

# 5

## Fundamentals of Cellular Communications

With the limitation in spectral width, the maximum number of users (capacity) that can be supported in a wireless system is an important performance measure. If the system is supported by a single base station, a high power transmitter is needed to support a large number of users. The system capacity (number of users) can be enlarged by arranging small cells, each requiring only a low power transmitter, in a cellular array. In this chapter, the rationale behind cellular operation as a means of increasing system capacity is explained. The increase in system capacity comes from the use of smaller cells, reuse of frequencies, and antenna sectoring. The ramifications of frequency reuse and the gain obtainable from cell splitting and antenna sectoring are described and discussed in detail.

### 5.1 INTRODUCTION

As discussed in Chapter 2, the wireless propagation channel exhibits impairments far more severe than those inherent in a guided wire. Severe impairments mean that the wireless channel yields a poorer signal-to-interference ratio (SIR) and, hence, higher transmission bit error rate at the output of the receiver compared with those in a wired channel. For example, the BER in an optical fiber is in the neighborhood of  $10^{-10}$  while that in a wireless channel with multipath fading may be in the range of  $10^{-2}$  to  $10^{-3}$ . This high error rate has the effect of limiting the amount of throughput and rendering signal reception unreliable. Impairments in the propagation channel will result in a reduction of usable spectral width. Mitigation of interference due to channel impairments has

the effect of enlarging the spectral width. A wider spectrum, in turn, means more bandwidth to support more users. Certain sophisticated modulation and coding techniques have built-in properties that the receiver can exploit to compensate for channel impairments. Some of the modulation and detection methods that are relevant to wireless communications are discussed in Chapters 3 and 4.

A wider spectrum will allow for greater sharing of the spectral width to support more users and increase the throughput. System capacity can be interpreted as the largest number of users that can be supported during any one use of the channel. A larger population will mean a larger geographical coverage. If a single transmitter were used to cover a large geographical area, a very high power transmitter and very high antenna would be required. With a single high power transmitter, all users will share the same set of frequencies, or radio channels. The system capacity, in terms of the maximum number of users that can be supported, offered by a single high power transmitter will hit a limit. The reason for this limitation is that the radio resources (i.e., frequencies) are not efficiently utilized. If the same set of radio resources were assigned to serve a smaller geographical area and then reused to serve another small geographical area, it would be possible to expand the system capacity. However, care must be taken to ensure that use of the same set of frequencies to serve more than one geographical area does not introduce reception interference between users in the two areas. Or, if there is some interference, it must be at an acceptable level. This means that the geographical regions that use the same set of radio frequencies must be physically separated from each other so that the power level of the signal that spills out from one region to a neighboring region does not produce unacceptable interference. This way of replicating identically structured and operated geographical regions gives rise to the concept of *cellular communications*.

A geographical region can be a single small area called a *cell* that is served by a single base station, or a cluster of cells. In cellular systems, each of the base stations in the cluster is connected to a mobile switching center (MSC) by wirelines. An MSC has more computing power and can perform many more functions than an individual base station. Therefore, most of the communications operations are handled by the MSC.

## 5.2 FREQUENCY REUSE AND MOBILITY MANAGEMENT

### 5.2.1 Cellular Communications and Frequency Reuse

If a given set of frequencies, or radio channels, can be reused without increasing the interference, then the large geographical area covered by a single high power transmitter can be divided into a number of small areas, each allocated a subset of frequencies. With a smaller geographical coverage, lower power transmitters with lower antennas can be used. Provided that the physical separation of two cells is sufficiently wide, the same subset of frequencies can be used in both cells. This is the concept of *frequency reuse* [92]. The ability to reuse the frequencies offers a means to expand the total system capacity without the need to employ high power transmitters. This plan of dividing the total large geographical coverage area into many small contiguous areas and using a low power transmitter with low antenna in each small area is referred to as *cellular communications*.

Each small area is served by a transmit/receive unit called a base station (BS). A base station is the common resource shared by multiple users. User-to-user communication passes through the base station. Users transmit their signals to the base station, and the base station then relays the information-bearing signals to their destinations. We say that the users access the base station so that the base station is also called an access point (AP). The propagation channel that handles the transmissions from the base station to the users is called the forward channel or downlink, while the channel used for transmissions from the users to the base station is called the reverse channel or uplink. Techniques for multiaccessing the base station will be discussed in Chapter 6.

The radio coverage by one base station is referred to as a cell, which is also called a footprint. Cells arranged in a two-dimensional array form a cellular structure. From a conceptual point of view, it is highly desirable to construct the cellular system such that the cells do not overlap, and are tightly packed without dead spots. This form of layout requires the use of regular topologies (e.g., a square, triangular or hexagonal topology). Because of the differences in terrain and population densities, real footprints are amorphous in nature. On the other hand, cellular layouts using irregular structures are inefficient and limit growth. For this reason, cellular layouts and performance studies are based on regular topologies, even though they may just be conceptual. Also, regular topologies allow for systematic growth. In this chapter, all our discussions will be based on a hexagonal topology. The design and performance obtained using regular topologies may not correspond to real environments, but these topologies should provide valuable information and guidelines for structuring practical layouts.

While the cellular approach allows the use of low power transmitters and frequency reuse to expand system capacity, these advantages do not come without a price. Since a salient feature of wireless communications is the flexibility to support user roaming, smaller geographical areas will mean that mobile users can move out of one cell and into another cell quite frequently. To maintain continuous operation of an ongoing session, when the mobile migrates out of its current serving base station into the footprint of another, the connection must be handed off from the serving base station to the new base station.

An effective and efficient handoff mechanism must be in place to maintain service continuity and preserve end-to-end QoS (quality of service) requirements. The procedure to perform and to manage handoff is referred to as handoff management.

The principle of cellular communications is that a mobile host (MH)<sup>1</sup> is assigned a home network. The MH is identified by an address, called its home address. An agent in the home network, referred to as the home agent, keeps track of the MH's current location to facilitate delivery of messages destined for the MH. As the MH migrates away from its home network, the association between the MH and its home agent must be maintained so that the home agent can keep track of the MH's current location for message delivery purposes. In cellular communications, the procedure to keep track of the user's current location so as to maintain the association between the MH and its home agent is referred to as location management. Both handoff management and location management are necessitated by user mobility. These management functions are considered as two components of mobility management.

<sup>1</sup>For mobility and resource management at the networking layer, the term mobile host (MH) is often used instead of mobile station (MS).

## 5.2.2 Mobility Management

Mobility management consists of handoff management and location management. While mobility management issues are dealt with in Chapters 7 and 8, the salient aspects of handoff and location operations are described in the following paragraphs.

**Handoff Management.** When a mobile moves into a different cell while a session is in progress, the session has to be transferred to a new channel belonging to the new cell. This operation is referred to as handoff. The handoff operation involves the identification of a new base station and the allocation of channels to support both data and control signals in the new base station. As alluded to above, the MSC has the computing power to perform many different functions. Therefore, the handoff operation is normally handled by the MSC. The MSC keeps track of the resource usage of all cells under its jurisdiction. When a mobile moves into a different cell while a conversation is in progress, the MSC determines the availability of the unused channels in the new cell and makes transfer decisions. If the new base station has channels that can be allocated to handle both information carrying and control signals to accommodate the handoff connection, the handoff takes place. Otherwise handoff cannot occur.

