Exercise 6:

a) We know that $p(x)$ is given by

$$p(x) = \frac{w(x)}{\int_a^b w(x)\,dx} = \frac{x^{-1/2}}{\int_0^1 x^{-1/2}\,dx} = \frac{1}{2\sqrt{x}}$$

as $w(x) = x^{-1/2}$.

Now in order to generate random numbers according to this distribution, we first find the cumulative density function

$$F(x) = \int_0^x p(x')\,dx' = \int_0^x \frac{1}{2\sqrt{x'}}\,dx' = \sqrt{x}$$

$$\Rightarrow F^{-1}(y) = y^2$$

$\therefore$ To generate a random number x according to the distribution $p(x)$ we can generate $y \in [0, 1]$ and perform the transformation:

$$x = F^{-1}(y) = y^2$$

Exercise 10:

a) Range of $\theta \in [0, \pi]$

Range of $\phi \in [0, 2\pi]$

$$\int_0^{\pi} p(\theta)\,d\theta = \int_0^{\pi} \frac{\sin\theta}{2}\,d\theta = -\frac{\cos\theta}{2}\Big|_0^{\pi} = \frac{2}{2} = 1$$

$$\int_0^{2\pi} p(\phi)\,d\theta = \int_0^{2\pi} \frac{1}{2\pi}\,d\phi = \frac{\phi}{2\pi}\Big|_0^{2\pi} = 1$$

Hence $p(\theta)$ and $p(\phi)$ are normalised.

b) Let $x = \int_0^{\theta} p(\theta)\,d\theta \Rightarrow 2x = 1 - \cos\theta$

$\therefore \theta = \cos^{-1}(1-2x)$

Let $y = \int_0^{\phi} p(\phi)\,d\phi = \dfrac{\phi}{2\pi}$

$\therefore \phi = 2\pi y$