23PHY114
Class Notes

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Contents

Chapter 1

Solids

1.1 Moment Of Area

The moment of inertia is used to help find the "resistance" to the force, given a specific axis or direction.

1.2 Resisting Force And Moments From Supports

There are three main kinds of supports -

1. Pin/Hinge - Fixes linear motion but leaves rotation free.
2. Roller - Fixes rotation but leaves motion free.
3. Clamp - Fixes both linear and rotational motion.

Take this case, with a bunch of forces being applied to the given load.

There are three main things we need to find for this figure.

1. The resultant force acting on this bar fixed to a hinge.
2. The support reaction force and moment.
3. The moment on the object (maximum)

The way to approach the problem is as always,

1. Free Body Diagram first.
2. Assuming $\Sigma f = 0$, because the object has no acceleration currently, because it is fixed.
3. Assuming $\Sigma M = 0$

4. The point at which the resultant force is applied is found by,

$$\frac{\int |r| dm}{\int dm}$$

1.3 Derivation Of The Uniaxial Formula

1.4 Code For Uniaxial Deformation

Main Subroutines For Uniaxial Deformation

1.4.1 Finding The Local Stiffness Matrix

```
function stiffnessLocal = localStiffnessGenerator(E,A,l,theta);
    stiffnessConstant = E*A/l;
    R = [cos(theta) -sin(theta); sin(theta) cos(theta)];
    stiffnessMatrix = [stiffnessConstant 0 -stiffnessConstant 0; zeros(1,4);-stiffnessConstant 0 stiffnessConstant 0; zeros(1,4)];
    R4 = [R zeros(2,2); zeros(2,2) R];
    stiffnessLocal = R4*stiffnessMatrix*R4';
end
```

1.4.2 Converting The Local Stiffness To A Global Stiffness Matrix

```
function stiffnessLocalGlobal = local2Global(stiffnessLocal,node1,node2,nodeCount)
    stiffnessLocalGlobal = zeros(2*nodeCount,2*nodeCount);
    i = 2*node1 - 1;
    j = 2*node2 - 1;
    stiffnessLocalGlobal(i:(i+1),i:(i+1)) = stiffnessLocal(1:2,1:2);
    stiffnessLocalGlobal(i:(i+1),j:(j+1)) = stiffnessLocal(1:2,3:4);
    stiffnessLocalGlobal(j:(j+1),i:(i+1)) = stiffnessLocal(3:4,1:2);
    stiffnessLocalGlobal(j:(j+1),j:(j+1)) = stiffnessLocal(3:4,3:4);
end
```

1.4.3 Main Loop Through Evaluating The Global Stiffness Matrix

```
 A = 0; theta = 0; l = 0; stiffnessLocal = zeros(4,4); nodeAxialForces = zeros(nodeCount,1);
for element = 1:elementCount % For first three bars
    A = areaVector(element);
    theta = angleVector(element);
    l = lengthVector(element);
    stiffnessLocal = localStiffnessGenerator(E,A,l,theta);
    nodeCounter = element*2 - 1;
```

```
    stiffnessLocalGlobal = local2Global(stiffnessLocal,nodeVector(nodeCounter),
     nodeVector(nodeCounter+1),nodeCount);
    stiffnessLocal
    stiffnessLocalGlobal
    stiffnessGlobal += stiffnessLocalGlobal;
end
```

1.4.4 Applying Boundary Conditions

```
selectedVector = [3 5:end]
forceEval = forceVector(selectedVector);
displacementEval = displacementVector(selectedVector);
stiffnessEval = stiffnessGlobal(selectedVector);
displacementEval = stiffnessEval\forceEval;
```

1.5 Deriving For Bending Deformation

Taking this example of a bar fixed to the wall, and examining small components of the bar and examining the forces at work on the component, we get a Tension, Moment and Stress.

Taking things along the $\hat{i}$ direction, we get

$$-T + T + \Delta T = 0$$

What does this tell us? Tension is constant.

While $-S + S + \Delta S - w\Delta x = 0$

$$\frac{dS}{dx} = \lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = w(x)$$

We get an expression for the effects of the distributed force on the shear force of an individual object.

Looking at the angular momentum balance,

$$\sum M_{/P} = 0$$

$$-M + (M + \Delta M)\hat{k} + (-\Delta x\hat{i} \times (-T\hat{i} - S\hat{j}) + (\frac{-\Delta x}{2}\hat{i} \times (-w(x)\Delta x\hat{j}))$$

$$\Rightarrow \frac{\Delta M}{\Delta x} + S - w\frac{\Delta x}{2} = 0$$

Taking the limit on the equations, we get,

$$\frac{dM}{dx} + S = 0$$

Thus we get,

$$\frac{dT}{dx} = 0$$

$$\frac{dS}{dx} = w(x)$$
$$\frac{dM}{dx} = -S$$

Now we get to the Finite Element Method for bending.

$$\theta(x) = v'(x)$$

We find by simplification,

$$c_1 = v_l$$
$$c_2 = \theta_l$$
$$c_3 = \frac{3}{l^2}(v_r - v_l) - \frac{1}{l}(2\theta_l + \theta_r)$$
$$c_4 = \frac{2}{l^3}(v_l - v_r) + \frac{\theta_l + \theta_r}{l^2}$$

$$v(x) = H_1(x)v_l + H_2(x)\theta_l(H_3(x))v_r + (H_4(x))\theta_r$$
$$H_1 = 2(\frac{x}{l}^3) - 3\frac{x}{l}^2 + 1$$
$$H_2 = x - 2\frac{x^2}{l} + \frac{x^3}{l^2}$$
$$H_3 = 3(\frac{x}{l})^2 - 2\frac{x}{l}^3$$
$$H_4 = \frac{x^3}{l^2} - \frac{x^2}{l}$$

