HOMEWORK 3

1-a) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{5} 3^n e^{-j\omega n} = 1 + 3e^{-j\omega} + 9e^{-2j\omega} + 27e^{-3j\omega} + 81e^{-4j\omega} + 243e^{-5j\omega}$

1-b) $X(\omega) = \sum_{n=-\infty}^{\infty} \delta[5-3n]e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta[5+3n]e^{-j\omega n}$,

$5-3n=0 \Rightarrow n=\frac{5}{3}$ $\qquad$ $5+3n=0 \Rightarrow n=-\frac{5}{3}$

not integers $\Rightarrow X(\omega)=0$

2-) a) $X(\omega) = 3\cos(3\omega) = \frac{3}{2}(e^{3j\omega} + e^{-3j\omega})$, $\quad \delta[n-n_0] \overset{F}{\longleftrightarrow} e^{-j\omega n_0}$

$\Rightarrow x[n] = \frac{3}{2}\delta[n+3] + \frac{3}{2}\delta[n-3]$

2-b) $a^n u[n] \overset{F}{\longleftrightarrow} \frac{1}{1-ae^{j\omega}}$, $\quad \begin{matrix} 1, & |n| \le M \\ 0, & |n| > M \end{matrix} \overset{F}{\longleftrightarrow} \frac{\sin\left(\frac{2M+1}{2}\omega\right)}{\sin(0{,}5\omega)}$

$X(\omega) = X_1(\omega) + X_2(\omega)$, $\quad X_1(\omega) = 0{,}75\, e^{-j6\omega} \cdot \frac{\sin(3{,}5\omega)}{\sin(0{,}5\omega)} \Rightarrow x_1[n] = \begin{cases} 0{,}75, & |n-6| \le 3 \\ 0, & |n-6| > 3 \end{cases}$

↳ time shift

$X_2(\omega) = e^{-j12\omega} \frac{1}{1-0{,}5\, e^{-j\omega}} \Rightarrow x_2[n] = 0{,}5^{n-12}\, u[n-12]$, $\quad x[n] = x_1[n] + x_2[n]$

$\Rightarrow x[n] = 0{,}75 \sum_{k=3}^{9} \delta[n-k] + 0{,}5^{n-12}\, u[n-12]$