Ch.5-Mathematical Induction and Binomial Theorem

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Section-A — JEE Advanced/IIT-JEE

I.Integer Value Correct Type

1) The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14. Then $n =$ (JEE Adv. 2013)
2) Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + ... + (1+x)^{49} + (1+x)^{50} + (1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is
(JEE Adv. 2016)
3) Let
$X = \left({}^{10}C_1{}^2\right) + 2\left({}^{10}C_2{}^2\right) + 3\left({}^{10}C_3{}^2\right) + ... + 10\left({}^{10}C_{10}{}^2\right)$, where ${}^{10}C_r, r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}$X is ______
(JEE Adv. 2018)
4) Suppose
$$\det\begin{pmatrix} \sum\limits_{k=0}^{n} k & \sum\limits_{k=0}^{n} k^{2n}C_k \\ \sum\limits_{k=0}^{n} {}^nC_k k & \sum\limits_{k=0}^{n} {}^nC_k 3^k \end{pmatrix} = 0$$
holds for some positive integer n. The $\sum\limits_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals ______
(JEE Adv. 2019)

Ch.18-Definite Integrals and Applications of Integrals

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Section-B — JEE Main / AIEEE

16) The area of the region bounded by the curves $y = |x-2|, x = 1, x = 3$ and the x-axis is (2004)
    a) 4
    b) 2
    c) 3
    d) 1
17) If $I_1 = \int_0^1 2^{x^2} \quad dx, I_2 = \int_0^1 2^{x^3} \quad dx, I_3 = \int_1^2 2^{x^2} \quad dx, I_4 = \int_1^2 2^{x^3} \quad dx$ then (2005)
    a) $I_2 > I_1$
    b) $I_1 > I_2$
    c) $I_3 = I_4$
    d) $I_3 > I_4$
18) The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is (2005)
    a) 1
    b) 2
    c) 3
    d) 4
19) The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom then $S_1 : S_2 : S_3$ is (2005)
    a) 1:2:1
    b) 1:2:3
    c) 2:1:2
    d) 1:1:1
20) Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the oordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\left(\beta \sin(\beta) + \frac{\pi}{4}\cos(\beta)\right)\right)$. Then $f\left(\frac{\pi}{2}\right)$ is (2005)
    a) $\frac{\pi}{4} + \sqrt{2} - 1$
    b) $\frac{\pi}{2} - \sqrt{2} + 1$
    c) $1 - \frac{\pi}{4} - \sqrt{2}$
    d) $1 - \frac{\pi}{4} + \sqrt{2}$
21) The value of $\int_{-\pi}^{\pi} \frac{\cos^2(x)}{1+a^x} \quad dx,\ a > 0$, is (2005)

a) π
b) $\frac{\pi}{2}$
c) $\frac{\pi}{a}$
d) 2π

22) The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} \quad dx$ is (2005)

a) $\frac{1}{2}$
b) $\frac{3}{2}$
c) 2
d) 1

23) $\int_0^{\pi} xf(\sin x) \quad dx$ is equal to (2006)

a) $\pi \int_0^{\pi} f(\cos x) \quad dx$
b) $\pi \int_0^{\pi} f(\sin x) \quad dx$
c) $\frac{\pi}{2} \int_0^{\frac{\pi}{2}} f(\sin x) \quad dx$
d) $\pi \int_0^{\frac{\pi}{2}} f(\cos x) \quad dx$

24) $\int_{\frac{-3\pi}{2}}^{\frac{-\pi}{2}} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right] \quad dx$ is equal to (2006)

a) $\frac{\pi^4}{32}$
b) $\frac{\pi^4}{32} + \frac{\pi}{2}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4} - 1$

25) The value of $\int_1^a [x]f'(x) \quad dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is (2006)

a) $af(a) - \{f(1) + f(2) + ...f([a])\}$
b) $[a]f(a) - \{f(1) + f(2) + ...f([a])\}$
c) $[a]f([a]) - \{f(1) + f(2) + ...f(a)\}$
d) $af([a]) - \{f(1) + f(2) + ...f(a)\}$

26) Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{logt}{1+t} \quad dt$, Then $F(e)$ equals (2006)

a) 1
b) 2
c) $\frac{1}{2}$
d) 0

27) The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{1}{t\sqrt{t^2-1}} \quad dt$ is (2007)

a) $\frac{\sqrt{3}}{2}$
b) $2\sqrt{2}$
c) 2
d) none

28) The area enclosed between the curves $y^2 = x$ and $y = |x|$ is (2007)

a) $\frac{1}{6}$
b) $\frac{1}{3}$
c) $\frac{2}{3}$
d) 1

29) Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\, dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\, dx$. Then which one of the following is true? (2007)

a) $I > \frac{2}{3}$ and $J > 2$
b) $I < \frac{2}{3}$ and $J < 2$
c) $I < \frac{2}{3}$ and $J > 2$
d) $I > \frac{2}{3}$ and $J < 2$

30) The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to (2008)

a) $\frac{5}{3}$
b) $\frac{1}{3}$
c) $\frac{2}{3}$
d) $\frac{4}{3}$