

**Course Outcome 5 (CO5): Convolutional and LDPC Codes**

1. Obtain the output codeword corresponding to the information sequence (1 1 0 1 1) for a convolutional encoder with rate  $\frac{1}{2}$  and constraint length 4, for generator sequences,  $g(1) = (1011)$  and  $g(2) = (1111)$ .
2. Explain the message-passing decoding algorithm for LDPC codes with respect to binary erasure channel.

**SYLLABUS****Module 1 – Entropy, Sources and Source Coding**

Entropy, Properties of Entropy, Joint and Conditional Entropy, Mutual Information, Properties of Mutual Information.

Discrete memoryless sources, Source code, Average length of source code, Bounds on average length, Uniquely decodable and prefix-free source codes. Kraft Inequality (with proof), Huffman code. Shannon's source coding theorem (both achievability and converse) and operational meaning of entropy.

**Module 2 – Channels and Channel Coding**

Discrete memoryless channels. Capacity of discrete memoryless channels. Binary symmetric channels (BSC), Binary Erasure channels (BEC). Capacity of BSC and BEC. Channel code. Rate of channel code. Shannon's channel coding theorem (both achievability and converse without proof) and operational meaning of channel capacity.

Modeling of Additive White Gaussian channels. Continuous-input channels with average power constraint. Differential entropy. Differential Entropy of Gaussian random variable. Relation between differential entropy and entropy. Shannon-Hartley theorem (with proof – mathematical subtleties regarding power constraint may be overlooked).

Inferences from Shannon Hartley theorem – spectral efficiency versus SNR per bit, power-limited and bandwidth-limited regions, Shannon limit, Ultimate Shannon limit.

**Module 3 – Introduction to Linear Block Codes**

Overview of Groups, Rings, Finite Fields, Construction of Finite Fields from Polynomial rings, Vector spaces.

Block codes and parameters. Error detecting and correcting capability. Linear block codes. Two simple examples -- Repetition code and single parity-check code. Generator and parity-check matrix. Systematic form.

Maximum likelihood decoding of linear block codes. Bounded distance decoding. Syndrome. Standard array decoding.

**Module 4 – A Few Important Classes of Algebraic codes**

Cyclic codes. Polynomial and matrix description. Interrelation between polynomial and matrix view point. Systematic encoding. Decoding of cyclic codes.

(Only description, no decoding algorithms) Hamming Codes, BCH codes, Reed-Solomon Codes.

**Module 5 – Convolutional and LDPC Codes**

Convolutional Codes. State diagram. Trellis diagram. Maximum likelihood decoding. Viterbi algorithm.

Low-density parity check (LDPC) codes. Tanner graph representation. Message-passing decoding for transmission over binary erasure channel.

**Text Books and References**

1. “Elements of Information Theory”, Joy A Thomas, Thomas M Cover, Wiley-Interscience.
2. “Information Theory, Inference and Learning Algorithms”, David JC McKay, Cambridge University Press
3. “Principles of digital communication”, RG Gallager, Cambridge University Press
4. “Digital Communication Systems”, Simon Haykin, Wiley.
5. “Introduction to Coding Theory”, Ron M Roth, Cambridge University Press
6. Shu Lin & Daniel J. Costello, Jr., Error Control Coding : Fundamentals and Applications, 2nd Edition.
7. Modern Coding Theory, Rüdiger Urbanke and TJ Richardson, Cambridge University Press.

**Course Contents and Lecture Schedule**

No	Topic	No. of Lectures
<b>1</b>	<b>Entropy, Sources and Source Coding</b>	
1.1	Entropy, Properties of Entropy, Joint and Conditional Entropy	2
1.2	Mutual Information, Properties of Mutual Information	2
1.3	Discrete memoryless sources, Source code, Average length of source code, Bounds on average length	2
1.4	Uniquely decodable and prefix-free source codes. Kraft Inequality (with proof)	2
1.5	Huffman code. Shannon’s source coding theorem and operational meaning of entropy	2
<b>2</b>	<b>Channels and Channel Coding</b>	
2.1	Discrete memoryless channels. Capacity of discrete memoryless channels	1
2.2	Binary symmetric channels (BSC), Binary Erasure channels (BEC). Capacity of BSC and BEC.	2

