Mortality modelling in R: an analysis of mortality trends by cause of death and socio-economic circumstances in England

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R in Insurance
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Agenda

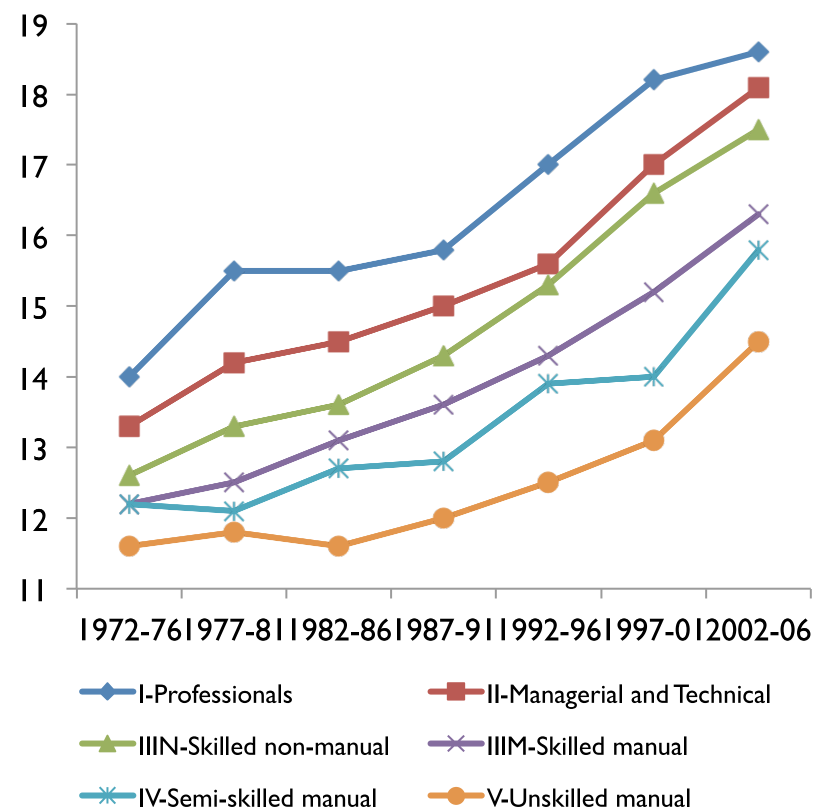
- Motivation
- Modelling mortality in R
- Modelling mortality by CoD and socio-economic stratification
- Case study: Mortality by deprivation in England
- Conclusions

Motivation

Socio-economic differences in mortality

- Well-documented relationship between mortality and socioeconomic variables
  - Education
  - Income
  - Occupation
- Important implications on social and financial planning
  - …
  - Public policy for tackling inequalities
  - Longevity risk management
  - …

Male life expectancy at age 65 by social class -England and Wales



Source: ONS Longitudinal Study

Motivation
Cause-specific mortality

- Forecasts of cause-specific mortality required for many purposes
  - E.g Estimation of health care costs

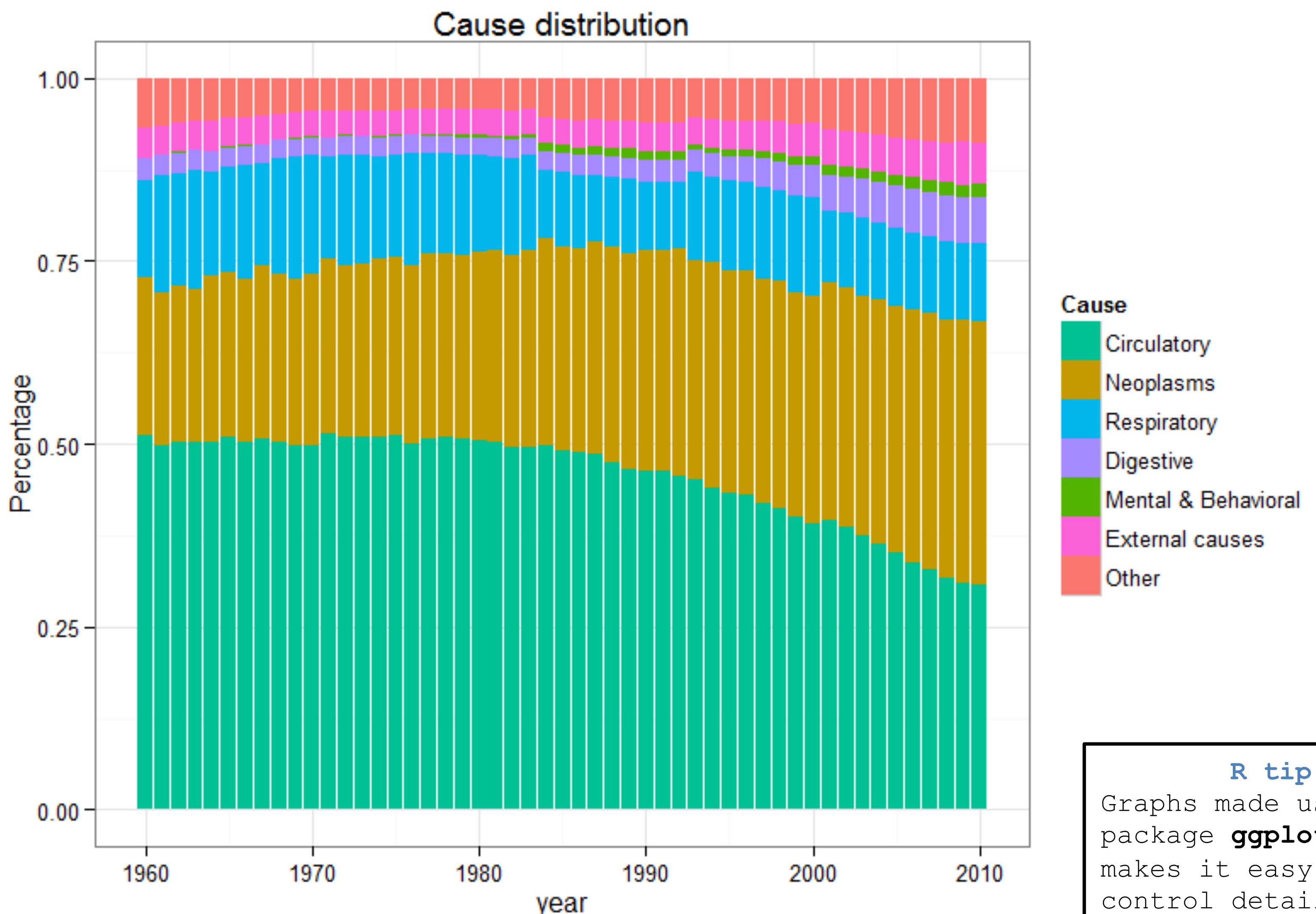
- Inform the assumptions underlying overall mortality projections

- Shed light on the drivers of mortality
  - Mortality change
  - Mortality differentials

Motivation
Why use R?

- Flexible tools for analysing mortality data and fitting mortality models
- Very good tools for presenting and communicating methods and results
- R is free

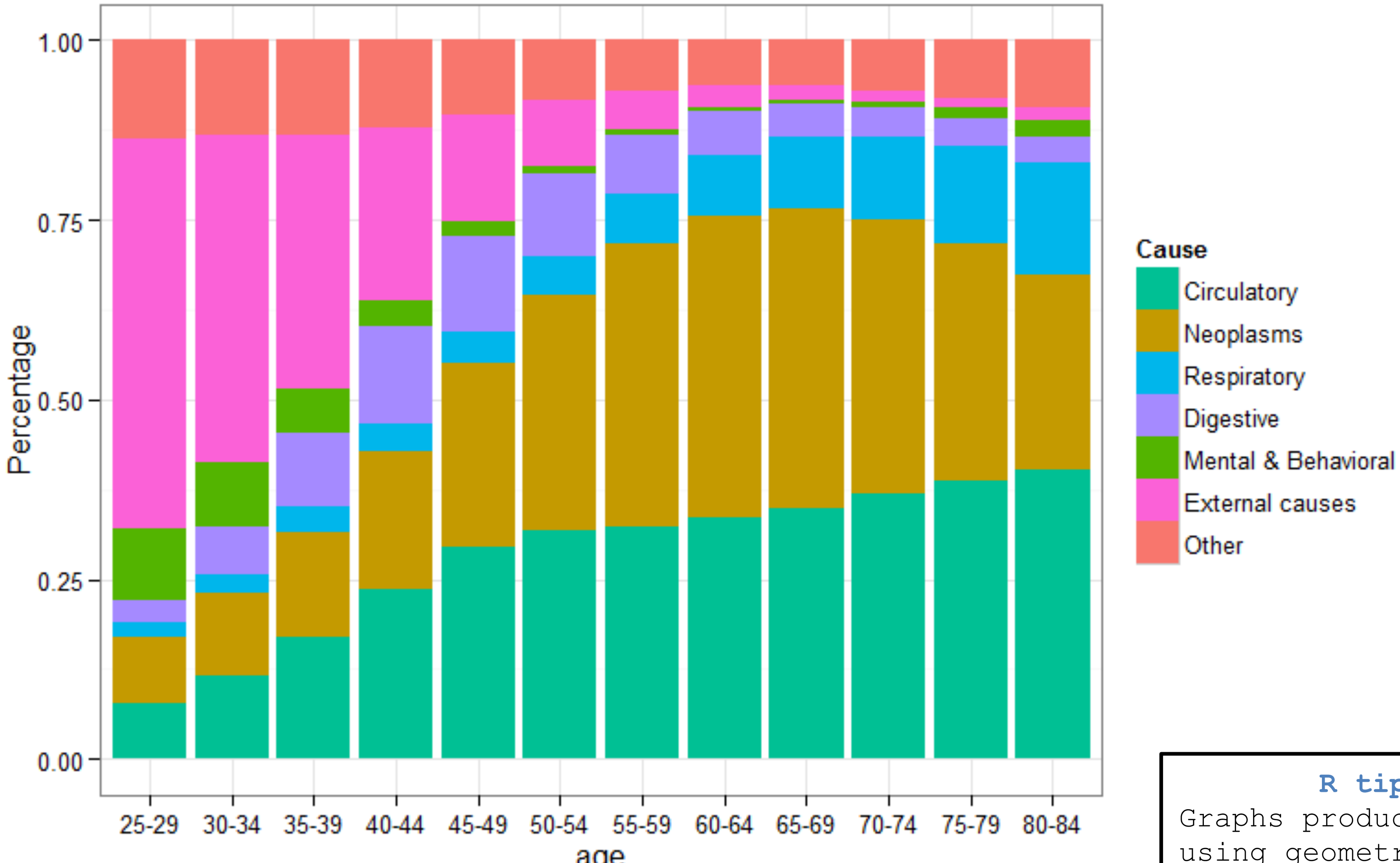
Causes of mortality in England and Wales
Causes distribution in time (ASDR males age 25-84)



R tip
Graphs made using package **ggplot2.** It makes it easy to control details (e.g legends, colours)

Causes of mortality in England and Wales

Causes distribution by age (males 2001-2010)

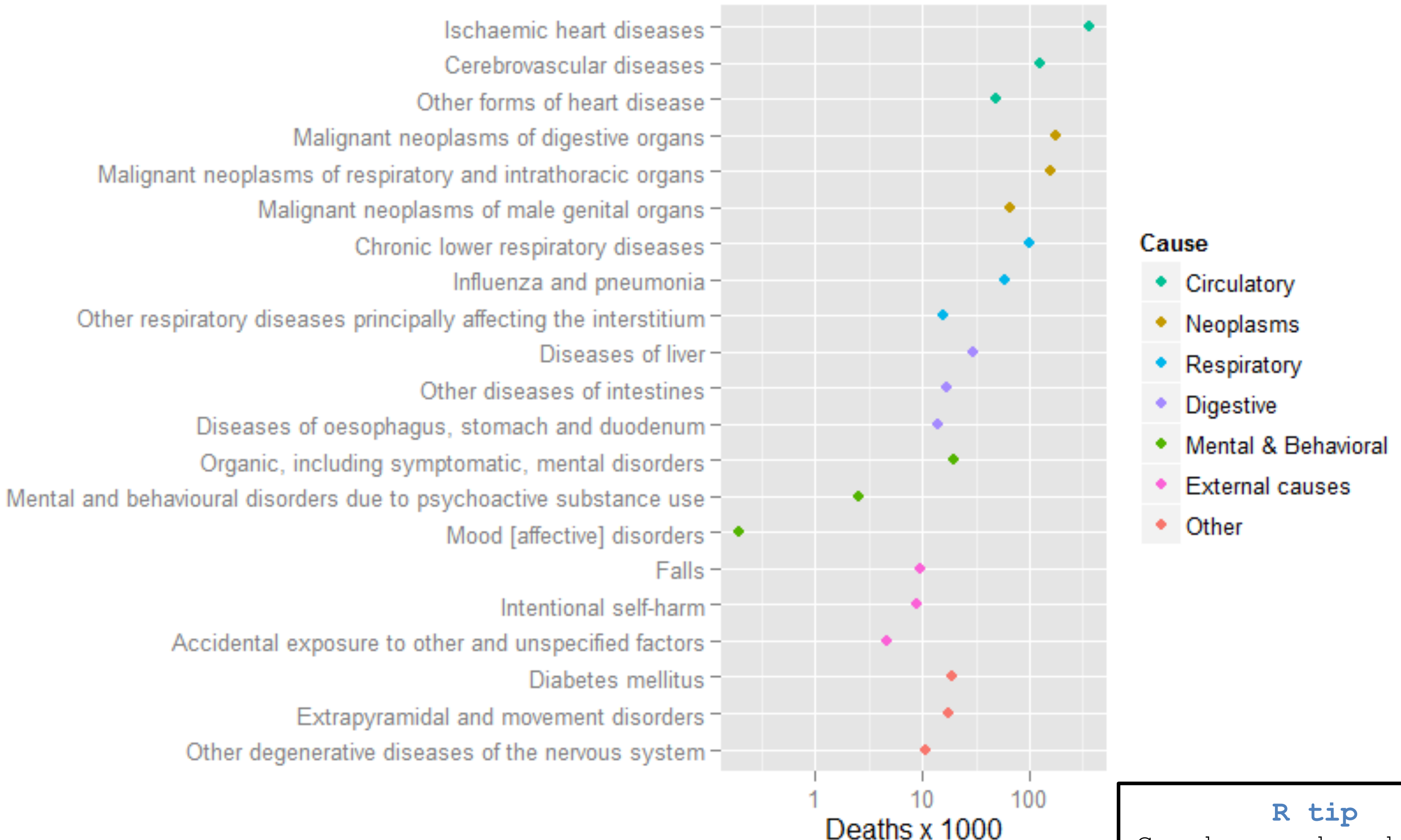


R tip

Graphs produced using geometry **geom_bar** from package **ggplot2**

Causes of mortality in England and Wales

Main causes for males aged 50-84 (2001-2010)

