Modelling Dynamics Of Vehicles In Multi-class Traffic Lacking Lane Discipline

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1 Introduction

With the rapid increase in the number of vehicles day by day and an increase in the number of people who prefer personal vehicles to go from place to place with minimal time, it's essential to understand how the flow of vehicles can be modelled effectively and how the flow can be optimized to minimize the time consumption.

Greenshield[12] did one of the foundational theories of traffic flow. The paper presented the results of a study undertaken to determine the approximate hourly traffic density on a roadway of a given width at which congestion begins and the amount by which traffic congestion is increased. Traffic with and without congestion is analyzed in the paper, and the equation for the relation between speed and density is derived, which is found to be linear. As the curve is a straight line, two points are sufficient to fix its direction. Free speed and a point near the maximum density are the most accurate, and observations showing these two traffic conditions could determine the effect of congestion for all different densities.Lighthill and Whitham[13] deduced a theory of propagation of changes in traffic distribution based on the relationship between flow and concentration. Hydrodynamic relationships inspire the theory and primarily apply to long, crowded roads. The speed density relationships further advanced later.Castillo and Benıtez[14]proposed a functional form of speed-density relationship. They nondimensionalize density and speed by taking jam density and kinematic wave speed at jam density as reference values. A generating function is defined with a non-dimensional rescaled spacing named equivalent spacing. They have found four types of generating functions: exponential, double exponential, rational and reciprocal-exponential families. By plotting the speed spacing curves generated by these families for several parameters, it is found that the exponential generating functions can reproduce the shape of these families. They derive the functional form of the speed density curve through two different arguments; one of the mathematical type where it is demonstrated that the proposed functional form is the only one that simplifies the problem of a continuous and differentiable function satisfying the set of properties that the volume-speed-density relationship should meet and the other based on the behaviour of driver where speed density relation is regarded as the equilibrium solution of a car following model. They define the sensitivity of the drivers with respect to the relative speed as a function, and the upper limit of all the sensitivity curves corresponding to the speed density curve is defined as the maximum sensitivity curve.Aw and Rascle[15]consider a conservation equation with no relaxation term involved, and pressure is regarded as a smoothly increasing density function. This leads to a hyperbolic system. A unique solution to the Rieman problem associated with the model exists. The solution satisfies several principles, such as the solution remains for all times with values in the invariant region and all the waves propagate at speed at most equal to the velocity of the corresponding state when solving the Rieman problem with arbitrary bounded nonnegative Riemann data in a suitable region of the plane density and velocity remain nonnegative and bounded from above, the solution to the Rieman problem agrees with qualitative properties that driver observes such as braking produces shock waves and accelerating produces rarefaction waves and model predicts instabilities for very light traffic. In this model, anticipation factor was added as a response to the convective derivative of density. This term describes how a driver reacts to variation of concentration with respect to space.Haque, Sultana, and Andallah[16]have numerically solved the second-order parabolic diffusion type traffic flow model as the analytical solution may cause huge errors as it depends on the initial value function over an integrodifferential equation. They solved the equation using the explicit upwind difference scheme, explicit centred difference scheme, and explicit second-order Lax-Wendroff scheme with exponential velocity density relation. They also obtained stability conditions for all the schemes using Von Neumann Stability analysis. Through the numerical simulation, they found that the Lax-Wendroff scheme gives a better solution and less error than other schemes and is suitable for congested areas.

When the distance between two vehicles is shorter than the safe distance, the driver of the following vehicle may not decelerate if the preceding vehicle travels faster than the following vehicle because the headway between the two vehicles will become larger. Many car following models couldn't explain this common driver behaviour in the real world.Jiang, Wu, and Zhu[17] propose a model that considers the effect of both the distance and the relative speed of two successive vehicles. They develop the corresponding macroscopic continuum model by transforming the discrete variables of individual vehicles into continuous flow variables. In the new continuum model, the speed gradient term replaces the density gradient term as the anticipation term, which solves the characteristic speed problem in the previous higher-order models and, therefore, enables the new model to satisfy the anisotropic property of traffic flow. To develop the macroscopic continuum version of the improved car following model, Gupta and Katiyar[18] transform the discrete variable of individual vehicles into the continuous flow variables and thus obtain the equation of motion and substitute the approximate expression of headway as a perturbation series into this. The traffic model includes some well-known non-equilibrium models as a special case. The model is isotropic, but the anisotropic factor β can control the isotropic factor; as beta tends to infinity, the model becomes anisotropic.However, the model is applicable only for single-lane flow with no overtaking for single species.