Since we know that,

$$\int_0^l EIq''v''dx$$
$$v = \underline{H}(x)^T v$$
$$q = H(x)$$

1.6 Quick Reference Notes

1.6.1 Derivations

The governing differential equation is

$$EIv^{(4)} = w(x)$$

Then we can form a linear relationship

$$
\begin{aligned}
v(x) &- \text{deflection} \\
v^{(2)}(x) = \theta(x) &- \text{slope} \\
EIv^{(2)}(x) = M(x) &- \text{bending moment related to curvature} \\
EIv^{(3)}(x) = \frac{dM}{dx} = V(x) &- \text{transverse shear} \\
EIv^{(4)}(x) = \frac{dV}{dx} = w(x) &- \text{load}
\end{aligned}
$$

Where, v is the deflection or displacement of the beam. E is the Young's Modulus and I is

1.6.2 Deriving Stiffness Matrix

Use Hermite's interpolation formula to derive cubic shape functions for the deflection of beams.

1.7 Final Equation

For a bar, with axial, bending and shear deformation... The final equation is, where R, S and M are Axial, Shear and Moment.

$$F = k\Delta x$$

$$
\begin{bmatrix} R_1 \\ S_1 \\ M_1 \\ R_2 \\ S_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & 0 & -C_1 & 0 & 0 \\ 0 & 12C_2 & 6C_2L & 0 & -12C_2 & 6C_2L \\ 0 & 6C_2L & 4C_2L^2 & 0 & -6C_2L & 2C_2L^2 \\ -C_1 & 0 & 0 & C_1 & 0 & 0 \\ 0 & -12C_2 & -6C_2L & 0 & 12C_2 & -6C_2L \\ 0 & 6C_2L & 2C_2L^2 & 0 & -6C_2L & 2C_2L^2 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix}
$$

Where, $C_1 = \frac{EA}{L}$ and $C_2 = \frac{EI}{L^3}$

Rotating the matrix by angle θ

$$R = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Chapter 2

Fluids

2.1 Main Contents

Computational Fluids Dynamics

1. Flow in channels and pipes
2. Flow over objects

Definition 2.1.1: Laminar Region

The region in a fluid where the flow is in a straight line is known as the laminar region

Definition 2.1.2: Turbulent Region

The region in a fluid where the flow is not smooth is known as the turbulent region

The motivation behind learning CFD

Computational Fluid Dynamics has been around as long as computers have been long. Using computation to calculate and visualize the flow of fluids is the basis behind wind tunnels. Trials can be done to help optimize solutions for the best aerodynamics.

Examples of data used is the velocity profile, the force and the acceleration. These fluid simulations can take months and be used to simulate the effects of fluids on a given shape.

2.2 Terminology

In engineering terms, fluids are deformable bodies. They take up the shape of its container. Viscosity, is defined as the friction to flow - μ. It dictates the rate of flow of a fluid.

The counterintuitive picture. Imagine a stack of papers on your palm. On tilting your palm, the first stack of paper goes out then the next then the next. It is dependent on the friction of the piece of paper below it. The last piece of paper has zero friction, and flows out the easiest. But a fluid does not work that way, when some water is placed on your palm, even after flowing out, some imprint of the water remains.

Think of fluids as layered.

Taking a co-ordinate system, plotting the horizontal and vertical velocity of a fluid, u and v.

Definition 2.2.1: No slip Condition

A condition where the part of the fluid at the base or in contact with the surface has no horizontal velocity.

$$U(x, 0) = 0$$

Unlike solids, fluids have a profile that is counterintuitive to the way we imagine physics.

Counterintuitively, a fluid's profile is parabolic, in a pipe. Closed on all sides, the middlemost component of the fluid moves with

Note:-

Lagrangian Method - The method of following a single point and discovering interesting properties about it is known as the Lagrangian Method
Eulerian Method - The method of following a single location and performing analyses on that location.

The study of fluids is performed using the Eulerian Method.

2.3 Derivation Of Equations

Taking the fluid flowing through the figure given. In the Eulerian Method, we focus on a given location. This location here is the outlined box.

The mass flow rate for a given location is

$$\begin{aligned}
\dot{m}_{in} - \dot{m}_{out} &= \frac{dm}{dt} \\
\dot{m} &= \frac{d\rho V}{dt} \\
\dot{m} &= \rho \frac{dV}{dt} \\
&= \rho Q && \text{(Q - volume flow rate)}
\end{aligned}$$

$$\rho \frac{dV}{dt} = \Delta y \Delta z.u$$

$$\begin{aligned}
(\dot{m}_{intx} + \dot{m}_{inty}) - (\dot{m}_{outx} + \dot{m}_{outy}) &= \frac{dm_{inside}}{dt} \\
(\rho \Delta Y \Delta Z u + \rho \Delta X \Delta Z v) - (\rho \Delta Y \Delta Z (U + \Delta U) + \rho \Delta X \Delta Z (v + \Delta v) &= \frac{d(\rho \Delta X \Delta Y \Delta Z)}{dt} \\
(\rho \Delta Y \Delta Z u + \rho \Delta X \Delta Z v) - (\rho \Delta Y \Delta Z (U + \Delta U) + \rho \Delta X \Delta Z (v + \Delta v) &= \frac{d(\rho \Delta X \Delta Y \Delta Z)}{dt} \\
-\rho \Delta Y \Delta Z \frac{\Delta U}{\Delta X} - \rho \Delta x \Delta z \frac{\Delta V}{\Delta Y} &= \Delta X \Delta Y \Delta Z \frac{d\rho}{dt} \\
-\rho \frac{\partial U}{\partial x} - \rho \frac{\partial v}{dy} &= \frac{d\rho}{dt}
\end{aligned}$$