Statistics Cheat Sheet

Population

The entire group one desires information about

Sample

A subset of the population taken because the entire population is usually too large to analyze
Its characteristics are taken to be representative of the population

Mean

Also called the arithmetic mean or average
The sum of all the values in the sample divided by the number of values in the sample/population
μ is the mean of the population; $\bar{x}$ is the mean of the sample

Median

The value separating the higher half of a sample/population from the lower half
Found by arranging all the values from lowest to highest and taking the middle one (or the mean of the middle two if there are an even number of values)

Variance

Measures dispersion around the mean
Determined by averaging the squared differences of all the values from the mean
Variance of a population is σ^2

$$\sigma^2 = \frac{\sum (x-\mu)^2}{n}$$

Can be calculated by subtracting the square of the mean from the average of the squared scores:

$$\sigma^2 = \frac{\sum x^2}{n} - \mu^2$$

Variance of a sample is s^2; note the *n-1*

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

Can be calculated by:

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

Standard Deviation

Square root of the variance
Also measures dispersion around the mean but in the same units as the values (instead of square units with variance)
σ is the standard deviation of the population and *s* is the standard deviation of the sample

Standard Error

An estimate of the standard deviation of the sampling distribution—the set of all samples of size *n* that can be taken from a population
Reflects the extent to which a statistic changes from sample to sample

For a mean, $\frac{s}{\sqrt{n}}$

For the difference between two means,

Assuming equal variances $\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$; unequal variances $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$

T-test

One-Sample

Tests whether the mean of a normally distributed population is different from a specified value

Null Hypothesis (H_0): states that the population mean is equal to some value (μ_0)
Alternative Hypothesis (H_a): states that the mean does not equal/is greater than/is less than μ_0
t-statistic: standardizes the difference between $\bar{x}$ and μ_0

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Degrees of freedom (df) = *n-1*

Read the table of t-distribution critical values for the p-value (probability that the sample mean was obtained by chance given μ_0 is the population mean) using the calculated t-statistic and degrees of freedom.

- H_a: $\mu > \mu_0$ → the t-statistic is likely positive; read table as given
- H_a: $\mu < \mu_0$ → the t-statistic is likely negative; the t-distribution is symmetrical so read the probability as if the t-statistic were positive
- Note: if the t-statistic is of the 'wrong' sign, the p-value is 1 minus the *p* given in the chart
- H_a: $\mu \neq \mu_0$ → read the p-value as if the t-statistic were positive and double it (to consider both less than and greater than)

If the p-value is less than the predetermined value for significance (called α and is usually 0.05), reject the null hypothesis and accept the alternative hypothesis.

Example:
You are experiencing hair loss and skin discoloration and think it might be because of selenium toxicity. You decide to measure the selenium levels in your tap water once a day for one week. Your results are given below. The EPA maximum contaminant level for safe drinking water is 0.05 mg/L. Does the selenium level in your tap water exceed the legal limit (assume α=0.05)?

Day	Selenium mg/L
1	0.051
2	0.0505
3	0.049
4	0.0516
5	0.052
6	0.0508
7	0.0506

H_0: $\mu = 0.05$; H_a: $\mu > 0.05$
Calculate the mean and standard deviation of your sample:

$$\bar{x} = 0.0508$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{(0.051 - 0.0508)^2 + (0.0505 - 0.0508)^2 + etc...}{6} = 9.15 \times 10^{-7}$$

$$s = \sqrt{s^2} = 9.56 \times 10^{-4}$$

The t-statistic is: $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{0.0508 - 0.05}{\frac{9.56 \times 10^{-4}}{\sqrt{7}}} = 2.17$ and the degrees of freedom are *n-1* = 7-1 = 6

Looking at the t-distribution of critical values table, 2.17 with 6 degrees of freedom is between *p*=0.05 and *p*=0.025. This means that the p-value is less than 0.05, so you can reject H_0 and conclude that the selenium level in your tap water exceeds the legal limit.

T-test

Two-Sample

Tests whether the means of two populations are significantly different from one another

Paired

Each value of one group corresponds directly to a value in the other group; ie: before and after values after drug treatment for each individual patient

Subtract the two values for each individual to get one set of values (the differences) and use $\mu_0 = 0$ to perform a one-sample t-test

Unpaired

The two populations are independent

H_0: states that the means of the two populations are equal ($\mu_1 = \mu_2$)

H_a: states that the means of the two populations are unequal or one is greater than the other ($\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$, $\mu_1 < \mu_2$)

t-statistic:

assuming equal variances: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ assuming unequal variances: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}}$

degrees of freedom = $(n_1\text{-}1)+(n_2\text{-}1)$

Read the table of t-distribution critical values for the p-value using the calculated t-statistic and degrees of freedom. Remember to keep the sign of the t-statistic clear (order of subtracting the sample means) and to double the p-value for an H_a of $\mu_1 \neq \mu_2$.

Example:

Consider the lifespan of 18 rats. 12 were fed a restricted calorie diet and lived an average of 700 days (standard deviation=21 days). The other 6 had unrestricted access to food and lived an average of 668 days (standard deviation=30 days). Does a restricted calorie diet increase the lifespan of rats (assume α=0.05)?

μ_1=700, s_1=21, n_1=12; μ_2=668, s_2=30, n_2=6

H_0: $\mu_1=\mu_2$

H_a: $\mu_1>\mu_2$ (because we are only asking if a restricted calorie diet increases lifespan)

We cannot assume that the variances of the two populations are equal because the different diets could also affect the variability in lifespan.

The t-statistic is: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}} = \dfrac{700 - 668}{\sqrt{\dfrac{21^2}{12} + \dfrac{30^2}{6}}} = 2.342$

Degrees of freedom = $(n_1\text{-}1)+(n_2\text{-}1)$ = (12-1)+(6-1)=16

From the t-distribution table, the p-value falls between 0.01 and 0.02, so we do reject H_0. The restricted calorie diet does increase the lifespan of rats.

Chi-Square Test

For Goodness of Fit

Checks whether or not an observed pattern of data fits some given distribution

H_0: the observed pattern fits the given distribution

H_a: the observed pattern does not fit the given distribution

The chi-square statistic is: $\chi^2 = \sum \dfrac{(O-E)^2}{E}$ (*O* is the observed value and *E* is the expected value)

Degrees of freedom = number of categories in the distribution – 1

Get the p-value from the table of χ^2 critical values using the calculated χ^2 and df values. If the p-value is less than α, the observed data does not fit the expected distribution. If $p>\alpha$, the data likely fits the expected distribution

***Example 1*:**

You breed puffskeins and would like to determine the pattern of inheritance for coat color and purring ability. Puffskeins come in either pink or purple and can either purr or hiss. You breed a purebred, pink purring male with a purebred, purple hissing female. All individuals of the F_1 generation are pink and purring. The F_2 offspring are shown below. Do the alleles for coat color and purring ability assort independently (assume α=0.05)?

Pink and Purring	Pink and Hissing	Purple and Purring	Purple and Hissing
143	60	55	18

Independent assortment means a phenotypic ratio of 9:3:3:1, so:

H_0: the observed distribution of F_2 offspring fits a 9:3:3:1 distribution

H_a: the observed distribution of F_2 offspring does not fit a 9:3:3:1 distribution

The expected values are:

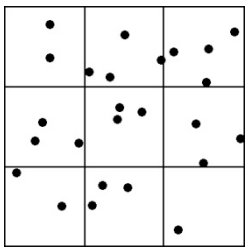
Pink and Purring	Pink and Hissing	Purple and Purring	Purple and Hissing
155.25	51.75	51.75	17.25

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(143-155.25)^2}{155.25} + \frac{(60-51.75)^2}{51.75} + \frac{(55-51.75)^2}{51.75} + \frac{(18-17.25)^2}{17.25} = 2.519$$

df=4-1=3

From the table of χ^2 critical values, the p-value is greater than 0.25, so the alleles for coat color and purring ability do assort independently in puffskeins.

Example 2:



You are studying the pattern of dispersion of king penguins and the diagram on the right represents an area you sampled. Each dot is a penguin. Do the penguins display a uniform distribution (assume α=0.05)?

H_0: there is a uniform distribution of penguins

H_a: there is not a uniform distribution of penguins

There are a total of 25 penguins, so if there is a uniform distribution, there should be 2.778 penguins per square. There actual observed values are 2, 4, 4, 3, 3, 3, 2, 3, 1, so the χ^2 statistic is:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(1-2.778)^2}{2.778} + 2\left(\frac{(2-2.778)^2}{2.778}\right) + 4\left(\frac{(3-2.778)^2}{2.778}\right) + 2\left(\frac{(4-2.778)^2}{2.778}\right) = 2.72$$

df=9-1=8

From the table of χ^2 critical values, the p-value is greater than 0.25, so we do not reject H_0. The penguins do display a uniform distribution.

Chi-Square Test

For Independence

Checks whether two categorical variables are related or not (independence)

H_0: the two variables are independent

H_a: the two variables are not independent

Does not make any assumptions about an expected distribution

The observed values ($\#_1$, $\#_2$, $\#_3$, and $\#_4$) are usually presented as a table. Each row is a category of variable 1 and each column is a category of variable 2.

		Variable 1		Totals
		Category *x*	Category *y*	
Variable 2	Category *a*	$\#_1$	$\#_2$	$\#_1+\#_2$
	Category *b*	$\#_3$	$\#_4$	$\#_3+\#_4$
Totals		$\#_1+\#_3$	$\#_2+\#_4$	$\#_1+\#_2+\#_3+\#_4$

The proportion of category *x* of variable 1 is the number of individuals in category *x* divided by the total number of individuals $\left(\frac{\#_1+\#_3}{\#_1+\#_2+\#_3+\#_4}\right)$. Assuming independence, the expected number of individuals that fall within category *a* of variable 2 is the proportion of category *x* multiplied by the number of individuals in category *a* $\left(\frac{\#_1+\#_3}{\#_1+\#_2+\#_3+\#_4}\right)(\#_1+\#_2)$. Thus, the expected value is:

$$E = \frac{(\#_1+\#_3)(\#_1+\#_2)}{\#_1+\#_2+\#_3+\#_4} = \frac{(row\ total)(column\ total)}{grand\ total}$$

Degrees of freedom = (*r*-1)(*c*-1) where *r* is the number of rows and *c* is the number of columns

The chi-square statistic is still $\chi^2 = \sum \frac{(O-E)^2}{E}$

Read the p-values from the table of χ^2 critical values.

Example:

Given the data below, is there a relationship between fitness level and smoking habits (assume α=0.05)?

	Fitness Level				
	Low	Medium-Low	Medium-High	High	
Never smoked	113	113	110	159	495
Former smokers	119	135	172	190	616
1 to 9 cigarettes daily	77	91	86	65	319
≥ 10 cigarettes daily	181	152	124	73	530
	490	491	492	487	1960

H_0: fitness level and smoking habits are independent
H_a: fitness level and smoking habits are not independent
First, we calculate the expected counts. For the first cell, the expected count is:

$$E = \frac{(row\ total)(column\ total)}{grand\ total} = \frac{(495)(490)}{1960} = 123.75$$

	Fitness Level			
	Low	Medium-Low	Medium-High	High
Never smoked	123.75	124	124.26	122.99
Former smokers	154	154.31	154.63	153.06
1 to 9 cigarettes daily	79.75	79.91	80.08	79.26
≥ 10 cigarettes daily	132.5	132.77	133.04	131.69

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(113-123.75)^2}{123.75} + \frac{(113-124)^2}{124} + \frac{(110-124.26)^2}{124.26} + etc... = 91.73$$

df=(*r*-1)(*c*-1)=(4-1)(4-1)=9

From the table of χ^2 critical values, the p-value is less than 0.001, so we reject H_0 and conclude that there is a relationship between fitness level and smoking habits.

Type I error

The probability of rejecting a true null hypothesis
Equals α

Type II error

The probability of failing to reject a false null hypothesis

Probability

Joint Probability

The probability of events A and B occurring
$P(A \text{ and } B) = P(A) \times P(B)$ when events A and B are independent

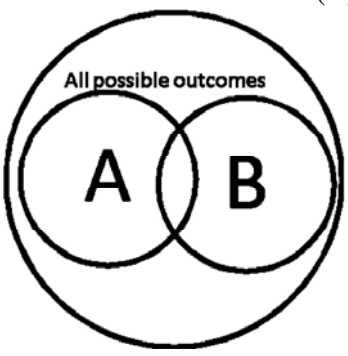
Union of Events

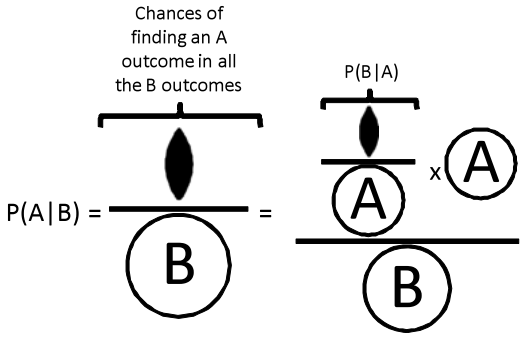
The probability of either event A or event B occurring
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Conditional Probability

The probability of event A occurring given that event B has occurred

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{or} \quad P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$





Example 1:
Assume that eye color is an autosomally inherited trait controlled by one gene with two alleles. Brown is dominant to blue. A brown-eyed man with genotype Bb and a blue-eyed woman have three children. The first has blue eyes. What is the probability that all three children have blue eyes?

Without considering the first child, the probability that the couple has three children with blue eyes is
$0.5 \times 0.5 \times 0.5 = 0.125 = P(A \text{ and } B) = P(2 \text{ children} = \text{bb and 1st child bb})$
With his parents, the probability that the 1st child is bb is: $P(B) = P(1\text{st child} = \text{bb}) = 0.5$
Therefore, $P(2 \text{ children} = \text{bb} \mid \text{1st child bb}) = P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.125}{0.5} 0.25$

Example 2:
Based on an analysis of her pedigree, it is determined that a woman has a 70% chance of being Zz and a 30% chance of being ZZ for a sex-linked trait, where Z is dominant to z. If she now has a son with the Z phenotype, what is the probability of her being Zz?

We're looking for: P(W=Zz|S=Z)
But it's hard to find P(W=Zz and S=Z) because the two events are not independent. Instead, let us use:
$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$
$P(S = Z \mid W = Zz) = 0.5$ (50% chance of passing on the Z allele)
$P(W = Zz) = 0.7$ (given)
$P(S = Z) = (0.7 \times 0.5) + (0.3 \times 1) = 0.65$ (son can be Z from the woman being either Zz or ZZ)
$P(W = Zz \mid S = Z) = \frac{0.5 \times 0.7}{0.65} = 0.538$

Multiple Experiments

Binomial distribution

For when you are not concerned about the order of the events, only that they occur

$$P(X = m) = \frac{n! \times p^m \times (1-p)^{(n-m)}}{m! \times (n-m)!}$$

for *m* outcomes of event X in *n* total trials with *p*=probability of X occurring once

Example:
What is the probability that a couple has one boy out of five children?

$$P(1 \text{ boy of 5 children}) = \frac{5! \times 0.5^1 \times 0.5^4}{1! \times (4)!} = 0.15625$$

Poisson distribution

The binomial distribution works for a small number of trials but as *n* gets too large, the factorials become unwieldy.
The Poisson distribution is an estimate of the binomial distribution for large *n*.

$$P(X = m) = \frac{e^{-np} \times (n \times p)^m}{m!}$$

Note: *np* is also known as the number of expected outcomes for event X

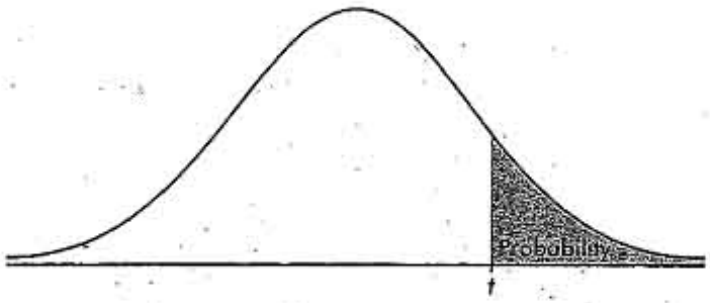


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

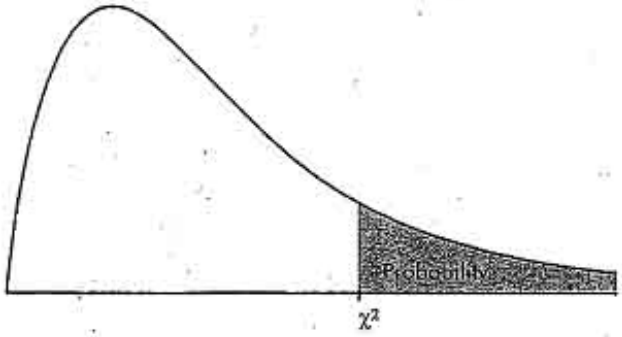
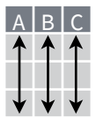


TABLE C: χ^2 CRITICAL VALUES

	Tail probability p										
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4

Data tidying with tidyr : : CHEAT SHEET

Tidy data is a way to organize tabular data in a consistent data structure across packages.
A table is tidy if:

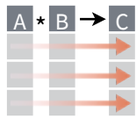


Each **variable** is in its own **column**

&

Each **observation**, or **case**, is in its own row

Access **variables** as **vectors**



Preserve **cases** in vectorized operations

Tibbles

AN ENHANCED DATA FRAME

Tibbles are a table format provided by the **tibble** package. They inherit the data frame class, but have improved behaviors:

- **Subset** a new tibble with], a vector with [[and $.
- **No partial matching** when subsetting columns.
- **Display** concise views of the data on one screen.

options(tibble.print_max = n, tibble.print_min = m, tibble.width = Inf**)** Control default display settings.

View() or **glimpse()** View the entire data set.

CONSTRUCT A TIBBLE

tibble(…**)** Construct by columns.
tibble(x = 1:3, y = c("a", "b", "c"))

tribble(…**)** Construct by rows.
tribble(~x, ~y,
1, "a",
2, "b",
3, "c")

Both make this tibble

```
A tibble: 3 × 2
      x     y
  <int> <chr>
1     1     a
2     2     b
3     3     c
```

as_tibble(x, …**)** Convert a data frame to a tibble.

enframe(x, name = "name", value = "value"**)** Convert a named vector to a tibble. Also **deframe()**.

is_tibble(x**)** Test whether x is a tibble.

Reshape Data - Pivot data to reorganize values into a new layout.

table4a

country	1999	2000
A	0.7K	2K
B	37K	80K
C	212K	213K

→

country	year	cases
A	1999	0.7K
B	1999	37K
C	1999	212K
A	2000	2K
B	2000	80K
C	2000	213K

pivot_longer(data, cols, names_to = "name", values_to = "value", values_drop_na = FALSE**)**

"Lengthen" data by collapsing several columns into two. Column names move to a new names_to column and values to a new values_to column.

pivot_longer(table4a, cols = 2:3, names_to ="year", values_to = "cases")

table2

country	year	type	count
A	1999	cases	0.7K
A	1999	pop	19M
A	2000	cases	2K
A	2000	pop	20M
B	1999	cases	37K
B	1999	pop	172M
B	2000	cases	80K
B	2000	pop	174M
C	1999	cases	212K
C	1999	pop	1T
C	2000	cases	213K
C	2000	pop	1T

→

country	year	cases	pop
A	1999	0.7K	19M
A	2000	2K	20M
B	1999	37K	172M
B	2000	80K	174M
C	1999	212K	1T
C	2000	213K	1T

pivot_wider(data, names_from = "name", values_from = "value"**)**

The inverse of pivot_longer(). "Widen" data by expanding two columns into several. One column provides the new column names, the other the values.

pivot_wider(table2, names_from = type, values_from = count)

Split Cells - Use these functions to split or combine cells into individual, isolated values.

table5

country	century	year
A	19	99
A	20	00
B	19	99
B	20	00

→

country	year
A	1999
A	2000
B	1999
B	2000

unite(data, col, …, sep = "_", remove = TRUE, na.rm = FALSE**)** Collapse cells across several columns into a single column.

unite(table5, century, year, col = "year", sep = "")

table3

country	year	rate
A	1999	0.7K/19M
A	2000	2K/20M
B	1999	37K/172M
B	2000	80K/174M

→

country	year	cases	pop
A	1999	0.7K	19M
A	2000	2K	20M
B	1999	37K	172
B	2000	80K	174

separate(data, col, into, sep = "[^[:alnum:]]+", remove = TRUE, convert = FALSE, extra = "warn", fill = "warn", …**)** Separate each cell in a column into several columns. Also **extract()**.

separate(table3, rate, sep = "/", into = c("cases", "pop"))

table3

country	year	rate
A	1999	0.7K/19M
A	2000	2K/20M
B	1999	37K/172M
B	2000	80K/174M

→

country	year	rate
A	1999	0.7K
A	1999	19M
A	2000	2K
A	2000	20M
B	1999	37K
B	1999	172M
B	2000	80K
B	2000	174M

separate_rows(data, …, sep = "[^[:alnum:].]+", convert = FALSE**)** Separate each cell in a column into several rows.

separate_rows(table3, rate, sep = "/")

Expand Tables

Create new combinations of variables or identify implicit missing values (combinations of variables not present in the data).

x

x1	x2	x3
A	1	3
B	1	4
B	2	3

→

x1	x2
A	1
A	2
B	1
B	2

expand(data, …**)** Create a new tibble with all possible combinations of the values of the variables listed in … Drop other variables.
expand(mtcars, cyl, gear, carb)

x

x1	x2	x3
A	1	3
B	1	4
B	2	3

→

x1	x2	x3
A	1	3
A	2	NA
B	1	4
B	2	3

complete(data, …, fill = list()**)** Add missing possible combinations of values of variables listed in … Fill remaining variables with NA.
complete(mtcars, cyl, gear, carb)

Handle Missing Values

Drop or replace explicit missing values (NA).

x

x1	x2
A	1
B	NA
C	NA
D	3
E	NA

→

x1	x2
A	1
D	3

drop_na(data, …**)** Drop rows containing NA's in … columns.
drop_na(x, x2)

x

x1	x2
A	1
B	NA
C	NA
D	3
E	NA

→

x1	x2
A	1
B	1
C	1
D	3
E	3

fill(data, …, .direction = "down"**)** Fill in NA's in … columns using the next or previous value.
fill(x, x2)

x

x1	x2
A	1
B	NA
C	NA
D	3
E	NA

→

x1	x2
A	1
B	2
C	2
D	3
E	2

replace_na(data, replace**)** Specify a value to replace NA in selected columns.
replace_na(x, list(x2 = 2))

Nested Data

A **nested data frame** stores individual tables as a list-column of data frames within a larger organizing data frame. List-columns can also be lists of vectors or lists of varying data types. Use a nested data frame to:

- Preserve relationships between observations and subsets of data. Preserve the type of the variables being nested (factors and datetimes aren't coerced to character).
- Manipulate many sub-tables at once with **purrr** funcitons like map(), map2(), or pmap() or with **dplyr** rowwise() grouping.

