Mithilesh singh

MATRICES

Matrices are a foundational element of linear algebra.

Linear Algebra is an essential field of mathematics, which defines the study of vectors, matrices, planes, mapping, and lines required for linear transformation.

Matrices are used throughout the field of machine learning such as processes the input data variable (X).

Applying the algorithm, training a Model.

Simple methods of solving system of linear equations.

Matrix simple expression

example 1 :-

Radha has 15 notebooks.

We can express it as [15]

with the understanding that the number inside [ ] is the number of notebooks that Radha has.

Example 2

Radha has 15 notebooks and 6 pens.

We can express it as [15 6]

with the understanding that first number inside [ ] is the number of notebooks while the other is the number of pens.

Example 3

Let us now suppose that we wish to express the information of possession of notebooks and pens by Radha and her two friends Fauzia and Simran which is as follows:

Radha has 15 notebooks and 6 pens,

Fauzia has 10 notebooks and 2 pens,

Simran has 13 notebooks and 5 pens.

Now this could be arranged in the tabular form as follows:

Radha has 15 notebooks and 6 pens,
Fauzia has 10 notebooks and 2 pens,
Simran has 13 notebooks and 5 pens.

Now this could be arranged in the tabular form as follows:

	Notebooks	Pens
Radha	15	6
Fauzia	10	2
Simran	13	5

and this can be expressed as

$$\begin{array}{cl} \begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix} & \begin{array}{l} \leftarrow \text{First row} \\ \leftarrow \text{Second row} \\ \leftarrow \text{Third row} \end{array} \\ \begin{array}{cc} \uparrow & \uparrow \\ \text{First Column} & \text{Second Column} \end{array} & \end{array}$$

An arrangement as above kind is called a matrix.

It is a 3x2 matrix it has 3 rows and 2 columns.

or

	Radha	Fauzia	Simran
Notebooks	15	10	13
Pens	6	2	5

which can be expressed as:

$$\begin{array}{cl} \begin{bmatrix} 15 & 10 & 13 \\ & & \\ 6 & 2 & 5 \end{bmatrix} & \begin{array}{l} \leftarrow \text{First row} \\ \\ \leftarrow \text{Second row} \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{First} & \text{Second} & \text{Third} \\ \text{Column} & \text{Column} & \text{Column} \end{array} & \end{array}$$

It is a 2x3 matrix it has 2 rows and 3 columns.

In general, an **m × n matrix** **has the following rectangular array:**

No. of columns=n

$$\begin{array}{cl} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}_{m \times n} & \begin{array}{l} \text{No. of} \\ \text{rows} \\ \text{m} \end{array} \end{array}$$

Properties:

- A specified number of rows and a specified number of columns
- Two numbers (rows x columns) describe the dimensions or size of the matrix.

TYPES OF MATRICES

1. **Column matrix**: Any no. of rows but the number of columns is only 1.-->(1xn) matrix

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

2. **Row matrix** Any number of columns but only one row. →(nx1)matrix

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

3. **Rectangular matrix** number of rows is not equal to the number of columns.-->(m x n) matrix)

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$

4. Square matrix The number of rows is equal to the number of columns .-->(m x m) matrix

m x m

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

5. Diagonal matrix A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

6. Unit or Identity matrix - I A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

7. Null (zero) matrix - 0 All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. Triangular matrix A square matrix whose elements above or below the main diagonal are all zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

8a. Upper triangular matrix

$$\begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 7 & 4 & 4 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

8b. Lower triangular matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

Splitting the square Matrix into two triangles is one example of **matrix decomposition** is also called matrix factorization.

Examples of imp Matrix Decomposition in ML

LU Decomposition- Lower Upper decomposition.

Factorization of a given square matrix into two triangular matrices, one upper triangular matrix and one lower triangular matrix, such that the product of these two matrices gives the original matrix.

Eigen Decomposition that decomposes a square matrix into eigen vectors and eigenvalues. This decomposition also plays a role in methods used in ML, such as in the Principal Component Analysis method or PCA A vector is an eigen vector of a matrix if it satisfies the following equation.

$A.v= \lambda v$

where λ is the eigen value and v is the eigen vector

Singular Value Decomposition (SVD)

A popular application of SVD is for dimensionality reduction. Data with a large number of features, such as more features (columns) than observations (rows) may be reduced to a smaller subset of features that are most relevant to the prediction problem.

Addition of matrices

Suppose You have two factories at places A and B. Each factory produces sport shoes for boys and girls in three different price categories labelled 1, 2 and 3. The quantities produced by each factory are represented as matrices given below: -

$$
\begin{array}{cc}
 & \text{Factory at A} \\
 & \begin{array}{cc} \text{Boys} & \text{Girls} \end{array} \\
\begin{array}{c} \text{In ca1} \\ \text{In ca2} \\ \text{In ca3} \end{array} & \begin{bmatrix} 80 & 60 \\ 75 & 65 \\ 90 & 85 \end{bmatrix}
\end{array}
\qquad
\begin{array}{cc}
 & \text{Factory at B} \\
 & \begin{array}{cc} \text{Boys} & \text{Girls} \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 90 & 50 \\ 70 & 55 \\ 75 & 75 \end{bmatrix}
\end{array}
$$

Total production of sport shoes in each category.

category 1 : for boys (80 + 90), for girls (60 + 50)
In category 2 : for boys (75 + 70), for girls (65 + 55)
In category 3 : for boys (90 + 75), for girls (85 + 75)

Addition of Matrix A & B

This can be represented in the matrix form as

$$
\begin{bmatrix} 80+90 & 60+50 \\ 75+70 & 65+55 \\ 90+75 & 85+75 \end{bmatrix}.
$$

Matrix Addiction follows the

Commutative Law: A + B = B + A

Associative Law: A + (B + C) = (A + B) + C = A + B + C

Multiplication of a matrix by a scalar

Now suppose that You have doubled the production at a factory A in all categories

$$2\text{x}\ \begin{array}{c c} & \begin{array}{cc} \text{Boys} & \text{Girls} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 80 & 60 \\ 75 & 65 \\ 90 & 85 \end{bmatrix} \end{array}$$

Revised quantities by factory A

$$\begin{array}{c c} & \begin{array}{cc} \text{Boys} & \text{Girls} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 2\times 80 & 2\times 60 \\ 2\times 75 & 2\times 65 \\ 2\times 90 & 2\times 85 \end{bmatrix} \end{array}$$

This can be represented as

$$\begin{bmatrix} 160 & 120 \\ 150 & 130 \\ 180 & 170 \end{bmatrix}$$

Scaling of Matrix

$$A = \begin{bmatrix} 3 & 1 & 1.5 \\ \sqrt{5} & 7 & -3 \\ 2 & 0 & 5 \end{bmatrix}, \text{then}$$

$$3A = 3\begin{bmatrix} 3 & 1 & 1.5 \\ \sqrt{5} & 7 & -3 \\ 2 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 4.5 \\ 3\sqrt{5} & 21 & -9 \\ 6 & 0 & 15 \end{bmatrix}$$

Please note: - Difference of matrices work similar as Addition of matrices.

Assignment1:-

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, then find $2A - B$.

Assignment 2:-

If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$. then find the matrix X, such that

$$2A + 3X = 5B$$

Assignment 3:-

Find X and Y, if $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

Assignment 4:-

Find the values of x and y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Assignment 5:-

Two farmers Ramkishan and Gurcharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The sale (in Rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

- **(i)** **Find the combined sales in September and October for each farmer in each variety.**
- **(ii)** **Find the decrease in sales from September to October.**
- **(iii)** **If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.**

Multiplication of matrices

Suppose Ajay and Nadeem are two friends. Ajay wants to buy 2 pens and 5 story books, while Nadeem needs 8 pens and 10 story books.

They both go to a shop to enquire about the rates which are quoted as follows: Pen – Rs 5 each, story book – Rs 50 each.

How much money does each need to spend?

Clearly, Ajay needs Rs (5 × 2 + 50 × 5) that is Rs 260, while Nadeem needs (8 × 5 + 50 × 10) Rs, that is Rs 540. In terms of matrix representation,

we can write the above information as follows:-

Requirements	Prices per piece (in Rupees)	Money needed (in Rupees)
$\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}$	$\begin{bmatrix} 5 \\ 50 \end{bmatrix}$	$\begin{bmatrix} 5\times 2+5\times 50 \\ 8\times 5+10\times 50 \end{bmatrix} = \begin{bmatrix} 260 \\ 540 \end{bmatrix}$

Suppose that they enquire about the rates from another shop, quoted as follows: pen – Rs 4 each, story book – Rs 40 each. Now, the money required by Ajay and Nadeem to make purchases will be respectively Rs (4 × 2 + 40 × 5) = Rs 208 and Rs (8 × 4 + 10 × 40) = Rs 432

Again, the above information can be represented as follows:

Requirements **Prices per piece (in Rupees)** **Money needed (in Rupees)**

$$\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 40 \end{bmatrix} \qquad \begin{bmatrix} 4\times 2+40\times 5 \\ 8\times 4+10\times 40 \end{bmatrix} = \begin{bmatrix} 208 \\ 432 \end{bmatrix}$$

Now, the information in both the cases can be combined and expressed in terms of matrices as follows:

Requirements **Prices per piece (in Rs)** **Money needed (in Rupees)**

$$\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix} \qquad \begin{bmatrix} 5 & 4 \\ 50 & 40 \end{bmatrix} \qquad \begin{bmatrix} 5\times 2+5\times 50 & 4\times 2+40\times 5 \\ 8\times 5+10\times 50 & 8\times 4+10\times 40 \end{bmatrix} = \begin{bmatrix} 260 & 208 \\ 540 & 432 \end{bmatrix}$$

Understanding the Matrix Multiplication: -

For example, if $C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix}$, then the product CD is defined

Entry in first row first column $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} (1)(2)+(-1)(-1)+(2)(5) & ? \\ ? & ? \end{bmatrix}$

Entry in first row second column $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & (1)(7)+(-1)(1)+2(-4) \\ ? & ? \end{bmatrix}$

Entry in second row first column $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & -2 \\ 0(2)+3(-1)+4(5) & ? \end{bmatrix}$

Entry in second row second column $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & -2 \\ 17 & 0(7)+3(1)+4(-4) \end{bmatrix}$

Thus $CD = \begin{bmatrix} 13 & -2 \\ 17 & -13 \end{bmatrix}$

Find AB, if $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}$.

Solution The matrix A has 2 columns which is equal to the number of rows of B. Hence AB is defined. Now

$$AB = \begin{bmatrix} 6(2)+9(7) & 6(6)+9(9) & 6(0)+9(8) \\ 2(2)+3(7) & 2(6)+3(9) & 2(0)+3(8) \end{bmatrix}$$

$$= \begin{bmatrix} 12+63 & 36+81 & 0+72 \\ 4+21 & 12+27 & 0+24 \end{bmatrix} = \begin{bmatrix} 75 & 117 & 72 \\ 25 & 39 & 24 \end{bmatrix}$$

Example:-

Non-commutativity of multiplication of matrices

Now, we shall see by an example that even if AB and BA are both defined, it is not necessary that AB = BA.

If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB, BA. Show that $AB \neq BA$.

Solution Since A is a 2×3 matrix and B is 3×2 matrix. Hence AB and BA are both defined and are matrices of order 2×2 and 3×3, respectively. Note that

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2-8+6 & 3-10+3 \\ -8+8+10 & -12+10+5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

and
$$BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$$

Clearly $AB \neq BA$

Example:-

In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three wavs: telephone, house calls, and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{array}{c} \textit{Cost per contact} \\ \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{array}$$

The number of contacts of each type made in two cities X and Y is given by

$$B = \begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{Housecall} & \text{Letter} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10,000 \end{bmatrix} \end{array} \begin{array}{l} \\ \to X \\ \to Y \end{array}$$

. Find the total amount spent by the group in the two cities X and Y.

Solution We have

$$BA = \begin{bmatrix} 40,000 + 50,000 + 250,000 \\ 120,000 + 100,000 + 500,000 \end{bmatrix} \begin{array}{l} \to X \\ \to Y \end{array}$$

$$= \begin{bmatrix} 340,000 \\ 720,000 \end{bmatrix} \begin{array}{l} \to X \\ \to Y \end{array}$$

So the total amount spent by the group in the two cities is 340,000 paise and 720,000 paise, i.e., Rs 3400 and Rs 7200, respectively.

Transpose of a Matrix

(Interchanging the Rows and columns)

$$\text{if } A = \begin{bmatrix} 3 & 5 \\ \sqrt{3} & 1 \\ 0 & \frac{-1}{5} \end{bmatrix}_{3\times 2}$$

Transpose of the matrix A

$$A' \text{ or } (A^T) = \begin{bmatrix} 3 & \sqrt{3} & 0 \\ 5 & 1 & \frac{-1}{5} \end{bmatrix}_{2\times 3}$$

Invertible Matrices or Inverse of Matrix

Invertible Matrices

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the *inverse* matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.

For example, let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.

Now

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Also $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$. Thus B is the inverse of A, in other words $B = A^{-1}$ and A is inverse of B, i.e., $A = B^{-1}$

Determinants of Matrix

A system of algebraic equations can be expressed in the form of matrices.

This means, a system of linear equations like

a1 x + b1 y = c 1

a2 x + b2 y = c 2

Can be represented as :-

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Now, this system of equations has a unique solution or not, is determined by the number a1 b2 – a2 b1 . (if a1 b2 – a2 b1 ≠ 0), then the system of linear equations has a unique solution). The number a1 b2 – a2 b1 which determines uniqueness of solution is called the Determinant of this Matrix.

determines of matrix $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$
or det A.

Determinant of a matrix of order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2 × 2,

then the determinant of A is defined as:

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Example 1 Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$.

Solution We have $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2(2) - 4(-1) = 4 + 4 = 8.$

Determinant of A Matrix of order 3

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22})$$

$$|A| = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{21}a_{12}a_{33} + a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} - a_{31}a_{13}a_{22}$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22}$$

LINEAR ALGEBRA

Algebra→ Branch of Mathematics.

Linear algebra → branch of algebra

Algebra as "generalized arithmetic."

Linear Algebra is the study of vector spaces and linear mappings between those spaces.

It is the study of linear sets of equations with transformation properties.

Two or more variables, then it becomes a linear equation in two variables or three variables and so

No variable in a linear equation is raised to a power greater than 1

Linear equation in two variables	Not a linear equation in two variables.
$2x + y = 4$	$xy + 2x = 5$ (Why?)
$-5x + \frac{1}{2} = y$	$\sqrt{x} + \sqrt{y} = 25$ (Why?)
$5x = 35y$	$x(x+1) = y$ (Why?)

Y=mx+c -→ Linear Equation.

If an equation has two variables each of which is in first degree such that the variables are not multiplied with each other, then it is a linear equation in two variables

Use of Vector in Machine Learning?

Linear Regression → supervised Machine Learning

→ finds the best fit linear line between the independent and dependent variable

→ finds the linear relationship between the dependent and independent variable.

→ linear**, that one variable increases or decreases a fixed amount for a unit increase or decrease in the other.**

Example

simple Linear equations , one independent variable and one dependent variable .

X	Y
2	4.21
3	620
4	8.21
5	10.21
6	12.20
7	14.21
8	16.22
9	18.20

f(x)=y = 2x + 0.21

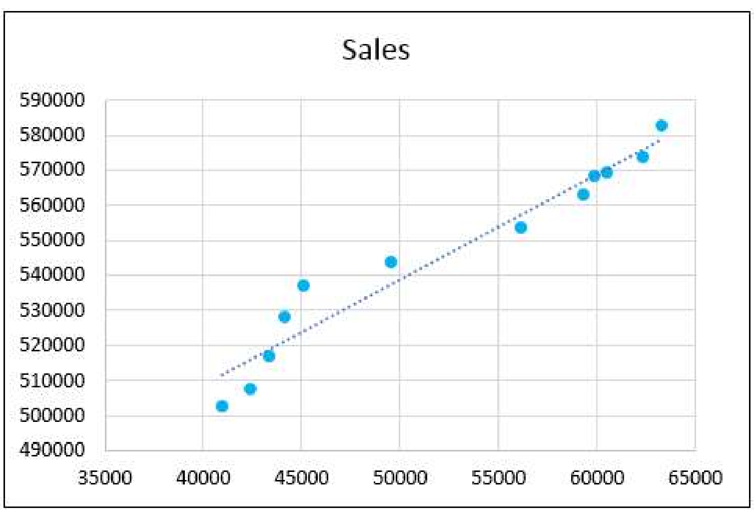
y=mx+c

here m=coefficient or slope =2

and c intercept with y axis.=0.21

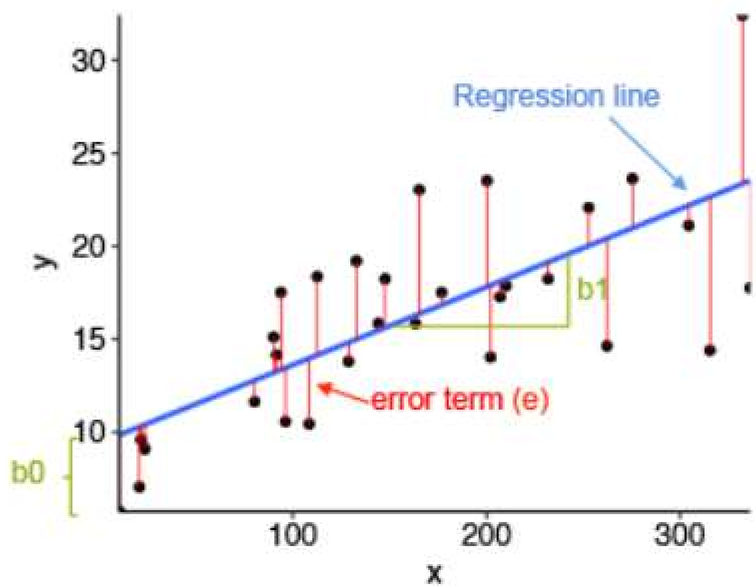
Linear regression : -

	A	B	C
1	Month	Advertising	Sales
2	Jan	40937	502729
3	Feb	42376	507553
4	Mar	43355	516885
5	Apr	44126	528347
6	May	45060	537298
7	Jun	49546	544066
8	Jul	56105	553664
9	Aug	59322	563201
10	Sep	59877	568657
11	Oct	60481	569384
12	Nov	62356	573764
13	Dec	63246	582746



Simple Linear Regression.(one variable)

Geometric Intuition: - best fit line



Y=mX+c
m=slope of x variable
c=intercept value

- **main aim to find the best fit linear line.**
- **optimal values of slop and intercept to minimize the error.**
- **error or residual =vertical distance between the data point and the regression line.**
- **Residual/Error=actual value-predicted value**

Sum of residual/error=sum(actual values-predicted values)

$$\sum e_i = \sum (Y_i - \hat{Y}_i)$$

Square of sum of residual/error=Square of (sum(actual values-predicted values))

$$\sum {e_i}^2 = \sum (Y_i - \hat{Y}_i)^2$$

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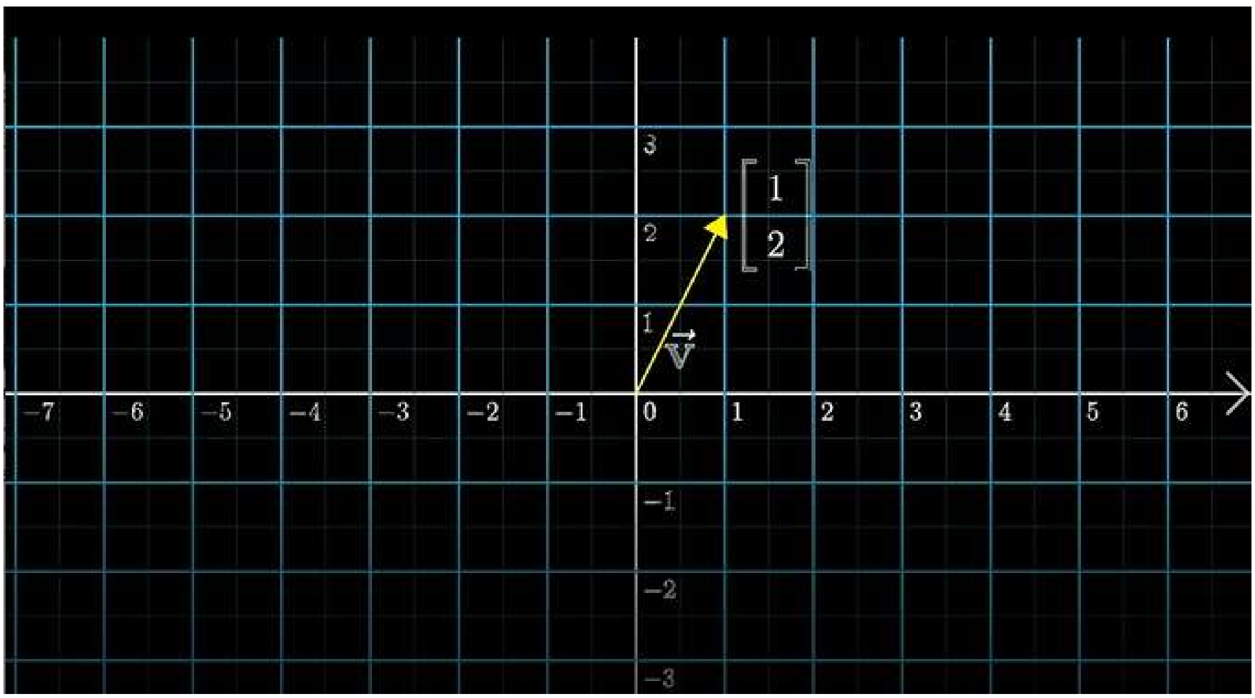
VECTOR

THE FUNDAMENTAL OF ANY LINEAR ALGEBRA IS VECTOR

What is the vector?



v is a vector having co-ordinate (1,2)

