Bonus Lecture - 6
Fenwick Tree

RECAP

Introduction

Given an Array. $(1 \le N \le 10^5,\ 1 \le a[i] \le 10^9)$

Process queries of 2 types: $(1 \le q \le 10^5)$

$\Rightarrow$ 1 l r $\Rightarrow$ print $a[l] + a[l+1]$ --- $a[r]$

$\Rightarrow$ 2 i X $\Rightarrow$ Increment the value of $a[i]$ by x. (i.e. $a[i] += X$).

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Intuition behind Fenwick Tree

$$arr \Rightarrow \begin{array}{ccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \end{array}$$
$$arr \Rightarrow [\,x,\ 5,\ 1,\ 6,\ 12,\ 8,\ 15,\ 10,\ 11,\ 2,\ 2\,]$$

$$0 \le g(i) < i$$

$$bit[i] \Leftarrow sum(arr[g(i)+1] \text{ ---- } arr[i])$$

Ex 1. $g(i) = 0$

$$bit[\,] = \{X, 5, 6, 12, 24, 32, 47, 57, 68, 70, 72\}$$

Ex 2 $g(i) = i-1$

$$bit[\,] = \{x, 5, 1, 6, 12, 8, 15, 10, 11, 2, 2\}$$

Query:→

```
ans = 0;
while(i > 0){
    ans += bit[i];
    i = g(i);
}
```

$g(i) = i-1 \Rightarrow O(N)$

$g(i) = 0 \Rightarrow O(1)$

Update:→ i x $(a[i] += x)$

for all j s.t. $g(j) < i \le j$:

$bit[j] \mathrel{+}= x$

$g(i) = 0 \Rightarrow O(N)$

$g(i) = i-1 \Rightarrow O(1)$

Magic is the way how $g(i)$ is chosen.

More detailed Explanation

$g(i) \Rightarrow p(i)$

$\longrightarrow par(i)$

For an i, if b is the rightmost set bit of i:

$\Rightarrow$ position of rsb

then $p(i) = i - 2^b$

$i = 44\ (101100) \Rightarrow p(i) = 40\ (101000)$

$i = 5\ (101) \Rightarrow p(i) = 4\ (100)$

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Range of responsibility

(acc. to rightmost set bit) $\Rightarrow$ rsb

$1 \Rightarrow [i, i] \Rightarrow p(i) = i-1$

$2 \Rightarrow [i-1, i] \Rightarrow p(i) \Rightarrow i-2$

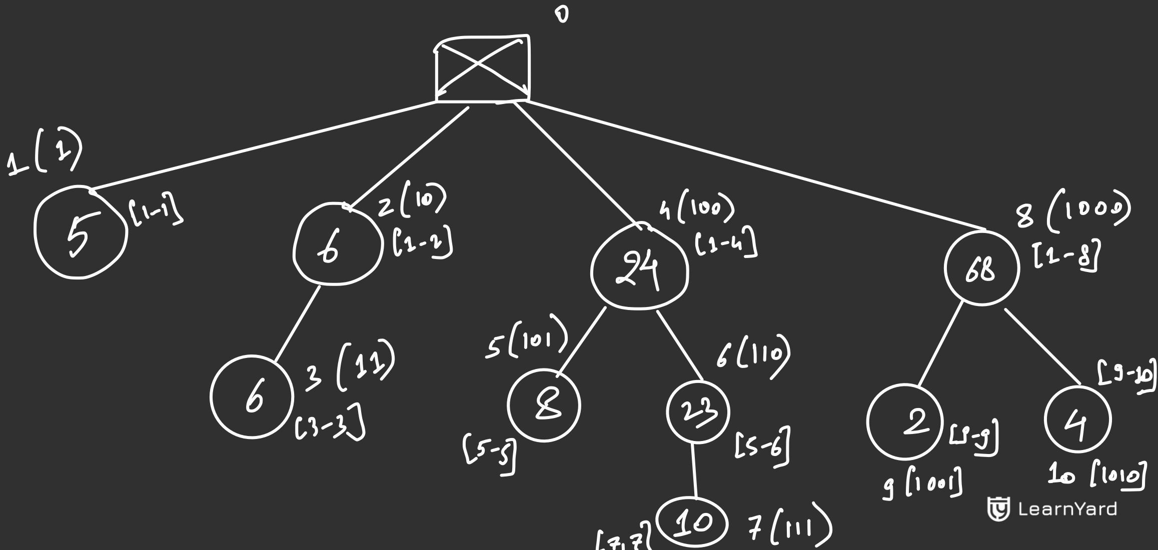
$4 \Rightarrow [i-3, i] \Rightarrow p(i) \Rightarrow i-4$

