

Recall  $y \in \{0, 1\} = Y$

We had a true system (not a model but sometimes be “the model.”)

$$y = t(z_1, z_2, z_3)$$

where  $z_1$  = has sufficient funds,  $z_2$  = unforeseen emergency and  $z_3$  = criminal intentions

Problem:  $\{z_1, z_2, z_3\}$  is unobserved (impossible to obtain). What to do?

Next best thing: Try to define and collect information “related” to  $\{z_1, z_2, z_3\}$

Thus use what you have (or what is easily available).

Let’s pretend we got the resources to “define and collect.”

- $x_1$ : salary - measured by average salary
- $x_2$ : previous loan repayment - did they ever miss previous loan payment?  $\in [0, 1]$
- $x_3$ : historical criminal record - previous crime type?  
 $\{\text{no crime, infraction, misdemeanor, felony}\}$

**Definition 0.1.** Process Assessment: use as much as you got and whatever is cheaply available

Example: use age.

Let’s say we have  $x_1, x_2, x_3$ . The idea is  $\{z_1, z_2, z_3\}$  which contains some info in  $\{x_1, x_2, x_3\}$ .

Let  $\vec{x} = [x_1 \ x_2 \ x_3]$  where the LHS is an observation/ record/ object/ input/ independent variable and the RHS is features/ attributes/ characteristics/ regressors/ covariances/ predictors.

Note that  $\dim \vec{x} = p$  or  $d$ .

$$\vec{x} = [x_1 \ x_2 \ x_3] \in X$$

where  $X$  is the covariance space.

Spaces:  $x_1 \in \mathbb{R}$ ,  $x_2 \in [0, 1]$  - binary ordinary variable,  $x_3$  - categorized variables with 4 “levels”

Two Ideas:

First to do: code is numerical, such as  $x_3 \in [0 \ 1 \ 2 \ 3]$  - this should only be done if predictor is “ordinal.”

Next to do: Take  $x_3$  and turn it into  $x_{3a}$  (binary no crime),  $x_{3b}$  - binary infraction,  $x_{3c}$ ,  $x_{3d}$ . This increases  $p$  from 3 to 6 - more variables to think about.

So, it is impossible to get  $\{z_1, z_2, z_3\}$  but we do have  $[x_1 \ x_2 \ x_3]$

GOAL: Do the best we can to explain  $y$  by creating a model  $f$ , the approximation - the best

relationship we can get. Does  $y = f(x_1, x_2, x_3)$ ? No.

In fact,

$$\begin{aligned} y &\approx f(x_1, x_2, x_3) \\ y &= f(x_1, x_2, x_3) + \delta \end{aligned}$$

where  $\delta = t(\vec{z}) - f(\vec{x})$ , which comes from ignorance.

How do we get  $f$ ? First note there is no analytical solution.

Example:  $h(x) = x^2$ . Find  $\min h$ .

Instead, use an “empirical solution.” An example of this is using data to learn from data.

**Definition 0.2.** Supervised Learning: uses historical examples of record and their responses

In this case, it requires 3 ingredients:

$$\mathcal{D} := \left\{ \langle \vec{x}_1, y_1 \rangle, \langle \vec{x}_2, y_2 \rangle, \langle \vec{x}_3, y_3 \rangle \right\}$$

where  $\vec{x}_1$  is Bill’s characteristics and  $y_1$  is whether or not he paid back loan,  $\vec{x}_2$  is Jill’s, etc.

Let

$$X = \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_n \end{bmatrix} \in X^n, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \in Y^n$$

where  $\dim(x) = n \cdot p$  and  $\dim(\vec{y}) = n$ .