23PHY114
Class Notes

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Contents

Chapter 1

Solids

1.1 Moment Of Area

The moment of inertia is used to help find the ”resistance” to the force, given a specific axis or direction.

1.2 Resisting Force And Moments From Supports

There are three main kinds of supports -

1. Pin/Hinge - Fixes linear motion but leaves rotation free.
2. Roller - Fixes rotation but leaves motion free.
3. Clamp - Fixes both linear and rotational motion.

Take this case, with a bunch of forces being applied to the given load.

There are three main things we need to find for this figure.

1. The resultant force acting on this bar fixed to a hinge.
2. The support reaction force and moment.
3. The moment on the object (maximum)

The way to approach the problem is as always,

1. Free Body Diagram first.
2. Assuming $\Sigma f = 0$, because the object has no acceleration currently, because it is fixed.
3. Assuming $\Sigma M = 0$

4. The point at which the resultant force is applied is found by,

$$\frac{\int |r| dm}{\int dm}$$

1.3 Derivation Of The Uniaxial Formula

1.4 Code For Uniaxial Deformation

Main Subroutines For Uniaxial Deformation

1.4.1 Finding The Local Stiffness Matrix

```
function stiffnessLocal = localStiffnessGenerator(E,A,l,theta);
    stiffnessConstant = E*A/l;
    R = [cos(theta) -sin(theta); sin(theta) cos(theta)];
    stiffnessMatrix = [stiffnessConstant 0 -stiffnessConstant 0; zeros(1,4);-stiffnessConstant 0 stiffnessConstant 0; zeros(1,4)];
    R4 = [R zeros(2,2); zeros(2,2) R];
    stiffnessLocal = R4*stiffnessMatrix*R4';
end
```

1.4.2 Converting The Local Stiffness To A Global Stiffness Matrix

```
function stiffnessLocalGlobal = local2Global(stiffnessLocal,node1,node2,nodeCount)
    stiffnessLocalGlobal = zeros(2*nodeCount,2*nodeCount);
    i = 2*node1 - 1;
    j = 2*node2 - 1;
    stiffnessLocalGlobal(i:(i+1),i:(i+1)) = stiffnessLocal(1:2,1:2);
    stiffnessLocalGlobal(i:(i+1),j:(j+1)) = stiffnessLocal(1:2,3:4);
    stiffnessLocalGlobal(j:(j+1),i:(i+1)) = stiffnessLocal(3:4,1:2);
    stiffnessLocalGlobal(j:(j+1),j:(j+1)) = stiffnessLocal(3:4,3:4);
end
```

1.4.3 Main Loop Through Evaluating The Global Stiffness Matrix

```
 A = 0; theta = 0; l = 0; stiffnessLocal = zeros(4,4); nodeAxialForces = zeros(nodeCount,1);
for element = 1:elementCount % For first three bars
    A = areaVector(element);
    theta = angleVector(element);
    l = lengthVector(element);
    stiffnessLocal = localStiffnessGenerator(E,A,l,theta);
    nodeCounter = element*2 - 1;
```

```
    stiffnessLocalGlobal = local2Global(stiffnessLocal,nodeVector(nodeCounter),
     nodeVector(nodeCounter+1),nodeCount);
    stiffnessLocal
    stiffnessLocalGlobal
    stiffnessGlobal += stiffnessLocalGlobal;
end
```

1.4.4 Applying Boundary Conditions

```
selectedVector = [3 5:end]
forceEval = forceVector(selectedVector);
displacementEval = displacementVector(selectedVector);
stiffnessEval = stiffnessGlobal(selectedVector);
displacementEval = stiffnessEval\forceEval;
```

1.5 Deriving For Bending Deformation

Taking this example of a bar fixed to the wall, and examining small components of the bar and examining the forces at work on the component, we get a Tension, Moment and Stress.

Taking things along the $\hat{i}$ direction, we get

$$-T + T + \Delta T = 0$$

What does this tell us? Tension is constant.

