Dynamisk Programmering (DP)

Plan:

- Introduktion
  - Hvad er dynamisk programmering?
  - Fibonacci-tal
- Stangudskæring (rod cutting)
- Længste fælles delsekvens (longest common subsequence)

Mikkel Abrahamsen

Dynamisk Programmering (DP)

Generel algoritmisk teknik
Smart udtømmende søgning
Genbrug af beregninger

Fibonacci-tal

$F_1 = F_2 = 1$
$F_n = F_{n-1} + F_{n-2}$, $n \geq 3$

Naiv rekursiv algoritme

fib(n)
  if $n \leq 2$
    return 1
  return fib($n-1$)+fib($n-2$)

Køretid: $T(n) = T(n-1) + T(n-2) + \Theta(1)$

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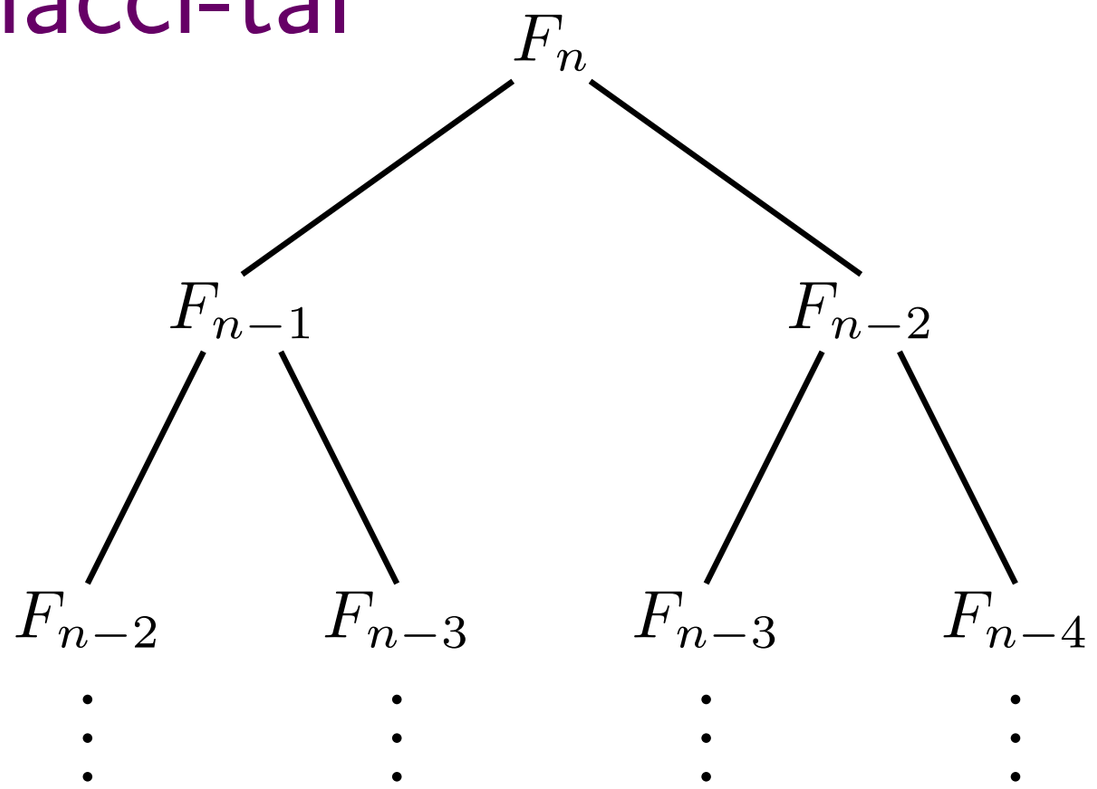

Alle blade: F_1 eller F_2

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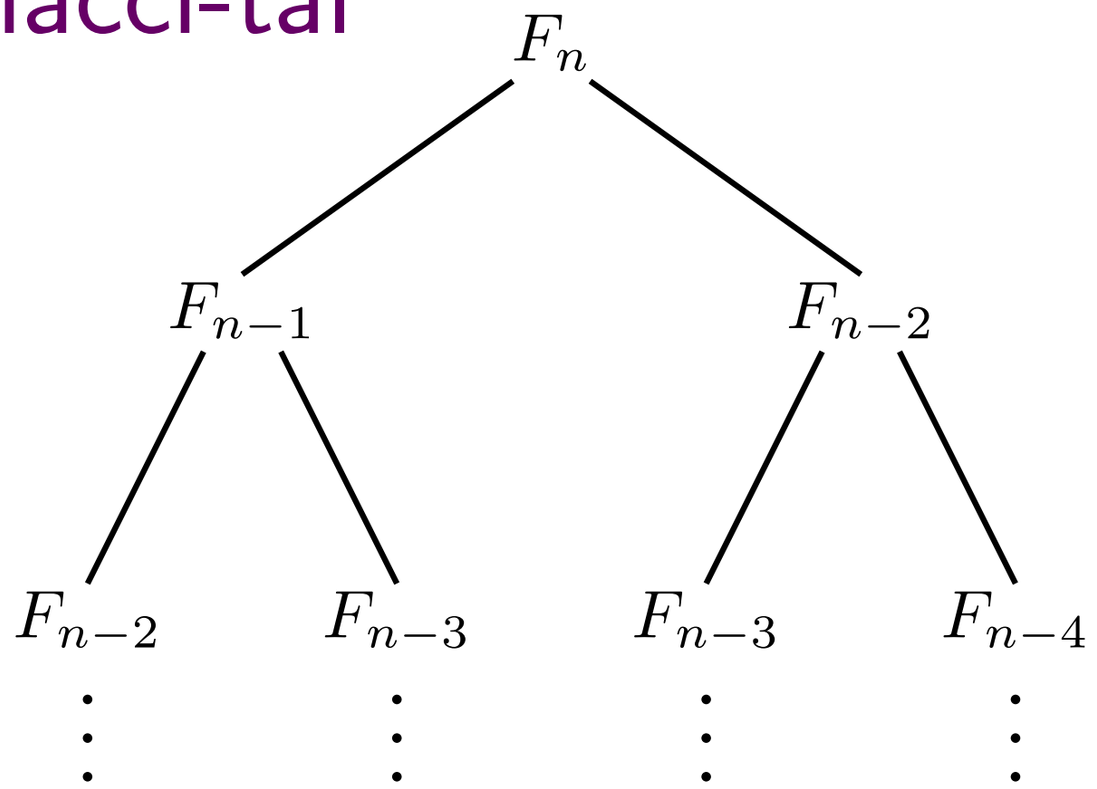

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$T(n) = \Omega(F_n) = \Omega(\varphi^n) = \Omega(1.61^n)$

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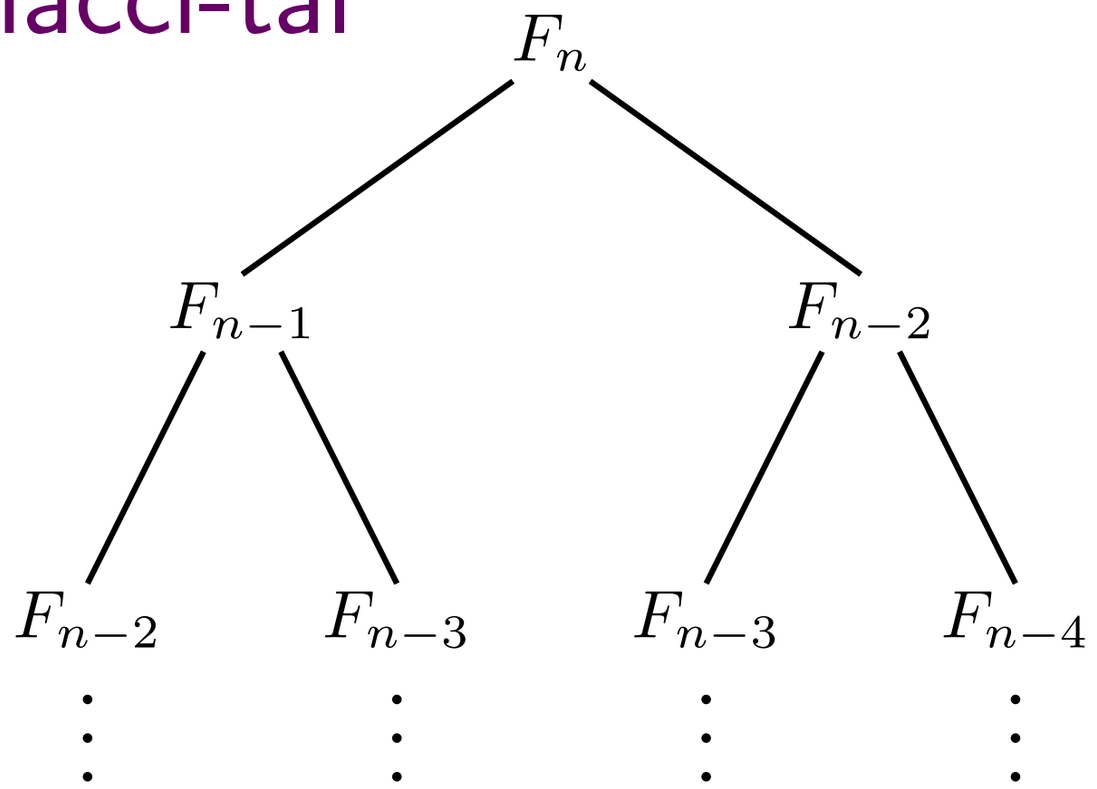