CREATE NESTED DATA

nest(data, …**)** Moves groups of cells into a list-column of a data frame. Use alone or with dplyr::**group_by()**:

1. Group the data frame with **group_by()** and use **nest()** to move the groups into a list-column.

```
n_storms <- storms %>%
  group_by(name) %>%
  nest()
```

2. Use **nest(new_col = c(x, y))** to specify the columns to group using dplyr::**select()** syntax.

```
n_storms <- storms %>%
  nest(data = c(year:long))
```

name	yr	lat	long
Amy	1975	27.5	-79.0
Amy	1975	28.5	-79.0
Amy	1975	29.5	-79.0
Bob	1979	22.0	-96.0
Bob	1979	22.5	-95.3
Bob	1979	23.0	-94.6
Zeta	2005	23.9	-35.6
Zeta	2005	24.2	-36.1
Zeta	2005	24.7	-36.6

nested data frame

name	data
Amy	<tibble [50x3]>
Bob	<tibble [50x3]>
Zeta	<tibble [50x3]>

"cell" contents

yr	lat	long
1975	27.5	-79.0
1975	28.5	-79.0
1975	29.5	-79.0

yr	lat	long
1979	22.0	-96.0
1979	22.5	-95.3
1979	23.0	-94.6

yr	lat	long
2005	23.9	-35.6
2005	24.2	-36.1
2005	24.7	-36.6

Index list-columns with [[]]. n_storms$data[[1]]

CREATE TIBBLES WITH LIST-COLUMNS

tibble::**tribble(**…**)** Makes list-columns when needed.

```
tribble( ~max, ~seq,
           3,   1:3,
           4,   1:4,
           5,   1:5)
```

max	seq
3	<int [3]>
4	<int [4]>
5	<int [5]>

tibble::**tibble(**…**)** Saves list input as list-columns.

```
tibble(max = c(3, 4, 5), seq = list(1:3, 1:4, 1:5))
```

tibble::**enframe(**x, name="name", value="value"**)** Converts multi-level list to a tibble with list-cols.

```
enframe(list('3'=1:3, '4'=1:4, '5'=1:5), 'max', 'seq')
```

OUTPUT LIST-COLUMNS FROM OTHER FUNCTIONS

dplyr::**mutate()**, **transmute()**, and **summarise()** will output list-columns if they return a list.

```
mtcars %>%
  group_by(cyl) %>%
  summarise(q = list(quantile(mpg)))
```

RESHAPE NESTED DATA

unnest(data, cols, ..., keep_empty = FALSE**)** Flatten nested columns back to regular columns. The inverse of nest().

```
n_storms %>% unnest(data)
```

unnest_longer(data, col, values_to = NULL, indices_to = NULL**)** Turn each element of a list-column into a row.

```
starwars %>%
  select(name, films) %>%
  unnest_longer(films)
```

name	films
Luke	<chr [5]>
C-3PO	<chr [6]>
R2-D2	<chr[7]>

name	films
Luke	The Empire Strik…
Luke	Revenge of the S…
Luke	Return of the Jed…
C-3PO	The Empire Strik…
C-3PO	Attack of the Cl…
C-3PO	The Phantom M…
R2-D2	The Empire Strik…
R2-D2	Attack of the Cl…
R2-D2	The Phantom M…

unnest_wider(data, col**)** Turn each element of a list-column into a regular column.

```
starwars %>%
  select(name, films) %>%
  unnest_wider(films)
```

name	films
Luke	<chr [5]>
C-3PO	<chr [6]>
R2-D2	<chr[7]>

name	..1	..2	..3
Luke	The Empire...	Revenge of...	Return of...
C-3PO	The Empire...	Attack of...	The Phantom...
R2-D2	The Empire...	Attack of...	The Phantom...

hoist(.data, .col, ..., .remove = TRUE) Selectively pull list components out into their own top-level columns. Uses purrr::pluck() syntax for selecting from lists.

```
starwars %>%
  select(name, films) %>%
  hoist(films, first_film = 1, second_film = 2)
```

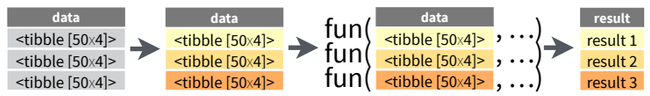
name	films
Luke	<chr [5]>
C-3PO	<chr [6]>
R2-D2	<chr[7]>

name	first_film	second_film	films
Luke	The Empire…	Revenge of…	<chr [3]>
C-3PO	The Empire…	Attack of…	<chr [4]>
R2-D2	The Empire…	Attack of…	<chr [5]>

TRANSFORM NESTED DATA

A vectorized function takes a vector, transforms each element in parallel, and returns a vector of the same length. By themselves vectorized functions cannot work with lists, such as list-columns.

dplyr::**rowwise(**.data, …**)** Group data so that each row is one group, and within the groups, elements of list-columns appear directly (accessed with [[ ), not as lists of length one. **When you use rowwise(), dplyr functions will seem to apply functions to list-columns in a vectorized fashion.**



Apply a function to a list-column and **create a new list-column.**

```
n_storms %>%
  rowwise() %>%
  mutate(n = list(dim(data)))
```

dim() returns two values per row

wrap with list to tell mutate to create a list-column

Apply a function to a list-column and **create a regular column.**

```
n_storms %>%
  rowwise() %>%
  mutate(n = nrow(data))
```

nrow() returns one integer per row

Collapse **multiple list-columns** into a single list-column**.**

```
starwars %>%
  rowwise() %>%
  mutate(transport = list(append(vehicles, starships)))
```

append() returns a list for each row, so col type must be list

Apply a function to **multiple list-columns.**

```
starwars %>%
  rowwise() %>%
  mutate(n_transports = length(c(vehicles, starships)))
```

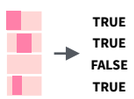
length() returns one integer per row

See **purrr** package for more list functions.

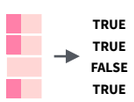
String manipulation with stringr : : **CHEAT SHEET**

The **stringr** package provides a set of internally consistent tools for working with character strings, i.e. sequences of characters surrounded by quotation marks.

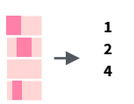
Detect Matches



str_detect(string, **pattern**, negate = FALSE**)** Detect the presence of a pattern match in a string. Also **str_like()**. str_detect(fruit, "a")

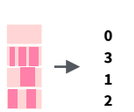


str_starts(string, **pattern**, negate = FALSE**)** Detect the presence of a pattern match at the beginning of a string. Also **str_ends()**. str_starts(fruit, "a")



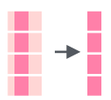
str_which(string, **pattern**, negate = FALSE**)** Find the indexes of strings that contain a pattern match. str_which(fruit, "a")

str_locate(string, **pattern**) Locate the positions of pattern matches in a string. Also **str_locate_all()**. str_locate(fruit, "a")

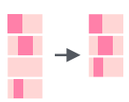


str_count(string, **pattern**) Count the number of matches in a string. str_count(fruit, "a")

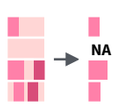
Subset Strings



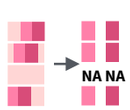
str_sub(string, start = 1L, end = -1L**)** Extract substrings from a character vector. str_sub(fruit, 1, 3); str_sub(fruit, -2)



str_subset(string, **pattern**, negate = FALSE**)** Return only the strings that contain a pattern match. str_subset(fruit, "p")

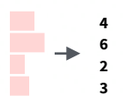


str_extract(string, **pattern**) Return the first pattern match found in each string, as a vector. Also **str_extract_all()** to return every pattern match. str_extract(fruit, "[aeiou]")

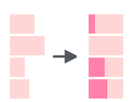


str_match(string, **pattern**) Return the first pattern match found in each string, as a matrix with a column for each ( ) group in pattern. Also **str_match_all()**. str_match(sentences, "(a|the) ([^ +])")

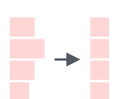
Manage Lengths



str_length(string**)** The width of strings (i.e. number of code points, which generally equals the number of characters). str_length(fruit)



str_pad(string, width, side = c("left", "right", "both"), pad = " "**)** Pad strings to constant width. str_pad(fruit, 17)

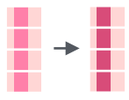


str_trunc(string, width, side = c("right", "left", "center"), ellipsis = "..."**)** Truncate the width of strings, replacing content with ellipsis. str_trunc(sentences, 6)

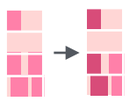
str_trim(string, side = c("both", "left", "right")**)** Trim whitespace from the start and/or end of a string. str_trim(str_pad(fruit, 17))

str_squish(string**)** Trim whitespace from each end and collapse multiple spaces into single spaces. str_squish(str_pad(fruit, 17, "both"))

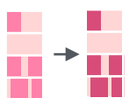
Mutate Strings



str_sub() <- value. Replace substrings by identifying the substrings with str_sub() and assigning into the results. str_sub(fruit, 1, 3) <- "str"



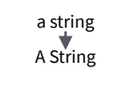
str_replace(string, **pattern**, replacement**)** Replace the first matched pattern in each string. Also **str_remove()**. str_replace(fruit, "p", "-")



str_replace_all(string, **pattern**, replacement**)** Replace all matched patterns in each string. Also **str_remove_all()**. str_replace_all(fruit, "p", "-")

str_to_lower(string, locale = "en"**)**[1] Convert strings to lower case. str_to_lower(sentences)

str_to_upper(string, locale = "en"**)**[1] Convert strings to upper case. str_to_upper(sentences)

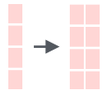


str_to_title(string, locale = "en"**)**[1] Convert strings to title case. Also **str_to_sentence()**. str_to_title(sentences)

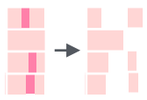
Join and Split

str_c(..., sep = "", collapse = NULL**)** Join multiple strings into a single string. str_c(letters, LETTERS)

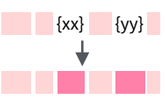
str_flatten(string, collapse = ""**)** Combines into a single string, separated by collapse. str_flatten(fruit, ", ")



str_dup(string, times**)** Repeat strings times times. Also **str_unique()** to remove duplicates. str_dup(fruit, times = 2)



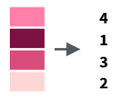
str_split_fixed(string, **pattern**, n**)** Split a vector of strings into a matrix of substrings (splitting at occurrences of a pattern match). Also **str_split()** to return a list of substrings and **str_split_n()** to return the nth substring. str_split_fixed(sentences, " ", n=3)



str_glue(..., .sep = "", .envir = parent.frame()**)** Create a string from strings and {expressions} to evaluate. str_glue("Pi is {pi}")

str_glue_data(.x, ..., .sep = "", .envir = parent.frame(), .na = "NA"**)** Use a data frame, list, or environment to create a string from strings and {expressions} to evaluate. str_glue_data(mtcars, "{rownames(mtcars)} has {hp} hp")

Order Strings

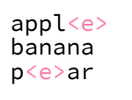


str_order(x, decreasing = FALSE, na_last = TRUE, locale = "en", numeric = FALSE, ...**)**[1] Return the vector of indexes that sorts a character vector. fruit[str_order(fruit)]

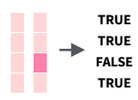
str_sort(x, decreasing = FALSE, na_last = TRUE, locale = "en", numeric = FALSE, ...**)**[1] Sort a character vector. str_sort(fruit)

Helpers

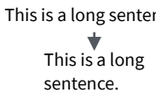
str_conv(string, encoding**)** Override the encoding of a string. str_conv(fruit,"ISO-8859-1")



str_view_all(string, **pattern**, match = NA**)** View HTML rendering of all regex matches. Also **str_view()** to see only the first match. str_view_all(sentences, "[aeiou]")



str_equal(x, y, locale = "en", ignore_case = FALSE, ...**)**[1] Determine if two strings are equivalent. str_equal(c("a", "b"), c("a", "c"))



str_wrap(string, width = 80, indent = 0, exdent = 0**)** Wrap strings into nicely formatted paragraphs. str_wrap(sentences, 20)

[1] See **bit.ly/ISO639-1** for a complete list of locales.

Need to Know

Pattern arguments in stringr are interpreted as regular expressions *after any special characters have been parsed.*

In R, you write regular expressions as *strings*, sequences of characters surrounded by quotes (**""**) or single quotes(**''**).

Some characters cannot be represented directly in an R string . These must be represented as **special characters**, sequences of characters that have a specific meaning., e.g.

Special Character	Represents
\\	\
\"	"
\n	new line

Run **?"'"** to see a complete list

Because of this, whenever a \ appears in a regular expression, you must write it as \\ in the string that represents the regular expression.

Use **writeLines()** to see how R views your string after all special characters have been parsed.

```
writeLines("\\.")
# \.

writeLines("\\ is a backslash")
# \ is a backslash
```

INTERPRETATION

Patterns in stringr are interpreted as regexs. To change this default, wrap the pattern in one of:

regex(pattern, ignore_case = FALSE, multiline = FALSE, comments = FALSE, dotall = FALSE, ...**)** Modifies a regex to ignore cases, match end of lines as well of end of strings, allow R comments within regex's , and/or to have . match everything including \n.
str_detect("I", regex("i", TRUE))

fixed() Matches raw bytes but will miss some characters that can be represented in multiple ways (fast). str_detect("\u0130", fixed("i"))

coll() Matches raw bytes and will use locale specific collation rules to recognize characters that can be represented in multiple ways (slow).
str_detect("\u0130", coll("i", TRUE, locale = "tr"))

boundary() Matches boundaries between characters, line_breaks, sentences, or words.
str_split(sentences, boundary("word"))

Regular Expressions -

Regular expressions, or *regexps*, are a concise language for describing patterns in strings.

MATCH CHARACTERS

see <- function(rx) str_view_all("abc ABC 123\t.!?\\(){}\n", rx)

string (type this)	regexp (to mean this)	matches (which matches this)	example	
	a (etc.)	a (etc.)	see("a")	abc ABC 123 .!?\(){}
\\.	**\.**	.	see("\\.")	abc ABC 123 .!?\(){}
\\!	**\!**	!	see("\\!")	abc ABC 123 .!?\(){}
\\?	**\?**	?	see("\\?")	abc ABC 123 .!?\(){}
\\\\	****	\	see("\\\\")	abc ABC 123 .!?\(){}
\\(	**\(**	(	see("\\(")	abc ABC 123 .!?\(){}
\\)	**\)**	)	see("\\)")	abc ABC 123 .!?\(){}
\\{	**\{**	{	see("\\{")	abc ABC 123 .!?\(){}
\\}	**\}**	}	see("\\}")	abc ABC 123 .!?\(){}
\\n	**\n**	new line (return)	see("\\n")	abc ABC 123 .!?\(){}
\\t	**\t**	tab	see("\\t")	abc ABC 123 .!?\(){}
\\s	**\s**	any whitespace (**\S** *for non-whitespaces*)	see("\\s")	abc ABC 123 .!?\(){}
\\d	**\d**	any digit (**\D** *for non-digits*)	see("\\d")	abc ABC 123 .!?\(){}
\\w	**\w**	any word character (**\W** *for non-word chars*)	see("\\w")	abc ABC 123 .!?\(){}
\\b	**\b**	word boundaries	see("\\b")	abc ABC 123 .!?\(){}
	[:digit:][1]	digits	see("[:digit:]")	abc ABC 123 .!?\(){}
	[:alpha:][1]	letters	see("[:alpha:]")	abc ABC 123 .!?\(){}
	[:lower:][1]	lowercase letters	see("[:lower:]")	abc ABC 123 .!?\(){}
	[:upper:][1]	uppercase letters	see("[:upper:]")	abc ABC 123 .!?\(){}
	[:alnum:][1]	letters and numbers	see("[:alnum:]")	abc ABC 123 .!?\(){}
	[:punct:][1]	punctuation	see("[:punct:]")	abc ABC 123 .!?\(){}
	[:graph:][1]	letters, numbers, and punctuation	see("[:graph:]")	abc ABC 123 .!?\(){}
	[:space:][1]	space characters (i.e. \s)	see("[:space:]")	abc ABC 123 .!?\(){}
	[:blank:][1]	space and tab (but not new line)	see("[:blank:]")	abc ABC 123 .!?\(){}
	.	every character except a new line	see(".")	abc ABC 123 .!?\(){}

[1] Many base R functions require classes to be wrapped in a second set of **[ ]**, e.g. **[[:digit:]]**

[:space:]
↵ new line

[:blank:]
space
tab

.

[:graph:]

[:punct:]
. , : ; ? ! / * @ #
- _ " ' [ ] { } ( )

[:symbol:]
| ` = + ^
~ < > $

[:alnum:]

[:digit:]
0 1 2 3 4 5 6 7 8 9

[:alpha:]

[:lower:]
a b c d e f
g h i j k l
m n o p q r
s t u v w x
y z

[:upper:]
A B C D E F
G H I J K L
M N O P Q R
S T U V W X
Y Z

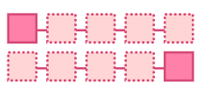
ALTERNATES

alt <- function(rx) str_view_all("abcde", rx)

regexp	matches	example	
ab|d	or	alt("ab|d")	abcde
[abe]	one of	alt("[abe]")	abcde
[^abe]	anything but	alt("[^abe]")	abcde
[a-c]	range	alt("[a-c]")	abcde

ANCHORS

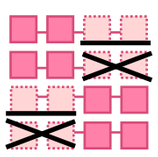
anchor <- function(rx) str_view_all("aaa", rx)



regexp	matches	example	
^a	start of string	anchor("^a")	aaa
a$	end of string	anchor("a$")	aaa

LOOK AROUNDS

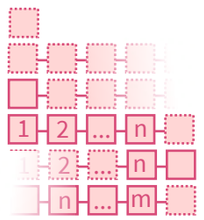
look <- function(rx) str_view_all("bacad", rx)



regexp	matches	example	
a(?=c)	followed by	look("a(?=c)")	bacad
a(?!c)	not followed by	look("a(?!c)")	bacad
(?<=b)a	preceded by	look("(?<=b)a")	bacad
(?<!b)a	not preceded by	look("(?<!b)a")	bacad

QUANTIFIERS

quant <- function(rx) str_view_all(".a.aa.aaa", rx)



regexp	matches	example	
a?	zero or one	quant("a?")	.a.aa.aaa
a*	zero or more	quant("a*")	.a.aa.aaa
a+	one or more	quant("a+")	.a.aa.aaa
a{n}	exactly **n**	quant("a{2}")	.a.aa.aaa
a{n, }	**n** or more	quant("a{2,}")	.a.aa.aaa
a{n, m}	between **n** and **m**	quant("a{2,4}")	.a.aa.aaa

GROUPS

ref <- function(rx) str_view_all("abbaab", rx)

Use parentheses to set precedent (order of evaluation) and create groups

regexp	matches	example	
(ab|d)e	sets precedence	alt("(ab|d)e")	abcde

Use an escaped number to refer to and duplicate parentheses groups that occur earlier in a pattern. Refer to each group by its order of appearance

string (type this)	regexp (to mean this)	matches (which matches this)	example (the result is the same as ref("abba"))	
\\1	**\1** (etc.)	first () group, etc.	ref("(a)(b)\\2\\1")	abbaab

Shiny : : CHEAT SHEET

Building an App

A **Shiny** app is a web page (**ui**) connected to a computer running a live R session (**server**).

Users can manipulate the UI, which will cause the server to update the UI's displays (by running R code).

Save your template as **app.R**. Keep your app in a directory along with optional extra files.

To generate the template, type **shinyapp** and press **Tab** in the RStudio IDE or go to **File > New Project > New Directory > Shiny Web Application**

In **ui** nest R functions to build an HTML interface

Customize the UI with **Layout Functions**

Add Inputs with ***Input()** functions

Add Outputs with ***Output()** functions

Tell the **server** how to render outputs and respond to inputs with R

Wrap code in **render*()** functions before saving to output

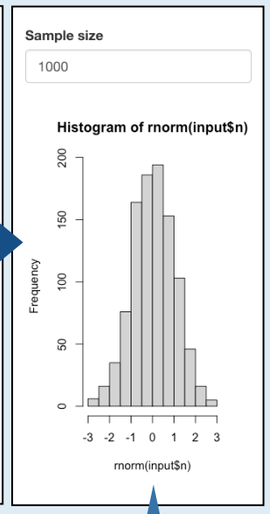
Refer to UI inputs with **input$<id>** and outputs with **output$<id>**

```
# app.R
library(shiny)

ui <- fluidPage(
  numericInput(inputId = "n",
    "Sample size", value = 25),
  plotOutput(outputId = "hist")
)

server <- function(input, output, session) {
  output$hist <- renderPlot({
    hist(rnorm(input$n))
  })
}

shinyApp(ui = ui, server = server)
```



Call **shinyApp()** to combine **ui** and **server** into an interactive app!

app-name
- app.R — The directory name is the app name
- DESCRIPTION — (optional) used in showcase mode
- README — (optional) directory of supplemental .R files that are sourced automatically, must be named "**R**"
- R/ — (optional) directory of files to share with web browsers (images, CSS, .js, etc.), must be named "**www**"
- www/

Launch apps stored in a directory with **runApp(**<path to directory>**)**.

See annotated examples of Shiny apps by running **runExample(**<example name>**)**. Run **runExample()** with no arguments for a list of example names.

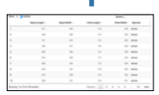
Share

Share your app in three ways:

1. **Host it on shinyapps.io**, a cloud based service from RStudio. To deploy Shiny apps:
   - Create a free or professional account at **shinyapps.io**
   - Click the Publish icon in RStudio IDE, or run: **rsconnect::deployApp(**"<path to directory>"**)**
2. **Purchase RStudio Connect**, a publishing platform for R and Python. **rstudio.com/products/connect/**
3. **Build your own Shiny Server** **rstudio.com/products/shiny/shiny-server/**

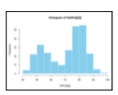
Outputs

render*() and *Output() functions work together to add R output to the UI.



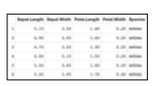
DT::**renderDataTable(**expr, options, searchDelay, callback, escape, env, quoted, outputArgs**)** — **dataTableOutput(**outputId**)**

renderImage(expr, env, quoted, deleteFile, outputArgs**)** — **imageOutput(**outputId, width, height, click, dblclick, hover, brush, inline**)**

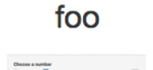


renderPlot(expr, width, height, res, …, alt, env, quoted, execOnResize, outputArgs**)** — **plotOutput(**outputId, width, height, click, dblclick, hover, brush, inline**)**

renderPrint(expr, env, quoted, width, outputArgs**)** — **verbatimTextOutput(**outputId, placeholder**)**



renderTable(expr, striped, hover, bordered, spacing, width, align, rownames, colnames, digits, na, …, env, quoted, outputArgs**)** — **tableOutput(**outputId**)**



renderText(expr, env, quoted, outputArgs, sep**)** — **textOutput(**outputId, container, inline**)**

renderUI(expr, env, quoted, outputArgs**)** — **uiOutput(**outputId, inline, container, …**)** **htmlOutput(**outputId, inline, container, …**)**

These are the core output types. See **htmlwidgets.org** for many more options.

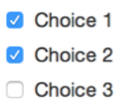
Inputs

Collect values from the user.

Access the current value of an input object with **input$<inputId>**. Input values are **reactive**.

actionButton(inputId, label, icon, width, …**)**

actionLink(inputId, label, icon, …**)**



checkboxGroupInput(inputId, label, choices, selected, inline, width, choiceNames, choiceValues**)**

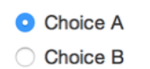
checkboxInput(inputId, label, value, width**)**

dateInput(inputId, label, value, min, max, format, startview, weekstart, language, width, autoclose, datesdisabled, daysofweekdisabled**)**

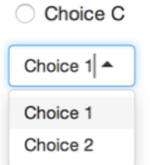
dateRangeInput(inputId, label, start, end, min, max, format, startview, weekstart, language, separator, width, autoclose**)**

fileInput(inputId, label, multiple, accept, width, buttonLabel, placeholder**)**

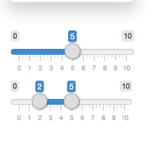
numericInput(inputId, label, value, min, max, step, width**)**



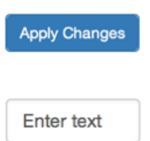
passwordInput(inputId, label, value, width, placeholder**)**



radioButtons(inputId, label, choices, selected, inline, width, choiceNames, choiceValues**)**

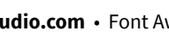


selectInput(inputId, label, choices, selected, multiple, selectize, width, size**)** Also **selectizeInput()**



sliderInput(inputId, label, min, max, value, step, round, format, locale, ticks, animate, width, sep, pre, post, timeFormat, timezone, dragRange**)**

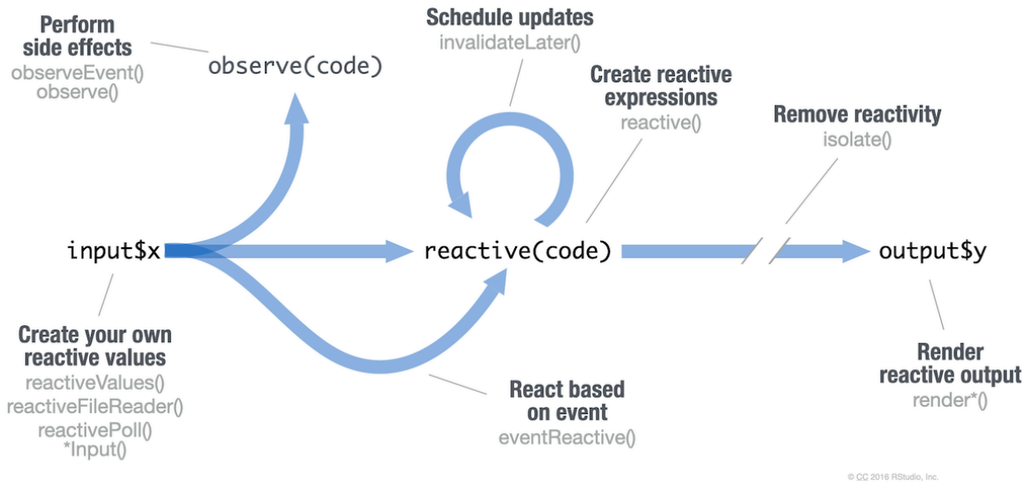
submitButton(text, icon, width**)** (Prevent reactions for entire app)



textInput(inputId, label, value, width, placeholder**)** Also **textAreaInput()**

Reactivity

Reactive values work together with reactive functions. Call a reactive value from within the arguments of one of these functions to avoid the error `Operation not allowed without an active reactive context.`



CREATE YOUR OWN REACTIVE VALUES

```
# *Input() example
ui <- fluidPage(
 textInput("a","","A")
)
```

```
#reactiveValues example
server <-
function(input,output){
 rv <- reactiveValues()
 rv$number <- 5
}
```

***Input() functions**
(see front page)

Each input function creates a reactive value stored as **input$<inputId>**.

reactiveValues(...)
Creates a list of reactive values whose values you can set.

