A

Design B

$$n_A = 80$$
$$x = 20$$

$$n_B = 20$$
$$y = 8$$

$$X \sim Binomial(n_A, p_A) \quad Y \sim Binomial(n_B, p_B)$$

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

Design B

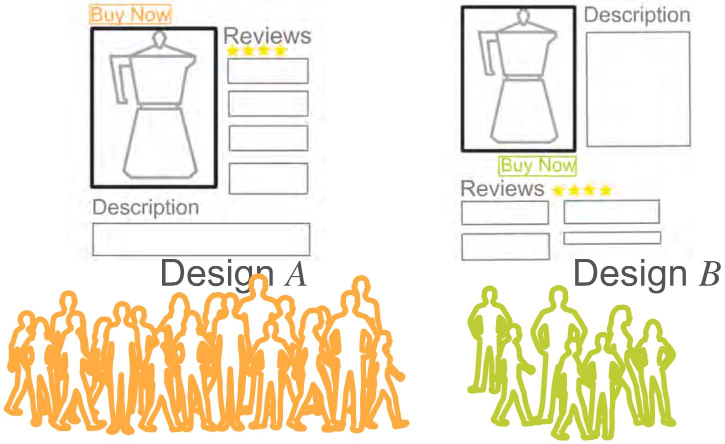
$n_B = 20$

$y = 8$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

A/B Testing: Conversion Rates



$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

A/B Testing: Conversion Rates

Design A

Design B

$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A + n_B}\right)}}\sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

Design A

Design B

$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

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$$z = \frac{\left(\frac{20}{80} - \frac{8}{20}\right) - 0}{\sqrt{(20 + 8)\left(1 - \frac{20 + 8}{80 + 20}\right)}}\sqrt{80\,20}$$

$z = -1.336$

A/B Testing: Conversion Rates

Design A

Design B

$n_A = 80$

$x = 20$

$n_B = 20$

$y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

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-2.07

A/B Testing: Conversion Rates

$n_A = 80$ $\quad$ $n_B = 20$

$x = 20$ $\quad$ $y = 8$

$X \sim Binomial(n_A, p_A)$ $\quad$ $Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$ $\quad$ If H_0 is true $\Rightarrow p_A = p_B = p$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}\left(0, 1^2\right)$$

$z = -1.336$

A/B Testing: Conversion Rates

Design A Design B

$n_A = 80$ $n_B = 20$

$x = 20$ $y = 8$

$X \sim Binomial(n_A, p_A)$ $Y \sim Binomial(n_B, p_B)$

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$z = -1.336$

p-value

-2.07

A/B Testing: Conversion Rates

Design A

$n_A = 80$

$x = 20$

$X \sim Binomial(n_A, p_A)$

Design B

$n_B = 20$

$y = 8$

$Y \sim Binomial(n_B, p_B)$

$H_0 : p_A - p_B = 0$ vs. $H_1 : p_A - p_B < 0$

$\alpha = 0.05$

If H_0 is true $\Rightarrow p_A = p_B = p$

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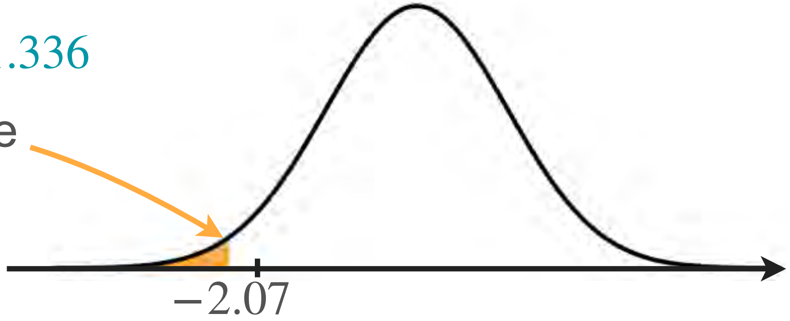
$z = -1.336$

p-value



A/B Testing: Conversion Rates

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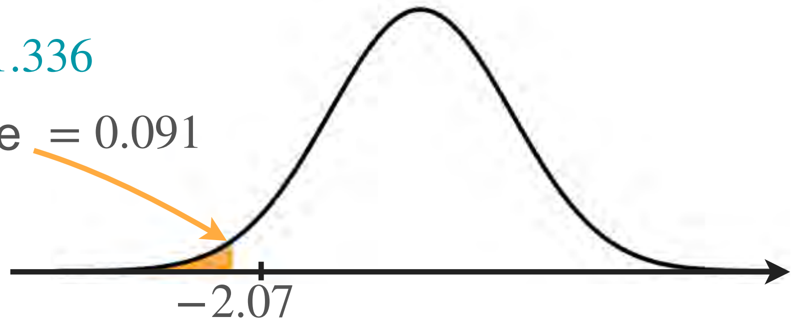
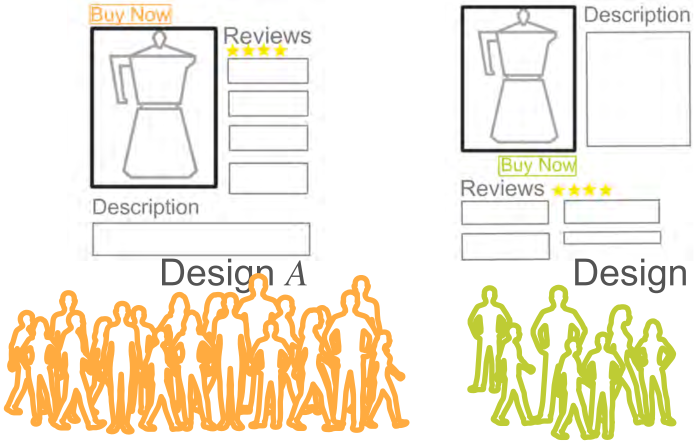
$z = -1.336$

p-value $= 0.091$



A/B Testing: Conversion Rates



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$y = 8$

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If H_0 is true $\Rightarrow p_A = p_B = p$

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$z = -1.336$

p-value $= 0.091$

Do not reject H_0