In developing countries, classic traffic models are not applicable due to lane indiscipline.Nair, Mahmassani, and Miller-Hooks[19] proposed a continuum model analogous to fluid flow through a porous medium, which describes a continuum heterogeneous traffic stream lacking lane discipline. A network of disordered space is defined by the vehicles in the traffic stream; each vehicle class has an associated critical pore size, and the speed distribution of each vehicle class is determined by the overall availability of pore spaces and vehicular characteristics associated with it. Using the pore space distribution, an equilibrium speed density relationship is derived through which the constituent vehicle classes interact. A size exclusion principle is used to model vehicle class-specific state variables. The incremental transfers principle is adapted to decompose the problem and compute the state variables of each class relatively independently for two vehicle classes.Agrawal, Kanagaraj, and Treiber[6]propose a two-dimensional LWR model in which the continuity equation is extended into two dimensions. The flux density function is based on hydrodynamics relationships, and lateral dynamics is defined to be distinctly different from longitudinal dynamics. Flux density in the lateral direction is proportional to an additional term, the density gradient, motivated by the Ficks law, which would reflect drivers' desire to go to regions of lesser density. In the longitudinal direction, the continuity equation is closed by Greenshields' and Castillo and Benitez's speed-density relation. At the same time, a novel model of exponential boundary repulsion is applied to close the lateral. Vikram, Mittal, and Chakroborty[20]propose a two-dimensional continuum model of traffic flow that incorporates driver behaviour in the longitudinal and lateral direction of the road. The model assumes that the driver accelerates in the direction of decreasing traffic density and the driver accelerates the vehicle in the longitudinal direction if traffic density decreases and decelerates if traffic density is in that direction. The model also includes the driver's sensitivity; the driver associated with high sensitivity responds with higher acceleration or deceleration to a spatial gradient of traffic density in a longitudinal direction compared to the driver with less sensitivity.

When traffic comprises several classes of varying speeds and sizes, and no lane discipline, microscopic traffic characteristics are challenging to measure. Density, the number of vehicles per unit length of the road, does not consider any heterogeneous traffic characteristics, and occupancy does not consider the width of vehicles, which is also inadequate in representing heterogeneous traffic.Mallikarjuna and Rao[21] propose a modified measure of occupancy termed area occupancy, which includes vehicle width beside its length. Area occupancy expresses how long a particular vehicle size is moving in that section of the road. Though area occupancy can be measured over space and time, as taking measurements over space is complex, the authors mention that in this paper, the term area occupancy refers to temporal measurements only.Arasan and Dhivya[8] introduce the concept of area occupancy, which is the proportion of time the observed vehicles occupy a particular stretch of the roadway, discuss the measurement of area occupancy and validate the model by comparing the observed and simulated area occupancy. They validate the concept by checking the logical correctness of the relation between flow, speed and area occupancy of homogeneous traffic and by comparing the values of area occupancy of heterogeneous traffic with different compositions. Mohan and Ramadurai[23]presented an extended AR model with area occupancy.Unlike the original AR model, the velocity dynamics equation of the extended model is dimensionally consistent. As AO replaces traffic concentration in the road section, the model gives an equilibrium speed AO relationship that is better suited for heterogeneous traffic than a conventional equilibrium speed-density relationship. The fundamental traffic flow equation is assumed to be valid separately for each vehicle type. The paper discusses the qualitative properties of the model and finds that the proposed extension of the AR model ensures the anisotropic behaviour of vehicles. It is found that the proposed model captures the order in which different vehicle types reduce their speeds effectively, captures bottleneck phenomena better than the other models and could better predict the gap-filling behaviour of heterogeneous traffic lacking lane discipline. For a given AO, the speed of a vehicle class depends upon the percentage composition of vehicles present. To address this effect, Mohan[22] redefines area occupancy as the area used by vehicle classes incorporating minimum lateral and front spacings required between the vehicles. The spatial distribution of vehicles is approximated as the intensity or degree of saturation with which vehicles are filled into each part of the road section. Incorporating this effect, area occupancy is formulated as perceived area occupancy. Linear, logarithmic, exponential and double exponential functional forms are chosen for fitting speed traffic concentration curves. For all the vehicles, the best fit is observed with a double exponential function; the fitting results justified the use of class-wise perceived area occupancy for modelling speed variations of vehicle classes. Compared to flow prediction from the speed density function, there is less error when using speed and class-wise perception of area occupancy. However, there is a significant violation of q=ku from the simulated flow. Mohan and Ramadurai[10] use the concept of area occupancy to represent the traffic concentration of multi-class flow. For traffic with N vehicle classes, the speed u_i and flow q_i of vehicle class i are functions of its density k_i and other vehicle class densities that can be expressed collectively in terms of area occupancy to obtain the three-dimensional representation of multi-class traffic flow. The propagation speed of small disturbances(PSSD) is reformulated for each class using a three-dimensional flow-density relationship. Using the formulated PSSD, they propose a macroscopic model for multi-class traffic with lane indiscipline. The speed-AO relationship reproduces the phenomena of gap-filling behaviour.

