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Question 1

- $X_1(e^{j\Omega}) = \sum_{-\infty}^{\infty} 3^n(u(t)-u(t-6))e^{-j\Omega n} = \sum_{0}^{5}(3e^{-j\Omega})^n = \frac{1-(3e^{-j\Omega})^6}{1-3e^{-j\Omega}}$
- $X_2(e^{j\Omega}) = \sum_{-\infty}^{\infty}(\delta(5-3n)+\delta(5+3n))e^{j\Omega n} = e^{-5j\Omega/3} + e^{5j\Omega/3}$

Question 2

- $3\cos(3\Omega) = 3/2\, e^{3j\Omega} + 3/2\, e^{-3j\Omega}$

$$x_1(n) = \frac{3}{4\pi}\int_{-\pi}^{\pi}\left(e^{j\Omega(3+n)} + e^{-j\Omega(3-n)}\right)d\Omega = \frac{3}{4\pi}\left(\frac{1}{j(3+n)}e^{j\Omega(3+n)} - \frac{1}{j(3-n)}e^{-j\Omega(3-n)}\right)\Bigg|_{-\pi}^{\pi} =$$

$$= \frac{3}{4\pi}\left(\frac{1}{j(3+n)}\left(e^{j\pi(3+n)} - e^{-j\pi(3+n)}\right) - \frac{1}{j(3-n)}\left(e^{-j\pi(3-n)} - e^{j\pi(3-n)}\right)\right) =$$

$$= \frac{3}{2\pi}\left(\frac{1}{3+n}\sin(3\pi+\pi n) + \frac{1}{3-n}\sin(3\pi-\pi n)\right) =$$

$$= \frac{3}{2\pi}\left(\frac{1}{3+n}\sin(-\pi n) + \frac{1}{3-n}\sin(\pi n)\right) = \frac{3\sin(\pi n)}{2\pi}\left(\frac{1}{3-n} - \frac{1}{3+n}\right) =$$

$$= \frac{3\sin(\pi n)}{2\pi}\cdot\frac{2n}{9-n^2} = \frac{3n\sin(\pi n)}{(9-n^2)\pi}$$

- Given: $a^n u[n] \xrightarrow{F} \frac{1}{1-ae^{-j\omega}}$ for $|a|<1$ (1)

$$x[n-n_0] \xrightarrow{F} e^{-j\Omega n_0}X(\Omega) \quad (2)$$

and $\begin{cases} 1, & |n|\leq N \\ 0, & |n|>N \end{cases} \xrightarrow{F} \frac{\sin((N+0.5)\Omega)}{\sin(0.5\Omega)}$ (3)

We can break $X(\Omega)$ into 2 parts:

$0.75\, e^{-j6\Omega}\frac{\sin(3.5\Omega)}{\sin(\Omega)}$ (a) and $e^{-j12\Omega}\frac{1}{1-0.5e^{-j\Omega}}$ (b)

in (a), we can use (2) & (3) to derive:

$$X_a(\Omega) \xrightarrow{F^{-1}} x_a[n] = \begin{cases} 0.75, & |n-6|\leq 3 \\ 0, & |n-6|>3 \end{cases}$$

in (b) we can use (1) & (2) to derive:

$$X_b(\Omega) \xrightarrow{F^{-1}} x_b[n] = 0.5^{n-12}u[n-12]$$

We know that $x_a + x_b \xrightarrow{F} X_a + X_b$, so

$$x[n] = \begin{cases} 0.75 + 0.5^{n-12}u[n-12], & |n-6|\leq 3 \\ 0.5^{n-12}u[n-12], & |n-6|>3 \end{cases}$$