Control Systems

G V V Sharma*

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

2.1. The characteristic equation of linear time invariant system is given by

$$\nabla(s) = s^4 + 3s^3 + 3s^2 + s + k = 0 \tag{2.1.1}$$

The system is BIBO stable if

$$A.0 < k < \frac{12}{9} \tag{2.1.2}$$

$$B.k > 3 \tag{2.1.3}$$

$$C.0 < k < \frac{8}{9} \tag{2.1.4}$$

$$D.k > 6 \tag{2.1.5}$$

2.2. **solution**

$$\nabla(s) = s^4 + 3s^3 + 3s^2 + s + k = 0 \tag{2.2.1}$$

For a system to be stable all coefficients of characteristic equation should lie on left half of s-plane because if any of the pole is in right half of the s-plane then there will be a component which is exponentially increasing in output,causing system to be unstable.This can be verified by Routh Array Criterion.
The Routh hurwitz criterion:-
This criterion is based on arranging the coefficients of characteristic equation into an array called Routh array.If all the coefficients in the first row of routh array are of same algebric sign then the system is stable.
For any characteristic equation q(s),

$$q(s) = a_0 s^n + a_1 s^{n-1} + \dots.. + a_{n-1} s + a_n = 0 \tag{2.2.2}$$

Routh array can be constructed as follows..,

$$\begin{pmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{pmatrix} \begin{pmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \dots \end{pmatrix}$$

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \tag{2.2.3}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \tag{2.2.4}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \tag{2.2.5}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \tag{2.2.6}$$

$$\begin{pmatrix} s^4 \\ s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 3 & k \\ 3 & 1 & 0 \\ \frac{8}{3} & k & 0 \\ \frac{\frac{8}{3}-3k}{\frac{8}{3}} & 0 & 0 \\ k & 0 & 0 \end{pmatrix} \tag{2.2.7}$$

Given system is stable if

$$\frac{\frac{8}{3} - 3k}{\frac{8}{3}} > 0, k > 0 \tag{2.2.8}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

$$\frac{8}{3} - 3k > 0 \tag{2.2.9}$$

$$3k < \frac{8}{3} \tag{2.2.10}$$

$$(0 < k < \frac{8}{9}) \tag{2.2.11}$$

for example the zeros of polynomial $s^4 + 3s^3 + 3s^2 + s + 0.5 = 0$ are

$$s1 = 0.08373 + 0.45773i \tag{2.2.12}$$

$$s2 = 0.08373 - 0.45773i \tag{2.2.13}$$

$$s3 = 1.41627 + 0.55075i \tag{2.2.14}$$

$$s4 = 1.41627 - 0.55075i \tag{2.2.15}$$

3 Compensators

4 Nyquist Plot