# Markov Chains and Algorithmic Applications: Mini-Project Supplementary Material

Hugo Aguettaz, Louis Amaudruz, Adrien Vandenbroucque

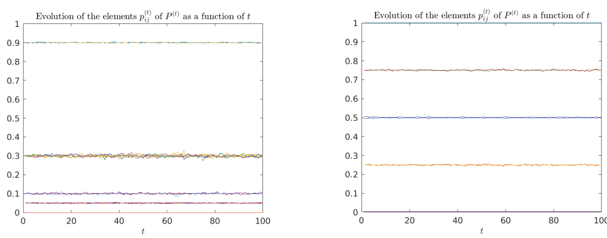
email: {hugo.aguettaz,louis.amaudruz,adrien.vandenbroucque}@epfl.ch

## I. Introduction

This document contains all the plots that are referenced but not shown in the main paper for the project.

## II. Section 1

Figure 1 plots the elements of the transition probability matrices at time $t$ of chains 1 and 2 respectively. As we can see, both chains are time-homogeneous due to their transition matrices not depending on time.




(a) Chain 1 (b) Chain 2

Figure 1: Transition probability matrix elements over time for chains 1 and 2

Chain 4 becomes time homogeneous after some time $t_0$, Figure 2 shows it's transition graph for $t > t_0$. The comments we make for this chain concern its behavior when it attains the time-homogeneous regime.

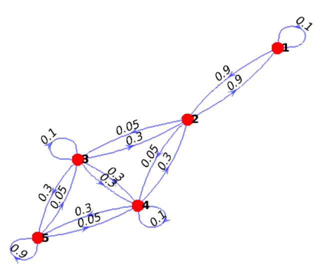
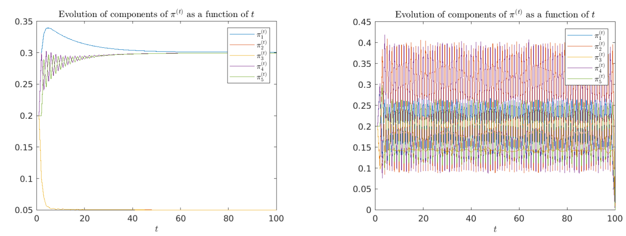


Figure 2: Transition graph for chain 4

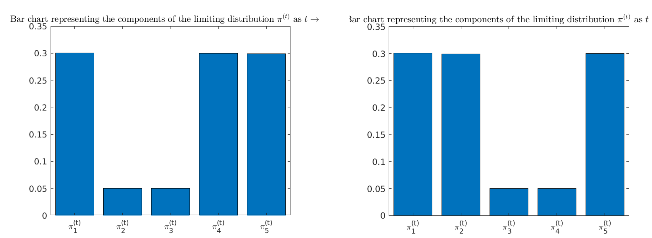
Starting from a uniform distribution, the probability distribution of chain 1 and 3 over time differ in their behaviour, chain 1 converges whereas chain 3 does not. Both of them are plotted in Figure 3.




(a) Chain 1 (b) Chain 3

Figure 3: Evolution of $\pi^{(t)}$ starting from uniform distribution for chains 1 and 3

Figure 4 depicts the limiting distribution elements found for chain 1 and 4.



(a) Chain 1 (b) Chain 4

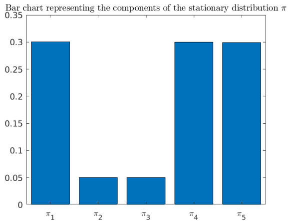
Figure 4: Limiting distribution $\pi$ for chain 1 and 4

Figure 5 shows the elements of the stationary distribution found for chain 1, 2 and 4 respectively. Notice how for chain 1 and 4, being ergodic, the stationary distribution matches the limiting one.

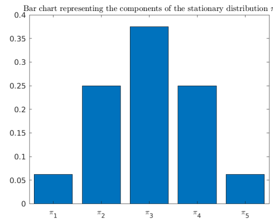
Figure 6 and 7 plot the Total Variation distance between the limiting distribution $\pi$ (which happens at large time $t_{max}$) and the probability distribution $\pi^{(t)}$ for $0 \leq t \leq t_{max}$, when starting from all possible initial states. The practical mixing time $T_\epsilon$ is also indicated in the vertical dotted line.

Figure 8 plots the Total Variation distance between the limiting distribution $\pi$ (which happens at large time $t_{max}$) and the probability distribution $\pi^{(t)}$ for $0 \leq t \leq t_{max}$, when starting from the uniform distribution for chain for chains 1 and 4.