**Location Management.** As previously mentioned, an MH is associated with a home network and its home address resides with its home agent. When the MH moves away from its home network, it enters a foreign territory. The network where the MH resides when it is away from home is called the foreign network. The MH has to register with its home agent, through the foreign agent, to let the home agent know its current location to facilitate message delivery. When an MH powers on, it registers with its home agent. When it moves to a foreign network, it has to register with its home agent, via the foreign agent. This means that there is an association between the home agent and the foreign agent. When the home agent has messages destined for the MH, it forwards them to the MH via the foreign agent. During the registration process, the home agent needs to ensure, from the identification conveyed through the foreign agent, that the mobile submitting a registration is the correct MH. The process of verifying that the identity provided during the registration process does belong to the correct MH is referred to as an authentication process.

## 5.3 CELL CLUSTER CONCEPT

In a wireless communications system, the channels used in the forward and reverse directions are separated in time or in frequency to permit duplexing. The total number of channels available in a cellular system is finite. The capacity of the cellular system is defined by the total number of channels available. The system capacity as a function of the total number of available channels depends on how the available channels are deployed. Specifically, if the available channels are reused for transmission of additional traffic, it is possible to expand the system capacity (i.e., support more users). Two or more different cells can use the same set of frequencies or radio channels if the nearest cells are separated such that the interference between cells at any given frequency is at an acceptable level. Cells which use the same set of frequencies are referred

to as cochannel cells, and the interference between cochannel cells is referred to as cochannel interference. Frequencies or channels represent radio resources. In this text, we will use the terms frequencies and channels interchangeably.

The space between adjacent cochannel cells can be filled with other cells that use different frequencies to provide frequency isolation. A group of cells that use a different set of frequencies in each cell is called a cell cluster. Let  $N$  be the cluster size in terms of the number of cells within it and  $K$  be the total number of available channels without frequency reuse. The  $N$  cells in the cluster would then utilize all  $K$  available channels. In this way, each cell in the cluster contains one- $N$ th of the total number of available channels. In this sense,  $N$  is also referred to as the frequency reuse factor of the cellular system.

### 5.3.1 Capacity Expansion by Frequency Reuse

Suppose each cell is allocated  $J$  channels ( $J \leq K$ ). If the  $K$  channels are divided among the  $N$  cells into unique and disjoint channel groups, each with  $J$  channels, then

$$K = JN. \quad J = K/N \quad (5.3.1)$$

Collectively the  $N$  cells in a cluster use the complete set of available frequencies. Since  $K$  is the total number of available channels, from Eq. (5.3.1), it can be seen that a decrease in the cluster size  $N$  is accompanied by an increase in the number of channels  $J$  allocated per cell. Thus, by decreasing the cluster size, it is possible to increase the capacity per cell.

The cluster can be replicated many times to form the entire cellular communications system. Let  $M$  be the number of times the cluster is replicated and  $C$  be the total number of channels used in the entire cellular system with frequency reuse.  $C$  is then the system capacity and is given by

$$C = MJN. \quad (5.3.2)$$

If  $N$  is decreased and  $J$  is proportionally increased so that Eq. (5.3.1) is satisfied, it is necessary to replicate the smaller cluster more times in order to cover the same geographical area. This means that the value of  $M$  has to be increased. Since  $JN (= K)$  remains constant and  $M$  is increased, Eq. (5.3.2) shows that the system capacity  $C$  is increased. That is, when  $N$  is minimized,  $C$  is maximized. We will see shortly that minimizing  $N$  will increase cochannel interference.

#### Example 5.1 Cellular System Capacity

Consider a cellular system in which there are a total of 1001 radio channels available for handling traffic. Suppose the area of a cell is  $6 \text{ km}^2$  and the area of the entire system is  $2100 \text{ km}^2$ .

- Calculate the system capacity if the cluster size is 7.
- How many times would the cluster of size 4 have to be replicated in order to approximately cover the entire cellular area?
- Calculate the system capacity if the cluster size is 4.
- Does decreasing the cluster size increase the system capacity? Explain.

**Solution** Given:

The total number of available channels  $K = 1001$   
Cluster size  $N = 7$

So decrease cluster size  
(not - No. of cells)  
increase no. of clusters  
will increase system capacity

Area of cell  $A_{cell} = 6 \text{ km}^2$

Area of cellular system  $A_{sys} = 2100 \text{ km}^2$

a. Since the number of channels per cell is  $J = K/N$ , then  $J = 1001/7 = 143$  channels/cell.  
The coverage area of a cluster is

$$A_{cluster} = N \times A_{cell} = 7 \times 6 = 42 \text{ km}^2.$$

The number of times that the cluster has to be replicated to cover the entire cellular system is  $M = A_{sys}/A_{cluster} = 2100/42 = 50$ . Therefore,

$$C = MJN = 50 \times 143 \times 7 = 50,050 \text{ channels.}$$

b. For  $N = 4$ ,  $A_{cluster} = 4 \times 6 = 24 \text{ km}^2$ . Therefore,

$$M = A_{sys}/A_{cluster} = 2100/24 = 87.5 \approx 87.$$

c. With  $N = 4$ ,  $J = 1001/4 \approx 250$  channels/cell. The system capacity is then

$$C = 87 \times 250 \times 4 = 87,000 \text{ channels.}$$

d. From (a) and (c), it is seen that a decrease in  $N$  from 7 to 4 is accompanied by an increase in  $M$  from 50 to 87, and the system capacity is increased from 50,050 channels to 87,000 channels. Therefore, decreasing the cluster size does increase the system capacity.

### 5.3.2 Cellular Layout for Frequency Reuse

As mentioned earlier, the discussion of cellular communications in this text is based on a two-dimensional chaining of hexagonal cells. The rule to find the nearest cochannel neighbor of a particular cell is as follows.

 Rule for Determining the Nearest Cochannel Neighbors. The following two-step rule can be used to determine the location of the nearest cochannel cell:

Step 1: Move  $i$  cells along any chain of hexagons;

Step 2: Turn 60 degrees counterclockwise and move  $j$  cells.

The method of locating cochannel cells in a cellular system using the preceding rule is shown in Figure 5.1 for  $i = 3$  and  $j = 2$ , where the cochannel cells are the shaded cells.

The cluster concept and frequency reuse in a cellular network are illustrated in Figure 5.2, where cells with the same number use the same set of frequencies. These are cochannel cells that must be separated by a distance such that the cochannel interference is below a prescribed QoS threshold. The parameters  $i$  and  $j$  measure the number of nearest neighbors between cochannel cells; the cluster size,  $N$ , is related to  $i$  and  $j$  by the equation

$$N = i^2 + ij + j^2. \quad (5.3.3)$$

For example, in Figure 5.2(b), we have  $i = 1$  and  $j = 2$ , so that  $N = 7$ . With a cluster size  $N = 7$ , the frequency reuse factor is seven since each cell contains one-seventh of the total number of available channels.

