
PROBLEM 1

The exact solution is $x - 2 + 3 \cdot \exp(-x/2)$

Euler's Method (h = 1)

[1. 0.5 0.75 1.375]

Euler's Method (h = 0.5)

[1. 0.75 0.6875 0.765625 0.94921875 1.21191406
1.53393555]

Euler's Method (h = 0.25)

[1. 0.875 0.796875 0.75976562 0.75854492 0.78872681
0.84638596 0.92808771 1.03082675 1.1519734 1.28922673 1.44057339
1.60425171]

Euler's Method (h = 0.125)

[1. 0.9375 0.88671875 0.84692383 0.81742859 0.7975893
0.78680247 0.78450232 0.79015842 0.80327352 0.82338143 0.85004509
0.88285477 0.92142635 0.9653997 1.01443722 1.06822239 1.12645849
1.18886734 1.25518813 1.32517637 1.39860285 1.47525267 1.55492438
1.6374291]

Modified Euler's Method (h = 1)

[1. 0.875 1.171875 1.73242188]

Modified Euler's Method (h = 0.5)

[1. 0.84375 0.83105469 0.93051147 1.11758709 1.37311491
1.68212103]

Modified Euler's Method (h = 0.25)

[1. 0.8984375 0.83807373 0.81408072 0.82219626 0.85865763
0.92014307 1.00372005 1.10679973 1.22709664 1.36259313 1.51150799
1.67226878]

Modified Euler's Method (h = 0.125)

[1. 0.94335938 0.89771652 0.86240556 0.83680093 0.82031493
0.81239547 0.81252387 0.82021286 0.83500466 0.85646922 0.88420253
0.91782503 0.95698016 1.00133292 1.05056862 1.10439162 1.16252416
1.22470531 1.29068995 1.36024778 1.43316247 1.50923076 1.58826171
1.67007594]

Taylor order 4 (h = 1)

Taylor order 4 (h = 0.5)

Taylor order 4 (h = 0.25)

Taylor order 4 (h = 0.125)

Runge-Kutta Order 4 (h = 1)

[1. 0.8203125 1.10451253 1.67018599]

Runge-Kutta Order 4 (h = 0.5)

[1. 0.83642578 0.81962848 0.9171423 1.1036826 1.35955749
1.66943076]

Runge-Kutta Order 4 (h = 0.25)

[1. 0.89749146 0.83640367 0.81186958 0.81959403 0.85578655
0.91710206 1.00058853 1.10364082 1.22395988 1.35951682 1.50852114
1.66939275]

Runge-Kutta Order 4 (h = 0.125)

[1. 0.94323921 0.89749075 0.86208742 0.83640243 0.81984698
0.81186794 0.81194569 0.8195921 0.8343486 0.85578442 0.88349487
0.9170998 0.95624208 1.00058621 1.04981703 1.10363847 1.16177241
1.22395755 1.28994845 1.35951453 1.43243919 1.50851893 1.58756259
1.66939061]

4-step Adam-Bashforth (h = 0.125)

[1. 0.94323921 0.89749075 0.86208742 0.83640329 0.81984853
0.81187016 0.81194847 0.81959536 0.83435228 0.85578845 0.8834992
0.91710437 0.95624685 1.00059114 1.04982208 1.10364361 1.16177761
1.22396278 1.2899537 1.35951977 1.4324444 1.50852409 1.5875677
1.66939565]

3-step Adam-Moulton (h = 0.125)

[1. 0.94323921 0.89749075 0.86208733 0.83640226 0.81984675
0.81186765 0.81194536 0.81959172 0.83434819 0.85578397 0.8834944
0.91709931 0.95624157 1.00058568 1.0498165 1.10363793 1.16177186
1.22395701 1.28994791 1.359514 1.43243865 1.5085184 1.58756207
1.6693901]

PROBLEM 2

[0. 3. -5.]

