Wireless Channel Impairments

- Characterize the wireless channel
  - by identifying the parameters of the corruptive elements that distort the information carrying signal
- Corruptive elements
  - multipath delay spread
  - doppler spread due to motion
  - signal fading
- Propagation channel modeled as
  - linear time-variant
  - then characterize by corruptive elements

Multipath propagation

- propagation channel contains objects (particles) which randomly scatter the energy of the transmitted signal
- multipath spread due to channel scattering
- multipath: a transmitted point source received as a multipoint source
- scatter introduces corruptive elements
- inherent background noise as thermal noise modeled as AWGN
- time dispersion:
  - multiple propagation paths have different propagation delays
  - scatters located on ellipses with transmitter and receiver as foci
  - one ellipse associated with one path length

- signals reflected by scatters located on the same ellipse will experience the same propagation delay result into indistinguishable at the receiver
- signals reflected by scatters located on different ellipses will arrive at the receiver with differential delays
- maximum differential delay spread is small compared with the symbol duration of the transmitted signal, channel exhibits *flat fading*
- differential delay is large compared with the symbol interval, the channel exhibits *frequency-selective fading*
- intersymbol interference (ISI): successive transmitted symbols overlap at receiver
- multipath components can be constructive or destructive depending on the carrier frequency and delay differences

Fading

- MS moves, the position of scatter may change with respect to transmitter and receiver
- received signal level fluctuates with time; called fading

$$r(t) = \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c (t - \tau))$$

$$r(t) = \alpha \cos(2\pi f_c t + \phi)$$

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1 \alpha_2 \cos(2\pi f_c \tau)}$$

$$\phi = -\tan^{-1}\left[\frac{\alpha_2 \sin(2\pi f_c t)}{\alpha_1 + \alpha_2 \cos(2\pi f_c t)}\right]$$

- as MS moves α_1, α_2, and τ change with time
- two paths add destructively; deep fade - received signal power is low, poor transmission quality, high transmission error rate
- as MS moves or scatterers move - propagation environment changes and hence, channel is time varying

Linear Time-Variant Channel Model

- N distinct scatterers
- path associated with nth scatterer is characterized by tuple$(\alpha_n(t), \tau_n(t))$
- $\alpha_n(t)$ amplitude fluctuation introduced to the transmitted signal by scatterer at t
- $\tau_n(t)$ associated propagation delay
- narrowband signal $\tilde{x}(t) = \Re\{x(t)e^{j2\pi f_c t}\}$

$$\tilde{r}(t) = \Re\left\{\sum_{n=1}^{N} \alpha_n(t)x(t-\tau_n(t))e^{j2\pi f_c(t-\tau_n(t))}\right\} = \Re\{r(t)e^{j2\pi f_c t}\}$$

$$r(t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j2\pi f_c \tau_n(t)}x(t-\tau_n(t))$$

Channel Impulse Response

$$r(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

- $h_1(t)$ and $h_2(t)$ channel responses to $\delta(t)$ and $\delta(t-t_1)$; as channel changes with time $(h_2(t) \neq h_1(t-t_1))$
- channel impulse response as a function of two time variables; one instant when impulse is applied to channel (initial time) and the other instant when channel output observed (final time)
- channel $h(\tau, t)$ - channel output at t in response to an impulse applied to the channel at $t-\tau$

$$r(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau, t)d\tau$$

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\theta_n(t)}\delta(\tau - \tau_n(t)) \quad \theta_n(t) = 2\pi f_c \tau_n(t)$$

$\theta_n(t)$ carrier phase distortion

Time-Variant Transfer Function

- $H(f,t)$ channel transfer function - Fourier transform of channel impulse response $h(\tau,t)$ with respect to τ
- as channel changes with t, frequency domain representation changes with t

$$H(f,t) = \mathcal{F}_\tau[h(\tau,t)] = \int_{-\infty}^{\infty} h(\tau,t)e^{-j2\pi f\tau}d\tau$$

$$R(f,t) = H(f,t)X(f) \quad r(t) = \int_{-\infty}^{\infty} R(f,t)e^{j2\pi ft}df$$

if channel is time invariant then the impulse response is a function of delay variable and independent of t

