Let's start at 9:05 PM

L71
DP with Counting

Join Discord - https://bit.ly/ly-discord

RECAP

1. Lost Array

N = 2, L = 1, R = 3

[1, 2]

[2, 1] ⇒ 3

[3, 3]

N = 3, L = 2, R = 2

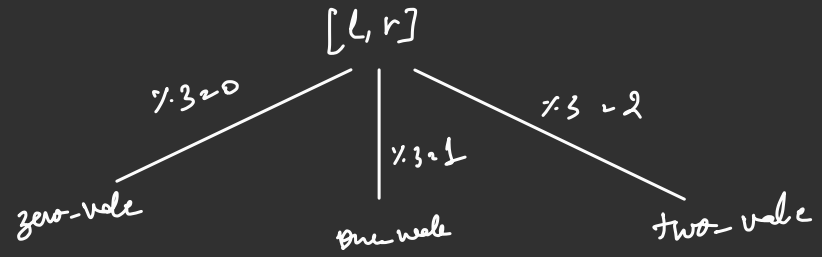
[2, 2, 2]

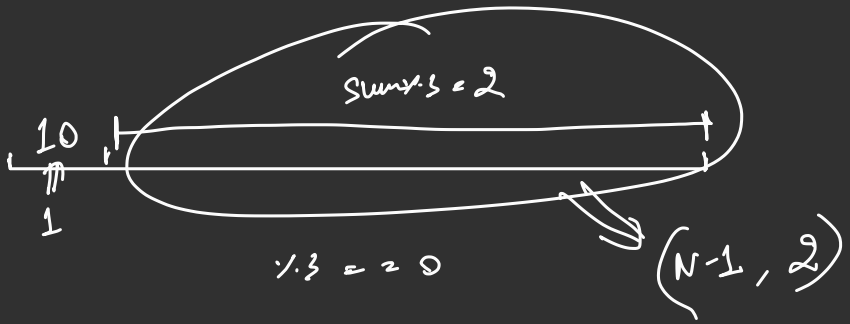
1

Intuition

$$(a_0 + a_1 + a_2 \cdots \cdots a_{n-1}) \% 3 = 0$$

$$((a_0 \% 3) + (a_1 \% 3) + (a_2 \% 3) \cdots (a_{n-1} \% 3)) \% 3 = 0$$





0/1/2

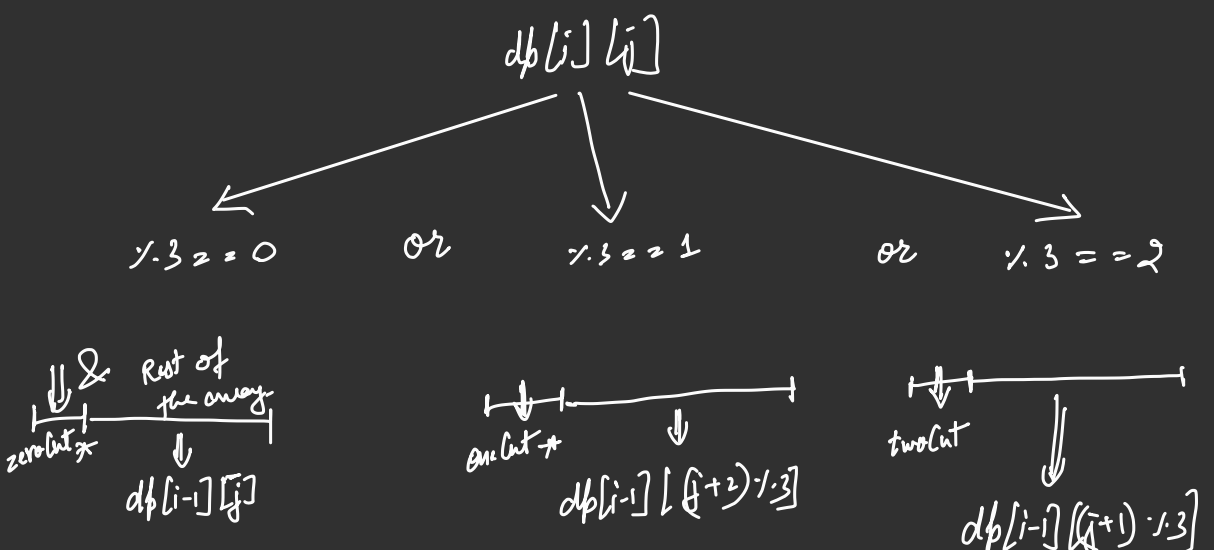
$f(i, j)$ ⇒ Number of arrays of size i s.t. when the sum of array elements is divided by 3, the remainder should be j.

