CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2000

Assignment #2 - Due date: Wednesday, 4/19/00

Problem 1. *(20 points)* Let $M = (Q, \Sigma, \delta, q_0, \{q_f\})$ be an ϵ-NFA such that there are no transitions into q_0 and there are no transitions out of q_f. Specify the language accepted by each of the following machines in terms of $L = L(M)$. You must provide a brief justification for your answer.
(a). The machine M_1 obtained from M by adding an ϵ-move from q_f to q_0.
(b). The machine M_2 obtained from M by adding an ϵ-move from q_0 to every state that is reachable from q_0.
(c). The machine M_3 obtained from M by adding ϵ-move to q_f from all states that can reach q_f.
(d). The machine M_4 obtained from M by adding the transitions described in both (b) and (c).

Problem 2. *(15 points)* Write a regular expression for the following language over the alphabet $\Sigma = \{0, 1\}$. Provide a brief explanation or justification that your regular expression is correct.

> The set of all strings in which every pair of adjacent 0's appears before any pair of adjacent 1's.

Problem 3. *(20 points)* Use the construction outlined in class and the course reader to convert the regular expression $(0 + 10)^*1^*$ into an ϵ-NFA.
(**Remarks:** Do you see how to simplify the ϵ-NFA? We recommend that you try to convert this machine into an NFA *without* ϵ-transitions as an exercise towards understanding the construction given in class. This suggested work is *not* a part of the homework assignment.)

Problem 4. *(5 points)* Prove or disprove the following identity for regular expressions R and S.

$$(R + S)^* = R^* + S^*.$$

(**Note:** Such an identity is said to be valid if the languages defined by the two expressions in the identity are the same, regardless of how we choose R and S. To disprove the identity you must demonstrate a string which is generated by one of the two expressions but not the other.)

Problem 5. *(40 points)* Use the Pumping Lemma to prove that the following languages over $\Sigma = \{0, 1\}$ are not regular.
a). The language L_1 consisting of all *palindromes*. A palindrome is a string that equals its own reverse, such as 00100 or 110011.
b). The language L_2 consisting of all strings in which the number of 1's is exactly three times the number of 0's.

Reading Assignment:

In Chapter 3 of the course reader, we have covered Sections 3.1 and 3.2 in detail. But we also recommend that you read Sections 3.3 and 3.4 for background material. We have covered portions of Section 3.4 in the class. (In the Hopcroft-Ullman book, the corresponding material can be found in Sections 2.5 and 2.8.)

Next, we will cover Sections 4.1, 4.2, and 4.3 in Chapter 4 of the course reader. We will only skim over the material in Section 4.4 before moving on to Chapter 5. (In the Hopcroft-Ullman book, these correspond to Chapters 3 and 4.)