Runge-Kutta Method

[[3.00000000e+00 -5.00000000e+00]
[2.52564583e+00 -4.48185417e+00]
[2.10402783e+00 -3.95059998e+00]
[1.73506269e+00 -3.43236152e+00]
[1.41653369e+00 -2.94411600e+00]
[1.14488509e+00 -2.49601257e+00]
[9.15809453e-01 -2.09320696e+00]
[7.24672260e-01 -1.73729612e+00]
[5.66809954e-01 -1.42742596e+00]
[4.37730897e-01 -1.16113448e+00]
[3.33243022e-01 -9.34982936e-01]
[2.49527116e-01 -7.45018933e-01]
[1.83170631e-01 -5.87108014e-01]
[1.31173663e-01 -4.57163346e-01]
[9.09360219e-02 -3.51297741e-01]
[6.02321666e-02 -2.65917363e-01]
[3.71790206e-02 -1.97772510e-01]
[2.02003485e-02 -1.43977516e-01]
[7.99028267e-03 -1.02009168e-01]
[-5.22220602e-04 -6.96907574e-02]
[-6.20684405e-03 -4.51671744e-02]
[-9.76260375e-03 -2.68750163e-02]
[-1.17454281e-02 -1.35105761e-02]
[-1.25925166e-02 -3.99772813e-03]
[-1.26435406e-02 2.54293781e-03]
[-1.21588569e-02 6.82288051e-03]
[-1.13349718e-02 9.41178031e-03]
[-1.03175270e-02 1.07609020e-02]
[-9.21209177e-03 1.12236149e-02]
[-8.09304189e-03 1.10731655e-02]
[-7.01079127e-03 1.05178789e-02]

```

[-5.99762285e-03  9.71401264e-03]
[-5.07234127e-03  8.77651024e-03]
[-4.24394469e-03  7.78790429e-03]
[-3.51448861e-03  6.80561486e-03]
[-2.88129036e-03  5.86787224e-03]
[-2.33860135e-03  4.99847459e-03]
[-1.87885374e-03  4.21056928e-03]
[-1.49357081e-03  3.50962480e-03]
[-1.17401450e-03  2.89573791e-03]
[-9.11630158e-04  2.36540066e-03]
[-6.98337176e-04  1.91283278e-03]
[-5.26704303e-04  1.53096804e-03]
[-3.90040552e-04  1.21216823e-03]
[-2.82425739e-04  9.48725166e-04]
[-1.98699322e-04  7.33200142e-04]
[-1.34421743e-04  5.58640337e-04]
[-8.58189304e-05  4.18703980e-04]
[-4.97177814e-05  3.07719156e-04]
[-2.34782568e-05  2.20695739e-04]
[-4.92598244e-06  1.53305409e-04]]

```

PROBLEM 3

```
[0.  1.25  0. ]
```

Runge-Kutta Method

```

[[ 1.25      0.      ]
 [ 1.22013136 -0.29733479]
 [ 1.13207262 -0.57956093]
 [ 0.99012213 -0.83458601]
 [ 0.80056866 -1.0546542 ]
 [ 0.57084402 -1.23592929]
 [ 0.3088503  -1.3773402 ]
 [ 0.02252151 -1.47947709]
 [-0.28042449 -1.54380686]
 [-0.59260866 -1.57219025]
 [-0.90703946 -1.56660848]
 [-1.21712077 -1.52901821]
 [-1.51663899 -1.4612801 ]
 [-1.79974024 -1.36512815]
 [-2.06090391 -1.2421609 ]
 [-2.29491568 -1.09384378]
 [-2.49684163 -0.92151703]
 [-2.6620042  -0.72640577]
 [-2.78596022 -0.5096306 ]
 [-2.86448105 -0.27221802]
 [-2.89353475 -0.01511001]]
[0.  0.  1.]

```

Runge-Kutta Method

```

[[ 0.      1.      ]
 [ 0.22013463 1.20260394]
 [ 0.48207145 1.42032999]

```

```
[ 0.79010835  1.66526909]
[ 1.15053425  1.94517197]
[ 1.57078146  2.26387079]
[ 2.05875251  2.62243451]
[ 2.62238182  3.02027208]
[ 3.26938753  3.45591635]
[ 4.00714888  3.92750655]
[ 4.84265757  4.43306152]
[ 5.78250979  4.97062468]
[ 6.83291924  5.53833544]
[ 7.99973984  6.13445985]
[ 9.28849224  6.75739943]
[10.70439079  7.40568876]
[12.25236942  8.07798763]
[13.93710573  8.77307095]
[15.76304289  9.48981812]
[17.73440954 10.22720267]
[19.85523765 10.98428261]]
```

Finite-Difference Method

```
[[ 1.31303543]
 [ 1.32296541]
 [ 1.28347176]
 [ 1.19995949]
 [ 1.07884802]
 [ 0.92700265]
 [ 0.75135659]
 [ 0.55869015]
 [ 0.35551604]
 [ 0.14802914]
 [-0.0579069 ]
 [-0.25675349]
 [-0.4432809 ]
 [-0.61254787]
 [-0.75987995]
 [-0.88084818]
 [-0.97124899]
 [-1.02708562]
 [-1.04455117]
 [-1.02001322]]
```