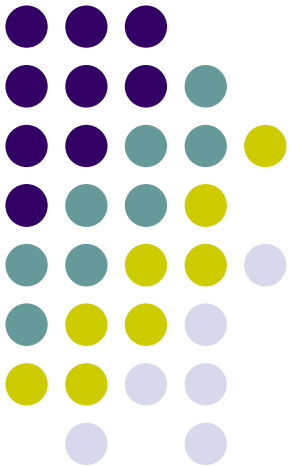
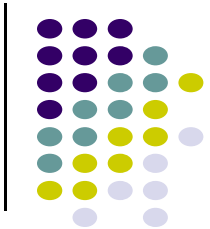
Derivation

Lecture - 6



Derivation



(1)
$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow id$

- $E \overset{E \rightarrow E+E}{\Rightarrow} E + E$
  $\overset{E \rightarrow id}{\Rightarrow}$ id + E
  $\overset{E \rightarrow id}{\Rightarrow}$ id + id

is a derivation of the terminal **string** *id* + *id* from E

- In a derivation, a production is applied at each step, **to replace a nonterminal** by the right-hand side of the corresponding production.
- The above derivation is represented in short as,

  $E \Rightarrow^*$ id + id,

and is read as

E derives id + id

Derivations: Example

- Grammar for palindromes: G = (N, T, P, S),
  - N = {S},
  - T = {0, 1},
  - P = {

$$S \rightarrow 0\ S\ 0 \mid 1\ S\ 1 \mid 0 \mid 1 \mid \varepsilon$$

}.

(2)
$S \rightarrow 0S0$
$S \rightarrow 1S1$
$S \rightarrow 0$
$S \rightarrow 1$
$S \rightarrow \epsilon$

- A derivation of the string **10101**:

$S \Rightarrow 1\ S\ 1$ (using $S \rightarrow 1S1$)

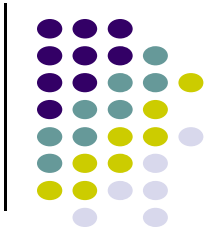
$\Rightarrow 1\ 0S0\ 1$ (using $S \rightarrow 0S0$)

$\Rightarrow 10101$ (using $S \rightarrow 1$)

Derivation Trees

- Derivations can be displayed as trees
- The internal nodes of the tree are all nonterminals and the leaves are all terminals
- Corresponding to each internal node A, there exists a production $\in$ P, with the RHS of the production being the list of children of A, read from left to right
- The **yield** of a derivation tree is the list of the labels of all the leaves read from left to right
- If α is the yield of some derivation tree for a grammar G, then $S \Rightarrow^* \alpha$ and conversely