2.3	Channel code. Rate of channel code. Shannon's channel coding theorem (without proof) and operational meaning of channel capacity.	2
2.4	Modeling of Additive White Gaussian channels. Continuous-input channels with average power constraint.	1
2.5	Differential entropy. Differential Entropy of Gaussian random variable. Relation between differential entropy and entropy	2
2.6	Shannon-Hartley theorem and its proof	1
2.7	Inferences from Shannon Hartley theorem – spectral efficiency versus SNR, power-limited and bandwidth-limited regions, Shannon limit, Ultimate Shannon limit.	2
<b>3</b>	<b>Introduction to Linear Block Codes</b>	
3.1	Overview of Groups, Rings, Finite Fields, Construction of Finite Fields from Polynomial rings, Vector spaces.	5
3.2	Block codes and parameters. Error detecting and correcting capability	1
3.3	Linear block codes. Generator and parity-check matrix. Systematic form. Two simple examples -- Repetition code and single parity-check code. General examples.	2
3.5	Maximum likelihood decoding of linear block codes. Bounded distance decoding. Syndrome. Standard array decoding.	3
<b>4</b>	<b>A Few Important Classes of Algebraic codes</b>	
4.1	Cyclic codes. Polynomial and matrix description. Interrelation between polynomial and matrix view point. Systematic encoding. Decoding of cyclic codes.	4
4.2	Hamming Codes,	1
4.3	BCH codes, Reed-Solomon Codes.	2
<b>5</b>	<b>Convolutional and LDPC Codes</b>	
5.1	Convolutional Codes.	1
5.2	State diagram. Trellis diagram.	2
5.3	Maximum likelihood decoding. Viterbi algorithm	
5.4	Low-density parity check (LDPC) codes. Tanner graph representation Message-passing decoding for transmission over binary erasure channel.	2

## ECT 306 Information Theory and Coding

### Simulation Assignments

The following simulation assignments can be done with Python/MATLAB/SCILAB/LabVIEW

#### 1. Source Coding - Huffman Code

1. Generate Huffman code for the source with symbol probabilities  $\{1/2, 1/3, 1/6\}$ .
2. Find the entropy, average codeword length and efficiency of the code.
3. Create the second order extended source by taking probabilities of 9 symbols in the extended source as the product of every possible combinations of two symbols from the original source.
4. Generate Huffman code for the extended source symbols and find the entropy, average codeword length and efficiency of the code.
5. Compare the two efficiencies and appreciate the Shannon's source coding theorem.

#### 2. Binary Symmetric Channel

1. Create a  $2 \times 2$  matrix,  $P(Y/X)$  for binary symmetric channel with channel transition probability,  $p < 0.5$ .
2. Let the input symbol probabilities corresponding to symbol 0 and 1 be  $\alpha$  and  $(1 - \alpha)$  respectively. For different values of  $\alpha$  ranging from 0 to 1, find the joint probability matrix,  $P(X,Y)$  and output probability,  $P(Y)$
3. Find mutual information,  $I(X; Y) = H(Y) - H(Y/X)$  for each value of  $\alpha$ . Plot the  $I(X; Y)$  versus  $\alpha$  graph and observe the channel capacity.