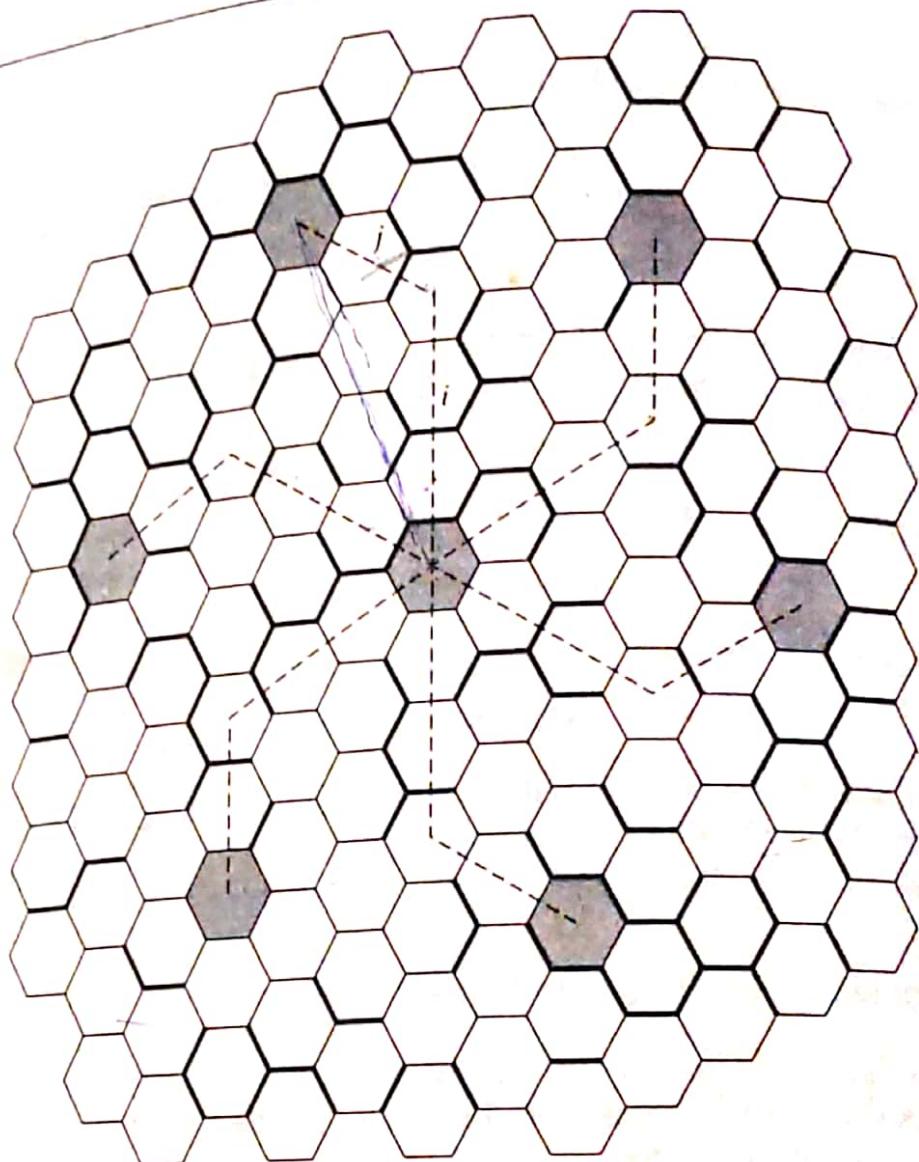


Figure 5.1 Locating cochannel cells in a cellular system.

**Advantages of Cellular Systems.** The advantages of operating in a cellular arrangement include:

- a. the use of low power transmitter and,
- b. an allowance for frequency reuse.

Frequency reuse needs to be structured so that cochannel interference is kept at an acceptable level. As the distance between cochannel cells increases, cochannel interference will decrease. If the cell size is fixed, the average signal-to-cochannel interference ratio will be independent of the transmitted power of each cell (discussed in Section 5.4). The distance between any two cochannel cells can be examined by making use of the geometry of hexagonal cells.

#### Example 5.2 Number of Frequency Channels

Consider a cellular system with a total bandwidth of 30 MHz which uses two 25 kHz simplex channels to provide full duplex voice and control channels. Assuming that the system uses a nine-cell reuse pattern and 1 MHz of the total bandwidth is allocated for control channels,

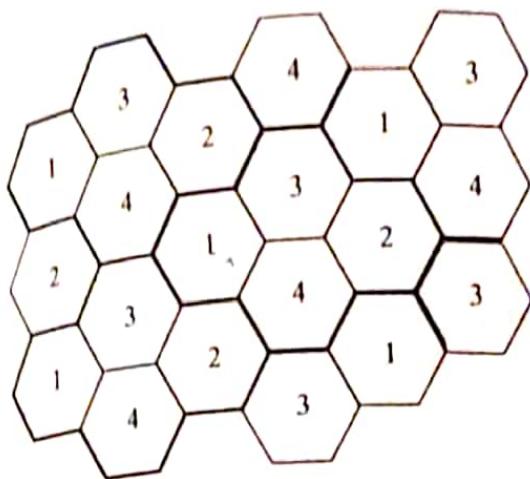
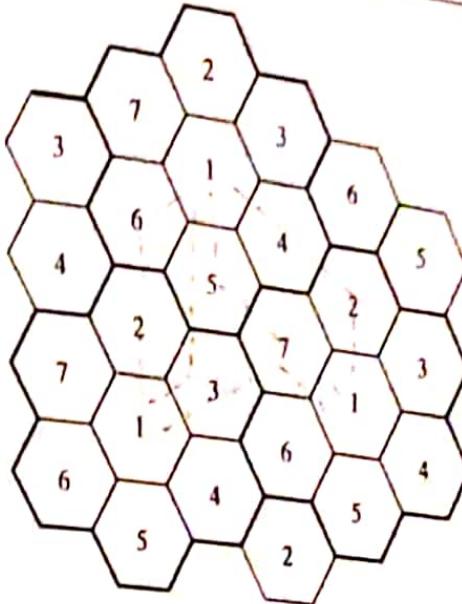
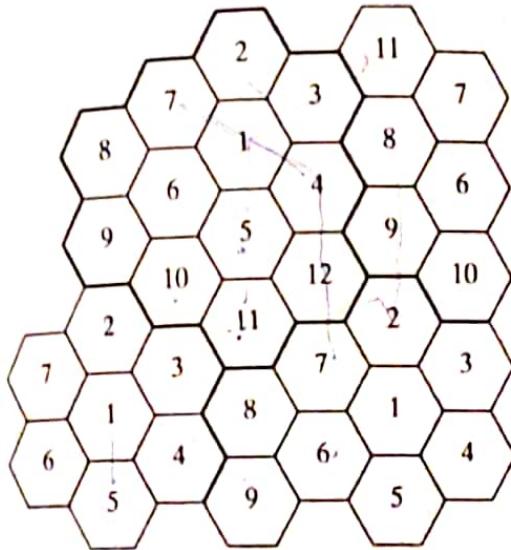
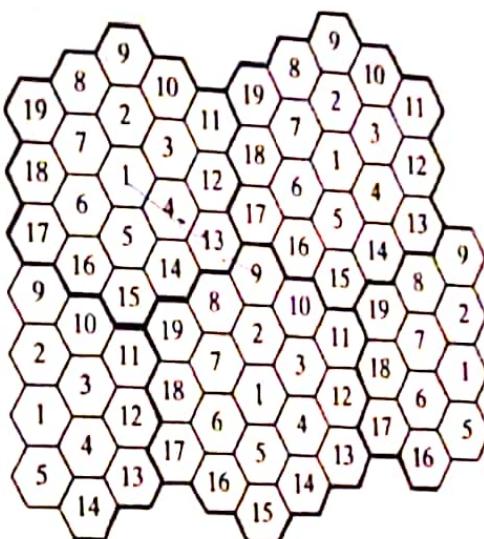
(a)  $i = 2$  and  $j = 0$ (b)  $i = 1$  and  $j = 2$ (c)  $i = 2$  and  $j = 2$ (d)  $i = 2$  and  $j = 3$ 

Figure 5.2 Cell clusters.

