Set membership question.

Question. Given a regular set $A \subseteq \Sigma^*$ and $x \in \Sigma^*$ is $x \in A$?

DFA M: Simulate M on the input x

$O(k)$ where $|x| = k$

NFA N: Is $x \in L(N)$?

$O(kn^2)$ $|x| = k$, $|Q| = n$.

Question. Given a CFL $A \subseteq \Sigma^*$ and $x \in \Sigma^*$ is $x \in A$?

Algorithm - Due to Cocke, Kasami and Younger

CKY - algorithm.

Runs in cubic-time

Determines for each substring y of x the set of all nonterminals that generate y.

Defined inductively on the length of y.

Assume G is in Chomsky normal form.

Example. $S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a, \quad B \to b, \quad C \to SB, \quad D \to SA$

$L(G)$ - all strings with equal number of a's and b's.

Let $x = aabbab$. Let $n = |x|$ (So $n = 6$ here)

	a	a	b	b	a	b
0	1	2	3	4	5	6

For $0 \leq i < j \leq n$, let x_{ij} denote the substring of x between i & j. Ex. $x_{0,3} = aab$, $x_{2,6} = bbab$
$x = x_{0,n}$.

0						
_	1					
_	_	2				
_	_	_	3			
_	_	_	_	4		
_	_	_	_	_	5	
_	_	_	_	_	_	6

Entry $T_{i,j}$ of T - Set of non-terminals of G that generates the substring x_{ij} of x.

Define by induction on the length of the substrings.

Example. $S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a, \quad B \to b, \quad C \to SB, \quad D \to SA$

$x =$ a a b b a b

0 1 2 3 4 5 6

Substring of length 1.

0						
A	1					
_	A	2				
_	_	B	3			
_	_	_	B	4		
_	_	_	_	A	5	
_	_	_	_	_	B	6

Example. $S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a, \quad B \to b, \quad C \to SB, \quad D \to SA$

$x \to$ a a b b a b

0 1 2 3 4 5 6

Substring of length 1.

	0	1	2	3	4	5
1	A					
2	_	A				
3	_	_	B			
4	_	_	_	B		
5	_	_	_	_	A	
6	_	_	_	_	_	B

Substring of length 2

	0	1	2	3	4	5
1	A					
2	ϕ	A				
3	_	S	B			
4	_	_	ϕ	B		
5	_	_	_	S	A	
6	_	_	_	_	S	B

For each $x_{i,i+2}$: split the string into two substrings $x_{i,i+1}$ and $x_{i+1,i+2}$. Select a non-terminal X from $T_{i,i+1}$ and Y from $T_{i+1,i+2}$.

Look for a production $Z \to XY$ in G.

Label $T_{i,i+2}$ with Z for each such production.

Example. $S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a, \quad B \to b, \quad C \to SB, \quad D \to SA$

$x \to$ a a b b a b

0 1 2 3 4 5 6

Substring of length 1.

0						
A	1					
_	A	2				
_	_	B	3			
_	_	_	B	4		
_	_	_	_	A	5	
_	_	_	_	_	B	6

Substring of length ≥ 3

0						
A	1					
ϕ	A	2				
ϕ	S	B	3			
S	C	ϕ	B	4		
D	S	ϕ	S	A	5	
S	C	ϕ	C	S	B	6

For each $x_{i,i+2}$,
break into 2 substring
$x_{i,i+1}$ & $x_{i+1,i+2}$
Select a nonterminal
X from $T_{i,i+1}$ and
Y from $T_{i+1,i+2}$
Look for production
$Z \to XY$ in G.
Label $T_{i,i+2}$ with Z
for each such production.

Example. $S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a, \quad B \to b, \quad C \to SB, \quad D \to SA$

$x \to$ a a b b a b

0 1 2 3 4 5 6

Substring of length 1.