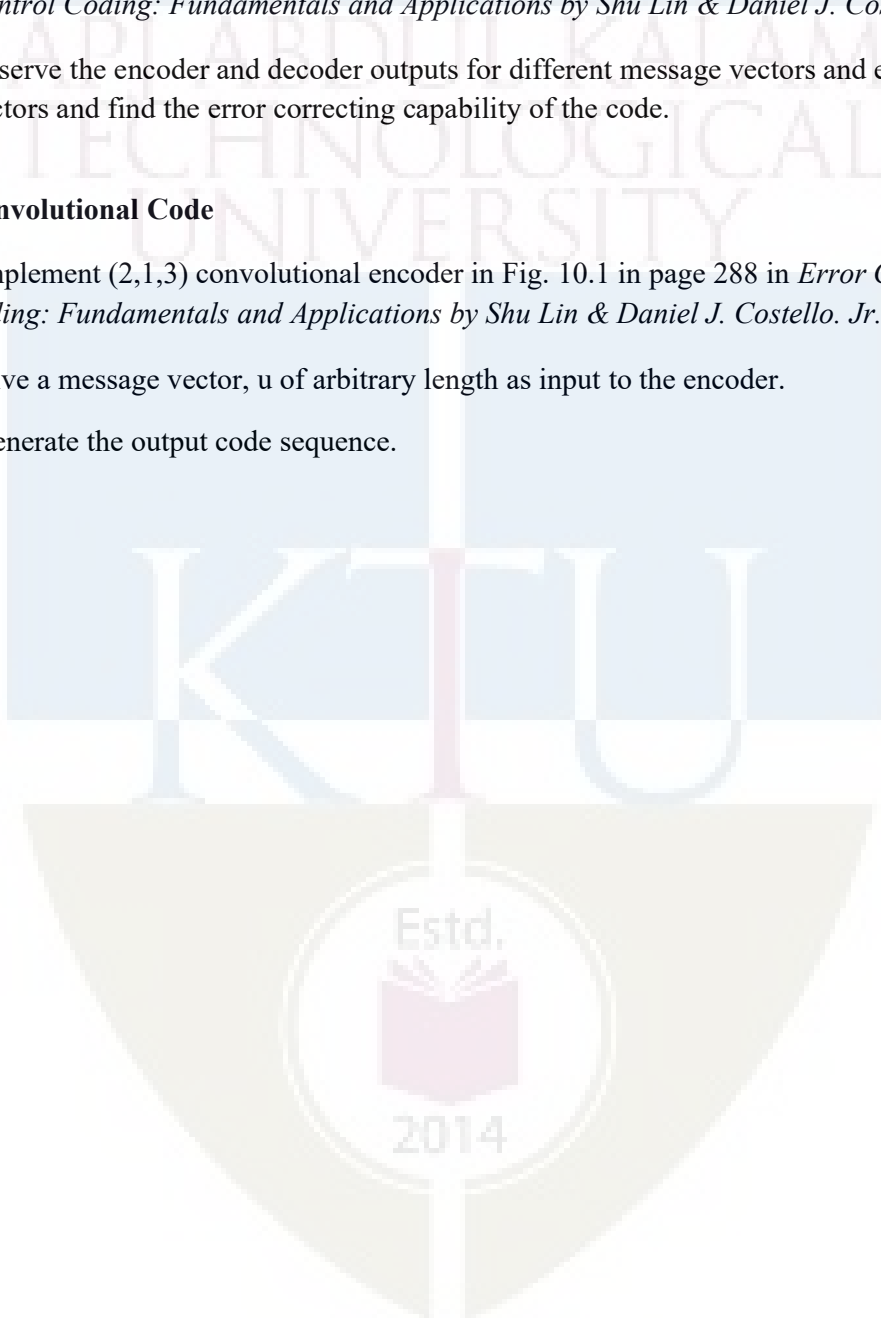
#### 3. Linear Block Code (LBC)

1. Create the  $k \times n$  generator matrix,  $G$  of  $(n, k)$  LBC.
2. Generate all possible codewords by multiplying the message vector of length,  $k$  with  $G$ .
3. Calculate the Hamming weight of all codewords and obtain the minimum distance,  $d_{\min}$  of the code.
4. Find its error correcting and detecting capability.

#### 5. Cyclic Code – Encoder & Decoder

1. Realize the encoder circuit for  $(7, 4)$  cyclic code in Fig. 4.2 in page 96 in *Error Control Coding: Fundamentals and Applications* by Shu Lin & Daniel J. Costello. Jr.

2. Create a random binary vector of length 4 as input message vector and generate the codeword.
  3. Create binary vector of length 7 with Hamming weight 1 as error vector and add it to the encoder output to generate the receiver output.
  4. Realize the decoder circuit for (7, 4) cyclic code in Fig. 4.9 in page 107 in *Error Control Coding: Fundamentals and Applications* by Shu Lin & Daniel J. Costello. Jr.
  5. Observe the encoder and decoder outputs for different message vectors and error vectors and find the error correcting capability of the code.
6. **Convolutional Code**
1. Implement (2,1,3) convolutional encoder in Fig. 10.1 in page 288 in *Error Control Coding: Fundamentals and Applications* by Shu Lin & Daniel J. Costello. Jr.
  2. Give a message vector,  $u$  of arbitrary length as input to the encoder.
  3. Generate the output code sequence.



**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**ECT 306 INFORMATION THEORY AND CODING**

Time: 3 hours

Max. Marks:100

**PART A**

Answer **all** questions. Each question carries **3 marks**.

