CS 106X

Lecture 18: Trees

Wednesday, February 22, 2017

Programming Abstractions (Accelerated)
Winter 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

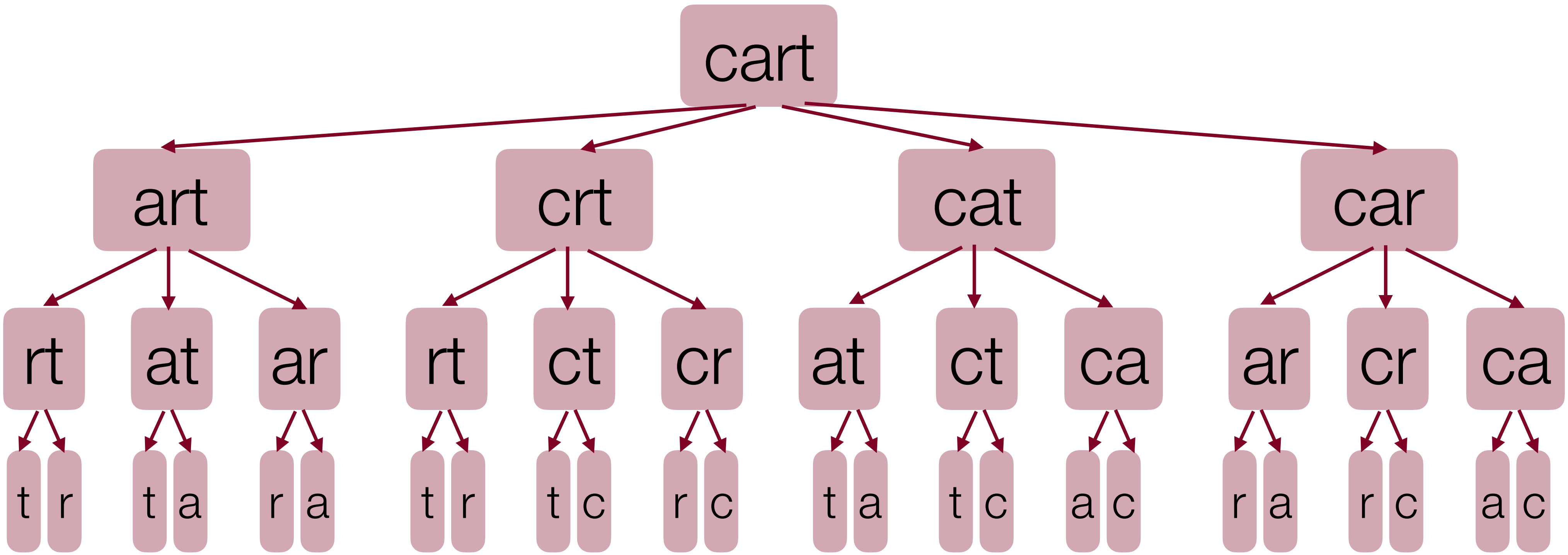
reading:
Programming Abstractions in C++, Section 16.1

Today's Topics

- Logistics
  - Midterm Tomorrow!
  - Midterm will cover up to and including Linked Lists
- Introduction to Trees

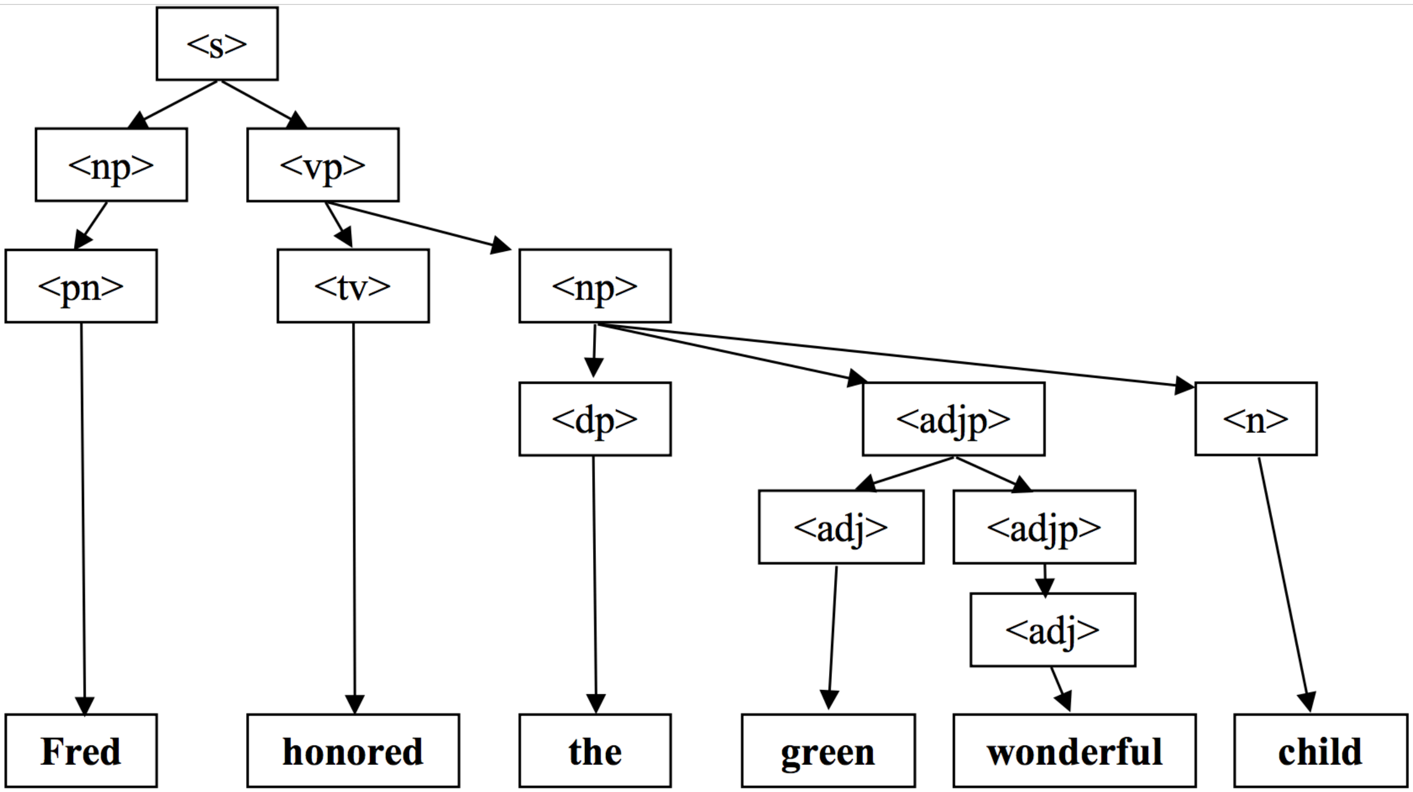
Trees

We have already seen trees in the class in the form of decision trees!