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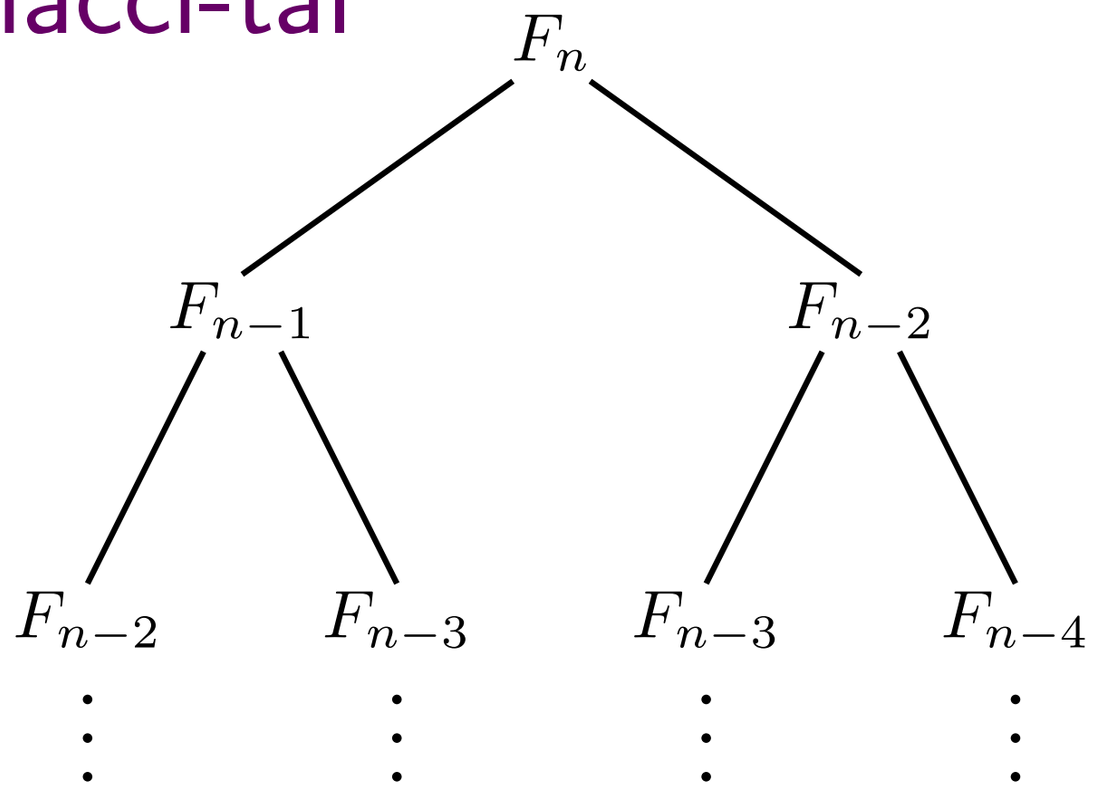

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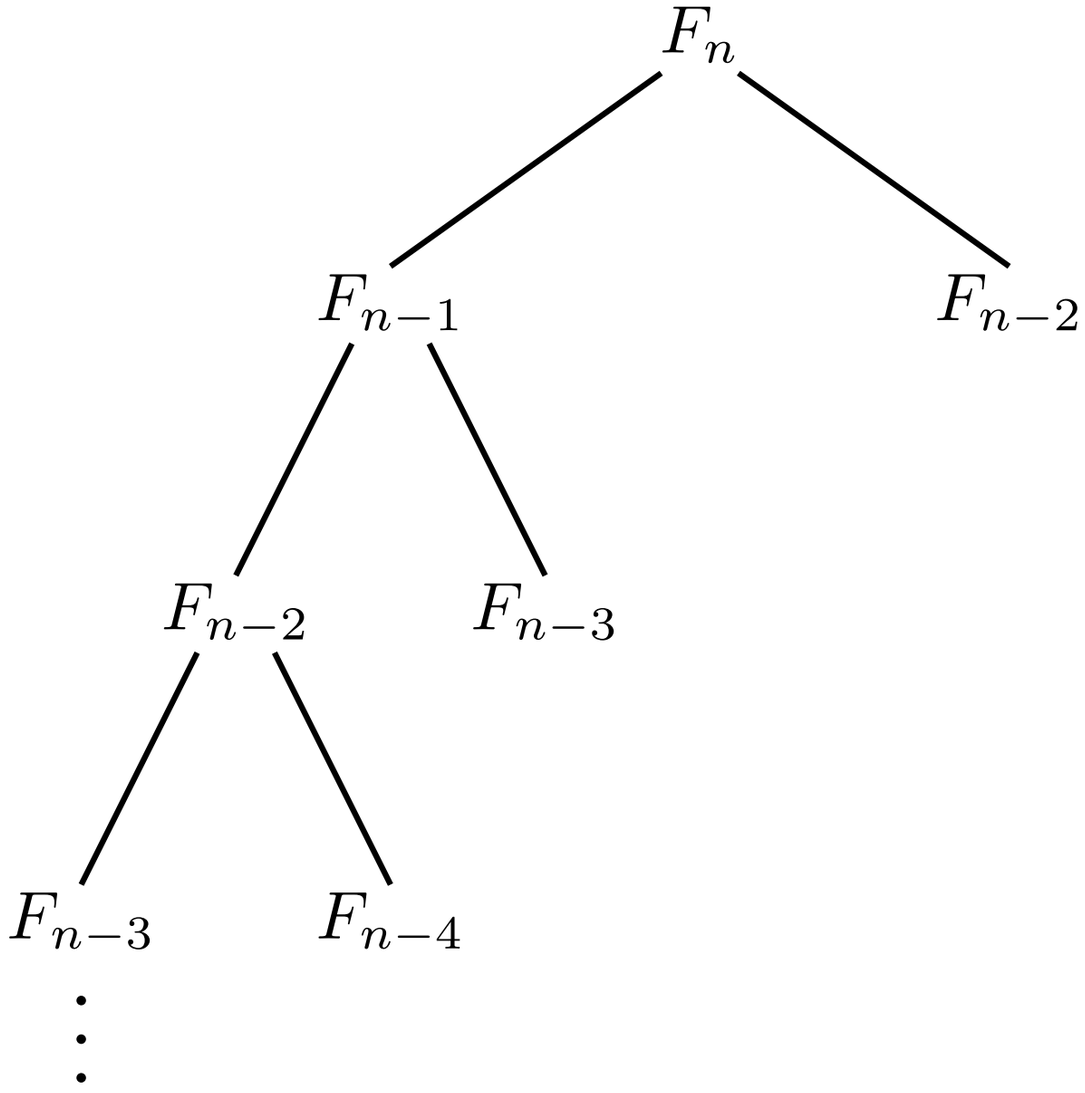
Hvad er ineffektivt?

Memoisering

Lad $F[1 \ldots N] = [1, 1, \underbrace{-1, -1, \ldots, -1}_{N-2}]$.

fibm(F, n)
  if $F[n] == -1$
    $F[n] =$fibm$(F, n-1)+$fibm$(F, n-2)$
  return $F[n]$



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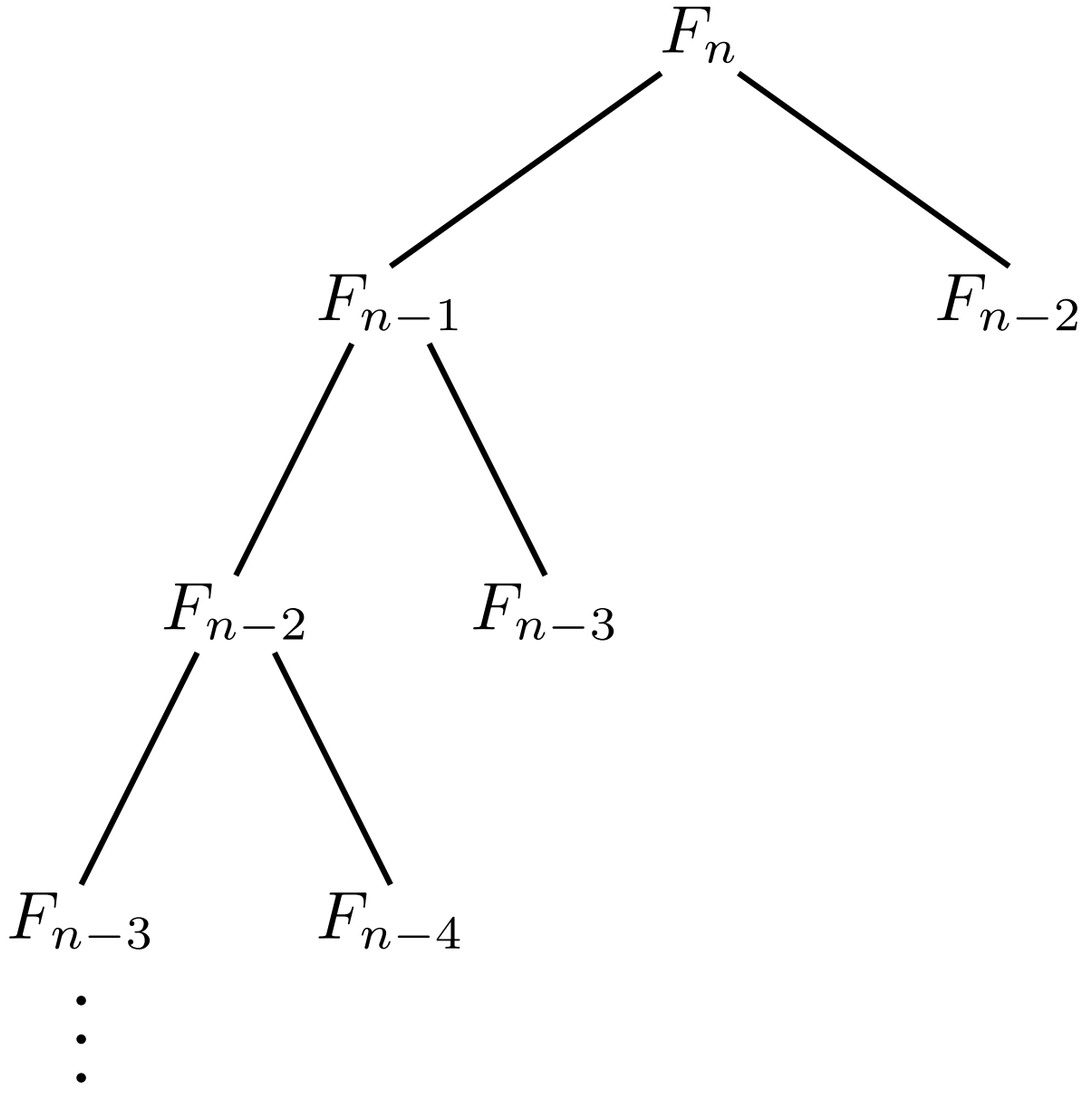
$T(n) = \Theta(n)$