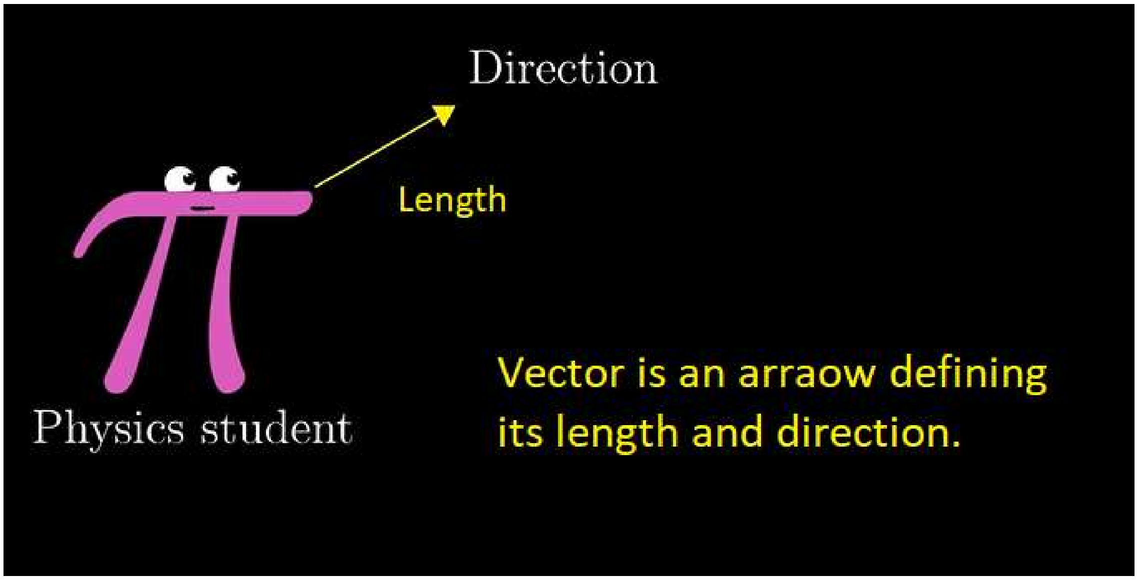
There are Three ideas of a Vector

(1) Physics prospective

(2) Computer Science Prospective

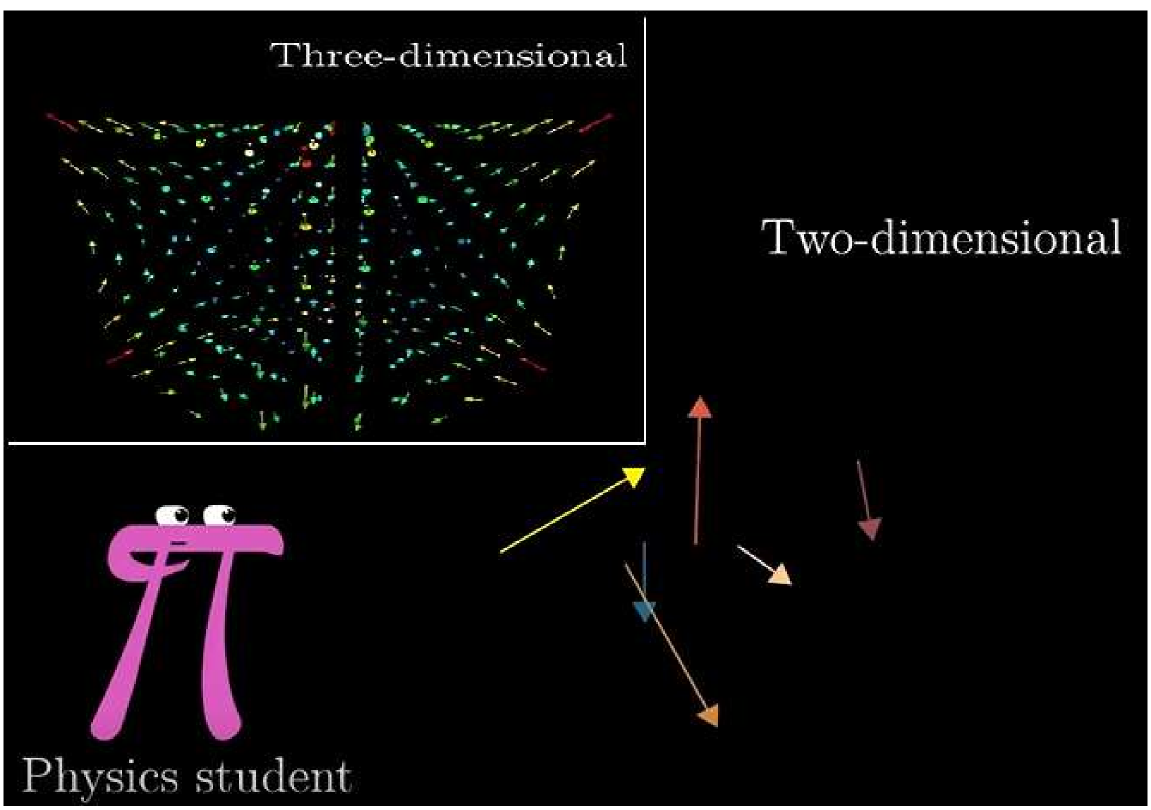
(3) Mathematician Prospective

PHYSICS STUDENTS



You can move this vector all around and it still the same vector.

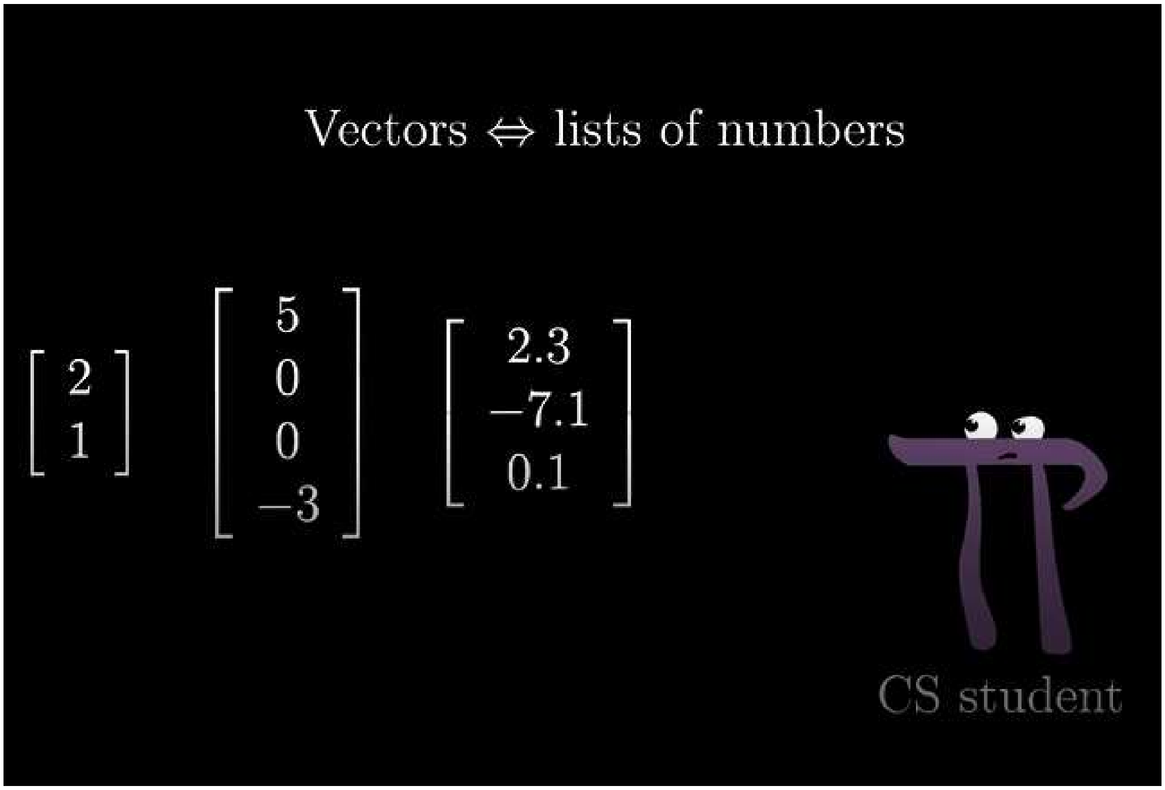
Vector Can be Two dimensional and Three dimensional also.

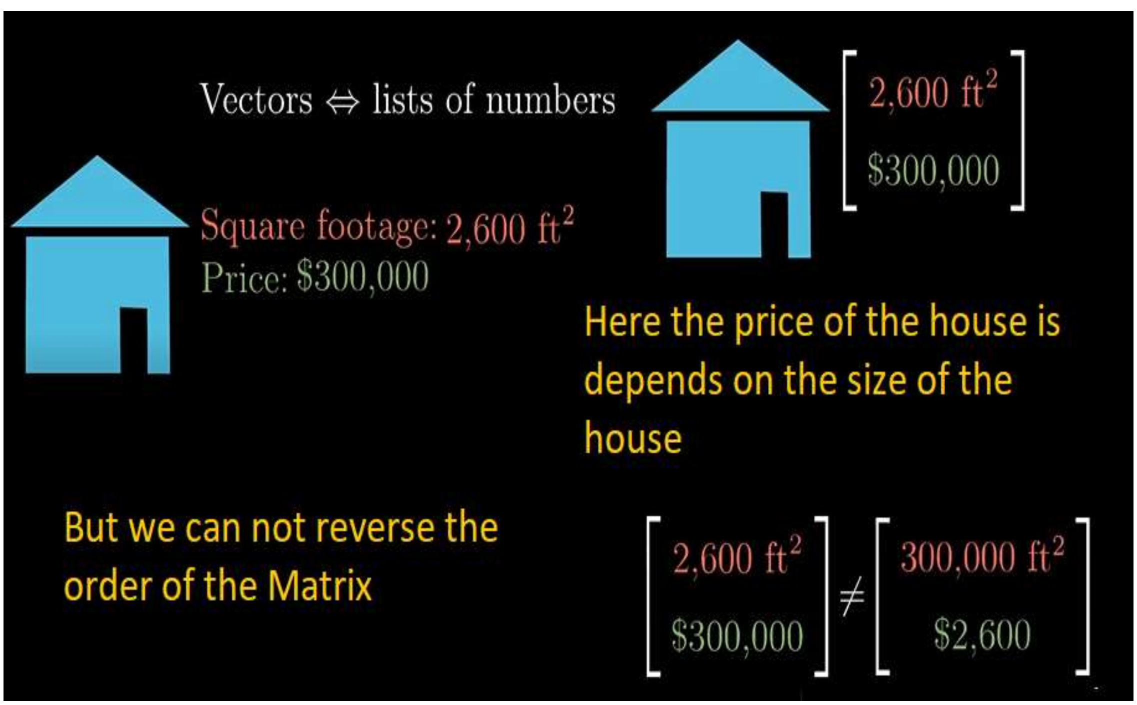


COMPUTER SCIENCE PROSPECTIVE:-

VECTOR IS A LIST OF NUMBERS.

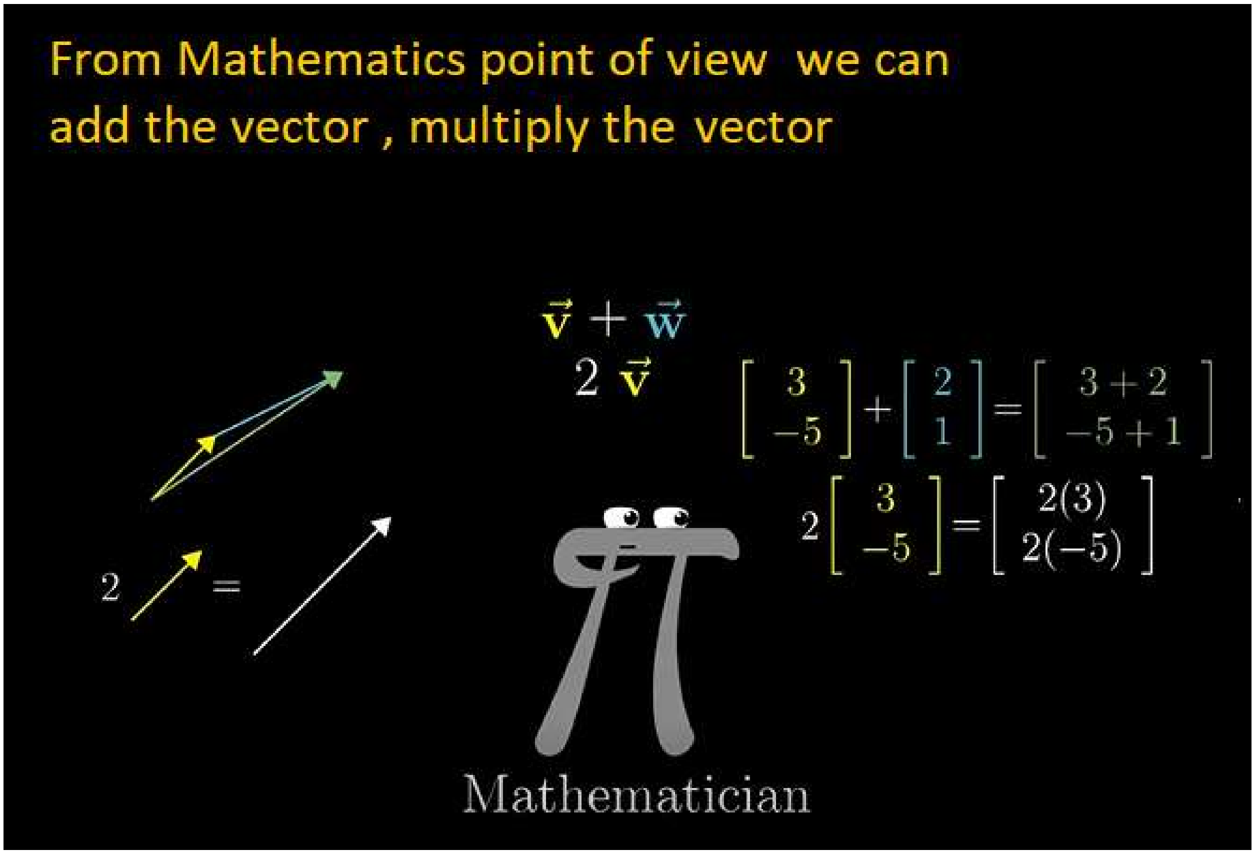
It is a type of metrics.





From Mathematics Point of View:-

We can do multiplication and addition with Vectors(Metrics).



The Multiplication of Vector and Addition of Vector Play very important role in Algebra.

Product of two vectors: -

Product of two number = Number.

Product of Two Matrix =Matrix

Multiplication of Two Vectors:-

(1) Scaler or Dot product→scaler

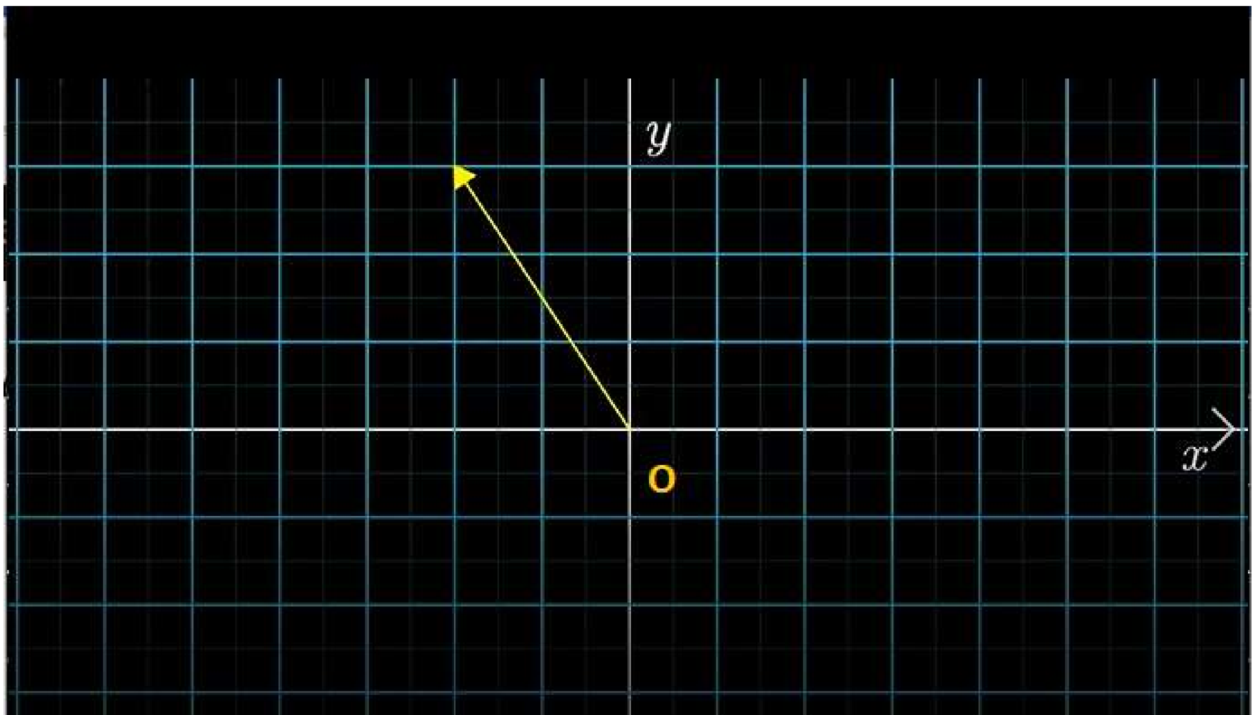
(2) Vector or Cross Products→vector

Scaler or Dot product: -

Vector or cross product: -

- **In Physics Vector can be anywhere in the space not necessary the Tail is at the Origin O.**

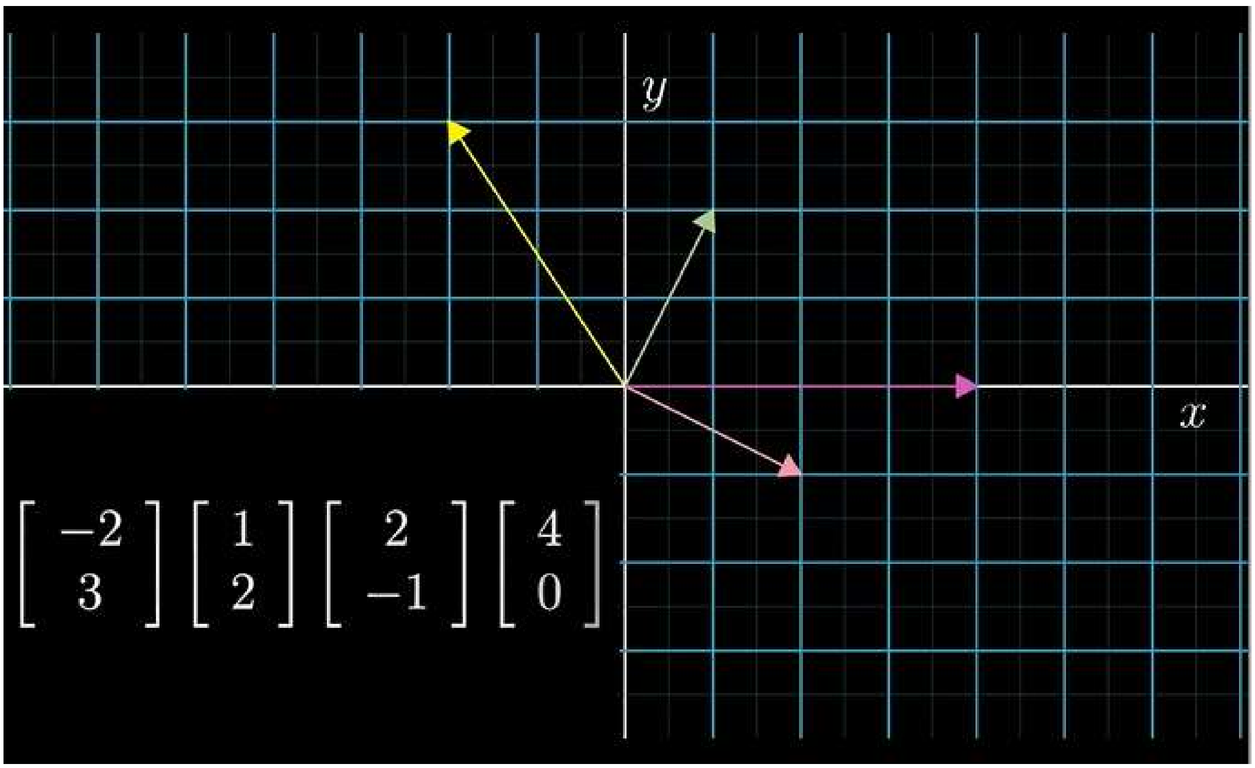
- **But in Linear Algebra the Tail of the Vector is considered always at the origin.**

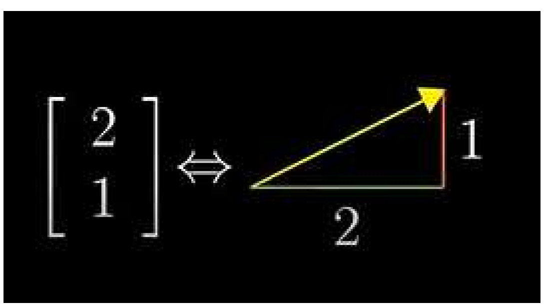


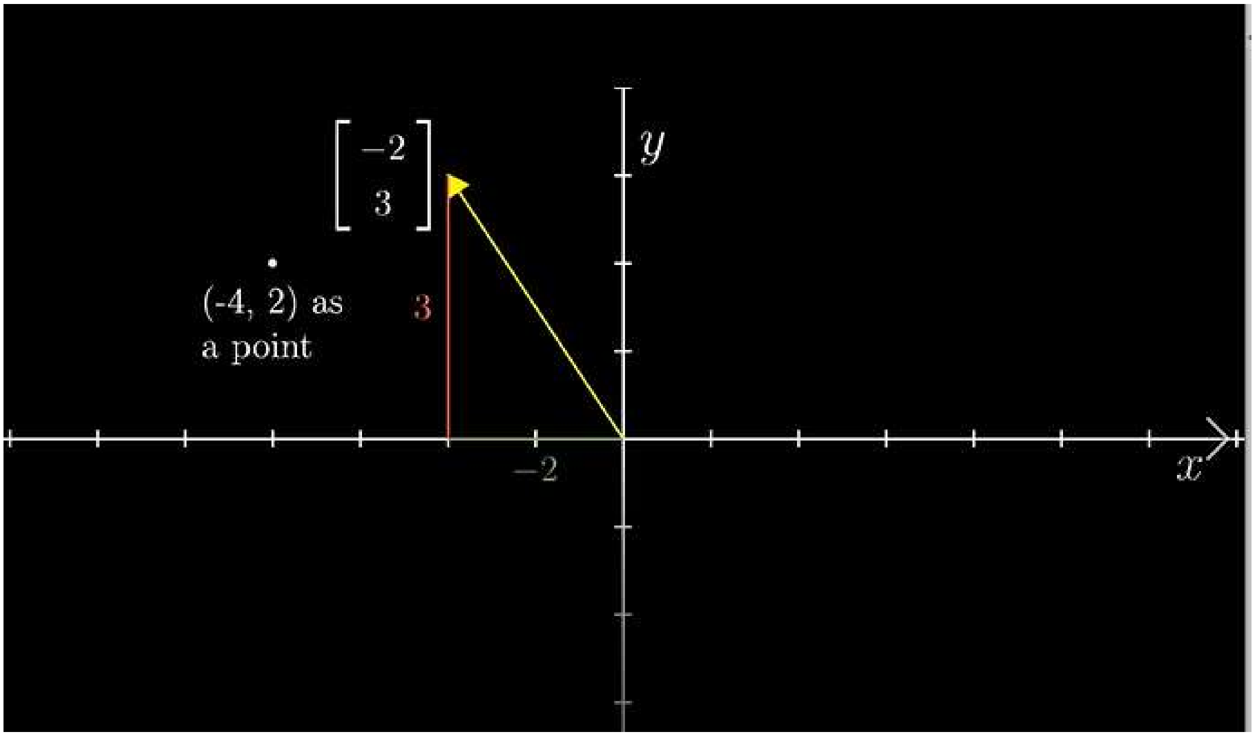
Consider this arrow inside a co-ordinate system in X,Y plane tail in the Origin

We can consider the direction of vectors as the co-ordinate of the system.

