6

Multiple Access Techniques

Multiple access is a signal transmission situation in which two or more users wish to simultaneously communicate with each other using the same propagation channel. This is precisely the uplink transmission situation in a wireless communications system. In the uplink or reverse channel, multiple users will want to transmit information simultaneously. Without proper coordination among the transmitting users, collisions will occur when two or more users transmit simultaneously. Access methods that incur collision are referred to as random access and variants of random access. This chapter discusses the throughput characteristics of two popular random access methods: Aloha and carrier-sense multiple access (CSMA). Multiple access strategies based on orthogonality among the competing transmissions are collision-free. Orthogonality can be in the form of frequency division, time division or code division. Techniques with built-in conflict resolution capability presented in this chapter are frequency-division multiple access (FDMA), time-division multiple access (TDMA) and code-division multiple access (CDMA). Performance analysis and evaluation of these conflict-free multiple access methods in terms of spectral efficiency and system capacity are described and discussed.

6.1 MULTIPLE ACCESS IN A RADIO CELL

In each radio cell, the transmission from the base station in the downlink can be heard by each and every mobile user in the cell. For this reason, this mode of transmission is referred to as *broadcasting*. On the other hand, transmissions from the mobile users in the uplink to the base station is many-to-one, and is referred to as *multiple access*. Figure 6.1 illustrates the uplink/downlink transmission scenarios.

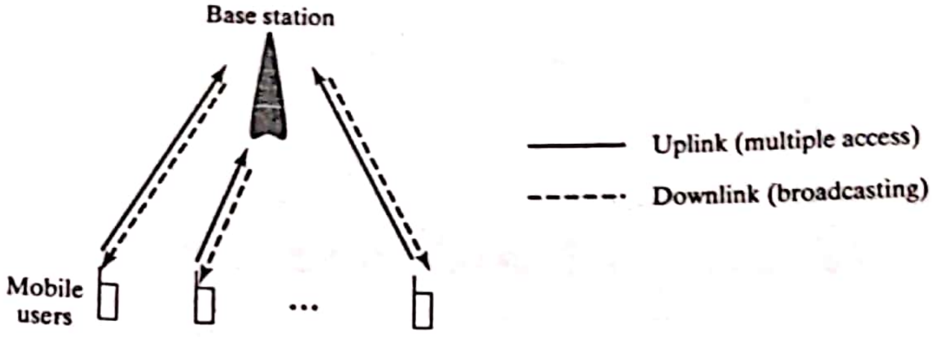

Figure 6.1 The uplink and downlink transmissions.

Transmissions in the uplink have the following attributes:

a. Multiple mobile users want to access the common resource (base station) simultaneously;
b. If the transmissions from two or more users arrive at the base station receiver at the same time, there will be destructive interference, unless the multiple arriving signals are mutually orthogonal;
c. Orthogonality between two signals $x_i(t)$ and $x_j(t)$, $t \in [0, T]$, means that their inner product over the signaling interval vanishes. That is

$$\int_0^T x_i(t)x_j(t)dt = 0, \quad \text{for } i \neq j. \tag{6.1.1}$$

The key element in multiple access is to make the transmitted signals from the different users orthogonal to each other. This raises the fundamental question of how this orthogonality condition should be mechanized.

Conflict-Free Multiple Access. Orthogonality can be mechanized using

a. space division multiple access (SDMA),
b. frequency division multiple access (FDMA),
c. time division multiple access (TDMA), or
d. code division multiple access (CDMA).

In theory, SDMA, FDMA, TDMA and CDMA are conflict-free multiple access techniques. The conflict-free property is achieved through coordination among all the participating users. In the case of SDMA, FDMA and TDMA, the coordination among all participating users is performed through fixed assignment. For example, in FDMA the system bandwidth, B_s, is partitioned into frequency bands and each user is assigned a unique frequency band for information transmission for the entire duration of the connection. CDMA is a spread spectrum technique. Each user is assigned a unique spreading function from a set of wideband orthogonal functions. Based on the orthogonality property given in Eq. (6.1.1), an individual user can transmit using the entire system bandwidth, B_s, during one use of the channel. Thus, FDMA is a narrowband multiple access plan while CDMA is wideband.

Random Access and Variants. In certain situations, depending on the traffic load and mixture, it may be advantageous to employ a non-conflict-free multiple access scheme. In non-conflict-free multiple access, transmissions by the different users are either uncoordinated or are only partially coordinated. A completely uncoordinated scheme is referred to as *random access*. In random access, a user contends for usage of the same resource, independent of any other users. For this reason, random access is also referred to as *contention access*. In a random access scheme, a user transmits whenever it has information to be transmitted, independent of the status of any other users.

In conflict-free multiple access systems, random access is often used for users to gain the initial access to the systems. For example, in GSM systems, there is a random access channel (RACH) among the control channels which provide the necessary control functions. The RACH is used by a mobile user to originate a call or to respond to a paging signal in the reverse link. The RACH uses a slotted Aloha access scheme. In responding to a call request from a mobile user via the RACH, the base station allocates a conflict-free channel to the user during the call connection.

6.2 RANDOM ACCESS

While we will not be dwelling much on non-conflict-free multiple access techniques in this chapter, it seems appropriate to briefly study some of the popular methods for non-conflict-free multiple access.

6.2.1 Aloha Systems

Random access was used by a research group from the University of Hawaii in the late 1960s and early 1970s for its satellite communications with the U.S. mainland at a transmission speed of 50 kbps [1, 2]. This system was called Aloha and the term Aloha has been used as a general name for random access. Aloha is a packet-switching system. The time interval required to transmit one packet is called a *slot*. When transmissions from two or more users overlap, they destroy each other, whether it is complete overlap or partial overlap. The maximum interval over which two packets can overlap and destroy each other is called the *vulnerable period*. The mode of random access in which users can transmit at anytime is called *pure Aloha*. In a pure Aloha system, where the packet length is a fixed constant, the vulnerable period is two slot times. A version in which users are restricted to transmit only from the instant corresponding to the slot boundary is referred to as *slotted Aloha*. The alignment of transmissions to coincide with the slot boundary means that packets can only experience complete overlap, so that the vulnerable period in slotted Aloha is one slot time. This means that the maximum throughput rate of slotted Aloha doubles that of pure Aloha.

Throughput of Aloha Systems. In the Aloha systems, a user can hear its own transmission or the transmissions by other users within the footprint of the serving satellite. A transmitting user, upon hearing a collision, backs off for a random delay interval and transmits again, until success is achieved. The transmission is successful when there are no other packet transmissions during the vulnerable period. Thus, the probability of successful transmission is defined as

$$\text{P[success]} = \text{P[no other packet transmission occurs within a vulnerable period].} \tag{6.2.1}$$

Let S be the throughput, defined as the successfully transmitted traffic load, and G be the total offered channel traffic load. Assuming that traffic generated for transmission obeys a Poisson distribution, then

$$\text{P[no other packet transmission occurs]} = e^{-\tau G}, \tag{6.2.2}$$

where τ is the vulnerable period. Using Eq. (6.2.2) in Eq. (6.2.1), we have

$$\text{P[success]} = e^{-\tau G}. \tag{6.2.3}$$

By definition, we also have

$$\text{P[success]} = \frac{S}{G}. \tag{6.2.4}$$

Combining Eqs. (6.2.3) and (6.2.4), we have the throughput equation of the Aloha systems given by

$$S = Ge^{-\tau G}. \tag{6.2.5}$$

From Eq. (6.2.5), we note that $S \longrightarrow 0$ in the limit as $G \longrightarrow \infty$. That is, the negative exponential decays faster than G increases.

The maximum value of S occurs when the slope of the throughput curve is 0 (i.e., $\frac{dS}{dG} = 0$). Taking the derivative of the right-hand side of Eq. (6.2.5) with respect to G and setting the result equal to zero, we get $G = 1/\tau$. For pure Aloha, the vulnerable period is $\tau = 2$ slots, while for slotted Aloha, it is $\tau = 1$ slot. The maximum throughput, for pure Aloha, is therefore

$$S_{\max} = \frac{1}{2e} \approx 0.184, \tag{6.2.6}$$

and that for slotted Aloha is

$$S_{\max} = \frac{1}{e} \approx 0.368. \tag{6.2.7}$$

Delay Throughput Characteristics of Aloha. Intuitively, the more traffic one tries to push through a system, the longer it will take to get through the system. That is, delay and throughput have paradoxical requirements. The delay experienced by a packet in the system is measured from the instant of the packet's arrival until the instant the sender receives confirmation. The packet delay is thus a function of the number of transmissions, the retransmission delay, and the time required for the sender to receive confirmation of successful transmission. Let

R be the number of slots to receive an acknowledgment,
$\bar{D}_{ret}$ be the mean retransmission delay, and
E be the mean number of transmissions until success.

The mean number of transmissions until success, E, is then

$$E = \frac{G}{S} = e^{\tau G}.$$

Hence, the average packet delay, $\bar{D}$, is given by

$$\bar{D} = R + (E - 1)(R + \bar{D}_{ret}) \tag{6.2.8}$$

for pure Aloha, and

$$\bar{D} = R + 0.5 + (E - 1)(\lceil R \rceil + \bar{D}_{ret}) \tag{6.2.9}$$

for slotted Aloha. In both Eqs. (6.2.8) and (6.2.9), R represents the time taken for the initial transmission and confirmation, and $(E - 1)$ represents the mean number of retransmissions. In Eq. (6.2.9), the term 0.5 represents the fact that, on average, an arrival is 1/2 slot time to the slot boundary, and the symbol $\lceil x \rceil$ denotes the smallest integer equal to or greater than x. Substituting $\tau = 2$ for pure Aloha and $\tau = 1$ for slotted Aloha, we have

$$\bar{D}_{pure} = R + (e^{2G} - 1)(R + \bar{D}_{ret}) \tag{6.2.10}$$

and

$$\bar{D}_{slotted} = R + 0.5 + (e^{G} - 1)(\lceil R \rceil + \bar{D}_{ret}). \tag{6.2.11}$$

In a random access environment, when two or more users transmit packets simultaneously, collisions will take place. An efficient scheme to resolve collisions will help to improve system throughput. One approach to constructing a collision resolution algorithm is by means of a *tree protocol* to resolve collisions using a divide-and-conquer approach. We discuss the tree collision resolution algorithm using the following example.

Example 6.1 Tree Protocol for Collision Resolution

Consider the situation in which there are 8 users in the radio cell. Suppose we number the users from 0 to 7. At a given epoch, users 0, 1, 2, 5 and 7 have packets ready for transmission, while users 3, 4 and 6 are idle. When the ready users transmit simultaneously, collisions will occur. The interval of time within which the collisions are resolved is referred to as the collision resolution interval (CRI). In the divide-and-conquer approach, the binary tree is divided into two halves; one half is searched to completion and then the other half is searched to resolve collisions. Collision resolution can be performed on a per collision resolution interval basis.

The channel states are described by the 3-tuple {idle, collision, success}. Assume that the users can detect the channel states.

a. Draw a binary tree showing that the users are located at the leaves of the tree, and label the intermediate nodes using the letters A through G, with A representing the root node.
b. Using the tree structure, describe, with diagrammatic illustration, a static approach to resolve collisions. What is the length of the collision resolution interval?

Solution

a. With 8 users, the binary tree has 4 levels, with the root at level 0 and the leaves at level 3, as shown in Figure 6.2, where ready users are indicated by the symbol ⊙.
b. Consider that, at the end of the $(i-1)$th collision resolution interval, and hence the start of the ith collision resolution interval, users 0, 1, 2, 5 and 7 are ready for packet transmission. At the root node A (level 0), collisions occur. Split the tree into two halves and resolve the left half first. Then at intermediate node B (level 1), users 0, 1, and 2 transmit, resulting in collision. Divide the subtree into two halves and search the left half. Users 0 and 1 transmit and collision occurs. Further divide the subtree, with node D as the root, into two halves. Since each of the halves only has the leaf node, transmissions by user 0, and then by user 1, will both be successful. This completes the search of the left half of the subtree with node D as the root. The algorithm next searches the right half of the subtree with node B as the root. The procedure is repeated until the entire tree has been searched and all ready users have successfully transmitted.
The ith collision resolution interval (CRI_i) is illustrated in Figure 6.3. The length of CRI_i is 9.

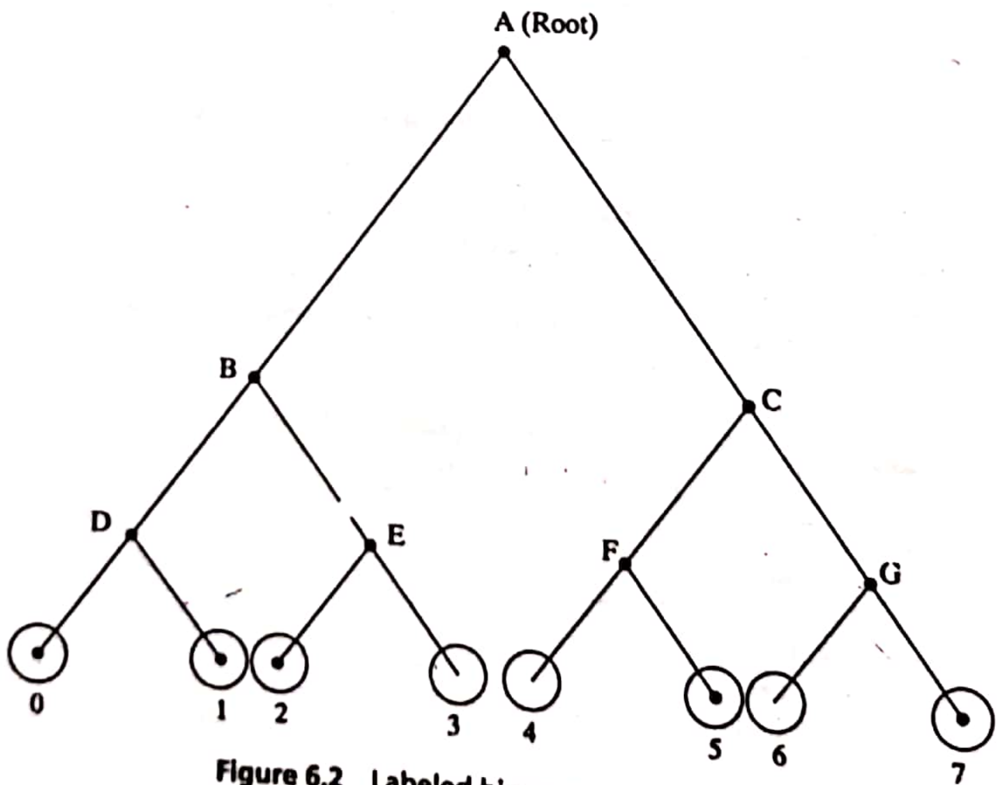

Figure 6.2 Labeled binary tree for 8 users.

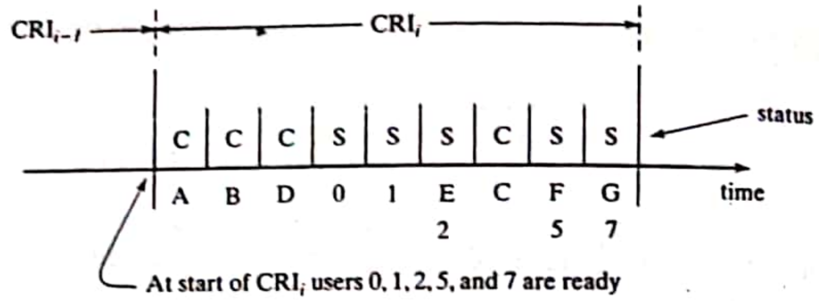

Figure 6.3 Collision resolution interval.