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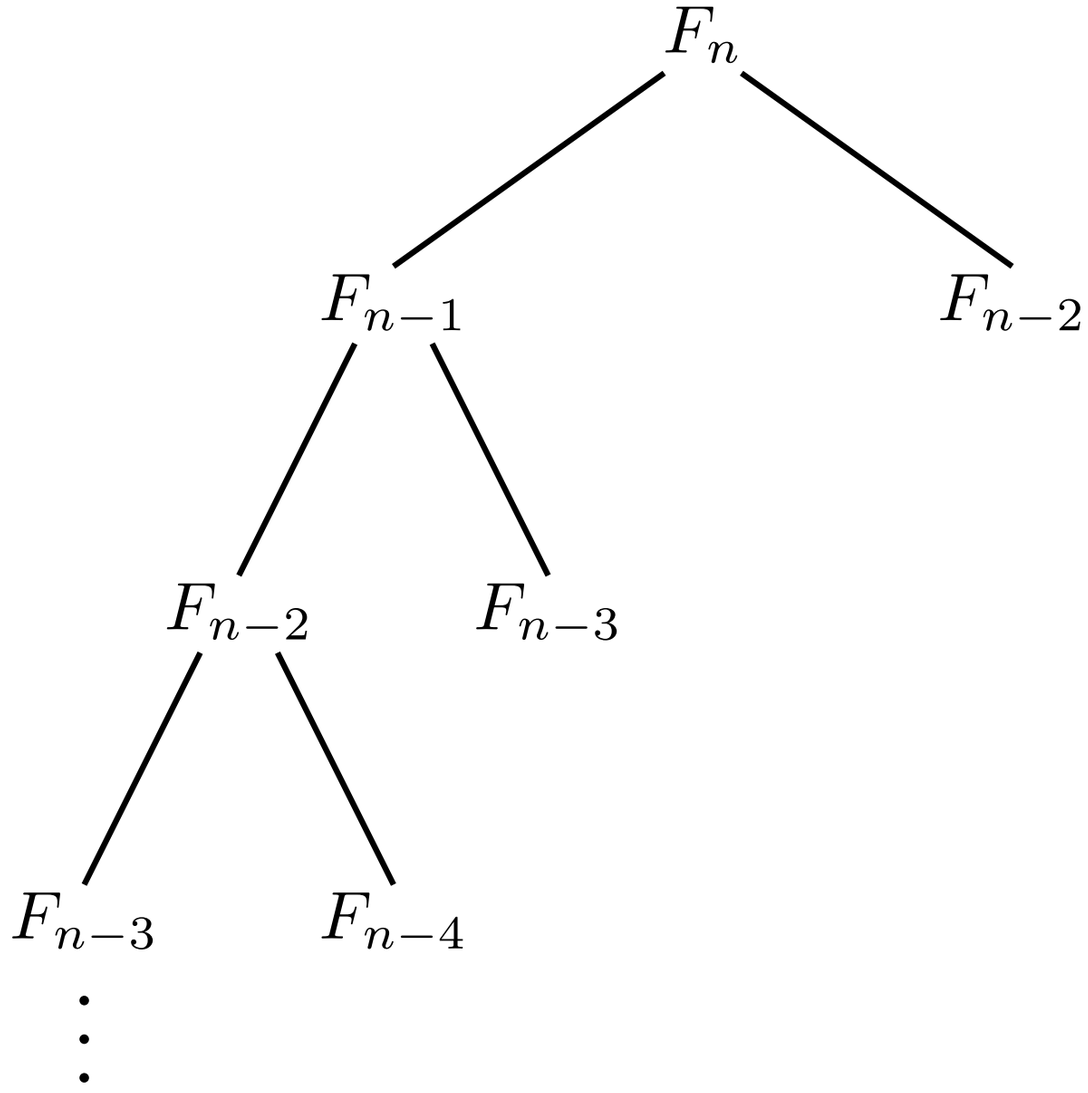
DP idé: gem og genbrug beregninger!



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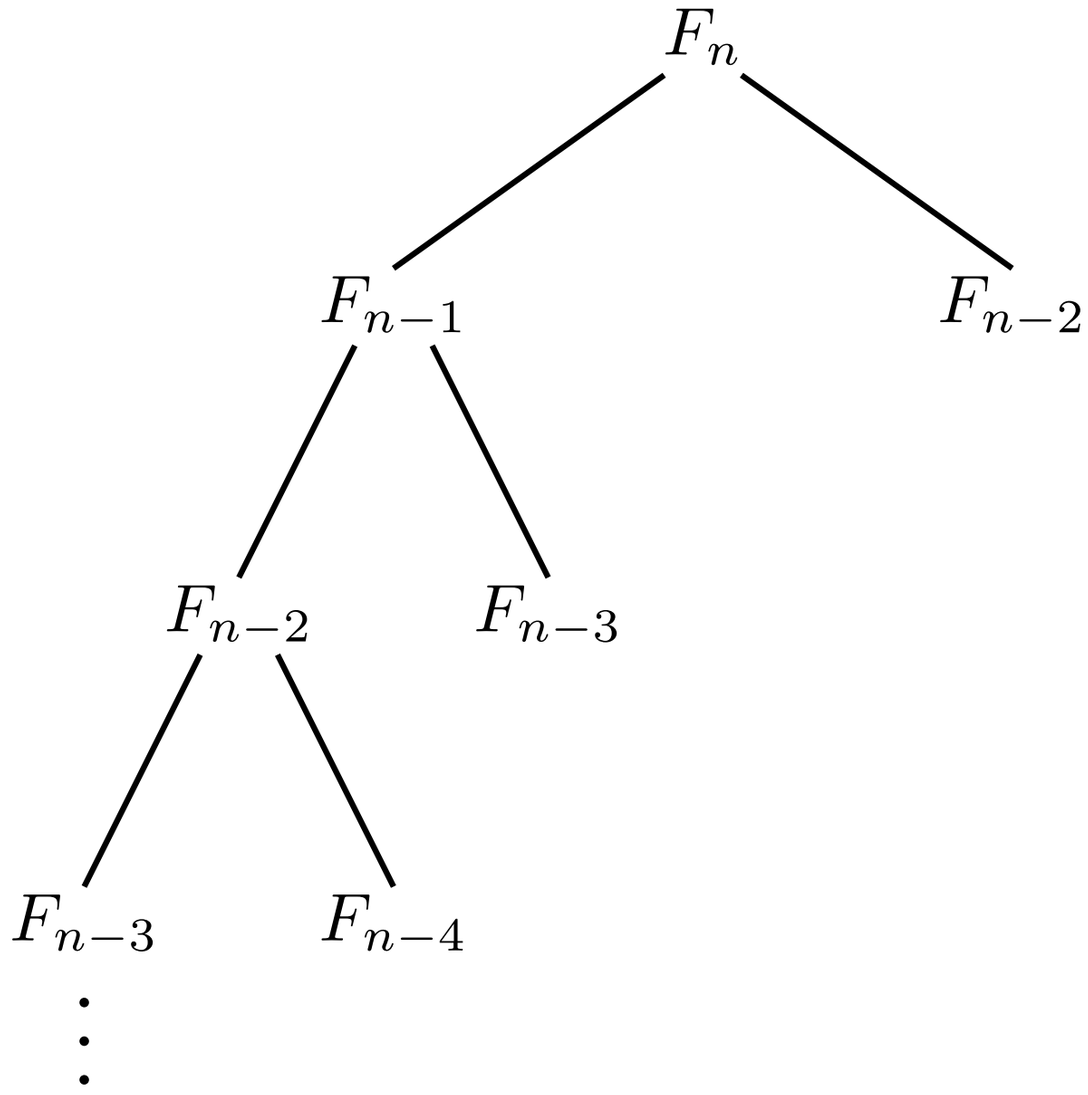
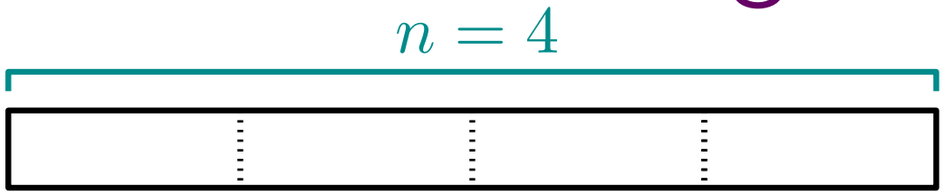
DP idé: gem og genbrug beregninger!

Obs: Fra ugeopgave 1 ved vi at F_n kan udregnes i $O(\log n)$ tid!



Stangudskæring (Rod cutting)



Værdier:

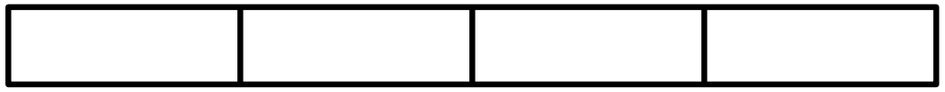
 $p_1 = 1$

 $p_2 = 5$

 $p_3 = 8$

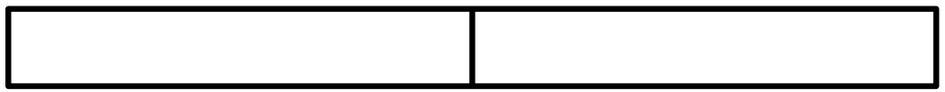
 $p_4 = 9$

Mulige udskæringer:

 4

 7

 9

 10

 9

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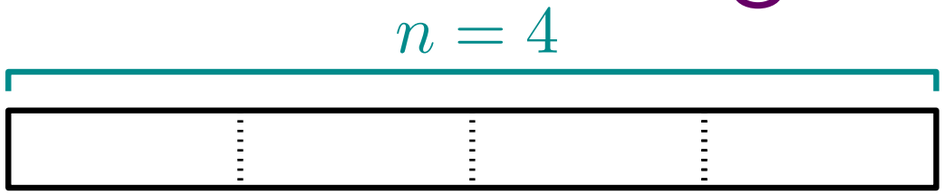

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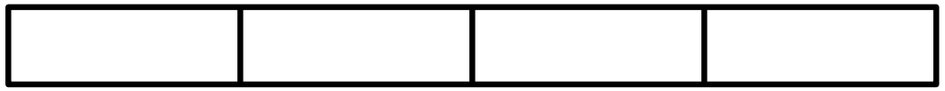 4

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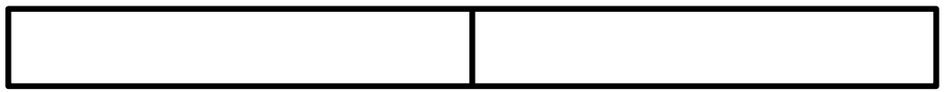 10

