LabWork #7
MAT 116E-Advanced Scientific and Engineering Computing (MATLAB)
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Main Task

Write a MATLAB program to find the minimum of the function

$$f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$$

where

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} -9 \\ -2 \end{bmatrix},\ c = \frac{51}{4},\ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

by using the following algorithms. Repeat the main steps of your algorithms until the desired accuracy is achieved, i.e. $|f(\mathbf{x}_{k+1}) - f(\mathbf{x}_k)| \leq \epsilon$. Take step size parameter as $\alpha = 0.0899$, initial guess as $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and absolute error bound as $\epsilon = 10^{-4}$ for every algorithm.

Gradient Descent Algorithm

Implement the gradient descent algorithm to find the minimum of the given function as follows:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha\nabla f(\mathbf{x}_k).$$

Accelerated Gradient Descent Algorithm

We can accelerate the above gradient descent algorithm by initial values $\mathbf{y}_0 = \mathbf{x}_0$ and $t_0 = 1$ as follows:

$$\begin{aligned} \mathbf{x}_{k+1} =& \mathbf{y}_k - \alpha\nabla f(\mathbf{y}_k), \\ t_{k+1} =& \frac{1}{2}(1 + \sqrt{1 + 4t_k^2}), \\ \mathbf{y}_{k+1} =& \mathbf{x}_{k+1} + \frac{t_k - 1}{t_{k+1}}(\mathbf{x}_{k+1} - \mathbf{x}_k). \end{aligned}$$

You may see sample realizations of these algorithms in the next page.

Submission Information

Any LabWork submitted after class will be subject to a 20-point deduction per 24 hour period. Extensions should be requested at least 3 days in advance and will only be granted for exceptional reasons (e.g., conference submission). You may work with your friends. Collaboration is strongly recommended. However, each student should be able to present his/her program.

Sample realizations of gradient descent and accelerated gradient descent algorithms.

