Assignment solⁿ :-

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Solⁿ (1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

By applying, $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -4 & 5 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \leftrightarrow R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & +3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ Ans

Rank of the Matrix is 3 Ans

Solⁿ (2) Max degree of polynomial $T = 2$

$\begin{bmatrix} x_1 & x_3 \\ x_3 & x_2 \end{bmatrix}$ Dimⁿ $(P_2) \to 3$

nullity = dim of null space
↳ subspace of N which gives 0 as output from P_2
for zero polynomial

$a-b=0 \to a=b$
$b-c=0 \to b=c$
$c-a=0 \to c=a$

$\therefore$ matrix can be written by 1 variable

$$k\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$\therefore$ Nullity = dimension of nullspace $\Rightarrow$ 1 Ans

Sol^n 3

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$$A = \begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$$

$$A - \lambda I = 0$$

$$\det\begin{bmatrix}2-\lambda & -1\\-1 & 2-\lambda\end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda) = \pm 1$$

$$\lambda = 1, 3$$

for $\lambda = 1$

$$\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$x - y = 0$$

$$x = y \qquad \text{let } x = t \; ; \; y = t$$

Eigen vector $v_1 = t\begin{bmatrix}1\\1\end{bmatrix}$

$\Rightarrow$ for $\lambda = 3$

$$\begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$-x - y = 0$$

$$x = -y$$

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Qsⁿ ④ Qⁿ:

let $x = t$

So, eigen value $V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

$y = -t$

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→ Now, A^{-1}

eigen values of A^{-1} will be $\frac{1}{\lambda_1}$ & $\frac{1}{\lambda_2}$ = $1, \frac{1}{3}$

⇒ Eigen vectors are same as of A

$V_1 = t\begin{bmatrix}1\\1\end{bmatrix}$ $V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Now, for $A + 4I$

eigen values for $A+4I$ will be $\lambda_1 + 4, \lambda_2 + 4 = 5, 7$

& eigen vectors are same as of A

$V_1 = t\begin{bmatrix}1\\1\end{bmatrix}$ $V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Solⁿ ④

$$eq^n = x^{k+1} = \frac{7.85 + 0.1y^k + 0.2z^k}{3}$$

$$y^{k+1} = \frac{-19.3 + 0.1x^{k+1} - 0.3z^k}{7}$$

$$\& \ z^{k+1} = \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

We know $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Iteration ① →

$$x(1) = \frac{7.85 + 0.1(0) + 0.2(0)}{3} = 2.6167$$

$$y(1) = \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{7} = 2.7945$$

$z(1) = \frac{71.4 - 0.3(2.6167) - 0.2(2.7945)}{10} = 70.056$

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Iteration 2 ⇒

$x \rightarrow \quad 3x - 0.1(2.7945) - 0.2(7.0056) = 7.85$

$3x = 8.97167$

$x = 2.990$

$y \rightarrow \quad 0.1(2.99) + 7y - 0.3(7.0056) = -19.3$

$7y = -17.49732$

$y = 2.49196$

$z \rightarrow \quad 0.3(2.99) - 0.2(2.4996) + 10z = 71.4$

$10z = 70.00308$

$z = 7.0003$

Iteration 3 =

Now, $3x - 0.1(2.4996) - 0.2(7.0003) = -19.3$

$3x = 9.0001$

$x = 3$ (approx)

Now, $0.1(3) + 7y - 0.3(7.0003) = -19.3$

$7y = -17.4999$

$y = -2.5$ (approx)

$0.3(3) - 0.2(-2.5) + 10z = 71.4$

$10z = 70$

$z = 7$

Hence $x = 3$, $y = -2.5$, $z = 7$

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Sol^n 5

$$x + 3y + 2z = 0 \quad -(1)$$
$$2x - y + 3z = 0 \quad -(2)$$
$$x + 17y + 4z = 0 \quad -(3)$$
$$3x - 5y + 4z = 0 \quad (4)$$

$\Rightarrow$ Consistent sys of linear eq^n $\Longrightarrow$ cond^n $\rho(A) = \rho(A:B)$

