PENGOLAHAN SINYAL DIGITAL

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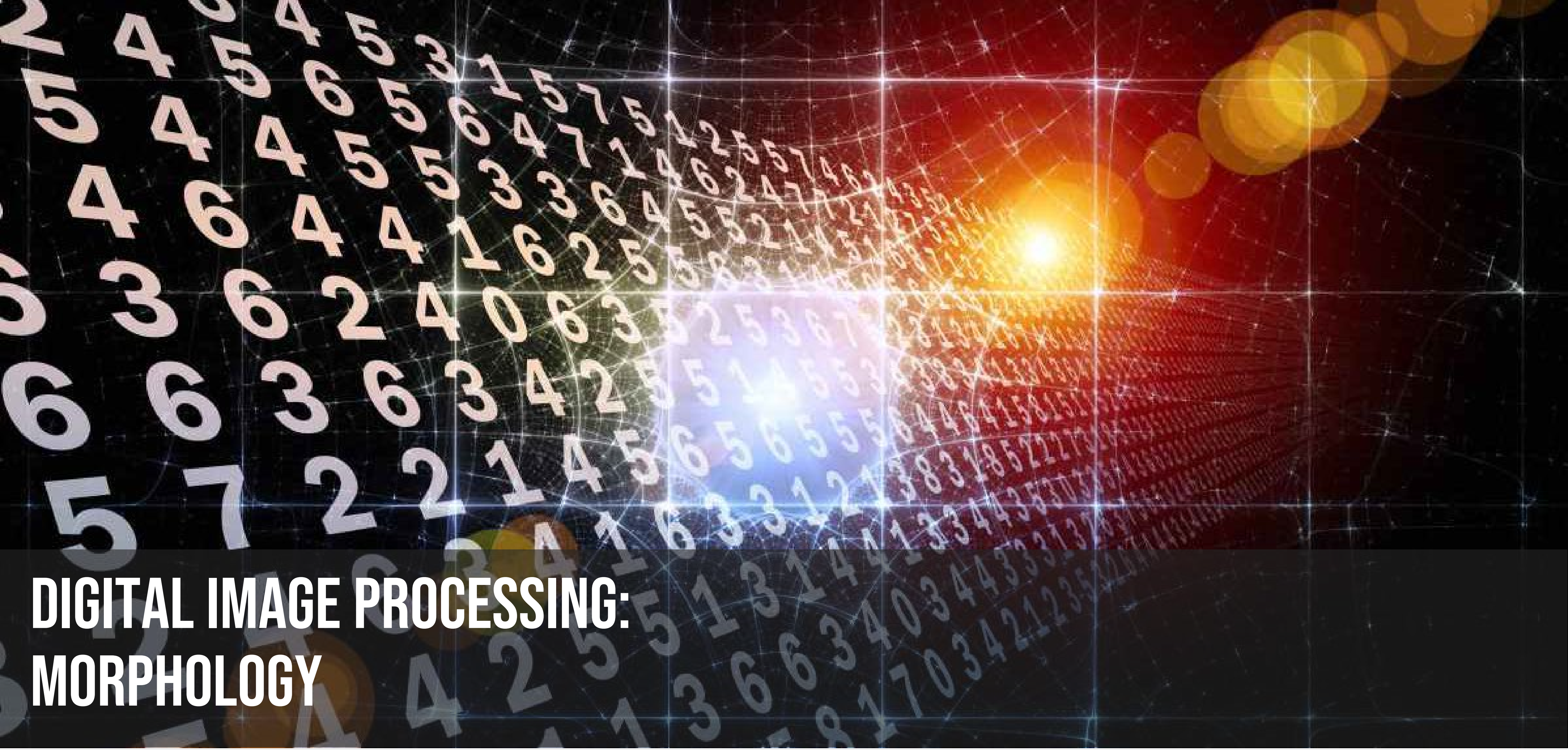

DIGITAL IMAGE PROCESSING: MORPHOLOGY

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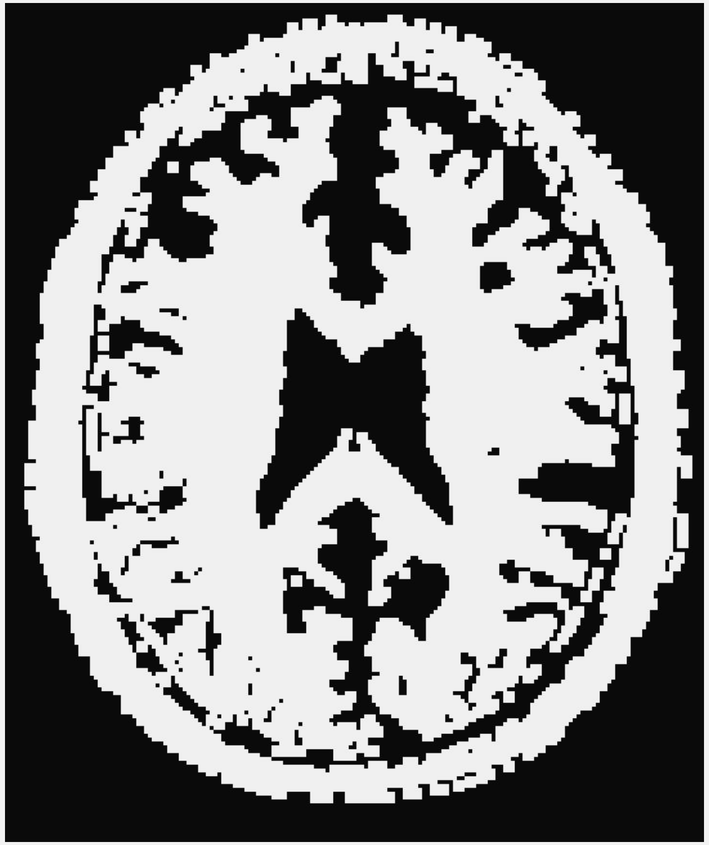
WHAT IS MORPHOLOGY?

- Back to previous lecture 1 …



Dilation
Erosion

PRELIMINARIES – SET THEORY

- Let A be a set in Z2. If a=(a1,a2) is an element of A, then we write $a \in A$ (if not, $a \notin A$)
- If A contains no element, it is called null or empty set $\varnothing$
- If every element of a set A is also an element of another set B, then A is said to be a subset of B, denoted as $A \subseteq B$
- More definitions:

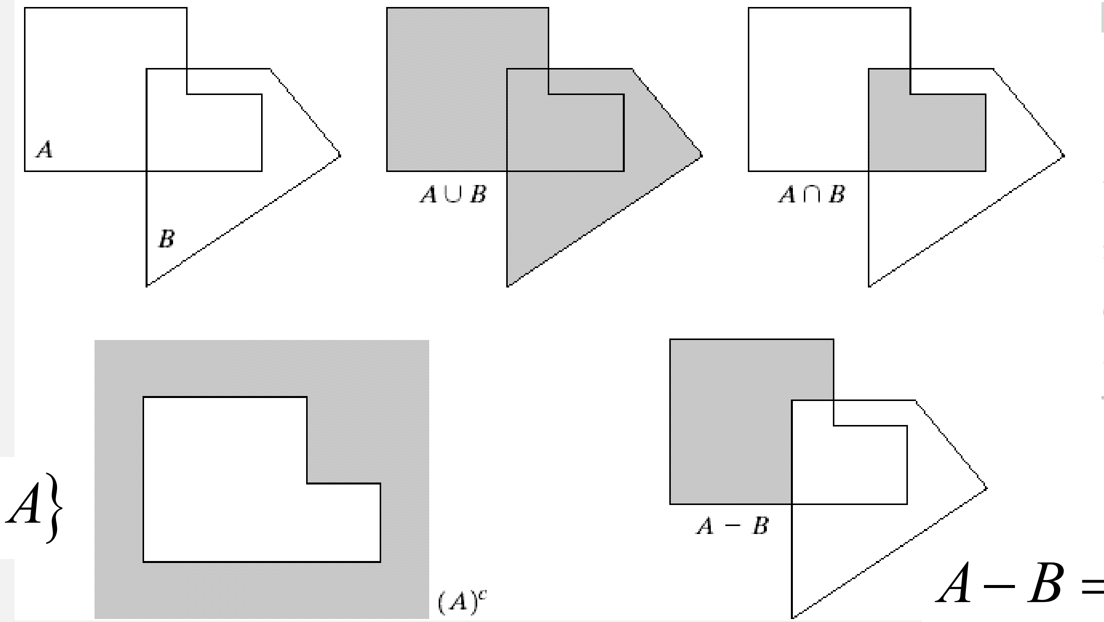


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

$$A^c = \{w \mid w \notin A\}$$

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

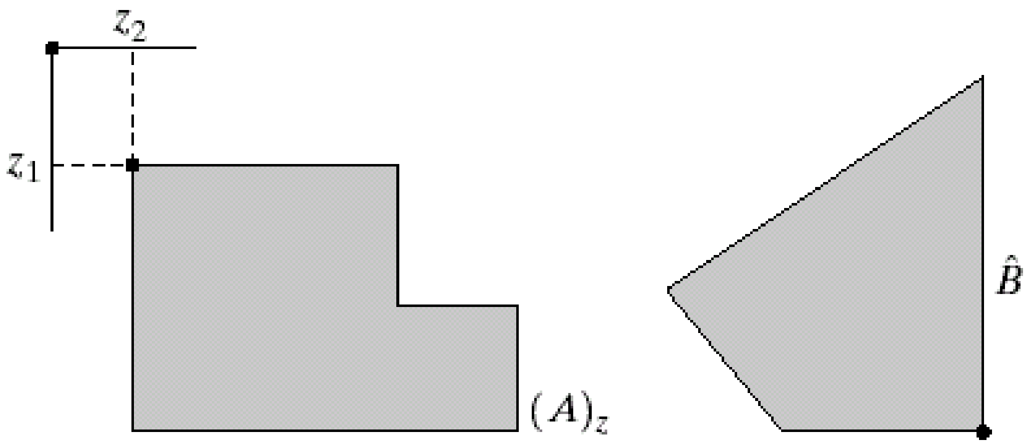
PRELIMINARIES – SET THEORY (MORE)

- Two more definitions particularly important to morphology:
  - *Reflection* of set *B*, denoted $\hat{B}$, is

$$\hat{B} = \{ w \mid w = -b, \quad \text{for} \quad b \in B \}$$

  - *Translation* of set *A* by point $z=(z_1,z_2)$, denoted $(A)_Z$, is

$$(A)_Z = \{ c \mid c = a + z, \quad \text{for} \quad a \in A \}$$



a b

FIGURE 9.2 (a) Translation of *A* by *z*. (b) Reflection of *B*. The sets *A* and *B* are from Fig. 9.1.

DILATION & EROSION

- Most primitive operations in morphology.
- All other operations are built from a combination of these two.

- Dilation is an operation that ”grows” or ”thickens” objects in an image
- Erosion ”shrinks” or ”thins” objects in a binary image

DILATION

- Dilation of A by B, denoted $A \oplus B$, is defined as

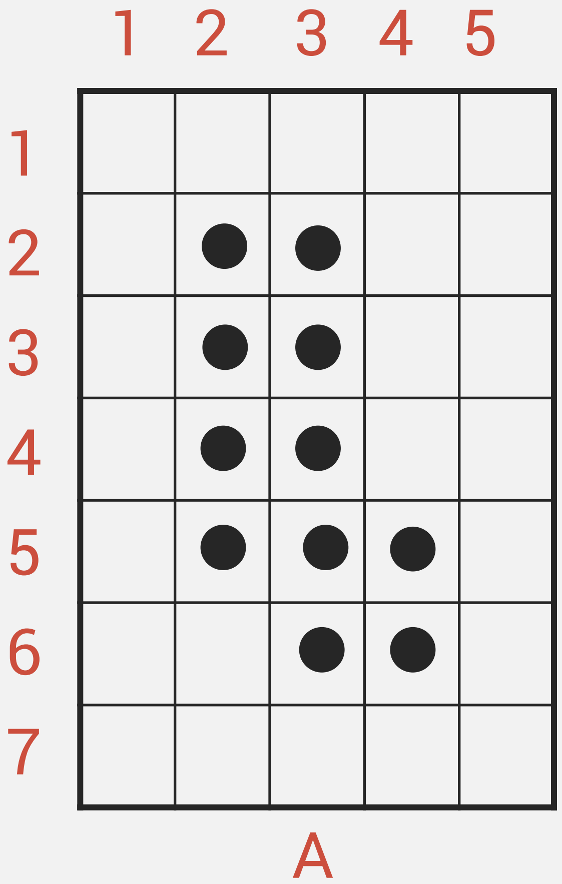
$$A \oplus B = \bigcup_{z \in B} A_z = \{(x,y)+(u,v) : (x,y) \in A, (u,v) \in B\}$$

- Intuition: for every point $z \in B$, we translate A by those coordinates. We then take the union of all these translation.
- From the definition, we see that dilation is commutative,

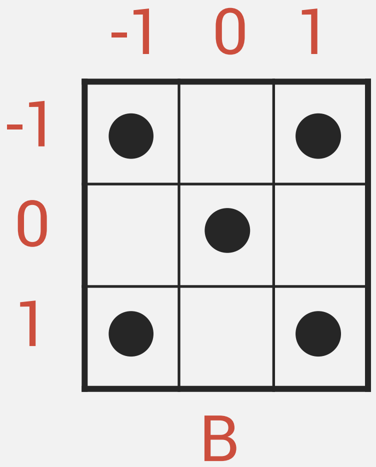
$$A \oplus B = B \oplus A$$

- B is referred to as a structuring element or as a kernel.