- calculate the total available channels,
- determine the number of control channels,
- determine the number of voice channels per cell, and
- determine an equitable distribution of control channels and voice channels in each cell.

**Solution** Given:

$$\text{Total bandwidth} = 30 \text{ MHz}$$

$$\text{Channel bandwidth} = 25 \text{ kHz} \times 2 = 50 \text{ kHz/duplex channel}$$

- The total number of available channels  $= \frac{30000}{50} = 600$ ,
- The number of control channels  $= \frac{1000}{50} = 20$ .

c. The number of voice channels per cell =  $\frac{600-20}{9} \simeq 64$ .

d. Since only a maximum of 20 channels can be used as control channels, for  $N = 9$ , one way to allocate is 7 cells with 2 control channels and 64 voice channels each, and 2 cells with 3 control channels and 66 voice channels each.

Note that the channel allocation performed in part (d) is not unique.

### 5.3.3 Geometry of Hexagonal Cells

The geometry of an array of hexagonal cells is depicted in Figure 5.3, where  $R$  is the radius of the hexagonal cell (from the center to a vertex). A hexagon has exactly six equidistant neighbors. As can be seen from Figure 5.3, in a cellular array the lines joining the centers of any cell and each of its neighbors are separated by multiples of 60 degrees. Note that in Figure 5.3 the 60° angle is bounded by the vertical line and the 30° line, both of which join centers of hexagonal cells.

The distance between the nearest cochannel cells in a hexagonal area can be calculated from the geometry shown in Figure 5.3. For notational convenience, the cell under consideration will be referred to as the candidate cell. The distance between the centers of two adjacent hexagonal cells is  $\sqrt{3}R$ . Let  $D_{\text{norm}}$  be the distance from the center of the candidate cell to the center of a

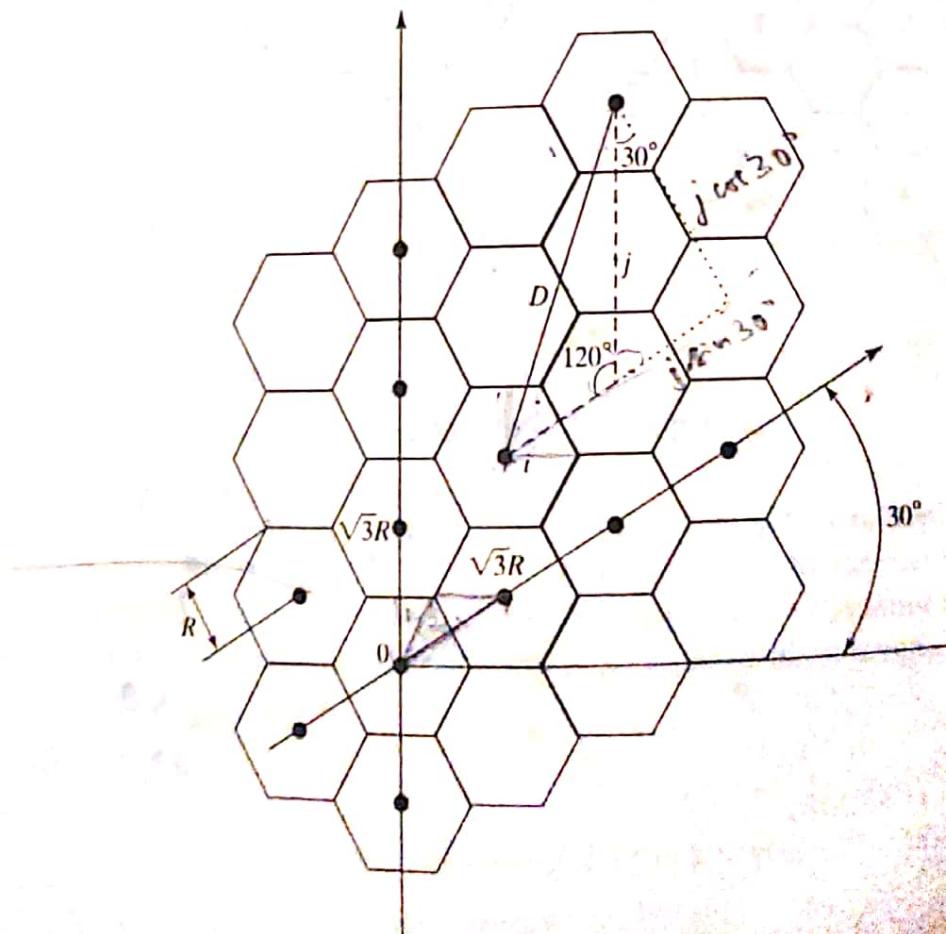


Figure 5.3 Distance between nearest cochannel cells.

nearest cochannel cell, normalized with respect to the distance between the centers of two adjacent cells,  $\sqrt{3}R$ . Note that the normalized distance between two adjacent cells (either with  $i = 1$  and  $j = 0$  or with  $i = 0$  and  $j = 1$ ) is unity. Let  $D$  be the actual distance between the centers of two adjacent cochannel cells.  $D$  is a function of  $D_{norm}$  and  $R$ . From the geometry shown in Figure 5.3 we readily have

$$\begin{aligned} D_{norm}^2 &= j^2 \cos^2(30^\circ) + (i + j \sin(30^\circ))^2 \\ &= i^2 + j^2 + ij. \end{aligned} \quad (5.3.4)$$

From Eqs. (5.3.4) and (5.3.3),

$$D_{norm} = \sqrt{N}.$$

With the actual distance,  $\sqrt{3}R$  between the centers of two adjacent hexagonal cells, the actual distance between the center of the candidate cell and the center of a nearest cochannel cell is

$$D = D_{norm} \times \sqrt{3}R = \sqrt{3NR}. \quad (5.3.5)$$

For hexagonal cells, there are six nearest cochannel neighbors to each cell. Cochannel cells are located in tiers. In general, a candidate cell is surrounded by  $6k$  cells in tier  $k$ . For cells with the same size, the cochannel cells in each tier lie on the boundary of the hexagon that chains all the cochannel cells in that tier. As  $D$  is the radius between two nearest cochannel cells, the radius of the hexagon chaining the cochannel cells in the  $k$ th tier is given by  $kD$ . For the frequency reuse pattern with  $i = 2$  and  $j = 1$  so that  $N = 7$ , the first two tiers of cochannel cells are illustrated in Figure 5.4. It can be readily observed from Figure 5.4 that the radius of the first tier is  $D$  and the radius of the second tier is  $2D$ .

### Example 5.3 Number of Cells in a Cluster

Verify that the cell cluster size is  $N = i^2 + ij + j^2$ , where  $i$  and  $j$  are the integer parameters determining the cochannel cells as illustrated in Figure 5.1.