$\hookrightarrow AX = B$

Inconsistent = having no sol^n

$\hookrightarrow \rho(A) \neq \rho(A:B)$ for $\rightarrow AX = B$

$\Rightarrow$ Now given eq^{n}s :-

$$A \quad X = B$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 1 & 17 & 4 \\ 3 & -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It's a homogeneous eq^n so always consistent $\Rightarrow \rho(A:B)$

echlon form of A :-

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_1]{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

$\rho(A) \neq 3$ (Dim^n)

$\therefore \infty$ many possible

Solⁿ (6) $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

(1) By additative

$T(u+v) = T(u) + T(v)$

$u = a_1 + b_1x + c_1x^2$

$v = a_2 + b_2x + c_2x^2$

$T(u+v) = T\{(a_1+a_2)+(b_1+b_2)x + (c_1+c_2)x^2\}$

$= (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2$

$= (a_1+1) + (b_1+1) + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$\rightarrow T(u) + T(v)$

Hence, Proved

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② Homogenity

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$T(ku) = kT(u)$

$= T(k(a+bx+cx^2))$
$= T(ka+kbx+kcx^2)$

$\Rightarrow (ka+kb+kc+1)+(ka+kb+kc+1)x+(ka+kb+kc+1)x^2$

$= k(a+1)+k(b+1)x+k(c+1)x^2$
$= kT(u)$

Hence, Proved.

Sol^n ⑦

$S = \{\overset{v_1}{(1,2,3)}, \overset{v_2}{(3,1,0)}, \overset{v_3}{(-2,1,3)}\}$

Now, $c_1v_1 + c_2v_2 + c_3v_3 = 0 \rightarrow$ then S spans $V_3(\mathbb{R})$

$c_1(1,2,3) + c_2(3,1,0) + c_3(-2,1,3) = 0$

$c_1 + 3c_2 - 2c_3 = 0$
$2c_1 + c_2 + c_3 = 0$
$3c_1 + 0c_2 + 3c_3 = 0$

$\longrightarrow \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} = 0$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$

$R_3 \rightarrow R_3 - \frac{9}{5}R_2$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

Rank (2) $\neq$ Dim^n (3)

$\therefore$ Not a Basis

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Dim^n of S = No. of unique vectors
$\hookrightarrow 3$

& Basis of subspace by $S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sol^n 8

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

with the initial values $x_0 = 1$, $y_0 = 1$, $z_0 = 1$

$$x = \frac{23 + 6y - 2z}{3}$$

$$y = -15 + 4x + z \quad ; \quad z = \frac{16 - x + 3y}{7}$$

Iteration 1

$$x = \frac{23 + 6(1) - 2(1)}{3} \quad ; \quad y = -15 + 4(1) + 1$$
$$= -10$$
$$= 9$$

$$z = \frac{16 - 1 + 3}{7} = 2.57$$

Iteration 2 $\quad x = \frac{23 + 6(-10) - 2(2.57)}{3} = -14.04$

$$y = \underline{-15 + 4(9) + 2.57} = 23.57$$

$$z = 16 - 9 + 3(-10) = -3.28$$

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Iteration (3)

$$x = \frac{23 + 6(23.57) - 2(3.28)}{3} \approx 57$$

$$y = -15 + 4(-14.04) + -3.28 = -74.4$$

$$z \Rightarrow 16 - (-14.04) + 3(23.57) = 100.75$$

Sol^n (9) Suppose we have a 2D ~~Matrix~~ image representation as grid or pixels. We can us AT Matrix to rotate around centro

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ rotation of image by θ

(1) Translation to origin
Translate the image, so that it's centre allign with origin.

(2) Rotation = apply rotation matrix.

Sol^n (10) Linear Trans^n for a rotation 2D images involve applying a rotation matrix to each pixel coordinate. This matrix rotates points counterclockwise by an angle around the origin. It preserves geometric properties like ll gram and distances. Rotation is essential in tasks like image allignment and object detection in computer vision.