# Permutation & Combination

## Permutation and combination

- **When the order does matter it is permutation**
- **When the order doesn't matter it is combination**

**Example:- Your fruit salad is a combination of apple, grapes & banana , if you say it is a combination of banana, apple grapes. It is the same fruit salad.**

**Example:- The secret code of your Bank locker is 472 here the order 724 will not work. So it is a permutation lock.**

**A permutation is ordered.**

**Permutation**

**(1) Repetition is allowed: - secret lock no. can be 333**

**(2) Repetition Not allowed: - out of 5 employees A,B,C,D,E you have to choose 3 for a training program. You can't choose A, A and C.**

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## (1) Permutation with repetition: -

**When a thing has n different types, choosing r something from n different types ..**

**Total possible permutation are**

**n x n x n x n….r times = n power r**

exp:- **In the lock there are 10 numbers to choose from 0,1,2,…9 and if we choose 3 from them .**

**no. of permutation = 10 power 3= 1000 no. of permutation can be possible.**

**When repetition allowed**

**Permutation = n power r.**

## (2)Permutation without Repetition:-

Example: -
What order could 10 pool balls can be chosen.

Condition: - Repetition not allowed, means if no. 8 chosen, we can’t choose it again.

First choice has 10 possibilities
Second chose has 9 possibilities
Third has 8…. And so on…..
Total possibilities= 10x9x8x7x….1=3,62,8,800.

But assume we just want to choose 3 balls

Than possibilities=10x9x8=720

## Factorial function

! :- factorial function.

$4! = 4x3x2x1$

$0! = 1$

$(10-6)! = 4! = 4x3x2x1$

$20!/18! = 20x19x18!/18! = 20x19$

$n! = nx(n-1)x(n-2)..........x2x1)$

**The combination (order doesn't matter) so the no. of possibilities is much less as compare to permutation.**

**Exercise:- (1)how many ways we can arrange ABC**

**(a)with repetition.**

**(b)without repetition.**

**Ans:-**

**(a)with repetition = n power r= 3 power 3= 3x3x3=27.**

**(b)without repetition = n!/(n-r)! =3!/(3-3)! = 3!/0!= 3x2x1/1= 6.**

**(2) how many ways we can arrange ABCDE**

**(a)with repetition.**

**(b)without repetition.**

**(3) Possibilities of 4 digit password creation in combination of different letters a to z , in small letter and capital letter & numbers 0 to 9 .**

**(a)Repetition allowed.**

**(b)Repetition not allowed.**

Hints**: - n= 26 char in small and 26 in caps, 10 numbers.**

**Answer:-**

**(4)How many 3 letters combination can be made from the letter(ABCDE), NO Repetition .**

**(5)BANANA how many times , it can be reproduce?**

**Ans:-**

**(6) In a company there are 20 people, need to make a vigilance team of 3 people. How many different group can be made?**

**Hint: - this is a combination problem, without repetition, because a person can't be chosen twice in the team, like RAM, RAM, MOHAN.**

**Ans :-**

## Restricted Combination: -

**(1)No. of ways of selecting r things out of n different things.**

**$nCr = n!/r!(n-r)!$**

**(2)No. of ways of selecting r things out of n different things . When p particular things are always be selected.**

**$(n-p)C(r-p)$**

**Example:- out of 30 players we have to select 11.**
**(a)wicket keeper must be selected.**
**Hint:- Here p=1,n=30,r=11**
**In fact, we are selecting 10 players out of 29 players**

**(b) wicket keeper & captain must be selected.**
**Hint:-Here p=2,n=30,r=11**

**(3)Out of 100 students , we need to select 10 students, but 2 students are highly notorious , they must be rejected from the selection.**

**Here p=2**

**Here finally we have to use only 98 no. of students.**

**(4)Number of selection of r things from n different things , when p particular things are not together in any selection :-**

For example:- **In an organization some husbands wives are working together. In any committee the Husband & wife both should not be there.**

$${}_{n}C_{r} - {}_{(n-p)}C_{(r-p)}$$

**Q . In an organization ABC there are 100 employees, A investigation committee of member 10 is required for form. There are 4 couples (husband and wives) in the company, the condition is both husband and wife should not be in the investigation committee. Find the no. of possibilities of selection?**

