

Let's begin at 9:02 PM

L72
Probability

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RECAP

Let's start with basics

Experiment \Rightarrow A procedure that yields one of the given set of possible outcomes.

Sample Space \Rightarrow The set of possible outcomes

Event \Rightarrow A subset of the sample space.

Probability \Rightarrow A number that reflects the chance or likelihood that a particular event will occur

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{No. of total outcomes}}$$

(given, that chance of each outcome is same)

Example 1

A dice is rolled, find the probability that the outcome is a composite number.

Total = $\{1, 2, 3, 4, 5, 6\}$ Favourable = $\{4, 6\}$

$$\text{ans} = \frac{2}{6} = \frac{1}{3}$$

Example 2

A pair of dice are rolled, find the probability that the sum of the outcomes is a prime number.

Total $\Rightarrow [(1,1), (1,2), (1,3), \dots, (1,6), (2,1), (2,2), \dots, (2,6) \dots]$
 $6 \times 6 = 36$

Favourable = [NEXT PAGE]

2, 3, 5, 7, 11

$$2 \Rightarrow (1, 1)$$

$$3 \Rightarrow (1, 2), (2, 1)$$

$$5 \Rightarrow (1, 4), (2, 3), (3, 2), (4, 1)$$

$$7 \Rightarrow (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$$

$$11 \Rightarrow (6, 5), (5, 6)$$

$$\text{ans} = 15/36 = 5/12$$

Mutually Exclusive

$$\hookrightarrow P(A \cap B) = 0$$

Independent

$$\hookrightarrow P(A|B) = P(A)$$

$$\& P(B|A) = P(B)$$

Some Important Properties

$$1) P(A) + P(\text{not } A) = 1$$

$$2) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$3) \text{ if } A \text{ \& } B \text{ are mutually exclusive } \Rightarrow P(A \cup B) = P(A) + P(B)$$

$$4) P(A \cap B) = P(A) * P(B|A) = P(B) * P(A|B)$$

$$5) \text{ If } A \text{ \& } B \text{ are independent, } P(A \cap B) = P(A) * P(B)$$

$$P(R) = 1/10$$

$$P(E.O) = 3/5$$

$$P(E.O|R) = \frac{1}{4}$$

$$P(R \& E.O) = \frac{1}{10} * \frac{1}{4} \Rightarrow \frac{1}{40}$$

$$P(E_0) \times P(R|E_0) = P(R) \times P(E_0|R)$$

$$\Rightarrow P(R|E_0) = \frac{P(R) \times P(E_0|R)}{P(E_0)}$$

$$\Rightarrow \frac{1}{10} \times \frac{1}{4} \times \frac{5}{3} \Rightarrow \frac{1}{24}$$

[Baye's Theorem]

Let's warmup a bit

Picking up balls from boxes

Box 1 \Rightarrow 3 W, 1 B

Box 2 \Rightarrow 2 W, 2 B

Box 3 \Rightarrow 1 W, 3 B

1 ball drawn at random from each box. What is the prob that we get 2 W & 1 B

W W B

$$\frac{3}{4} \times \frac{2}{4} \times \frac{1}{4}$$

W B W

$$\frac{3}{4} \times \frac{1}{4} \times \frac{2}{4}$$

B W W

$$+ \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{26}{64} \Rightarrow \frac{13}{32}$$

we ended up getting 2W & 1B balls, what is the probability that we got B ball from 2nd box.

$$P(B_2 | 2W1B) * P(2W1B) = P(B_2) * P(2W1B | B_2)$$

$$\text{ans} = \frac{1}{2} * \frac{3}{16} * \frac{\overset{1}{32}}{13} \Rightarrow \frac{3}{13}$$

Monty Hall Problem



$\uparrow\uparrow$
 $\frac{1}{3}$



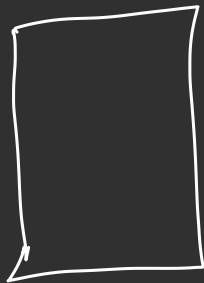
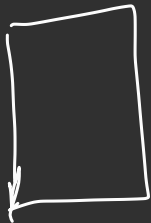
$\underbrace{\hspace{10em}}_{\frac{2}{3}}$

$$p(W) \Rightarrow \frac{1}{3}$$

$$p(L) \Rightarrow \frac{2}{3}$$

Don't switch

\Rightarrow winning $\Rightarrow 1/3$ losing $\Rightarrow 2/3$



Switch



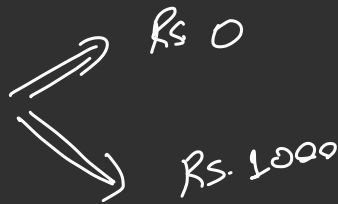
Gamble or not?