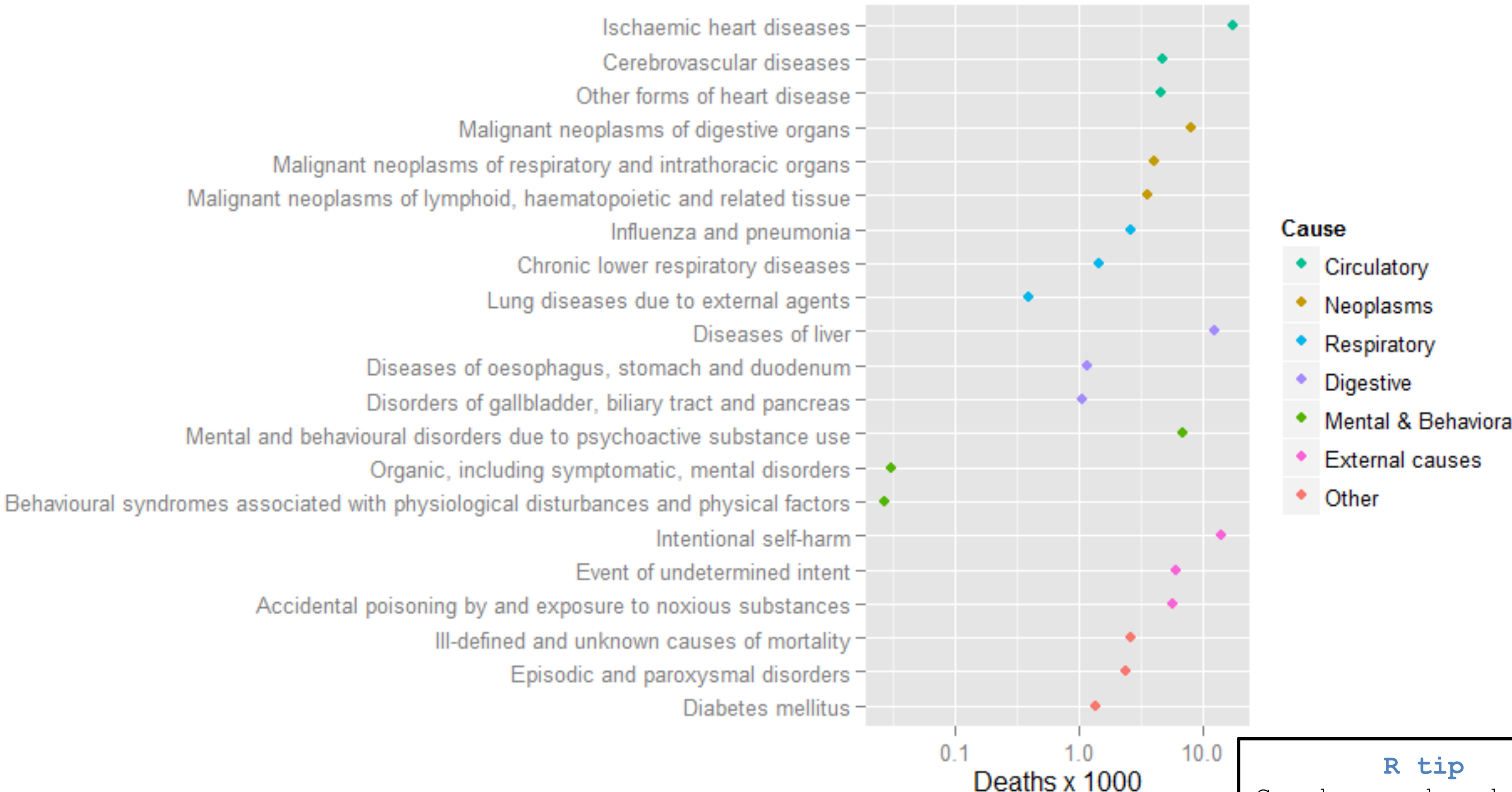


R tip

Graphs produced using geometry **geom_point** from package **ggplot2**

Causes of mortality in England and Wales

Main causes for males aged 25-49 (2001-2010)



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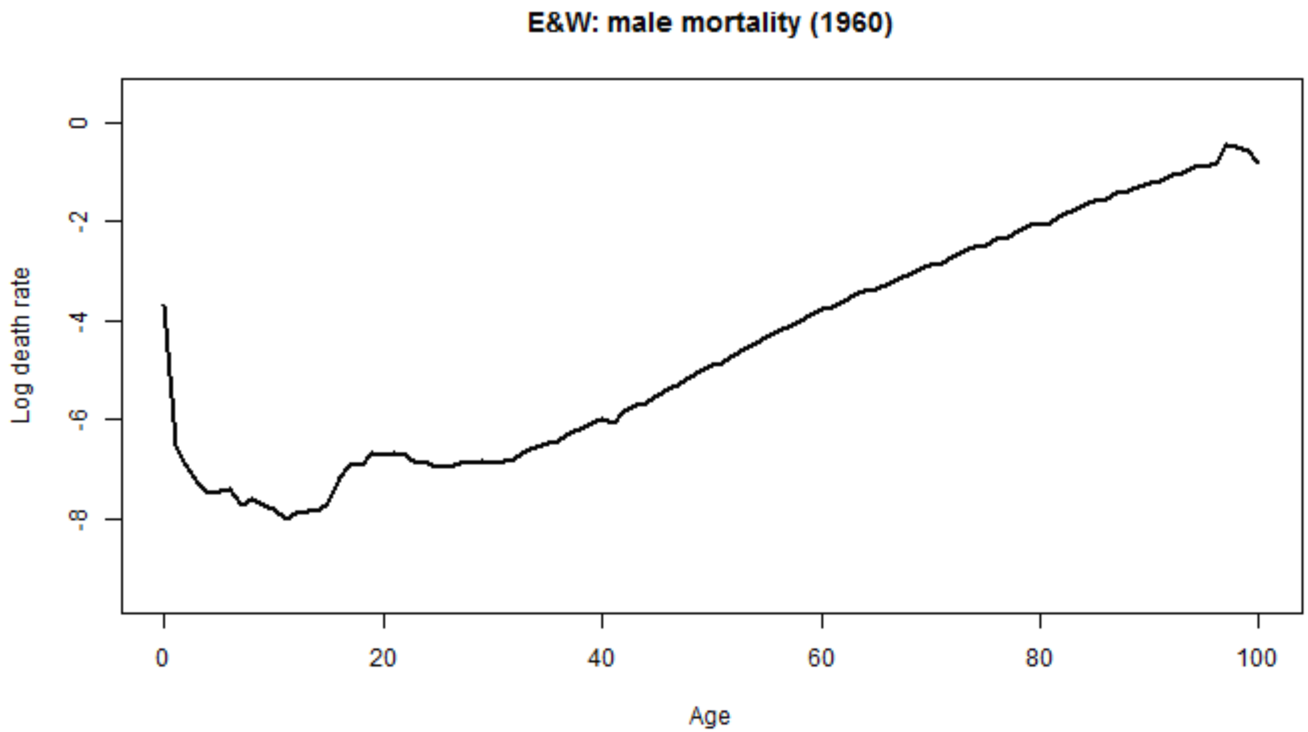
Modelling mortality

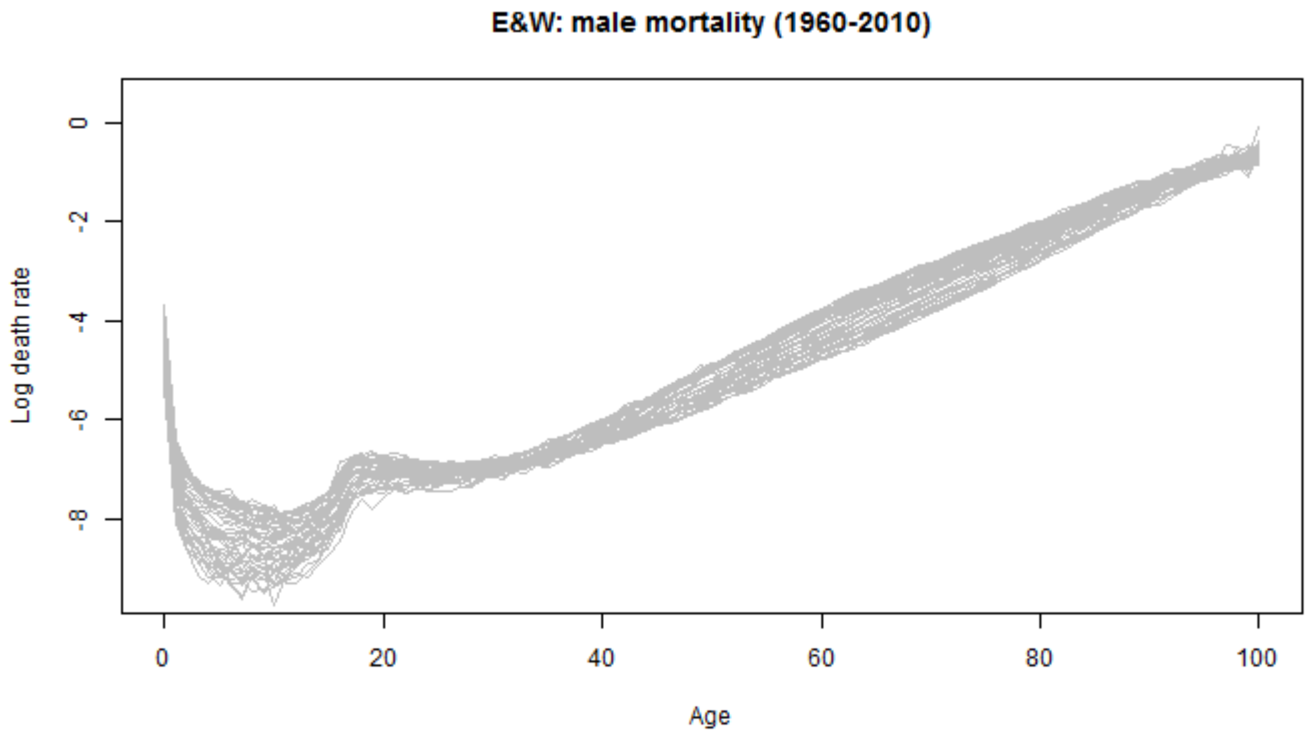
Lee-Carter model



R tip

Use function **saveGIF** from package **animation** to produce the animation

Modelling mortality
Lee-Carter model



$$\log \mu_{xt} \quad =$$

Modelling mortality
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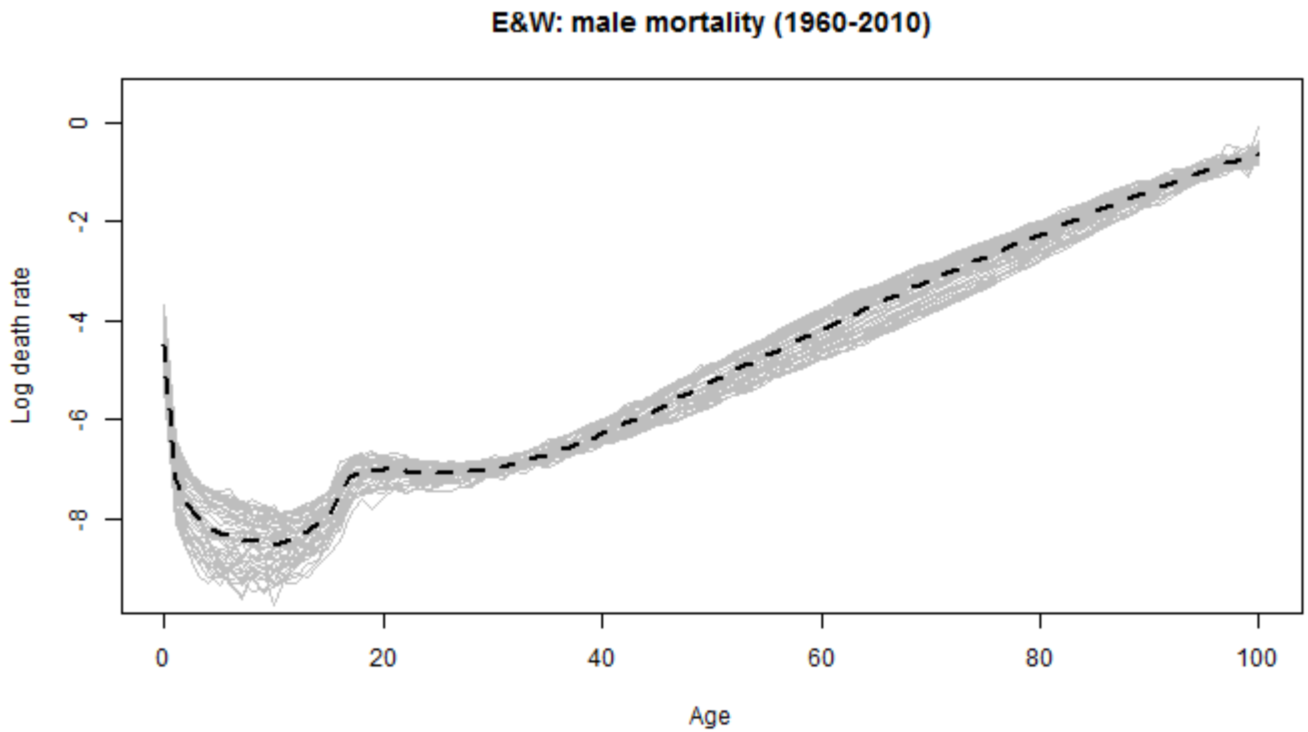

