2023-April Session-04-11-2023 Shift 1

AI24BTECH11016 - Jakkula Adishesh Balaji

1 16-30 (Math)

1) If the equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$, then $\alpha + \beta + \gamma$ is:
   a) 15
   b) 18
   c) 17
   d) 16
2) An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to a total of 60 men and only five men got medals in all the three events, then how many received medals in exactly two of three events?
   a) 15
   b) 9
   c) 21
   d) 10
3) Let $y = y(x)$ be a solution curve of the differential equation $\left(1 - x^2y^2\right)dx = ydx + xdy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then the value of α is:
   a) $\frac{1+3e^2}{2(3e^2-1)}$
   b) $\frac{1-3e^2}{2(3e^2+1)}$
   c) $\frac{3e^2}{2(3e^2-1)}$
   d) $\frac{3e^2}{2(3e^2+1)}$
4) Let $\begin{pmatrix}\alpha & \beta & \gamma\end{pmatrix}$ be the image of the point $\mathbf{P} = \begin{pmatrix}2 & 3 & 5\end{pmatrix}$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to:
   a) 5
   b) 9
   c) 10
   d) 12
5) Let $f(x) = \left[x^2 - x\right] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then f is:
   a) not continuous at $x = 0$ and $x = 1$
   b) continuous at $x = 0$ and $x = 1$
   c) continuous at $x = 1$, but not continuous at $x = 0$

d) continuous at $x = 0$, but not continuous at $x = 1$

6) The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)$ is:

7) The number of ordered triplets of the truth values of p, q, and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \implies (q \vee r)$ is true, is equal to:

8) Let $A = \begin{pmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{pmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$ where $n \in \mathbb{N}$, then n is equal to:

9) For $m, n > 0$, let $\alpha(m,n) = \int_0^2 t^m (1+3t)^n \, dt$. If $11\alpha(10,6) + 18\alpha(11,5) = p14^6$, then p is equal to:

10) Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $16S - \left(25^{-54}\right)$ is equal to:

11) Let $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$, $n \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21l$ is equal to:

12) The mean of the coefficients of x, x^2,..., x^7 in the binomial expansion of $(2+x)^9$ is:

13) If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to:

14) In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways in which none of the students sits on the allotted seat is:

15) Let a line l pass through the origin and be perpendicular to the lines
$l_1 : \mathbf{r} = \hat{i} - 11\hat{j} - 7\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right), \lambda \in \mathbb{R}$ and
$l_2 : \mathbf{r} = -\hat{i} + \hat{k} + \mu\left(2\hat{i} + 2\hat{j} + \hat{k}\right), \mu \in \mathbb{R}$ If $\mathbf{P}$ is the point of intersection of l and l_1, and $\mathbf{Q}\left(\alpha \quad \beta \quad \gamma\right)$ is the foot of perpendicular from $\mathbf{P}$ on l_2, then $9(\alpha + \beta + \gamma)$ is equal to