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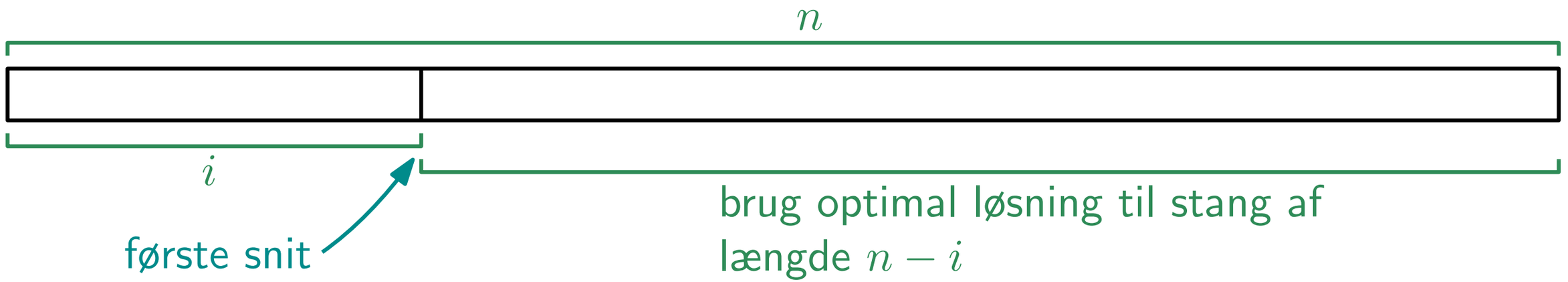
Naiv algoritme:
Prøv alle 2^{n-1} udskæringer.

Optimal delstruktur



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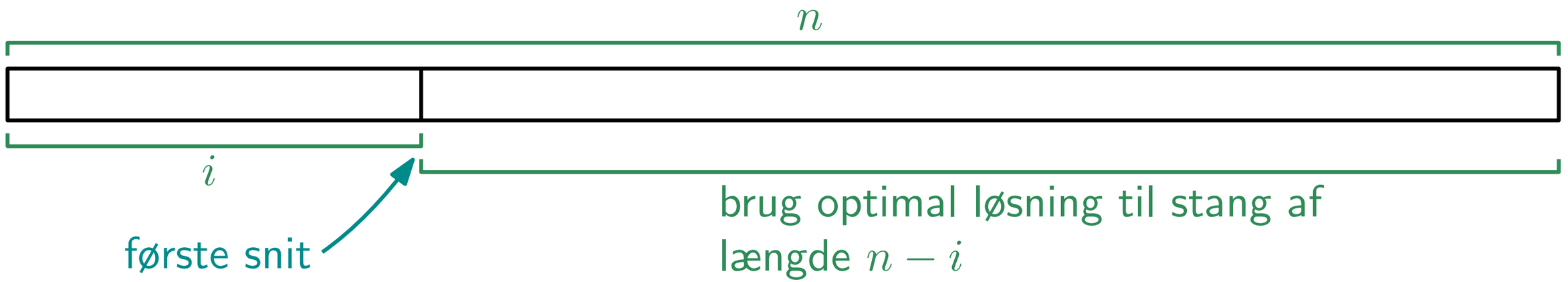


```
CUT-ROD(p, n)
1  if n == 0
2       return 0
3  q = -∞
4  for i = 1 to n
5       q = max {q, p[i] + CUT-ROD(p, n - i)}
6  return q
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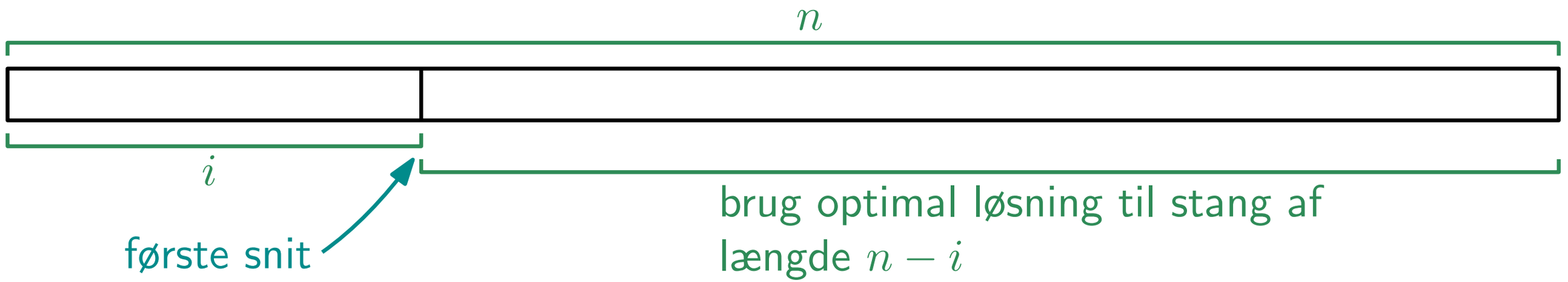


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Prøver alle muligheder $\Rightarrow T(n) = \Omega(2^{n-1})$

Memoisering

$r[n]$: Værdi af bedste udskæring af stang af længde n.

```
MEMOIZED-CUT-ROD(p, n)
1  let r[0:n] be a new array          // will remember solution values in r
2  for i = 0 to n
3      r[i] = −∞
4  return MEMOIZED-CUT-ROD-AUX(p, n, r)

MEMOIZED-CUT-ROD-AUX(p, n, r)
1  if r[n] ≥ 0                       // already have a solution for length n?
2      return r[n]
3  if n == 0
4      q = 0
5  else q = −∞
6      for i = 1 to n                // i is the position of the first cut
7          q = max{q, p[i] + MEMOIZED-CUT-ROD-AUX(p, n − i, r)}
8  r[n] = q                          // remember the solution value for length n
9  return q
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$r[n]$: Værdi af bedste udskæring af stang af længde n.

$O(1)$ tid, domineret af linje 7

$O(n)$ tid pr. gang (ekskl. tid til rekursive kald)
$n+1$ gange $\Rightarrow O(n^2)$ tid i alt.

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Top-down algoritme (start med størrelse n og lav rekursion)

Bottom-up

Start med små instanser og udvid

$$1 + 2 + 3 + \ldots + n = \Theta(n^2)$$

```
BOTTOM-UP-CUT-ROD(p, n)
1  let r[0:n] be a new array          // will remember solution values in r
2  r[0] = 0
3  for j = 1 to n                     // for increasing rod length j
4      q = -∞
5      for i = 1 to j                 // i is the position of the first cut
6          q = max{q, p[i] + r[j - i]}
7      r[j] = q                       // remember the solution value for length j
8  return r[n]
```

Har vi allerede udregnet!

$r[n]$: Værdi af bedste udskæring af stang af længde n.

Find udskæringen (ikke kun værdien)

```
EXTENDED-BOTTOM-UP-CUT-ROD(p, n)
 1  let r[0:n] and s[1:n] be new arrays
 2  r[0] = 0
 3  for j = 1 to n                    // for increasing rod length j
 4      q = -∞
 5      for i = 1 to j                // i is the position of the first cut
 6          if q < p[i] + r[j - i]
 7              q = p[i] + r[j - i]
 8              s[j] = i              // best cut location so far for length j
 9      r[j] = q                      // remember the solution value for length j
10  return r and s

PRINT-CUT-ROD-SOLUTION(p, n)
1  (r, s) = EXTENDED-BOTTOM-UP-CUT-ROD(p, n)
2  while n > 0
3      print s[n]          // cut location for length n
4      n = n - s[n]        // length of the remainder of the rod
```

$s[j] == i$: Skær stykke af længde i fra stang af længde j.

Hvad er s og r?

$n = 5$
$p[1 \ldots 5] = [1, 5, 8, 9, 10]$

```
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```

A: $r = [0, 1, 5, 8, 9, 10]$, $s = [1, 2, 3, 4, 5]$

B: $r = [0, 1, 5, 8, 10, 13]$, $s = [1, 2, 3, 2, 3]$

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C: $r = [1, 5, 8, 10, 13]$, $s = [1, 2, 3, 2, 2]$

D: $r = [0, 1, 5, 8, 10, 13]$, $s = [1, 2, 3, 2, 2]$

E: $r = [0, 1, 8, 8, 13, 16]$, $s = [1, 3, 3, 2, 3]$

Hvordan skal stangen udskæres?

$n = 10$

Når algoritmen er færdig:

$s[1 \ldots 10] = [1, 2, 3, 2, 5, 3, 7, 2, 9, 2]$

```
EXTENDED-BOTTOM-UP-CUT-ROD(p, n)
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```

$4, 6$

A

$2, 2, 2, 2, 2$

B

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$1, 3, 3, 3$

C

$1, 9$

D

$2, 2, 3, 3$

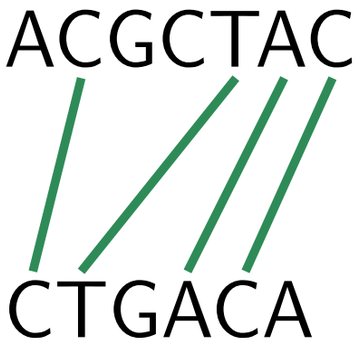
E

Længste fælles delsekvens (LCS)

Længste fælles delsekvens: CGCA
LCS(ACGCTAC,CTGACA) $= 4$

Motivation:
Mål for lighed mellem strenge

Længste fælles delsekvens (LCS)



Ikke unik løsning

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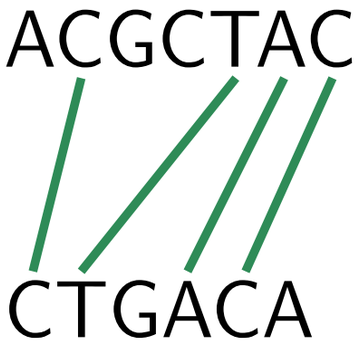

