Channel Estimation

TEQIP-II Sponsored Short Term Training Program on
Wireless Network and Mobile Computing
organized by Computer Engineering Department
Sardar Vallabhbhai National Institute of Technology, Surat
16-20 May 2016

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Channel Estimation

- channel state information (CSI) channel properties of a communication link
- describes how a singal propagates from transmitter to receiver
- makes it possible to adapt transmissions to current channel conditions
- crucial for achieving reliable communication with high data rates
- channel estimation classified into three classes:
- training based, blind and semi-blind

Channel Estimation

- channel model as mathematical representation of transfer characteristics of physical medium
- channel esitmation as the process of characterising the effect of physical channel on the input sequence
- receiver to approximate impulse response of the channel
- once model established, its parameter need to be updated
- estimated in order to minimize the error as the channel changes
- minimize MMSE $E[e^2(n)]$
- if receiver has a-priori knowledge of information being sent over the channel
- it can utilize this knowledge to obtain an accurate estiamte of impulse response of the channel
- this method is called training sequence based channel estimation

Channel Estimation

- wasteful of bandwidth - training sequence transmitted for channel estimation
- most systems send information lumped frames, after receipt of frame
- channel estimate can be extracted from the embedded training sequence
- for fast fading channels not adequate - since coherence time of channel might be shorter that the frame time
- blind methods - no training sequence
- utilize underlying mathematical information about the kind of data being transmitted
- bandwidth efficient, slow to converge (more than 1000 symbols may be required for an FIR channel with 10 coefficient).
- computationally intensive - impractical to implement in real time system

Channel Estimation

- training based
  - long training for reliable channel estimate
  - reduces bandwidth efficeincy
- blind methods
  - no training
  - CSI acquired by relying on the received signal statistics
  - using statistical information - solve convergence problem
  - achieves high system throughput
  - high computational complexity
- semi-blind
  - combination of two procedures
  - few training symbols along with blind

Channel Estimation: Pilot based estimation

- pilot based - training based channel estimation
- pilot symbols are used with information symbols in the transmission frame
- pilots are fixed set of symbols which are known at the receiver
- from the received output of pilot symbols, estimation of channel can be performed
- employed for detection of the information symbols transmitted subsequently
- benefit of robust estimate and low computational complexity
- drawback - pilot symbols carry no information - overhead on communication system
- results in wastage of bandwidth - bandwidth inefficient
- least square, minimum mean square error, maximum likelihood, maximum a posteriori can be employed

Channel Estimation: Blind estimation

- does not require pilot symbols - bandwidth efficient
- channel can be estimated using statistical knowledge of received output symbols
- if transmitter employs a symmetric transmit constellation with equal priori probabilities
- then the received symbol stream has a statistical mean of zero
- with knowledge of covariance of the input information symbol
- the computed covariance of output information symbols can be employed to estimate at least part of channel
- computationally complex and having convergence problem
- not attractive where robustness of estimate and computational complexity are critical
- widely known techqunice is subspace method using second order statistics (SOS)

Channel Estimation

- in subspace method, autocorrelation matrix of received signal is decomposed into the signal and noise subspaces
- due to orthogonally of the noise anc signal subspace,
- the channel estimates can be calculated based on the noise subpsace
- decomposition of autocorrelation fucntion via eigen value decomposition or singular value decomposition used
- QR decomposition restricts direct matrix inversion and convert full rank channel matrix into simple form - low complexity
- Semi-blind estimation
- combination of both pilot and blind channel estimation
- low complexity with robustness by using limited number of pilot symbol
- and bandwidth efficiency by using statistical blind informtation

Channel Estimation

- quality of channel estimate is enhanced by employing statistical information to aid estimation process or
- minimize the number of pilot symbols transmitted by employing statstical information
- to improve the nature of channel estiammte, increasing bandwidth efficiency
- CSI generated by channel estimation, sent into detection block or fed back to transmitter side to construct beam forming weight vector
- different pilot symbol arrangements:
- estimator with block type pilot (training based)
- estimator with comb type pilot (pilot symbol aided modulation)

Channel Estimation: LS and MMSE

- estimator takes measurement data as inputs and produces estimated values of parameters

$$Y = XH + \eta$$

rewriting

$$Y = Xh + \eta$$

- H and h are unknown vectors, X is known matrix, Y is measurement matrix
- LS channel estimation
- channel estimation $\hat{h}$ for equation $X\hat{h} \approx Y$
- in LS minimization of Euclidean norm squared to the residual $X\hat{h} - Y$

$$\arg_h \min \| X\hat{h} - Y \|^2$$

Channel Estimation: LS and MMSE

$$\| X\hat{h} - Y \|^2 = \left(X\hat{h} - Y\right)^H (X\hat{h} - Y)$$

$$= \left(X\hat{h}\right)^H \left(X\hat{h}\right) - Y^H X\hat{h} - \left(X\hat{h}\right)^H Y + Y^H Y$$

- minimum is found at the zero of derivative with respect to $\hat{h}$

$$2X^H X\hat{h} - 2X^H Y = 0 \Rightarrow X^H X\hat{h} = X^H Y$$

$$\hat{h} = \left(X^H X\right)^{-1} X^H Y$$

Channel Estimation: LS and MMSE

- MMSE channel estimation
- optimal result by exploiting statistical dependence between measured data and estimated parameters
- signal source $\rightarrow$ mutlipath channel $\rightarrow$ + noise $\rightarrow$
- receiver filter $\rightarrow$ channel estimator $\rightarrow$ MMSE detector
- minimizing $E[(h - \hat{h}_{MMSE})^2]$

$$\hat{h} = R_{hY} R_{YY}^{-1} Y$$

- R_{hY} and R_{YY} are cross covariance matrices between h and Y and autocovariance matrix of Y respectively

Channel Estimation: LS and MMSE

$$R_{hY} = E[hY^H] = E[h(Xh+\eta)^H] = R_{hh}X^H$$
$$R_{YY} = E[YY^H] = E[(Xh+\eta)(Xh+\eta)^H]$$
$$= E[Xh(Xh)^H + Xh\eta^H + \eta(Xh)^H + \eta\eta^H]$$
$$= XR_{hh}X^H + \sigma_n^2 I$$

- $R_{hh} = E[hh^H]$ is autocovariance matrix of h
- σ_n^2 noise covariance $E[\eta\eta^H]$
- these two quantities are assumed to be known at the estimator
- channel estimate can be written as

$$\hat{h} = R_{hh}X^H(XR_{hh}X^H + \sigma_n^2 I)^{-1}Y$$

Channel Estimation: Maximum a posteriori (MAP)

- requires knowledge of the training sequence, the channel covariance, and the noise covariance at the receiver
- system model described for LS estimation applies to MAP estimation
- maximizes $p(H|Y,X)$ with respect to H
- MAP estimate for H satisfy

$$\frac{\partial \ln(P(H|Y,X))}{\partial H}|H = \hat{H}_{MAP} = 0$$

using Bayes' rule

$$P(H|Y,X) = \frac{p(Y|H,X)p(H,X)}{p(Y|X)}$$

$$\hat{H}_{MAP} = (X^H C_n^{-1} X + C_H)^{-1} X^H C_n^{-1} Y$$

Channel Estimation

- noise covariance $C_n = R_{\eta\eta} = E[\eta\eta^H]$
- channel covariance $C_H = R_{HH} = E[HH^H]$
- for independent Rayleigh fading channels, C_H can be approximated as an identity matrix
- block type pilot - continuous pilot blocks to obtain channel impulse resposne on all sub-carriers
- the length of training block is fixed to the number of sub-carriers in the block
- comb-tye pilot - channel changes even from one block to subsequent one

Channel Estimation using Pilot

- $y = Mh + n$
- channel impluse response $h = [h_0 h_1 \dots h_L]^T$
- within each transmission burst the transmitter sends a unique training sequence which divided into
- a reference length of P and guard period of L bits
- $m = [m_0 m_1 \dots m_{P+L-1}]^T$ bipolar elements $m_i \in \{-1, +1\}$
- circulant training sequence matrix M is formed as

$$M = \begin{bmatrix} m_L & \dots & m_1 & m_0 \\ m_{L+1} & \dots & m_2 & m_1 \\ \ddots & \ddots & \ddots & \ddots \\ m_{L+P-1} & \dots & m_P & m_{P-1} \end{bmatrix}$$

- LS channel estimates $\hat{h} = \arg\min_h \| y - Mh \|^2$
- assuming white Gaussian noise

$$\hat{h}_{LS} = (M^H M)^{-1} M^H y$$

Channel Estimation

- signal multipath - multi propagation paths, separate phase, attenuation, delay and
- doppler frequency - they add up destructively - called fading
- $y(t) = \sum_{i=1}^{N} \alpha_i s(t - \tau_i(t))$
- N paths arriving at receiver, α and τ attenuation and delay

$$s(t) = \text{real part of } \{\tilde{s}(t)e^{j2\pi f_c t}\}$$

$$\tilde{y}(t) = \sum_{i=1}^{N} \tilde{\alpha}_i \tilde{s}(t - \tau_i(t))$$

- f_c carrier frequency, $\tilde{\alpha}_i = \alpha_i e^{j2\pi f_c t}$ time varying complex attenuation of each path
- time varying discrete multipath channel by time varying complex impulse response

$$\tilde{h}(\tau; t) = \sum_{i=1}^{N} \tilde{\alpha}_i \delta(t - \tau_i(t))$$

Channel Estimation

- modelling channel tap gain as an auto regressive process
- complex gaussian random process can be represented by a general auto regressive model
- any stationay random process can be represented as an infinite tap AR process
- infinite tap AR process model is impractical, truncated to N-tap form
- AR process represented by a difference equation

$$S(n) = \sum_{i=1}^{N} \phi_i S(n - i) + w(n)$$

- $S(n)$ complex gaussian process, ϕ_i parameters of the model
- N number of delays in the autoregressive model
- $w(n)$ sequence of identically distributed zero-mean complex gaussian random variables