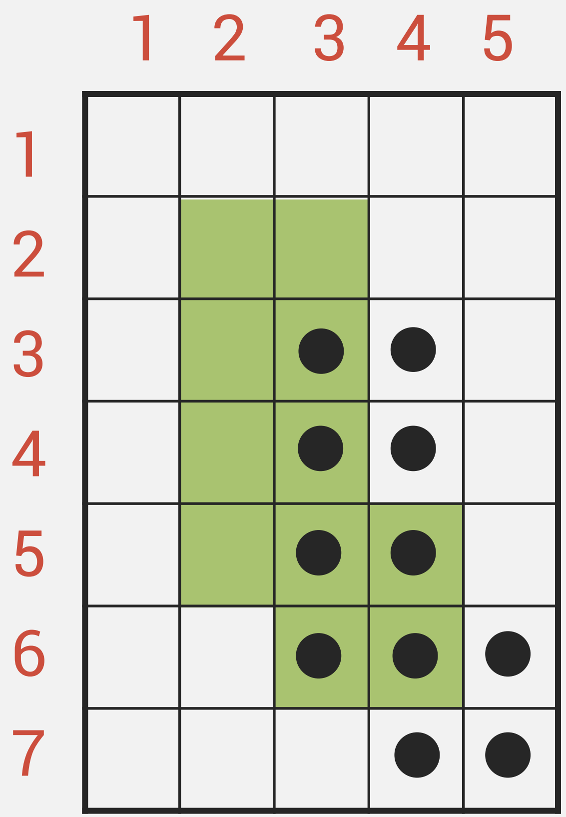
EXAMPLE



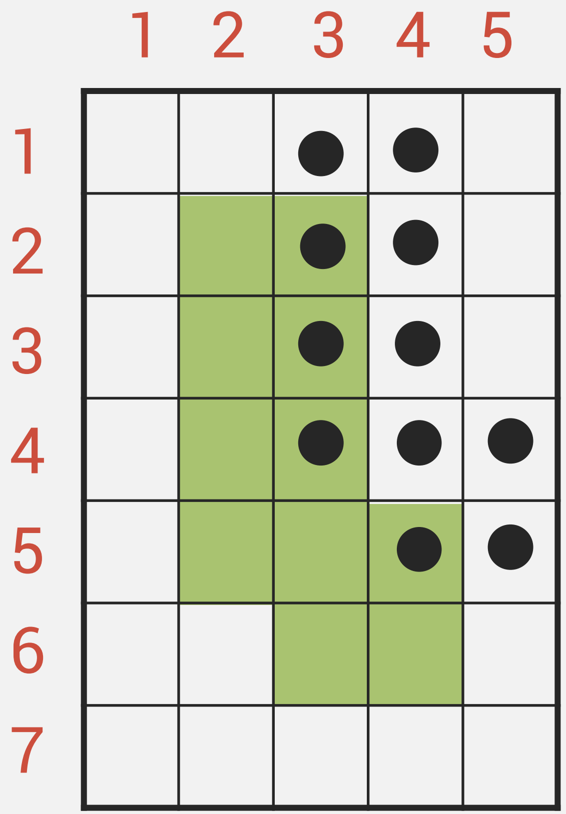
$A_{(0,0)}$ is itself



$A_{(1,1)}$ & $A_{(-1,1)}$

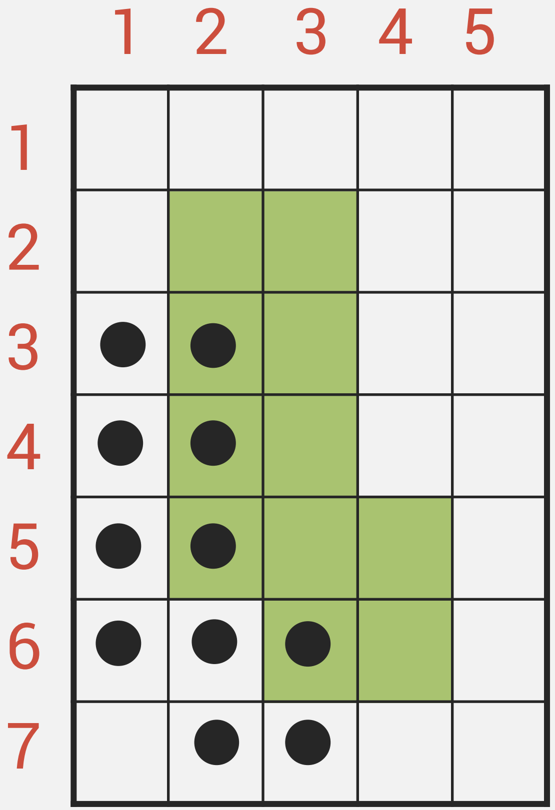


$A_{(1,1)}$

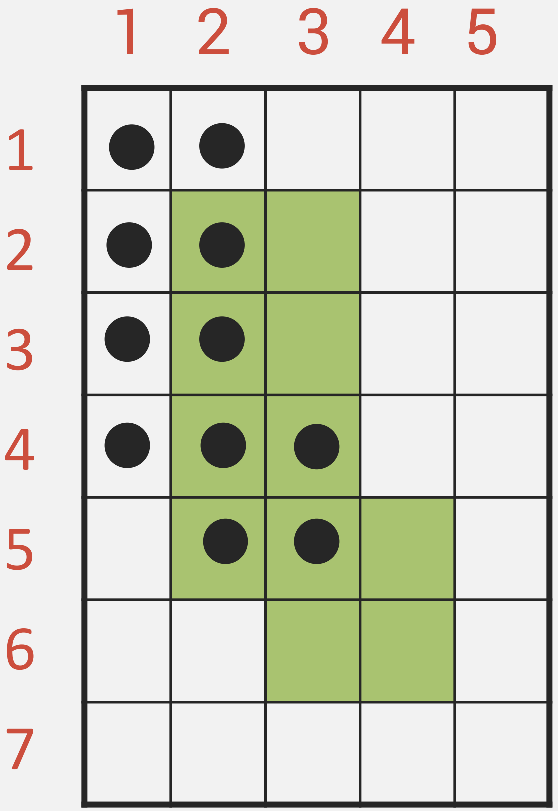


$A_{(-1,1)}$

$A_{(1,-1)}$ & $A_{(-1,-1)}$

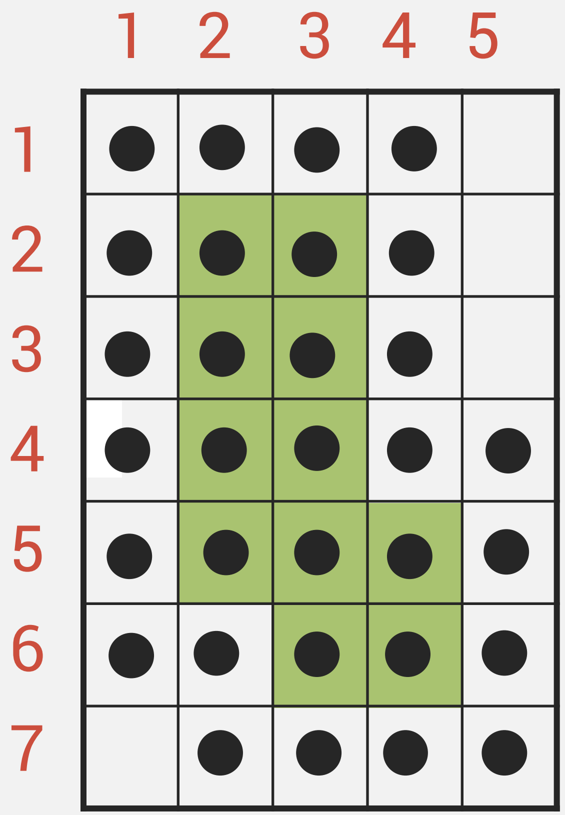


$A_{(1,-1)}$



$A_{(-1,-1)}$

RESULT



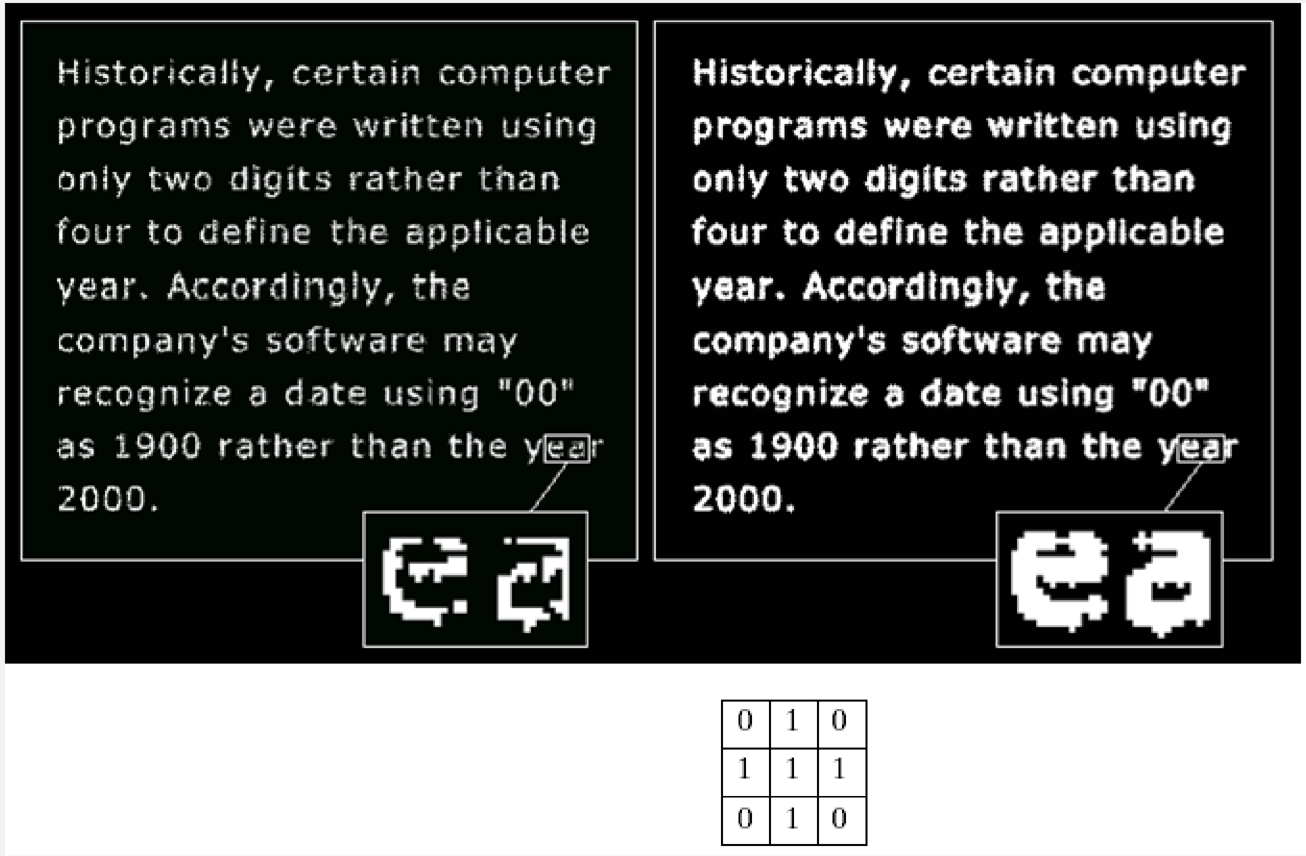
$A \oplus B$

SOME REMARKS

- Dilation has the effect of increasing the size of an object.
- However, it is not necessarily true that the original object A will lie within its dilation.
    - Try out B={(7,3),(6,2),(6,4),(8,2),(8,4)}

APPLICATION

- A simple application is bridging gaps.
    - Max length of break is 2 pixels



a c
b

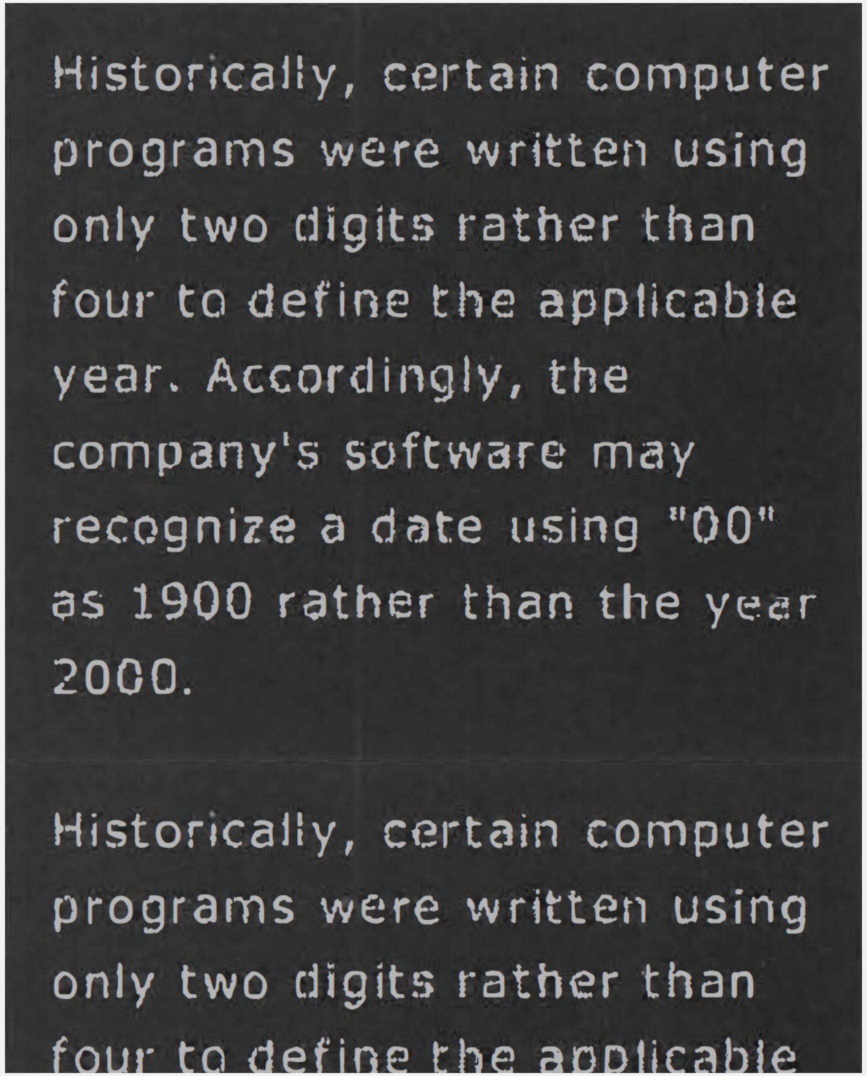
FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