6.2.2 Carrier Sense Multiple Access (CSMA)

In CSMA, users listen before transmission. This listening is referred to as sensing the channel. If a user senses the channel idle, it transmits. Otherwise, the ready user takes one of the following actions [73]:

a. Defers transmission and tries again after a random delay. This mode of retry is called non-persistent CSMA;
b. Waits until the channel becomes idle and then transmits again. This mode of retry is called 1-persistent CSMA.

Throughput of CSMA. Here we derive the throughput for unslotted CSMA. To facilitate the derivation, we quantify the parameters as follows. For fixed packet lengths, the packet transmission time is one slot. The propagation delay between two users is τ s, and the normalized propagation delay, a, is given by

$$a = \frac{\tau}{l/C} = \frac{\tau C}{l},$$

where C is the channel capacity in bits/s and l is the packet length in bits. The status of the channel alternates with busy and idle periods. Let

B_n be the busy period in the nth cycle,
I_n be the idle period in the nth cycle, and
U be the time that the channel is used without collision.

Let $\bar{X}$ denote the mean value of X. The success rate S is then given by

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}}. \tag{6.2.12}$$

The parameters $\bar{U}$, $\bar{I}$ and $\bar{B}$ are determined as follows. Let

$$\begin{aligned} P_s &= P[\text{a busy period has a single packet transmission}] \\ &= P[\text{no transmission in normalized interval } a] \\ &= e^{-aG}. \end{aligned} \tag{6.2.13}$$

The mean time that the channel is used without collision is given by

$$\begin{aligned}\bar{U} &= \text{packet transmission time} \times P_s \\ &= 1 \times P_s.\end{aligned}$$

The mean idle period is $\bar{I} = 1/G$. It remains to determine $\bar{B}$.

A busy period can have multiple packet transmissions. Let Y be the time between the first and last packet transmissions in a busy period. $Y = 0$ means that there is only a single packet transmission in the busy period so that the transmission is successful. If Y is equal to or longer than the normalized propagation delay a, there is no collision. Collisions take place when $0 < Y < a$. For this case, the cumulative distribution function of Y is

$$\begin{aligned}F_Y(y) &= P[Y \leq y] \\ &= P[\text{no packet arrives in } (y, a)] \\ &= e^{-(a-y)G} \qquad 0 \leq y \leq a.\end{aligned} \tag{6.2.14}$$

The probability density function of Y, $f_Y(y)$, is obtained by differentiating Eq. (6.2.14)

$$f_Y(y) = e^{-aG}\delta(y) + Ge^{-(a-y)G}, \quad 0 \leq y \leq a. \tag{6.2.15}$$

The mean value of Y is thus given by

$$\begin{aligned}\bar{Y} &= \int_0^{\infty} y f_Y(y) dy \\ &= a - \frac{1 - e^{-aG}}{G}.\end{aligned} \tag{6.2.16}$$

The mean busy period is given by the sum of the packet transmission time, the normalized propagation delay, and the mean interval between the first and last packet transmissions instants. That is

$$\bar{B} = 1 + a + \bar{Y}. \tag{6.2.17}$$

The throughput for unslotted non-persistent CSMA is obtained by substitution of $\bar{U}$, $\bar{I}$ and $\bar{B}$ in Eq. (6.2.12), yielding

$$S_{np-CSMA} = \frac{Ge^{-aG}}{G(1+2a) + e^{-aG}}. \tag{6.2.18}$$

In the limit when $a \longrightarrow 0$, $S \longrightarrow G/(1+G)$. The throughput for unslotted 1-persistent CSMA can be derived in a similar manner.

$$S_{1p-CSMA} = \frac{Ge^{-G(1+2a)}[1 + G + aG(1 + G + aG/2)]}{G(1+2a) - (1 - e^{-aG}) + (1 + aG)e^{-G(1+a)}}. \tag{6.2.19}$$

In the limit as $a \longrightarrow 0$, $S \longrightarrow \frac{Ge^{-aG}(1+G)}{G+e^{-aG}}$. The mean packet delay of CSMA is approximately given by

$$\bar{D}_{CSMA} = R + \left(\frac{G}{S} - 1\right)(R + \bar{D}_r), \tag{6.2.20}$$

where the parameters R and $\bar{D}_r$ are as defined earlier.

6.3 CONFLICT-FREE MULTIPLE ACCESS TECHNOLOGIES

The methods commonly used in mobile wireless cellular systems are FDMA, TDMA and CDMA. As discussed in Chapter 1, the first generation wireless systems use FDMA while those in the second generation use TDMA and CDMA. CDMA is the targeted multiple access technology for the third generation (3G) wireless communications systems. FDMA and TDMA are fixed capacity allocation schemes in that an individual user is assigned a frequency band (in FDMA) or a time slot (in TDMA) for the duration of the connection. With sufficiently well designed filters (in the FDMA case) and slot synchronizers (in the TDMA case), there should be no, or a minimal amount of, spectral overlap (in FDMA) or timing jitter (in TDMA). In this way, FDMA and TDMA would be conflict-free multiple access schemes.

CDMA is a spread spectrum technique. Orthogonality between any pair of transmitted signals in CDMA is based on algebraic properties. However, practically generated wideband spreading functions are not truly orthogonal. The cross correlation between any pair of transmitted signals represents interference. Hence, CDMA is an interference limited multiple access strategy.

6.3.1 FDMA

In FDMA, the total bandwidth is divided into non-overlapping frequency subbands. Each user is allocated a unique frequency subband for the duration of the connection, whether the connection is in an active or idle state. Orthogonality among transmitted signals from different mobile users is achieved by bandpass filtering in the frequency domain. This type of multiple access support is narrowband, and is not suitable for multimedia communications with various transmission rates. In addition, it incurs a waste of bandwidth when the user is in a dormant state.

FDMA is relatively simple to implement. However, the power amplifiers and the power combiners used are nonlinear, and tend to generate intermodulation frequencies, resulting in intermodulation distortion. To minimize the effects of intermodulation distortion, stringent RF filters are required to reject intermodulation distortion. RF filters are heavy, cumbersome, and costly.

To provide interference-free transmissions between the uplink and the downlink channels, the frequency allocations have to be separated by a sufficient amount. The frequency separation can be achieved using two antennas operating at different frequencies, or one antenna with frequency division duplexing. That is, the uplink and downlink channels of FDMA operate at distinctly different frequency bands. Therefore, the channel impairments seen at the cell-site receiver are different from those seen at the receiver of each of the mobile users.

6.3.2 TDMA

In a TDMA system, the channel time is partitioned into frames. The length of a frame is long enough so that every user in service has an opportunity to transmit once per frame. To achieve this, a TDMA frame is further partitioned into time slots. Users have to transmit in their assigned slots from frame to frame. The slot assignment can be fixed or dynamic. If the assigned slot is fixed from frame to frame for the duration of the connection, the users have to synchronize to their respective assigned slots. This mode of TDMA is referred to as synchronous TDMA (STDMA). With packet-switched transmission, it is more efficient to allow a user to transmit only when it has a packet to send. In this case, a user is not assigned a fixed time slot for the duration of its connection. Transmission slots are dynamically assigned from frame to frame. This mode of TDMA is referred to as asynchronous TDMA (ATDMA). In STDMA, the frame length is fixed by the number of users, whether or not they are active. In ATDMA, the frame length varies from frame to frame, depending on the number of active users in the frame. In ATDMA, dynamic assignment of slots is performed through a reservation access procedure. This subsection will discuss the ramifications of both STDMA and ATDMA methods.

STDMA. Depending on the manner in which frequency is allocated, STDMA can be wideband or narrowband. It is called wideband TDMA if the channel time is divided into slots, and an individual user is allowed to use the entire available channel bandwidth to transmit its information in the assigned slot, as shown in Figure 6.4(a) where B_s is the total frequency band allocated to the uplink transmission. It is called narrowband TDMA if the overall bandwidth is first divided into frequency bands, and the channel time corresponding to each frequency band is divided into time slots for packet transmission, as shown in Figure 6.4(b). In this way an individual user can only transmit at a rate governed by the allocated frequency subband.

In STDMA, the channel time is divided into contiguous slots, each of which is long enough to transmit or receive one information unit. STDMA operates on a frame-by-frame basis. A group of N_{slot} slots plus a header and a trailer form a TDMA frame, as illustrated in Figure 6.5. In addition to carrying the information data, a slot also includes other fields, such as trailer bits, synchronization (sync) bits, guard bits, etc. A slot can only be used by one user to transmit or receive during one use of the transmission channel.

With TDMA, the receiver must be able to synchronize to the received signal within a slot time. This means that timing information has to be extracted from the observed signal. A conventional approach to extract timing information is to use matched filtering or correlation detection to achieve synchronization within a time slot.

To provide the required separation between the transmissions in the uplink and downlink channels, TDMA can use TDD or FDD. FDD provides two simplex channels at the same time, while TDD provides two simplex time slots on the same frequency band. The manner in which FDD and TDD provide duplex operation is shown in Figure 6.6. As mentioned previously, in FDD systems, the channel disturbances in the uplink and downlink channels are different. On the other hand, in a TDD system, the uplink and downlink channels operate at the same frequency band. In this case, the cell-site receiver and the user's receiver see approximately the same propagation channel if the channel coherence time is much larger than the frame duration. From Figure 6.6, it

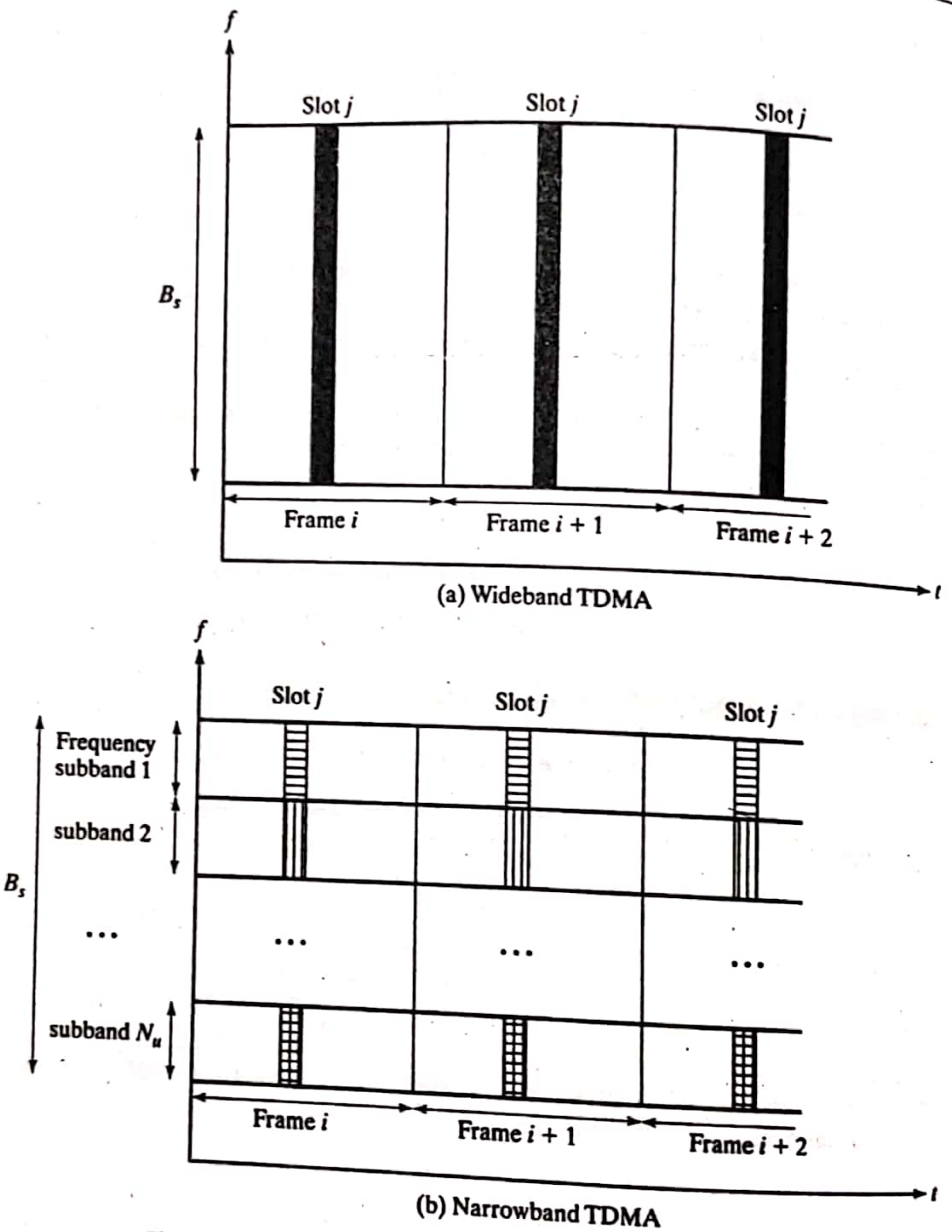

Figure 6.4 Wideband TDMA and narrowband TDMA.

is observed that uplink and downlink propagations are separated by an *off* interval (either in time or in frequency).

In summary, STDMA has a wideband version and a narrowband version. In *wideband TDMA*, transmission in each slot uses the entire frequency band, while in *narrowband TDMA*, since the whole frequency band is divided into subbands, transmission in each slot only uses the frequency width of one subband. The number of contiguous slots during one use of the channel (whether it is the entire band or a subband) constitute a frame.

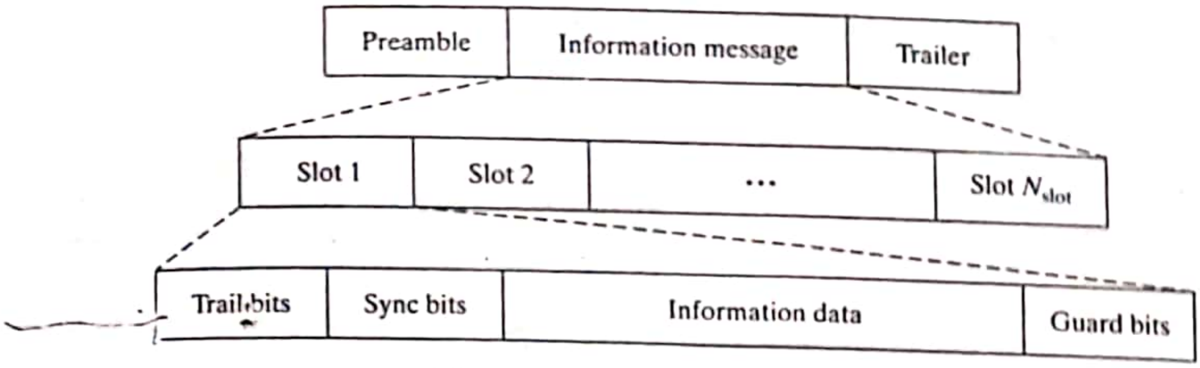

Figure 6.5 TDMA frame structure.