**Ans:-**

**Q. There are 5 professors, 4 Doctors, 3 Teachers, 2 Bankers.**

**Make a 5-member committee: -**

**There must be 1 professor, 1 teacher, 2 Banker, 1 Doctor.**

**How many different committee, we can make?**

**Ans:-**

**Q. There are 7 Male and 6 Female in a group.**

**Make a 5-member committee so that at least 3 males are there in the committee. In how many ways it can be done?**

# PROBABILITY

## Randomness

## Random: -True Random means no prediction Power. We can’t predict if it is true Random.

**Example: - Flipping a Coin.**

**Probability of Head=**

**Probability of Tail=**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| H | H | T | T | T | H | H | T | T | T |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| H | H | H | T | H | H | T | H | T | H |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 100 |
| H | H | T | T | T | H | H | T | T | T | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10000 |
| H | H | T | T | T | H | H | T | T | T | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20000 |
| H | H | T | T | T | H | H | T | T | T | |

**Randomness**

**(1) Deterministic**

**(2) Non Deterministic**

**Non Deterministic**

**(1)Random Stochastic Phenomenon →**

**Unable to predict the outcome but in long run (large no. of trial) the outcome exhibits statically regularity.**

**(2)Haphazard Random Phenomenon→**

**Unpredictable outcome. but even in long run (large no. of trial or experiments) the still the outcome exhibit statically irregular.**

**Probability & Inference: - are used everywhere.**

**Inference → A conclusion reached on the basis of evidence & Reasoning.**

## Probability and Likelihood: -

- **Probability refers to the chance that a particular outcome occurs based on the values of parameters in a model.**
- **Likelihood refers to how well a sample provides support for particular values of a parameter in a model.**
- **The term Likelihood refers to the process of determining the best data distribution given a specific situation in the data.**
- **When calculating the probability of a given outcome, you assume the model's parameters are reliable.**
- **Probability is used to finding the chance of occurrence of a particular situation,**
- **whereas Likelihood is used to generally maximizing the chances of a particular situation to occur.**

**P(H)=1/2 # probability of getting head when toss a coin.**

**P(T)= 1/2 # probability of getting head when toss a coin.**

**# what is the probability of getting 2 heads when we toss two coins together?**

**Q.A box contains 20 cards, numbered from 1 to 20. A card is drawn from the box at random. Find the probability that the number on the card drawn is (i) even (ii) prime and (iii) multiple of 3.**

**Ans:-**

**Even number = 2,4,6,8,10,12,14,16,18,20**

**Prime Number = 2,3,5,7,11,13,17,19**

**Multiple of 3 = 3,6,9,12,15,18**

(i)Probability of getting an Even Number

$$= \frac{\textbf{Number of Even Numbers}}{\textbf{Total Numbers}} = \frac{10}{20}$$

(ii)Probability of getting a prime number

$$= \frac{\textbf{Number of prime Numbers}}{\textbf{Total Numbers}} = \frac{8}{20}$$

(iii)Probability of getting a multiple of 3

$$= \frac{\textbf{Number of mu;ltiples of 3}}{\textbf{Total numbers}} = \frac{6}{20}$$

## Event & Sample space

**In a random experiment**

**Set of all possible outcomes→ Sample space S**

**Set of desired possible outcome→Event E**

**Event E is the Subset of Sample Space S.**

**Example:-**

**Tossing a dice., getting even no.**

**S=6# (1,2,3,4,5,6)**

**E=3# getting even no.(2,4,6)**

**Probability=E/S=3/6=1/2.**

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**EVENTS**

**(1) Dependent Events or conditional**

**(2) Independent Events.**

**Dependent Event: - A card is chosen out of 52 cards( no replacement after drawn).**

**4/52 → probability first card is Queen.**

**4/51→ Probability next card is Jack.**

**P(Queen and Jack)→ 4/52 x 4/51=16/2652=4/663.**

**P(Queen OR Jack)→4/52 + 4/51.**

**Independent Event: - A card is chosen out of 52 cards (replacement done after drawn).**

**4/52 → probability first card is Queen.**

**4/52→ Probability next card is Jack.**

**P(Queen and Jack)→ 4/52 x 4/52.**

**P(Queen OR Jack)→4/52 + 4/52.**

## Exercise:-

The table below shows data on the monthly income of employed people in two residential areas. Representative samples were used in the collection of the data.

| MONTHLY INCOME (IN RS) | Area 1 | Area 2 | TOTAL |
|---|---|---|---|
| $x < 3\,200$ | 173 | 101 | 274 |
| $3\,200 \leq x < 25\,600$ | 218 | 709 | 927 |
| $x \geq 25\,600$ | 476 | 1 214 | 1 690 |
| **Total** | 867 | 2 024 | 2 891 |

What is the probability that a person chosen randomly from the entire sample will be:

**(1)A person who earns at least Rs 3200 but less than Rs 25600 per month?**

**(2)A person from Area 2 who earns at least Rs 3200 but less than Rs 25600 per month?**

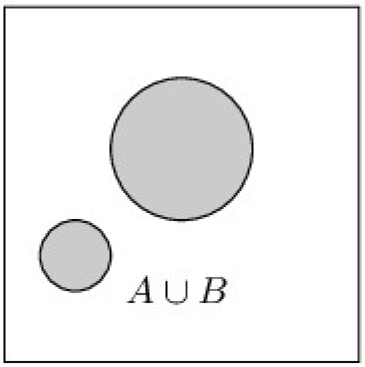
**(3)A person from Area 2 who earns at least Rs 3200 per month?**

**ANS:-**

# Union and Intersection

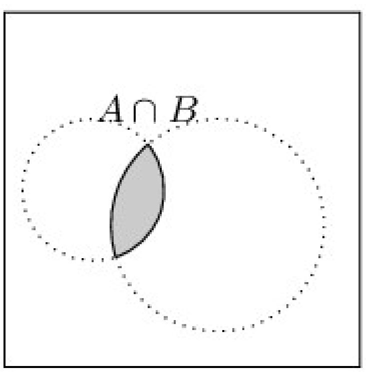
### Union

The union of two EVENTS that contains all of the elements that are in at least one of the two sets. The union is written as A∪B it is **"A or B"**.
AUB occurs if the event A occurs or the Event B occurs.




### Intersection

The intersection of two sets is a new set that contains all of the elements that are in both sets. The intersection is written as A∩B it is **"A and B"**.
A∩B occurs means Event A and Event B both occurs.




**$P(A \cup B)=P(A)+P(B)-P(A \cap B)$**

**$P(A \cap B)= P(A)+P(B)- P(A \cup B)$**

## Mutually exclusive

**Two events A & B are called mutually exclusive if**

**$A \cap B = \emptyset$ null event→ Event A and B can't occur together**

**Exp:- flipping a coin , Head and Tail can't occurs together**

**$P(A \cup B) = P(A)+P(B)$**

## Compliment :-

**Compliment of A →Probability(Not A)=1-P(A)**

**Q. In a class of 250 students, 150 like orange Juice, 90 like Apple Juice, 60 like both Juice.**

**(a)How many students like either orange or Apple Juice?**

**(b)How many don't like any juice?**

**Ans:-**

## Conditional probability or dependent probability: -

**If we are told that event B has occurred, what is the probability that Event A will also occur.?**

**P(A/B)→ Probability of A , when B already occurred.**

# P(A/B) = P(A∩B)/P[B]

**Q. If P(A)=7/13, P(B)=9/13, P(A∩B)=4/13.**

**What is P(A/B)?**

**P(A/B)→Probability of A when B has already occurred.**

**Ans:-P(A/B)=P(A∩B)/P(B)= (4/13)/(9/13)=4/9.**

**Q.A family has two children , what is the probability that both the children are boys. Given that at least one of them is a boy.**

**Ans:- Recall the Formula**

**P(A/B)→ Probability of A , when B already occurred.**

$$P(A/B) = P(A\cap B)/P[B]$$

**E→ both the children are boy.**

**F→at least one of them is a boy.**

**Sample space={bb,bg,gb,gg}**

**$P(E)=1/4$**

**$P(F)=3/4$**

**$(E\cap F)=\{bb\}$**

**$P(E\cap F)=1/4$**

**P(E/F)→ probability that both children are boy , given that one child is boy.**

**$P(E/F)= P(E\cap F)/P(F)=(1/4)/3/4)=1/3.$**

**Q.** **A movie is released, for a specific married couple the probability that husband will watch the movie is 80%.The probability that wife watch the movie is 65 %. The probability that both will watch the movie is 60%.**

**If the husband is watching the movie,**

**what is the probability that wife is also watching the movie?**

**Ans:- Recall the Formula**

**P(A/B)→ Probability of A , when B already occurred.**

$$P(A/B) = P(A \cap B)/P[B]$$

Q. In a school there are 1000 students, out of which 430 are girls, it is known that out of 430 girls 10% girls are studying in class 12 .

A student chosen randomly from the school,

what is the probability that chosen one is a student of class 12?.

It is given that the chosen student is a girl.

