



**PES University, Bangalore**

(Established under Karnataka Act No. 16 of 2013)

**APRIL 2021: IN SEMESTER ASSESSMENT (ISA) B.TECH. IV SEMESTER**

**UE19MA251- LINEAR ALGEBRA**

**Mathematics Lab**

**Session: Jan-May 2021**

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**Branch : Electronics & Communication**

**Semester & Section : Semester IV Section A**

**FOR OFFICE USE ONLY:**

**Marks : 

/05
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**Name of the Course Instructor : Prof. Deepthi Rao**

**Signature of the Course Instructor : \_\_\_\_\_**

## Topic: Gaussian Elimination

Example 1: Solve the system of equations  $2x + 4y + 6z = 14$ ,  $3x - 2y + z = -3$  and  $4x + 2y - z = -4$  using Gaussian Elimination. Also, identify the pivots.

Solution:

```
clc;
clear;
close;
A=[2,4,6;3,-2,1;4,2,-1],b=[14;-3;-4]
A_aug=[A b]
a=A_aug
n=3;
for i=2:n
    for j=2:n+1
        a(i,j)=a(i,j)-a(1,j)*a(i,1)/a(1,1);
    end
    a(i,1)=0;
end
for i=3:n
    for j=3:n+1
        a(i,j)=a(i,j)-a(2,j)*a(i,2)/a(2,2);
    end
    a(i,2)=0;
end
x(n)=a(n,n+1)/a(n,n);
for i=n-1:-1:1
    sumk=0;
    for k=i+1:n
        sumk=sumk+a(i,k)*x(k);
    end
    x(i)=(a(i,n+1)-sumk)/a(i,i);
end
disp('The values of x,y,z are ',x(1),x(2),x(3));
disp('The pivots are',a(1,1),a(2,2),a(3,3));
```

The screenshot displays the Scilab 6.1.0 interface. The left pane shows the Command History with the following text:

```
"The values of x,y,z are "
```

```
-1.
```

```
1.
```

```
2.
```

```
"The pivots are"
```

```
2.
```

```
-8.
```

```
-7.
```

```
--> |
```

The right pane shows the Variable Browser with the following table:

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
A_aug	3x4	Double	local	304 B
a	3x4	Double	local	304 B
b	[14; -3...	Double	local	232 B
i	1	Double	local	216 B
j	4	Double	local	216 B
k	3	Double	local	216 B
n	3	Double	local	216 B

Below the Variable Browser is the Command History pane, which shows a list of commands and their execution times, such as:

```
// -- 17/04/2021 13:44:45 -- //
```

```
// -- 17/04/2021 13:45:39 -- //
```

```
// -- 21/04/2021 12:15:31 -- //
```

```
// -- 22/04/2021 19:37:16 -- //
```

```
// -- 22/04/2021 22:58:06 -- //
```

```
// -- 23/04/2021 19:27:28 -- //
```

```
// -- 23/04/2021 21:17:06 -- //
```

```
// -- 23/04/2021 22:57:59 -- //
```

```
// -- 24/04/2021 00:04:30 -- //
```

```
// -- 24/04/2021 00:13:47 -- //
```

```
// -- 24/04/2021 09:53:22 -- //
```

```
// -- 24/04/2021 09:53:22 -- //
```

Example 2: Solve the system of equations  $2x + 3y - z = 5$ ,  $4x + 4y - 3z = 3$  and  $-2x + 3y - z = 1$  by Gaussian Elimination.

Solution:

```
clc
clear
A=[2 3 -1;4 4 -3;-2 3 -1]; //coefficient
B=[5;3;1]; //constant
n=length(B);
Aug=[A,B];
//forward elimination
for j=1:n-1
    for i=j+1:n
        Aug(i,j:n+1)=Aug(i,j:n+1)-(Aug(i,j)/Aug(j,j)*Aug(j,j:n+1));
    end
end
//backward substitution
x=zeros(n,1);
x(n)=Aug(n,n+1)/Aug(n,n);
for i=n-1:-1:1
    x(i)=(Aug(i,n+1)-Aug(i,i+1:n)*x(i+1:n))/Aug(i,i);
end
disp('The values of x,y,z are',x(1),x(2),x(3))
```

