Part II - Matlab

Part - 1

a

$$\phi(x,y) = A\cos(2\pi\omega_x x)\cos(2\pi\omega_y y)$$

$$A = 5000, \omega_x = 5, \omega_y = 3$$

The given signal has a maximal value of

$$\phi_H = \phi(0,0) = A\cos 0\cos 0 = A$$

And a minimal value of

$$\phi_L = \phi\left(\frac{1}{2\omega_x}, 0\right) = A\cos(\pi)\cos 0 = -A$$

and therefore the range of values is of size: $\phi_H - \phi_L = 2A = 10,000$.

$$Energy\left(\phi'_x\right) = \int_0^1\int_0^1 \left(\frac{\partial}{\partial x}\phi(x,y)\right)^2 dxdy$$

$$= \int_0^1\int_0^1 \left(\frac{\partial}{\partial x}A\cos(2\pi\omega_x x)\cos(2\pi\omega_y y)\right)^2 dxdy$$

$$= \int_0^1\int_0^1 \left(-2\pi\omega_x A\sin(2\pi\omega_x x)\cos(2\pi\omega_y y)\right)^2 dxdy$$

$$4\pi^2\omega_x^2 A^2 \int_0^1 \sin^2(2\pi\omega_x x)\,dx \int_0^1 \cos^2(2\pi\omega_y y)\,dy$$

$$= 4\pi^2\omega_x^2 A^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \pi^2\omega_x^2 A^2$$

$$\Downarrow$$

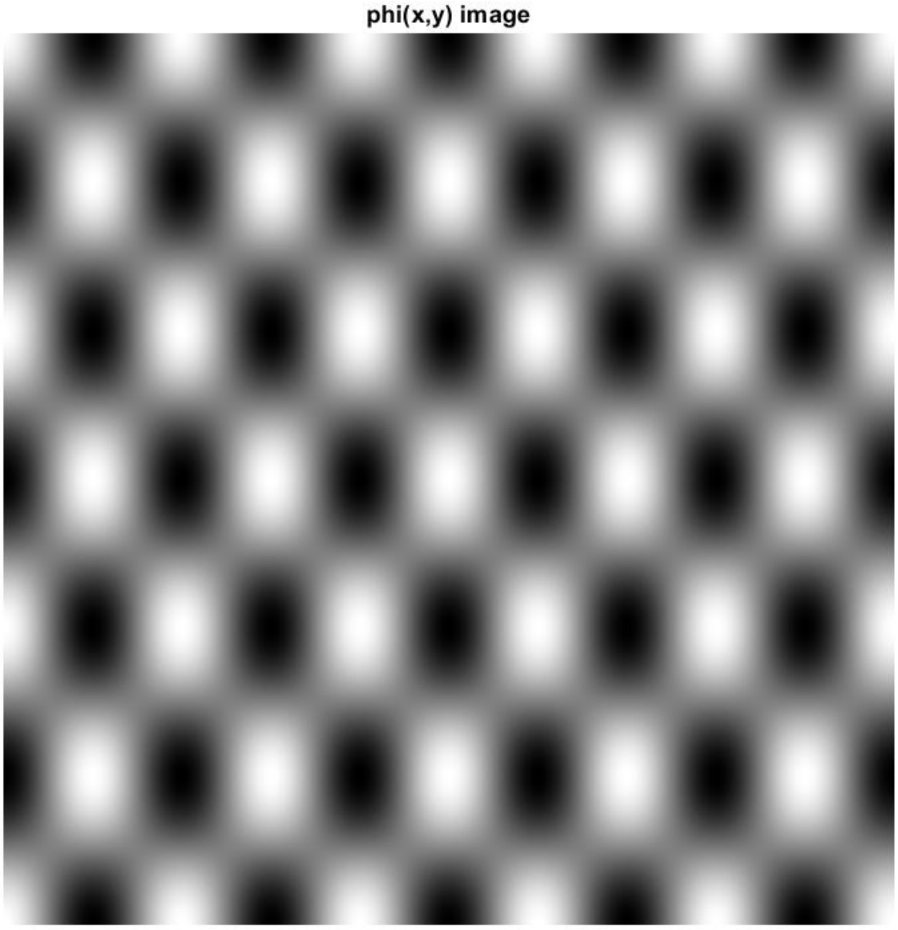
$$Energy\left(\phi'_x\right) = \pi^2\omega_x^2 A^2\left(= 6.1685\cdot 10^9\right)$$