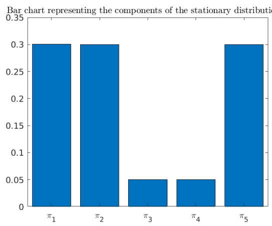
Figure 9 depicts again the Total Variation distance between the limiting distribution $\pi$ (which happens at large




(a) Chain 1




(b) Chain 2




(c) Chain 4

Figure 5: Bar charts for the stationary distribution of chain 1, 2 and 4

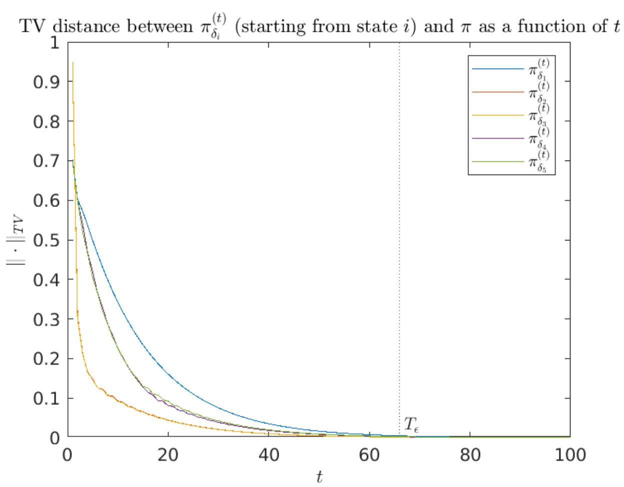



Figure 6: Total variation distance between limiting distribution and $\pi^{(t)}$ of chain 1

time $t_{max}$) and the probability distribution $\pi^{(t)}$ for $0 \leq t \leq t_{max}$, when starting from the uniform distribution for chain 4. However this time, it also include the theoretical upper bound, together with both practical and theoretical $T_\epsilon$.

## III. Section 2

The plots from Figure 10 to 18 depict our result from section 2: the limiting distribution computed from each possible initial state and for each Metropolis chains, and also the evolution of the Total Variation distance between $\pi^{(t)}$ and the limiting one, starting from all possible initial states.

One can observe that we are able to sample from the

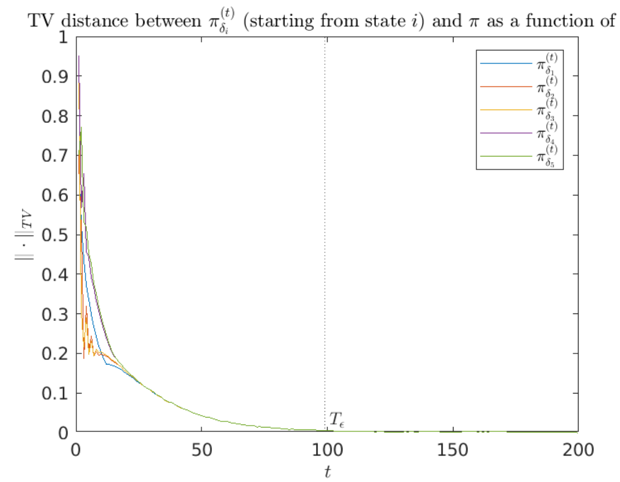
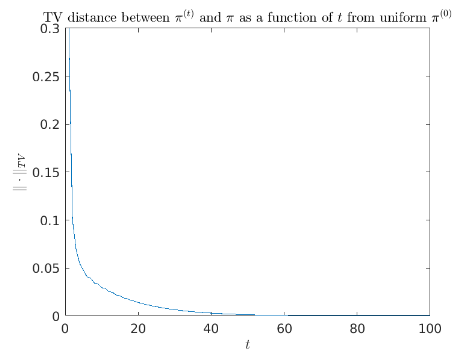
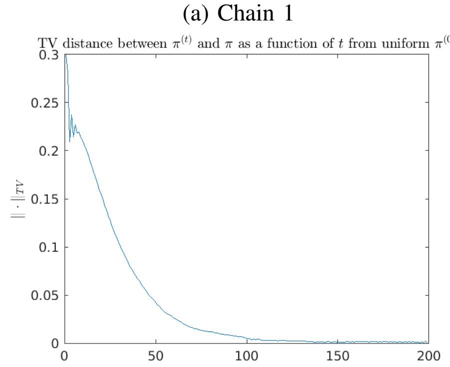



Figure 7: Total variation distance between limiting distribution and $\pi^{(t)}$ of chain 4




(a) Chain 1




(b) Chain 4

Figure 8: Total variation distance between limiting distribution and $\pi^{(t)}$ with uniform $\pi^{(0)}$

desired distributions all the time, and starting from different states give sometime slightly different results.

