Assignment-2

Adil Salfi - CS20BTECH11031

Download all python codes from

```
https://github.com/AdilSalfi/AI1103/tree/main/
    Assignment−2/Codes
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and latex-tikz code from

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https://github.com/AdilSalfi/AI1103/tree/main/
    Assignment−2
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Problem

GATE-EC Question 59 :
Let X be a random variable having the distribution function :

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \le x < 1 \\ \frac{1}{3} & 1 \le x < 2 \\ \frac{1}{2} & 2 \le x < \frac{11}{3} \\ 1 & x \ge \frac{11}{3} \end{cases}$$

Then $E(X)$ is equal to :

Solution

→The Cumulative Distribution Function F(x) is a step function and Proabability can be calculated using :

$$\Pr(X = a) = F(a) - F(a^-) \tag{1}$$

→When F(x) is continuos it takes a constant value. Therefore, Using (1) the probability when F(X) is continuos is 0. Hence these events never occur.

→When F(x) is discontinuos there is a jump in the value taken by F(x). Therefore, Using (1) the probability when F(X) is discontinuos is finite.

→As the points of discontinuity are discrete, F(x) represents a cdf of a discrete Random Variable.

→The points of discontinuity are $\{0, 1, 2, \frac{11}{3}\}$. Therefore, Random Variable $X \in \{0, 1, 2, \frac{11}{3}\}$

1) Calculation of $\Pr(X = 0)$:
Using (1),

$$\rightarrow \Pr(X = 0) = F(0) - F(0^-) = \frac{1}{4} - 0$$

$$\therefore \Pr(X = 0) = \frac{1}{4} \tag{2}$$

2) Calculation of $\Pr(X = 1)$:
Using (1),

$$\rightarrow \Pr(X = 1) = F(1) - F(1^-) = \frac{1}{3} - \frac{1}{4}$$

$$\therefore \Pr(X = 1) = \frac{1}{12} \tag{3}$$

3) Calculation of $\Pr(X = 2)$:
Using (1),

$$\rightarrow \Pr(X = 2) = F(2) - F(2^-) = \frac{1}{2} - \frac{1}{3}$$

$$\therefore \Pr(X = 2) = \frac{1}{6} \tag{4}$$

4) Calculation of $\Pr\left(X = \frac{11}{3}\right)$:
Using (1),

$$\rightarrow \Pr\left(X = \frac{11}{3}\right) = F(\frac{11}{3}) - F(\frac{11}{3}^-) = 1 - \frac{1}{2}$$

$$\therefore \Pr\left(X = \frac{11}{3}\right) = \frac{1}{2} \tag{5}$$

For Discrete Random Variables

$$E(X) = \sum_{i=1}^{n} x \Pr(x) \tag{6}$$

Using (2),(3),(4) and (5)

$$= (0 \times \frac{1}{4}) + (1 \times \frac{1}{12}) + (2 \times \frac{1}{6}) + (\frac{11}{3} \times \frac{1}{2})$$
$$= \frac{27}{12}$$

$$\therefore E(X) = 2.25 \tag{7}$$

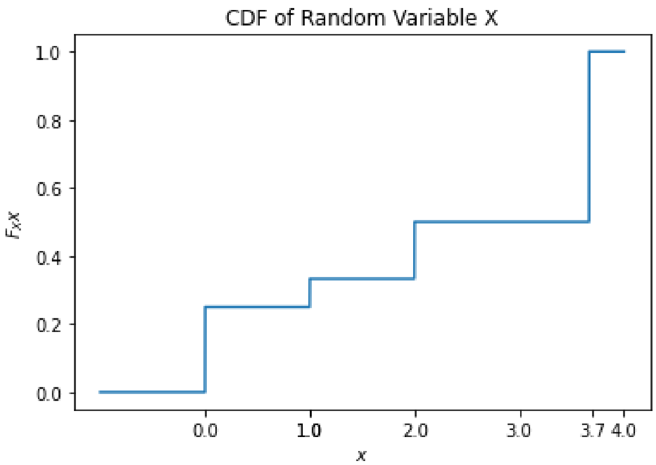

Fig. 1:CDF of X

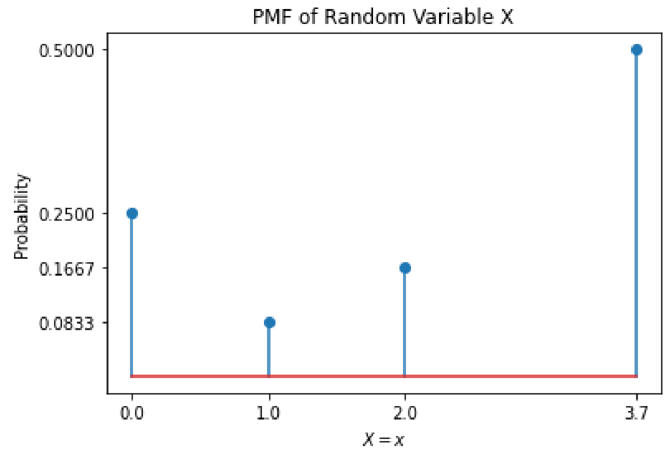

Fig. 2:PMF of X