Channel Estimation

- $E\{w(n)\} = 0$

$$E\{w(n)w(j)\} = \left\{ \begin{array}{ll} s_n^2 & \text{for } n = j \\ 0 & n \neq j \end{array} \right\}$$

$$f_{w(n)}(\lambda) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{\frac{-\lambda^2}{2\sigma_n^2}}$$

- sequence $w(n)$ white gaussian noise because its spectrum is broad and uniform over an infinite frequency range
- AR process is another name for a linear difference equation model driven by gaussian noise
- Nth order difference equation can be reduced to a state model in the vector form

$$\bar{S}(n) = F\bar{S}(n-1) + \bar{W}(n)$$

- $\bar{S}$ and $\bar{W}(n)$ column vectors of size $N \times 1$ and F is $N \times N$ matrix

Channel Estimation

- mean

$$\mu_s = E[S(n)] = E\left[\sum_{i=1}^{N} \phi_i S(n - i) + w(n)\right] = 0$$

- variance

$$\sigma_S^2 = E\{S(n)S(n)\} = E\left\{S(n)\left(\sum_{i=1}^{N} \phi_i S(n - i) + w(n)\right)\right\}$$

$$= \sum_{i=1}^{N} \phi_i R_{SS}(i) + \sigma_n^2$$

- autocorrelation

$$R_{SS}(m) = E\{S(n-m)S(n)\} = E\left\{\left[\sum_{i=1}^{N} \phi_i S(n - i) + w(n)\right] S(n - m)\right\}$$

$$= \sum_{i=1}^{N} \phi_i R_{SS}(m - i)$$

Channel Estimation

- autocorrelation coefficeint

$$r_{SS}(m) = \frac{R_{SS}(m)}{s_X^2} = \sum_{i=1}^{N} \phi_i r_{SS}(m-i)$$

$m \geq 1$

- N th order difference equation can be solved for desired AR coefficients

$$\begin{bmatrix} 1 & r_{SS}(1) & \dots & r_{SS}(N-1) \\ r_{SS}(1) & 1 & \dots & r_{SS}(N-2) \\ \ddots & \ddots & \dots & \ddots \\ r_{SS}(N-1) & \dots & \dots & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} = \begin{bmatrix} r_{SS}(1) \\ r_{SS}(2) \\ \vdots \\ r_{SS}(N) \end{bmatrix}$$

matrix equation is known as Yule-Walker equation

- $\bar{R}\phi = \bar{r}_{SS}$ $\phi = \bar{R}^{-1}\bar{r}_{SS}$
- matrix of AR coefficeints that models the complex gaussian process
- given autocorrelation of the process using Yule-Walker equation calculate AR coeffcients

Channel Estimation

- data based channel estimator uses training sequences sent over the channel to estimate the impulse response of the channel
- channel estimation using correlation method
- say, training seuqnece of length M known to receiver is sent over the channel
- assumed that the channel does not change over the span of data sent

$$\bar{x} = [x_0 x_1 \dots x_{M-1}]^T$$

- this bit sequence mapped to unit energy symbols bit 0 → +1 symobl and bit 1 → -1 symbol to simulate BPSK modulation
- channel impulse response when the training sequence is sent over the channel

$$\bar{h} = [\bar{h}_0 \bar{h}_1 \bar{h}_2 \dots \bar{h}_{L-1}]^T$$

- L is channel impulse response length or the number of processes tracked

Channel Estimation

- signal received is the convolution sum of the signal sent and the impulse response received

$$\bar{y} = \bar{x} * \bar{h} + \bar{n}_c$$

$$y(n) = \sum_{m=0}^{L-1} h(m)x(n-m) + n_c$$

matrix form

$$\bar{y} = \begin{bmatrix} x_0 & 0 & . & 0 \\ x_1 & x_0 & . & . \\ . & x_1 & . & 0 \\ x_{M-1} & . & . & x_1 \\ 0 & x_{M-1} & . & . \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & . & . x_{M-1} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix} + \begin{bmatrix} n_{c_0} \\ n_{c_1} \\ \vdots \\ n_{c_{L+M-1}} \end{bmatrix}$$

$$\bar{Y} = \bar{X}\bar{h} + \bar{n}_c$$

Channel Estimation

- $\bar{X}$ Toeplitz matrix containing delayed versions of the training sequence sent
- gaussian channel noise variance σ_c^2 of $\bar{n}_c$
- $E_b = 1$ SNR of channel given by $\frac{E_b}{N_o} = \frac{1}{2\sigma_c^2}$
- following general linear regression method, the estimate of channel is given by

$$\hat{\bar{h}} = (\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{Y})$$

$$\hat{\bar{h}} = (\bar{X}^T\bar{X})^{-1}(\bar{X}^T(\bar{X}\bar{h} + \bar{n}_c))$$

$$= (\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{X})\bar{h} + (\bar{X}^T\bar{X}^{-1}(\bar{X}^T\bar{n}_c)$$

$$= \bar{h} + (\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)$$

- error $\tilde{\bar{h}} = \hat{\bar{h}} - \bar{h} = (\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)$

Channel Estimation

- expection of estimation error

$$E\left[\tilde{\bar{h}}\right] = E\left[(\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)\right] = (\bar{X}^T\bar{X})^{-1}(\bar{X}^T E[\bar{n}_c])$$

- channel noise is zero mean $E\left[\tilde{\bar{h}}\right] = 0$
- estimator is unbiased
- error covariance

$$P_D = E\left[\tilde{\bar{h}}\left(\tilde{\bar{h}}\right)^H\right]$$

$$= E\left\{\left[(\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)\right]\left[(\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)\right]^H\right\}$$

$$P_D = E\left\{\left[(\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)\right]\left[(\bar{X}^T\bar{n}_c)^H((\bar{X}^T\bar{X})^{-1})^H\right]\right\}$$

$$= E\left\{(\bar{X}^T\bar{X})^{-1}(\bar{X}^T\bar{n}_c)(\bar{n}_c^H\bar{X})(\bar{X}^T\bar{X})^{-1}\right\}$$

$$= (\bar{X}^T\bar{X})^{-1}\bar{X}^T E\left(\bar{n}_c\bar{n}_c^H\right)\bar{X}(\bar{X}^T\bar{X})^{-1}$$

Channel Estimation

$$= (\bar{X}^T\bar{X})^{-1}\bar{X}^T\{\sigma_c^2 I\}\bar{X}(\bar{X}^T\bar{X})^{-1}$$

$$= \sigma_c^2\left[(\bar{X}^T\bar{X})^{-1}\left(\bar{X}^T\bar{X}\right)(\bar{X}^T\bar{X})^{-1}\right]$$

$$= \sigma_c^2(\bar{X}^T\bar{X})^{-1}$$

H is hermetian transpose of a matrix defined as complex conjugate of standard transpose

$(\bar{X}^T\bar{X}) =$

$$\begin{bmatrix} x_0 & x_1 & . & . & x_{M-1} & 0 & . & 0 \\ 0 & x_0 & x_1 & . & . & x_{M-1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & . & 0 & x_0 & x_1 & . & . & x_{M-1} \end{bmatrix} \begin{bmatrix} x_0 & 0 & . & 0 \\ x_1 & x_0 & . & . \\ . & x_1 & . & 0 \\ x_{M-1} & . & . & x_1 \\ 0 & x_{M-1} & . & . \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & . & x_{M-1} \end{bmatrix}$$

Channel Estimation

$$= \begin{bmatrix} \sum_{i=0}^{M-1} x_i^2 & \sum_{i=0}^{M-1} x_i x_{i-1} & . & \sum_{i=0}^{M-1} x_i x_{i-L+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{M-1} x_i x_{i-L+1} & . & \sum_{i=0}^{M-1} x_i x_{i-1} & \sum_{i=0}^{M-1} x_i^2 \end{bmatrix}$$

$x_i = \pm 1$, $\sum_{i=0}^{M-1} x_i^2 = M$ and $(\bar{X}^T\bar{X}) =$

$$\begin{bmatrix} M & \sum_{i=0}^{M-1} x_i x_{i-1} & . & \sum_{i=0}^{M-1} x_i x_{i-L+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{M-1} x_i x_{i-L+1} & . & \sum_{i=0}^{M-1} x_i x_{i-1} & M \end{bmatrix}$$

$$= M\begin{bmatrix} 1 & \sum_{i=0}^{M-1} x_i x_{i-1} & . & \sum_{i=0}^{M-1} x_i x_{i-L+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{M-1} x_i x_{i-L+1} & . & \sum_{i=0}^{M-1} x_i x_{i-1} & 1 \end{bmatrix}$$

$$r_{xx}(k) = \frac{1}{R_{xx}(0)}\sum_{i=0}^{M-1} x_i x_{i-k} = \frac{1}{M}\sum_{i=0}^{M-1} x_i x_{i-k}$$

Channel Estimation

- $(\bar{X}^T\bar{X})$ $L \times L$ matrix containing delayed versions of training sequence autocorrelation

$$(\bar{X}^T\bar{X}) = M\begin{bmatrix} 1 & r_{xx}(1) & . & r_{xx}(L-1) \\ \vdots & \vdots & \vdots & \vdots \\ r_{xx}(L-1) & . & r_{xx}(1) & 1 \end{bmatrix}$$

- $r_{xx}(\tau)$ normaized training sequence autocorrelation
- for ideal auto correlation $P_D = \frac{\sigma_c^2}{M}[I]$
- for a single process estimate $L = 1$ error covariance is $P_D = \frac{\sigma_c^2}{M}$
- inverse relationship between length of training sequence and the covariance of the data estimate
- data estimate worsens as noise in the channel increases

Multi Input and Multi Output (MIMO)