$$\log \mu_{xt} = \alpha_x$$

Modelling mortality

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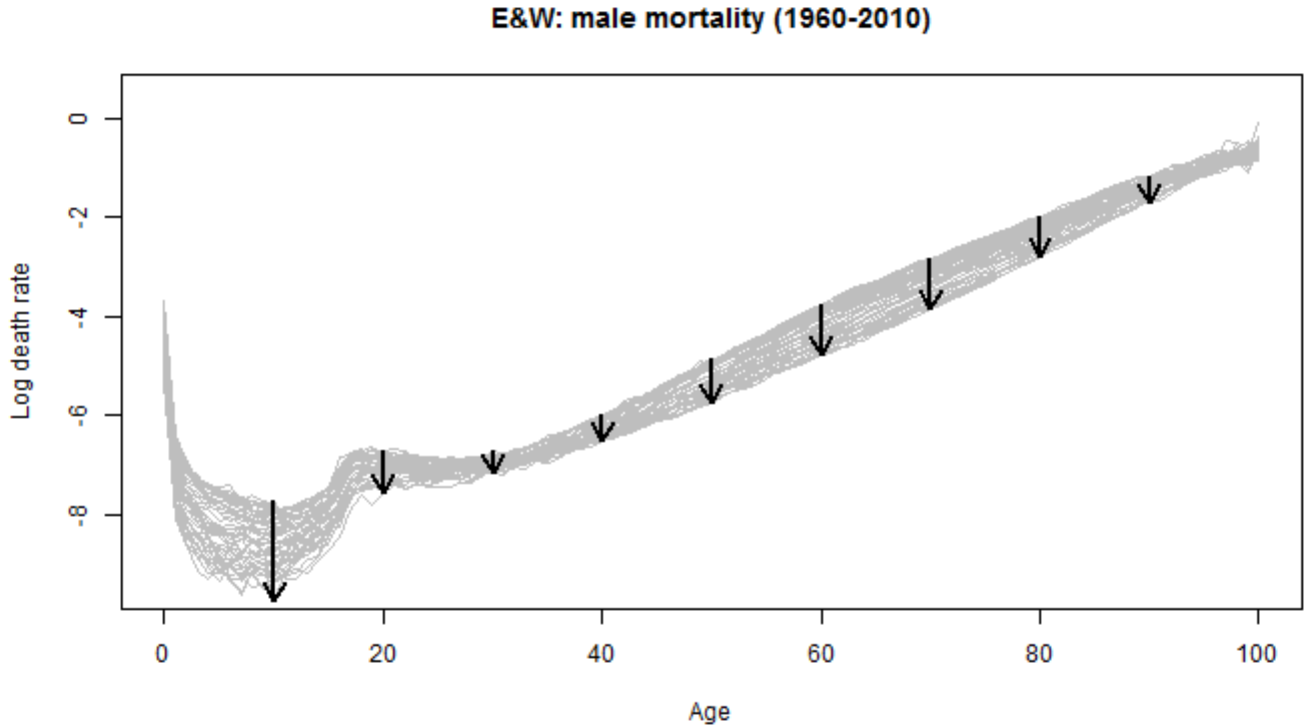
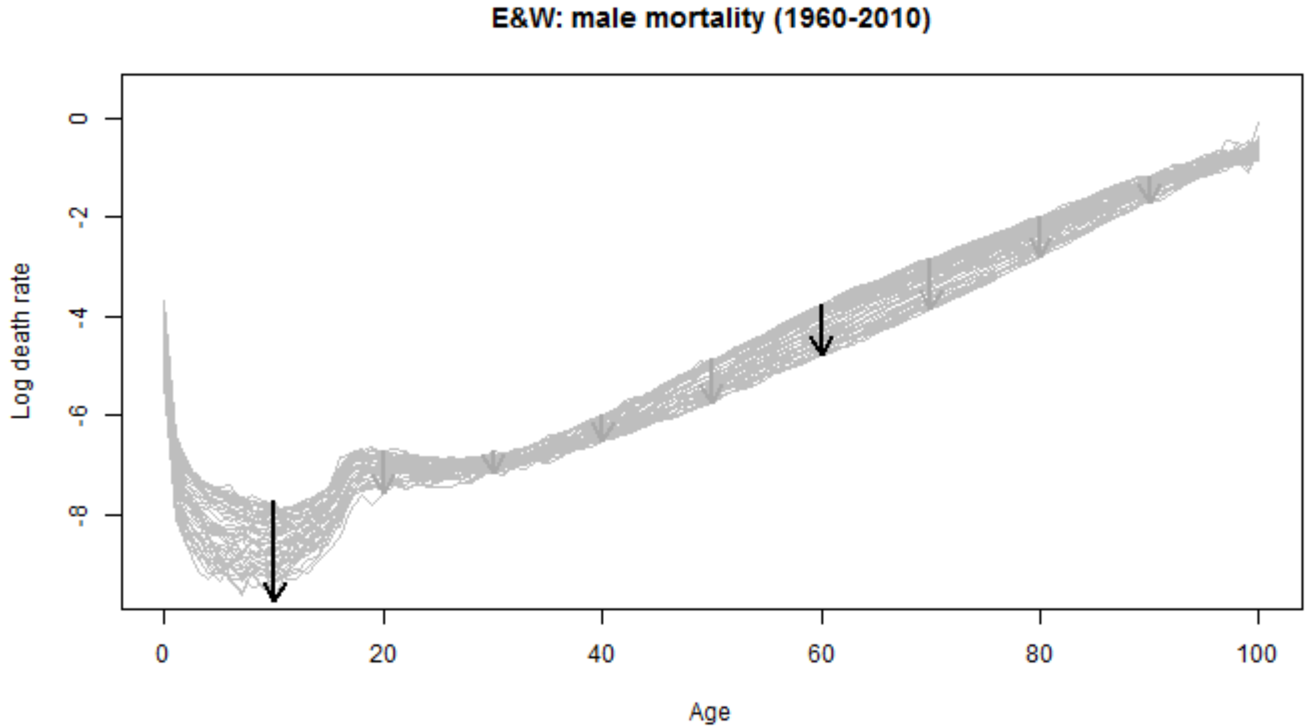


$$\log \mu_{xt} = \alpha_x + \kappa_t$$

Modelling mortality
Lee-Carter model



$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t$$

Modelling mortality

Lee-Carter model in R

Package **demography** (Hyndman, 2012)

- Implements several variants of the Lee-Carter model
- Great for using data from the Human Mortality Data Base
- Easy plotting of results and projection of mortality using package **forecast**
- Not easily extendable

LifeMetrics software (JP Morgan)

- Implements the Lee-Carter models and other popular mortality models (Renshaw and Haberman (2006), APC, CBD and extensions)
- Excel interface
- Not easily extendable

Package **gnm** (Turner and Firth, 2012)

- General purpose package for fitting generalised non-linear models
- Can be used to fit a large number of mortality models including the Lee-Carter model
- Easily extendable (e.g cohort effects, multipopulation models, logit/binomial framework)

Modelling mortality

Lee-Carter model with **gnm**

Lee Carter in a Poisson regression framework (Brouhns et al, 2002)

$$D_{xt} \sim Poisson(E_{xt}\ \mu_{xt})$$

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$$\sum_t \kappa_t = 0 \quad \sum_x \beta_x = 1$$

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gnmLC <- gnm(D~offset(log(E)) -1 + factor(age) + Mult(factor(age),factor(year)),
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#Extract the coefficients
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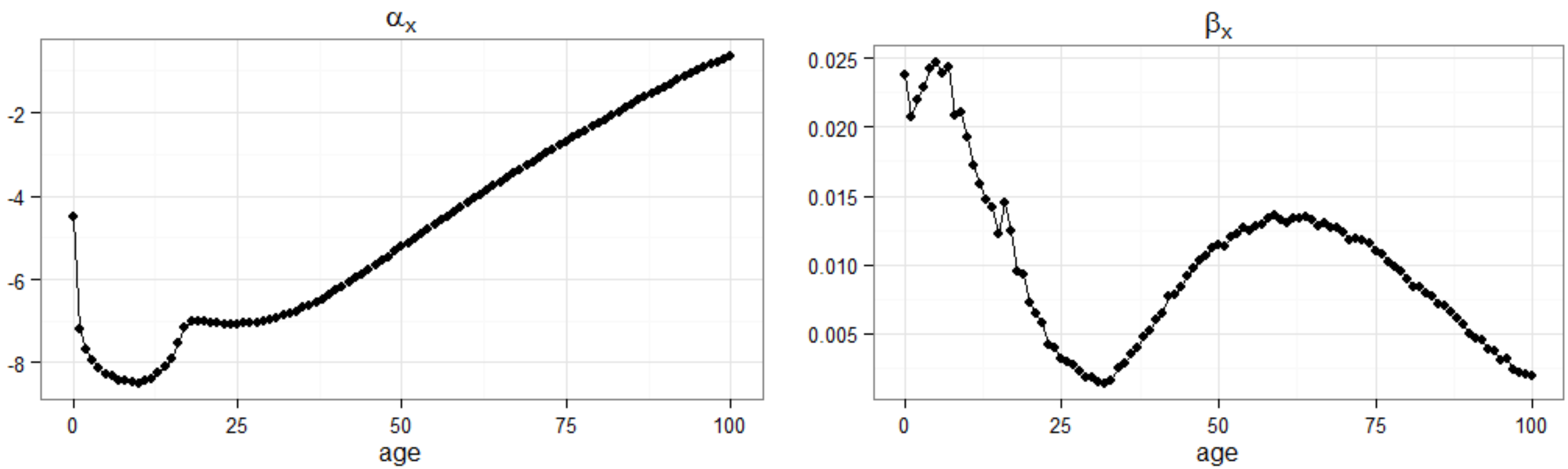
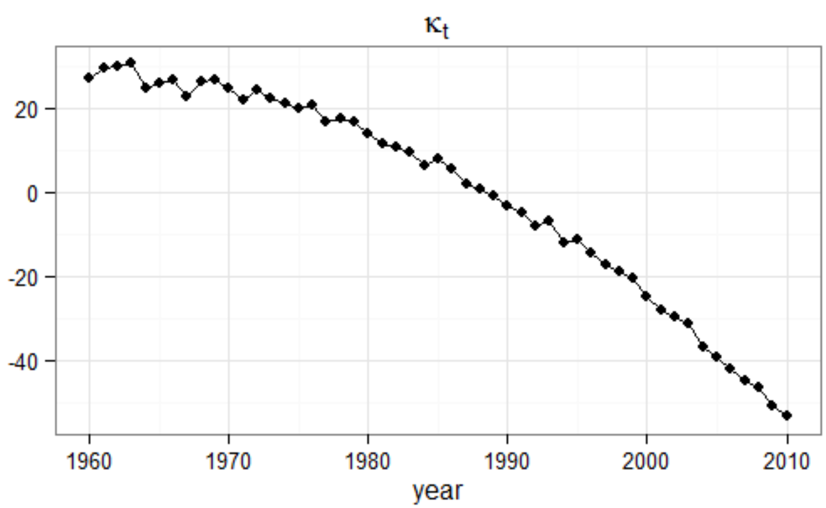
Modelling mortality

Lee-Carter model with **gnm** (E&W males 1960-2010)





R tip

Use packages **`grid`** and **`gridExtra`**

For controlling the layout of graphics

Modelling mortality by cause of death

Lee-Carter model with coding changes

- Challenges
  - …
  - Changes in classification of causes of death difficult the analysis of trends
  - …

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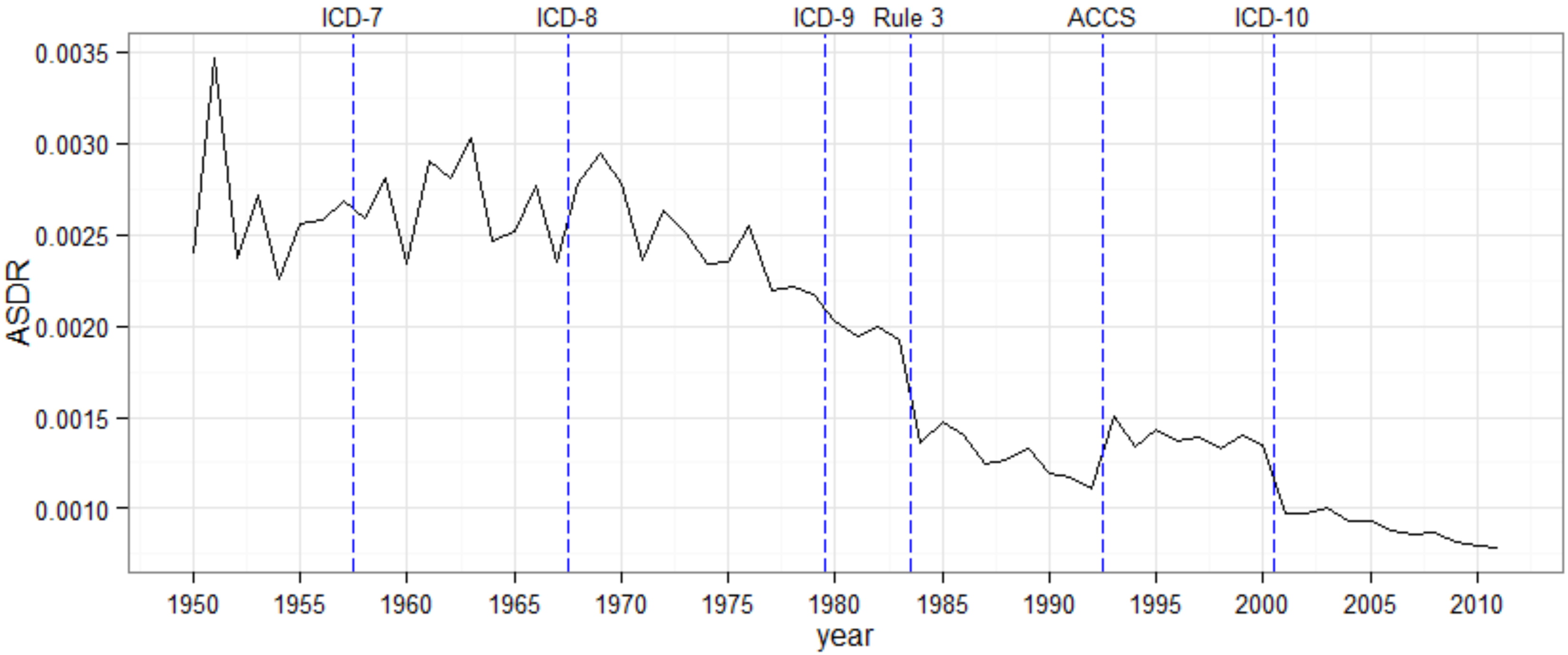
Age-standardised mortality rate for respiratory diseases
(Male age 25-84 – England and Wales)



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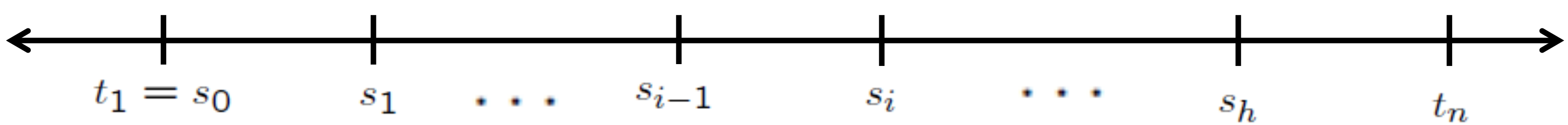

