CORRECT MODEL INADEQUESIES

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ASSIGNMENT - 4

Problem 5.14

Linear Regression Model:-

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$Var(\varepsilon_i) = \sigma^2 x_i^2$$

$\boxed{a}$ $y' = y/x$ $\quad x' = 1/x$

$\Rightarrow$ Linear Regression $\Rightarrow$ $y_i' = \beta_0 x_i' + \beta_1 + \frac{\varepsilon_i}{x_i}$

$\Rightarrow y_i' = \beta_0 x_i' + \beta_1 + \varepsilon_i x_i'$ $\Rightarrow$ $\boxed{Var(y') = \frac{1}{x^2} Var(y_i) = \sigma^2}$

$\Rightarrow$ Thus, $\boxed{\text{YES}}$ it is a valid transformation

$\boxed{b}$ From above equation.

$y_i' = \beta_0' + \beta_1' x_i' + \varepsilon_i'$ where

$\beta_0' = \beta_1$ & $\beta_1' = \beta_0$ $\Rightarrow$ Their roles are reversed.

$\boxed{c}$ Suppose we use weights $w_i = \frac{1}{x_i^2}$, the least square ~~it will be~~ approximation is same except the parameters are reversed as per $\boxed{B}$.

Problem 15.15

Regression Model $\Rightarrow$ $y = \beta x + \varepsilon$.

$\boxed{a}$ $S(\beta) = \sum_{i=1}^{n} w_i (y_i - \beta x_i)^2$

$$\frac{dS(\beta)}{d\beta} = \sum_{i=1}^{n} 2w_i (y_i - \beta x_i)(-x_i) = 0$$

$$\Rightarrow \boxed{\hat{\beta} = \frac{\sum_{i=1}^{n} w_i x_i y_i}{\sum_{i=1}^{n} w_i x_i^2}}$$

b $Var(\hat{\beta}) = \left(\frac{1}{\sum_{i=1}^{n} w_i x_i^2}\right)^2 \sum_{i=1}^{n} w_i^2 x_i^2 Var(y_i)$

$= \left(\frac{1}{\sum_{i=1}^{n} w_i x_i^2}\right)^2 \sum_{i=1}^{n} w_i^2 x_i^2 \sigma^2 / w_i$

$\Rightarrow Var(\hat{\beta}) = \boxed{\frac{\sigma^2}{\sum_{i=1}^{n} w_i x_i^2}}$

c $Var(y_i) = c x_i \quad \Rightarrow w_i = 1/x_i$

$\Rightarrow \hat{\beta} = \frac{\sum_{i=1}^{n} w_i y_i x_i}{\sum_{i=1}^{n} w_i x_i^2} = \frac{\sum_{i=1}^{n} 1/x_i \, y_i x_i}{\sum_{i=1}^{n} 1/x_i \, x_i^2} = \boxed{\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}}$

$\boxed{Var(\hat{\beta}) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i}}$

d $Var(y_i) = c x_i^2 \Rightarrow w_i = 1/x_i^2$

$\Rightarrow \hat{\beta} = \frac{\sum_{i=1}^{n} \frac{1}{x_i^2} y_i x_i}{\sum_{j=1}^{n} \frac{1}{x_i^2} \times x_i^2} = \boxed{\frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{x_i}}$

$\boxed{Var(\hat{\beta}) = \frac{\sigma^2}{n}}$

5. Problem 5.16

Model:- $Y = B_1 X_1 + B_2 X_2 + \varepsilon$, $E[\varepsilon] = 0$ $Var(\varepsilon) = V\sigma^2$

$H_0 = B_2 = 0$, $H_1 = B_1 \neq 0$

Let $\beta = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$ $K' = \begin{bmatrix} 0 & I \end{bmatrix}$, $m = 0$

Let rank $(K') = P_2$

Note, $\boxed{K'\hat{\beta} = \hat{B}_2}$

$F_0 \quad \frac{(K'\hat{\beta} - m)' (K'(X'X)^{-1}K)^{-1} (K'\beta - m)}{P_2 \, MSE}$

For H_0, F_0 has Central distribution. In F
For H_{01}, F_0 will have non-central distribution in F.

Problem 5.17

Model:- $y = X\beta + \varepsilon$ $\quad E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma^2 V$, where
$V \to$ known, $\sigma^2 \to$ Unknown

The equation in previous question in non-quadratic form is as follows:-

$$y' \underbrace{\left[V^{-1} - V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}\right]}_{\text{Let this matrix be depicted as } A} y$$

From appendix C,

$$E(y'Ay) = \text{trace}\left[(A)(\sigma^2 V)\right] + \mu' A \mu,$$

where $\mu = E[y] = 0$

This implies rank $[AV]$ = trace $[AV] = n-p$.

Thus,

$$E[y'Ay] = (n-p)\sigma^2$$

Thus, $\left(y'V^{-1}y - y'V^{-1}X\left[X'V^{-1}X\right]^{-1}X'V^{-1}y\right)/n-p$

is an unbiased estimator of σ^2.