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Wireless System

- demand of higher data rates - performance of wireless system
- data rate 1 gigabit/second
- capacity = BW $\log_2(1 + SNR)$ shannon's law
- channel capacity in bits per second
- higher data rates by increasing bandwidth
- higher bandwidth - more data
- boosting power - more range
- increase in SNR marginal gain in channel throughput
- spectrum become a scarce and expensive resource
- limited bandwidth and tranmit power
- goal: high data rate and high link quality

Technology for Higher Demand

- increasing singal bandwidth of channel by increasing symbol rate of a modulated carrier
- increases its suceptibility to multipath fading
- to meet this chanllenge - MIMO
- Multiple antennas and multiple signal streams to boost data rate, range, connection reliability
- higher data rate over longer distances motivations behind MIMO OFDM
- 802.11n, 802.1ac Wi-Fi, WiMAX, LTE (Long Term Evolution) 4G supports MIMO
- supports enhanced data throughput under interference, fading and multipath

Multiple Input Multiple Output (MIMO)

- 1984 Jack Winters at Bell lab wrote a patent on wireless communication using multiple antennas
- 1993 Paulraj and Kailath concept of spatial multiplexing using MIMO
- 2006 802.11n
- boost system capacity and enhance reliability of link uses several diversity schemes
- N_T tranmit antennas and N_R receive antennas - MIMO system model
- $x_1 - T_{x_1}$ $x_2 - T_{x_2} \rightarrow H(N_R \times N_T) \rightarrow R_{x_1} - y_1$ $R_{x_2} - y_2$ ---
- Antennas $T_{x_1}, \ldots, T_{x_{N_T}}$ respectively send signal $x_1, \ldots, x_{N_T}$ to receive antennas $R_{x_1}, \ldots, R_{x_{N_R}}$
- received signals $y_1, \ldots, y_{N_R}$ respectively
- each receive antenna combines incoming signal which coherently add up

Multiple Input Multiple Output (MIMO)

- offers significant increases in data throughput
- link range without additional bandwidth or increased transmit power
- achieves by spreading the same total transmit power over the antennas
- to achieve an array gain that improves the spectral efficiency of bandwidth - more bits per second per hertz
- to improve link reliability - reducing fading by achieving diversity gain

Multiple Input Multiple Output (MIMO)

- time and frequency domain processing are at limits but space is not - MIMO
- MIMO improves BER or data rate using multiple TX/RX
- core scheme of MIMO: space time coding (STC)
- two main functions of STC: diversity and multiplexing
- maximum peformance needs tradeoff between diversity and multiplexing
- N TXs and M RXs; $y(k) = Hx(k) + v(k)$
- multiath propagation, multiple antennas at transmitter and receiver
- establish multiple parallel channels operate on same frequency band at same total radiated power

Shannon's Theorem

- Shannon bound for SISO

$$C = \log_2(1 + SNR) \quad SNR = P_T/\sigma_v^2$$

- each extra bps/Hz requires a doubling of TX power
- 1 bps/Hz to 11 bps/Hz TX power must be increased by 1000 times
- MISO - transmit diversity against fading
- slow logarithmic growth of capacity with no. of antennas

$$C = \log_2(1 + SNR) \text{ for TX1} \quad C = \log_2(1 + SNR) \text{ for TXN also}$$

- SIMO - receive diversity against fading

$$C = \log_2(1 + SNR) \text{ for RX1} \quad C = \log_2(1 + SNR) \text{ for RXM}$$

- MIMO - transmit and receive diversity

$$C = \log_2(1 + SNR) \text{ MN channels}$$

- parallel spatial channels

$$C = Q\log_2(1 + SNR) \quad Q = \min(M, N)$$

Wireless System

- quality - minimize probability of error P_e
- minimize complexity, transmission power, and bandwidth
- smart modulation, coding, and multiplexing
- maximum error free transmission rate

$$C = \log_2(1 + SNR) \text{ bits/s/Hz}$$

- R data rate bits/symbol, R_s - symbol rate symbols/second, W - allotted BW Hz
- spectral effiecency = number of bits transmitted per second per Hz

$$R \times R_s/W \text{ bits/s/Hz}$$

- as a result of filtering / signal reconstruction requirements $R_s \leq W$
- spectral efficiency = R if $R_s = W$
- if transmit data at rate of $R \leq C$ - achieve low P_e

Wireless System

- BPSK 1 b/s/Hz, QPSK 2 b/s/Hz, 16-QAM 4 b/s/Hz, 64-QAM 6 b/s/Hz

$$\mathbf{y} = H\mathbf{s} + \mathbf{n}$$

- h_{ij} complex gaussian random variable that models fading gain between ith transmit and jth receive
- fading is produced by mutliple replicas of transmitted signal at the receiving antenna
- multiple reflections and mutliple communication paths between two radio terminals cause
- fading - amplitude and phase changes
- ellipsoidal fading model - replica by random reflection
- constructive or destructive interference
- worst case 180 degree phase - casue signal cancellation

Symbol Period and Spread

- T_{sym} to be symbol time period
- T_{spread} to be the time at which the last reflection arrives
- frequency selective $T_{spread} > T_{sym}$; $1/T_{sym}$ is longer
- occurs for wideband signal (small T_{sym}); tough to deal it
- frequency flat $T_{spread} < T_{sym}$; $1/T_{sym}$ is less
- occurs for narrowband signal (larger T_{sym})
- easier fading gain is complex gaussian
- multipath not resolvable
- time spread τ - channel time variance t

$$H(\tau, t) = \begin{bmatrix} h_{1,1}(\tau,t) & h_{1,2}(\tau,t) & \dots & h_{1,M_T}(\tau,t) \\ h_{2,1}(\tau,t) & h_{2,2}(\tau,t) & \dots & h_{2,M_T}(\tau,t) \\ \vdots & \vdots & \vdots & \vdots \\ h_{M_R,1}(\tau,t) & h_{M_R,2}(\tau,t) & \dots & h_{M_R,M_T}(\tau,t) \end{bmatrix}$$

Fading

- signal fading, selective and non-selective fading
- non-selective fading where frequency components over the signal bandwidth are
- dynamically attenuated by the same amount and do not create any signal distortion but only temporal signal loss
- selective fading the case where smaller frequency segments of the signal's spectrum are
- attenuated relative to other remaining frequency segments
- when this occurs, the signal specturm is distorted and in turn creates communcation impairment
- non-selective fading can be countered by providing more signal level margin, or
- using selection diversity techniques to select the best antenna inp based on the relative signal strength

Exploiting Multipath

- strongest signal is chosen to make the connection
- all other signals filtered out
- multipath reflections from objects, weaker and arrive at different times
- different antenna pick up different signals, filter noise and recombine to generate stronger signal
- basic MIMO - data transmitted scrambled, interleaved and
- divided up into parallel data streams, each which modulates a separate transmitter
- multiple antennas capture different streams, have different phases, travelled different routes and combine into one

Exploiting Multipath

- each multipath route treated as a separate channel
- separate antennas take advantage of this to transfer more data
- multiplying throughput, range is increased due to antenna diversity
- each receive antenna has a measurement of each transmitted data stream
- maximum data rate per channel grows linearly with the number of different data streams
- that are transmitted in the same channel, providing scalability and reliable link

MIMO-OFDM

- wide bandwidth channel - solution - use a series of narrowband overlapping subcarriers
- use of overlapping OFDM subcarriers improve spectral efficiency
- lower symbol rates used by narrowband subcarriers reduces the impact of multipath signal products
- interesting solution to multipath challenge by requiring multiple signal paths
- multiple signal paths to gain knowledge of communication channel
- receiver can recover independent streams from each of transmitter's antennas
- 2 x 2 MIMO produces two spatial streams effectively double maximum data rate what achieved in 1 x 1 channel
- maximum channel capacity of MIMO - as a function of N spatial streams Capacity $= N$ BW $\log_2(1 + SNR)$

MIMO-OFDM

- for Wi-Fi, OFDM using BPSK, QPSK, 16 phase QAM, depending on data rate
- signals from different antennas combined to reinforce one another, improving SNR
- smart antenna uses beam forming to focus transmitted signal energy toward the receiver to strengthen the signal - may provide better range
- beyond 4 by 4 configuration, very little additional gain is achieved
- 2 transmitter and three receivers seems to be the most popular
- transmitting two or more data streams in the same bandwidth multiplies data rate by the number of streams used
- MIMO 11n allows two 20 MHz channels bonded together into a single 40-MHz channel provide higher data rate

Multiple Transmit and Multiple Receive: Generic View

- in general, m transmit and n receive antennas
- by using the same channel every antenna receives
- not only the direct components intended for it but also
- the indirect components intended for the other antennas
- time dependent narrowband channel is assumed
- direct connection from antenna 1 to 1 is h_{11}
- indirect connection from antenna 1 to 2 is h_{21}
- h_{rt} transmission matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1m} \\ h_{21} & h_{22} & \dots & h_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & n_{n2} & \dots & h_{nm} \end{bmatrix}$$

$$y = Hx + n$$

MIMO Example: SU MIMO

- SU - MIMO - data rate is to be increased for a single UE, called single user MIMO
- base station - two transmit antennas, UE - two receive antennas
- data to be transmitted is divided into independent data streams
- number of streams M is alwasy less than or equal to the number of antennas
- in the case of asymmetrical ($m \neq n$) antenna constellations,
- it is always smaller or equal the minimum number of antennas
- 4×4 transmit four or fewer streams
- 3×2 transmit two or fewer streams
- theoretically the capacity C increases linearly with the number of streams M

$$C = MB \log_2(1 + \frac{S}{N})$$

MIMO Example: MU MIMO

- Multi user MIMO MU - MIMO
- individual streams are assigned to various users
- this mode useful in the uplink because the complexity on UE side can be kept at a mimimum by one - only one transmit antenna
- called collaborative MIMO
- base station - two transmit antennas
- two UE -UE1 and UE2 two receive antnnas
- Cyclic delay diversity - CDD
- introduces virtual echoes into OFDM systems
- increases frequency selectivity at the reciever
- in CDD, signals are transmitted by the individual antennas with a time delay introduces additional diversity
- useful as addition to spatial multiplexing