In this work, flux density is expressed as the sum of mean motion, which is advection and the random velocity component of the driver that leads to flow away from density gradient, which is diffusion, which is substituted into the two-dimensional flow conservation equation, which is then expressed in terms of area occupancy in section 2.3. In section 2.4, characteristic speed is obtained in terms of area occupancy. The model properties are simulated for a 2 km road in the presence of a traffic signal at 1 km in section 3.

2 Model Development

2.1 Definitions

Flow$(q_i(x,t))$

$$q_i(x,t) = \lim_{dt \to 0} \frac{N_i([x,[t+dt])}{dt} \tag{1}$$

with $N_i(x,[t,t+dt])$ the number of vehicles of class i that cross location x between or at times t and $t+dt$. Since traffic flow is assumed to be a continuum flow, N_i is supposed to be continuous and differentiable.
Class-specific density$(\rho_i(x,y,t))$
The density $(\rho_i(x,y,t))$ is defined as the number of vehicles per square area (veh/m^2)

Class-specific flux density(Q**)**
Q_x and Q_y represents flux density[veh/ms] in the longitudinal(x) and lateral(y) directions respectively
Class-specific vehicle velocity$(u_i(x,t))$

$$u_i(x,t) = \frac{Q_i(x,t)}{\rho_i(x,t)} \quad (2)$$

Area Occupancy

$$Area\ Occupancy = \frac{\Sigma t_m a_m}{TA} \quad (3)$$

Where the numerator is the product of the time during which the stretch of roadway is occupied by vehicle m (occupancy time) and the area of the road space occupied by the vehicle m, and the denominator is the product of the area of the whole of the road stretch and total observation period.
If the observation period T is too small such that even vehicles with the highest speed cannot pass the section in this time period, equation (4) converges to the area occupancy measured over space and can be expressed as

$$Area\ Occupancy = \frac{\Sigma a_m}{A} = \frac{\Sigma \rho_i A a_i}{A} = \Sigma \rho_i a_i \quad (4)$$

which is the fractional area occupied on a road section by all vehicle classes, where a_i is the area of vehicle class i and A is the area of the road section. The congestion level experienced by different vehicle classes will vary depending on the vehicle's size. This is satisfied if the maximum area occupancy (AO_{max}) values for different vehicle classes are different.

2.2 Continuity equation in traffic flow

The system can be described by the set of macroscopic state variables $\rho_i(x,t)$,$Q_i(x,t)$ and $u_i(x,t)$ as $Q_i = \rho_i.u_i$ which is defined as the fundamental equation for traffic flow.
Consider that vehicles are going in the x direction. Two parallel surfaces of unit area are drawn perpendicular to the x-axis and separated by a distance Δx
Let $\rho_i(x,t)$ be the number of vehicles per unit length at point x at time t.
Suppose that the flux density of vehicles of class i across the unit surface located at x is $Q_i(x,t)$; then, using the Taylor series, the flux density of vehicles across the unit surface located at $x + \Delta x$ will be

$$Q_i(x + \Delta x) = Q_i(x) + \frac{\partial Q_i(x)}{\partial x} \Delta x \quad (5)$$

$$Q_i(x) - Q_i(x + \Delta x) = -\frac{\partial Q_i(x)}{\partial x} \Delta x \quad (6)$$

similarly

$$Q_i(y) - Q_i(y + \Delta y) = -\frac{\partial Q_i(y)}{\partial y} \Delta y \quad (7)$$

This must be equal to the rate of change of the number of vehicles over that space, assuming that the number of vehicles is conserved. Hence

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial\ Q_{xi}}{\partial x} + \frac{\partial\ Q_{yi}}{\partial y} = 0 \quad (8)$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot\ Q_i = 0 \quad (9)$$

Which can be termed as the mass balance equation or continuity equation in traffic flow