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Gradient Descent Algorithm
k=1, x1=0.000000, x2=0.000000, f(x)=12.750000
k=2, x1=0.809100, x2=0.179800, f(x)=6.619836, abs. error=6.130164
k=3, x1=1.246428, x2=0.521240, f(x)=3.638429, abs. error=2.981406
k=4, x1=1.570643, x2=0.774380, f(x)=1.999773, abs. error=1.638656
k=5, x1=1.811005, x2=0.962049, f(x)=1.099126, abs. error=0.900647
k=6, x1=1.989202, x2=1.101181, f(x)=0.604107, abs. error=0.495018
k=7, x1=2.121311, x2=1.204328, f(x)=0.332033, abs. error=0.272075
k=8, x1=2.219252, x2=1.280799, f(x)=0.182494, abs. error=0.149539
k=9, x1=2.291863, x2=1.337491, f(x)=0.100303, abs. error=0.082190
k=10, x1=2.345694, x2=1.379521, f(x)=0.055129, abs. error=0.045174
k=11, x1=2.385603, x2=1.410681, f(x)=0.030300, abs. error=0.024829
k=12, x1=2.415189, x2=1.433782, f(x)=0.016654, abs. error=0.013647
k=13, x1=2.437124, x2=1.450908, f(x)=0.009153, abs. error=0.007500
k=14, x1=2.453386, x2=1.463605, f(x)=0.005031, abs. error=0.004122
k=15, x1=2.465442, x2=1.473018, f(x)=0.002765, abs. error=0.002266
k=16, x1=2.474380, x2=1.479996, f(x)=0.001520, abs. error=0.001245
k=17, x1=2.481006, x2=1.485170, f(x)=0.000835, abs. error=0.000684
k=18, x1=2.485918, x2=1.489005, f(x)=0.000459, abs. error=0.000376
k=19, x1=2.489560, x2=1.491849, f(x)=0.000252, abs. error=0.000207
k=20, x1=2.492260, x2=1.493957, f(x)=0.000139, abs. error=0.000114
k=21, x1=2.494262, x2=1.495520, f(x)=0.000076, abs. error=0.000062
Accelerated Gradient Descent Algorithm
k=1, x1=0.000000, x2=0.000000, f(x)=12.750000
k=2, x1=0.809100, x2=0.179800, f(x)=6.619836, abs. error=6.130164
k=3, x1=1.246428, x2=0.521240, f(x)=3.638429, abs. error=2.981406
k=4, x1=1.661991, x2=0.845703, f(x)=1.625969, abs. error=2.012461
k=5, x1=2.012450, x2=1.119332, f(x)=0.550368, abs. error=1.075600
k=6, x1=2.276527, x2=1.325517, f(x)=0.115629, abs. error=0.434739
k=7, x1=2.451552, x2=1.462173, f(x)=0.005435, abs. error=0.110194
k=8, x1=2.548285, x2=1.537700, f(x)=0.005398, abs. error=0.000037
```

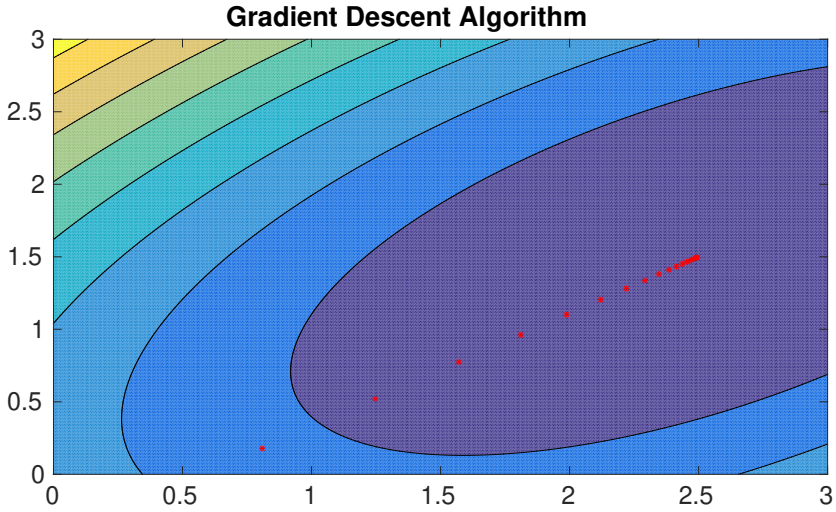

Figure 1: Gradient Descent Algorithm.

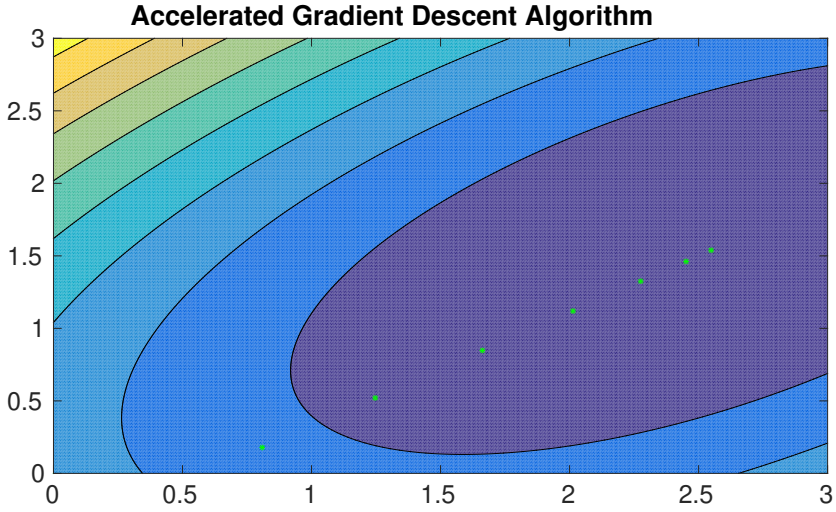

Figure 2: Accelerated Gradient Descent Algorithm.