Chapter 4

Matriks 4x4 – Determinan Ekspansi Laplace

Objective

- Mahasiswa mampu menjelaskan Determinan & Invers matriks 4x4
- Mampu menyelesaikan Invers matriks
- Mampu menyelesaikan Sistem Persamaan Linier (SPL) menggunakan invers matriks

Definisi

- Jika A dan B matriks bujur sangkar sedemikian rupa sehingga A B = B A = I , maka B disebut balikan atau *invers* dari A dan dapat dituliskan $B = A^{-1}$ (B sama dengan *invers* A).
- Matriks B juga mempunyai *invers* yaitu A maka dapat dituliskan $A = B^{-1}$.
- Jika tidak ditemukan matriks B, maka A dikatakan **matriks tunggal** (singular).
- Jika matriks B dan C adalah *invers* dari A maka B = C.

Matriks 4x4

- Hitung Determinan :

$$A = \begin{pmatrix} 2 & 0 & -3 & 0 \\ 0 & 5 & 2 & 3 \\ -2 & 3 & -1 & 4 \\ 3 & 6 & 0 & 5 \end{pmatrix}$$

Determinan 4x4

$$A = \begin{pmatrix} 2 & 0 & -3 & 0 \\ 0 & 5 & 2 & 3 \\ -2 & 3 & -1 & 4 \\ 3 & 6 & 0 & 5 \end{pmatrix}$$

- Perhatikan baris pertama, terdapat 2 nol, kita gunakan Ekspansi Laplace baris pertama
- Det(A) $= a_{11} \times c_{11} - a_{12} \times c_{12} + a_{13} \times c_{13} - a_{14} \times c_{14}$

$$= 2 \begin{vmatrix} 5 & 2 & 3 \\ 3 & -1 & 4 \\ 6 & 0 & 5 \end{vmatrix} - 0 + (-3) \begin{vmatrix} 0 & 5 & 3 \\ -2 & 3 & 4 \\ 3 & 6 & 5 \end{vmatrix} - 0$$

$= 2\ ((-25+48+0) - ((-18) - 0 + 30) - 0\ +$

$(-3)\ ((0+60-36) - (27 - 0 - 50) - 0$

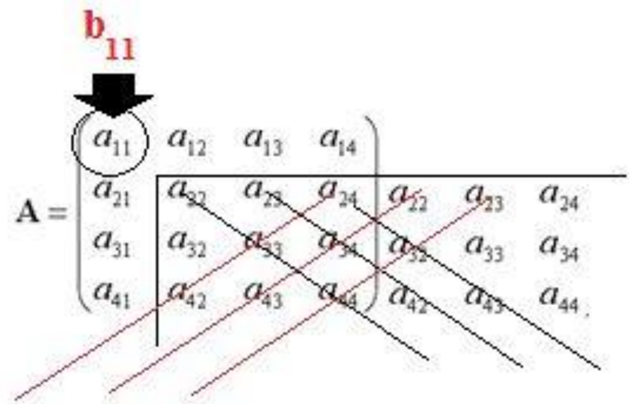
$= 2\ (23 + 18 - 30) + (-3)(24 - (-23))$