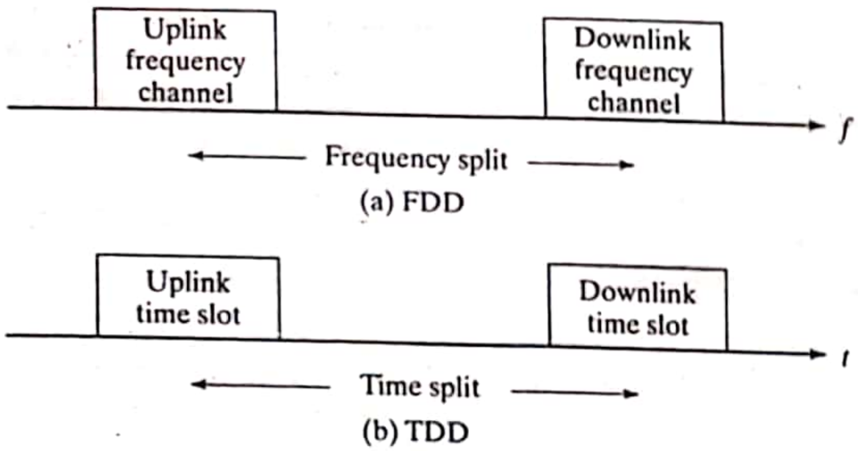

Figure 6.6 FDD and TDD methods for duplex operation.

ATDMA. Asynchronous TDMA is implemented using a reservation access mechanism. An ATDMA frame contains two segments: a leading segment and a trailing segment. The leading segment contains mini request slots for active users to submit requests for slot allocation. The trailing segment contains information slots for packet transmission. Since the leading segment is used for reservation, it represents an overhead on the system. Requests submitted in the request slots in the current uplink frame will be received by the base station. Based on the information contained in the request vector, the base station assigns information slots to requesting users in the next downlink frame. Thus, in an ATDMA system, requests submitted in the current frame will be accommodated in the next frame.

The simplest way to structure an ATDMA frame is to allocate a request slot to each user on a permanent basis. If there are N users, the request segment has to contain N request slots. In this way, each user owns a request slot and requests can be submitted on a conflict-free basis. However, if the user population is overly large, the overhead can be quite large. If the traffic load is very heavy such that every user is active, this way of structuring the request slot segment would be fine. But, if the system were not that heavily loaded, it would be more efficient to use STDMA.

Under normal operating conditions, the average traffic load would never be 100%. Consider voice transmission for example. On average, a voice process is in a talk state only 40% of the time; the other 60% of the time, it is in a silent state. So, the average load would be 0.4. In this case, the number of request slots in the ATDMA frame can be fewer than the number of users. However, if the number of request slots is fewer than the number of users, then how should request submissions be performed? One approach is to let active users choose a request slot randomly to submit requests. This can lead to collision in transmission of requests. The uncollided requests will be successful and those users will receive slot allocation in the next frame. A collided request will have to be retransmitted, again by randomly choosing a request slot in the next uplink frame.

In TDMA, each user collects the low-rate source information data in the previous frame and transmits the data at a much higher rate in the allocated time slot of the current time frame. As the frame duration is usually very small (e.g., 5 ms or 10 ms), the small delay in the transmission may not be noticed by the end user.

6.3.3 CDMA

CDMA is a spread spectrum multiple access method [155, 80]. The principle of spread spectrum communications is that the bandwidth of the baseband information-carrying signals from the different users is spread by different signals with a bandwidth much larger than that of the baseband signals. Ideally, the spreading signals used for the different users are orthogonal to each other. Thus, at the receiver, the same spreading signal is used as the despreading signal to coherently extract the baseband signal from the target user, while suppressing the transmissions from any other users. In spread spectrum communications, the spreading signals have to be derived from a set of orthogonal functions. Orthogonal functions with an infinitely large bandwidth will look like white noise to each other. However, white noise cannot be practically generated. Thus, truly orthogonal functions with an infinitely large bandwidth are difficult, if not impossible, to generate in practice.

Sequences, or functions, that can be generated have a deterministic feature. These sequences are referred to as pseudorandom noise (PN) sequences. The generation of PN sequences using linear shift registers is shown in Appendix E. CDMA in which the spread spectrum is achieved by directly multiplying the user's baseband signal with a high rate PN sequence is referred to as *direct sequence* CDMA (DS-CDMA). Different users in a DS-CDMA system use different spreading signals, therefore, they may use the same carrier frequency, f_c, and transmit the spread signals simultaneously. In a DS-CDMA system, each user is assigned its own PN sequence, which is approximately orthogonal to all other sequences assigned to other users. The users may be randomly located within the footprint of the base station. Even if the different users transmit at the same power level, because the distances from the various users to the cell-site differ, the power levels received from the different users at the cell-site receiver will differ. This phenomenon is called the near–far problem. The near–far problem can be avoided through power control. In a CDMA-based system, power control is implemented to provide the same received signal power level (or desired power levels) at the base station receiver from the different users, independent of the location of each mobile user within the cell area [8, 134, 94].

Since the spreading PN sequences are not truly orthogonal, as a multiple access technology, CDMA is interference limited. This means that the capacity of CDMA has a soft limit, and is a function of the service quality required.

Direct Sequence Spread Spectrum. Consider a radio cell with a population of K users. Each of the mobile users is assigned a unique spreading sequence. Each symbol in the PN sequence is called a chip. Figure 6.7 shows the functional block diagram of the transmitter and receiver for the kth user, $k = 1, 2, \ldots, K$. For simplicity, consider real-valued binary spreading waveforms. The information-carrying baseband signal $d_k(t)$ is

$$d_k(t) = \sum_i s_{k,i} \Pi\left(\frac{t - iT_b}{T_b}\right),$$

where $s_{k,i} \in \{-1, +1\}$ is the ith binary information bit, T_b is the information bit interval, and $\Pi(t/T_b)$ is the rectangular pulse

$$\Pi\left(\frac{t}{T_b}\right) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases}.$$

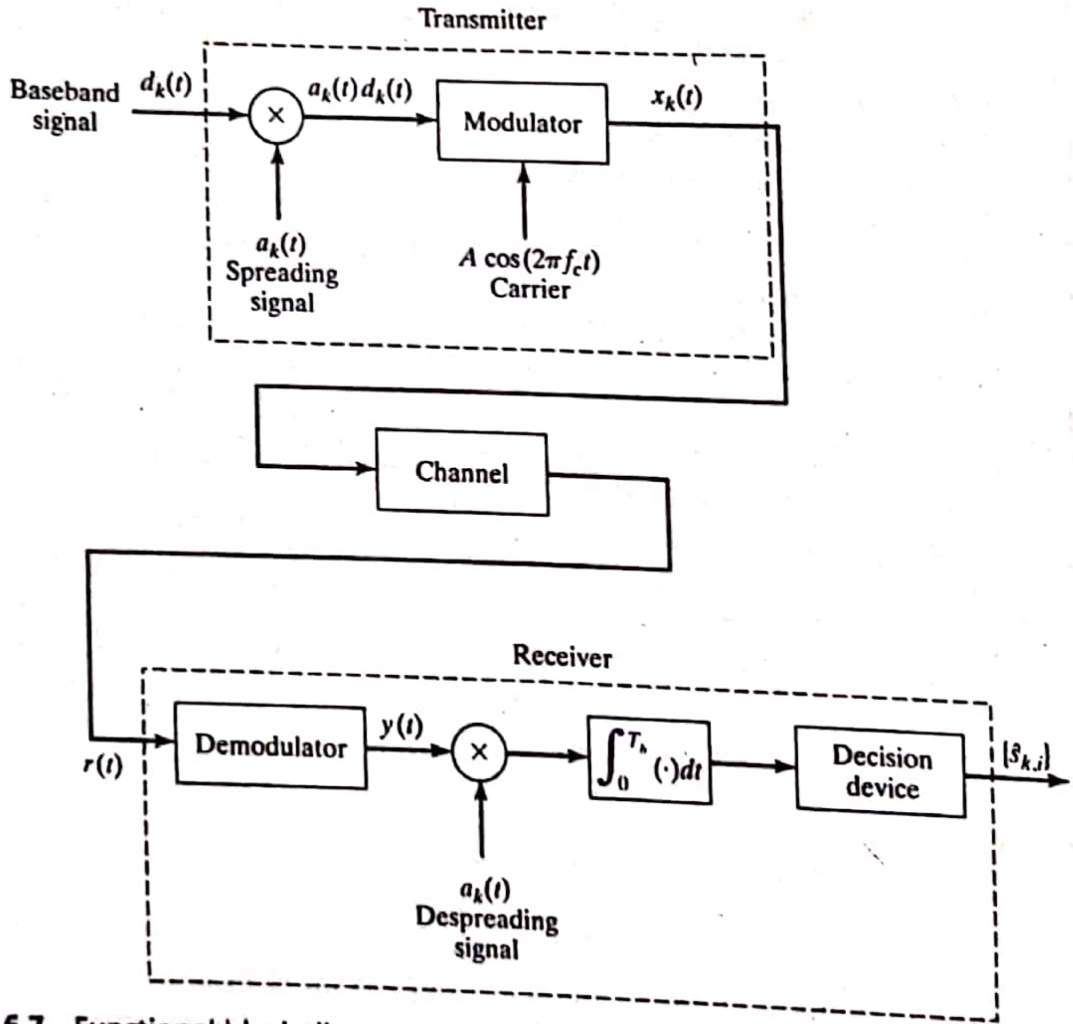

Figure 6.7 Functional block diagram of user k transmitter and receiver in a DS-CDMA system.

The spreading signal of the kth user is $a_k(t)$ and can be represented as

$$a_k(t) = \sum_l a_{k,l} P_{T_c}(t - lT_c),$$

where $a_{k,l} \in \{-1, +1\}$ is the lth chip of the binary PN sequence assigned to user k, $P_{T_c}(t)$ is the chip pulse waveform depending on baseband pulse shaping, and T_c is the chip interval, corresponding to a chip rate of $1/T_c$. The spreading process is to modulate $a_k(t)$ onto $d_k(t)$, which gives the spread signal $a_k(t)d_k(t)$. The spreading function $a_k(t)$ is often phase shift keyed onto the baseband signal $d_k(t)$ (i.e., the product operation, $a_k(t)d_k(t)$, is performed by the phase shift keying process, as discussed in Chapter 3). The spread signal $a_k(t)d_k(t)$ is then modulated with carrier frequency f_c ($\gg 1/T_c$), resulting in the bandpass signal $x_k(t)$ with amplitude A_c and bit interval T_b, for transmission. Normally, $T_b = LT_c$, where L is an integer. The principle of signal spreading is that $L \gg 1$ (i.e., $T_b \gg T_c$). Since the spreading signal has a chip rate much larger than the transmitted information symbol rate[1], the bandwidth of the spread signal is much larger than the bandwidth of the baseband information signal, hence, the name spread spectrum modulation. In the following, for simplicity, we consider $P_{T_c}(t) = \Pi(t/T_c)$ and BPSK for the passband modulation with coherent demodulation. If the PN sequences are periodic, with period L, then the transmitted signal is

$$x_k(t) = A_c \left[\sum_i s_{k,i} \sum_{l=1}^{L} a_{k,l} \Pi\left(\frac{t - iT_b - lT_c}{T_c}\right)\right] \cos(2\pi f_c t).$$

Normally, the information bits and the PN sequence chips are completely independent. Then, from Section 3.4, the psd of the transmitted signal is

$$\Phi_1(f) = \frac{E_c}{2}\{\mathrm{sinc}^2[(f - f_c)T_c] + \mathrm{sinc}^2[(f + f_c)T_c]\}, \tag{6.3.1}$$

where $E_c = \int_0^{T_c}[x_k(t)]^2 dt = \frac{1}{2}A_c^2 T_c$ is the chip energy. For comparison, without spread spectrum, the transmitted signal would be $A_c d_k(t)\cos(2\pi f_c t)$ and the corresponding psd would be

$$\Phi_2(f) = \frac{E_b}{2}\{\mathrm{sinc}^2[(f - f_c)T_b] + \mathrm{sinc}^2[(f + f_c)T_b]\}, \tag{6.3.2}$$

where $E_b = \int_0^{T_c}[A_c a_k(t)\cos(2\pi f_c t)]^2 dt = \frac{1}{2}A_c^2 T_b = LE_c$ is the bit energy. Figure 6.8 illustrates the normalized psd without spreading, $\Phi_2(f)/(E_b/2)$, and with spreading, $\Phi_1(f)/(E_c/2)$, for $f > 0$.

The output emerging from the channel, which is also the input to the receiver, is a superposition of the spread signals from all users in the same radio cell, plus background noise and interference from neighboring cells. Let $r(t)$ denote the received signal. Under the assumption that all the K users in the cell are synchronized in time and have the same received signal power

[1] For binary signaling, the symbol rate equals the bit rate; the terms symbol and bit are synonymous.

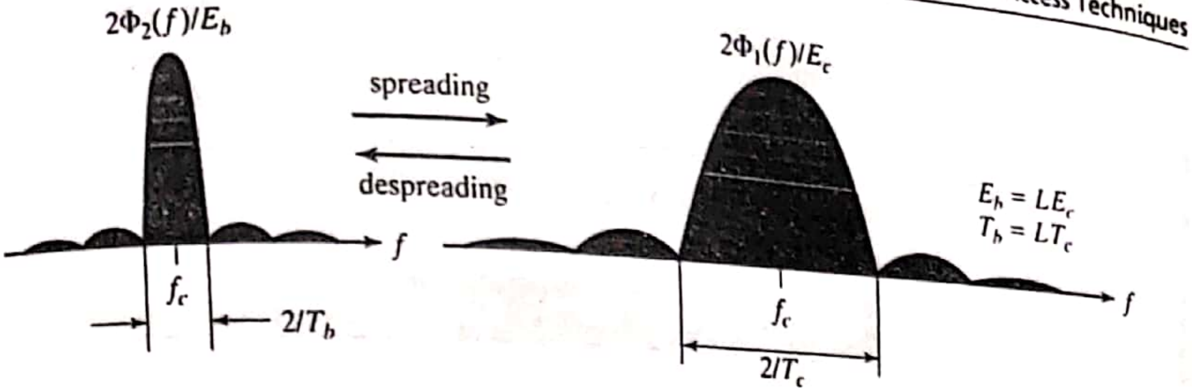

Figure 6.8 Illustration of signal power spectral density without and with spreading.