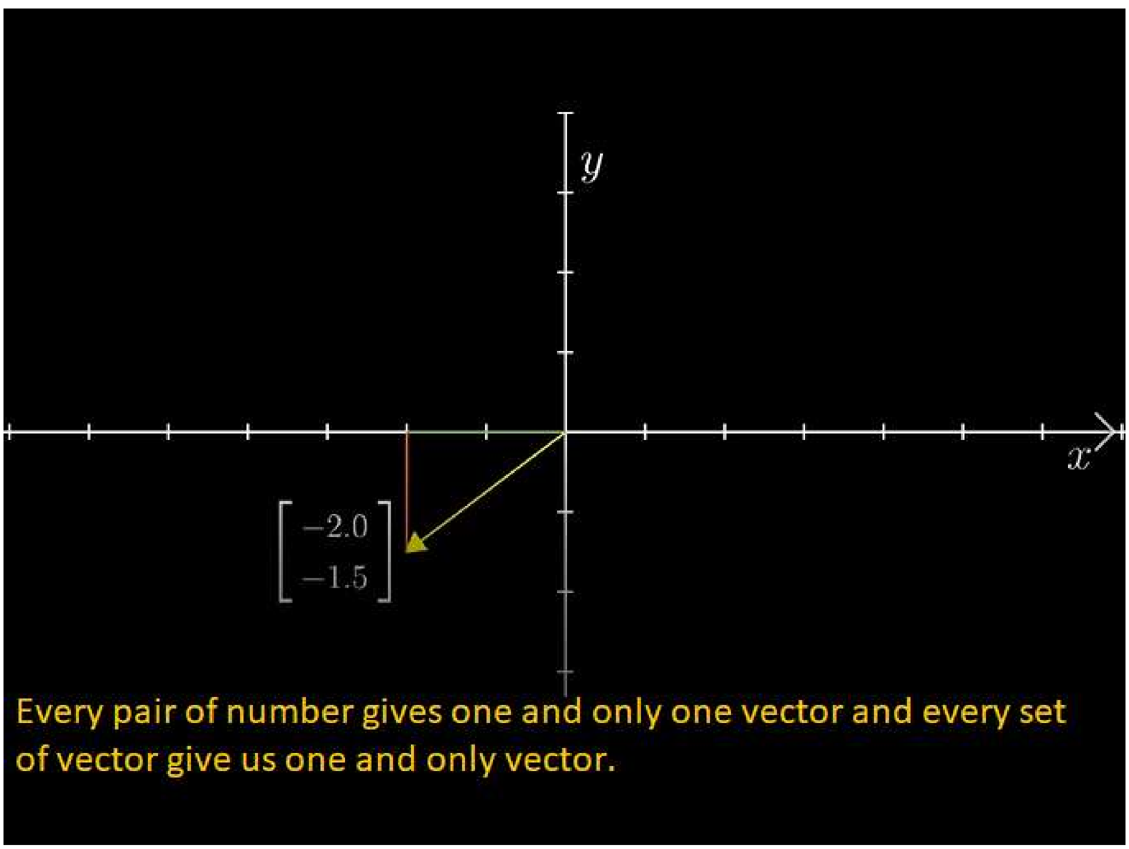
Understanding Vector Co-ordinate system



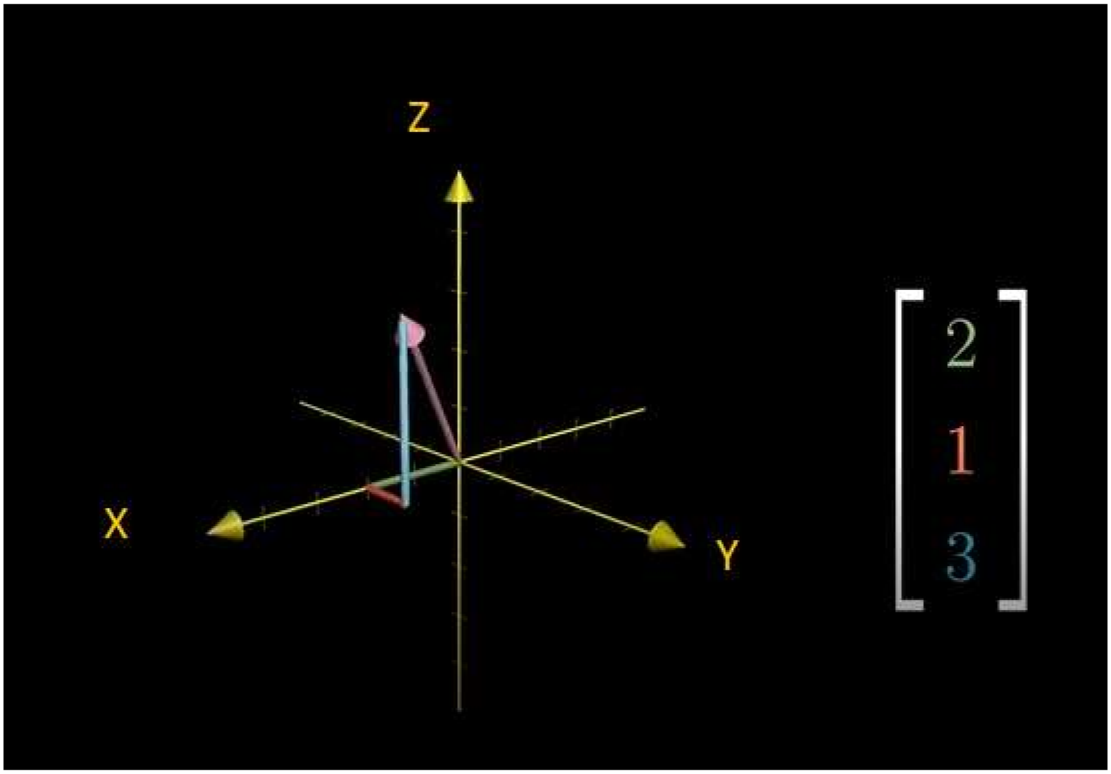




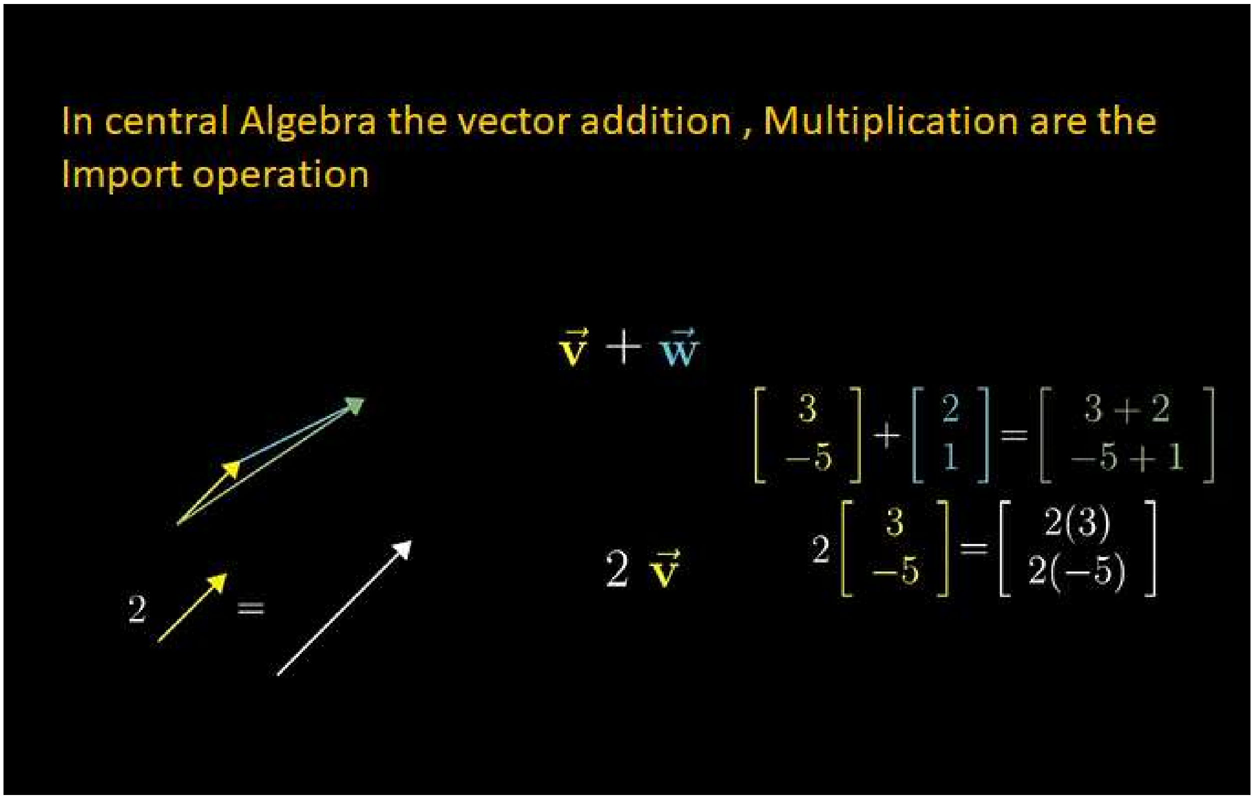
Every vector is unique & every pair of number is also unique.



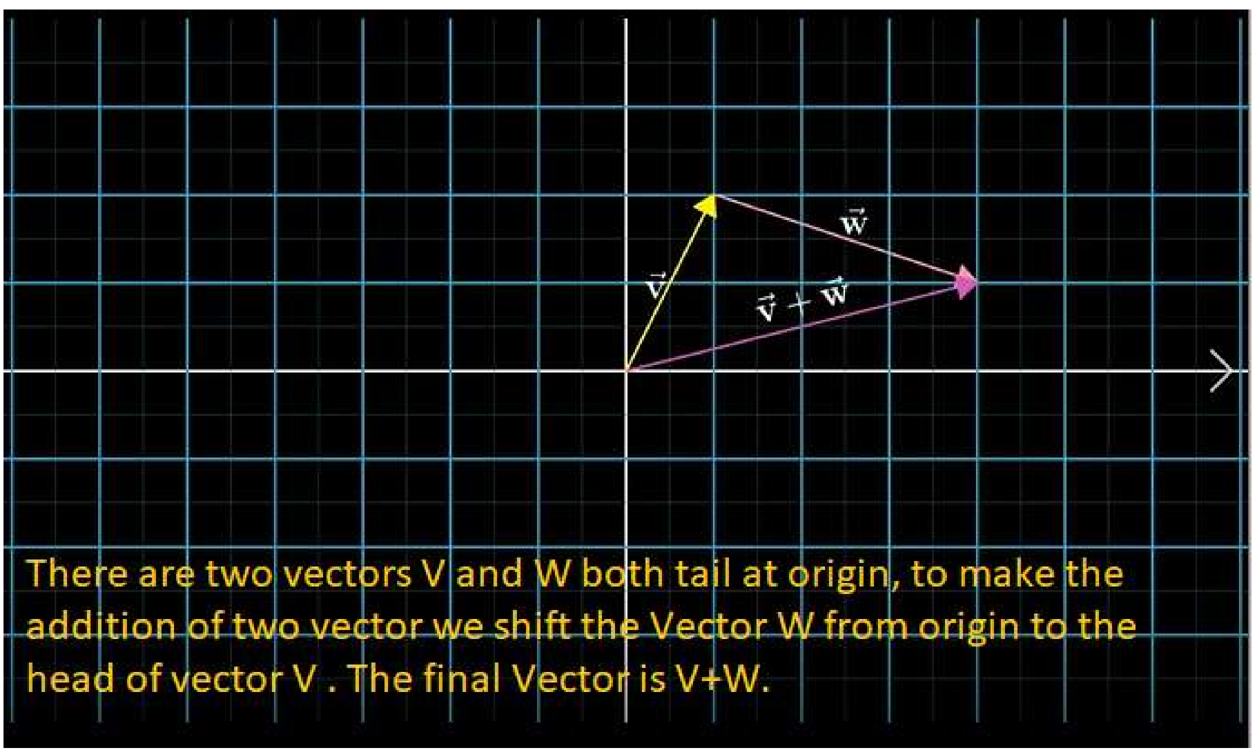
Three Dimension Plane



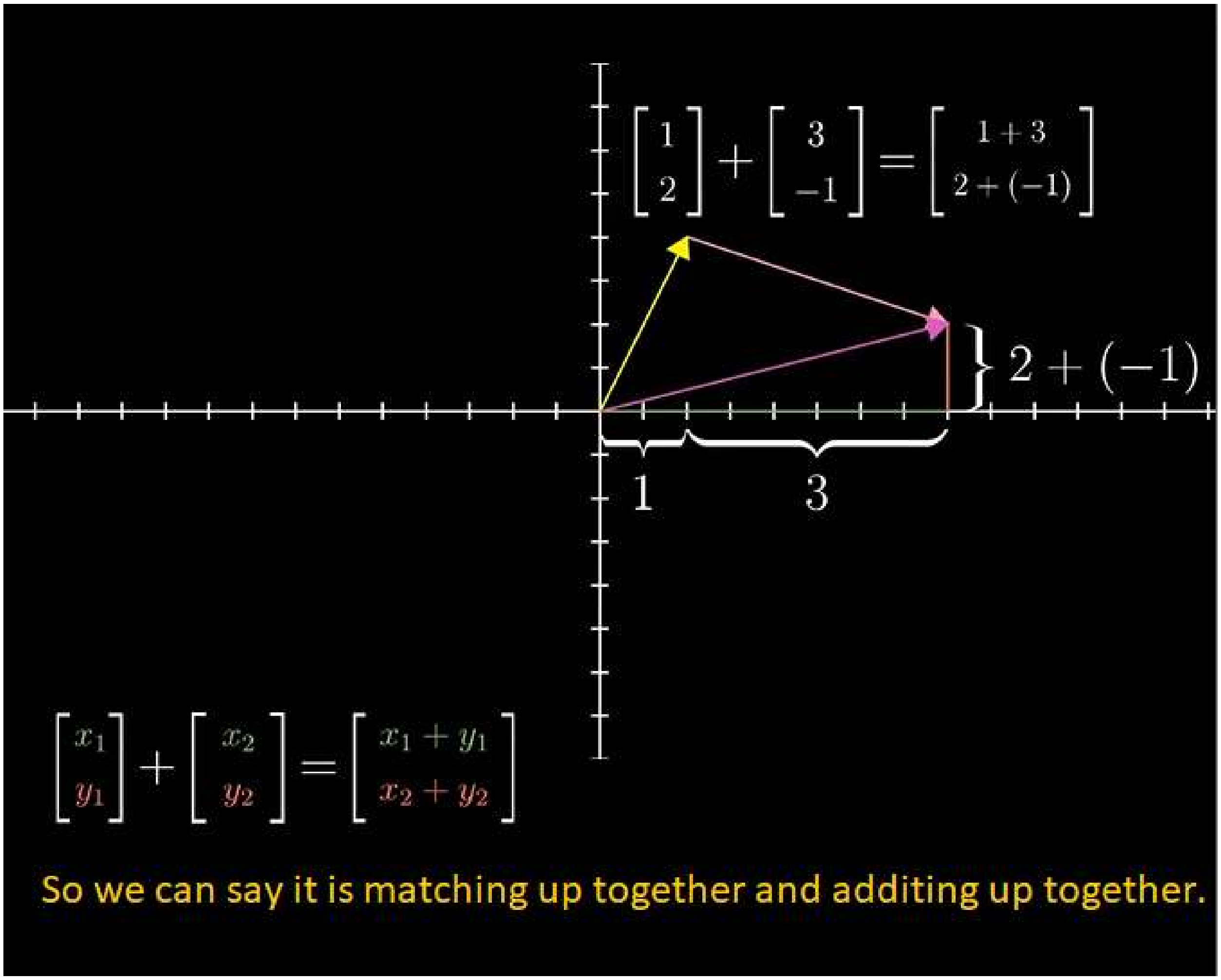
Here X=2,Y=1,Z=3.



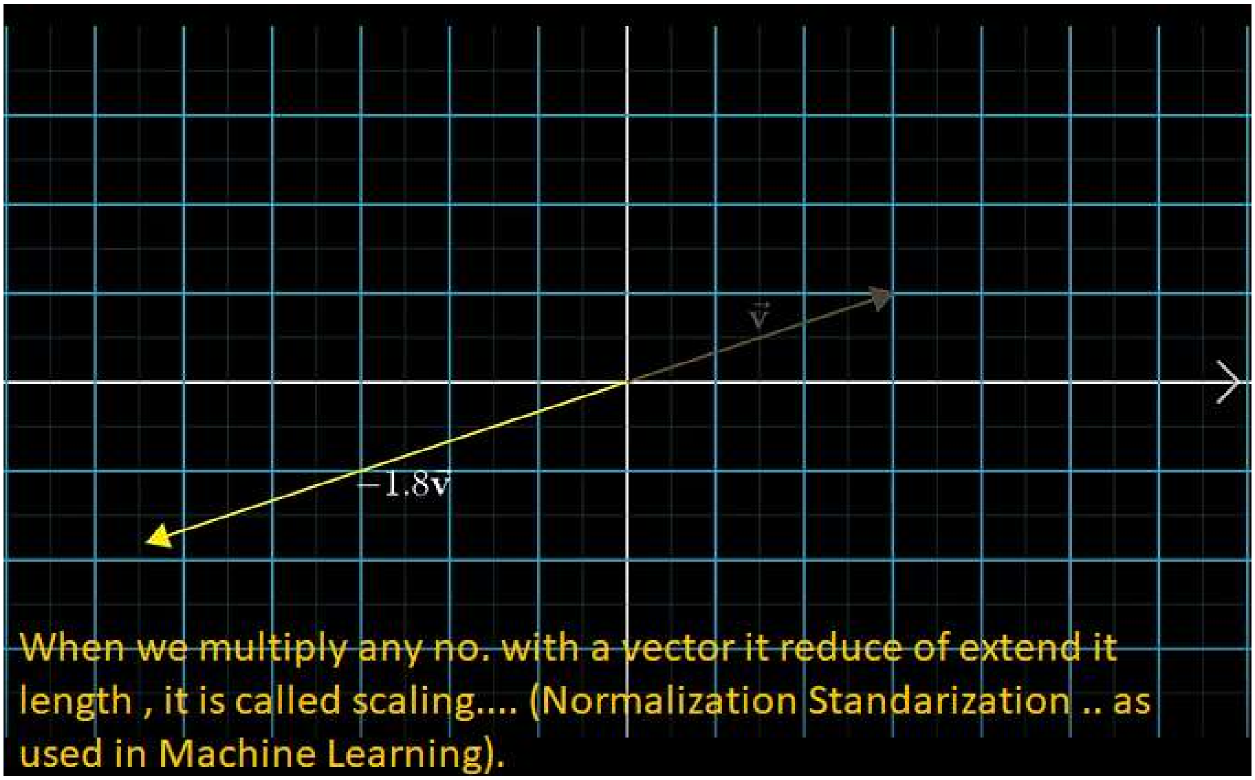
Addition of vectors



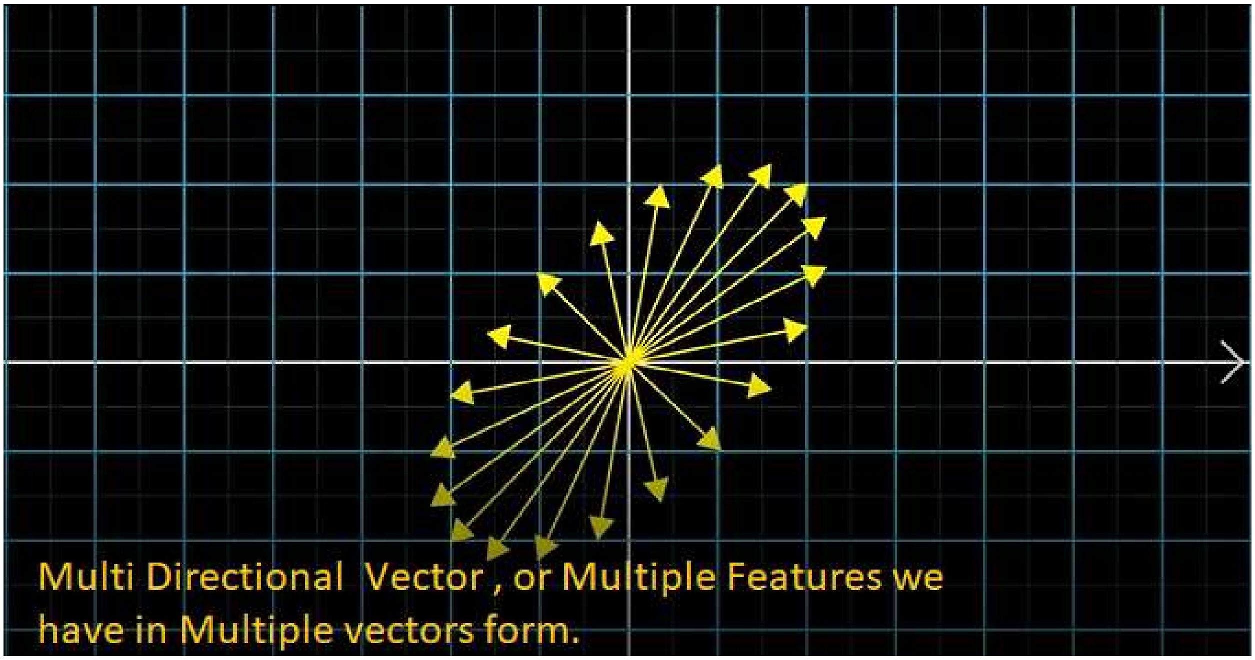
Vector/metrics Co-ordinate system in addition operation



Scaling a Vector



DATA SCIENCE POINT OF VIEW multiple vectors /metrix



$$\begin{bmatrix} 2.00 \\ 2.00 \end{bmatrix} \begin{bmatrix} 1.59 \\ 2.21 \end{bmatrix} \begin{bmatrix} 1.03 \\ 2.21 \end{bmatrix} \begin{bmatrix} 0.36 \\ 1.98 \end{bmatrix} \begin{bmatrix} -0.34 \\ 1.56 \end{bmatrix}$$

$$\begin{bmatrix} -1.01 \\ 0.99 \end{bmatrix} \begin{bmatrix} -1.58 \\ 0.32 \end{bmatrix} \begin{bmatrix} -1.99 \\ -0.38 \end{bmatrix} \begin{bmatrix} -2.21 \\ -1.04 \end{bmatrix} \begin{bmatrix} -2.21 \\ -1.60 \end{bmatrix}$$