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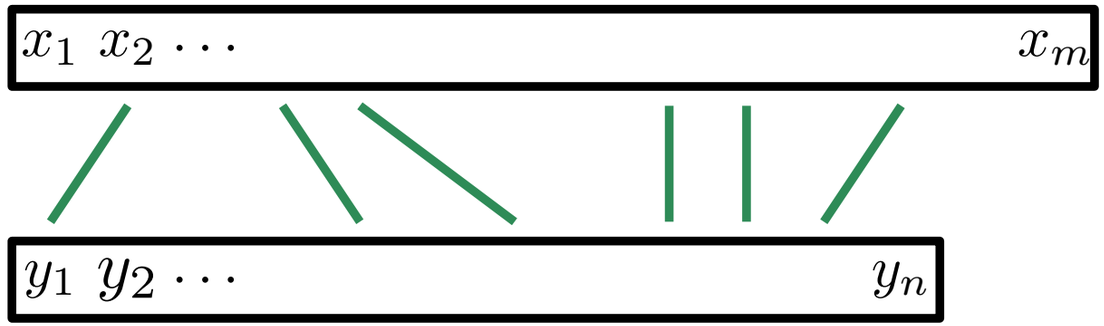
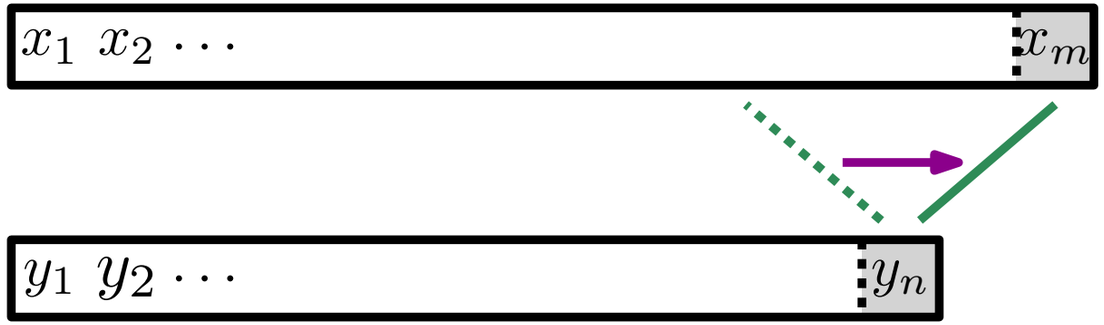


Find største ikke-krydsende parring

Optimal delstruktur

$X_i = \langle x_1, x_2, \ldots, x_i \rangle$
$Y_i = \langle y_1, y_2, \ldots, y_i \rangle$
Hele input: X_m og Y_n.

Tilfælde 1, $x_m == y_n$: Par x_m og y_n og brug største parring af X_{m-1} og Y_{n-1}.



Find LCS(X_{m-1}, Y_{n-1})

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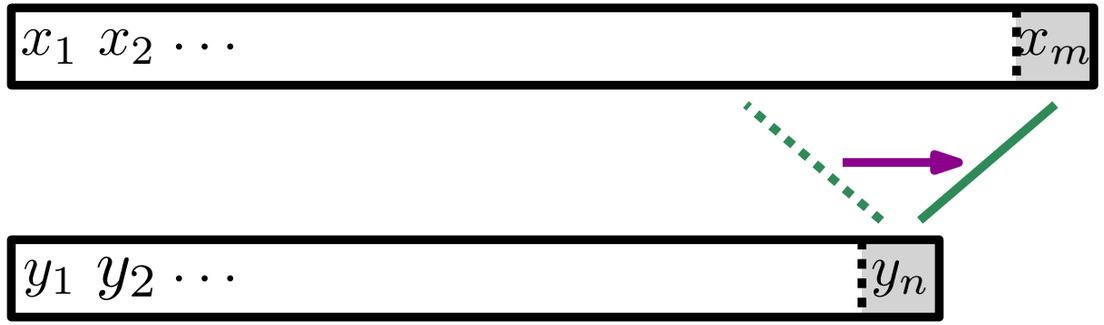


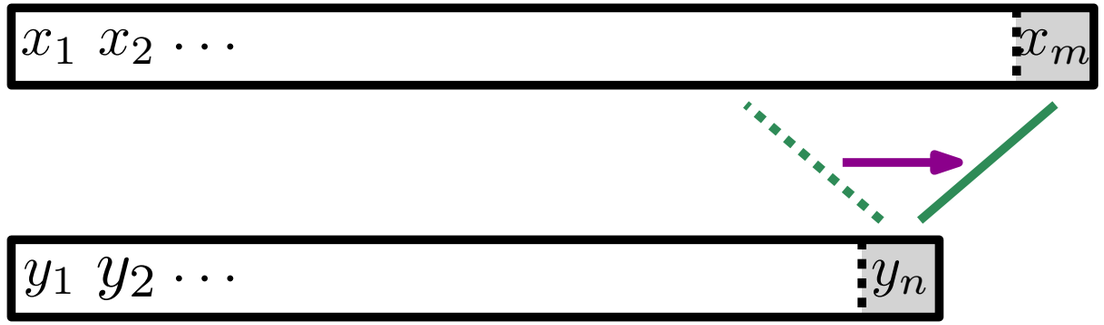
Find LCS(X_{m-1}, Y_{n-1})

LCS$(X_m, Y_n) = 1+$LCS(X_{m-1}, Y_{n-1})

Optimal delstruktur

$X_i = \langle x_1, x_2, \ldots, x_i \rangle$
$Y_i = \langle y_1, y_2, \ldots, y_i \rangle$
Hele input: X_m og Y_n.

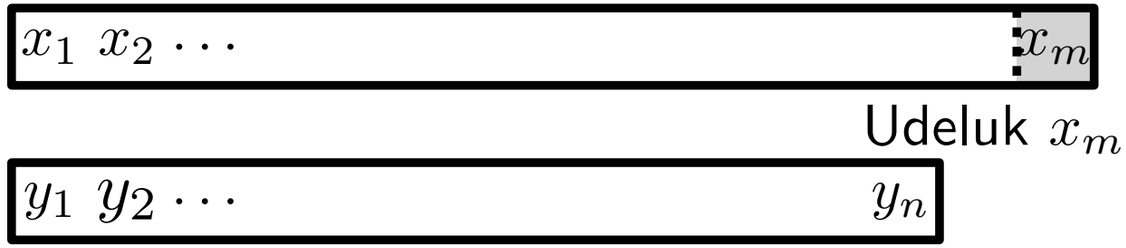
Tilfælde 1, $x_m == y_n$: Par x_m og y_n og brug største parring af X_{m-1} og Y_{n-1}.



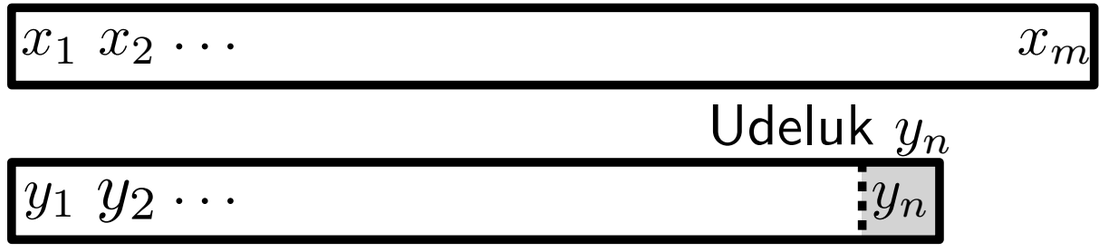
Find LCS(X_{m-1}, Y_{n-1})

LCS$(X_m, Y_n) = 1$+LCS(X_{m-1}, Y_{n-1})

Tilfælde 2, $x_m \neq y_n$: Udeluk x_m eller y_n. Find største parring i hvert tilfælde



Find LCS(X_{m-1}, Y_n)

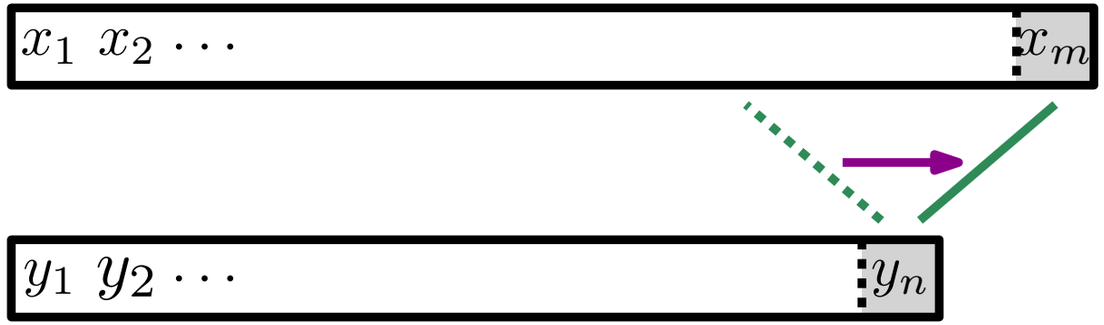


Find LCS(X_m, Y_{n-1})

Optimal delstruktur

$X_i = \langle x_1, x_2, \ldots, x_i \rangle$
$Y_i = \langle y_1, y_2, \ldots, y_i \rangle$
Hele input: X_m og Y_n.

Tilfælde 1, $x_m == y_n$: Par x_m og y_n og brug største parring af X_{m-1} og Y_{n-1}.



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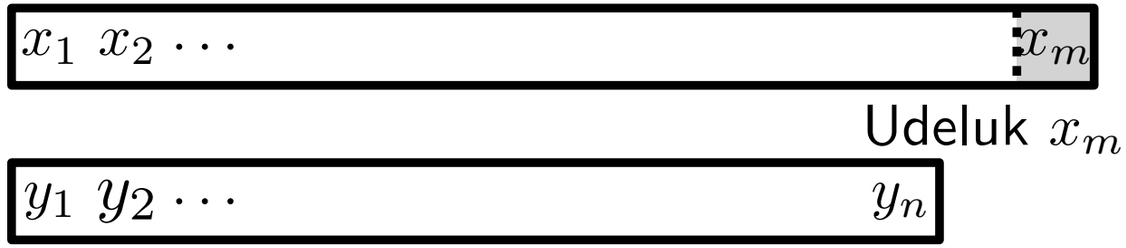
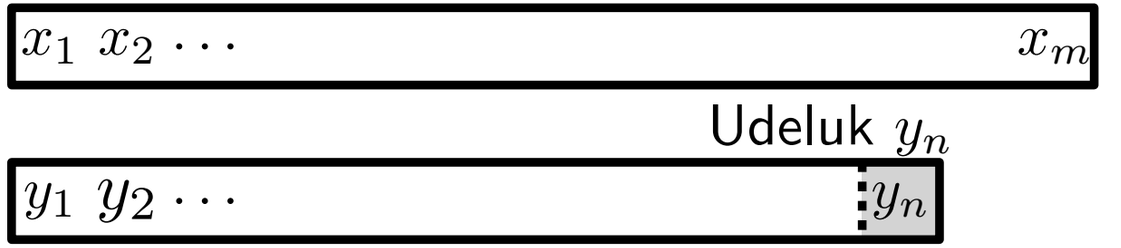