EXAMPLE

```
img = imread('dsp_broken_text.jpg');
B   = [ 0 1 0 ; 1 1 1 ; 0 1 0 ] ;
imgDilate =  imdilate( img , B ) ;
figure;
subplot(121); imshow(img); title(‘Original Broken Image');
subplot(122); imshow(imgDilate); title('Dilated Broken Image');
```

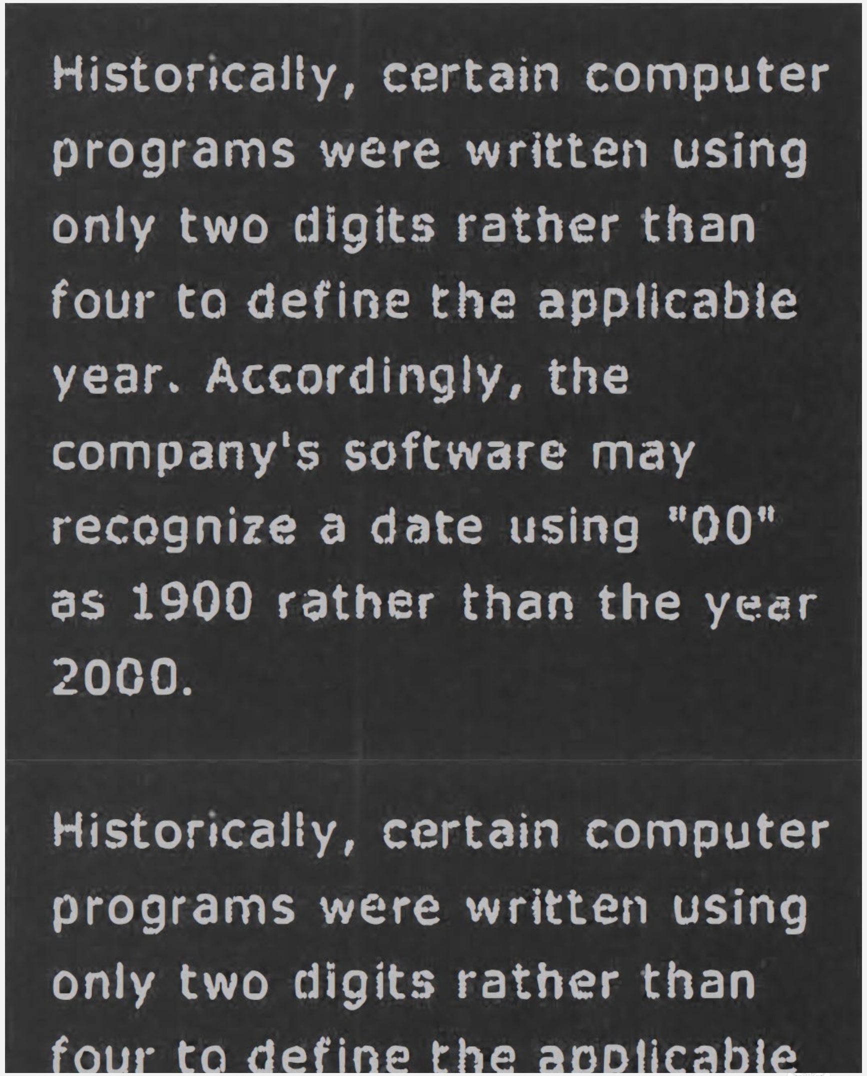
EXAMPLE

Iriginal Broken Image


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable

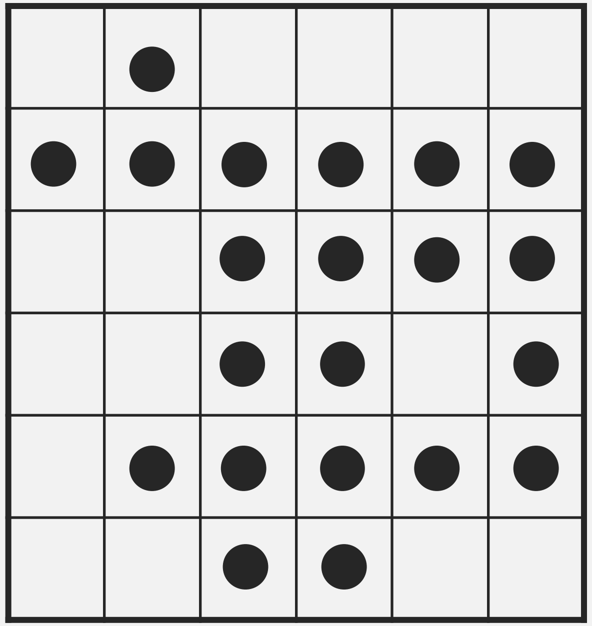
Dilated Broken Image


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

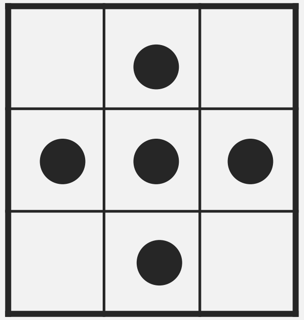
Historically, certain computer programs were written using only two digits rather than four to define the applicable

EROSION

- Erosion of *A* by *B* is $A \Theta B = \{w : B_w \subseteq A\}$
- Intuition: all point *w=(x,y)* for which B_w is in *A*.
- Consider *A* and *B* below:

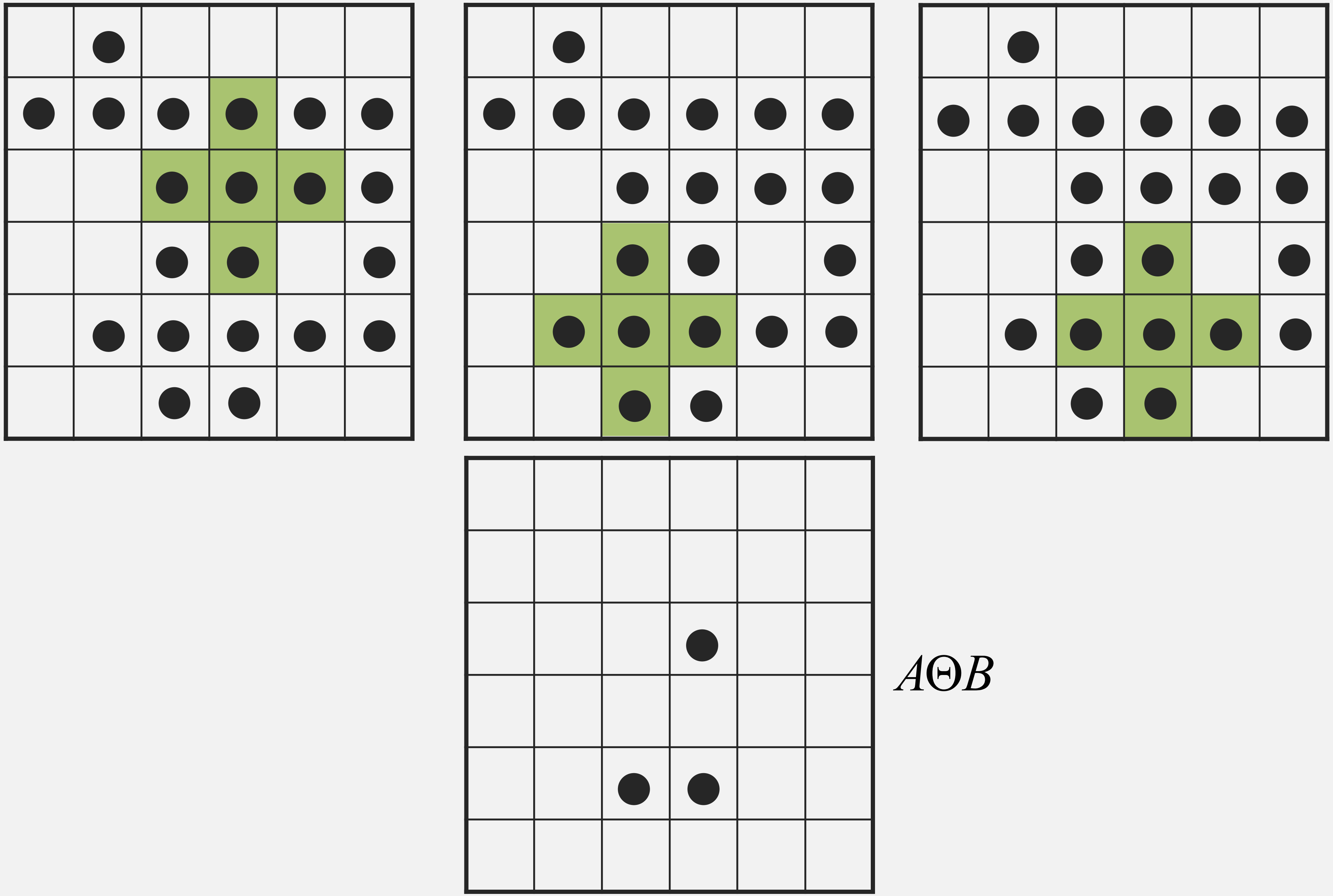


A



B

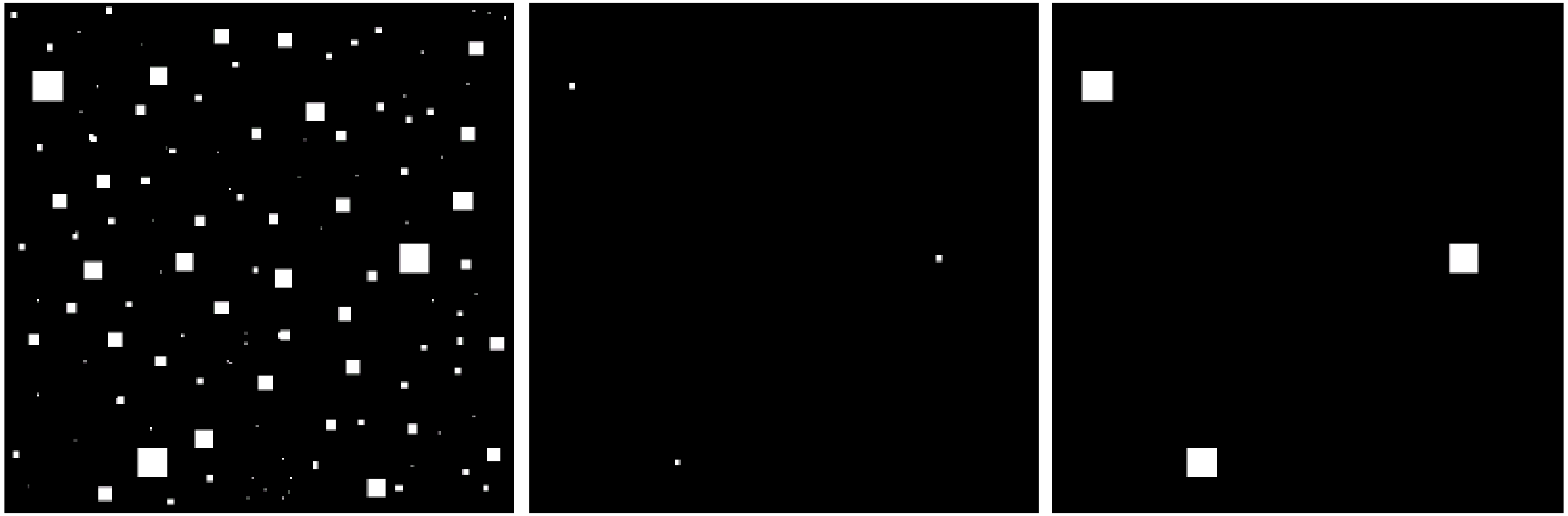
MOVING B AROUND



$A \Theta B$

APPLICATION

- One simple application is eliminating irrelevant detail from a binary image.



