Assignment-2

Adil Salfi - CS20BTECH11031

Download all python codes from

```
https://github.com/AdilSalfi/AI1103/tree/main/
    Assignment−2/Codes
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and latex-tikz code from

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https://github.com/AdilSalfi/AI1103/tree/main/
    Assignment−2
```

Problem

GATE-EC Question 59 :
Let X be a random variable having the distribution function :

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{1}{3} & 1 \leq x < 2 \\ \frac{1}{2} & 2 \leq x < \frac{11}{3} \\ 1 & x \geq \frac{11}{3} \end{cases}$$

Then $E(X)$ is equal to :

Solution

Definition 1 (Heaviside step function)**.** *Heaviside step function $u(x)$ is*

$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

Using the Heaviside step function $u(x)$, a function $F(t)$ can be obtained whose output is $f(t)$ for the interval $[a, b)$ and 0 everywhere else

$$F(t) = f(t)[u(t-a) - u(t-b)] \tag{1}$$

Definition 2 (Dirac delta function)**.** *Dirac delta function is the derivative of the Heaviside step function $u(x)$*

$$\delta(x) = \frac{du(x)}{dx} \tag{2}$$

An important property of the Dirac delta function is

$$\int_{-\infty}^{\infty} f(x)\delta(x - x_0)dx = f(x_0) \tag{3}$$

Using Definition 1, we get CDF $F(x)$ in terms of Heaviside step function $u(x)$

$$F(x) = \frac{1}{4}\{u(x) - u(x-1)\} + \frac{1}{3}\{u(x-1) - u(x-2)\} + \frac{1}{2}\left\{u(x-2) - u\left(x - \frac{11}{3}\right)\right\} + u\left(x - \frac{11}{3}\right)$$

$$\implies F(x) = \frac{u(x)}{4} + \frac{u(x-1)}{12} + \frac{u(x-2)}{6} + \frac{u\left(x - \frac{11}{3}\right)}{2} \tag{4}$$

Differentiating (4) and using Definition 2, we obtain PDF $f(x)$

$$f(x) = \frac{\delta(x)}{4} + \frac{\delta(x-1)}{12} + \frac{\delta(x-2)}{6} + \frac{\delta\left(x - \frac{11}{3}\right)}{2} \tag{5}$$

Using (5), we can state that Random Variable X is discrete and it takes values at the points where $f(x) \to \infty$

$$\therefore X \in \left\{0, 1, 2, \frac{11}{3}\right\} \tag{6}$$

To obtain the PMF($p_X(k)$) we use the formula

$$(p_X(k)) = \lim_{x \to k} \int_k^x f(x)dx \tag{7}$$

Definition 3 (PMF of Random Variable X)**.** *The PMF($p_X(k)$) using (7) is :*

$$(p_X(k)) = \begin{cases} \frac{1}{4} & \text{if } k = 0 \\ \frac{1}{12} & \text{if } k = 1 \\ \frac{1}{6} & \text{if } k = 2 \\ \frac{1}{2} & \text{if } k = \frac{11}{3} \\ 0 & \text{otherwise} \end{cases}$$

To obtain $E(x)$ we use the formula

$$E(X) = \sum x \times (p_X(k)) \tag{8}$$

Therefore, using PMF we get

$$E(X) = \left(0 \times \frac{1}{4}\right) + \left(1 \times \frac{1}{12}\right) + \left(2 \times \frac{1}{6}\right) + \left(\frac{11}{3} \times \frac{1}{2}\right)$$

$$\implies E(X) = 2.25$$

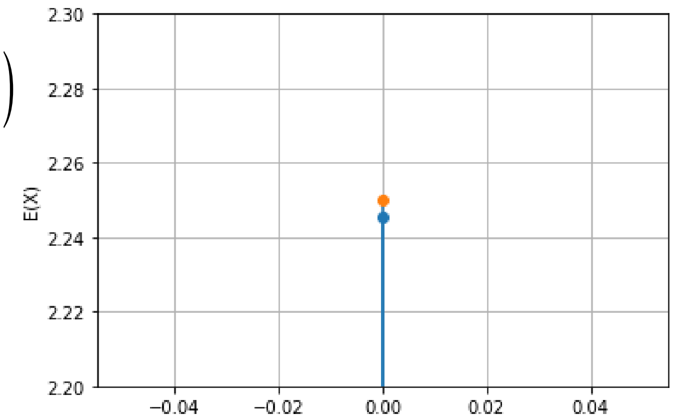

Fig. 1:Plot comparing Simulated and Theoretical value of $E(x)$