2.3 Transport equation

Suppose that the traffic is moving with a velocity u. We call the transport by the mean motion of the traffic as advection.
Each vehicle-driver unit has a random velocity component that leads to flow away from the density gradient (Fick's law) and to a diffusion term.
Suppose the density of each vehicle class is ρ_i and diffusion coefficient is D_i.
The flux density through a unit area in the YZ plane by the velocity component in the X direction is the quantity $(u\rho)$
The total rate of mass transport is the advective $(u\rho)$ plus the diffusive flux density $(-D_i.\frac{\partial \rho}{\partial x})$.

$$\begin{aligned} Q_{ix} = u_{ix}\rho_i - D_i.\frac{\partial \rho_i}{\partial x} \; and \\ Q_{iy} = u_{yi}\rho_i - D_i.\frac{\partial \rho_i}{\partial y} \end{aligned} \tag{10}$$

Where the diffusion coefficient is defined for each vehicle class i.
On substituting Eq.(13) in Eq.(9)

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial(u_{ix}\rho_i - D_i.\frac{\partial \rho_i}{\partial x})}{\partial x} + \frac{\partial(u_{yi}\rho_i - D_i.\frac{\partial \rho_i}{\partial y})}{\partial y} = 0 \tag{11}$$

We get

$$\frac{\partial \rho_i}{\partial t} = -\frac{\partial u_{xi}\rho_i}{\partial x} + D_i\frac{\partial^2 \rho_i}{\partial x^2} - \frac{\partial u_{yi}\rho_i}{\partial y} + D_i\frac{\partial^2 \rho_i}{\partial y^2} \tag{12}$$

In general form

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \; (u_i\rho_i) = D_i\nabla^2\rho_i \tag{13}$$

For traffic lacking lane discipline, area occupancy is a better measure of concentration. From Eq. (5)

$$\rho_i = \frac{(AO - AO_{-i})}{a_i} \tag{14}$$

Using area occupancy as a better measure of concentration, from Eq.(12) and Eq.(14), we get

$$\frac{\partial \rho_i}{\partial t} = -\frac{1}{a_i}(\frac{\partial((AO - AO_{-i})u_{ix})}{\partial x} + \frac{\partial(u_{iy}(AO - AO_{-i}))}{\partial y}) + D_i\frac{1}{a_i}(\frac{\partial^2(AO - AO_{-i})}{\partial x^2} + \frac{\partial^2(AO - AO_{-i})}{\partial y^2}) \tag{15}$$

Suppose $AO - AO_{-i} = AO_i$ and assume there are four vehicle classes; then we will get the system of equations

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} &= -\frac{1}{a_1}(\frac{\partial(AO_1u_{1x})}{\partial x} + \frac{\partial(u_{1y}AO_1)}{\partial y}) + D_1\frac{1}{a_1}(\frac{\partial^2 AO_1}{\partial x^2} + \frac{\partial^2 AO_1}{\partial y^2}) \\ \frac{\partial \rho_2}{\partial t} &= -\frac{1}{a_2}(\frac{\partial(AO_2u_{2x})}{\partial x} + \frac{\partial(u_{2y}AO_2)}{\partial y}) + D_2\frac{1}{a_2}(\frac{\partial^2 AO_2}{\partial x^2} + \frac{\partial^2 AO_2}{\partial y^2}) \\ \frac{\partial \rho_3}{\partial t} &= -\frac{1}{a_3}(\frac{\partial(AO_3u_{3x})}{\partial x} + \frac{\partial(u_{3y}AO_3)}{\partial y}) + D_3\frac{1}{a_3}(\frac{\partial^2 AO_3}{\partial x^2} + \frac{\partial^2 AO_3}{\partial y^2}) \\ \frac{\partial \rho_4}{\partial t} &= -\frac{1}{a_4}(\frac{\partial(AO_4u_{4x})}{\partial x} + \frac{\partial(u_{4y}AO_4)}{\partial y}) + D_4\frac{1}{a_4}(\frac{\partial^2 AO_4}{\partial x^2} + \frac{\partial^2 AO_4}{\partial y^2}) \end{aligned} \tag{16}$$

2.4 Characteristic Speed Of The Flow

The characteristic speed is the change in flow corresponding to a small change in traffic concentration. The speed of small disturbance in heterogeneous traffic can be defined as

$$c_i = \frac{\partial Q_i}{\partial AO}\frac{\partial AO}{\partial \rho_i} = \frac{\partial(Q_{ix} + Q_{iy})}{\partial AO}\frac{\partial AO}{\partial \rho_i} \tag{17}$$