CREATE REACTIVE EXPRESSIONS

```
library(shiny)
ui <- fluidPage(
 textInput("a","","A"),
 textInput("z","","Z"),
 textOutput("b"))

server <-
function(input,output){
 re <- reactive({
paste(input$a,input$z)})
 output$b <- renderText({
   re()
  })
}
shinyApp(ui, server)
```

reactive(x, env, quoted, label, domain**)**

Reactive expressions:
- **cache** their value to reduce computation
- can be called elsewhere
- notify dependencies when invalidated

Call the expression with function syntax, e.g. `re()`.

REACT BASED ON EVENT

```
library(shiny)
ui <- fluidPage(
 textInput("a","","A"),
 actionButton("go","Go"),
 textOutput("b")
)

server <-
function(input,output){
 re <- eventReactive(
  input$go,{input$a})
 output$b <- renderText({
   re()
  })
}
shinyApp(ui, server)
```

eventReactive(eventExpr, valueExpr, event.env, event.quoted, value.env, value.quoted, ..., label, domain, ignoreNULL, ignoreInit**)**

Creates reactive expression with code in 2nd argument that only invalidates when reactive values in 1st argument change.

RENDER REACTIVE OUTPUT

```
library(shiny)
ui <- fluidPage(
 textInput("a","","A"),
 textOutput("b")
)

server <-
function(input,output){
 output$b <-
  renderText({
   input$a
  })
}
shinyApp(ui, server)
```

render*() functions
(see front page)

Builds an object to display. Will rerun code in body to rebuild the object whenever a reactive value in the code changes.

Save the results to **output$<outputId>**.

PERFORM SIDE EFFECTS

```
library(shiny)
ui <- fluidPage(
 textInput("a","","A"),
 actionButton("go","Go")
)

server <-
function(input,output){
 observeEvent(input$go,{
   print(input$a)
  })
}

shinyApp(ui, server)
```

observeEvent(eventExpr, handlerExpr, event.env, event.quoted, handler.env, handler.quoted, ..., label, suspended, priority, domain, autoDestroy, ignoreNULL, ignoreInit, once**)**

Runs code in 2nd argument when reactive values in 1st argument change. See **observe()** for alternative.

REMOVE REACTIVITY

```
library(shiny)
ui <- fluidPage(
 textInput("a","","A"),
 textOutput("b")
)

server <-
function(input,output){
 output$b <-
  renderText({
   isolate({input$a})
  })
}

shinyApp(ui, server)
```

isolate(expr**)**

Runs a code block. Returns a **non-reactive** copy of the results.

UI - An app's UI is an HTML document.

Use Shiny's functions to assemble this HTML with R.

```
fluidPage(
  textInput("a","")
)
## <div class="container-fluid">
##  <div class="form-group shiny-input-container">
##     <label for="a"></label>
##     <input id="a" type="text"
##        class="form-control" value=""/>
##   </div>
## </div>
```

Returns HTML

Add static HTML elements with **tags**, a list of functions that parallel common HTML tags, e.g. **tags$a()**. Unnamed arguments will be passed into the tag; named arguments will become tag attributes.

Run **names(tags)** for a complete list.
`tags$h1("Header") -> <h1>Header</h1>`

The most common tags have wrapper functions. You do not need to prefix their names with **tags$**

```
ui <- fluidPage(
  h1("Header 1"),
  hr(),
  br(),
  p(strong("bold")),
  p(em("italic")),
  p(code("code")),
  a(href="", "link"),
  HTML("<p>Raw html</p>")
)
```

To include a CSS file, use **includeCSS()**, or
1. Place the file in the **www** subdirectory
2. Link to it with:

```
tags$head(tags$link(rel = "stylesheet",
  type = "text/css", href = "<file name>"))
```

To include JavaScript, use **includeScript()** or
1. Place the file in the **www** subdirectory
2. Link to it with:

```
tags$head(tags$script(src = "<file name>"))
```

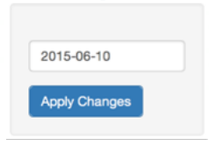
IMAGES

To include an image:
1. Place the file in the **www** subdirectory
2. Link to it with `img(src="<file name>")`

Layouts

Combine multiple elements into a "single element" that has its own properties with a panel function, e.g.

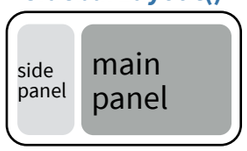
```
wellPanel(
  dateInput("a", ""),
  submitButton()
)
```



absolutePanel()
conditionalPanel()
fixedPanel()
headerPanel()
inputPanel()
mainPanel()
navlistPanel()
sidebarPanel()
tabPanel()
tabsetPanel()
titlePanel()
wellPanel()

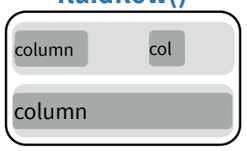
Organize panels and elements into a layout with a layout function. Add elements as arguments of the layout functions.

sidebarLayout()




```
ui <- fluidPage(
  sidebarLayout(
    sidebarPanel(),
    mainPanel()
  )
)
```

fluidRow()




```
ui <- fluidPage(
  fluidRow(column(width = 4),
           column(width = 2, offset = 3)),
  fluidRow(column(width = 12))
)
```

Also **flowLayout()**, **splitLayout()**, **verticalLayout()**, **fixedPage()**, **and fixedRow()**.

Layer tabPanels on top of each other, and navigate between them, with:

```
ui <- fluidPage( tabsetPanel(
    tabPanel("tab 1", "contents"),
    tabPanel("tab 2", "contents"),
    tabPanel("tab 3", "contents")))

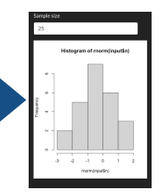
ui <- fluidPage( navlistPanel(
    tabPanel("tab 1", "contents"),
    tabPanel("tab 2", "contents"),
    tabPanel("tab 3", "contents")))

ui <- navbarPage(title = "Page",
    tabPanel("tab 1", "contents"),
    tabPanel("tab 2", "contents"),
    tabPanel("tab 3", "contents"))
```

Themes

Use the **bslib** package to add existing themes to your Shiny app ui, or make your own.

```
library(bslib)
ui <- fluidPage(
  theme = bs_theme(
    bootswatch = "darkly",
    ...
  )
)
```

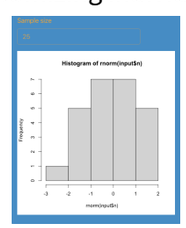


bootswatch_themes() Get a list of themes.

Build your own theme by customizing individual arguments.

bs_theme(bg = "#558AC5", fg = "#F9B02D", ...**)**

?bs_theme for a full list of arguments.



bs_themer() Place within the server function to use the interactive theming widget.

rmarkdown : : CHEAT SHEET

What is rmarkdown?

.Rmd files • Develop your code and ideas side-by-side in a single document. Run code as individual chunks or as an entire document.

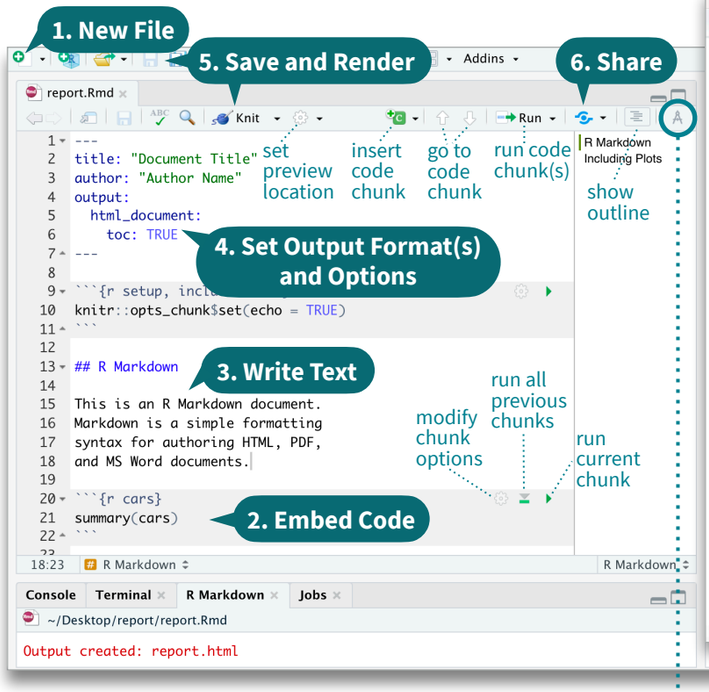
Dynamic Documents • Knit together plots, tables, and results with narrative text. Render to a variety of formats like HTML, PDF, MS Word, or MS Powerpoint.

Reproducible Research • Upload, link to, or attach your report to share. Anyone can read or run your code to reproduce your work.

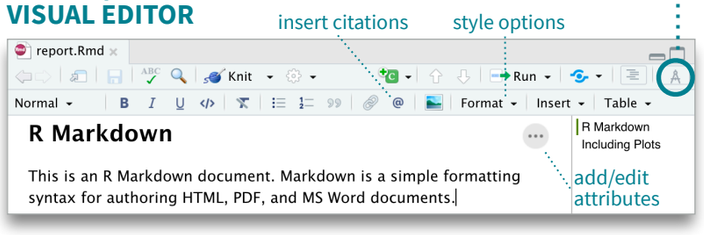
Workflow

1. Open a **new .Rmd file** in the RStudio IDE by going to *File > New File > R Markdown*.
2. **Embed code** in chunks. Run code by line, by chunk, or all at once.
3. **Write text** and add tables, figures, images, and citations. Format with Markdown syntax or the RStudio Visual Markdown Editor.
4. **Set output format(s) and options** in the YAML header. Customize themes or add parameters to execute or add interactivity with Shiny.
5. **Save and render** the whole document. Knit periodically to preview your work as you write.
6. **Share your work!**

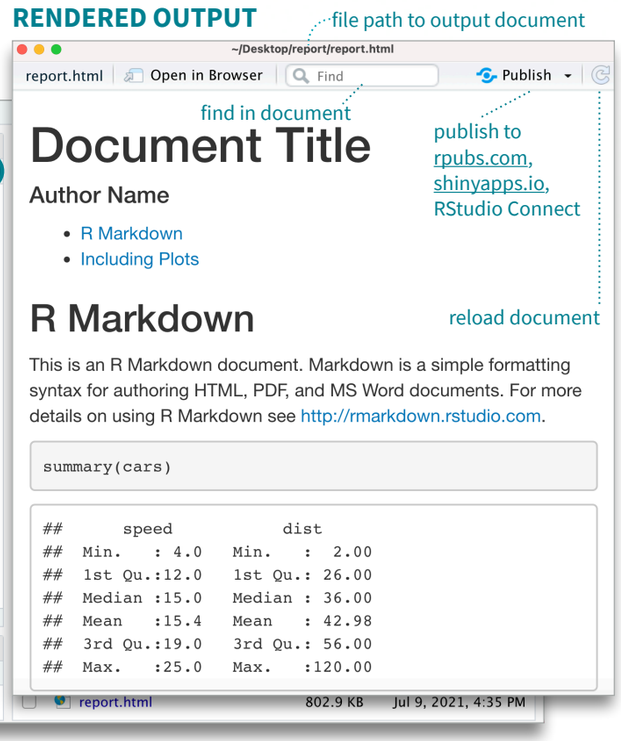
SOURCE EDITOR



VISUAL EDITOR



RENDERED OUTPUT



Embed Code with knitr

CODE CHUNKS

Surround code chunks with ```` ```{r} ```` and ```` ``` ```` or use the Insert Code Chunk button. Add a chunk label and/or chunk options inside the curly braces after **r**.

````
```{r chunk-label, include=FALSE}
summary(mtcars)
```
````

SET GLOBAL OPTIONS

Set options for the entire document in the first chunk.

````
```{r include=FALSE}
knitr::opts_chunk$set(message = FALSE)
```
````

INLINE CODE

Insert **`r <code>`** into text sections. Code is evaluated at render and results appear as text.

"Built with `` `r getRversion()` `` " --> "Built with 4.1.0"

OPTION	DEFAULT	EFFECTS
echo	TRUE	display code in output document
error	FALSE	TRUE (display error messages in doc) FALSE (stop render when error occurs)
eval	TRUE	run code in chunk
include	TRUE	include chunk in doc after running
message	TRUE	display code messages in document
warning	TRUE	display code warnings in document
results	"markup"	"asis" (passthrough results) "hide" (don't display results) "hold" (put all results below all code)
fig.align	"default"	"left", "right", or "center"
fig.alt	NULL	alt text for a figure
fig.cap	NULL	figure caption as a character string
fig.path	"figure/"	prefix for generating figure file paths
fig.width & fig.height	7	plot dimensions in inches
out.width		rescales output width, e.g. "75%", "300px"
collapse	FALSE	collapse all sources & output into a single block
comment	"##"	prefix for each line of results
child	NULL	files(s) to knit and then include
purl	TRUE	include or exclude a code chunk when extracting source code with knitr::purl()

See more options and defaults by running **str(knitr::opts_chunk$get())**

Insert Citations

Create citations from a bibliography file, a Zotero library, or from DOI references.

BUILD YOUR BIBLIOGRAPHY

- Add BibTeX or CSL bibliographies to the YAML header.

```
---
title: "My Document"
bibliography: references.bib
link-citations: TRUE
---
```

- If Zotero is installed locally, your main library will automatically be available.
- Add citations by DOI by searching "from DOI" in the **Insert Citation** dialog.

INSERT CITATIONS

- Access the **Insert Citations** dialog in the Visual Editor by clicking the **@** symbol in the toolbar or by clicking **Insert > Citation**.
- Add citations with markdown syntax by typing **[@cite]** or **@cite**.

Insert Tables

Output data frames as tables using **kable(**data, caption**)**.

````
```{r}
data <- faithful[1:4, ]
knitr::kable(data,
             caption = "Table with kable")
```
````

Table with kable

eruptions	waiting
3.600	79
1.800	54
3.333	74
2.283	62

Other table packages include **flextable**, **gt**, and **kableExtra**.

Write with Markdown

The syntax on the left renders as the output on the right.

Plain text. End a line with two spaces to start a new paragraph. Also end with a backslash\ to make a new line.	Plain text. End a line with two spaces to start a new paragraph. Also end with a backslash to make a new line.
italics and **bold** superscript^2^/subscript~2~ ~~strikethrough~~ escaped: * _ \\ endash: --, emdash: ---	*italics* and **bold** superscript^2/subscript$_2$ ~~strikethrough~~ escaped: * _ \ endash: –, emdash: —
# Header 1 ## Header 2 ... ###### Header 6	**Header 1** **Header 2** ... **Header 6**
- unordered list - item 2 - item 2a (indent 1 tab) - item 2b 1. ordered list 2. item 2 - item 2a (indent 1 tab) - item 2b	• unordered list • item 2 • item 2a (indent 1 tab) • item 2b 1. ordered list 2. item 2 • item 2a (indent 1 tab) • item 2b
<link url> [This is a link.](link url) [This is another link][id]. At the end of the document: [id]: link url ![Caption](image.png) or ![Caption][id2] At the end of the document: [id2]: image.png	http://www.rstudio.com/ This is a link. This is another link.  Caption.
`verbatim code` ``` multiple lines of verbatim code ```	`verbatim code` ``` multiple lines of verbatim code ```
> block quotes	> block quotes
equation: $e^{i \pi} + 1 = 0$ equation block: $$E = mc^{2}$$	equation: $e^{i\pi} + 1 = 0$ equation block: $$E = mc^2$$
horizontal rule: ---	horizontal rule:
| Right | Left | Default | Center | |-------:|:------|----------|:--------:| | 12 | 12 | 12 | 12 | | 123 | 123 | 123 | 123 | | 1 | 1 | 1 | 1 |	Right / Left / Default / Center 12 / 12 / 12 / 12 123 / 123 / 123 / 123 1 / 1 / 1 / 1
HTML Tabsets # Results {.tabset} ## Plots text text ## Tables more text	**Results** Plots | Tables text

Set Output Formats and their Options in YAML

Use the document's YAML header to set an **output format** and customize it with **output options**.

```
---
title: "My Document"
author: "Author Name"
output:
  html_document:
    toc: TRUE
---
```

Indent format 2 characters, indent options 4 characters

OUTPUT FORMAT	CREATES
html_document	.html
pdf_document*	.pdf
word_document	Microsoft Word (.docx)
powerpoint_presentation	Microsoft Powerpoint (.pptx)
odt_document	OpenDocument Text
rtf_document	Rich Text Format
md_document	Markdown
github_document	Markdown for Github
ioslides_presentation	ioslides HTML slides
slidy_presentation	Slidy HTML slides
beamer_presentation*	Beamer slides

* Requires LaTeX, use **tinytex::install_tinytex()**

Also see **flexdashboard**, **bookdown**, **distill**, and **blogdown**.

IMPORTANT OPTIONS	DESCRIPTION	HTML	PDF	MS Word	MS PPT
anchor_sections	Show section anchors on mouse hover (TRUE or FALSE)	X			
citation_package	The LaTeX package to process citations ("default", "natbib", "biblatex")		X		
code_download	Give readers an option to download the .Rmd source code (TRUE or FALSE)	X			
code_folding	Let readers to toggle the display of R code ("none", "hide", or "show")	X			
css	CSS or SCSS file to use to style document (e.g. "style.css")	X			
dev	Graphics device to use for figure output (e.g. "png", "pdf")	X	X		
df_print	Method for printing data frames ("default", "kable", "tibble", "paged")	X	X	X	X
fig_caption	Should figures be rendered with captions (TRUE or FALSE)	X	X	X	X
highlight	Syntax highlighting ("tango", "pygments", "kate", "zenburn", "textmate")	X	X	X	
includes	File of content to place in doc ("in_header", "before_body", "after_body")	X	X		
keep_md	Keep the Markdown .md file generated by knitting (TRUE or FALSE)	X	X	X	X
keep_tex	Keep the intermediate TEX file used to convert to PDF (TRUE or FALSE)		X		
latex_engine	LaTeX engine for producing PDF output ("pdflatex", "xelatex", or "lualatex")		X		
reference_docx/_doc	docx/pptx file containing styles to copy in the output (e.g. "file.docx", "file.pptx")			X	X
theme	Theme options (see Bootswatch and Custom Themes below)	X			
toc	Add a table of contents at start of document (TRUE or FALSE)	X	X	X	X
toc_depth	The lowest level of headings to add to table of contents (e.g. 2, 3)	X	X	X	X
toc_float	Float the table of contents to the left of the main document content (TRUE or FALSE)	X			

Use **?<output format>** to see all of a format's options, e.g. **?html_document**

Render

When you render a document, rmarkdown:

1. Runs the code and embeds results and text into an .md file with knitr.
2. Converts the .md file into the output format with Pandoc.

Save, then **Knit** to preview the document output. The resulting HTML/PDF/MS Word/etc. document will be created and saved in the same directory as the .Rmd file.

Use **rmarkdown::render()** to render/knit in the R console. See **?render** for available options.

Share

Publish on RStudio Connect

to share R Markdown documents securely, schedule automatic updates, and interact with parameters in real time.

rstudio.com/products/connect/

More Header Options

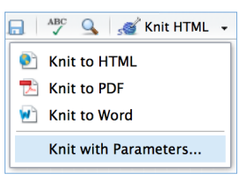
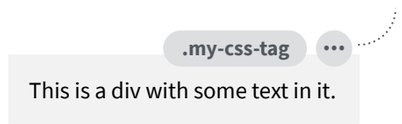
PARAMETERS

Parameterize your documents to reuse with new inputs (e.g., data, values, etc.).

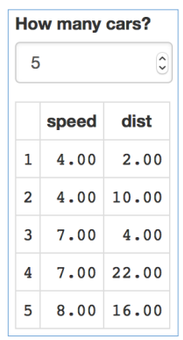
1. **Add parameters** in the header as sub-values of params.

```
---
params:
  state: "hawaii"
---
```

2. **Call parameters** in code using params$<name>.

```
```{r}
data <- df[, params$state]
summary(data)
```
```

3. **Set parameters** with Knit with Parameters or the params argument of **render()**.



REUSABLE TEMPLATES

1. **Create a new package** with a inst/rmarkdown/templates directory.
2. **Add a folder** containing **template.yaml** (below) and **skeleton.Rmd** (template contents).

```
---
name: "My Template"
---
```

3. **Install** the package to access template by going to **File > New R Markdown > From Template**.

BOOTSWATCH THEMES

Customize HTML documents with Bootswatch themes from the **bslib** package using the theme output option.

Use **bslib::bootswatch_themes()** to list available themes.


```
---
title: "Document Title"
author: "Author Name"
output:
  html_document:
    theme:
      bootswatch: solar
---
```

CUSTOM THEMES

Customize individual HTML elements using bslib variables. Use **?bs_theme** to see more variables.

```
---
output:
  html_document:
    theme:
      bg: "#121212"
      fg: "#E4E4E4"
      base_font:
        google: "Prompt"
---
```

More on **bslib** at **pkgs.rstudio.com/bslib/**.

STYLING WITH CSS AND SCSS

Add CSS and SCSS to your document by adding a path to a file with the **css** option in the YAML header.

```
---
title: "My Document"
author: "Author Name"
output:
  html_document:
    css: "style.css"
---
```

Apply CSS styling by writing HTML tags directly or:

- Use markdown to apply style attributes inline.

Bracketed Span
A [green]{.my-color} word.

Fenced Div
::: {.my-color}
All of these words
are green.
:::

A green word.

All of these words are green.

- Use the Visual Editor. Go to **Format > Div/Span** and add CSS styling directly with Edit Attributes.



INTERACTIVITY

Turn your report into an interactive Shiny document in 4 steps:

1. Add **runtime: shiny** to the YAML header.
2. Call Shiny input functions to embed input objects.
3. Call Shiny render functions to embed reactive output.
4. Render with **rmarkdown::run()** or click **Run Document** in RStudio IDE.

```
---
output: html_document
runtime: shiny
---
```

```
```{r, echo = FALSE}
numericInput("n",
  "How many cars?", 5)

renderTable({
  head(cars, input$n)
})
```
```



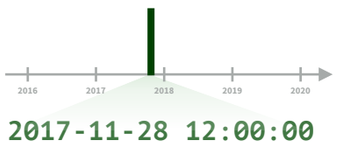
	speed	dist
1	4.00	2.00
2	4.00	10.00
3	7.00	4.00
4	7.00	22.00
5	8.00	16.00

Also see Shiny Prerendered for better performance. **rmarkdown.rstudio.com/authoring_shiny_prerendered**

Embed a complete app into your document with **shiny::shinyAppDir()**. More at **bookdown.org/yihui/rmarkdown/shiny-embedded.html**.

