Exercise 1

Part A

Using the formula for finding the root of a quadratic equation: x = (-b ± sqrt(b**2 - 4*a*c))/(2*a)

```
Enter the value of 'a' in ax^2 + bx + c :0.001
Enter the value of 'b' in ax^2 + bx + c :1000
Enter the value of 'c' in ax^2 + bx + c :0.001
The roots of the quadratic equation 0.001x^2 + 1000.0x + 0.001 are -9.999894245993346e-07 and -999999.999999
```

Part B

```
Enter the value of 'a' in ax^2 + bx + c :0.001
Enter the value of 'b' in ax^2 + bx + c :1000
Enter the value of 'c' in ax^2 + bx + c :0.001
The roots of the quadratic equation 0.001x^2 + 1000.0x + 0.001 are -9.999894245993346e-07 and -999999.999999
The roots of the quadratic equation 0.001x^2 + 1000.0x + 0.001 using the new method are -1.000000000001e-06 and -1000010.5755125057

f(-9.999894245993346e-07) using method given in Part A is 1.0575401665491313e-08
f(-999999.999999) using method given in Part A is 7.247924804689582e-08
f(-1.000000000001e-06) using method given in Part B is 0.0
f(-1000010.5755125057) using method given in Part B is 10575.62534720993"""
```

The value of the function f(-1000010.5755125057) diverges from 0 by a very big margin because (-b + sqrt(b**2 - 4*a*c)) in the eqauation x = (2*c)/(-b ∓ sqrt(b**2 - 4*a*c)) is a very small number which causes trunctation error when dividing it, hence the root diverges.

Part c

Thus the roots obtained using the new modified method are (-1.000000000001e-06, -999999.999999) we check the function values
f(-999999.999999) using method given in Part A is 1.0575401665491313e-08
f(-1.000000000001e-06) using method given in Part B is 0.0

Exercise 2

Part A

The derivative of f at x = 1 computed using this method is 1.010000000000001.
The derivative of f at x = 1 computed analytically is 1. We can observe that the two values agree upto 2 significant figures. They don't agree perfectly because the analytic solution is the one at which Delta tends to 0, here our Delta is 0.01 which is considerably greater than 0. Hence they dont agree perfectly.

Part B

The derivative of f at x = 1 computed using this method with delta 1e-2 is 1.010000000000001
The derivative of f at x = 1 computed using this method with delta 1e-4 is 1.0000999999998899
The derivative of f at x = 1 computed using this method with delta 1e-6 is 1.0000009999177333
The derivative of f at x = 1 computed using this method with delta 1e-8 is 1.0000000039225287
The derivative of f at x = 1 computed using this method with delta 1e-10 is 1.000000082840371
The derivative of f at x = 1 computed using this method with delta 1e-12 is 1.0000889005833413
The derivative of f at x = 1 computed using this method with delta 1e-14 is 0.9992007221626509

The value closest to the actual value is when delta is 1e-8 with the derivative being 1.0000000039225287, we can the values deviating from 1 in an increasing fashion, this caused because of the truncation error which arises as a result of dividing with very small values namely 1e-10, 1e-12, 1e-14.

Exercise 3

Part A

The solution of this equation found using fixed point iteration is: 0.7968133630966887. Obtained in 13 iterations.

Part B

The solution of this equation found using fixed point iteration is: 0.7968133630966887
Root obtained at 0th iteration is 0.8144387474091372 with error 0.014517961973149333
Root obtained at 1th iteration is 0.7999207854359879 with error 0.0025356062279175643
Root obtained at 2th iteration is 0.7973851792080703 with error 0.00046660573598257955
Root obtained at 3th iteration is 0.7969185734720877 with error 8.66440000362326e-05
Root obtained at 4th iteration is 0.7968319294720515 with error 1.611561775200535e-05
Root obtained at 5th iteration is 0.7968158138542994 with error 2.998396626344378e-06
Root obtained at 6th iteration is 0.7968128154576731 with error 5.578996155163993e-07
The solution of this equation found using accelerated fixed point iteration is: 0.7968128154576731

Exercise 4

Roots computed using fixed point iteration are x = 0.636383990038212 and y = 1.999928171652484
Roots computed analytically are x = 2 and y = 0.4
Thus we can see that even after 100 iterations the roots don't converge to the analytic value hence this method is not very efficient.