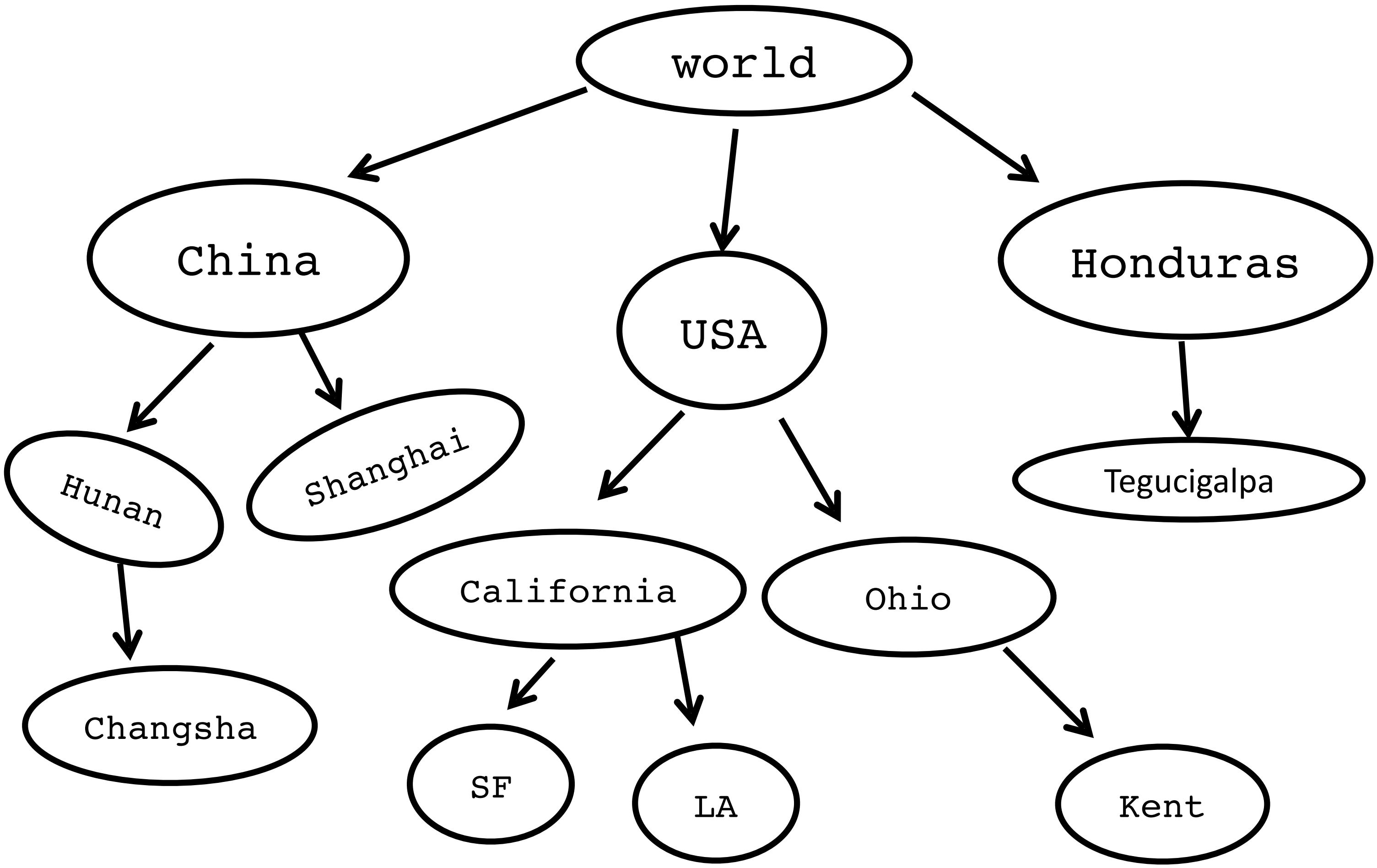
Trees

You've coded trees for recursive assignments!

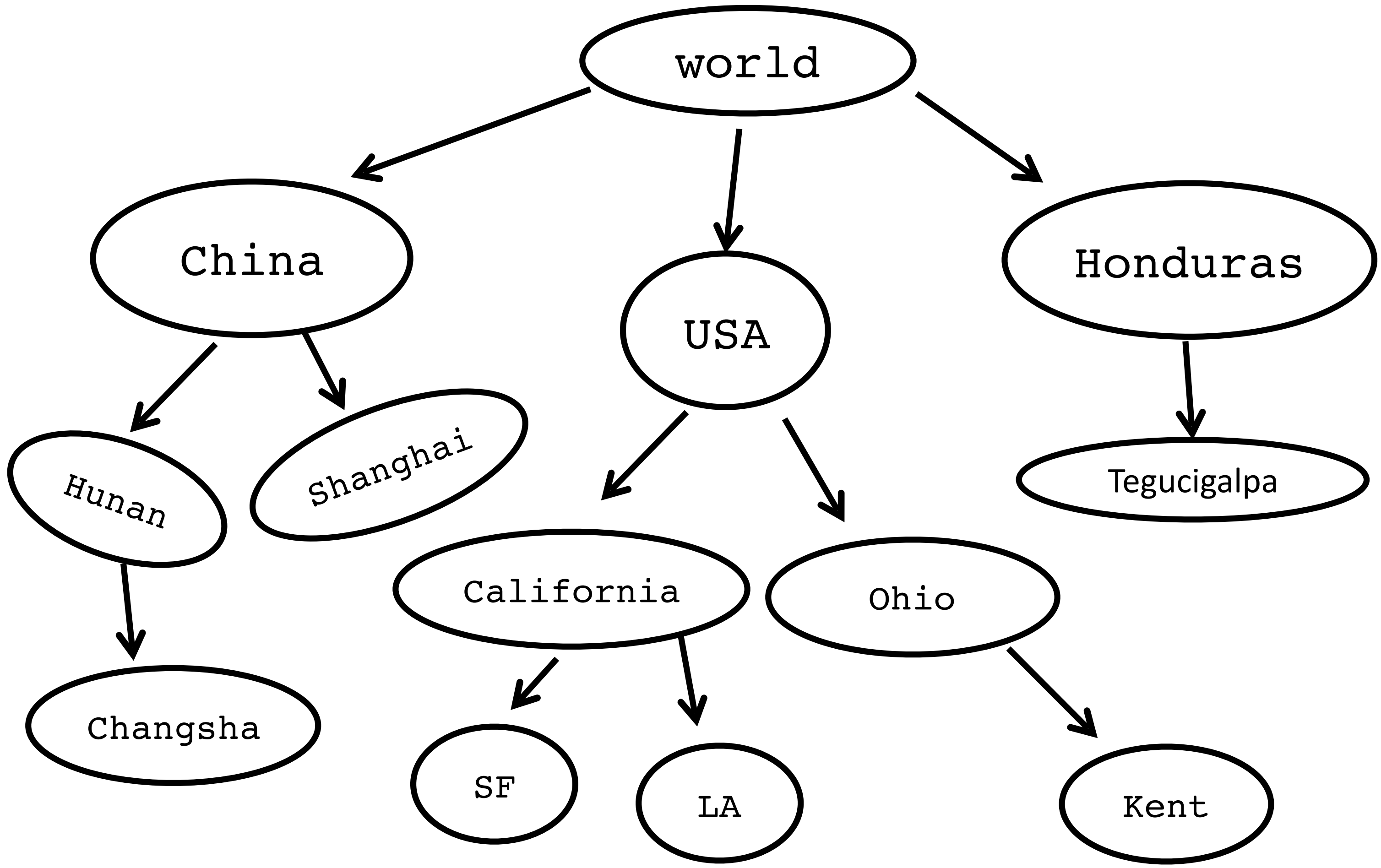


Random expansion from `sentence.txt` *grammar for symbol* "<s>"

