WMNC assignment

3) What is equivalent MIMO system? Derive the channel capacity for MIMO system.

Ans: Equivalent MIMO system is based on the idea of selecting the channel matrix. It is in such a way that optimizers the channel output, thereby increasing signal-to-noise-ratio (SNR).

Since SNR is not easy to calculate for MIMO, capacity of the channel is very complicated to determine

$$\boxed{y = Hx + n}$$

where,

y = received signal $(N \times 1)$ | x = sent signal $(M \times 1)$
H = channel matrix $(N \times M)$ | n = noise $(N \times 1)$

using singular value decomposition (SVD)

$$\boxed{H = UDV^H}$$

where,

H = channel matrix $(N \times M)$
U = Unitary matrix $UU^H = U^H U = I_N$
D = Diagonal matrix $(N \times M)$
V^H = Complex conjugate transpose / Hermitian transpose

of MxM unitary matrix

$$VV^H = V^H V = I_M$$

Diagonal elements of D are called sigular values of H.

There are non-negative square roots of the eigen values λ, of the following equation

$$(HH^H)X = \lambda X \quad \text{if } N < M$$
$$(H^H H)X = \lambda X \quad \text{if } N \geq M$$

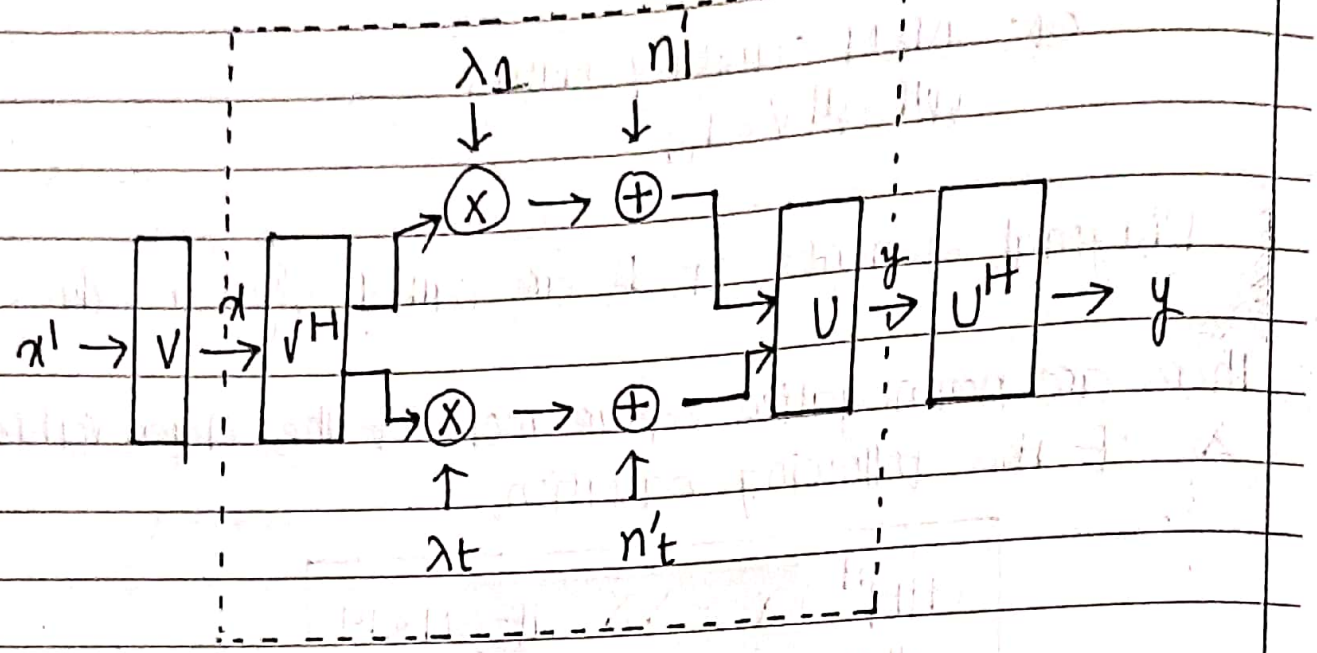
X is Nx1, eigenvector associated with λ

$$y = UDV^H x + n$$

consider transformations,

$$y' = U^H y$$
$$x' = V^H x$$
$$n' = U^H n$$
$$U^H y = U^H U D V^H x + U^H n$$
$$y' = DV^H x + n'$$

$$y' = Dx' + n'$$ called Equivalent MIMO system.



where $t=\min(M,N)$, $D=[\lambda_{ij}]$ $i=j$

fig: Conversion of MIMO to equivalent MIMO through SVD

System has the same total input power, total output power and total noise power as the original system.

The o/p SNR of equivalent MIMO is sames as actual MIMO.

Therefore, channel capacity is same.

No, of nonzero eigen values of matrix HH^T is equal to rank of matrix H denoted by r..

$$y_i^1 = \sqrt{\lambda_i}\, x_i^1 + n_i^1 \qquad i = 1, 2, \ldots r$$

$$y_i^1 = 0 + n_i^1 \qquad r = r+1, \ldots N$$

channels of equivalent MIMO are uncoupled and parallel

channel capacity can be calculated by a summation of individual capacities of parallel channel

$$C = B \sum_{i=1}^{r} \log_2 \left(1 + P_{y_i'} / \sigma^2\right)$$

where;

C = Capacity of channels

B = bandwidth of channels

$P_{y_i'}$ = Power received at i^{th} receiving antenna

σ^2 = power of noise received