The screenshot displays the Scilab 6.1.0 interface. The Console window on the left shows the output of the code: "The values of x,y,z are" followed by three numbered lines (1., 2., 3.) where the numerical values of x, y, and z would appear. The Variable Browser on the right lists the variables created during execution: A (3x3 Double, 280 B), Aug (3x4 Double, 304 B), B (5x1 Double, 232 B), i (1 Double, 216 B), j (2 Double, 216 B), n (3 Double, 216 B), and x (1x3 Double, 232 B). The Command History window at the bottom shows the sequence of commands executed, including the code from the example and the 'disp' command.

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
Aug	3x4	Double	local	304 B
B	[5; 3; 1]	Double	local	232 B
i	1	Double	local	216 B
j	2	Double	local	216 B
n	3	Double	local	216 B
x	[1; 2; 3]	Double	local	232 B

## Topic: LU decomposition of a matrix

Example 3: Find the triangular factors L and U for the matrix  $A = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$

Solution:

```
clc;
clear;
close;
A=[3,-0.1,-0.2;0.1,7,-0.3;0.3,-0.2,10];
U=A;
disp('The given matrix is A=',A)
m=det(U(1,1));
n=det(U(2,1));
a=n/m;
U(2,:)=U(2,:)-U(1,:)/(m/n);
b=n/m;
U(3,:)=U(3,:)-U(1,:)/(m/n);
m=det(U(2,2));
n=det(U(3,2));
c=n/m;
U(3,:)=U(3,:)-U(2,:)/(m/n);
disp('The upper triangular matrix is U=',U)
L=[1,0,0;a,1,0;b,c,1];
disp('The lower triangular matrix is L=',L)
```

The screenshot displays the Scilab 6.1.0 interface. The left pane shows the Command Window with the execution of the provided code. The right pane shows the Variable Browser with a table of variables and their properties.

**Scilab 6.1.0 Console**

```
"The given matrix is A="
3.   -0.1  -0.2
0.1   7.   -0.3
0.3  -0.2  10.

"The upper triangular matrix is U="
3.   -0.1  -0.2
0.   7.   -0.29
0.2  0.   10.

"The lower triangular matrix is L="
1.    0.    0.
0.03  1.    0.
0.03 -0.03  1.

--> |
```

**Variable Browser**

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
L	3x3	Double	local	280 B
U	3x3	Double	local	280 B
a	0.0333	Double	local	216 B
b	0.0333	Double	local	216 B
c	-0.0281	Double	local	216 B
m	7	Double	local	216 B
n	-0.197	Double	local	216 B

**Command History**

```
// -- 17/04/2021 13:44:45 -- //
// -- 17/04/2021 13:45:39 -- //
// -- 21/04/2021 12:15:31 -- //
// -- 22/04/2021 19:37:16 -- //
// -- 22/04/2021 22:58:06 -- //
// -- 23/04/2021 19:27:28 -- //
// -- 23/04/2021 21:17:06 -- //
// -- 23/04/2021 22:57:59 -- //
// -- 24/04/2021 00:04:30 -- //
// -- 24/04/2021 00:13:47 -- //
// -- 24/04/2021 09:53:22 -- //
// -- 24/04/2021 09:53:22 -- //
```

