Mobility Management and Resource Management
and
Mobile Client-Server Computing

M. A. Zaveri
Computer Engineering Department
Sardar Vallabhbhai National Institute of Technology, Surat
mazaveri@coed.svnit.ac.in

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Cellular Communications

- Performance measure: with limited spectral width support maximum number of users
- Poor signal to interference ratio, higher transmission BER due to channel impairment result in a reduction of usable spectral width
- Parameters: wider spectrum, powerful transmitter, larger geographical coverage, high antenna, cell size, frequency reuse
- Cellular communications: dividing the total large geographical coverage area into many small contiguous area and using a low power transmission with low antenna in each small area

Terminology

- BS: small area is served by a transmit/receive unit
- user access the base station so called access point (AP)
- radio coverage by one base station is referred to as a cell or called a footprint
- forward channel (downlink) BS to user
- reverse channel (uplink) user to BS
- cells arrangement: square, triangular, hexagonal - no dead spot and no overlap
- due to difference in terrain and population densities, cells are amorphous in nature
- irregular structure or topology result into inefficient and limited growth

Other Issues

- support user roaming: continuous operation of an ongoing session and preserve end-to-end QoS; effective and efficient handoff management
- handoff operation: identification of a new base station, allocation of channel to support data and control signal
- MSC having larger computing power handles handoff operation
- MH (MS) is assigned a home network
- MH is identified by address, called home address
- An agent in home network, called home agent keeps track of MH's current location to facilitate delivery of messages destined for MH
- foreign territory and foreign network

- As MH migrates away from its home network, the association between MH and its home agent must be maintained so that the home agent can keep track of MH's current location for message delivery
- MH has to register with its home agent, through the foreign agent, to let the home agent know its current location
- When MH powers on, it registers with its home agent so when it moves to a foreign network, it has to register with its home agent via foreign agent
- requires association between the home agent and the foreign agent
- home agent needs to ensure the registration process through the foreign agent is the correct MH called as authentication process

Cell Cluster

- forward channel and reverse channel separated in time or in frequency for duplexing
- channel capacity is the total number of channels available which are finite
- also depends on how the available channels are deployed
- reuse for additional traffic and expand the system capacity
- different cells can reuse radio channels if cells are separated and minimum interference between cells; called cochannel cells and cochannel interference
- a group of cells that use a different set of frequencies in each cell is called a cell cluster
- N cluster size (number of cells) and K total number of available channels without frequency reuse

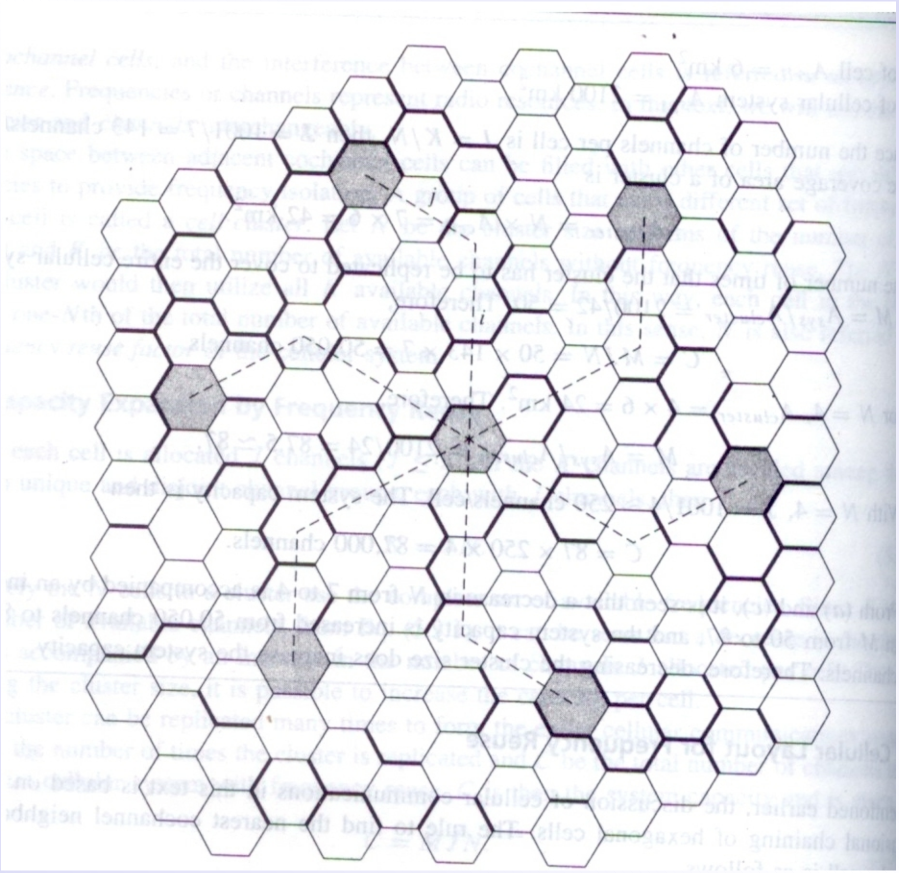
- N cells utilize all K available channels
- each cell in the cluster contains one-Nth of the total number of available channels
- N is referred to as the frequency reuse factor of the cellular system
- Let, each cell is allocated J channels and if K channels are divided among the N cells into unique and disjoint channel group $K = JN$
- decrease in the cluster size $N \rightarrow$ increase in the number of channels J per cell; increase capacity per cell
- cluster replicated say M times and C the total number of channels used in the entire cellular system with frequency reuse
- System capacity $C = MJN$

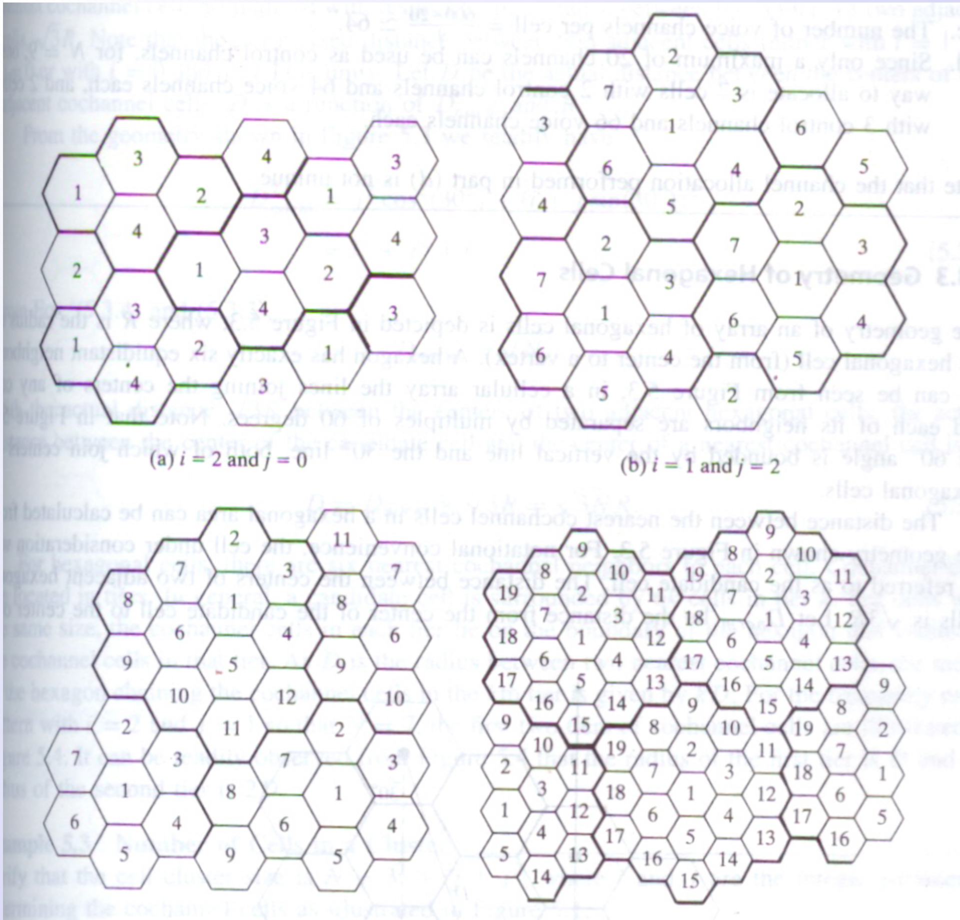
- replicate smaller cluster more times to cover the same geographical area with $K = JN$ constant and M is increased then C is increased
- when N is minimized, C is maximized
- minimizing N will increase cochannel interference
- Say 1001 radio channel available, area of a cell is $A_{cell} = 6$ km^2 and the entire system is $A_{sys} = 2100$ km^2
- Say if cluster size is $N = 7$ and $K = 1001$; $J = 1001/7 = 143$ channels/cell
- $A_{cluster} = N \times A_{cell} = 7 \times 6 = 42$km^2 so $M = A_{sys}/A_{cluster} = 2100/42 = 50$
- $C = MJN = 50 \times 143 \times 7 = 50,050$ channels

