Problem 3:

Satellite No.	Satellite Position *x* (Mm)	Satellite Position *y* (Mm)	Satellite Position *z* (Mm)
1	101	16	207
2	52	21	302
3	17	53	350
4	-15	159	208

Equations used for solving the problem include:

Here the GPS coordinates are (x, y, z, t_i^v) and Satellite system coordinates are (x_i, y_i, z_i, t_i^c).

Distance between the GPS unit and one satellite is,

$$d_i = c\Delta t_i = c(t_i^v - t_i^c - b) \qquad b\text{: clock bias between GPS and satellite}$$

The difference in distance between them should be zero,

$$\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - c(t_i^v - t_i^c - b) = 0$$

Hence, we must find the minimum of the below-cost function to find the coordinates of the GPS,

$$(x, y, z, b) = \underset{(x,y,z,b)}{\arg\min} \sum_i \left(\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - c(t_i^v - t_i^c - b)\right)^2 \qquad i = 1, 2, \ldots.$$

The GPS location was solved using fmincon, a nonlinear programming algorithm in MATLAB, and the initial values for X, Y, Z coordinates were equal to the radius of earth and 0 for the time difference in the clock.

Output:

The coordinates of the GPS unit are as follows:

X coordinate of GPS unit	5.185537 Mm
Y coordinate of GPS unit	7.578706 Mm
Z coordinate of GPS unit	8.456620 Mm
Clock difference between GPS and satellite	0.299846 s

MATLAB script:

```
%%% Author - Adithya Suresh %%%
%%% Professor - Dr. Yunyi Jia %%%
%%% Homework 1 - 3rd problem solution %%%

clear all
clc
%% Loading time data
load('st');
load('rt');
%% Data from question
x = [101,52,17,-15];
y = [16,21,53,159];
z = [207,302,350,208];
radiowave_speed = 300;
earth_radius = 6.378;
%% Time taken for individual satellite to receive the signal
time_diff = rt - st;
%% Function to calculate position of GPS unit
fun = @(a)((sqrt(((a(1)-x(1)).^(2))+((a(2)-y(1)).^(2))+((a(3)-z(1)).^(2)))-
(radiowave_speed.*(time_diff(1)-a(4)))).^(2))...
 +((sqrt(((a(1)-x(2)).^(2))+((a(2)-y(2)).^(2))+((a(3)-z(2)).^(2)))-
(radiowave_speed.*(time_diff(2)-a(4)))).^(2))...
 +((sqrt(((a(1)-x(3)).^(2))+((a(2)-y(3)).^(2))+((a(3)-z(3)).^(2)))-
(radiowave_speed.*(time_diff(3)-a(4)))).^(2))...
 +((sqrt(((a(1)-x(4)).^(2))+((a(2)-y(4)).^(2))+((a(3)-z(4)).^(2)))-
(radiowave_speed.*(time_diff(4)-a(4)))).^(2));

initial_data = [earth_radius,earth_radius,earth_radius,0];
A = [];
B = [];
C = [];
D = [];
E = [];
F = [];
func_variable = @equation;
GPS_position = fmincon(fun, initial_data, A, B, C, D, E, F, func_variable);
```

```
Local minimum possible. Constraints satisfied.


fmincon stopped because the size of the current step is less than

the value of the step size tolerance and constraints are

satisfied to within the value of the constraint tolerance.
```

```
<stopping criteria details>
```

```
%% Output
fprintf('X coordinate position of the GPS is %f Mm \n',GPS_position(1));
```

```
X coordinate position of the GPS is 5.185537 Mm
```

```
fprintf('Y coordinate position of the GPS is %f Mm \n',GPS_position(2));
```

```
Y coordinate position of the GPS is 7.578706 Mm
```

```
fprintf('Z coordinate position of the GPS is %f Mm \n',GPS_position(3));
```

```
Z coordinate position of the GPS is 8.456620 Mm
```

```
fprintf('Clock difference between GPS unit clock and Satelite system clock is %f s \n',GPS_position(4));
```

```
Clock difference between GPS unit clock and Satelite system clock is 0.299846 s
```

Equation function:

```
function [c,ceq] = equation(m)
c = [5-m(1) m(1)-20 5-m(2) m(2)-20 5-m(3) m(3)-20 -10-m(4) m(4)-10];
ceq=[];
```