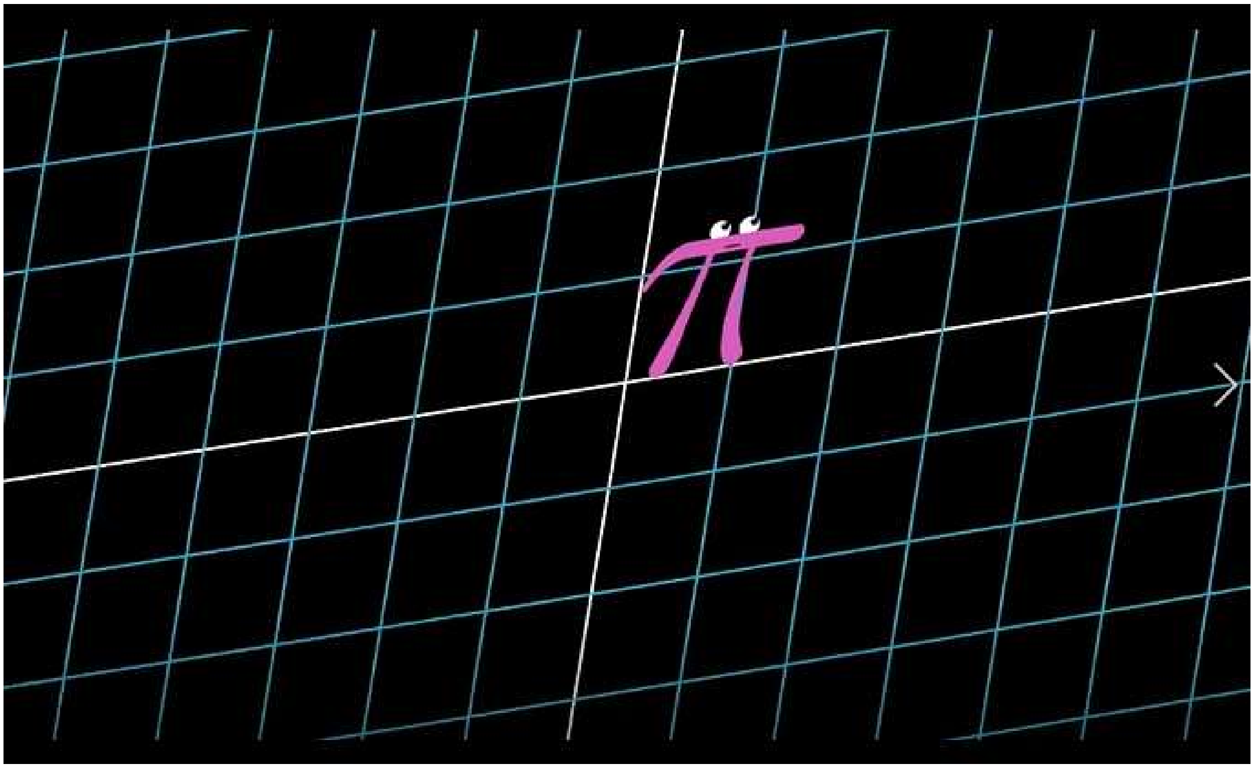
$$\begin{bmatrix} -1.99 \\ -2.01 \end{bmatrix} \begin{bmatrix} -1.58 \\ -2.21 \end{bmatrix} \begin{bmatrix} -1.01 \\ -2.20 \end{bmatrix} \begin{bmatrix} -0.34 \\ -1.97 \end{bmatrix} \begin{bmatrix} 0.36 \\ -1.55 \end{bmatrix}$$

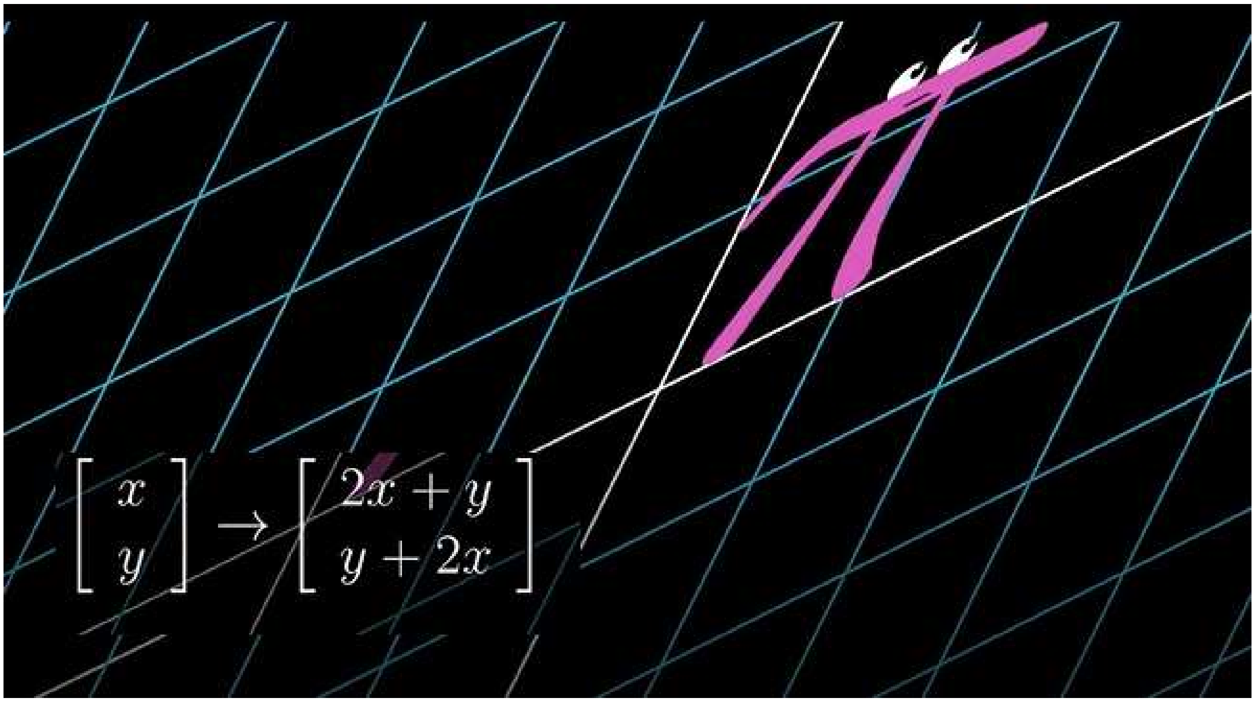
$$\begin{bmatrix} 1.03 \\ -0.97 \end{bmatrix} \begin{bmatrix} 1.59 \\ -0.30 \end{bmatrix} \begin{bmatrix} 2.00 \\ 0.40 \end{bmatrix} \begin{bmatrix} 2.21 \\ 1.06 \end{bmatrix} \begin{bmatrix} 2.21 \\ 1.62 \end{bmatrix}$$

From the Point of View of ML

In Computer and Graphics Program

We Manipulate the space rotates it in different angle// to represent and calculate the new dimensions

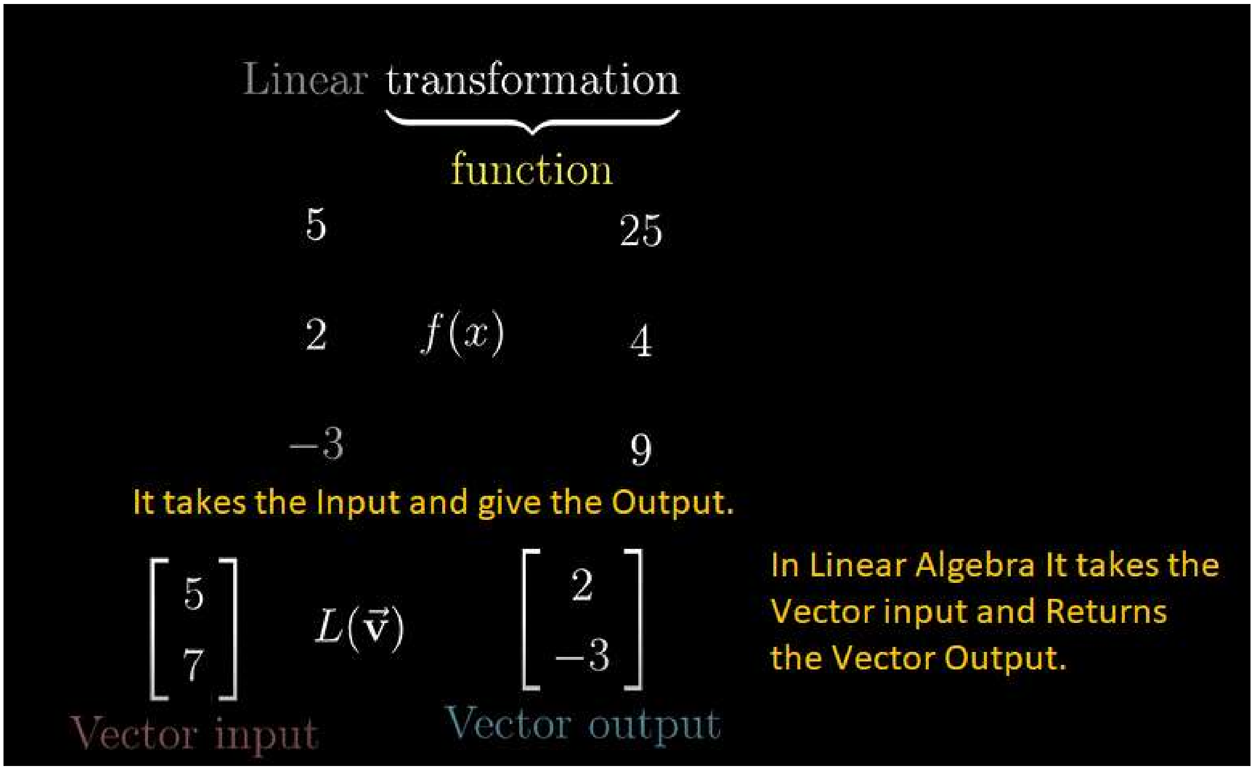




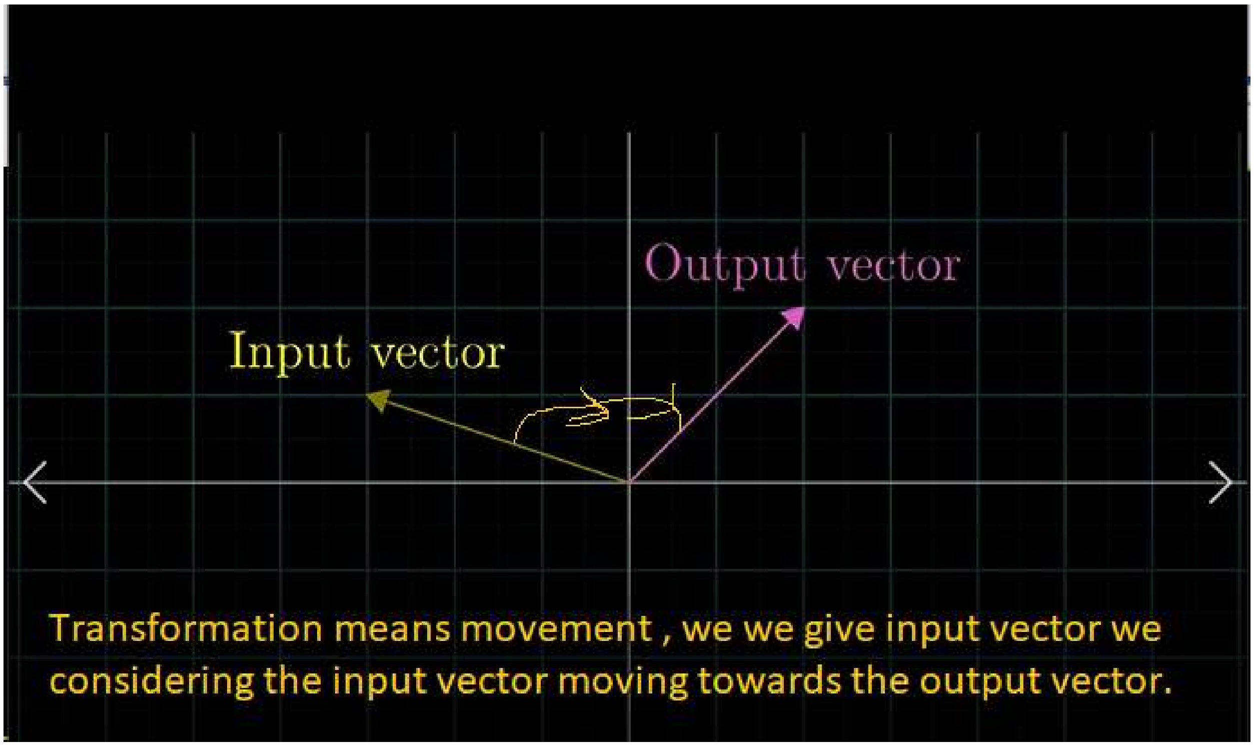
Matrix Multiplication.

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} (1)(5) + (-3)(7) \\ (2)(5) + (4)(7) \end{bmatrix}$$

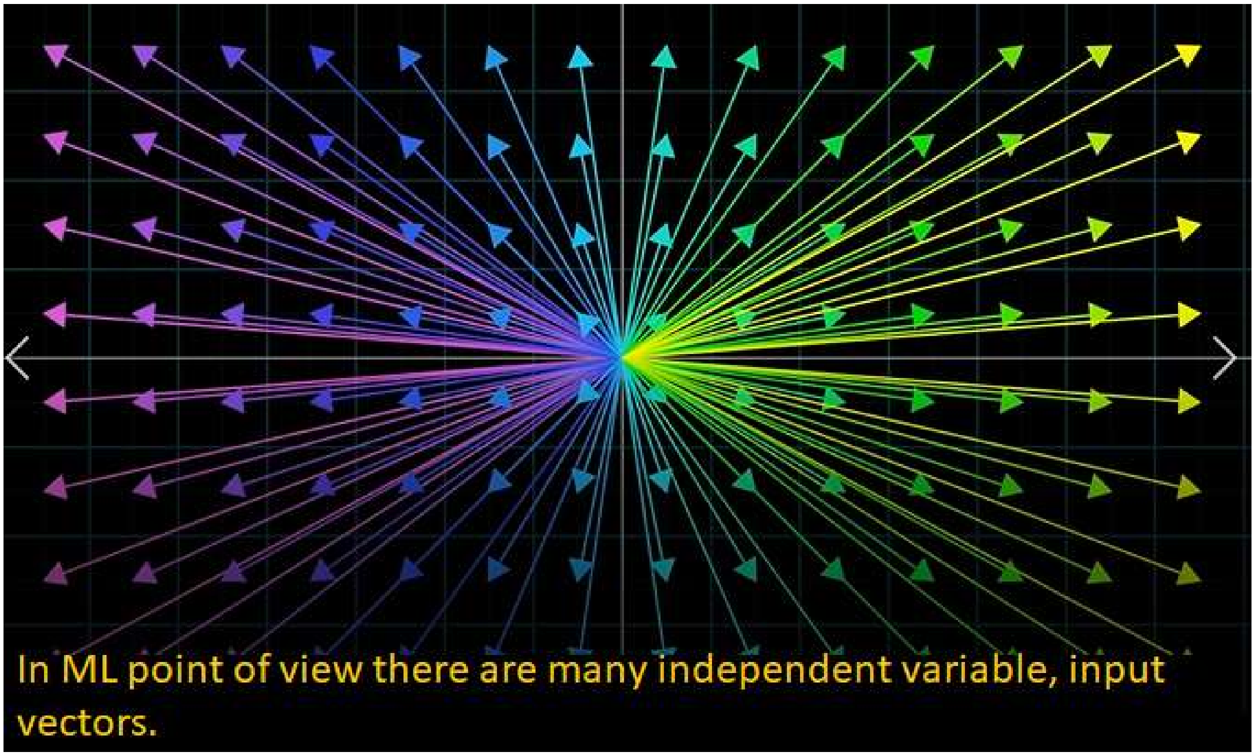
What is Transformation?

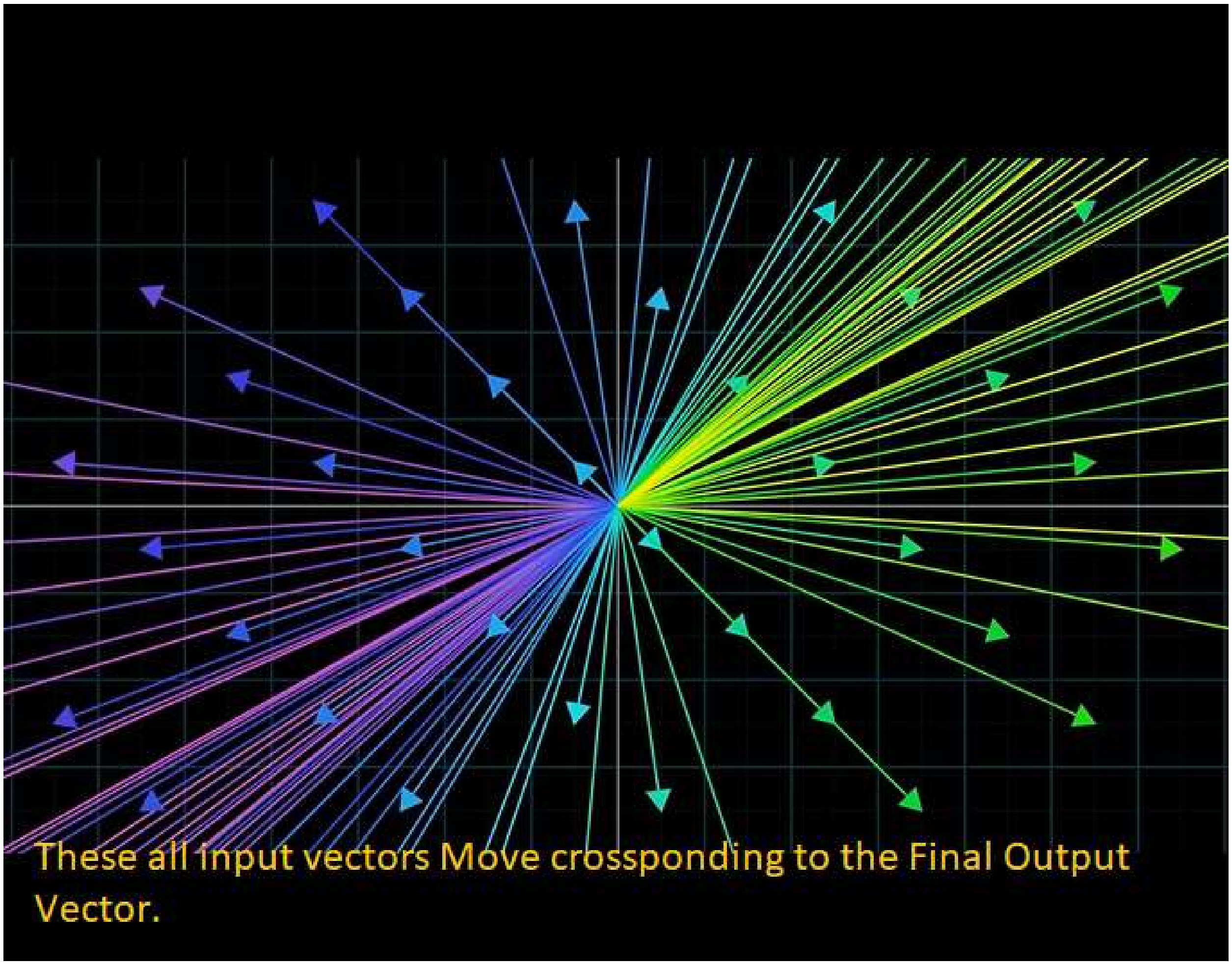


TRANSFORMATION ---- MOVEMENT –FLOW



Transformation become more complicated in Multi Dimensions





In Machine learning and Deep Learning we will use many a times words…..

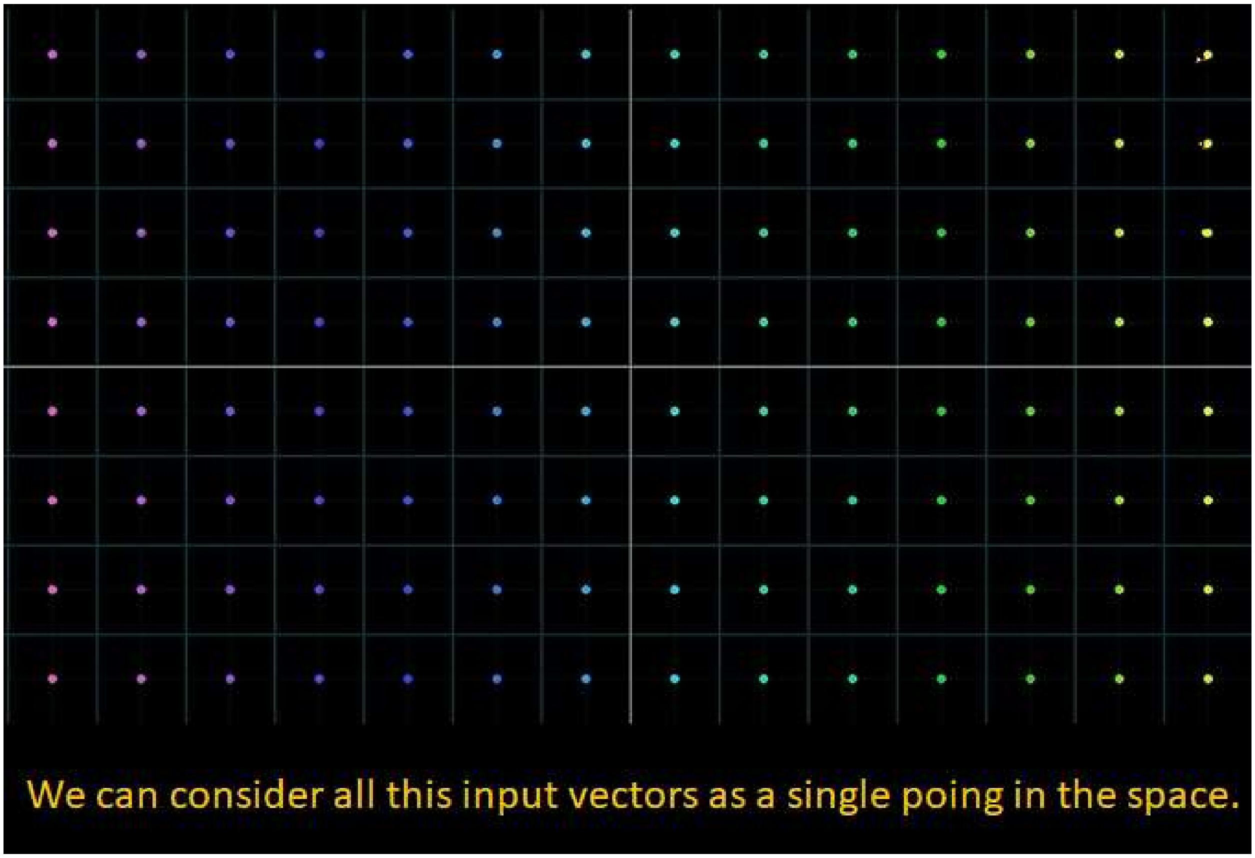
- **Transformation**
- **TENSOR FLOW Networking …**
- **FLOW OF Tensor,**

Tensor is an object which transform as a project of vectors under rotation.

