Memory Polynomial Model for Digital Predistortion of Broadband Solid-State Radar Amplifiers

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***Abstract*—After a discussion on the potential benefits of the use of digital predistortion (DPD) on high power amplifiers (HPA) in phased array radar systems, current work is presented on amplifier and predistorter modeling towards this goal. Simulated HPA performance results utilizing DPD demonstrate a significant decrease in spectral spreading versus non-predistorted waveforms, and testing DPD on a physical HPA verifies these findings. The demonstrated DPD method has great potential for application in coded waveform phased array and MIMO radar systems.**

I. Introduction

In radar systems utilizing coded waveforms, the fidelity of the transmitted signal is crucial to the ultimate fidelity of the analyzed results of the received signal. With this in mind, the transmit chain of a radar system, specifically the main high power amplifier (HPA), must be fully characterized so that the final transmitted waveform can be known relative to the desired output waveform. However, physical HPAs exhibit non-linear behavior over their input power range, which becomes more extreme as the amplifier nears its compression region where the power added efficiency of the amplifier is maximized [1,2]. In addition, this non-linear behavior also varies as a function of input frequency. As a result, HPA modeling can take several forms. The Volterra series is a convenient and "compact" mathematical model that is capable of modeling systems with both non-linear behavior and memory effects. Non-linear variations over frequency can be viewed, and thus modeled, as a result of memory effects. With certain assumptions about the system to be modeled, even more compact forms of the Volterra model can be applied. One of these such models, convenient for use with non-linear and memory dependent systems with complex input and output data, is the memory polynomial (MP) model. Utilizing the MP model, it is possible to accurately model non-linear behavior and non-linear variation across input frequency.

While knowledge of the actual, distorted output waveform is useful in and of itself, it would be better to use this knowledge to alter the input waveform so that the final distorted output waveform is equal to the desired waveform. The inversion of the HPA model for the purpose of linearizing and equalizing the total system is known generically as predistortion [3]. *Digital* alteration of an input signal so that the output distorted signal of a system equals the true desired signal is known as digital predistortion (DPD). The model governing DPD for a given radar transmit chain is matched to each individual HPA. Seeing that the DPD must be a non-linear function of input power and input frequency, DPD can also be modeled with the MP model. DPD allows a given amplifier to output signals that appear to be linearly amplified with minimal distortion, even while the amplifier is operated in its compression region, thus maximizing power added efficiency and significantly reducing spectral regrowth.

II. Application In Phased Array Radar

Recent advancements in the fields of solid-state amplifiers have led to the practical implementation of the active array architecture. This is in contrast to a traditional passive array, where all of the antenna's elements are connected to one power amplifier [4-8]. Therefore, with each antenna element having its own HPA, maximizing the utility and output of HPAs on an element-by-element basis is crucial to maximizing the capability of the overall radar system. Digital predistortion would be ideal for use in this type of architecture, as allowing each amplifier to operate in its compression region without distortion of the output waveform leads to the maximization of each amplifier's power added efficiency for waveforms with a constant modulus while giving similar output characteristics of a larger amplifier for waveforms requiring linear amplification. By utilizing smaller amplifiers to their full potential, the benefits of lower cost, lower weight, smaller hardware footprint, easier heat management, and smaller and less expensive power supplies and other associated hardware is quickly recognized. When these benefits are multiplied over the number of antenna elements and transmit chains in a given radar system, it can be seen that DPD can have a tremendous effect on the utility of the overall radar system without making drastic and expensive changes to the system's hardware to achieve similar performance.

While digital predistortion has been utilized in communication systems, as in [9-11], it does not have widespread use in phased array radar applications. This is especially true for wideband radar systems, which is the primary focus of this paper. Whereas DPD in wideband radar applications must apply over a much larger bandwidth than in communication systems, adaptive predistortion of unpredictable and constantly changing waveforms is not needed. Radar systems only have a few discrete waveforms to be used during operation, and the waveform to be transmitted is always known

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prior to transmission. As a result, adaptive predistortion is not needed at every transmit pulse. Instead, the predistorted version of the input waveforms can be saved into memory to be played out during operation. As a result, the desired waveform is produced at the output of the amplifer even though this step only requires the same amount of computational power as playing the non-predistorted waveform from memory. With this in mind, it can be seen that while conservative implementation of DPD on an active phased array radar requires digitization at the element level, it does not require significant excess computational power. This makes it attractive for modern and emerging architectures.