On substituting Q_{ix} and Q_{iy} from Eq.(10) with area occupancy as a better measure of concentration, we get c_i as

$$c_i = -\frac{1}{a_i} \frac{\partial(AO_i u_{ix} - D_i.\frac{\partial AO_i}{\partial x} + u_{yi} AO_i - D_i.\frac{\partial AO_i}{\partial y})}{\partial AO} \frac{\partial AO}{\partial \rho_i} \tag{18}$$

From Eq.(9) we get

$$\frac{\partial \rho_i}{\partial t} = -(\frac{\partial\, Q_i}{\partial \rho_i}\frac{\partial \rho_i}{\partial x} + \frac{\partial\, Q_i}{\partial \rho_i}\frac{\partial \rho_i}{\partial y}) = -\frac{1}{a_i}(\frac{\partial(AO_i u_{ix} - D_i.\frac{\partial AO_i}{\partial x} + u_{yi} AO_i - D_i.\frac{\partial AO_i}{\partial y})}{\partial AO} \frac{\partial AO}{\partial \rho_i}(\frac{\partial \rho_i}{\partial x} + \frac{\partial \rho_i}{\partial y})$$
$$= -c_i(\frac{\partial \rho_i}{\partial x} + \frac{\partial \rho_i}{\partial y}) \tag{19}$$

where $Q_i = Q_{ix} + Q_{iy}$
Eq.(19) can be expressed in matrix form as

$$\begin{bmatrix} \frac{\partial \rho_1}{\partial t} \\ \frac{\partial \rho_2}{\partial t} \\ \frac{\partial \rho_3}{\partial t} \\ \frac{\partial \rho_4}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial q_1}{\partial AO}\frac{\partial AO}{\partial \rho_1} & 0 & 0 & 0 \\ 0 & \frac{\partial q_2}{\partial AO}\frac{\partial AO}{\partial \rho_2} & 0 & 0 \\ 0 & 0 & \frac{\partial q_3}{\partial AO}\frac{\partial AO}{\partial \rho_3} & 0 \\ 0 & 0 & 0 & \frac{\partial q_4}{\partial AO}\frac{\partial AO}{\partial \rho_4} \end{bmatrix} \times \begin{bmatrix} \frac{\partial \rho_1}{\partial x} + \frac{\partial \rho_1}{\partial y} \\ \frac{\partial \rho_2}{\partial x} + \frac{\partial \rho_2}{\partial y} \\ \frac{\partial \rho_3}{\partial x} + \frac{\partial \rho_3}{\partial y} \\ \frac{\partial \rho_4}{\partial x} + \frac{\partial \rho_4}{\partial y} \end{bmatrix}$$

The matrix J = $\frac{\partial q}{\partial \rho_i}\frac{\partial AO}{\partial \rho_i}$ is

$$J= \begin{bmatrix} \frac{\partial q_1}{\partial AO}\frac{\partial AO}{\partial \rho_1} & 0 & 0 & 0 \\ 0 & \frac{\partial q_2}{\partial AO}\frac{\partial AO}{\partial \rho_2} & 0 & 0 \\ 0 & 0 & \frac{\partial q_3}{\partial AO}\frac{\partial AO}{\partial \rho_3} & 0 \\ 0 & 0 & 0 & \frac{\partial q_4}{\partial AO}\frac{\partial AO}{\partial \rho_4} \end{bmatrix}$$

where
$q_1 = \frac{1}{a_1}(u_{1x}AO_{1x} + u_{1y}AO_{1y} - D_1(\frac{\partial AO_1}{\partial x} + \frac{\partial AO_1}{\partial y}))$
$q_2 = \frac{1}{a_2}(u_{2x}AO_{2x} + u_{2y}AO_{2y} - D_2(\frac{\partial AO_2}{\partial x} + \frac{\partial AO_2}{\partial y}))$
$q_3 = \frac{1}{a_3}(u_{3x}AO_{3x} + u_{3y}AO_{3y} - D_3(\frac{\partial AO_3}{\partial x} + \frac{\partial AO_3}{\partial y}))$
$q_4 = \frac{1}{a_4}(u_{4x}AO_{4x} + u_{4y}AO_{4y} - D_4(\frac{\partial AO_4}{\partial x} + \frac{\partial AO_4}{\partial y}))$

$$= \begin{bmatrix} J_{11} & 0 & 0 & 0 \\ 0 & J_{22} & 0 & 0 \\ 0 & 0 & J_{33} & 0 \\ 0 & 0 & 0 & J_{44} \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