**Solution** A candidate cell has 6 nearest cochannel cells. By joining the centers of the 6 nearest neighboring cochannel cells, we form a large hexagon, as shown in Figure 5.5. This large hexagon has radius equal to  $D$ , which is also the cochannel cell separation. With the cell radius  $R$ , from Eq. (5.3.4), we have

$$D = \sqrt{3}RD_{norm} = \sqrt{3(i^2 + ij + j^2)}R.$$

In general, the area of a hexagon is proportional to the square of its radius. Let  $\beta$  ( $\approx 2.598$ ) be the proportional constant. Then the area of the large hexagon with radius  $D$  is

$$A_{\text{large}} = \beta D^2 = \beta[3(i^2 + ij + j^2)R^2]$$

and the area of a cell (the small hexagon) with radius  $R$  is

$$A_{\text{small}} = \beta R^2.$$

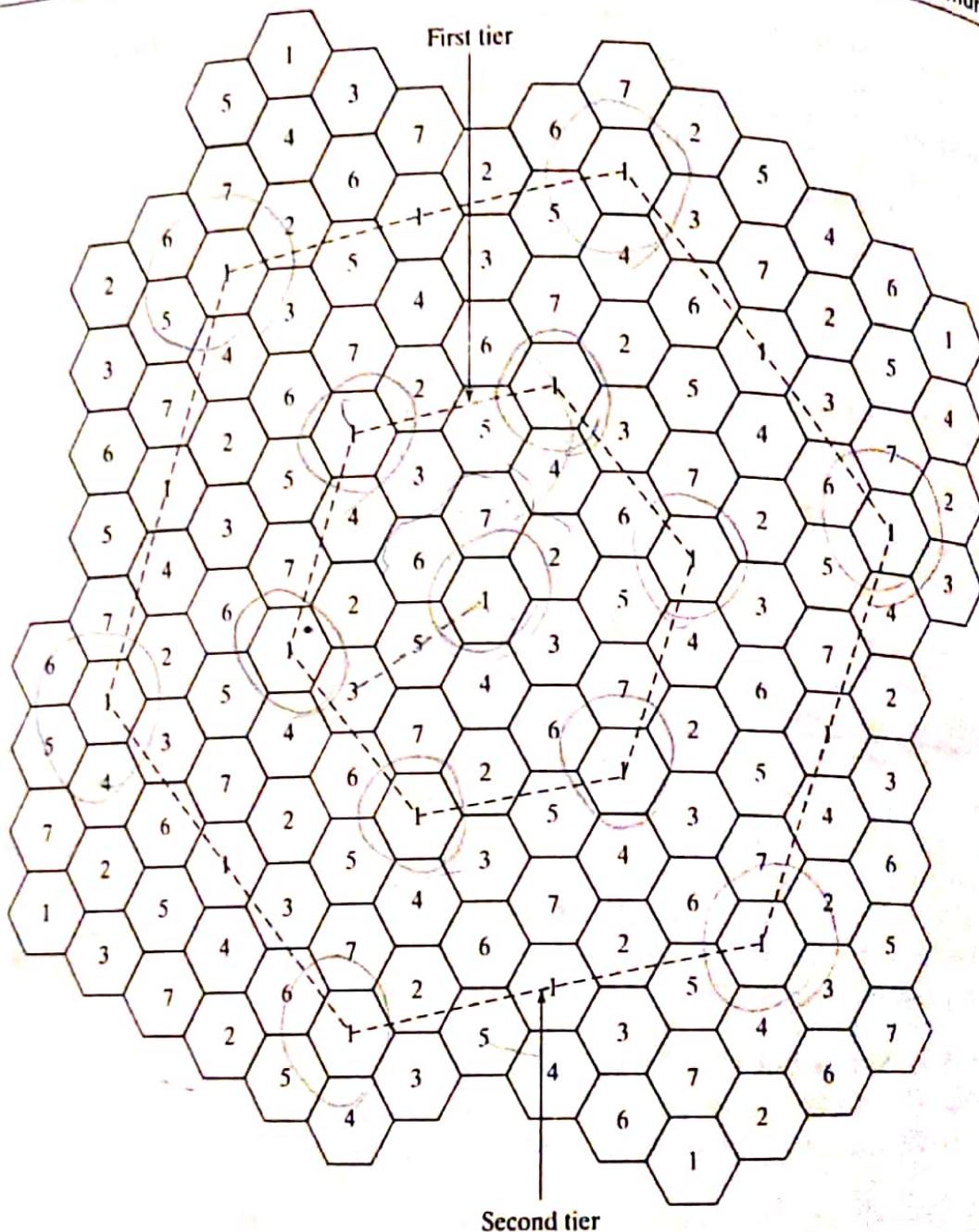


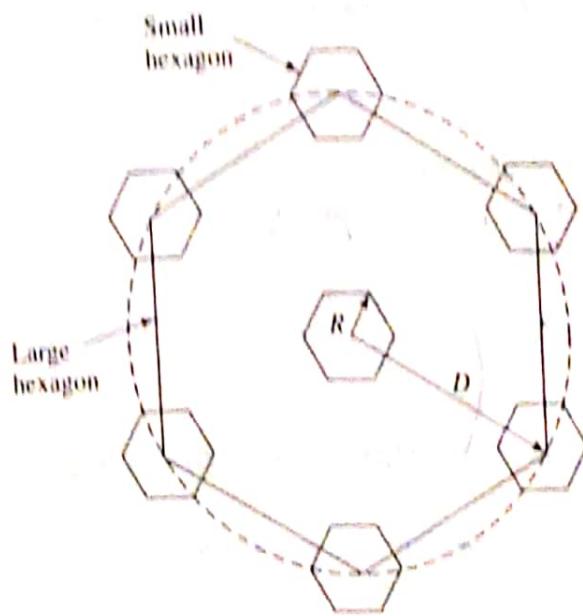
Figure 5.4 Two tiered interfering cells with  $N = 7$ .

The number of cells in the large hexagon is then

$$\frac{A_{\text{large}}}{A_{\text{small}}} = 3(i^2 + ij + j^2). \quad (5.3.6)$$

On the other hand, from the geometry (e.g., Figure 5.4) it can be seen that, in general, the large hexagon encloses the center cluster of  $N$  cells plus  $1/3$  the number of the cells associated with six other peripheral large hexagons. Hence, the total number of cells enclosed by the large hexagon is

$$N + 6 \left( \frac{1}{3} N \right) = 3N. \quad (5.3.7)$$



**Figure 5.5** First tier cochannel interfering cells.

From Eqs. (5.3.6) and (5.3.7), we have

$$N = i^2 + ij + j^2,$$

#### 5.3.4 Frequency Reuse Ratio

The frequency reuse ratio,  $q$ , is defined as

$$q \triangleq \frac{D}{R} = \frac{\sqrt{3}NR}{R}$$
(5.3.8)

Because frequency reuse leads to cochannel cell operation,  $q$  is also referred to as the cochannel reuse ratio.

**Table 5.1** Frequency Reuse Ratio and Cluster Size

Frequency Reuse Pattern ( $i, j$ )	Cluster Size $N$	Frequency Reuse Ratio $q$
(1, 1)	3	3.00
(2, 0)	4	3.46
(2, 1)	7	4.58
(3, 0)	9	5.20
(2, 2)	12	6.00
(3, 1)	13	6.24
(3, 2)	19	7.55
(4, 1)	21	7.94
(3, 3)	27	9.00
(4, 2)	28	9.17
(4, 3)	37	10.54

Substituting Eq. (5.3.5) into Eq. (5.3.8), the frequency reuse ratio  $q$  is related to the cluster size (or frequency reuse factor)  $N$  by

$$q = \sqrt{3N}.$$

(5.3.9)

Since  $q$  increases with  $N$  and since a smaller value of  $N$  has the effect of increasing the capacity of the cellular system (see Subsection 5.3.1) and, at the same time, increasing cochannel interference, the choice of  $q$  or  $N$  has to be made such that the signal-to-cochannel interference ratio is at an acceptable level. Several frequency reuse patterns, together with the corresponding cluster sizes and frequency reuse ratios, are tabulated in Table 5.1 for easy reference.

