

Let's begin at 9:05 PM

L98

Prefix Function and KMP Algorithm

Join Discord - <https://bit.ly/ly-discord>

RECAP

Problem is the same as the previous class

Given two strings `needle` and `haystack`, return the index of the first occurrence of `needle` in `haystack`, or `-1` if `needle` is not part of `haystack`.

Of course, there is a brute force way.

Of course, there is an efficient Rabin
Karp way also.

But how to be a 100% sure that
answer will be correct?

Prefix Function

Prefix fun. (π) of a string is array of length N where $\pi[i]$ represents the length of the longest proper prefix of $s[0 \dots i]$ which is also its suffix.

eg. $abcabcd \Rightarrow [0, 0, 0, 1, 2, 3, 0]$

"a" $\rightarrow 0$

"abcabc" $\rightarrow 3$

"ab" $\rightarrow 0$

"abcabcd" $\rightarrow 0$

"abc" $\rightarrow 0$

"abca" $\rightarrow 1$

"abcab" $\rightarrow 2$

lps(n, 0)

Brute Force

```
for(i = 0; i < n; ++i) {  
    str = s[0 --- i];  
    for(l = 1; l <= i; ++l)  
        if (str[0 --- l-1] == str[i-l+1 --- i])  
            lps[i] = l;  
}  
return lps;
```

$O(N^3)$ time

An observation

$lps[i]$ can't be greater than $lps[i-1] + 1$

$s_0 s_1 s_2 s_3 \dots s_{i-3} s_{i-2} s_{i-1} s_i$

4

$$lps[i] \leq lps[i-1] + 1$$

Optimisation 1

$O(N^2)$

time

```
for (i = 1; i < n; ++i) {
```

```
    int pos = lps[i-1] + 1;
```

```
    string str = s[0...i]
```

```
    while (pos > 0 && s[0...pos-1] != s[i-pos+1...i])  
        pos--;
```

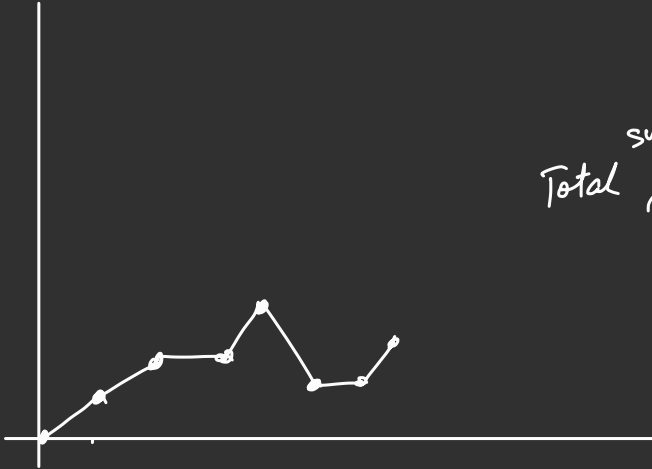
```
    lps[i] = pos;
```

```
}
```

Total ^{sum of} _n increments in
lbs value $\leq N$

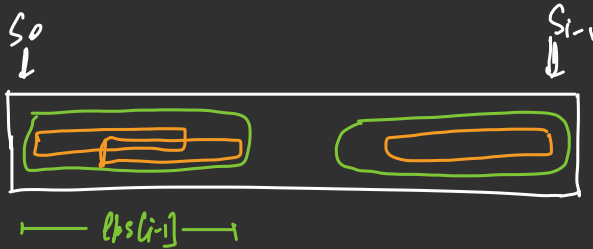


Total ^{sum of} _n decrements in
lbs value $\leq N$



Let's understand the final optimisation

$$\boxed{s_0 s_1 s_2 s_3 \dots s_{i-2} s_{i-1} s_i} \quad lps[i-1] + 1$$



ababab efg h a g b ababab | a

ⁱ⁻¹
d

j. lps[i-1] // 6

j. lps[j-1] \Rightarrow ans[i] = 5

Let's implement this

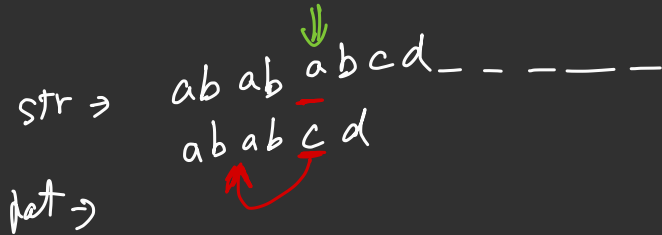
2 Practice Problems (Let's try)

1. Pattern Find

Intuition / Solution

$\Pi \Rightarrow$ pattern

str \Rightarrow ab ab a b c d _ _ _ _
pat \Rightarrow ab ab c d

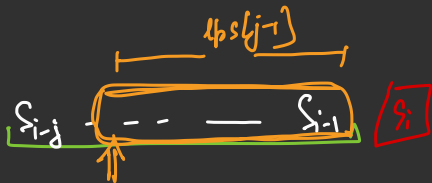


$i = 4$
 $j = 4$

$\underbrace{s_0 \dots s_{i-1}}_{s_i}$

$\underbrace{p_0 \dots p_{j-1}}_{p_j}$

$i \Rightarrow$



$j \Rightarrow$



$i = 0, j = 0$

while ($i < l_1$) {

if ($s[i] == s[j]$)

$i++, j++$

else if ($j == 0$)

$i++$

else { // $j == 0$

$j = lps[j-1]$

}

Let's implement

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!