Let's start at 9:02 PM

L49

Introduction to Sieve of Eratosthenes

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# RECAP

# Number Theory Starts

# Some Basic Terminologies

(I) Integers ⇒ . . . . . . . . −10, −9, −8, . . . −1, 0, 1, 2, . . . 5 . . .

(W) Whole numbers ⇒ 0, 1, 2, 3, . . .

(N) Natural numbers ⇒ 1, 2, 3, . . .

Composite

Prime.

Given a number N, find $\sum_{d \mid N} d^2 \Rightarrow$ 24

$\Downarrow$

1, 2, 3, 4, 6, 8, 12, 24

$| \Rightarrow$ divides

d divides N $\Rightarrow$ d is a factor of N ($n \% d == 0$)

N is a multiple of d.

# Fundamental Theorem of Arithmetic

$N = 24$

$\rightarrow 2^3 * 3^1$

$N = 47$

$\rightarrow 47$

2 divides 24
2 divides 12
2 divides 6
3 divides 3
so 2:3 and 3:1
only required.
not rest numbers
like 4,6,8 etc
composite
numbers
required to divide
24

Every +ve integer can be uniquely represented as a product of primes.

this is because, eg. x = 24
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24
now, if we divide numbers starting
from 2 to 24, only prime numbers will
come, because rest composite
numbers will be cancelled while

Let’s do a puzzle:

Given a number, can we find a formula to find its number of divisors?

$$N = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \cdots\cdots p_k^{\alpha_k}$$

$$N = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \cdots\cdots p_k^{\alpha_k}$$

$$d(N) = (\alpha_1+1) * (\alpha_2+1) * (\alpha_3+1) \cdots\cdots * (\alpha_k+1)$$

$$600 = 2^3 * 3^1 * 5^2$$

$$\boxed{2^0 / 2^1 / 2^2 / 2^3} \quad \boxed{3^0 / 3^1} \quad \boxed{5^0 / 5^1 / 5^2}$$

$$4 * 2 * 3 = 24$$

# Finding if a number is prime

```
if (n == 1) return false;
for (i = 2; i < n; ++i) {
    if (n % i == 0)
        return false;
}
return true;
```

Time ⇒ $O(N)$

```
if (N == 1) return false;
for (i = 2; i * i <= N; ++i)
    if (n % i == 0)
        return false;

return true;
```

Time ⇒ $O(\sqrt{N})$

$(a<N)$ $(b<N)$

$$N = a * b$$

$>\sqrt{N}$ $>\sqrt{N}$

$\Downarrow$

How can their product be equal to N?

$$N = (\sqrt{N}) * (\sqrt{N})$$

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# Doing the prime factorization of a number

$600 \Rightarrow \{2:3,\ 3:1,\ 5:2\}$

$i = 2,\ 3,\ \cancel{4},\ 5,\ 6 \ldots$

$i = 5,\ c = \cancel{0}$

2

$\cancel{1}$

$N = \cancel{600}$

$\cancel{300}$

$\cancel{150}$

$\cancel{75}$

$\cancel{25}$

$\cancel{5}$

1

N = 11

2, 3, 4

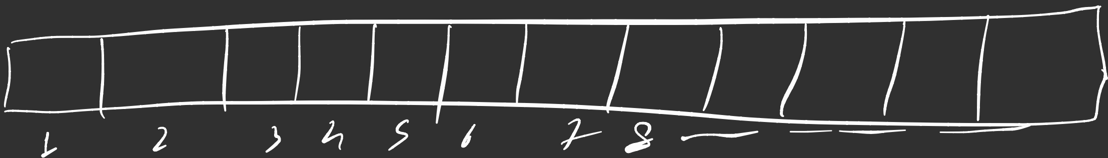
```
map<int, int>  pfact (int n) {

    mii  ans;

    for (i=2; i*i <= n; ++i) {
        if (n%i)
            continue;

        int c=0;
        while (n%i==0)
            n/=i, c++;

        ans.put(i, c);
    }

  if (n>1) ans.put(n, 1);
  return ans;

}
```

Time = $O(\sqrt{N})$

# Sieve of Eratosthenes : Introduction

N = 100

isPrime.



Let’s visualise

# Intuition

# Pseudo-Code

# What's the time taken?

$$N + \frac{N}{2} + \frac{N}{3} + \frac{N}{4} - - - - - \frac{N}{N}$$

$$\Rightarrow N * \underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - - - \frac{1}{N}\right)}_{\log_2 N}$$

$$\frac{1}{1} + \underbrace{\frac{1}{2} + \frac{1}{3}}_{2\text{ terms}} + \underbrace{\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}}_{4\text{ terms}} + \underbrace{\frac{1}{8} + \frac{1}{9} + \frac{1}{10} \cdots \frac{1}{15}}_{8\text{ terms}}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\le 1 \qquad \le 1 \qquad \le 1$$

$$1 + 1 + 1 - \cdots 1 \Rightarrow (\le \log N)$$

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Some optimisations?

Let’s implement

# Improved Time Complexity

$$O(N \log \log N)$$

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Let’s do a problem! (Hai himmat?)

# Counting Primes

# *Thank You!*

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!