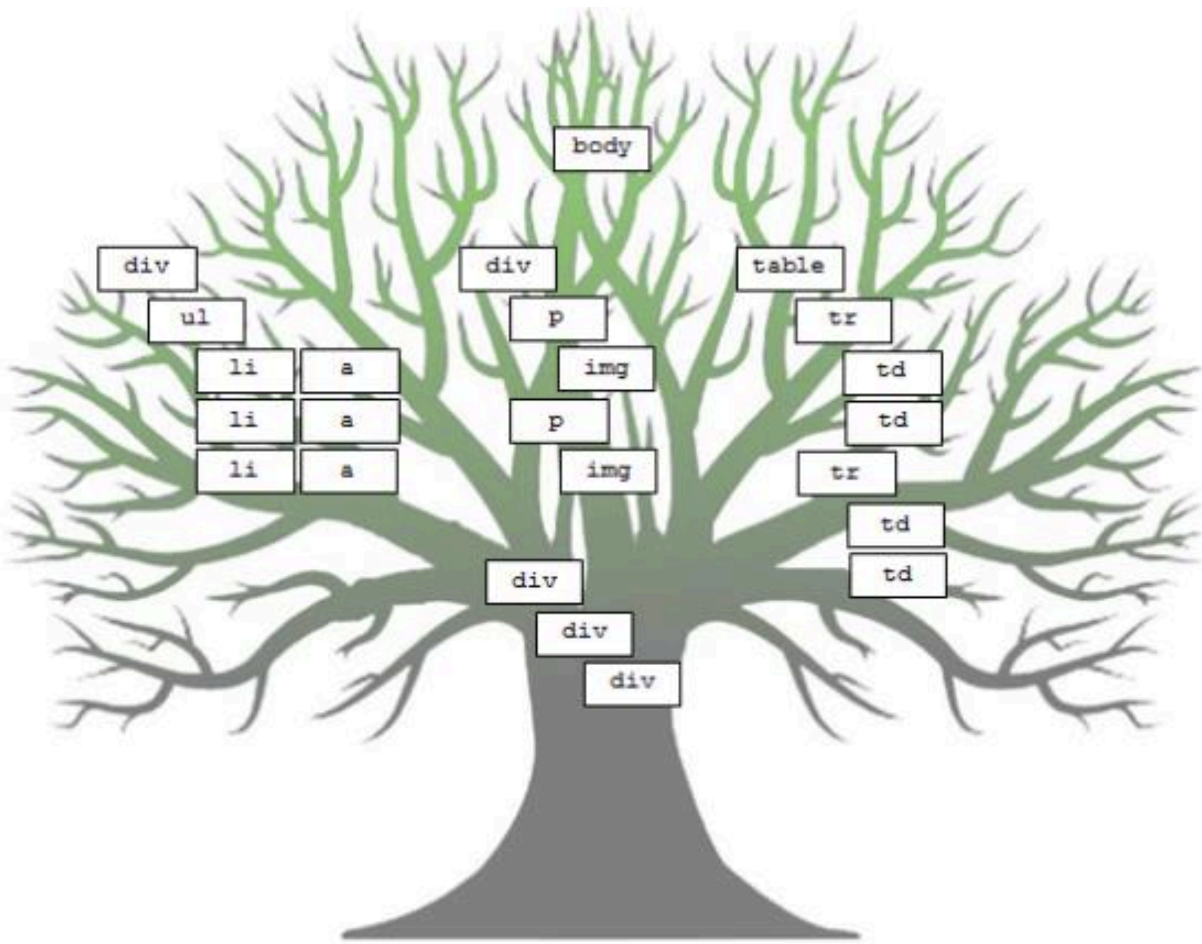
Trees Can Describe Hierarchies



Trees Can Describe Hierarchies

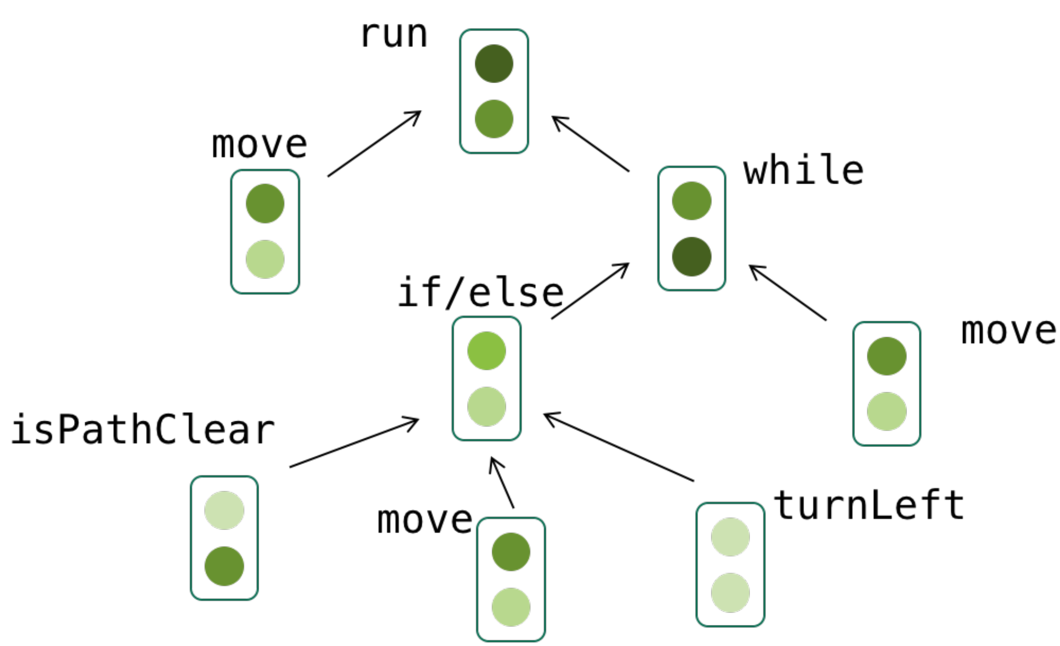


Trees Can Describe Websites (HTML)



Trees Can Describe Programs

```
// Example student solution
function run() {
   // move then loop
   move();
   // the condition is fixed
   while (notFinished()) {
      if (isPathClear()) {
         move();
      } else {
         turnLeft();
      }
      // redundant
      move();
   }
}
```

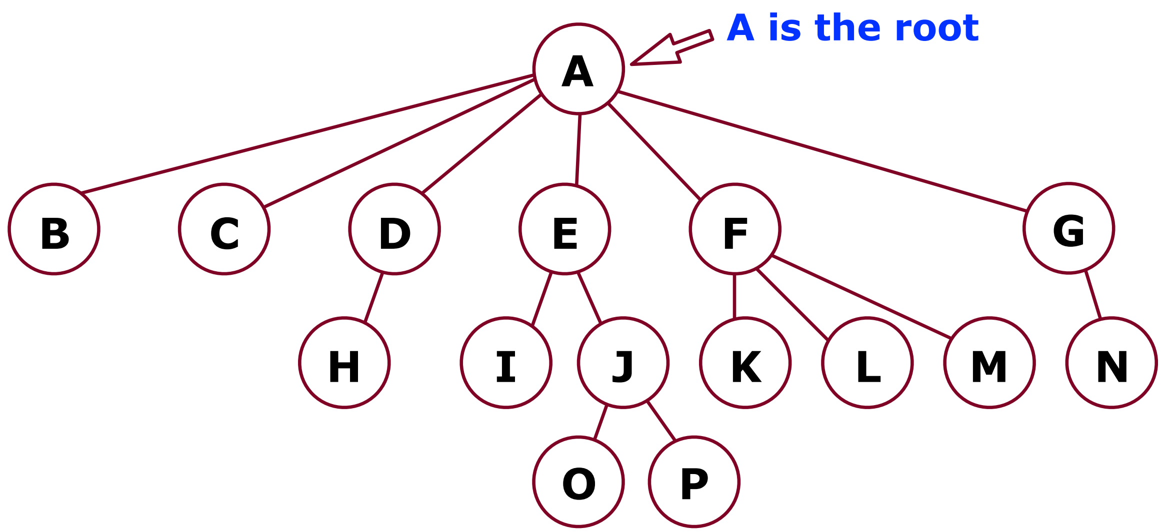


* This is a figure in an academic paper written by a recent CS106 student!

Trees are inherently recursive

What is a Tree (in Computer Science)?

- A tree is a collection of **nodes**, which can be empty. If it is not empty, there is a "root" node, ***r***, and ***zero or more non-empty subtrees***, T_1, T_2, …, T_k, whose roots are connected by a directed edge from ***r***.



Tree Terminology

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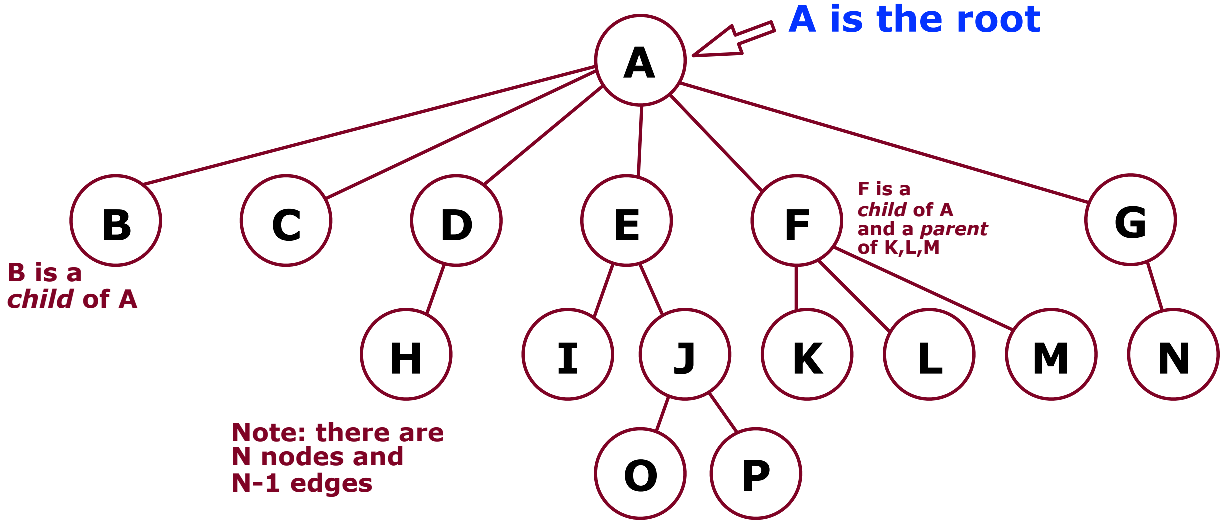