- Now, $N = 4$ $A_{cluster} = 4 \times 6 = 24$ km^2
- $M = A_{sys}/A_{cluster} = 2100/24 = 87.5$
- $J = 1001/4 \approx 250$ channels/cell
- Channel capacity $C = 87 \times 250 \times 4 = 87,000$ channels
- $N : 7 \longrightarrow 4 \longrightarrow M : 50 \longrightarrow 87$
- results into $C : 50,050 \longrightarrow 87,000$ channels
- decreasing the cluster size does increase the system capacity

Cellular Layout for Frequency Reuse

- Rule for determining the nearest Cochannel neighbors
  - Move i cells along any chain of hexagons
  - Turn 60 degrees counterclockwise and move j cells
- each cell is numbered and cells with the same number use the same set of frequencies (Co-channel cells)
- must be separated by a distance such that the cochannel interference is below a prescribed QoS threshold
- i and j measure the number of nearest neighbors between cochannel cells; the cluster size $N = i^2 + ij + j^2$
- with $i = 1$ and $j = 2$, $N = 7$, *i.e*, the frequency reuse factor is $N = 7$ since each cell contains one-seventh of the total number of available channels





(a) $i = 2$ and $j = 0$

(b) $i = 1$ and $j = 2$

(c) $i = 2$ and $j = 2$

(d) $i = 2$ and $j = 3$

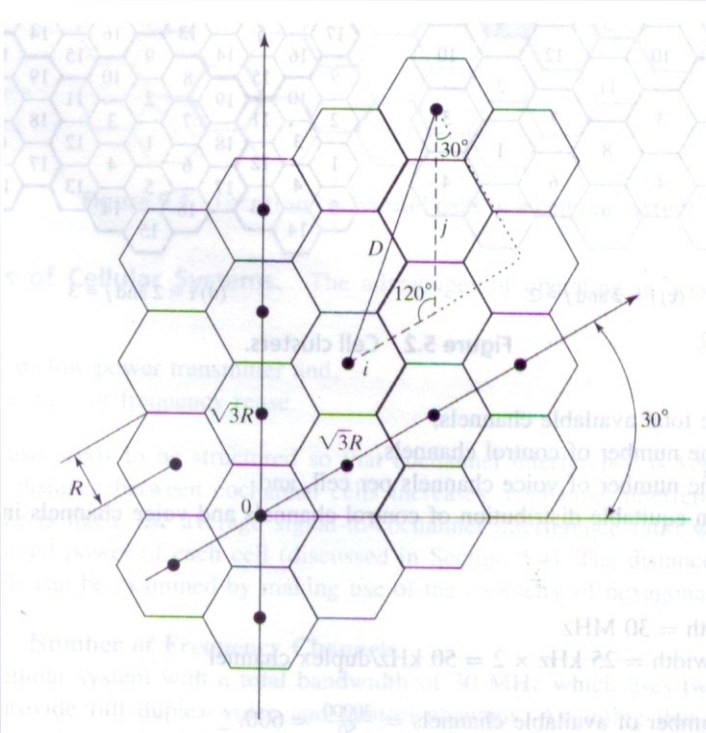
- feature: low power transmitter and frequency reuse
- distance between cochannel cells increase, cochannel interference will decrease
- Example: bandwidth 30 MHz, uses two 25 kHz simplex channels for fullduplex voice and control channels
- $N = 9$ and 1 MHz of total bandwidth for control channels
- Channel bandwidth = 50 kHz/duplex channel
- number of available channels = 30000/50 = 600
- number of control channels = 1000/50 = 20
- number of voice channels per cell = $(600 - 20)/9 \approx 64$
- 7 cells with 2 control channels, 64 voice channels
- 2 cells with 3 control channels, 66 voice channels

Hexagonal Cells

- R radius of hexagonal cell
- in a cellular array the lines joining the centers of any cell and each of its neighbors are separated by multiples of 60 degrees
- distance between the centers of two adjacent hexagonal cells is $\sqrt{3}R$
- D_{norm} distance from the center of the candidate cell (cell under consideration) to the center of a nearest cochannel cell, normalized with respect to the distance between the centers of two adjacent cells, $\sqrt{3}R$
- normalized distance between two adjacent cells ($i = 1, j = 0$ or $i = 0, j = 1$) is unity
- D actual distance between the centers of two adjacent cochannel cells

$$D_{norm}^2 = j^2 \cos^2(30^\circ) + (i + j \sin(30^\circ))^2 = i^2 + j^2 + ij$$