Find LCS(X_{m-1}, Y_n)



Find LCS(X_m, Y_{n-1})

LCS$(X_m, Y_n) =$
$\max\{$LCS(X_{m-1}, Y_n),LCS$(X_m, Y_{n-1})\}$

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
 3      c[i, 0] = 0
 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
 7      for j = 1 to n
 8          if x_i == y_j
 9              c[i, j] = c[i − 1, j − 1] + 1
10              b[i, j] = "↖"
11          elseif c[i − 1, j] ≥ c[i, j − 1]
12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0									
1	A								
2	B								
3	C								
4	B								
5	D								
6	A								
7	B								

Bottom-up algoritme

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0						
2	B		0						
3	C		0						
4	B		0						
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0					
2	B		0						
3	C		0						
4	B		0						
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0				
2	B		0						
3	C		0						
4	B		0						
5	D		0						
6	A		0						
7	B		0						

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Køretid: $O(mn)$

i	j	0	1	2	3	4	5	6
			B	D	C	A	B	A
0		0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0			
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

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15              b[i, j] = "←"
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```

Køretid: $O(mn)$

i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1		
2 B	0						
3 C	0						
4 B	0						
5 D	0						
6 A	0						
7 B	0						

Bottom-up algoritme

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```
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```

Køretid: $O(mn)$

i	j	0	1	2	3	4	5	6
			B	D	C	A	B	A
0		0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

Bottom-up algoritme

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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0						
3	C		0						
4	B		0						
5	D		0						
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Bottom-up algoritme

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1					
3	C		0						
4	B		0						
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```

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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1				
3	C		0						
4	B		0						
5	D		0						
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Bottom-up algoritme

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1			
3	C		0						
4	B		0						
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1		
3	C		0						
4	B		0						
5	D		0						
6	A		0						
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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	
3	C		0						
4	B		0						
5	D		0						
6	A		0						
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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0						
4	B		0						
5	D		0						
6	A		0						
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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
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12              c[i, j] = c[i − 1, j]
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14          else c[i, j] = c[i, j − 1]
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16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1				
4	B		0						
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
 3      c[i, 0] = 0
 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m          // compute table entries in row-major order
 7      for j = 1 to n
 8          if x_i == y_j
 9              c[i, j] = c[i − 1, j − 1] + 1
10              b[i, j] = "↖"
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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2			
4	B		0						
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

$c[i,j]$: $\mathsf{LCS}(X_i, Y_j)$

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2		
4	B		0						
5	D		0						
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Bottom-up algoritme

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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	
4	B		0						
5	D		0						
6	A		0						
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Bottom-up algoritme

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```
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 6  for i = 1 to m            // compute table entries in row-major order
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Køretid: $O(mn)$

i \ j	0	1	2	3	4	5	6
		B	D	C	A	B	A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0						
5 D	0						
6 A	0						
7 B	0						

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1					
5	D		0						
6	A		0						
7	B		0						

Bottom-up algoritme

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```
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15              b[i, j] = "←"
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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
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Bottom-up algoritme

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2			
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Bottom-up algoritme

$c[i,j]$: $\mathsf{LCS}(X_i, Y_j)$

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2		
5	D		0						
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Bottom-up algoritme

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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	
5	D		0						
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Bottom-up algoritme

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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
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Bottom-up algoritme

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```

Køretid: $O(mn)$

i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5 D	0	↑ 1	↖ 2				
6 A	0						
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Bottom-up algoritme

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 6  for i = 1 to m              // compute table entries in row-major order
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Køretid: $O(mn)$

i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5 D	0	↑ 1	↖ 2	↑ 2			
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Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2		
6	A		0						
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Bottom-up algoritme

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i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	
6	A		0						
7	B		0						

Bottom-up algoritme

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LCS-LENGTH(X, Y, m, n)
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 7      for j = 1 to n
 8          if x_i == y_j
 9              c[i, j] = c[i − 1, j − 1] + 1
10              b[i, j] = "↖"
11          elseif c[i − 1, j] ≥ c[i, j − 1]
12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0						
7	B		0						

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
 3      c[i, 0] = 0
 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
 7      for j = 1 to n
 8          if x_i == y_j
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10              b[i, j] = "↖"
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12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1					
7	B		0						

Bottom-up algoritme

$c[i,j]$: $\mathsf{LCS}(X_i, Y_j)$

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
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 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
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12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2				
7	B		0						

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
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13              b[i, j] = "↑"
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15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2			
7	B		0						

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
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10              b[i, j] = "↖"
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14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5 D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6 A	0	↑ 1	↑ 2	↑ 2	↖ 3		
7 B	0						

Bottom-up algoritme

$c[i,j]$: $\mathsf{LCS}(X_i, Y_j)$

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
 3      c[i, 0] = 0
 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
 7      for j = 1 to n
 8          if x_i == y_j
 9              c[i, j] = c[i − 1, j − 1] + 1
10              b[i, j] = "↖"
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12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	
7	B		0						

Bottom-up algoritme

$c[i,j]$: $\mathsf{LCS}(X_i, Y_j)$

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
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 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
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 8          if x_i == y_j
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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0						

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
 3      c[i, 0] = 0
 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m          // compute table entries in row-major order
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14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j 0	1 B	2 D	3 C	4 A	5 B	6 A
0		0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B	0	↖ 1					

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2				

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2			

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
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 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
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 5      c[0, j] = 0
 6  for i = 1 to m              // compute table entries in row-major order
 7      for j = 1 to n
 8          if x_i == y_j
 9              c[i, j] = c[i − 1, j − 1] + 1
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16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2	↑ 3		

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
 2  for i = 1 to m
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 4  for j = 0 to n
 5      c[0, j] = 0
 6  for i = 1 to m          // compute table entries in row-major order
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 8          if x_i == y_j
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16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

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14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

Bottom-up algoritme

$c[i,j]$: LCS(X_i, Y_j)

```
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16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
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12              c[i, j] = c[i − 1, j]
13              b[i, j] = "↑"
14          else c[i, j] = c[i, j − 1]
15              b[i, j] = "←"
16  return c and b
```

Køretid: $O(mn)$

i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5 D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6 A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7 B	0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

Bottom-up algoritme

$c[i, j]$: LCS(X_i, Y_j)

```
LCS-LENGTH(X, Y, m, n)
 1  let b[1:m, 1:n] and c[0:m, 0:n] be new tables
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 4  for j = 0 to n
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 6  for i = 1 to m              // compute table entries in row-major order
 7      for j = 1 to n
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 9              c[i, j] = c[i − 1, j − 1] + 1
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11          elseif c[i − 1, j] ≥ c[i, j − 1]
12              c[i, j] = c[i − 1, j]
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14          else c[i, j] = c[i, j − 1]
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16  return c and b
```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B		0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

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2 B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
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i \ j	0	1 B	2 D	3 C	4 A	5 B	6 A
0	0	0	0	0	0	0	0
1 A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
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3 C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4 B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5 D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
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Køretid: $O(mn)$

i	j	0	1	2	3	4	5	6
			B	D	C	A	B	A
0		0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
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```

Køretid: $O(mn)$

i		j	0	1	2	3	4	5	6
				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D		0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A		0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
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Køretid: $O(mn)$

i	j	0	1	2	3	4	5	6
			B	D	C	A	B	A
0		0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
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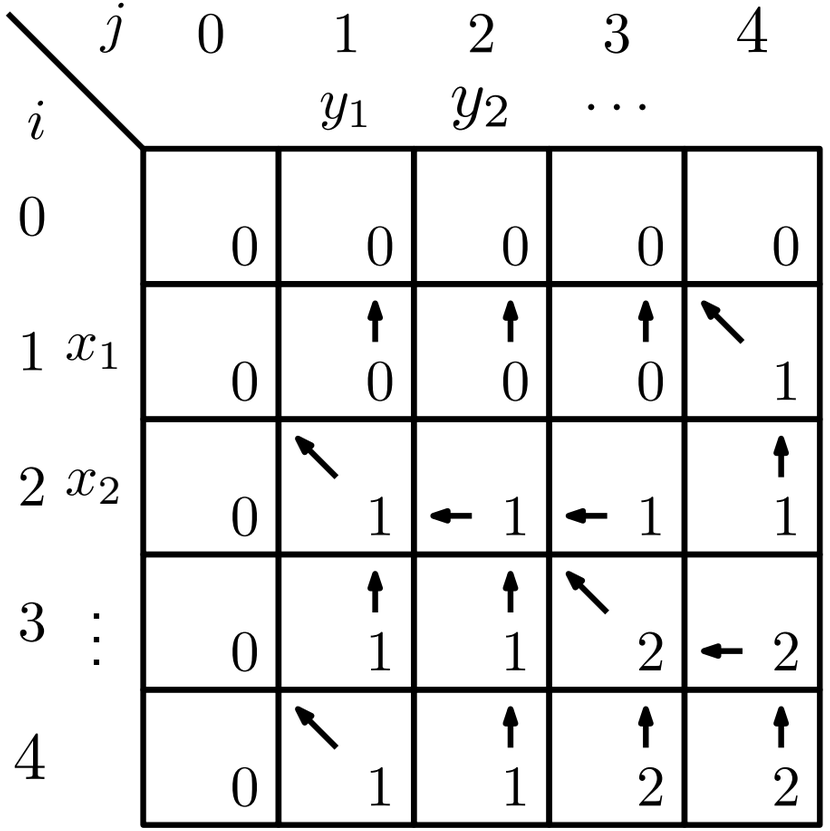
6	A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B	0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

Hvilke strenge passer til tabellen?

