APS-ASSIGNMENT 2

1 a) $1-\left(\frac{5}{6}\right)^4$ → 5/6 is the probability of not getting a six in a roll of die
since we rolle 4 times we take $(5/6)^4$ and to calculate the probability of rolling the die and getting a 6 we take the complement $1-(5/6)^4$

$=\frac{671}{1296}$

$=0.518$

b) $1-\left(\frac{35}{36}\right)^{24}$ → 35/36 is the probability of not getting atleast one 12 in a roll of pair of dice.

→ To find the probability of the single 12 appearing we take the complement $1-\left(35/36\right)^{24}$

$=0.491$

Atleast one 6 in 4 rolls of a die is more likely.

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2)a) Taking one from the pile

$1\times\frac{10}{18}=0.55$

Taking 2 from the pile

$\frac{10}{18}\times\frac{8}{17}+\frac{8}{18}\times\frac{10}{17}=0.52$

Taking 1 from the 2 and 1 from the pile is better as the probability of serious health issue not being there is more.

2b) $\frac{10\times10}{{}^{20}C_2}=0.53$ → probability of taking from the pile when both are mixed.

2c)
$$\frac{\overset{\nearrow A}{{}^{100}C_{10}} \times \overset{\nearrow B}{{}^{100}C_{10}} + \overset{\nearrow A}{{}^{100}C_{9}} \times \overset{\nearrow B}{{}^{100}C_{11}} + \overset{\nearrow A}{{}^{100}C_{11}} \times \overset{\nearrow B}{{}^{100}C_{9}}}{{}^{200}C_{20}}$$

$$= \underline{4.213 \times 10^{22}}$$

choose 10 A and 10B tablets or choose 9A and 11B tablets or choose 11A and 9B tablets.

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3) U = Event that a person is a drug user

P - Event that the test is positive

$P(U) = 0.05$ $\quad P(P|U) = P(P^c|U^c) = \underline{\underline{0.95}}$

To find $P(U|P)$?

By Bayes theorem

$$P(U|P) = \frac{\overset{\nearrow 0.95}{P(P|U)} \cdot \overset{\nearrow 0.05}{P(U)}}{P(P)}$$

$$\swarrow$$

$$\underset{\substack{\downarrow \\ 0.95}}{P(P|U)} \cdot \underset{\substack{\downarrow \\ 0.05}}{P(U)} + P(P|U^c) \cdot \underset{\substack{\downarrow \\ 0.95 \\ 1-P(U)}}{P(U^c)}$$

$1-(P^c|U^c)$

$1-0.95$

$= \underline{\underline{0.05}}$

$$= \underline{\underline{0.5}}$$

4a

		Income			
		<25k	$25k-$50k	>$50k	
	<25	952	1050	53	2055
Age	25-45	456	2055	1570	4081
	>45	54	952	1008	2014
		1462	4057	2631	

4 a) $\frac{2055}{8150}$ → less than 25 years old

b) $\frac{6095}{8150}$ → more than 25 years old

c) $\frac{5519}{8150}$ → earn less than 70k.

d) $\frac{53}{2055}$ → less than 25 years earns more than \$70k

e) $\frac{456}{1462}$ → Less than \$25 ~~years~~ and b/w 25-45 years

f) $\frac{1006}{2014}$ → more than 45 years and earns less than \$70k.

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5 a) $0.4 + 0.2 = \underline{\underline{0.6}}$ → probability that she forecasts rain. → P(A)

b) Probability of making a mistake

$0.15 + 0.2 = \underline{\underline{0.35}}$

c) F → event of forecast rain

R → It actually rains

$P(R|F) = ?$

given :- $P(F \cap R) = 0.4$ $P(F \cap R^c) = 0.2$

$P(F^c \cap R) = 0.15$ $P(F^c \cap R^c) = 025$

$$P(R|F) = \frac{P(R \cap F)}{P(F)} = \frac{0.4}{0.6} = \underline{\underline{0.67}}$$

d) $$P(F|R) = \frac{\overset{\nearrow 0.67}{P(R|F)} \cdot \overset{\nearrow 0.6}{P(F)}}{P(R)}$$

$\swarrow$

$P(F \cap R) + P(F^c \cap R) = 0.4 + 0.15 = 0.55$

$$= \frac{0.67 \times 0.6}{0.5} = \underline{\underline{0.731}}$$

c) E :- Event of atleast one string hashed to each bucket.
F_i :- event of atleast one string hashed to ith bucket.

$P(E) = F(F_1 \cap F_2 \cap F_3 \cap F_4) = 1 - P(E^c)$

$= 1 - P[(F_1 \cap F_2 \cap F_3 \cap F_4)^c] = 1 - P(F_1^c \cup F_2^c \cup F_3^c \cup F_4^c)$

$= 1 - [P(F_1^c) + P(F_2^c) + P(F_3^c) + P(F_4^c) -$
$P(F_1^c \cap F_2^c) - P(F_1^c \cap F_3^c) - P(F_1^c \cap F_4^c) -$
$P(F_1^c \cap F_3^c - P(F_2^c \cap F_4^c) - P(F_3^c \cap F_4^c) +$
$P(F_1^c \cap F_2^c \cap F_3^c) + P(F_1^c \cap F_2^c \cap F_4^c) + P(F_2^c + F_3^c \cap F_4^c) +$
$P(F_1^c \cap F_3^c \cap F_4^c) - P(F_1^c \cap F_2^c \cap F_3^c \cap F_4^c)]$

$P(F_1^c) = (1-P_i)^6$ - compliment of none of the 6 strings being hashed.

$P(F_1^c \cap F_2^c)$ - probability of none of the strings hashed to 1 & 2
$= [1-(P_1+P_2)]^6$ and so on for each term

$P(E) = 1 - [(1-P_1)^6 + (1-P_2)^6 + (1-P_3)^6 + (1-P_4)^6 - (1-P_1-P_2)^6 -$
$(1-P_1-P_3)^6 - (1-P_1-P_4)^6 - (1-P_2-P_3)^6 - (1-P_2-P_4)^6$
$- (1-P_3-P_4)^6 + (1-P_1-P_2-P_3)^6 + (1-P_1-P_2-P_4)^6$
$+ (1-P_2-P_3-P_4)^6 + (1-P_1-P_3-P_4)^6 - (1-P_1-P_2-P_3-P_4)^6]$

substituing P_1, P_2, P_3, P_4 we get $P(E) = \underline{\underline{0.217}}$.