## 5.4 COCHANNEL AND ADJACENT CHANNEL INTERFERENCE

Consider the performance of the candidate cell in a cellular array. A given base station provides the capacity to handle the services of many mobile users. The signal received from a target user at the cell-site (base station) receiver is subject to interference from transmissions of other mobiles in the same cell, background noise, and interference from transmissions by mobiles in neighboring cells. Under the assumption of a proper separation between uplink transmissions and downlink transmissions, either in the time domain (i.e., time division duplexing) or in the frequency domain (i.e., frequency division duplexing), interference from transmissions in the other link can be neglected. Interference from other mobiles at the cell-site receiver in the same cell is intracell interference. Interference from other cells is intercell interference. Intercell interference in the downlink that affects the reception at the individual mobile hosts may be more of a problem than uplink interference at the cell-site receiver. The reason for this can be attributed to the fact that the cell-site receiver may be more sophisticated than the receivers of the individual mobile users.

If the different cells in the entire cellular system were to use different sets of frequencies, intercell interference would be kept at a minimum. However, the system capacity would be limited; deployment of frequency reuse (see Subsection 5.3.1) is necessary to enlarge the system capacity. On the other hand, frequency reuse will introduce cochannel interference from cells using the same set of frequencies. Therefore, frequency reuse needs to be carefully planned so that cochannel interference is kept at an acceptable level.

### 5.4.1 Cochannel Interference

#### Intercell

As mentioned in Chapter 1, wireless channels are interference limited. Except for the cochannel cells, other neighboring cells will operate at frequencies different from those of the candidate cell so that interference from noncochannel cells will be minimal. Intercell interference is thus dominated by cochannel interference. It is thus of interest to assess system performance, taking into consideration interference from cochannel cells. For simplicity in the following analysis, we will consider only the average channel quality as a function of the distance dependent path loss without going into details of channel statistics due to propagation shadowing and multipath fading.

We will use the symbols  $S$  and  $I$  to denote respectively the power of the desired signal and the power of the cochannel interference at the output of the receiver demodulator. Let  $N_c$  be the number of cochannel interfering cells and  $I_i$  be the interference power caused by the  $i$ -th

from the  $i$ th interfering cochannel cell base station. The signal-to-cochannel interference ratio ( $S/I$ ) at the desired mobile receiver is given by

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{N_I} I_i}$$

The average received signal strength at any point decays as a power law of the distance between the transmitter and the receiver, as discussed in Section 2.4.

Let  $D_i$  be the distance between the  $i$ th interferer and the mobile. The received interference,  $I_i$ , at a given mobile due to the  $i$ th interfering cell is proportional to  $(D_i)^{-\kappa}$ , where  $\kappa$  is the path loss exponent. The path loss exponent  $\kappa$  is normally determined by measurement. In many cases it is in the range  $2 \leq \kappa \leq 5$ .

In addition to cochannel interference, there is always the inherent background noise. However, in an interference dominated environment, we may neglect the background noise. It is noted that the desired received signal power  $S$  is proportional to  $r^{-\kappa}$ , where  $r$  is the distance between the mobile and the serving base station. When the transmit powers from all base stations are equal, and the path loss exponent is the same throughout the geographical coverage area, the cochannel interference from the  $i$ th cochannel cell,  $I_i$ , for all  $i$ , depends on  $D_i$  and  $\kappa$  only. The  $S/I$  at a typical mobile receiver can be approximated by

$$\frac{S}{I} = \frac{r^{-\kappa}}{\sum_{i=1}^{N_I} D_i^{-\kappa}}. \quad (5.4.1)$$

The degree of cochannel interference is a function of the location of the mobile within the cell of the serving base station. When the mobile is located at the cell boundary (i.e.,  $r = R$ ), the worst case cochannel interference occurs as the power of the desired signal is minimum. With hexagon shaped cellular systems, there are always six cochannel interfering cells in the first tier. If we neglect cochannel interference from the second and other higher tiers, this means that  $N_I = 6$ . In the case that  $r = R$  and using  $D_i \approx D$  for  $i = 1, 2, \dots, N_I$ ,

$$\frac{S}{I} = \frac{(D/R)^\kappa}{N_I} = \frac{q^\kappa}{N_I} = \frac{(\sqrt{3}N)^\kappa}{N_I}. \quad (5.4.2)$$

Thus, the frequency reuse ratio can be expressed as

$$q = \left( N_I \times \frac{S}{I} \right)^{1/\kappa} = \left( 6 \times \frac{S}{I} \right)^{1/\kappa}. \quad \frac{S}{I} = q^\kappa + \frac{1}{6} \quad (5.4.3)$$

For the U.S. AMPS analog FM system, a value of  $S/I = 18$  dB or greater is acceptable. With a path loss exponent of  $\kappa = 4$ , the frequency reuse ratio  $q$  is determined as

$$q = (6 \times 10^{1.8})^{1/4} = (6 \times 63.1)^{0.25} \approx 4.41.$$

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Therefore, the cluster size  $N$  should be

$$N = q^2/3 = 6.49 \approx 7.$$

#### Example 5.4 S/I Ratio versus Cluster Size

Suppose the acceptable signal-to-cochannel interference ratio in a certain cellular communications situation is  $S/I = 20$  dB or 100. Also, from measurements, it is determined that  $\kappa = 4$ . What is the minimum cluster size?

**Solution** The frequency reuse ratio can be calculated, using Eq. (5.4.3), as

$$q = (6 \times 100)^{1/4} = 4.9492.$$

Then, from Eq. (5.3.9), the cluster size is given by

$$N = q^2/3 = 8.165 \approx 9.$$

In this case, a 9-reuse pattern is needed for an  $S/I$  ratio of at least 20 dB. Since

$$q = D/R \quad \text{or} \quad D = qR.$$

$D$  can be determined, given the cell radius  $R$ , and vice versa. Note that if  $N$  is less than 9, the  $S/I$  value would be below the acceptable level of 20 dB.

Consider that the mobile is at the cell boundary, where it experiences worst case cochannel interference on the forward channel. If we use a better approximation of the distance between the mobile and the first tier interfering base stations as illustrated in Figure 5.6, then from Eq. (5.4.1) the  $S/I$  ratio can be expressed as (see Problem 5-3)

$$\frac{S}{I} = \frac{R^{-\kappa}}{2(D-R)^{-\kappa} + 2D^{-\kappa} + 2(D+R)^{-\kappa}}. \quad (5.4.4)$$

Recall that  $D/R = q$ . With a path loss exponent of  $\kappa = 4$ , Eq. (5.4.4) can be written as

$$\frac{S}{I} = \frac{1}{2(q-1)^{-4} + 2q^{-4} + 2(q+1)^{-4}}. \quad (5.4.5)$$

#### Example 5.5 Worst Case Cochannel Interference

Consider a cellular system that requires an  $S/I$  ratio of 18 dB.

- For a frequency reuse factor of 7, calculate the worst-case signal-to-cochannel interference ratio.
- Is a frequency reuse factor of 7 acceptable in terms of cochannel interference? If not, what would be a better choice of frequency reuse factor?