Dates and times with lubridate : : **CHEAT SHEET**

Date-times



`2017-11-28 12:00:00`
A **date-time** is a point on the timeline, stored as the number of seconds since 1970-01-01 00:00:00 UTC

dt <- ***as_datetime(****1511870400****)***
"2017-11-28 12:00:00 UTC"

`2017-11-28`
A **date** is a day stored as the number of days since 1970-01-01

d <- ***as_date(****17498****)***
"2017-11-28"

`12:00:00`
An hms is a **time** stored as the number of seconds since 00:00:00

t <- *hms::****as.hms(****85****)***
00:01:25

PARSE DATE-TIMES (Convert strings or numbers to date-times)

1. Identify the order of the year (**y**), month (**m**), day (**d**), hour (**h**), minute (**m**) and second (**s**) elements in your data.
2. Use the function below whose name replicates the order. Each accepts a tz argument to set the time zone, e.g. ymd(x, tz = "UTC").

`2017-11-28T14:02:00` **ymd_hms(), ymd_hm(), ymd_h().** ymd_hms("2017-11-28T14:02:00")

`2017-22-12 10:00:00` **ydm_hms(), ydm_hm(), ydm_h().** ydm_hms("2017-22-12 10:00:00")

`11/28/2017 1:02:03` **mdy_hms(), mdy_hm(), mdy_h().** mdy_hms("11/28/2017 1:02:03")

`1 Jan 2017 23:59:59` **dmy_hms(), dmy_hm(), dmy_h().** dmy_hms("1 Jan 2017 23:59:59")

`20170131` **ymd(), ydm().** ymd(20170131)

`July 4th, 2000` **mdy(), myd().** mdy("July 4th, 2000")

`4th of July '99` **dmy(), dym().** dmy("4th of July '99")

`2001: Q3` **yq()** Q for quarter. yq("2001: Q3")

`07-2020` **my(), ym().** my("07-2020")

`2:01` hms::**hms()** Also lubridate::**hms()**, **hm()** and **ms()**, which return periods.* hms::hms(sec = 0, min= 1, hours = 2, roll = FALSE)

`2017.5` **date_decimal(**decimal, tz = "UTC"**)** date_decimal(2017.5)

now(tzone = ""**)** Current time in tz (defaults to system tz). now()

today(tzone = ""**)** Current date in a tz (defaults to system tz). today()

fast_strptime() Faster strptime. fast_strptime('9/1/01', '%y/%m/%d')

parse_date_time() Easier strptime. parse_date_time("9/1/01", "ymd")

GET AND SET COMPONENTS

Use an accessor function to get a component.

```
d ## "2017-11-28"
day(d) ## 28
```

Assign into an accessor function to change a component in place.

```
day(d) <- 1
d ## "2017-11-01"
```

`2018-01-31 11:59:59` **date(**x**)** Date component. date(dt)

`2018-01-31 11:59:59` **year(**x**)** Year. year(dt)
isoyear(x**)** The ISO 8601 year.
epiyear(x**)** Epidemiological year.

`2018-01-31 11:59:59` **month(**x, label, abbr**)** Month. month(dt)

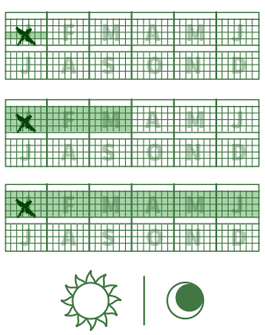
`2018-01-31 11:59:59` **day(**x**)** Day of month. day(dt)
wday(x, label, abbr**)** Day of week.
qday(x**)** Day of quarter.

`2018-01-31 11:59:59` **hour(**x**)** Hour. hour(dt)

`2018-01-31 11:59:59` **minute(**x**)** Minutes. minute(dt)

`2018-01-31 11:59:59` **second(**x**)** Seconds. second(dt)

`2018-01-31 11:59:59 UTC` **tz(**x**)** Time zone. tz(dt)



week(x**)** Week of the year. week(dt)
isoweek() ISO 8601 week.
epiweek() Epidemiological week.

quarter(x**)** Quarter. quarter(dt)

semester(x, with_year = FALSE**)** Semester. semester(dt)

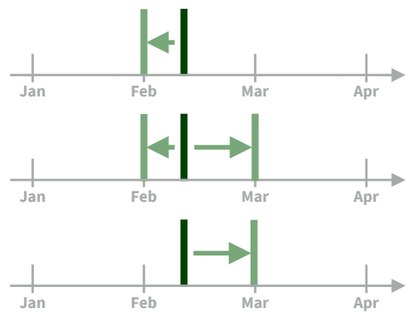
am(x**)** Is it in the am? am(dt)
pm(x**)** Is it in the pm? pm(dt)

dst(x**)** Is it daylight savings? dst(d)

leap_year(x**)** Is it a leap year? leap_year(d)

update(object, ..., simple = FALSE**)** update(dt, mday = 2, hour = 1)

Round Date-times



floor_date(x, unit = "second"**)**
Round down to nearest unit.
floor_date(dt, unit = "month")

round_date(x, unit = "second"**)**
Round to nearest unit.
round_date(dt, unit = "month")

ceiling_date(x, unit = "second", change_on_boundary = NULL**)**
Round up to nearest unit.
ceiling_date(dt, unit = "month")

Valid units are second, minute, hour, day, week, month, bimonth, quarter, season, halfyear and year.

rollback(dates, roll_to_first = FALSE, preserve_hms = TRUE**)** Roll back to last day of previous month. Also **rollforward()**. rollback(dt)

Stamp Date-times

stamp() Derive a template from an example string and return a new function that will apply the template to date-times. Also **stamp_date()** and **stamp_time()**.

1. Derive a template, create a function
   sf <- stamp("Created Sunday, Jan 17, 1999 3:34")
2. Apply the template to dates
   sf(ymd("2010-04-05"))
   ## [1] "Created Monday, Apr 05, 2010 00:00"

Tip: use a date with day > 12

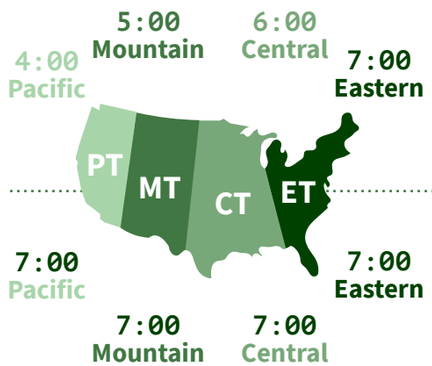
Time Zones

R recognizes ~600 time zones. Each encodes the time zone, Daylight Savings Time, and historical calendar variations for an area. R assigns *one* time zone per vector.

Use the **UTC** time zone to avoid Daylight Savings.

OlsonNames() Returns a list of valid time zone names. OlsonNames()

Sys.timezone() Gets current time zone.



with_tz(time, tzone = ""**)** Get the **same date-time** in a new time zone (a new clock time). Also **local_time(**dt, tz, units**)**.
with_tz(dt, "US/Pacific")

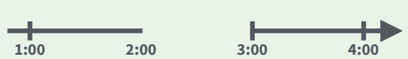
force_tz(time, tzone = ""**)** Get the **same clock time** in a new time zone (a new date-time). Also **force_tzs()**.
force_tz(dt, "US/Pacific")

Math with Date-times – Lubridate provides three classes of timespans to facilitate math with dates and date-times.

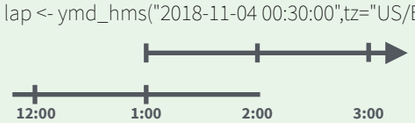
Math with date-times relies on the **timeline**, which behaves inconsistently. Consider how the timeline behaves during:

A normal day
nor <- ymd_hms("2018-01-01 01:30:00",tz="US/Eastern")

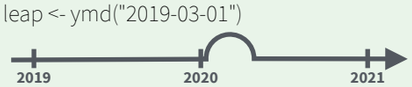
The start of daylight savings (spring forward)
gap <- ymd_hms("2018-03-11 01:30:00",tz="US/Eastern")



The end of daylight savings (fall back)
lap <- ymd_hms("2018-11-04 00:30:00",tz="US/Eastern")

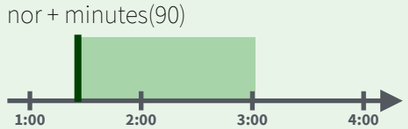


Leap years and leap seconds
leap <- ymd("2019-03-01")

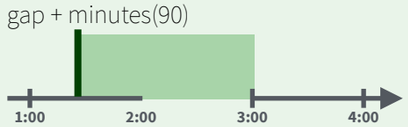


Periods track changes in clock times, which ignore time line irregularities.

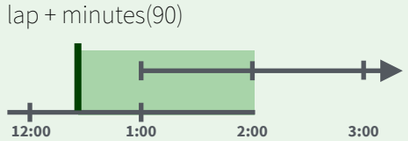
nor + minutes(90)



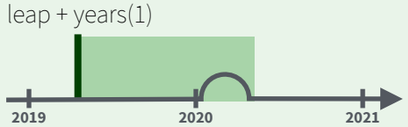
gap + minutes(90)



lap + minutes(90)

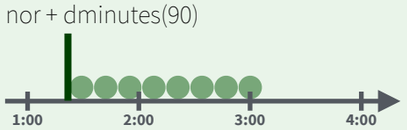


leap + years(1)

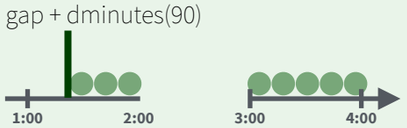


Durations track the passage of physical time, which deviates from clock time when irregularities occur.

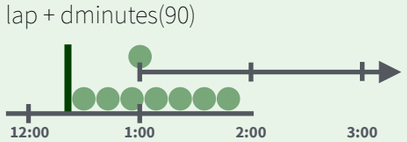
nor + dminutes(90)



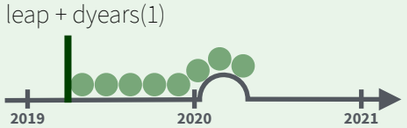
gap + dminutes(90)



lap + dminutes(90)

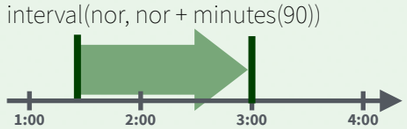


leap + dyears(1)

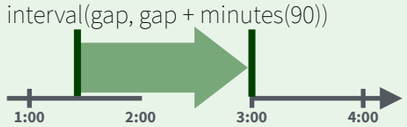


Intervals represent specific intervals of the timeline, bounded by start and end date-times.

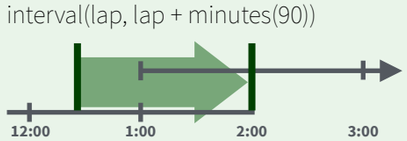
interval(nor, nor + minutes(90))



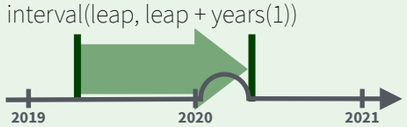
interval(gap, gap + minutes(90))



interval(lap, lap + minutes(90))



interval(leap, leap + years(1))



Not all years are 365 days due to **leap days**.

Not all minutes are 60 seconds due to **leap seconds**.

It is possible to create an imaginary date by adding **months**, e.g. February 31st

```
jan31 <- ymd(20180131)
jan31 + months(1)
## NA
```

%m+% and **%m-%** will roll imaginary dates to the last day of the previous month.

```
jan31 %m+% months(1)
## "2018-02-28"
```

add_with_rollback(e1, e2, roll_to_first = TRUE**)** will roll imaginary dates to the first day of the new month.

```
add_with_rollback(jan31, months(1),
roll_to_first = TRUE)
## "2018-03-01"
```

PERIODS

Add or subtract periods to model events that happen at specific clock times, like the NYSE opening bell.

Make a period with the name of a time unit ***pluralized***, e.g.

```
p <- months(3) + days(12)
p
"3m 12d 0H 0M 0S"
```

Number of months | Number of days | etc.

years(x = 1**)** x years.
months(x**)** x months.
weeks(x = 1**)** x weeks.
days(x = 1**)** x days.
hours(x = 1**)** x hours.
minutes(x = 1**)** x minutes.
seconds(x = 1**)** x seconds.
milliseconds(x = 1**)** x milliseconds.
microseconds(x = 1**)** x microseconds
nanoseconds(x = 1**)** x nanoseconds.
picoseconds(x = 1**)** x picoseconds.

period(num = NULL, units = "second", ...**)** An automation friendly period constructor. period(5, unit = "years")

as.period(x, unit**)** Coerce a timespan to a period, optionally in the specified units. Also **is.period()**. as.period(i)

period_to_seconds(x**)** Convert a period to the "standard" number of seconds implied by the period. Also **seconds_to_period()**. period_to_seconds(p)

DURATIONS

Add or subtract durations to model physical processes, like battery life. Durations are stored as seconds, the only time unit with a consistent length. **Difftimes** are a class of durations found in base R.

Make a duration with the name of a period prefixed with a ***d***, e.g.

```
dd <- ddays(14)
dd
"1209600s (~2 weeks)"
```

Exact length in seconds | Equivalent in common units

dyears(x = 1**)** 31536000x seconds.
dmonths(x = 1**)** 2629800x seconds.
dweeks(x = 1**)** 604800x seconds.
ddays(x = 1**)** 86400x seconds.
dhours(x = 1**)** 3600x seconds.
dminutes(x = 1**)** 60x seconds.
dseconds(x = 1**)** x seconds.
dmilliseconds(x = 1**)** x $\times 10^{-3}$ seconds.
dmicroseconds(x = 1**)** x $\times 10^{-6}$ seconds.
dnanoseconds(x = 1**)** x $\times 10^{-9}$ seconds.
dpicoseconds(x = 1**)** x $\times 10^{-12}$ seconds.

duration(num = NULL, units = "second", …**)** An automation friendly duration constructor. duration(5, unit = "years")

as.duration(x, …**)** Coerce a timespan to a duration. Also **is.duration()**, **is.difftime()**. as.duration(i)

make_difftime(x**)** Make difftime with the specified number of units. make_difftime(99999)

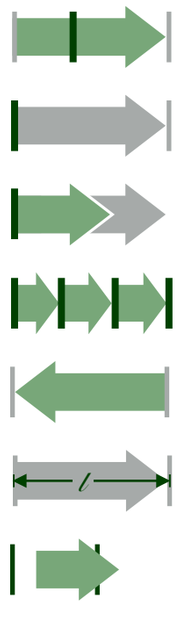
INTERVALS

Divide an interval by a duration to determine its physical length, divide an interval by a period to determine its implied length in clock time.

Make an interval with **interval()** or **%--%**, e.g.

Start Date | End Date

```
i <- interval(ymd("2017-01-01"), d)
j <- d %--% ymd("2017-12-31")
## 2017-01-01 UTC--2017-11-28 UTC
## 2017-11-28 UTC--2017-12-31 UTC
```



a **%within%** b Does interval or date-time *a* fall within interval *b*? now() %within% i

int_start(int**)** Access/set the start date-time of an interval. Also **int_end()**. int_start(i) <- now(); int_start(i)

int_aligns(int1, int2**)** Do two intervals share a boundary? Also **int_overlaps()**. int_aligns(i, j)

int_diff(times**)** Make the intervals that occur between the date-times in a vector. v <-c(dt, dt + 100, dt + 1000); int_diff(v)

int_flip(int**)** Reverse the direction of an interval. Also **int_standardize()**. int_flip(i)

int_length(int**)** Length in seconds. int_length(i)

int_shift(int, by**)** Shifts an interval up or down the timeline by a timespan. int_shift(i, days(-1))

as.interval(x, start, …**)** Coerce a timespan to an interval with the start date-time. Also **is.interval()**. as.interval(days(1), start = now())

Leaflet Cheat Sheet

Leaflet for R

an open-source JavaScript library for mobile-friendly interactive maps

Quick Start

Installation

Use **install.packages("leaflet")** to installize the package or directly from Github *devtools::install_github("rstudio/leaflet")*.

First Map

```
m <- leaflet() %>%      # leaflet works with the pipe operator
  addTiles() %>%        # setup the default OpenStreetMap map tiles
  addMarkers(lng = 174.768, lat = -36.852, popup = "The birthplace of R")
                        # add a single point layer
m
```

Map Widget

Initialization

m <- leaflet(options = leafletOptions(...))

center	Initial geographic center of the map
zoom	Initial map zoom level
minZoom	Minimum zoom level of the map
maxZoom	Maximum zoom level of the map

Map Methods

m %>% setView(lng, lat, zoom, options = list()**)**
Set the view of the map (center and zoom level)

m %>% fitBounds(lng1, lat1, lng2, lat2**)**
Fit the view into the rectangle [lng1, lat1] - [lng2, lat2]

m %>% clearBounds()
Clear the bound, automatically determine from the map elements

Data Object

Both **leaflet()** and the **map** layers have an optional data parameter that is designed to receive spatial data with the following formats:

Base R — *The arguments of all layers take normal R objects:*

```
df <- data.frame(lat = ..., lng = ... )
leaflet(df) %>% addTiles() %>% addCircles()
```

sp *package* — library(sp) *Useful functions:*
SpatialPoints, SpatialLines, SpatialPolygons, ...

maps *package* — library(maps) *Build a map of states with colors:*

```
mapStates <- map("state", fill = TRUE, plot = FALSE)
leaflet(mapStates) %>% addTiles() %>%
  addPolygons(fillColor = topo.colors(10, alpha = NULL), stroke = FALSE)
```

Markers

Use markers to call out points, express locations with latitude/longitude coordinates, appear as icons or as circles.
Data come from vectors or assigned data frame, or **sp** package objects.

Icon Markers

Regular Icons: default and simple
addMarkers(lng, lat, popup, label**)** *add basic icon markers*
makeIcon/icons (iconUrl, iconWidth, iconHeight, iconAnchorX, iconAnchorY, shadowUrl, shadowWidth, shadowHeight, ... **)** *customize marker icons*
iconList() *create a list of icons*
Awesome Icons: customizable with colors and icons
addAwesomeMarkers, makeAwesomeIcon, awesomeIcons, awesomeIconList
Marker Clusters: option of addMarters()
clusterOptions = markerClusterOptions()
freezeAtZoom Freeze the cluster at assigned zoom level

Circle Markers

addCircleMarkers(color, radius, stroke, opacity, ... **)**
Customize their color, radius, stroke, opacity

Popups and Labels

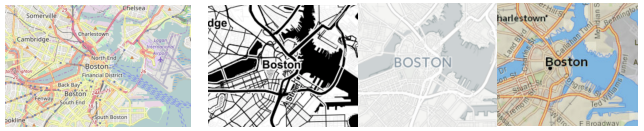
addPopups(lng, lat, ...content..., options) *Add standalone popups*
options = popupOptions(closeButton=FALSE)
addMarkers(..., popup, ...) *Show popups with markers or shapes*
addMarkers(..., label, labelOptions...) *Show labels with markers or shapes*
labelOptions = labelOptions(noHide, textOnly, textsize, direction, style)
addLabelOnlyMarkers() *Add labels without markers*

Lines and Shapes

Polygons and Polylines
addPolygons(color, weight=1, smoothFactor=0.5, opacity=1.0, fillOpacity=0.5, fillColor= ~colorQuantile("YlOrRd", ALAND)(ALAND), highlightOptions, ... **)**
highlightOptions(color, weight=2, bringToFront=TRUE**)** *highlight shapes*
Use **rmapshaper::ms_simplify** to simplify complex shapes
Circles **addCircles(**lng, lat, weight=1, radius, ... **)**
Rectangles **addRectangles(**lng1, lat1, lng2, lat2, fillColor="transparent", ... **)**

Basemaps

addTiles() providers$Stamen.Toner, CartoDB.Positron, Esri.NatGeoWorldMap



Default Tiles Third-Party Tiles **addProviderTiles()**
Use **addTiles()** to add a custom map tile URL template, use **addWMSTiles()** to add WMS (Web Map Service) tiles

GeoJSON and TopoJSON

There are two options to use the GeoJSON/TopoJSON data.
* To read into **sp** objects with the **geojsonio** or **rgdal** package:
geojsonio::geojson_read(...,what="sp") *rgdal::readOGR(...,"OGRGeoJSON")*
* Or to use the **addGeoJSON()** and **addTopoJSON()** functions:
addTopoJSON/addGeoJSON(... weight, color, fill, opacity, fillOpacity...)
Styles can also be tuned separately with a *style: {...}* object.

Other packages including **RJSONIO** and **jsonlite** can help fast parse or generate the data needed.

Shiny Integration

To integrate a Leaflet map into an app:
* In the UI, call **leafletOutput**("name")
* On the server side, assign a **renderLeaflet**(...) call to the output
* Inside the **renderLeaflet** expression, return a Leaflet map object

Modification

To modify an existing map or add incremental changes to the map, you can use **leafletProxy()**. This should be performed in an observer on the server side.

Other useful functions to edit your map:
fitBounds(0, 0, 11, 11**)** *similar to setView*
fit the view to within these bounds
addCircles(1:10, 1:10, layerId = LETTERS[1:10]**)**
create circles with layerIds of "A", "B", "C"...
removeShape(c("B", "F")**)** *remove some of the circles*
clearShapes() *clear all circles (and other shapes)*

Inputs/Events

Object Events
Object event names generally use this pattern:
input$*MAPID_OBJCATEGORY_EVENTNAME*.
Triger an event changes the value of the Shiny input at this variable.
Valid values for *OBJCATEGORY* are *marker*, *shape*, *geojson* and *topojson*.
Valid values for *EVENTNAME* are *click*, *mouseover* and *mouseout*.

All of these events are set to either *NULL* if the event has never happened, or a *list()* that includes:
* lat The latitude of the object, if available; otherwise, the mouse cursor
* lng The longitude of the object, if available; otherwise, the mouse cursor
* id The layerId, if any

GeoJSON events also include additional properties:
* featureId The feature ID, if any
* properties The feature properties

Map Events

input$*MAPID*_click	*when the map background or basemap is clicked value -- a list with lat and lng*
input$*MAPID*_bounds	*provide the lat/lng bounds of the visible map area value -- a list with north, east, south and west*
input$*MAPID*_zoom	*an integer indicates the zoom level*

Kejia Shi @ Data Science Institute, Columbia University in the City of New York, Kejia.Shi@Columbia.edu

Adapted from Rstudio materials https://rstudio.github.io/leaflet/ 2017/05/01

Data visualization with ggplot2 : : CHEAT SHEET

Basics

ggplot2 is based on the **grammar of graphics**, the idea that you can build every graph from the same components: a **data** set, a **coordinate system**, and **geoms**—visual marks that represent data points.

To display values, map variables in the data to visual properties of the geom (**aesthetics**) like **size**, **color**, and **x** and **y** locations.

Complete the template below to build a graph.

```
ggplot (data = <DATA>) +                                  ] required
<GEOM_FUNCTION>(mapping = aes(<MAPPINGS>),
 stat =<STAT>, position =<POSITION>) +                    ] Not required, sensible defaults supplied
<COORDINATE_FUNCTION> +
<FACET_FUNCTION> +
<SCALE_FUNCTION> +
<THEME_FUNCTION>
```

ggplot(data = mpg, **aes(**x = cty, y = hwy**))** Begins a plot that you finish by adding layers to. Add one geom function per layer.

last_plot() Returns the last plot.

ggsave("plot.png", width = 5, height = 5**)** Saves last plot as 5' x 5' file named "plot.png" in working directory. Matches file type to file extension.

Aes

Common aesthetic values.

color and **fill** - string ("red", "#RRGGBB")

linetype - integer or string (0 = "blank", 1 = "solid", 2 = "dashed", 3 = "dotted", 4 = "dotdash", 5 = "longdash", 6 = "twodash")

lineend - string ("round", "butt", or "square")

linejoin - string ("round", "mitre", or "bevel")

size - integer (line width in mm)

shape - integer/shape name or a single character ("a")

RStudio

Geoms

Use a geom function to represent data points, use the geom's aesthetic properties to represent variables. Each function returns a layer.

