

1

Show that the class NP is closed under union and concatenation of languages.

2

Show that the set of incompressible strings contains no infinite Turing-recognizable subset.

3

Show that given a CNF-formula F with c clauses and m variables, show you can construct in polynomial time an NFA with $O(cm)$ states which accepts exactly the assignments which do not satisfy F , represented as length- m binary strings.

Conclude that NFAs cannot be minimized in polynomial time, unless $P = NP$.

4

Define the “super-halting problem” recursively as follows. First, define the base case to be the regular halting problem for Turing machines, $\text{SUPERHALTS}_0 = \text{HALT}_{\text{TM}}$. Next, define each SUPERHALTS_{n+1} to be the halting problem for oracle Turing machines with oracles for SUPERHALTS_n .

Show that for each $n \in \mathbb{N}$, SUPERHALTS_n is m -complete in Σ_{n+1}^0 .

Conclude that the arithmetic hierarchy does not collapse.

Hint: The definition of the arithmetic hierarchy using oracles is more useful here than the one using quantifiers.

5

Include a References section. Cite all sources that you used and people, including yourself, that you collaborated with on this homework.