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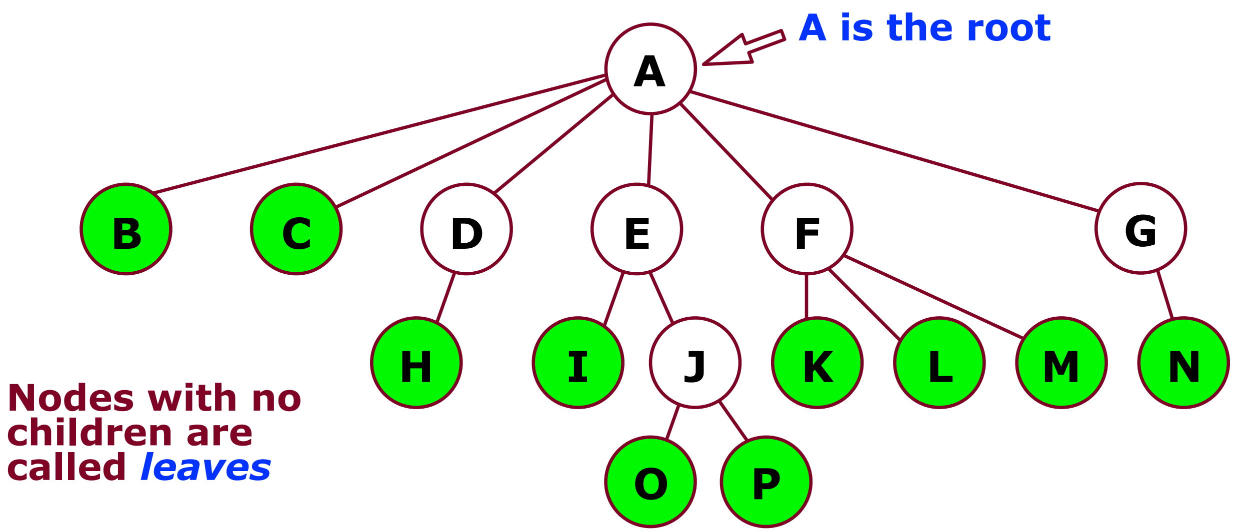

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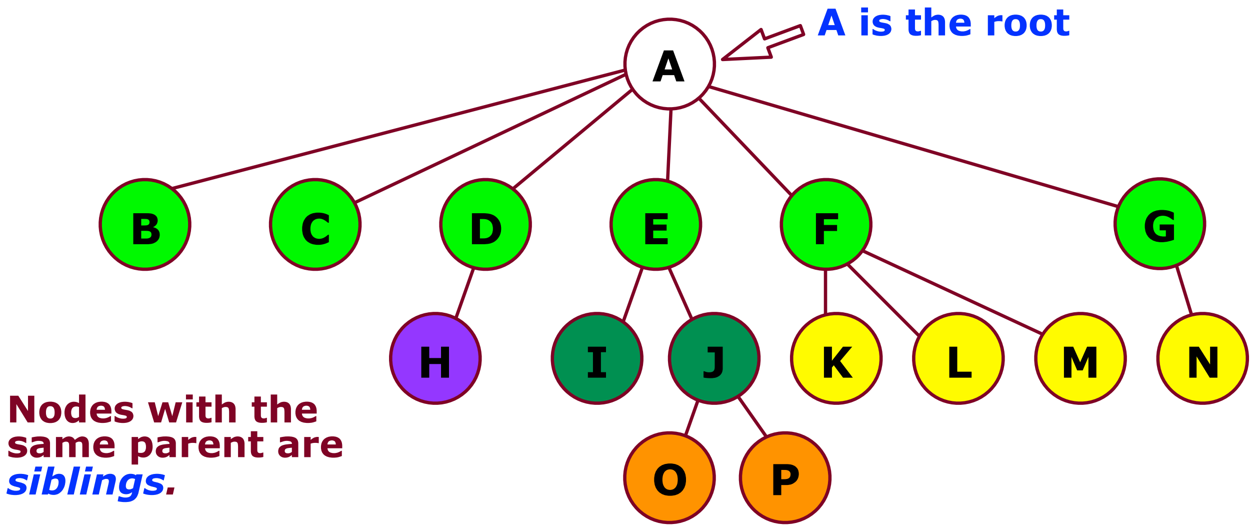
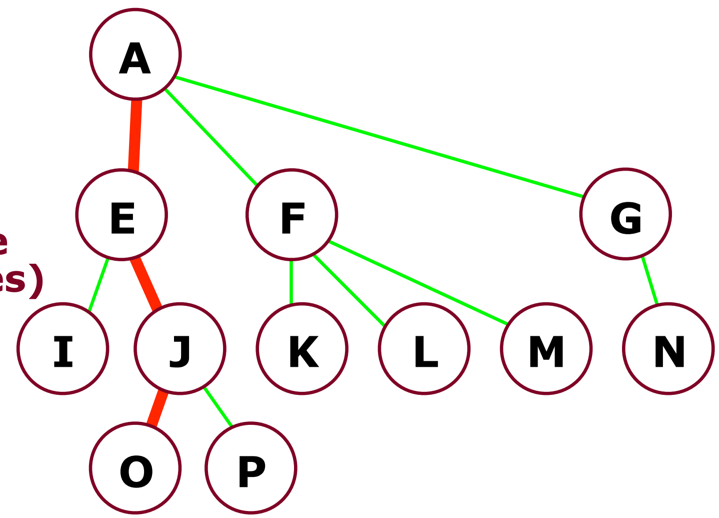


Tree Terminology

We can define a *path* from a parent to its children.

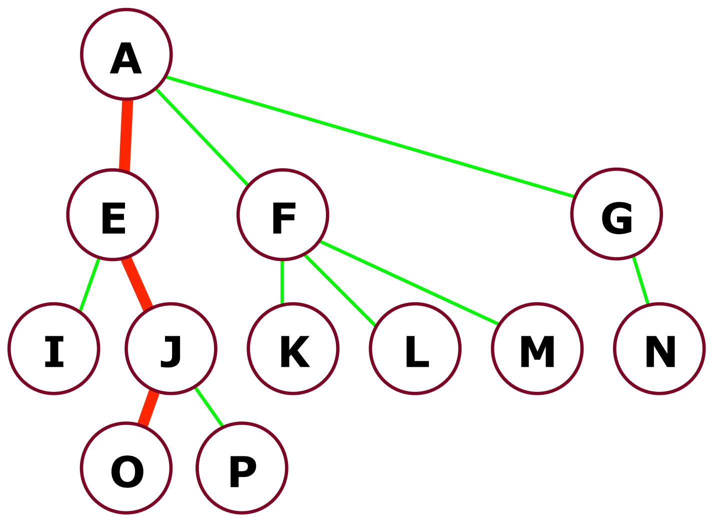
The path A-E-J-O has a *length* of three (the number of edges)



Tree Terminology

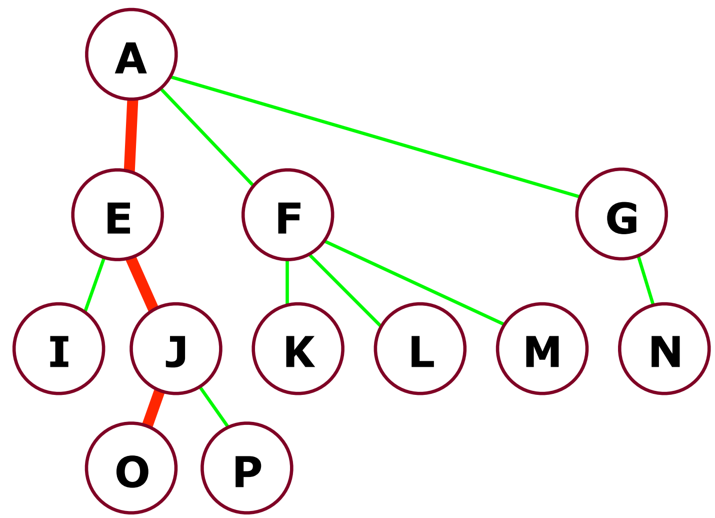
The ***depth*** of a node is the length from the root. The depth of node J is 2. The depth of the root is 0.