III. Technical Approach to Modelling Amplifier and Predistorter

A. Volterra Series and MP Model

As stated in the introduction, the Volterra series is useful for modeling systems with both non-linearities and memory effects, and it is ideal for modelling the output of HPAs and their associated predistortion models [12-15]. The general form of the discrete Volterra series is given by

$$y(n) = \sum_{k=1}^{K} y_k(n) \tag{1}$$

where

$$y_k(n) = \sum_{m_1}^{M-1} \dots \sum_{m_k}^{M-1} h_k(m_1, \dots, m_k) \prod_{l=1}^{k} x(n - m_l) \tag{2}$$

where $y(n)$ is the output sample, $x(n)$ is the input sample, K is the order of non-linearity of the system, M is the order of memory of the system, and $h_k(m_1, \dots, m_k)$ is a coefficient with set values as a function of k and m_1 though m_k. The general form of the Volterra series is capable of modeling non-linear systems with memory effects due to the extensive number of coefficients paired with every combination of input sample and delayed input sample combinations within the bounds of the specified non-linear order and memory order of the model. The general form of the Volterra series is therefore able to model systems with both large non-linearities and drastic memory effects, and higher precision and more accurate modeling can be provided by simply raising the non-linear and memory orders of the model. However, the number of coefficients, and thus computational complexity in calculating the coefficients, increases at a substantial rate as either the non-linear order or the memory order of the model is increased. With this in mind, many simplifications of the full Volterra model, with less coefficients and easier calculations, have been devised. One of these simplified Volterra based models that has had previous success modelling physical amplifiers at complex baseband is the MP model [16-19]. The MP model is given by

$$y_{MP}(n) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} h_{km} x(n-m) |x(n-m)|^k \tag{3}$$

where $y_{MP}(n)$ is the output sample, $x(n)$ is the input sample, K is the order of non-linearity of the system, M is the order of memory of the system, and h_{km} is a coefficient with set values as a function of k and m.

Due to the MP model being composed of a summation of coefficients paired with varying combinations of input terms that are both delayed and not delayed, the MP model output can be represented efficiently in matrix form as

$$\boldsymbol{y} = \boldsymbol{X}\Theta_{\boldsymbol{MP}}, \tag{4}$$

which can be expanded to be shown as

$$\begin{bmatrix} y(n) \\ \vdots \\ y(\mathrm{n}+\mathrm{N}) \end{bmatrix} = \tag{5}$$

$$\begin{bmatrix} x(n) & \cdots & x(n-m)|x(n-m)|^k \\ \vdots & \ddots & \vdots \\ x(n+N) & \cdots & x(n+N-m)|x(n+N-m)|^k \end{bmatrix} \begin{bmatrix} h_{00} \\ \vdots \\ h_{km} \end{bmatrix}$$

where $\boldsymbol{y}$ is an [A x 1] array containing the calculated outputs of the MP model, $\Theta_{\boldsymbol{MP}}$ is a [B x 1] array containing all the coefficients h_{km}, and $\boldsymbol{X}$ is an [A x B] matrix called the delay matrix. It should be noted that B is equal to the number of coefficients for a given non-linear order and memory order of the MP model, while $A = length(input\ array) - (M-1)$, which insures that any generated output in $\boldsymbol{y}_{MP}$ was calculated with populated values for the necessary delayed input terms. The delay matrix is composed of the varying combinations of input terms that are both delayed and not delayed associated with the B coefficients for each of the A output samples to be created. This means that the delay matrix can be created given only the input sample array and the order of non-linearity and memory order of the MP model to be used. The [B x 1] model coefficient array $\Theta_{\boldsymbol{MP}}$ contains the unknown coefficients h_{km} that collectively capture HPA behavior over the power and frequency ranges of interest. To estimate the coefficients in y $\Theta_{\boldsymbol{MP}}$, one requires high fidelity measurements of the output of the amplifier excited by a strategically chosen and well known input signal. This pair of data is known as the calibration data, and it should be chosen so that the input signal excites across the entire bandwidth and power ranges over which the amplifier is to be modeled. A good way to capture as much of the non-linearity effects and memory effects as possible is to generate a random signal spanning the desired power range before filtering the signal to the desired bandwidth. This random signal approach creates many combinations of input power and frequencies that help to excite the amplifier in as many different ways as possible, leading to more behavior of the amplifier being recorded. As a result, when using the frequency filtered random signal approach to generating calibration data, the longer the signal is in time, the better the system overall will be characterized. With this in mind, it is useful to pick lengthy calibration data, but the length of the calibration input signal is practically limited by the computational power needed to analyze the calibration data when calculating the MP model coefficients.