In analog way we will get for $Energy\left(\phi'_y\right)$:

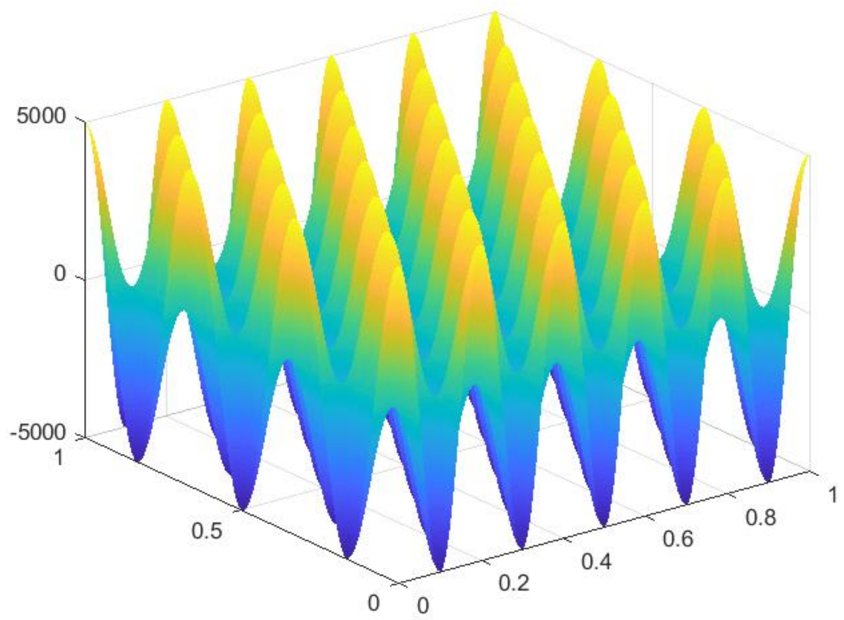
$$Energy\left(\phi'_y\right) = \pi^2\omega_y^2 A^2\left(= 2.2206\cdot 10^9\right)$$

b

The signal as image histogram:



This signal as graph:



c

The values we got from the numerically calculation is:

Value range: 2A(=10,000)

$Energy\left(\phi'_x\right) = 6.1803 \cdot 10^9$

$Energy\left(\phi'_y\right) = 2.225 \cdot 10^9$

The value range is exactly the same, and we got pretty good approximations for $Energy\left(\phi'_x\right)$ and $Energy\left(\phi'_y\right)$.

d+e

The values we got for $B_{low} = 5000$:

$N_x = 50.4571$

$N_y = 30.275$

$b = 3.2731$

and indeed $N_x N_y b \approx 5000$

The values we got for $B_{high} = 50000$:

$N_x = 129.9571$

$N_y = 77.9763$

$b = 4.9341$

and indeed $N_x N_y b \approx 50000$

f+g

The values we got for $B_{low} = 5000$:

