Let's start at 9:02 PM

L70
Combinatorics - 2

Join Discord - https://bit.ly/ly-discord

RECAP

Let’s do some actual problems today

1. Non Overlapping Subarrays

A-) Let's say you're given an array of size N.
How many non-empty subarrays will that array have?

$[1, 2, 3, 4]$

$[1], [2], [3], [4] \Rightarrow n$

$[1, 2], [2, 3], [3, 4] \Rightarrow n-1$

$[1, 2, 3], [2, 3, 4] \Rightarrow n-2$

$\vdots$

$$\Rightarrow n + (n-1) + (n-2) \cdot \text{———} 1 \Rightarrow \frac{n*(n+1)}{2}$$

Part 2) How many pairs of subarrays are there?

$N = 3 \Rightarrow [1, 2, 3]$

$[1]$ $[2]$ $[3]$ $[1, 2]$ $[2, 3]$ $[1, 2, 3]$

$$ans = {}^{\frac{n(n+1)}{2}}C_2$$

LearnYard

Part 3) How many pairs of non-overlapping subarrays are there? (Assume all the numbers are unique)

Eg. $N = 3 \Rightarrow [1, 2, 3]$

$[1]$ $[2]$ $[3]$ $[1, 2]$ $[2, 3]$ $[1, 2, 3]$

$\{[1] [2, 3]\}$ $\{[1], [2]\}$ $\{[1], [3]\}$ $\{[2] [3]\}$ $\Rightarrow 5$

$\{[1, 2] [3]\}$

LearnYard

. _ _ _ _ _ _ i $(i+1)$ _ _ _ _ _ _ $n-1$

$\Uparrow$

$(i+1)$

$O(1) \Rightarrow$

Case 1 : $len(s1) >= 2$ & $len(s2) >= 2$

$^{N}C_4$

Case 2 $\Rightarrow$ $len(s1) = 1$ & $len(s2) >= 2$

$^{N}C_3$

Case 3 $\Rightarrow$ $len(s1) >= 2$ & $len(s2) = 1$

LearnYard

Case 4 : Both have length = 1

$${}^{N}C_2$$

$$ans = {}^{N}C_2 + 2 * {}^{N}C_3 + {}^{N}C_4$$

Intuition

Solution

Let’s implement

2. K - Banned

[HW]

3. Count the Arrays

Your task is to calculate the number of arrays such that:

- each array contains n elements;
- each element is an integer from 1 to m;
- for each array, there is **exactly** one pair of equal elements;
- for each array a, there exists an index i such that the array is **strictly ascending** before the i-th element and **strictly descending** after it (formally, it means that $a_j < a_{j+1}$, if $j < i$, and $a_j > a_{j+1}$, if $j \geq i$).

$N = 3, \quad M = 4 \quad [1, 2, 3, 4]$

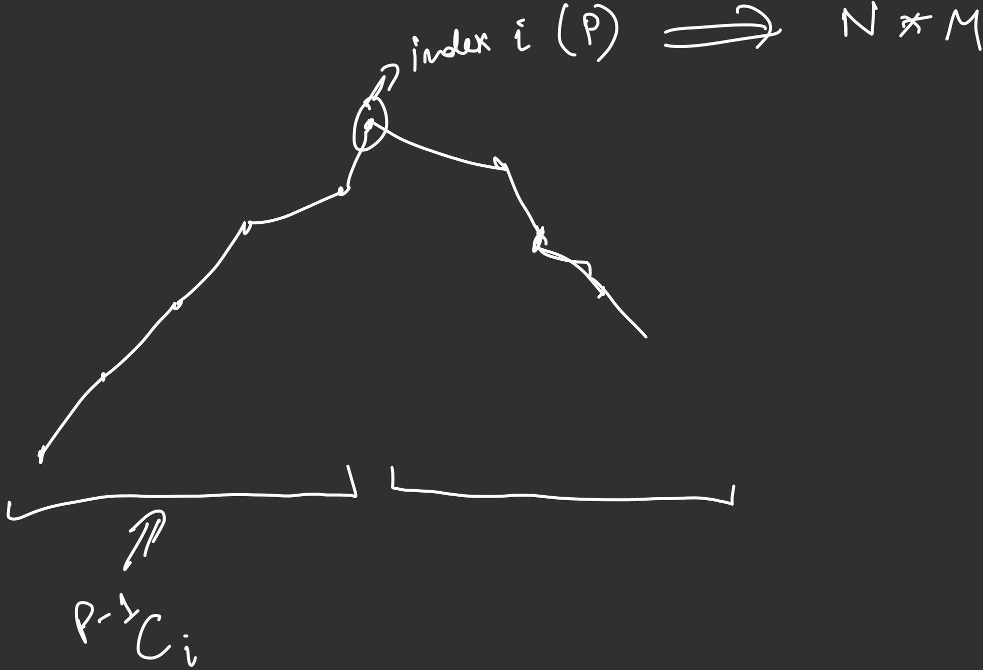
arr = [1, 2, 1] [2, 3, 2]

[1, 3, 1] [2, 4, 2]

[1, 4, 1] [3, 4, 3]

$\Rightarrow$ $\boxed{\text{6 Arrays}}$

LearnYard


index i (P) ⟹ N * M
$^{P-1}C_i$

1 to M $\Longrightarrow$ M-1

$N-2 \Longrightarrow (M-1)^{N-2}$

LearnYard

Intuition

[1 to M]

Step 1. What are the N valid set of numbers that will be present in the array?

Step 2. What will be their order?

Set ⇒ [1, 2, 3, 3, 4]

[1, 3, 4, 3, 2]

[2, 3, 4, 3, 1]

[1, 2, 3, 4, 3]

[3, 4, 3, 2, 1]

Set = [2, 2, 3, 5, 6]

[2, 3, 5, 6, 2]

[2, 3, 6, 5, 2]

[2, 5, 6, 3, 2]

[2, 6, 5, 3, 2]

LearnYard

$$^{M}C_{N-1} * (N-2)$$

$N=5$

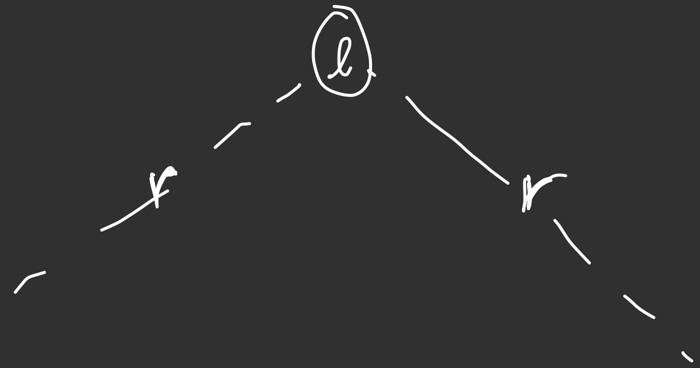
[1, 2, 3, 4]

[1, 1, 2, 3, 4]

[1, 2, 2, 3, 4]

[1, 2, 3, 3, 4]

Step 2:



$\Rightarrow$ N-3 are left. $\Longrightarrow$ 2^{N-3}

Solution

$$ans = \underbrace{{}^{M}C_{N-1}}_{\text{modulo inverse}} * (N-2) * \underbrace{2^{N-3}}_{\text{modulo exponentiation}}$$

LearnYard

Let’s implement

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!