$(A_c^2/2)$, $r(t)$ is given by

$$r(t) = \sum_{k=1}^{K} x_k(t) + I(t) + w(t), \tag{6.3.3}$$

where $I(t)$ represents intercell interference and $w(t)$ represents background white Gaussian noise with zero mean and two-sided psd $N_0/2$. Both intracell interference and intercell interference are due to multiple access and, therefore, are called multiple access interference (MAI). Suppose we tag the signal transmitted by user 1 as the desired signal and the transmitted signals from all other users as interference. For the purpose of detecting user 1's signal at the receiver, it is convenient to isolate the desired signal component on the right-hand side of Eq. (6.3.3) and express $r(t)$ as

$$\begin{aligned} r(t) &= A_c a_1(t) d_1(t) \cos(2\pi f_c t) \\ &\quad + \sum_{k=2}^{K} A_c a_k(t) d_k(t) \cos(2\pi f_c t) + I(t) + w(t). \end{aligned} \tag{6.3.4}$$

The first block in the receiver of Figure 6.7 is the demodulator, which translates the received signal centered at frequency f_c to baseband centered at frequency zero. This is done by a correlator or a matched filter in coherent demodulation. The output of the demodulator at the end of the lth chip interval is

$$\frac{1}{T_c} \int_{lT_c}^{(l+1)T_c} r(t) \cos(2\pi f_c t) dt.$$

Over each chip interval, $a_k(t)d_k(t)$ $(k = 1, 2, \ldots, K)$ is a constant. The demodulator output as a function of time, $y(t)$, can be written as

$$y(t) = \frac{A_c}{2} a_1(t) d_1(t) + \sum_{k=2}^{K} \frac{A_c}{2} a_k(t) d_k(t) + n(t),$$

where

$$n(t) = \frac{1}{T_c} \sum_l \left\{ \int_{lT_c}^{(l+1)T_c} [I(t) + w(t)] \cos(2\pi f_c t) dt \right\} \Pi \left(\frac{t - lT_c}{T_c} \right)$$

is due to the intercell interference and additive background noise. In general, the integral can be approximated by a zero-mean Gaussian random variable and is independent from chip to chip. As a result, $n(t)$ is baseband Gaussian noise with bandwidth approximately equal to $1/T_c$. To extract user 1's transmitted signal from the demodulator output in the receiver shown in Figure 6.7, $a_1(t)$ should be used as the despreading signal. Despreading is achieved by first multiplying the demodulator output, $y(t)$, with $a_1(t)$ and then integrating the product over each symbol interval. Since $a_1^2(t) = 1$ at any t, we have

$$a_1(t)y(t) = \frac{A_c}{2} d_1(t) + \sum_{k=2}^{K} \frac{A_c}{2} a_1(t) a_k(t) d_k(t) + a_1(t) n(t),$$

where the first term represents the desired signal component and is a constant over each symbol interval. It is clearly observed that the despreading process indeed recovers the original baseband signal $d_1(t)$ from the spread signal $d_1(t)a_1(t)$. The second term represents the effect of the intracell interference, where $a_1(t)a_k(t)$ can be viewed as a new spreading signal with the same chip rate. With a very high chip rate, the power of the interference is approximately uniformly distributed over the frequency band $[0, \frac{1}{T_c}]$. For the intercell interference plus noise term, since (a) $a_{1,l}$ takes on the values of -1 and $+1$ with equal likelihood, (b) $n(t)$ in each chip interval is a zero-mean Gaussian random variable, and (c) $a_1(t)$ and $n(t)$ have the same bandwidth, the statistical behavior of $a_1(t)n(t)$ is the same as that of $n(t)$. Therefore, the power of the intercell interference and noise is approximately uniformly distributed over the frequency band $[0, \frac{1}{T_c}]$.

To suppress the interference and noise, the next step in the receiver is to integrate the despread signal over each information symbol interval over which the desired signal component is a constant. The output of the integrator at the end of the ith symbol is

$$\frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} a_1(t) y(t) dt = \frac{A_c T_b}{2} \alpha_1 d_{1,i} + \frac{A_c T_b}{2} \left[\sum_{k=2}^{K} \alpha_k d_{k,i} \right] + n_i,$$

where

$$\alpha_1 = \frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} [a_1(t)]^2 dt = 1$$

is the autocorrelation of the spreading signal $a_1(t)$ over the symbol interval and

$$\alpha_k = \frac{1}{T_b} \int_{iT_b}^{(i+1)T_b} a_1(t) a_k(t) dt, \quad k = 2, 3, \ldots, K$$

is the crosscorrelation between the spreading signals $a_1(t)$ and $a_k(t)$ over the symbol interval. If all the spreading signals $a_k(t)$, $k = 1, 2, \ldots, K$, are orthogonal in the symbol interval, then there no intracell interference in the recovered baseband signal. The effect of the intercell interference d background noise on the signal detection is given by the last term n_i, which is

$$n_i = \frac{1}{T_b}\int_{iT_b}^{(i+1)T_b} a_1(t)n(t)dt.$$

the frequency domain, the integrator is a low-pass filter with bandwidth approximately equal $1/T_b$. The LPF lets the desired signal component $\frac{A_c}{2}d_1(t)$ go through without distortion and eatly reduces the interference and noise power. The despreading process significantly improves e signal-to-interference plus noise ratio (SINR).

The spread spectrum system performance is measured by the processing gain, G_p, defined as e SINR improvement achieved by despreading. That is

$$G_p \triangleq \frac{\text{SINR after despreading}}{\text{SINR before despreading}}.$$

Figure 6.9 plots the psd of the signals before and after despreading to illustrate the SINR nprovement achieved by despreading, where the sinc function shape of the psd of the signals, s given in Eqs. (6.3.1) and (6.3.2), is approximated by a uniform psd with cut-off frequency t the first null point of the sinc function, $W = 1/T_c$. Note that W is the spreading bandwidth nd $R_s = 1/T_s$ is the baseband data symbol rate. From the simplified illustration, it is observed hat (*a*) the desired signal power (the area in the figure for user 1) remains unchanged in the lespreading process, and (*b*) the power of the interference and noise is reduced by W/R_s times. As a result, the processing gain is given by

$$G_p = \frac{W}{R_s} = \frac{T_s}{T_c} = L.$$

For binary signaling, R_s ($= R_b$) is the bit rate, and T_s ($= T_b$) is the bit interval.

Transmission Performance. In the absence of path loss, the received signal energy per bit is E_b. For transmission in an AWGN channel without MAI (by using truly orthogonal spreading sequences), the BER for the DS-CDMA user in additive white Gaussian noise of zero mean and two-sided psd $N_0/2$ is the same as that without spread spectrum modulation, due to the fact that (*a*) for the desired signal, the functions of spreading at the transmitter and despreading at the receiver cancel each other, and (*b*) the despreading process does not change the statistics of the noise component at the decision device input. For example, if BPSK is used, the BER for the DS-CDMA user with coherent detection is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

If the spreading sequences are not orthogonal, the MAI from all other mobile users in the system will increase the transmission error rate. When the number of mobile users in the system is large and the interferences from all other users are independent and have similar stochastic behavior,

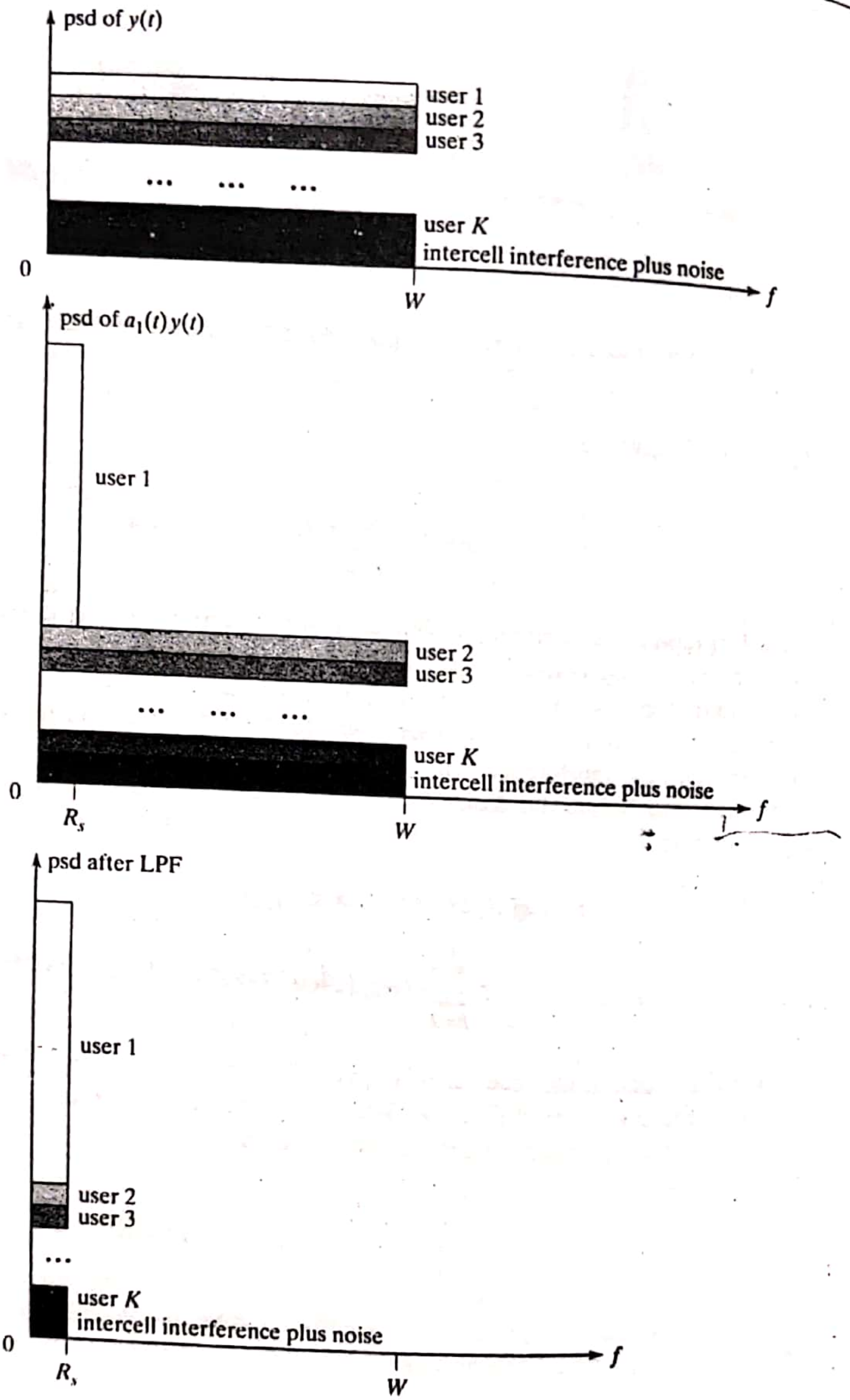

Figure 6.9 Illustration of signal power spectral density before and after despreading.

MAI - Multiple Access Interference.

from the central limit theorem, the MAI can be approximated as a Gaussian process. Furthermore, with a large spread spectrum bandwidth W, the psd of the interference is approximately uniform over the bandwidth. As a result, the effect of the MAI on the transmission performance can be treated in the same way as the additive white Gaussian noise, and the BER is

$$P_b = Q\left(\sqrt{\frac{2E_b}{I_0 + N_0}}\right),$$

where I_0 is the two-sided psd of the MAI over the spread spectrum bandwidth. Even though spread spectrum does not provide performance gain over an AWGN channel where the interference (noise) is wideband with an infinite power, it does achieve a processing gain over narrowband interference (jamming) as shown in Example 6.2.

Example 6.2 Suppression of Narrowband Jamming in DS-CDMA

Consider a DS spread spectrum system using BPSK. The channel introduces a single-tone jamming signal with power P_J. The jamming signal is synchronized with the desired signal both in frequency and in the initial phase. Under the assumptions of (a) accurate PN code synchronization and carrier phase synchronization at the receiver and (b) rectangular chip pulses, find the signal-to-interference ratio (SIR) at the demodulator output in the absence of background noise.

Solution Let P_d denote the received power of the desired signal, T_b the symbol interval, and T_c the chip interval. The received signal plus interference can be represented as

$$r(t) = \sqrt{2P_d}d(t)a(t)\cos(2\pi f_c t + \theta) + \sqrt{2P_J}\cos(2\pi f_c t + \theta),$$

where $d(t) = \sum_i s_i \Pi\left(\frac{t-iT_b}{T_b}\right)$, s_i ($\in \{-1, +1\}$) represents the ith binary information digit, $a(t) = \sum_l a_l \Pi\left(\frac{t-lT_c}{T_c}\right)$, a_l ($\in \{-1, +1\}$) represents the lth binary chip, f_c is the carrier frequency, and θ is the carrier phase at $t = 0$. For simplicity, assume that a_l takes on the values of $+1$ and -1 with equal likelihood and all the chips are independent of each other. Without loss of generality, consider the detection of symbol s_1 over $t \in [0, T_b]$. With accurate PN code and carrier phase synchronization, the demodulator output at $t = T_b$ is

$$\begin{aligned}
&\frac{1}{T_b}\int_0^{T_b} r(t)[a(t)\cos(2\pi f_c t + \theta)]dt \\
&= \frac{1}{T_b}[\sqrt{2P_d}\int_0^{T_b} s_1 a^2(t)\cos^2(2\pi f_c t + \theta)dt + \sqrt{2P_J}\int_0^{T_b} a(t)\cos^2(2\pi f_c t + \theta)dt] \\
&\approx \frac{1}{2}\sqrt{2P_d}s_1 + \frac{1}{2T_b}\sqrt{2P_J}\int_0^{T_b} a(t)dt \\
&= (\sqrt{P_d/2})s_1 + \frac{1}{2G_p}\sqrt{2P_J}\sum_{l=1}^{G_p} a_l,
\end{aligned}$$

where $G_p = T_b/T_c$ (assumed to be an integer) is the processing gain. The desired signal component, $(\sqrt{P_d/2})s_1$, has a power of $P_d/2$. The interference component due to the jamming signal, $\frac{1}{2G_p}\sqrt{2P_J}\sum_{l=1}^{G_p} a_l$, has a power of

$$E\left\{\left[\frac{1}{2G_p}\sqrt{2P_J}\sum_{l=1}^{G_p} a_l\right]^2\right\} = \frac{P_J}{2}\left(\frac{1}{G_p}\right)^2 E\left[\left(\sum_{l=1}^{G_p} a_l\right)^2\right]$$

$$= \frac{1}{2G_p}P_J.$$

The SIR at the demodulator output is

$$\frac{P_d/2}{P_J/(2G_p)} = G_p(P_d/P_J).$$

That is, the SIR is increased by the processing gain G_p, as compared with the SIR at the demodulator input. Note that, without spread spectrum, the SIR at the demodulator output would remain the same as that at the demodulator input.

Example 6.3 Transmission Performance in Multiple Access Interference

Consider the forward link transmission of a single-cell DS-CDMA system using BPSK, where the multiple access interference due to other users in the cell is synchronized with the desired signal both in chip timing and in carrier phase. It is assumed that (*a*) all the PN code sequences are independent of each other, and chip values "+1" and "−1" in each sequence are equally likely and are independent of each other; (*b*) rectangular pulses are used for the spreading waveforms, (*c*) the receiver uses coherent detection, (*d*) the number of users in the cell is large, and (*e*) the received signal power levels from all the mobiles are the same at the base station receiver. Derive the probability of bit error in an AWGN channel in terms of the processing gain G_p, the number K of users in the cell, the bit energy E_b of the received signal, and the one-sided noise psd N_0.