GRAPHICAL PRIMITIVES

a <- ggplot(economics, aes(date, unemploy))
b <- ggplot(seals, aes(x = long, y = lat))

a + geom_blank() and **a + expand_limits()**
Ensure limits include values across all plots.

b + geom_curve(aes(yend = lat + 1, xend = long + 1), curvature = 1**)** - x, xend, y, yend, alpha, angle, color, curvature, linetype, size

a + geom_path(lineend = "butt", linejoin = "round", linemitre = 1**)**
x, y, alpha, color, group, linetype, size

a + geom_polygon(aes(alpha = 50)**)** - x, y, alpha, color, fill, group, subgroup, linetype, size

b + geom_rect(aes(xmin = long, ymin = lat, xmax = long + 1, ymax = lat + 1)**)** - xmax, xmin, ymax, ymin, alpha, color, fill, linetype, size

a + geom_ribbon(aes(ymin = unemploy - 900, ymax = unemploy + 900)**)** - x, ymax, ymin, alpha, color, fill, group, linetype, size

LINE SEGMENTS

common aesthetics: x, y, alpha, color, linetype, size

b + geom_abline(aes(intercept = 0, slope = 1)**)**
b + geom_hline(aes(yintercept = lat)**)**
b + geom_vline(aes(xintercept = long)**)**

b + geom_segment(aes(yend = lat + 1, xend = long + 1)**)**
b + geom_spoke(aes(angle = 1:1155, radius = 1)**)**

ONE VARIABLE continuous

c <- ggplot(mpg, aes(hwy)); c2 <- ggplot(mpg)

c + geom_area(stat = "bin"**)**
x, y, alpha, color, fill, linetype, size

c + geom_density(kernel = "gaussian"**)**
x, y, alpha, color, fill, group, linetype, size, weight

c + geom_dotplot()
x, y, alpha, color, fill

c + geom_freqpoly()
x, y, alpha, color, group, linetype, size

c + geom_histogram(binwidth = 5**)**
x, y, alpha, color, fill, linetype, size, weight

c2 + geom_qq(aes(sample = hwy)**)**
x, y, alpha, color, fill, linetype, size, weight

discrete

d <- ggplot(mpg, aes(fl))

d + geom_bar()
x, alpha, color, fill, linetype, size, weight

TWO VARIABLES both continuous

e <- ggplot(mpg, aes(cty, hwy))

e + geom_label(aes(label = cty), nudge_x = 1, nudge_y = 1**)** - x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

e + geom_point()
x, y, alpha, color, fill, shape, size, stroke

e + geom_quantile()
x, y, alpha, color, group, linetype, size, weight

e + geom_rug(sides = "bl"**)**
x, y, alpha, color, linetype, size

e + geom_smooth(method = lm**)**
x, y, alpha, color, fill, group, linetype, size, weight

e + geom_text(aes(label = cty), nudge_x = 1, nudge_y = 1**)** - x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

one discrete, one continuous

f <- ggplot(mpg, aes(class, hwy))

f + geom_col()
x, y, alpha, color, fill, group, linetype, size

f + geom_boxplot()
x, y, lower, middle, upper, ymax, ymin, alpha, color, fill, group, linetype, shape, size, weight

f + geom_dotplot(binaxis = "y", stackdir = "center"**)**
x, y, alpha, color, fill, group

f + geom_violin(scale = "area"**)**
x, y, alpha, color, fill, group, linetype, size, weight

both discrete

g <- ggplot(diamonds, aes(cut, color))

g + geom_count()
x, y, alpha, color, fill, shape, size, stroke

e + geom_jitter(height = 2, width = 2**)**
x, y, alpha, color, fill, shape, size

continuous bivariate distribution

h <- ggplot(diamonds, aes(carat, price))

h + geom_bin2d(binwidth = c(0.25, 500)**)**
x, y, alpha, color, fill, linetype, size, weight

h + geom_density_2d()
x, y, alpha, color, group, linetype, size

h + geom_hex()
x, y, alpha, color, fill, size

continuous function

i <- ggplot(economics, aes(date, unemploy))

i + geom_area()
x, y, alpha, color, fill, linetype, size

i + geom_line()
x, y, alpha, color, group, linetype, size

i + geom_step(direction = "hv"**)**
x, y, alpha, color, group, linetype, size

visualizing error

df <- data.frame(grp = c("A", "B"), fit = 4:5, se = 1:2)
j <- ggplot(df, aes(grp, fit, ymin = fit - se, ymax = fit + se))

j + geom_crossbar(fatten = 2**)** - x, y, ymax, ymin, alpha, color, fill, group, linetype, size

j + geom_errorbar() - x, ymax, ymin, alpha, color, group, linetype, size, width
Also **geom_errorbarh()**.

j + geom_linerange()
x, ymin, ymax, alpha, color, group, linetype, size

j + geom_pointrange() - x, y, ymin, ymax, alpha, color, fill, group, linetype, shape, size

maps

data <- data.frame(murder = USArrests$Murder, state = tolower(rownames(USArrests)))
map <- map_data("state")
k <- ggplot(data, aes(fill = murder))

k + geom_map(aes(map_id = state), map = map**)**
+ expand_limits(x = map$long, y = map$lat**)**
map_id, alpha, color, fill, linetype, size

THREE VARIABLES

seals$z <- with(seals, sqrt(delta_long^2 + delta_lat^2)); l <- ggplot(seals, aes(long, lat))

l + geom_contour(aes(z = z)**)**
x, y, z, alpha, color, group, linetype, size, weight

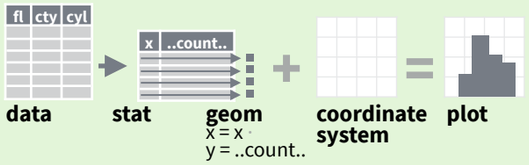
l + geom_contour_filled(aes(fill = z)**)**
x, y, alpha, color, fill, group, linetype, size, subgroup

l + geom_raster(aes(fill = z), hjust = 0.5, vjust = 0.5, interpolate = FALSE**)**
x, y, alpha, fill

l + geom_tile(aes(fill = z)**)**
x, y, alpha, color, fill, linetype, size, width

Stats An alternative way to build a layer.

A stat builds new variables to plot (e.g., count, prop).



Visualize a stat by changing the default stat of a geom function, **geom_bar(stat="count")** or by using a stat function, **stat_count(geom="bar")**, which calls a default geom to make a layer (equivalent to a geom function). Use **..name..** syntax to map stat variables to aesthetics.

geom to use | stat function | geommappings

i + stat_density_2d(aes(fill = ..level..), geom = "polygon"**)**

variable created by stat

c + stat_bin(binwidth = 1, boundary = 10**)**
x, y | ..count.., ..ncount.., ..density.., ..ndensity..

c + stat_count(width = 1**) x, y** | ..count.., ..prop..

c + stat_density(adjust = 1, kernel = "gaussian"**)**
x, y | ..count.., ..density.., ..scaled..

e + stat_bin_2d(bins = 30, drop = T**)**
x, y, fill | ..count.., ..density..

e + stat_bin_hex(bins = 30**) x, y, fill** | ..count.., ..density..

e + stat_density_2d(contour = TRUE, n = 100**)**
x, y, color, size | ..level..

e + stat_ellipse(level = 0.95, segments = 51, type = "t"**)**

l + stat_contour(aes(z = z)**) x, y, z, order** | ..level..

l + stat_summary_hex(aes(z = z), bins = 30, fun = max**)**
x, y, z, fill | ..value..

l + stat_summary_2d(aes(z = z), bins = 30, fun = mean**)**
x, y, z, fill | ..value..

f + stat_boxplot(coef = 1.5**)**
x, y | ..lower.., ..middle.., ..upper.., ..width.. , ..ymin.., ..ymax..

f + stat_ydensity(kernel = "gaussian", scale = "area"**) x, y**
| ..density.., ..scaled.., ..count.., ..n.., ..violinwidth.., ..width..

e + stat_ecdf(n = 40**) x, y** | ..x.., ..y..

e + stat_quantile(quantiles = c(0.1, 0.9),
formula = y ~ log(x), method = "rq"**) x, y** | ..quantile..

e + stat_smooth(method = "lm", formula = y ~ x, se = T,
level = 0.95**) x, y** | ..se.., ..x.., ..y.., ..ymin.., ..ymax..

ggplot() + xlim(-5, 5**) + stat_function(**fun = dnorm,
n = 20, geom = "point"**) x** | ..x.., ..y..

ggplot() + stat_qq(aes(sample = 1:100)**)**
x, y, sample | ..sample.., ..theoretical..

e + stat_sum() x, y, size | ..n.., ..prop..

e + stat_summary(fun.data = "mean_cl_boot"**)**

h + stat_summary_bin(fun = "mean", geom = "bar"**)**

e + stat_identity()

e + stat_unique()

Scales Override defaults with **scales** package.

Scales map data values to the visual values of an aesthetic. To change a mapping, add a new scale.

n <- d + geom_bar(aes(fill = fl))

scale_ | aesthetic to adjust | prepackaged scale to use | scale-specific arguments

n + scale_fill_manual(
values = c("skyblue", "royalblue", "blue", "navy"),
limits = c("d", "e", "p", "r"), breaks =c("d", "e", "p", "r"),
name = "fuel", labels = c("D", "E", "P", "R")**)**

range of values to include in mapping | title to use in legend/axis | labels to use in legend/axis | breaks to use in legend/axis

GENERAL PURPOSE SCALES

Use with most aesthetics

scale_*_continuous() - Map cont' values to visual ones.
scale_*_discrete() - Map discrete values to visual ones.
scale_*_binned() - Map continuous values to discrete bins.
scale_*_identity() - Use data values as visual ones.
scale_*_manual(values = c()**)** - Map discrete values to manually chosen visual ones.
scale_*_date(date_labels = "%m/%d"),
date_breaks = "2 weeks"**)** - Treat data values as dates.
scale_*_datetime() - Treat data values as date times. Same as scale_*_date(). See ?strptime for label formats.

X & Y LOCATION SCALES

Use with x or y aesthetics (x shown here)

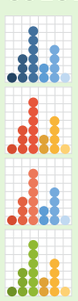
scale_x_log10() - Plot x on log10 scale.
scale_x_reverse() - Reverse the direction of the x axis.
scale_x_sqrt() - Plot x on square root scale.

COLOR AND FILL SCALES (DISCRETE)

n + scale_fill_brewer(palette = "Blues"**)**
For palette choices:
RColorBrewer::display.brewer.all()

n + scale_fill_grey(start = 0.2,
end = 0.8, na.value = "red"**)**

COLOR AND FILL SCALES (CONTINUOUS)



o <- c + geom_dotplot(aes(fill = ..x..))

o + scale_fill_distiller(palette = "Blues"**)**

o + scale_fill_gradient(low="red", high="yellow"**)**

o + scale_fill_gradient2(low = "red", high = "blue",
mid = "white", midpoint = 25**)**

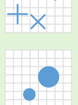
o + scale_fill_gradientn(colors = topo.colors(6)**)**
Also: rainbow(), heat.colors(), terrain.colors(),
cm.colors(), RColorBrewer::brewer.pal()

SHAPE AND SIZE SCALES

p <- e + geom_point(aes(shape = fl, size = cyl))

p + scale_shape() + scale_size()
p + scale_shape_manual(values = c(3:7)**)**

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25



p + scale_radius(range = c(1,6)**)**
p + scale_size_area(max_size = 6**)**

Coordinate Systems

r <- d + geom_bar()

r + coord_cartesian(xlim = c(0, 5)**)** - xlim, ylim
The default cartesian coordinate system.

r + coord_fixed(ratio = 1/2**)**
ratio, xlim, ylim - Cartesian coordinates with fixed aspect ratio between x and y units.

ggplot(mpg, aes(y = fl)**) + geom_bar()**
Flip cartesian coordinates by switching x and y aesthetic mappings.

r + coord_polar(theta = "x", direction=1**)**
theta, start, direction - Polar coordinates.

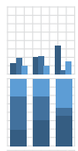
r + coord_trans(y = "sqrt"**)** - x, y, xlim, ylim
Transformed cartesian coordinates. Set xtrans and ytrans to the name of a window function.

π + coord_quickmap()
π + coord_map(projection = "ortho", orientation = c(41, -74, 0)**)** - projection, xlim, ylim
Map projections from the mapproj package (mercator (default), azequalarea, lagrange, etc.).

Position Adjustments

Position adjustments determine how to arrange geoms that would otherwise occupy the same space.

s <- ggplot(mpg, aes(fl, fill = drv))



s + geom_bar(position = "dodge")
Arrange elements side by side.

s + geom_bar(position = "fill")
Stack elements on top of one another, normalize height.

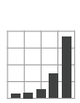
e + geom_point(position = "jitter")
Add random noise to X and Y position of each element to avoid overplotting.

e + geom_label(position = "nudge")
Nudge labels away from points.

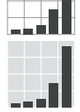
s + geom_bar(position = "stack")
Stack elements on top of one another.

Each position adjustment can be recast as a function with manual **width** and **height** arguments:
s + geom_bar(position = position_dodge(width = 1))

Themes

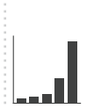


r + theme_bw()
White background with grid lines.



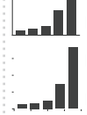
r + theme_gray()
Grey background (default theme).

r + theme_dark()
Dark for contrast.



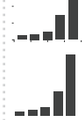
r + theme_classic()

r + theme_light()



r + theme_linedraw()

r + theme_minimal()
Minimal theme.



r + theme_void()
Empty theme.

r + theme() Customize aspects of the theme such as axis, legend, panel, and facet properties.
r + ggtitle("Title") + theme(plot.title.postion = "plot")
r + theme(panel.background = element_rect(fill = "blue"))

Faceting

Facets divide a plot into subplots based on the values of one or more discrete variables.

t <- ggplot(mpg, aes(cty, hwy)) + geom_point()

t + facet_grid(cols = vars(fl))
Facet into columns based on fl.

t + facet_grid(rows = vars(year))
Facet into rows based on year.

t + facet_grid(rows = vars(year), cols = vars(fl))
Facet into both rows and columns.

t + facet_wrap(vars(fl))
Wrap facets into a rectangular layout.

Set **scales** to let axis limits vary across facets.

t + facet_grid(rows = vars(drv), cols = vars(fl),
scales = "free")
x and y axis limits adjust to individual facets:
"free_x" - x axis limits adjust
"free_y" - y axis limits adjust

Set **labeller** to adjust facet label:

t + facet_grid(cols = vars(fl), labeller = label_both)

fl: c	fl: d	fl: e	fl: p	fl: r

t + facet_grid(rows = vars(fl),
labeller = label_bquote(alpha ^ .(fl)**))**

α^c α^d α^e α^p α^r

Labels and Legends

Use **labs()** to label the elements of your plot.

t + labs(x = "New x axis label", **y** = "New y axis label",
title ="Add a title above the plot",
subtitle = "Add a subtitle below title",
caption = "Add a caption below plot",
alt = "Add alt text to the plot",
<AES> = "New <AES> legend title"**)**

t + annotate(geom = "text", x = 8, y = 9, label = "A"**)**
Places a geom with manually selected aesthetics.

p + guides(x = guide_axis(n.dodge = 2)**)** Avoid crowded or overlapping labels with guide_axis(n.dodge or angle).

n + guides(fill = "none"**)** Set legend type for each aesthetic: colorbar, legend, or none (no legend).

n + theme(legend.position = "bottom"**)**
Place legend at "bottom", "top", "left", or "right".

n + scale_fill_discrete(name = "Title",
labels = c("A", "B", "C", "D", "E")**)**
Set legend title and labels with a scale function.

Zooming

Without clipping (preferred):
t + coord_cartesian(xlim = c(0, 100), ylim = c(10, 20)**)**

With clipping (removes unseen data points):
t + xlim(0, 100**) + ylim(**10, 20**)**

t + scale_x_continuous(limits = c(0, 100)**) +**
scale_y_continuous(limits = c(0, 100)**)**

Data transformation with dplyr : : **CHEAT SHEET**

dplyr functions work with pipes and expect **tidy data**. In tidy data:

Each **variable** is in its own **column**

&

Each **observation**, or **case**, is in its own **row**

pipes

x %>% f(y) becomes **f(x, y)**

Summarise Cases

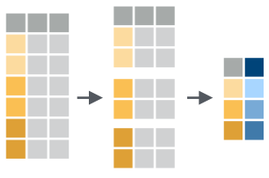
Apply **summary functions** to columns to create a new table of summary statistics. Summary functions take vectors as input and return one value (see back).

summarise(.data, …**)**
Compute table of summaries.
summarise(mtcars, avg = mean(mpg))

count(.data, …, wt = NULL, sort = FALSE, name = NULL**)** Count number of rows in each group defined by the variables in … Also **tally()**.
count(mtcars, cyl)

Group Cases

Use **group_by(**.data, …, .add = FALSE, .drop = TRUE**)** to create a "grouped" copy of a table grouped by columns in ... dplyr functions will manipulate each "group" separately and combine the results.



```
mtcars %>%
  group_by(cyl) %>%
  summarise(avg = mean(mpg))
```

Use **rowwise(**.data, …**)** to group data into individual rows. dplyr functions will compute results for each row. Also apply functions to list-columns. See tidyr cheat sheet for list-column workflow.

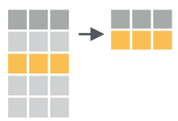
```
starwars %>%
  rowwise() %>%
  mutate(film_count = length(films))
```

ungroup(x, …**)** Returns ungrouped copy of table.
ungroup(g_mtcars)

Manipulate Cases

EXTRACT CASES

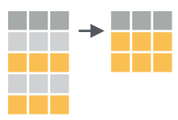
Row functions return a subset of rows as a new table.



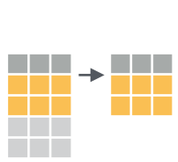
filter(.data, …, .preserve = FALSE**)** Extract rows that meet logical criteria.
filter(mtcars, mpg > 20)

distinct(.data, …, .keep_all = FALSE**)** Remove rows with duplicate values.
distinct(mtcars, gear)

slice(.data, …, .preserve = FALSE**)** Select rows by position.
slice(mtcars, 10:15)



slice_sample(.data, …, n, prop, weight_by = NULL, replace = FALSE**)** Randomly select rows. Use n to select a number of rows and prop to select a fraction of rows.
slice_sample(mtcars, n = 5, replace = TRUE)



slice_min(.data, order_by, …, n, prop, with_ties = TRUE**)** and **slice_max()** Select rows with the lowest and highest values.
slice_min(mtcars, mpg, prop = 0.25)

slice_head(.data, …, n, prop**)** and **slice_tail()** Select the first or last rows.
slice_head(mtcars, n = 5)

Logical and boolean operators to use with filter()

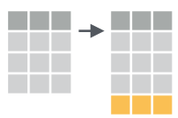
==	<	<=	is.na()	%in%	|	xor()
!=	>	>=	!is.na()	!	&	

See **?base::Logic** and **?Comparison** for help.

ARRANGE CASES

arrange(.data, …, .by_group = FALSE**)** Order rows by values of a column or columns (low to high), use with **desc()** to order from high to low.
arrange(mtcars, mpg)
arrange(mtcars, desc(mpg))

ADD CASES



add_row(.data, …, .before = NULL, .after = NULL**)** Add one or more rows to a table.
add_row(cars, speed = 1, dist = 1)

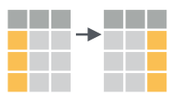
Manipulate Variables

EXTRACT VARIABLES

Column functions return a set of columns as a new vector or table.

pull(.data, var = -1, name = NULL, …**)** Extract column values as a vector, by name or index.
pull(mtcars, wt)

select(.data, …**)** Extract columns as a table.
select(mtcars, mpg, wt)

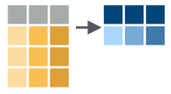


relocate(.data, …, .before = NULL, .after = NULL**)** Move columns to new position.
relocate(mtcars, mpg, cyl, .after = last_col())

Use these helpers with select() and across()
e.g. select(mtcars, mpg:cyl)

contains(match**)**	**num_range(**prefix, range**)**	**:**, e.g. mpg:cyl
ends_with(match**)**	**all_of(**x**)**/**any_of(**x, …, vars**)**	**-**, e.g, -gear
starts_with(match**)**	**matches(**match**)**	**everything()**

MANIPULATE MULTIPLE VARIABLES AT ONCE

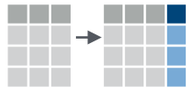


across(.cols, .funs, …, .names = NULL**)** Summarise or mutate multiple columns in the same way.
summarise(mtcars, across(everything(), mean))

c_across(.cols**)** Compute across columns in row-wise data.
transmute(rowwise(UKgas), total = sum(c_across(1:2)))

MAKE NEW VARIABLES

Apply **vectorized functions** to columns. Vectorized functions take vectors as input and return vectors of the same length as output (see back).



mutate(.data, …, .keep = "all", .before = NULL, .after = NULL**)** Compute new column(s). Also **add_column(), add_count(),** and **add_tally().**
mutate(mtcars, gpm = 1 / mpg)

transmute(.data, …**)** Compute new column(s), drop others.
transmute(mtcars, gpm = 1 / mpg)

rename(.data, …**)** Rename columns. Use **rename_with()** to rename with a function.
rename(cars, distance = dist)

Vectorized Functions

TO USE WITH MUTATE ()

mutate() and **transmute()** apply vectorized functions to columns to create new columns. Vectorized functions take vectors as input and return vectors of the same length as output.

vectorized function

OFFSET

dplyr::**lag()** - offset elements by 1
dplyr::**lead()** - offset elements by -1

CUMULATIVE AGGREGATE

dplyr::**cumall()** - cumulative all()
dplyr::**cumany()** - cumulative any()
cummax() - cumulative max()
dplyr::**cummean()** - cumulative mean()
cummin() - cumulative min()
cumprod() - cumulative prod()
cumsum() - cumulative sum()

RANKING

dplyr::**cume_dist()** - proportion of all values <=
dplyr::**dense_rank()** - rank w ties = min, no gaps
dplyr::**min_rank()** - rank with ties = min
dplyr::**ntile()** - bins into n bins
dplyr::**percent_rank()** - min_rank scaled to [0,1]
dplyr::**row_number()** - rank with ties = "first"

MATH

+, - , *, /, ^, %/%, %% - arithmetic ops
log(), log2(), log10() - logs
<, <=, >, >=, !=, == - logical comparisons
dplyr::**between()** - x >= left & x <= right
dplyr::**near()** - safe == for floating point numbers

MISCELLANEOUS

dplyr::**case_when()** - multi-case if_else()

```
starwars %>%
  mutate(type = case_when(
    height > 200 | mass > 200 ~ "large",
      species == "Droid"      ~ "robot",
      TRUE                    ~ "other")
    )
```

dplyr::**coalesce()** - first non-NA values by element across a set of vectors
dplyr::**if_else()** - element-wise if() + else()
dplyr::**na_if()** - replace specific values with NA
pmax() - element-wise max()
pmin() - element-wise min()

Summary Functions

TO USE WITH SUMMARISE ()

summarise() applies summary functions to columns to create a new table. Summary functions take vectors as input and return single values as output.

summary function

COUNT

dplyr::**n()** - number of values/rows
dplyr::**n_distinct()** - # of uniques
sum(!is.na()) - # of non-NA's

POSITION

mean() - mean, also **mean(!is.na())**
median() - median

LOGICAL

mean() - proportion of TRUE's
sum() - # of TRUE's

ORDER

dplyr::**first()** - first value
dplyr::**last()** - last value
dplyr::**nth()** - value in nth location of vector

RANK

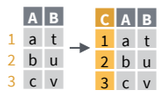
quantile() - nth quantile
min() - minimum value
max() - maximum value

SPREAD

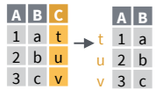
IQR() - Inter-Quartile Range
mad() - median absolute deviation
sd() - standard deviation
var() - variance

Row Names

Tidy data does not use rownames, which store a variable outside of the columns. To work with the rownames, first move them into a column.



tibble::**rownames_to_column()**
Move row names into col.
a <- rownames_to_column(mtcars, var = "C")



tibble::**column_to_rownames()**
Move col into row names.
column_to_rownames(a, var = "C")

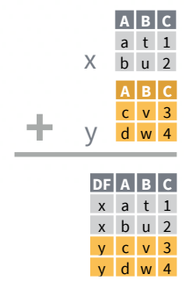
Also tibble::**has_rownames()** and tibble::**remove_rownames()**.