$8 \Rightarrow [i-7, i] \Rightarrow p(i) = i-8$

$16 \Rightarrow [i-15, i] \Rightarrow p(i) = i-16$

$$\begin{array}{lccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ arr \Rightarrow & [x, & 5, & 1, & 6, & 12, & 8, & 15, & 10, & 11, & 2, & 2] \end{array}$$

$$bit \Rightarrow [x, 5, 6, 6, 24, 8, 23, 10, 68, 2, 4]$$



$i = 0000000000101100$

$-i = (\text{com of } i)100$

$i \& (-i) \Rightarrow (100)_2$

$i \longrightarrow f(i)$

just do $i -= i \& (-i)$

$-i \Rightarrow$ 2's complement of i

$\Rightarrow$ 1's com of i

$+ 1$

$$
\begin{array}{r}
1111111111010011 \\
1 \\
\hline
1111\ldots 0100
\end{array}
$$

Querying

$$
\begin{array}{r}
101100 \\
-\ 100 \\
\hline
101000
\end{array}
\Rightarrow p(i)
$$

i

$- i \& (-i)$

```
prefSum(int i){
    int ans = 0;
    while(i > 0){
        ans += bit[i];
        i -= i&(-i);
}   }
```

$\Longrightarrow O(\log_2 N)$

Updation $\Rightarrow$ i x $\Rightarrow$ $a[i] \mathrel{+}= x$

Go to all j s.t. $p(j) < i \leq j$

$\Downarrow$

and do $bit[j] \mathrel{+}= x$

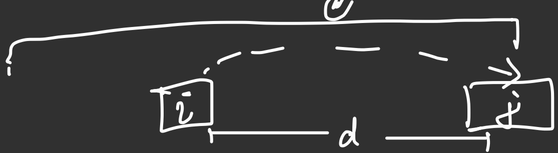
$$i \longrightarrow j_1 \longrightarrow j_2 \longrightarrow \leq N$$

$i = (101100)_2$

$$\underline{p(j)} < i \leq j$$

$$\Rightarrow j - 2^{rsb(j)} < i \leq j$$

$ror(j) > d$



$$\Rightarrow i + d - 2^{rsb(i+d)} < i \leq i + d$$

$$\Rightarrow i - 2^{rsb(i+d)} < i - d \leq i$$

$$\Rightarrow 2^{rsb(j)} > d$$

$i = 10011000$

$\Big\Downarrow$ + smallest d

$res \Rightarrow 2^{rsb(res)} > d$

$j_1 \Rightarrow i + 2^{rsb(i)}$

$j_2 \Rightarrow j_1 + 2^{rsb(j_1)}$

$j_3 \Rightarrow j_2 + 2^{rsb(j_2)}$

$$
\begin{array}{r}
i \Rightarrow 10011000 \\
1000 \\
\hline
10100000
\end{array}
$$

$j_1 = i + 2^{rsb(i)}$

$rsb(j_1) > rsb(i)$

$$
\begin{array}{r}
j_1 \Rightarrow 10100000 \\
100000 \\
\hline
j_1 \Rightarrow 11000000 \\
\hline
\end{array}
$$

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$$\left[j_1 + 2^{rsb(j_1)} \right.$$

$$rsb(j_2) \geq rsb(j_1) + 1$$

$$2^{rsb(j_2)} \geq 2^{rsb(j_1)+1}$$

$$\left(\because ror(j_1) \geq ror(i) \right.$$

$$ror(j_2) \geq 2 * ror(j_1)$$

$$\Rightarrow$$

$$\Rightarrow ror(j_2) > \underset{\Uparrow\, d}{ror(j_1)} + \underset{\Downarrow\, 2^{rsb(i)}}{ror(i)}$$

```
update(i, x) {

    while(i <= n) {
        bit[i] += x;
        i += i & (-i);
    }
}
```

$O(\log_2 N)$

Preprocess

Initialize $\rightarrow$ bit $= \{0 \ldots 0\}$

array

$i \Rightarrow 0$

$x \Rightarrow a[i]$

$a[i]$

Time

$\Downarrow$

$O(N \log N)$

Let’s do some problems!

1. Range Sum Queries

Let’s implement

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!