WMNC assignment.

2) Derive the necessary equations for extracting one bit message from the received signal using maximum likelihood approach for multiuser detection scenario.

Ans: consider the scenario for single user i.e; $K=1$.
channel impulse response q_1 is known to receiver
cochannel interference (CCI); $I=0$;
the ambient noise is modelled as additive, white Gaussian noise (AWGN) with special height σ^2

now, received signal is

$$\boxed{r(t) = \sum_{i=0}^{M-1} b_1[i]\, \Gamma_{i,1}(t) + n(t)}$$

$$\boxed{\Gamma_{i,1}(t) = \int_{-\infty}^{\infty} q_1(t,u)\, w_{i,1}(u)\, du}$$

But for a single symbol ; $M=1$

$$\boxed{\begin{aligned} r(t) &= b_1[0]\, \Gamma_{0,1}(t) + n(t) \\ &= b_1[0] \int_{-\infty}^{\infty} q_1(t,u)\, w_{0,1}(u)\, du \end{aligned}}$$

optimal inferences about the symbol $b_1[0]$ can be derived (or) determined using the likelihood function of observations conditional on the symbol $b_1[0]$

$$\boxed{\mathcal{L}(r|b_1[0]) = \exp\left\{\frac{1}{\sigma^2}\left(2R\left\{b_1^*[0]\int_{-\infty}^{\infty} f_{0,1}^*(t)\,dt\right\} - |b_1[0]|^2\int_{-\infty}^{\infty}|f_{0,1}|^2\,dt\right)\right\}}$$

where;

z^*, represents complex conjugate of z

$R(z)$, represents real part of z

if the symbol alphabet is A, then we have to maximize $\mathcal{L}(r/b_1[0])$ such that (~~$b_1[0]=A$~~) $b_1[0] \leftarrow A$ i.e; using maximum likelihood that maximises $\mathcal{L}(r/b_1[0])$ over the symbol alphabet A.

$$\boxed{\begin{aligned} b_1^n[0] &= \operatorname{argmax}\ \mathcal{L}(r/b_1[0]=b) \\ &= \operatorname*{argmax}_{b\in A}\left(2R\left\{b^*\int_{-\infty}^{\infty} f_{0,1}^*(t)\,r(t)\,dt\right\} - |b|^2\int_{-\infty}^{\infty}|f_{0,1}(t)|^2\,dt\right)\end{aligned}}$$

on solving we get the symbol estimate as

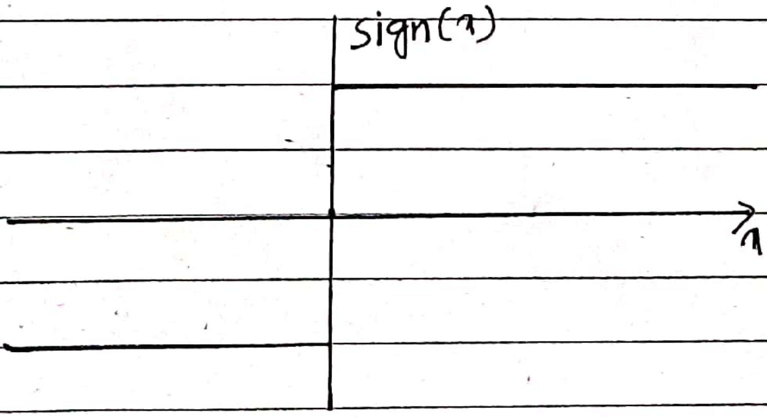
$$\boxed{\begin{aligned} \hat{b}_1[0] &= \underset{b \in A}{\operatorname{argmax}} \, |b - z|^2 \\ &= \underset{b \in A}{\operatorname{argmax}} \left| b - \frac{\int_{-\infty}^{\infty} p^*_{0,1}(t)\, r(t)\, dt}{\int_{-\infty}^{\infty} |p_{0,1}(t)|^2\, dt} \right|^2 \end{aligned}}$$

$$\boxed{\text{where, } \quad z = \frac{\int_{-\infty}^{\infty} p^*_{0,1}(t)\, r(t)\, dt}{\int_{-\infty}^{\infty} |p_{0,1}(t)|^2\, dt}}$$

Clearly maximum likelihood estimate $\hat{b}_1[0]$ is the closest point in the symbol estimate to the observable z.

For binary phase shift keying, max likelihood estimate is $\hat{b}_1[0] = \text{sign}\{\mathcal{R}\{z\}\}$

$$\boxed{\hat{b}_1[0] = \text{sign}\left\{\mathcal{R}\left\{\int p^*_{0,1}(t)\, r(t)\, dt\right\}\right\}}$$



where

$$\text{sign}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$