

Trafficflow_Simulation

July 2, 2024

```
[4]: import numpy as np
import matplotlib.pyplot as plt

# Define time parameters
T = 180.0 # total simulation time
dt = 1 # time step
time_steps = int(T / dt) + 1
time = np.arange(0, T + dt, dt) # array of times

# Define constants
D_A, D_B, D_C, D_D = 0.1, 0.2, 0.2, 0.1 # Diffusion coefficients

# Initialize storage for eigenvalues and maximum velocities
eigenvalues = np.zeros((time_steps, 4))
max_velocities = np.zeros(time_steps)

# Initialize densities
rho_A = np.zeros((50, 100))
rho_B = np.zeros((50, 100))
rho_C = np.zeros((50, 100))
rho_D = np.zeros((50, 100))

# Assign initial densities based on the condition  $x < 25$  or  $x > 25$ 
for x in range(50):
    if x < 25:
        rho_A[x, :] = 100
        rho_B[x, :] = 50
        rho_C[x, :] = 200
        rho_D[x, :] = 25
    else:
        rho_A[x, :] = 1200
        rho_B[x, :] = 400
        rho_C[x, :] = 500
        rho_D[x, :] = 100

# Initialize velocity fields
u_x_A = np.ones((50, 100)) * 45.0
u_y_A = np.ones((50, 100)) * 2.5
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u_x_B = np.ones((50, 100)) * 42.1
u_y_B = np.ones((50, 100)) * 1.4
u_x_C = np.ones((50, 100)) * 52.9
u_y_C = np.ones((50, 100)) * 5.6
u_x_D = np.ones((50, 100)) * 47.2
u_y_D = np.ones((50, 100)) * 4.3

# Velocities and diffusion coefficients
u_x = [u_x_A, u_x_B, u_x_C, u_x_D]
u_y = [u_y_A, u_y_B, u_y_C, u_y_D]
D = [D_A, D_B, D_C, D_D]

# Define inflow values
q_A_inflow = 500
q_B_inflow = 100
q_C_inflow = 50
q_D_inflow = 10

# Define the finite difference gradient function
def finite_difference_gradient(f, dx, dy):
    grad_x = (f[2:, 1:-1] - f[:-2, 1:-1]) / (2 * dx)
    grad_y = (f[1:-1, 2:] - f[1:-1, :-2]) / (2 * dy)
    return grad_x, grad_y

# Function to compute the flux q
def compute_q(grad_rho_x, grad_rho_y, rho, u_x, u_y, D):
    q = u_x[1:-1, 1:-1] * rho[1:-1, 1:-1] + u_y[1:-1, 1:-1] * rho[1:-1, 1:-1] -
    ↪D * (grad_rho_x + grad_rho_y)
    return q

# Function to compute the Jacobian matrix
def compute_jacobian(rho_A, rho_B, rho_C, rho_D, dx, dy):
    J = np.zeros((4, 4)) # Initialize Jacobian matrix

    # Compute gradients using finite differences
    grad_rho_A_x, grad_rho_A_y = finite_difference_gradient(rho_A, dx, dy)
    grad_rho_B_x, grad_rho_B_y = finite_difference_gradient(rho_B, dx, dy)
    grad_rho_C_x, grad_rho_C_y = finite_difference_gradient(rho_C, dx, dy)
    grad_rho_D_x, grad_rho_D_y = finite_difference_gradient(rho_D, dx, dy)

    # Compute Jacobian using numerical differentiation
    epsilon = 1e-5
    for i, rho_i in enumerate([rho_A, rho_B, rho_C, rho_D]):
        for j, rho_j in enumerate([rho_A, rho_B, rho_C, rho_D]):
            perturbed_rho_j = np.copy(rho_j)
            perturbed_rho_j[1:-1, 1:-1] += epsilon

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        perturbed_grad_x, perturbed_grad_y =
→finite_difference_gradient(perturbed_rho_j, dx, dy)
        perturbed_flux = compute_q(perturbed_grad_x, perturbed_grad_y,
→perturbed_rho_j, u_x[j], u_y[j], D[j])
        grad_x, grad_y = finite_difference_gradient(rho_j, dx, dy)
        flux = compute_q(grad_x, grad_y, rho_j, u_x[j], u_y[j], D[j])
        J[i, j] = np.mean((perturbed_flux - flux) / epsilon)

    return J

# Function to update the density field
def update_density(rho, u_x, u_y, D, dt, dx, dy, q_inflow):
    rho_new = np.copy(rho)
    for j in range(1, rho.shape[0] - 1):
        for i in range(1, rho.shape[1] - 1):
            rho_new[j, i] = (rho[j, i]
→rho[j, i] * dt / dx * (rho[j, i] - rho[j, i-1])
→rho[j, i] * dt / dy * (rho[j, i] - rho[j-1, i])
→rho[j, i+1] - 2 * rho[j, i] +
→rho[j-1, i]))
            + D * dt / dx**2 * (rho[j, i+1] - 2 * rho[j, i] +
→rho[j-1, i]))
            + D * dt / dy**2 * (rho[j+1, i] - 2 * rho[j, i] +
→rho[j-1, i]))

    # Apply inflow
    rho_new[:, 0] += q_inflow * dt / rho_new[:, 0]

    return rho_new

dx=1
dy=1
# Simulate densities and compute eigenvalues and velocities over time
for t in range(time_steps):
    # Update densities
    rho_A = update_density(rho_A, u_x_A, u_y_A, D_A, dt, dx, dy, q_A_inflow)
    rho_B = update_density(rho_B, u_x_B, u_y_B, D_B, dt, dx, dy, q_B_inflow)
    rho_C = update_density(rho_C, u_x_C, u_y_C, D_C, dt, dx, dy, q_C_inflow)