While $-S + S + \Delta S - w\Delta x = 0$

$$\frac{dS}{dx} = \lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = w(x)$$

We get an expression for the effects of the distributed force on the shear force of an individual object.

Looking at the angular momentum balance,

$$\sum M_{/P} = 0$$

$$-M + (M + \Delta M)\hat{k} + (-\Delta x\hat{i} \times (-T\hat{i} - S\hat{j}) + (\frac{-\Delta x}{2}\hat{i} \times (-w(x)\Delta x\hat{j}))$$

$$\Rightarrow \frac{\Delta M}{\Delta x} + S - w\frac{\Delta x}{2} = 0$$

Taking the limit on the equations, we get,

$$\frac{dM}{dx} + S = 0$$

Thus we get,

$$\frac{dT}{dx} = 0$$

$$\frac{dS}{dx} = w(x)$$
$$\frac{dM}{dx} = -S$$

Now we get to the Finite Element Method for bending.

$$\theta(x) = v'(x)$$

We find by simplification,

$$c_1 = v_l$$
$$c_2 = \theta_l$$
$$c_3 = \frac{3}{l^2}(v_r - v_l) - \frac{1}{l}(2\theta_l + \theta_r)$$
$$c_4 = \frac{2}{l^3}(v_l - v_r) + \frac{\theta_l + \theta_r}{l^2}$$

$$v(x) = H_1(x)v_l + H_2(x)\theta_l(H_3(x))v_r + (H_4(x))\theta_r$$
$$H_1 = 2(\frac{x}{l}^3) - 3\frac{x}{l}^2 + 1$$
$$H_2 = x - 2\frac{x^2}{l} + \frac{x^3}{l^2}$$
$$H_3 = 3(\frac{x}{l})^2 - 2\frac{x}{l}^3$$
$$H_4 = \frac{x^3}{l^2} - \frac{x^2}{l}$$

Since we know that,

$$\int_0^l EIq''v''dx$$
$$v = \underline{H}(x)^T v$$
$$q = H(x)$$

1.6 Quick Reference Notes

1.6.1 Derivations

The governing differential equation is

$$EIv^{(4)} = w(x)$$

Then we can form a linear relationship

$$
\begin{aligned}
v(x) &- \text{deflection} \\
v^{(2)}(x) = \theta(x) &- \text{slope} \\
EIv^{(2)}(x) = M(x) &- \text{bending moment related to curvature} \\
EIv^{(3)}(x) = \frac{dM}{dx} = V(x) &- \text{transverse shear} \\
EIv^{(4)}(x) = \frac{dV}{dx} = w(x) &- \text{load}
\end{aligned}
$$

Where, v is the deflection or displacement of the beam. E is the Young's Modulus and I is

1.6.2 Deriving Stiffness Matrix

Use Hermite's interpolation formula to derive cubic shape functions for the deflection of beams.

1.7 Final Equation

For a bar, with axial, bending and shear deformation... The final equation is, where R, S and M are Axial, Shear and Moment.

$$F = k\Delta x$$

$$
\begin{bmatrix} R_1 \\ S_1 \\ M_1 \\ R_2 \\ S_2 \\ M_2 \end{bmatrix} =
\begin{bmatrix}
C_1 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 12C_2 & 6C_2L & 0 & -12C_2 & 6C_2L \\
0 & 6C_2L & 4C_2L^2 & 0 & -6C_2L & 2C_2L^2 \\
-C_1 & 0 & 0 & C_1 & 0 & 0 \\
0 & -12C_2 & -6C_2L & 0 & 12C_2 & -6C_2L \\
0 & 6C_2L & 2C_2L^2 & 0 & -6C_2L & 2C_2L^2
\end{bmatrix}
\begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix}
$$

Where, $C_1 = \frac{EA}{L}$ and $C_2 = \frac{EI}{L^3}$

Rotating the matrix by angle θ

$$R = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$