$N_x = 52$

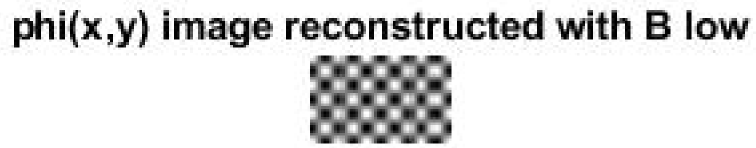
$N_y = 32$

$b = 3$

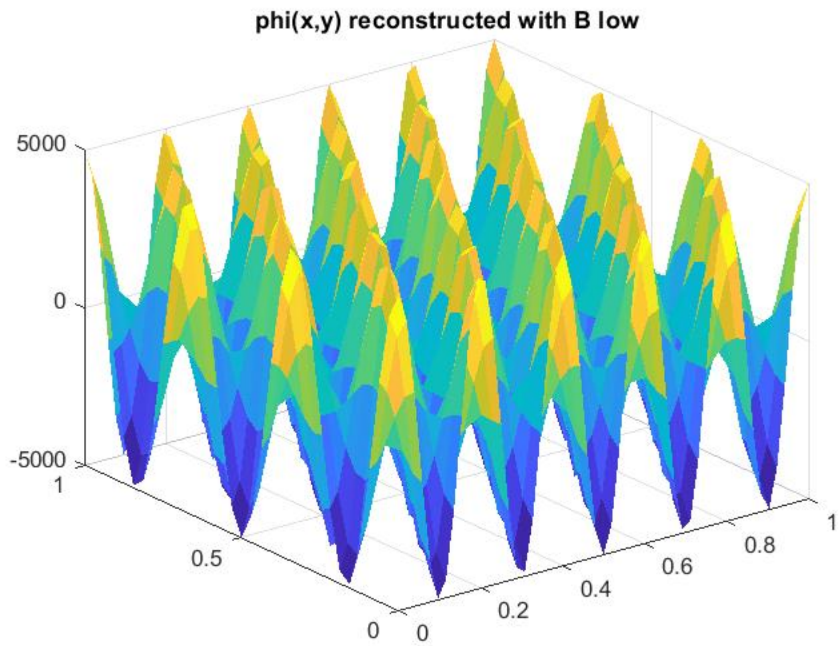
and indeed $N_x N_y b \approx 5000$, and the results we got in this part are pretty close to the results from section e.

The reconstructed signal using B_low

as histogram image:



as graph:



The values we got for $B_{high} = 50000$:

$N_x = 128$

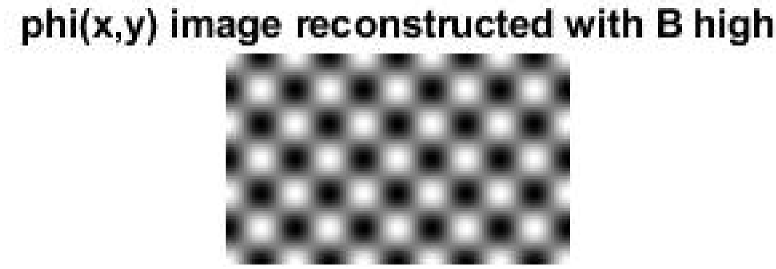
$N_y = 78$

$b = 5$

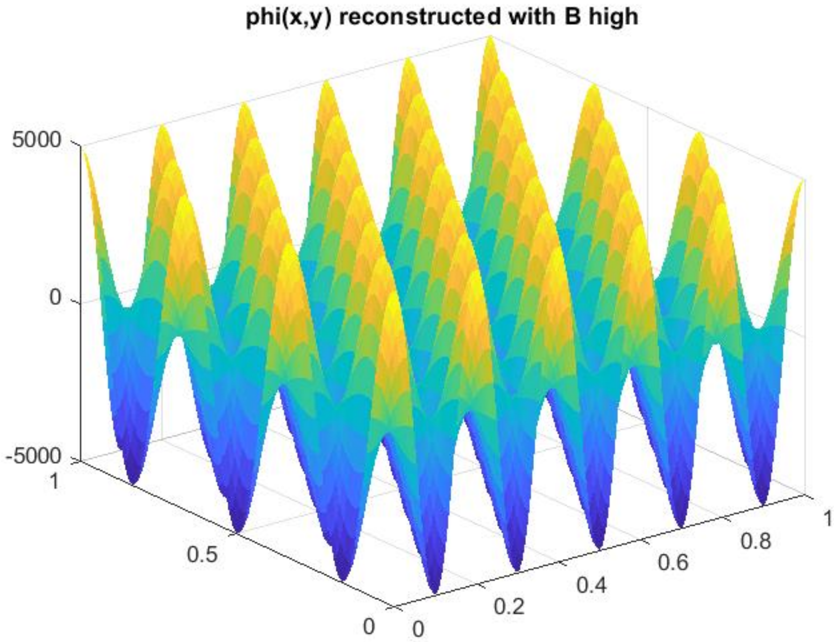
and indeed $N_x N_y b \approx 50000$, and the results we got in this part are pretty close to the results from section e.