where
$J_{11} = a = \frac{1}{a_1}(\frac{\partial(u_{1x}AO_1 + u_{1y}AO_1)}{\partial AO}\frac{\partial AO}{\partial \rho_1} - D_1(\frac{\partial^2 AO_1}{\partial AO \partial x}\frac{\partial AO}{\partial \rho_1} + \frac{\partial^2 AO1}{\partial AO \partial y}\frac{\partial AO}{\partial \rho_1}))$
$J_{22} = f = \frac{1}{a_2}(\frac{\partial(u_{2x}AO_2 + u_{2y}AO_2)}{\partial AO}\frac{\partial AO}{\partial \rho_2} - D_2(\frac{\partial^2 AO_2}{\partial AO \partial x}\frac{\partial AO}{\partial \rho_2} + \frac{\partial^2 AO2}{\partial AO \partial y}\frac{\partial AO}{\partial \rho_2}))$
$J_{33} = k = \frac{1}{a_3}(\frac{\partial(u_{3x}AO_3 + u_{3y}AO_3)}{\partial AO}\frac{\partial AO}{\partial \rho_3} - D_3(\frac{\partial^2 AO_3}{\partial AO \partial x}\frac{\partial AO}{\partial \rho_3} + \frac{\partial^2 AO3}{\partial AO \partial y}\frac{\partial AO}{\partial \rho_3}))$
$J_{44} = p = \frac{1}{a_4}(\frac{\partial(u_{4x}AO_4 + u_{4y}AO_4)}{\partial AO}\frac{\partial AO}{\partial \rho_4} - D_4(\frac{\partial^2 AO_4}{\partial AO \partial x}\frac{\partial AO}{\partial \rho_4} + \frac{\partial^2 AO4}{\partial AO \partial y}\frac{\partial AO}{\partial \rho_4}))$

The eigenvalues of J will be the characteristic speed of the flow.
Eigenvalues (λ) of J can be found by setting

$$|[J] - \lambda[I]| = 0 \tag{20}$$

which leads us to the characteristic equation of the Jacobian matrix J

$$(a - \lambda)(f - \lambda)(k - \lambda)(p - \lambda) = 0 \tag{21}$$

On solving the equation, we get the eigenvalues as

$$\begin{aligned}\lambda_1 &= a\\ \lambda_2 &= f\\ \lambda_3 &= k\\ \lambda_4 &= p\end{aligned} \tag{22}$$

which are the characteristic speeds

3 Numerical Simulation

A 2 km road with a 7.5 m width and four vehicle classes are simulated for testing model properties. The simulation was done for 30 minutes. The road was divided into several grids, with 100 grids along the length of the road and 10 grids along the width of the road. Density fields, class-specific flux density, and velocity fields are initialized with random values scaled to average densities and velocities, respectively. The time step was calculated to satisfy the CFL conditions and ensure stability. To ensure convective stability, it is ensured that

$$\begin{aligned}dt \leq \frac{dx}{u_{xf}}\ and\\ dt \leq \frac{dy}{u_{yf}}\end{aligned} \tag{23}$$

To ensure diffusive stability

$$\begin{aligned}dt \leq \frac{dx^2}{2D}\ and\\ dt \leq \frac{dy^2}{2D}\end{aligned} \tag{24}$$

Finally, the time step adopted should be less than or equal to the time step given by CFL criteria

$$dt \leq min(\frac{dx}{u_{xf}}, \frac{dy}{u_{yf}}, \frac{dx^2}{2D}, \frac{dy^2}{2D}) \tag{25}$$

A constant inflow of each vehicle class is given uniformly to each cell at the beginning of the road throughout the simulation. The spatial domain is discretized into finite cells (i, j) in the x and y directions. It is ensured that no vehicle is coming in or exiting the side of the road by setting the lateral flux at the lateral boundaries to zero. Density is updated using the continuity equation.

$$\frac{\rho[i,j,t+\Delta t] - \rho[i,j,t]}{\Delta t} = -\frac{(Q_x[i,j,t] - Q_x[i-1,j,t])}{\Delta x} - \frac{(Q_y[i,j,t] - Q_y[i,j-1,t])}{\Delta y} \tag{26}$$

Based on the density and area of each vehicle class area, occupancy is updated over time. Then, the flux density is updated using the transport equation.