    rho_D = update_density(rho_D, u_x_D, u_y_D, D_D, dt, dx, dy, q_D_inflow)

    # Update velocities based on the new densities and inflow q = rho * u
    u_x_A = q_A_inflow / rho_A
    u_y_A = q_A_inflow / rho_A
    u_x_B = q_B_inflow / rho_B
    u_y_B = q_B_inflow / rho_B
    u_x_C = q_C_inflow / rho_C
    u_y_C = q_C_inflow / rho_C
    u_x_D = q_D_inflow / rho_D
    u_y_D = q_D_inflow / rho_D

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# Compute Jacobian
J = compute_jacobian(rho_A, rho_B, rho_C, rho_D, dx, dy)

# Compute eigenvalues
eigs = np.linalg.eigvals(J)
eigenvalues[t] = np.real(eigs) # Store real part of eigenvalues
print(f"Eigenvalues at time step {t}:\n{eigenvalues[t]}\n")

# Compute maximum velocity (max of magnitudes of all velocity vectors)
max_velocity = max(np.max(np.sqrt(u_x_A**2 + u_y_A**2)),
                    np.max(np.sqrt(u_x_B**2 + u_y_B**2)),
                    np.max(np.sqrt(u_x_C**2 + u_y_C**2)),
                    np.max(np.sqrt(u_x_D**2 + u_y_D**2)))
max_velocities[t] = max_velocity
print(f"Maximum velocity at time step {t}:\n{max_velocity}\n")

# Plot both eigenvalues and maximum velocity over time in a single plot
fig, ax1 = plt.subplots()

# Plot eigenvalues
for i in range(4):
    ax1.plot(time, eigenvalues[:, i], label=f'Eigenvalue {i+1}', linestyle='-',
             marker='o')

ax1.set_xlabel('Time')
ax1.set_ylabel('Eigenvalue')
ax1.legend(loc='upper left')
ax1.set_title('Eigenvalues and Maximum Velocity Over Time')

# Create a second y-axis for maximum velocity
ax2 = ax1.twinx()
ax2.plot(time, max_velocities, color='orange', label='Maximum Velocity',
         linestyle='-', marker='o')
ax2.set_ylabel('Maximum Velocity')
ax2.legend(loc='upper right')

plt.grid(True)
plt.show()

# Plot each eigenvalue over time in separate plots
for i in range(4):
    plt.figure()
    plt.plot(time, eigenvalues[:, i], label=f'Eigenvalue {i+1}', linestyle='-',
             marker='o')
    plt.xlabel('Time')
    plt.ylabel('Eigenvalue')
    plt.legend()
    plt.title(f'Eigenvalue {i+1} of Jacobian Matrix Over Time')

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    plt.ylim(np.min(eigenvalues) - 1, np.max(eigenvalues) + 1) # Adjust y-axis
    ↪ limits for better visibility
    plt.grid(True)
    plt.show()

# Plot maximum velocity over time
plt.figure()
plt.plot(time, max_velocities, label='Maximum Velocity', linestyle='-',
    ↪ marker='o')
plt.xlabel('Time')
plt.ylabel('Maximum Velocity')
plt.legend()
plt.title('Maximum Velocity Over Time')
plt.grid(True)
plt.show()

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Eigenvalues at time step 0:

```
[2.84217094e-14 2.01000000e+02 9.86076132e-32 9.38797315e-16]
```

Maximum velocity at time step 0:

```
7.0710678118654755
```

Eigenvalues at time step 1:

```
[0.00000000e+00 2.01000000e+02 0.00000000e+00 1.72963019e-15]
```

Maximum velocity at time step 1:

```
47.50588194018135
```

Eigenvalues at time step 2:

```
[ 0.00000000e+00  2.01000000e+02 -1.58377333e-16  2.26797510e-30]
```

Maximum velocity at time step 2:

```
32.053835278925334
```

Eigenvalues at time step 3:

```
[0.00000000e+00 2.01000000e+02 3.94430453e-31 3.48176677e-16]
```

Maximum velocity at time step 3:

```
28.754834806403988
```

Eigenvalues at time step 4:

```
[ 2.84217094e-14  2.01000000e+02 -2.08924880e-30 -1.50543099e-17]
```

Maximum velocity at time step 4:

```
189.5459181443251
```

Eigenvalues at time step 5:

[0.00000000e+00 2.01000000e+02 -1.23259516e-31 -3.09901144e-16]

Maximum velocity at time step 5:

134.93917531738833

Eigenvalues at time step 6:

[-2.84217094e-14 2.01000000e+02 -1.93586730e-15 -7.38778232e-33]

Maximum velocity at time step 6:

684.0470357163342

Eigenvalues at time step 7:

[2.84217094e-14 2.00999996e+02 1.48586648e-15 -9.86076132e-32]

Maximum velocity at time step 7:

45.746870434427464

Eigenvalues at time step 8:

[0.00000000e+00 2.01000000e+02 -4.19082356e-31 2.73776455e-16]

Maximum velocity at time step 8:

24.548224372861373

Eigenvalues at time step 9:

[2.84217094e-14 2.00999964e+02 1.02588911e-15 -1.97215226e-31]

Maximum velocity at time step 9:

46.432773008914126

Eigenvalues at time step 10:

[0.00000000e+00 2.00999884e+02 1.38818605e-19 -6.21523027e-28]