Solution Let K denote the number of mobile users in the system. Consider the detection process at mobile user 1's receiver. The received signal $r(t)$ is

$$r(t) = \sqrt{2P_d}\left[a_1(t)d_1(t) + \sum_{k=2}^{K} a_k(t)d_k(t)\right]\cos(2\pi f_c t + \theta_0) + w(t),$$

where P_d is the desired signal power of the kth user and is independent of k, $d_k(t) = \sum_i s_{k,i}\Pi\left(\frac{t-iT_b}{T_b}\right)$, $a_k(t) = \sum_l a_{k,l}\Pi\left(\frac{t-lT_c}{T_c}\right)$, $s_{k,i}$ $(\in \{-1, +1\})$ is the ith information binary digit and $a_{k,l}$ $(\in \{-1, +1\})$ is the lth chip of the PN code sequence for the kth user, $k = 1, 2, \ldots, K$, T_b and T_c are the bit and chip intervals respectively, f_c is the carrier frequency, θ_0 is the carrier phase at $t = 0$, $w(t)$ is the additive white Gaussian noise with zero mean and two-sided psd $N_0/2$ (i.e., $E[w(t)w(s)] = (N_0/2)\delta(t-s)$). Consider the signal detection over $t \in [0, T_b]$. The output of the coherent demodulator is

$$\frac{1}{T_b}\int_0^{T_b} r(t)[a_1(t)\cos(2\pi f_c t+\theta_0)]dt.$$

The output consists of three components:

(1) the desired signal component, $(\sqrt{P_d/2})s_{1,1}$, which is a constant given $s_{1,1}$;
(2) the noise component

$$(1/T_b)\int_0^{T_b} w(t)a_1(t)\cos(2\pi f_c t+\theta_0)dt,$$

which is a Gaussian random variable with zero mean and variance

$$E\left[\frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} w(t)w(s)a_1(t)a_1(s)\cos(2\pi f_c t+\theta_0)\cos(2\pi f_c s+\theta_0)dtds\right]$$

$$=\frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E[w(t)w(s)]E[a_1(t)a_1(s)]\cos(2\pi f_c t+\theta_0)\cos(2\pi f_c s+\theta_0)dtds$$

$$=\frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\delta(t-s)E[a_1(t)a_1(s)]\cos(2\pi f_c t+\theta_0)\cos(2\pi f_c s+\theta_0)dtds$$

$$=\frac{N_0}{2T_b^2}\int_0^{T_b} E[a_1^2(t)]\cos^2(2\pi f_c t+\theta_0)dt$$

$$\approx\frac{N_0}{4T_b};\text{ and}$$

(3) the intracell interference

$$(\sqrt{P_d/2})(1/G_p)\sum_{k=2}^{K} s_{k,1}\sum_{l=1}^{G_p} a_{1,l}a_{k,l},$$

where $G_p = T_b/T_c$ (assumed to be an integer) is the processing gain. If $s_{k,1}$ takes on the values of -1 and $+1$ with equal likelihood, and is independent for different k values, $s_{k,1}\sum_{l=1}^{G_p} a_{1,l}a_{k,l}$ are iid random variables for different k values. If $K \gg 1$, then based on the central limit theorem, the interference component can be modeled as a Gaussian random variable with zero mean and variance

$$\frac{P_d}{2G_p^2}E\left\{\left[\sum_{k=2}^{K} s_{k,1}\sum_{l=1}^{G_p} a_{1,l}a_{k,l}\right]\left[\sum_{k'=2}^{K} s_{k',1}\sum_{l'=1}^{G_p} a_{1,l'}a_{k',l'}\right]\right\}$$

$$=\frac{P_d}{2G_p^2}\sum_{k=2}^{K} E[s_{k,1}^2]\sum_{l=1}^{G_p} E[a_{1,l}^2]E[a_{k,l}^2]$$

$$=\frac{(K-1)P_d}{2G_p}.$$

In summary, the demodulator output is a Gaussian random variable with mean $(\sqrt{P_d/2})s_{1,1}$ and variance $\frac{N_0}{4T_b} + \frac{(K-1)P_d}{2G_p}$, given that $s_{1,1}$ was sent. Similar to the derivation of the BER for coherent BPSK in an AWGN channel in Section 3.5, the probability of bit error is

$$P_b = Q\left(\frac{\sqrt{P_d/2}}{\sqrt{\frac{N_0}{4T_b} + \frac{(K-1)P_d}{2G_p}}}\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0 + 2(K-1)E_b/G_p}}\right).$$

where E_b $(= P_d T_b)$ is the received signal bit energy. If there is no multiple access interference (i.e., $K = 1$), the probability of error is the same as BPSK in AWGN with coherent detection as given in Section 3.5. On the other hand, if there is no background noise (i.e., $N_0 = 0$), then the probability of error is $Q(\sqrt{G_p/(K-1)})$, where the effective signal-to-interference ratio is the product of the processing gain G_p and the ratio of the desired signal power P_d to the total interference power $(K-1)P_d$. Given the background noise, the maximum number of users allowed in the system, K, can be determined from the transmission accuracy requirement.

Characteristics of CDMA. Based on the preceding analysis of the DS-CDMA system, we can summarize some properties of DS-CDMA as follows:

a. *Universal frequency reuse* – As CDMA achieves the orthogonality among the transmitted signals from the mobile users by using the orthogonal, or approximately orthogonal, PN sequences in spreading the signals, the total frequency bandwidth allocated to the system can be reused from cell to cell. As a result, we achieve the minimum cell cluster size $(N = 1)$ and maximum frequency reuse. This significantly reduces the complexity of frequency planning in cellular system design;

b. *Soft handoff* – Because of the universal frequency reuse, a mobile user can simultaneously communicate with several nearby base stations using the same frequency band and the same spreading signal in each link. When the mobile user is at the cell boundary, it can establish a connection with the new base station before terminating the connection with the old base station. This will improve handoff performance;

c. *High transmission accuracy* – With spread spectrum, as discussed in Chapter 4, we can use Rake receivers to mitigate the fading dispersive channel impairments and, therefore, improve transmission accuracy, especially during soft handoff;

d. *Soft capacity* – As in practice, the PN sequences are not truly orthogonal, MAI will degrade the transmission BER performance. The maximum number of users that can be supported in each cell depends on the required quality of service (QoS) and is limited by MAI, to be discussed in Subsection 6.4.3. As a result, unlike TDMA and FDMA, there is no hard limit on the number of users in each cell. During peak traffic hours, if the users can tolerate a lower QoS to a certain degree, the system can accommodate more users to satisfy the high service demands in that period;

e. *Flexibility* – As CDMA is interference limited, if a user does not transmit, it does not generate any interference with other active users and, therefore, does not use the system resources. This feature translates to a high resource utilization via statistical multiplexing for on-off voice traffic and bursty data traffic. Even though TDMA can make use of the traffic activity factor to increase resource utilization, with CDMA it is easier to implement the statistical multiplexing. In addition, CDMA has more flexibility than TDMA in supporting multimedia services (with various time-varying traffic rates).

The advantages of the CDMA systems are not achieved without paying a price. First, CDMA requires stringent power control to achieve high capacity. For example, with voice services, the cell capacity is maximized when the signals received at the base station from all the mobiles in the cell have the same minimum power level. Second, with a large processing gain, the maximum transmission rate in each code channel (using a unique PN sequence in the signal spreading) is limited as compared with TDMA. This limitation can be overcome by parallel transmissions of information from/to one mobile user, with each transmission using a unique PN sequence. Third, the CDMA systems operate at a high chip rate and require accurate PN synchronization at the receiver. The complexity of the transmitter and (Rake) receiver is higher than that of TDMA and FDMA systems.

6.4 SPECTRAL EFFICIENCY

Because of the severe channel impairments, the spectrum of the wireless channel is interference limited. Techniques commonly used to enhance the spectrum utilization in a mobile communication system include

a. data compression to reduce the transmission rate,
b. bandwidth reduction,
c. channel assignment, and
d. choice of multiple access method.

The overall spectral efficiency of a mobile communication system can be estimated based on a knowledge of

a. channel spacing in kHz,
b. cell area in km^2,
c. frequency reuse factor, and
d. multiple access scheme used.

η_{sys} η_{acess}

Factors (*a*)–(*c*) are attributed to the system parameters, including the modulation method used. Note that the modulation scheme used does not depend on the choice of multiple access technology.

The spectral efficiency of a mobile communication system can be represented as a combination of two independent components: one component that depends on the system parameters, and the other component that depends on the multiple access method used. Let η_{sys} denote the spectral efficiency component that depends on the system parameters, and η_{access} denote the spectral efficiency component that depends on the multiple access scheme used. The overall system spectral efficiency for a mobile communications system, η, can then be expressed as

$$\eta = \eta_{sys} \times \eta_{access}.$$

Depending on the units used, we can define η in two different ways. The unit commonly used for η is channels/MHz/km^2 or Erlangs/MHz/km^2. Here, Erlang is a measure of the traffic load, as discussed in Section 5.5. η is represented in terms of channels/MHz per km^2 or Erlangs/MHz per km^2 in order to capture the frequency reuse in the service coverage area of the system. Accordingly, η may be defined in the following ways:

Definition 6.1

$$\eta \triangleq \frac{\text{Total number of channels available for data in system}}{\text{(system bandwidth)(total coverage area)}} \quad \text{Channels/MHz/km}^2.$$

Definition 6.2

$$\eta \triangleq \frac{\text{Total traffic carried by the system}}{\text{(system bandwidth)(total coverage area)}} \quad \text{Erlangs/MHz/km}^2.$$

6.4.1 FDMA Systems

Spectral Efficiency of FDMA (η_{FDMA}). The AMPS system (in America and Australia) is based on FDMA/FDD. Here, a single user occupies a single channel while the call is in progress. When a call is finished or handed off to another base station, the channel is vacated so that another subscriber may use it. The single channel is actually two simplex channels which are frequency duplexed, as shown in Figure 6.6(a), with a 45 MHz split. A guard band of width B_g is used in each of the edges, as shown in Figure 6.10.

The number of channels, N_s, that can be simultaneously supported is

$$N_s = \frac{B_s - 2B_g}{B_c},$$

where B_s is the total frequency spectrum bandwidth for transmissions in one direction (uplink or downlink). The above can be rearranged as

$$B_s = N_s B_c + 2B_g. \tag{6.4.1}$$

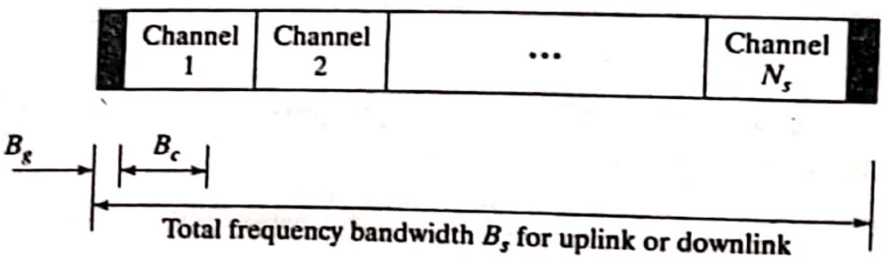

Figure 6.10 Channel spacing and guard bands in FDMA uplink or downlink.

Let N_{ctl} be the number of allocated control channels and N_{data} be the number of data channels in the system. Then the total number of available channels is

$$N_s = N_{data} + N_{ctl}. \tag{6.4.2}$$

Since each user in service is assigned a data channel, N_{data} is also the maximum number of simultaneous users in each cell cluster. Substituting Eq. (6.4.2) into Eq. (6.4.1), we get,

$$B_s = N_{data} B_c + N_{ctl} B_c + 2B_g,$$

from which we have the inequality

$$\boxed{N_{data} B_c < B_s,}$$

where $N_{data} B_c$ is the total bandwidth available for data transmission.

As discussed in Chapter 5, to increase the system capacity, the entire cellular array of cells is partitioned into clusters. All available frequencies, or radio channels, are allocated to the cells in a single cluster, and the same frequencies are then reused in each and every cluster. For the purpose of discussing spectral efficiency, we will consider one cluster as the system. The spectral efficiency of FDMA is defined as

$$\eta_{FDMA} = \frac{\text{bandwidth available for data transmission}}{\text{system bandwidth}} = \frac{N_{data} B_c}{B_s} < 1. \tag{6.4.3}$$

Example 6.4 Spectral Efficiency of FDMA

In the AMPS system, the system bandwidth is 12.5 MHz, the channel spacing is 30 kHz, and the edge guard spacing is 10 kHz. The number of channels allocated for control signaling is 21. Find

a. the number of channels available for message transmission, and
b. the spectral efficiency of FDMA.

Solution We have $B_s = 12.5$ MHz, $B_c = 30$ kHz, and $B_g = 10$ kHz. Therefore,

a. the number of available channels is

$$N_s = \frac{B_s - 2B_g}{B_c} = \frac{12.5 \times 1000 - 20}{30} = 416 \text{ channels, and}$$

b. the spectral efficiency of this FDMA system is

$$\eta_{FDMA} = \frac{30 \times (416 - 21)}{12.5 \times 1000} = 0.948.$$

System Spectral Efficiency. To express η_{sys} in terms of mathematical symbols, we will use the following notation:

B_s = system bandwidth in MHz

B_c = channel spacing in MHz

$$
\begin{aligned}
B_g &= \text{the guard-band bandwidth in MHz} \\
N &= \text{cluster size (i.e., the number of cells in a cluster)} \\
&= \text{the frequency reuse factor} \\
N_{ch/cell} &= \text{number of available channels per cell} \\
N_{data/cell} &= \text{number of available data channels per cell} \\
N_{ctl/cell} &= \text{number of control channels per cell} \\
N_{ch/cluster} &= \text{number of available channels per cluster} \\
N_{data/cluster} &= \text{number of available data channels per cluster} \\
N_{ctl/cluster} &= \text{number of control channels per cluster} \\
A_{cell} &= \text{cell area in km}^2
\end{aligned}
$$

As discussed in detail in Chapter 5, frequency reuse is employed to increase the capacity of the entire cellular system. This is achieved by allocating the entire set of available frequencies to a single cluster. Therefore, the number of available channels per cluster is given by

$$N_{ch/cluster} = \frac{B_s - 2B_g}{B_c}.$$

Of the $N_{ch/cluster}$ channels in the cluster, $N_{ctl/cluster}$ are allocated as control channels. The total number of channels available for data traffic per cluster is thus given by

$$N_{data/cluster} = N_{ch/cluster} - N_{ctl/cluster} = \frac{B_s - 2B_g}{B_c} - N_{ctl/cluster}. \tag{6.4.4}$$

The total number of channels available for data traffic per cell is given by

$$N_{data/cell} = \frac{N_{data/cluster}}{N} = \frac{\dfrac{B_s - 2B_g}{B_c} - N_{ctl/cell}}{N}, \tag{6.4.5}$$

which is also the cell capacity, N_c, defined as the maximum number of mobile stations that can be served at one time in each cell.

Therefore, from Definition 6.1, we can express η as

$$\eta = \frac{\text{number of data channels per cluster}}{\text{system bandwidth times area of the cluster}} = \frac{N_{data/cluster}}{B_s \times (N \times A_{cell})}. \tag{6.4.6}$$

Substituting Eq. (6.4.4) into Eq. (6.4.6), we get

$$\eta = \frac{1 - \dfrac{B_c}{B_s} \times \left(N_{ctl/cluster} + \dfrac{2B_g}{B_c} \right)}{B_c \times N \times A_{cell}} \text{ channels/MHz/km}^2. \tag{6.4.7}$$

Equation (6.4.7) can be rearranged to yield the following form

Overhead

$$\eta = \frac{1}{B_c \times N \times A_{cell}} - \frac{N_{ctl/cluster} + \frac{2B_g}{B_c}}{B_s \times N \times A_{cell}} \text{ channels/MHz/km}^2. \quad (6.4.8)$$

In Eq. (6.4.8), the second term on the right-hand side accounts for the overhead in FDMA, due to the guard bands and control channels.