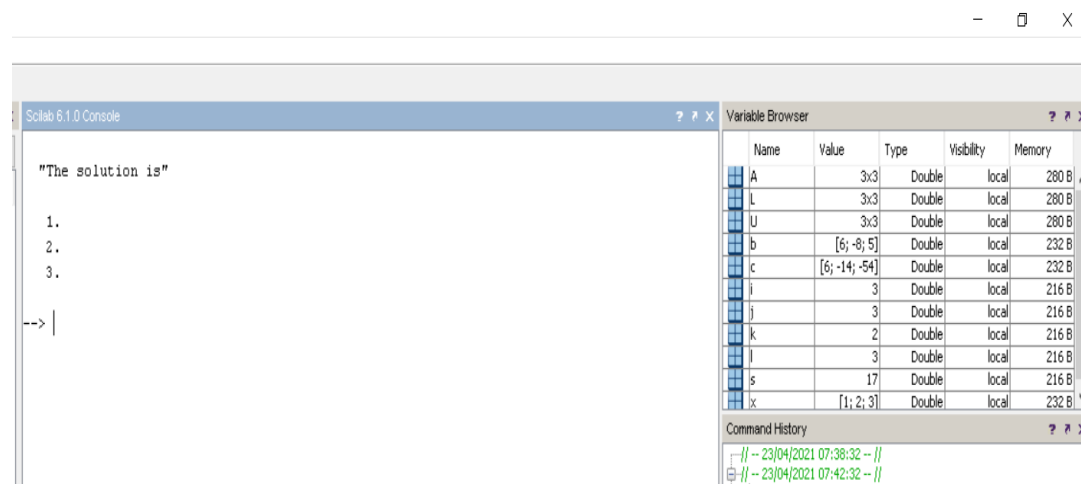
Example 4: Solve the system of equations  $4x - 2y + 2z = 6$ ,  $4x - 3y - 2z = -8$  and  $2x + 3y - z = 5$  by LU decomposition.

Solution:

```
clear;
close;
clc;
format('v',5);
A= {4 -2 2;4 -3 -2;2 3 -1};
for l=1:3
    L(l,l)=1;
end

for i=1:3
    for j=1:3
        s=0;
        if (j>=i)
            for k=1:i-1
                s=s+(L(i, k)*U(k, j));
            end
            U(i,j)=A(i,j)-s;
        else
            for k=1:j-1
                s=s+(L(i,k)*U(k,j));
            end
            L(i,j)=(A(i,j)-s)/U(j,j);
        end
    end
end

b=[6;-8;5];
c=L\b;
x=U\c;
disp( "Solution of the given equation is: ", x)
```



## Topic: The Gauss - Jordan method of calculating $A^{-1}$

Example 5: Find the inverse of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$

by Gauss – Jordan method.

Solution:

```
clc;
clear;
A=[1 1 1;4 3 -1;3 5 3];
n=length(A(1,:));
Aug=[A,eye(n,n)];
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
    end
end
for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
end
for j=1:n
    Aug(j,:)=Aug(j,:)/Aug(j,j);
end
B=Aug(:,n+1:2*n);
disp('The Inverse of A is',B);
```

The screenshot displays the Scilab 6.1.0 interface. The console window on the left shows the output of the code: "The Inverse of A is" followed by a 3x3 matrix of values: 1.4, 0.2, -0.4; -1.5, 0., 0.5; 1.1, -0.2, -0.1. The Variable Browser on the right lists the variables created: A (3x3 Double, 280 B), Aug (3x6 Double, 352 B), B (3x3 Double, 280 B), i (3 Double, 216 B), j (3 Double, 216 B), and n (3 Double, 216 B). The Command History at the bottom shows the sequence of commands executed, including the code for calculating the inverse of matrix A.

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
Aug	3x6	Double	local	352 B
B	3x3	Double	local	280 B
i	3	Double	local	216 B
j	3	Double	local	216 B
n	3	Double	local	216 B