Maximum velocity at time step 10:

33.511686071483204

Eigenvalues at time step 11:

[0.00000000e+00 2.01000116e+02 -2.82430699e-15 -1.19897232e-32]

Maximum velocity at time step 11:

32.488836836434515

Eigenvalues at time step 12:

[2.84217094e-14 2.01003914e+02 1.54155976e-15 0.00000000e+00]

Maximum velocity at time step 12:

35.0303957987246

Eigenvalues at time step 13:

[0.00000000e+00 2.00984942e+02 -1.73610818e-16 1.03537994e-30]

Maximum velocity at time step 13:

117.92816871067016

Eigenvalues at time step 14:

[0.00000000e+00 2.00972175e+02 1.57959050e-15 9.86076132e-32]

Maximum velocity at time step 14:

384.7637316158766

Eigenvalues at time step 15:

[0.00000000e+00 2.00838680e+02 4.93038066e-32 -3.75936358e-16]

Maximum velocity at time step 15:

144.9158158918444

Eigenvalues at time step 16:

[-2.84217094e-14 2.00532462e+02 -3.04630266e-15 -1.44016488e-33]

Maximum velocity at time step 16:

70.8860172006178

Eigenvalues at time step 17:

[-2.84217094e-14 2.00489130e+02 0.00000000e+00 -5.82413052e-16]

Maximum velocity at time step 17:

159.87982913595863

Eigenvalues at time step 18:

[0.00000000e+00 2.00404050e+02 -1.92894545e-15 2.11187662e-32]

Maximum velocity at time step 18:

30.676519901822804

Eigenvalues at time step 19:

[0.00000000e+00 2.00278288e+02 6.70174379e-16 0.00000000e+00]

Maximum velocity at time step 19:

970.7567075387217

Eigenvalues at time step 20:

[0.00000000e+00 2.00000277e+02 1.36577249e-15 -9.86076132e-32]

Maximum velocity at time step 20:

75.81605076438323

Eigenvalues at time step 21:

[-2.84217094e-14 1.99877607e+02 -2.30168070e-15 8.63917833e-33]

Maximum velocity at time step 21:

90.11117343089329

Eigenvalues at time step 22:

[0.00000000e+00 1.99468552e+02 1.97215226e-31 -2.27245215e-15]

Maximum velocity at time step 22:

368.31352855286815

Eigenvalues at time step 23:

[0.00000000e+00 1.99292259e+02 1.34038819e-15 0.00000000e+00]

Maximum velocity at time step 23:

133.30186453422075

Eigenvalues at time step 24:

[3.69102162e+02 -1.57073902e-14 -3.86298286e-17 -3.50365175e-30]

Maximum velocity at time step 24:

77.47282586345447

Eigenvalues at time step 25:

[0.00000000e+00 1.98999464e+02 -1.30087039e-15 0.00000000e+00]

Maximum velocity at time step 25:

19.28076732187358

Eigenvalues at time step 26:

[0.00000000e+00 1.98993872e+02 -1.97215226e-31 2.33038364e-15]

Maximum velocity at time step 26:

43.37063351275928

Eigenvalues at time step 27:

[0.00000000e+00 1.98724483e+02 2.69129691e-28 -4.70839231e-19]

Maximum velocity at time step 27:

162.41640184206545

Eigenvalues at time step 28:

[0.00000000e+00 1.98271600e+02 1.84293818e-15 -9.84506955e-33]

Maximum velocity at time step 28:

53.89816804378304

Eigenvalues at time step 29:

[2.84217094e-14 1.98020663e+02 -1.97215226e-31 -1.94236741e-15]

Maximum velocity at time step 29:

154.79742839039668

Eigenvalues at time step 30:

[0.00000000e+00 1.97827168e+02 2.18011287e-15 0.00000000e+00]

Maximum velocity at time step 30:

581.8259244963192

Eigenvalues at time step 31:

[-2.84217094e-14 1.97692966e+02 -8.20772236e-16 -4.93038066e-32]

Maximum velocity at time step 31:

855.2086696239088

Eigenvalues at time step 32:

[0.00000000e+00 1.97461743e+02 -4.93038066e-32 4.77658651e-16]

Maximum velocity at time step 32:

195.264051659255

Eigenvalues at time step 33:

[0.00000000e+00 1.97502317e+02 -1.35921787e-15 0.00000000e+00]

Maximum velocity at time step 33:

413.54819656709867

Eigenvalues at time step 34:

[0.00000000e+00 1.96923419e+02 -4.63843069e-17 -1.90435953e-30]

Maximum velocity at time step 34:

77.1194861849431

Eigenvalues at time step 35:

[0.00000000e+00 1.96555161e+02 1.45573498e-15 0.00000000e+00]

Maximum velocity at time step 35:

92.61898029659015

Eigenvalues at time step 36:

[0.00000000e+00 1.96567961e+02 8.83424291e-16 0.00000000e+00]

Maximum velocity at time step 36:

149.60235665450625

Eigenvalues at time step 37:

[0.00000000e+00 1.96083922e+02 1.40034357e-16 2.46519033e-32]

Maximum velocity at time step 37:

3791.1191885194594

Eigenvalues at time step 38:

[2.84217094e-14 1.95854561e+02 2.49980824e-15 0.00000000e+00]

Maximum velocity at time step 38:

63.20416125617893

Eigenvalues at time step 39:

[0.00000000e+00 1.95480898e+02 -2.75793168e-31 -2.32018500e-17]

Maximum velocity at time step 39:

84.694113040768

Eigenvalues at time step 40:

[2.84217094e-14 1.95162303e+02 1.29080172e-15 0.00000000e+00]

Maximum velocity at time step 40:

235.36355078824934

Eigenvalues at time step 41:

[0.00000000e+00 1.94891571e+02 0.00000000e+00 1.48918672e-15]

Maximum velocity at time step 41:

85.97702243460503

Eigenvalues at time step 42:

[0.00000000e+00 1.94506609e+02 1.03835110e-16 6.16297582e-33]

Maximum velocity at time step 42:

176.34394775467192

Eigenvalues at time step 43:

[0.00000000e+00 1.94266925e+02 -2.43869270e-15 -9.98024141e-33]

Maximum velocity at time step 43:

492.4773672559718

Eigenvalues at time step 44:

[0.00000000e+00 1.93907563e+02 -3.20474743e-31 -2.27545152e-16]

Maximum velocity at time step 44:

265.5028099123249

Eigenvalues at time step 45:

[0.00000000e+00 1.93498163e+02 9.86076132e-32 -9.56388327e-16]

Maximum velocity at time step 45:

212.4299467633022

Eigenvalues at time step 46:

[0.00000000e+00 1.93097772e+02 -2.46519033e-31 2.82122118e-16]

Maximum velocity at time step 46:

73.7149343684658

Eigenvalues at time step 47:

[0.00000000e+00 1.92941603e+02 8.62816615e-32 1.27542114e-16]

Maximum velocity at time step 47:

630.2226952944004

Eigenvalues at time step 48:

[2.84217094e-14 1.97826885e+02 2.58307544e-15 1.32958793e-31]

Maximum velocity at time step 48:

319.7733698956603

Eigenvalues at time step 49:

[0.00000000e+00 1.92354267e+02 4.93038066e-32 7.36537143e-16]

Maximum velocity at time step 49:

219.05855967496552

Eigenvalues at time step 50:

[0.00000000e+00 1.91522724e+02 9.49929754e-17 5.54667824e-32]

Maximum velocity at time step 50:

31.995009317710508

Eigenvalues at time step 51:

[0.00000000e+00 1.91155055e+02 -1.26897264e-15 0.00000000e+00]

Maximum velocity at time step 51:

80.39259979221144

Eigenvalues at time step 52:

[0.00000000e+00 1.91143034e+02 1.90590187e-16 1.14631350e-30]

Maximum velocity at time step 52:

194.04943751357644

Eigenvalues at time step 53:

[0.00000000e+00 1.90616488e+02 -9.48144444e-16 0.00000000e+00]

Maximum velocity at time step 53:

2387.1619317387735

Eigenvalues at time step 54:

[-2.84217094e-14 1.90325846e+02 -2.94360963e-15 -1.71576739e-32]

Maximum velocity at time step 54:

307.7743172290198

Eigenvalues at time step 55:

[0.00000000e+00 1.89919463e+02 -1.97215226e-31 -1.40270796e-15]

Maximum velocity at time step 55:

238.12393553475167

Eigenvalues at time step 56:

[0.00000000e+00 1.89383868e+02 -1.66414278e-15 9.86076132e-32]

Maximum velocity at time step 56:

533.0803379873958

Eigenvalues at time step 57:

[0.00000000e+00 1.89012675e+02 1.97215226e-31 -2.82856794e-15]

Maximum velocity at time step 57:

58.540680414801365

Eigenvalues at time step 58:

[0.00000000e+00 1.88512056e+02 0.00000000e+00 -7.42324356e-16]

Maximum velocity at time step 58:

100.11012776016787

Eigenvalues at time step 59:

[0.00000000e+00 1.88297199e+02 -1.47630387e-15 0.00000000e+00]

Maximum velocity at time step 59:

86.20867515273432

Eigenvalues at time step 60:

[2.84217094e-14 1.87745658e+02 -7.39557099e-32 3.66824233e-16]

Maximum velocity at time step 60:

111.60912361812171

Eigenvalues at time step 61:

[0.00000000e+00 1.87418733e+02 -4.17146164e-15 4.94125415e-32]

Maximum velocity at time step 61:

124.87466423949785

Eigenvalues at time step 62:

[0.00000000e+00 1.86617923e+02 9.86076132e-32 2.33195326e-16]

Maximum velocity at time step 62:

75.38491016198434

Eigenvalues at time step 63:

[0.00000000e+00 1.85911870e+02 4.93038066e-32 4.88557102e-16]

Maximum velocity at time step 63:

148.39656475370896

Eigenvalues at time step 64:

[0.00000000e+00 1.85859901e+02 -2.13631478e-15 -4.48292317e-32]

Maximum velocity at time step 64:

328.2150493936841

Eigenvalues at time step 65:

[0.00000000e+00 1.85105944e+02 2.35203203e-16 -2.46519033e-32]

Maximum velocity at time step 65:

252.39365362124673

Eigenvalues at time step 66:

[0.00000000e+00 1.84476447e+02 4.93038066e-32 5.25149414e-16]

Maximum velocity at time step 66:

347.54594156929653

Eigenvalues at time step 67:

[0.00000000e+00 1.84115994e+02 2.22488814e-16 -3.99360833e-30]