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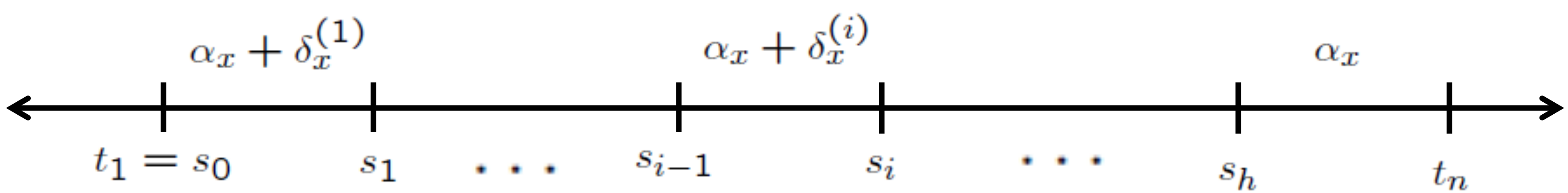
$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t + \sum_{i=1}^{h} \delta_x^{(i)} f^{(i)}(t)$$

$$f^{(i)}(t) = \mathcal{I}_{\{s_{i-1} \le t < s_i\}}$$

Adjustment for coding changes



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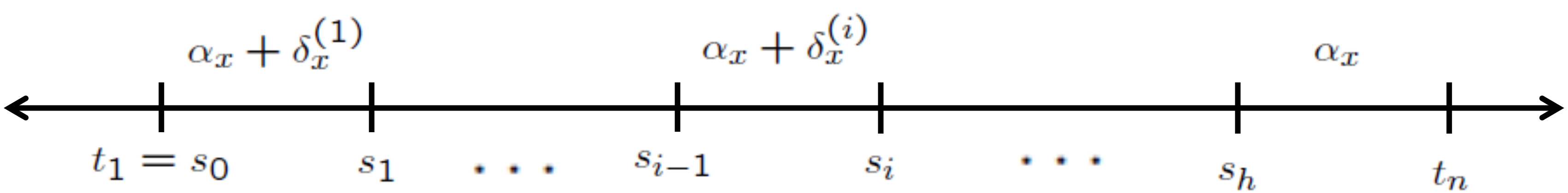
Adjustment for coding changes

R implementation

This extension can be implemented using **gnm** combined with the package **mgcv** in order to ensure smoothness at the times of coding changes



Modelling by CoD and socio-economic stratification

Three-way Lee-Carter model (Russolillo et al, 2011)

$$\log \mu_{xtg}^{c} = \alpha_x^{c} \qquad\qquad + \beta_x^{c} \quad \kappa_t^{c} + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

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Level differentials

Modelling by CoD and socio-economic stratification

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$$\log \mu_{xtg}^{c} = \alpha_x^c + \underbrace{\alpha_{xg}^c}_{\text{Level differentials}} + \underbrace{\beta_x^c \lambda_g^c \kappa_t^c}_{\text{Improvement differentials}} + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

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Level differentials

Improvement differentials

R implementation

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$$\log \mu_{xtg}^{c} = \alpha_x^c + \underbrace{\alpha_{xg}^c}_{\text{Level differentials}} + \underbrace{\beta_x^c \lambda_g^c \kappa_t^c}_{\text{Improvement differentials}} + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$

R implementation

- The standard three-way Lee-Carter and other **multipopulation extensions of the Lee-Carter** can be easily fitted using **gnm**

```
gnm(D~offset(log(E)) -1 + factor(age:sec) +
Mult(factor(age), factor(sec),factor(year)),.....)
```

- The three-way Lee-Carter with cause of death **coding changes** can be fitted with **gnm** in a **two stage** estimation procedure with a **reference population**

Case study: Mortality by deprivation in England

Application data

Subpopulation data

- England population disaggregated by IMD 2007 quintile
- Ages: 25-29,30-34,…,80-84
- Period: 1981-2007

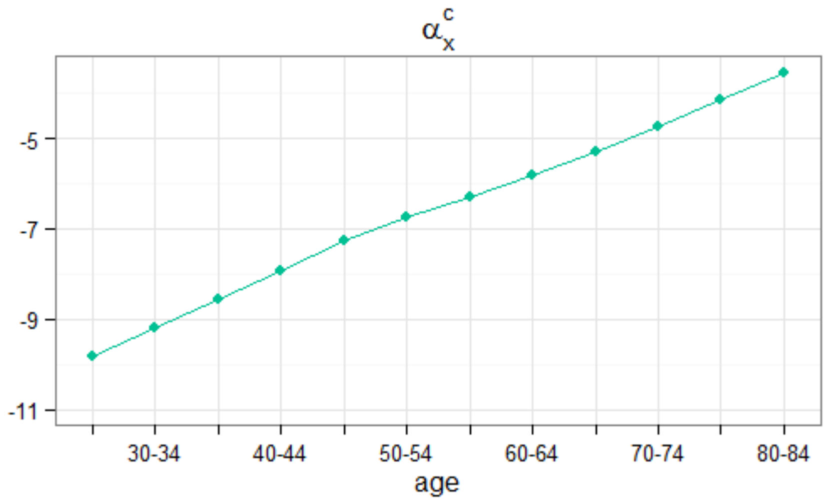
Reference population data

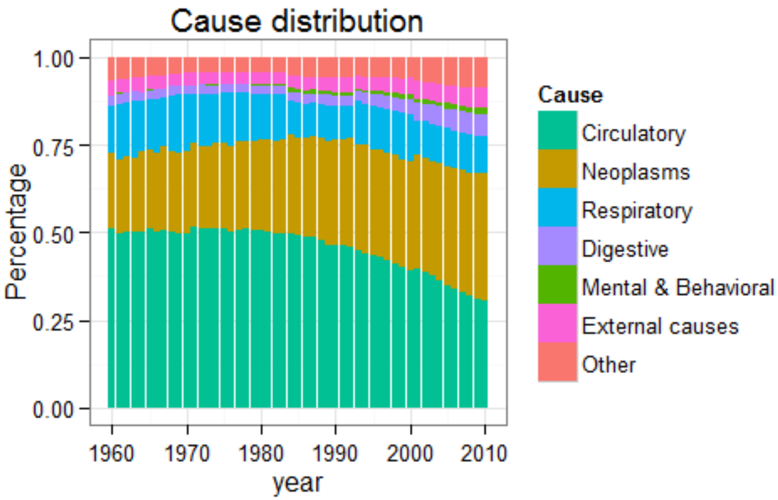
- England and Wales population
- Ages: 25-29,30-34,…,80-84
- Period: 1960-2009

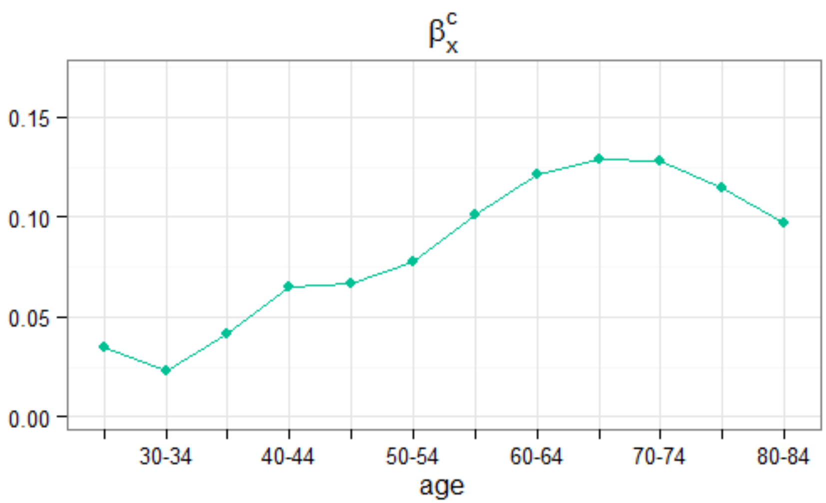
Case study: Mortality by deprivation in England

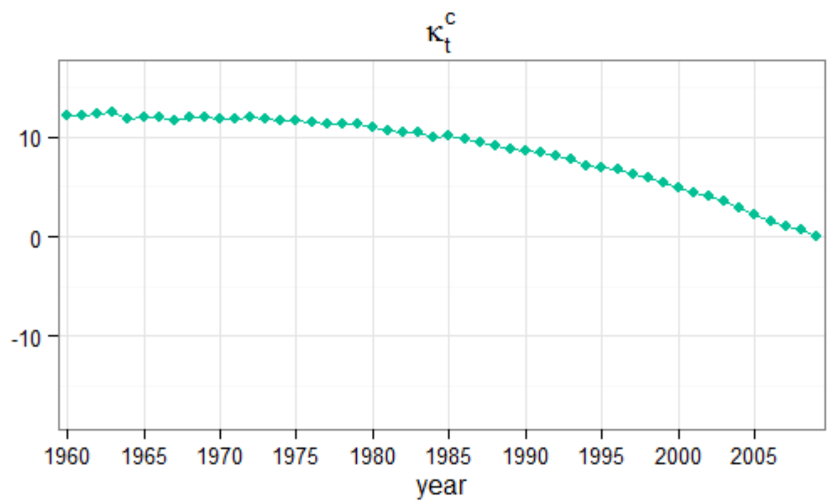
England and Wales Male population parameters

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$





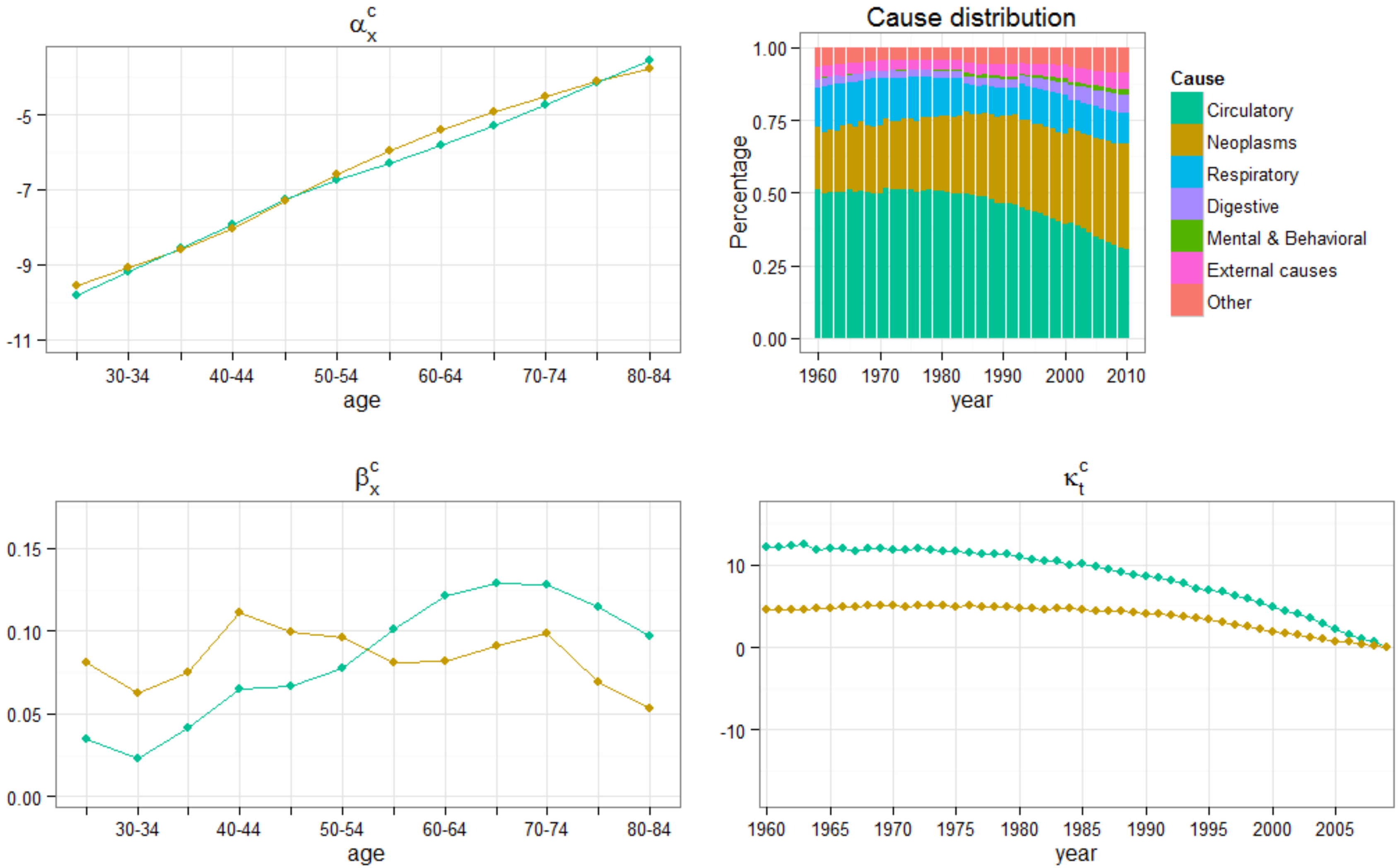




Case study: Mortality by deprivation in England

England and Wales Male population parameters

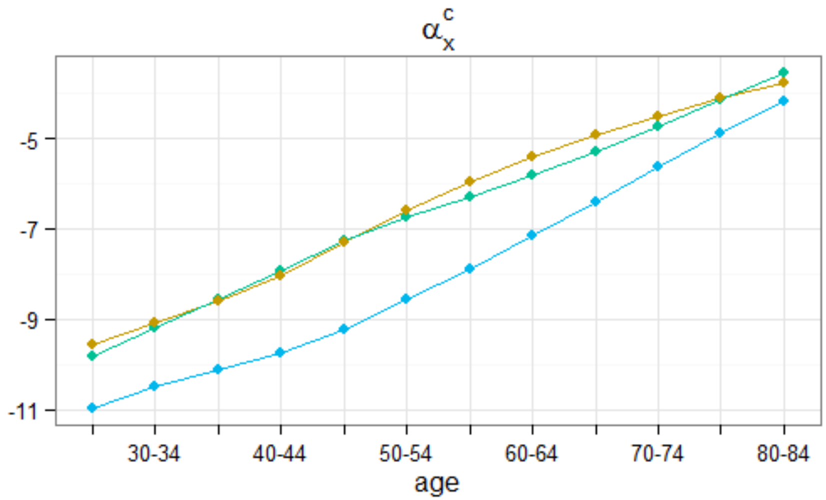
$$\log \mu^{c}_{xtg} = \alpha^{c}_{x} + \alpha^{c}_{xg} + \beta^{c}_{x}\lambda^{c}_{g}\kappa^{c}_{t} + \sum_{i=1}^{h} \delta^{c,(i)}_{x} f^{(i)}(t)$$