In any multiple access system there is a finite probability that some of the access traffic is blocked. Let η_t be the trunk efficiency in each cell, which is a function of the blocking probability and the total number of available channels per cell, $N_{data/cell}$, as discussed in Section 5.5. Hence, the total traffic carried in a cluster, in Erlangs, is $\eta_t \times N_{data/cluster}$. Using Definition 6.2, we can express η as

$$\eta = \frac{\eta_t \times N_{data/cluster}}{B_s \times N \times A_{cell}} \text{ Erlangs/MHz/km}^2.$$

Substituting Eq. (6.4.4) for $N_{data/cluster}$ in the above equation, we have

$$\eta = \frac{\eta_t}{B_c \times N \times A_{cell}} - \frac{\eta_t \times \left(N_{ctl/cluster} + \frac{2B_g}{B_c}\right)}{B_s \times N \times A_{cell}} \text{ Erlangs/MHz/km}^2, \quad (6.4.9)$$

where the second term on the right-hand side is due to the overhead in FDMA.

Example 6.5 System Spectral Efficiency in Channels/MHz/km^2

Suppose a cellular system in which the one-way bandwidth of the system is 12.5 MHz, the channel spacing is 30 kHz, and the guard band at each boundary of the spectrum is 10 kHz. If (1) the cell area is 6 km^2, (2) the frequency reuse factor (cluster size) is 7, and (3) 21 of the available channels are used to handle control signaling, calculate

a. the total number of available channels per cluster,
b. the number of available data channels per cluster,
c. the number of available data channels per cell, and
d. the system spectral efficiency in units of channels/MHz/km^2.

Solution We allocate all of the available frequencies to one cluster and these frequencies, or channels, are distributed evenly among the N cells in the cluster.

a. The total number of available channels in the cluster is

$$N_{ch/cluster} = \frac{B_s - 2B_g}{B_c} = \frac{12.5 - 2 \times 0.01}{0.03} = 416.$$

b. The number of available data channels per cluster is

$$N_{data/cluster} = N_{ch/cluster} - N_{ctl/cluster} = 416 - 21 = 395.$$

c. The number of available data channels per cell is

$$N_{data/cell} = N_{data/cluster}/N = 395/7 \approx 56.$$

d. The overall spectral efficiency of the system is

$$\eta = \frac{N_{data/cell}}{B_s \times A_{cell}} = \frac{56}{12.5 \times 6} = 0.747 \text{ channels/MHz/km}^2.$$

Example 6.6 System Spectral Efficiency in Erlangs/MHz/km²

Suppose that the system parameter values are the same as those in Example 6.5. In addition, there are the following other specifications:

(4) The area of the entire cellular system is 3024 km²
(5) The average number of calls per user during a busy hour is 1.5
(6) The average channel holding time of a call is 180 s
(7) The trunk efficiency, η_t, is 0.95

Calculate the following parameter values:

a. the number of cells in the system
b. the number of calls per hour per cell
c. the average number of users per hour per cell
d. the system spectral efficiency in Erlangs/MHz/km²

$\frac{3024}{7} =$

Solution

a. The number of cells in the system = $\frac{3024}{6} = 504$.
b. The number of calls per hour per cell, N_{call}, is

$$N_{call} = \frac{N_{data/cluster}}{N} \times \eta_t \times \text{ number of calls per hour}$$
$$= 56 \times 0.95 \times \frac{3600}{180}$$
$$= 1064 \text{ calls/hour/cell.}$$

c. The average number of users per hour per cell, N_{users}, is

$$N_{users} = \frac{\text{number of calls per hour per cell}}{\text{average number of calls per user per hour}}$$
$$= 1064/1.5$$
$$\approx 709 \text{ users/hour/cell.}$$

d. The system spectral efficiency is, from Example 6.5,

$$\eta = \eta_t \times 0.747$$
$$= 0.710 \text{ Erlangs/MHz/km}^2.$$

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6.4.2 TDMA Systems

TDMA can operate as wideband or narrowband. In wideband TDMA (W-TDMA), the entire frequency spectrum is available to any individual user. In narrowband TDMA (N-TDMA), the total available frequency spectrum is divided into a number of subbands, with each subband operating as a TDMA system. An individual user only uses the allocated subband. Thus, in narrowband TDMA, both frequency and time are partitioned.

Spectral Efficiency of Wideband TDMA (η_{W-TDMA}). The spectral efficiency of wideband TDMA, η_{W-TDMA}, is defined as the percentage of the time duration used for transmitting information data symbols in each frame. For the frame structure shown in Figure 6.5, let

τ_p = the time duration for the preamble

τ_t = the time duration for the trailer

T_f = the frame duration

L_d = the number of information data symbols in each slot

L_s = the total number of symbols in each slot.

Then, we have

$$\eta_{W-TDMA} = \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s}. \tag{6.4.10}$$

In the preceding equation, the first term on the right-hand side takes into account the overhead at the frame level (due to the frame header/trailer), and the second term takes into account overhead at the slot level (due to the trailer bits, synchronization bits, and guard bits, as shown in Figure 6.5). The overhead corresponds to the overhead due to control channels in FDMA, which are necessary to coordinate the multiple access.

Spectral Efficiency of Narrowband TDMA (η_{N-TDMA}). As mentioned earlier, in narrowband TDMA, the system bandwidth is divided into a number of subbands, and each subband is partitioned into time slots. Let

B_c = the bandwidth of an individual user

N_u = the number of subbands

B_g = the guard spacing.

Then, the number of subbands is

$$N_u = \frac{B_s - 2B_g}{B_c}.$$

The channel time in each subband is divided into time slots, numbered $1, 2, \cdots, N_{slot}$, as shown in Figure 6.11. In this way, any one of the numbered time slots, say slot 1, can be used by N_u users. Therefore, the actual usable bandwidth for information transmission is $N_u B_c$ $(= B_s - 2B_g)$.

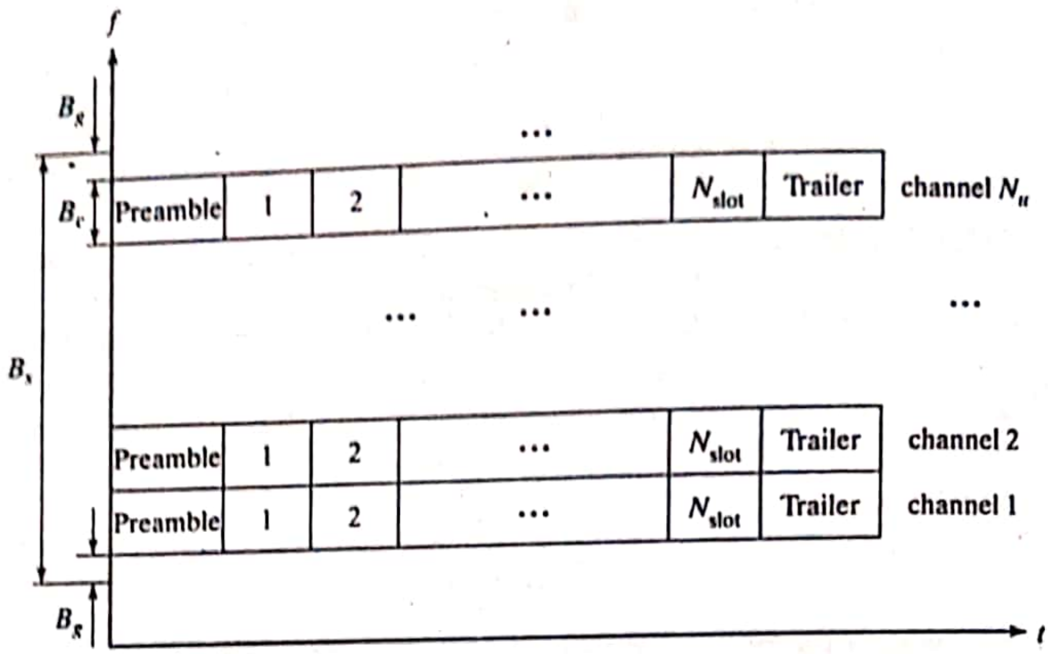

Figure 6.11 Narrowband TDMA format.

The spectral efficiency of narrowband TDMA is proportional to that of wideband TDMA. The proportionality constant is the ratio of the available information transmission bandwidth to the system bandwidth. Thus, the proportionality constant is

$$\alpha = \frac{N_u B_c}{B_s} \leq 1,$$

since $N_u B_c \leq B_s$. The spectral efficiency of narrowband TDMA is then given by

$$\begin{aligned}\eta_{N-TDMA} &= \eta_{W-TDMA} \times \frac{B_c N_u}{B_s} \\ &= \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s} \times \frac{B_s - 2B_g}{B_s}.\end{aligned} \tag{6.4.11}$$

Cell Capacity of TDMA Systems. The cell capacity is defined as the maximum number of mobile users that can be supported simultaneously in each cell. With TDMA, the maximum number of simultaneous users that can be accommodated during one use of the available frequency spectrum is

$$N_s = N_u \times N_{slot}, \tag{6.4.12}$$

where

$$N_u = \begin{cases} 1, & \text{for W-TDMA} \\ \dfrac{B_s - 2B_g}{B_c} > 1, & \text{for N-TDMA} \end{cases}$$

max
no of users in Both W-TDMA = N-TDMA

With the same total available bandwidth, B_s, the bandwidth of an individual user, B_c, in W-TDMA is approximately N_u times of that in N-TDMA. If the same modulation scheme is used, then the transmission bit rate, R_b, in W-TDMA is N_u times that in N-TDMA. Therefore, with the same source traffic characteristics from mobile users, the number of time slots, N_{slot}, in each frame in W-TDMA is actually N_u times that in N-TDMA. As a result, the maximum number of simultaneous users in W-TDMA is the same as that in N-TDMA. However, in practical implementation, due to the difference in transmission rate, W-TDMA and N-TDMA have different pros and cons.

N_s is the total number of TDMA channels available for the entire cellular system without frequency reuse. With frequency reuse, all the N_s channels can be allocated to a single cell cluster. Then, the cell capacity, N_c, in a TDMA system with frequency reuse factor N is

$$\boxed{N_c = \frac{N_u \times N_{slot}}{N}.}$$

In general, the source stream of a user is unlikely to be always in an *on* state. Let s_f denote the source activity factor, defined as the percentage of time that a connected mobile user is actually generating information data for transmission. For example, in voice transmission the voice stream alternates between a talk spurt (*on*) state and a silence (*off*) state. If, on average, 40% of the time it stays in the *on* state and 60% of the time in the *off* state, then $s_f = 0.4$. During the silence interval, the capacity can be used by other active users (in ATDMA). Therefore, the cell capacity of the system, N_c, should be modified by the source activity factor to represent the number of connected users per cell. Assuming perfect statistical multiplexing, the cell capacity of a TDMA system is given by

$$N_c = \frac{N_u \times N_{slot}}{s_f \times N} \text{ users/cell.} \tag{6.4.13}$$

In a synchronous TDMA system where the resources corresponding to the *off* state of a source stream are not allocated for use by other users, the source activity factor equals 1 (i.e., $s_f = 1$).

System Spectral Efficiency. The cell capacity depends on the number of slots, N_{slot}, which is a function of factors that govern the efficiency of the system usage, including the modulation scheme used. The overall frequency efficiency of a TDMA system in bits/s/Hz/cell is derived in the following.

The maximum number of radio channels in an N-TDMA system is given by the system bandwidth divided by the bit rate of each user. If each user transmits at a basic bit rate R_b and the system bandwidth is B_s, then the maximum number of users that can be supported during one use of the system bandwidth is

$$N_s = \frac{B_s}{R_b}, \quad \text{(Bit rate)}$$

N_s NTDMA

under the assumption that the required bandwidth for transmitting 1 bps is 1 Hz. For example, if we use QPSK and the transmission bandwidth is equal to the width of the main psd lobe as shown in Figure 3.23 (bandpass filtering may be necessary to suppress out-of-band radiation). with a frequency reuse factor of N, the cell capacity is

$$N_c = \frac{N_s}{N}.$$

This value is reduced because of inefficiency in both bandwidth usage due to the guard bands and modulation scheme. Let ϵ_{bw} denote the modulation efficiency, defined as the maximum transmission rate in bits per second that the modulation can accommodate over one hertz bandwidth. For example, if BPSK is used and the transmission bandwidth is equal to the width of the main psd lobe as shown in Figure 3.23 (bandpass filtering may be necessary to suppress out-of-band radiation), then $\epsilon_{bw} = 0.5$. The effective number of users, $N_{effective}$, that can be supported per cell is N_c, modified by the modulation efficiency factor. Therefore,

$\epsilon_{bw} = 0.5$

$$N_{effective} = \epsilon_{bw} \times \frac{B_s}{R_b \times N}.$$

If we consider the overhead necessary for TDMA, from Eq. (6.4.11), the effective number should be further modified to

TDMA

$$N_{effective} = \epsilon_{bw} \times \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s} \times \frac{B_s - 2B_g}{R_b \times N},$$

where, for a W-TDMA system, $B_g = 0$. In fact, there may exist guard bands in a W-TDMA system. The overhead due to the guard bands can be captured in the modulation efficiency ϵ_{bw}. Under the assumption of perfect statistical multiplexing, a channel corresponds to a constant bit rate of R_b. The overall spectral efficiency of the system in bits/unit time/unit bandwidth/cell, η, can be expressed as

W-TDMA
$B_g = 0$

$$\eta = N_{effective} \times \frac{R_b}{B_s}$$

$$= \epsilon_{bw} \times \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s} \times \frac{B_s - 2B_g}{B_s} \times \frac{1}{N} \text{ bits/s/Hz/cell.} \quad (6.4.14)$$

Example 6.7 Spectral Efficiency of the IS-54 System

Consider IS-54 (updated as IS-136), which is a synchronous N-TDMA/FDD system that uses a one way bandwidth of 25 MHz for the forward (or reverse) channel. The system bandwidth is divided into radio channels of 30 kHz, each supporting transmission at a rate of 16.2 kbps. Guard bands with $B_g = 20$ kHz are used. The frame duration is 40 ms, consisting of 6 time slots. A single radio channel supports 3 full-rate speech channels, each using 2 slots in a frame. Each slot consists of 324 bits, among which 260 bits are for information data and the remaining 64 bits are overhead for access control. The speech codec rate is 7.95 kbps, which corresponds to a gross bit rate of 13.0 kbps with channel encoding. If the frequency reuse factor is 7, find

a. the number of simultaneous users that can be accommodated in each cell cluster,
b. the cell capacity,
c. the spectral efficiency η_{N-TDMA} of TDMA, and
d. the overall spectral efficiency.