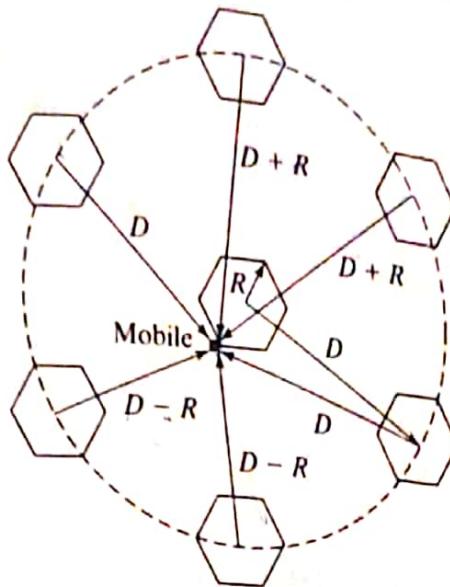


Figure 5.6 Worst case cochannel interference scenario for  $N = 7$ .

**Solution**

- a. For  $N = 7$ , the frequency reuse ratio is  $q = \sqrt{3N} = 4.6$ . Assuming a path loss exponent of  $\kappa = 4$ , then, from Eq. (5.4.5), the worst-case signal-to-cochannel interference ratio is

$$\frac{S}{I} = 54.3 \quad \text{or} \quad 17.3 \text{ dB.} \quad 10 \log 54.3$$

- b. The value of  $S/I$  for a 7 frequency reuse factor is below the acceptable level of 18 dB. To increase  $S/I$ , we need to decrease  $I$ . This can be achieved by increasing the frequency reuse factor,  $N$ . For  $N = 9$ , we have  $q = \sqrt{3N} = 5.20$ , so that  $S/I = 95.66$  or 19.8 dB. This value of  $S/I$  is above the acceptable level of 18 dB. Therefore, a frequency reuse factor of 9 is a suitable choice.

Although an increase of frequency reuse factor from 7 to 9 in Example 5.5 yields an acceptable  $S/I$  level, this increase in  $N$  is accompanied by a decrease in system capacity, since a 9-cell reuse offers a spectrum utilization of  $1/9$  within each cell, whereas a 7-cell reuse offers a spectrum utilization of  $1/7$ . The capacity reduction may not be tolerable. From an operational point of view, catering to the worst-case scenario, which rarely occurs, may not be desirable. The designer may want to seek the best tradeoff by accepting the fact that, with a small but nonzero probability, the worst-case scenario may occur and, hence, degrade the performance to a level below specification during some interval of the holding time of a call.

From the preceding analysis, it is clear that cochannel interference determines link performance which, in turn, dictates the frequency reuse plan and the overall capacity of cellular systems.

#### 5.4.2 Adjacent Channel Interference

Adjacent channel interference (ACI) results from signals which are adjacent in frequency to the desired signal. ACI is mainly due to imperfect receiver filters which allow nearby frequencies to

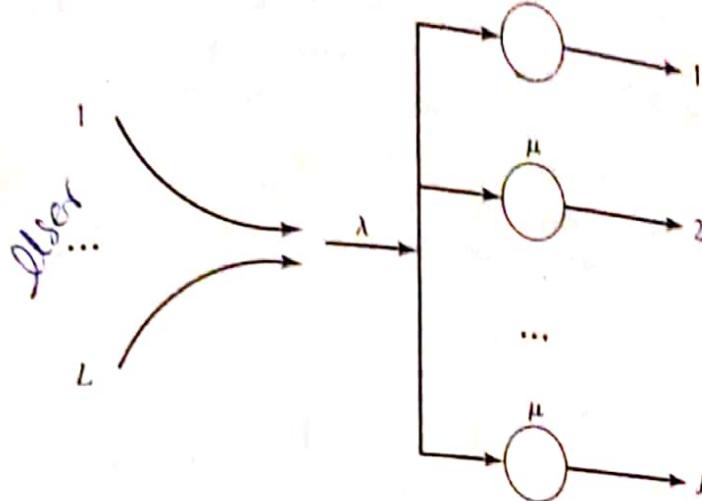
leak into the passband. Consider the uplink transmissions from two mobile users using adjacent channels, one very close to the base station and the other very close to the cell boundary. Without proper transmission power control, the received power from the mobile close to the base station is much larger than that from the other mobile far away. This near-far effect can significantly increase the ACI from the strong received signal to the weak received signal. To reduce ACI, we should: (a) use modulation schemes which have low out-of-band radiation (e.g., MSK is better than QPSK and GMSK); (b) carefully design the bandpass filter at the receiver front end; (c) use proper channel interleaving by assigning adjacent channels to different cells; (d) avoid using adjacent channels in adjacent cells to further reduce ACI if the cell cluster size is large enough; and (e) separate the uplink and downlink properly by TDD or FDD.

## 5.5 CALL BLOCKING AND DELAY AT THE CELL-SITE

Signal-to-interference ratio, which determines the transmission bit error rate, is a QoS factor at the output of the cell-site receiver. From the user's perspective, quality of service is more than an acceptable transmission accuracy. In fact, there are two crucial questions: (a) How successfully can a new user get a connection established? (b) After connection establishment, how successfully will the connection be maintained as the user moves from one cell to another? The first question refers to the admission of new calls, while the second question refers to the admission of handoff calls. The performance measure is the probability that a call (new or handoff) is blocked.

To formulate the probability of call blocking, consider a radio cell which has been allocated  $J$  channels. Assume a large population size of mobile users in the cell. Suppose during the connection time of a call, each user occupies one channel. If the number of active users during any epoch equals  $J$ , all available channels will be occupied. Then, with probability 1, a call request will be denied (i.e., blocked). If the number of ongoing calls is fewer than  $J$ , a call will be blocked with probability smaller than 1. This is equivalent to the condition that the trunk traffic load in Erlangs is less than  $J$ , the number of available channels. One Erlang represents the amount of traffic load carried by a channel that is completely occupied, such as one call-hour per hour. If a channel is busy for 30 minutes during a one hour period, then the channel is said to carry 0.5 Erlangs of traffic. Offered traffic refers to the amount of traffic sent by the users, while carried traffic refers to the amount of traffic served.

To obtain an expression for call blocking probability, we model the cell-site as a bufferless system. With no buffer, blocked calls are lost. Assume that (a) there are  $L (\gg 1)$  users in the system; (b) the aggregate arrival traffic is Poisson distributed with rate  $\lambda$ ; (c) the duration of a call is exponentially distributed with parameter  $\mu_1$ ; and (d) the residence time of each user in a cell is exponentially distributed with parameter  $\mu_2$ . As an exponential random variable is memoryless, the channel holding time is the minimum of the call duration and the cell residence time, which is also exponentially distributed with parameter  $\mu = \mu_1 + \mu_2$ . That is, the mean channel holding time of the call is  $\mu^{-1}$ , corresponding to a mean service rate of  $\mu$  for the call. Since the channel holding time is exponentially distributed, the service time of each of the servers is also exponentially distributed. With Poisson arrivals and exponential service times, the underlying queuing process is Markovian.

