6.336: Introduction to Numerical Modeling and Simulation

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October 20, 2020

Problem Definition

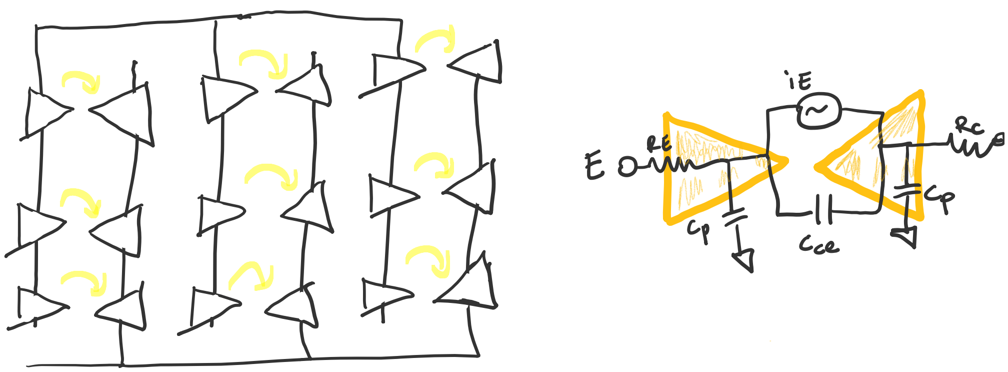

Figure 1: Problem definition: (left) top view of sensor array. (right) Circuit model of an individual bow-tie element

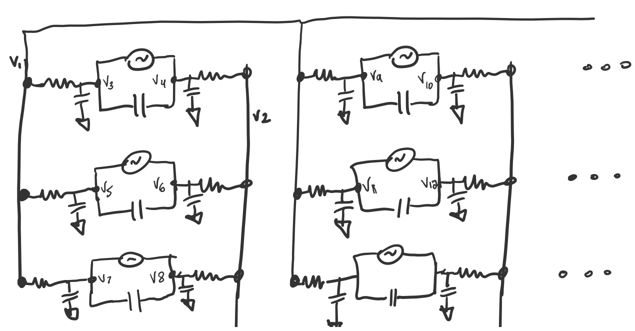

Figure 2: Problem definition: (left) top view of sensor array. (right) Circuit model of an individual bow-tie element

The system of interest is an array of bow-tie transistors detectors. When a burst of light hits a bow-tie, current is generated.

The system nodes are the the main bias wires and internal nodes inside the bow-tie devices. The nodal quantities are the nodal voltages. While the bias voltage on the main wires is real and measurable, the voltage on the internal nodes is an artifact created for convenience. The components in the system are the circuit branches. The quantities associated with each branch is the current flowing through it. The conservation equations governing the current through each junction are as follows.

For i is odd:

$$\frac{1}{R_E}(v_1 - v_i) - C_p\frac{dv_i}{dt} - C_{CE}\frac{d(v_i - v_{i+1})}{dt} - J(v_i - v_{i+1}) + i_{in} = 0 \tag{1a}$$

For i is even:

$$-\frac{1}{R_C}(v_i - v_2) - C_p\frac{dv_i}{dt} + C_{CE}\frac{d(v_{i-1} - v_i)}{dt} + J(v_{i-1} - v_i) + i_{in} = 0 \tag{1b}$$

Where $J(v)$ is the diode current defined by the Fowler–Nordheim tunneling for the nano-tip structure. In the circuit model it is modeled as a highly non-linear voltage dependent current source. C_p is the parasitic capacitance of each terminal, C_{CE} is the coupling capacitance between the collector and the emitter nodes, R_E and R_C are the emitter and collector resistances respectively. i_{in} is the short burst of input current to each triangle of the bow-tie caused by light incident.

The current in a capacitors with capacitance C positioned between node a and node b is described by:

$$i_C = C\frac{d(v_a - v_b)}{dt} \tag{2}$$

The current in a resistor of resistance R positioned between node a and node b is described by:

$$i_R = \frac{1}{R}(v_a - v_b) \tag{3}$$

The current in the voltage dependent current source, $J(v)$ is described by:

$$J = A\frac{a}{\phi}\frac{\beta^2 v^2}{d^2}\left(t(y) + \frac{\phi^{3/2}d}{e\beta vR}\psi(y)\right)\exp\left(-\frac{b\phi^{3/2}d}{\beta v}\left(V(y) + \frac{\phi d}{e\beta vR}\right)\right) \tag{4}$$

where ϕ, β, e, R, A, d, and b are physical parameters of the device. y is a function of the electric field (and therefore of the voltage too).

Mathematical formulation

The state vector v contains the nodal voltages in the system.

$$v = [v_1, v_2, \ldots, v_n]^T$$

The parameter vector contains the resistances of the collector and emitter, the capacitance between the collector and emitter, the parasitic capacitance, and all the physical parameters of the device (shape, material work function, gap).

$$p = [R_e, R_c, C_{CE}, C_p, R, \phi, b, \beta, d, A, a, e]^T$$

We can represent the voltage bias on v_1 as a current source in parallel with R_E. This choice leads to a change in equation 1a.

$$i_{bias} + i_{excitation} - \frac{v_i}{R_E} - C_{EC}\frac{d(v_i - v_{i+1})}{dt} - C_p\frac{dv_i}{dt} - J(v_i - v_{i+i}) = 0 \tag{5}$$

Where $i_{bias} = v_{bias}/R_E$. The sources are the voltage sources used to bias the main wires and the injected current that results from a photon hitting the sensor.

$$u = [v_{bias,1}/R_E, v_{bias,2}/R_C, i_{ex,1}, i_{ex,2}, \ldots, i_{ex,n}] \tag{6}$$

For odd i, the function is

$$\frac{dv_i}{dt} = \frac{1}{C_{EC} + C_p} \left(-\frac{v_i}{R_E} + C_{EC} \frac{dv_{i+1}}{dt} - J(v_i - v_{i+1}) + \tilde{I} \right) \tag{7}$$

Where $\tilde{I} = i_{excitation} + v_{bias}/R_E$.
For even i, the function is

$$\frac{dv_i}{dt} = \frac{1}{C_{EC} + C_p} \left(-\frac{v_i}{R_C} + C_{EC} \frac{dv_{i-1}}{dt} + J(v_{i-1} - v_i) + \tilde{I} \right) \tag{8}$$

Substituting in to get rid of the other derivatives we get

$$\frac{dv_i}{dt} = \frac{\frac{1}{C_{EC}+C_p} \left(-\frac{v_i}{R_E} - J(v_i - v_{i+1}) + \tilde{I}_i \right) + \frac{C_{EC}}{(C_{EC}+C_p)^2} \left(-\frac{v_{i+1}}{R_C} + J(v_i - v_{i+1}) \right) + \tilde{I}_{i+1}}{1 - C_{EC}^2/(C_{EC} + C_p)^2} \tag{9}$$

for i is odd. and

$$\frac{dv_i}{dt} = \frac{\frac{1}{C_{EC}+C_p} \left(-\frac{v_i}{R_C} + J(v_{i-1} - v_i) + \tilde{I}_i \right) + \frac{C_{EC}}{(C_{EC}+C_p)^2} \left(-\frac{v_{i-1}}{R_E} - J(v_{i-1} - v_i) \right) + \tilde{I}_{i-1}}{1 - C_{EC}^2/(C_{EC} + C_p)^2} \tag{10}$$

for i is even.