Amrita Vishwa Vidyapeetham

School of Artificial Intelligence, Bengaluru Campus

B. Tech. | Artificial Intelligence and Data Science (AID) | 2023–2027

23PHY114 Computational Mechanics II

Homework 1 (06 March 2024)

Due date: 12 March 2024 Maximum marks: 3%

Q1. For all the following three trusses (Figures 1 to 3), first draw the free body diagram of the structure and write the equations for LMB and the AMB about point A. Then write a finite element method code in Octave to calculate the reaction forces at the supports. Finally, plot the deformed shape of the truss on top of the original shape. Crosscheck your results with the answers given below the figures. (10 marks each)

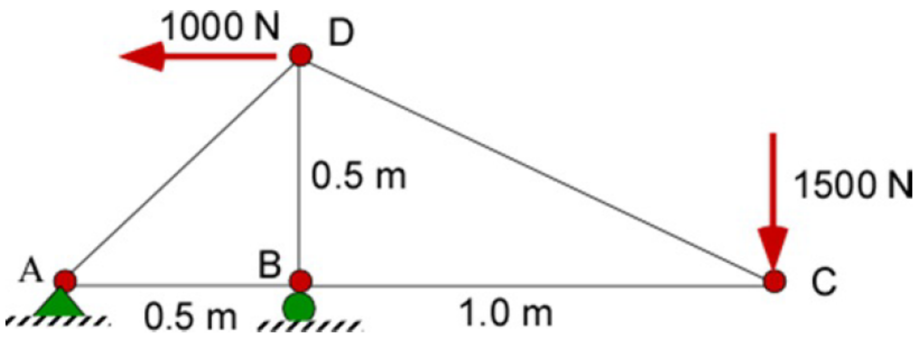

Figure 1: **Given:** Young's Modulus $E = 11.4$ MPa and cross-section areas of all the bars $A = 50 \times 10^{-4}$ m^2. **Answers:** Reactions forces $R_{Ax} = 1000$ N, $R_{Ay} = -2000$ N, $R_{By} = 3500$ N. Displacements $u_B = -0.026316$ m, $u_C = -0.078947$ m, $v_C = -0.41503$ m, $u_D = 0.080323$ m, $v_D = -0.030702$ m.

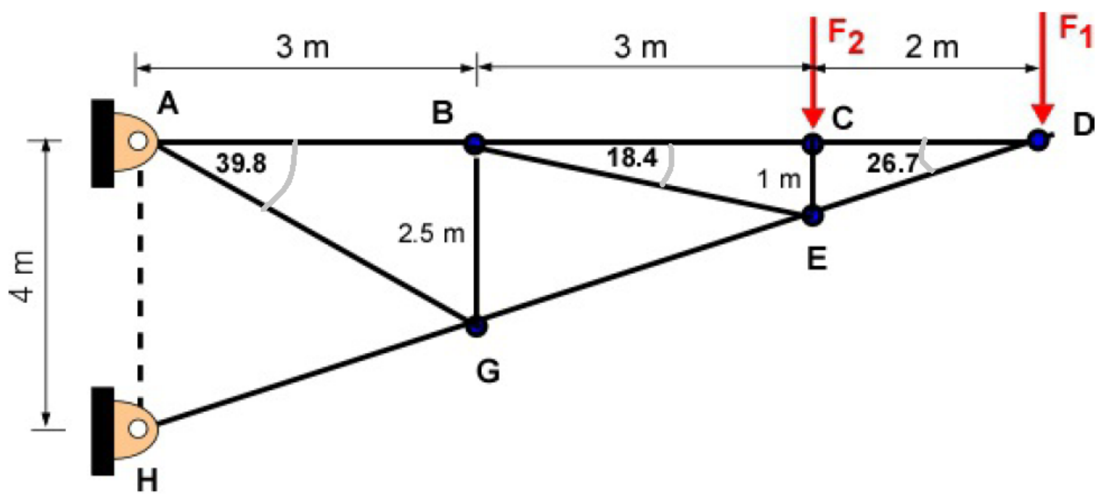

Figure 2: **Given:** Forces $F_1 = 12$ kN, $F_2 = 1.5$ kN; Coordinates of points A(0,4), B(3,4), C(6,4), D(8,4), H(0,0), G(3,1.5); Young's Modulus $E = 200$ MPa and cross-section areas of all the bars $A = 8 \times 10^{-3}$ m^2. **Answers:** Reactions forces $R_{Ax} = -6250$ N, $R_{Ay} = 375$ N, $R_{Hx} = 6250$ N, $R_{Hy} = 3125$ N. Displacements $u_B = 0.01087$ m, $v_B = -0.01462$ m, $u_C = 0.01837$ m, $v_C = -0.06247$ m, $u_D = 0.02337$ m, $v_D = -0.12388$ m, $u_E = -0.00081$ m, $v_E = -0.06153$ m, $u_G = -0.00953$ m, $v_G = -0.01367$ m.