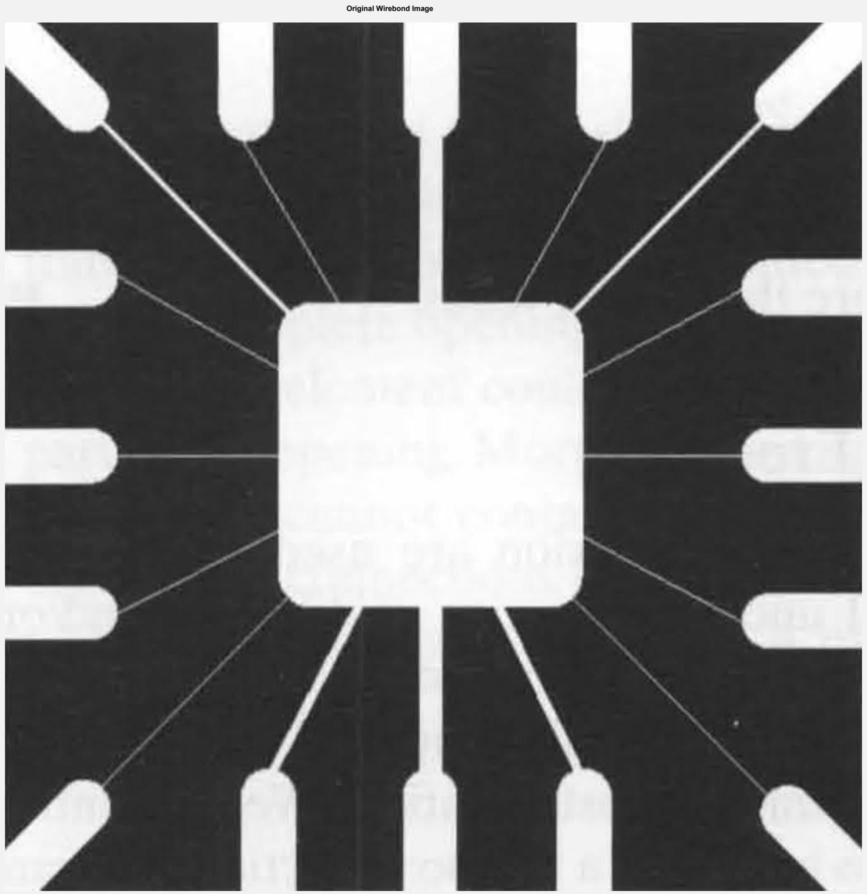
a b c

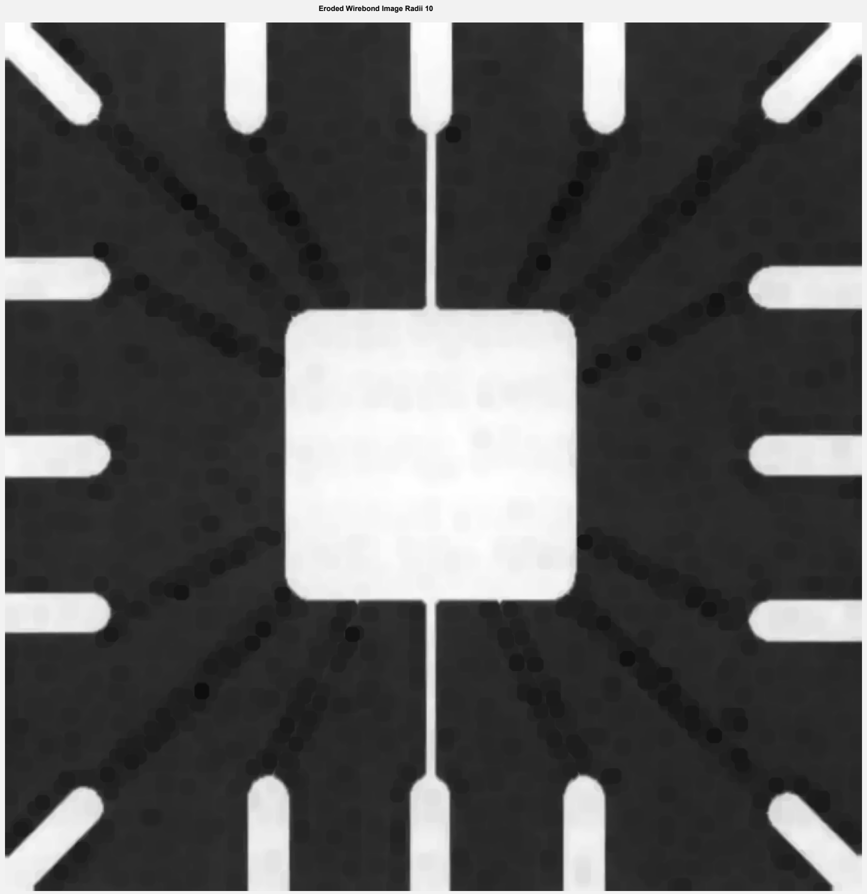
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

EXAMPLE

```
img = imread('dsp_wirebond_mask.jpg');
se = strel('disk' , 10);
imgErode =  imerode( img , se ) ;

figure;
subplot(121); imshow(img);
title('Original Wirebond Image');
subplot(122); imshow(imgErode);
title('Eroded Wirebond Image Radii 10');
```

EXAMPLE





COMBINING DILATION AND EROSION

- In image-processing applications, dilation and erosion are used most often in various combinations.

- An image will undergo a series of dilations and/or erosions using the same, or sometimes different, structuring elements

OPENING AND CLOSING

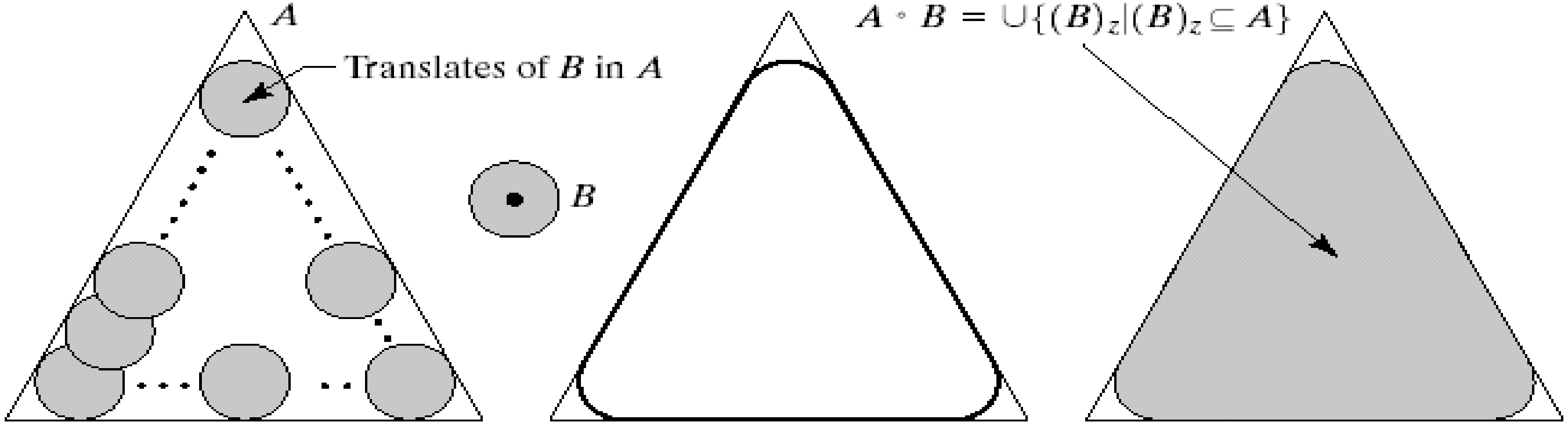
- Some combination of dilation and erosion
- *Opening*: breaking narrow is thmuses, and eliminating protrusions.

$$A \circ B = \left(A \Theta B \right) \oplus B$$

- *Closing*: fusing narrow breaks and long thin gulfs.

$$A \bullet B = \left(A \oplus B \right) \Theta B$$

GEOMETRIC INTERPRETATION: OPENING

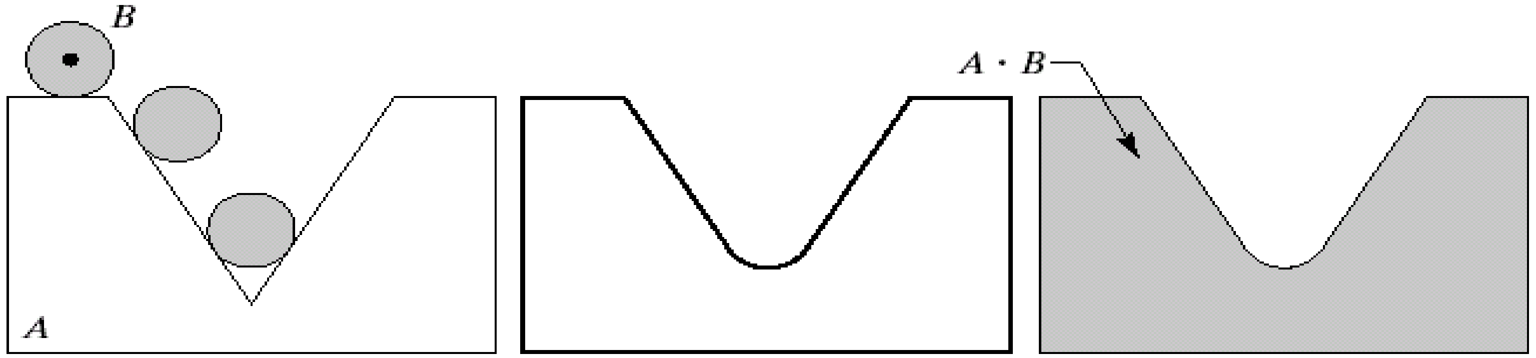


a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening of A by B is obtained by taking the union of all translates of B that fit into A

GEOMETRIC INTERPRETATION: CLOSING



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$x \in A \bullet B$ if all translation B_w that contain x have nonempty intersections with A

ANOTHER ILLUSTRATION

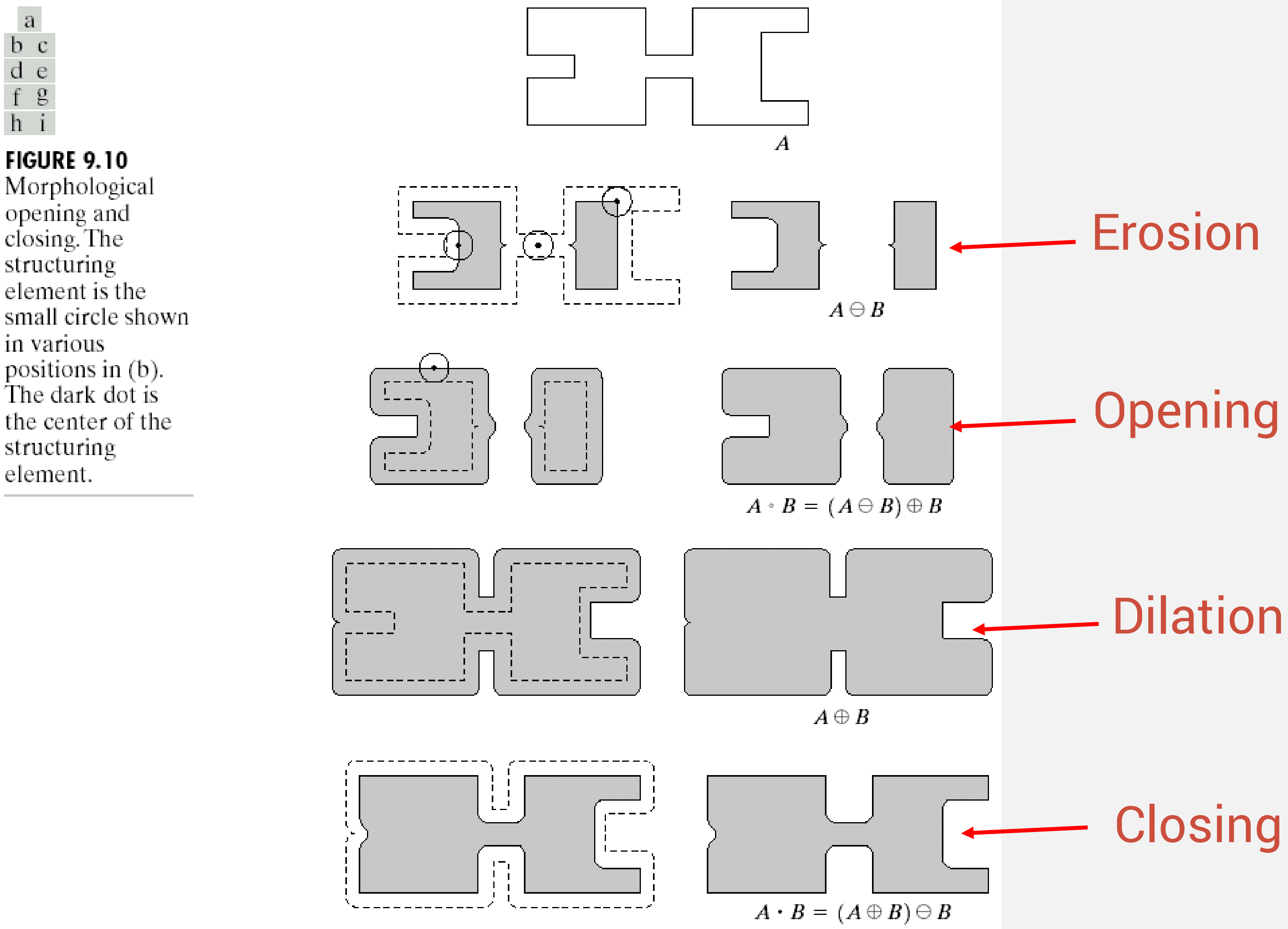


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

PROPERTIES OF OPENING & CLOSING

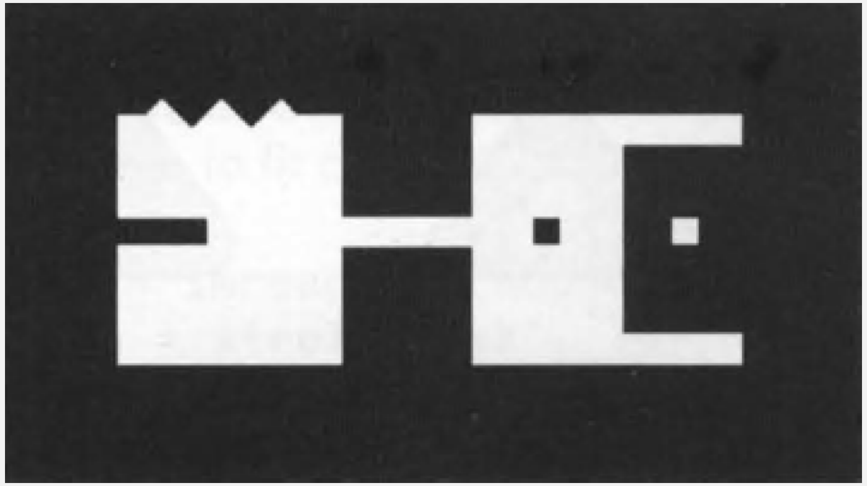
- Opening:
  - $A \circ B$ is a subset (subimage) of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$ (idempotence, meaning opening cannot be done multiple times)
- Closing:
  - A is a subset (subimage) of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  - $(A \bullet B) \bullet B = A \bullet B$ (same as opening)