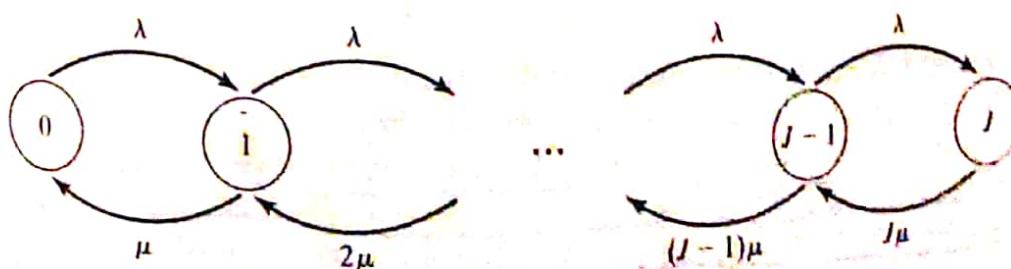
Figure 5.7  $J$ -server bufferless model.

The cell-site receiver can be modeled as a  $J$ -server system, in which each server serves traffic at a mean rate  $\mu$ . The  $J$ -server system for a population of size  $L$  and aggregate arrival rate  $\lambda$  is illustrated in Figure 5.7. The state transition rate diagram of the multiserver system with Poisson arrival and exponential service time is shown in Figure 5.8. As shown in the diagram, when the system is in state  $j$ , for  $j = 0, 1, \dots, J$ , there are  $j$  ongoing calls, and  $j$  servers, each with mean service rate  $\mu$ , are being engaged. When the system is in state  $j = J$ , all  $J$  servers are engaged and new requests will be blocked. Note that, if  $L \leq J$ , the number of users is fewer than the number of servers, there will be no blocking. But this would not be the best way to utilize the available resources.

// The traffic intensity is defined as  $\rho \triangleq \lambda/\mu$ . In terms of the traffic intensity  $\rho$ , the probability of blocking is given by the *Erlang loss formula*

$$\text{Erlang B} \quad \text{Erlang Loss}, P(\text{blocking}) = \frac{\rho^J / J!}{\sum_{j=0}^J \rho^j / j!}. \quad (5.5.1)$$

Equation (5.5.1) is also known as the Erlang-B loss formula. The call blocking probability is considered to be the grade of service (GoS) parameter for the Erlang-B system. The traffic intensity in Erlangs for a prescribed GoS is tabulated in Appendix F.

Figure 5.8 State transition rate diagram of a  $J$ -server system.

In the bufferless  $J$ -server system, the delay for any request is zero. The request is either granted or blocked. If some queueing delay can be tolerated, the  $J$ -server system in Figure 5.7 can also have a common buffer for queueing requests when all  $J$  servers are fully engaged. Upon arrival, a request (new or handoff), finding all  $J$  servers are busy, can join the queue. When a server becomes available, the head of the queue will then receive service. In this case, the queueing delay would be nonzero. The reason for queueing requests is to decrease the call blocking probability. The probability of queueing is given by [31, 158]

$$P(\text{queueing}) = \frac{\frac{J\rho^J}{J!(J-\rho)}}{\left[ \frac{J\rho^J}{J!(J-\rho)} \right] + \sum_{j=0}^{J-1} \left( \frac{\rho^j}{j!} \right)}. \quad (5.5.2)$$

Queueing gives rise to delay. The probability of nonzero delay is given by

$$P(\text{delay} > 0) = \frac{\rho^J}{\rho^J + J! \left( 1 - \frac{\rho}{J} \right) \sum_{j=0}^{J-1} \frac{\rho^j}{j!}}. \quad (5.5.3)$$

Equations (5.5.2) and (5.5.3) are basically the same. Both equations are known as the Erlang-C formula. The probability of nonzero delay that can be tolerated is also considered to be the GoS parameter for the Erlang-C system. The offered loads in Erlangs corresponding to the number of channels available for different 'GoS', given by the Erlang-C formula, are tabulated in Appendix F.

A given request for network access can only tolerate a finite amount of delay. It is of interest to know the probability that the delay exceeds a given delay threshold. Let  $t$  be the delay threshold. Given that the mean channel holding time of a call is  $\mu^{-1}$ , the probability that the delay exceeds  $t$  is given by

$$\begin{aligned} P(\text{delay} > t) &= P(\text{delay} > 0) \times P(\text{delay} > t | \text{delay} > 0) \\ &= P(\text{delay} > 0) \exp[-(J - \rho)\mu t]. \end{aligned} \quad (5.5.4)$$

The average delay,  $\bar{D}$ , for all calls in the queueing system is given by

$$\bar{D} = P(\text{delay} > 0) \times \frac{1}{\mu(J - \rho)}. \quad (5.5.5)$$

In the Erlang-B or Erlang-C system, a channel is allocated to a user on a call by call basis, thus allowing the system to serve a number of subscribers much greater than the number of available channels. The system performance can be described by *trunking efficiency*. Trunking efficiency is defined as the carried traffic intensity in Erlangs per channel, which is a value between zero and one, and is a function of the number of channels per cell and the specified GoS parameters, such as call blocking rate in the Erlang-B system and average queueing delay in the Erlang-C system.

For a comparison of the Erlang-B and Erlang-C systems, given (a) the same number of channels per cell and (b) the call blocking rate in the Erlang-B system being the same as the probability of nonzero queueing delay in the Erlang-C system, the offered traffic intensity of the Erlang-B system is larger than that of the Erlang-C system. For example, with  $J = 20$ ,  $P(\text{blocking}) = P(\text{queueing}) = 0.1$ , from Appendix F, the offered traffic intensity is 17.613 Erlangs in the Erlang-B system and is 14.116 Erlangs in the Erlang-C system. This is because, in the Erlang-B system, the blocked calls are cleared and are not served by the system; while in the Erlang-C system, the new calls are buffered (not dropped) when all the channels are busy and are then served when a channel becomes available. As a result, some offered traffic (the blocked calls) in the Erlang-B system is not carried by the system, but all the offered traffic in the Erlang-C system is carried by the system, under the assumption that the incoming calls can tolerate any queueing delay.

### Example 5.6 Traffic Load and Call Blocking Probability

Consider a cellular system with 416 radio channels available for handling traffic. Suppose 21 of these channels are designated as control channels. Let the average channel holding time of a call be 3 minutes, the blocking probability during busy hours be 2%, and the frequency reuse factor be 9.

- Determine the number of calls per cell per hour.
- Determine the signal-to-cochannel interference ratio,  $S/I$ , in dB.

#### Solution

- In a cellular system with frequency reuse, all the available radio channels are allocated to handle services in a single cluster of cells. Therefore, the number of voice channels in a cluster is  $416 - 21 = 395$ . With a frequency reuse factor of 9, the effective number of voice channels per cell  $= \frac{395}{9} \approx 44$ . With 44 available channels and a 2% blocking probability, the traffic load is (from the Erlang-B table in Appendix F) 34.683 Erlangs.

The number of calls per cell per unit time  $= \frac{\text{number of calls per cell (Traffic)}}{\text{average holding time of a call}}$

Therefore,

$$\begin{aligned} \text{the number of calls per cell per unit time} &= \frac{34.683}{3} \text{ calls per cell per minute} \\ &= \frac{34.683}{3} \times 60 \approx 693 \text{ calls per cell per hour.} \end{aligned}$$

- The frequency reuse ratio  $q = \sqrt{3N} = \sqrt{3 \times 9} = 5.1962$ . For  $\kappa = 4$ , the signal-to-cochannel interference ratio is

$$S/I = \frac{1}{6} \times q^\kappa = \frac{1}{6} \times (5.1962)^4 = 121.5 \text{ or } 20.845 \text{ dB}$$