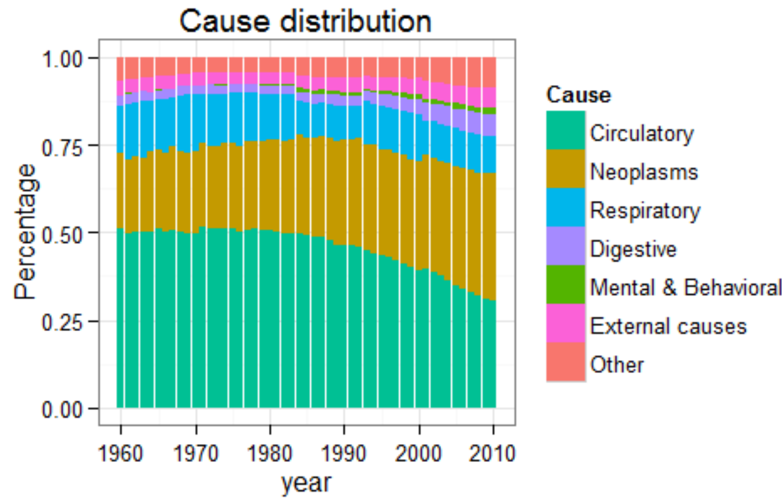


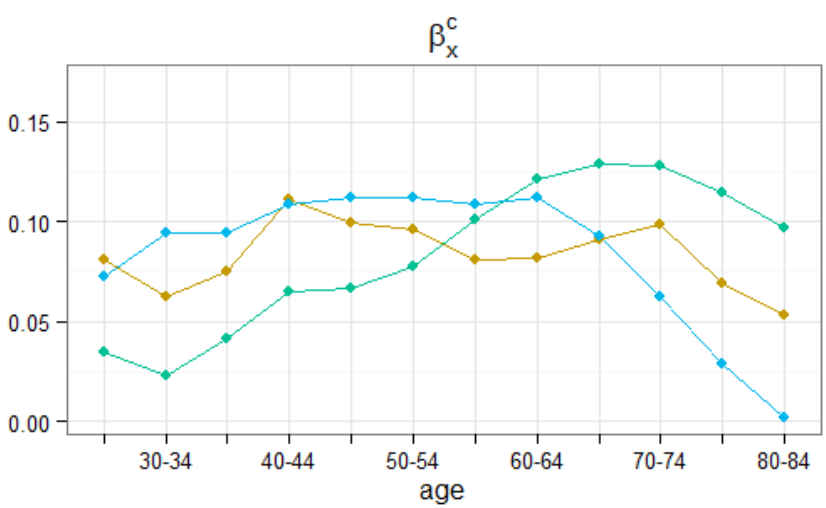
Case study: Mortality by deprivation in England

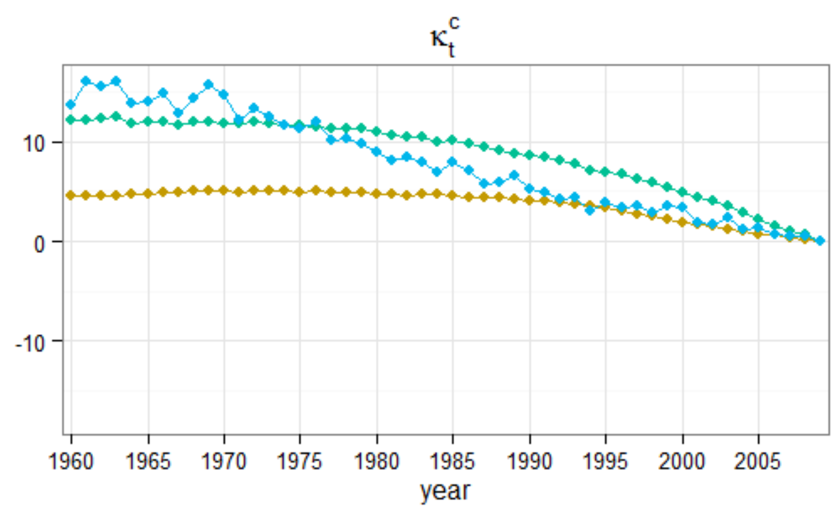
England and Wales Male population parameters

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$









Case study: Mortality by deprivation in England

England and Wales Male population parameters

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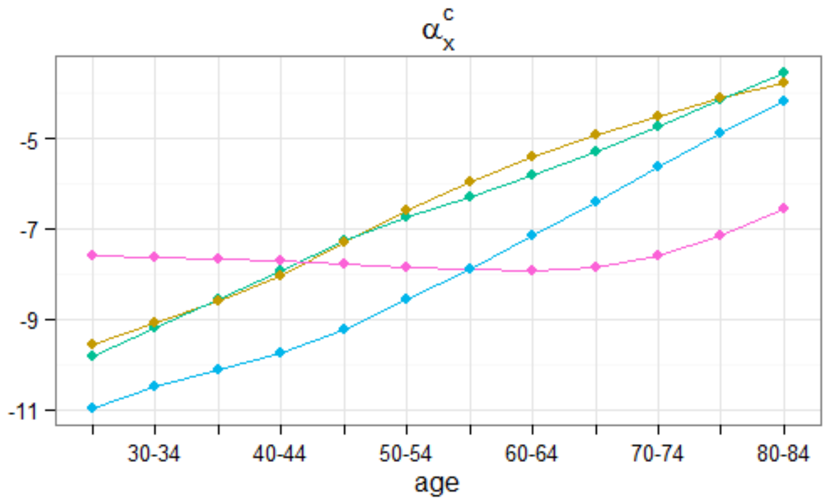

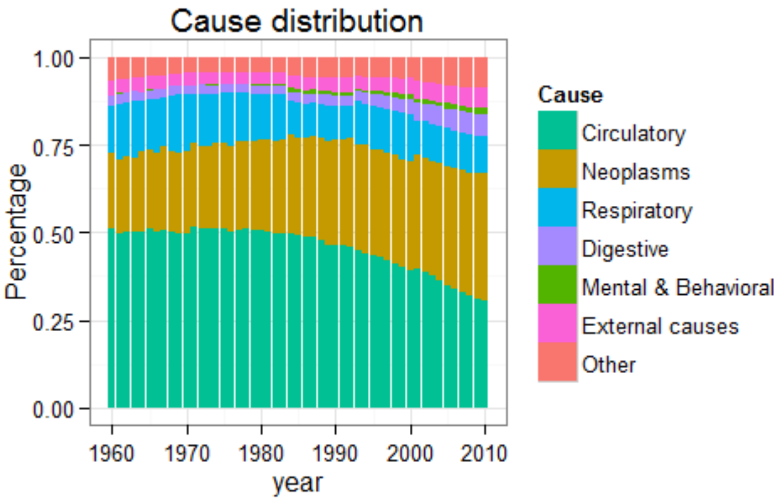

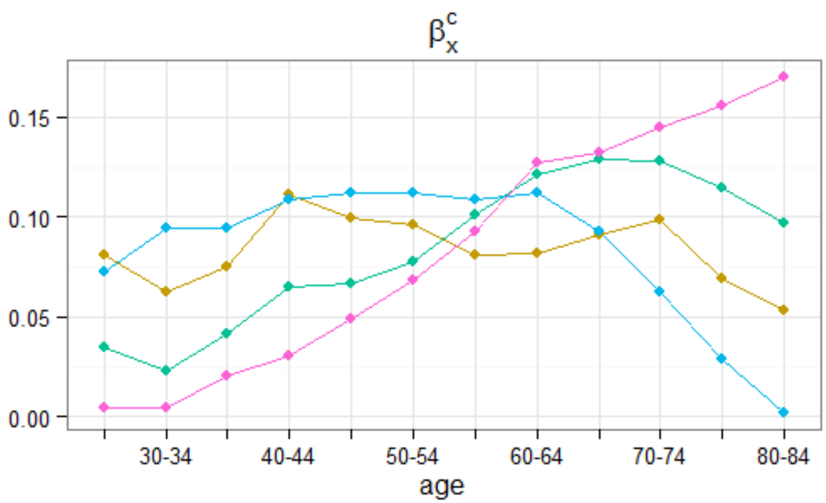

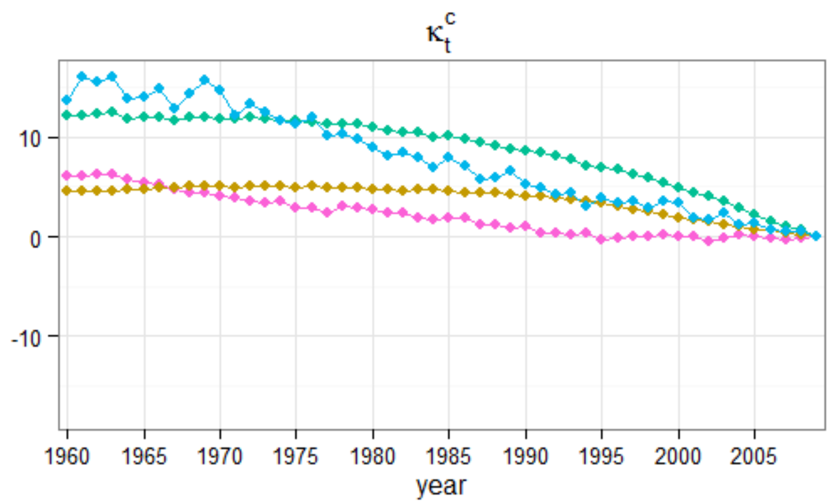

Case study: Mortality by deprivation in England

England and Wales Male population parameters

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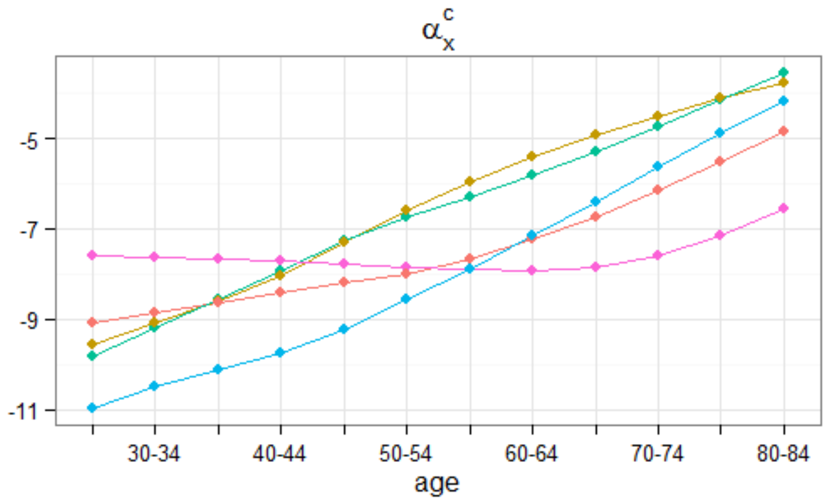
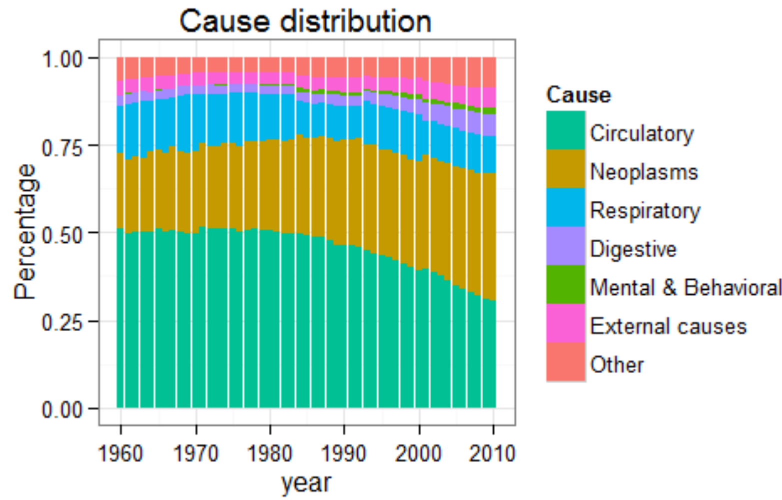
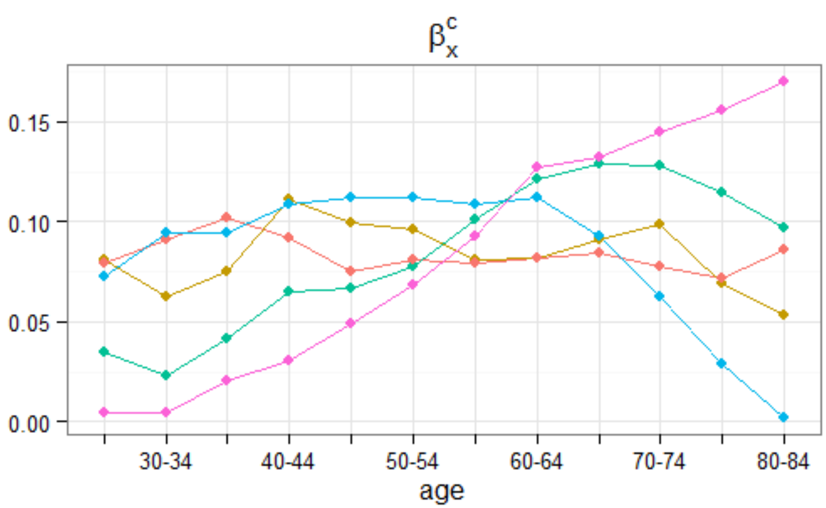
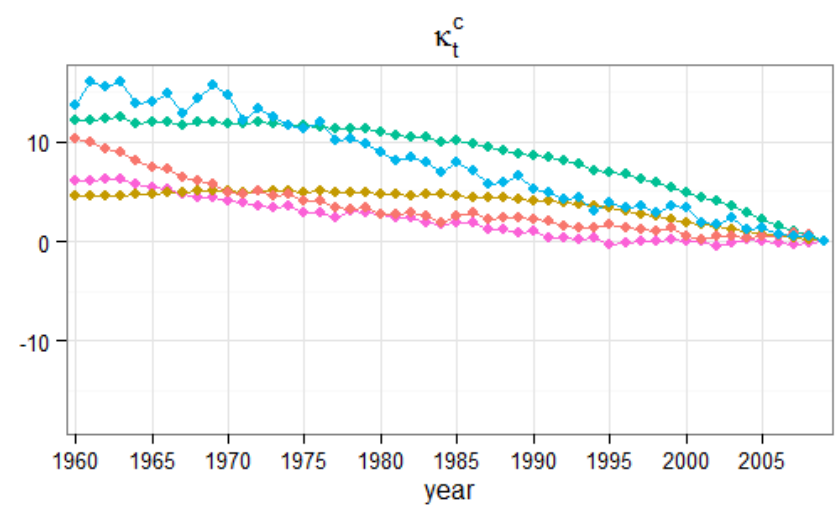