Thus the solution otained by rearranging the equations:
Roots computed using fixed point iteration are x = 2.0 and y = 0.4
Roots computed analytically are x = 2 and y = 0.4
This completely agrees with the analytic solution

Exercise 5

Part A

The wavelength which emits maximum radiation is 0.00289788818359375 m
The value of Wein's constant obtained is 0.00289788818359375 K-m

Part B

The temperature of the surface of sun estimated using this method is 5772.685624688745 K
The literature value of sun's temperature is: T_Sun = 5778 K

```
Exercise 6

Energy of 0 state is: 0.3179326435890676 eV
Energy of 1 state is: 5.050013926619766 eV
Energy of 2 state is: 7.849075165687786 eV
Energy of 3 state is: 19.132874959416142 eV
The number of bound states for the given system is  4
```

Exercise 7

Part A

The rough values of the roots are 0.033, 0.17, 0.379, 0.619, 0.830, 0.965

Part B

The value of the six roots computed using Newton's method are:
0.033765242898423975
0.16939530676505093
0.38069040695770584
0.6193095930415944
0.8306046932349631
0.9662347571015738

Exercise 8

Part A

We plot the graph to get an idea of where the root of the equation lies and choose two values which seem to be reasonably close to the root and they bracket the root in order to reduce the computation time required to compute root, eventhough it can be done for any two arbitrary points.

The root of the above equation lies between 3.822e+8 and 3.825e+8

Using the Secant method:
The approximate solution obtained in 9 iteration is 326050065.3643446
The distance of the Lagrange point from the center of the Earth is 3.261e+08 m

Exercise 9

The values of the voltages are, V1 = 3.446971806506484 Volts and V2 = 2.829542290240274 Volts

The potential difference across the diode is V1-V2 = 0.6174295162662098 Volts. Thus this is close to the approximation, 0.6 Volts, used by engineers.

Exercise 10

Part B

Thus the potential of each of the nodes are:
V0 = 3.0 Volts
V1 = 1.6666666666666665 Volts
V2 = 3.3333333333333335 Volts
V3 = 2.0 Volts

Exercise 11

Part A

[3.   1.66666667 3.33333333 2.   ]. Which is the same result obtained in previous problem using gaussian elimination without pivoting:
V0 = 3.0 Volts
V1 = 1.6666666666666665 Volts
V2 = 3.3333333333333335 Volts
V3 = 2.0 Volts

Part B

[1.61904762 -0.42857143 -1.23809524 1.38095238]
Whereas without pivoting we get the out put "None".

Exercise 12

The solution to the given system of equations is [3.   1.66666667
3.33333333 2.   ], hence it is consistent with the solution
obtained in problem 10

Exercise 13

Amplitude of V1 = 1.7014390658777336 Volts and the phase is = -5.469094970111936 degrees
Amplitude of V1 = 1.4806053465364062 Volts and the phase is = 11.583418604687067 degrees
Amplitude of V1 = 1.8607693200562134 Volts and the phase is = -4.164672651865924 degrees

Exercise 14

Part A

V1 = 3.725490196078432 volts
V2 = 3.4313725490196085 volts
V3 = 2.745098039215687 volts
V4 = 2.2549019607843146 volts
V5 = 1.5686274509803926 volts
V6 = 1.2745098039215692 volts

