

# Assignment-2

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Download all python codes from

<https://github.com/AdilSalfi/AI1103/tree/main/Assignment-2/Codes>

and latex-tikz code from

<https://github.com/AdilSalfi/AI1103/tree/main/Assignment-2>

## PROBLEM

GATE-EC Question 59 :

Let  $X$  be a random variable having the distribution function :

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{1}{3} & 1 \leq x < 2 \\ \frac{1}{2} & 2 \leq x < \frac{11}{3} \\ 1 & x \geq \frac{11}{3} \end{cases}$$

Then  $E(X)$  is equal to :

## SOLUTION

**Definition 1** (Heaviside step function). *Heaviside step function  $u(x)$  is*

$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \quad (1)$$

*Using the Heaviside step function  $u(x)$ , a function  $F(t)$  can be obtained whose output is  $f(t)$  for the interval  $[a, b)$  and 0 everywhere else*

$$F(t) = f(t)[u(t-a) - u(t-b)] \quad (2)$$

**Definition 2** (Dirac delta function). *Dirac delta function is the derivative of the Heaviside step function  $u(x)$*

$$\delta(x) = \frac{du(x)}{dx} \quad (3)$$

*An important property of the Dirac delta function is*

$$\int_{-\infty}^{\infty} f(x)\delta(x-x_0)dx = f(x_0) \quad (4)$$

→To obtain the CDF  $F(x)$  in terms of Heaviside step function  $u(x)$ , we use (2)

$$F(x) = \frac{1}{4}[u(x) - u(x-1)] + \frac{1}{3}[u(x-1) - u(x-2)] + \frac{1}{2}[u(x-2) - u(x-\frac{11}{3})] + u(x-\frac{11}{3})$$

$$\Rightarrow F(x) = \frac{u(x)}{4} + \frac{u(x-1)}{12} + \frac{u(x-2)}{6} + \frac{u(x-\frac{11}{3})}{2} \quad (5)$$

→To obtain PDF  $f(x)$  we differentiate (5) and using (3), we get

$$f(x) = \frac{\delta(x)}{4} + \frac{\delta(x-1)}{12} + \frac{\delta(x-2)}{6} + \frac{\delta(x-\frac{11}{3})}{2} \quad (6)$$

→Using (6), we can state that Random Variable  $X$  is discrete and it takes values at the points where  $f(x) \rightarrow \infty$

$$\therefore X \in \{0, 1, 2, \frac{11}{3}\}$$

→To obtain the PMF( $P(x)$ ) we use the formula

$$\Pr(X=a) = \lim_{x \rightarrow a} \int_a^x f(x)dx \quad (7)$$

→Using (7), The PMF( $P(x)$ ) is :

$$P(x) = \begin{cases} \frac{1}{4} & \text{if } x = 0 \\ \frac{1}{12} & \text{if } x = 1 \\ \frac{1}{6} & \text{if } x = 2 \\ \frac{1}{2} & \text{if } x = \frac{11}{3} \\ 0 & \text{otherwise} \end{cases}$$

→To obtain  $E(x)$  we use the formula

$$E(X) = \sum_{\text{all } x} x \Pr(X) \quad (8)$$

→Therefore, using PMF we get

$$E(X) = (0 \times \frac{1}{4}) + (1 \times \frac{1}{12}) + (2 \times \frac{1}{6}) + (\frac{11}{3} \times \frac{1}{2})$$

$$\Rightarrow E(X) = 2.25$$

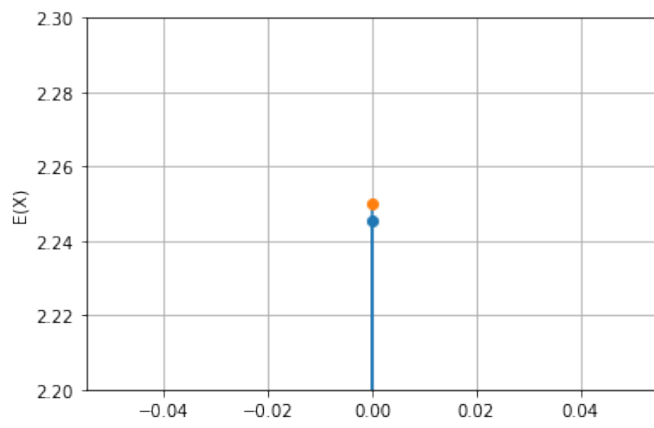


Fig. 1:Plot comparing Simulated and Theoretical value of  $E(x)$