Multi User MIMO

- received signal at antenna R_{x_q}

$$y_q = \sum_{p=1}^{N_T} h_{qp} \cdot x_p + b_q$$

$q = 1, \ldots, N_R$

- flat fading MIMO described by

$$y = H \cdot x + b$$

H complex channel matrix

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_T} \\ h_{21} & h_{22} & \cdots & h_{2N_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N_R1} & h_{N_R2} & \cdots & h_{N_RN_T} \end{pmatrix}$$

$p = 1, \ldots, N_T$ $q = 1, \ldots, N_R$

- h_{qp} is complex channel gain which links transmit antenna T_{x_p} to receive antenna R_{x_q}

Multi User MIMO

- $x = [x_1, \ldots, x_{N_T}]^T$ is $N_T \times 1$ complex transmitted signal vector
- $y = [y_1, \ldots, y_{N_R}]^T$ is $N_R \times 1$ complex received signal vector
- $b = [b_1, \ldots, b_{N_R}]^T$ is $N_R \times 1$ complex additive noise signal vector
- continuous time delay MIMO channel model of $N_R \times N_T$
- MIMO channel H associated with time delay τ and noise signal $b(t)$

$$y(t) = \int_{\tau} H(t, \tau) x(t - \tau) d\tau + b(t)$$

- $y(t)$ spatio temporal output, $x(t)$ input and $b(t)$ noise signal
- MIMO channel modeling - physical and analytical models
- physical models - deterministic models and geometry based stochastic channel models
- analytical models - correlation based models, statiscal based mod[els] propagation based models

Multi User MIMO

- physical models - channel impulse response evaluted based on experimental measurements for extracting
- channel propagation parameters including antenna configuration at transmitter and receiver, antenna polarization, scatters
- propagation parameters such as delay spread, angular spread, spatial correlation and cross polarization discrimination
- determinstic models define a channel model according to prediction of propagation signal
- geometry based has immediate relation with physical characteristics of propagation channel
- clusters of scatters distributed around transmitter and receiver
- scatters result in discrete channel path and can involve statistical characterization of propagation parameters

Multi User MIMO

- analytical models - based on statistical properties through measurement
- distribution of direction of departure, distribution of direction of arrival
- MIMO - improves capacity of communication link without the need of increase the transmitter power
- MIMO system capacity evaluated according to following scenarios
- when no channel state information (CSI) available at transmitter
- the power is equally split between N_T transmit antennas
- instantaneous channel capacity (bits/s/Hz)

$$C(H) = \log_2 \left[det(I_{N_R} + \frac{\gamma}{N_T} HH^*) \right]$$

- γ denotes SNR, $(\cdot)^*$ stands for conjugate transpose operator

Multi User MIMO

- when CSI is available at receiver, SVD is used to derive MIMO channel capacity

$$C_{SVD}(H) = R \log_2 \left[det(1 + \frac{\gamma}{N_T} HH^*) \right]$$

$R = min(N_R, N_T)$ rank of channel matrix H

- when CSI is available at both transmitter and receiver
- the channel capacity is computed by performing water-filling algorithm

$$C_{WF}(H) = \sum_{p=1}^{R} \log_2 \left[\left(\frac{\lambda_{H,p}\mu}{\sigma_b^2} \right)^+ \right)$$

- $a^+ = max(a, 0)$, $\lambda_{H,p}$ p-th singlular value of channel matrix H
- μ constant scalar satisfies total power constraint, σ_b^2 noise signal power

MIMO: Diversity

- introducing higher modulation types or providing larger bandwidth - achieved by MIMO
- MIMO refers to channel - transmitter is channel input and the receiver channel output
- different diversity modes
- time diversity - differnt timeslots and channel coding
- frequency diversity - different channels, spread spectrum and OFDM
- spatial diversity - use of multiple antennas at the transmitter or receiver end
- MIMO increase data rate - spatial multiplexity

MIMO: Antenna Diversity

- the effect of attenuation can be compensated by
- receiving the signal at two different receiver antenna placement location and
- selecting the best antenna based on the signal strength or some other receiver performance measure
- the signal strengths associated with the antennas are generally not correlated in time and/or space and
- when one signal is in a null, the other can be found near a maximum and selected for communication
- the selection of one antenna over the other based on signal quality or
- selection diversity is technique that works well for non-selective fading in that
- the antenna selection essentially is a signal strength restoration technique

MIMO: Antenna Diversity

- selection diversity and other signal strength restoration techniques are not enough to
- equalize the impairments imposed by the multipath channel due to selective fading
- selective fading occurs when the signal bandwidth exceeds the coherence bandwidth of channel
- the coherence bandwidth of the channel is approximately the reciprocal of delay spread of the channel
- the delay spread is the rms average of delay times of the complex impulse response of the channel h
- h is the received complex envelope at a particular point in space assuming that a carrier signal is modulated by dirac impulse
- the arrival of various signal components can be modeled by FIR f

MIMO: Eqaulizer

- ideally, channel transfer function only adds
- a flat fading process (constant attenuation versus frequency) and a single fixed time delay (linear phase)
- actually, frequency dependent fading of amplitudes and nonlinear phase response
- if the sidebands of a complex modulated signal where distributed across the transfer function,
- there would be signal distortion
- to compensate these effects the transfer function encountered via communication channel
- must be equalized at the receiver

MIMO: Eqaulizer

- ideally the inverse of transfer function applied to signal prior to detection to flatten
- the amplitude response and to linearize phase the signal impairments would be eliminated
- there are several different types of transversal equalizers that
- operate on time domain representation of the baseband signal to achieve equalization
- many rely on training sequences and training periods
- that creates communication overhead

OFDM and Channel Equalization

- a means of spectrally efficient communications,
- also a means of channel equalization in the frequency domain
- OFDM spectrum 48 data subcarriers and 4 pilot subcarriers
- pilot subcarriers are not information bearing but are used
- to help maintain the carrier and timing tracking in low SNR since there are always BPSK modulated
- OFDM provides a means of sampling the magnitude and phase of the channel
- at any or all of the subcarrier frequencies,
- since the carrier phase tracked at the receiver and all of the subcarrier phases are coherent to the main carrier
- in practice, performing continuous subcarrier phase tracking on each of subcarrier is not exectuted, rather,

OFDM and Channel Equalization

- short and long training symbols with known patterns are transmitted with each packet
- such that the receiver can determine the channel transfer function, invert it and then
- apply the inverse to the received spectrum in order to equalize the cahnnel on a packet by packet basis
- how the channel transfer function due to selective fading weights the received spectrum at the receiver
- the magnitude and phase of each of subcarriers provide an estimate of channel transfer funcion $H_c()$
- equalization transfer function $H_e()$ can be applied to the channel weighted spectrum
- to re-normalize the spectrum such that the effects of the channel be reduced or eliminated

OFDM and Channel Equalization

$$S_{RX}(\omega) = H_c(\omega)S_{TX}(\omega)$$

$$H_c(\omega) = \frac{S_{RX}(\omega)}{S_{TX}(\omega)}$$

$$H_r(\omega) = H_c(\omega)H_e(\omega)$$

$$H_e(\omega) = H_c^{-1}(\omega)$$

$$H_r(\omega) = H_c(\omega)H_c^{-1}(\omega) = 1$$

Exploiting Diversity

- using MIMO - additional paths can be used to advantage - by increasing data capacity
- two main formats for MIMO
- spatial diversity - refers to transmit and receive diversity
- provide improvment in SNR, improving realiability of the system with respect to various forms of fading
- spatial multiplexing - provide additional data capacity by utilizing
- different paths to carry additional traffic, increasing data througput capability
- the maximum amount of data carried by radio channel limited by shannon's law

Exploiting Diversity

- antenna selection diversity is a technique for improving the performance in a non-selective fading channel
- OFDM provides for channel equalization against frequency selective fading
- SISO systmes do not provide any type of robustness or capacity improvment
- SIMO systems provide receive-side diversity and additional robustness but no capacity improvement
- receivers with selection diversity schemes or linear maximal ratio combining scheme

Exploiting Diversity

- MISO - provides transmitter diversity
- it couples to the channel at a different point in space such that
- the links will not have the same fading characterisitcs though to the receive antenna and
- spatial sum of the signals will be dominated by the stronger of the two signals or
- transmitter signal design can be such that the combining at the single receiver
- can be made in an optimal fashion as is done in space-time coding techniques

MIMO: Spatial Multiplexing

- MIMO systems de-multiplex the source data stream into multiple independent channel streams
- provides both redundancy and channel capacity improvement
- MIMO referred to as Spatial Division Multiplexing
- a single source data stream is multiplexed between two spatial streams
- direct links and cross links between two transmitters and two receivers, for example,
- there are four different communication channels that connect the two terminals,
- the channel characterization has a higher complexity

MIMO: Spatial Multiplexing

- another method to minimize the effects of a multipath communication channel through its redundancy and channel equalization properties
- MIMO - to transfer more data at the same time
- take advantage of a radio wave phenomenon called multipath where
- transmitted information bounces off walls, ceilings and other objects
- reaching the receiving antenna multiple times via differnt angle and at slightly differnt times
- allow mutiple antennas to send and receive multiple spatial streams at the same time

MIMO: Spatial Multiplexing

- combine data streams arriving from different paths and at different times to effectively
- increase receiver signal capturing power
- if there are more antennas than spatial streams; the additional antennas can add receiver diversity and increase range
- increase the capacity of a given channel
- increasing the number of receiver and transmit antennas
- it is possible to linearly increase the througput of the channel with every pair of antennas added to the system
- SISO there is no diversity and no additional processing required
- performance limited by interference and fading
- throughput depends upon the channel bandwidth and SNR