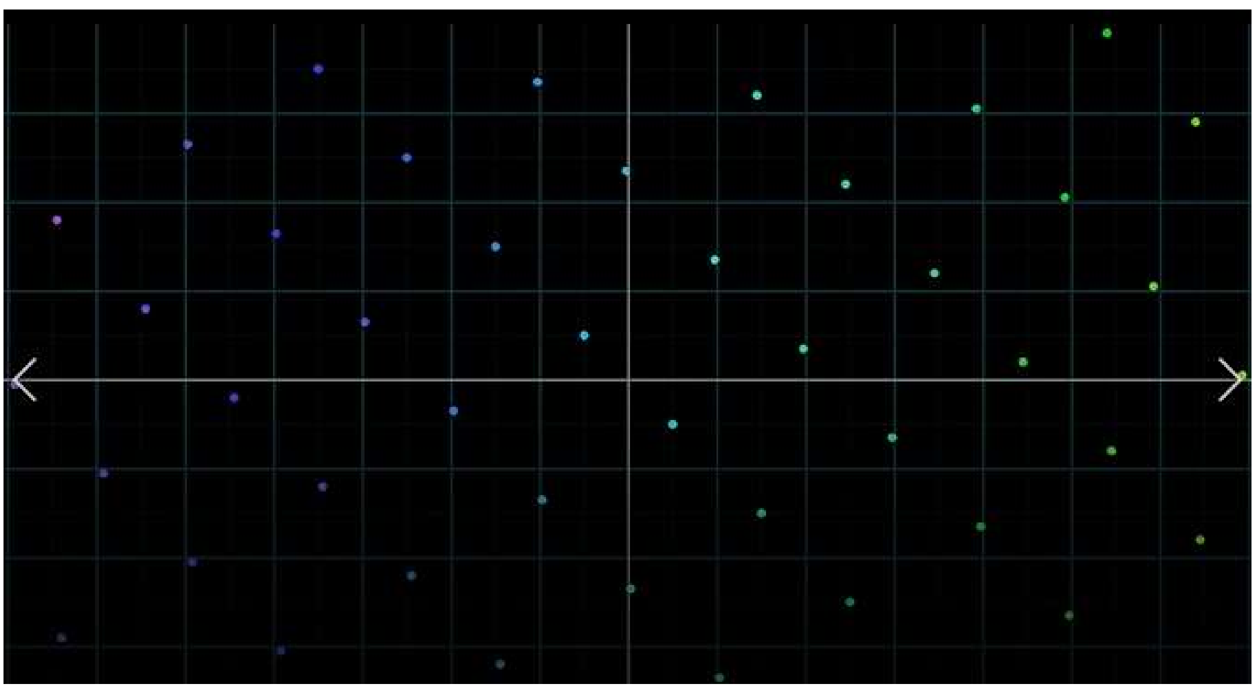
- **Input as Tensor means input of VECTOR… MATRIXS….**

Giving input means ….INPUT OF Vector /Matrix.

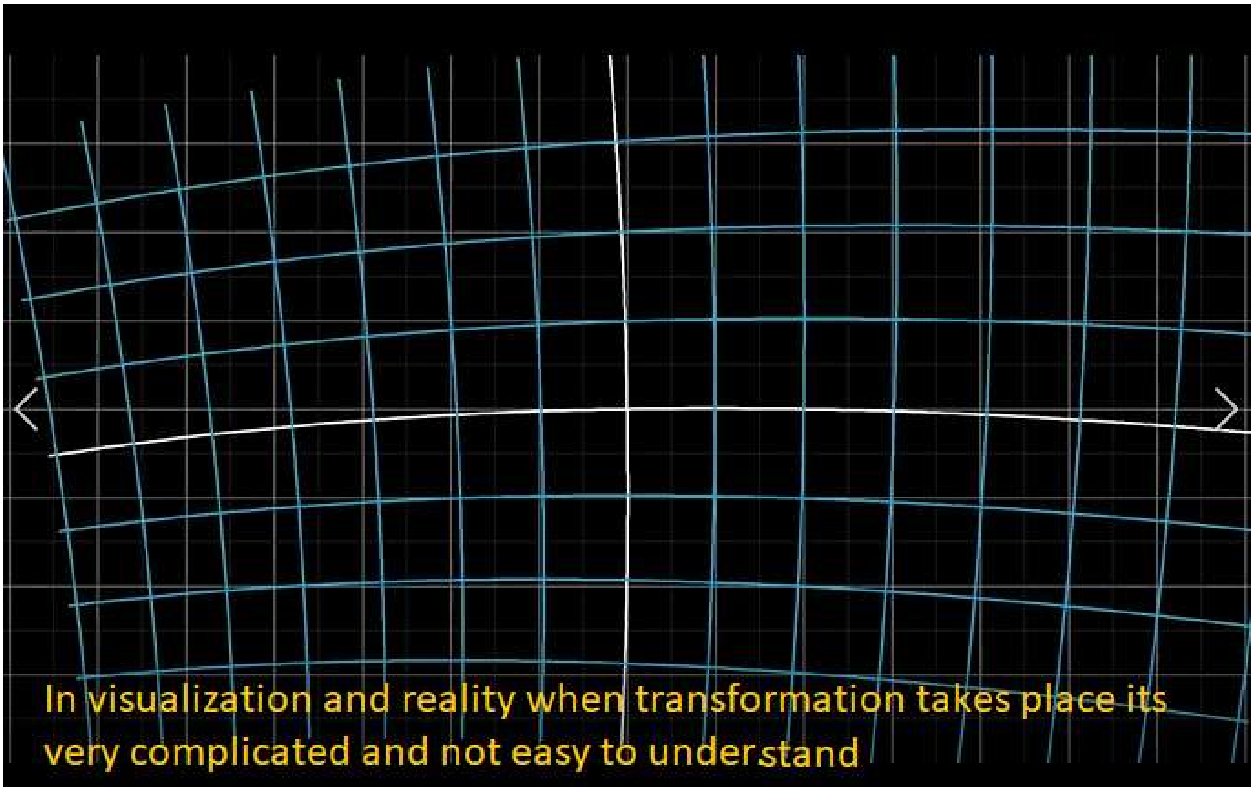
CONSIDERING VETORS AS POINT IN THE SPACE…..



Vectors are in transformation … means Points are in Transformation.. Points moving one point to some other points.



Transformation in Multi dimensional space…

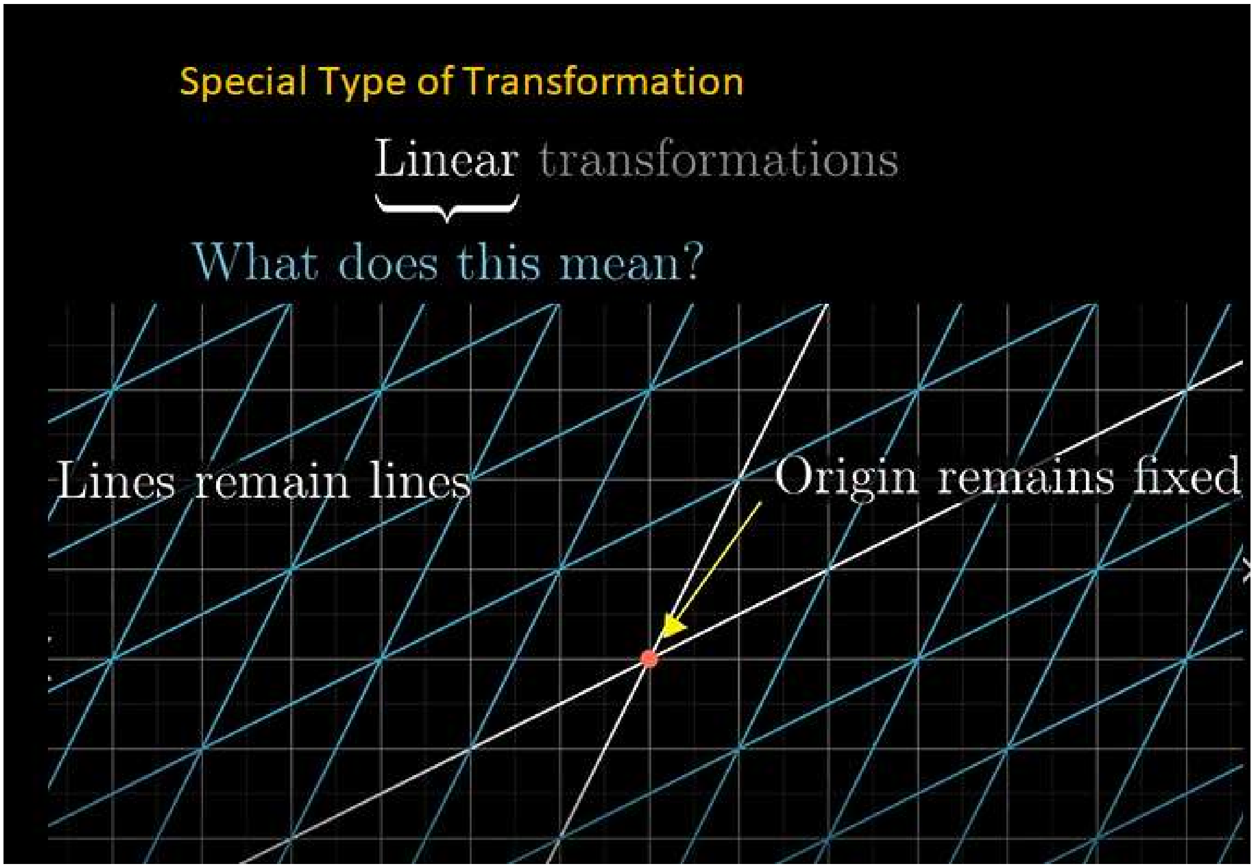


Moving around all the points in the multi-dimensional space.

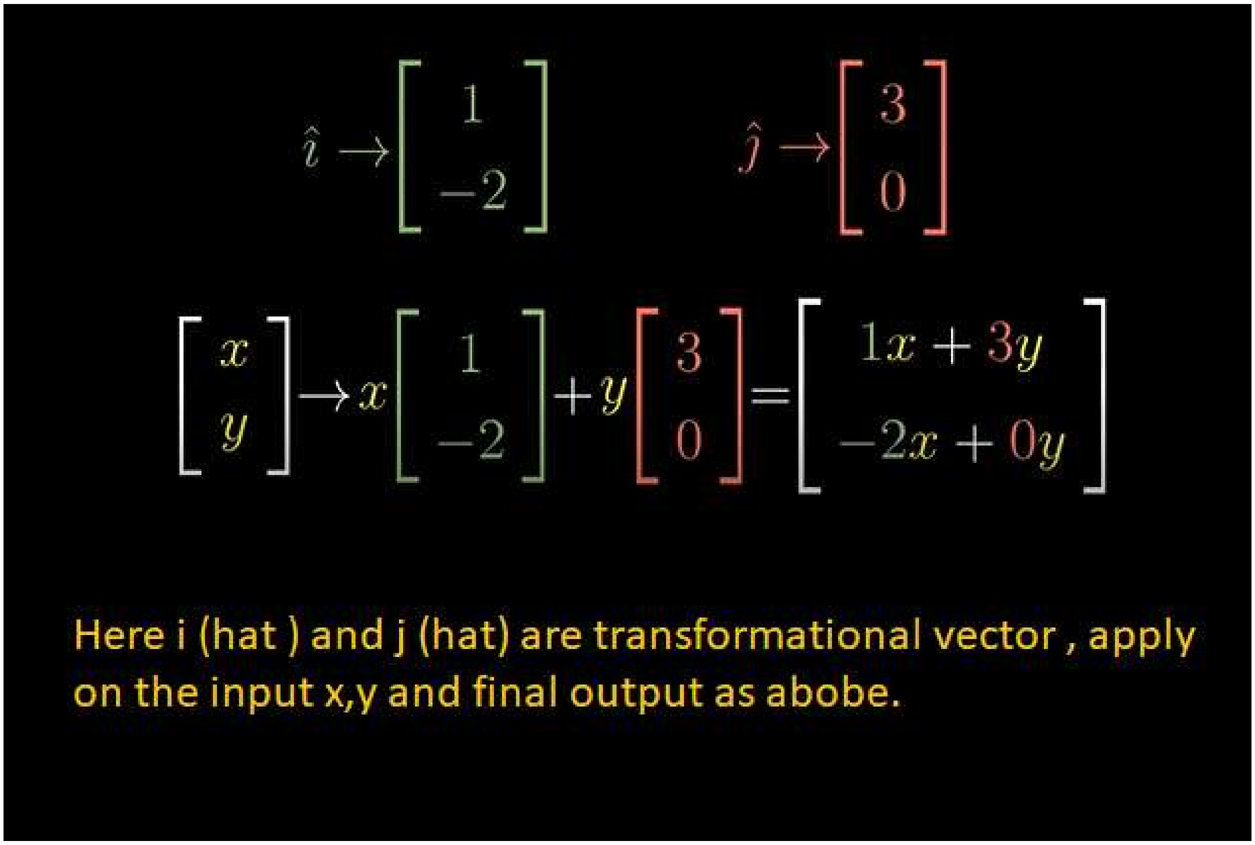
arbitrary Transformation----(based **on random choice)- That is interesting (but difficult to understand) about 4D rotations. Looks very complicated.**

For BETTER VISUALIZATION & UNDERSTANDING , SOME assumption has been made. Which bring the concept …

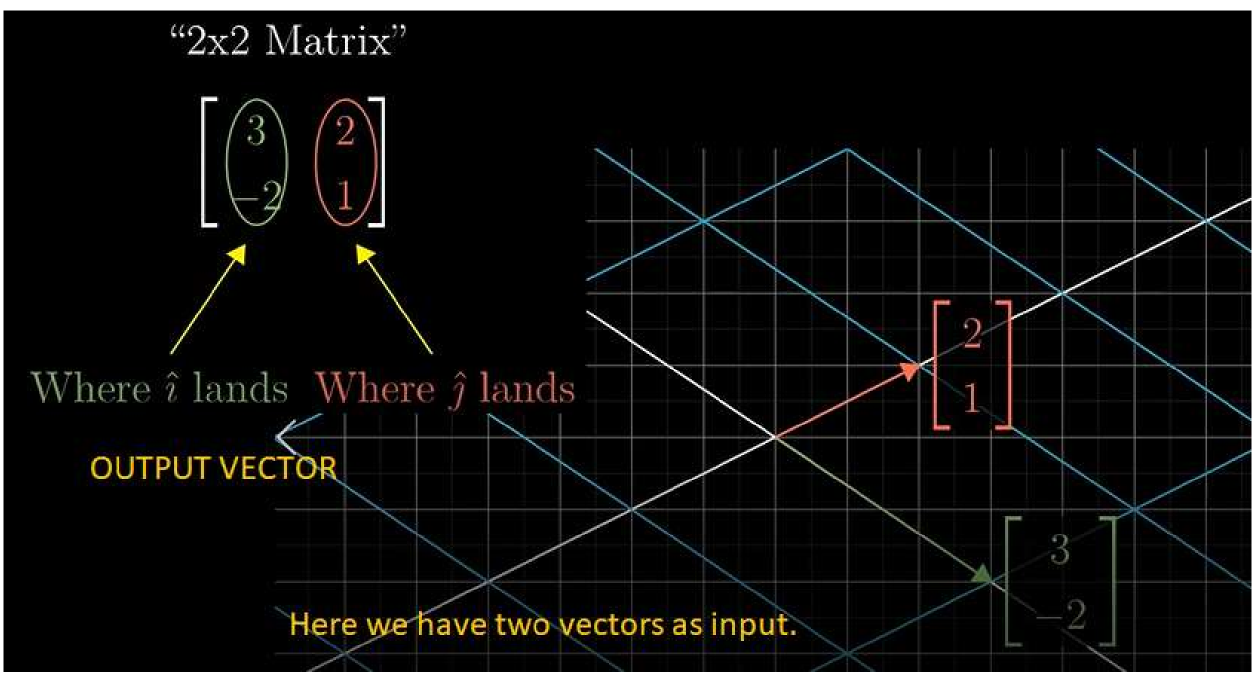
LINEAR TRANSFORMATION



FINALLY, GRID LINE REMAIN PARALLEL AND EVENLY SPACED.



Here i(hat) and j(hat) are the basis vector(unit vector).

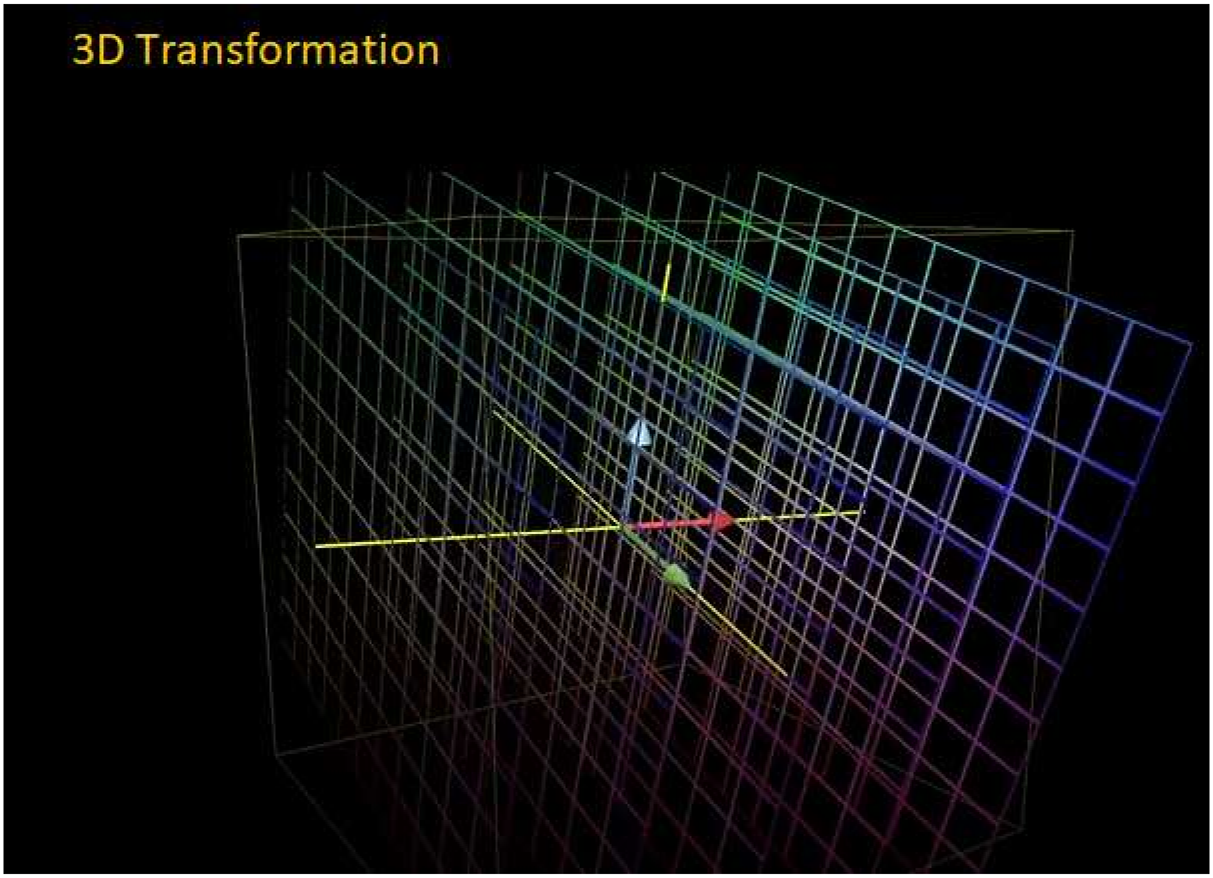


Take the example of Two Inputs ;
.. the final output will be as below.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