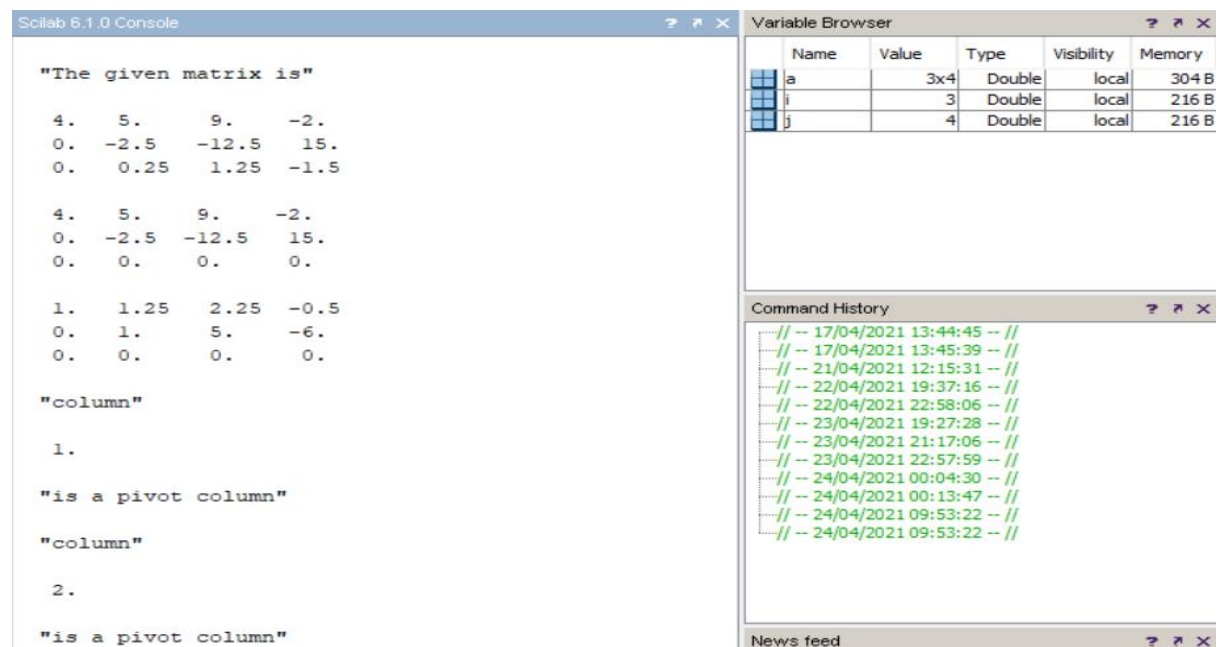
## Topic: Span of the Column Space of A

Example 6: Identify the columns that are in the column space of A where

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$$

Solution:

```
clc;
clear;
close;
disp('The given matrix is')
a=[4 5 9 -2;6 5 1 12;3 4 8 -3]
a(2,:)=a(2,:)-(a(2,1)/a(1,1))*a(1,:)
a(3,:)=a(3,:)-(a(3,1)/a(1,1))*a(1,:)
disp(a)
a(3,:)=a(3,:)-(a(3,2)/a(2,2))*a(2,:)
disp(a)
a(1,:)=a(1,:)/a(1,1)
a(2,:)=a(2,:)/a(2,2)
disp(a)
for i=1:3
    for j=1:4
        if(a(i,j)<>0)
            disp('column',j,'is a pivot column')
            break
        end
    end
end
end
```



## Topic: The Four Fundamental Subspaces

Example 7: Find the four fundamental subspaces of  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

Solution:

```
clear;
close;
clc;
A=[0 1 0;0 0 1;0 0 0];
disp('A=',A);
[m,n]=size(A);
disp('m=',m);
disp('n=',n);
[v,pivot]=rref(A);
disp(rref(A));
disp(v);
r=length(pivot);
disp('rank=',r);
cs=A(:,pivot);
disp('Column Space=',cs);
ns=kernel(A);
disp('Null Space=',ns);
rs=v(1:r,:);
disp('Row Space=',rs)
lns=kernel(A');
disp('Left Null Space=',lns );
```



Scilab 6.1.0 Console
Variable Browser

```

"A="

0.  1.  0.
0.  0.  1.
0.  0.  0.

"m="

3.

"n="

3.

0.  1.  0.
0.  0.  1.
0.  0.  0.

0.  1.  0.
0.  0.  1.
0.  0.  0.

"rank="

2.

"Column Space="

1.  0.
0.  1.
0.  0.

"Null Space="

1.
0.
0.

"Row Space="

0.  0.
1.  0.
0.  1.

"Left Null Space="

0.
0.
1.