- $D_{norm} = \sqrt{N}$ and $D = D_{norm} \times \sqrt{3}R = \sqrt{3N}R$

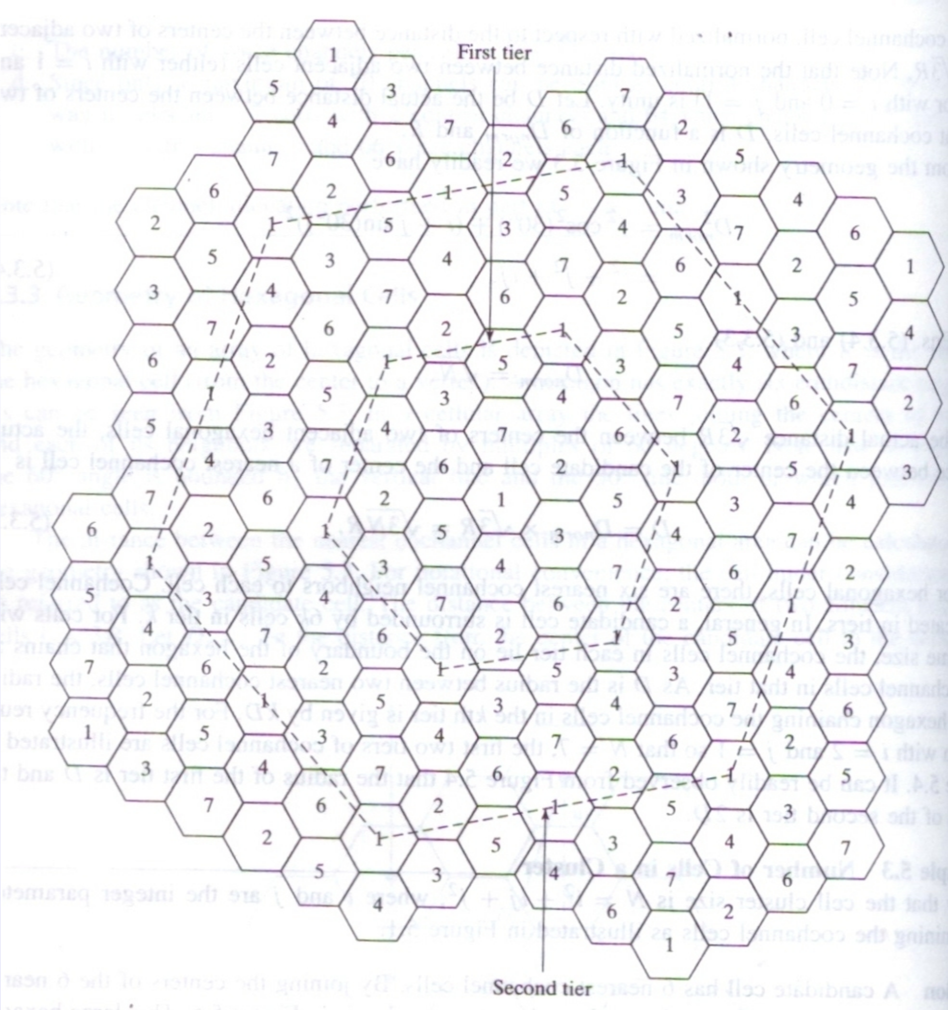


30°
j
D
120°
i
$\sqrt{3}R$
$\sqrt{3}R$
R
0
30°

- six nearest cochannel neighbors to each cell
- cochannel cells are located in tiers
- a candidate cell is surrounded by $6k$ cells in tier k
- cochannel cells in each tier lie on the boundary of the hexagon that chains all the cochannel cells in that tier
- D is the radius between two nearest cochannel cells, the radius of the hexagon chaining the cochannel cells in the kth tier is given by kD
- candidate cell has 6 nearest cochannel cells, join the centers of the 6 nearest neighboring cochannel cells gives large hexagon having radius D, is also cochannel cell separation

$$D = \sqrt{3}RD_{norm} = \sqrt{3(i^2 + ij + j^2)}R$$

- area of hexagon is proportional to square of its radius
- $A_{large} = \beta D^2$ and $A_{small} = \beta R^2$


First tier
Second tier

- number of cells in the large hexagon = $3(i^2 + ij + j^2)$
- from geometry, the large hexagon encloses the center cluster of N cells plus 1/3 the number of the cells associated with six other peripheral large hexagons
- the total number of cells enclosed by the large hexagon is $N + 6\left(\frac{1}{3}N\right) = 3N$, i.e. $N = i^2 + ij + j^2$
- *Frequency reuse ratio* $q \triangleq \frac{D}{R} = \sqrt{3N}$ *(cochannel reuse ratio)*
- *For example,* $(i, j) = (1, 1)$ $N = 3$ $q = 3.0$
- q *increases with* N*, a smaller value of* N *has the effect of increasing the capacity of the cellular system but increases cochannel interference*

Mobility Management

- Roaming support, handoff connection for continuity
- frequency of cell crossing increases with the speed of the user
- new location of the mobile station must be known
- backbone network, wired and wireless links need routers
- base station as a hub, also known as access point
- MSC - controller to handle the handoff functions and wired with other MSCs
- MSC responsible for collecting and accumulating information all cells in the cluster
- handoff management - connection transferred to one BS to another BS

- user subscribe to a regional subnetwork (home network)
- subscriber's identity (permanent address) resides in subscriber's home location register (HLR)
- in foreign network, it must update its registration with the HLR through its VLR to facilitate message delivery to its new location
- procedure of maintaining an association between the mobile and its HLR when it is away from its home network is referred to as location management
- Capacity of base station is number of basic channels; capable of handling the information transmission in one connection (N_c)
- total capacity is used to handle ongoing connections and to admit new and handoff requests
- call admission control (CAC) - oversees the admission of new and handoff calls to protect the integrity of the network and satisfy user QoS requirements

- QoS - packet level factor - packet loss rate, packet delay, packet delay variation, and throughput rate
- GoS - call level factor - new call blocking probability (NCBP) , handoff call dropping probability (HCDP) and connection forced termination probability (CFTP)
- CAC ensures network integrity by restricting access to the network so as to avoid overload and congestion and to ensure QoS
- new request - new call or handoff call - admitted or denied depending upon resources available with QoS
- CAC algorithm needs to determine the amount of unallocated capacity - number of basic channels available for accepting new and handoff requests
- handoff call request is given preference to new call requests

- handoff requests should be offered a higher admission priority than new requests
- P_n new call blocking probability and P_h handoff call dropping probability

$$\text{GoS} \quad = P_n + \alpha P_h$$

- $\alpha > 1$ balancing factor between new call and handoff call
- to manage the admission of requests based on priority reserve capacity for admitting handoff requests
- N_g number of basic channels reserved for admitting handoff requests - common technique is guard channel method
- N_{ua} number of unallocated channels

- with guard channel method, the admission rule
  - if $N_{ua} > N_g$ admit a new or handoff request
  - if $N_{ua} < N_g$ admit a handoff request only
- fixed reservation or dynamic reservation for guard channel method
- if MSC knows exactly the number (rate) of handoff requests during any epoch, it can determine N_g exactly
- handoff procedure - initiation phase and execution phase
- initiation phase - decision making based on received signal level with or without hysteresis
  - without hysteresis: handoff initiated *as soon as* average signal level from new BS exceeds that from the current BS
  - with hysteresis: *hysteresis level (threshold) specified*
- execution phase - channel assignment and the exchange of control messages

- MSC obtains status information for all the base stations - channel occupancy
- MSC handles handoff initiation and execution phases
- intraswitch and interswitch handoff
  - first one - no need to copy connection identification states
  - later needs to transfer state information
- depending on the information used and the action taken to initiate the hand-off
  - mobile controlled handoff (MCHO)
  - network controlled handoff (NCHO)
  - mobile assisted handoff (MAHO)

- MCHO - reduces burden on network but increases complexity of MS
- NCHO - APs monitor the signal quality from MS and report to MSC; MSC responsible for handoff and MS is passive
- MAHO - variant of NCHO and employed by GSM;
  - MS measures the signal levels from various APs using a periodic beacon generated by the APs (to keep track of the locations of the mobiles)
  - MS feeds the power levels from different APs back to MSC via AP
- Hard handoff - break before make - one radio link with only one AP
- Soft handoff - make before break e.g. CDMA
- Backward handoff - handoff is predicted ahead of time and initiated via the existing radio link
- Forward handoff - handoff is initiated via the new radio link associated with the candidate AP

- Issues
  - during transition time some packets leftover with old serving AP
  - order of packets delivered should be preserved
- handoff: fast and lossless, minimal number of control signal exchanges, scalable with network size, capable of recovering from link failures
- low handoff delay, low cell loss, small buffer required and efficient use of resources
- Feedback based MAHO
  - on the downlink from BS to MS, APs broadcast pilot (beacon) signals to the mobile
  - the MS sends the strengths of beacon received from APs to MSC via serving AP; creates a profile of signal strengths and sends the profile
  - AP computes and sends the distance information to MSC (direction of movement can be known)