Exploiting Diversity

- SIMO - receive diversity
- receives signals from a number of independent sources to combat the effects of fading
- processing is required in the receiver
- receiver located in mobile device, limited by size, cost and battery drain
- switched diversity SIMO - looks for strongest signal and switches to that antenna
- maximum ratio combining SIMO - takes both signals and sums them to give a combination
- signals from both antennas contribute to overall signal

MIMO: Spatial Diversity

- to make the transmission more roubust
- no increase in data rate; mode uses redundant data on different paths
- receive diversity - RX diversity
- RX diversity uses more antennas on the receiver side than on the transmitter side
- simplest scenario consists of two RX and TX SIMO 1×2
- special coding methods are not needed, this scenario easy to implement only two RF paths are needed for the receiver
- RX $\leftarrow$ A and RX $\leftarrow$ B; two paths A and B
- switched diversity $C = \max(A, B)$ - uses stronger signal
- maximum ratio combining $C = (A + B)$ - uses the sum sigmal from two signals
- different transmission paths, the receiver sees two differently faded signals
- by appropriate method in the reciver, SNR can be increased

MIMO: TX Diversity

- there are more TX than RX antennas
- two TX and RX MISO 2 by 1
- the same data is transmitted redundantly over two antennas
- advantage that the multiple antennas and redundancy coding is moved from the mobile UE to the base station, where
- these technologies are simpler and cheapter to implement
- to generate a redundant signal, space time codes are used
- Alamouti developed the first codes for two antennas
- space time codes improves performance and make spatial diversity usable
- signal copy is transmitted not only from a different antenna but also at a different time
- delayed transmission is called delayed diversity
- space-time codes combine spatial and temporal signal copies

Scanned by CamScanner

Exploiting Diversity

- MISO - termed transmit diversity
- the same data is transmitted redundantly from the two transmitter
- receiver is able to receive the optimum signal which it can use to extract required data
- processing is moved from receiver to transmitter
- MIMO - a variety of signal paths to carry the data,
- choosing separate paths for each antenna to enable multiple signal paths to be used
- space-time signal processing
- mutiple spatially distributed antennas located at different points
- signal can take many paths - as a result of number of objects introduce interference

MIMO: Spatial Multiplexing

- signaling technique combine with properties of OFDM modulation leads to efficient channel estimation and equalization scheme
- even signals are transmitted on TX0 and odd symbols are transmitted on TX1
- two receivers RX0 and RX1 receive both the transmitted streams
- through 4 possible channel transfer functions $h_{00}, h_{01}, h_{10}, h_{11}$ forms a channel matrix
- in order to equalize and extract the source symbol stream,
- estimate of channel matrix must be made transmitting both streams using
- a coordinated system of pilot subcarriers and null carriers

MIMO Channel Equalization

- the received symbols at each of receive inputs can be computed as follows:

$$r_0 = h_{00}s_0 + h_{10}s_1 \qquad \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{10} \\ h_{01} & h_{11} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = [R] = [H][S]$$
$$r_1 = h_{10}s_0 + h_{11}s_1$$

- for case of pilot subcarriers relative to null subcarriers
- $r_{00} = h_{00}s_0$ $r_{10} = h_{10}s_1$ $r_{01} = h_{01}s_0$ $r_{11} = h_{11}s_1$
- the magnitude and phase of pilot subcarriers is known,
- the the s-terms can be factored out and the elements of the channel matrix are determined
- given channel matrix the received signals can be equalized and restored

MIMO Channel Equalization

$$[S] = [H]^{-1}[R] = \left[\frac{adj[H]}{|H|}\right][R] = \left[\frac{\begin{bmatrix} h_{11} & -h_{01} \\ -h_{10} & h_{00} \end{bmatrix}^T}{h_{00}h_{11} - h_{10}h_{01}}\right][R]$$

$$[S] = \left[\frac{1}{h_{00}h_{11} - h_{10}h_{01}}\right] \begin{bmatrix} h_{11} & -h_{10} \\ -h_{01} & h_{00} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}$$

- transmitted signal can be restored perfectly if channel matrix can be estimated perfectly
- does not work due to singularity in the matrix inversion where all channel elements are equal or correlated

MIMO: Types of space time code

- spatial diversity
  - ST block code - provides diversity gain, no coding gain
  - ST trellis code - diversity and coding gain
- spatial multiplexing
  - layered ST code - coding gain and diversity gain, bandwidth efficiency
- BLAST: Bell labs layered space time
- V-BLAST and D-BLAST
- vertical BLAST; it is $1 : N$ demux on channel cc0, cc1, cc2, ccN
- TX1 $\rightarrow$ encoder $\alpha \rightarrow \alpha_0\ \alpha_1 \ldots$
- TX2 $\rightarrow$ encoder $\beta \rightarrow \beta_0\ \beta_1 \ldots$
- TX3 $\rightarrow$ encoder $\gamma \rightarrow \gamma_0\ \gamma_1 \ldots$
- space vs. time
- simple and low complexity, lower capacity than shannon bound
- Data rate $\propto N$

MIMO: Spatial diversity

- diagonal BLAST
- TX1 $\rightarrow$ encoder $\alpha \rightarrow \alpha_0\ \beta_0\ \gamma_0 \ldots$
- TX2 $\rightarrow$ encoder $\beta \rightarrow \ldots\ \alpha_1\ \beta_1\ \gamma_1 \ldots$
- TX3 $\rightarrow$ encoder $\gamma \rightarrow \ldots\ \ldots\ \alpha_2 \ldots$
- closer to shannon bound due to spatial switch diversity gain
- high complexity, coding constraints
- Beamforming schemes
- beamformer - spatial filter that combines array inputs
- form a beam in space to recive desired signal while suppressing interference and noise
- transmit beamformer and receive beamformer

MIMO: Beamforming

- antenna technologies are the key in increasing network capacity
- started with sectorized antennas
- antennas illuminate 60 or 120 degrees and operate as one cell
- in GSM the capacity can be tripled, by 120 degree antennas
- adaptive antenna arrays intensity spatial multiplexing using narrow beams
- smart antennas belong to adaptive antenna arrays but differ in their smart direction of arrival estimation
- smart antennas can form a user specific beam
- optional feedback can reduce complexity of array system
- beamforming is the method used to create the radiation pattern of an antenna array applied to MIMO system

MIMO: Beamforming

- smart antennas divided into two groups
- phased array systems (switched beamforming) with a finite number of fixed predefined patterns
- adaptive array systems (adaptive beamforming) with an infinite number of patterns adjusted to the scenario in realtime
- switched beamformers electrically calculate the DoA and switch on the fixed beam
- the user only has the optimum signal strength along the center of the beam
- the adaptive beamformer deals with that problem and adjusts the beam in realtime to the moving UE
- complexity and cost higher than switched beamformer
- various mobile radio and network standards use MIMO
- all standards use TX diversity and spatial multiplexing

MIMO: Alamouti coding

- signals S_0 and S_1 are multiplexed in two data chains
- after that a signal replication is added to create the alamouti space-time block code

$$[S_0 \;\; S_1] \rightarrow \begin{bmatrix} S_0 \\ S_1 \end{bmatrix} \rightarrow \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- -TX1 and -TX2 space and time
- coding can also be handled in frequency domain; called space-frequency coding
- spatial multiplexing
- not intended to make transmission more robust; rather it increases data rate data is divided into separate streams
- the streams are transmitted independently via separate antennas

MIMO: Alamouti coding

- MIMO transmits via the same channel, transmissions using cross components
- MIMO 2 × 2 $H = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix}$
- if transmission matrix H is known the cross components can be calculated on the receiver
- in open-loop method, the transmission includes the special sections that are also known to the receiver
- the receiver can perform a channel estimation
- in closed-loop method, the receiver reports the channel status to the transmitter via a special feedback channel
- makes it possible to respond to changing circumstances

MIMO: Transmit diversity

- a simple transmit diversity technique for wireless communications
- Alamouti IEEE journal on selected areas in communication Oct 1998
- ST block code

$$[S_0 \;\; S_1] -> \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- encoding and tramission sequence: at a given symbol period
- two signals are simultaneously transmitted from the two antennas
- space vs. time; transmit

$$X = \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- ST block codeword property

$$X^H X = \begin{bmatrix} |S_0|^2 + |S_1|^2 & 0 \\ 0 & |S_0|^2 + |S_1|^2 \end{bmatrix}$$
$$= (|S_0|^2 + |S_1|^2)\, I_2$$

- codeword X that satisfies property is called orthogonal design

MIMO: Transmit diversity

- fading is constant across two consecutive symbols

$$h_0(t) = h_0(t+T) = h_0 = |\alpha_0| e^{j\theta}$$
$$h_1(t) = h_1(t+T) = h_1 = |\alpha_1| e^{j\theta}$$

- T is symbol duration
- received signal can be expressed as

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1$$
$$\mathbf{R} = \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix} + \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix} = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

MIMO: Transmit diversity

- $\bar{\mathbf{H}}$ satisfies

$$\bar{\mathbf{H}}^H\bar{\mathbf{H}} = (|h_0|^2 + |h_1|^2)\,\mathbf{I}_2 = \rho \cdot \mathbf{I}_2$$

- $\rho = (|h_0|^2 + |h_1|^2)$ is diversity gain
- combining scheme - combiner builds two combined signals that are sent to the ML detector

$$\tilde{\mathbf{S}} = \begin{bmatrix} \tilde{S}_0 \\ \tilde{S}_1 \end{bmatrix} = \bar{\mathbf{H}}^H\mathbf{R} = \bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{S} + \bar{\mathbf{H}}^H\mathbf{n} = \rho\mathbf{S} + \bar{\mathbf{H}}^H\mathbf{n}$$

$$= \rho \begin{bmatrix} S_0 \\ S_1 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