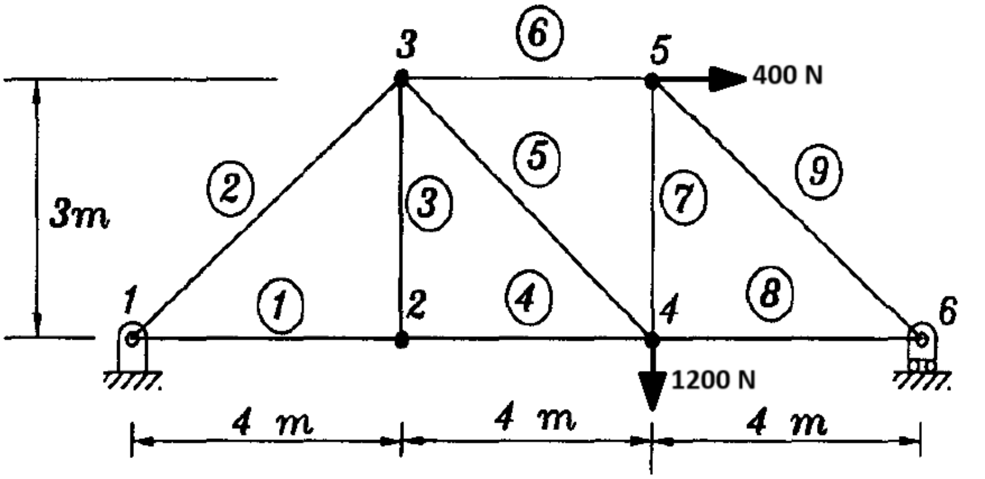

Figure 3: **Given:** Material of the bars is steel of Young's Modulus $E = 200$ GPa and cross-section areas of all the bars $A = 2.5 \times 10^{-3}$ m^2. **Answers:** Displacements $u_1 = 0.0 \times 10^{-5}$ m, $v_1 = 0.0 \times 10^{-5}$ m, $u_2 = 0.32 \times 10^{-5}$ m, $v_2 = -1.57 \times 10^{-5}$ m, $u_3 = 0.865 \times 10^{-5}$ m, $v_3 = -0.157 \times 10^{-5}$ m, $u_4 = 0.64 \times 10^{-5}$ m, $v_4 = -2.2867 \times 10^{-5}$ m, $u_5 = 0.545 \times 10^{-5}$ m, $v_5 = -2.0167 \times 10^{-5}$ m, $u_6 = 1.12 \times 10^{-5}$ m, $v_6 = 0.0 \times 10^{-5}$ m.

Q2. For all the above three trusses (Figures 1 to 3), using the same code calculate the axial forces (tensions) in every bar.
HINT: Calculate first the forces at every node, i.e the global force vector, from the above code. Then create a function named 'tension' which takes input as the forces along $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ directions at the Left and Right nodes (i.e. f_{iL}, f_{jL}, f_{iR}, and f_{jR}) for a given bar and rotates them back to axial forces (i.e. f_L and f_R) at the nodes. This is exactly reverse of how we obtained the $[u_1\, v_1\, u_2\, v_2]$ displacements from the axial displacement $[u_L\, u_R]$, or how we obtained the K_ℓ matrix from K_e matrix. The resultant of the axial forces is the tension in the bar (or compression, depending on the sign). Alternatively, calculate $[u_L\, u_R]$ from $[u_1\, v_1\, u_2\, v_2]$ and then mulitply K_e to it to find $[f_L\, f_R]$.
(10 marks each)

Q3. For all the above three trusses (Figures 1 to 3), using the same code and the above tension function to calculate the axial (tensile/compressive) stresses and strains in every bar.
HINT: Recall the definitions of stress and strains from the early classes. (10 marks each)

Q4. What should be the cross-section area of each bar in Figure 3 if the maximum allowable stress is 300 MPa? Use these new cross-section areas to recalculate stiffness matrix of each bar and, therefore, recalculate the correct deformed shape of the structure, nodal and reaction forces, tension force and tensile stresses and strains in each bar (i.e. repeat Q1 and Q2 for the new cross-section areas). (10 marks each)

Q5. If the truss in Figure 3 was to be made of some wood ($E = 13$ GPa and allowable tensile stress 80 MPa) instead of steel then what would be the reaction forces, nodal forces, tension force and strains in each bar, and cross-section areas of each bar?

Q6. In Figure 4 (overleaf) the blue bar is made of steel of Young's Modulus $E = 200$ GPa, length $L_1 = 25$ cm, and cross-section diameter 50 mm. The orange bar is made of brass with $E = 200$ GPa, length $L_2 = 30$ cm, and diameter 35 mm. The two circular bars are welded together at point B and the entire pillar is fixed between the ceiling and the floor of a machine. A vertical load of 25 kN is hung on the pillar at the welded junction. *Note* that the overall distance between the ceiling and the floor cannot change. Calculate (a) the reaction forces at the ceiling and the floor, (b) the axial forces, stresses, and strains in the steel and brass bars. Are these tensile or compressive? (c) Do the stresses exceed the maximum allowable stresses (250 MPa for steel and 410 MPa for brass)?

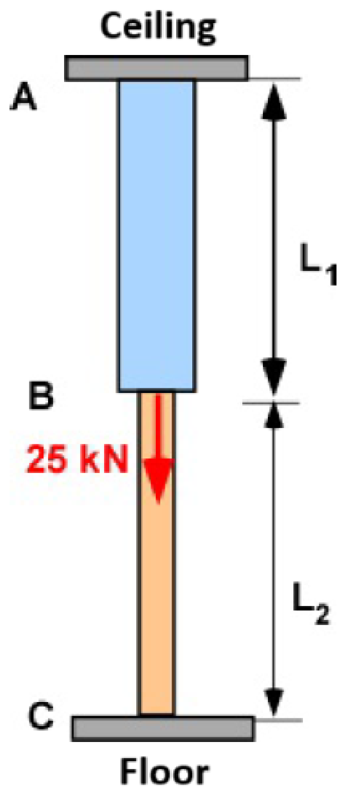

Figure 4: A composite pillar