Bonus Lecture - 1
Digit DP

Introduction

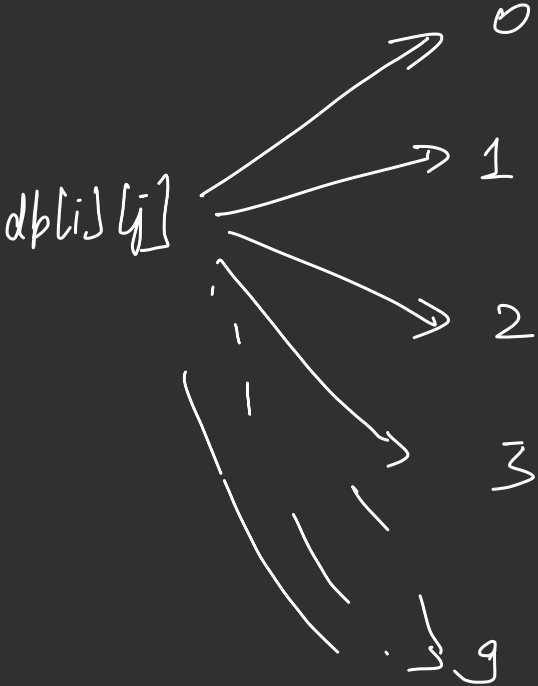
Given a length N, find the number of arrays of size N such that all the array elements are b/w 0 and 9, and the sum of array elements is divisible by K.

dp[N][K]

dp[i][j]

No. of ways to fill arr till index i

sum[0---i] % K = j


dp[i][j]
0
1
2
3
9

i, j

```
dp[i][j] = 0;
for(d = 0; d <= 9; ++d)
  dp[i][j] += dp[i-1][(j-d+k)%k];
```

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Let’s say N = 5, don’t you think that the solution will be equal to number of whole numbers less than or equal to 99999 having digit sum divisible by K?

0-9 0-9 0-9 0-9 0-9 $\Rightarrow$ sum of digits has to be a multiple of K.

[0, 0, 5, 1, 2] → 5K

[1, 0, 9, 1, 1] $\Rightarrow$ 10,911

What if we are asked to find number of whole numbers less than or equal to NUM, having digit sum divisible by K?

NUM $\Rightarrow$ 50239

numArr $\Rightarrow$ [5, 0, 2, 3, 9]

No. of arrays of size 5 where $a[i] \in [0, 9]$ & sum of array elements div. by K, & are lexicographically $\leq$ numArr.

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Let’s do some problems!

1. Digit Sum

Intuition / Solution

num = 5 0 2 3 9

n = numDigits(num)

K ⇒ divisor

dp[N][K][2]

- N ⇒ index until which digits have been filled
- K ⇒ digSum % K value
- 2 ⇒ The prefix so far == numPrefix? or not

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$dp[i][j][0] \Rightarrow$ The no. of prefices of num $(0 - - - i)$ that they are $<$ corresp pref & digSum % K $== j$

$dp[i][j][1] \Rightarrow$ The no. of prefices of num $(0 - - - i)$ that they are $==$ corresp pref & digSum % K $== j$

$\Downarrow$

0 or 1

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(0 - - - - n-1) (digSum % k == 0) (< && = = both are fine)

$$ans = (dp[n-1][0][0] + dp[n-1][0][1]) \% mod;$$

return (ans - 1 + mod) % mod;

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$\underbrace{0 \text{ - - - - - } i-1}$, (i, j) $\Rightarrow$ equal

j^{th} digit of num $\Rightarrow d_i$

$dp[i][j][1] \rightarrow dp[i-1][(j - d_i + 2) \% k][1]$

$\underline{0 - - - - - - i-1}$ $(i,j) \rightarrow d_i \Rightarrow 5 \rightarrow$ less

$$dp[i][j][0] \xrightarrow{d} d < d_i$$

$$\Downarrow$$

$$dp[i][j][0] \mathrel{+}= dp[i-1][(j-d+k)\%k][0]$$

$$dp[i][j][0] \mathrel{+}= dp[i-1][(j-d+k)\%k][1]$$

$$dp[i][j][0] \xrightarrow{d} d \geq d_i$$

$$\Rightarrow dp[i][j][0] \mathrel{+}= dp[i-1][(j-d+k)\%k][0]$$

Let’s implement

2. Stepping Number

getRank(n) $\Rightarrow$ Number of whole numbers $\leq n$ that are stepping numbers

