

### PROBLEM 4.21

To find:-

$$E[MSPE] = ?$$

$$E[MS_{LOF}] = ?$$

Working:-

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$$E[MS_{PE}] = \frac{1}{n-m} E\left[\sum_{i=1}^m \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2\right] \text{ where } \quad (1)$$

$$E[MSPE] = \frac{1}{n-m} E \left[ \sum_{i=1}^m \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2 \right]$$

$$E \left[ \sum_{i=1}^m \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2 \right] = E \left[ \sum_{i=1}^m \sum_{j=1}^n (y_{ij}^2 - 2y_{ij}\bar{y}_i + \bar{y}_i^2) \right]$$

$$= \sum_{j=1}^m \sum_{i=1}^n \left\{ E(y_{ij}^2) - 2E\left(y_{ij} \frac{z_i}{\sum_{j=1}^m z_i}, \frac{y_{ij}}{n_i}\right) + E(y_{ij}^2) \right\}$$

$$= \sum_{i=1}^3 \sum_{j=1}^n \left\{ -2 - 2E \left[ y_{ij} \frac{y_{ij}}{n_i} \right] + \frac{2}{n_i} \right\}$$

$$= n\sigma^2 + m\sigma^2 - 2 \sum_{i=1}^m \left( \sum_{j=1}^n \sum_{j'=1}^{n_i} \frac{y_{ij} y_{ij'}}{n_i} \right)$$

$$- n\sigma^2 + m\sigma^2 - \sum_{i=1}^m \frac{n_i\sigma^2}{n_i} = n\sigma^2 + m\sigma^2 - 2m\sigma^2$$

$$= -(n-m)s^2 \quad \text{--- (2)}$$

Sub ② in ① :-

Sub (2) in (1) :-  
 $E[MSPE] = \frac{1}{n-m} \times (n-m)\sigma^2 = \sigma^2 \rightarrow E[MSPE] = \sigma^2$   
 $SS_{\text{LOF}} = SS_{\text{res}} - SS_{\text{PE}} \rightarrow \frac{m}{n} (E(Y_i) - \bar{Y})^2$

② in ① :-  $\frac{1}{n-m} \times (n-m)s^2 = s^2$

$MSPE = \frac{1}{n-m} \times (n-m)s^2 = s^2$

$SS_{Res} = SS_{LOF} + SS_{PE}$

$SS_{LOF} = SS_{Res} - SS_{PE}$

$MSPE = (n-2)s^2 + \sum_{i=1}^m (y_i - \hat{y}_i)^2$

$$\begin{aligned} E[MSPE] &= \frac{1}{n-m} \times \dots \\ SS_{Res} &= SS_{LOF} + SS_{PE} \Rightarrow SS_{LOF} = SS_{Res} - SS_{PE} \\ E[SS_{LOF}] &= E[SS_{Res}] - E[SS_{PE}] = (n-2)s^2 + \sum_{i=1}^m (E(y_i) - \beta_0 - \beta_1 x_i)^2 - (n-m)s^2 \end{aligned}$$

$$= (m-2)\sigma^2 + \sum_{i=1}^m [\mathbb{E}[y_i] - \beta_0 - \beta_1 x_i]^2$$

$$\Rightarrow E[SS_{LOF}] = E[SS_{KFS}] - (n-m)\sigma^2$$

$$= (m-2)\sigma^2 + \sum_{i=1}^m [E(y_i) - \beta_0 - \beta_1 x_i]^2$$

$$\Rightarrow E[MS_{LOF}] = \frac{E[SS_{LOF}]}{m-2} = \sigma^2 + \frac{\sum_{i=1}^m [E(y_i) - \beta_0 - \beta_1 x_i]^2}{m-2}$$