S_0 and S_1 are detected independently

MIMO Modelling

- received signal vector y channel matrix **H**

$$y = Hx + n$$

$$y = [y_1\ y_2\ \dots y_N]^T$$

received signal vector

$$X = [x_1\ x_2\ \dots x_M]^T$$

transmitted signal vector

$$n = [n_1\ n_2\ \dots n_M]^T$$

noise vector

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N1} & h_{N2} & \dots & h_{NM} \end{bmatrix}$$

transmitted power $P_T = X^H X = |x_1|^2 + |x_2|^2 + \dots + |x_M|^2 \le C$

MIMO Modelling

- covariance matrices

$$R_{xx} = E\left\{XX^H\right\} \quad R_{yy} = E\left\{yy^H\right\} = E\left\{HXX^HH^H\right\} + E\left\{nn^H\right\}$$

- traces of R_{xx} and R_{yy} give total powers of transmitted and received signals
- offdiagonal elements of R_{xx} and R_{yy} give correlations between the signals at different antennas elements
- consider a symbol period of time T_s for transmitted signals such that $R_{xx} = E\left\{XX^H\right\} = I_M$ and within T_s

$$R_{yy} = E\left\{HXX^HH^H\right\} + E\left\{nn^H\right\}$$

$$= HE\left\{XX^H\right\}H^H + R_{nn} = HH^H + R_{nn}$$

R_{nn} is noise covariance

- over a longer period of time ($> T_s$) average received signal covariance matrix is

$$R_{yy} = E\left\{HH^H\right\} + R_{nn}$$

Equivalent MIMO

- select H to optimize the channel output SNR so as the capacity of MIMO
- for MIMO calculation of the capacity is more complicated due to determination of SNR
- channel matrix H $(N \times M)$ $y = Hx + n$
- using SVD singular value decomposition $H = UDV^H$
- D is $N \times M$ a diagonal matrix with non-negative elements
- U is $N \times N$ unitary matrix; V is $M \times M$ unitary matrix
- $UU^H = U^HU = I_N$ and $VV^H = V^HV = I_M$
- diagonal elements of D are called singular values of H
- there are non negative square roots of the eigenvalues λ of the following equation

$$\left(HH^H\right)X = \lambda X \quad if \quad N < M$$

$$\left(H^HH\right)X = \lambda X \quad if \quad N \geq M$$

Equivalent MIMO

- x is $N \times 1$ eigenvector associated with λ

$$y = UDV^H X + n$$

- consider transformations $y' = U^H y \quad X' = V^H X \quad n' = U^H n$

$$U^H y = U^H UDV^H X + U^H n$$

$$y' = DV^H X + n' \quad y' = Dx' + n'$$

- called equivalent MIMO system

$$R_{y'y'} = E\left\{y'y'^H\right\} = E\left\{U^H yy^H U\right\} = U^H R_{yy} U$$

$$R_{x'x'} = E\left\{x'x'^H\right\} = E\left\{V^H xx^H V\right\} = V^H R_{xx} V$$

$$R_{n'n'} = E\left\{n'n'^H\right\} = E\left\{U^H nn^H U\right\} = U^H R_{nn} U$$

- $tr\{R_{y'y'}\} = tr\{R_{yy}\}$, $tr\{R_{x'x'}\} = tr\{R_{xx}\}$, $tr\{R_{n'n'}\} = tr\{R_{nn}\}$

Equivalent MIMO

- system has the same total input power, total output power and total noise power as actual MIMO
- the output SNR of equivalent MIMO is same as actual MIMO
- it means channel capacity of equivalent MIMO is same as that of actual MIMO capacity is function of output SNR
- channels all decoupled, N channels are parallel to each other with
- channel gains given by diagonal elements of $D \rightarrow \sqrt{\lambda_i}\ i = 1, \ldots, N$
- number of nonzero eigenvalues of matrix HH^H is equal to rank of matrix H denoted by r

$$y_i' = \sqrt{\lambda_i} x_i' + n_i'$$

for $i = 1, 2, \ldots, r$

$$y_i' = 0 + n_i'$$

for $i = r, \ldots, N$

Equivalent MIMO

- MIMO has more transmitting antennas than receiving antennas $M > N$
- than H is a horizontal mattrix with a maximum rank N
- maximum number of uncoupled equivalent MIMO channel is N
- remaining $M - N$ transmitting antennas will become redundant with no receiving antennas
- if $M < N$ - more receiving antennas than transmitting antennas
- H is vertical matrix with maximum rank M
- maximum number of uncoupled equivalent MIMO channels is M
- remaining $N - M$ receiving antennas become redundant with no received signals
- maximum number of uncoupled equivalent channels is $\min(N, M)$

MIMO Channel Capacity

- channels of equivalent MIMO are uncoupled and parallel
- channel capacity can be calculated by a summation of individual capacities of parallel channel

$$C = B \sum_{i=1}^{r} \log_2 \left[1 + \frac{P_{y_i'}}{\sigma^2}\right]$$

- B channel bandwidth, $P_{y_i'}$ power received at the i th receiving antenna σ^2 noise power at receiving antenna
- r rank of H
- classify MIMO according availability of channel knowledge to transmitter or receiver

MIMO Channel Capacity

- channel state information CSI known to receiver only
- transmitter does not know CSI, the best strategy transmit power equally from all its transmitting antennas
- equivalent MIMO - making all elements of X' have same power
- received power $P_{y_i'} = \lambda_i \frac{P}{M}$
- P total transmitting power

$$C = B\sum_{i=1}^{r} \log_2\left(1+\lambda_i \frac{P}{M\sigma^2}\right) = B\log_2 \prod_{i=1}^{r}\left(1+\lambda_i \frac{P}{M\sigma^2}\right)$$

- eigenvalue λ_i can be expressed in terms of matrix HH^H or $H^H H$

$$C = \begin{cases} B\log_2 det\left(I_N + \frac{P}{M\sigma^2}HH^H\right) & N < M \\ B\log_2 det\left(I_M + \frac{P}{M\sigma^2}H^H H\right) & N \geq M \end{cases}$$

MIMO Channel Capacity

- total transmitting power P not known, average received powers P_r at receiving antennas are same

$$P_r = P \times P_{loss}$$

- P_{loss} average path loss form the transmitter to receiver

$$C = \begin{cases} B\log_2 det\left(I_N + \frac{P_r}{M\sigma^2}\frac{HH^H}{P_{loss}}\right) & N < M \\ B\log_2 det\left(I_M + \frac{P_r}{M\sigma^2}\frac{H^H H}{P_{loss}}\right) & N \geq M \end{cases}$$

MIMO Channel Capacity

- CSI known to both transmitter and receiver
- if transmitter knows CSI, channel matrix H
- the best strategy is to transmit more power along those channels whose channel gains are larger and
- to transmit less power or along those channels with a smaller channel gain
- this is called water-filling principle
- the transmitting power P_i for the ith channel in equivalent MIMO

$$P_i = \left(\mu - \frac{\sigma^2}{\lambda_i}\right)$$

$i = 1, \ldots, r$ $r = rank(H)$

MIMO Channel Capacity

- if P_i is negative it will be set to zero
- μ is determined by satisfying transmitting power constraint

$$P = \sum_{i=1}^{r} P_i$$

- received powers

$$P_{y_i'} = \lambda_i P_i = (\lambda_i\mu - \sigma^2)$$

- channel capacity is

$$C = B\sum_{i=1}^{r}\log_2\left[1+\frac{1}{\sigma^2}(\lambda_i\mu - \sigma^2)\right]$$

Different Standards and MIMO support

- 3GPP mobile radio standard - UMTS undergone phases of development
- starting with WCDMA data acceleration methods have been introduced,
- including HSDPA and HSUPA
- newest releases cover HSPA+ and Long Term Evolution (LTE)
- HSPA+ (3GPP release 7/8)
- transmit diversity mode introduced in release 99 (WCDMA)
- release 7 of 3GPP specification (HSPA+) expanded this approach to MIMO and increased data rate with respect to release 6 (HSDPA)
- introduction of 64QAM modulation and MIMO in the downlink makes a peak data rate of 28 Mbps
- MIMO was introduced in the form of double transmit antenna arr for high speed downlink shared channel (HS-DSCH)

Different Standards and MIMO support

- LTE (3GPP release 8) - UMTS LTE introduced in 3GPP release 8
- objective is high data rate, low latency and packet otpimized radio access technology
- LTE is referred as E-UTRA (Evolved UMTS Terrestrial Radio Access) or
- E-UTRAN (Evolved UMTS Terrestrial Radio Access Network)
- the basic concept for LTE is downlink is OFDMA (uplink: SC-FDMA)
- while MIMO is integral part of LTE
- modulation modes are QPSK, 16QAM, and 64QAM
- peak data rates of up to 300 Mbps (4 X 4 MIMO) and up to 150 Mbps (2 X 2 MIMO) in the downlink and
- upto 75 Mbps in the uplink are specified

Different Standards and MIMO support

- downlink - transmission modes possible in LTE
  - single antenna transmission, no MIMO
  - transmit diversity
  - open-loop spatial multiplexing, no UE feedback required
  - closed-loop spatial multiplexing, UE feedback required
  - Multi user MIMO (more than one UE is assigned to same resource block)
  - beamforming
  - closed-loop precoding for rank=1 (no spatial multiplexing, but precoding is used)
- WiMAX 802.16e-2005
  - peak data rate of 74 Mbps at a bandwidth of upto 20 MHz
  - modulation QPSK, 16QAM, 64QAM
- WLAN 802.11n
  - peak data rate of up to 600 Mbps at a bandwidth of 40 MHz
  - modulation BPSK, QPSK, 16QAM, 64QAM

Thank You

Multi User Detection

TEQIP-II Sponsored Short Term Training Program on Wireless Network and Mobile Computing
organized by Computer Engineering Department
Sardar Vallabhbhai National Institute of Technology, Surat
16-20 May 2016