$$\begin{aligned}Q_x[i,j] &= u_x[i,j](AO_i[i,j]) - \frac{D(AO_i[i,j]) - AO_i[i-1,j]}{a\Delta x},\\ Q_y[i,j] &= u_y[i,j](AO_i[i,j]) - \frac{D(AO_i[i,j]) - AO_i[i,j-1]}{a\Delta y}.\end{aligned} \tag{27}$$

while velocity is updated with time using the equation

$$u_i = u_{fi}\left(1 - \exp\left(1 - \exp\left(\frac{r_i(AO_{\max,i})}{AO} - r_i\right)\right)\right) \tag{28}$$

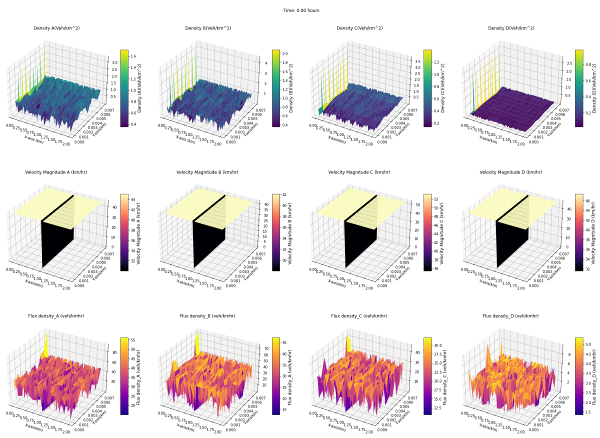
The parameters used for simulation are listed in Table (1)

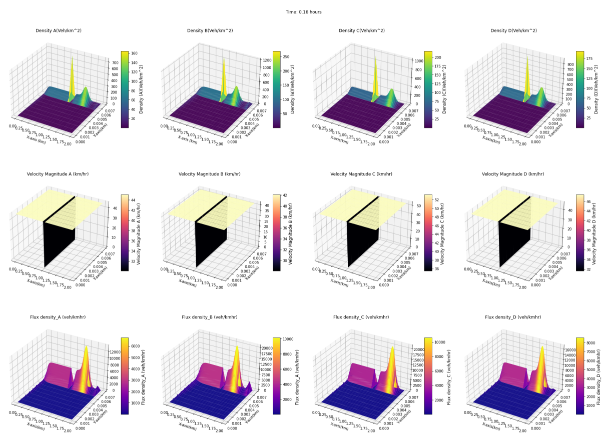
Vehicle Class	Class A	Class B	Class C	Class D
Length(m) × Width(m)	1.8 × 0.6	2.6 × 1.4	4 × 1.6	6.5 × 2.2
AOmax	0.80	0.78	0.68	0.50
r	0.89	0.78	0.74	0.50
q inflow (veh/hr)	50	80	70	60
Initial flux density (veh/kmhr)	570	661	469	463
D (km$^2/hr$)	0.00006	0.00005	0.00004	0.00003
Initial density (veh/km^2)	12	15	8	9
Initial resultant velocity (km/hr)	45	42	53	47