EXAMPLE

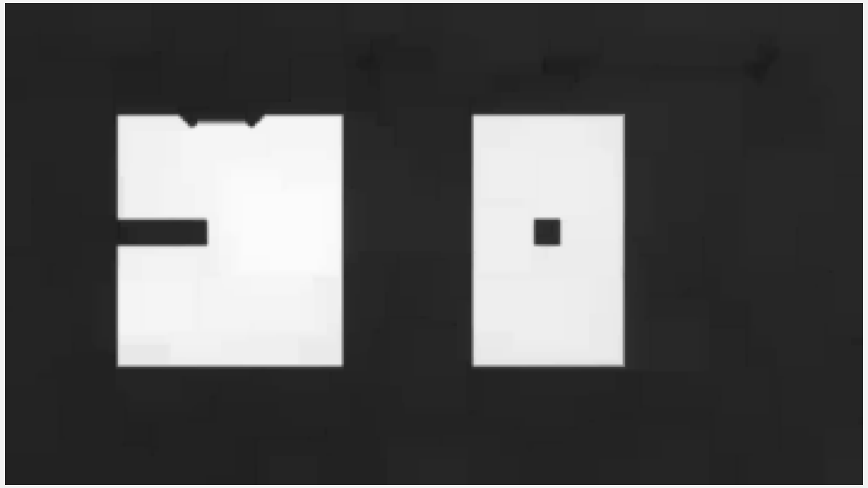
```
img = imread('dsp_shapes.jpg');
img = imresize(img, [200 NaN]);
se = strel('square' , 20);
imgOpen =  imopen( img , se ) ;

figure;
subplot(121); imshow(img); title('Original Wirebond Image');
subplot(122); imshow(imgOpen); title('Opened Image Radii 20');
```

EXAMPLE

Original Wirebond Image



Opened Image Radii 20

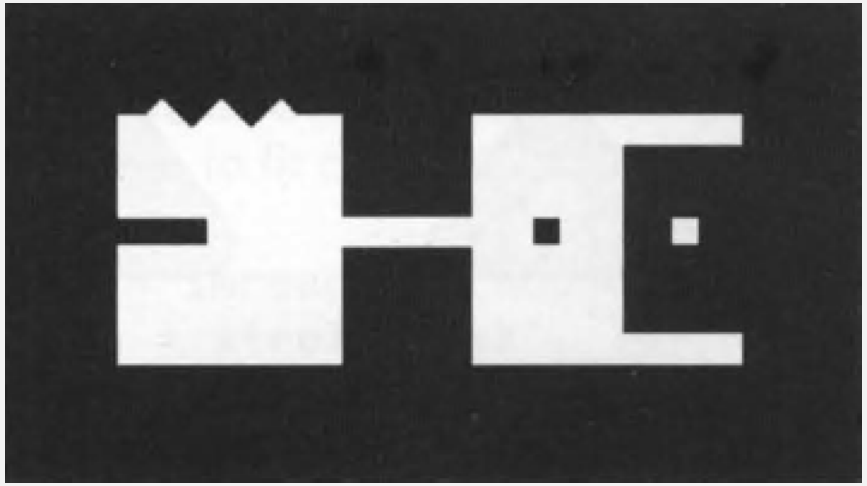


EXAMPLE

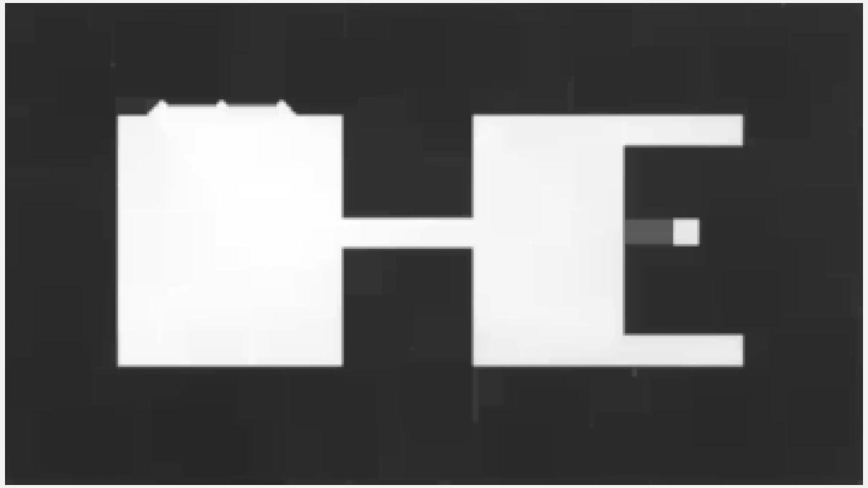
```
img = imread('dsp_shapes.jpg');
img = imresize(img, [200 NaN]);
se = strel('square' , 20);
imgClose=  imclose( img , se ) ;

figure;
subplot(121); imshow(img); title('Original Wirebond Image');
subplot(122); imshow(imgClose); title('Closed Image Radii 20');
```

EXAMPLE

Original Wirebond Image



Closed Image Radii 20

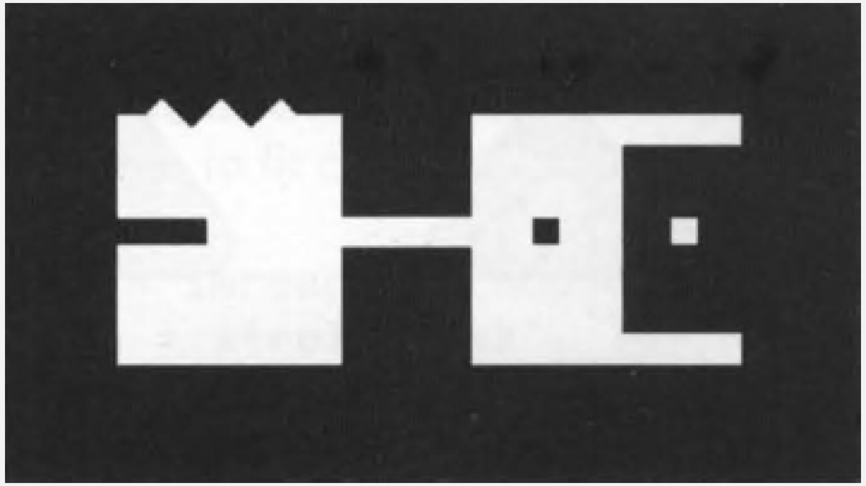


EXAMPLE

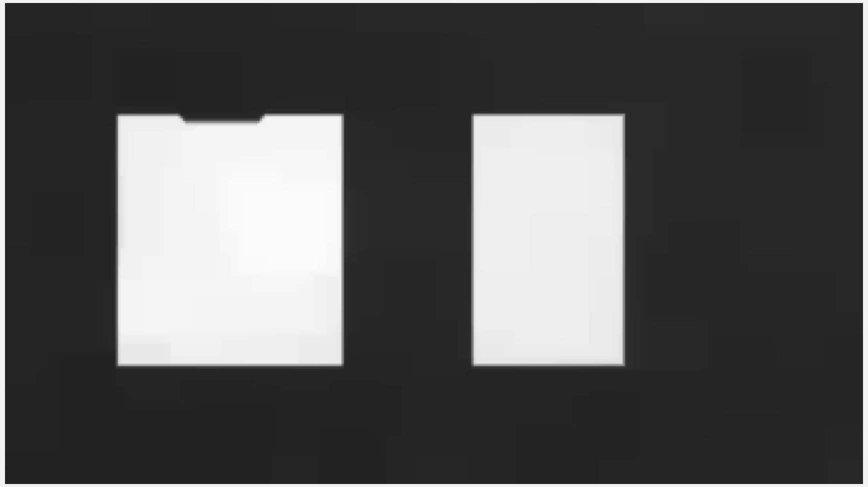
```
img = imread('dsp_shapes.jpg');
img = imresize(img, [200 NaN]);
se = strel('square' , 20);
imgOpen =  imopen( img , se ) ;
imgClose2 =  imclose( imgOpen , se ) ;

figure;
subplot(121); imshow(img); title('Original Wirebond Image');
subplot(122); imshow(imgClose2); title('Closed Image Radii 20');
```

EXAMPLE

Original Wirebond Image



Closed Image Radii 20



APPLICATION: NOISE REMOVAL

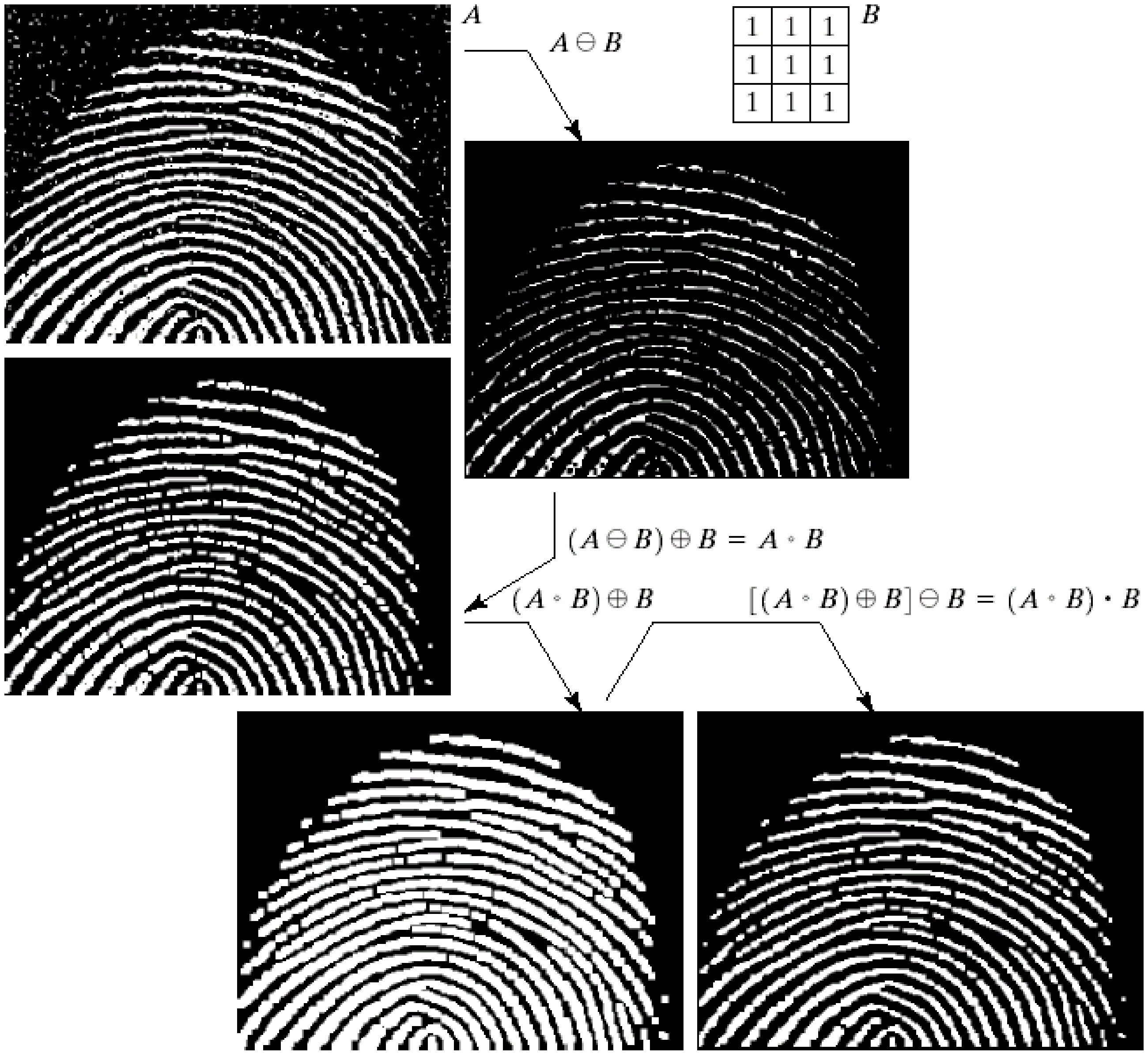
- Similar in concept to the spatial filtering
- Let's take a look at the fingerprint image corrupted by noise.
- What morphological operations can be done to remove the black and white dots?