Maximum velocity at time step 67:

78.68460231707611

Eigenvalues at time step 68:

[0.00000000e+00 1.83459677e+02 -1.64354013e-15 0.00000000e+00]

Maximum velocity at time step 68:

1276.2434991392895

Eigenvalues at time step 69:

[0.00000000e+00 1.83174857e+02 7.70486127e-19 3.00245256e-29]

Maximum velocity at time step 69:

193.1346275024927

Eigenvalues at time step 70:

[0.00000000e+00 1.82280015e+02 -3.11954645e-15 1.09241339e-33]

Maximum velocity at time step 70:

1140.958184261595

Eigenvalues at time step 71:

[0.00000000e+00 1.81389375e+02 -1.14078452e-15 -1.97215226e-31]

Maximum velocity at time step 71:

209.47780217153536

Eigenvalues at time step 72:

[0.00000000e+00 1.80628070e+02 2.61725117e-15 -8.31669099e-32]

Maximum velocity at time step 72:

270.41221658636823

Eigenvalues at time step 73:

[2.84217094e-14 1.80121055e+02 1.23259516e-30 6.86964163e-16]

Maximum velocity at time step 73:

116.16363626953225

Eigenvalues at time step 74:

[0.00000000e+00 1.79675916e+02 -1.61438827e-15 -9.86076132e-32]

Maximum velocity at time step 74:

229.28632003094413

Eigenvalues at time step 75:

[0.00000000e+00 1.78688464e+02 -4.93038066e-32 -7.14843411e-16]

Maximum velocity at time step 75:

672.4320248340042

Eigenvalues at time step 76:

[0.00000000e+00 1.77596693e+02 0.00000000e+00 -4.74314308e-16]

Maximum velocity at time step 76:

41.81076865157595

Eigenvalues at time step 77:

[0.00000000e+00 1.76776022e+02 -1.39075575e-15 0.00000000e+00]

Maximum velocity at time step 77:

184.34877831726163

Eigenvalues at time step 78:

[0.00000000e+00 1.76383660e+02 1.28256502e-15 0.00000000e+00]

Maximum velocity at time step 78:

907.849895888184

Eigenvalues at time step 79:

[0.00000000e+00 1.75720476e+02 -1.06399057e-15 0.00000000e+00]

Maximum velocity at time step 79:

133.7782205475774

Eigenvalues at time step 80:

[0.00000000e+00 1.74864115e+02 -2.47339666e-15 -6.64370253e-32]

Maximum velocity at time step 80:

183.61838727243014

Eigenvalues at time step 81:

[0.00000000e+00 1.74264261e+02 -1.73685248e-15 0.00000000e+00]

Maximum velocity at time step 81:

54.817990709697426

Eigenvalues at time step 82:

[2.84217094e-14 1.73572162e+02 2.74702899e-15 -1.18311442e-31]

Maximum velocity at time step 82:

4904.754592780371

Eigenvalues at time step 83:

[0.00000000e+00 1.73843433e+02 -1.49912441e-15 0.00000000e+00]

Maximum velocity at time step 83:

103.23500338789333

Eigenvalues at time step 84:

[0.00000000e+00 1.72038706e+02 1.44029954e-15 -9.86076132e-32]

Maximum velocity at time step 84:

243.96436904103558

Eigenvalues at time step 85:

[2.84217094e-14 1.71454393e+02 1.40515849e-30 4.22027989e-16]

Maximum velocity at time step 85:

172.2064062669175

Eigenvalues at time step 86:

[0.00000000e+00 1.70936748e+02 -2.31600577e-15 3.94430453e-31]

Maximum velocity at time step 86:

163.41807518023228

Eigenvalues at time step 87:

[2.84217094e-14 1.70204562e+02 2.94018960e-16 1.55306991e-30]

Maximum velocity at time step 87:

81.29227774411409

Eigenvalues at time step 88:

[0.00000000e+00 1.69624228e+02 2.06967820e-15 1.97215226e-31]

Maximum velocity at time step 88:

37.4249217798049

Eigenvalues at time step 89:

[0.00000000e+00 1.68815689e+02 1.62548734e-15 0.00000000e+00]

Maximum velocity at time step 89:

262.0103245500876

Eigenvalues at time step 90:

[0.00000000e+00 1.68311771e+02 0.00000000e+00 -1.08427003e-15]

Maximum velocity at time step 90:

166.4988047215411

Eigenvalues at time step 91:

[0.00000000e+00 1.67590709e+02 0.00000000e+00 1.27377462e-15]

Maximum velocity at time step 91:

551.7874079991691

Eigenvalues at time step 92:

[0.00000000e+00 1.66744622e+02 2.77699594e-15 -3.01280229e-32]

Maximum velocity at time step 92:

53.29452549509546

Eigenvalues at time step 93:

[0.00000000e+00 1.66323977e+02 -4.71944709e-16 4.93038066e-32]

Maximum velocity at time step 93:

1593.2629002284405

Eigenvalues at time step 94:

[0.00000000e+00 1.65525423e+02 2.91740456e-15 -1.78028576e-31]

Maximum velocity at time step 94:

75.79805099250946

Eigenvalues at time step 95:

[0.00000000e+00 1.64924087e+02 1.97215226e-31 -6.60715754e-16]

Maximum velocity at time step 95:

36.32927800420647

Eigenvalues at time step 96:

[0.00000000e+00 1.64502635e+02 -5.09240613e-16 4.93038066e-32]

Maximum velocity at time step 96:

57.27088748884264

Eigenvalues at time step 97:

[0.00000000e+00 1.63899638e+02 -1.75429375e-15 0.00000000e+00]

Maximum velocity at time step 97:

374.0284851897695

Eigenvalues at time step 98:

[0.00000000e+00 1.62958494e+02 -1.09500806e-15 2.95822839e-31]

Maximum velocity at time step 98:

43.5912515613074

Eigenvalues at time step 99:

[0.00000000e+00 1.62771476e+02 0.00000000e+00 -2.91231793e-15]

Maximum velocity at time step 99:

485.8772784767978

Eigenvalues at time step 100:

[0.00000000e+00 1.62052130e+02 -1.97215226e-31 -5.09373306e-16]

Maximum velocity at time step 100:

89.82032701392416

Eigenvalues at time step 101:

[0.00000000e+00 1.61318723e+02 5.30364596e-16 -4.93038066e-32]

Maximum velocity at time step 101:

1232.535119493353

Eigenvalues at time step 102:

[0.00000000e+00 1.60975551e+02 3.10260803e-15 1.94977725e-32]

Maximum velocity at time step 102:

112.10460759107946

Eigenvalues at time step 103:

[0.00000000e+00 1.60136830e+02 -1.85224202e-15 1.97215226e-31]

Maximum velocity at time step 103:

1830.078437729578

Eigenvalues at time step 104:

[0.00000000e+00 1.59671286e+02 -3.62617463e-15 -2.43654753e-32]

Maximum velocity at time step 104:

662.3271566759305

Eigenvalues at time step 105:

[0.00000000e+00 1.59020086e+02 2.26308829e-15 7.88860905e-31]

Maximum velocity at time step 105:

371.9860314461543

Eigenvalues at time step 106:

[0.00000000e+00 1.58335068e+02 0.00000000e+00 9.88636359e-16]

Maximum velocity at time step 106:

55.96524517824526

Eigenvalues at time step 107:

[0.00000000e+00 1.57843483e+02 2.93985125e-15 3.63633443e-34]

Maximum velocity at time step 107:

202.5156004132057

Eigenvalues at time step 108:

[0.00000000e+00 1.57220493e+02 -2.31111593e-32 -2.62007114e-17]

Maximum velocity at time step 108:

641.5987578401802

Eigenvalues at time step 109:

[0.0000000e+00 1.5672150e+02 -3.2207782e-15 0.0000000e+00]

Maximum velocity at time step 109:

286.4381703074382

Eigenvalues at time step 110:

[0.0000000e+00 1.5616534e+02 2.9179501e-15 5.96289880e-32]

Maximum velocity at time step 110:

197.00144141953564

Eigenvalues at time step 111:

[0.0000000e+00 1.55396730e+02 -1.86285281e-16 8.62816615e-32]

Maximum velocity at time step 111:

91.71517540946186

Eigenvalues at time step 112:

[0.0000000e+00 1.54958606e+02 2.69171655e-15 1.97215226e-31]

Maximum velocity at time step 112:

217.19227446283355

Eigenvalues at time step 113:

[0.0000000e+00 1.54510942e+02 2.89194394e-15 -1.97215226e-31]

Maximum velocity at time step 113:

2708.6089112776676

Eigenvalues at time step 114:

[0.0000000e+00 1.53650206e+02 -2.45216598e-15 -7.36885899e-32]

Maximum velocity at time step 114:

115.32615839236308

Eigenvalues at time step 115:

[2.84217094e-14 1.53213076e+02 -3.94430453e-31 3.27582538e-15]

Maximum velocity at time step 115:

64.11350172546437

Eigenvalues at time step 116:

[2.84217094e-14 1.52494747e+02 2.09283244e-15 1.97215226e-31]

Maximum velocity at time step 116:

531.8386386160586

Eigenvalues at time step 117:

[2.84217094e-14 1.52136245e+02 -1.47911420e-31 -5.15403361e-16]

Maximum velocity at time step 117:

141.92781534408525

Eigenvalues at time step 118:

[0.00000000e+00 1.51298640e+02 6.70813711e-16 9.86076132e-32]

Maximum velocity at time step 118:

49.69501283537795

Eigenvalues at time step 119:

[0.00000000e+00 1.50810597e+02 1.59004897e-15 0.00000000e+00]

Maximum velocity at time step 119:

25.6209543472994

Eigenvalues at time step 120:

[-1.42108547e-14 1.50199463e+02 9.86076132e-32 1.32877807e-15]

Maximum velocity at time step 120:

177.62783535050417

Eigenvalues at time step 121:

[-1.42108547e-14 1.49721579e+02 1.95206164e-15 0.00000000e+00]

Maximum velocity at time step 121:

15.507039932878099

Eigenvalues at time step 122:

[0.00000000e+00 1.49239533e+02 9.50820409e-16 0.00000000e+00]

Maximum velocity at time step 122:

130.5873163183357

Eigenvalues at time step 123:

[1.42108547e-14 1.48615250e+02 -1.95129309e-15 -1.97215226e-31]

Maximum velocity at time step 123:

642.9556424735568

Eigenvalues at time step 124:

[-1.42108547e-14 1.47776298e+02 -1.04770589e-31 -1.01439284e-16]