getRank(ans) $-$ getRank(n) $\Rightarrow$ K

rankDiff $\Rightarrow$ K

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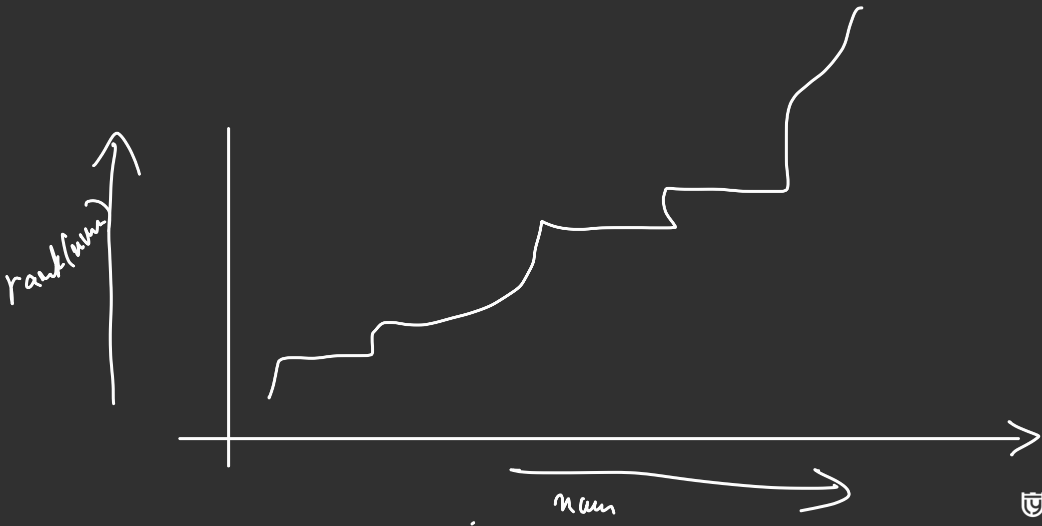
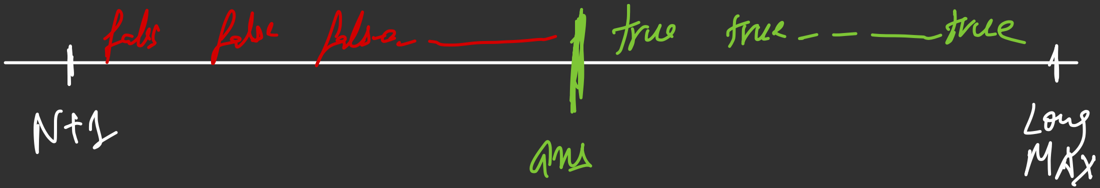
Intuition / Solution

$N = 7, \quad K = 6$



find the smallest number

s.t. getRank(ans) - getRank(n) $\geq K$



$\Rightarrow n = \text{numDigits}(num)$

$\leq num$

every consec. digits

$abs(d_i - d_{i+1}) \leq 1$

$dp[n][10][2]$

equal.

$$dp[i][d][1] \Rightarrow d == d_i$$

$$\&\& \ (abs(d_i - d_{l_i}) \le 1)$$

$$dp[i][d][1] = dp[i-1][d_{l_i}][1]$$

$dp[i][d][0]$

less

d

$dl = d-1$ $\quad$ $dl = d$ $\quad$ $dl = d+1$

dl

$(i-1)$ $\boxed{d}$

dl $\quad$ d

$i-1$ $\quad$ i

$abs(diff) \le 1$

$dp[i][d][0]$

i

$dp[i-1][dl][0/1]$

$i-1$

$d < d_i \implies i^{th}$ smaller

$d \geq d_i \implies$ should've already been smaller $(i-1)$

```
dp[i][d][0] += dp[i-1][dl][0];
if (d < di)
    dp[i][d][0] += dp[i-1][dl][1];
```

$\underbrace{5432235}$

$\underbrace{0054456} \Rightarrow 54456$

$\cancel{54056}$

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num = 543218 $\Rightarrow$ n = 6

$(i = 0) \Rightarrow$ 1 to 9

Find all the st—

digits ≤ 5

Let’s implement

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!