**Ans:- Recall the Formula**

**P(A/B)→ Probability of A , when B already occurred.**

$$P(A/B) = P(A \cap B)/P[B]$$

Q.

| owner | Probability Have pet animal | Probability Don't have Pet animal | total |
|---|---|---|---|
| male | 0.41 | 0.08 | 0.49 |
| female | 0.45 | 0.06 | 0.51 |
| total | 0.86 | 0.14 | 1 |

What is the probability that the randomly selected person is a male?

Given that the selected person has a Pet animal?

**Ans:- Recall the Formula**

**P(A/B)→ Probability of A , when B already occurred.**

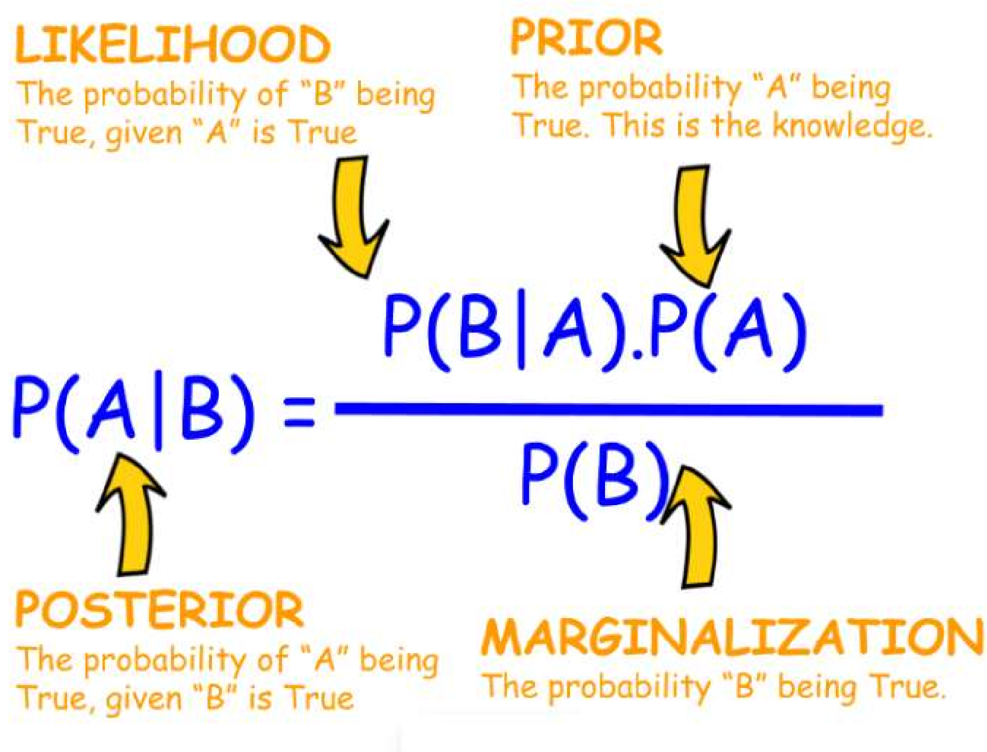
$$P(A/B) = P(A \cap B)/P[B]$$

## Bayes Theorem

**Named after Thomas Bayes.**

**Describes the probability of an event based on the prior knowledge of conditions that might be related to the event.**

**The conditional Probability is known as hypothesis. The Hypothesis is calculated through previous evidence or knowledge.**



**Deriving Bayes Equation:-**

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B|A)\ P(A)$$

$$P(A \cap B) = P(A|B)\ P(B)$$

**Solving above two equations we get Bayes Theorem.**

Besides statistics, the Bayes' theorem is also used in various disciplines, with medicine and pharmacology as the most notable examples. In addition, the theorem is commonly employed in different fields of finance. Some of the applications include but are not limited to, modeling the risk of lending money to borrowers or forecasting the probability of the success of an investment.

## These type of computations are actually used for SPAM Filters. we need to know a conditional probability.

**For Example,**

**The probability that the word MONEY word appears in an email, given that the email is spam is 8%.**

**Probability that an email can be a spam =20%**

**Probability that Money can appear in an email=2.4%**

**M→Event of Money, S→ Even of SPAM**

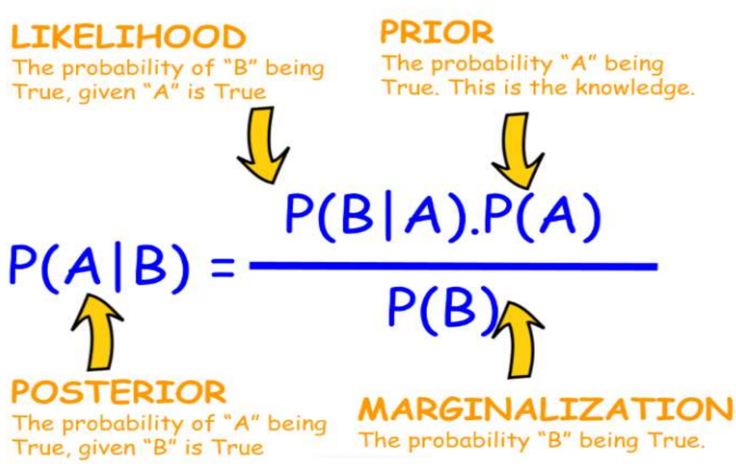
**P(Money/Spam)=0.08, P(Spam)=0.2 , P(Money)=0.024**

