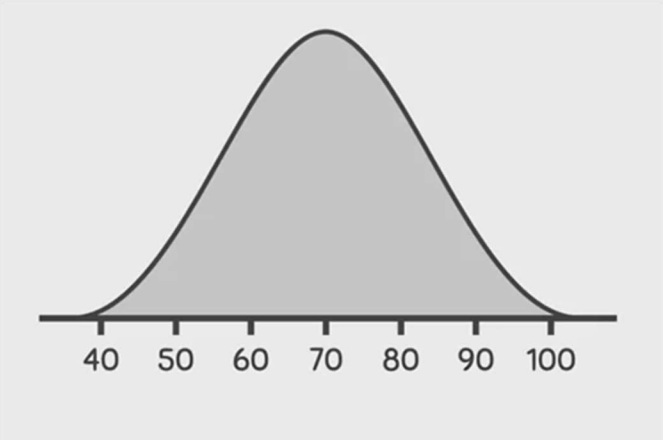
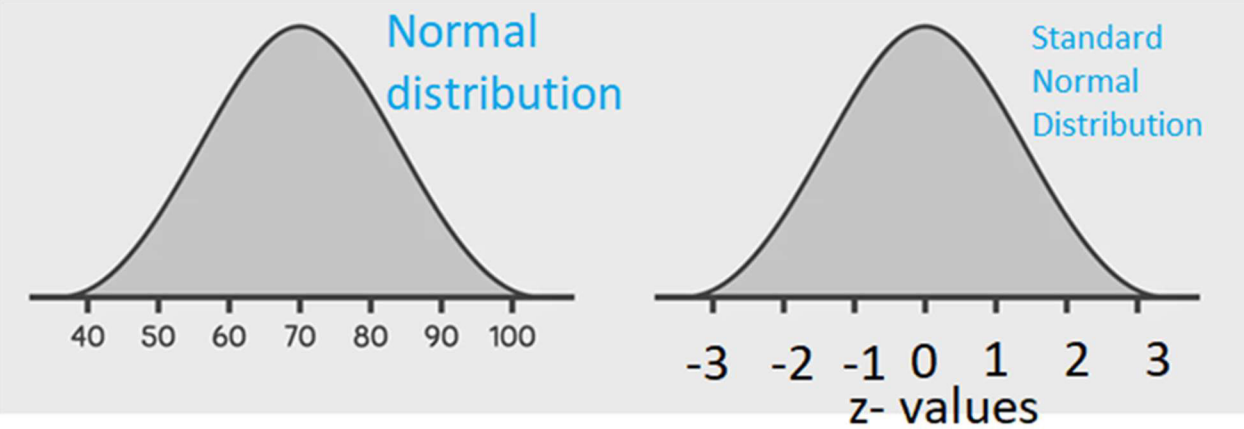
z-score

Q1. For a recent final written statistics exam for a "Data scientist" job selection process, the mean was 70 with a standard deviation of 10. If you scored 76 marks. What is your percentile or (area in the Normal distribution)?.



Normal distribution to standard Normal Distribution Conversion

$$Z = \frac{(observed\ value\ - mean)}{SD}$$



Z-score is also known as standard score gives us an idea of how far a data point is from the mean. It indicates how many standard deviations an element is from the mean. Hence, Z-Score is measured in terms of standard deviation from the mean.

- If the number of elements in the set is large, about 68% of the elements have a z-score between -1 and 1; about 95% have a z-score between -2 and 2 and about 99% have a z-score between -3 and 3.

Q1. For a recent final written statistics exam for a "Data scientist" job selection process, the mean was 70 with a standard deviation of 10. If you scored 76 marks. What is your percentile or (area in the Normal distribution)?.

Ans:-
Here, mean=70
SD=10.