Combine Tables

COMBINE VARIABLES

bind_cols(…, .name_repair**)** Returns tables placed side by side as a single table. Column lengths must be equal. Columns will NOT be matched by id (to do that look at Relational Data below), so be sure to check that both tables are ordered the way you want before binding.

COMBINE CASES



bind_rows(…, .id = NULL**)** Returns tables one on top of the other as a single table. Set .id to a column name to add a column of the original table names (as pictured).

RELATIONAL DATA

Use a "**Mutating Join**" to join one table to columns from another, matching values with the rows that they correspond to. Each join retains a different combination of values from the tables.

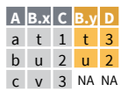
left_join(x, y, by = NULL, copy = FALSE, suffix = c(".x", ".y"), …, keep = FALSE, na_matched = "na"**)** Join matching values from y to x.

right_join(x, y, by = NULL, copy = FALSE, suffix = c(".x", ".y"), …, keep = FALSE, na_matches = "na"**)** Join matching values from x to y.

inner_join(x, y, by = NULL, copy = FALSE, suffix = c(".x", ".y"), …, keep = FALSE, na_matches = "na"**)** Join data. Retain only rows with matches.

full_join(x, y, by = NULL, copy = FALSE, suffix = c(".x", ".y"), …, keep = FALSE, na_matches = "na"**)** Join data. Retain all values, all rows.

COLUMN MATCHING FOR JOINS

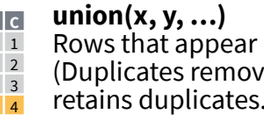


Use **by = c("col1", "col2", …)** to specify one or more common columns to match on.
left_join(x, y, by = "A")

Use a named vector, **by = c("col1" = "col2")**, to match on columns that have different names in each table.
left_join(x, y, by = c("C" = "D"))

Use **suffix** to specify the suffix to give to unmatched columns that have the same name in both tables.
left_join(x, y, by = c("C" = "D"), suffix = c("1", "2"))

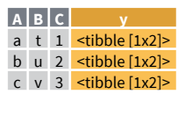
Use a "**Filtering Join**" to filter one table against the rows of another.



semi_join(x, y, by = NULL, copy = FALSE, …, na_matches = "na"**)** Return rows of x that have a match in y. Use to see what will be included in a join.

anti_join(x, y, by = NULL, copy = FALSE, …, na_matches = "na"**)** Return rows of x that do not have a match in y. Use to see what will not be included in a join.

Use a "**Nest Join**" to inner join one table to another into a nested data frame.



nest_join(x, y, by = NULL, copy = FALSE, keep = FALSE, name = NULL, …**)** Join data, nesting matches from y in a single new data frame column.

SET OPERATIONS

intersect(x, y, …)
Rows that appear in both x and y.

setdiff(x, y, …)
Rows that appear in x but not y.

union(x, y, …)
Rows that appear in x or y. (Duplicates removed). **union_all()** retains duplicates.

Use **setequal()** to test whether two data sets contain the exact same rows (in any order).

SQL CHEAT SHEET http://www.sqltutorial.org

QUERYING DATA FROM A TABLE

```
SELECT c1, c2 FROM t;
```
Query data in columns c1, c2 from a table

```
SELECT * FROM t;
```
Query all rows and columns from a table

```
SELECT c1, c2 FROM t
WHERE condition;
```
Query data and filter rows with a condition

```
SELECT DISTINCT c1 FROM t
WHERE condition;
```
Query distinct rows from a table

```
SELECT c1, c2 FROM t
ORDER BY c1 ASC [DESC];
```
Sort the result set in ascending or descending order

```
SELECT c1, c2 FROM t
ORDER BY c1
LIMIT n OFFSET offset;
```
Skip *offset* of rows and return the next n rows

```
SELECT c1, aggregate(c2)
FROM t
GROUP BY c1;
```
Group rows using an aggregate function

```
SELECT c1, aggregate(c2)
FROM t
GROUP BY c1
HAVING condition;
```
Filter groups using HAVING clause

QUERYING FROM MULTIPLE TABLES

```
SELECT c1, c2
FROM t1
INNER JOIN t2 ON condition;
```
Inner join t1 and t2

```
SELECT c1, c2
FROM t1
LEFT JOIN t2 ON condition;
```
Left join t1 and t1

```
SELECT c1, c2
FROM t1
RIGHT JOIN t2 ON condition;
```
Right join t1 and t2

```
SELECT c1, c2
FROM t1
FULL OUTER JOIN t2 ON condition;
```
Perform full outer join

```
SELECT c1, c2
FROM t1
CROSS JOIN t2;
```
Produce a Cartesian product of rows in tables

```
SELECT c1, c2
FROM t1, t2;
```
Another way to perform cross join

```
SELECT c1, c2
FROM t1 A
INNER JOIN t2 B ON condition;
```
Join t1 to itself using INNER JOIN clause

USING SQL OPERATORS

```
SELECT c1, c2 FROM t1
UNION [ALL]
SELECT c1, c2 FROM t2;
```
Combine rows from two queries

```
SELECT c1, c2 FROM t1
INTERSECT
SELECT c1, c2 FROM t2;
```
Return the intersection of two queries

```
SELECT c1, c2 FROM t1
MINUS
SELECT c1, c2 FROM t2;
```
Subtract a result set from another result set

```
SELECT c1, c2 FROM t1
WHERE c1 [NOT] LIKE pattern;
```
Query rows using pattern matching %, _

```
SELECT c1, c2 FROM t
WHERE c1 [NOT] IN value_list;
```
Query rows in a list

```
SELECT c1, c2 FROM t
WHERE  c1 BETWEEN low AND high;
```
Query rows between two values

```
SELECT c1, c2 FROM t
WHERE  c1 IS [NOT] NULL;
```
Check if values in a table is NULL or not

SQL CHEAT SHEET http://www.sqltutorial.org

MANAGING TABLES

```
CREATE TABLE t (
    id INT PRIMARY KEY,
    name VARCHAR NOT NULL,
    price INT DEFAULT 0
);
```
Create a new table with three columns

```
DROP TABLE t ;
```
Delete the table from the database

```
ALTER TABLE t ADD column;
```
Add a new column to the table

```
ALTER TABLE t DROP COLUMN c ;
```
Drop column c from the table

```
ALTER TABLE t ADD constraint;
```
Add a constraint

```
ALTER TABLE t DROP constraint;
```
Drop a constraint

```
ALTER TABLE t1 RENAME TO t2;
```
Rename a table from t1 to t2

```
ALTER TABLE t1 RENAME c1 TO c2 ;
```
Rename column c1 to c2

```
TRUNCATE TABLE t;
```
Remove all data in a table

USING SQL CONSTRAINTS

```
CREATE TABLE t(
   c1 INT, c2 INT, c3 VARCHAR,
   PRIMARY KEY (c1,c2)
);
```
Set c1 and c2 as a primary key

```
CREATE TABLE t1(
   c1 INT PRIMARY KEY,
   c2 INT,
   FOREIGN KEY (c2) REFERENCES t2(c2)
);
```
Set c2 column as a foreign key

```
CREATE TABLE t(
   c1 INT, c1 INT,
   UNIQUE(c2,c3)
);
```
Make the values in c1 and c2 unique

```
CREATE TABLE t(
  c1 INT, c2 INT,
  CHECK(c1> 0 AND c1 >= c2)
);
```
Ensure c1 > 0 and values in c1 >= c2

```
CREATE TABLE t(
    c1 INT PRIMARY KEY,
    c2 VARCHAR NOT NULL
);
```
Set values in c2 column not NULL

MODIFYING DATA

```
INSERT INTO t(column_list)
VALUES(value_list);
```
Insert one row into a table

```
INSERT INTO t(column_list)
VALUES (value_list),
       (value_list), ....;
```
Insert multiple rows into a table

```
INSERT INTO t1(column_list)
SELECT column_list
FROM t2;
```
Insert rows from t2 into t1

```
UPDATE t
SET c1 = new_value;
```
Update new value in the column c1 for all rows

```
UPDATE t
SET c1 = new_value,
    c2 = new_value
WHERE condition;
```
Update values in the column c1, c2 that match the condition

```
DELETE FROM t;
```
Delete all data in a table

```
DELETE FROM t
WHERE condition;
```
Delete subset of rows in a table

SQL CHEAT SHEET http://www.sqltutorial.org

MANAGING VIEWS

```
CREATE VIEW v(c1,c2)
AS
SELECT c1, c2
FROM t;
```
Create a new view that consists of c1 and c2

```
CREATE VIEW v(c1,c2)
AS
SELECT c1, c2
FROM t;
WITH [CASCADED | LOCAL] CHECK OPTION;
```
Create a new view with check option

```
CREATE RECURSIVE VIEW v
AS
select-statement -- anchor part
UNION [ALL]
select-statement; -- recursive part
```
Create a recursive view

```
CREATE TEMPORARY VIEW v
AS
SELECT c1, c2
FROM t;
```
Create a temporary view

```
DROP VIEW view_name;
```
Delete a view

MANAGING INDEXES

```
CREATE INDEX idx_name
ON t(c1,c2);
```
Create an index on c1 and c2 of the table t

```
CREATE UNIQUE INDEX idx_name
ON t(c3,c4);
```
Create a unique index on c3, c4 of the table t

```
DROP INDEX idx_name;
```
Drop an index

SQL AGGREGATE FUNCTIONS

AVG returns the average of a list

COUNT returns the number of elements of a list

SUM returns the total of a list

MAX returns the maximum value in a list

MIN returns the minimum value in a list

MANAGING TRIGGERS

```
CREATE OR MODIFY TRIGGER trigger_name
WHEN EVENT
ON table_name TRIGGER_TYPE
EXECUTE stored_procedure;
```
Create or modify a trigger

WHEN
- **BEFORE** – invoke before the event occurs
- **AFTER** – invoke after the event occurs

EVENT
- **INSERT** – invoke for INSERT
- **UPDATE** – invoke for UPDATE
- **DELETE** – invoke for DELETE

TRIGGER_TYPE
- **FOR EACH ROW**
- **FOR EACH STATEMENT**

```
CREATE TRIGGER before_insert_person
BEFORE INSERT
ON person FOR EACH ROW
EXECUTE stored_procedure;
```
Create a trigger invoked before a new row is inserted into the person table

```
DROP TRIGGER trigger_name;
```
Delete a specific trigger

Super VIP Cheatsheet: Machine Learning

Afshine Amidi and Shervine Amidi

October 6, 2018

Contents

1 Supervised Learning

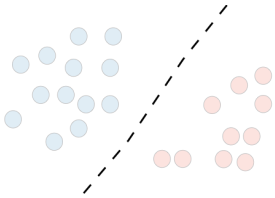
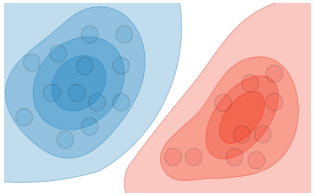
1.1 Introduction to Supervised Learning

Given a set of data points $\{x^{(1)},...,x^{(m)}\}$ associated to a set of outcomes $\{y^{(1)},...,y^{(m)}\}$, we want to build a classifier that learns how to predict y from x.

❒ **Type of prediction** – The different types of predictive models are summed up in the table below:

	Regression	Classifier
Outcome	Continuous	Class
Examples	Linear regression	Logistic regression, SVM, Naive Bayes

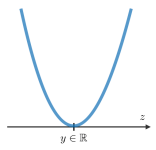
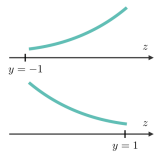
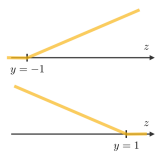
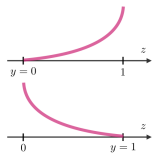
❒ **Type of model** – The different models are summed up in the table below:

	Discriminative model	Generative model
Goal	Directly estimate $P(y\|x)$	Estimate $P(x\|y)$ to deduce $P(y\|x)$
What's learned	Decision boundary	Probability distributions of the data
Illustration		
Examples	Regressions, SVMs	GDA, Naive Bayes

1.2 Notations and general concepts

❒ **Hypothesis** – The hypothesis is noted h_θ and is the model that we choose. For a given input data $x^{(i)}$, the model prediction output is $h_\theta(x^{(i)})$.

❒ **Loss function** – A loss function is a function $L:(z,y)\in\mathbb{R}\times Y\longmapsto L(z,y)\in\mathbb{R}$ that takes as inputs the predicted value z corresponding to the real data value y and outputs how different they are. The common loss functions are summed up in the table below:

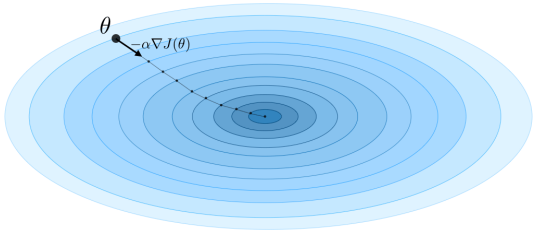
Least squared	Logistic	Hinge	Cross-entropy
$\frac{1}{2}(y-z)^2$	$\log(1+\exp(-yz))$	$\max(0,1-yz)$	$-\left[y\log(z)+(1-y)\log(1-z)\right]$
			
Linear regression	Logistic regression	SVM	Neural Network

❒ **Cost function** – The cost function J is commonly used to assess the performance of a model, and is defined with the loss function L as follows:

$$\boxed{J(\theta)=\sum_{i=1}^{m}L(h_\theta(x^{(i)}),y^{(i)})}$$

❒ **Gradient descent** – By noting $\alpha\in\mathbb{R}$ the learning rate, the update rule for gradient descent is expressed with the learning rate and the cost function J as follows:

$$\boxed{\theta\longleftarrow\theta-\alpha\nabla J(\theta)}$$



Remark: Stochastic gradient descent (SGD) is updating the parameter based on each training example, and batch gradient descent is on a batch of training examples.

❒ **Likelihood** – The likelihood of a model $L(\theta)$ given parameters θ is used to find the optimal parameters θ through maximizing the likelihood. In practice, we use the log-likelihood $\ell(\theta)=\log(L(\theta))$ which is easier to optimize. We have:

$$\boxed{\theta^{\text{opt}}=\underset{\theta}{\arg\max}\ L(\theta)}$$

❒ **Newton's algorithm** – The Newton's algorithm is a numerical method that finds θ such that $\ell'(\theta)=0$. Its update rule is as follows:

$$\boxed{\theta\leftarrow\theta-\frac{\ell'(\theta)}{\ell''(\theta)}}$$

Remark: the multidimensional generalization, also known as the Newton-Raphson method, has the following update rule:

$$\theta\leftarrow\theta-\left(\nabla^2_\theta\ell(\theta)\right)^{-1}\nabla_\theta\ell(\theta)$$

1.3 Linear models

1.3.1 Linear regression

We assume here that $y|x;\theta\sim\mathcal{N}(\mu,\sigma^2)$

❒ **Normal equations** – By noting X the matrix design, the value of θ that minimizes the cost function is a closed-form solution such that:

$$\boxed{\theta=(X^TX)^{-1}X^Ty}$$

❒ **LMS algorithm** – By noting α the learning rate, the update rule of the Least Mean Squares (LMS) algorithm for a training set of m data points, which is also known as the Widrow-Hoff learning rule, is as follows:

$$\boxed{\forall j, \quad \theta_j \leftarrow \theta_j + \alpha \sum_{i=1}^{m} \left[y^{(i)} - h_\theta(x^{(i)})\right] x_j^{(i)}}$$

Remark: the update rule is a particular case of the gradient ascent.

❒ **LWR** – Locally Weighted Regression, also known as LWR, is a variant of linear regression that weights each training example in its cost function by $w^{(i)}(x)$, which is defined with parameter $\tau \in \mathbb{R}$ as:

$$\boxed{w^{(i)}(x) = \exp\left(-\frac{(x^{(i)} - x)^2}{2\tau^2}\right)}$$

1.3.2 Classification and logistic regression

❒ **Sigmoid function** – The sigmoid function g, also known as the logistic function, is defined as follows:

$$\forall z \in \mathbb{R}, \quad \boxed{g(z) = \frac{1}{1+e^{-z}} \in]0,1[}$$

❒ **Logistic regression** – We assume here that $y|x;\theta \sim \text{Bernoulli}(\phi)$. We have the following form:

$$\boxed{\phi = p(y=1|x;\theta) = \frac{1}{1+\exp(-\theta^T x)} = g(\theta^T x)}$$

Remark: there is no closed form solution for the case of logistic regressions.

❒ **Softmax regression** – A softmax regression, also called a multiclass logistic regression, is used to generalize logistic regression when there are more than 2 outcome classes. By convention, we set $\theta_K = 0$, which makes the Bernoulli parameter ϕ_i of each class i equal to:

$$\boxed{\phi_i = \frac{\exp(\theta_i^T x)}{\sum_{j=1}^{K} \exp(\theta_j^T x)}}$$

1.3.3 Generalized Linear Models

❒ **Exponential family** – A class of distributions is said to be in the exponential family if it can be written in terms of a natural parameter, also called the canonical parameter or link function, η, a sufficient statistic $T(y)$ and a log-partition function $a(\eta)$ as follows:

$$\boxed{p(y;\eta) = b(y)\exp(\eta T(y) - a(\eta))}$$

Remark: we will often have $T(y) = y$. Also, $\exp(-a(\eta))$ can be seen as a normalization parameter that will make sure that the probabilities sum to one.

Here are the most common exponential distributions summed up in the following table:

Distribution	η	$T(y)$	$a(\eta)$	$b(y)$
Bernoulli	$\log\left(\frac{\phi}{1-\phi}\right)$	y	$\log(1+\exp(\eta))$	1
Gaussian	μ	y	$\frac{\eta^2}{2}$	$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)$
Poisson	$\log(\lambda)$	y	e^η	$\frac{1}{y!}$
Geometric	$\log(1-\phi)$	y	$\log\left(\frac{e^\eta}{1-e^\eta}\right)$	1

❒ **Assumptions of GLMs** – Generalized Linear Models (GLM) aim at predicting a random variable y as a function fo $x \in \mathbb{R}^{n+1}$ and rely on the following 3 assumptions:

(1) $\boxed{y|x;\theta \sim \text{ExpFamily}(\eta)}$ (2) $\boxed{h_\theta(x) = E[y|x;\theta]}$ (3) $\boxed{\eta = \theta^T x}$

Remark: ordinary least squares and logistic regression are special cases of generalized linear models.

1.4 Support Vector Machines

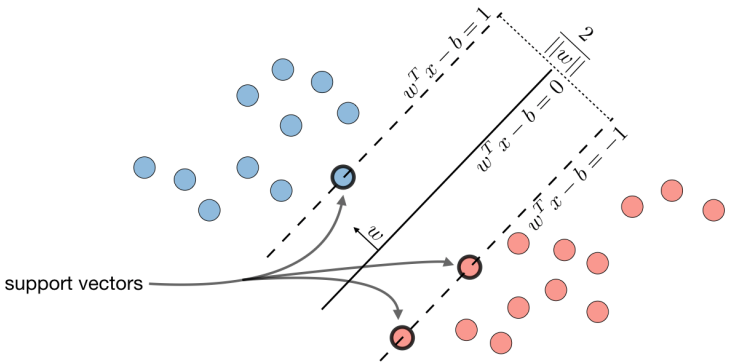
The goal of support vector machines is to find the line that maximizes the minimum distance to the line.

❒ **Optimal margin classifier** – The optimal margin classifier h is such that:

$$\boxed{h(x) = \text{sign}(w^T x - b)}$$

where $(w,b) \in \mathbb{R}^n \times \mathbb{R}$ is the solution of the following optimization problem:

$$\boxed{\min \frac{1}{2}||w||^2} \quad \text{such that} \quad \boxed{y^{(i)}(w^T x^{(i)} - b) \geqslant 1}$$



Remark: the line is defined as $\boxed{w^T x - b = 0}$.

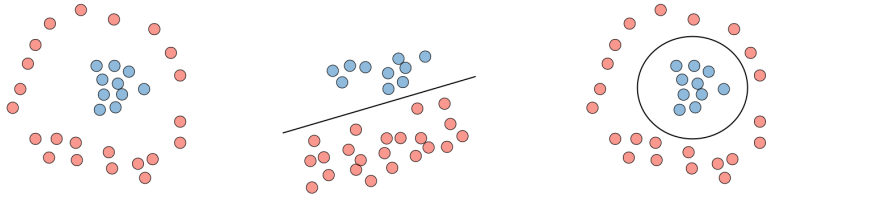
❒ **Hinge loss** – The hinge loss is used in the setting of SVMs and is defined as follows:

$$\boxed{L(z,y) = [1-yz]_+ = \max(0, 1-yz)}$$

❒ **Kernel** – Given a feature mapping ϕ, we define the kernel K to be defined as:

$$\boxed{K(x,z) = \phi(x)^T\phi(z)}$$

In practice, the kernel K defined by $K(x,z) = \exp\left(-\frac{||x-z||^2}{2\sigma^2}\right)$ is called the Gaussian kernel and is commonly used.



Remark: we say that we use the "kernel trick" to compute the cost function using the kernel because we actually don't need to know the explicit mapping ϕ, which is often very complicated. Instead, only the values $K(x,z)$ are needed.

❒ **Lagrangian** – We define the Lagrangian $\mathcal{L}(w,b)$ as follows:

$$\boxed{\mathcal{L}(w,b) = f(w) + \sum_{i=1}^{l}\beta_i h_i(w)}$$

Remark: the coefficients β_i are called the Lagrange multipliers.

1.5 Generative Learning

A generative model first tries to learn how the data is generated by estimating $P(x|y)$, which we can then use to estimate $P(y|x)$ by using Bayes' rule.

1.5.1 Gaussian Discriminant Analysis

❒ **Setting** – The Gaussian Discriminant Analysis assumes that y and $x|y=0$ and $x|y=1$ are such that:

$$\boxed{y \sim \text{Bernoulli}(\phi)}$$

$$\boxed{x|y=0 \sim \mathcal{N}(\mu_0,\Sigma)} \quad \text{and} \quad \boxed{x|y=1 \sim \mathcal{N}(\mu_1,\Sigma)}$$

❒ **Estimation** – The following table sums up the estimates that we find when maximizing the likelihood:

$\widehat{\phi}$	$\widehat{\mu_j}$ $(j=0,1)$	$\widehat{\Sigma}$
$\frac{1}{m}\sum_{i=1}^{m} 1_{\{y^{(i)}=1\}}$	$\frac{\sum_{i=1}^{m} 1_{\{y^{(i)}=j\}}x^{(i)}}{\sum_{i=1}^{m} 1_{\{y^{(i)}=j\}}}$	$\frac{1}{m}\sum_{i=1}^{m}(x^{(i)}-\mu_{y^{(i)}})(x^{(i)}-\mu_{y^{(i)}})^T$

1.5.2 Naive Bayes

❒ **Assumption** – The Naive Bayes model supposes that the features of each data point are all independent:

$$\boxed{P(x|y) = P(x_1,x_2,...|y) = P(x_1|y)P(x_2|y)... = \prod_{i=1}^{n} P(x_i|y)}$$

❒ **Solutions** – Maximizing the log-likelihood gives the following solutions, with $k \in \{0,1\}$, $l \in [\![1,L]\!]$

$$\boxed{P(y=k) = \frac{1}{m} \times \#\{j|y^{(j)}=k\}} \quad \text{and} \quad \boxed{P(x_i=l|y=k) = \frac{\#\{j|y^{(j)}=k \text{ and } x_i^{(j)}=l\}}{\#\{j|y^{(j)}=k\}}}$$

Remark: Naive Bayes is widely used for text classification and spam detection.

1.6 Tree-based and ensemble methods

These methods can be used for both regression and classification problems.

❒ **CART** – Classification and Regression Trees (CART), commonly known as decision trees, can be represented as binary trees. They have the advantage to be very interpretable.

❒ **Random forest** – It is a tree-based technique that uses a high number of decision trees built out of randomly selected sets of features. Contrary to the simple decision tree, it is highly uninterpretable but its generally good performance makes it a popular algorithm.

Remark: random forests are a type of ensemble methods.