The ***height*** of a node is the longest path from the node to a leaf. The height of node F is 1. The height of all leaves is 0.



Tree Terminology

The height of a tree is the height of the root (in this case, the height of the tree is 3.



Tree Terminology

Trees can have only one parent, and cannot have *cycles*

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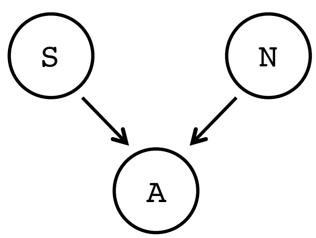

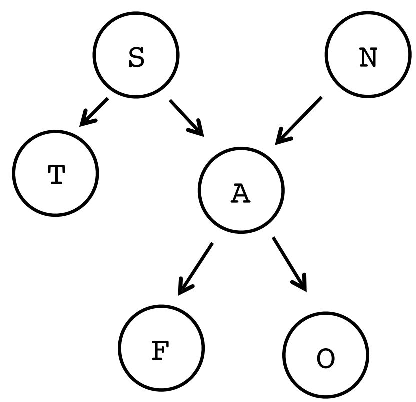

Tree Terminology

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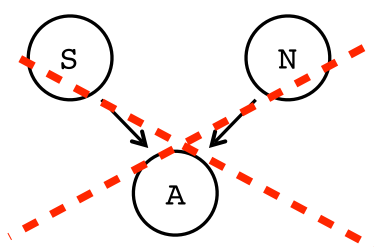
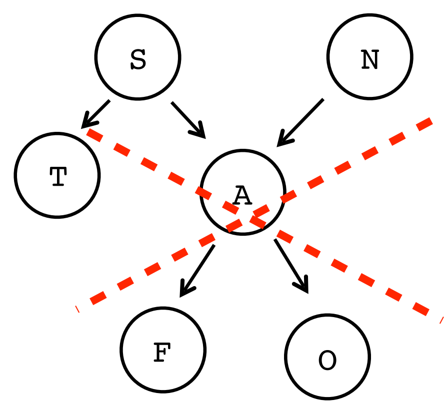


Node A has two parents



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Tree Terminology

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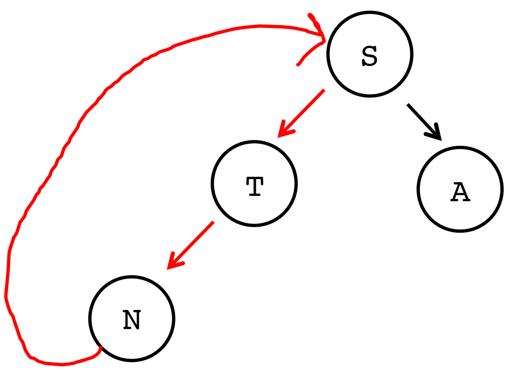

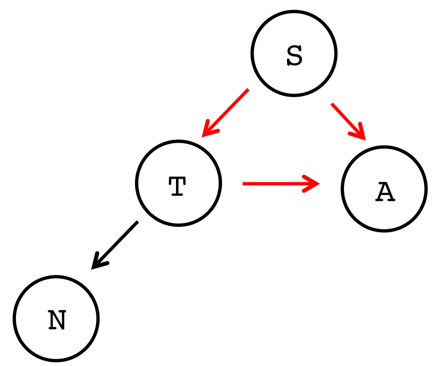

Tree Terminology

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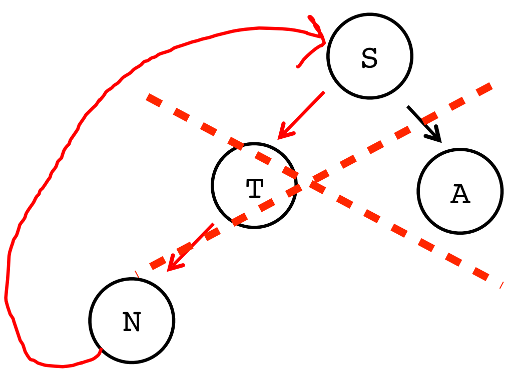
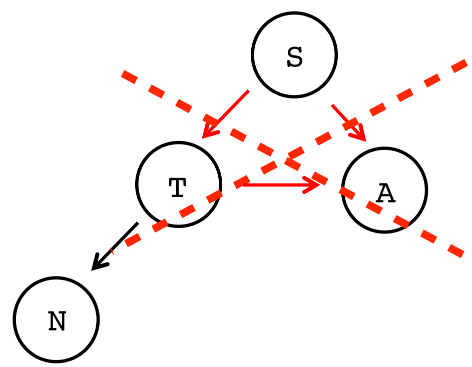
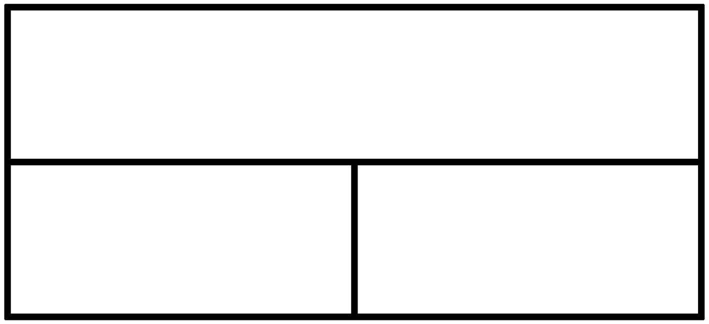




not a tree: the red edges make a cycle

How can we build trees programmatically?



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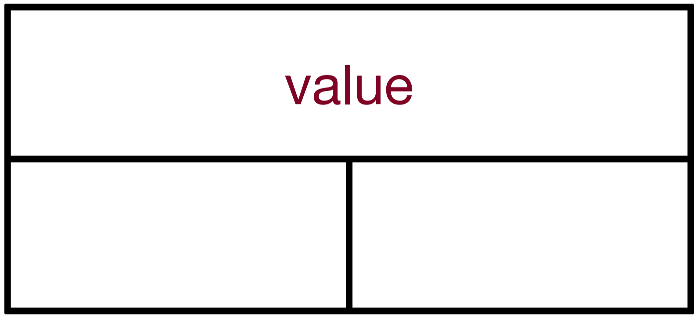
Binary Tree:



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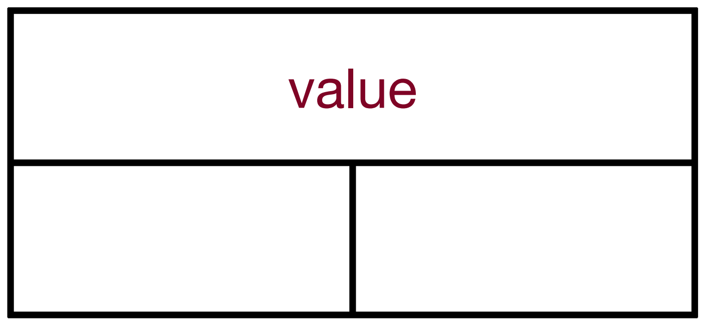
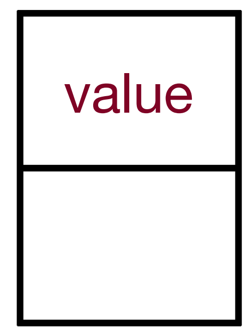
Binary Tree:



Linked List



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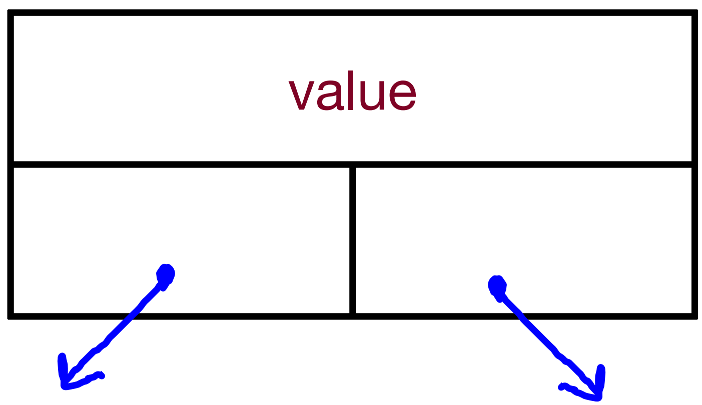
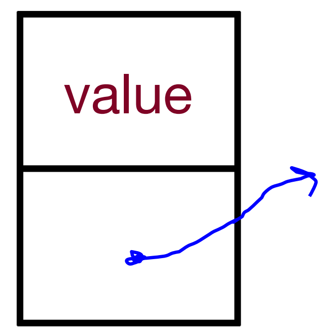


Linked List



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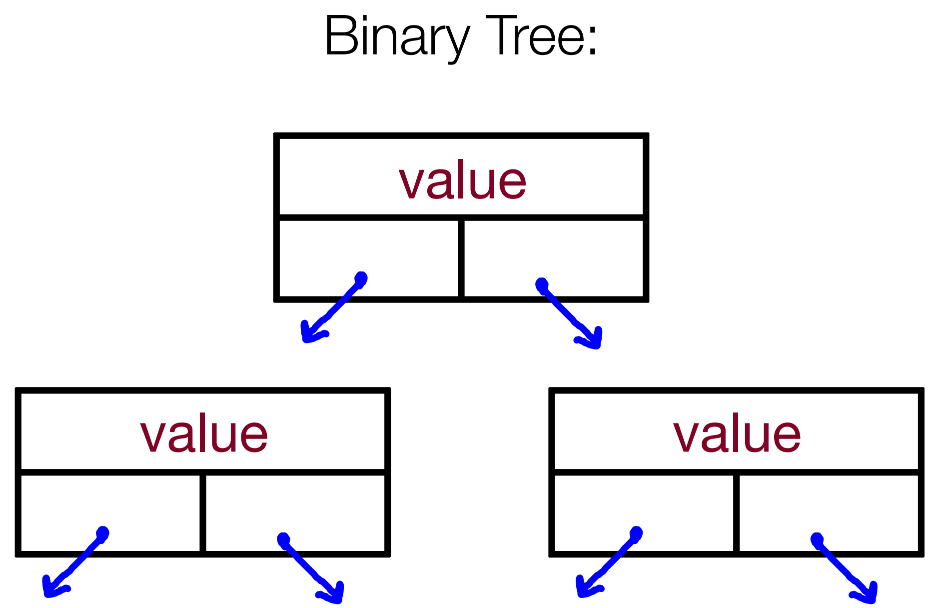

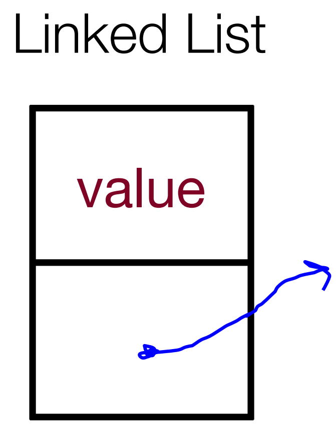

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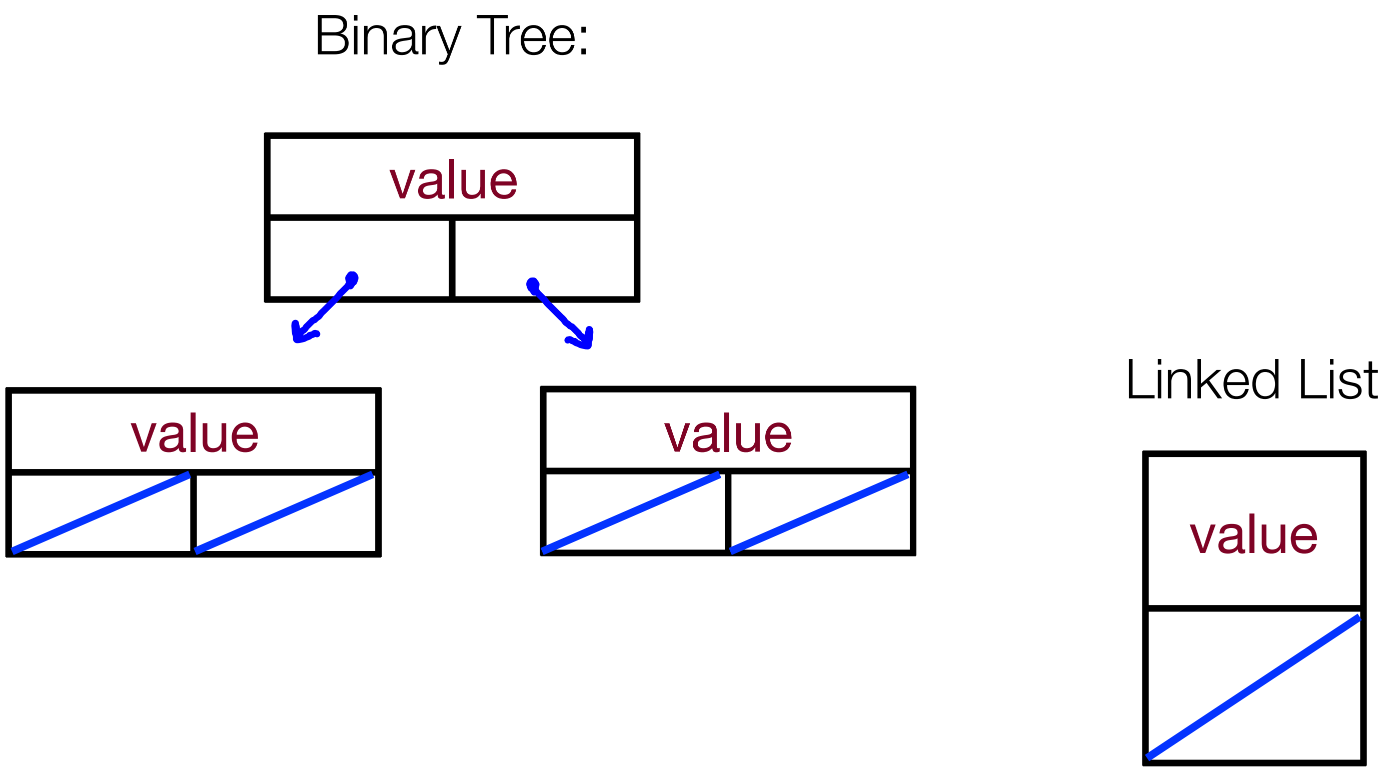
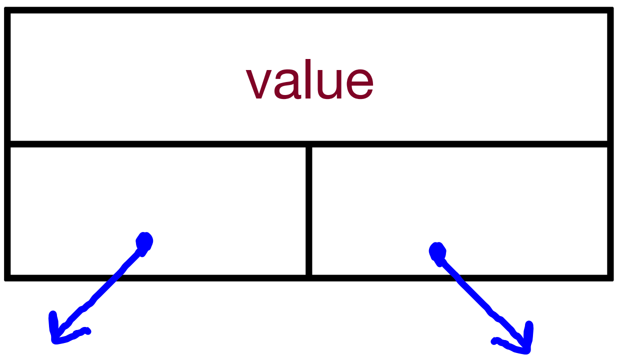


The Most Important Slide

Binary Tree:

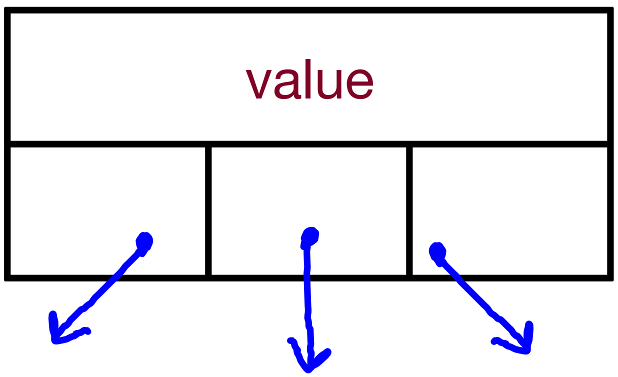
```
struct Tree {
    string value;
    Tree *left;
    Tree *right;
};
```