- feedback profiles is overhead on system; small profiles desirable
- MS receives equal signal strengths from at most three APs - highest signal strength being sufficient for decision making
- MS is located in the middle of the boundary between two cells
- three scenarios
  - three APs are located in the same cluster
  - three APs are divided between two clusters
  - three APs are divided among three clusters
- necessary to identify a single cluster or at most three adjacent clusters uniquely to the current serving MSC
- each MSC (or cluster) has six adjacent MSCs

- 3-bit AP/MSC identification - 3-bit codeword uniquely identify a supercluster of 7 MSCs
- MSC maintains a list of all adjacent MSCs
- identification process can continue until all the tiers are exhausted
- in order to have equal size identities, if MSCs in tier L (the highest tier) have a K bit identity (3 of the K bits identifying the MSCs and the other $(K-3)$ bits for tier identification)
- with 7 APs per MSC, and additional 3 bits are needed for AP identification so $(K+3)$ bits required for identification in the profile
- profile has to carry three power fields (signal strength)

- λ_d and λ_p - data generation rate and profile generation rate
- MS transceiver compares data rate λ_d with the profile rate λ_p
- if $\lambda_d > \lambda_p$ the profile is queued in the profile buffer
- on next data packet, the profile is dequeued and piggy-backed - only one profile in the queue at any time
- if $\lambda_d < \lambda_p$ the profile is transmitted separately if there is no overlap
- degradation if the data rate is small
- performance measure: buffer requirement, mean handoff delay, feedback interval and system resource utilization

Mobility Model

- critical to design of handoff algorithm
- mobility pattern defined by the speed and the direction of movement
- monitor and measure or model based on assumed statistics
- two variable: speed of travel and change in direction of movement relative to the MS' current direction of movement
- most important is the expected time interval between the time a mobile initiates a call in a cell and the time mobile reaches the cell boundary referred as sojourn time t_s
- X random variable - the distance traveled by the MS before reaching the cell boundary and Y random variable - the velocity of MS
- X is uniformly distributed between 0 and D_{max} and Y is uniformly distributed between V_{min} and Y_{max}

$$t_s = X/Y$$

- pdf of sojourn time t_s

$$\begin{aligned} f_{t_s}(t) &= \frac{(\alpha+\beta)}{2}[u(t)-u(t-\beta)] + \left[\frac{\alpha\beta}{2t^2(\alpha-\beta)} - \frac{\beta}{2\alpha(\alpha-\beta)}\right] \\ &\quad \times [u(t-\beta)-u(t-\alpha)] \end{aligned}$$

$$E[t_s] = \frac{\alpha\beta}{2(\alpha-\beta)} \ln\left(\frac{\alpha}{\beta}\right)$$

$$\lim_{\frac{\alpha}{\beta}\to 1} E[t_s] = \alpha/2 \quad \textbf{and} \quad \lim_{\frac{\alpha}{\beta}\to\infty} E[t_s] = \beta/2$$

$$\beta = \frac{D_{max}}{V_{max}} \quad \alpha = \frac{D_{max}}{V_{min}} \quad \frac{\beta}{\alpha} = \frac{V_{min}}{V_{max}}$$

- performance depends on the frequency with which the MS feeds information back to MSC
- too frequently feedback reduces efficiency and too infrequently degrades effectiveness
- I_f average feedback interval and $r_p = 1/I_f$ rate of transmitting profiles
- N_p mean number of profiles sent by the MS before the MS moves to the cell boundary $\approx E[t_s] \times r_p$

Intraswitch Handoff: Example

- MSC send message NEW-AP-READY to mobile via AP-0: indicates that the candidate AP-1 is ready
- AP-0 responds with the message LAST-PKT which contains the sequence number of the packet sent (or waiting to be sent) to the mobile prior to receiving (1) message; MSC calculates the sequence number of the last downlink packet transmitted to AP-0 by MSC and MSC sends the next number of the sequence number of the following downlink packet to AP-1 using message SEQ-PKT
- HO-MUST indicates the last downlink packet from the MSC to AP-0 and also indicates the end of uplink packets from the mobile; AP-0 sends all uplink packets to the MSC and flags the termination of the connection by sending the message UP-NO-MORE to MSC

- the mobile switches its operating frequency and sends the message READY to AP-1 and continues uplink transmission via the new connection; it contains the sequence number of the last packet correctly received by the mobile
- AP-1 starts the downlink transmission and buffers all uplink packets arriving from the mobile. It sends the message LAST-UP to MSC requesting for the approval of uplink transmission
- MSC waits for the message NO-MORE from AP-0 and then MSC switches the uplink connection from AP-0 to AP-1 and sends message UP-READY to indicate that the uplink flow can resume

Location Management

- registration process: association between the mobile and its home network
- database of a location register: identity of the mobile
- update registration with HLR when the mobile moves to foreign or visitor network through VLR
- HLR performs authentication process using cryptographic messages
- footprint covered by all the cells in a subnet of the cellular network constitutes a registration area (RA)
- no need to update registration with its HLR when moving within a RA, update registration only when it moves into a new RA
- signal flows for location update and call delivery are supported by a specific network architecture and control signals are supported by a specific control signaling network

SS7 Network and Common Channel Signaling

- SS7 network carries out network signaling exchange operation using common channel signaling (CCS)
- CCS is a digital communications techniques that provides simultaneous transmission of user data, signaling data, and other related traffic throughout a network
- using out-of-band signaling channels that logically separate the network control data from the user information on the same channel
- CCS is used to pass user data and control signals between the MS and BS, between BS and MSC and between MSCs
- In brief, identify mobile RA, then location update and call setup
- on moving into a new RA, it must update the registration with its HLR
- Mobile is identified by Mobile Identification Number (MIN)
- registration request contains MIN
- base station in RA periodically broadcast beacon signals

Location update

- Mobile listens to pilot signals from base stations in RA and uses these to identify its current RA
- in new RA, mobile sends registration request to the base station of the current cell
- base station forwards the request to MSC which in turn launches a registration query to its associated VLR
- VLR adds an entry in its records on the location of the mobile and forwards the registration request to the HLR
- HLR performs the necessary authentication routing and records the identity of the new serving VLR in its database entry for the mobile and HLR sends a ACK to new VLR
- HLR sends registration cancellation message to old VLR
- old VLR removes the record and sends ACK to HLR

Call Setup

- when network (network access point) receives a call for a mobile, must locate called mobile and deliver the call to it
- calling mobile sends the call request to the MSC in the registration area via its serving base station
- MSC which handles the call request is referred to as the calling MSC
- calling MSC locates the called MSC/VLR combination through its association with the HLR
- HLR routes the call request to the called MSC which in turn pages the called mobile to set up the call

- calling BS receives a call request having MIN, forwards to the VLR of its RA and VLR forwards the request to its MSC
- calling MSC sends a location lookup request to the HLR of the called mobile
- HLR of the called mobile determines the current VLR of the called mobile and sends a routing request to the VLR and VLR forwards the message to its MSC
- called MSC allocates a temporary local directory number to the called mobile and sends a reply to the HLR together with the TLDN
- HLR forwards the TLDN to the MSC of the calling mobile
- using TLDN the MSC of calling mobile initiates a connection request to the called MSC through the SS7 network

- call setup procedure establishes a linkage between the calling MSC and the called MSC
- the called MSC knows that the called mobile is in the RA under its jurisdiction but does not know in which cell the called mobile is located
- paging procedure is needed to locate the called mobile within a registration area
- polling: called MSC broadcasts a polling message, which contains the MIN of the called mobile to all cells within the RA
- BS of each cells relays the paging message to the mobile terminals
- only the called mobile responds to the called MSC through its current BS with the base station ID of its current cell
- the called MSC then knows where to forward the call

