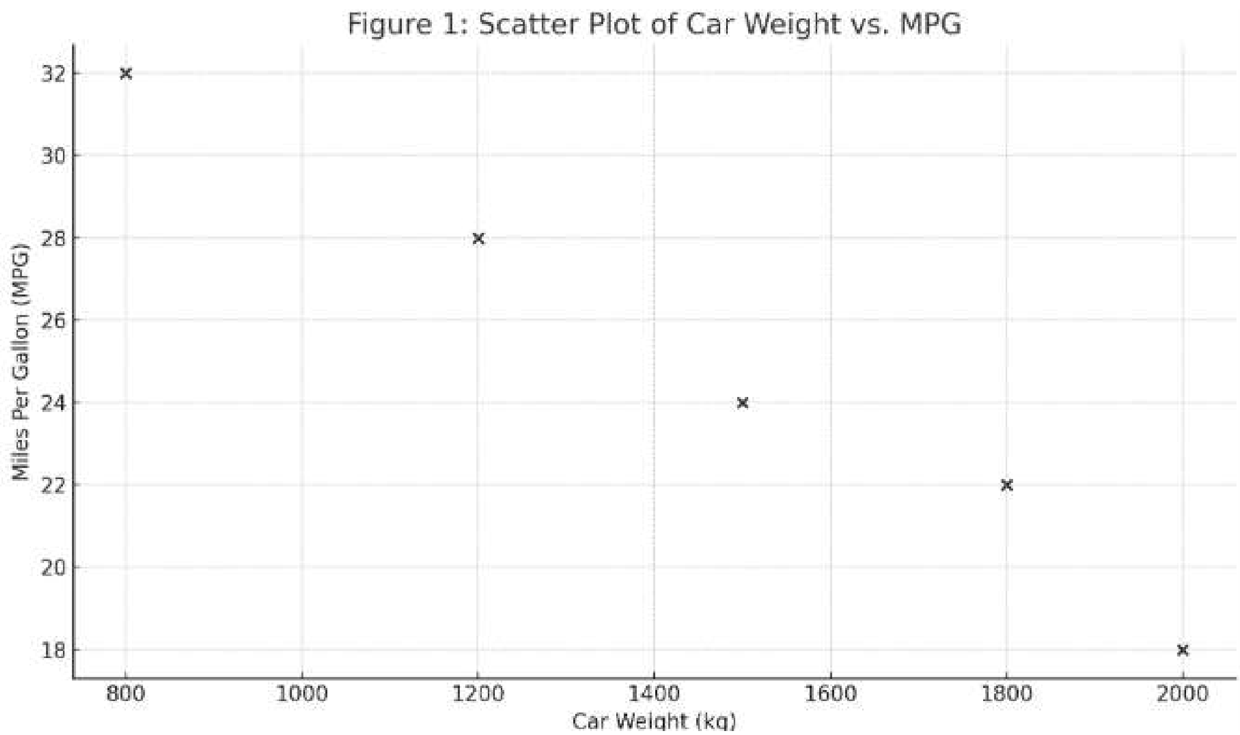
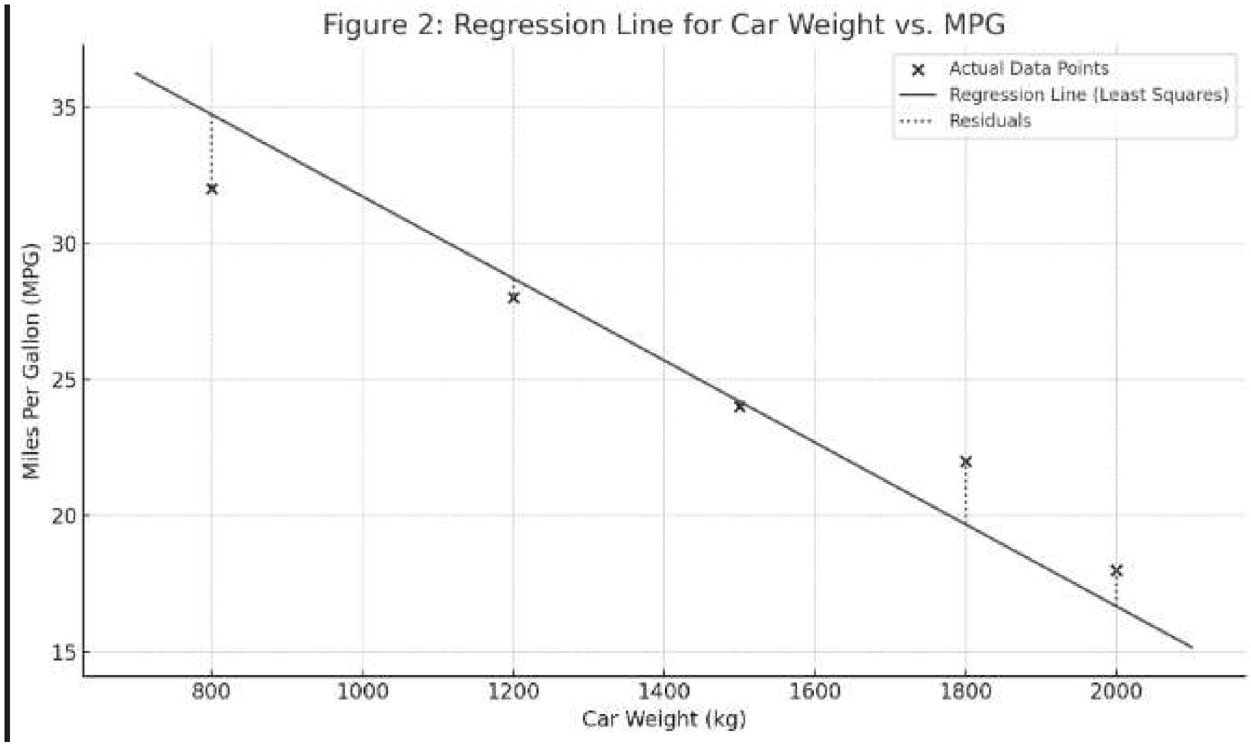
TITLE: Predicting Car Fuel Efficiency (MPG) Based on Car Weight Using the Least Squares Method