A

1	1	1
1	1	1
1	1	1

B

APPLICATION: NOISE REMOVAL



a b
d c
e f

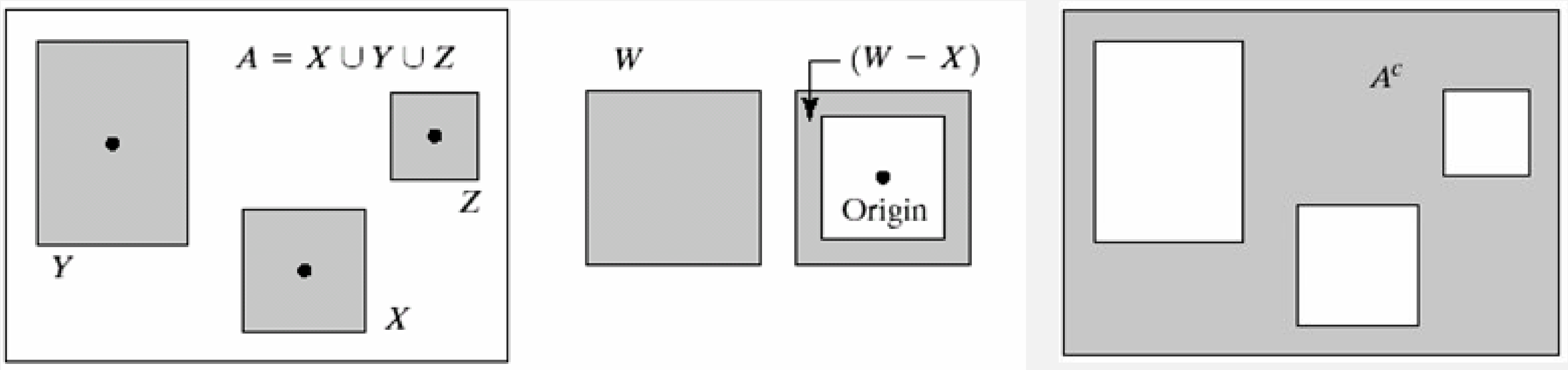
FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

EXAMPLE

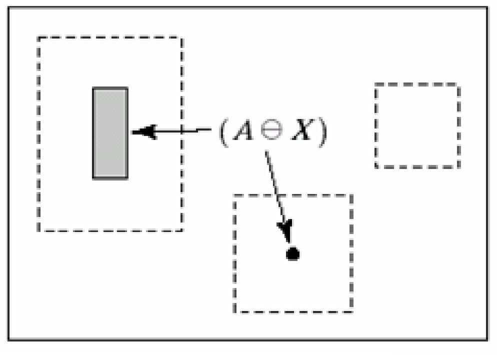
```
img = imread('dsp_fingerprint.jpg');
img = imresize(img, [200 NaN]);
img = im2bw(img);
se = strel('square' , 2);
imgOpen       = imopen(img,se) ;
imgClose      = imclose(imgOpen,se) ;


figure;
subplot(131); imshow(img); title('Original Wirebond Image');
subplot(132); imshow(imgOpen); title('Opening Image');
subplot(133); imshow(imgClose); title('Opening followed by
closing');
```

HIT-OR-MISS TRANSFORMATION

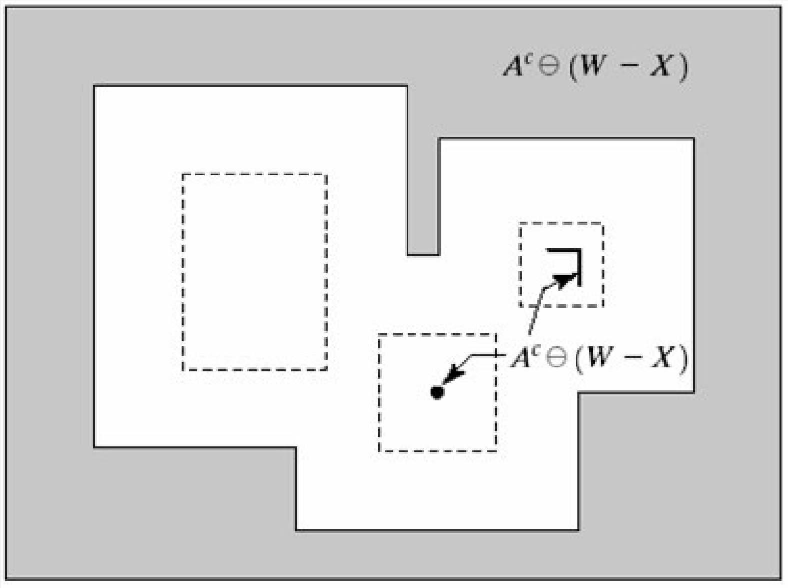
- A powerful method for finding objects of a specific shape in an image.
- Only need erosion.
- Now consider set A below, and we want to find subset X using structuring element B, the same as X



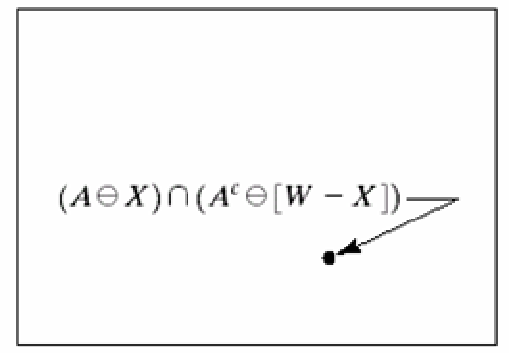
HIT-OR-MISS TRANSFORMATION: EXAMPLE



Step 1



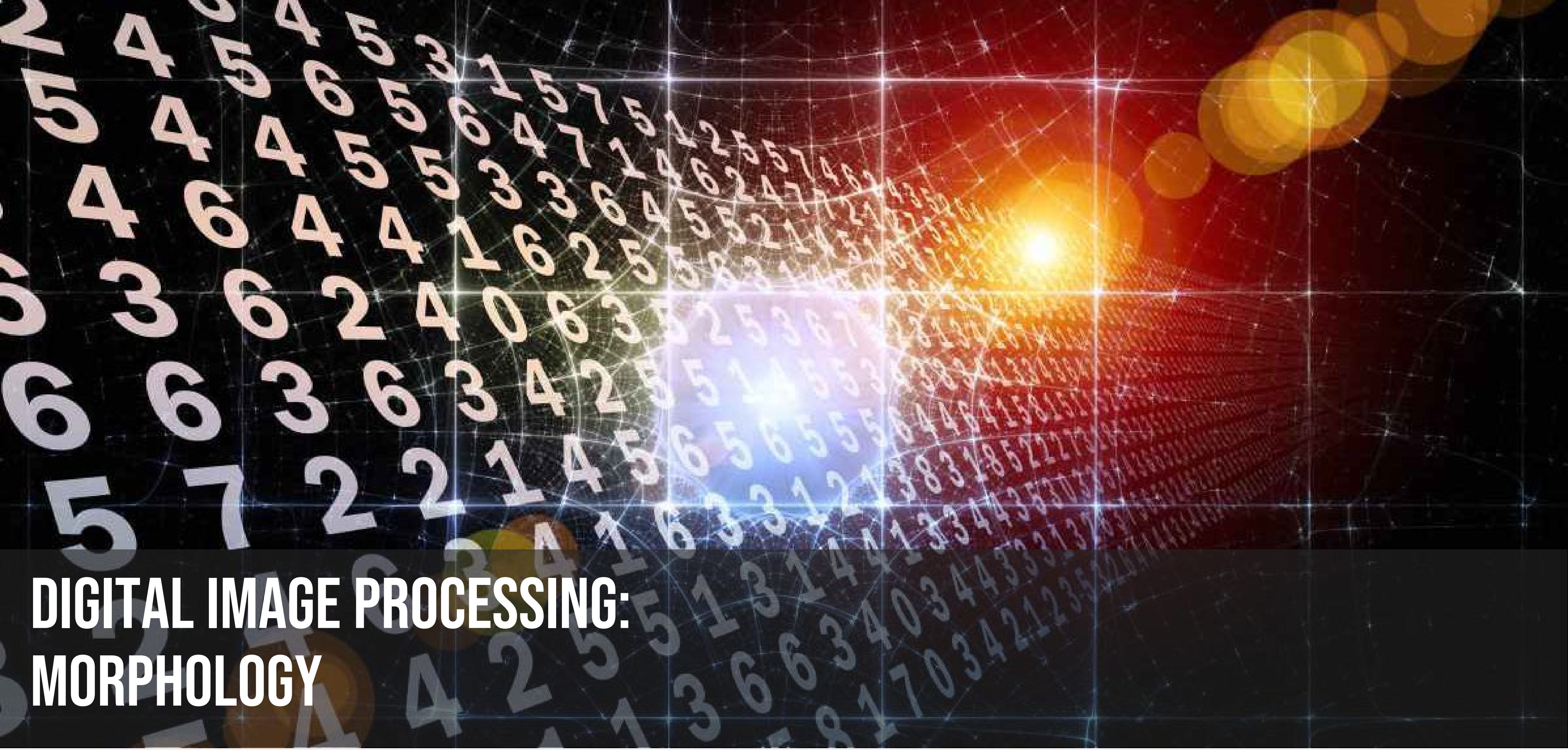
Step 2



Final

SUMMARY

- Morphological operations:
  - Dilation
  - Erosion
  - Opening (erosion then dilation)
  - Closing (dilation then erosion)
- Morphological algorithm
  - Hit-or-Miss transform



DIGITAL IMAGE PROCESSING: MORPHOLOGY

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OVERVIEW

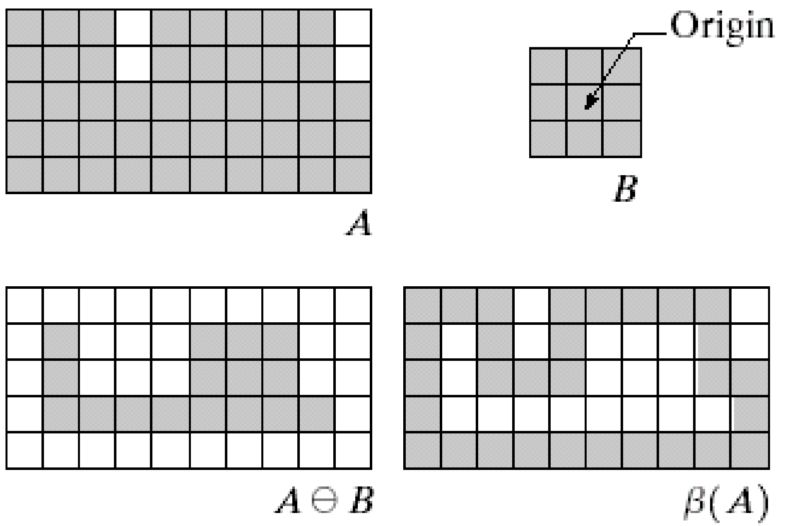
- Extracting image components that are useful in the representation and description of shape.
- We'll consider:
    - Boundary extraction
    - Region filling
    - Extraction of connected components
    - Skeletonization

BOUNDARY EXTRACTION

- Denote the boundary of set A by $\beta(A)$
- Step1: eroding A by the structuring element B
- Step2: taking the difference between A and its erosion.

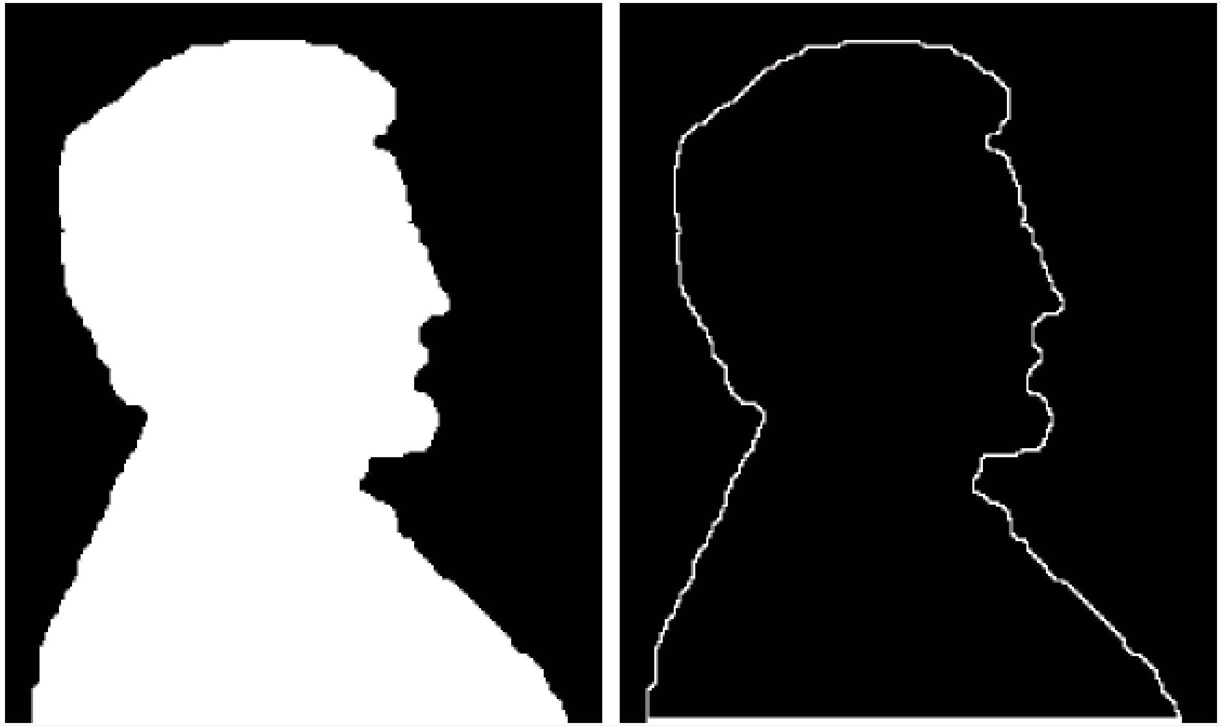
a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



$$\beta(A) = A - (A \Theta B)$$

ANOTHER ILLUSTRATION



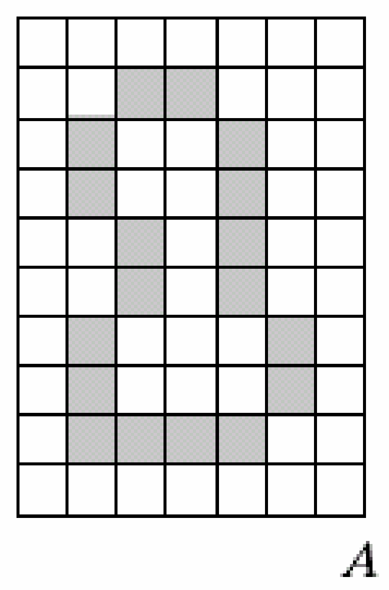
a b

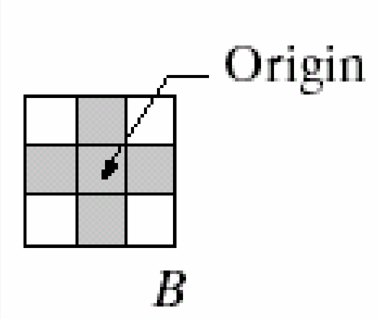
FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Using *B* as the structuring element, so boundary is 1 pixel thick.