We Can Have Ternary Trees (or any number, *n*)

Ternary Tree:

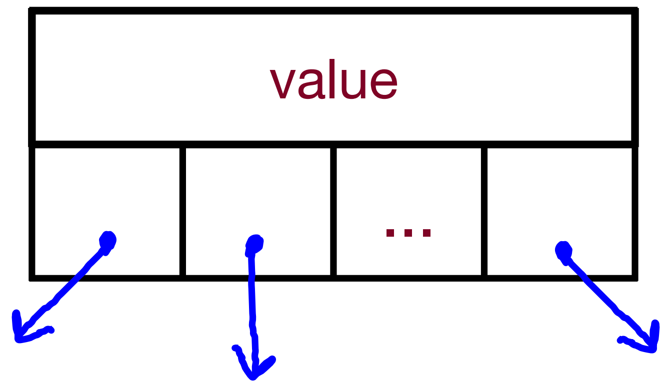
```
struct Tree {
    string value;
    Tree *left;
    Tree *middle;
    Tree *right;
};
```



We Can Have Ternary Trees (or any number, *n*)

N-ary Tree:

```
struct Tree {
    string value;
    Vector<Tree *> children;
};
```



Trees can be defined as either structs or classes

```
struct Tree {
    string value;
    Tree * left;
    Tree * right;
};
```

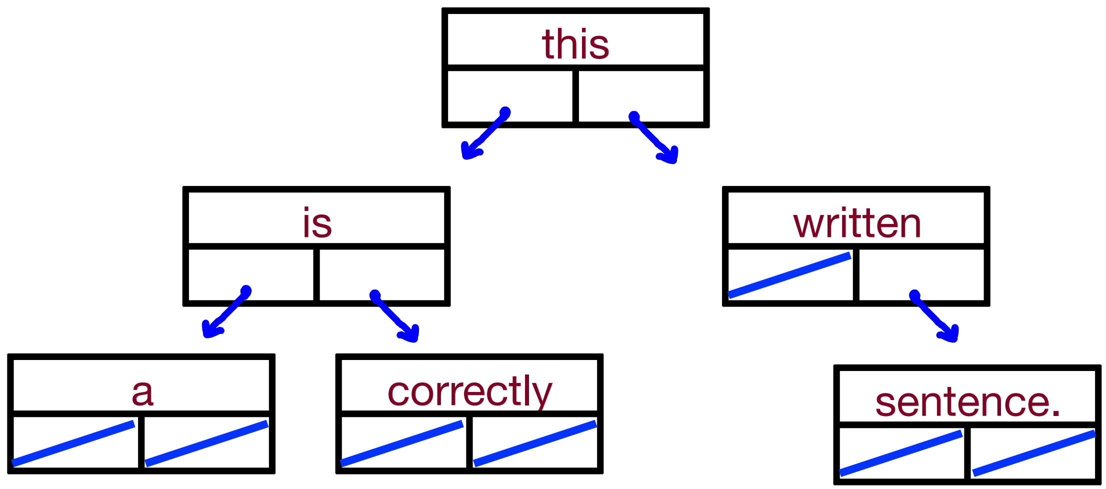
```
class Tree {
private:
    string value;
    Vector<Tree *> children;
};
```

Let's write some code to "traverse" the tree

```
struct Tree {
   string value;
   Tree * left;
   Tree * right;
};
```

There are multiple ways to traverse the nodes in a binary tree:

1. Pre-order
2. In-order
3. Post-order
4. Level-order



Let's write some code to "traverse" the tree

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struct Tree {
   string value;
   Tree * left;
   Tree * right;
};
```

There are multiple ways to traverse the nodes in a binary tree:

1. Pre-order
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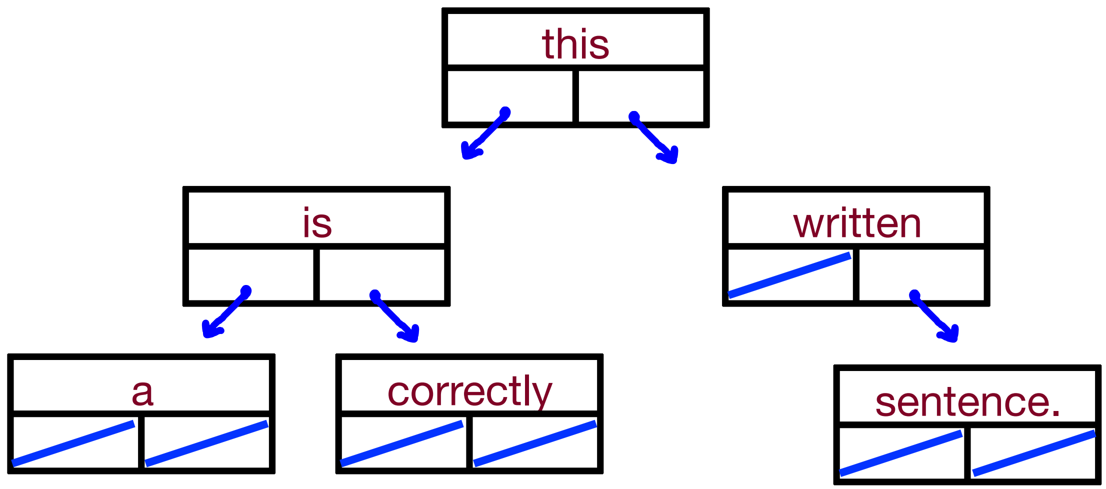
4. Level-order

1. Do something
2. Go left
3. Go right



Let's write some code to "traverse" the tree

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struct Tree {
   string value;
   Tree * left;
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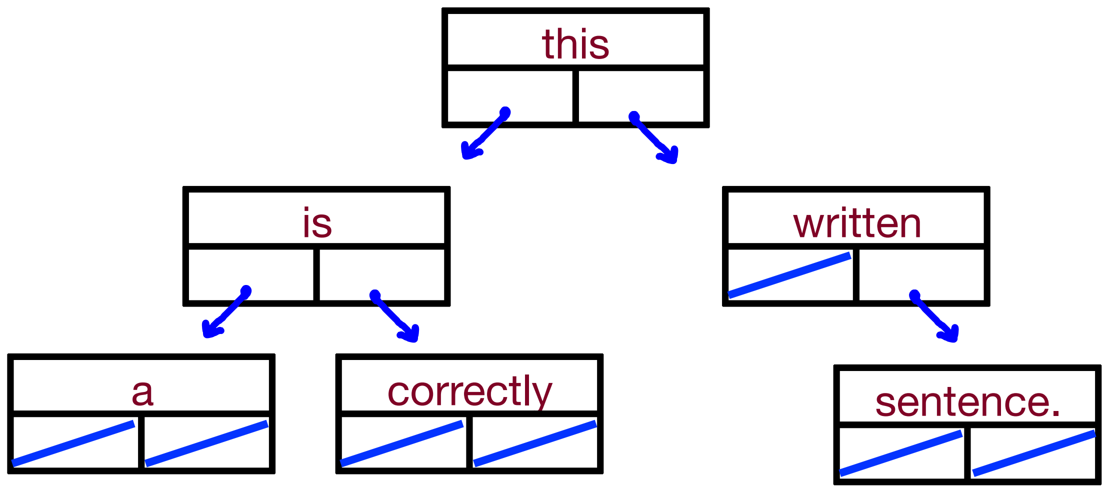

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1. Pre-order
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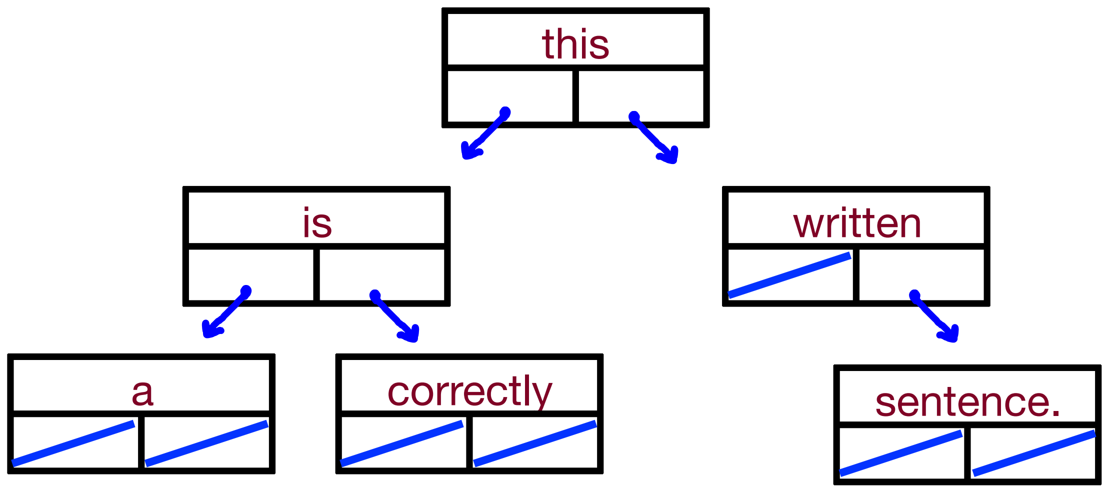
1. Go left
2. Go right
3. Do something



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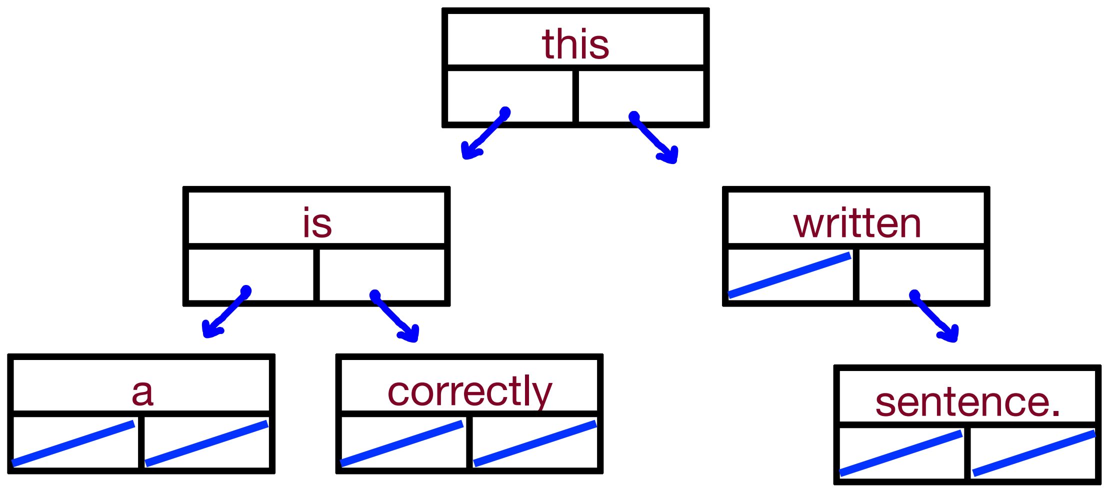

Hmm...can we do this recursively?
We want to print the levels: 0, 1, 2 from left-to-right order

Let's write some code to "traverse" the tree

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struct Tree {
   string value;
   Tree * left;
   Tree * right;
};
```
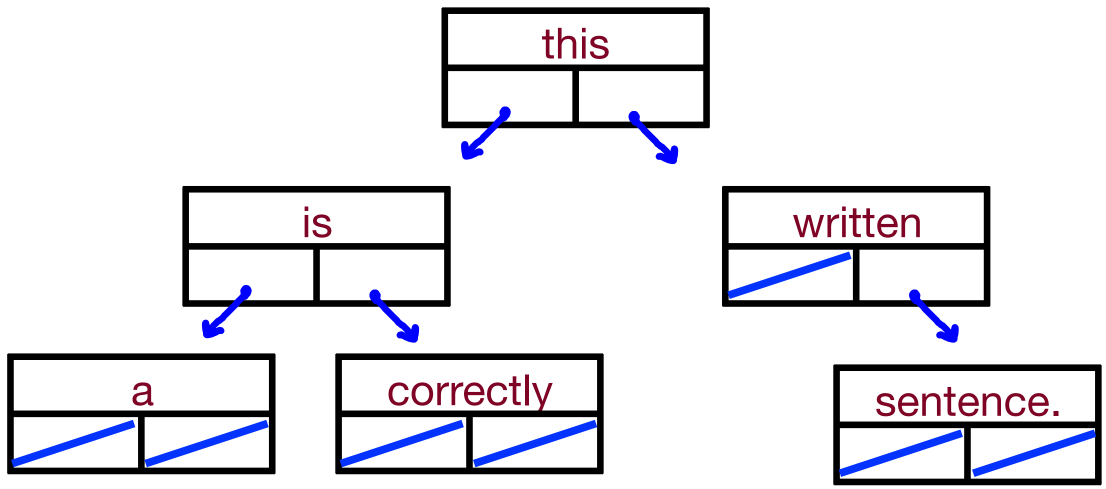