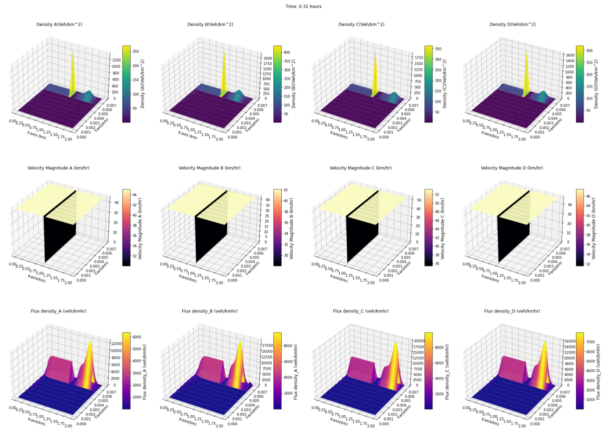
Table 1: Parameters for simulation

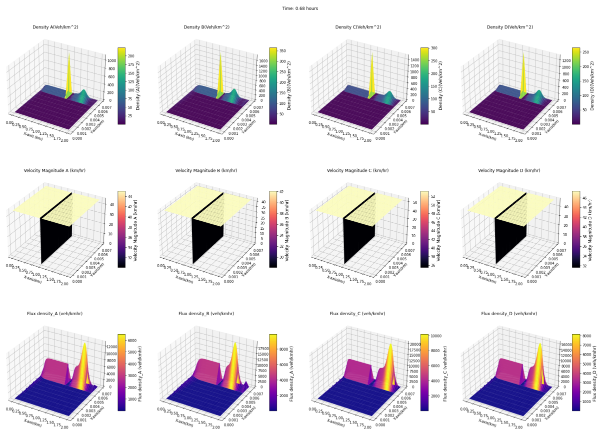
3.1 Case 1

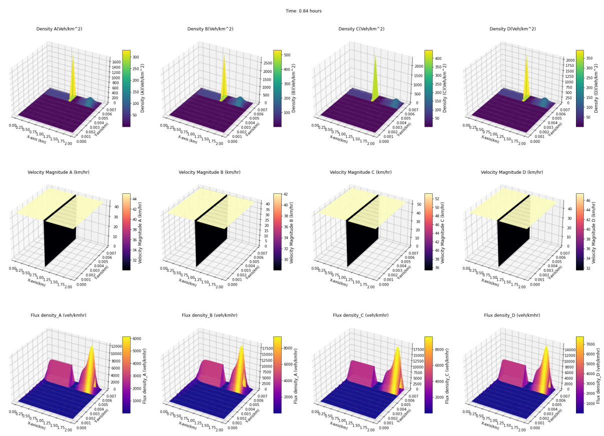
At 1 km for a 30-meter distance, the velocity and flux density of all vehicle classes is zero in every alternative 35 seconds, indicating the red traffic lights.
Variations of the magnitude of velocity, flux density, and density of each vehicle class with the length and breadth of the road at various instants of time are plotted.
In the plots, a low-velocity zone at a distance of 1 km along the length of the road indicates the stop signal.

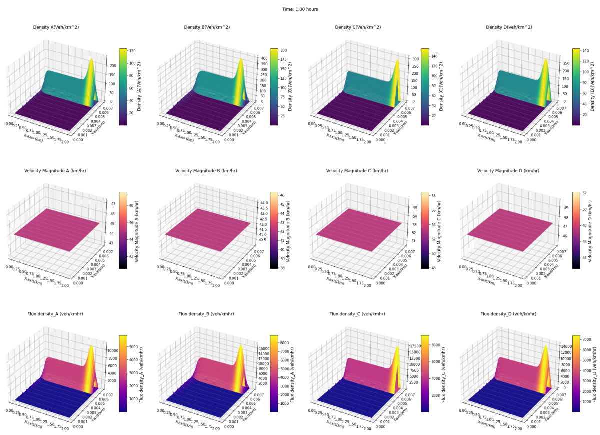





Time: 0.32 hours
Density A(Veh/km^2)
Density B(Veh/km^2)
Density C(Veh/km^2)
Density D(Veh/km^2)
Velocity Magnitude A (km/hr)
Velocity Magnitude B (km/hr)
Velocity Magnitude C (km/hr)
Velocity Magnitude D (km/hr)
Flux density_A (veh/kmhr)
Flux density_B (veh/kmhr)
Flux density_C (veh/kmhr)
Flux density_D (veh/kmhr)


Time: 0.68 hours
Density A(Veh/km^2)
Density B(Veh/km^2)
Density C(Veh/km^2)
Density D(Veh/km^2)
Velocity Magnitude A (km/hr)
Velocity Magnitude B (km/hr)
Velocity Magnitude C (km/hr)
Velocity Magnitude D (km/hr)
Flux density_A (veh/kmhr)
Flux density_B (veh/kmhr)
Flux density_C (veh/kmhr)
Flux density_D (veh/kmhr)


Time: 0.84 hours
Density A(Veh/km^2)
Density B(Veh/km^2)
Density C(Veh/km^2)
Density D(Veh/km^2)
Velocity Magnitude A (km/hr)
Velocity Magnitude B (km/hr)
Velocity Magnitude C (km/hr)
Velocity Magnitude D (km/hr)
Flux density_A (veh/kmhr)
Flux density_B (veh/kmhr)
Flux density_C (veh/kmhr)
Flux density_D (veh/kmhr)



4 Conclusions

A two-dimensional second-order model is formulated using flow conservation in terms of area occupancy, which is a better measure of concentration in traffic flow compared to density. For this, flux density is expressed as the sum of mean motion, which is advection and the random velocity component of the driver that leads to flow away from density gradient, which is diffusion, which is substituted into the two-dimensional flow conservation equation, which is then expressed in terms of area occupancy. The system's characteristic flow speed is then obtained in terms of area occupancy. The model properties are simulated for a 2 km road in the presence of a traffic signal at 1 km.

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