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1 1-15(Math)

1) Suppose the vectors $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ are the solutions of the system of linear equations, $Ax = \mathbf{b}$ when the vector $\mathbf{b}$ on the right side is equal to $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{b_3}$ respectively. If $\mathbf{x_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{x_2} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{x_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{b_2} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, $\mathbf{b_3} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$, then the determinant of A is equal to
   a) 2
   b) $\frac{1}{2}$
   c) $\frac{3}{2}$
   d) 4
2) If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$ then $a+b$ is equal to:
   a) 33
   b) 57
   c) 9
   d) 24
3) The distance of the point $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:
   a) $\frac{1}{7}$
   b) 7
   c) $\frac{7}{5}$
   d) 1
4) Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t-x} = 0$
   If $f(x) = 1$, then x is equal to:
   a) e
   b) 2e
   c) $\frac{1}{e}$
   d) $\frac{2}{e}$
5) Contrapositive of the statement :
   If a function f is differentiable at a, then it is also continuous at a, is:
   a) If a function f is not continuous at a, then it is not differentiable at a.

b) If a function f is continuous at a, then it is differentiable at a.
c) If a function f is continuous at a, then it is not differentiable at a.
d) If a function f is continuous at a, then it is not differentiable at a.

6) The minimum value of $2^{\sin(x)} + 2^{\cos(x)}$ is:
a) $2^{1-\sqrt{2}}$
b) $2^{1-\frac{1}{\sqrt{2}}}$
c) $2^{-1+\sqrt{2}}$
d) $2^{-1+\frac{1}{\sqrt{2}}}$

7) If the perpendicular bisector of the line segment joining the points $\mathbf{P}\begin{pmatrix}1\\4\end{pmatrix}$ and $\mathbf{Q}\begin{pmatrix}k\\3\end{pmatrix}$ has y-intercept equal to -4, then a value of k is:
a) -2
b) $\sqrt{15}$
c) $\sqrt{14}$
d) -4

8) The area (in sq. units) of the largest rectangle $ABCD$ whose vertices **A** and **B** lie on the $x-axis$ and vertices **C** and **D** lie on the parabola, $y = x^2 - 1$ below the $x-axis$, is:
a) $\frac{2}{3\sqrt{3}}$
b) $\frac{4}{3}$
c) $\frac{1}{3\sqrt{3}}$
d) $\frac{4}{3\sqrt{3}}$

9) The integral
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x \sin^2 3x \left(2 \sec^2 x \sin^2 3x + 3 \tan x \sin 6x\right) dx$ is equal to
a) $\frac{9}{2}$
b) $\frac{-1}{18}$
c) $\frac{-1}{9}$
d) $\frac{7}{18}$

10) If the system of equations $x + y + z = 2$
$2x + 4y - z = 6$
$3x + 2y + \lambda z = \mu$
has infinitely many solutions, then
a) $\lambda - 2\mu = -5$
b) $2\lambda + \mu = 14$
c) $\lambda + 2\mu = 14$
d) $2\lambda - \mu = 5$

11) In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :
a) $\frac{5}{31}$

b) $\frac{31}{61}$
c) $\frac{30}{61}$
d) $\frac{5}{6}$

12) If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :

a) 792
b) 252
c) 462
d) 330

13) The function
$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x & \text{if } |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & \text{if } |x| > 1 \end{cases}$$

a) both continuous and differentiable on $\mathbb{R} - \{-1\}$
b) continuous on $\mathbb{R} - \{-1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$
c) continuous on $\mathbb{R} - \{1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$
d) both continuous and differentiable on $\mathbb{R} - \{1\}$

14) The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e y+3x} + 3 = 0$ is: where c is a constant of integration

a) $x - \log_e y + 3x = c$
b) $x - \frac{1}{2}\left(\log_e y + 3x^2\right) = c$
c) $x - 2\log_e y + 3x = c$
d) $y + 3x - \frac{1}{2}\log_e x^2 = c$

15) Let $\lambda \neq 0$ be in $\mathbb{R}$. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to:

a) 27
b) 9
c) 18
d) 36