Let's start at 9:05 PM

L69
Combinatorics - 1

Fancy word for counting (in my opinion)

Join Discord - https://bit.ly/ly-discord

RECAP

Let’s start with fundamental
rules of counting

Sum Rule

If $A \Rightarrow n(A)$

& $B \Rightarrow n(B)$

A or $B \Rightarrow n(A) + n(B)$

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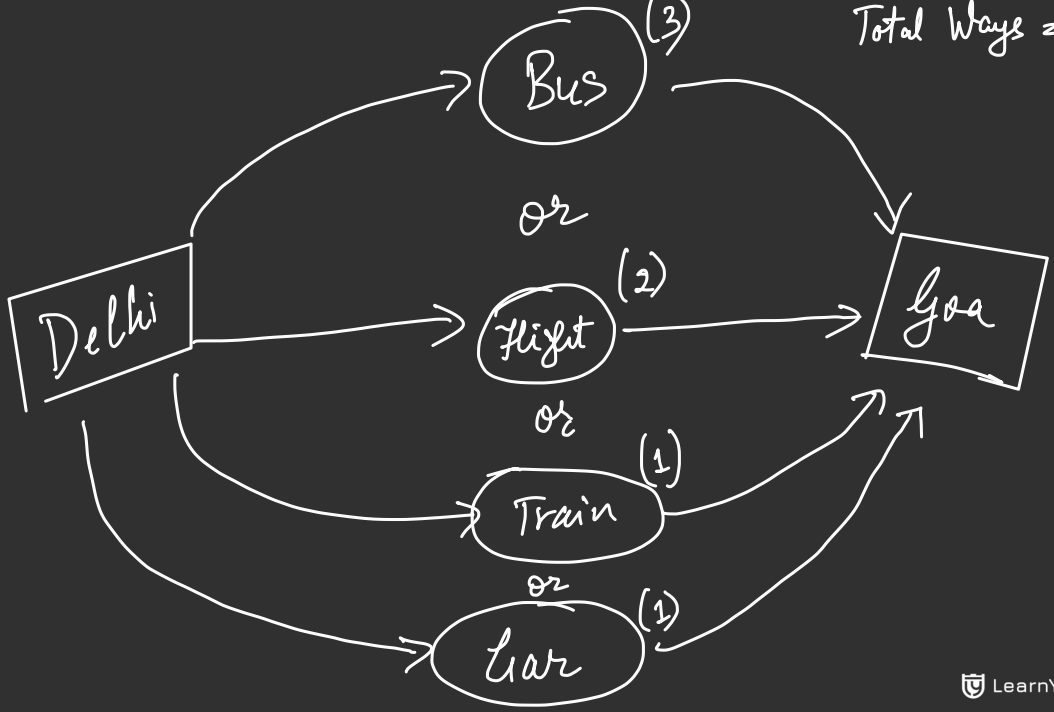

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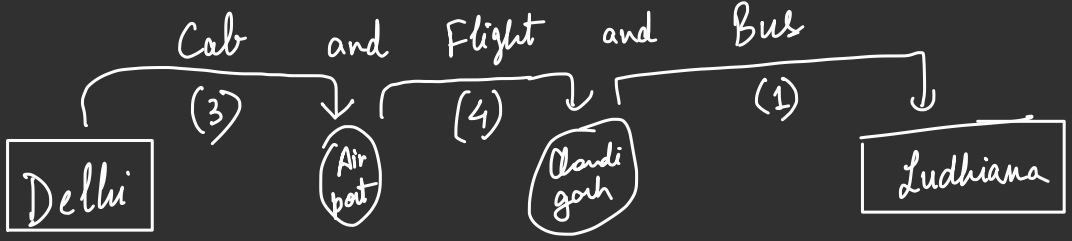
Product Rule

Total Ways = 12



C_1 C_2 C_3

F_1 F_2 F_3 F_4

B_1

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Number of Permutations

Given an array of size N, contains N distinct numbers. No. of permutations of the array?

Eg. A = [3, 5, 6]

[3, 5, 6], [3, 6, 5], [5, 3, 6], [5, 6, 3], [6, 3, 5], [6, 5, 3]

⇒ 6 perm.

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_ _ _ _ _ _ _ _ _ _ (n indices)

$N * (N-1) * (N-2) * (N-3) * (N-4) \cdots \cdots 1$

$$P(N) = N!$$

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Permutations with repetitions

arr = [1, 2, 2]

[1, 2, 2] , [2, 1, 2] , [2, 2, 1]

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arr = [1, 2, 2*]

[1, 2, 2*] , [2, 1, 2*] , [2, 2*, 1]

[1, 2*, 2] , [2*, 1, 2] , [2*, 2, 1]