```

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
cs	3x2	Double	local	256 B
ins	[0; 0; 1]	Double	local	232 B
m	3	Double	local	216 B
n	3	Double	local	216 B
ns	[1; 0; 0]	Double	local	232 B
pivot	[2, 3]	Double	local	224 B
r	2	Double	local	216 B
rs	3x2	Double	local	256 B
v	3x3	Double	local	280 B

Command History

```

// -- 17/04/2021 13:44:45 -- //
// -- 17/04/2021 13:45:39 -- //
// -- 21/04/2021 12:15:31 -- //
// -- 22/04/2021 19:37:16 -- //
// -- 22/04/2021 22:58:06 -- //
// -- 23/04/2021 19:27:28 -- //
// -- 23/04/2021 21:17:06 -- //
// -- 23/04/2021 22:57:59 -- //
// -- 24/04/2021 00:04:30 -- //
// -- 24/04/2021 00:13:47 -- //
// -- 24/04/2021 09:53:22 -- //
// -- 24/04/2021 09:53:22 -- //

```

News feed
March 2021: IIT Bombay Work...

### March 2021: IIT Bombay Workshop

#### Empower your students

From March 4th to 6th, the [FOSSEE Project](#) team from IIT Bombay will be leading a workshop around Scilab.

#### Content

News feed
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#### Content

## Topic: Projections by Least Squares

Example 8: Find the solution  $x = (C, D)$  of the system  $Ax = b$  and the line of best fit

$C + Dt = b$  given  $b = (1, 1, 3)$ ,  $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ ,

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ ,

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$

Solution:

```
clear;
close;
clc;
A=[1 -1;1 1;1 2];
disp('A=',A);
b=[1;1;3];
disp('b=',b);
x=(A'*A)\(A'*b);
disp('x=',x);
C=x(1,1);
D=x(2,1);
disp('C=',C);
disp('D=',D);
disp('The line of best fit is b=C+Dt');
```

The screenshot displays the Scilab 6.1.0 interface. The Console window on the left shows the execution of the provided code, with output for each variable: A (3x2 matrix), b (3x1 vector), x (2x1 vector), C (scalar), D (scalar), and the final line of best fit equation. The Variable Browser on the right provides a summary of the variables, including their names, values, types, visibility, and memory usage. The Command History window at the bottom right lists the commands executed, timestamped. The News feed at the very bottom shows a message about March 2021 IIT Bombay work.

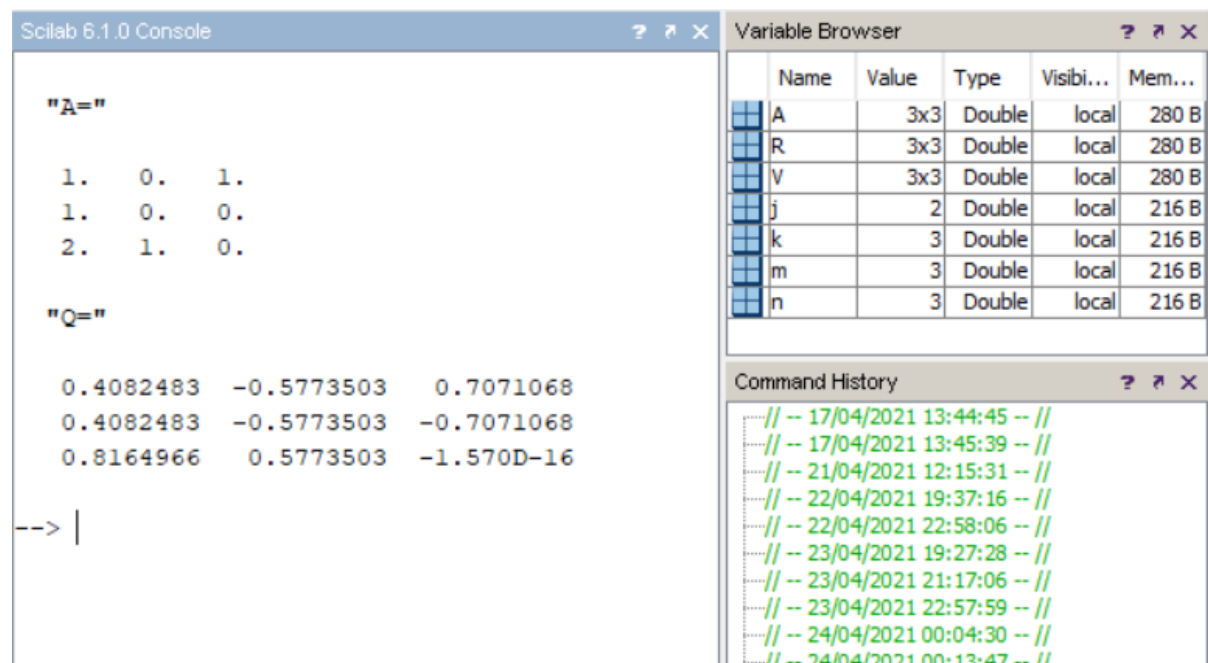
Name	Value	Type	Visibility	Memory
A	3x2	Double	local	256 B
C	1.29	Double	local	216 B
D	0.571	Double	local	216 B
b	[1; 1; 3]	Double	local	232 B
x	[1.29; ...]	Double	local	224 B

