Assignment-2

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Download all python codes from

https://github.com/AdilSalfi/AI1103/tree/main/Assignment−2/Codes

and latex-tikz code from

https://github.com/AdilSalfi/AI1103/tree/main/Assignment−2

Problem

GATE-EC Question 59 :
Let X be a random variable having the distribution function :

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \le x < 1 \\ \frac{1}{3} & 1 \le x < 2 \\ \frac{1}{2} & 2 \le x < \frac{11}{3} \\ 1 & x \ge \frac{11}{3} \end{cases}$$

Then $E(X)$ is equal to :

Solution

→As the Cumulative Distribution Function $F(x)$ is discontinuos and is similar to a step function, it represents the CDF of a discrete Random Variable.
→As the CDF is discontinuos when $x = (0, 1, 2, \frac{11}{3})$, Random Variable $X \in \{0, 1, 2, \frac{11}{3}\}$.
→The probabilites for each value of X are calculated below using the formula:

$$\Pr(X = i) = F(b) - F(a), \ i \in [a, b] \tag{1}$$

1) Calculation of $\Pr(X = 0)$:

Using (1),

$$\rightarrow \Pr(X = 0) = F(0.5) - F(-0.5) = \frac{1}{4} - 0$$

$$\therefore \Pr(X = 0) = \frac{1}{4} \tag{2}$$

2) Calculation of $\Pr(X = 1)$:

Using (1),

$$\rightarrow \Pr(X = 1) = F(1.5) - F(-0.5) = \frac{1}{3} - \frac{1}{4}$$

$$\therefore \Pr(X = 1) = \frac{1}{12} \tag{3}$$

3) Calculation of $\Pr(X = 2)$:

Using (1),

$$\rightarrow \Pr(X = 2) = F(2.5) - F(1.5) = \frac{1}{2} - \frac{1}{3}$$

$$\therefore \Pr(X = 2) = \frac{1}{6} \tag{4}$$

4) Calculation of $\Pr\left(X = \frac{11}{3}\right)$:

Using (1),

$$\rightarrow \Pr\left(X = \frac{11}{3}\right) = F(4) - F(3) = 1 - \frac{1}{2}$$

$$\therefore \Pr\left(X = \frac{11}{3}\right) = \frac{1}{2} \tag{5}$$

For Discrete Random Variables

$$E(X) = \sum_{i=1}^{n} x \Pr(x) \tag{6}$$

Using (2),(3),(4) and (5)

$$= (0 \times \frac{1}{4}) + (1 \times \frac{1}{12}) + (2 \times \frac{1}{6}) + (\frac{11}{3} \times \frac{1}{2})$$
$$= \frac{27}{12}$$
$$= 2.25$$

$$\therefore E(X) = 2.25 \tag{7}$$

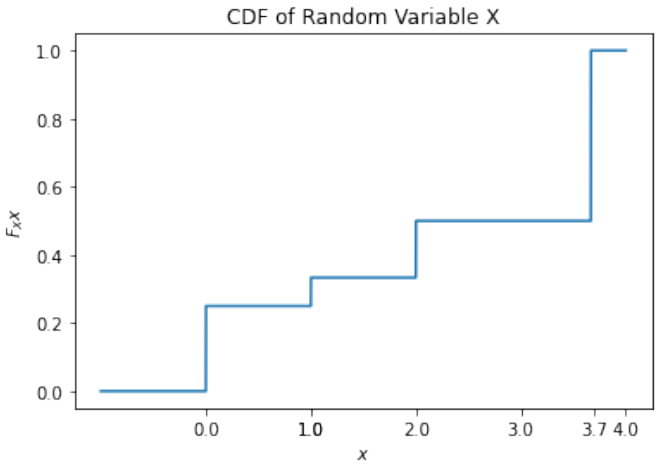

Fig. 0: CDF of X

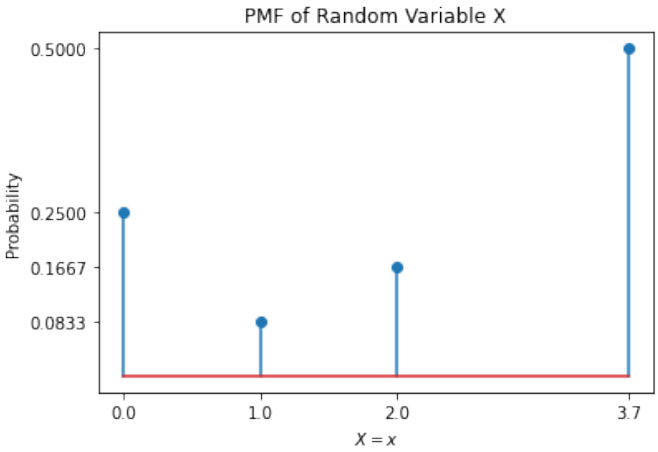

Fig. 0: PMF of X