Chapter 4

Determinan Matriks

Objective

- Mahasiswa mampu menjelaskan determinan matriks
- Mampu menyelesaikan determinan matriks menggunakan sifat-sifat matriks

DETERMINAN MATRIKS

- Setiap matriks persegi atau bujur sangkar memiliki nilai determinan
- Nilai determinan dari suatu matriks merupakan suatu skalar.
- Jika nilai determinan suatu matriks sama dengan nol, maka matriks tersebut disebut matriks singular.
- **Determinan adalah suatu fungsi tertentu yang menghubungkan suatu bilangan real dengan suatu matriks bujursangkar.**

NOTASI DETERMINAN

- Misalkan matriks A merupakan sebuah matriks bujur sangkar
- Fungsi determinan dinyatakan oleh det (A)
- Jumlah det(A) disebut determinan A
- det(A) sering dinotasikan |A|

NOTASI DETERMINAN

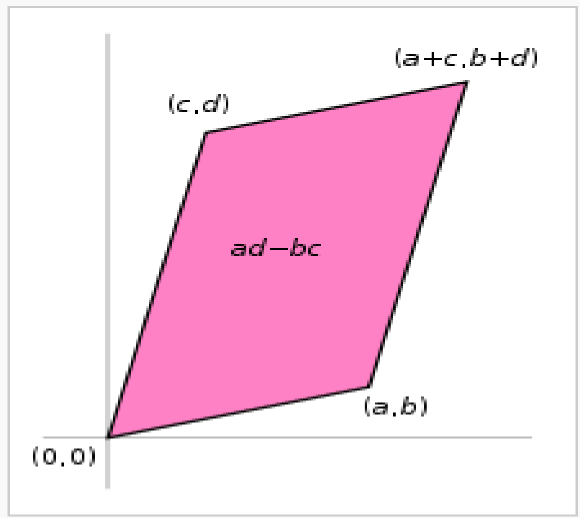
- Pada matriks 2x2 cara menghitung nilai determinannya adalah :

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \quad \det(A) = a_{11}a_{22} - a_{21}a_{12}$$

- Contoh :

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \quad \det(A) = 6 - 5 = 1$$

$$\det(A) = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$$



The area of the parallelogram is the absolute value of the determinant of the matrix formed by the vectors representing the parallelogram's sides.

METODE SARRUS

- Pada matriks 3x3 cara menghitung nilai determinannya adalah menggunakan Metode Sarrus
- Metode Sarrus hanya untuk matrix berdimensi 3x3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad \left[\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}\right] \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{array}$$

$$\det(A) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

METODE SARRUS

- Contoh :

$$A = \begin{pmatrix} -2 & 2 & -3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}$$

- Nilai Determinan dicari menggunakan metode Sarrus

det(A) = (-2·1 ·-1) + (2 ·3 ·2) + (-3 ·-1 ·0) – (-3 ·1 ·2) –(-2 ·3 ·0)-(2 ·-1 ·-1)

= 2 +12+0+6-0-2

= 18

MINOR

- Yang dimaksud dengan MINOR unsur a_{ij} adalah determinan yang berasal dari determinan orde ke-n tadi dikurangi dengan baris ke-i dan kolom ke-j.
- Dinotasikan dengan M_{ij}
- Contoh Minor dari elemen a_{11}

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \qquad M_{11} = \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}$$

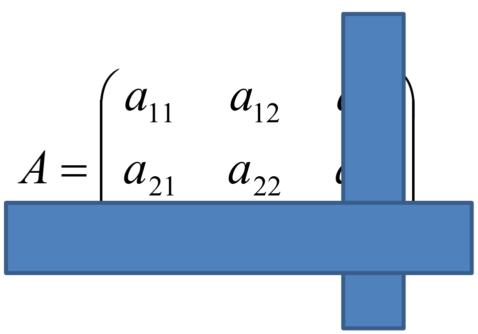
MINOR

- Minor-minor dari Matrik A (ordo 3x3)

$$|M_{11}| = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad |M_{21}| = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \qquad |M_{31}| = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|M_{12}| = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \qquad |M_{22}| = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \qquad |M_{32}| = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

$$|M_{13}| = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \qquad |M_{23}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \qquad |M_{33}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$



KOFAKTOR MATRIKS

- Kofaktor dari baris ke-i dan kolom ke-j dituliskan dengan

$$c_{ij} = (-1)^{i+j} M_{ij}$$

- Contoh :

  Kofaktor dari elemen a_{23}

$$c_{23} = (-1)^{2+3} M_{23} = -M_{23}$$

$$\begin{bmatrix} + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ + & - & + & - & + & \cdots \\ - & + & - & + & - & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

TEOREMA LAPLACE

- Determinan dari suatu matriks sama dengan jumlah perkalian elemen-elemen dari sembarang baris atau kolom dengan kofaktor-kofaktornya

Ekspansi Baris

$$|A| = \sum_{j=1}^{n} a_{ij}c_{ij} = a_{i1}c_{i1} + a_{i2}c_{i2} + \ldots\ldots\ldots\ldots\ldots + a_{in}c_{in}$$

Ekspansi Kolom

$$|A| = \sum_{j=1}^{n} a_{ij}c_{ij} = a_{1j}c_{1j} + a_{2j}c_{2j} + \ldots\ldots\ldots\ldots\ldots + a_{nj}c_{nj}$$

TEOREMA LAPLACE

Determinan dengan Ekspansi Kofaktor Pada Baris

- Misalkan ada sebuah matriks A berordo 3x3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

- Determinan Matriks A dengan metode ekspansi kofaktor baris pertama

$$\begin{aligned} |A| &= a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} \\ &= a_{11}|M_{11}| - a_{12}|M_{12}| + a_{13}|M_{13}| \\ &= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \end{aligned}$$

