

Ch.5-Mathematical Induction and Binomial Theorem

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Section-A — JEE Advanced/IIT-JEE

I.Integer Value Correct Type

1) The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14. Then $n =$ (JEE Adv. 2013)

2) Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+x)^{50} + (1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integer n . Then the value of n is (JEE Adv. 2016)

3) Let $X = \binom{10}{1}C_1^2 + 2\binom{10}{2}C_2^2 + 3\binom{10}{3}C_3^2 + \dots + 10\binom{10}{10}C_{10}^2$, where ${}^{10}C_r, r \in \{1, 2, \dots, 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is _____ (JEE Adv. 2018)

4) Suppose

$$\det \begin{vmatrix} \sum_{k=0}^n k & \sum_{k=0}^n k^{2n} C_k \\ \sum_{k=0}^n {}^nC_k k & \sum_{k=0}^n {}^nC_k 3^k \end{vmatrix} = 0$$

holds for some positive integer n . The $\sum_{k=0}^n \frac{{}^nC_k}{k+1}$ equals _____ (JEE Adv. 2019)

Ch.18-Definite Integrals and Applications of Integrals

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Section-B — JEE Main / AIEEE

- 16) The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is (2004)
- 4
 - 2
 - 3
 - 1
- 17) If $I_1 = \int_0^1 2x^2 dx$, $I_2 = \int_0^1 2x^3 dx$, $I_3 = \int_1^2 2x^2 dx$, $I_4 = \int_1^2 2x^3 dx$ then (2005)
- $I_2 > I_1$
 - $I_1 > I_2$
 - $I_3 = I_4$
 - $I_3 > I_4$
- 18) The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is (2005)
- 1
 - 2
 - 3
 - 4
- 19) The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom then $S_1 : S_2 : S_3$ is (2005)
- 1:2:1
 - 1:2:3
 - 2:1:2
 - 1:1:1
- 20) Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\left(\beta \sin(\beta) + \frac{\pi}{4} \cos(\beta)\right)\right)$. Then $f\left(\frac{\pi}{2}\right)$ is (2005)
- $\frac{\pi}{4} + \sqrt{2} - 1$
 - $\frac{\pi}{2} - \sqrt{2} + 1$
 - $1 - \frac{\pi}{4} - \sqrt{2}$
 - $1 - \frac{\pi}{4} + \sqrt{2}$
- 21) The value of $\int_{-\pi}^{\pi} \frac{\cos^2(x)}{1+a^x} dx$, $a > 0$, is (2005)

- a) π
- b) $\frac{\pi}{2}$
- c) $\frac{\pi}{a}$
- d) 2π

22) The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is (2005)

- a) $\frac{1}{2}$
- b) $\frac{3}{2}$
- c) 2
- d) 1

23) $\int_0^\pi x f(\sin x) dx$ is equal to (2006)

- a) $\pi \int_0^\pi f(\cos x) dx$
- b) $\pi \int_0^\pi f(\sin x) dx$
- c) $\frac{\pi}{2} \int_0^{\frac{\pi}{2}} f(\sin x) dx$
- d) $\pi \int_0^{\frac{\pi}{2}} f(\cos x) dx$

24) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$ is equal to (2006)

- a) $\frac{\pi^4}{32}$
- b) $\frac{\pi^4}{32} + \frac{\pi}{2}$
- c) $\frac{\pi^4}{2}$
- d) $\frac{\pi}{4} - 1$

25) The value of $\int_1^a [x] f'(x) dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is (2006)

- a) $a f(a) - \{f(1) + f(2) + \dots f([a])\}$
- b) $[a] f(a) - \{f(1) + f(2) + \dots f([a])\}$
- c) $[a] f([a]) - \{f(1) + f(2) + \dots f(a)\}$
- d) $a f([a]) - \{f(1) + f(2) + \dots f(a)\}$

26) Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$, Then $F(e)$ equals (2006)

- a) 1
- b) 2
- c) $\frac{1}{2}$
- d) 0

27) The solution for x of the equation $\int_{\sqrt{2}}^x \frac{1}{t\sqrt{t^2-1}} dt$ is (2007)

- a) $\frac{\sqrt{3}}{2}$
- b) $2\sqrt{2}$
- c) 2
- d) none

28) The area enclosed between the curves $y^2 = x$ and $y = |x|$ is (2007)

- a) $\frac{1}{6}$
- b) $\frac{1}{3}$
- c) $\frac{2}{3}$
- d) 1

29) Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true?

(2007)

- a) $I > \frac{1}{2}$ and $J > 2$
- b) $I < \frac{1}{2}$ and $J < 2$
- c) $I < \frac{1}{2}$ and $J > 2$
- d) $I > \frac{1}{2}$ and $J < 2$

30) The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(2008)

- a) $\frac{5}{3}$
- b) $\frac{1}{3}$
- c) $\frac{2}{3}$
- d) $\frac{14}{3}$