The reconstructed signal using B_high

as histogram image:



as graph:



h

h-a

$$\phi(x, y) = A \cos\left(2\pi\omega_x x\right) \cos\left(2\pi\omega_y y\right)$$
$$A = 5000, \omega_x = 5, \omega_y = 7$$

The given signal has a maximal value of

$$\phi_H = \phi(0, 0) = A \cos 0 \cos 0 = A$$

And a minimal value of

$$\phi_L = \phi\left(\frac{1}{2\omega_x}, 0\right) = A\cos(\pi)\cos 0 = -A$$

and therefore the range of values is of size: $\phi_H - \phi_L = 2A = 10,000$.

$$Energy\left(\phi'_x\right) = \int_0^1\int_0^1 \left(\frac{\partial}{\partial x}\phi(x,y)\right)^2 dxdy$$

$$= \int_0^1\int_0^1 \left(\frac{\partial}{\partial x} A\cos\left(2\pi\omega_x x\right)\cos\left(2\pi\omega_y y\right)\right)^2 dxdy$$

$$= \int_0^1\int_0^1 \left(-2\pi\omega_x A\sin\left(2\pi\omega_x x\right)\cos\left(2\pi\omega_y y\right)\right)^2 dxdy$$

$$4\pi^2\omega_x^2 A^2 \int_0^1 \sin^2\left(2\pi\omega_x x\right)dx \int_0^1 \cos^2\left(2\pi\omega_y y\right)dy$$

$$= 4\pi^2\omega_x^2 A^2 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \pi^2\omega_x^2 A^2$$

$$\Downarrow$$

$$Energy\left(\phi'_x\right) = \pi^2\omega_x^2 A^2 \left(= 6.1685\cdot 10^9\right)$$

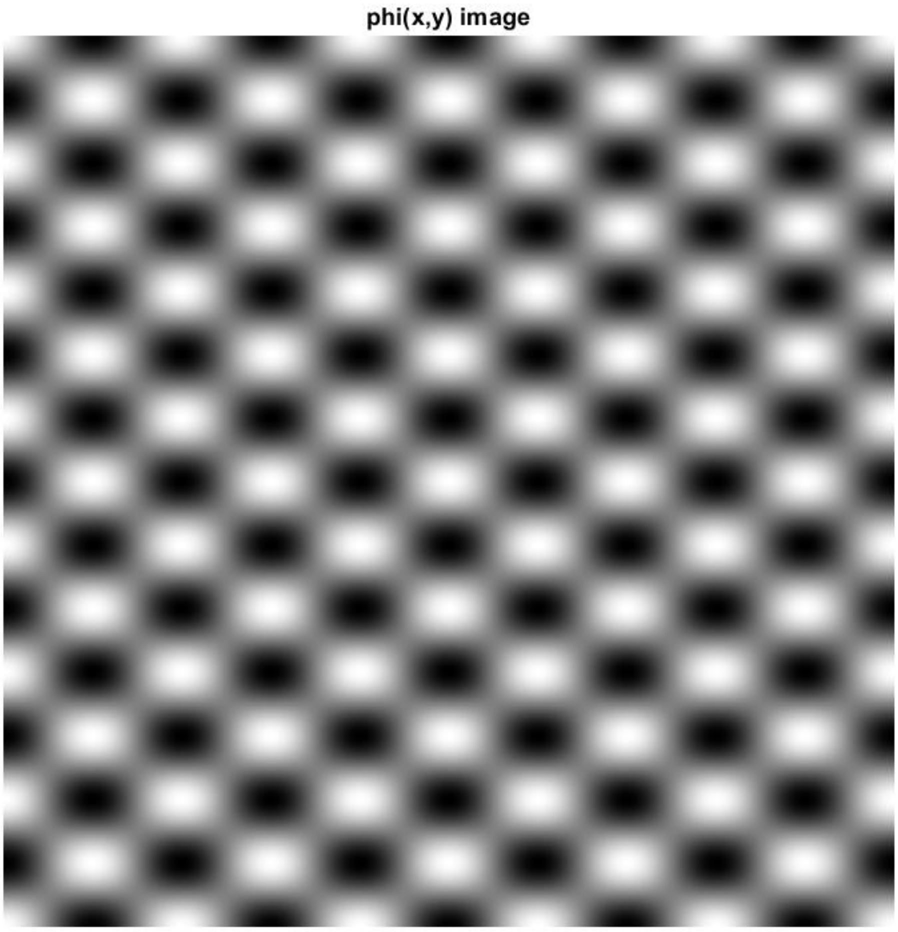
In analog way we will get for $Energy\left(\phi'_y\right)$:

$$Energy\left(\phi'_y\right) = \pi^2\omega_y^2 A^2 \left(= 1.209\cdot 10^{10}\right)$$

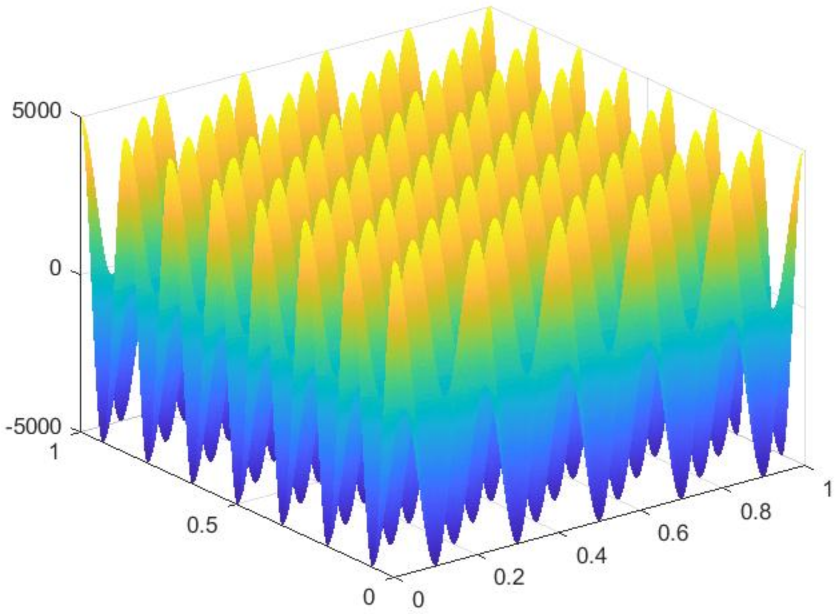
We can see that the energy in y direction increased, as we could expected because ω_y was increased and thus the function frequency increased.

h-b

The signal as image histogram:



This signal as graph:



We can see the signal has higher frequency compare to the last one.

h-c

The values we got from the numerically calculation is:
Value range: 2A(=10,000)
$Energy\left(\phi'_x\right) = 6.1803 \cdot 10^9$
$Energy\left(\phi'_y\right) = 1.12113 \cdot 10^{10}$
The value range is exactly the same, and we got pretty good approximations for $Energy\left(\phi'_x\right)$ and $Energy\left(\phi'_y\right)$.

h-d+e

The values we got for $B_{low} = 5000$:
$N_x = 36.6292$
$N_y = 51.2788$
$b = 2.662$
and indeed $N_x N_y b \approx 5000$
The values we got for $B_{high} = 50000$:
$N_x = 90.8949$
$N_y = 127.2479$
$b = 4.3229$
and indeed $N_x N_y b \approx 50000$
As we could expect, N_y got larger amount of the bits because y is the higher frequency now, compare to the function before.