1. A source emits one of four symbols,  $s_0, s_1, s_2, s_3$  with probabilities  $1/3, 1/6, 1/4$  and  $1/4$  respectively. The successive symbols emitted by the source are statistically independent. Calculate the entropy of the source.
2. Identify the instantaneous codes from the code sets listed below.

Symbol	Code I	Code II	Code III	Code IV
$s_0$	0	0	0	00
$s_1$	10	01	01	01
$s_2$	110	001	011	10
$s_3$	1110	0010	110	110
$s_4$	1111	0011	111	111

3. State Shannon's channel coding theorem. What is its significance in digital communication system?
4. An analog signal band limited to 'B' Hz is sampled at Nyquist rate. The samples are quantized into 4 levels. The quantization levels are assumed to be independent and occur with probabilities:  $p_1 = p_4 = 1/8, p_2 = p_3 = 3/8$ . Find the information rate of the source assuming  $B = 100\text{Hz}$ .
5. List the properties of group. Give an example.
6. Show that  $C = \{0000, 1100, 0011, 1111\}$  is a linear code. What is its minimum distance?
7. Explain generation of systematic cyclic code using polynomial description.
8. List the features of Reed Solomon code.
9. Draw a (3,2,1) convolutional encoder with generator sequences,  $g_1^{(1)} = (11), g_1^{(2)} = (01), g_1^{(3)} = (11)$  and  $g_2^{(1)} = (01), g_2^{(2)} = (10), g_2^{(3)} = (10)$ .
10. Draw the Tanner graph of rate  $1/3$  LDPC code for the given parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

**PART B**

Answer **any one** question from each module. Each question carries 14 marks.

**MODULE I**

11 (a) The joint probability of a pair of random variables is given below. Determine  $H(X, Y)$ ,  $H(X/Y)$ ,  $H(Y/X)$  and  $I(X, Y)$ . Verify the relationship between joint, conditional and marginal entropies.

$$P(X, Y) = \begin{bmatrix} 1/3 & 1/3 \\ 0 & 1/3 \end{bmatrix}$$

(10 marks)

11 (b) Explain uniquely decodable and prefix-free property of source code. (4 marks)

12 (a) Find the binary Huffman code for the source with probabilities  $\{1/3, 1/5, 1/5, 2/15, 2/15\}$ . Also find the efficiency of the code. (9 marks)

12 (b) Prove that  $H(Y) \geq H(Y/X)$ . (5 marks)

**MODULE II**

13 (a) A voice grade channel of the telephone network has a bandwidth of 3.4 KHz. Calculate channel capacity of the telephone channel for signal to noise ratio of 30 dB. Also determine the minimum SNR required to support information transmission through the telephone channel at the rate of 4800 bits/sec.

(7 marks)

13 (b) Derive the expression for channel capacity for binary erasure channel. (7 marks)

14 (a) A binary channel has the following noise characteristic.

$$P(Y/X) = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

If the input symbols are transmitted with probabilities  $3/4$  and  $1/4$  respectively, find  $I(X; Y)$ . Also compute channel capacity and what are the input symbol probabilities that correspond to the channel capacity. (9 marks)

14 (b) State Shannon Hartley theorem and explain the significance of Shannon limit. (6 marks)

**MODULE III**

15 (a) The parity check matrix of (7,4) linear block code is given as

$$H = \begin{bmatrix} 1 & 00 & 1 & 01 & 1 \\ 0 & 10 & 1 & 11 & 0 \\ 0 & 01 & 0 & 11 & 1 \end{bmatrix}$$

Compute the minimum distance of the code and find its error detection and correcting capability. Suppose that the received codeword,  $r = (1001111)$ . Determine whether the received codeword is in error? If so, form the decoding table and obtain the correct codeword. (9 marks)

16 (b) List the properties of vector space. Define subspace. (5 marks)

17 (a) The parity bits of a (8, 4) linear systematic block code are generated by



$$c_5 = d_1 + d_2 + d_4$$

$$c_6 = d_1 + d_2 + d_3$$

$$c_7 = d_1 + d_3 + d_4$$

$$c_8 = d_2 + d_3 + d_4$$

(+ sign denotes modulo-2 addition)

where  $d_1, d_2, d_3$  and  $d_4$  are message bits and  $c_5, c_6, c_7$  and  $c_8$  are parity bits. Find generator matrix  $G$  and parity check matrix  $H$  for this code. Draw the encoder circuit (7 marks)

17 (b) Explain the construction of finite field from polynomial ring with the help of an ex-ample.