Once a good calibration dataset is acquired for an HPA and the order of non-linearity and order of memory for the MP model is chosen, the coefficient array $\Theta_{\boldsymbol{MP}}$ can be calculated. The coefficients are found by using the well-known method of minimizing the least-squares error through the Moore-Penrose Pseudoinverse, where the error is defined as the difference between the calculated calibration output samples and the measured calibration output samples [20]. The Moore-Penrose Pseudoinverse is executed and the coefficient array Θ_{MP} is found by

$$\Theta_{MP} = \left(X_{cal}^H X_{cal}\right)^{-1} X_{cal}^H y_{cal} \tag{6}$$

where y_{cal} is the measured output signal array of the calibration data, X_{cal} is the delay matrix formed by the calibration data input signal, and the superscript ^{H} is the conjugate transpose operator. Once the coefficients of the MP model have been found, if an adequate order of non-linearity and order of memory were selected, and the range of input power levels and range of frequencies present in an input signal array fall within those represented by the calibration data input signal, then the realistic output of the HPA can be simulated. It should be noted that if Θ_{MP} remains unchanged throughout simulations following calibration, the method used to assemble the calibration data delay matrix given the calibration data's input signal should be used to assemble all future delay matrices given a desired input signal. This will insure that the MP model coefficients for the given model are always correctly paired with their associated delayed and non-delayed input signal combinations.

It is desired not only to create a realistic model of a HPA, but also a model of an associated predistortion function that can be used in conjunction with the HPA to create an overall linearly behaving system. More specifically, when the DPD is paired with the HPA, the desired signal is input into the DPD, and this signal is then reproduced at the output of the HPA, multiplied only by the gain corresponding with the linear region of the HPA. Therefore, the model of the DPD is basically an inverse of the model of the HPA, and because the HPA exhibits both non-linearities and memory effects, the DPD will need to account for both non-linearities and memory effects. However, because the HPA is successfully modeled by the MP model, this also means that the HPA's associated DPD can also be successfully modeled using the MP model, given by

$$y = X\Theta_{PD} \tag{7}$$

where y is the DPD output signal array, X is the delay matrix formed by the DPD input signal, and Θ_{PD} is an array containing all the MP model coefficients of the DPD. The order of non-linearity and order of memory for the DPD may differ from the order of non-linearity and order of memory used in the model of the HPA. The coefficients of the MP model for the DPD can be calculated similarly to the method used to find Θ_{MP} using the Moore-Penrose Pseudoinverse and a set of calibration data. In order to calculate the values of the coefficient array Θ_{PD} so that the DPD will be a match with the given amplifier, the same set of calibration data is used, but it is scaled and used in reverse order. The calibration data input signal array x_{cal} remains unscaled, but the calibration data output signal y_{cal} is rescaled so that the maximum magnitude equals the magnitude that when multiplied by the HPA's linear region gain equals the maximum magnitude of the measured calibration data input signal. The Moore-Penrose Pseudoinverse is executed and the coefficient array Θ_{PD} is found by

$$\Theta_{PD} = \left(Y_{cal}^H Y_{cal}\right)^{-1} Y_{cal}^H x_{cal} \tag{8}$$

where x_{cal} is the unscaled input signal array of the calibration data, Y_{cal} is the delay matrix formed by the rescaled calibration data output signal y_{cal}. Once the coefficients of the MP model for the DPD have been found, if an adequate order of non-linearity and order of memory were selected, and the range of input power levels and range of frequencies present in the input signal array fall within those allowed by the calibration data, then the necessary predistorted signal can be simulated. It should be noted that the allowed range of frequencies for the input signal is equal to the range of frequencies represented by the calibration data, but the allowed range of amplitudes is limited by the maximum magnitude of the rescaled calibration data output signal y_{cal} that was used to solve for Θ_{PD} , which was previously decided to be the magnitude that, when multiplied by the HPA's linear region gain, equals the maximum magnitude of the calibration data input signal. It should also be noted that in the way the DPD coefficient array Θ_{PD} was found, these coefficients are actually the necessary coefficients for a MP model post-inverse filter. However, due to the inherent quality of the Volterra series that the pth order post-inverse of a Volterra series is equal to the pth order pre-inverse of a Volterra series, the coefficients found for the post-inverse model can be used as the coefficients of a pre-inverse model instead [21]. Therefore, by using two different realizations of the MP model simplification of the Volterra series and a single set of calibration data, it is possible to not only successfully model the output of a HPA given an input signal, but it is also possible to find the necessary DPD model that, when used in series with the HPA, will make the overall transmit chain behave as a linear system.