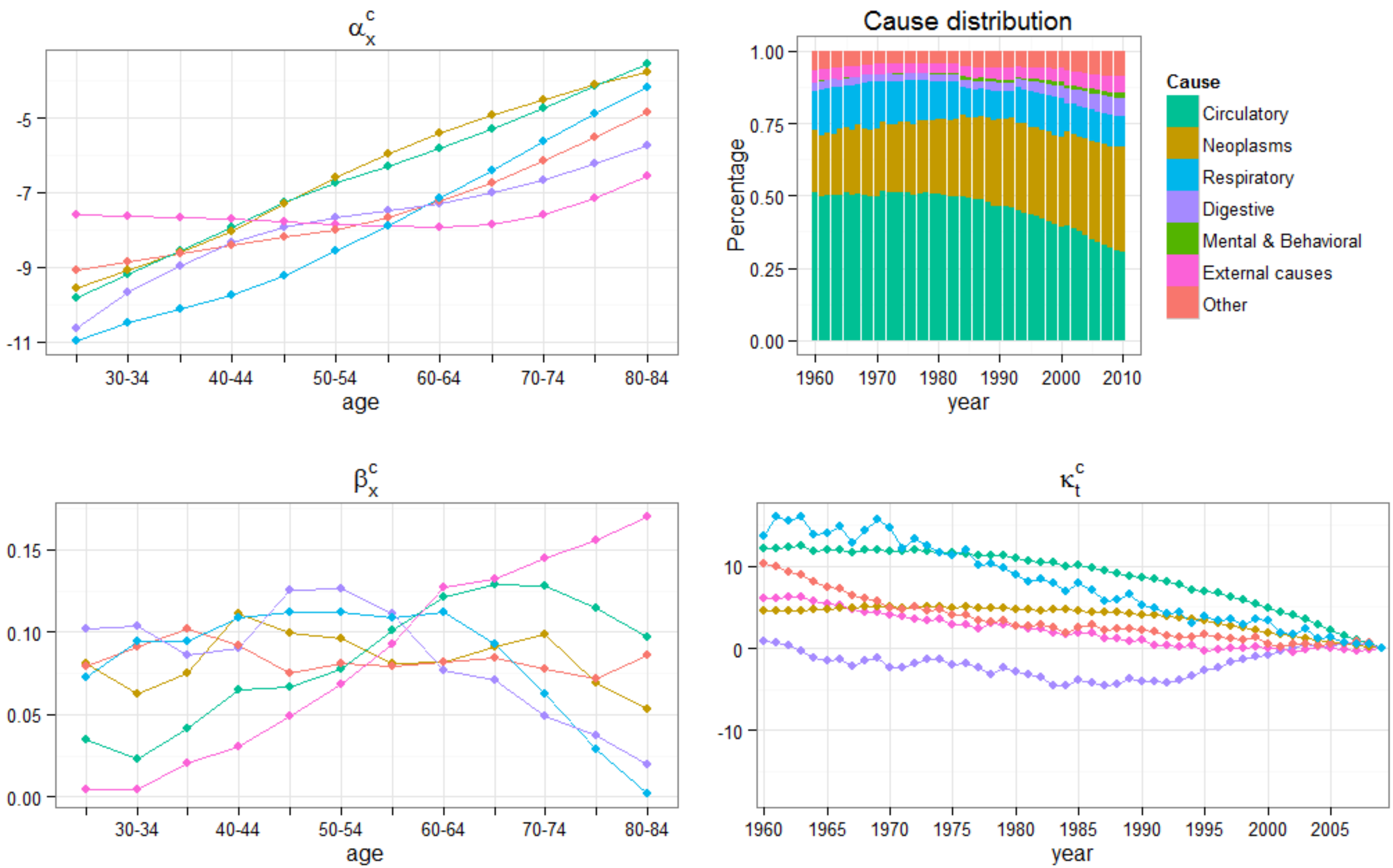




Case study: Mortality by deprivation in England

England and Wales Male population parameters

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$



Case study: Mortality by deprivation in England

England and Wales Male population parameters

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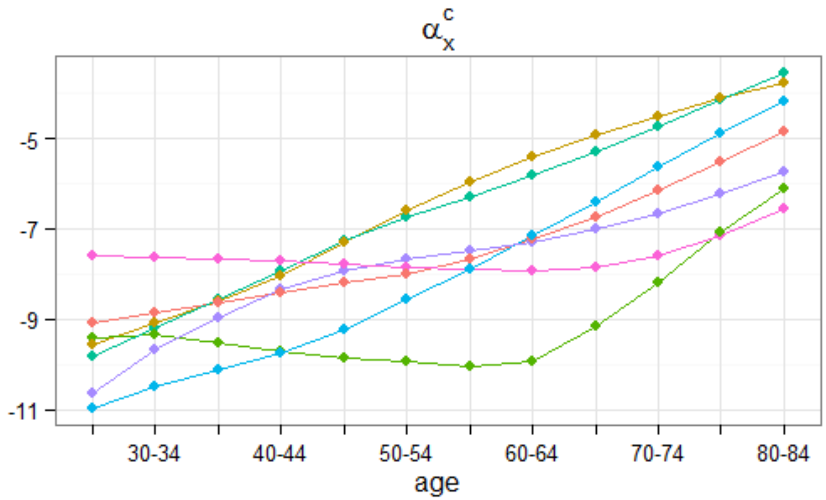
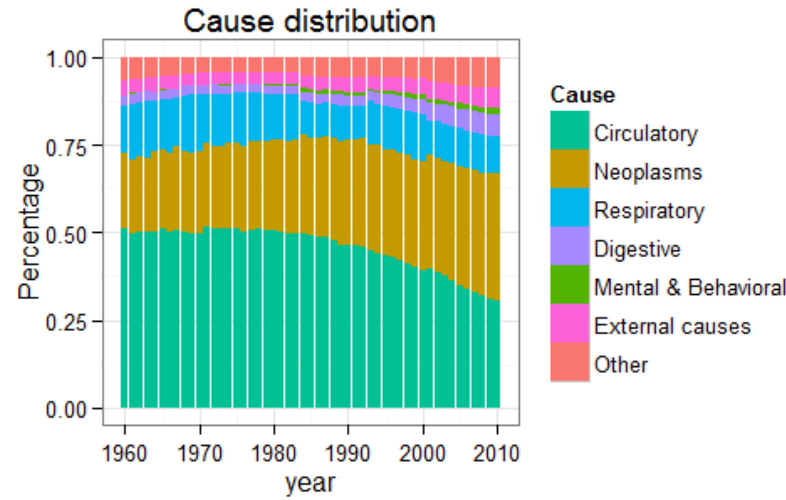
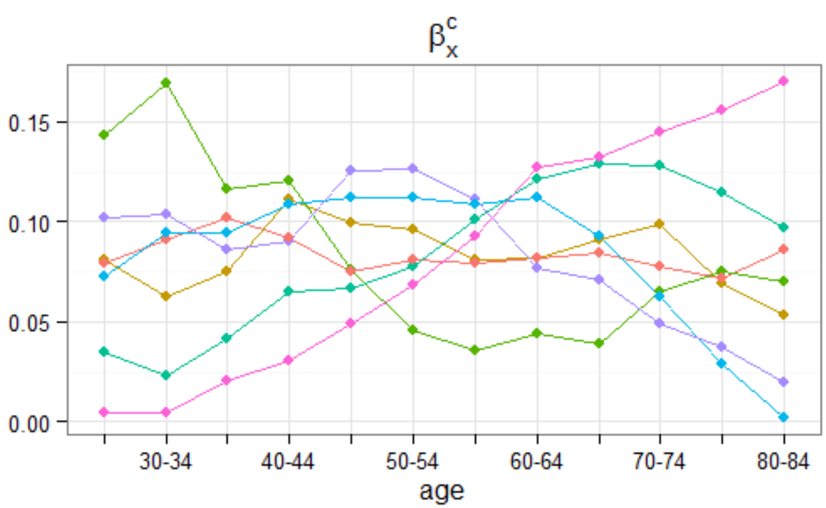
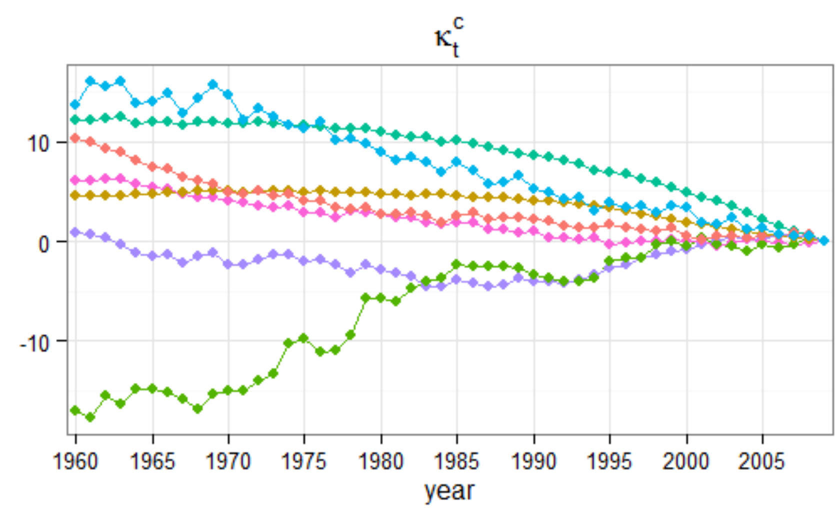