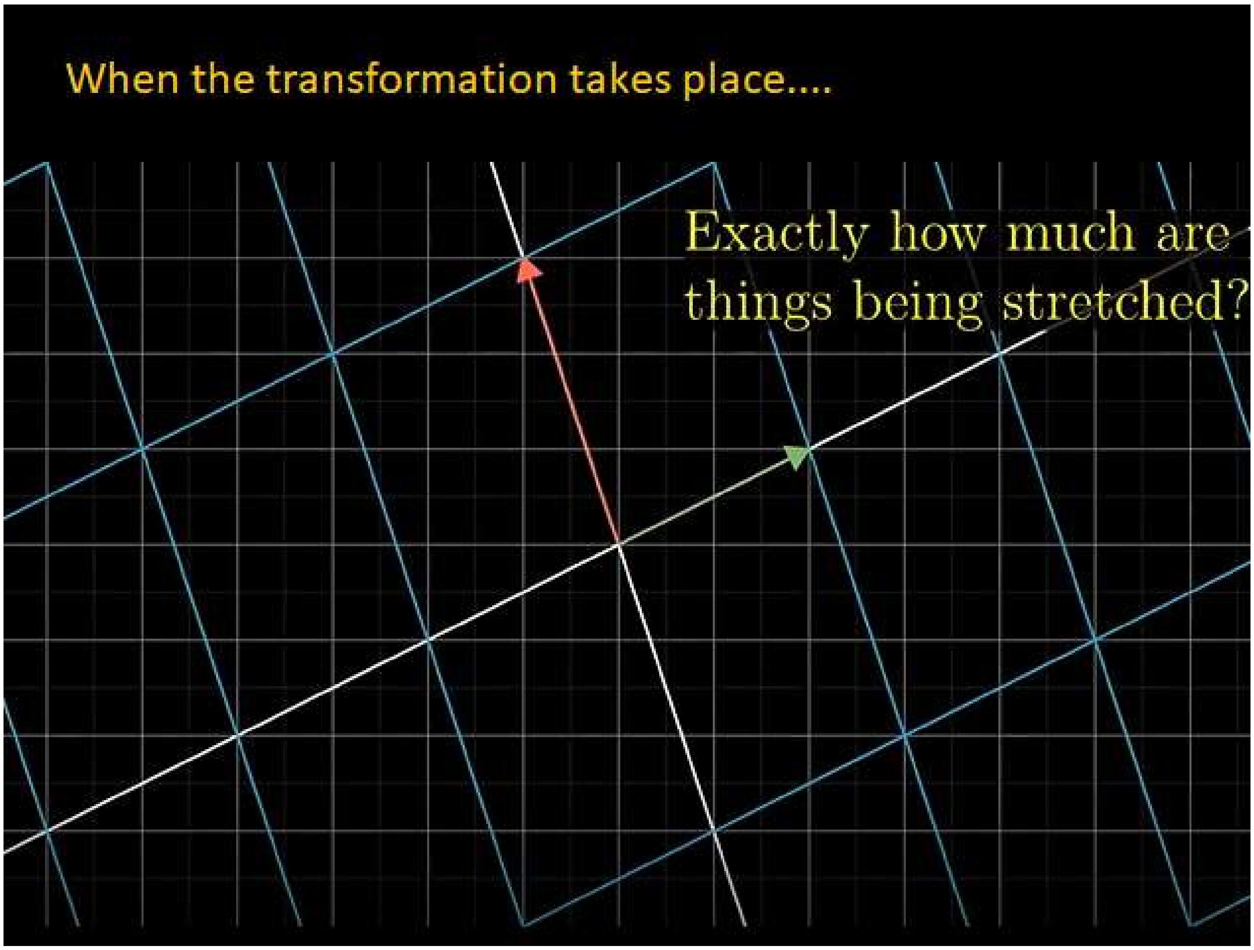
Three-dimensional transformation

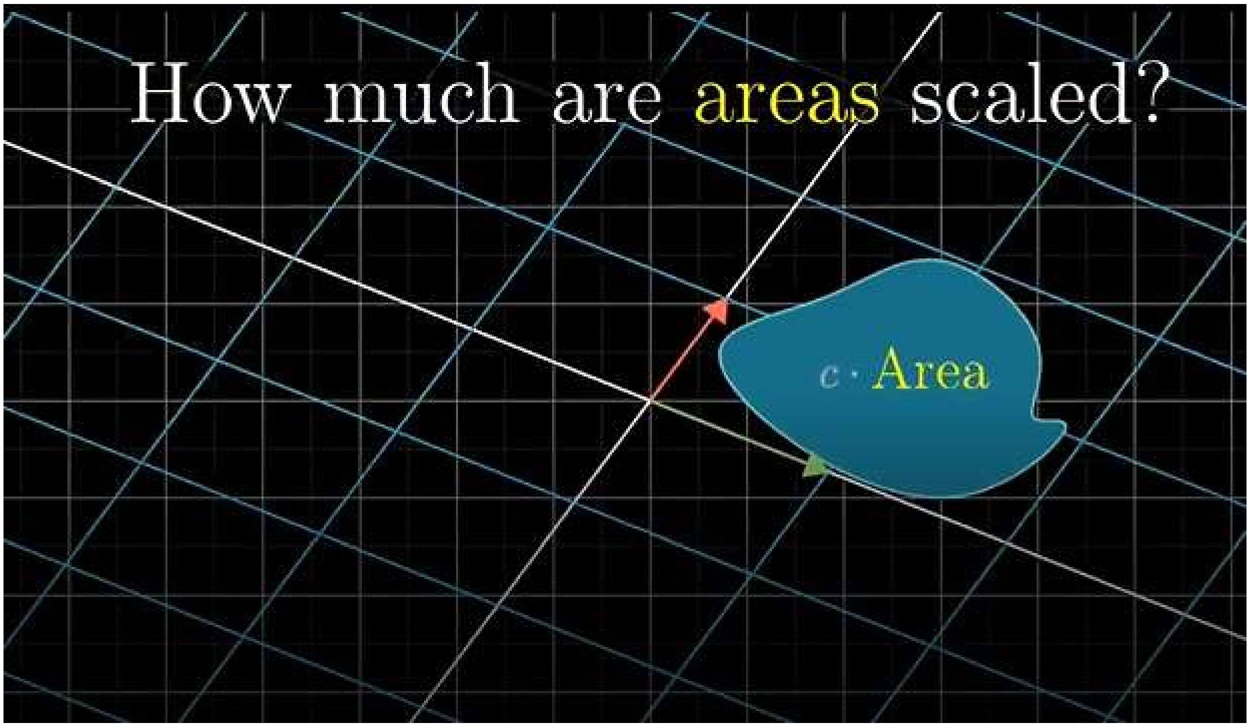
$$\underset{\text{Input}}{\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}} \qquad L(\vec{\mathbf{v}}) \qquad \underset{\text{Output}}{\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}}$$



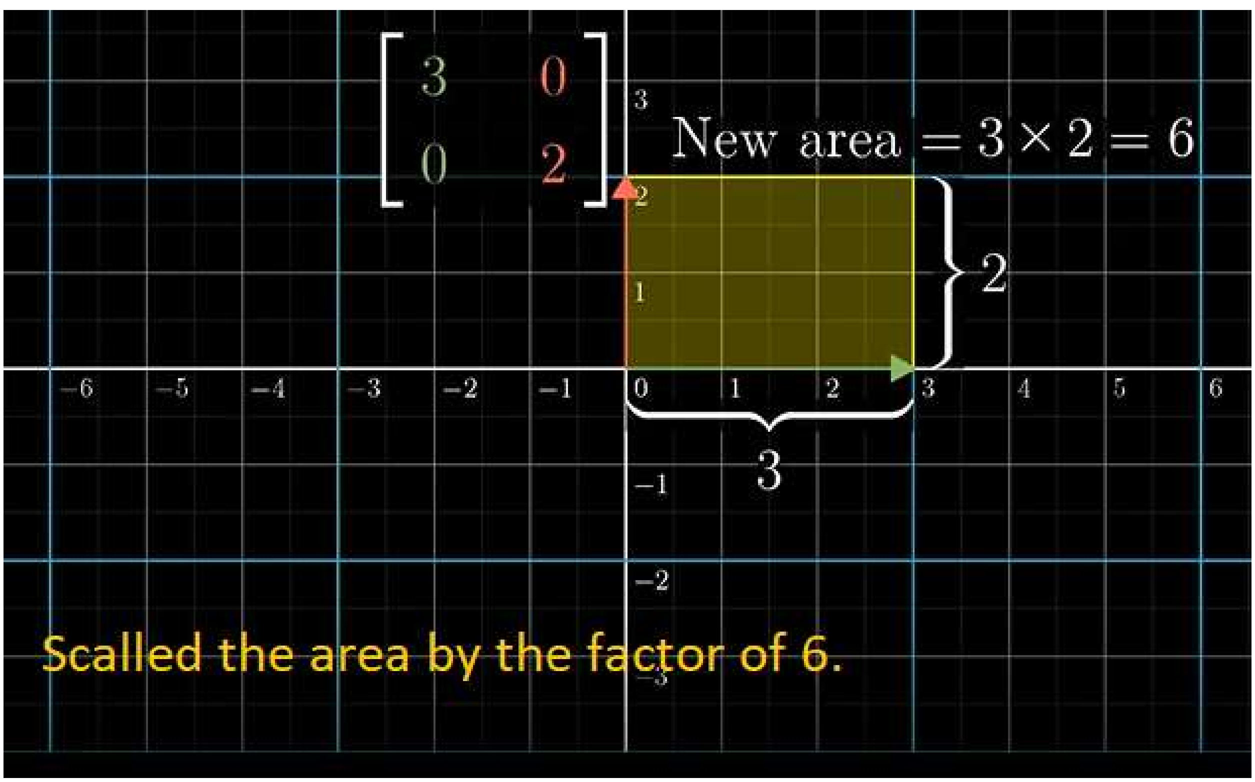
Multi-dimensional Input output calculation is not easy to visualize and understand.

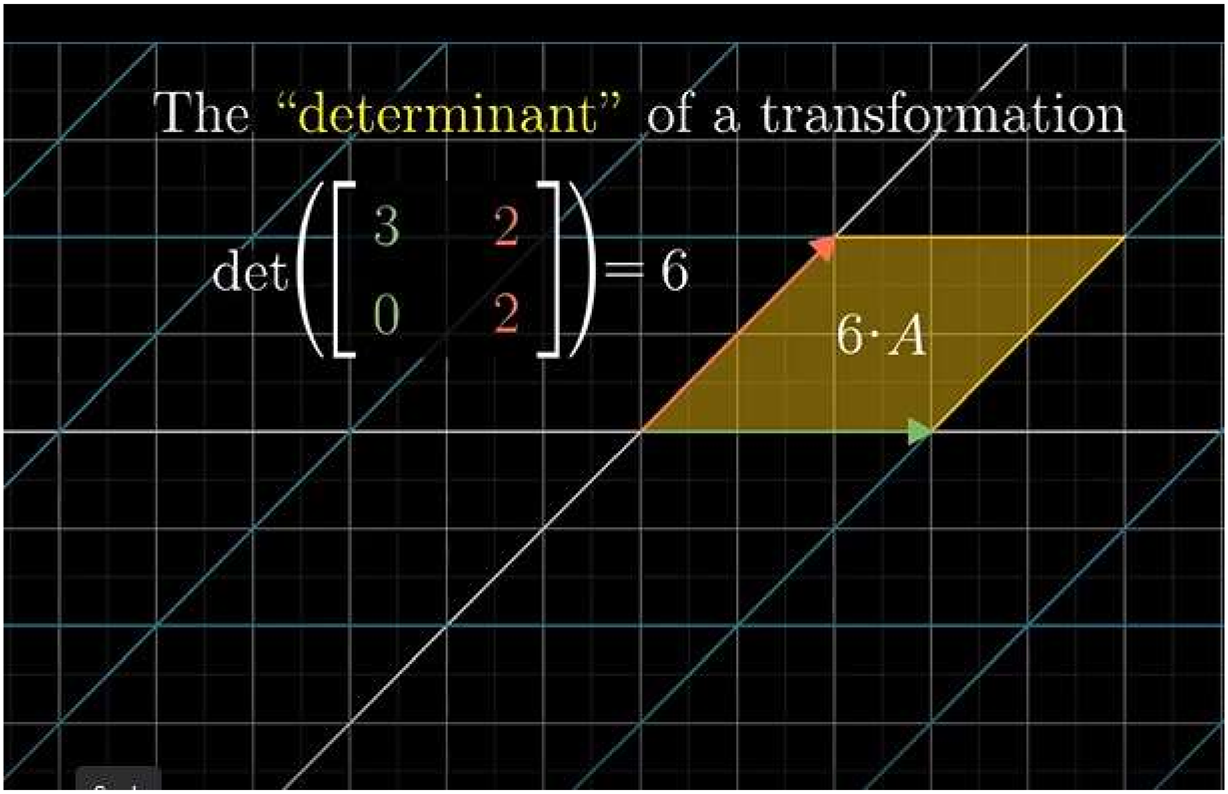
Determinant of Transformation Metrix

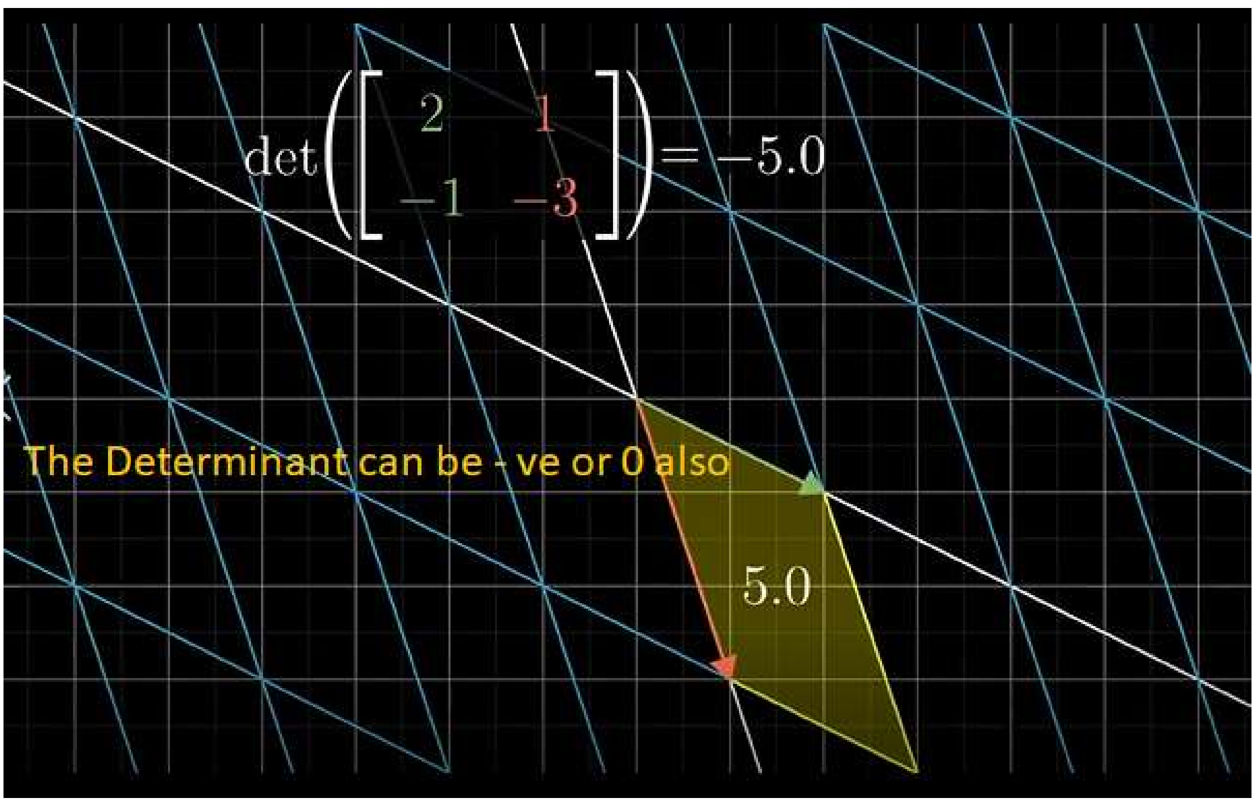


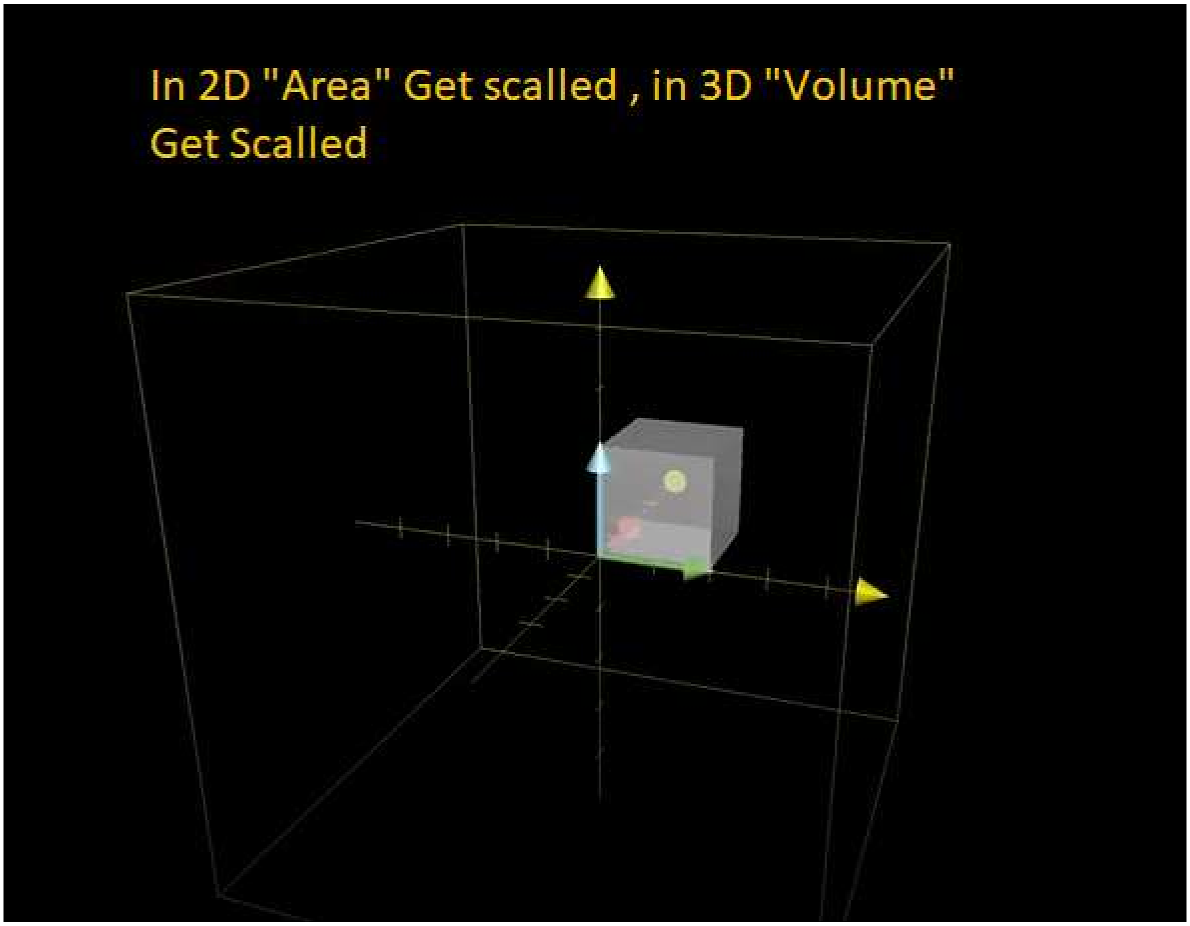


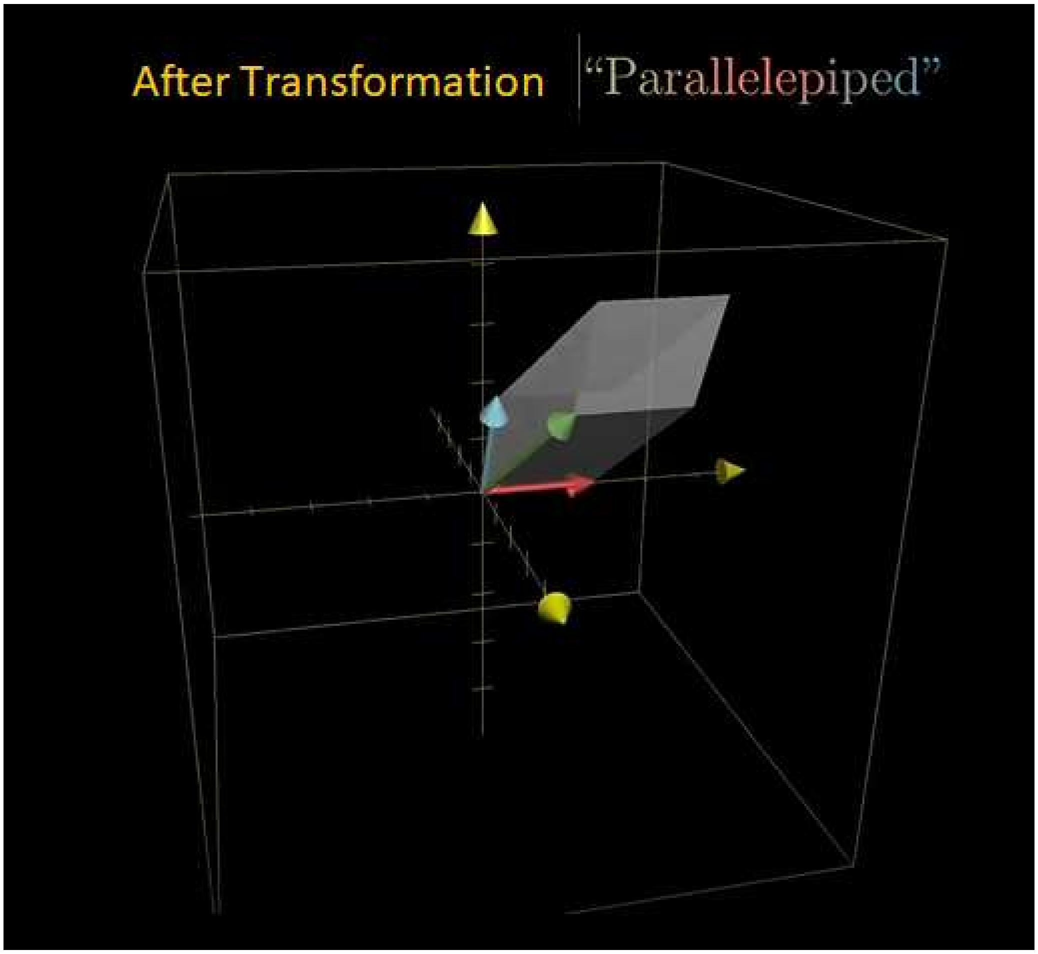
Take the example of this matrix as input variable

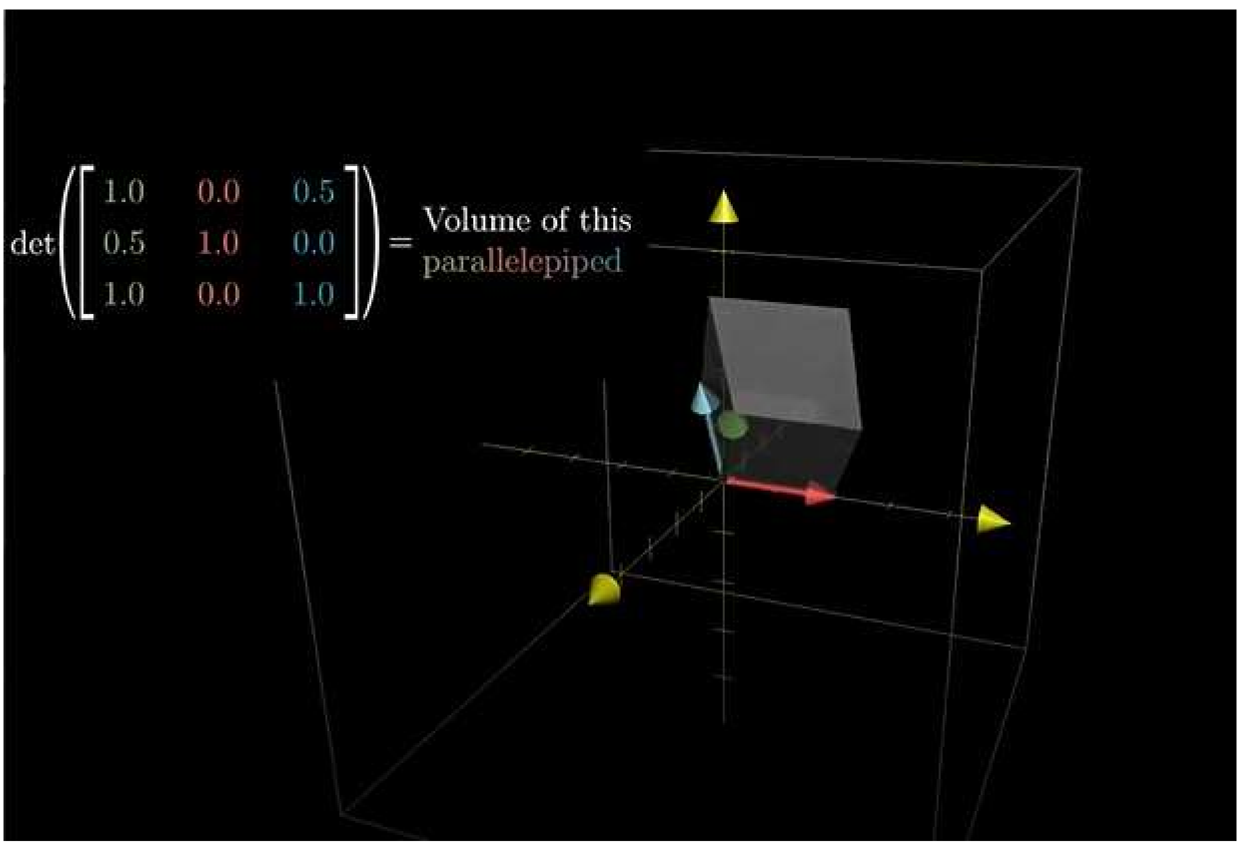



The "determinant" of a transformation
$\det\left(\begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}\right) = 6$
$6 \cdot A$