There are multiple ways to traverse the nodes in a binary tree:

1. Pre-order
2. In-order
3. Post-order
4. Level-order

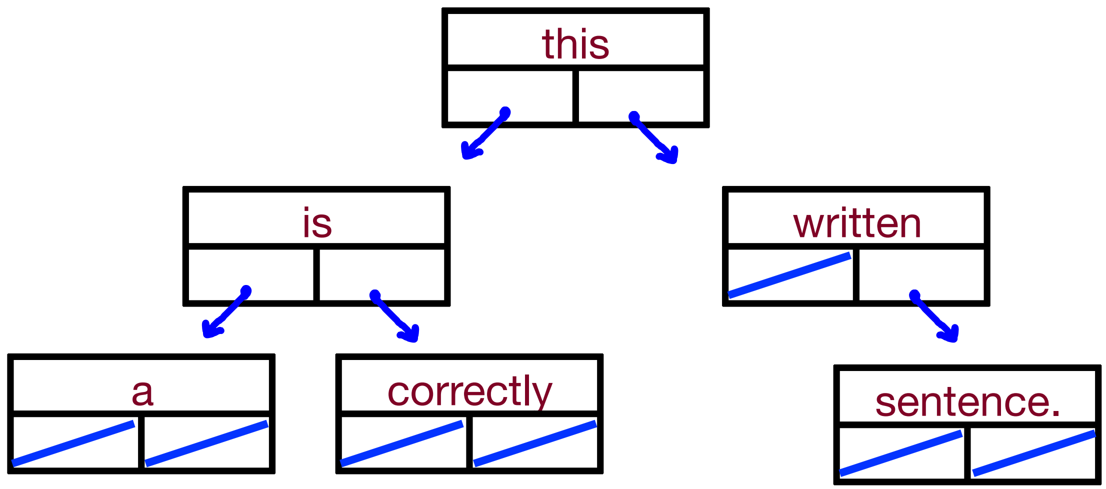
Not easy recursively...let's use a queue!

1. Enqueue root
2. While queue is not empty:
    a. dequeue node
    b. do something with node
    c. enqueue left child of node if it exists
    d. enqueue right child of node if it exists

should look familiar...word ladder?

Let's write some code

```
struct Tree {
   string value;
   Tree * left;
   Tree * right;
};

void preOrder(Tree * tree) {
  if(tree == NULL) return;
  cout<< tree->value <<" ";
  preOrder(tree->left);
  preOrder(tree->right);
}

void inOrder(Tree * tree) {
  if(tree == NULL) return;
  inOrder(tree->left);
  cout<< tree->value <<" ";
  inOrder(tree->right);
}

void postOrder(Tree * tree) {
  if(tree == NULL) return;
  postOrder(tree->left);
  postOrder(tree->right);
  cout<< tree->value << " ";
}
```




```
void levelOrder(Tree *tree) {
    Queue<Tree *>treeQueue;
    treeQueue.enqueue(tree);
    while (!treeQueue.isEmpty()) {
        Tree *node = treeQueue.dequeue();
        cout << node->value << " ";

       if (node->left != NULL) {
            treeQueue.enqueue(node->left);
        }
        if (node->right != NULL) {
            treeQueue.enqueue(node->right);
        }
    }
}
```

References and Advanced Reading

- **References:**
  - https://en.wikipedia.org/wiki/Tree_(data_structure)
  - http://pages.cs.wisc.edu/~vernon/cs367/notes/8.TREES.html

- **Advanced Reading:**
  - http://www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/trees.html
  - Great set of tree-type questions:
    - http://cslibrary.stanford.edu/110/BinaryTrees.html