Location Management for PCS Network

- communication by anyone, from anywhere and at anytime
- higher transmission rate and higher frequency spectrum (1.85 to 1.99 GHz)
- smaller cell sizes to enlarge the system capacity, hence smaller registration area and increase in the number of times that HLR is accessed
- large distances introduce signaling delay
- to reduce delay in call setup and delivery
- overlay approach: enhance two tiered architecture reducing the number of HLR accesses using pointer forwarding
- setup pointer from old VLR to new VLR eliminating signaling exchanges between the HLR and the VLR's

- need to traverse the link chain from the initial VLR to the Mobile's current VLR
- disadvantage: chain may be long and delay becomes intolerable
- upper bound on the cost of call setup incorporated
- K limit on length of the link chain; if chain length exceeds K, update to HLR is forced and link chain is reset to NULL
- rate of call arrivals and the rate of RA crossing have impact on overlay approach

$$\text{Call-to-mobility ration} = \frac{\text{rate of call arrivals}}{\text{rate of RA crossings}}$$

- pointer forwarding scheme works well if CMR is relatively low

- Local Anchor Approach: create virtual HLR in the form of a local anchor
- when mobile crosses RA, register with the LA which has an association with the HLR and plays the role of HLR (virtual HLR)
- MSC of the newly entered RA registers the mobile location at VLR called LA
- need to assess performance of both handoff and location management
- rate of boundary crossing by the mobile is an important parameter
- mobility model, network architecture and traffic parameters
- calculate intraswitch handoff rate, interswitch handoff rate and the location update rate

Traffic Calculation

- model based or measurement based
- analytical model constructed based on known distributions
- does not reflect real scenario but provide guidance
- Say, cluster of N hexagonal cells

$$A_{cell} = 2.598R^2$$

$$A_{cluster} \approx N \times 2.598R^2 \approx 2.598R^2_{cluster}$$

$$R_{cluster} \approx \sqrt{N}R$$

- to calculate intracluster handoff rate - number of boundary crossing per cell per unit time
- for intercluster handoff rate - number of cell crossing per cluster per unit time
- calculate handoff rate based on network parameter, traffic parameter and Mobile's movement pattern

- population density of mobiles = ρ (number of mobiles/km^2)
- speed of mobile V km/hr
- traffic load per mobile station = λ_{ar} (Erlangs/MS)
- percentage of powered stations = δ
- percentage of active mobiles among the powered stations = ϵ
- Number of crossing per cell per unit time = μ_{cell}
- Number of crossing per cluster per unit time = $\mu_{cluster}$
- mobile powered - listening mode / active - ongoing connection
- the percentage of the total population that is active is given by $\epsilon\delta$
- number of handoff per unit time is smaller than the number of boundary crossing per unit time

Mobility Model based on active mobiles

- mobiles are uniformly distributed in the cell
- direction of travel of all mobiles relative to the cell boundary is uniformly distributed on $[0, 2\pi]$
- mobile travels with constant velocity V in a particular given direction
- the rate of boundary crossing - the rate at which a randomly chosen mobile crosses the cell boundary time the number of mobiles per cell
- it is function of cell size, ρ, and V
- say, perimeter of the cell $L_{cell} = 6R$

- rate of mobiles departing the cell μ_{cell}

$$\mu_{cell} = \frac{\rho V L_{cell}}{\pi} = \frac{6\rho V R}{\pi}$$

- by the flow conservation law, the number of departing a cell per unit time equals the number of entering per unit time
- rate of mobiles departing the cluster $\mu_{cluster}$

$$\mu_{cluster} = \frac{\rho V L_{cluster}}{\pi}$$

- perimeter of cluster $L_{cluster}$

$$L_{cluster} \approx 6R_{cluster} = 6\sqrt{N}R$$

$$\mu_{cluster} = \frac{6\rho V \sqrt{N} R}{\pi}$$

$\lambda_{ho\_cluster}$ - total handoff rate of a cluster (number of handoffs in the cluster both within the cluster and to neighboring clusters per unit time)

$$\begin{aligned}\lambda_{ho\_cluster} &= \text{number of cells in the cluster} \quad \times \\ &\quad \text{number of crossings per cell per unit time} \quad \times \%\text{active mobiles} \\ &\quad \times\%\text{powered stations} \quad \times \text{ traffic load per MS} \\ &= N \times \mu_{cell} \times \epsilon \times \delta \times \lambda_{ar}\end{aligned}$$

$\lambda_{ho\_intercluster}$ - total handoff rate of a cluster includes both intracluster handoff rate and intercluster handoff rate

$$\begin{aligned}\lambda_{ho\_intercluster} &= \text{number of crossings per cluster per unit time} \quad \times \%\text{active mobiles} \\ &\quad \times\%\text{powered stations} \quad \times \text{ traffic load per MS} \\ &= \mu_{cluster} \times \epsilon \times \delta \times \lambda_{ar}\end{aligned}$$

$$\lambda_{ho\_intracluster} = \lambda_{ho\_cluster} - \lambda_{ho\_intercluster}$$

Receiver Signal Processing

- transmitting a sequence $\{b[0], b[1], \ldots, b[M-1]\}$
- $(b[i] = \pm 1)$ or a finite alphabet of complex numbers
- linear modulation using a signaling waveform

$$x(t) = \sum_{i=0}^{M-1} b[i] w_i(t)$$

$w_i(\cdot)$ modulation waveform associated with the i^{th} symbol , for example

$$w_i(t) = Ap(t - iT)e^{j(w_c + \phi)}$$

$A > 0$ $\phi \in (- = pi, \pi)$ and $p(\cdot)$ baseband pulse shape

$$p(t) = p_T(t) \triangleq \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

or spreading waveform for DSSP system

$$p(t) = \sum_{j=0}^{N-1} c_j \psi(t - jT_c)$$

N is the spreading gain and $c_0, c_1, \ldots, c_{N-1}$ is a pseudorandom spreading code $(c_j \in \{+1, -1\})$, $\psi(\cdot)$ chip waveform and $T_c \triangleq T/N$ chip interval

- chip waveform may be a unit-energy rectangular pulse of duration T_c: $\psi(t) = p_{T_c}(t)$
- repeat the same spreading code in every symbol interval
- system with long spreading codes, the periodicity is much longer than a single symbol interval and varies spreading code from symbol to symbol

$$p_i(t) = \sum_{j=0}^{N-1} c_j^i \psi(t - jT_c)$$

- spread spectrum modulation can take the form of frequency hopping
- carrier frequency is changed over time according to a pseudorandom pattern
- carrier frequency changes at a rate much slower than the symbol rate - slow frequency hopping
- fast hopping - the carrier changes within a symbol interval
- multicarrier system by choosing $\{w_i(\cdot)\}$ with different frequencies

$$w_i(t) = Ap(t)e^{j(w_i t+\phi_i)}$$

- individual carrier can also be direct-spread - multicarrier CDMA
- OFDM: baseband pulse shape is a unit pulse p_T, intercarrier spacing is $1/T$ cycles per second, and the phases are orthogonal at this spacing