Derivation Tree Example



S → aAS | a
A → SbA | SS | ba

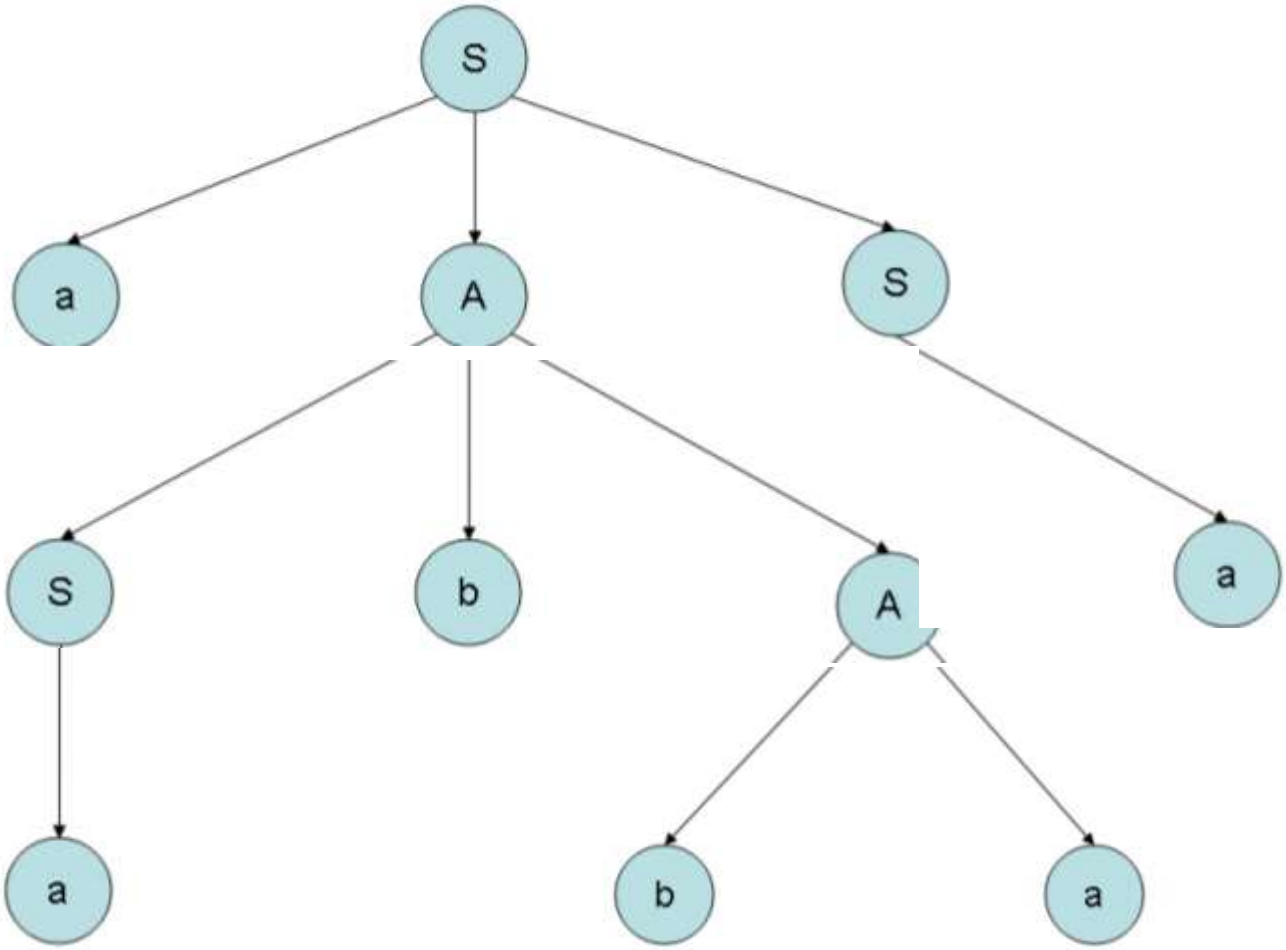
S => aAS

=> aSbAS

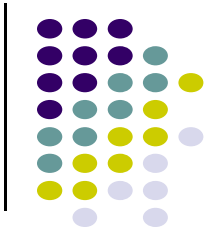
=> aabAS

=> aabbaS

=> aabbaa



Leftmost and Rightmost Derivations



- A *leftmost derivation* is one where, at each step, the leftmost nonterminal is replaced.

  (analogous for *rightmost derivation*)

- *Example*: a grammar for arithmetic expressions:

  $E \rightarrow E + E \mid E * E \mid \mathbf{id}$

Leftmost derivation:

$E \Rightarrow E * E$
$\Rightarrow E + E * E$
$\Rightarrow \mathbf{id} + E * E$
$\Rightarrow \mathbf{id} + \mathbf{id} * E$
$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$

Rightmost derivation:

$E \Rightarrow E + E$
$\Rightarrow E + E * E$
$\Rightarrow E + E * \mathbf{id}$
$\Rightarrow E + \mathbf{id} * \mathbf{id}$
$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$

Context-free Grammars: L(G)

A grammar for palindromic bit-strings:

G = (N, T, P, S), where:

- V = { S, B }
- T = {0, 1}
- P = { $S \rightarrow B$,
  $S \rightarrow \varepsilon$,
  $S \rightarrow 0\ S\ 0$,
  $S \rightarrow 1\ S\ 1$,
  $B \rightarrow 0$,
  $B \rightarrow 1$
  }

- $L(G) = \{ w \mid w \in T^* \text{ and } S \Rightarrow^* w \}$.

L(G) = { 0, 1, 00,11,101,010, 111, 000……. }