B. *Weighting for Numerical Stability*

When the specified order of non-linearity of the HPA or DPD model is large, numerical instabilities can begin to arise when solving for the model coefficients by executing the Moore-Penrose pseudo-inverse. While other approaches have been proposed to make the pseudo-inverse more numerically stable for large order polynomials by modifying the Memory Polynomial model itself [22], it was decided in this context to normalize the delay matrices of both the HPA model and the DPD model by dividing each individual term of the delay matrix by the maximum magnitude of the calibration input signal for that particular delay matrix to the power corresponding with the order of non-linearity of the particular term in the delay matrix. This can be represented by introducing a weighting matrix W, substituting X_w for X in the previous equations, where

$$X_w = XW. \tag{9}$$

The elements $W_{i,j}$ of the square weighting matrix W of dimensions [B x B] are represented by

$$W_{i,j} = \begin{cases} \dfrac{1}{\left(x_{cal,max}\right)^{K_j}} & for\ i = j \\ 0 & for\ i \neq j \end{cases} \tag{10}$$

where $x_{cal,max}$ is the maximum instantaneous magnitude present in the calibration data input signal, and K_j is the order of non-linearity associated with the $\boldsymbol{j}^{\text{th}}$ coefficient term in the selected MP model.

This results in each individual term of the delay matrix being scaled so that, for a given input time signal, the range of typical values spanned by the varying non-linear terms is reduced by several orders of magnitude. As a result, when this normalized delay matrix is used in the Moore-Penrose Pseudo-inverse for finding the coefficients of the model, the least-squares solution for the coefficients is produced with a much more balanced importance being placed on each term as the order of non-linearity associated with that term changes. This normalization process effectively creates a weighted least-squares approach, with more accurate and numerically stable HPA simulation and DPD results than previous approaches, such as [15].

IV. Predistortion of Digitally Coded Waveforms

Simulations and tests were conducted on a representative waveform that spans the power and bandwidth of a possible wideband radar waveform, and this waveform was both non-predistorted and predistorted utilizing the methods described in Section III. This was done to test the ability of the DPD to match the final distorted output waveform to the desired output waveform, and to quantify the effect on spectral spreading behavior while the amplifier is operating in its compression region. Through trial and error, it was found that $K = 9$ and $M = 5$ for the MP DPD orders provided optimal performance for the given test datasets.

A. Band-Limited Noise Waveform Simulation

A simulated system was analyzed using calibration data measured through a Mini Circuits ZFL-500HLN amplifier and a test signal composed of a randomly generated complex signal filtered to a bandwidth of 20 MHz, a pulse width of 500 μs, and a maximum input voltage equal to the maximum allowed magnitude as specified by analysis of the calibration data. Using an HPA model with $K = 9$ and $M = 5$, it was seen that the digitally predistorted signal was nearly an exact match to the desired output signal, whereas the non-predistorted signal, which was equal to the DPD input signal and scaled so that its maximum magnitude corresponded with the predistorted signal's maximum magnitude, experienced non-linear distortion and compression across the span of the waveform.

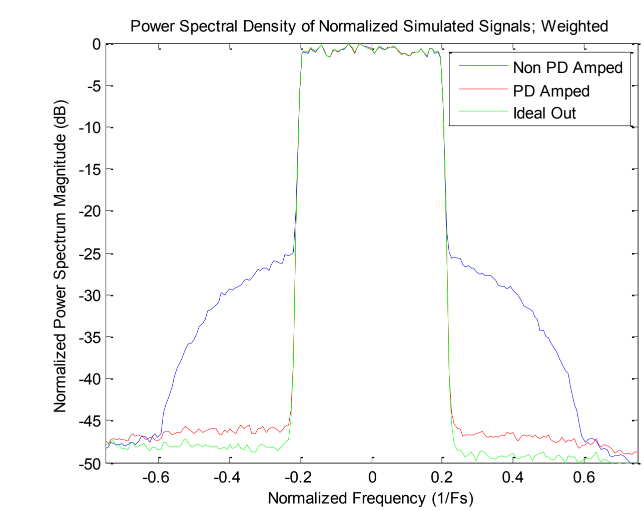

Figure 1. Power Spectral Density of Simulated Predistorted and Non-Predistorted Signals after HPA Distortion

It was also seen that the spectral spreading of the non-predistorted output signal was much larger than that of the predistorted signal after being distorted by the HPA. This is seen in Fig. 1. With these results in mind, it can be seen that using the DPD model on the input waveform leads to a much more desirable simulated result, closely matching the ideal output signal, and with a much smaller degree of spectral spreading than would be present without digital predistortion.