socrative.com → Student login,
Room name: ABRAHAMSEN3464



$X = ABCA$
$Y = BDCA$

A

$X = ABCB$
$Y = BDCA$

B

$X = ABCB$
$Y = BDC$

C

$X = BBCB$
$Y = BDCA$

D

$X = ADCB$
$Y = BDCA$

E

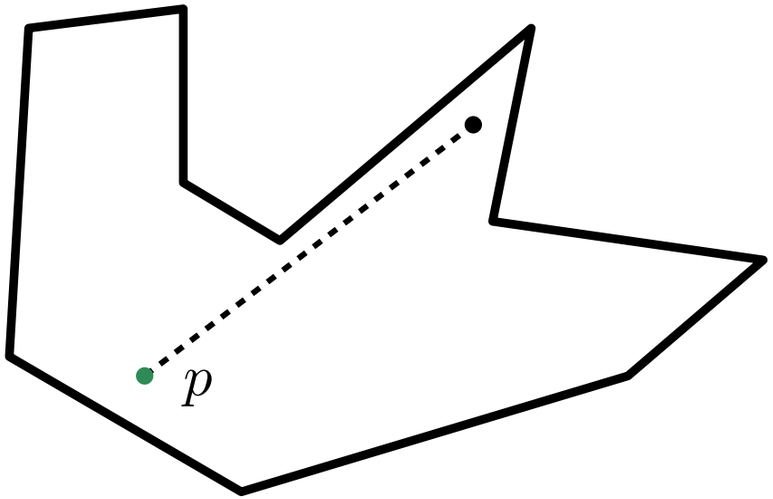
Top down vs. bottom up

Bottom up:
Undgår rekursion
For-løkker hurtigere end rekursive kald (mindre konstanter i O-notation)

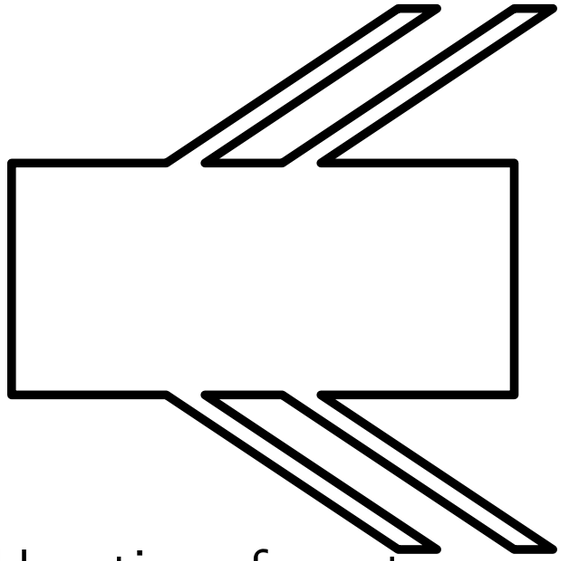
Top down:
Ofte simplere
Løser kun nødvendige delproblemer

Konklusion:
Kommer an på det konkrete problem og personlige præferencer.

Minimum stjerne-partition

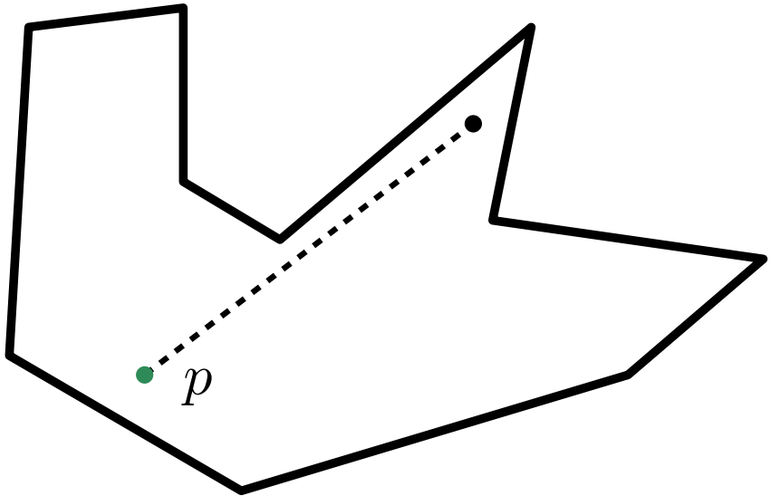


Stjerneformet, p kan "se" alle andre punkter

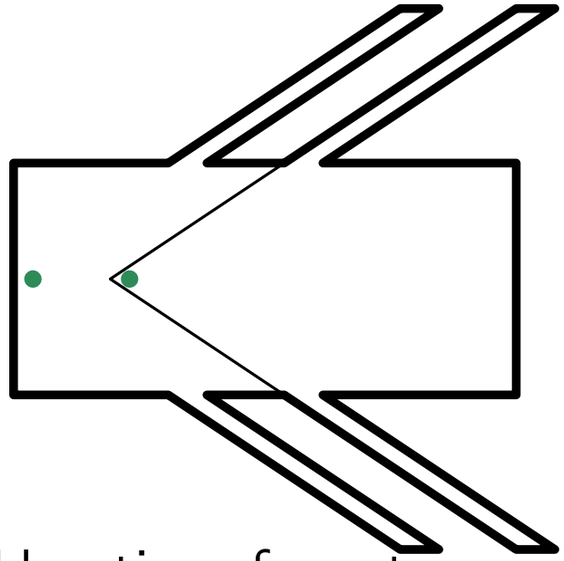


Ikke stjerneformet

Minimum stjerne-partition

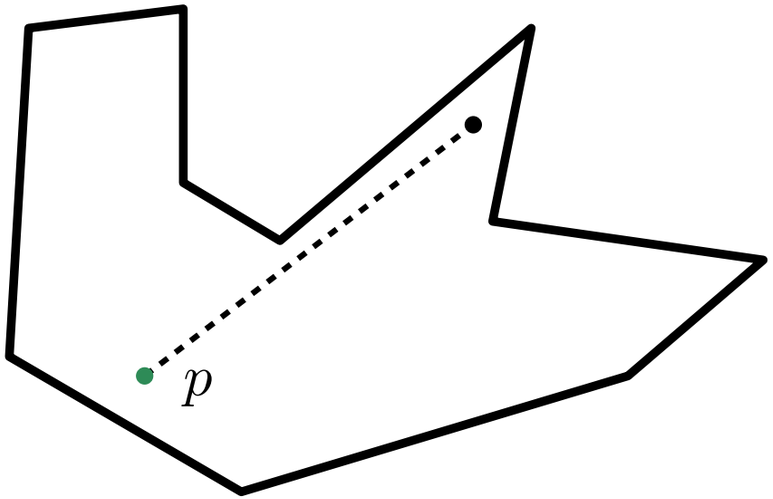


Stjerneformet, p kan "se" alle andre punkter



Ikke stjerneformet

Minimum stjerne-partition



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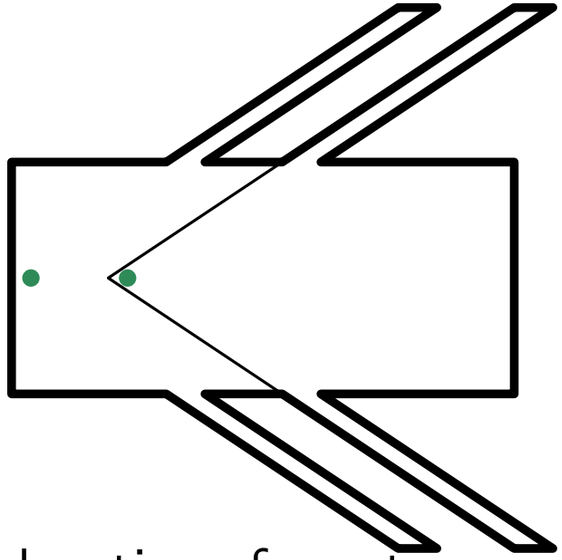
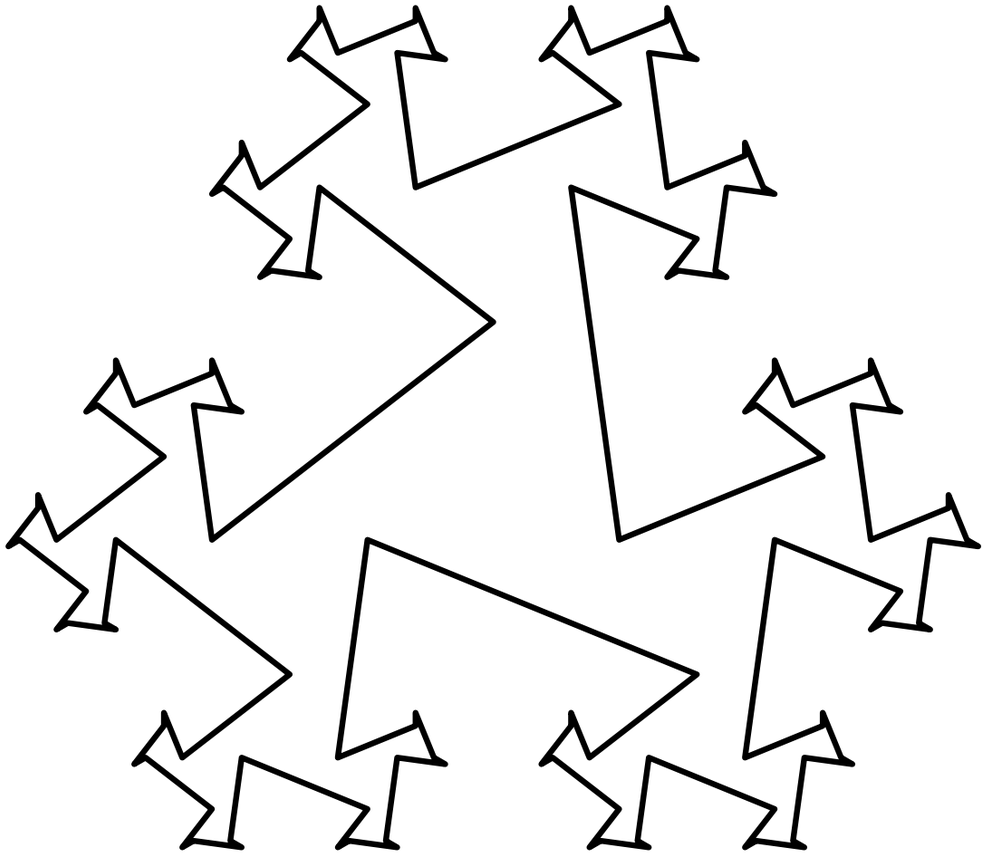
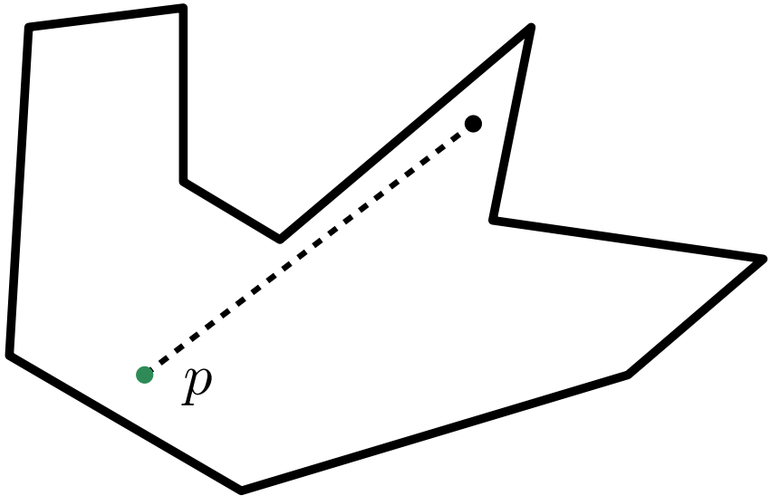