$\det\left(\begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}\right) = -5.0$
The Determinant can be - ve or 0 also
5.0

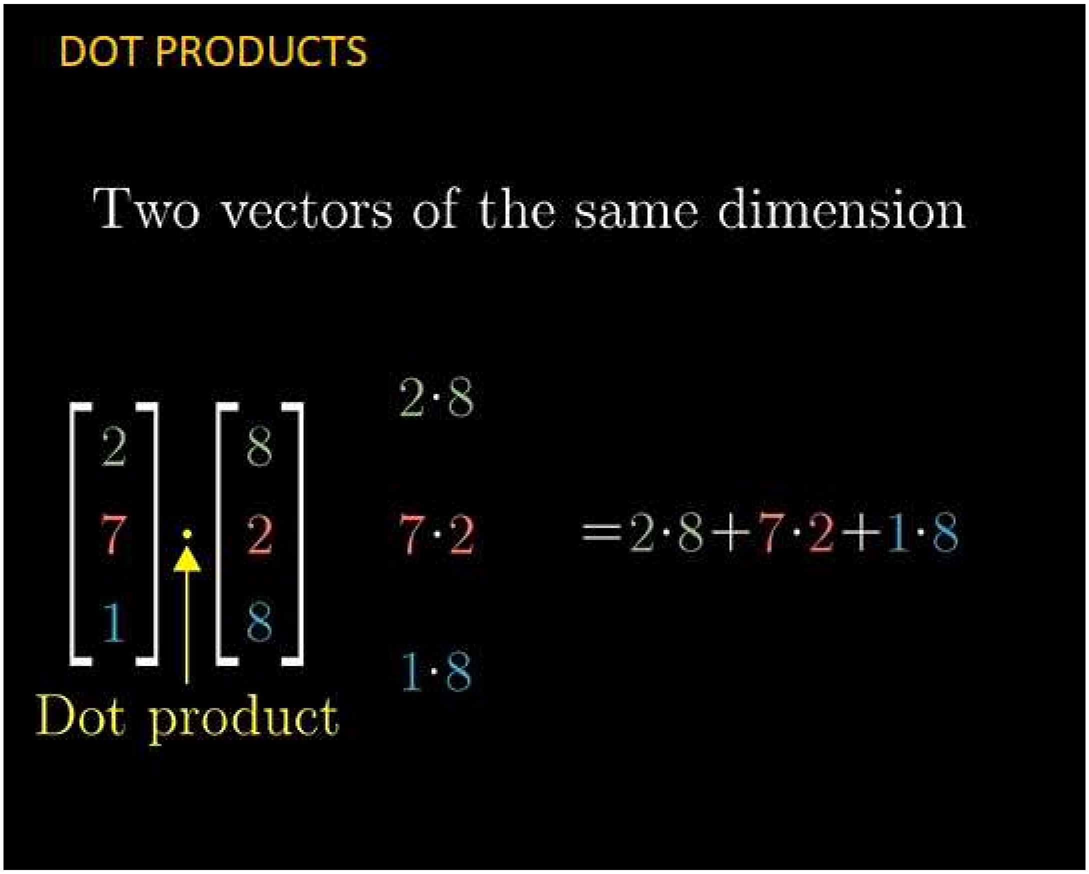

In 2D "Area" Get scalled , in 3D "Volume"
Get Scalled

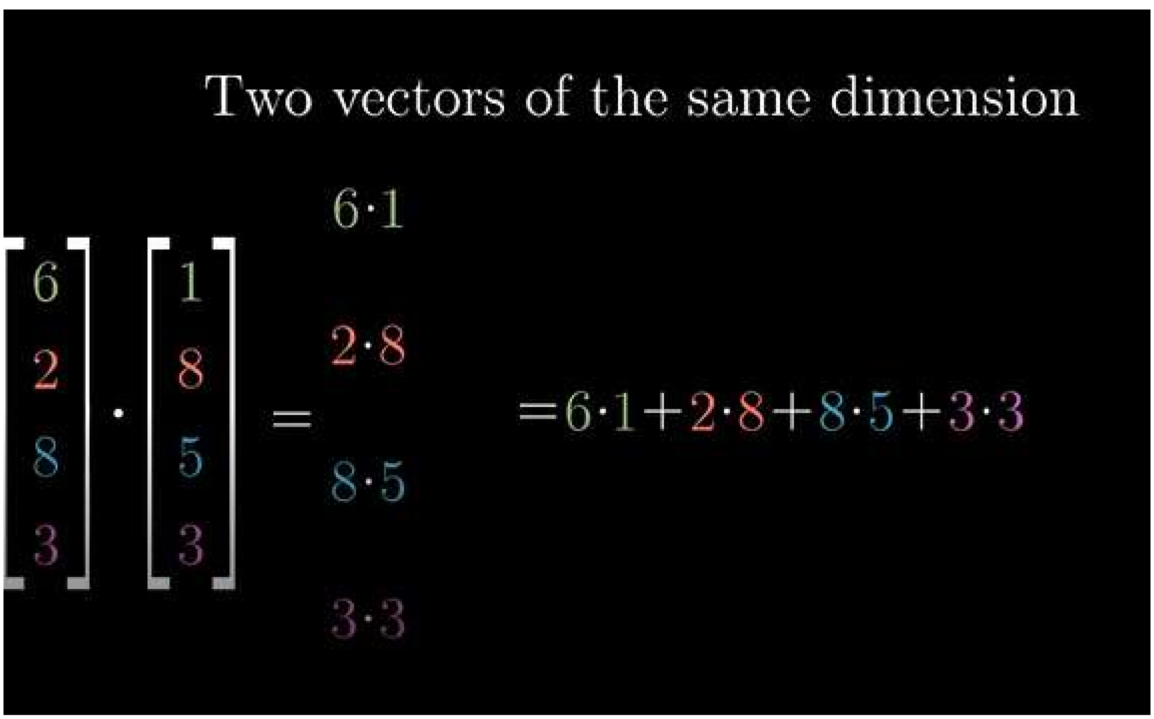




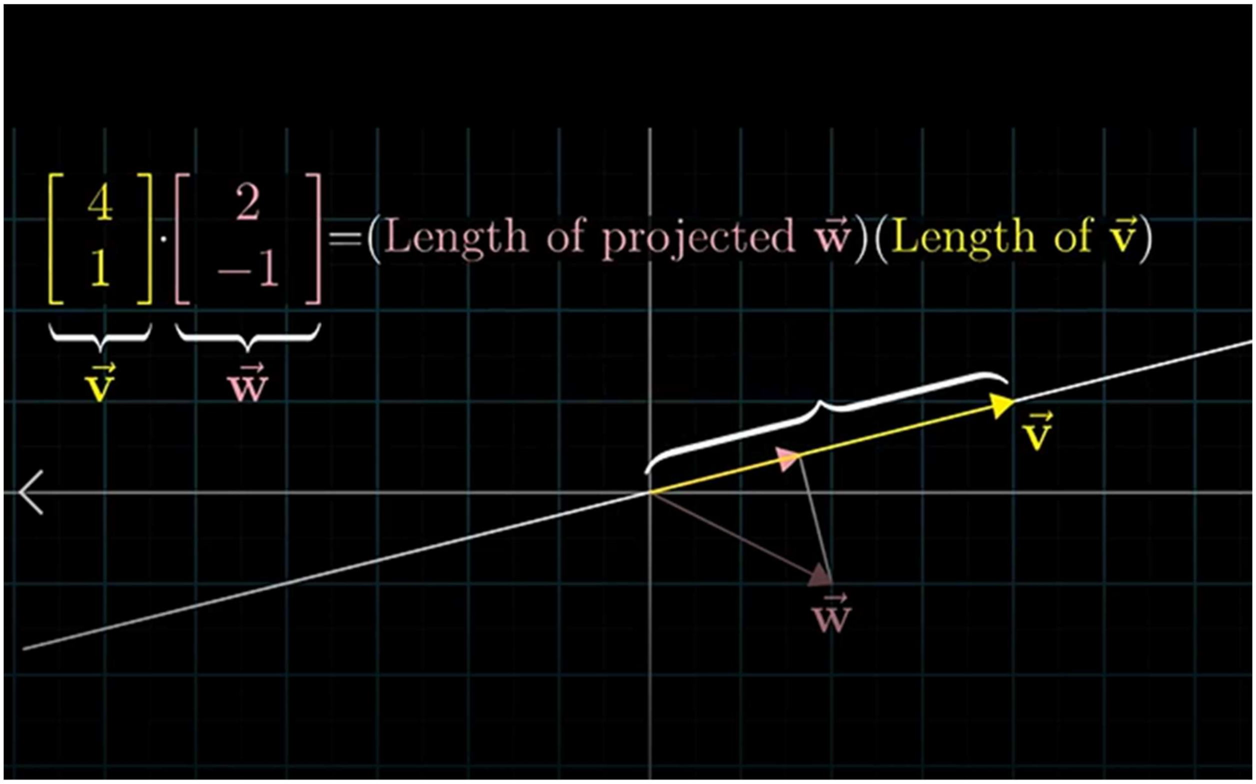
Determinant is the volume of parallelepiped

DOT PRODUCTS BETWEEN MATRICS





DOT PRODUCT GRAPHICAL REPRESENTATION



 # If two vectors are in same direction the dot product is +ve.

 # if two vectors are in opposite direction the dot product is –ve.

 # dot product can be 0 also.

 # dot product application used internally finding the independent variables co-linearity, significance ,PCA(principal component analysis

#dot products between two matrices, or two vectors means—how much they the pulling or pushing in some direction.

If Dot product +ve , they are pulling in same direction.

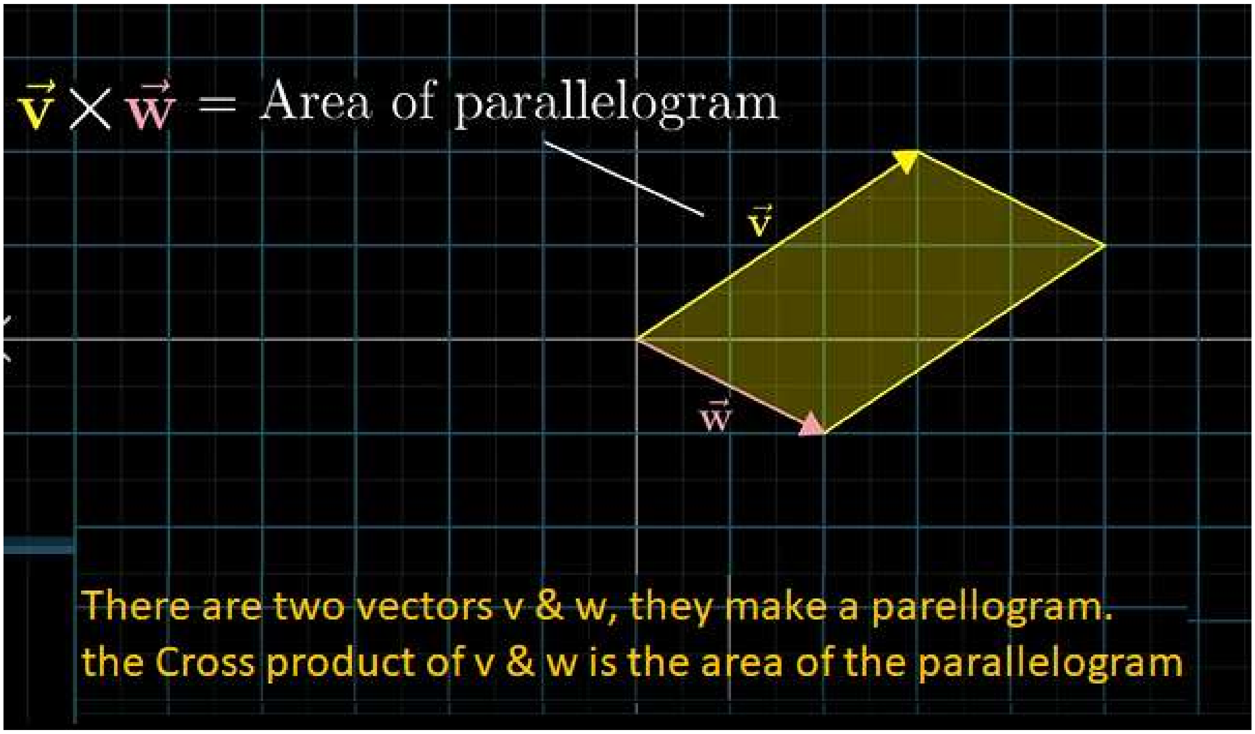
If Dot product –ve , they are pushing in different direction.

If Dot product is +1 they are same value , in same direction, , equal significance.

If Dot product is -1 they are same value , in opposite direction.

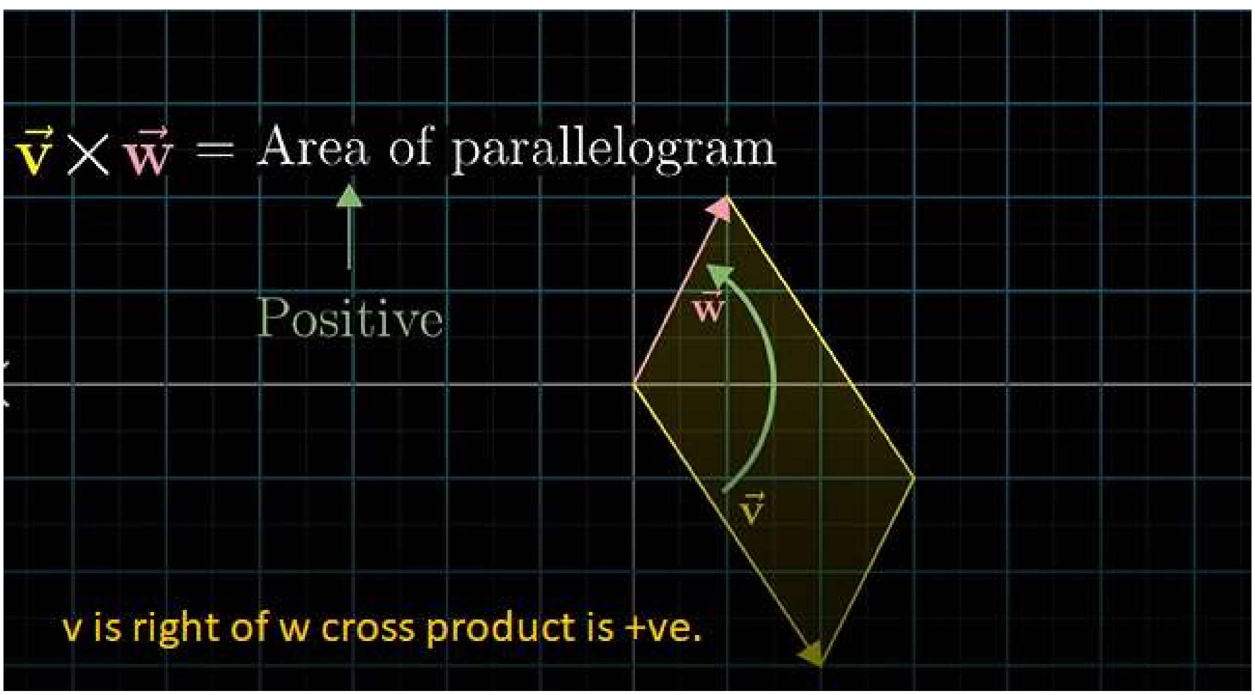
If Dot product is 0 no pulling or pushing , No impact.

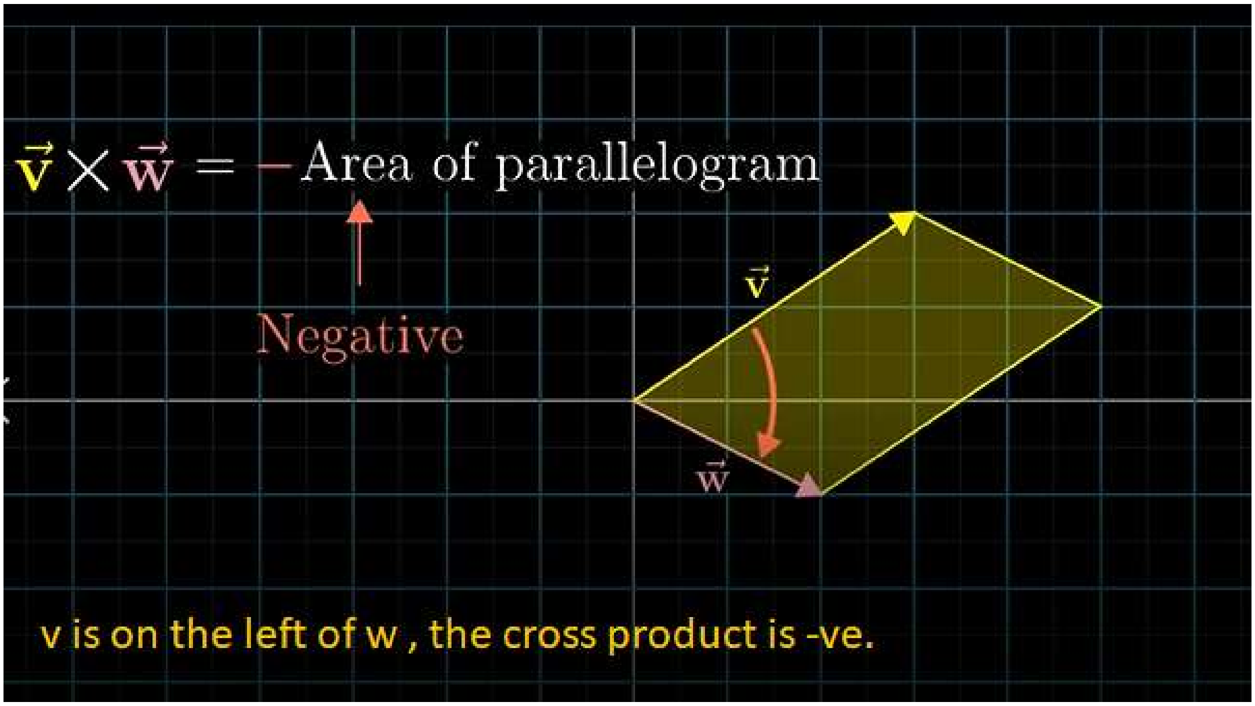
CROSS PRODUCT



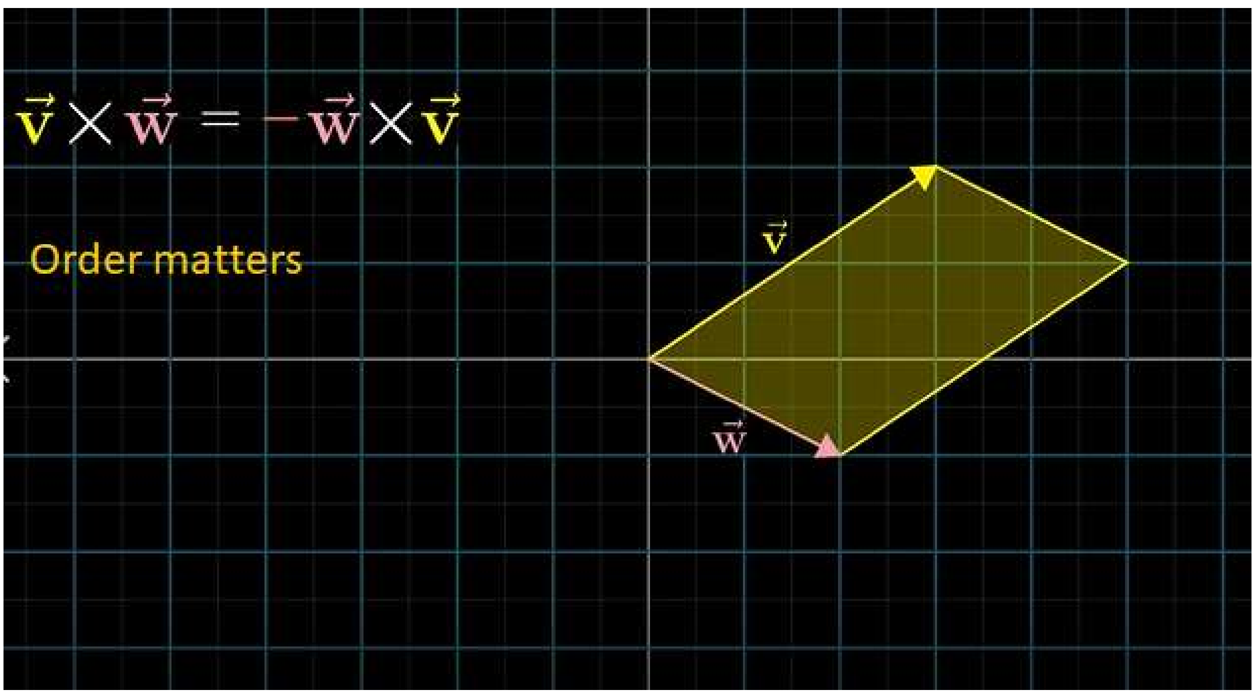
CROSS PRODUCT IS APPLIED FINDING AREA, VOLUME …

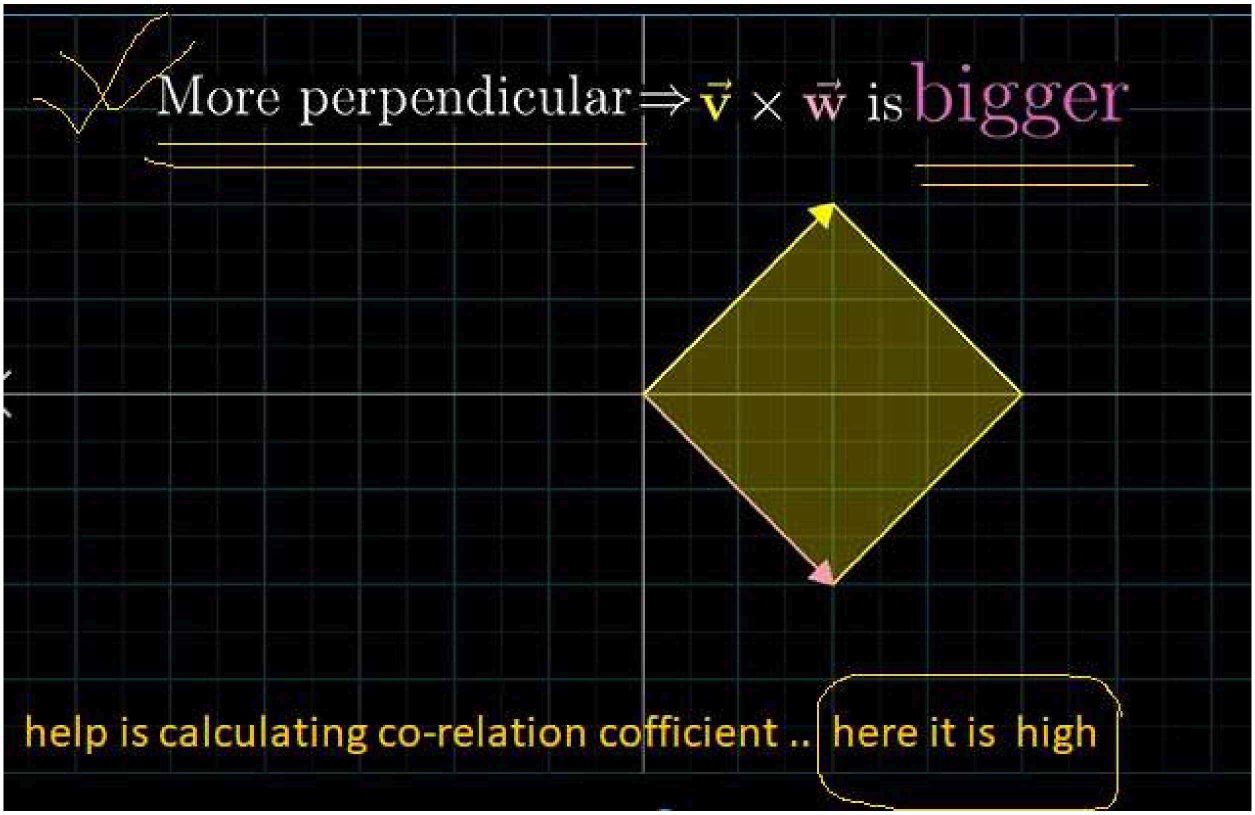
Determinant of Final Matrices… is Area, Volume.