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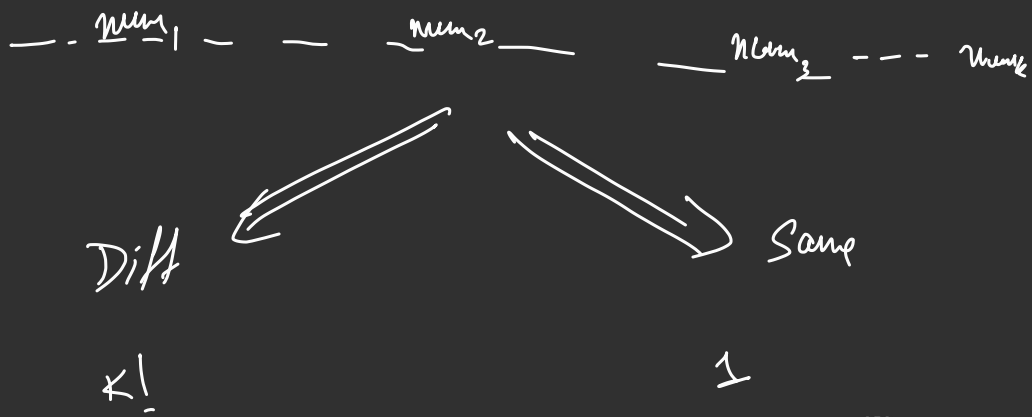
num ⇒ k times



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N

$num_1 \Rightarrow r_1$

$num_2 \Rightarrow r_2$

$\vdots$

$num_k \Rightarrow r_k$

$[r_1 + r_2 + r_3 \cdots r_k] = N$

$$\text{Permutations} = \frac{N!}{r_1! * r_2! * r_3! \cdots r_k!}$$

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Combinations (nCr)

Given N different items $[I_1, I_2 \ldots I_N]$, what are the no. of ways to pick R items of these?

N = 5
R = 3

1, 2, 3	1, 3, 4	2, 3, 4	3, 4, 5
1, 2, 4	1, 3, 5	2, 3, 5	
1, 2, 5	1, 4, 5	2, 4, 5	

$\Rightarrow 10$

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I_5, I_5, I_3

I_4, I_5, I_3

I_3, I_4, I_5

I_5, I_3, I_4

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$$N*(N-1)*(N-2)\ \text{-----}\ *(N-R+1)$$

$$\frac{N\ \text{--}\ \text{---}\ (N-R+1)*\underbrace{[(N-R)*(N-R-1)\ \text{--}\ \text{---}\ 1]}}{(N-R)!}$$

$$\Rightarrow \frac{N!}{R!*(N-R)!} = {}^{N}C_{R}$$

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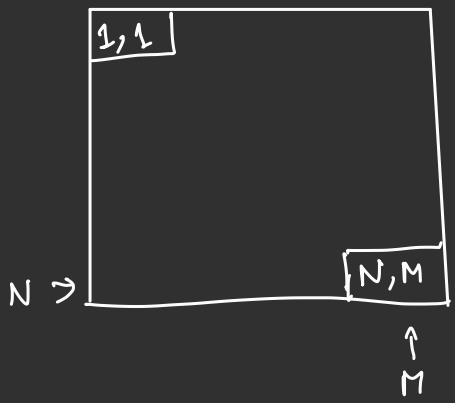
Let’s warmup a bit

Number of ways from (1, 1) to (N, M) in a grid



1 step ⇒ right $(i, j+1)$

Down $(i+1, j)$

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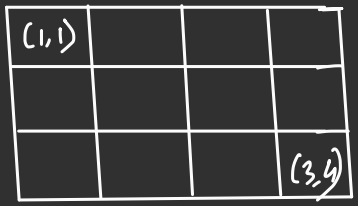
N = 3

M = 4



R R R D D

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Row 1 $\Rightarrow$ Row N $\Rightarrow$ $(N-1)$ down

Col 1 $\Rightarrow$ Col M $\Rightarrow$ $(M-1)$ rights

$$ANS = \frac{(N+M-2)!}{(N-1)!\,(M-1)!}$$

$$= {}^{N+M-2}C_{N-1}$$

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Number of ways to form a ICPC team out of N people such that:

1. At least 2 people to be chosen.
2. No more than N - 2 people to be chosen

$P_1, P_2, P_3 \dots P_N$

$ans = {}^{N}C_2 + {}^{N}C_3 \dots {}^{N}C_{N-2}$

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0 | 1 | 2 - —— N-2 | N-1 | N

$$2^N - {}^NC_0 - {}^NC_1 - {}^NC_{N-1} - {}^NC_N$$

$$\Rightarrow 2^N - 1 - N - N - 1$$

$$\Rightarrow 2^N - 2N - 2 \quad \text{(ans)}$$

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mod = $10^9 + 7$

Let's write some code?

Q queries:

1) N & M.

2) Print no. of ways to reach from (1,1) to (N,M) % mod.

Constraints :

$1 \leq Q \leq 10^5$

$1 \leq N, M \leq 10^5$

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if mod is prime

$$a^{-1} \equiv a^{mod-2} \pmod{m}$$

[Fermat's Little Theorem]

out of scope

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Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!