Solution Given:

$$B_s = 25 \text{ MHz} = 25000 \text{ kHz}$$

$$B_c = 30 \text{ kHz}$$

$$B_g = 20 \text{ kHz}$$
$$R_b = 7.95 \text{ kbps}$$
$$N_{\text{slot}} = 6/2 = 3$$
$$T_f = 40 \text{ ms}$$
$$\tau_s = 0 \text{ ms}$$
$$\tau_t = 0 \text{ ms}$$
$$L_d = 260 \text{ bits}$$
$$L_s = 324 \text{ bits}$$
$$\epsilon_{bw} = 7.95 \text{ kbps}/30 \text{ kHz} = 0.265$$
$$s_f = 1$$
$$N = 7$$

a. The number of simultaneously transmitting users that can be accommodated in each cell cluster is

$$N_u = \frac{B_s - 2B_g}{B_c} = \frac{25 \times 1000 - 2 \times 20}{30} = 832 \text{ users/cell cluster.}$$

b. The cell capacity of the synchronous TDMA system is

$$N_c = \frac{N_u N_{\text{slot}}}{s_f N} = \frac{832 \times 3}{1 \times 7} \approx 356 \text{ users/cell.}$$

c. The spectral efficiency of the N-TDMA is

$$\eta_{N-TDMA} = \eta_{W-TDMA} \times \frac{B_c N_u}{B_s} = \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s} \times \frac{B_s - 2B_g}{B_s} \approx 0.8.$$

d. The overall spectral efficiency is

$$\begin{aligned}\eta &= \epsilon_{bw} \times \frac{T_f - \tau_p - \tau_t}{T_f} \times \frac{L_d}{L_s} \times \frac{B_s - 2B_g}{B_s} \times \frac{1}{N} \\ &= 0.265 \times \frac{40 - 0 - 0}{40} \times \frac{260}{324} \times \frac{25000 - 2 \times 20}{25000} \times \frac{1}{7} \\ &\approx 0.0303 \text{ bits/s/Hz/cell.}\end{aligned}$$

6.4.3 DS-CDMA Systems

Before attempting to define the spectral efficiency of a DS-CDMA system, let us first consider the cell capacity.

Cell Capacity of DS-CDMA Systems

cell capacity

Definition 6.3 The cell capacity, N_c, of a DS-CDMA system is defined as the maximum number of mobile stations that can be supported during one use of the wireless channel in a single cell, under the constraint that quality of service requirements are met.

The cell capacity of a DS-CDMA system is a function of many system-related factors, as follows:

- Energy/bit — E_b = energy of transmitted signal per information bit
- interference density — I_0 = one-sided interference-plus-noise power spectral density
- noise power — P_n = background noise power
- signal received — S = signal power received at the cell-site receiver
- gain processing — G_p = signal processing gain
- reuse efficiency — η_f = frequency reuse efficiency (to be defined)
- capacity degradation — c_d = capacity degradation factor due to imperfect power control
- sector — Q = number of cell sectors
- source activity factor — s_f = source activity factor

As a function of the preceding parameters, the number of mobile stations, N_{MS}, that can be supported by a DS-CDMA system can be expressed as

$$\left\{ N_{MS} = 1 + \frac{c_d \eta_f}{s_f}\left[\frac{QG_p}{E_b/I_0} - \frac{P_n}{S}\right]. \right\} \qquad (6.4.15)$$

No of mobile stations

Derivation of Eq. (6.4.15). Let us now derive the expression for N_{MS} as given by Eq. (6.4.15). Consider a single base station supporting multiple mobile stations, as shown in Figure 6.12. We will first obtain an expression for the number of mobile stations as a function of E_b/I_0, on the basis that (*a*) each base station uses an omnidirectional antenna, (*b*) there is perfect power control, and (*c*) sources are persistently transmitting information.

Under the assumption that the uplink (from mobile users to the base station) has accurate automatic power control, the signal power received by the base station from each and every

(1) E_b/I_0
(2) omni direction

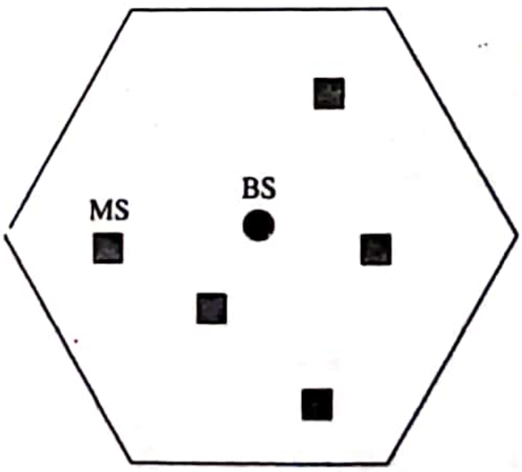

Figure 6.12 Single hexagonal cell.

mobile in the cell is the same, independent of their location within the cell. This power control is designed to circumvent the near–far problem.

The signal power received from the candidate mobile station is

$$S = R_b E_b,$$ // Received signal power

Bitrate

where R_b is the bit rate. The interference power is

$$I = B_s I_0,$$ //Interference power is

where B_s is the one-way system bandwidth. Therefore, the signal-to-interference ratio, S/I, is given by

$$\frac{S}{I} = \frac{R_b E_b}{B_s I_0}. \tag{6.4.16}$$

In the preceding equation, I_0 is the only variable. Recall that the processing gain is $G_p = B_s/R_b$. Then we can write Eq. (6.4.16) as

as $G_p = B_s/R_b$

$$\frac{S}{I} = \frac{1}{G_p}\frac{E_b}{I_0}$$ — QoS

or

(QoS)

$$\frac{E_b}{I_0} = G_p \frac{S}{I}. \tag{6.4.17}$$ — QoS

Both S/I and E_b/I_0 represent the quality of service factor at the cell-site receiver.

Interference is a combination of intracell interference and intercell interference. In addition, there is the background noise. Consider the signal reception from a desired user, say user 1. Transmissions from the other $N_{MS}-1$ users represent intracell interference. Under the assumption of perfect power control, the signal power received from each and every user in the cell is S. Then, the intracell interference power from the $(N_{MS}-1)$ other users is $(N_{MS}-1)S$. Intracell interference

Consider now the interference from neighboring cells due to frequency reuse. In CDMA, the frequency reuse factor is 1, so that intercell interference comes from all neighbors. We can consider interference from different tiers of the cellular array. The first two tiers of interference cells are illustrated in Figure 6.13. Let $k_i, i = 1, 2, 3, \cdots$, denote the normalized interference contribution from all mobiles in a tier i cell due to frequency reuse, normalized with respect to the intracell interference from within the target cell.

The number of neighbors in the different tiers are, six in tier 1, 12 in tier 2, 18 in tier 3, etc. In general, tier i has $6i$ cells. Define the interference factor as

Interference factor as

Total interference Caused ← $$\kappa_f = 1 + 6k_1 + 12k_2 + 18k_3 + \cdots.$$

κ_f is the total intracell and intercell interference (on average) normalized to the total intracell interference. Therefore, the total interference power, including both intracell and intercell interference, is

$$I = [(N_{MS}-1)S] \times \kappa_f.$$ Total Interference Power Because of Intercell & Intracell

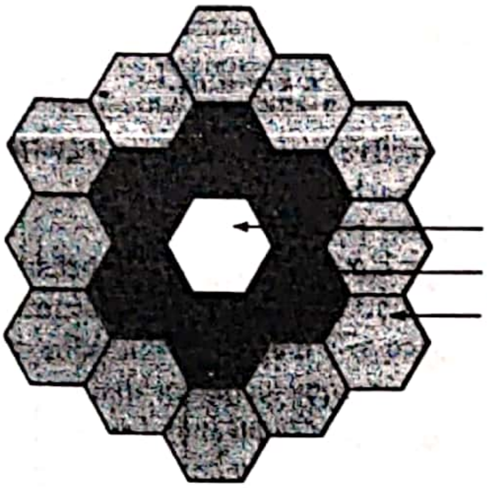

Figure 6.13 Interfering neighbors in hexagonal array.

In addition, there is always background and spurious noise. Let P_n denote the noise power. Then, the total power of interference plus noise is $[(N_{MS}-1)S \times \kappa_f] + P_n$. The SINR can then be expressed as

$$\frac{S}{I} = \frac{1}{(N_{MS}-1)\kappa_f + P_n/S}. \qquad (6.4.18)$$

$\eta_f = 1/\kappa_f$

Define $\eta_f = 1/\kappa_f$ as the frequency reuse efficiency factor. Note that $\eta_f = 1$ for a single-cell system and $\eta_f < 1$ for a multiple-cell system due to intercell interference. Substituting Eq. (6.4.18) into Eq. (6.4.17), the energy-to-interference spectral density ratio can be written as

Energy to interference

$$\frac{E_b}{I_0} = G_p \times \frac{S}{I} = \frac{G_p \times \eta_f}{(N_{MS}-1) + \eta_f P_n/S}. \qquad (6.4.19)$$

Equation (6.4.19) can be rearranged to yield an expression for N_{MS} as

$$\boxed{N_{MS} = 1 + \eta_f \left[\frac{G_p}{E_b/I_0} - \frac{P_n}{S}\right].} \qquad (6.4.20)$$

The derivation of Eq. (6.4.20) assumes

a. perfect automatic power control,
b. unity source activity factor (i.e., persistent transmissions),
c. the cell-site antenna has only one sector (i.e., omnidirectional antenna), and
d. every cell handles the same type of traffic.

If items (*a*) through (*c*) were to be relaxed, the number of mobile stations given in Eq. (6.4.20) would have to be modified by the capacity degradation factor c_d due to imperfect power control, source activity factor s_f, and the number, Q, of cell sectors resulting from use of directive antennas in the cell-site.

Taking into account the source activity factor, s_f, both intracell interference and intercell interference are reduced, but the background noise power remains the same. The average power factor s_f

of total interference and noise is now

$$I' = s_f[(N_{MS} - 1)S \times \kappa_f] + P_n.$$

With Q cell sectors resulting from use of directive antennas, the average power of interference and noise seen at the base station receiver for each sector is reduced to

directive antenna, Q cell sectors

$$I'' = \{s_f[(N_{MS} - 1)S \times \kappa_f] + P_n\}/Q.$$

The SINR is now S/I'', from which we get

$$N_{MS} = 1 + \frac{\eta_f}{s_f}\left[\frac{QG_p}{E_b/I_0} - \frac{P_n}{S}\right]. \tag{6.4.21}$$

With the capacity degradation factor c_d due to imperfect power control, Eq. (6.4.21) is modified to become

$$N_{MS} = 1 + \frac{c_d\eta_f}{s_f}\left[\frac{QG_p}{E_b/I_0} - \frac{P_n}{S}\right],$$

capacity degradation (cd) imperfect power control

which is Eq. (6.4.15). N_{MS} decrease as E_b/I_0 increases

From Eq. (6.4.15), we see that the number of mobile stations that can be supported decreases as the E_b/I_0 value increases. The BER of any modulation system is a function of E_b/I_0. Smaller values of E_b/I_0 correspond to larger values of BER. To achieve an acceptable BER, the E_b/I_0 has to exceed a prescribed threshold. For example, with an efficient modem, powerful channel coding (using convolutional code) and two-branch diversity, to achieve satisfactory transmission quality specified as a BER of 10^{-3} on the uplink, the required E_b/I_0 is 7 dB. The cell capacity, N_c, should therefore be defined in terms of an E_b/I_0 requirement. Let $(E_b/I_0)^*$ be the QoS specification. Acceptable BER performance requires that $E_b/I_0 \geq (E_b/I_0)^*$. The cell capacity, from Definition 6.3, is then

Erlang capacity

$$\boxed{N_c = 1 + \frac{c_d\eta_f}{s_f}\left[\frac{QG_p}{(E_b/I_0)^*} - \frac{P_n}{S}\right].} \tag{6.4.22}$$

N_c, as given by Eq. (6.4.22), is known as the Erlang capacity. The right-hand side of Eq. (6.4.22) is derived on the basis that each user requires the same E_b/I_0^* to satisfy the same target BER requirement under all propagation conditions, where I_0^* is the maximum total acceptable interference power spectral density and E_b is a constant. In addition, the number of users in the cell is assumed fixed. In a cellular system in which the users are in motion, the number of users in a cell at a given epoch is a random variable. The actual number of active users that can be supported in a cell will be smaller than the Erlang capacity given by Eq. (6.4.22).

Definition 6.4 The system utilization is defined as the number of users, U_{MS}, that can be supported under the constraint that $E_b/I_0 \geq (E_b/I_0)^*$.

No of users

The system utilization is then given by

$$U_{MS} = 1 + \frac{c_d\eta_f}{s_f}\left[\frac{QG_p}{E_b/I_0} - \frac{P_n}{S}\right], \quad \text{with } E_b/I_0 \geq (E_b/I_0)^*. \tag{6.4.23}$$

As CDMA systems are highly complex, the overall capacity depends on many factors, such as power control error, soft handoff, etc., whose effects on the capacity are difficult to model. As a result, the preceding derivation of the cell capacity and system utilization of the CDMA system is a first approximation.

Example 6.8 Capacity and Utilization of CDMA System

Consider a DS-CDMA system whose one-way system bandwidth is 25 MHz. Suppose the data rate per user is 8 kbps. Assume perfect power control, one sector antenna, and persistent transmissions.

a. Calculate the E_b/I_0 specification required to support a maximum number of 250 users/cell if the signal-to-background noise ratio is 26 dB and the frequency reuse efficiency is 0.9.
b. If the actual operating value of E_b/I_0 is 12 dB, calculate the system utilization for the parameter values specified in part (*a*).

Solution

a. With the assumptions given in the problem, the E_b/I_0 specification can be written as

$$(E_b/I_0)^* = \frac{\eta_f G_p}{N_c - 1 + \eta_f P_n/S}.$$

It is given that $S/P_n = 26$ dB or 400, $\eta_f = 0.9$, $G_p = B_s/R_b = 25 \times 1000/8 = 3125$, and $N_c = 250$. Substituting these values in the preceding equation for $(E_b/I_0)^*$ yields

$$(E_b/I_0)^* = \frac{0.9 \times 3125}{250 - 1 + 0.9/400} \approx 11.25 \text{ or } 10.5 \text{ dB}.$$

b. The utilization can be expressed as

$$U_{MS} = 1 + \eta_f \times \left[\frac{G_p}{E_b/I_0} - \frac{P_n}{S}\right], \quad \text{with } E_b/I_0 \geq 10.5 \text{ dB}.$$

If the actual operating value of E_b/I_0 is 12 dB, or 15.85, then the system utilization is

$$U_{MS} = 1 + 0.9 \times \left[\frac{3125}{15.85} - 1/400\right] \approx 178.$$

Spectral Efficiency of DS-CDMA Systems. The spectral efficiency, η, of a DS-CDMA system can be represented as a unitless quantity or in bits/unit time/unit bandwidth. It is defined as

$$\eta = \frac{U_{MS}}{N_c} \div \frac{(E_b/I_0)^*}{E_b/I_0}, \quad E_b/I_0 \geq (E_b/I_0)^* \tag{6.4.24}$$

or

$$\eta = U_{MS} \times \frac{s_f R_b}{B_s} \text{ bits/s/Hz}, \tag{6.4.25}$$

where R_b is the constant bit rate (in units of bits/s/user) when a user is in an *on* state, and B_s is the one-way system bandwidth in Hz. Note that, unlike the FDMA and TDMA systems, it is not necessary to represent the efficiency η in terms of bits/s/Hz per km^2, as in DS-CDMA the frequency spectrum is reused from cell to cell.

Example 6.9 Spectral Efficiency of DS-CDMA

Consider the CDMA system in Example 6.8, with the same assumptions about parameter values.

a. If the actual operating value of E_b/I_0 is 12 dB, find the spectral efficiency for this CDMA system.
b. Suppose the actual operating value of E_b/I_0 is now 11 dB. Find the spectral efficiency.
c. Discuss the impact of operating at a lower value of E_b/I_0 in terms of spectral efficiency and system utilization. What would be the benefit, if any, to the service provider?