h- f+g

The values we got for $B_{low} = 5000$:

$N_x = 34$

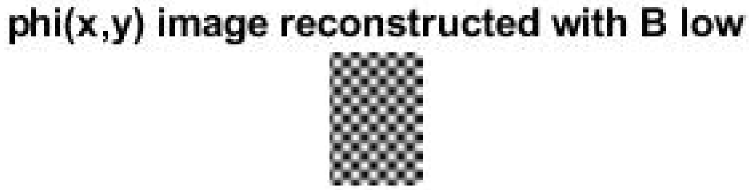
$N_y = 49$

$b = 3$

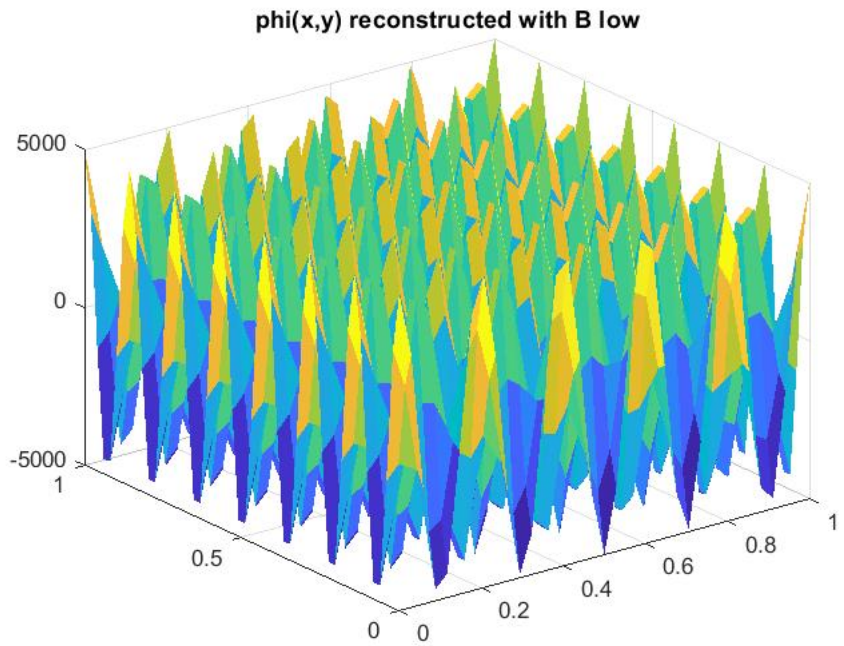
and indeed $N_x N_y b \approx 5000$, and the results we got in this part are pretty close to the results from section e.

The reconstructed signal using B_low

as histogram image:



as graph:



The values we got for $B_{high} = 50000$:

$N_x = 96$

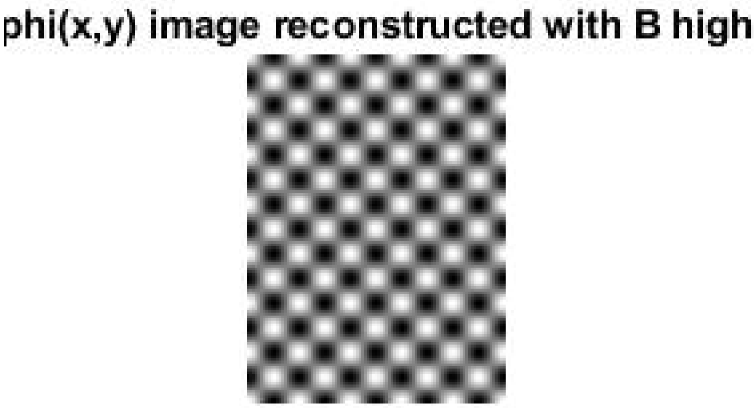
$N_y = 130$

$b = 4$

and indeed $N_x N_y b \approx 50000$, and the results we got in this part are pretty close to the results from section e.

The reconstructed signal using B_high

as histogram image:



as graph: