

L31

Quick Sort and more

*If interested, check out the System Design course.
Early Bird discount ends TONIGHT.*

Join Discord - <https://bit.ly/ly-discord>

RECAP

Let's play a game

Nuts & Bolts Problem



$B_1 \quad B_2 \quad B_3 \quad B_4$
 $N_1 \quad N_2 \quad N_3 \quad N_4$

$B_1 \quad B_2 \quad B_3 \quad B_4$
 $N_1 \quad N_3 \quad N_1 \quad N_2$

Basically, K nuts are given & K bolts are given:

1. Each nut has a unique perfect bolt.
2. Our Target is to find the matches for all.

But, there are a few constraints as well:

1. We can't compare 2 nuts directly.
2. Nor can we compare 2 bolts directly.

Then, what can we do? xD
We can compare a nut and a bolt in 1 operation.

$$N_i \leftrightarrow B_j$$

Possible Outcomes :

1) $N_i < B_j$

2) $N_i = B_j$

3) $N_i > B_j$

Brute Force

$$N_1 \Rightarrow B_{m1} (k)$$

$$N_2 \Rightarrow B_{m2} (k-1)$$

:

$$(k-2)$$

:

{

Operations
↓

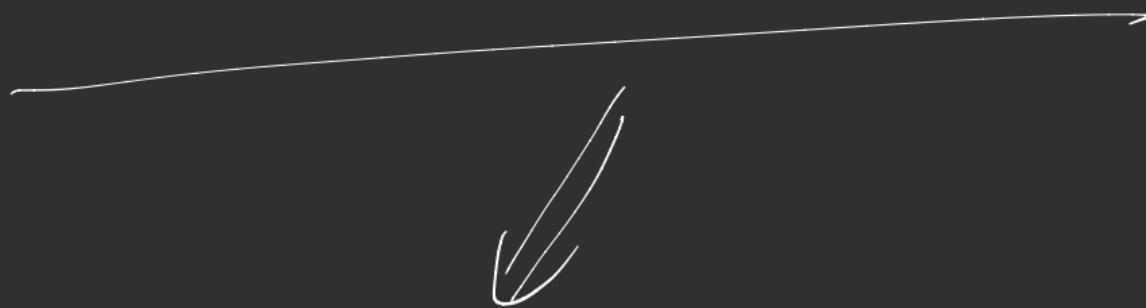
$$\mathcal{O}(k^2)$$

$$\left. \begin{array}{c} Bm_1 \\ | \\ N_1 \end{array} \right\} < N_1$$
$$\left. \begin{array}{c} \\ \\ \end{array} \right\} \geq N_1$$

$$\left. \begin{array}{c} Bm_2 \\ | \\ N_2 \end{array} \right\} < N_1$$
$$\left. \begin{array}{c} BM \\ | \\ N_1 \end{array} \right\} \geq N_1$$

N_3

$$1 + 2 + 3 + 4. \dots - (k-1)$$



$$O(k^2)$$

K = 8

$$\underbrace{B_8 \quad B_1 \quad B_5 \quad B_4}_{N_2 \quad N_5 \quad N_3 \quad N_8}$$

$$B_6 \quad \underbrace{B_2 \quad B_7 \quad B_3}_{N_4 \quad N_7 \quad N}$$

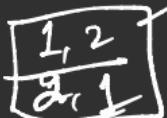
$$N = \boxed{6, 4, 3, 1, 2, 5}$$
$$B = \boxed{2, 1, 4, 5, 3, 6}$$



$$N = \boxed{\begin{matrix} 6 & 3 & 1 \\ 2 & 1 & 4 \end{matrix}, 2, 5}$$
$$B = \boxed{2, 1, 4, 5, 3}$$



$$N = \boxed{\begin{matrix} 3 & 1 & 2 \\ 2 & 1 & 3 \end{matrix}}$$
$$B = \boxed{(3, 3)}$$



$$\begin{array}{c} N_2 \\ \hline B_2 \end{array} \left[\begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array} \right]$$

3 1 ✓



$$\begin{array}{c} N \\ \hline B_2 \end{array} \left[\begin{array}{cc} 2 \\ 2 \end{array} \right] \checkmark$$

Let's come to Quick Sort now

[5, 4, 10, 2, 9, 7, 8, 6]

1, 2, 4, 4, 5, 10, 7, 8

Comparison b/w Merge & Quick Sort

$O(1)$
Divide $\Rightarrow \frac{N}{2} \quad \frac{N}{2}$

Conquer \Rightarrow
Recursion sort(first half)
sort(sec half)

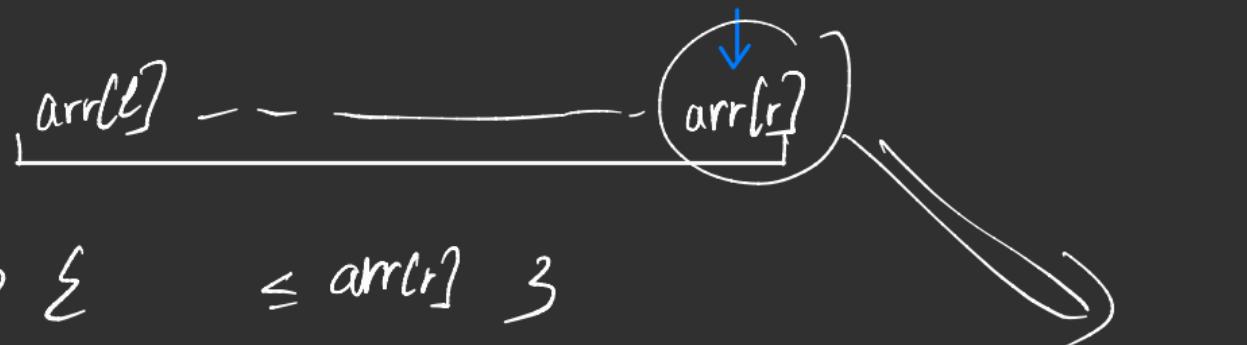
Combine \Rightarrow Main logic
 $O(N)$

$O(N)$
Divide \Rightarrow Main logic

Conquer \Rightarrow sort(left part)
sort(right part)

Combine \Rightarrow Nothing to do

Let's first get the partition function out of the way



Small $\Rightarrow \{ \quad \leq \text{arr}[r] \}$

big $\Rightarrow \{ \quad > \text{arr}[r] \}$

temp $\leftarrow \text{arr}[r]$

$\left[\text{small}[0], \text{small}[1], \dots, \text{temp}, \text{big}[0], \text{big}[1], \dots \right]$

Now, let's understand the time complexity

Best Case

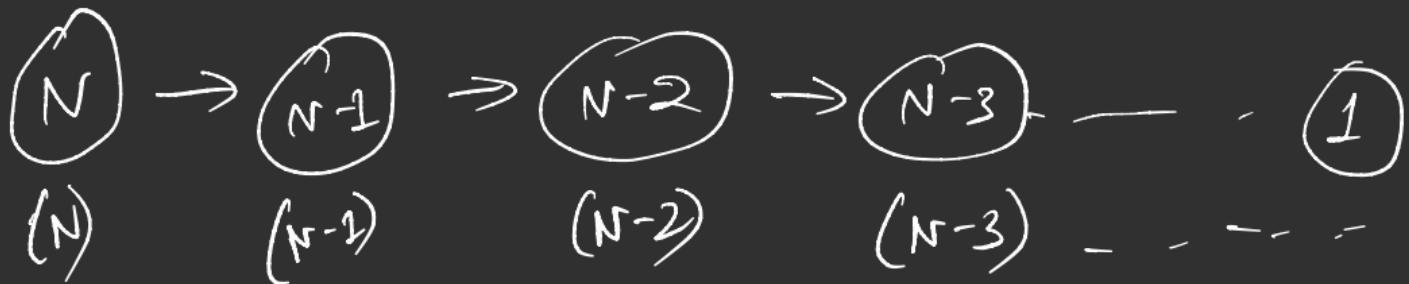


$$T(N) = 2 * T(N/2) + N$$

Time $\Rightarrow O(N \log N)$

Worst Case

$$\Rightarrow T(N) = T(N-1) + N$$
$$T(N-1) = T(N-2) + (N-1)$$

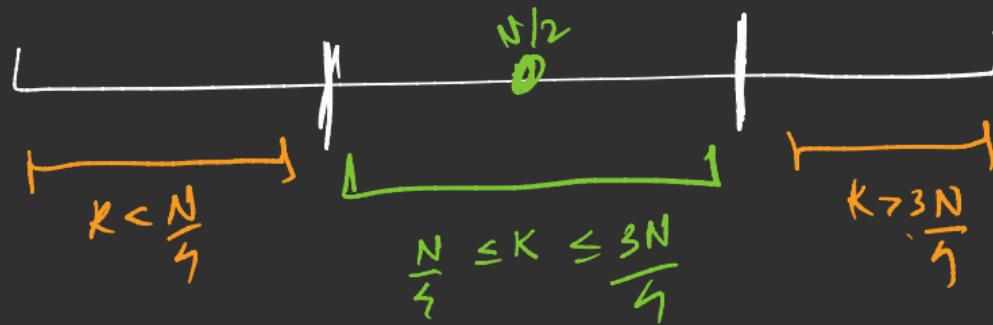


Time = $O(N^2)$

$$T(N) = T(K) + T(N-K) + N$$



Average Case



0.5 \Rightarrow good Scenario \Rightarrow

0.5 \Rightarrow Bad Scenario

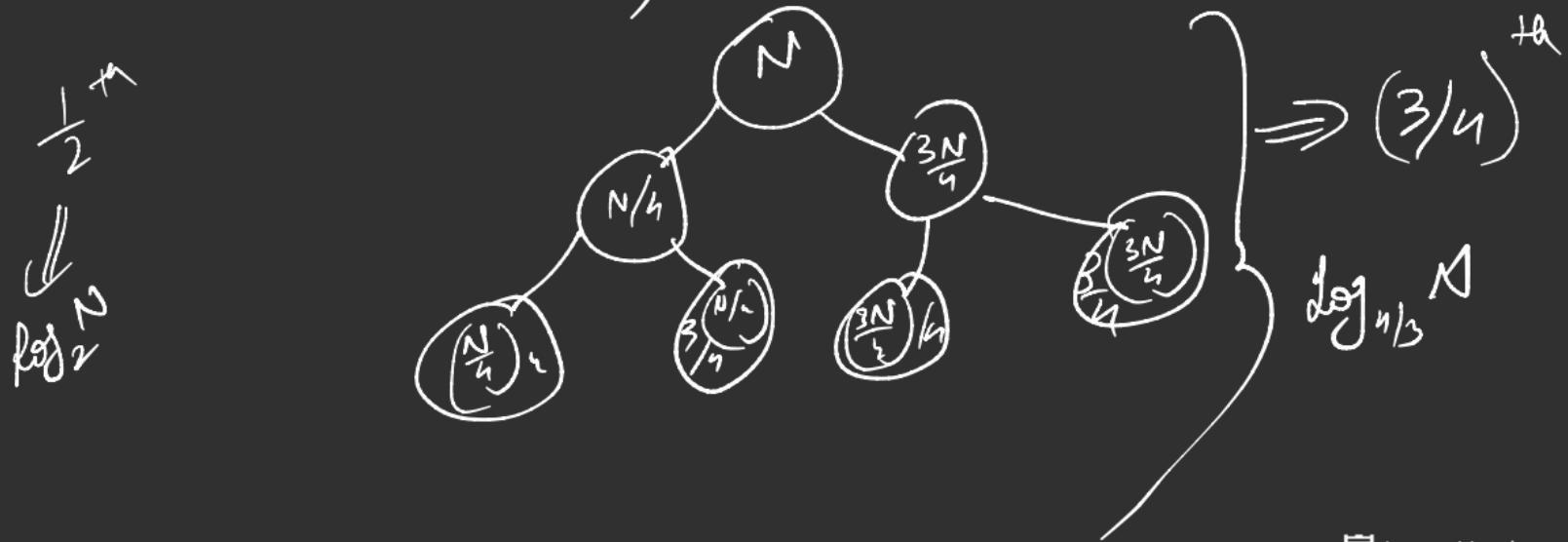
Proving the Average Case Time Complexity

good Scenario

$$T(N) = T\left(\frac{N}{4}\right) + T\left(\frac{3N}{4}\right) + N$$

$$K \geq N/\frac{1}{4}$$

$$K \leq \frac{3N}{4}$$



$$N \times \log_{1/3} N \xrightarrow{\quad} 2 \cdot 6 N \log_3 N$$

$$\Theta(N \log^2 N)$$

(Avg. Case)

for 2 steps , $N \xrightarrow{2 \text{ steps}} \frac{3N}{4}$

2.6 ~~$N \log N$~~

\Downarrow Time
 $S \cdot 2 N \log_2 N \Rightarrow \downarrow$
 $O(N \log_2 N)$

Pseudo-code Comparison b/w MS & QS

ms(arr, l, r) {

 left = arr(l, mid);

 right = arr(mid+1, r);

 ms(left)

 ms(right),

merge(left, right);

qs(arr, l, r) {

 id = partition(arr, l, r);

 qs(l, id-1);

 qs(id+1, r);

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!