Solution

a. For $E_b/I_0 = 12$ dB, the number of users that can be supported is 178 (Example 6.8), and the system capacity is 250. Therefore, the spectral efficiency is

$$\eta = 178/250 = 71.2\%$$

or

$$\eta = 178 \times \frac{8}{25 \times 1000} = 0.005696 \text{ bits/s/Hz}.$$

b. If the value of E_b/I_0 is now 11 dB or 12.59, the system utilization is

$$U_{MS} = 1 + 0.9 \times \left[\frac{3125}{12.59} - \frac{1}{400}\right] \approx 224.$$

Therefore, the spectral efficiency is

$$\eta = 224/250 = 89.6\%$$

or

$$\eta = 224 \times \frac{8}{25 \times 1000} = 0.007168 \text{ bits/s/Hz}.$$

c. A reduction in the operating value of E_b/I_0 from 12 dB to 11 dB is accompanied by an increase in spectral efficiency from 71.2% to 89.6%. With a 1 dB decrease in E_b/I_0, the system utilization is increased from 178 to 224 users. This is significant from the revenue point of view for the service provider.
With the system operating at $E_b/I_0 = 11$ dB, which is 0.5 dB above the QoS requirement, the resultant BER would be acceptable to the mobile users.

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SUMMARY

Wireless communication systems are characterized by multiple access in the uplink (reverse channel) and broadcast in the downlink (forward channel). The capacity of the system, in terms of the number of users that can be supported during any one use of the wireless channel, depends on the multiple access technology used. The commonly used multiple access technologies are of the conflict-free type or, at worst, exhibit minimal interference. These technologies include FDMA, TDMA and CDMA. FDMA is narrowband and has been used as the multiple access technology in the first generation wireless standards. TDMA, which can be narrowband or wideband, is the multiple access technology of the second generation wireless standards. CDMA, an interference-limited multiple access technology, has been used as a second generation wireless standard (IS-95). This chapter has discussed the implementation issues and the spectral efficiency of each of these three multiple access techniques. As random access is often used for users to gain initial access to conflict-free multiple access systems, two random access schemes (i.e., Aloha and CSMA) have also been studied.

The first and second generation wireless systems mainly support voice communications. The information rate of these systems, as tabulated in Tables 1.1 and 1.2, are relatively low. Third generation wireless systems are expected to support multimedia services, with transmission rates up to 2 Mbps. CDMA has been targeted as the multiple access technology for the third generation wireless standards.

ENDNOTES

1. For key developments in the theory and practice of multiple user communication channels, see the book edited by Abramson [3]. In particular, for Aloha random access, see the papers by Abramson [1, 2] and by Namislo [104]; for CSMA, see the paper by Kleinrock and Tobagi [73]; for TDMA, see the book chapter by Campanella and Schaefer [21] and papers by Dill [38] and by Falconer, Adachi and Gudmundson [49].
2. For principles of spread spectrum systems, see the papers by Scholtz [135] and by Pickholtz, Schilling, and Milstein [116], and textbooks by Dixon [40] and by Peterson, Ziemer, and Borth [114].
3. As the capacity of CDMA systems is interference limited, an essential approach to achieve maximal capacity is multiuser detection, where information about multiple users is used to improve detection of each individual user [100]. The main challenge in multiuser detection is the complexity of the optimal receiver, which increases exponentially with the number of active users and with the delay spread of the channel. Various techniques have been proposed to reduce the receiver complexity while achieving near-optimum performance. For details, see the book by Verdu [154], the special issue of the *IEEE Journal on Selected Areas in Communications* [28], and papers by Divsalar, Simon and Raphaeli [39], Honig and Xiao [64], Müller [96], and Tse and Hanly [152].
4. In Subsection 6.4.3 on the cell capacity of DS-CDMA systems, the goal of accurate automatic power control is to equalize the received signal power levels from all the mobile users in the cell in order to increase the cell capacity. Power equalization (also referred to as channel

inversion) is common in spread spectrum systems to overcome the near–far problem and is simple to implement. However, the approach may not achieve the maximum capacity in extreme fading (such as Rayleigh fading) environments. When side information about the current channel state is available to both the transmitter and receiver, the optimal adaptive transmission scheme should use water-pouring in time for power adaptation. For details, see the papers by Goldsmith and Varaiya [55] and by Shamai and Verdú [138].

5. In DS-CDMA systems, spread spectrum is achieved by modulation using PN sequences. Prior to data transmission, the receiver needs to accurately synchronize the locally generated PN sequence waveform with the incoming PN sequence waveform. As a result, PN sequence synchronization is an essential part of CDMA systems. The synchronization is usually achieved in two steps (i.e., acquisition and tracking). The acquisition process is to achieve coarse alignment within some fraction of one chip interval between the two PN waveforms, and the tracking process is to achieve fine alignment between the PN waveforms which further reduces synchronization error to an allowed limit. For details, see the papers by Polydoros and Weber [118, 119], Sourour and Gupta [144, 145], Spilker, Jr. and Magill [67], Meyr [97], and Sheen and Stuber [137].
6. For CDMA in wireless communications, see the book by Viterbi [155] and papers by Kohno, Meidan and Milstein [75] and Lee [80]. For further details on Erlang capacity in CDMA systems, see the papers by Gilhousen *et al.* [54] and the book by Viterbi [155]; for spectrum efficiency, see the paper by Lee [79]; and for power control and power allocation, see the papers by Ariyavistakul and Chang [8], by Sampath, Kumar and Holtzman [134], and by Mark and Zhu [94].
7. The three pure multiple access schemes can be combined for hybrid multiple access, such as hybrid TDMA/CDMA and multi-carrier (MC)-CDMA. In hybrid TDMA/CDMA, statistical multiplexing can be achieved in both time and code domains [65, 66, 6]. Time is partitioned into frames and further into slots as in TDMA. Within each time slot, multiple packet transmissions are possible by using orthogonal PN codes as in CDMA. There are two types of MC-CDMA [58, 74]. One is a combination of orthogonal frequency division multiplexing (OFDM) and CDMA, where the spread sequence is serial-to-parallel converted and each chip modulates a different carrier frequency. The other is a parallel transmission scheme of narrowband DS waveforms in the frequency domain, where the available frequency spectrum is divided into a number (typically much smaller than the processing gain) of equal-width frequency bands and each frequency band is used to transmit narrowband DS-CDMA signals.
8. For the evolution toward wideband CDMA in third generation wireless communication systems, see the paper by Prasad and Ojanperä [121].
9. For in-depth analysis of CDMA system performance, see the textbook by Stüber [147].

PROBLEMS

P6-1 Verify that, with orthogonal spreading sequences, transmission accuracy in AWGN with spread spectrum modulation is the same as that without spread spectrum, assuming that coherent BPSK is used.

P6-2 Random access is a contention medium access control protocol used in the Aloha systems.

a. Show that the maximum throughput of pure Aloha and slotted Aloha are given respectively by Eqs. (6.2.6) and (6.2.7).
b. Plot the throughput S versus offered load G graphs for both pure Aloha and slotted Aloha, and comment on the properties characterized by the $S \sim G$ curves.
c. Plot the delay versus throughput curves for pure Aloha and slotted Aloha, and describe the behavior displayed by these curves.

P6-3 The tree search for the collision resolution algorithm described in Example 6.1 can be implemented by a software program to facilitate the handling of large user populations.

a. Write a program to implement the collision resolution algorithm.
b. Test your program for correctness by calculating the length of the collision resolution interval in Example 6.1, and then do the same for a larger population, say 100 users, for the ith collision resolution interval by assuming that a subset of the users have packets ready for transmission in the ith collision resolution interval.

P6-4 CSMA is a contention access protocol with partial coordination among the multiple users.

a. Explain why and how CSMA should provide better throughput performance than the Aloha systems. Plot the throughput curves and derive your explanation from a comparison of the characteristics displayed by the curves.
b. Compare the delay-throughput characteristics of Aloha and CSMA. The implementation of CSMA is slightly more complex compared to Aloha. At the same throughput, say 0.3, which protocol (slotted Aloha or unslotted CSMA) incurs a larger delay?

P6-5 Consider digital transmission using spread spectrum with BPSK in an AWGN channel. There is a pulse noise jammer in the system, which transmits pulses of white Gaussian noise within the system frequency spectrum (W Hz) with an average power of P_J. The ratio of the average *on* period to the average *off* period of the jamming signal is α. The one-sided psd of the background noise is N_0. The received signal is detected coherently.

a. Derive the BER.
b. Determine the α value which results in the maximal BER.
c. Determine the maximal BER using the α value obtained in (*b*).
d. Does the spread spectrum offer any advantage in suppressing the jamming signal?

P6-6 Consider the forward link transmission of a single-cell DS-CDMA system using BPSK, where the processing gain is 256. The transmitted signals for all the users are synchronized both in chip and in carrier phase with the same power level. Assume that background noise is negligible when compared with the multiple access interference. Determine the maximum number of users allowed in the system in order to achieve a BER of 10^{-3}, 10^{-4}, and 10^{-5}, respectively, with coherent detection.

P6-7 Consider a metropolitan city with a total area of 1500 km^2 to be covered by a hexagonal cellular system with 7-cell reuse pattern. Suppose each cell has a radius (R) of 5 km and the city is allocated 25 MHz of spectrum, with a full-duplex channel bandwidth of 30 kHz and total 40 kHz guard bands. The system uses FDMA with 14 control channels. Determine

a. the number of cells in the service area,
b. the number of channels without frequency reuse,
c. the cell capacity, and
d. the system spectral efficiency in channels/MHz/km^2.

P6-8 In the system of Problem 6-7, it is known that, on average, each user makes two calls in a busy hour and each call lasts 2 minutes. Given that the trunking efficiency is 0.9, calculate

a. the number of calls per hour per cell,
b. the average number of users served per hour per cell, and
c. the system spectral efficiency in Erlangs/MHz/km^2.

P6-9 The Personal Digital Cellular (PDC) TDMA system uses a 42 kbps data rate to support 3 users per frame. Each user occupies 2 of the 6 time slots per frame.

a. What is the raw data rate provided for each user?
b. If the frame efficiency is 80% and the frame duration is 6.667 ms, determine the number of information bits sent to each user per frame. Assume no overhead at the slot level.
c. If speech coding that reduces the data rate by half is used, then 6 users per frame can be accommodated. Determine the number of information bits provided for each user per frame.
d. What is the information rate per user in this half-rate compressed PDC system?

P6-10 In GSM, the total system bandwidth (50 MHz) is divided into 25 MHz for uplink and 25 MHz for downlink. There are a total of 125 duplex RF carriers, each having a bandwidth of 2 × 200 kHz with a frequency separation of 45 MHz between the uplink and downlink channels. Each carrier supports 8 users using TDMA with a frame duration of 4.615 ms. Using GMSK, the bandwidth efficiency is 1.35 bits/s/Hz. The speech codec rate is 13 kbps. Channel coding results in a coded bit rate of 22.8 kbps. Consider only the normal speech frames. Each frame consists of 8 slots, and each time slot has 156.25 bits–3 start bits, 116 coded speech bits, 26 training bits, 3 stop bits, and 8.25 bits for guard period. Assume a frequency reuse factor of 7. Determine

a. the number of simultaneous users that can be accommodated in each cell cluster,
b. the cell capacity,
c. the spectral efficiency η_{N-TDMA} of TDMA, and
d. the overall spectral efficiency in bits/s/Hz/cell.

P6-11 A GSM system has a service area of 4500 km^2. Each cell has a coverage area of 7.5 km^2. During a busy hour, on average, each user makes 2 calls and each call lasts 5 minutes. If the trunking efficiency is 0.85, using the GSM system parameters described in Problem 6-10, determine

a. the number of cells in the system,
b. the number of calls per hour per cell,
c. the average number of users served per hour per cell, and
d. the overall system spectral efficiency in Erlangs/MHz/km^2.

P6-12 In an omnidirectional CDMA cellular system, the required E_b/I_0 is 10 dB. If 250 users, each with a baseband data rate of 13 kbps, are to be accommodated, determine the minimum channel chip rate of the spread spectrum sequence for the following two cases:

a. ignoring voice activity considerations,
b. the voice activity factor is 0.4.

P6-13 Consider a DS-CDMA cellular system.

a. Discuss why power control is needed in a CDMA system.
b. Discuss how each of the following factors influence the capacity of the DS-CDMA system: (1) the processing gain G_p, (2) antenna sectorization, (3) the required E_b/N_o value, (4) imperfection in automatic power control.
c. If the following factors are specified, determine the system capacity and the spectral efficiency of the CDMA system: (1) frequency reuse efficiency = 0.55, (2) capacity degradation due to imperfect power control = 0.9, (3) E_b/N_o = 10 dB, (4) information bit rate = 16.2 kbps, (5) system bandwidth = 12.5 MHz, (6) neglecting all other sources of interference.

P6-14 Consider a CDMA cellular system using hexagonal cells with BPSK modulation for voice services. The allocated radio spectrum for reverse link has a bandwidth of W Hz. The data transmission rate for each user is R_b bps. The additive background noise component is negligible. To provide satisfactory service quality, it is required that the received signal energy per bit to interference density ratio should be at least γ_r. The source activity factor is s_f. Assuming perfect power control, derive the cell capacity of the CDMA system.

P6-15 Suppose 61 users share a CDMA system, and each user has a processing gain of 480. Assume that (1) the power control on the uplink is perfect, (2) BPSK modulation is used, (3) the signal transmitted by each user is a Gaussian process, and (4) the system is interference limited so that the background noise can be neglected. Determine the probability of bit error for each user.

P6-16 On the basis that the total interference seen at the cell-site receiver is given by the sum of intracell interference, intercell interference, and background noise, show that the Erlang capacity of a CDMA-based radio cell is given by Eq. (6.4.22). To facilitate your derivation, make the same assumptions as those used in the alternate derivation of the Erlang capacity as given by Eq. (6.4.22). (Hint: Let I_0 be the power spectral density of the total interference and W be the spread spectrum bandwidth. Then the total interference is $I_0 W$. Let I_0^* be the power spectral density of the acceptable total interference. Then, to satisfy the quality of service requirement, the total interference $I_0 W$ at anytime must satisfy the inequality $I_0 W \le I_0^* W$.)

P6-17 Consider the uplink transmission of voice signals in a single-cell DS-CDMA system with a processing gain of 256. Here, we are interested in studying the effect of power control error on the cell capacity by computer simulation. For simplicity, the following assumptions are made:

(1) The required transmission accuracy is specified by a required E_b/I_0 of 7 dB.
(2) The background additive noise is negligible when compared with the multiple access interference.

(3) The transmission is frame based with a frame duration of 5 ms. All the transmissions from mobile users are synchronized in frame.
(4) Different users have independent power control errors. For each user, the power control error is independent from frame to frame. Over each frame duration, the power control error for each user remains constant.
(5) Each power control error (in dB) can be modeled as a Gaussian random variable with zero mean and standard deviation σ (also in dB).

Due to the power control error, there are chances that the required E_b/I_0 cannot be guaranteed from time to time. It is required that the outage probability, defined as $P(E_b/I_0 < 7\text{dB})$, should be kept below α.

a. Given $\alpha = 5\%$, determine the cell capacity for $\sigma = 1$ dB, 2 dB, and 3 dB, respectively.
b. Given $\sigma = 1$ dB, determine the cell capacity for $\alpha = 1\%$, 2%, and 5%, respectively.
c. Compute the cell capacity under the assumption of perfect power control.
d. Comment on the effects of the power control error and the required outage probability bound on the cell capacity.

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