It costs Rs. 100 to play a game where you flip a coin 4 times, and if get 4 straight heads, you'll get Rs. 1000 as prize

Should we gamble?

$$P(W) = 1/16$$

$$P(L) = 15/16$$



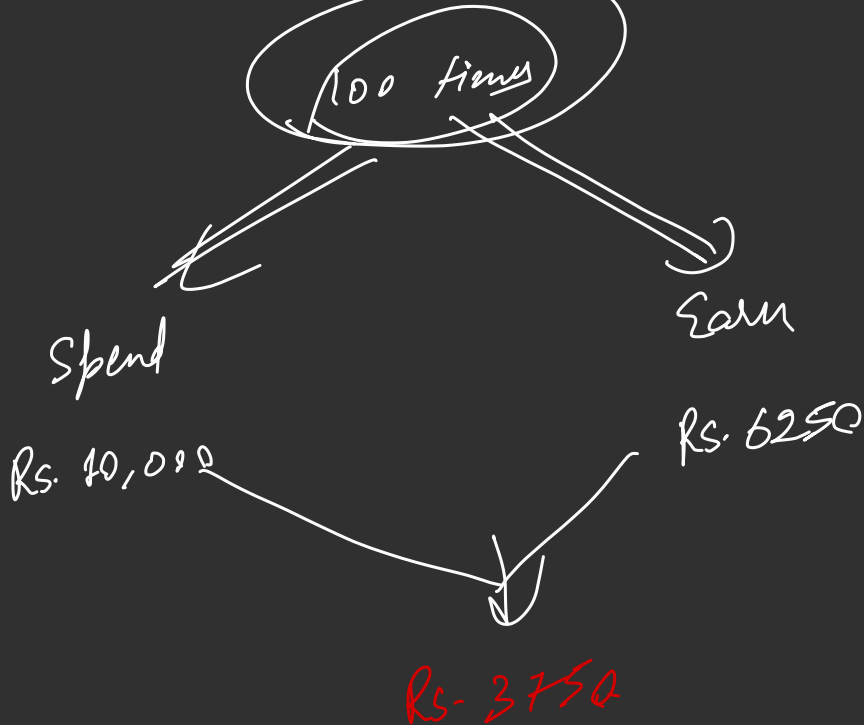
$$E(x) = \sum_{\text{all outcomes}} P(o) * o$$

Rs 37.5

loss



$$\Rightarrow 15/16 * 0 + \frac{1}{16} * 1000 = \text{Rs. } 62.5$$



1st prize was Rs. 2000

$$\text{Exp.} \Rightarrow 0 \times \frac{15}{16} + \frac{1}{16} \times 2000 = \text{Rs. } 125$$

Some real Problem Solving

1. Is it a giveaway?

Intuition

Solution

Let's implement

2. Archer

Intuition

$a \Rightarrow$ Small
 $b \Rightarrow$ Zeros

$$W_a \Rightarrow a/b$$

$$L_a \Rightarrow 1 - W_a$$

$$W_b \approx c/d$$

$$L_b \approx 1 - W_b$$

Solution

(1)

$$W_a$$

(2)

$$L_a * L_b * W_a$$

(3)

$$L_a^2 * L_b^2 * W_a$$

(4)

$$L_a^3 * L_b^3 * W_a$$

$$\Rightarrow W_a * \left[1 + L_a L_b + (L_a L_b)^2 + (L_a L_b)^3 \dots \right]$$

$$\Rightarrow \frac{W_a}{1 - L_a * L_b} \quad (\text{ans})$$

Let's implement

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!