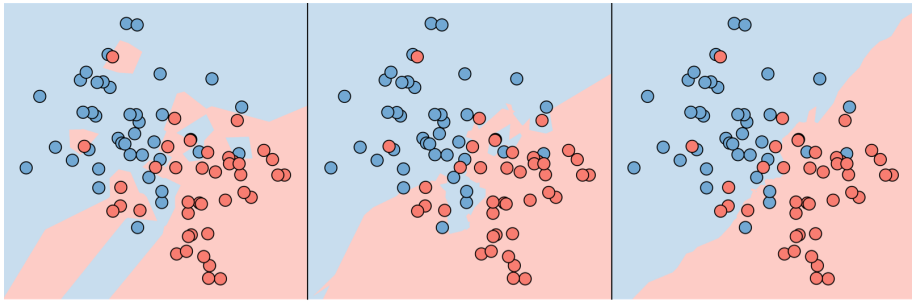
❒ **Boosting** – The idea of boosting methods is to combine several weak learners to form a stronger one. The main ones are summed up in the table below:

Adaptive boosting	Gradient boosting
- High weights are put on errors to improve at the next boosting step - Known as Adaboost	- Weak learners trained on remaining errors

1.7 Other non-parametric approaches

❒ **k-nearest neighbors** – The k-nearest neighbors algorithm, commonly known as k-NN, is a non-parametric approach where the response of a data point is determined by the nature of its k neighbors from the training set. It can be used in both classification and regression settings.

Remark: The higher the parameter k, the higher the bias, and the lower the parameter k, the higher the variance.

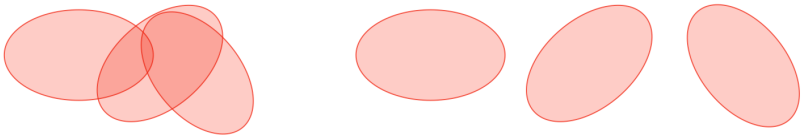


$k = 1$ $k = 3$ $k = 11$

1.8 Learning Theory

❒ **Union bound** – Let $A_1, ..., A_k$ be k events. We have:

$$\boxed{P(A_1 \cup ... \cup A_k) \leqslant P(A_1) + ... + P(A_k)}$$



$A_1 \cup A_2 \cup A_3$ A_1 A_2 A_3

❒ **Hoeffding inequality** – Let $Z_1, .., Z_m$ be m iid variables drawn from a Bernoulli distribution of parameter ϕ. Let $\widehat{\phi}$ be their sample mean and $\gamma > 0$ fixed. We have:

$$\boxed{P(|\phi - \widehat{\phi}| > \gamma) \leqslant 2\exp(-2\gamma^2 m)}$$

Remark: this inequality is also known as the Chernoff bound.

❒ **Training error** – For a given classifier h, we define the training error $\widehat{\epsilon}(h)$, also known as the empirical risk or empirical error, to be as follows:

$$\boxed{\widehat{\epsilon}(h) = \frac{1}{m}\sum_{i=1}^{m} 1_{\{h(x^{(i)}) \neq y^{(i)}\}}}$$

❒ **Probably Approximately Correct (PAC)** – PAC is a framework under which numerous results on learning theory were proved, and has the following set of assumptions:

- the training and testing sets follow the same distribution
- the training examples are drawn independently

❒ **Shattering** – Given a set $S = \{x^{(1)},...,x^{(d)}\}$, and a set of classifiers $\mathcal{H}$, we say that $\mathcal{H}$ shatters S if for any set of labels $\{y^{(1)}, ..., y^{(d)}\}$, we have:

$$\boxed{\exists h \in \mathcal{H}, \quad \forall i \in [\![1,d]\!], \quad h(x^{(i)}) = y^{(i)}}$$

❒ **Upper bound theorem** – Let $\mathcal{H}$ be a finite hypothesis class such that $|\mathcal{H}| = k$ and let δ and the sample size m be fixed. Then, with probability of at least $1 - \delta$, we have:

$$\boxed{\epsilon(\widehat{h}) \leqslant \left(\min_{h \in \mathcal{H}} \epsilon(h)\right) + 2\sqrt{\frac{1}{2m}\log\left(\frac{2k}{\delta}\right)}}$$

❒ **VC dimension** – The Vapnik-Chervonenkis (VC) dimension of a given infinite hypothesis class $\mathcal{H}$, noted $\text{VC}(\mathcal{H})$ is the size of the largest set that is shattered by $\mathcal{H}$.

Remark: the VC dimension of $\mathcal{H} = \{$set of linear classifiers in 2 dimensions$\}$ is 3.

❒ **Theorem (Vapnik)** – Let $\mathcal{H}$ be given, with $\text{VC}(\mathcal{H}) = d$ and m the number of training examples. With probability at least $1 - \delta$, we have:

$$\boxed{\epsilon(\widehat{h}) \leqslant \left(\min_{h \in \mathcal{H}} \epsilon(h)\right) + O\left(\sqrt{\frac{d}{m}\log\left(\frac{m}{d}\right) + \frac{1}{m}\log\left(\frac{1}{\delta}\right)}\right)}$$

2 Unsupervised Learning

2.1 Introduction to Unsupervised Learning

❒ **Motivation** – The goal of unsupervised learning is to find hidden patterns in unlabeled data $\{x^{(1)},...,x^{(m)}\}$.

❒ **Jensen's inequality** – Let f be a convex function and X a random variable. We have the following inequality:

$$\boxed{E[f(X)] \geqslant f(E[X])}$$

2.2 Clustering

2.2.1 Expectation-Maximization

❒ **Latent variables** – Latent variables are hidden/unobserved variables that make estimation problems difficult, and are often denoted z. Here are the most common settings where there are latent variables:

Setting	Latent variable z	$x\|z$	Comments
Mixture of k Gaussians	Multinomial(ϕ)	$\mathcal{N}(\mu_j,\Sigma_j)$	$\mu_j \in \mathbb{R}^n, \phi \in \mathbb{R}^k$
Factor analysis	$\mathcal{N}(0,I)$	$\mathcal{N}(\mu+\Lambda z,\psi)$	$\mu_j \in \mathbb{R}^n$

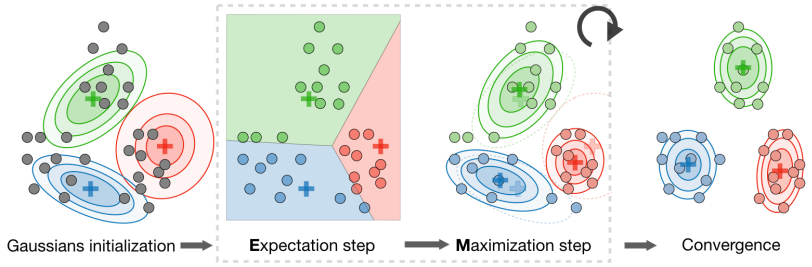
❒ **Algorithm** – The Expectation-Maximization (EM) algorithm gives an efficient method at estimating the parameter θ through maximum likelihood estimation by repeatedly constructing a lower-bound on the likelihood (E-step) and optimizing that lower bound (M-step) as follows:

- E-step: Evaluate the posterior probability $Q_i(z^{(i)})$ that each data point $x^{(i)}$ came from a particular cluster $z^{(i)}$ as follows:

$$\boxed{Q_i(z^{(i)}) = P(z^{(i)}|x^{(i)};\theta)}$$

- M-step: Use the posterior probabilities $Q_i(z^{(i)})$ as cluster specific weights on data points $x^{(i)}$ to separately re-estimate each cluster model as follows:

$$\boxed{\theta_i = \underset{\theta}{\text{argmax}} \sum_i \int_{z^{(i)}} Q_i(z^{(i)}) \log\left(\frac{P(x^{(i)},z^{(i)};\theta)}{Q_i(z^{(i)})}\right) dz^{(i)}}$$

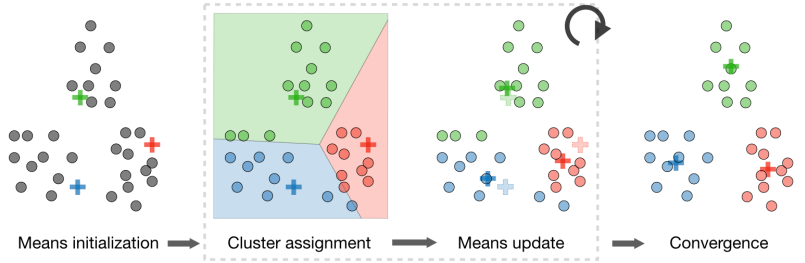


2.2.2 k-means clustering

We note $c^{(i)}$ the cluster of data point i and μ_j the center of cluster j.

❒ **Algorithm** – After randomly initializing the cluster centroids $\mu_1,\mu_2,...,\mu_k \in \mathbb{R}^n$, the k-means algorithm repeats the following step until convergence:

$$\boxed{c^{(i)} = \underset{j}{\arg\min}||x^{(i)}-\mu_j||^2} \quad \text{and} \quad \boxed{\mu_j = \frac{\sum_{i=1}^m 1_{\{c^{(i)}=j\}} x^{(i)}}{\sum_{i=1}^m 1_{\{c^{(i)}=j\}}}}$$



❒ **Distortion function** – In order to see if the algorithm converges, we look at the distortion function defined as follows:

$$\boxed{J(c,\mu) = \sum_{i=1}^m ||x^{(i)} - \mu_{c^{(i)}}||^2}$$

2.2.3 Hierarchical clustering

❒ **Algorithm** – It is a clustering algorithm with an agglomerative hierarchical approach that build nested clusters in a successive manner.

❒ **Types** – There are different sorts of hierarchical clustering algorithms that aims at optimizing different objective functions, which is summed up in the table below:

Ward linkage	Average linkage	Complete linkage
Minimize within cluster distance	Minimize average distance between cluster pairs	Minimize maximum distance of between cluster pairs

2.2.4 Clustering assessment metrics

In an unsupervised learning setting, it is often hard to assess the performance of a model since we don't have the ground truth labels as was the case in the supervised learning setting.

❒ **Silhouette coefficient** – By noting a and b the mean distance between a sample and all other points in the same class, and between a sample and all other points in the next nearest cluster, the silhouette coefficient s for a single sample is defined as follows:

$$\boxed{s = \frac{b-a}{\max(a,b)}}$$

❒ **Calinski-Harabaz index** – By noting k the number of clusters, B_k and W_k the between and within-clustering dispersion matrices respectively defined as

$$B_k = \sum_{j=1}^{k} n_{c^{(i)}}(\mu_{c^{(i)}} - \mu)(\mu_{c^{(i)}} - \mu)^T, \qquad W_k = \sum_{i=1}^{m}(x^{(i)} - \mu_{c^{(i)}})(x^{(i)} - \mu_{c^{(i)}})^T$$

the Calinski-Harabaz index $s(k)$ indicates how well a clustering model defines its clusters, such that the higher the score, the more dense and well separated the clusters are. It is defined as follows:

$$\boxed{s(k) = \frac{\text{Tr}(B_k)}{\text{Tr}(W_k)} \times \frac{N-k}{k-1}}$$

2.3 Dimension reduction

2.3.1 Principal component analysis

It is a dimension reduction technique that finds the variance maximizing directions onto which to project the data.

❒ **Eigenvalue, eigenvector** – Given a matrix $A \in \mathbb{R}^{n\times n}$, λ is said to be an eigenvalue of A if there exists a vector $z \in \mathbb{R}^n \backslash \{0\}$, called eigenvector, such that we have:

$$\boxed{Az = \lambda z}$$

❒ **Spectral theorem** – Let $A \in \mathbb{R}^{n\times n}$. If A is symmetric, then A is diagonalizable by a real orthogonal matrix $U \in \mathbb{R}^{n\times n}$. By noting $\Lambda = \text{diag}(\lambda_1,...,\lambda_n)$, we have:

$$\boxed{\exists \Lambda \text{ diagonal}, \quad A = U\Lambda U^T}$$

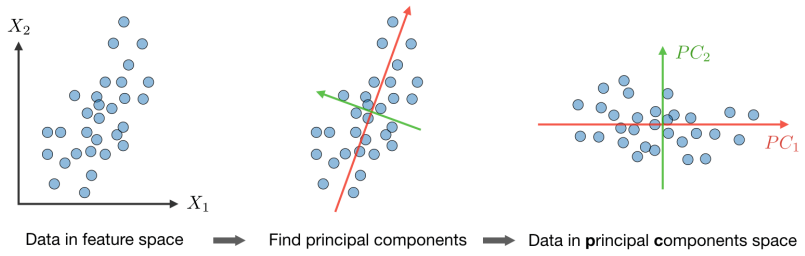
Remark: the eigenvector associated with the largest eigenvalue is called principal eigenvector of matrix A.

❒ **Algorithm** – The Principal Component Analysis (PCA) procedure is a dimension reduction technique that projects the data on k dimensions by maximizing the variance of the data as follows:

- Step 1: Normalize the data to have a mean of 0 and standard deviation of 1.

$$\boxed{x_j^{(i)} \leftarrow \frac{x_j^{(i)} - \mu_j}{\sigma_j}} \quad \text{where} \quad \boxed{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}} \quad \text{and} \quad \boxed{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2}$$

- Step 2: Compute $\Sigma = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}x^{(i)^T} \in \mathbb{R}^{n\times n}$, which is symmetric with real eigenvalues.
- Step 3: Compute $u_1, ..., u_k \in \mathbb{R}^n$ the k orthogonal principal eigenvectors of Σ, i.e. the orthogonal eigenvectors of the k largest eigenvalues.
- Step 4: Project the data on $\text{span}_{\mathbb{R}}(u_1,...,u_k)$. This procedure maximizes the variance among all k-dimensional spaces.



2.3.2 Independent component analysis

It is a technique meant to find the underlying generating sources.

❒ **Assumptions** – We assume that our data x has been generated by the n-dimensional source vector $s = (s_1,...,s_n)$, where s_i are independent random variables, via a mixing and non-singular matrix A as follows:

$$\boxed{x = As}$$

The goal is to find the unmixing matrix $W = A^{-1}$ by an update rule.

❒ **Bell and Sejnowski ICA algorithm** – This algorithm finds the unmixing matrix W by following the steps below:

- Write the probability of $x = As = W^{-1}s$ as:

$$p(x) = \prod_{i=1}^{n} p_s(w_i^T x) \cdot |W|$$

- Write the log likelihood given our training data $\{x^{(i)}, i \in [\![1,m]\!]\}$ and by noting g the sigmoid function as:

$$l(W) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} \log\left(g'(w_j^T x^{(i)})\right) + \log|W|\right)$$

Therefore, the stochastic gradient ascent learning rule is such that for each training example $x^{(i)}$, we update W as follows:

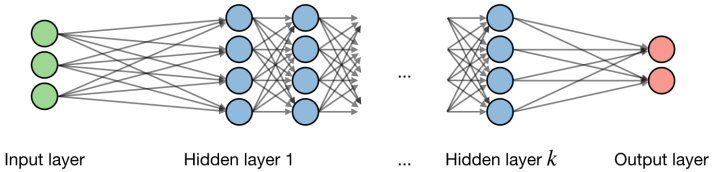
$$\boxed{W \longleftarrow W + \alpha\left(\begin{pmatrix} 1 - 2g(w_1^T x^{(i)}) \\ 1 - 2g(w_2^T x^{(i)}) \\ \vdots \\ 1 - 2g(w_n^T x^{(i)}) \end{pmatrix} x^{(i)^T} + (W^T)^{-1}\right)}$$

3 Deep Learning

3.1 Neural Networks

Neural networks are a class of models that are built with layers. Commonly used types of neural networks include convolutional and recurrent neural networks.

❒ **Architecture** – The vocabulary around neural networks architectures is described in the figure below:

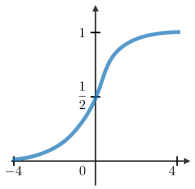
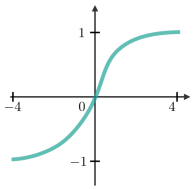
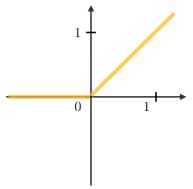
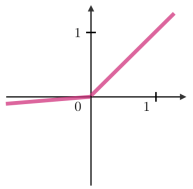


By noting i the i^{th} layer of the network and j the j^{th} hidden unit of the layer, we have:

$$\boxed{z_j^{[i]} = {w_j^{[i]}}^T x + b_j^{[i]}}$$

where we note w, b, z the weight, bias and output respectively.

❒ **Activation function** – Activation functions are used at the end of a hidden unit to introduce non-linear complexities to the model. Here are the most common ones:

Sigmoid	Tanh	ReLU	Leaky ReLU
$g(z) = \dfrac{1}{1+e^{-z}}$	$g(z) = \dfrac{e^z - e^{-z}}{e^z + e^{-z}}$	$g(z) = \max(0,z)$	$g(z) = \max(\epsilon z, z)$ with $\epsilon \ll 1$
			

❒ **Cross-entropy loss** – In the context of neural networks, the cross-entropy loss $L(z,y)$ is commonly used and is defined as follows:

$$\boxed{L(z,y) = -\Big[y\log(z) + (1-y)\log(1-z)\Big]}$$

❒ **Learning rate** – The learning rate, often noted η, indicates at which pace the weights get updated. This can be fixed or adaptively changed. The current most popular method is called Adam, which is a method that adapts the learning rate.

❒ **Backpropagation** – Backpropagation is a method to update the weights in the neural network by taking into account the actual output and the desired output. The derivative with respect to weight w is computed using chain rule and is of the following form:

$$\boxed{\frac{\partial L(z,y)}{\partial w} = \frac{\partial L(z,y)}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w}}$$

As a result, the weight is updated as follows:

$$\boxed{w \longleftarrow w - \eta \frac{\partial L(z,y)}{\partial w}}$$

❒ **Updating weights** – In a neural network, weights are updated as follows:

- Step 1: Take a batch of training data.
- Step 2: Perform forward propagation to obtain the corresponding loss.
- Step 3: Backpropagate the loss to get the gradients.
- Step 4: Use the gradients to update the weights of the network.

❒ **Dropout** – Dropout is a technique meant at preventing overfitting the training data by dropping out units in a neural network. In practice, neurons are either dropped with probability p or kept with probability $1-p$.

3.2 Convolutional Neural Networks

❒ **Convolutional layer requirement** – By noting W the input volume size, F the size of the convolutional layer neurons, P the amount of zero padding, then the number of neurons N that fit in a given volume is such that:

$$\boxed{N = \frac{W - F + 2P}{S} + 1}$$

❒ **Batch normalization** – It is a step of hyperparameter γ, β that normalizes the batch $\{x_i\}$. By noting μ_B, σ_B^2 the mean and variance of that we want to correct to the batch, it is done as follows:

$$\boxed{x_i \longleftarrow \gamma \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} + \beta}$$

It is usually done after a fully connected/convolutional layer and before a non-linearity layer and aims at allowing higher learning rates and reducing the strong dependence on initialization.

3.3 Recurrent Neural Networks

❒ **Types of gates** – Here are the different types of gates that we encounter in a typical recurrent neural network:

Input gate	Forget gate	Output gate	Gate
Write to cell or not?	Erase a cell or not?	Reveal a cell or not?	How much writing?

❒ **LSTM** – A long short-term memory (LSTM) network is a type of RNN model that avoids the vanishing gradient problem by adding 'forget' gates.

3.4 Reinforcement Learning and Control

The goal of reinforcement learning is for an agent to learn how to evolve in an environment.

❒ **Markov decision processes** – A Markov decision process (MDP) is a 5-tuple $(S,A,\{P_{sa}\},\gamma,R)$ where:

- $\mathcal{S}$ is the set of states
- $\mathcal{A}$ is the set of actions
- $\{P_{sa}\}$ are the state transition probabilities for $s \in \mathcal{S}$ and $a \in \mathcal{A}$
- $\gamma \in [0,1[$ is the discount factor
- $R : \mathcal{S} \times \mathcal{A} \longrightarrow \mathbb{R}$ or $R : \mathcal{S} \longrightarrow \mathbb{R}$ is the reward function that the algorithm wants to maximize

❒ **Policy** – A policy π is a function $\pi : \mathcal{S} \longrightarrow \mathcal{A}$ that maps states to actions.

Remark: we say that we execute a given policy π if given a state s we take the action $a = \pi(s)$.

❒ **Value function** – For a given policy π and a given state s, we define the value function V^π as follows:

$$\boxed{V^\pi(s) = E\Big[R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + ...|s_0 = s,\pi\Big]}$$

❒ **Bellman equation** – The optimal Bellman equations characterizes the value function V^{π^*} of the optimal policy π^*:

$$\boxed{V^{\pi^*}(s) = R(s) + \max_{a\in\mathcal{A}} \gamma \sum_{s'\in S} P_{sa}(s')V^{\pi^*}(s')}$$

Remark: we note that the optimal policy π^* for a given state s is such that:

$$\boxed{\pi^*(s) = \underset{a\in\mathcal{A}}{\text{argmax}} \sum_{s'\in\mathcal{S}} P_{sa}(s')V^*(s')}$$

❒ **Value iteration algorithm** – The value iteration algorithm is in two steps:

- We initialize the value:

$$\boxed{V_0(s) = 0}$$

- We iterate the value based on the values before:

$$\boxed{V_{i+1}(s) = R(s) + \max_{a\in\mathcal{A}} \left[\sum_{s'\in\mathcal{S}} \gamma P_{sa}(s')V_i(s')\right]}$$

❒ **Maximum likelihood estimate** – The maximum likelihood estimates for the state transition probabilities are as follows:

$$\boxed{P_{sa}(s') = \frac{\#\text{times took action } a \text{ in state } s \text{ and got to } s'}{\#\text{times took action } a \text{ in state } s}}$$

❒ **Q-learning** – Q-learning is a model-free estimation of Q, which is done as follows:

$$\boxed{Q(s,a) \leftarrow Q(s,a) + \alpha\Big[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)\Big]}$$

4 Machine Learning Tips and Tricks

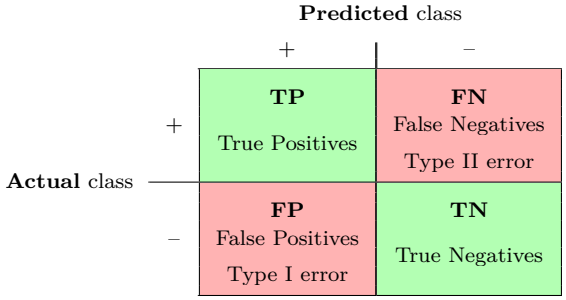
4.1 Metrics

Given a set of data points $\{x^{(1)}, ..., x^{(m)}\}$, where each $x^{(i)}$ has n features, associated to a set of outcomes $\{y^{(1)}, ..., y^{(m)}\}$, we want to assess a given classifier that learns how to predict y from x.

4.1.1 Classification

In a context of a binary classification, here are the main metrics that are important to track to assess the performance of the model.