REGION FILLING

- A is a set containing a subset whose elements are 8-connected boundary points of a region.
- Goal: fill the entire region with 1's.





REGION FILLING (CON'D)

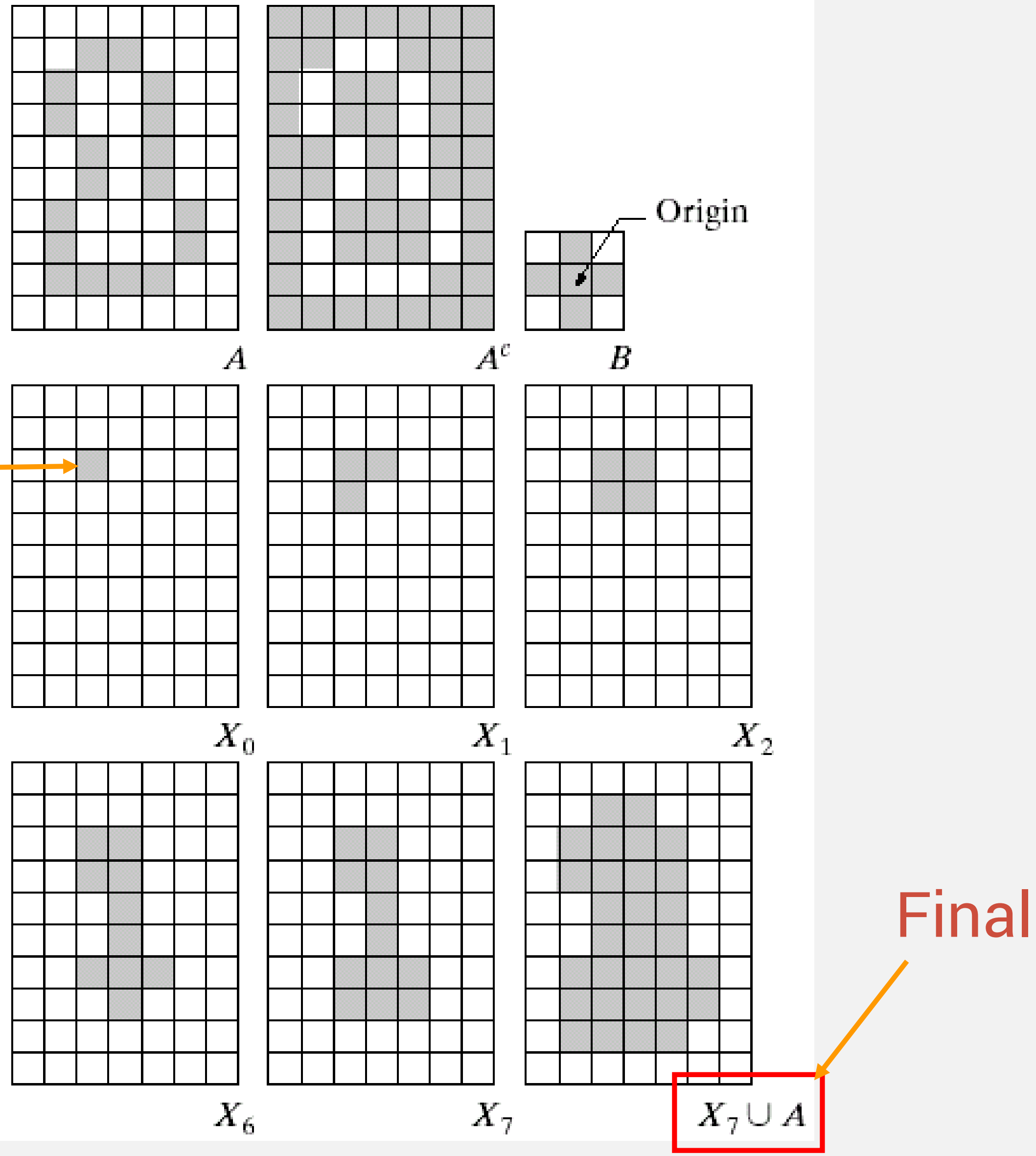


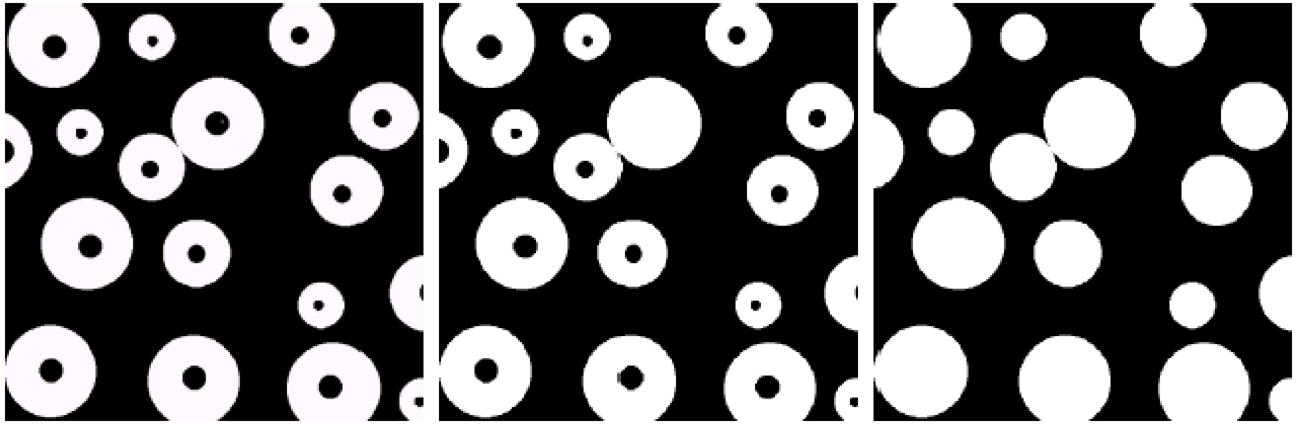
FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c$$

Terminate when $X_k = X_{k-1}$

SOME REMARKS

- The dilation process would fill the entire area if left unchecked.
- $\cap A^c$ limits the result to inside the region of interest.
- Conditional dilation
- Applicable to any finite number of such subsets, assuming that a point inside each boundary is given.



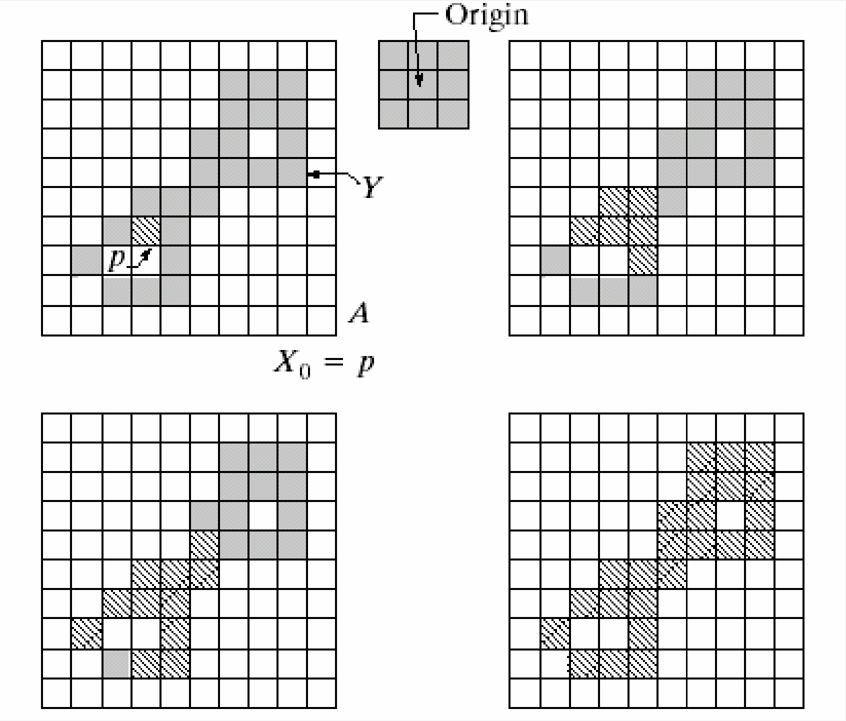
a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

CONNECTED COMPONENTS

- Extraction of connected components in a binary image is central to many automated image analysis applications.
- Let Y be a connected component in set A and p a point of Y.

$$X_k = \left(X_{k-1} \oplus B\right) \cap A$$



APPLICATION OF CONNECTED COMPONENT

a
b
c d

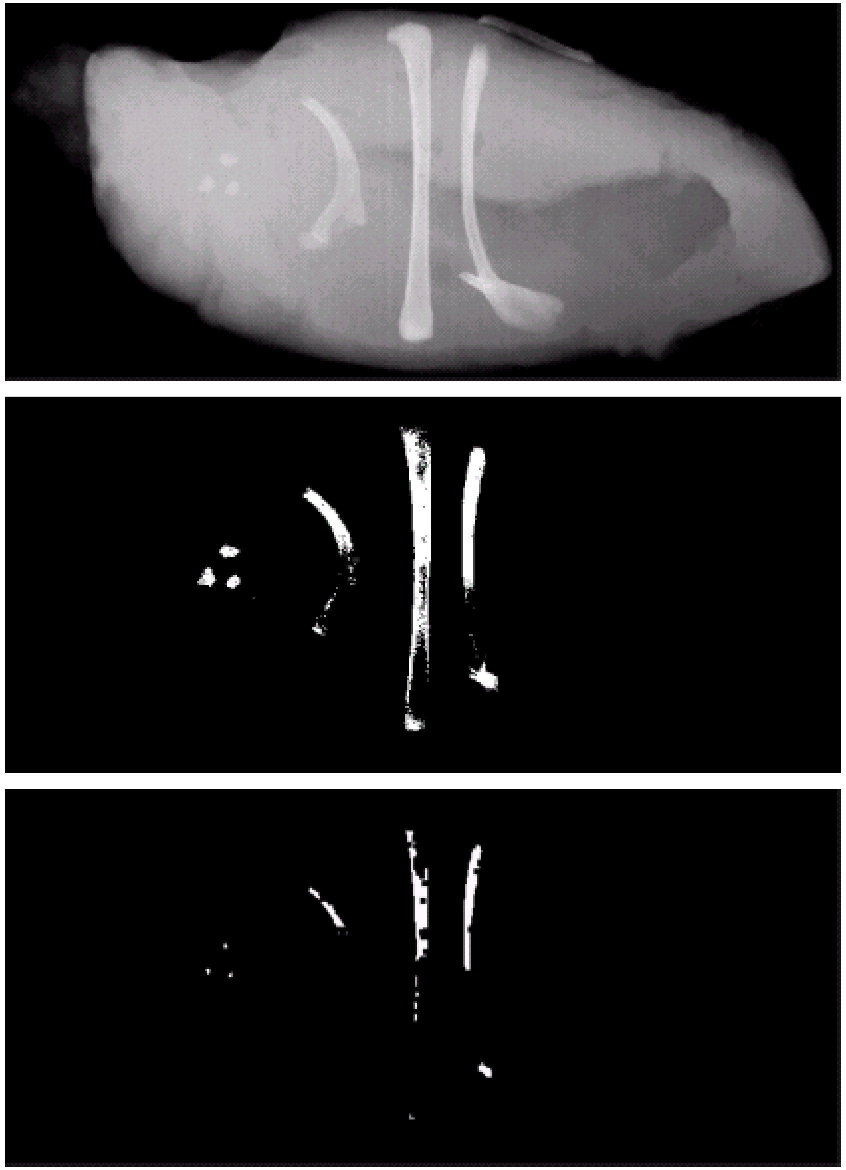


FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's.
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

SKELETONIZATION

- We have seen some algorithms for skeletonization when discussing topology.
- Review: skeleton of a binary object is a collection of lines and curves that describe the size and shape of the object.
- Different algorithms and many possible different skeletons of the same object.
- Here we use a combination erosion and opening operations

(CON'D)