Ikke stjerneformet



Problem: Give polygon P med n hjørner, inddel P i minimum antal stjerneformede polygoner

Minimum stjerne-partition



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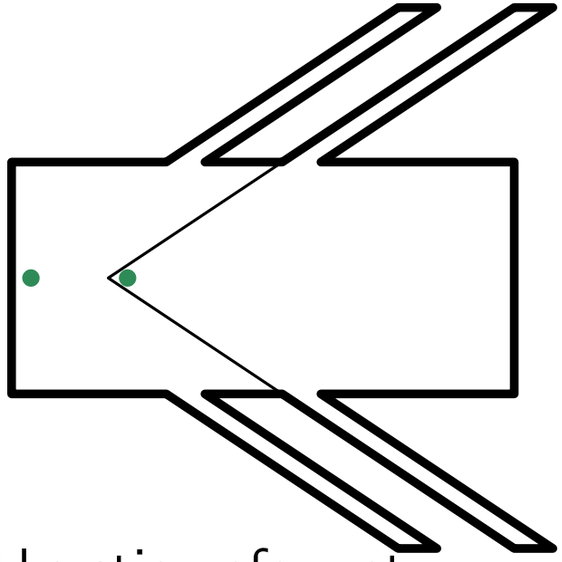
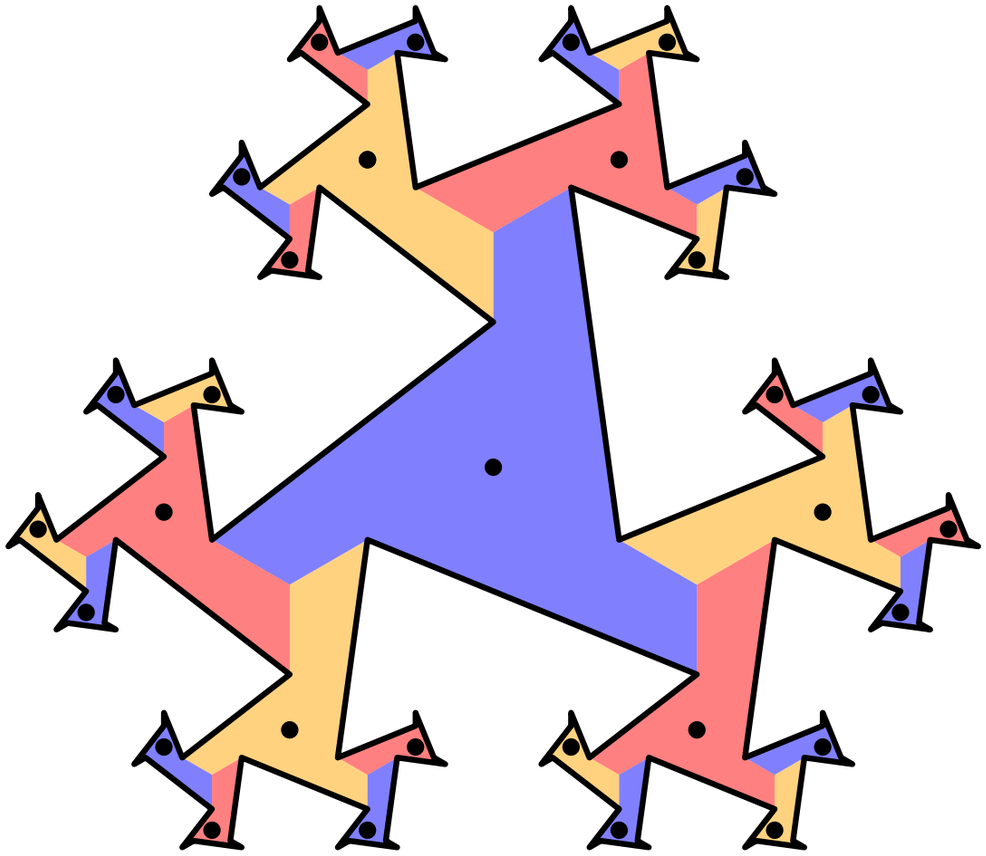


Ikke stjerneformet



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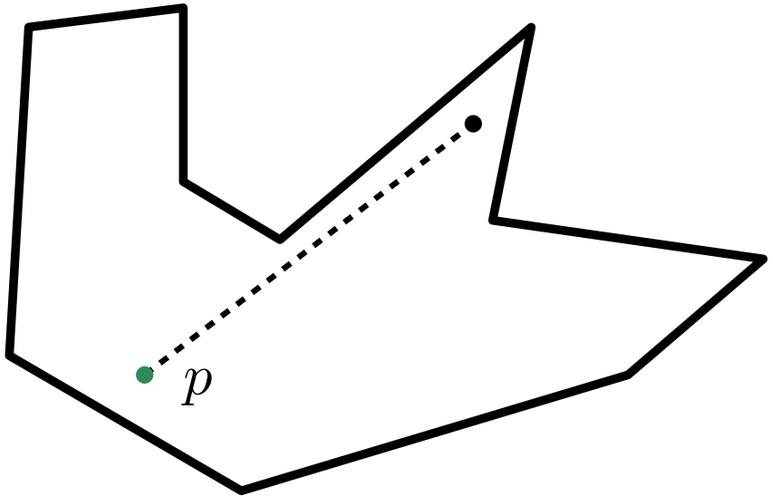
Minimum stjerne-partition



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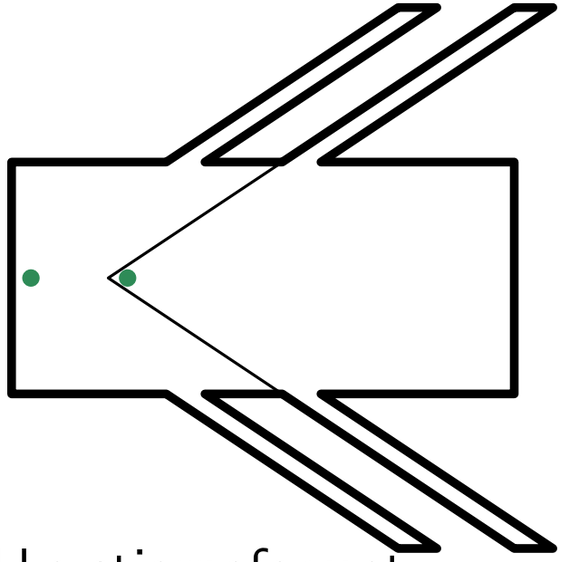
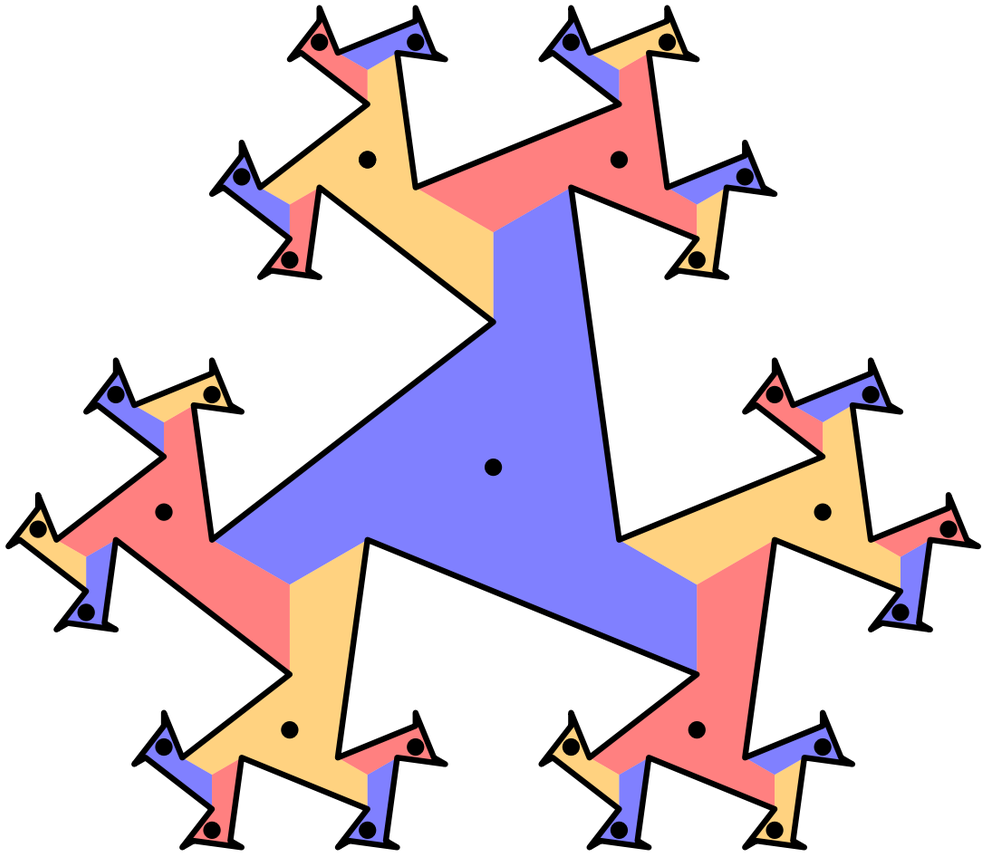


Ikke stjerneformet



Problem: Give polygon P med n hjørner, inddel P i minimum antal stjerneformede polygoner

Resultat: Kan gøres i $O(n^{107})$ tid vha. DP!
Vi løser $O(n^{70})$ delproblemer.

Abrahamsen, Blikstad, Nusser, Zhang: Minimum Star Partitions of Simple Polygons in Polynomial Time