(7 marks)

#### MODULE IV

18 (a) Consider a (7, 4) cyclic code with generator polynomial,  $g(x) = 1 + x + x^3$ . Express the generator matrix and parity-check matrix in systematic and non-systematic form

(8 Marks)

18 (b) Find the generator polynomial for single, double and triple error correcting BCH code of block length,  $n = 15$ .

(6 marks)

19 (a) Draw syndrome circuit for a (7,4) cyclic code generated by  $g(x)=1+x+x^3$ . If the received vector  $r$  is [0010110] what is the syndrome of  $r$ ? Explain the circuit with a table showing the contents of the syndrome register.

(8 Marks)

19 (b) What are the features of Hamming code? Find the parity check matrix for (15, 11) Hamming code.

(6 marks)

#### MODULE V

20 (a) Draw the state diagram of a convolution encoder with rate  $1/3$  and constraint length

3 for generator sequences  $g^{(1)} = (1 \ 0 \ 0)$ ,  $g^{(2)} = (1 \ 0 \ 1)$ ,  $g^{(3)} = (1 \ 1 \ 1)$ .

(7 marks)

20 (b) Explain message passing decoding algorithm for LDPC codes with the help of an example.

(7 marks)

21 For a (2,1,2) convolutional encoder with generator sequences  $g^{(1)} = (1 \ 1 \ 1)$  and  $g^{(2)} = (1 \ 0 \ 1)$ . Draw Trellis and perform Viterbi decoding on this trellis for the received sequence {01, 10, 10, 11, 01, 01, 11} and obtain the estimate of the transmitted sequence.

(14 marks)



ECT308	COMPREHENSIVE COURSE WORK	CATEGORY	L	T	P	CREDIT
		PCC	1	0	0	1

**Prerequisite:**

- Course Outcomes:** After the completion of the course the student will be able to

<b>CO 1</b>	Apply the knowledge of circuit theorems and solid state physics to solve the problems in electronic Circuits
<b>CO 2</b>	Design a logic circuit for a specific application
<b>CO 3</b>	Design linear IC circuits for linear and non-linear circuit applications.
<b>CO 4</b>	Explain basic signal processing operations and Filter designs
<b>CO 5</b>	Explain existent analog and digital communication systems

### Mapping of course outcomes with program outcomes

[illegible]

**Assessment Pattern**

<b>Bloom's Category</b>	<b>End Semester Examination</b>
Remember	10
Understand	20
Apply	20
Analyse	
Evaluate	
Create	

**Mark distribution**

<b>Total Marks</b>	<b>CIE</b>	<b>ESE</b>	<b>ESE Duration</b>
50	0	50	1 hour

**End Semester Examination Pattern:** Objective Questions with multiple choice (Four). Question paper include Fifty Questions of One mark each covering the five identified courses.

**Syllabus****Full Syllabus of all five selected courses****Course Contents and Lecture Schedule**

No	Topic	No. of Lectures
<b>1</b>	<b>Analog Circuits</b>	
1.1	Mock Test on Module 1 and Module 2	1
1.2	Mock Test on Module 3, Module 4 and Module 5	1
1.3	Feedback and Remedial	1
<b>2</b>	<b>Logic Circuit design</b>	
2.1	Mock Test on Module 1, Module 2 and Module 3	1
2.2	Mock Test on Module 4 and Module 5	1
2.3	Feedback and Remedial	1
<b>3</b>	<b>Linear IC</b>	
3.1	Mock Test on Module 1 and Module 2	1
3.2	Mock Test on Module 3, Module 4 and Module 5	1
3.3	Feedback and Remedial	1
<b>4</b>	<b>Digital Signal Processing</b>	
4.1	Mock Test on Module 1, Module 2 and Module 3	1
4.2	Mock Test on Module 4 and Module 5	1
4.3	Mock Test on Module 1, Module 2 and Module 3	1
<b>5</b>	<b>Analog and Digital Communication</b>	
5.1	Mock Test on Module 1, Module 2 and Module 3	1
5.2	Mock Test on Module 4 and Module 5	1
5.3	Feedback and Remedial	1