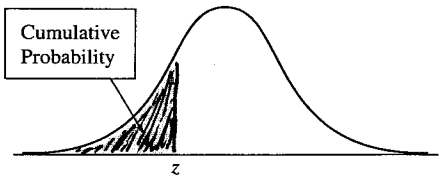
$$Z = \frac{(observed\ value\ - mean)}{SD}$$

Mean value 70 Z score = (70-70)/10=0
80 marks Z score = (80-70)/10=1
60 marks Z score = (60-70)/10=-1
76 marks Z score =(76-70)/10=0.60

In the z table the value of 0.60 is 0.7257.

This is the value of area under curve or the percentile.

APPENDIX A

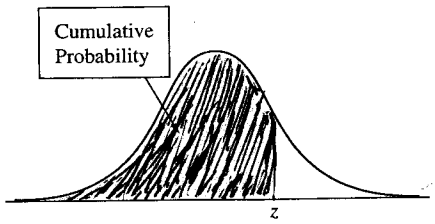


Cumulative probability for z is the area under the standard normal curve to the left of z

z	.00
−5.0	.000000287
−4.5	.00000340
−4.0	.0000317
−3.5	.000233

TABLE A Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative probability for z is the area under the standard normal curve to the left of z

TABLE A Standard Normal Cumulative Probabilities *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

z	.00
3.5	.999767
4.0	.9999683
4.5	.9999966
5.0	.999999713

z-score

A normal curve is determined by mean and SD . If the data follow the normal curve , than knowing mean and SD means knowing the whole histogram.

To compute area under the normal curve , we first standardize the data using Z score formula.

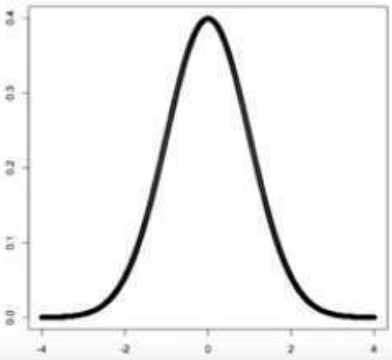
$$Z = \frac{(observed\ value\ - mean)}{SD}$$

Here Z is called the standardized value or Z scaore.

Z has no unit (the observed value, mean,SD have the unit for example meter , inches etc.)

When we standarized the data we have mean 0 and standard deviation equal to 1 →this is the point of stansardization.

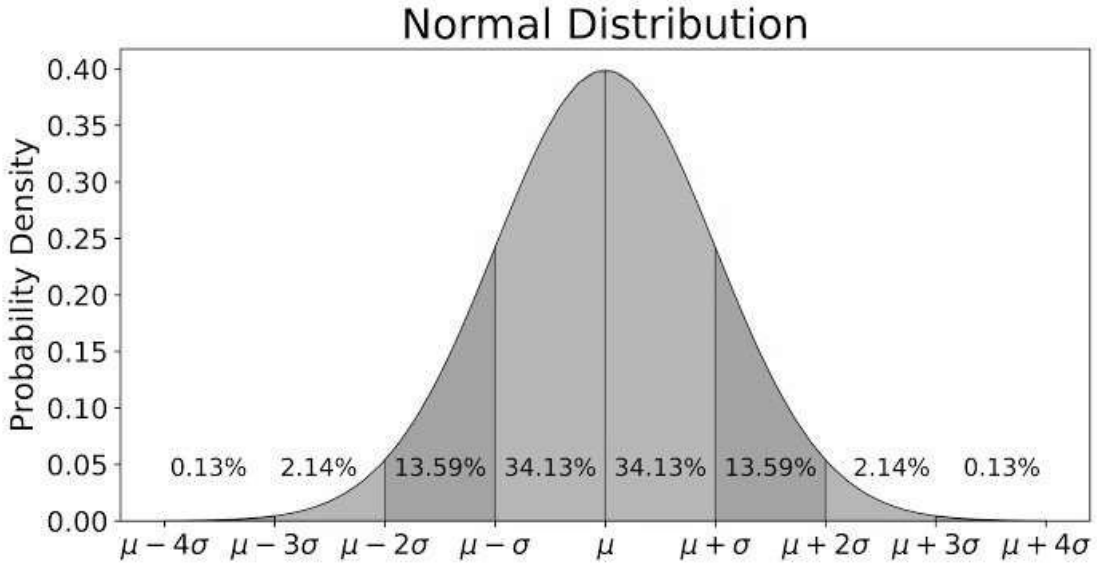
We can convert any normal distribution into the standard normal distribution