LIST OF FIGURES

1. Figure 1: Scatter Plot of Car Weight vs. MPG



2. Figure 2: Regression Line for Car Weight vs. MPG



LIST OF SYMBOLS AND ABBREVIATIONS

- **X**: Car Weight (in kg)
- **Y**: Miles Per Gallon (MPG)
- **a**: Intercept of the regression line
- **b**: Slope of the regression line
- **n**: Number of data points

1.0 INTRODUCTION

In this report, we aim to predict a car's fuel efficiency (miles per gallon, MPG) based on its weight. This problem is relevant in the automotive industry, where understanding the relationship between a car's weight and its fuel efficiency can influence design choices and consumer decisions. The least squares method is a simple and widely-used technique to find a linear relationship between two variables.

2.0 THE MODEL

2.1 Sub Section 2

The least squares method finds the line of best fit for a set of data points. The equation for the line is:

Y=a+bX

Where:

- **Y** is the dependent variable (MPG),
- **X** is the independent variable (Car Weight),
- **a** is the intercept, and
- **b** is the slope of the line.

The slope **b** is calculated using the formula:

$$b = \frac{n\sum(XY) - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

And the intercept **a** is calculated by:

$$a = \frac{\sum Y - b\sum X}{n}$$

2.2 Sub Section 2

For this specific case, we used the following data set for car weight (X) and MPG (Y):

Car Weight (X)	MPG (Y)
800	32
1200	28
1500	24
1800	22
2000	18

We then applied the least squares method to calculate the line of best fit.

3.0 METHOD OF SOLUTIONS

3.1 Sub Section 3

Using the given data, we performed the necessary calculations to find the slope **b** and intercept **a**.

1. Calculate the summations:

- $\Sigma X = 800 + 1200 + 1500 + 1800 + 2000 = 7300$
- $\Sigma Y = 32 + 28 + 24 + 22 + 18 = 124$
- $\Sigma XY = (800\times32) + (1200\times28) + (1500\times24) + (1800\times22) + (2000\times18) = 25600 + 33600 + 36000 + 39600 + 36000 = 170800$
- $\Sigma X^2 = (800^2) + (1200^2) + (1500^2) + (1800^2) + (2000^2) = 640000 + 1440000 + 2250000 + 3240000 + 4000000 = 11340000$
- $n = 5$ (the number of data points)

2. Calculate the slope **b**:

$$b = \frac{5 \times 170800 - 7300 \times 124}{5 \times 11340000 - 7300^2} = \frac{854000 - 905200}{56700000 - 53290000} = \frac{-51200}{3401000} \approx -0.01505$$

3. Calculate the intercept **a**:

$$a = \frac{124 - (-0.01505 \times 7300)}{5} = \frac{124 + 109.87}{5} = \frac{233.87}{5} \approx 46.77$$

Thus, the regression equation is:

$Y=46.77-0.01505X$

4.0 RESULT AND DISCUSSION

4.1 Sub Section 4

The equation derived from the least squares method is:

MPG=46.77−0.01505×Car Weight

This equation suggests that for every additional kilogram of car weight, the car’s fuel efficiency (MPG) decreases by approximately 0.015. For example, if a car weighs 1000 kg, we can predict its MPG as:

MPG=46.77−0.01505×1000=46.77−15.05=31.72MPG

This model provides a good fit for predicting MPG based on car weight. However, the slope indicates that as car weight increases, fuel efficiency decreases, which is typical in real-world scenarios.

5.0 CONCLUSION

Using the least squares method, we derived a linear model to predict a car's fuel efficiency (MPG) based on its weight. The model’s equation is:

MPG=46.77−0.01505×Car Weight

This model shows a clear negative correlation between car weight and fuel efficiency, which can help car manufacturers and consumers make more informed decisions regarding car design and fuel consumption.

6.0 FUTURE SCOPE

In the future, this model could be enhanced by including more variables such as engine size, type of fuel, or aerodynamics, which might provide a more accurate prediction of fuel efficiency. Additionally, collecting a larger data set would help improve the robustness of the model and reduce errors in predictions