1

Let $c_0 + c_1 x + \cdots + c_{n-1} x^{n-1} + c_n x^n$ be a polynomial with a root at $x = x_0$. Let $c_{\max}$ be the largest absolute value of any c_i. Show that

$$|x_0| < (n+1) \frac{c_{\max}}{|c_n|}$$

(Note: This shows that determining if a polynomial has an integer root is decidable, since there are only finitely many integers to check.)

2

A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

3

Show that a language L is Turing-recognizable iff $L \leqslant_m A_{TM}$.

4

Consider the language

$$\textsc{DoubleStack} = \{\langle M \rangle \mid \text{M is a PDA and for some } w \in \{0,1\}^*, \text{M accepts } ww\}.$$

Use the computation history method to show that this problem is undecidable.

5

Include a References section. Cite all sources that you used and people, including yourself, that you collaborated with on this homework.