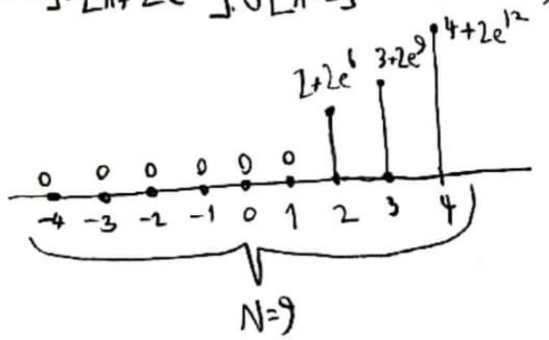
BLG354E HW2 Solutions

Yusuf H. Şahin

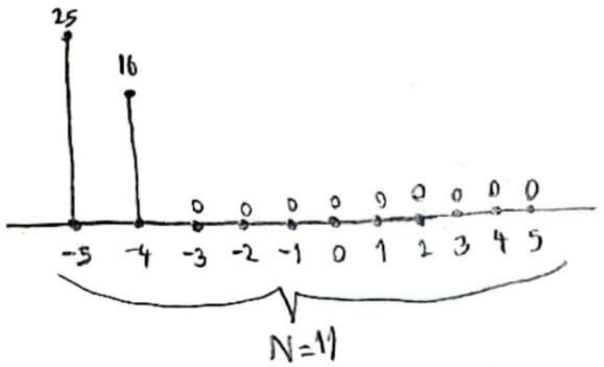
1-) a) $x[n] = [n + 2e^{3n}] . u[n-2] \quad -5 < n < 5, \ N = 9$



$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] . e^{-jk\Omega_0 n} \qquad \Omega_0 = \frac{2\pi}{9}$$

$$= \frac{1}{9} \sum_{n=2}^{4} x[n] . e^{-j\frac{2\pi}{9}n} = \frac{1}{9}\left[(2+2e^6) . e^{-\frac{4\pi jk}{9}} + (3+2e^9) . e^{-\frac{6\pi jk}{9}} + (4+2e^{12}) . e^{-\frac{8\pi jk}{9}}\right]$$

b-)



$$a_k = \frac{1}{11} \sum_{n=-5}^{-4} x[n] . e^{-jk\Omega_0 n} \qquad \Omega_0 = \frac{2\pi}{11}$$

$$= \frac{1}{11}\left[25 . e^{\frac{10jk\pi}{11}} + 16 . e^{\frac{8jk\pi}{11}}\right]$$

c-)

$$z[-3] = \sum_{m=-5}^{-1} m.e^m = -0,89$$

$$z[-2] = \sum_{m=-4}^{0} m.e^m = -0,86$$

$$z[-1] = \sum_{m=-3}^{1} m.e^m = 1,93$$

$$z[0] = \sum_{m=-2}^{2} m.e^m = 16,85$$

$$z[1] = \sum_{m=-1}^{3} m e^m = 77,38$$

$$z[2] = \sum_{m=0}^{4} m.e^m = 296.14$$

$$z[3] = \sum_{m=1}^{5} m e^m = 1038$$

$$a_k = \frac{1}{7} \sum_{n=-3}^{3} x[n] . e^{-jk\frac{2\pi}{7}n}$$

$$= \frac{1}{7}\left[-0.89 . e^{jk\frac{6\pi}{7}} - 0,86 . e^{jk\frac{4\pi}{7}} + 1,93 . e^{jk\frac{2\pi}{7}} + 16.85 + 77,38 . e^{-jk\frac{2\pi}{7}} + 296.14 . e^{-jk\frac{4\pi}{7}} + 1038 . e^{-jk\frac{6\pi}{7}}\right]$$

3-) $x(t) = 2\sin(2\pi t) + 4\cos(3\pi t)$

$$= 2\cdot\left(\frac{e^{2j\pi t} - e^{-2j\pi t}}{2j}\right) + 4\cdot\left(\frac{e^{3j\pi t} + e^{-3j\pi t}}{2}\right)$$

$$= -j \cdot e^{2j\pi t} + j\, e^{-2j\pi t} + 2.e^{j3\pi t} + e^{-j3\pi t}$$

$a_2 = -j \quad a_{-2} = j \quad a_3 = 2 \quad a_{-3} = 2$

4-) a-) $x(t) = e^{2t}.u(-t)$

$$X(w) = \int_{-\infty}^{0} e^{2t}.e^{-jwt}\,dt = \int_{-\infty}^{0} e^{t(2-jw)}\,dt = \left.\frac{e^{t(2-jw)}}{2-jw}\right|_{t=-\infty}^{0} = \frac{1}{2-jw}$$

b-) $x(t) = e^{2|t|}$

$$X(w) = \int_{-\infty}^{\infty} e^{2|t|}.e^{-jwt}\,dt = \int_{-\infty}^{0} e^{-2t}\,e^{-jwt}\,dt + \int_{0}^{\infty} e^{2t}.e^{-jwt}\,dt$$

$$= \int_{-\infty}^{0} e^{-t(2+jw)}\,dt + \int_{0}^{\infty} e^{t(2-jw)}\,dt$$

$$= \underbrace{\left.\frac{-e^{-t(2+jw)}}{2+jw}\right|_{t=-\infty}^{0}} + \underbrace{\left.\frac{e^{t(2-jw)}}{2-jw}\right|_{t=0}^{\infty}}$$

$$\underbrace{\frac{-1}{2+jw} + \frac{e^{\infty}}{2+jw}}_{\text{Could not converge}} \quad \underbrace{\frac{e^{\infty}}{2-jw} - \frac{1}{2-jw}}_{\text{Could not converge}}$$

The signal doesn't have a FT.

5-) a-) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 3\delta(w-4)\,e^{jwt}\,dw = \frac{3e^{4jt}}{2\pi}$

b-) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi e^{-|w|}\,e^{jwt}\,dw = \frac{1}{2}\left(\int_{-\infty}^{0} e^{w}.e^{jwt}\,dw + \int_{0}^{\infty} e^{-w}.e^{jwt}\,dw\right)$

$$= \frac{1}{2}\left(\left.\frac{1}{1+jt}\cdot e^{w(jt+1)}\right|_{-\infty}^{0} + \left.\frac{1}{-1+jt}\cdot e^{w(jt-1)}\right|_{0}^{\infty}\right) = \frac{1}{2}\left(\frac{1}{1+jt} + \frac{1}{1-jt}\right) = \frac{1}{1+t^2}$$