Maximum velocity at time step 124:

54.736614910010616

Eigenvalues at time step 125:

[0.00000000e+00 1.47515777e+02 4.03973183e-15 -1.70818677e-32]

Maximum velocity at time step 125:

25.372843957365358

Eigenvalues at time step 126:

[0.00000000e+00 1.46867914e+02 1.09969288e-15 0.00000000e+00]

Maximum velocity at time step 126:

161.73873383695306

Eigenvalues at time step 127:

[1.42108547e-14 1.46678857e+02 8.75142567e-31 -1.64430706e-16]

Maximum velocity at time step 127:

293.53625106111696

Eigenvalues at time step 128:

[1.42108547e-14 1.46004189e+02 2.57034573e-15 -3.38571894e-31]

Maximum velocity at time step 128:

369.37806614497606

Eigenvalues at time step 129:

[1.42108547e-14 1.45303822e+02 2.35348639e-30 -9.82706538e-18]

Maximum velocity at time step 129:

21.56812315142733

Eigenvalues at time step 130:

[0.00000000e+00 1.44902760e+02 -3.86119543e-16 0.00000000e+00]

Maximum velocity at time step 130:

51.54287209653843

Eigenvalues at time step 131:

[0.00000000e+00 1.43991819e+02 -3.17437014e-15 2.16509708e-31]

Maximum velocity at time step 131:

37.02321395269815

Eigenvalues at time step 132:

[1.42108547e-14 1.43627054e+02 1.54482413e-15 0.00000000e+00]

Maximum velocity at time step 132:

218.26283661128917

Eigenvalues at time step 133:

[1.42108547e-14 1.43106128e+02 -2.18421350e-15 1.97215226e-31]

Maximum velocity at time step 133:

150.37856144221632

Eigenvalues at time step 134:

[0.00000000e+00 1.42724553e+02 -1.17368588e-15 -9.86076132e-32]

Maximum velocity at time step 134:

82.87488166764794

Eigenvalues at time step 135:

[0.00000000e+00 1.41963316e+02 4.74394627e-15 -1.41993142e-32]

Maximum velocity at time step 135:

1042.0339180217184

Eigenvalues at time step 136:

[-1.42108547e-14 1.41811455e+02 3.11942119e-15 3.28207993e-31]

Maximum velocity at time step 136:

290.98867423285276

Eigenvalues at time step 137:

[-1.42108547e-14 1.41162712e+02 1.60834670e-15 0.00000000e+00]

Maximum velocity at time step 137:

131.74263304401595

Eigenvalues at time step 138:

[-1.42108547e-14 1.40720931e+02 4.93038066e-31 1.30380993e-15]

Maximum velocity at time step 138:

23.543723341987402

Eigenvalues at time step 139:

[-1.42108547e-14 1.40220737e+02 -3.05256659e-15 -5.91645679e-31]

Maximum velocity at time step 139:

1753.6366464611897

Eigenvalues at time step 140:

[0.00000000e+00 1.39231136e+02 3.26831542e-16 -2.46519033e-32]

Maximum velocity at time step 140:

21.174978490151688

Eigenvalues at time step 141:

[0.00000000e+00 1.38848399e+02 1.44545532e-15 -1.97215226e-31]

Maximum velocity at time step 141:

64.64413600592515

Eigenvalues at time step 142:

[0.00000000e+00 1.38505085e+02 3.47413462e-16 1.23259516e-31]

Maximum velocity at time step 142:

453.9201428812217

Eigenvalues at time step 143:

[0.00000000e+00 1.37873419e+02 4.22524553e-15 1.03420909e-31]

Maximum velocity at time step 143:

25.920702794079165

Eigenvalues at time step 144:

[-1.42108547e-14 1.37319985e+02 -7.78347227e-16 -4.93038066e-32]

Maximum velocity at time step 144:

36.81132233913202

Eigenvalues at time step 145:

[-1.42108547e-14 1.36777006e+02 4.75726908e-15 1.21849341e-31]

Maximum velocity at time step 145:

74.1679407557926

Eigenvalues at time step 146:

[0.00000000e+00 1.36233198e+02 -4.88266629e-15 3.85410165e-31]

Maximum velocity at time step 146:

56.76740099676936

Eigenvalues at time step 147:

[0.00000000e+00 1.35508753e+02 3.27327842e-15 1.97215226e-31]

Maximum velocity at time step 147:

177.79602398396932

Eigenvalues at time step 148:

[1.42108547e-14 1.35166158e+02 8.20945367e-16 1.97215226e-31]

Maximum velocity at time step 148:

27.702956515237545

Eigenvalues at time step 149:

[0.00000000e+00 1.34781274e+02 7.33872160e-16 0.00000000e+00]

Maximum velocity at time step 149:

40.59765077076948

Eigenvalues at time step 150:

[0.00000000e+00 1.34259794e+02 2.95822839e-31 1.29173122e-15]

Maximum velocity at time step 150:

46.662127058786815

Eigenvalues at time step 151:

[0.00000000e+00 1.33525585e+02 1.61535882e-15 -9.86076132e-32]

Maximum velocity at time step 151:

54.75352342195751

Eigenvalues at time step 152:

[0.00000000e+00 1.32944312e+02 3.42060351e-15 -4.93459768e-31]

Maximum velocity at time step 152:

40.124127199833495

Eigenvalues at time step 153:

[0.00000000e+00 1.32524375e+02 0.00000000e+00 2.09419873e-15]

Maximum velocity at time step 153:

28.47621719142925

Eigenvalues at time step 154:

[0.00000000e+00 1.31915395e+02 5.17625039e-17 -1.32503980e-31]

Maximum velocity at time step 154:

37.939271362057625

Eigenvalues at time step 155:

[0.00000000e+00 1.31471373e+02 2.56709881e-15 0.00000000e+00]

Maximum velocity at time step 155:

49.87392748243603

Eigenvalues at time step 156:

[0.00000000e+00 1.31039865e+02 2.78736822e-15 0.00000000e+00]

Maximum velocity at time step 156:

94.80419395977368

Eigenvalues at time step 157:

[0.00000000e+00 1.30535949e+02 2.58430132e-16 -3.25405123e-30]

Maximum velocity at time step 157:

131.88551901109292

Eigenvalues at time step 158:

[-1.42108547e-14 1.29696052e+02 -1.94070378e-15 -1.97215226e-31]

Maximum velocity at time step 158:

77.44007944416201

Eigenvalues at time step 159:

[0.00000000e+00 1.29187838e+02 4.31545255e-15 3.28113548e-32]

Maximum velocity at time step 159:

21.86871063327028

Eigenvalues at time step 160:

[0.00000000e+00 1.29042323e+02 4.49426101e-15 6.10588073e-32]

Maximum velocity at time step 160:

31.69438432996781

Eigenvalues at time step 161:

[0.00000000e+00 1.28256129e+02 8.99632189e-16 0.00000000e+00]

Maximum velocity at time step 161:

36.76201210182138

Eigenvalues at time step 162:

[0.00000000e+00 1.28005470e+02 -3.64279369e-15 -1.48615930e-31]

Maximum velocity at time step 162:

38.47749456269425

Eigenvalues at time step 163:

[0.00000000e+00 1.27401239e+02 5.49485710e-16 -4.93038066e-32]

Maximum velocity at time step 163:

65.30353326869361

Eigenvalues at time step 164:

[1.42108547e-14 1.26909586e+02 6.28199645e-15 8.20834197e-33]

Maximum velocity at time step 164:

23.46528873638525

Eigenvalues at time step 165:

[1.42108547e-14 1.26127886e+02 -6.92561547e-16 -9.86076132e-32]

Maximum velocity at time step 165:

27.582320428919076

Eigenvalues at time step 166:

[0.00000000e+00 1.25968716e+02 -1.02305399e-30 -2.46202723e-17]

Maximum velocity at time step 166:

52.137504785104994

Eigenvalues at time step 167:

[-1.42108547e-14 1.25420425e+02 -2.84482720e-15 -1.97215226e-31]

Maximum velocity at time step 167:

37.653498150676775

Eigenvalues at time step 168:

[0.00000000e+00 1.24505745e+02 -4.93038066e-32 5.73494760e-16]

Maximum velocity at time step 168:

981.5021410199686

Eigenvalues at time step 169:

[0.00000000e+00 1.24475460e+02 3.00011331e-15 0.00000000e+00]

Maximum velocity at time step 169:

274.4422234730324

Eigenvalues at time step 170:

[0.00000000e+00 1.24100175e+02 -5.87280728e-15 2.09756162e-31]

Maximum velocity at time step 170:

51.37292915855126

Eigenvalues at time step 171:

[0.00000000e+00 1.23433874e+02 -1.97215226e-31 -3.04035103e-16]

Maximum velocity at time step 171:

50.24975446234779

Eigenvalues at time step 172:

[0.00000000e+00 1.22741877e+02 -2.90583114e-15 -1.97215226e-31]

Maximum velocity at time step 172:

16.286985919250153

Eigenvalues at time step 173:

[-1.42108547e-14 1.22099273e+02 8.22556200e-17 6.53275437e-31]

Maximum velocity at time step 173:

24.571909525076777

Eigenvalues at time step 174:

[0.00000000e+00 1.21686703e+02 -1.33848971e-15 -9.86076132e-32]

Maximum velocity at time step 174:

34.51398802789087

Eigenvalues at time step 175:

[0.00000000e+00 1.21327334e+02 -5.44312069e-15 -2.81899972e-31]

Maximum velocity at time step 175:

67.09611395982138

Eigenvalues at time step 176:

[0.00000000e+00 1.20969165e+02 2.67532889e-15 0.00000000e+00]

Maximum velocity at time step 176:

116.3595874435396

Eigenvalues at time step 177:

[0.00000000e+00 1.20090327e+02 -1.82956002e-15 0.00000000e+00]

Maximum velocity at time step 177:

189.96649481250518

Eigenvalues at time step 178:

[0.00000000e+00 1.19921851e+02 -2.87367135e-16 9.86076132e-32]

Maximum velocity at time step 178:

43.225967431882175

Eigenvalues at time step 179:

[0.00000000e+00 1.19439206e+02 0.00000000e+00 2.22689015e-15]

Maximum velocity at time step 179:

64.6902040599611

Eigenvalues at time step 180:

[0.00000000e+00 1.19323017e+02 -2.87194673e-30 -1.76909593e-16]

Maximum velocity at time step 180:

61.130674534941676