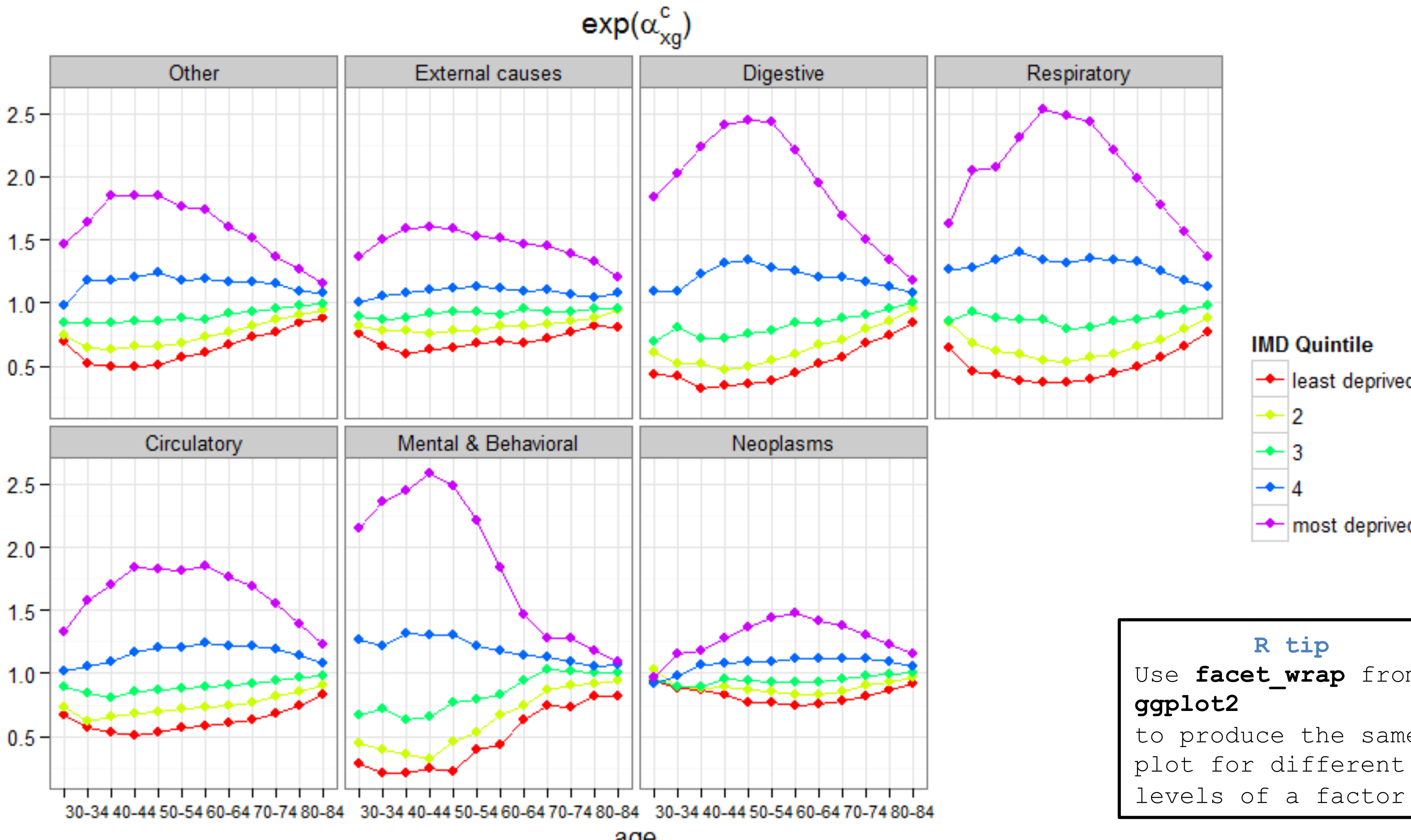




Case study: Mortality by deprivation in England

Level differences by deprivation quintile

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$



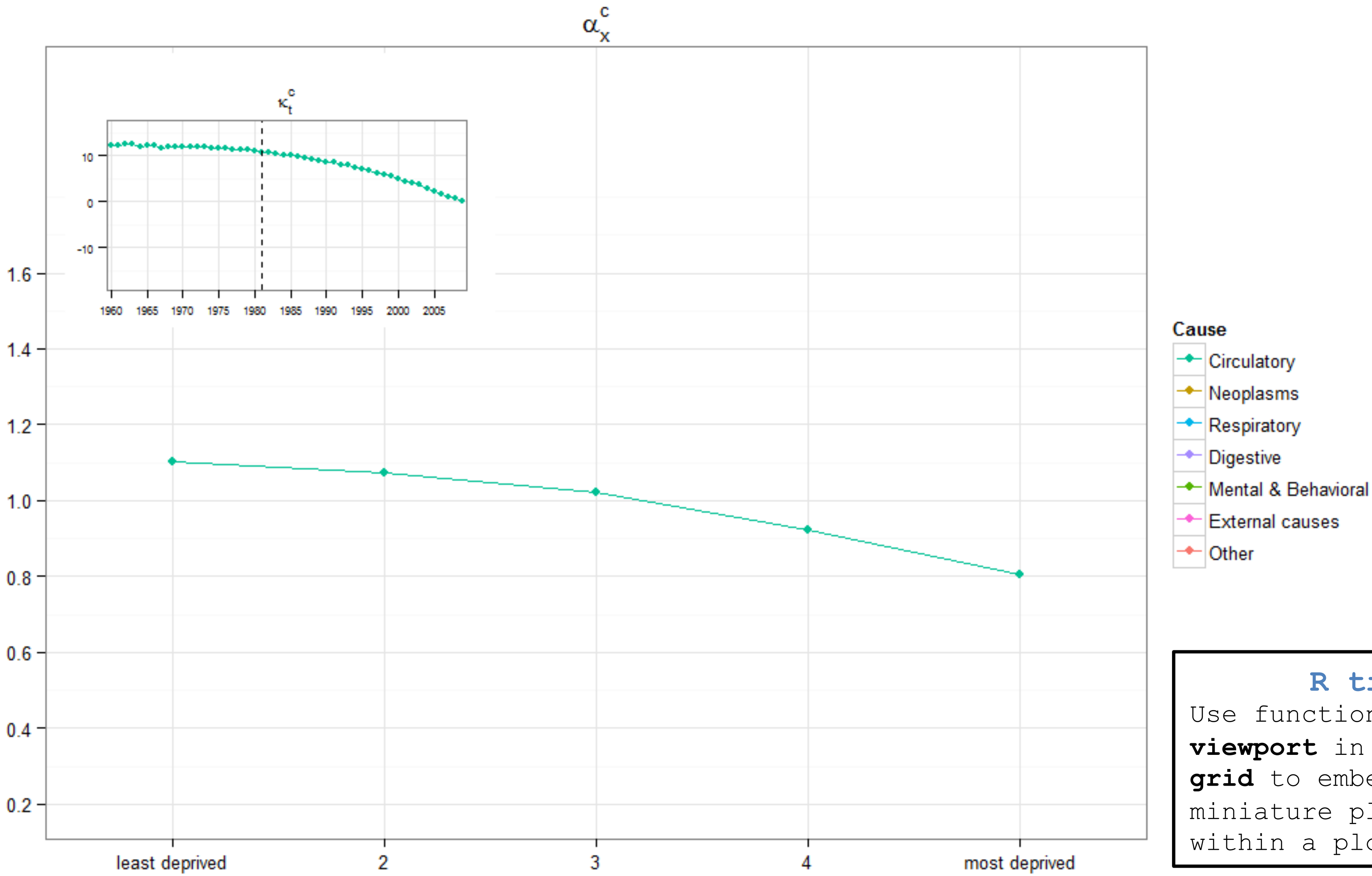
R tip

Use **facet_wrap** from **ggplot2** to produce the same plot for different levels of a factor

Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

$$\log \mu_{xtg}^{c} = \alpha_x^c + \alpha_{xg}^c + \beta_x^c \lambda_g^c \kappa_t^c + \sum_{i=1}^{h} \delta_x^{c,(i)} f^{(i)}(t)$$



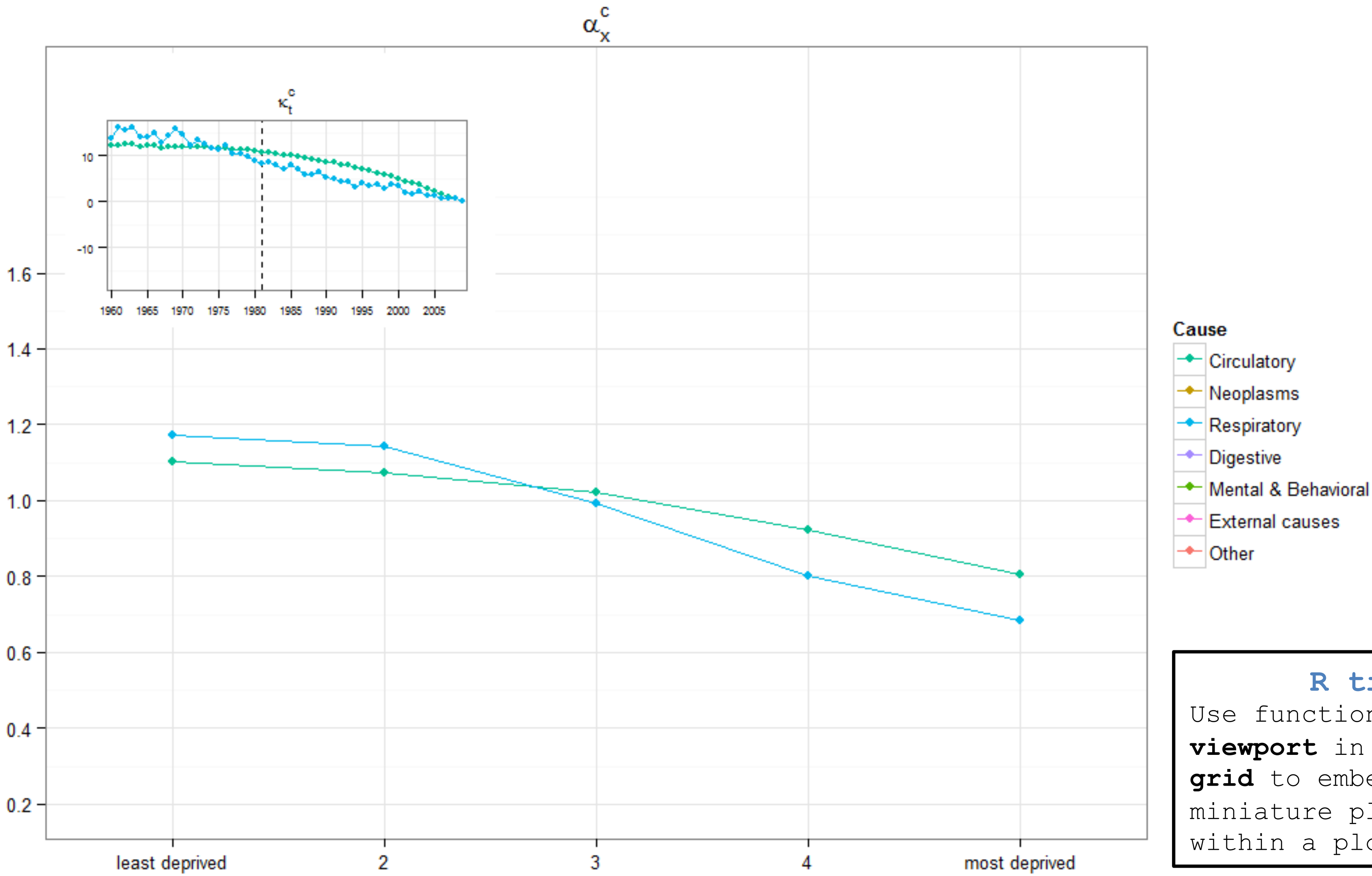
R tip

Use function **viewport** in package **grid** to embed a miniature plot within a plot

Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$



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Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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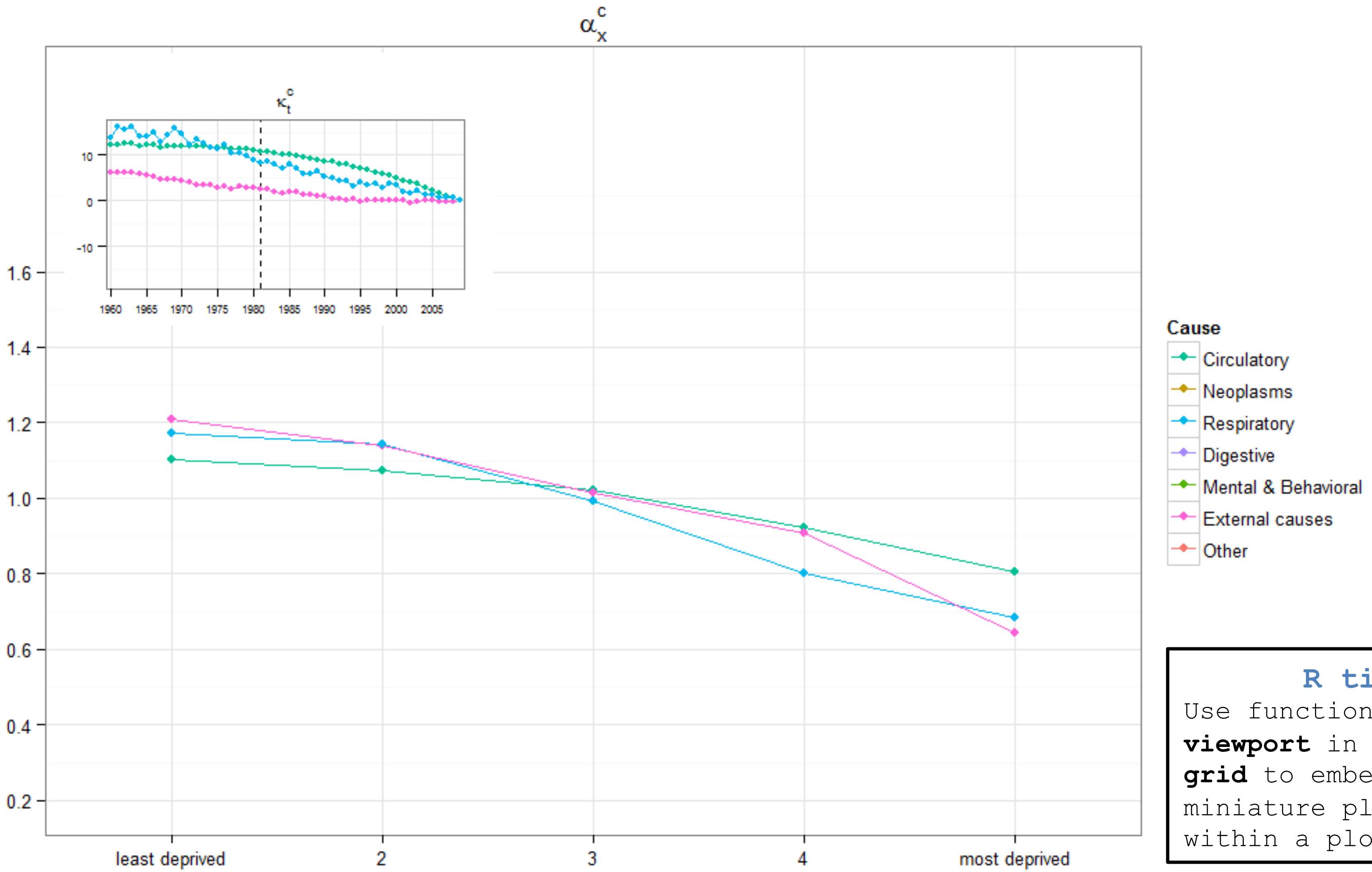

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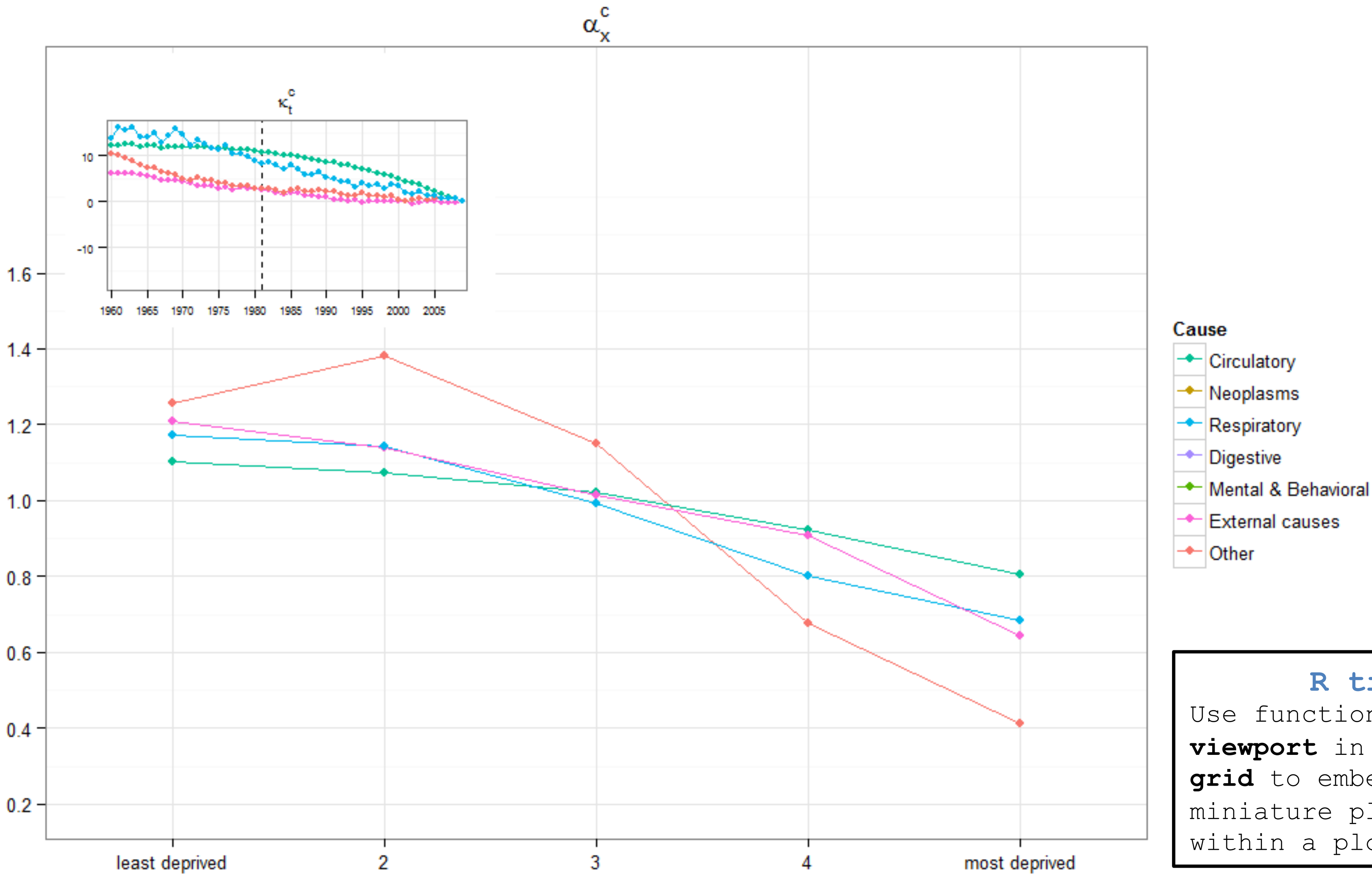
Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

$$\log \mu^c_{xtg} = \alpha^c_x + \alpha^c_{xg} + \beta^c_x \lambda^c_g \kappa^c_t + \sum_{i=1}^{h} \delta^{c,(i)}_x f^{(i)}(t)$$



