APS ASSIGNMENT

1. $5_{C_2} * 6_{C_2} * 4_{C_3} = 600$ ways.

We have to choose 2 assistant proffessor out of 5 so we take 6_{C_2} AND 5_{C_2} AND 2 out of 6 associate proffessors so it is
3 proffessors out of 4 so it is 4_{C_3}

2a. $\frac{9!}{3!\,4!\,2!} = 1260$ ways

All the computers must be serviced so we take 9!. As there are matching i,e indistinguishable items so we have divide 9! by 3! for 3 PC's, 4! for MACs and 2! for linux machines so that it can be considered distinguishable.

2b. $5 * \left(\frac{4!}{3!}\right) + 5 * \left(\frac{4!}{2! \times 2!}\right) = 50$ ways

The first 5 serviced computers must include all MAC's so therefore we take it as 5 the remaining 4 positions can be taken by 3PC's or 2 Linux machines.

In first case i,e 1 Linux machine is placed in the first 5 then we take $\frac{4!}{3!}$ for repeation we divide by 3!

In the second case where 1 PC is placed in the first 5 then we take $\frac{4!}{2! \times 2!}$ for repeatations we divide by 2! for PC's and 2! for Linux machines

2c) $^3C_2 * \frac{6!}{4! \times 2!} * {}^3C_1 = 135$ ways

We have to service 2 PC's in the first 3 positions so we take $^3C_2$ AND the last PC must be serviced at the last 3 positions so we take it as $^3C_1$ AND finally the remaining 6 positions can be taken in 6! ways since the machines are identical we divide by 4! and 2!.

3a) Initial payments need to be done to 4 companies so we take $1+2+3+4 = 10$ Lakshs.

$n = 20-10 = 10$ lakhs

$r = 4$

$$\binom{n+r-1}{r-1} = \binom{13}{3} = {}^{13}C_3 = 286$$

We have 10 lakh remaining which can be divided to any of the 4 companies, it is with replacement and order does not matter so we use the above formula and get $^{13}C_3$.

3b)

C_1	C_2	C_3	C_4		Remaining	$n=14$ $r=3$
1	2	3	0	$= 6$	14	$\binom{14+3-1}{3-1} = {}^{16}C_2$
0	2	2	4	$= 9$	11	$\binom{11+3-1}{2} = {}^{13}C_2$
1	2	0	4	$= 7$	13	$\binom{15}{2} = {}^{15}C_2$
1	0	3	4	$= 8$	12	$\binom{14}{C_2} = {}^{14}C_2$

Scanned with CamScanner

Since we have to give investment to atleast 3 companies and we calculate the initial payment for the companies and subtract it from the ~~[illegible]~~ total amount. So for each case we get

$^{16}C_2 * {}^{13}C_2 * {}^{15}C_2 * {}^{14}C_2$

So the final answer is $^{13}C_3 + {}^{16}C_2 + {}^{15}C_2 + {}^{14}C_2 = \underline{680}$

4a) $^{2}C_1 * \left[({}^{5}C_4 * {}^{4}C_3 * {}^{3}C_3) + ({}^{5}C_3 * {}^{4}C_3 * {}^{3}C_3) + ({}^{5}C_4 * {}^{4}C_2 * {}^{3}C_3) \right]$

$= 2(20+40+30) = \underline{180}$

First we chose 1 goalie from 2 so $^{2}C_1$

Then we assume 1 goalie for midfield so we get

$^{5}C_4 * {}^{4}C_3 * {}^{3}C_3 = 20$

In case 2 we take 1 goalie for defense so we get

$^{5}C_3 * {}^{4}C_3 * {}^{3}C_3 = \underline{40}$

In case 3 we get 1 goalie for forward so we get

$^{5}C_4 + {}^{4}C_2 + {}^{3}C_3 = \underline{30}$

5a) no. of servers $= n$

total requests $= r$

mi is the minimum request for ith server

$m = m_1 + m_2 + \dots m_n$

remaining request $= r - m$

Sample → with replacement, order does not matter

$$\binom{(r-m)+n-1}{n-1} = {}^{(r-m+n-1)}C_{(n-1)}$$

6. No. of steps is n

No. of points is K

No. of up points denoted by U_p

No. of downpoints denoted by D_n

Total steps: no. of Ups + no. of Downs

$$n = U_p + D_n \quad \text{—} \ (1)$$

Total points: no. of Ups - no. of downs

$$K = U_p - D_n \quad \text{—} \ (2)$$

Adding eqn (1) and (2)

$$2U_p = n + k$$

$$\boxed{U_p = \frac{n+k}{2}}$$

To get k points U must be $\frac{n+k}{2}$

no. of ways $\frac{n+k}{2}$ can be chosen from n is $\binom{n}{\frac{n+k}{2}}$