0						
A	1					
_	A	2				
_	_	B	3			
_	_	_	B	4		
_	_	_	_	A	5	
_	_	_	_	_	B	6

Substring of length ≥ 3

0						
A	1					
ϕ	A	2				
ϕ	S	B	3			
S	C	ϕ	B	4		
D	S	ϕ	S	A	5	
S	C	ϕ	C	S	B	6

For each $x_{i,i+2}$, break into 2 substring $x_{i,i+1}$ & $x_{i+1,i+2}$

Select a nonterminal X from $T_{i,i+1}$ and Y from $T_{i+1,i+2}$

Look for production $Z \to XY$ in G.

Label $T_{i,i+2}$ with Z for each such production.

$S \in T_{0,6}$ indicates $S \xrightarrow[G]{*} x_{0,6} = x$: the input string.

Therefore $x \in L(G)$.

Algorithm.

```
For i := 0 to n-1 do                                   [length 1 strings].
  begin
        T_{i,i+1} = φ                                  [Initialise to φ]
        For A → a , a production in G do
           if a = x_{i,i+1} then T_{i,i+1} = T_{i,i+1} ∪ {A}
  end

For m := 2 to n do                                     [length m ≥ 2]
    for i := 0 to n-m do                               [for each substring of
                                                          length m]
      begin
         T_{i,i+m} := φ                                [Initialise]
         For j = i+1 to i+m-1 do                       [All possible splits]
           For A → BC , a production in G do
              if B ∈ T_{i,j} and C ∈ T_{j,i+m}
               then
                     T_{i,i+m} = T_{i,i+m} ∪ {A}
    end
```

Running Time - $O(n^3 p)$ where $p = |P|$ and $n = |x|$.

Ambiguous Grammar.

Consider the grammar $S \to S+S \mid S*S \mid (S) \mid A$

$A \to a \mid b$

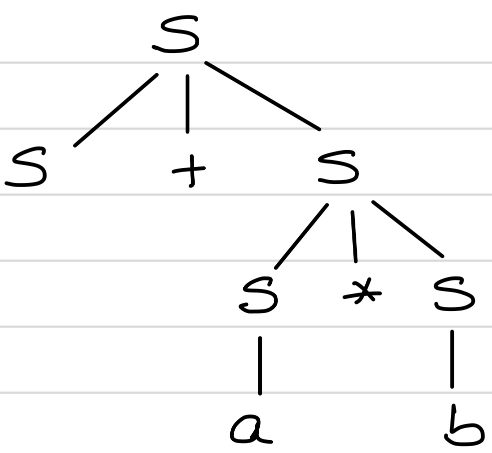
Consider the string $a+a*b$

Derivation 1. $S \to S+S \to a+S \to a+S*S \to a+a*b$

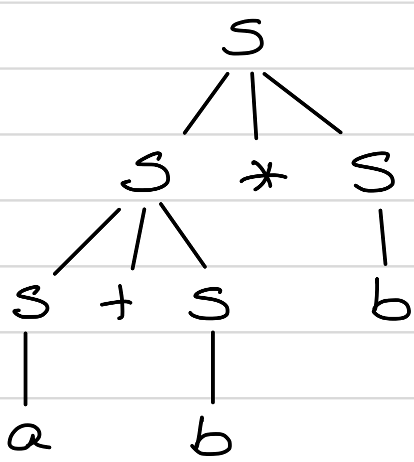
Derivation 2. $S \to S*S \to S+S*S \to a+S*S \to a+a*b$

Parse Tree

Derivation 1



Derivation 2.



A CFG G is ambiguous if $\exists x \in L(G)$ for which there are two different parse trees.

→ Not two different derivations.

Definition

A string x is derived ambiguously in a CFG G if it has two different leftmost derivation.

Grammar G is ambiguous if it generates some string ambiguously.

G is unambiguous if G is not ambiguous.

A CFL $A \subseteq \Sigma^*$ is inherently ambiguous if $\forall$ CFG G s.t $L(G) = A$, G is ambiguous.

Note. There are inherently ambiguous CFLs.

DCFLs – CFLS that can be accepted by a DPDA.

DCFLS always admit an unambiguous grammar.

DCFLs $\subsetneq$ unambiguous CFLS.