Multiple-Access Techniques

- radio resources shared among multiple users
- FDMA - frequency band available is divided into subbands and allocated to individual user, users do not transmit signals within other subbands
- TDMA - time is divided into equal-length intervals, each user is allowed to transmit throughout the entire allocated frequency band during a given slot
- FDMA allows each user to use part of the spectrum all of the time, TDMA allows each user to use all of the spectrum part of the time
- FDMA and TDMA systems are intended to assign orthogonal channels to all active users by giving each, for exclusive use, a slice of the available frequency band or transmission time
- channels are said to be orthogonal because interference between user does not

- Code-division multiple access (CDMA) assigns channels in a way that allows all users to use all of the available time and frequency resources simultaneously, through the assignment of a pattern or code to each user that specifies the way in which these resources will be used by that user
- spread spectrum modulation - pattern is the pseudorandom code that determines the spreading sequence in the case of direct sequence or the hopping pattern in the case of frequency hopping
- channel is defined by a particular pseudorandom code, and each user is assigned a channel by being assigned a pseudorandom code
- for a system of K users

$$x_k(t) = \sum_{i=0}^{M-1} b_k[i] w_{i,k}(t) \qquad k = 1, 2, \ldots, K$$

$w_{i,k}(\cdot)$ represents ith modulation waveform of user k

- each user in a multiple-access system can be modeled in the same way as in a single-user system
- if the waveforms $\{w_{i,k}(\cdot)\}$ are of the form of sinusoidal with different carrier frequencies $\{w_k\}$ - FDMA
- if they are with time-slotted amplitude pulses $\{p_k(\cdot)\}$ - TDMA
- if they are spread-spectrum signals of this form but with different pseudo-random spreading codes or hopping patterns - CDMA
- Another aspect of wireless network
  - ambient noise, propagation losses, multipath, interference
  - properties arising from the use of multiple antennas
- ambient noise - thermal motion of electrons on the antenna and the receiver electronics and from background radiation sources
- modeled as a very wide bandwidth and no particular deterministic structure (e.g. AWGN)

- Propagation losses: diffusive losses and shadow fading
- diffusive losses due to open nature of wireless channel, energy decreases with the square of the distance between antenna and source
- shadow fading results from the presence of objects, modeled by an attenuation in signal amplitude that follows a log-normal distribution
- multipath - multiple copies of a transmitted signal are received at the receiver
- multipath is manifested in several ways
  - degree of path difference relative to the wavelength of propagation
  - degree of path difference relative to the signaling rate
  - relative motion between the transmitter and receiver
  - results into Rayleigh fading or frequency-selective fading or time-selective fading

- multipath from scatterers that are spaced very close together will cause a random change in the amplitude of the received signal
- resulting received amplitude is often modeled as being a complex Gaussian random variable
- random amplitude whose envelope has a Rayleigh distribution - termed as Rayleigh fading
- When the scatterers are spaced so that the difference in their corresponding path lengths are significant relative to a wavelength of the carrier and add constructively or destructively
- this is a fading depends on the wavelength of radiation - frequency-selective fading
- when there is relative motion between the transmitter and receiver, this fading depends on time - time-selective fading
- when the difference in path lengths in such that time delay of arrival along different paths is significant relative to a symbol interval, results in dispersion of the transmitted signal and causes ISI

- wideband signaling methods such as spread spectrum - a countermeasure to frequency-selective fading
- dividing a high-rate signal into many parallel lower-rate signal - OFDM mitigates channel dispersion on high-rate signals
- multiple access interference - arising from other signals in the same network as the signal of interest (if signals received are not orthogonal to one another)
- co-channel interference - due to signals different networks but operating in same frequency band
- the above phenomena can be incorporated into a general analytical model for a wireless multiple-access channel

$$r(t) = \sum_{k=0}^{K} \sum_{i=0}^{M-1} b_k[i] \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du + i(t) + n(t)$$

- $g_k(t, u)$ impulse response of a linear filter representing the channel between the kth transmitter and the receiver
- $i(\cdot)$ co-channel interference and $n(\cdot)$ ambient noise, in general, all are random processes
- co-channel interference and channel impulse responses are structured and can be parameterized
- pure multipath channel

$$g_k(t, u) = \sum_{l=1}^{L_k} \alpha_{l,k} \delta(t - u - \tau_{l,k})$$

  L_k number of paths between user k and the receiver, α and τ gain and delay (lth path of kth user)

- model includes frequency-selective fading; relative delays will cause constructive and destructive interference at the receiver, depending on the wavelength of propagation and Rayleigh fading also using path gains

- composite modulation waveform associated with $b_k[i]$:

$$f_{i,k}(t) = \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du$$

- if these waveforms are not orthogonal for different values of i, ISI will result
- higher-rate transmission are more likely to encounter ISI than are lower-rate transmission
- if the composite waveform for different values of k are not orthogonal, MAI will result
- this can happen in CDMA when pseudorandom code sequences used by different users are not orthogonal, this happens in FDMA and TMDA due to the effects of multipath or asynchronous transmission
- model can be generalized for multiple antennas at the receiver

$$\mathbf{r}(t) = \sum_{k=1}^{K} b_k[i] \int_{-\infty}^{\infty} \mathbf{g}_k(t,u) w_{i,k}(u) du + \mathbf{i}(t) + \mathbf{n}(t)$$

- pth component of $\mathbf{g}_k(t,u)$ is the impulse response of the channel between user k and the pth element of the receiving array

$$\mathbf{g}_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k}\delta(t-u-\tau_{l,k})$$

- multiple antennas at both the transmitter and receiver called multiple-input/multiple-output (MIMO) systems
- channel transfer functions are matrices with the number of rows equal to the number of receiving antennas and the number of columns equal to the number of transmitting antennas at each source

Mobile Computing

- new paradigm of computing
  - advances in wireless data networking
  - portable information appliances
- Ubiquitous computing / wireless computing / Nomadic computing
- mobile (non-static) and ubiquitous (everywhere)
- connect from different access points through wireless links and might want to stay connected while on the move, despite possible intermittent disconnection
- Applications: Multimedia, Field service, PDA's, Healthcare, Industrial, Managerial, Sales

Challenges and Constraints

- Security Privacy issue - personal information is being captured and stored
- Disconnection, low bandwidth, bandwidth variability, heterogeneity of the network
- Address migration, mobile routing, location dependent information, low power design
- Restricted memory, disk, and user interfaces
- Consistent replicated data, session guarantees
- Cache validation, reduce contention
- Data access model and dynamic network architecture
- Open file system, fault-tolerance, performance
- Distributed control: Distributed object management, Distributed file system (Coda)

Challenges and Constraints

- Mobile host, static hosts as gateways, adhoc network
- Bluetooth - leave and join, active and passive
- Decentralized location query service
- Optimization code, Multiple file systems
- Development
  - Creating new environments and user interfaces for Mobile Computing
  - Agent based remote procedure
  - Web application - WML, Intelligent WAP system, forms manipulation
  - Application-specific knowledge to address mobile resource constraints
  - Video transmission - dynamically adapt the bandwidth

Client-Server Computing in Mobile Environments

- Mobile Vs. Traditional Client-server computing
- Mobility of user and their computers
- Mobile resource constraints
- Goal: a collection of trusted information servers connected via a fixed network to provide information services to a collection of untrusted mobile clients over wireless and mobile networks
- Mobile client-server computing can be categorized into
  - mobile aware adaption
  - extended client-server model
  - mobile data access

Client-Server Computing

- Mobile aware adaption - how systems and applications respond to the environmental changes
  - laisse-faire adaptation - responsibility of individual applications, avoids the need for system support
  - application-transparent adaptation - places responsibility on the system
    * file system proxy, disconnected operations, isolation only transactions
    * automatically detect read/write conflicts, caching, differencing
    * web proxy - client slide intercept, server side intercept, header reduction
  - application-aware adaptation - collaborative adaptation between applications and system
    * applications can decide how to best adapt to the changing environment while
    * the system provides support through the monitoring of resources and the enforcing of resource allocation decisions