TEOREMA LAPLACE

- Determinan Matriks A dengan metode ekspansi kofaktor baris kedua

$$
\begin{aligned}
|A| &= a_{21}c_{21} + a_{22}c_{22} + a_{23}c_{23} \\
&= -a_{21}\left|M_{21}\right| + a_{22}\left|M_{22}\right| - a_{23}\left|M_{23}\right| \\
&= -a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{22}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{23}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}
\end{aligned}
$$

- Determinan Matriks A dengan metode ekspansi kofaktor baris ketiga

$$
\begin{aligned}
|A| &= a_{31}c_{31} + a_{32}c_{32} + a_{33}c_{33} \\
&= a_{31}\left|M_{31}\right| - a_{32}\left|M_{32}\right| + a_{33}\left|M_{33}\right| \\
&= a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + a_{33}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}
\end{aligned}
$$

TEOREMA LAPLACE

Determinan dengan Ekspansi Kofaktor Pada Kolom

- Misalkan ada sebuah matriks A berordo 3x3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

- Determinan Matriks A dengan metode ekspansi kofaktor kolom pertama

$$\begin{aligned} |A| &= a_{11}c_{11} + a_{21}c_{21} + a_{31}c_{31} \\ &= a_{11}|M_{11}| - a_{21}|M_{21}| + a_{31}|M_{31}| \\ &= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \end{aligned}$$

TEOREMA LAPLACE

- Determinan Matriks A dengan metode ekspansi kofaktor kolom kedua

$$
\begin{aligned}
|A| &= a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32} \\
&= a_{12}|M_{12}| - a_{22}|M_{22}| + a_{32}|M_{32}| \\
&= a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} - a_{22}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}
\end{aligned}
$$

- Determinan Matriks A dengan metode ekspansi kofaktor kolom ketiga

$$
\begin{aligned}
|A| &= a_{13}c_{13} + a_{23}c_{23} + a_{33}c_{33} \\
&= a_{13}|M_{13}| - a_{23}|M_{23}| + a_{33}|M_{33}| \\
&= a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - a_{23}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + a_{33}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}
\end{aligned}
$$

DET MATRIKS SEGITIGA

- Jika A adalah matriks segitiga bujur sangkar berupa segitiga atas atau segitiga bawah maka nilai det(A) adalah hasil kali diagonal matriks tersebut

$$\det(A) = a_{11} \cdot a_{22} \cdot a_{33} \cdots dst$$

- Contoh

$$\begin{bmatrix} 2 & 7 & -3 & 8 & 3 \\ 0 & -3 & 7 & 5 & 1 \\ 0 & 0 & 6 & 7 & 6 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \qquad \det(A) = 2 \cdot (-3) \cdot 6 \cdot 9 \cdot 4 = -1296$$

Latihan , dengan menggunakan minor-kofaktor tentukan Determinan :

$$A = \begin{pmatrix} -2 & 2 & -3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}$$

REFERENSI

1. Discrete Mathematics and its Applications; Kenneth H. Rosen; McGraw Hill; sixth edition; 2007
2. http://p4tkmatematika.org/
3. http://www.idomaths.com/id/matriks.php

Hitung determinan

1 (a) $A = \begin{bmatrix} 6 & 5 \\ 2 & 3 \end{bmatrix}$, (b) $B = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$, (c) $C = \begin{bmatrix} 4 & -5 \\ -1 & -2 \end{bmatrix}$, (d) $D = \begin{bmatrix} t-5 & 6 \\ 3 & t+2 \end{bmatrix}$

2 (a) $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, (b) $B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{bmatrix}$, (c) $C = \begin{bmatrix} 1 & 3 & -5 \\ 3 & -1 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

3 (a) $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}$, (b) $B = \begin{bmatrix} 4 & -6 & 8 & 9 \\ 0 & -2 & 7 & -3 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$, (c) $C = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{1}{3} \\ \frac{3}{4} & \frac{1}{2} & -1 \\ 1 & -4 & 1 \end{bmatrix}$.

Tunjukkan apakah matriks tersebut Singular atau non singular

4 (a) $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

5 (a) $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 2 \\ 0 & 1 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 4 & 1 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 & 0 \\ 3 & -2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Buktikan

6 $$\det\left(\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}\right) = (b-a)(c-a)(c-b)$$

7 $$\det\left(\begin{pmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{pmatrix}\right) = 0 \qquad x_1 \neq x_2$$

KUIS I
20 Sept

1. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 5 \\ 3 & -2 & 3 \\ 2 & 0 & 1 \end{bmatrix}.$$

Evaluate

(a) a_{11} (b) a_{13} (c) a_{31} (d) $\sum_{i=1}^{3} a_{ii}$

2. Consider the matrix

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 0 & 1 & 2 \\ 2 & -1 & 4 & 1 \\ 0 & -3 & 1 & 3 \end{bmatrix}.$$

Evaluate

(a) b_{12} (b) b_{21} (c) b_{23} (d) $\sum_{i=1}^{4} b_{ii}$

3. Consider the matrices

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & -2 \\ 4 & 2 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 6 & 8 & 5 \\ 4 & -2 & 7 \\ 3 & 1 & 2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 3 \\ 2 & -4 \\ 5 & -2 \end{bmatrix}.$$

Calculate the following when they exist.

(a) $\mathbf{A}^T$ (b) $\mathbf{C}^T$ (c) $\mathbf{A} + \mathbf{B}$

(d) $\mathbf{A} + \mathbf{C}$ (e) $(\mathbf{A} + \mathbf{B})^T$ (f) $\mathbf{A}^T + \mathbf{B}^T$

(g) $\mathbf{B} + \mathbf{B}^T$ (h) $\mathbf{C} + \mathbf{C}^T$