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Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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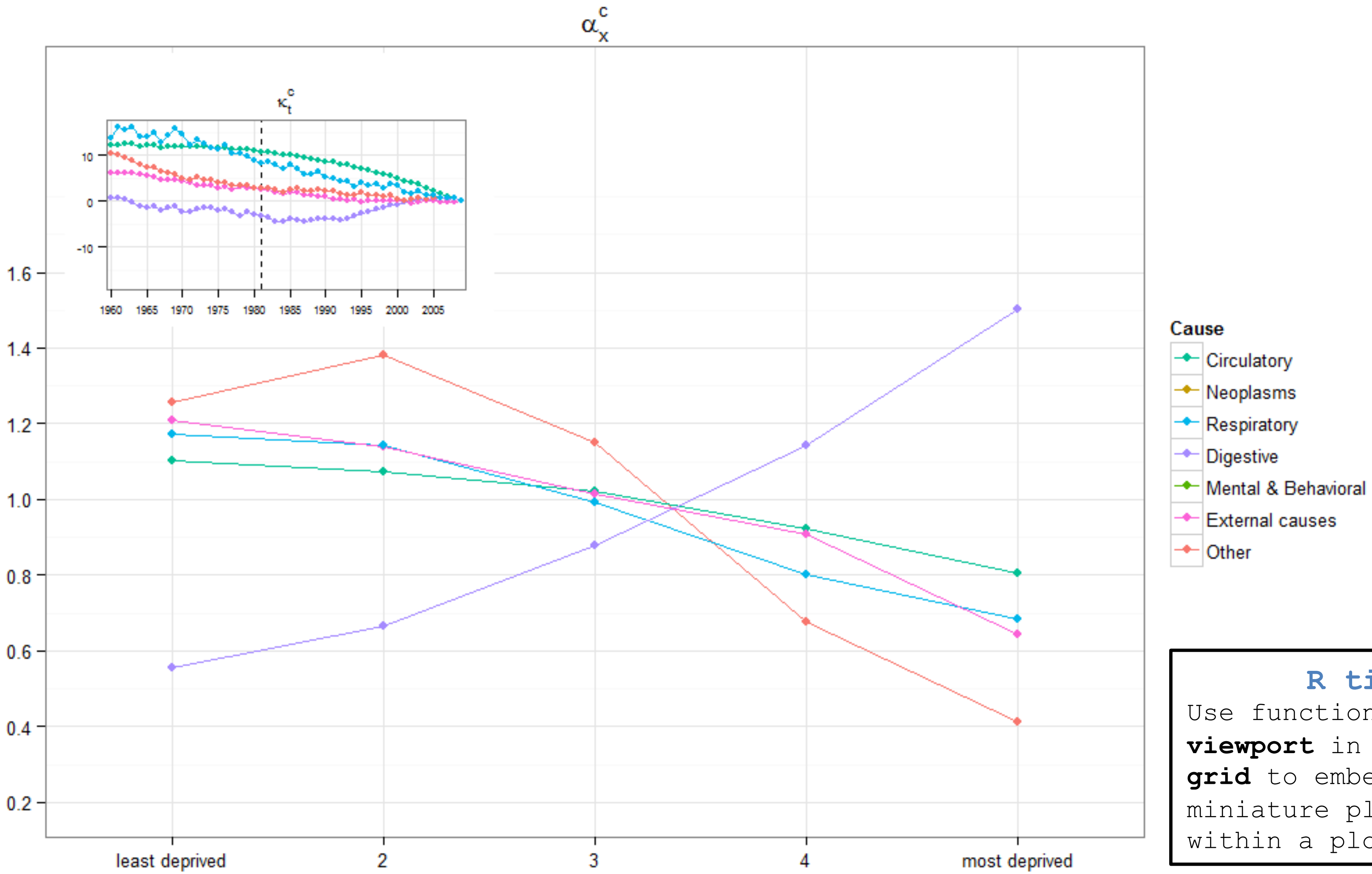

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Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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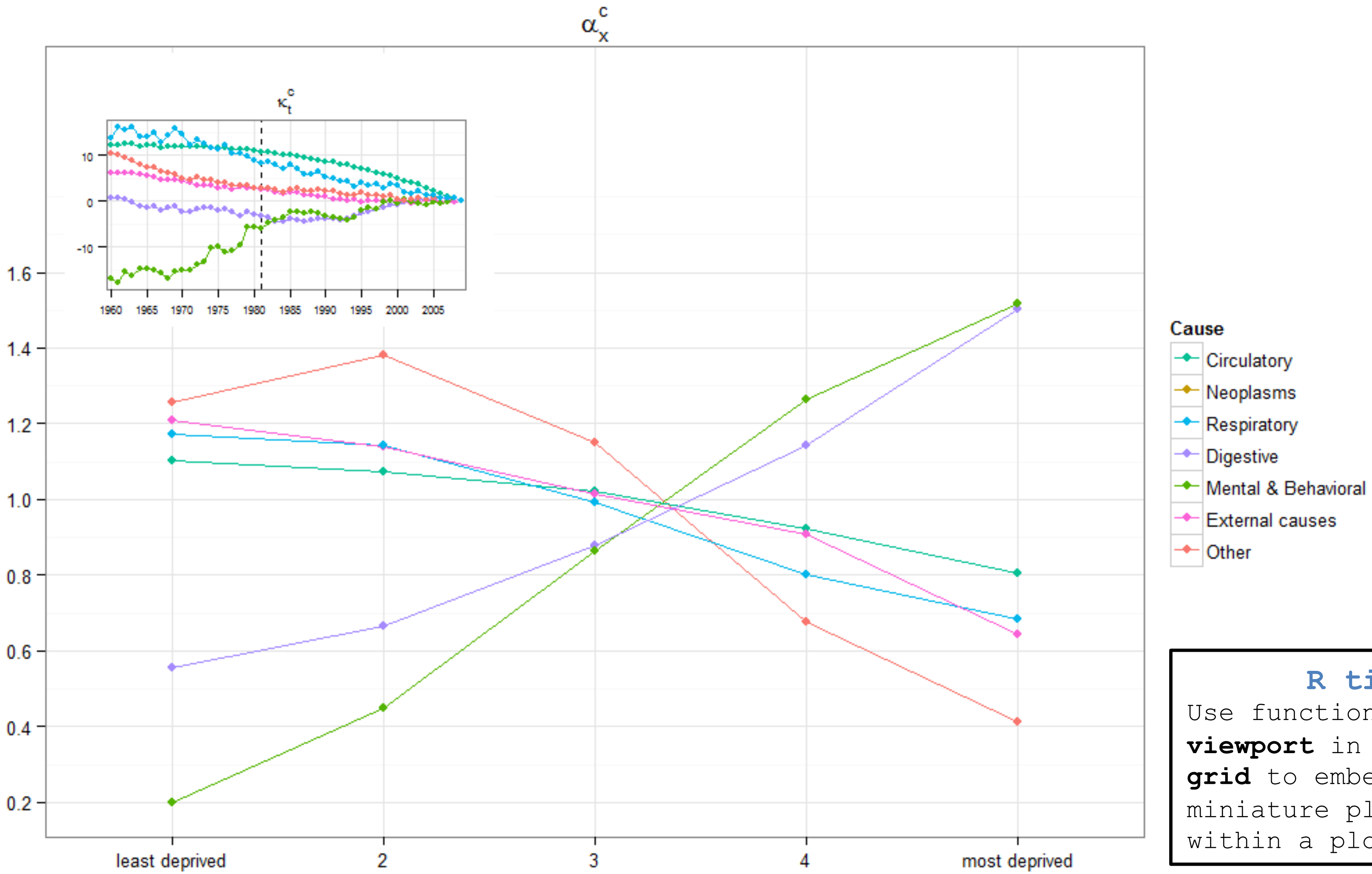

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Case study: Mortality by deprivation in England

Trend differences by deprivation quintile

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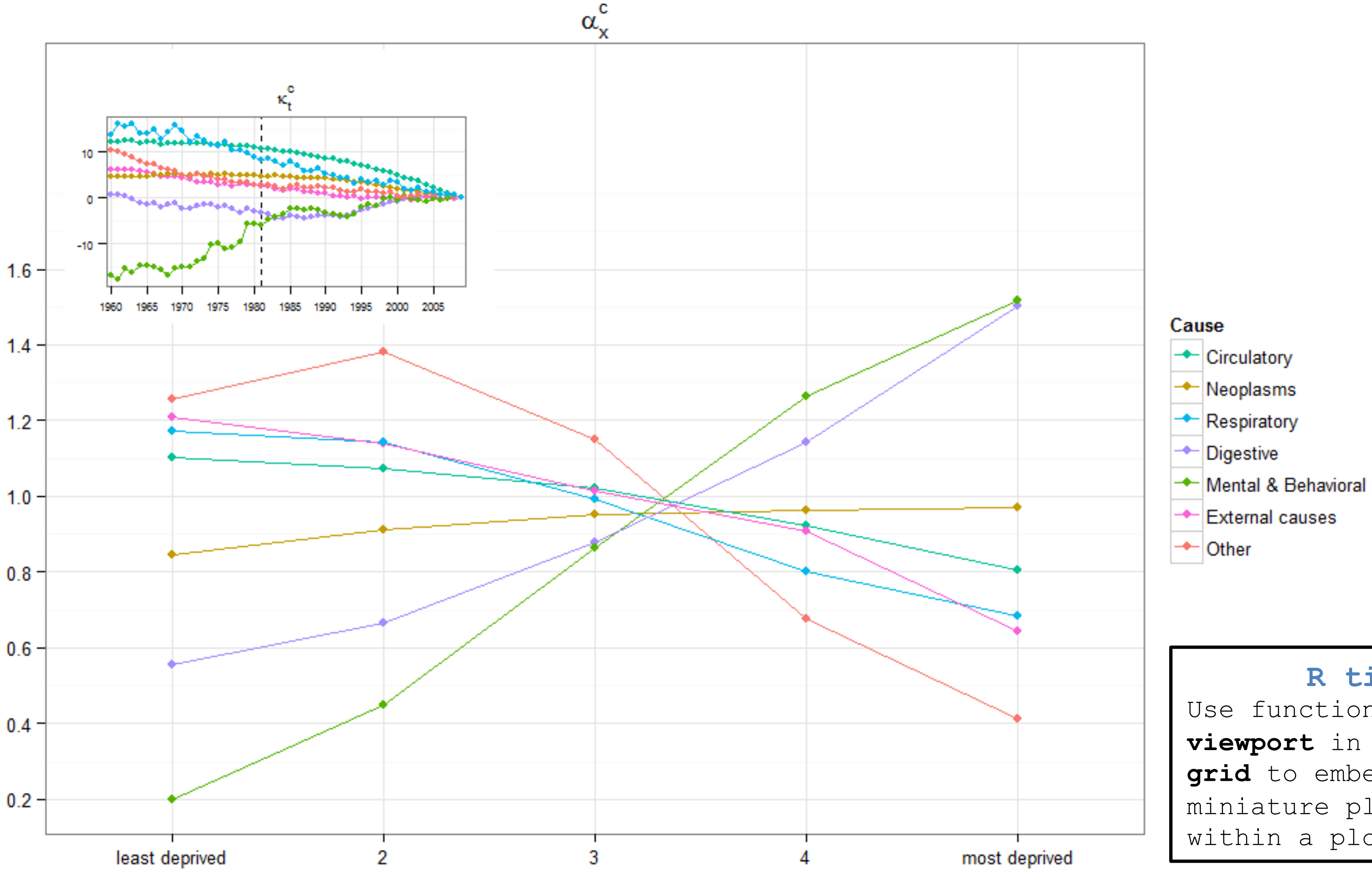

R tip

Use function **viewport** in package **grid** to embed a miniature plot within a plot

Conclusions

- **R** and in particular the package **gnm** are flexible tools for the fitting of
  - Standard mortality models including the Lee-Carter model
  - Complex extensions of the Lee-Carter model

- **R** offers compelling tools for communicating modelling results

- Application in the analysis of the extent of mortality differentials across deprivation subgroups in England for the period 1981- 2007
  - Clear inverse relationship between area deprivation and mortality for all causes
  - Reduction of differentials in cancer mortality
  - Offset of this reduction by marked differentials in digestive, respiratory and mental and behavioural diseases

Useful references

- Turner, H., Firth, D., 2012. Generalized nonlinear models in R: an overview of the **gnm** package. (http://cran.r-project.org/web/packages/gnm/index.html)
- Hyndman, R. J. (2012), **demography**: Forecasting mortality, fertility, migration and population data. with contributions from Heather Booth, Leonie Tickle and John Maindonald. (http://cran.r-project.org/web/packages/demography/index.html)
- **LifeMetrics (**http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics/software)
- Camarda, C. G. (2012), **MortalitySmooth**: An R package for smoothing Poisson counts with P-splines. Journal of Statistical Software 50(1), 1-24. (http://www.jstatsoft.org/v50/i01)
  - Package for fitting P-Splines to mortality data
- Booth, Hyndman and Tickle (2013). Prospective life tables. In: Computational Actuarial Science with R. Edited by Arthur Charpentier (http://robjhyndman.com/chapters/prospective-life-tables/)
  - Examples of the use of packages **demography**, **MortalitySmooth and LifeMetric functions**
- Charpentier , Dutang (2012) . Chapter 5 (Les tables prospectives) from L'Actuariat avec R. (http://cran.r-project.org/doc/contrib/Charpentier_Dutang_actuariat_avec_R.pdf)
  - Examples of the use of packages **demography**, **gnm and LifeMetric functions**
- Wickham, H. (2009). **ggplot2**: elegant graphics for data analysis. Springer New York. (http://cran.r-project.org/web/packages/ggplot2/index.html)
- Xie, Y. (2013). **animation**: An R Package for Creating Animations and Demonstrating Statistical Methods. Journal of Statistical Software 53(1), 1-27 (http://www.jstatsoft.org/v53/i01/paper)
- http://robjhyndman.com/hyndsight/animations/: Rob Hyndman explanation of animations in R with an example for mortality data