CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2000

Assignment #1 - Due date: Wednesday, 4/12/00

Problem 1. *(10 points)* Construct DFA's to recognize the following language over the alphabet $\Sigma = \{0, 1\}$.

$$L = \{\text{all strings that do not end in } 11\}.$$

Problem 2. (30 points) Let M be a DFA such that $M = (Q, \Sigma, \delta, q_0, F)$ with $F = \{q_f\}$ and

$$\forall\, a \in \Sigma\,,\ \delta(q_f, a) = \delta(q_0, a)$$

(a). Show that for all *non-empty* strings $w \in \Sigma^*$, it must be the case that $\widehat{\delta}(q_f, w) = \widehat{\delta}(q_0, w)$.
(b). Let $w \in L(M)$ be any string in the language of M. Prove *by induction* that for all $k > 0$, the string $w^k \in L(M)$.

Problem 3. *(30 points)*
(a). Let $L \subset \{0, 1\}^*$ be the language of all strings $w \in \{0, 1\}^*$ such that there is a pair of 0's in w that are separated by a string of length $5k$, for some integer $k > 0$. Construct an NFA for this language.
(b). You should be glad that I did not ask you to construct a DFA for this language. To truly appreciate this, prove that any DFA for this problem must have at least 2^5 states. How does this compare with your NFA? What are the implications for the relative power of NFAs and DFAs?

Problem 4. *(30 points)* Consider the NFA $N = (Q, \Sigma, \delta, p, F)$ which has $Q = \{p, q, r\}$, $\Sigma = \{0, 1\}$, $F = \{r\}$, and the transition function as defined in the following table.

δ	0	1
p	$\{p, q\}$	$\{p\}$
q	$\{q, r\}$	$\emptyset$
r	$\emptyset$	$\{r\}$

(a). Convert this NFA into a DFA using the subset construction described in the class. Your solution should consist of a transition diagram with *only* the essential states.
(b). Describe the language defined by this DFA and give a regular expression for it.

Reading Assignment:

Each homework will specify a set of readings from the course reader — these are generally a required reading to follow the material presented in the lectures. We will also give some recommended (but not required) reading from the Hopcroft-Ullman book.

1. Read Chapter 1; you should be familiar with the background material in this chapter. (In the Hopcroft-Ullman book, we suggest that you read Chapter 1.)

2. We have already covered Chapter 2 of the course reader, so make sure that you read this right away. We will be covering portions of Chapters 3 and 4 in the next one week and you should try to read the relevant sections in conjunction with the lectures. (In the Hopcroft-Ullman book, we have covered Chaptter 2.1-2.5, and are now moving on to Chapter 3.)