Doppler Spread Function

- LTI does not have frequency components different from those of the input signal
- for nonlinear and time varying system introduce new frequency components
- due to the mobility of user/scatterer, channel is linear but time varying - introduces frequency shift to transmitted signal called doppler effect (doppler shift)
- pilot signal from BS at frequency f_c to MS moving with velocity V in x direction and $\theta(t)$ angle viewed from MS to BS
- received signal at MS at time t has a frequency of $f_c + v(t)$. $v(t)$ is doppler shift

$$v(t) = \frac{V f_c}{c} \cos\theta(t)$$

Channel being time varying in the time domain $\longrightarrow$ a channel introducing doppler shift in the frequency domain

- wireless channel introduces continuous doppler shift
- the effect of the channel on the transmitted signal in the frequency domain is more spectral broadening than a simple spectral shift
- Let, delay approximated by its mean value $\bar{\tau}$ and it does not change with time, then time variant impulse response of channel

$$h(\tau,t) \approx Z(t)\delta(\tau-\bar{\tau}) \quad Z(t) = \sum_{n=1}^{N} \alpha_n(t)\exp[-j2\pi f_c \tau_n(t)]$$

$$r(t) = \int_{-\infty}^{\infty} h(\tau,t)x(t-\tau)d\tau = \int_{-\infty}^{\infty} [Z(t)\delta(\tau-\bar{\tau})]x(t-\tau)d\tau = Z(t)x(t-\bar{\tau})$$

provide complex time-varying gain and delay

$$\begin{aligned} R(f) &= \mathcal{F}[r(t)] = \mathcal{F}[Z(t)x(t-\bar{\tau})] \\ &= \mathcal{F}[Z(t)] \star \mathcal{F}[x(t-\bar{\tau})] = \mathcal{F}[Z(t)] \star [X(f)e^{-j2\pi f\bar{\tau}}] \end{aligned}$$

- $Z(t)$ changes with time, its Fourier transform has a finite nonzero pulse width in the frequency domain and the pulse width of $R(f)$ is larger than the pulse width of $X(f)$ due to convolution
- the channel broadens the transmitted signal spectrum by introducing new frequency components called *frequency dispersion*
- Doppler Spread Function $H(f,v)$ relationship between input and output in the frequency domain

$$R(f) = \int_{-\infty}^{\infty} X(f-v)H(f-v,v)dv$$

v doppler shift, $H(f,t)$ - time variant transfer function and $H(f,v)$ - channel gain associated with v at frequency f; both can be used to describe the same channel

$$H(f,v) = \mathcal{F}_t[H(f,t)] = \int_{-\infty}^{\infty} H(f,t)e^{-j2\pi vt}dt$$

$$H(f,t) = \mathcal{F}_v^{-1}[H(f,v)] = \int_{-\infty}^{\infty} H(f,v)e^{j2\pi vt}dv$$

f treated as parameter

- time variant in time domain can be equivalently described by having doppler shift in the frequency domain
- Delay-Doppler Spread Function

$$H(\tau,v) = \mathcal{F}_t[h(\tau,t)] = \int_{-\infty}^{\infty} e^{-j2\pi vt}dt$$

$$r(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)H(\tau,v)e^{j2\pi vt}dv d\tau$$

Four channel functions $h(\tau,t)$, $H(f,t)$, $H(\tau,v)$, and $H(f,v)$ characterize the relation between the transmitted signal and the received signal

Channel Functions

Impulse response $h(\tau, t)$

$\mathcal{F}_t : H(\tau, v) = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi v t} dt \swarrow \nearrow \mathcal{F}_v^{-1}$ $\mathcal{F}_f^{-1} \nwarrow \searrow \mathcal{F}_\tau : H(f, t) = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi f \tau} d\tau$

Delay-Doppler spread $H(\tau, v)$ i Transfer function $H(f, t)$

$\mathcal{F}_\tau : H(f, v) = \int_{-\infty}^{\infty} H(\tau, v) e^{-j2\pi f \tau} d\tau \searrow \nwarrow \mathcal{F}_f^{-1}$ $\mathcal{F}_v^{-1} \nearrow \swarrow \mathcal{F}_t : H(f, v) = \int_{-\infty}^{\infty} H(f, t) e^{-j2\pi v t} dt$

Doppler spread $H(f, v)$