$= 2\ (11) - (-3)(47) = 22 - 141 = -119$

Invers 4x4

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$



$$b_{11} = a_{22}a_{33}a_{44} + a_{23}a_{34}a_{42} + a_{24}a_{32}a_{43} - a_{22}a_{34}a_{43} - a_{23}a_{32}a_{44} - a_{24}a_{33}a_{42}$$
$$b_{12} = a_{12}a_{34}a_{43} + a_{13}a_{32}a_{44} + a_{14}a_{33}a_{42} - a_{12}a_{33}a_{44} - a_{13}a_{34}a_{42} - a_{14}a_{32}a_{43}$$
$$b_{13} = a_{12}a_{23}a_{44} + a_{13}a_{24}a_{42} + a_{14}a_{22}a_{43} - a_{12}a_{24}a_{43} - a_{13}a_{22}a_{44} - a_{14}a_{23}a_{42}$$
$$b_{14} = a_{12}a_{24}a_{33} + a_{13}a_{22}a_{34} + a_{14}a_{23}a_{32} - a_{12}a_{23}a_{34} - a_{13}a_{24}a_{32} - a_{14}a_{22}a_{33}$$
$$b_{21} = a_{21}a_{34}a_{43} + a_{23}a_{31}a_{44} + a_{24}a_{33}a_{41} - a_{21}a_{33}a_{44} - a_{23}a_{34}a_{41} - a_{24}a_{31}a_{43}$$
$$b_{22} = a_{11}a_{33}a_{44} + a_{13}a_{34}a_{41} + a_{14}a_{31}a_{43} - a_{11}a_{34}a_{43} - a_{13}a_{31}a_{44} - a_{14}a_{33}a_{41}$$
$$b_{23} = a_{11}a_{24}a_{43} + a_{13}a_{21}a_{44} + a_{14}a_{23}a_{41} - a_{11}a_{23}a_{44} - a_{13}a_{24}a_{41} - a_{14}a_{21}a_{43}$$
$$b_{24} = a_{11}a_{23}a_{34} + a_{13}a_{24}a_{31} + a_{14}a_{21}a_{33} - a_{11}a_{24}a_{33} - a_{13}a_{21}a_{34} - a_{14}a_{23}a_{31}$$
$$b_{31} = a_{21}a_{32}a_{44} + a_{22}a_{34}a_{41} + a_{24}a_{31}a_{42} - a_{21}a_{34}a_{42} - a_{22}a_{31}a_{44} - a_{24}a_{32}a_{41}$$
$$b_{32} = a_{11}a_{34}a_{42} + a_{12}a_{31}a_{44} + a_{14}a_{32}a_{41} - a_{11}a_{32}a_{44} - a_{12}a_{34}a_{41} - a_{14}a_{31}a_{42}$$
$$b_{33} = a_{11}a_{22}a_{44} + a_{12}a_{24}a_{41} + a_{14}a_{21}a_{42} - a_{11}a_{24}a_{42} - a_{12}a_{21}a_{44} - a_{14}a_{22}a_{41}$$
$$b_{34} = a_{11}a_{24}a_{32} + a_{12}a_{21}a_{34} + a_{14}a_{22}a_{31} - a_{11}a_{22}a_{34} - a_{12}a_{24}a_{31} - a_{14}a_{21}a_{32}$$
$$b_{41} = a_{21}a_{33}a_{42} + a_{22}a_{31}a_{43} + a_{23}a_{32}a_{41} - a_{21}a_{32}a_{43} - a_{22}a_{33}a_{41} - a_{23}a_{31}a_{42}$$
$$b_{42} = a_{11}a_{32}a_{43} + a_{12}a_{33}a_{41} + a_{13}a_{31}a_{42} - a_{11}a_{33}a_{42} - a_{12}a_{31}a_{43} - a_{13}a_{32}a_{41}$$
$$b_{43} = a_{11}a_{23}a_{42} + a_{12}a_{21}a_{43} + a_{13}a_{22}a_{41} - a_{11}a_{22}a_{43} - a_{12}a_{23}a_{41} - a_{13}a_{21}a_{42}$$
$$b_{44} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

Mencari Determinan dan Invers

$$A=\begin{pmatrix} -4 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -4 & 4 & 4 & 0 \\ -1 & -9 & -1 & 1 \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix} -\frac{1}{4} & 0 & 0 & 0 \\ -\frac{1}{8} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{8} & -\frac{1}{2} & \frac{1}{4} & 0 \\ -\frac{3}{2} & 4 & \frac{1}{4} & 1 \end{pmatrix}$$

Cari invers dari matriks

$$\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ -1 & 3 & 0 \end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix} -3 & 6 & -7 \\ -1 & 2 & -1 \\ 5 & -6 & 5 \end{pmatrix}$$

Mencari w, x, y, z dengan matriks

- $2w + 2x - 5y + z = -16$
- $-w + x + 6y - z = 15$
- $2w - x + y + 6z = 3$
- $w + x + 2y - z = 7$

$(w,x,y,z) = (62/85, 207/85, 24/5, 98/17)$

Cari x,y,z dengan bentuk matriks

$$-2x + 5y - z = 8$$

$$3x - 4y + 2z = 5$$

$$x + 5y - 3z = 6$$

Chapter 6

Invers Matriks 4x4