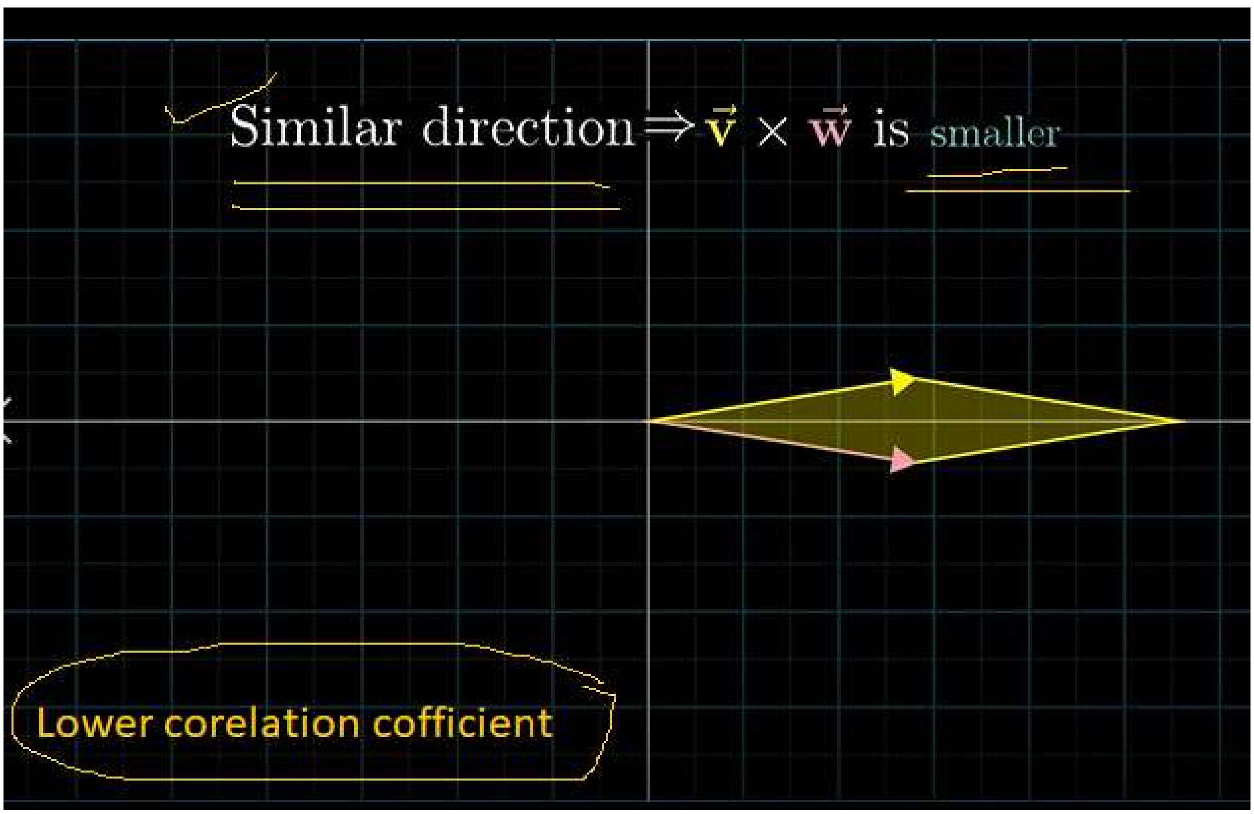




Vector order matters in cross product. (sign getchanged + ve to -ve)



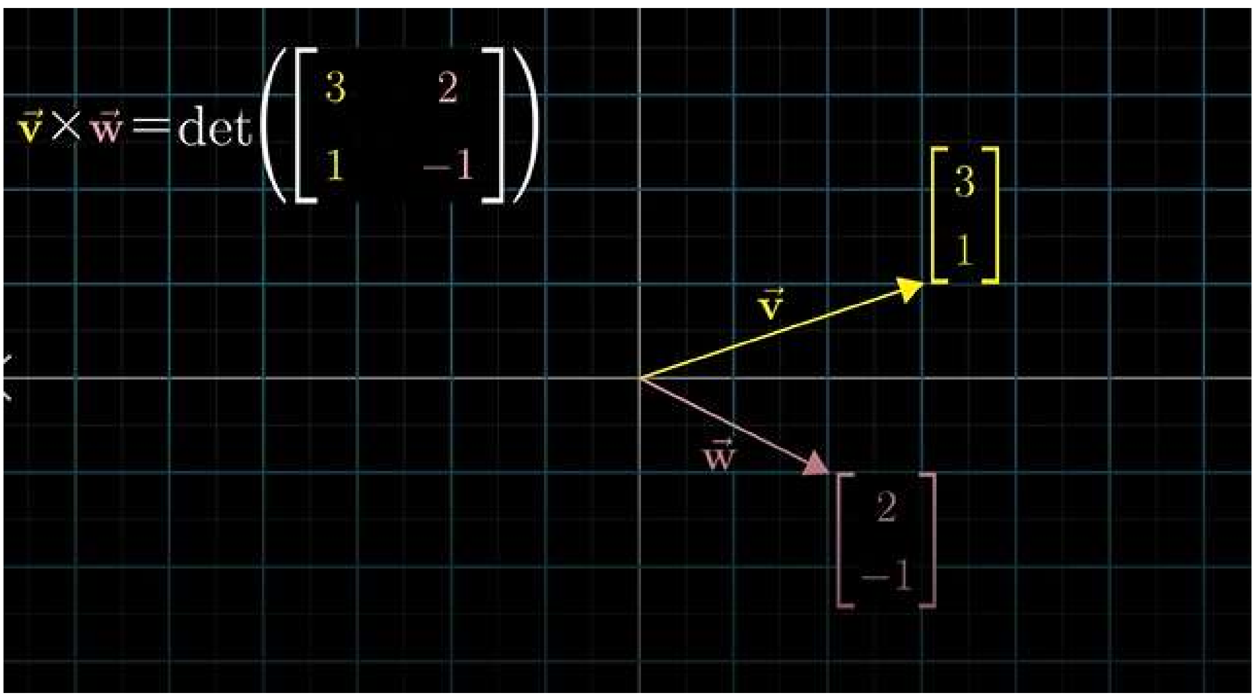

More perpendicular $\Rightarrow \vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is bigger
help is calculating co-relation cofficient .. here it is high

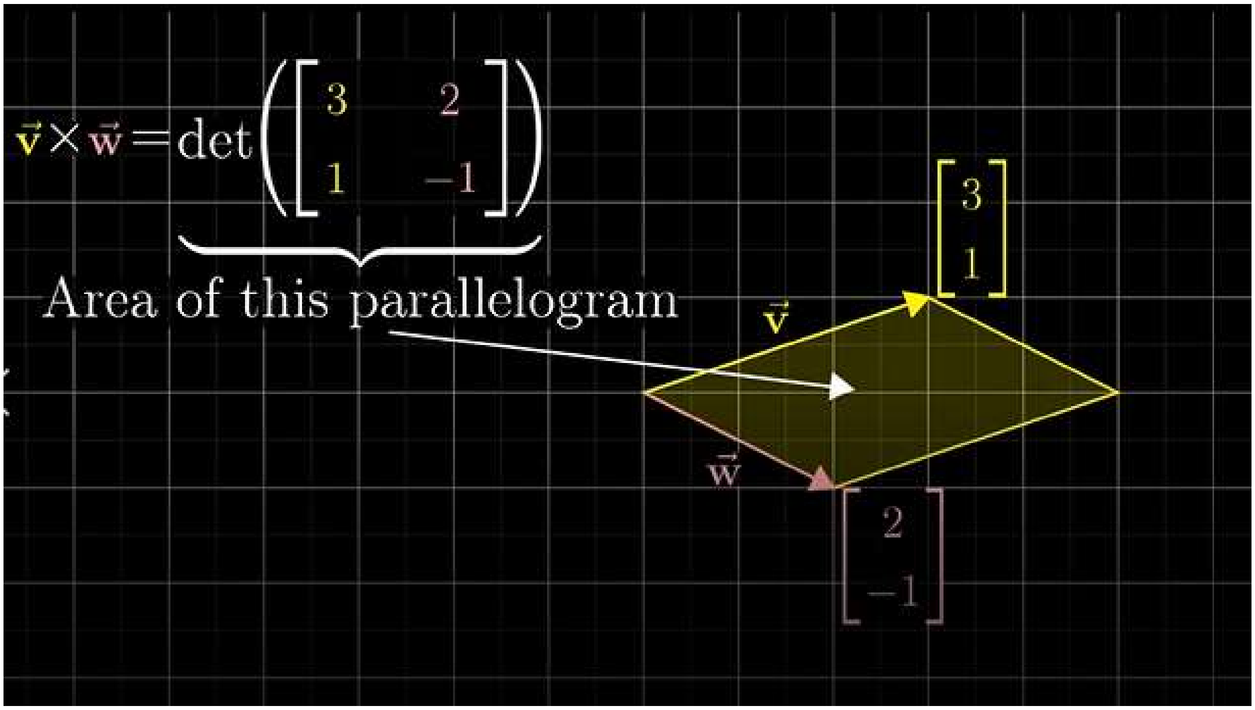

Similar direction $\Rightarrow \vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is smaller
Lower corelation cofficient

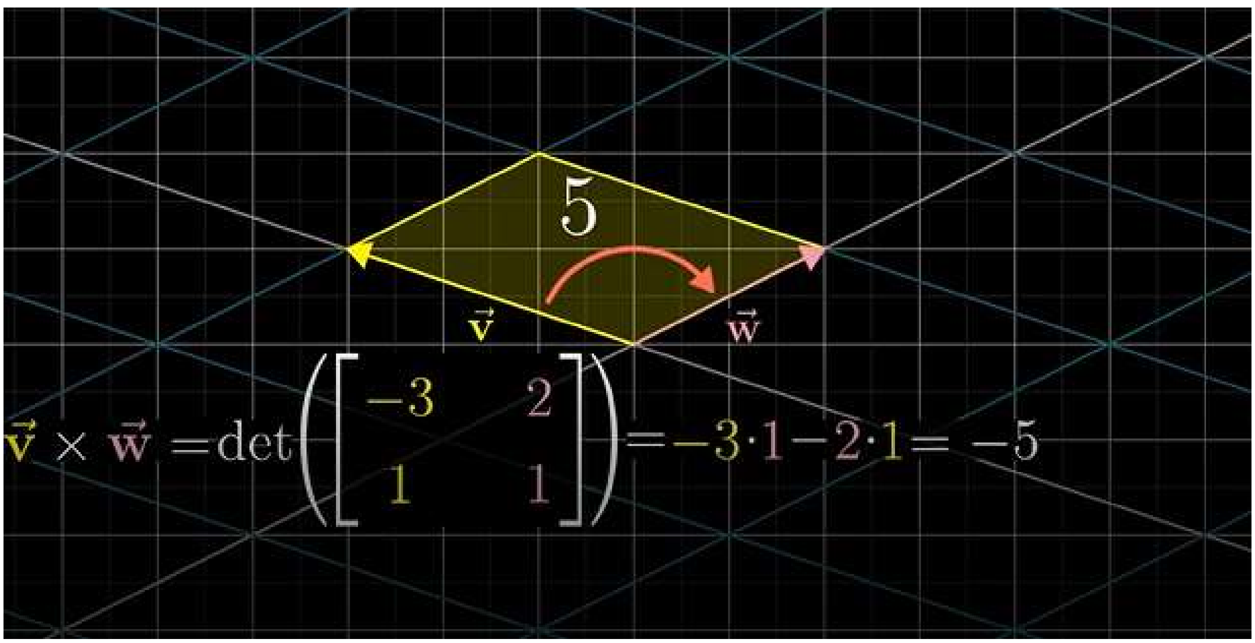
Question is how to calculate cross product (area, volume.) of two more 3,4,5…..n ..number of Vectors?.

Determinant of Transformation comes in picture……..

<u>Just refresh ….recall Matrix and our last pages…</u>

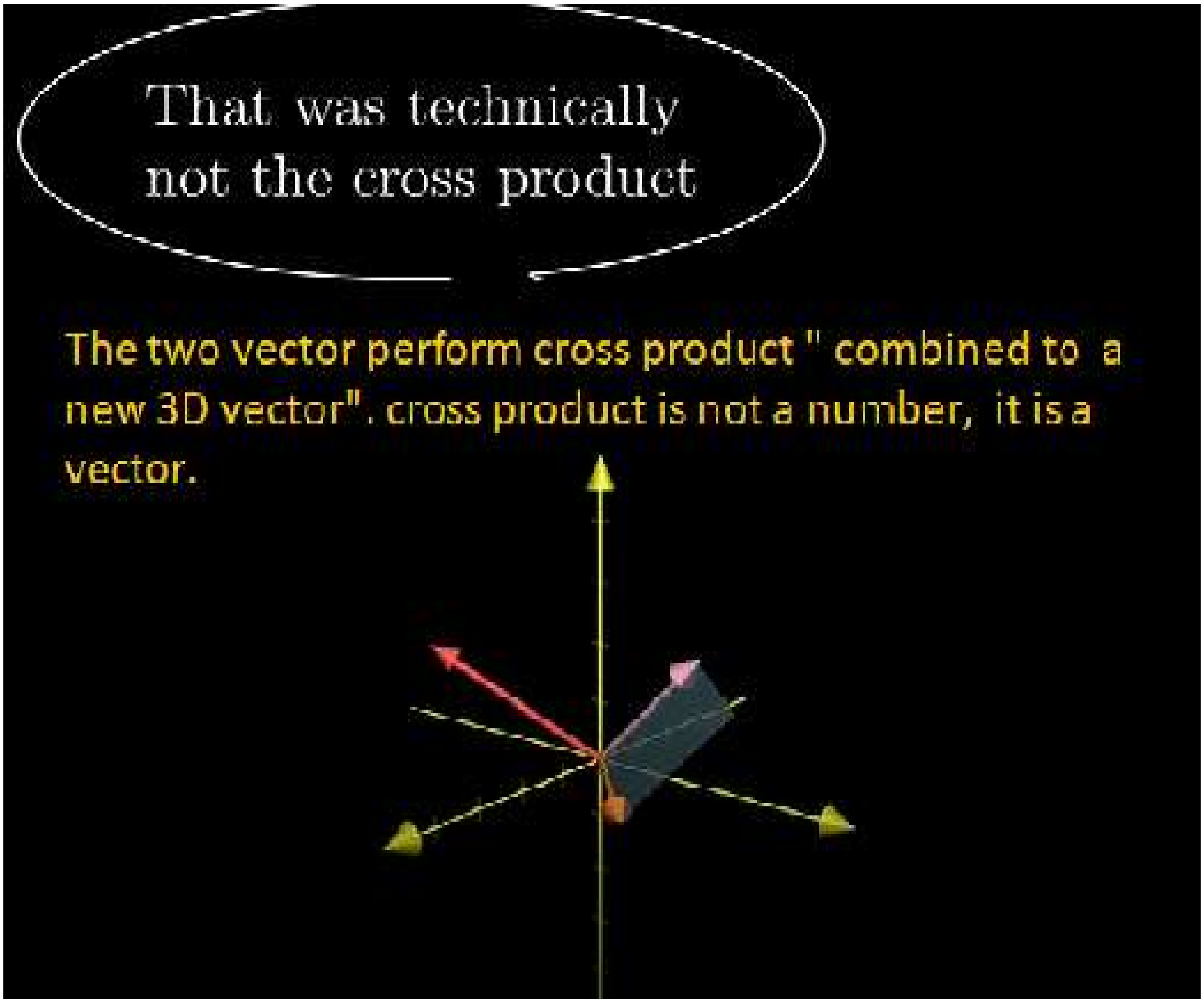






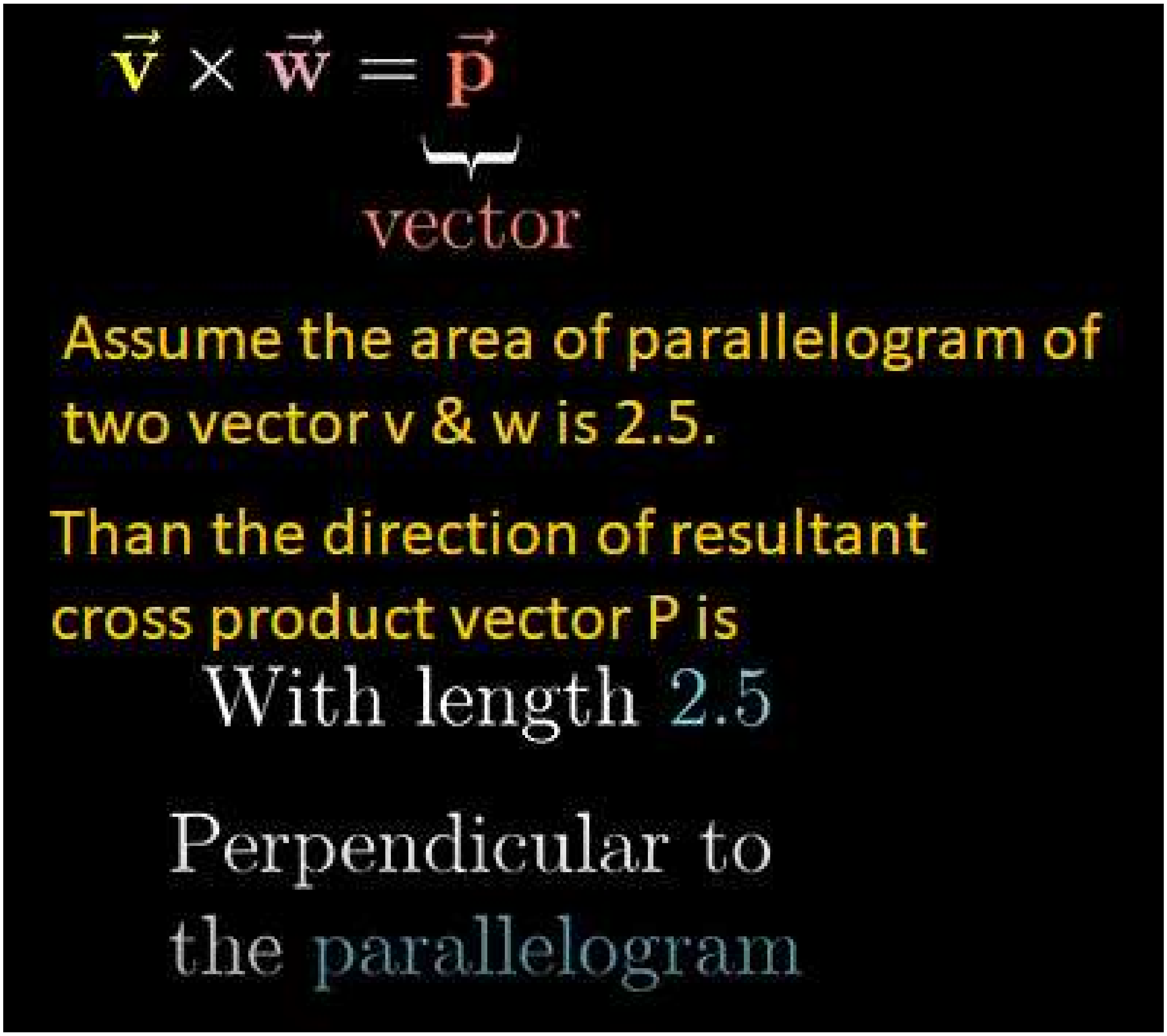
Cross product

TECHNICALLY determinant is not only the output of cross product....it is incomplete information.



- **The Two vector perform cross product , it combined to a new 3D Vector**
- **Cross Product is not a Number only (Area as calculated) But Cross product is a new Vector.**

How to define completely the Cross Product



How to find the Direction of the Cross Product:-