Part B

Refer the file potential.txt

Exercise 15

Part A

```
A =  [[1 4 8 4]
 [4 2 3 7]
 [8 3 6 9]
 [4 7 9 2]]
Q =  [[ 0.10153462  0.558463    0.80981107  0.1483773 ]
 [ 0.40613847 -0.10686638 -0.14147555  0.8964462 ]
 [ 0.81227693 -0.38092692  0.22995024 -0.37712564]
 [ 0.40613847  0.72910447 -0.5208777  -0.17928924]]
R =  [[ 9.8488578   6.49821546 10.55960012 11.37187705]
 [ 0.          5.98106979  8.4234836  -0.484346  ]
 [ 0.          0.          2.74586406  3.27671222]
 [ 0.          0.          0.          3.11592335]]
QR =  [[1. 4. 8. 4.]
 [4. 2. 3. 7.]
 [8. 3. 6. 9.]
 [4. 7. 9. 2.]]
```

Hence A and QR are the same.

Part B

```
Eigenvalues of A are: [20.99999999999998, -7.999999999999982, -3.0,
0.9999999999999997]
Eigenvectors of A are:  [[ 0.43151698 -0.38357064 -0.77459666 -
0.25819889]
 [ 0.38357063  0.43151698 -0.2581989   0.77459667]
 [ 0.62330228  0.52740965  0.25819889 -0.51639778]
 [ 0.52740965 -0.62330227  0.51639779  0.25819889]]
```

Exercise 16

Part C

```
Calculating the first ten enrgy levels using 10x10 matrix method
The 0 state energy is : 5.836376902706757 eV
The 1 state energy is : 11.18109290629752 eV
The 2 state energy is : 18.662891578380517 eV
The 3 state energy is : 29.14419775590048 eV
The 4 state energy is : 42.655074845333516 eV
The 5 state energy is : 59.185257819333145 eV
The 6 state energy is : 78.72936018850429 eV
The 7 state energy is : 101.28548383437379 eV
The 8 state energy is : 126.8513857454442 eV
The 9 state energy is : 155.55532885350138 eV
```

Part D

```
Calculating the first ten enrgy levels using 100x100 matrix method
The 0 state energy is : 5.8363765020361384 eV
The 1 state energy is : 11.181091583031925 eV
The 2 state energy is : 18.66288970736844 eV
The 3 state energy is : 29.144188956249522 eV
The 4 state energy is : 42.65506572522523 eV
The 5 state energy is : 59.18520523633897 eV
The 6 state energy is : 78.72930836144975 eV
The 7 state energy is : 101.28485290329287 eV
The 8 state energy is : 126.8505534197182 eV
The 9 state energy is : 155.42570638517662 eV

The relative difference between the two methods are:
For the 0 state energy the difference between 10x10 and 100x100 method is
: 4.0067061846116303e-07 eV
For the 1 state energy the difference between 10x10 and 100x100 method is
: 1.3232655948769434e-06 eV
For the 2 state energy the difference between 10x10 and 100x100 method is
: 1.8710120771143402e-06 eV
For the 3 state energy the difference between 10x10 and 100x100 method is
: 8.79965095634816e-06 eV
For the 4 state energy the difference between 10x10 and 100x100 method is
: 9.120108288129813e-06 eV
For the 5 state energy the difference between 10x10 and 100x100 method is
: 5.258299417221224e-05 eV
For the 6 state energy the difference between 10x10 and 100x100 method is
: 5.182705453421477e-05 eV
For the 7 state energy the difference between 10x10 and 100x100 method is
: 0.0006309310809200497 eV
For the 8 state energy the difference between 10x10 and 100x100 method is
: 0.0008323257260087757 eV
For the 9 state energy the difference between 10x10 and 100x100 method is
: 0.12962246832475444 eV
```

Hence we can observe that 10x10 method agrees very closely with 100x100 method with only a considerable discrepancy at higher energy levels.

Part E

```
The value of integral psi**2 for the 0 state is 0.9999999984333351
The value of integral psi**2 for the 1 state is 0.999999999846115
```

The value of integral psi**2 for the 2 state is 0.999999976569353.

Hence the wavefunction is normalized.