For the Total Variation plots, we see that the starting state often affects the convergence, and that in every case the practical convergence is always upper bounded by the

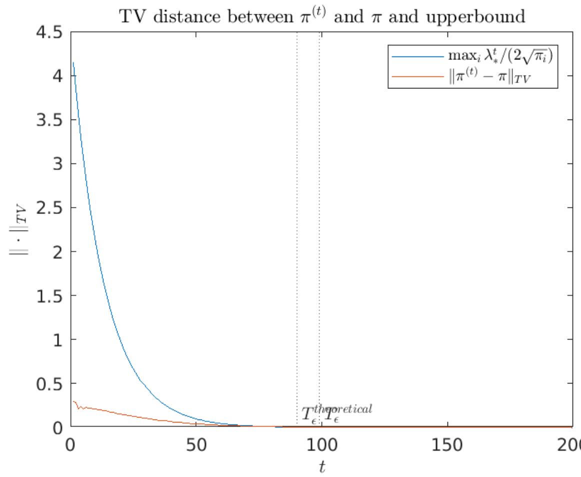



Figure 9: Total variation distance between limiting distribution and $\pi^{(t)}$ for chain 4, together with the theoretical upper bound

theoretical upper bound derived in class.

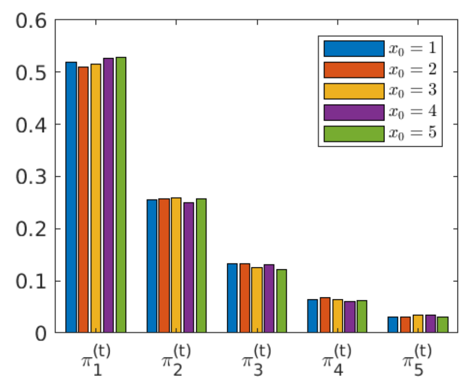



Figure 10: Limiting distribution computed starting from each possible state with base chain 2 to sample from distribution 1

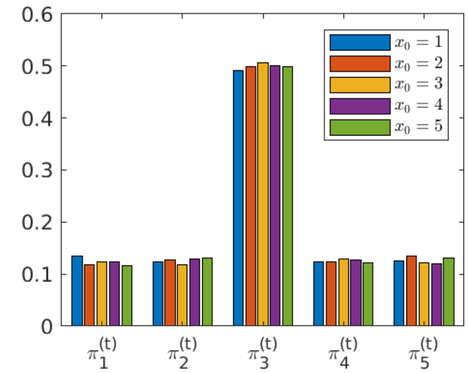



Figure 11: Limiting distribution computed starting from each possible state with base chain 1 to sample from distribution 2

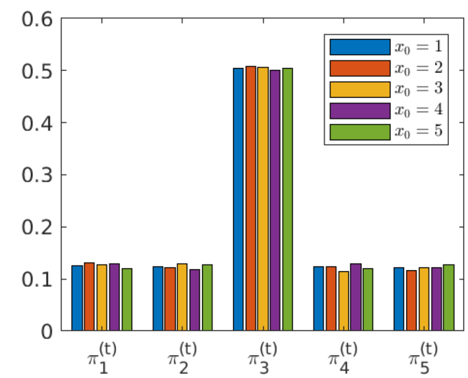



Figure 12: Limiting distribution computed starting from each possible state with base chain 2 to sample from distribution 2

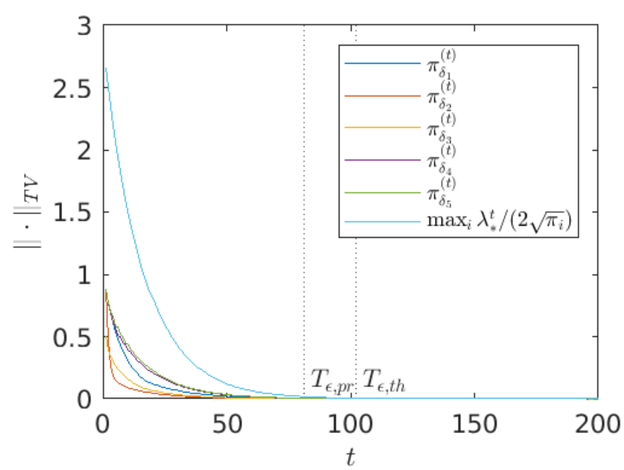



Figure 13: Total variation distance between limiting distribution and $\pi^{(t)}$ with chain 1 as base chain for sampling distribution 2, together with the theoretical upper bound