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Single User System

- transmitting a sequence $\{b[0], b[1], \ldots, b[M-1]\}$
- $(b[i] = \pm 1)$ or a finite alphabet of complex numbers
- linear modulation using a signaling waveform

$$x(t) = \sum_{i=0}^{M-1} b[i] w_i(t)$$

$w_i(\cdot)$ modulation waveform associated with the i^{th} symbol, for example

$$w_i(t) = Ap(t - iT)e^{j(w_c + \phi)}$$

$A > 0$ $\varphi \in (- = pi, \pi)$ and $p(\cdot)$ baseband pulse shape

$$p(t) = p_T(t) \triangleq \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

Single User System

- or spreading waveform for DSSP system

$$p(t) = \sum_{j=0}^{N-1} c_j \psi(t - jT_c)$$

N is the spreading gain and

- $c_0, c_1, \ldots, c_{N-1}$ is a pseudorandom spreading code ($c_j \in \{+1, -1\}$),
- $\psi(\cdot)$ chip waveform and $T_c \triangleq T/N$ chip interval
- chip waveform may be a unit-energy rectangular pulse of duration T_c: $\psi(t) = p_{T_c}(t)$
- repeat the same spreading code in every symbol interval
- system with long spreading codes, the periodicity is much longer than a single symbol interval and varies spreading code from symbol to symbol

$$p_i(t) = \sum_{j=0}^{N-1} c_j^i \psi(t - jT_c)$$

Single User System

- spread spectrum modulation can take the form of frequency hopping
- carrier frequency is changed over time according to a pseudorandom pattern
- carrier frequency changes at a rate much slower than the symbol rate - slow frequency hopping
- fast hopping - the carrier changes within a symbol interval
- multicarrier system by choosing $\{w_i(\cdot)\}$ with different frequencies

$$w_i(t) = Ap(t)e^{j(w_i t + \phi_i)}$$

- individual carrier can also be direct-spread - multicarrier CDMA
- OFDM: baseband pulse shape is a unit pulse p_T.
- intercarrier spacing is $1/T$ cycles per second, and the phases are orthogonal at this spacing

Multiple-Access Techniques

- radio resources shared among multiple users
- FDMA - frequency band available is divided into subbands and
- allocated to individual user, users do not transmit signals within other subbands
- TDMA - time is divided into equal-length intervals,
- each user is allowed to transmit throughout the entire allocated frequency band during a given slot
- FDMA allows each user to use part of the spectrum all of the time
- TDMA allows each user to use all of the spectrum part of the time
- FDMA and TDMA systems are intended to assign orthogonal channels to all active users by giving each, for exclusive use, a slice of the available frequency band or transmission time
- channels are said to be orthogonal because interference between u... does not

Multiple-Access Techniques

- code-division multiple access (CDMA) assigns channels in a way that
- allows all users to use all of the available time and frequency resources simultaneously,
- through the assignment of a pattern or code to each user
- that specifies the way in which these resources will be used by that user
- spread spectrum modulation - pattern is the pseudorandom code
- that determines the spreading sequence in the case of direct sequence or the hopping pattern in the case of frequency hopping
- channel is defined by a particular pseudorandom code, and
- each user is assigned a channel by being assigned a pseudorandom code
- for a system of K users

$$x_k(t) = \sum_{i=0}^{M-1} b_k[i] w_{i,k}(t) \quad k = 1, 2, \ldots, K$$

$w_{i,k}(\cdot)$ represents ith modulation waveform of user k

Multi User System

- each user in a multiple-access system can be modeled in the same way as in a single-user system
- if the waveforms $\{w_{i,k}(\cdot)\}$ are of the form of sinusoidal with different carrier frequencies $\{w_k\}$ - FDMA
- if they are with time-slotted amplitude pulses $\{p_k(\cdot)\}$ - TDMA
- if they are spread-spectrum signals of this form but with different pseudorandom spreading codes or hopping patterns - CDMA
- another aspect of wireless network
  - ambient noise, propagation losses, multipath, interference
  - properties arising from the use of multiple antennas
- ambient noise - thermal motion of electrons on the antenna and the receiver electronics and from background radiation sources
- modeled as a very wide bandwidth and no particular deterministic structure (e.g. AWGN)

Multi User System

- propagation losses: diffusive losses and shadow fading
- diffusive losses due to open nature of wireless channel, energy decreases with the square of the distance between antenna and source
- shadow fading results from the presence of objects, modeled by an attenuation in signal amplitude that follows a log-normal distribution
- multipath - multiple copies of a transmitted signal are received at the receiver
- multipath is manifested in several ways
  - degree of path difference relative to the wavelength of propagation
  - degree of path difference relative to the signaling rate
  - relative motion between the transmitter and receiver
  - results into Rayleigh fading or frequency-selective fading or time-selective fading

Multi User System

- multipath from scatterers that are spaced very close together will cause a random change in the amplitude of the received signal
- resulting received amplitude is often modeled as being a complex Gaussian random variable
- random amplitude whose envelope has a Rayleigh distribution - termed as Rayleigh fading
- when the scatterers are spaced so that the difference in their corresponding path lengths are significant relative to a wavelength of the carrier and add constructively or destructively
- this is a fading depends on the wavelength of radiation - frequency-selective fading
- when there is relative motion between the transmitter and receiver, this fading depends on time - time-selective fading
- when the difference in path lengths in such that time delay of arrival along different paths is significant relative to a symbol interval, re[illegible] in dispersion of the transmitted signal and causes ISI

Multi User System

- wideband signaling methods such as spread spectrum - a countermeasure to frequency-selective fading
- dividing a high-rate signal into many parallel lower-rate signal - OFDM mitigates channel dispersion on high-rate signals
- multiple access interference (MAI) - arising from other signals in the same network as
- the signal of interest (if signals received) are not orthogonal to one another
- co-channel interference - due to signals different networks but operating in same frequency band
- the above phenomena can be incorporated into a general analytical model for a wireless multiple-access channel

$$r(t) = \sum_{k=0}^{K} \sum_{i=0}^{M-1} b_k[i] \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du + i(t) + n(t)$$

Multi User System

- $g_k(t,u)$ impulse response of a linear filter representing the channel between the kth transmitter and the receiver
- $i(\cdot)$ co-channel interference and $n(\cdot)$ ambient noise, in general, all are random processes
- co-channel interference and channel impulse responses are structured and can be parameterized
- pure multipath channel

$$g_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k} \delta(t - u - \tau_{l,k})$$

L_k number of paths between user k and the receiver, α and τ gain and delay (lth path of kth user)

- model includes frequency-selective fading;
- relative delays will cause constructive and destructive interference at the receiver, depending on the wavelength of propagation and
- Rayleigh fading also using path gains

Multi User System

- composite modulation waveform associated with $b_k[i]$:

$$f_{i,k}(t) = \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du$$

- if these waveforms are not orthogonal for different values of i, ISI will result
- higher-rate transmission are more likely to encounter ISI than are lower-rate transmission
- if the composite waveform for different values of k are not orthogonal, MAI will result
- this can happen in CDMA when pseudorandom code sequences used by different users are not orthogonal,
- this happens in FDMA and TMDA due to the effects of multipath asynchronous transmission

Multi User System: MIMO

- model can be generalized for multiple antennas at the receiver

$$\mathbf{r}(t)=\sum_{k=1}^{K} b_k[i]\int_{-\infty}^{\infty}\mathbf{g}_k(t,u)w_{i,k}(u)du+\mathbf{i}(t)+\mathbf{n}(t)$$

- pth component of $\mathbf{g}_k(t,u)$ is the impulse response of the channel between user k and the pth element of the receiving array

$$\mathbf{g}_k(t,u)=\sum_{l=1}^{L_k}\alpha_{l,k}\delta(t-u-\tau_{l,k})$$

- multiple antennas at both the transmitter and receiver called multiple-input/multiple-output (MIMO) systems
- channel transfer functions are matrices with the number of rows equal to the number of receiving antennas and the number of columns equal to the number of transmitting antennas at each source

Multi User Detection

- basic receiver signal processing
- matched filter / RAKE receiver
- say single user $K=1$ channel impulse $g_1(\cdot,\cdot)$ is known to receiver,
- there CCI $i(\cdot)=0$, the ambient noise is AWGN with spectral height σ^2

$$r(t)=\sum_{i=0}^{M-1} b_1[i]f_{i,1}(t)+n(t)$$

$$f_{i,1}(t)=\int_{-\infty}^{\infty} g_1(t,u)w_{i,1}(u)du$$

say, there is a single symbol to be transmitted $M=1$ received waveform

$$r(t)=b_1[0]f_{0,1}(t)+n(t)$$

- optimal inferences about the symbol $b_1[0]$ using likelihood function observations conditional on the symbol $b_1[0]$

Multi User Detection

$$\mathcal{L}(r(\cdot)|b_1[0])=\exp\left\{\frac{1}{\sigma^2}\left[2\Re\left\{b_1^*[0]\int_{-\infty}^{\infty}f_{0,1}^*(t)dt\right\}-|b_1[0]|^2\int_{-\infty}^{\infty}|f_{0,1}|^2dt\right]\right.$$

asterik - complex conjugation $\Re(\cdot)$ real part of argument

- optimal inferences about $b_1[0]$ can be made using ML or MAP
- using ML that maximizes $\mathcal{L}(r(\cdot)|b_1[0])$ over the symbol alphabet $\mathcal{A}$

$$\hat{b}_1[0]=\arg\left\{\max_{b\in\mathcal{A}}\mathcal{L}(r(\cdot)|b_1[0]=b)\right\}$$

$$=\arg\left\{\max_{b\in\mathcal{A}}\left[2\Re\left\{b^*\int_{-\infty}^{\infty}f_{0,1}^*(t)r(t)dt\right\}-|b|^2\int_{-\infty}^{\infty}|f_{0,1}(t)|^2dt\right]\right\}$$

- the symboel estimate - the soultion to the problem is $\min_{b\in\mathcal{A}}|b-z|^2$

$$z=\frac{\int_{-\infty}^{\infty}f_{0,1}^*(t)r(t)dt}{\int_{-\infty}^{\infty}|f_{0,1}(t)|^2dt}$$