Client-Server Computing

- Extended client-server model
  - dynamic partitioning of client-server functionality and responsibilities
  - thin client architecture, flexible client-server architecture - mobile objects
- Mobile data access
  - how data over wireless and mobile networks is structured consistency of client cache
  - type of communication links, connectivity of mobile hosts
  - server data dissemination, client cache management

PCS network architecture

- Personnel communication systems can provide wireless communication services to user on the move
- It is necessary to have an efficient way to locate the mobile user to deliver services
- Cell, Base station, Registration area (RA)
  - An RA consists of an aggregation of a number of cells, forming a contiguous geographical region
- Signaling network
  - used to setup calls and it is distinct from the network used to actually transport the information contents of the calls

Common Channel Signaling (CCS)

- CCS network is used to set up calls which use the signaling systems No. 7 (SS7) protocols
- All base stations are connected via a wire-line network to an end-office switch or Service Switching Point (SSP)
- Each SSP serves as an RA
- All the SSPs of different RAs are connected to higher hierarchical Local Signaling Transfer Points (LSTP), which are connected to a Regional STP (RSTP)
- An RSTP connects to all the LSTPs in one region
- The RSTPs are also connected to a Service Control Point (SCP)
- Each SCP is equipped with a HLR database

- Each VLR is associated with one Mobile Switching Center (MSC)
- MSC connects the BSs and backbone communication infrastructure (PSTN)
- MSC, SSP, and VLR database are associated together to serve to an RA
- Two standards for PCS location management: IS-41 and GSM MAP
- Both use a two-tier system of HLR and VLR databases
- A mobile user performs location update (registration) at the HLR every time the user crosses the boundary of a RA and deregisters at the previous VLR
- Location management does through location registration and paging
- the objective is to reduce signaling traffic
- Methods used - Local Anchor scheme, per-user caching scheme, pointer forwarding scheme

Location Management Schemes

- Local anchor (LA) scheme: a VLR close to the user is selected as the LA for the user and whenever a user moves from one RA to another, it will perform location update to the LA
- A LA for a mobile will change only when a call request to the mobile arrives;
- at the same time HLR is also updated via the registration process
- When a call request terminating at this user is received by the HLR, the user can be traced to the LA
- The LA scheme avoids update to HLR completely at the expense of the increase in local signaling traffic
- The drawback of LA scheme is that when the user keeps moving constantly without receiving any call, the updates to LA may become costly

Location Management Schemes

- per-user pointer forwarding scheme: some updates to the HLR can be avoided by setting up a forwarding pointer from the previous VLR to the new VLR
- When a call request to a mobile user arrives, the PCS network first queries the user's HLR to determine the VLR, which the user was visiting at the previous location update, the follows a chain of forwarding pointers to the user's current VLR to find the mobile user
- The traffic to the HLR is decreased by using the pointer chain; but the penalty is the time delay for tracking the user when a call to the user arrives
- The longer the pointer chain, the less the signaling traffic, the longer the setup delay for finding the user
- To avoid long setup delay, a threshold of the length of the pointer chain is used
- The user needs to perform registration to the HLR after the chain threshold is reached

Location Management Schemes

- Two Level Pointer Forwarding Strategy for Location Management in PCS Networks, *IEEE Transaction on Mobile Computing Vol. 1 No. 1 Jan-Mar 2002*
- Objective: New location management scheme - mitigate the signaling traffic and reduce the tracking delay
- Solution - a set of VLRs traversed by users as the mobility agents (MA), which forms another level of management in order to make some registration signaling traffic localized, instead of updating HLR, MAs are updated
- two kinds of pointers are used; some VLRs are selected as the mobility agents, which will be responsible for location management in a larger area comparing to the RAs and can be geographically distributed
- MA can form the virtual management network (VMN)
- The pointers between MAs are level-1 pointers
- those between VLRs in the charging domain of MAs are level-2 pointers are set

- When the user crosses the boundaries of RAs, the level-2 pointers are set
- If the level-2 pointer chain threshold is reached, the current RA is selected as an MA for this user and a level-1 pointer is set up from previous MA
- Calls to a given user will query the HLR first and follow the level-1 pointer chain to the current MA, then reach the user current VLR by tracking the level-2 pointer chain
- The user does not need to update the HLR until the level-1 pointer chain threshold is reached
- two level pointer scheme chain can be longer than that in simple pointer forwarding scheme, but can have shorter call setup delay due to the level-1 pointer chain
- two operations are defined:
  - MOVE: PCS user moves from one RA to another and
  - FIND: determination of the RA where the PCS user is currently located

- IS-41 Location Management Scheme: Basic MOVE()
  - The mobile terminal detects that it is in a new RA
  - The MS sends a registration message to the new VLR
  - The new VLR sends a registration message to the user's HLR
  - The HLR sends a registration cancellation message to the old VLR
  - The old VLR sends a cancellation confirmation message to the HLR
  - The HLR sends a registration confirmation message to the new VLR
- IS-41 Location Management Scheme: Basic FIND()
  - Call to a PCS user is detected at the local switch, if the called party is in the same RA, then return;
  - Switch queries the called party's HLR; HLR queries the called party's current VLR, V; VLR V returns the called party's location to HLR; HLR returns the location to the calling party;

Two-Level Pointer Forwarding Scheme

- When user moves from one RA to another; it informs the switch (and the VLR) at the new RA about the old RA
- it also informs the new RA about the previous MA it was registered
- the switch at new RA determines whether to invoke the BasicMOVE or the TwolevelFwdMOVE
- In TwolevelMOVE: the new VLR exchange message with the old VLR or the old MA to setup a forwarding pointer from the old VLR to the new VLR; if a pointer is set up from the previous MA, the new VLR is selected as the current MA; it does not involve HLR
- level-2 pointers are built from the old VLR to the new VLR
- when the chain threshold for level-2 pointer is reached so the new RA is selected as the user's new MA
- and a level-1 pointer is set up from the old MA to the new MA and level-2 pointer chain is reset

Performance Analysis

- Analytic model - different parameters for different classes of users
- characterize the classes of users according to their call-to-mobility ratio (CMR)
- CMR of a user is defined as the expected number of calls to a user during the period that the user visits an RA
- if calls are received by the user at a mean rate λ and the time the user resides in a given RA has a mean $1/\mu$ then the CMR denoted as $p = \lambda/\mu$
- For comparison model the basic procedures used in IS-41
  - A user crosses several RAs between two consecutive calls; if basic user location strategy is used the user's HLR is updated every time the user moves to a new RA
  - if two level pointer forwarding strategy is used, the HLR is updated only every $K_1 \cdot K_2$
  - K_1 and K_2 are the level-1 and level-2 pointer chain length threshold, while forwarding pointers are set up for all other moves

- C_B and C_F to be the total costs of maintaining the location information (location updating) and locating the user (location tracking) between two consecutive calls for the basic strategy and two level forwarding strategy
- m the cost of a single invocation of BasicMOVE
- M the total cost of all the BasicMOVES between two consecutive calls F the cost of a single BasicFIND
- M' the expected cost of all TwoLevelFwdMOVEs between two consecutive calls
- F' the average cost of the TwoLevelFwdFIND
- S_1 the cost of setting up a forwarding pointer (level-1 pointer) between MAs during a TwoLevelFwdMOVE
- S_2 the cost of setting up a forwarding pointer (level-2 pointer) between VLRs during a TwoLevelFwdMOVE
- T_1 the cost of traversing a forwarding pointer (level-1 pointer) between MAs during a TwoLevelFwdFIND
- T_2 the cost of traversing a forwarding pointer (level-2 pointer) between VLRs during a TwoLevelFwdFIND