❒ **Confusion matrix** – The confusion matrix is used to have a more complete picture when assessing the performance of a model. It is defined as follows:



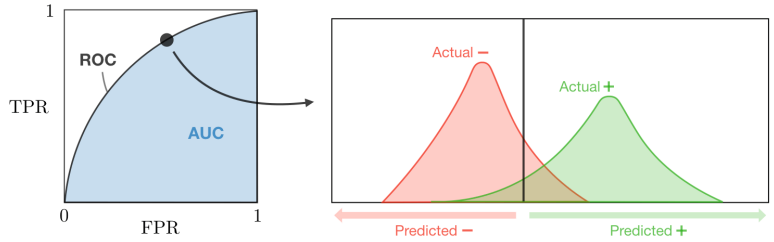
❒ **Main metrics** – The following metrics are commonly used to assess the performance of classification models:

Metric	Formula	Interpretation
Accuracy	$\frac{\text{TP}+\text{TN}}{\text{TP}+\text{TN}+\text{FP}+\text{FN}}$	Overall performance of model
Precision	$\frac{\text{TP}}{\text{TP}+\text{FP}}$	How accurate the positive predictions are
Recall Sensitivity	$\frac{\text{TP}}{\text{TP}+\text{FN}}$	Coverage of actual positive sample
Specificity	$\frac{\text{TN}}{\text{TN}+\text{FP}}$	Coverage of actual negative sample
F1 score	$\frac{2\text{TP}}{2\text{TP}+\text{FP}+\text{FN}}$	Hybrid metric useful for unbalanced classes

❒ **ROC** – The receiver operating curve, also noted ROC, is the plot of TPR versus FPR by varying the threshold. These metrics are are summed up in the table below:

Metric	Formula	Equivalent
True Positive Rate TPR	$\frac{\text{TP}}{\text{TP}+\text{FN}}$	Recall, sensitivity
False Positive Rate FPR	$\frac{\text{FP}}{\text{TN}+\text{FP}}$	1-specificity

❒ **AUC** – The area under the receiving operating curve, also noted AUC or AUROC, is the area below the ROC as shown in the following figure:



4.1.2 Regression

❒ **Basic metrics** – Given a regression model f, the following metrics are commonly used to assess the performance of the model:

Total sum of squares	Explained sum of squares	Residual sum of squares
$\text{SS}_{\text{tot}} = \sum_{i=1}^{m}(y_i - \overline{y})^2$	$\text{SS}_{\text{reg}} = \sum_{i=1}^{m}(f(x_i) - \overline{y})^2$	$\text{SS}_{\text{res}} = \sum_{i=1}^{m}(y_i - f(x_i))^2$

❒ **Coefficient of determination** – The coefficient of determination, often noted R^2 or r^2, provides a measure of how well the observed outcomes are replicated by the model and is defined as follows:

$$\boxed{R^2 = 1 - \frac{\text{SS}_{\text{res}}}{\text{SS}_{\text{tot}}}}$$

❒ **Main metrics** – The following metrics are commonly used to assess the performance of regression models, by taking into account the number of variables n that they take into consideration:

Mallow's Cp	AIC	BIC	Adjusted R^2
$\frac{\text{SS}_{\text{res}} + 2(n+1)\widehat{\sigma}^2}{m}$	$2\big[(n+2) - \log(L)\big]$	$\log(m)(n+2) - 2\log(L)$	$1 - \frac{(1-R^2)(m-1)}{m-n-1}$

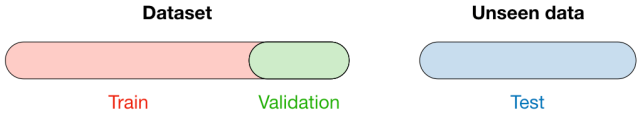
where L is the likelihood and $\widehat{\sigma}^2$ is an estimate of the variance associated with each response.

4.2 Model selection

❒ **Vocabulary** – When selecting a model, we distinguish 3 different parts of the data that we have as follows:

Training set	Validation set	Testing set
- Model is trained - Usually 80% of the dataset	- Model is assessed - Usually 20% of the dataset - Also called hold-out or development set	- Model gives predictions - Unseen data

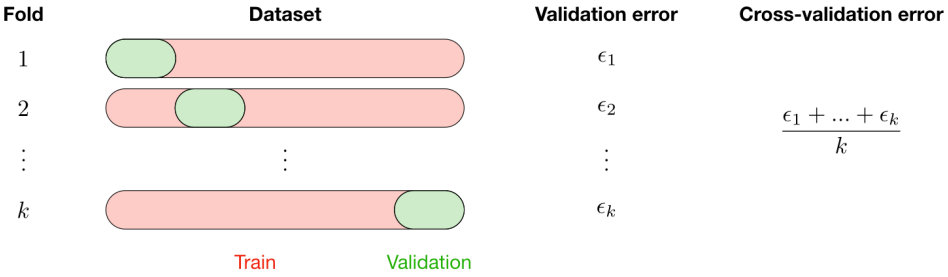
Once the model has been chosen, it is trained on the entire dataset and tested on the unseen test set. These are represented in the figure below:



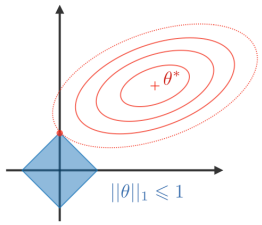
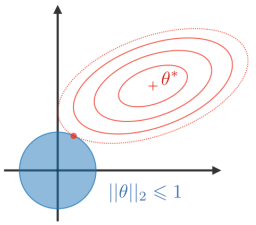
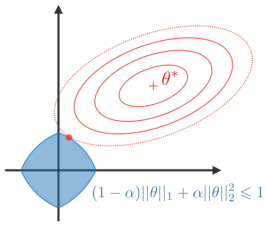
❒ **Cross-validation** – Cross-validation, also noted CV, is a method that is used to select a model that does not rely too much on the initial training set. The different types are summed up in the table below:

k-fold	Leave-p-out
- Training on $k-1$ folds and assessment on the remaining one - Generally $k = 5$ or 10	- Training on $n-p$ observations and assessment on the p remaining ones - Case $p = 1$ is called leave-one-out

The most commonly used method is called k-fold cross-validation and splits the training data into k folds to validate the model on one fold while training the model on the $k-1$ other folds, all of this k times. The error is then averaged over the k folds and is named cross-validation error.



❒ **Regularization** – The regularization procedure aims at avoiding the model to overfit the data and thus deals with high variance issues. The following table sums up the different types of commonly used regularization techniques:

LASSO	Ridge	Elastic Net
- Shrinks coefficients to 0 - Good for variable selection	Makes coefficients smaller	Tradeoff between variable selection and small coefficients
		
$... + \lambda\|\|\theta\|\|_1$ $\lambda \in \mathbb{R}$	$... + \lambda\|\|\theta\|\|_2^2$ $\lambda \in \mathbb{R}$	$... + \lambda\left[(1-\alpha)\|\|\theta\|\|_1 + \alpha\|\|\theta\|\|_2^2\right]$ $\lambda \in \mathbb{R}$, $\alpha \in [0,1]$

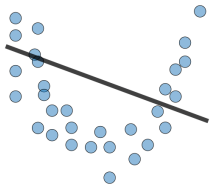
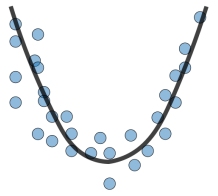
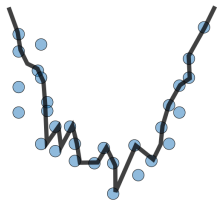
❒ **Model selection** – Train model on training set, then evaluate on the development set, then pick best performance model on the development set, and retrain all of that model on the whole training set.

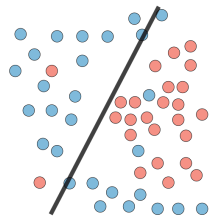
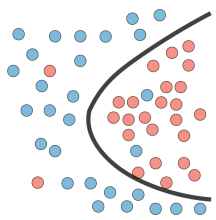
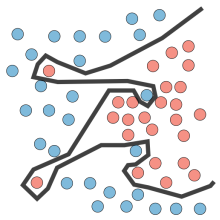
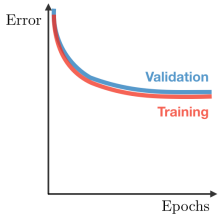
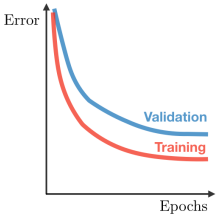
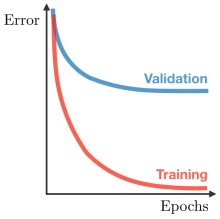
4.3 Diagnostics

❒ **Bias** – The bias of a model is the difference between the expected prediction and the correct model that we try to predict for given data points.

❒ **Variance** – The variance of a model is the variability of the model prediction for given data points.

❒ **Bias/variance tradeoff** – The simpler the model, the higher the bias, and the more complex the model, the higher the variance.

	Underfitting	Just right	Overfitting
Symptoms	- High training error - Training error close to test error - High bias	- Training error slightly lower than test error	- Low training error - Training error much lower than test error - High variance
Regression			

Classification			
Deep learning			
Remedies	- Complexify model - Add more features - Train longer		- Regularize - Get more data

❒ **Error analysis** – Error analysis is analyzing the root cause of the difference in performance between the current and the perfect models.

❒ **Ablative analysis** – Ablative analysis is analyzing the root cause of the difference in performance between the current and the baseline models.

5 Refreshers

5.1 Probabilities and Statistics

5.1.1 Introduction to Probability and Combinatorics

❒ **Sample space** – The set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by S.

❒ **Event** – Any subset E of the sample space is known as an event. That is, an event is a set consisting of possible outcomes of the experiment. If the outcome of the experiment is contained in E, then we say that E has occurred.

❒ **Axioms of probability** – For each event E, we denote $P(E)$ as the probability of event E occuring. By noting $E_1,...,E_n$ mutually exclusive events, we have the 3 following axioms:

$$(1) \quad \boxed{0 \leqslant P(E) \leqslant 1} \quad (2) \quad \boxed{P(S) = 1} \quad (3) \quad \boxed{P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)}$$

❒ **Permutation** – A permutation is an arrangement of r objects from a pool of n objects, in a given order. The number of such arrangements is given by $P(n,r)$, defined as:

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

❒ **Combination** – A combination is an arrangement of r objects from a pool of n objects, where the order does not matter. The number of such arrangements is given by $C(n,r)$, defined as:

$$\boxed{C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}}$$

Remark: we note that for $0 \leqslant r \leqslant n$, we have $P(n,r) \geqslant C(n,r)$.

5.1.2 Conditional Probability

❒ **Bayes' rule** – For events A and B such that $P(B) > 0$, we have:

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

Remark: we have $P(A \cap B) = P(A)P(B|A) = P(A|B)P(B)$.

❒ **Partition** – Let $\{A_i, i \in [\![1,n]\!]\}$ be such that for all i, $A_i \neq \varnothing$. We say that $\{A_i\}$ is a partition if we have:

$$\boxed{\forall i \neq j, A_i \cap A_j = \emptyset \quad \text{and} \quad \bigcup_{i=1}^{n} A_i = S}$$

Remark: for any event B in the sample space, we have $P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$.

❒ **Extended form of Bayes' rule** – Let $\{A_i, i \in [\![1,n]\!]\}$ be a partition of the sample space. We have:

$$\boxed{P(A_k|B) = \frac{P(B|A_k)P(A_k)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}}$$

❒ **Independence** – Two events A and B are independent if and only if we have:

$$\boxed{P(A \cap B) = P(A)P(B)}$$

5.1.3 Random Variables

❒ **Random variable** – A random variable, often noted X, is a function that maps every element in a sample space to a real line.

❒ **Cumulative distribution function (CDF)** – The cumulative distribution function F, which is monotonically non-decreasing and is such that $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to+\infty} F(x) = 1$, is defined as:

$$\boxed{F(x) = P(X \leqslant x)}$$

Remark: we have $P(a < X \leqslant B) = F(b) - F(a)$.

❒ **Probability density function (PDF)** – The probability density function f is the probability that X takes on values between two adjacent realizations of the random variable.

❒ **Relationships involving the PDF and CDF** – Here are the important properties to know in the discrete (D) and the continuous (C) cases.

Case	CDF F	PDF f	Properties of PDF
(D)	$F(x) = \sum_{x_i \leqslant x} P(X = x_i)$	$f(x_j) = P(X = x_j)$	$0 \leqslant f(x_j) \leqslant 1$ and $\sum_j f(x_j) = 1$
(C)	$F(x) = \int_{-\infty}^{x} f(y)dy$	$f(x) = \frac{dF}{dx}$	$f(x) \geqslant 0$ and $\int_{-\infty}^{+\infty} f(x)dx = 1$

❒ **Variance** – The variance of a random variable, often noted $\text{Var}(X)$ or σ^2, is a measure of the spread of its distribution function. It is determined as follows:

$$\boxed{\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2}$$

❒ **Standard deviation** – The standard deviation of a random variable, often noted σ, is a measure of the spread of its distribution function which is compatible with the units of the actual random variable. It is determined as follows:

$$\boxed{\sigma = \sqrt{\text{Var}(X)}}$$

❒ **Expectation and Moments of the Distribution** – Here are the expressions of the expected value $E[X]$, generalized expected value $E[g(X)]$, k^{th} moment $E[X^k]$ and characteristic function $\psi(\omega)$ for the discrete and continuous cases:

Case	$E[X]$	$E[g(X)]$	$E[X^k]$	$\psi(\omega)$
(D)	$\sum_{i=1}^{n} x_i f(x_i)$	$\sum_{i=1}^{n} g(x_i) f(x_i)$	$\sum_{i=1}^{n} x_i^k f(x_i)$	$\sum_{i=1}^{n} f(x_i) e^{i\omega x_i}$
(C)	$\int_{-\infty}^{+\infty} x f(x)dx$	$\int_{-\infty}^{+\infty} g(x) f(x)dx$	$\int_{-\infty}^{+\infty} x^k f(x)dx$	$\int_{-\infty}^{+\infty} f(x) e^{i\omega x}dx$

Remark: we have $e^{i\omega x} = \cos(\omega x) + i\sin(\omega x)$.

❒ **Revisiting the k^{th} moment** – The k^{th} moment can also be computed with the characteristic function as follows:

$$\boxed{E[X^k] = \frac{1}{i^k}\left[\frac{\partial^k \psi}{\partial \omega^k}\right]_{\omega=0}}$$

❒ **Transformation of random variables** – Let the variables X and Y be linked by some function. By noting f_X and f_Y the distribution function of X and Y respectively, we have:

$$\boxed{f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|}$$

❒ **Leibniz integral rule** – Let g be a function of x and potentially c, and a, b boundaries that may depend on c. We have:

$$\boxed{\frac{\partial}{\partial c}\left(\int_a^b g(x)dx\right) = \frac{\partial b}{\partial c} \cdot g(b) - \frac{\partial a}{\partial c} \cdot g(a) + \int_a^b \frac{\partial g}{\partial c}(x)dx}$$

❒ **Chebyshev's inequality** – Let X be a random variable with expected value μ and standard deviation σ. For $k, \sigma > 0$, we have the following inequality:

$$\boxed{P(|X - \mu| \geqslant k\sigma) \leqslant \frac{1}{k^2}}$$

5.1.4 Jointly Distributed Random Variables

❒ **Conditional density** – The conditional density of X with respect to Y, often noted $f_{X|Y}$, is defined as follows:

$$\boxed{f_{X|Y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)}}$$

❒ **Independence** – Two random variables X and Y are said to be independent if we have:

$$\boxed{f_{XY}(x,y) = f_X(x) f_Y(y)}$$

❒ **Marginal density and cumulative distribution** – From the joint density probability function f_{XY}, we have:

Case	Marginal density	Cumulative function
(D)	$f_X(x_i) = \sum_j f_{XY}(x_i,y_j)$	$F_{XY}(x,y) = \sum_{x_i \leqslant x} \sum_{y_j \leqslant y} f_{XY}(x_i,y_j)$
(C)	$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x,y)dy$	$F_{XY}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x',y')dx'dy'$

❒ **Distribution of a sum of independent random variables** – Let $Y = X_1 + ... + X_n$ with $X_1, ..., X_n$ independent. We have:

$$\boxed{\psi_Y(\omega) = \prod_{k=1}^{n} \psi_{X_k}(\omega)}$$

❒ **Covariance** – We define the covariance of two random variables X and Y, that we note σ^2_{XY} or more commonly $\text{Cov}(X,Y)$, as follows:

$$\boxed{\text{Cov}(X,Y) \triangleq \sigma^2_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = E[XY] - \mu_X\mu_Y}$$

❒ **Correlation** – By noting σ_X, σ_Y the standard deviations of X and Y, we define the correlation between the random variables X and Y, noted ρ_{XY}, as follows:

$$\boxed{\rho_{XY} = \frac{\sigma^2_{XY}}{\sigma_X\sigma_Y}}$$

Remarks: For any X, Y, we have $\rho_{XY} \in [-1,1]$. If X and Y are independent, then $\rho_{XY} = 0$.

❒ **Main distributions** – Here are the main distributions to have in mind:

Type	Distribution	PDF	$\psi(\omega)$	$E[X]$	$\text{Var}(X)$
(D)	$X \sim \mathcal{B}(n,p)$ Binomial	$P(X=x) = \binom{n}{x} p^x q^{n-x}$ $x \in [\![0,n]\!]$	$(pe^{i\omega}+q)^n$	np	npq
	$X \sim \text{Po}(\mu)$ Poisson	$P(X=x) = \frac{\mu^x}{x!}e^{-\mu}$ $x \in \mathbb{N}$	$e^{\mu(e^{i\omega}-1)}$	μ	μ
(C)	$X \sim \mathcal{U}(a,b)$ Uniform	$f(x) = \frac{1}{b-a}$ $x \in [a,b]$	$\frac{e^{i\omega b}-e^{i\omega a}}{(b-a)i\omega}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
	$X \sim \mathcal{N}(\mu,\sigma)$ Gaussian	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $x \in \mathbb{R}$	$e^{i\omega\mu - \frac{1}{2}\omega^2\sigma^2}$	μ	σ^2
	$X \sim \text{Exp}(\lambda)$ Exponential	$f(x) = \lambda e^{-\lambda x}$ $x \in \mathbb{R}_+$	$\frac{1}{1-\frac{i\omega}{\lambda}}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

5.1.5 Parameter estimation

❒ **Random sample** – A random sample is a collection of n random variables $X_1, ..., X_n$ that are independent and identically distributed with X.

❒ **Estimator** – An estimator $\hat{\theta}$ is a function of the data that is used to infer the value of an unknown parameter θ in a statistical model.

❒ **Bias** – The bias of an estimator $\hat{\theta}$ is defined as being the difference between the expected value of the distribution of $\hat{\theta}$ and the true value, i.e.:

$$\boxed{\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta}$$

Remark: an estimator is said to be unbiased when we have $E[\hat{\theta}] = \theta$.

❒ **Sample mean and variance** – The sample mean and the sample variance of a random sample are used to estimate the true mean μ and the true variance σ^2 of a distribution, are noted $\overline{X}$ and s^2 respectively, and are such that:

$$\boxed{\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i} \quad \text{and} \quad \boxed{s^2 = \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

❒ **Central Limit Theorem** – Let us have a random sample $X_1, ..., X_n$ following a given distribution with mean μ and variance σ^2, then we have:

$$\boxed{\overline{X} \underset{n\to+\infty}{\sim} \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)}$$

5.2 Linear Algebra and Calculus

5.2.1 General notations

❒ **Vector** – We note $x \in \mathbb{R}^n$ a vector with n entries, where $x_i \in \mathbb{R}$ is the i^{th} entry:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$

❒ **Matrix** – We note $A \in \mathbb{R}^{m\times n}$ a matrix with m rows and n columns, where $A_{i,j} \in \mathbb{R}$ is the entry located in the i^{th} row and j^{th} column:

$$A = \begin{pmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & & \vdots \\ A_{m,1} & \cdots & A_{m,n} \end{pmatrix} \in \mathbb{R}^{m\times n}$$

Remark: the vector x defined above can be viewed as a $n \times 1$ matrix and is more particularly called a column-vector.

❒ **Identity matrix** – The identity matrix $I \in \mathbb{R}^{n\times n}$ is a square matrix with ones in its diagonal and zero everywhere else:

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

Remark: for all matrices $A \in \mathbb{R}^{n\times n}$, we have $A \times I = I \times A = A$.

❒ **Diagonal matrix** – A diagonal matrix $D \in \mathbb{R}^{n\times n}$ is a square matrix with nonzero values in its diagonal and zero everywhere else:

$$D = \begin{pmatrix} d_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & d_n \end{pmatrix}$$

Remark: we also note D as $diag(d_1,...,d_n)$.

5.2.2 Matrix operations

❒ **Vector-vector multiplication** – There are two types of vector-vector products:

- inner product: for $x,y \in \mathbb{R}^n$, we have:

$$\boxed{x^T y = \sum_{i=1}^{n} x_i y_i \in \mathbb{R}}$$

- outer product: for $x \in \mathbb{R}^m, y \in \mathbb{R}^n$, we have:

$$\boxed{xy^T = \begin{pmatrix} x_1y_1 & \cdots & x_1y_n \\ \vdots & & \vdots \\ x_my_1 & \cdots & x_my_n \end{pmatrix} \in \mathbb{R}^{m\times n}}$$

❒ **Matrix-vector multiplication** – The product of matrix $A \in \mathbb{R}^{m\times n}$ and vector $x \in \mathbb{R}^n$ is a vector of size $\mathbb{R}^m$, such that:

$$\boxed{Ax = \begin{pmatrix} a_{r,1}^T x \\ \vdots \\ a_{r,m}^T x \end{pmatrix} = \sum_{i=1}^{n} a_{c,i} x_i \in \mathbb{R}^m}$$

where $a_{r,i}^T$ are the vector rows and $a_{c,j}$ are the vector columns of A, and x_i are the entries of x.

❒ **Matrix-matrix multiplication** – The product of matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times p}$ is a matrix of size $\mathbb{R}^{n\times p}$, such that:

$$\boxed{AB = \begin{pmatrix} a_{r,1}^T b_{c,1} & \cdots & a_{r,1}^T b_{c,p} \\ \vdots & & \vdots \\ a_{r,m}^T b_{c,1} & \cdots & a_{r,m}^T b_{c,p} \end{pmatrix} = \sum_{i=1}^{n} a_{c,i} b_{r,i}^T \in \mathbb{R}^{n\times p}}$$

where $a_{r,i}^T, b_{r,i}^T$ are the vector rows and $a_{c,j}, b_{c,j}$ are the vector columns of A and B respectively.

❒ **Transpose** – The transpose of a matrix $A \in \mathbb{R}^{m\times n}$, noted A^T, is such that its entries are flipped:

$$\boxed{\forall i,j, \quad A_{i,j}^T = A_{j,i}}$$

Remark: for matrices A,B, we have $(AB)^T = B^T A^T$.

❒ **Inverse** – The inverse of an invertible square matrix A is noted A^{-1} and is the only matrix such that:

$$\boxed{AA^{-1} = A^{-1}A = I}$$

Remark: not all square matrices are invertible. Also, for matrices A,B, we have $(AB)^{-1} = B^{-1}A^{-1}$

❒ **Trace** – The trace of a square matrix A, noted $\mathrm{tr}(A)$, is the sum of its diagonal entries:

$$\boxed{\mathrm{tr}(A) = \sum_{i=1}^{n} A_{i,i}}$$

Remark: for matrices A,B, we have $tr(A^T) = tr(A)$ and $tr(AB) = tr(BA)$

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$$\boxed{\det(A) = |A| = \sum_{j=1}^{n} (-1)^{i+j} A_{i,j} |A_{\backslash i,\backslash j}|}$$

Remark: A is invertible if and only if $|A| \neq 0$. Also, $|AB| = |A||B|$ and $|A^T| = |A|$.

5.2.3 Matrix properties

❒ **Symmetric decomposition** – A given matrix A can be expressed in terms of its symmetric and antisymmetric parts as follows:

$$\boxed{A = \underbrace{\frac{A + A^T}{2}}_{\text{Symmetric}} + \underbrace{\frac{A - A^T}{2}}_{\text{Antisymmetric}}}$$

❒ **Norm** – A norm is a function $N : V \longrightarrow [0, +\infty[$ where V is a vector space, and such that for all $x,y \in V$, we have:

- $N(x + y) \leqslant N(x) + N(y)$
- $N(ax) = |a|N(x)$ for a scalar
- if $N(x) = 0$, then $x = 0$

For $x \in V$, the most commonly used norms are summed up in the table below:

Norm	Notation	Definition	Use case
Manhattan, L^1	$\|\|x\|\|_1$	$\sum_{i=1}^{n} \|x_i\|$	LASSO regularization
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❒ **Linearly dependence** – A set of vectors is said to be linearly dependent if one of the vectors in the set can be defined as a linear combination of the others.
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❒ **Matrix rank** – The rank of a given matrix A is noted $\text{rank}(A)$ and is the dimension of the vector space generated by its columns. This is equivalent to the maximum number of linearly independent columns of A.

❒ **Positive semi-definite matrix** – A matrix $A \in \mathbb{R}^{n\times n}$ is positive semi-definite (PSD) and is noted $A \succeq 0$ if we have:

$$\boxed{A = A^T} \quad \text{and} \quad \boxed{\forall x \in \mathbb{R}^n, \quad x^T A x \geqslant 0}$$

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❒ **Eigenvalue, eigenvector** – Given a matrix $A \in \mathbb{R}^{n\times n}$, λ is said to be an eigenvalue of A if there exists a vector $z \in \mathbb{R}^n \backslash \{0\}$, called eigenvector, such that we have:

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VIP Refresher: Linear Algebra and Calculus

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