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Question 1

1. $\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{9}$

$$a_k = \frac{1}{9}\sum_{n=-4}^{4}(n+2e^{3n})\,u(n-2)\,e^{-j2\pi kn/9} = \frac{1}{9}\sum_{n=2}^{4}(n+2e^{3n})\,e^{-j2\pi kn/9} =$$

$$= \frac{1}{9}\left((2+e^{6})\,e^{-j4\pi k/9} + (3+e^{9})\,e^{-j6\pi k/9} + (4+e^{12})\,e^{-j8\pi k/9}\right)$$

2. $\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{11}$

$$a_k = \frac{1}{11}\sum_{n=-5}^{4} n^2 e^{-j2\pi nk/11} = \frac{1}{11}\left(16\,e^{j8\pi k/11} + 25e^{j10\pi k/11}\right)$$

3. $\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{7}$

$$a_k = \frac{1}{7}\sum_{n=-3}^{3}\left(\sum_{m=n-2}^{n+2} m e^{m}\right)e^{-j2\pi nk/7}$$

Question 3

$x(t) = \frac{1}{j}e^{j2\pi t} - \frac{1}{j}e^{-j2\pi t} + 2e^{j3\pi t} + 2e^{-j3\pi t}$ $\quad \omega_0 = \pi \Rightarrow T_0 = 2$

$$a_n = \frac{1}{2}\int_0^2\left(\frac{1}{j}e^{j2\pi t} - \frac{1}{j}e^{-j2\pi t} + 2e^{j3\pi t} + 2e^{-j3\pi t}\right)e^{-jn\pi t}\,dt =$$

$$= \frac{1}{2}\int_0^2\left(\frac{1}{j}e^{jt(2\pi-n\pi)} - \frac{1}{j}e^{jt(-2\pi-n\pi)} + 2e^{jt(3\pi-n\pi)} + 2e^{j(-3\pi-n\pi)}\right)dt$$

$\int_{T_0} e^{jm\omega_0 t}\,dt = \begin{cases} 0 & m\neq 0 \\ T_0 & m=0 \end{cases} \Rightarrow$ Therefore, above expression produces nonzero values only for $n=-3,-2,2,3$.

$a_{-3}=2 \quad a_{-2}=-\frac{1}{j} \quad a_2=\frac{1}{j} \quad a_3=2$

Question 4

1. $x(t) = e^{2t}u(-t)$

$$X(\omega) = \int_{-\infty}^{\infty} e^{2t}u(-t)\,e^{-j\omega t}\,dt = \int_{-\infty}^{0} e^{t(2-j\omega)}\,dt = \frac{1}{2-j\omega}e^{t(2-j\omega)}\Big|_{-\infty}^{0} =$$

$$= \frac{1}{2-j\omega}(1-0) = \frac{1}{2-j\omega}$$

2. $x(t) = e^{2|t|}$

$$X(\omega) = \int_{-\infty}^{\infty} e^{2|t|}e^{-j\omega t}\,dt = \int_{-\infty}^{0} e^{-2t}e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{2t}e^{-j\omega t}\,dt =$$

$$= \int_{-\infty}^{0} e^{t(2-\omega t)}\,dt + \int_{0}^{\infty} e^{t(2-\omega t)}\,dt = \frac{-1}{2+\omega t}e^{t(2-\omega t)}\Big|_{-\infty}^{0} + \frac{1}{2-\omega t}e^{t(2+\omega t)}\Big|_{0}^{\infty} =$$

$$= -\frac{1}{2+\omega t}(1-\infty) + \frac{1}{2-\omega t}(\infty-1) \Rightarrow \infty;\ \text{no CTFT}$$

Question 5

1. $X(\omega) = 3\delta(\omega-4)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 3\delta(\omega-4)\,e^{j\omega t}\,d\omega = \frac{1}{2\pi}\,3e^{4jt}$$

2. $X(\omega) = \pi e^{-|\omega|}$

$$x(t) = \frac{1}{2\pi}\pi\left(\int_{-\infty}^{0} e^{\omega}e^{j\omega t}\,d\omega + \int_{0}^{\infty} e^{-\omega}e^{j\omega t}\,d\omega\right) = \frac{1}{2}\left(\int_{-\infty}^{0} e^{\omega(1+jt)} + \int_{0}^{\infty} e^{\omega(jt-1)}\right) =$$

$$= \frac{1}{2}\left(\frac{1}{1+jt}e^{\omega(1+jt)}\Big|_{-\infty}^{0} + \frac{1}{jt-1}e^{\omega(jt-1)}\Big|_{0}^{\infty}\right) = \frac{1}{2}\left(\frac{1}{1+jt} + \frac{1}{jt-1}\right) = -\frac{jt}{t^2+1}$$