- K_1 the threshold of level-1 pointer chain
- K_2 the threshold of level-2 pointer chain
- $\alpha(i)$ the probability that there are i RA crossings between two consecutive calls
- $C_B = M + F = m/p + F$
- $C_F = M' + F'$
- Say a user crosses i RA boundaries between two consecutive calls - the HLR is updated $\left\lfloor \frac{i}{K_1 K_2} \right\rfloor$ times
- $\left\lfloor \frac{i}{K_2} \right\rfloor - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor$ level-1 pointer creations - every K_2 moves may require a level-1 pointer creation
- level-2 pointers are created for all the rest $i - \left\lfloor \frac{i}{K_2} \right\rfloor$ moves

$$M' = \sum_{i=0}^{\infty} \left\{ \left\lfloor \frac{i}{K_1 K_2} \right\rfloor m + \left(\left\lfloor \frac{i}{K_2} \right\rfloor - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor \right) S_1 + \left(i - \left\lfloor \frac{i}{K_2} \right\rfloor \right) S_2 \right\} \alpha(i)$$

- After the last BasicMove operations, the user traverses $\left\lfloor \frac{i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2}{K_2} \right\rfloor$ level-1 pointers and
- $i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2 - \left\lfloor \frac{i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2}{K_2} \right\rfloor K_2$ level-2 pointers
- To evaluate $\alpha(i)$
  - the call arrivals to a user form a Poisson process with arrival rate λ
  - the residence time of a user at a registration area is a random variable with a general density function
  - the expected residence time of a user at an RA is $\frac{1}{\mu}$
  - for demonstration purpose, RA residence time of a user is Gamma distributed with mean $\frac{1}{\mu}$

$$F' = F + \sum_{i=0}^{\infty} \left\{ \left\lfloor \frac{i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2}{K_2} \right\rfloor T_1 + \left(i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2 - \left\lfloor \frac{i - \left\lfloor \frac{i}{K_1 K_2} \right\rfloor K_1 K_2}{K_2} \right\rfloor K_2 \right) T_2 \right\} \alpha(i)$$

- updating the HLR and performing a BasicFIND involve the same number of messages between HLR and VLR databases, so we can choose $m = F$
- without loss of generality, normalize $m = 1$
- assume the cost of setting up a forwarding pointer is about twice the cost of traversing it since twice as many messages are involved, $S_1 = 2T_1$ and $S_2 = 2T_2$, $S_2 = \delta$ $\delta < 1$ and
- level-1 point is more expensive than level-2 pointer in terms of setup cost, $S_1 = KS_2$ $K \geq 1$

$$\frac{C_F}{C_B} = \frac{p}{1+p}\left\{1 + \frac{3\delta}{2p} + \frac{(K-1)\delta}{(1+p)^{K_2}-1} + \frac{1-(K+\frac{1}{2}K_1K_2)\delta}{(1+p)^{K_1K_2}-1} + \right.$$
$$\left. \frac{\delta(K-K_2)[(1+p)^{K_1K_2} - K_1(1+p)^{K_2} + K_1 - 1]}{2[(1+p)^{K_1K_2}-1][(1+p)^{K_2}-1]}\right\}$$

Location Estimation Method

- Location aware computing - personal security, navigation, tourism, and entertainment
- Known as geolocation, location identification, localization, positioning
- geometric approach based on angle and distance estimates from which a location estimate is deduced using standard geometry
- problem of inaccurate measurements
- cellular system - complex propagation of radio waves, obstructions, reflecting objects, interference
- explore the dependency between the location of the receiver and observable signal properties, signal attenuation, reflection, diffraction and interference

Statistical Modeling Approach

- the distribution of received signal power - received power is observed
- other signal properties - angle of arrival, and propagation delay treated as random variables;
- these are statistically dependent on the locations of the transmitter, the receiver and the propagation environment
- A Statistical Modeling Approach to Location Estimation *IEEE Transactions on Mobile Computing Vol.1 No. 1 January-March 2002*
- Signal Propagation Model - predicts some properties of a radio signal at a given location
- Signal Transmitter Model - log loss model - linear regression model

- zero mean Gaussian distribution with a constant variance is used for the error term e
- two regression parameters β_0 and β_1 define the mean value of the received power at a given distance
- the variance of e - σ^2, p is the transmitted power in decibels, d is the transmitter receiver distance
- θ denotes the set of parameters

$$\mu(d, p, \theta) = p + \beta_0 + \beta_1 \ln d$$

- direction of transmission to which the transmitted power is higher than to other directions
- log-loss model can be improved by adding a term which depends on the deviation between the direction of the receiver and the direction of transmission - deviation δ

$$\mu(d, p, \theta) = p + \beta_0 + \beta_1 \ln d + \beta_2 \delta \ln d$$

- the distribution of r received signal power

$$f(r|d,\delta,p,\theta) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{r-\mu(d,\delta,p,\theta)}{\sigma}\right)^2\right)$$

- multiple transmitters: r_j denote the received power of channel j and c_i denote the channel of transmitter i
- each transmitter i has location, denoted by l_i, direction of transmission α_i and transmitted power p_i
- g_j p.d.f. of the received power on channel j, measurement is performed at location l

$$g_j(r|l,\theta) = f(r|d(l,l_i),\delta(l,l_i,\alpha_i),p_i,\theta)$$

the index i is chosen so that it maximizes the mean received power

$$i = argmax_{\{i:c_i=j\}}\mu(d(l,l_i),\delta(l,l_i,\alpha_i),p_i,\theta)$$

- received signal power $\mathbf{r} = (r^{(1)}, \ldots, r^{(n)})$
- distances between the transmitter and the receiver $\mathbf{d} = (d^{(1)}, \ldots, d^{(n)})$
- deviations between the direction of transmission and the direction of the receiver $\delta = (\delta^{(1)}, \ldots, \delta^{(n)})$
- transmitted powers $\mathbf{p} = (p^{(1)}, \ldots, p^{(n)})$
- maximum likelihood from complete data

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} f(r^{(i)} | d^{(i)}, \delta^{(i)}, p^{(i)}, \theta)$$

$$\text{SSE} = \sum_{i=1}^{n} (r^{(i)} - \mu^{(i)})^2$$

- maximum likelihood from incomplete data
- EM can be applied - evaluate the expected value of the logarithm of the complete data log-likelihood

- E-step

$$Q(\theta, \theta_t) = E_{\theta_t} \ln \mathcal{L}(\theta)$$

- M-step: maximize the expected log-likelihood

$$\theta_{t+1} = argmax_{\theta} Q(\theta, \theta_t)$$

- Location Estimation

$$p(l|\mathbf{r}, \hat{\theta}) = \frac{g(\mathbf{r}|l, \hat{\theta})\pi(l)}{\int g(\mathbf{r}|l', \hat{\theta})\pi(l')dl'}$$

- $\mathbf{r}$ vector consisting of the received power values r_j for each channel j
- and $g(\mathbf{r}|l, \hat{\theta})$ is the likelihood function given by

$$g(\mathbf{r}|l, \hat{\theta}) = \prod_j g_j(r_j|l, \hat{\theta})$$

- π is the prior p.d.f. of the location variable

Thanks