

Let's begin at 9:02 PM

L73

DP with Probability

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RECAP

Some real Problem Solving

## 1. The Last Ball

There are  $W$  white balls &  $B$ <sup>black</sup> balls in a box. We repeat a procedure until there is only 1 ball left in the box.

find the probability that the last 1 ball is the WHITE ball.

In each turn, we'll pick 2 balls randomly:

a) If both of them have same colour, we'll throw those 2 balls away & put a black ball back in the box.

b) If the colour of balls is different, we'll throw those 2 balls away & put a white ball back in the box.

Last  $\Rightarrow$  1W 1B

$P_W \Rightarrow$  Probability to pick white  $\Rightarrow \frac{W}{W+B}$

$P_B \Rightarrow$  Probability to pick black  $\Rightarrow \frac{B}{W+B}$

$P_{WB}$

$(W, B)$

$$P_B = \frac{C(B, 2)}{C(W+B, 2)}$$



--- or ---

$$P_W = \frac{C(W, 2)}{C(W+B, 2)}$$



--- or ---

$$P_{WB} = 1 - P_W - P_B$$



$(W, B-1)$



$(W-2, B+1)$



$(W, B-1)$

$$\text{Prob}(W, B) = P(BB \& f_w) + P(WW \& f_w) + P(WB \& f_w)$$

$$\text{Prob}(W, B) = P(BB) * P(f_w | BB) + P(WW) * P(f_w | WW) + P(WB) * P(f_w | WB)$$

$$\text{Prob}(W, B) = P_B * \text{Prob}(W, B-1) + P_w * \text{Prob}(W-2, B+1) + (1 - P_B - P_w) * \text{Prob}(W, B-1)$$



$$W < 0 \parallel B < 0 \Rightarrow 0.0$$

$$W \geq 1, B \geq 0 \Rightarrow 1.0$$

$$W \geq 0, B \geq 1 \Rightarrow 0.0$$

DP imp. fact

$$f(i) = \frac{f(i-1) + f(i+1)}{2}$$



Intuition

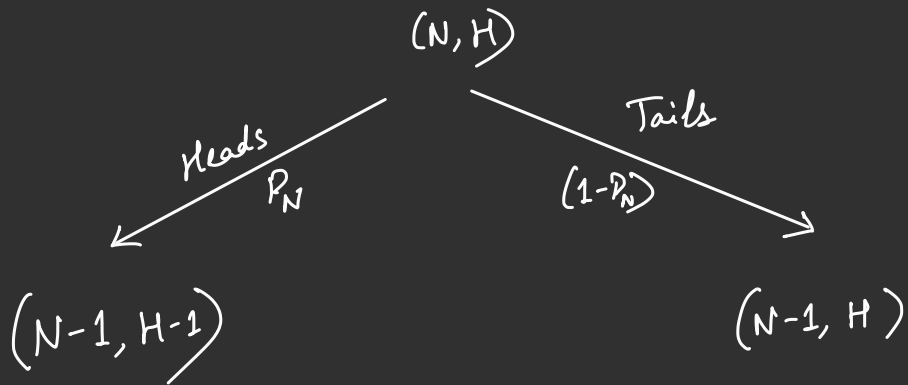
Solution

Let's implement

## 2. Coins

## Intuition

$\text{prob}(N, h) \Rightarrow$  What is the probability of getting exactly  $h$  heads out of  $N$  throws?





$$\text{Prob}(H \text{ out of } N) = P(\text{Head}) \times P(H-1 \text{ out of } N-1) + P(\text{Tail}) \times P(H \text{ out of } N)$$

## Solution

$$\text{Prob}(N, H) = P_N * \text{Prob}(N-1, H-1) + (1-P_N) * \text{Prob}(N-1, H)$$

Base Cases  $\Rightarrow N=0 \& H>0 \Rightarrow 0$

$N \geq 0 \& H=0 \Rightarrow 1$

Let's implement

# *Thank You!*

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!