This curve is given by the function

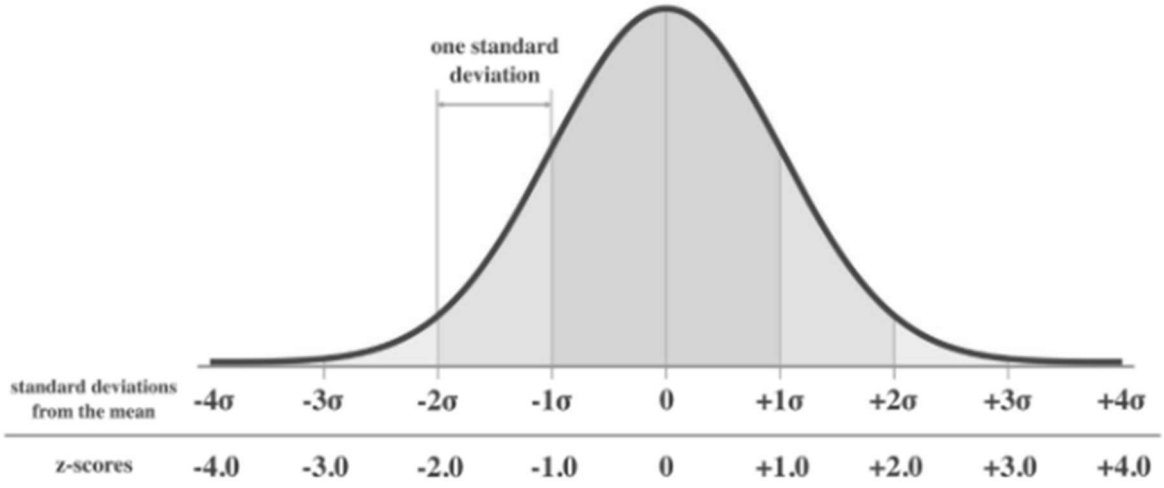
$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

No need to Remember this formula it is just for an understanding.



Standard Normal Distribution

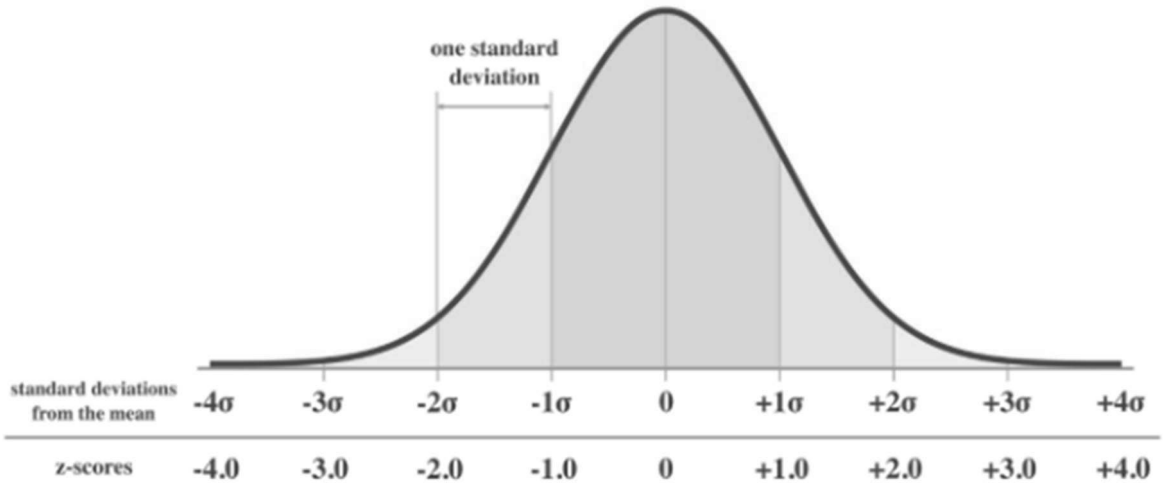
The (z-value/ z-score / z / standard score) represents the number of standard deviations an observation is from the mean for a set of data.



