

Exercise 12 (a):

We are given with the expression for the intensity as a function of ω :

$$I(\omega) = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1} \quad (1)$$

Now we know that amount of thermal energy per second equals to $I(\omega)d\omega$. Thus:

$$dW = I(\omega)d\omega \quad (2)$$

$$W = \int_0^\infty I(\omega)d\omega \quad (3)$$

$$W = \int_0^\infty \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1} d\omega \quad (4)$$

Now we perform change of variables $\frac{\hbar\omega}{k_B T} \rightarrow x \Rightarrow \frac{\hbar}{k_B T} d\omega \rightarrow dx$

$$W = \int_0^\infty \frac{\hbar}{4\pi^2 c^2} \frac{k_B^3 T^3}{\hbar^3} \frac{x^3}{e^x - 1} \frac{k_B T}{\hbar} dx \quad (5)$$

Thus we have the required equation:

$$\boxed{W = \frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{e^x - 1} dx} \quad (6)$$