Formulation:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with

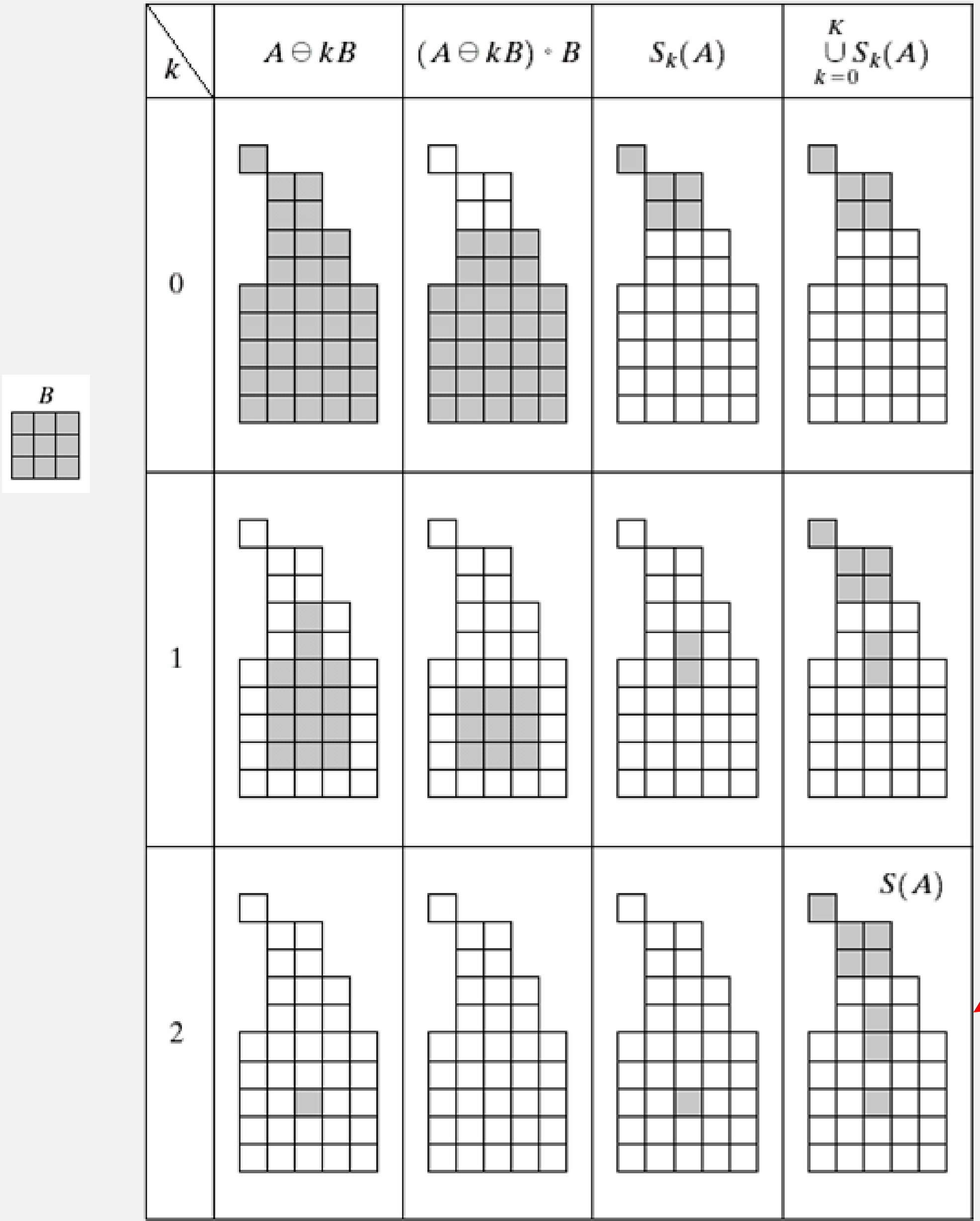
$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$$

where

$$(A \Theta kB) = \underbrace{(\ldots(A \Theta B) \Theta B) \Theta \ldots \quad) \Theta B}_{\text{k time}}$$

The big K is the last iterative step before A erodes to an empty set.

SKELETONIZATION: DEMO



Final skeleton

GRAYSCALE MORPHOLOGY – DILATION (ADVANCE)

- General formulation:

$$(f \oplus b)(x, y) = \max\left\{ f(x+s, y+t) + b(s,t) \mid (s,t) \in D_b \right\}$$

  - where f is the grayscale image and b is the structuring element.
- In other words, the value of dilation at (x,y) is the maximum of the sum of f and b in the interval spanned by b.

EXAMPLE

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5\ (x) \\
\hline
1 & 10 & 20 & 20 & 20 & 30 \\
2 & 20 & 30 & \boxed{30} & \boxed{40} & \boxed{50} \\
3 & 20 & 30 & \boxed{30} & \boxed{50} & \boxed{60} \\
4 & 20 & 40 & \boxed{50} & \boxed{50} & \boxed{60} \\
5 & 30 & 50 & 60 & 60 & 70 \\
\hline
(y) & & & & &
\end{array}
$$

f

$$
\begin{array}{c|ccc|}
 & -1 & 0 & 1 \\
\hline
-1 & 1 & 2 & 3 \\
0 & 4 & 5 & 6 \\
1 & 7 & 8 & 9 \\
\hline
\end{array}
$$

b

What is $(f \oplus b)(4,3)$?

EFFECT OF DILATION

- Two effects:
  - If all values of the structuring element are positive, the output image tends to be brighter.
  - Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element.

GRAYSCALE MORPHOLOGY - EROSION

$$(f \Theta b)(x,y) = \min\left\{ f(x+s, y+t) - b(s,t) \mid (s,t) \in D_b \right\}$$

f

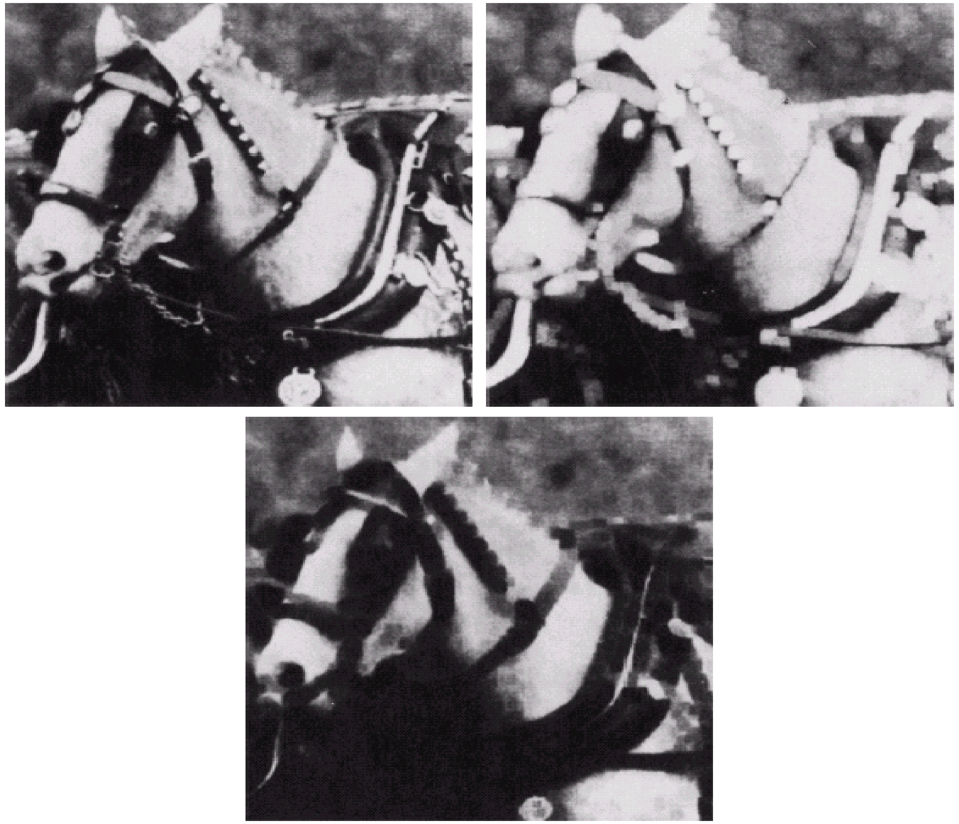
(y) \ (x)	1	2	3	4	5
1	10	20	20	20	30
2	20	30	30	40	50
3	20	30	30	50	60
4	20	40	50	50	60
5	30	50	60	60	70

b

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

What is $(f \Theta b)(4,3)$?

DEMO



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

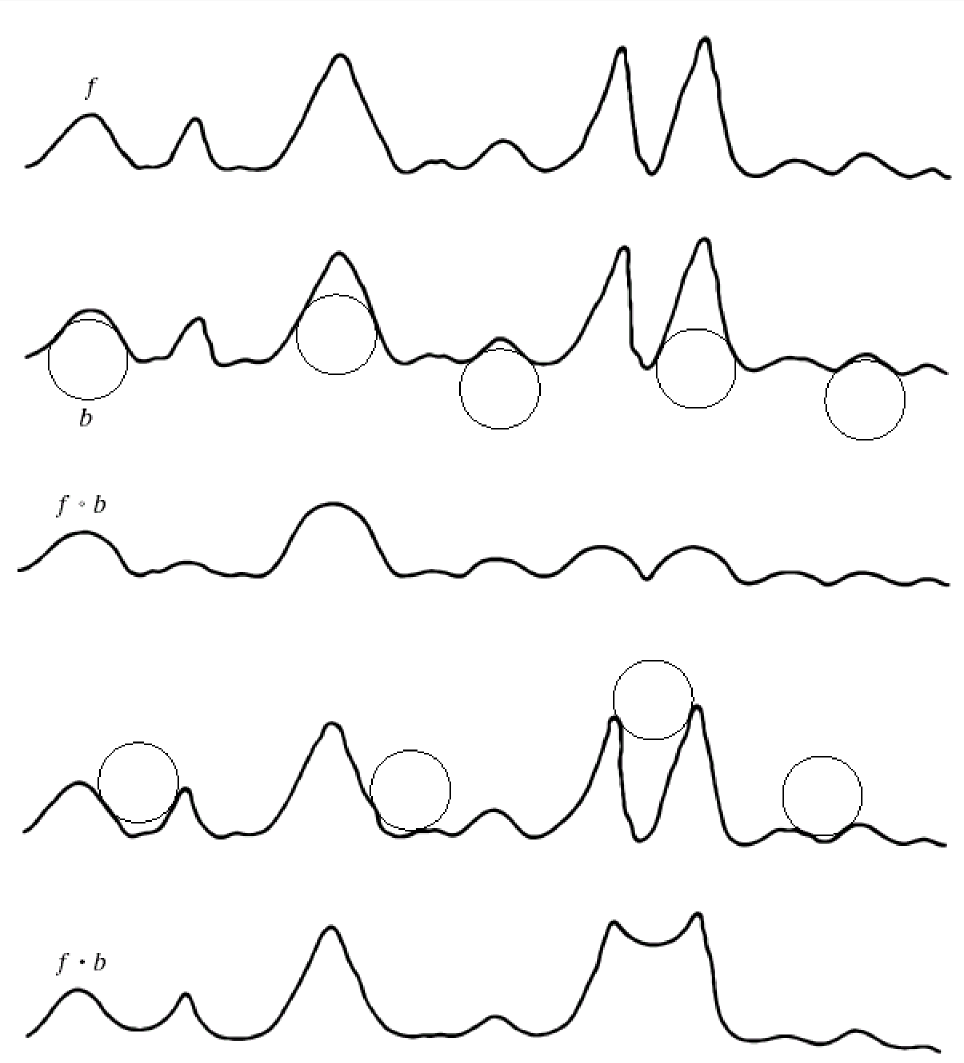
OPENING & CLOSING

Opening:

$$f \circ b = \left(f \Theta b \right) \oplus b$$

Closing:

$$f \circ b = \left(f \oplus b \right) \Theta b$$



a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

DEMO

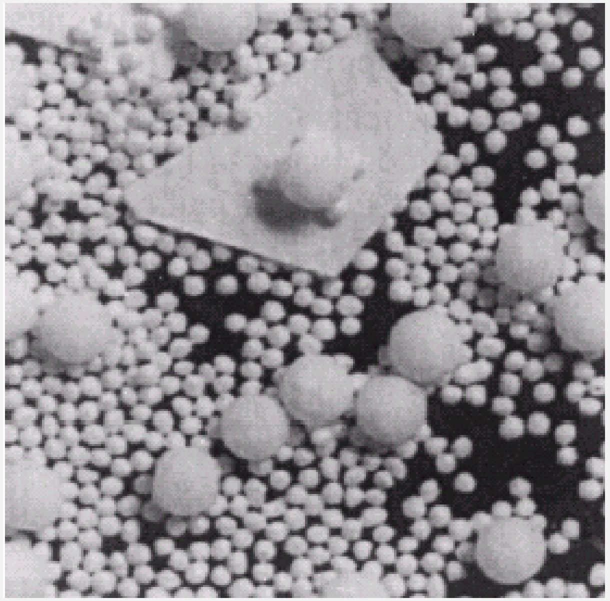
original

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

APPLICATION: GRANULOMETRY

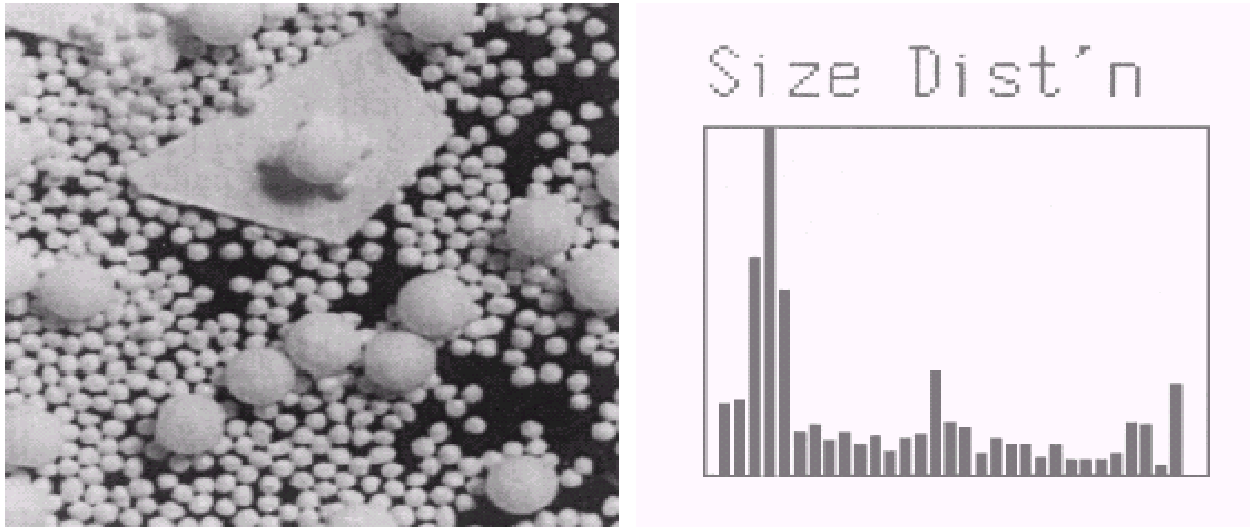
- Definition: determining the size distribution of particles in an image
  - Useful when objects are overlapping and clustered.
  - Detection of individual particles are hard.



(CON'D)

- Opening operations with structuring elements of increasing size are performed on the original image.
- Motivation: opening operations of a particular size have the most effect on regions containing particles of similar size.
- Method:
  - For every structuring element, compute the difference btw the I_{m-1} and I_m, where m is the index of the structuring element.
  - At the end, normalize the differences => a histogram of particle-size distribution

DEMO



a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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