CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2000

Assignment #4 - Due date: Wednesday, 5/3/00

Midterm Exam

The midterm examination will be held in class 3:15–4:30 pm on Wednesday, May 10. (For the SITN students, we will send the midterm to you via the SITN courier or electronically.) The exam will be open-book and open-notes, i.e., you will be allowed to consult any of the class handouts, your notes, and the text-book/reader. You are not permitted to refer to any other source during the exam. The exam will cover the material presented in class up to, but not including, the Pumping Lemma for Context-free Languages.

CS 154N Students

We expect to start discussing the topics from CS 154N, starting with Turing machines, around Monday, May 1. We suggest that you start attending the lectures starting now.

Reading Assignment

There was a typo in the last reading assignment – we said that Section 5.4 would not be covered in detail, but what we meant to say was that Section 6.4 would not be covered in detail. We are done with Chapter 6 of the course reader by now and will be covering Chapter 7 in the next couple of classes. (In the Hopcroft-Ullman book, this corresponds to Chapter 6.)

Problem Set

Problem 1. *(20 points)*
a). Suppose G is a context-free grammar in the Chomsky Normal Form. Let w be a string of length n and in $L(G)$. What is the length of a derivation of w in G?
b). Can every CFL without ϵ be generated by a CFG all of whose productions are of the forms $A \to BCD$ and $A \to a$?

Problem 2. [20 points] Let G be the context-free grammar with the productions

$$\begin{aligned} S &\to aAa \mid bBb \mid \epsilon \\ A &\to a \mid C \\ B &\to b \mid C \\ C &\to CDE \mid \epsilon \\ D &\to ab \mid A \mid B \end{aligned}$$

Simplify this grammar by removing useless symbols, unit productions and ϵ-productions (in the right order); finally, obtain a grammar in the Chomsky normal form. You only need to show the resulting grammar after each step of the simplification process.

Problem 3. [20 points]
Consider the PDA M with the following transitions:

$$\begin{aligned}
\delta(q_0, 0, Z_0) &= \{(q_0, AZ_0)\} \\
\delta(q_0, 0, A) &= \{(q_0, AA)\} \\
\delta(q_0, 1, A) &= \{(q_0, \epsilon)\}
\end{aligned}$$

(a). Describe the *final state* language $L(M)$ in English, assuming that q_0 is a final state.
(b). Suppose we add the transition

$$\delta(q_0, \epsilon, Z_0) = \{(q_0, \epsilon)\}$$

to the PDA M defined above. Describe the *empty stack* language $N(M)$ in English.

Problem 4. [30 points] Consider the following language

$$L_{NEQ} = \{0^i 1^j \mid i \neq j \text{ and } i, j \geq 0\}.$$

(a). Provide a context-free grammar for L_{NEQ}.
(b). Provide an NPDA M such that $N(M) = L_{NEQ}$. You must provide a transition diagram and a brief explanation of how your NPDA works.
(**Hint:** Consider separately the two cases where $i > j$ and $i < j$.)

Problem 5. *[10 points]* Consider the context free grammar G with the productions:

$$\begin{aligned}
S &\longrightarrow a \mid b \mid XX \mid XY \mid YX \\
X &\longrightarrow a \mid XX \\
Y &\longrightarrow b \mid YY
\end{aligned}$$

Construct an NPDA M such that $N(M) = L(G)$ using the construction given in class.

Suggested Exercise. The following is *not* a part of the homework and is only included as a practice problem.

Let M be a PDA with the empty stack language $L = N(M)$ such that $\epsilon \notin L$. Describe how you would modify M so as to make its empty stack language equal $L \cup \{\epsilon\}$. Justify your construction.