

**Exercise 10 (a):**

By energy conservation we have

$$E = T + V \quad (1)$$

where  $T = \frac{1}{2}m(\frac{dx}{dt})^2$ , i.e., the kinetic energy and  $V$  is the potential energy.

Since we start at  $x = a$  and we have zero initial velocity  $E = V(a)$ . We also know that the oscillator can reach  $x = a$  to  $x = 0$  in  $T/4$  time, where  $T$  is the time period of oscillation. Thus:

$$\frac{2}{m}(E - V) = \frac{dx^2}{dt} \quad (2)$$

$$dt = -\frac{dx}{\sqrt{\frac{2}{m}(E - V)}} \quad (3)$$

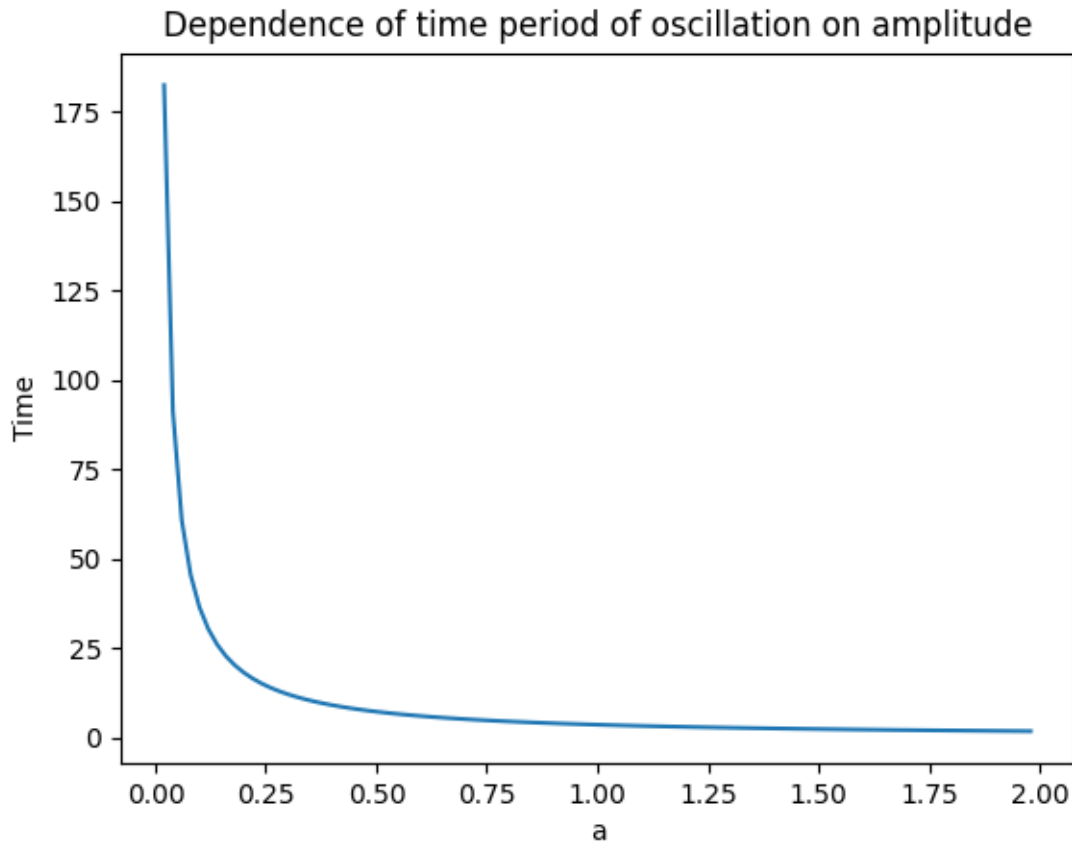
-ve sign is due to the fact that  $x$  goes from  $a$  to  $0$

$$\int_0^{\frac{T}{4}} dt = \int_a^0 -\frac{dx}{\sqrt{\frac{2}{m}(E - V)}} \quad (4)$$

$$\int_0^{\frac{T}{4}} dt = \int_0^a \frac{dx}{\sqrt{\frac{2}{m}(E - V)}} \quad (5)$$

$$\boxed{T = 4 \int_0^a \frac{dx}{\sqrt{\frac{2}{m}(E - V)}}} \quad (6)$$

**Exercise 10 (b):**



Plot of the dependence of time period on amplitude for anharmonic oscillator

**Exercise 10 (c):**

The restorative force grows as  $x^3$  with distance  $x$  but the distance travelled is just amplitude  $a$ , so the particle is pulled back to  $x = 0$  faster for larger  $a$ . Thus time decreases with  $a$ .