Multi User Detection

- ML symbol estimate is the closet point in the symbol alphabet to the observable z
- for BPSK ML symbol estimate is

$$\hat{b}_i[0]=sign\{\Re\{z\}\}=sign\left\{\Re\left\{f_{0,1}^*(t)r(t)dt\right\}\right\}$$

$sign\{\cdot\}$ denotes signum function:

$$sign\{x\}=\begin{cases}-1 & x<0\\ 0 & x=0\\ +1 & x>0\end{cases}$$

- choices of symbol alphabet are M-ary phase shift keying MPSK and quadrature amplitude modulation QAM

Multi User Detection

- MPSK symbol alphabet is

$$\mathcal{A} = \left\{ e^{j2\pi m/M} | m \in \{0, 1, \ldots, M-1\} \right\}$$

or some rotation of this set around the unit circle
- QAM symbol alphabet containing $M \times N$ values is

$$\mathcal{A} = \{ b_R + jb_I | b_R \in \mathcal{A}_R and b_I \in \mathcal{A}_I \}$$

$\mathcal{A}_R$ and $\mathcal{A}_I$ are discrete sets of amplitudes containing M and N points respectively, with $M = N$

$$\mathcal{A}_R = \mathcal{A}_I = \left\{ \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots, \pm\frac{M}{4} \right\}$$

or a scaled version of this choice
- BPSK $\mathcal{A} = \{-1, +1\}$
- for MPSK ML symbol choice that whose angle is closest to the anlge of complex number z
- for QAM ML symbol estimate are decoupled with $\Re\{b\}$
- being chosen to be the closest element of $\mathcal{A}_R$ to $\Re\{z\}$

Multi User Detection

- MAP symbol detection $b_1[0]$ is random variable taking values in $\mathcal{A}$ with known probabilities
- MAP estimate via Bayes' formula is

$$P(b_1[0] = b | r(\cdot)) = \frac{\mathcal{L}(r(\cdot)|b_1[0] = b) P(b_1[0] = b)}{\sum_{a \in \mathcal{A}} \mathcal{L}(r(\cdot)|b_1[0] = a) P(b_1[0] = a)}$$

- MAP estimate

$$\hat{b}_1[0] = \arg \left\{ \max_{b \in \mathcal{A}} P(b_1[0] = b | r(\cdot)) \right\}$$

$$= \arg \left\{ \max_{b \in \mathcal{A}} [\mathcal{L}(r(\cdot)|b_1[0] = b) P(b_1[0] = b)] \right\}$$

- if symbol are equiprobabile ML and MAP estimates are same

Multi User Detection

- single user, single symbol, known channel case
- the receiver signal processing task is to compute the term

$$y_1[0] = \int_{-\infty}^{\infty} f_{0,1}^* r(t) dt$$

- this structure is called a correlator because
- it correlates the received signal $r(\cdot)$ with knwon composite signaling waveform $f_{1,0}(\cdot)$
- this structure can be implemented by sampling the output of a time invariant linear filter

$$\int_{-\infty}^{\infty} f_{0,1}^* r(t) dt = (h * r)(0)$$

- convolution between h and r
- h is impulse response of the time invariant linear filter

$$h(t) = f_{0,1}^*(-t)$$

- this structure is called a matched filter,

Multi User Detection

- the impulse response is matched to the composite waveform on which the symbol is received
- the composite signaling waveform has a finite duration so that
- $h(t) = 0$ for $t < -D \leq 0$
- the matched filter receiver can be implemented by sampling at time D
- the output of the causal filter with the impulse response

$$h_D(t) = \begin{cases} f_{0,1}^*(D-1) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- if signaling waveform $s_{0,1}(t)$ has duration $[0, T]$ and the channel has delay spread τ_d
- the composite signaling waveform will have this property with $D = T + \tau_d$

Multi User Detection

- a special case of correlator - a pure multipath channel in ...
- the channel impulse response is

$$g_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k}\delta(t-u-\tau_{l,k})$$

- the composite function

$$f_{0,1}(t) = \sum_{l=1}^{L_1} \alpha_{l,1} s_{0,1}(t-\tau_{l,1})$$

- the correlator output

$$y_1[0] = \sum_{l=1}^{L_1} \alpha_{l,1}^* \int_{-\infty}^{\infty} s_{0,1}^*(t-\tau_{l,1}) r(t) dt$$

- a configuration known as RAKE receiver

Multi User Detection

- Equalization
- there is more than one symbol in the frame $M > 1$
- likelihood function of observations $r(\cdot)$ conditioned on the entire frame of symbols $b_1[0], b_1[1], \ldots, b_1[M-1]$
- $\mathcal{L}(r(\cdot)|b_1[0], b_1[1], \ldots, b_1[M-1])$

$$= \exp\left\{\frac{1}{\sigma^2}\left[2\Re\left\{\mathbf{b}_1^H \mathbf{y}_1\right\} - \mathbf{b}_1^H \mathbf{H}_1 \mathbf{b}_1\right]\right\}$$

- H conjugate transpose - Hermitian transpose, $\mathbf{b}_1$ column vector whose ith component is $b_1[i]$, $i = 0, 1, \ldots, M-1$
- $\mathbf{y}_1$ its i th component

$$y_1[i] = \int_{-\infty}^{\infty} f_{i,1}^*(t) r(t) dt$$

- $\mathbf{H}_1$ $M \times M$ whose (i,j)th element is cross correlation between $f_{i,1}(t)$ and $f_{j,1}(t)$

Multi User Detection

$$\mathbf{H}_1[i,j] = \int_{-\infty}^{\infty} f_{i,1}^*(t) f_{j,1}(t) dt$$

- likelihood function depends on $r(\cdot)$ through vector $\mathbf{y}_1$ of correlator outputs
- this vector is sufficient statistic for making inferences about the $\mathbf{b}_1$
- maximum likelihood detection

$$\hat{\mathbf{b}}_1 = \arg\left\{\max_{\mathbf{b}\in\mathcal{A}^M}\left[2\Re\left\{\mathbf{b}^H \mathbf{y}_1\right\} - \mathbf{b}^H \mathbf{H}_1 \mathbf{b}\right]\right\}$$

- if $\mathbf{H}_1$ is a diagonal matrix (all of its off-diagonal elements are zero) decouples into a set of M independent problems of single symbol type
- the solution in this case

$$\hat{b}_1[i] = \arg\max_{b\in\mathcal{A}} |b - z_1[i]|^2$$

$$z_1[i] = \frac{y_1[i]}{\int_{-\infty}^{\infty} |f_{i,1}(t)|^2 dt}$$

Multi User Detection

- in general case there is intersymbol interference, will not decouple
- the optimization must take place over the entire frame
- a problem known as sequence detection
- MAP estimate very high complexity

$$P\left(b_1[0] = b|r(\cdot)\right) = \frac{\sum_{\{\mathbf{a}\in\mathcal{A}^M|a_i=b\}} \mathcal{L}(r(\cdot)|\mathbf{b}_1 = \mathbf{a})P(\mathbf{b}_1 = \mathbf{a})}{\sum_{\{\mathbf{a}\in\mathcal{A}^M\}} \mathcal{L}(r(\cdot)|\mathbf{b}_1 = \mathbf{a})P(\mathbf{b}_1 = \mathbf{a})}$$

- the dynamic programming solution krwon as maximum likelihood sequence detector
- a number of lower-complexity algorithms have been devised
- examining sufficient statistic vector $\mathbf{y}_1$, can be written as

$$\mathbf{y}_1 = \mathbf{H}_1 \mathbf{b}_1 + \mathbf{n}_1$$

Multi User Detection

- $\mathbf{n_1}$ is complex Guassian random vecotr with indepedent real and imaginary parts having
- identical $\mathcal{N}(\mathbf{0}, \frac{\sigma^2}{2}\mathbf{H_1})$ distributions
- above equation describes a linear model and the goal of equalization is to fit this model with data vector $\mathbf{b_1}$
- ML and MAP are two ways but exponential complexity with exponent equal to bandwidth of $\mathbf{H_1}$
- the vector $\mathbf{b_1}$ takes on values form a discrete set
- one way is to fit linear model without constraining $\mathbf{b_1}$ to be discrete and then to
- quantize the resulting (continuous) estimate of $\mathbf{b_1}$ into symbol estimates

Multi User Detection

- linear fit $\mathbf{My_1}$ as continuous estimate of $\mathbf{b_1}$
- $\mathbf{M}$ is $M \times M$ matrix
- i the symbol decision is $\hat{b}_1[i] = q([\mathbf{My_1}]_i)$
- $[\mathbf{My_1}]_i$ denotes i the component of $\mathbf{My_1}$
- $q(\cdot)$ dentoes quantizer mapping the complex numbers to the symbol alphabet $\mathcal{A}$
- various choce of $\mathbf{M}$ lead to different linear equalizers
- $\mathbf{M} = \mathbf{I_M}$ $M \times M$ identity matrix
- the resulting linear detector is the common matched filter, which is optimal in the absence of ISI
- matched filter ignores ISI
- if $\mathbf{H_1}$ is invertible choice $\mathbf{M} = \mathbf{h_1^{-1}}$
- forces the ISI to zero

$$\mathbf{H_1^{-1}y_1} = \mathbf{b_1} + \mathbf{H_1^{-1}n_1}$$

Multi User Detection

- is knwon as zero forcing equalizer ZFE
- it would be optimal, perfect decision in absence of AWGN
- a tradeoff between the extremes is effected by minimum mean square error MMSE linear equalizer
- which chooses $\mathbf{M}$ to give an MMSE fit of the model assuming the symbols are independent of the noise
- this results in choice

$$\mathbf{M} = (\mathbf{H_1} + \sigma^2 \sum_b^{-1})^{-1}$$

- $\sum_b$ denotes covariance matrix of the symbol $\mathbf{b_1}$ (this will be in the form of a constant time $\mathbf{I_M}$)
- for multi user detection symbols sorted by symbol number and th by user number

Thank You