**But We are interested to know the probability that the email is spam, given that Money appears in the email.**

**P(Spam/Money)=?**

**P(Spam/Money)=P(Money/Spam)x P(Spam)/P(Money)**

**=0.08x0.2/0.024=0.67=67%.**



## Example 1

What is the probability of a patient having liver disease if they are alcoholic?
Given data(**Prior Information): -**
(1)As per earlier records of the clinic, it states that 10% of the patient's entering the clinic are suffering from liver disease.
(2)Earlier records of the clinic showed that 5% of the patients entering the clinic are alcoholic.
(3)Earlier records of the clinic showed, 7% out of the patient's that are diagnosed with liver disease, are alcoholics.
This defines the B|A: probability of a patient being alcoholic, given that they have a liver disease is 7%.
**What is the probability that Patient Being Alcoholic, chances that he is having a liver disease?**
**Ans:**

P(A)=Probability that Patient having liver disease =0.10
P(B)= Probability that Patient is alcoholic =0.05
P(A|B)= Probability that Patient having liver disease, it is known that he is alcoholic =?
P(B|A)= Probability that Patient is alcoholic having liver disease=0.07.
As, per Bayes theorem formula,
$P(A|B) = (P(B|A) * P(A))/ P(B)$
$P(A|B) = (0.07 * 0.1)/0.05 = 0.14$
Therefore, for a patient being alcoholic, the chances of having a liver disease is 0.14 (14%).
This is a large increase from 10% suggested by past data.

## Example 2

In a particular pain clinic , 10% of patients are prescribed narcotic pain killers. Overall 5% of clinic patients are addicted to narcotics (including pain killers and illegal substances)

Out of all the people prescribed pain killers 8% are addicted.

If a patient is addicted what is the probability that he is prescribed pain killer?

ANS:-

## Example 3

Black box manufactures for aircraft

A →75% production → defect 4%.

B →15% production → defect 6%.

C →10% production → defect 8%.

A defective Black box is randomly chosen, what is the probability that it was manufactured by company A?

Ans:-

## Example 4

**A factory produces an item using three machines A,B & C which account for 20%, 30% and 50% of its output respectively.**

**Defective Item manufactured by machine A, B & C are 5 % , 3 % and 1% respectively. If a randomly selected item is defective , what is the probability is was produced by machine C?**

**Ans:-**

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**Please Note:-** **There are several forms of Baye's theorem out there, and they are all equivalent (they are written in slightly different ways)**

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|\text{not } B)\,P(\text{not } B)}$$

**Example Q.**

**1% of people have genetic defect.**

**90% of test correctly detect the Genetic defect (True Positive).**

**9.6% of the test are False Positive.**

**If a person gets a positive result, what is the probability that actually the person has the genetic defect?**

**Ans:-**

**Assume A=Positive test report**

**B=Person having Genetic Disorder.**

**P(A/B)=Probability of finding the Positive report when the person having the Genetic disorder =90%.**

**P(B)=Probability the person has genetic defect=1%**

**P(A/~B)=Probability with positive test result person has not genetic defect=9.6%**

**P(~B)=1-P(B)=100-01=99% =Probability that Person has not genetic defect=99%**

**P(B/A)=probability that actually the person has the genetic defect when the test showing the Positive Report.**

**P(B/A)=?**

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|\text{not } B)\,P(\text{not } B)}$$

**P(B/A)=(0.9x0.01)/[(0.9x0.01)+(0.096x0.99)]**

**=0.09/[0.09+0.09504]**

**=0.09/0.18504**

**=0.4863=48.63 % chance that in spite of having a positive test report person is patient of Genetic disorder.**

## Try Yourself:-

**Question :- 1% of the population has certain disease. If an infected person is tested, then there is a 95% chance that the test gives an erroneous positive result ("False Positive").**

**Given that a person tests positive, what are the chances that he has the disease?**

**Answer:-**