Z-scores range from -3 standard deviations (which would fall to the far left of the normal distribution curve) up to +3 standard deviations (which would fall to the far right of the normal distribution curve). In order to use a z-score, you need to know the mean μ and also the standard deviation σ.

- **Z-scores can be positive or negative.**

- **The sign tells you whether the observation is above or below the mean.**
- **z-score of +2 indicates that the data point two standard deviations above the mean,**
- **z-score of -2 signifies it is two standard deviations below the mean.**



$$Z = \frac{(observed\ value\ - mean)}{SD}$$

we can use the z-value.

```
import scipy.stats as stats

stats.zscore(data=df, axis=1)

or
df.apply(stats.zscore)
```

Normal approximation:-

Finding areas under the normal curve is called normal approximation.

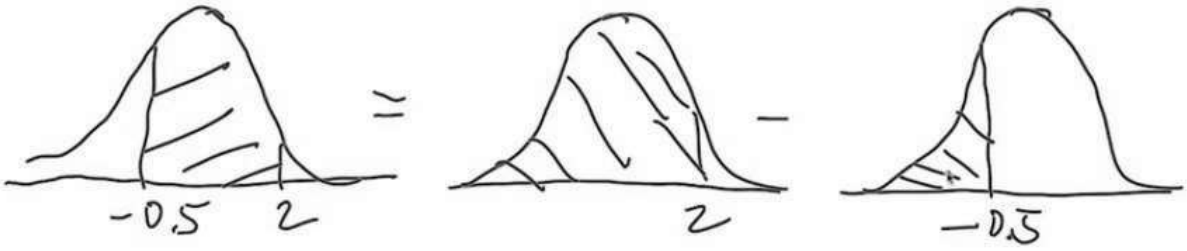
Q. fathers' heights follow the normal curve with a mean of 68.3 inches and a standard deviation of 1.8 inches.
What percentage of fathers have heights between 67.4 inch and 71.9 inch?

1. Standardize:

$$\frac{67.4 \text{ in} - 68.3 \text{ in}}{1.8 \text{ in}} = -0.5 \qquad \frac{71.9 \text{ in} - 68.3 \text{ in}}{1.8 \text{ in}} = 2$$

desired area in a form that can be computed by looked up in a table:

Typically we can look up the area to the left of a given value.



Use a table to find these values: $97.7\% - 30.9\% = 66.8\%$

Q. What is the 30 percentile of the Father's height?

Ans:-

Q. One student score 80 marks in Mathematics and 75 Marks in English.
At this point we can say he has performed excellent in Math as compare to English.
Consider the average class score of Mathematics is 90, SD is 5.
Average class score of English is 60 , SD is 5.
Verify the performance?

Ans:- lets calculate the Z values
Zm=(80-90)/5= -2
Ze=(75-60)/5=3

Z score value is -3 to +3
if it is close to -3 it means lower performance
if z score close to +3 means excellent performance.

By the use of Z-table we can find the Area under the curve / percentile value.

Q3. If you know that -0.1 corresponds to approximately 46 % and 1.8 corresponds to approximately 96.4 % (both percentages are areas under the curve to the left of the value), what percentage of fathers will have heights between 68.1 inc and 71.5 in?

Ans:-

Q5. What proportion of students are between 5.81 feet & 6.3 feet height. Given Mean=5.5, sd=0.5 feet.

Q6. mean height of Gurkhas is 146 cm with Sd of 3 cm . what is the probability of

(a)Height having greater than 152 cm.

(b)Height between 140 and 150 cm.

Q7. Mean demand of an oil is 1000 ltr per month with SD Of 250 ltr.

(a)if 1200 ltrs are stocked , what is the satisfaction level?

(b)For an assurance of 95%. what stock must be kept?