B. Band-Limited Noise Waveform Test

A 20 MHz band-limited randomly generated noise signal centered at 150 MHz was amplified through a Mini Circuits ZFL-500HLN amplifier and tested both with and without DPD. Using a pulse width of 500 μs and a maximum input voltage equal to the maximum allowed magnitude as specified by analysis of the calibration data, it can be seen in Fig. 2 that the measured predistorted signal closely matches the ideal linear gain across the output power range. The large vertical width of the measured non-predistorted signal for a given input power is due to the wideband nature of the waveform in conjunction with the frequency dependency of the amplifier. The vertical narrowing of the predistorted signal is due to frequency dependency correction.

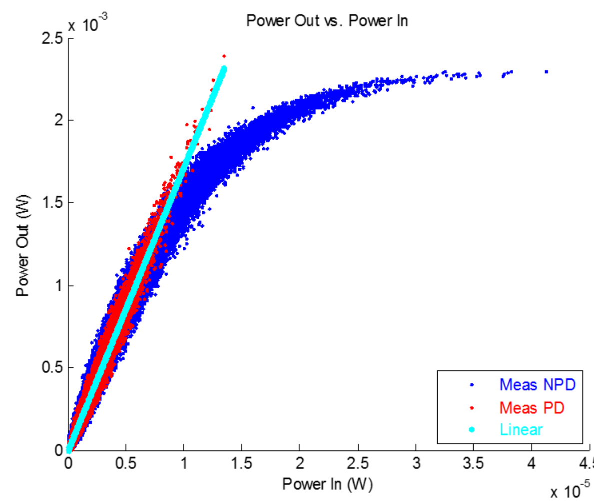

Figure 2. Power Out vs. Power In of Measured Predistorted and Non-Predistorted Signals after HPA Distortion

It is seen in Fig. 3 that the digitally predistorted signal had much lower spectral spreading than the non-predistorted signal. It can also be seen in Fig. 3 that the magnitude of the predistorted signal in the spectrum outside the main signal band, although roughly constant in magnitude, is a few dB larger than the magnitude of the non-predistorted signal at frequencies far from the center frequency. This roughly constant magnitude outside the main signal is actually the noise floor of the testing equipment, and it makes sense that the noise floor appears higher for the predistorted case on a normalized power spectral density graph as a predistorted signal, when having the same maximum magnitude as the non-predistorted version of the same signal, will always have a lower average power than the non-predistorted signal.

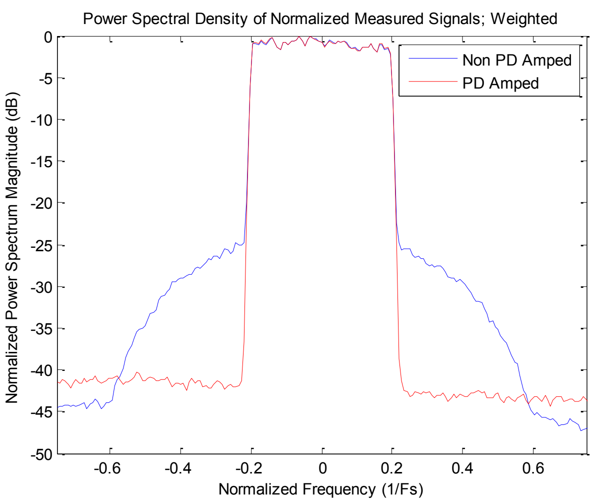

Figure 3. Power Spectral Density of Measured Predistorted and Non-Predistorted Signals after HPA Distortion

V. Conclusion

In conclusion, it has been shown that the Volterra series, more specifically the MP model, is able to accurately model the non-linear effects and memory effects of both high power amplifiers and their associated predistortion models in wideband radar systems. The predistortion model can be implemented digitally in a system, and when used in conjunction with phased array radar architectures utilizing solid-state amplifiers and waveform generators at every antenna element, significant improvements in radar spectral performance can be achieved. DPD allows the radar system to linearly amplify and transmit waveforms over the entire HPA's output power range, maximizing power added efficiency, and have an overall performance similar to that of a much larger and more costly radar system that does not utilize digital predistortion.

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