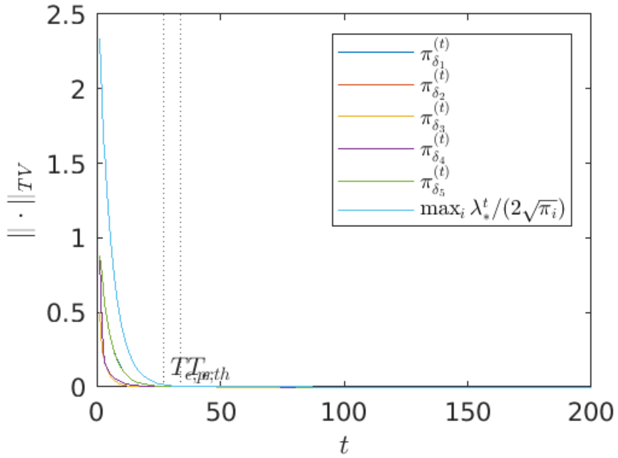



Figure 14: Total variation distance between limiting distribution and $\pi^{(t)}$ with chain 2 as base chain for sampling distribution 2, together with the theoretical upper bound

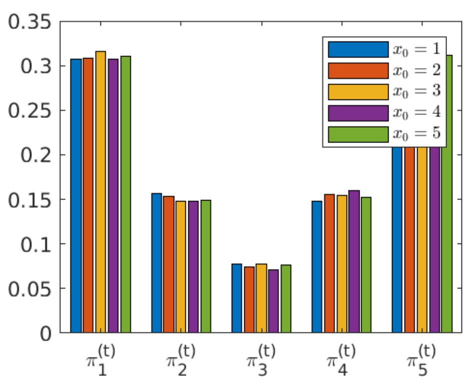



Figure 15: Limiting distribution computed starting from each poosible state with base chain 1 to sample from distribution 3

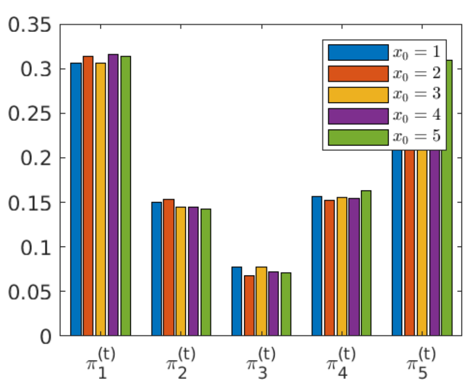



Figure 16: Limiting distribution computed starting from each possible state with base chain 2 to sample from distribution 3

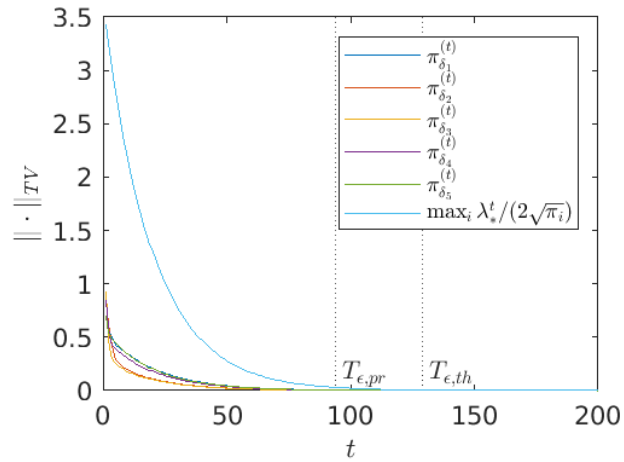



Figure 17: Total variation distance between limiting distribution and $\pi^{(t)}$ with chain 1 as base chain for sampling distribution 3, together with the theoretical upper bound

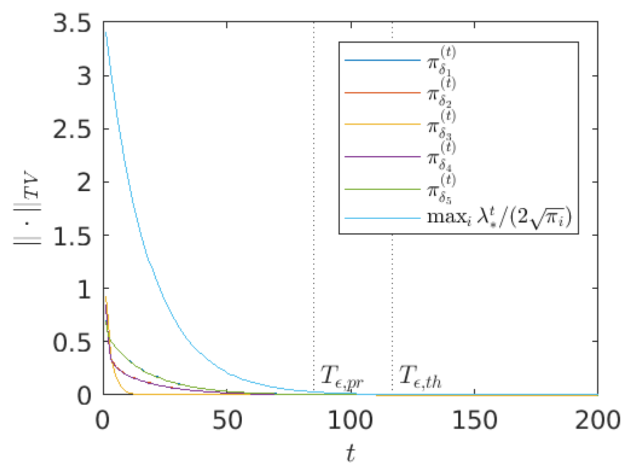



Figure 18: Total variation distance between limiting distribution and $\pi^{(t)}$ with chain 2 as base chain for sampling distribution 3, together with the theoretical upper bound