EE3025 Assignment-1

Adithya Vardhan - EE18BTECH11008

Download all python codes from

https://github.com/Adithya-Vardhan/HW1/tree/main/codes

and latex-tikz codes from

https://github.com/Adithya-Vardhan/HW1

1 Problem

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.0.1}$$

and $H(k)$ using h(n).

2 Solution

Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{2.0.1}$$

and the given difference equation is

$$y(n) + \frac{1}{2} y(n-1) = x(n) + x(n-2) \tag{2.0.2}$$

By applying Z-transform to the above equation we get,

$$Y(z) + \frac{1}{2} z^{-1} Y(z) = X(z) + z^{-2} X(z) \tag{2.0.3}$$

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)} X(z) \tag{2.0.4}$$

Therefore H(z) is

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.0.5}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.0.6}$$

$$H(z) = z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{2.0.7}$$

By applying inverse z-transform we get,

$$h(n) = \left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.0.8}$$

By using equation 1.0.1

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.9}$$

the above equation can be written as,where $\omega = e^{\frac{-j2\pi}{N}}$

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \cdots & \omega^{(N-1)(N-1)} \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{pmatrix} \tag{2.0.10}$$

and

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.11}$$

the above equation can be written as,

$$\begin{pmatrix} H(0) \\ H(1) \\ H(2) \\ \vdots \\ H(N-1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \cdots & \omega^{(N-1)(N-1)} \end{pmatrix} \begin{pmatrix} h(0) \\ h(1) \\ h(2) \\ \vdots \\ h(N-1) \end{pmatrix} \tag{2.0.12}$$

from above mentioned python codes we get the following plots

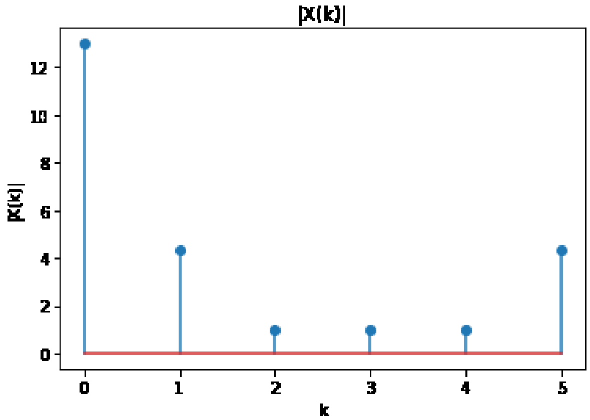

Fig. 0: Magnitude of X(k)

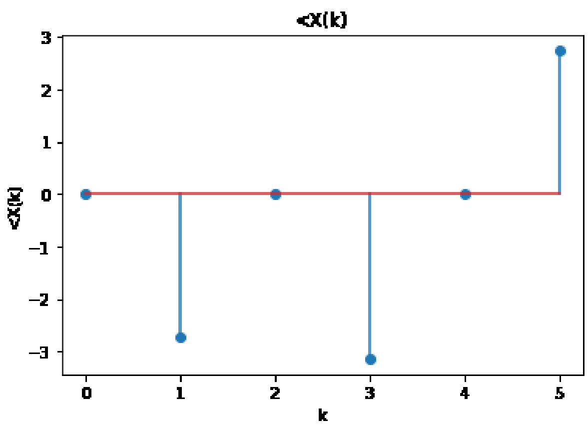

Fig. 0: Phase of X(k)

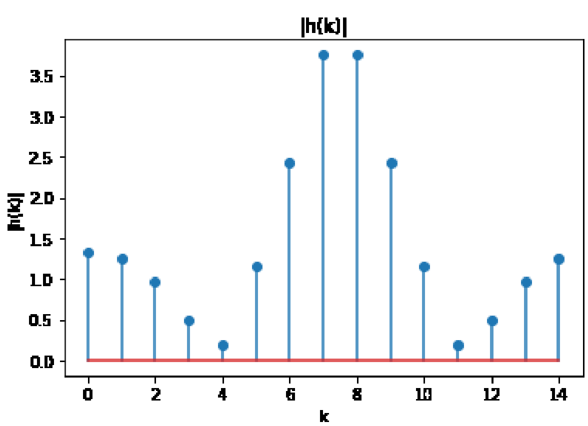

Fig. 0: Magnitude of h(k)

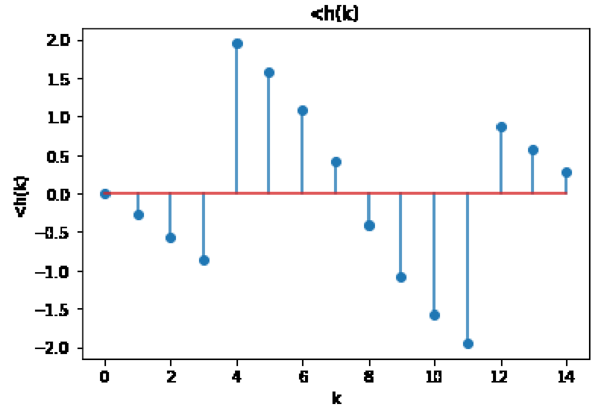

Fig. 0: Phase of h(k)