## Topic: The Gram- Schmidt Orthogonalization

Example 9: Apply the Gram – Schmidt process to the vectors  $(1, 0, 1)$ ,  $(1, 0, 0)$  and  $(2, 1, 0)$  to produce a set of orthonormal vectors.

Solution:

```
clear;
close;
clc;
A=[1 0 1;1 0 0; 2 1 0];
disp('A=',A);
[m,n]=size(A);
for k=1:n
    V(:,k)=A(:,k);
    for j=1:k-1
        R(j,k)=V(:,j)'*A(:,k);
        V(:,k)=V(:,k)-R(j,k)*V(:,j);
    end
    R(k,k)=norm(V(:,k));
    V(:,k)=V(:,k)/R(k,k);
end
disp('Q=',V);
```



## Topic: Eigen values and Eigen vectors of a given square matrix

Example 10: Find the Eigen values and the corresponding Eigen vectors of

$$A = \begin{bmatrix} 3 & -2 & 5 \\ -2 & 3 & 6 \\ 5 & 6 & 4 \end{bmatrix}$$

Solution:

```
clear;
close;
clc;
format('v',5);
A= {4 -2 2;4 -3 -2;2 3 -1 };
for l=1:3
    L(l,l)=1;
end

for i=1:3
    for j=1:3
        s=0;
        if (j>=i)
            for k=1:i-1
                s=s+(L(i, k)*U(k, j));
            end
            U(i,j)=A(i,j)-s;
        else
            for k=1:j-1
                s=s+(L(i,k)*U(k,j));
            end
            L(i,j)=(A(i,j)-s)/U(j,j);
        end
    end
end

b=[6;-8;5];
c=L\b;
x=U\c;
disp( "Solution of the given equation is: ", x)
```

"The characteristice Matrix is"

```
3 -lam -2      5
-2      3 -lam  6
5      6      4 -lam
```

"The characteristic polynomial is"

```
283 -32lam -10lam^2 +lam^3
```

"The eigen values of A are"

```
-5.4409348
 4.9650189
10.475916
```

"The eigen vectors of A are"

```
-0.5135977  0.7711676  0.3761887
-0.5746266 -0.6347298  0.5166454
 0.6371983  0.0491799  0.7691291
```

	Name	Value	Type	Visibi...	Mem...
	A	3x3	Double	local	280 B
	ans	[]	Double	local	208 B
	charMat	3x3	Polyn...	local	408 B
	charPoly	1x1	Polyn...	local	280 B
	lam	[-5.44...	Double	local	232 B
	x	3x3	Double	local	280 B

```
// -- 17/04/2021 13:44:45 -- //
// -- 17/04/2021 13:45:39 -- //
// -- 21/04/2021 12:15:31 -- //
// -- 22/04/2021 19:37:16 -- //
// -- 22/04/2021 22:58:06 -- //
// -- 23/04/2021 19:27:28 -- //
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// -- 23/04/2021 22:57:59 -- //
// -- 24/04/2021 00:04:30 -- //
// -- 24/04/2021 00:13:47 -- //
// -- 24/04/2021 09:53:22 -- //
// -- 24/04/2021 13:10:06 -- //
```



Internet Of Things wit...