dp[i][j]

$dp[N+1][3]$

↳ $dp[i][j]$

Number of arrays of size i s.t. when the sum of array elements is divided by 3, the remainder should be j.



$$dp[i][j] = zeroCut * dp[i-1][j] + oneCut * dp[i-1][(j+2)\%3] + twoCut * dp[i-1][(j+1)\%3]$$

$$zeroCut = R/3 - (L-1)/3$$

$$oneCut = \frac{(R-1)}{3} - (L-2)/3$$

$$twoCut = (R-L+1) - zeroCut - oneCut$$

$$dp[i][j] = zeroCut * dp[i-1][j] + oneCut * dp[i-1][(j+2)\%3] + twoCut * dp[i-1][(j+1)\%3]$$

$$Ans = dp[N][0]$$

Solution

$$zeroCut = R/3 - (L-1)/3$$

$$oneCut = (R-1)/3 - (L-2)/3$$

$$twoCut = (R-L+1) - zeroCut - oneCut$$

Let’s implement

2. Two Arrays

$N = 2$, $M = 2$

$A = [1, 1] \Rightarrow B = [1,1]\ [2,1]\ [1,2]\ [2,2]$

$[1, 2] \Rightarrow B = [2,2]$

$[2, 2] \Rightarrow B = [2,2]$

ANS = 5

Intuition

$$a_0 \le a_1 \le a_2 \text{ - - - } \le a_{m-1}$$

$$b_0 \ge b_1 \ge b_2 \text{ - - - } \ge b_{m-1}$$

1.) a is non-dec. $\Rightarrow$ (inc $\Rightarrow$ A)

2.) b is non-inc $\Rightarrow$ (dec $\Rightarrow$ B)

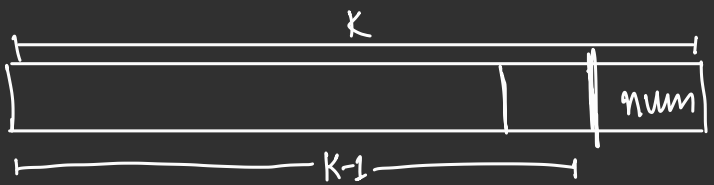
3.) $b[m-1] \ge a[m-1]$

$1 \leq N \leq 1000$

$1 \leq M \leq 10$

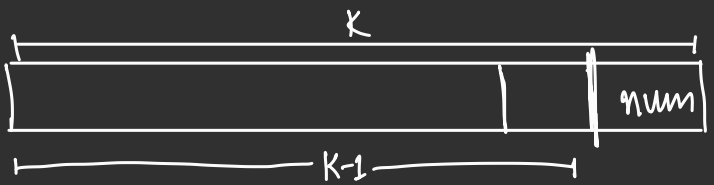
$dp[len][n_1][n_2] \Rightarrow$ no. of pairs s.t. length of
& last element in a is n_1
& last element in b is n_2

$inc[K][num]$ $\Rightarrow$ No. of non-decreasing sequences of length **K** s.t. the last element is equal to **num**



$$inc[K][num] = \sum_{l=1}^{num} inc[K-1][l]$$

$dec[K][num] \Rightarrow$ No. of non-increasing sequences of length **K** s.t. the last element is equal to **num**



$$dec[K][num] = \sum_{l=num}^{N} dec[K-1][l]$$

Solution

```
ans = 0;
for (a = 1; a <= n; ++a)
    for (b = a; b <= n; ++b) {
        ans += inc[m][a] * dec[m][b];
        ans %= mod;
    }
print(ans);
```

Let’s implement
(HW)

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!