CS154: Homework #1

Due: Wednesday, July 11, 2012 by 5PM

Problem 1. *Given 8 distinct natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference (the pairs need not be disjoint as sets).*

- **Note:** Recall that the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$.

Problem 2. *Prove that the number* $111\ldots11$ *(243 ones) is divisible by 243.*

- **Hint:** Try proving that a number written with 3^n ones is divisible by 3^n, for all $n \geq 1$.

Problem 3. *Draw a DFA to show that the language* L *over the alphabet* $\{0, 1\}$ *consisting of runs of even numbers of* 0*'s and odd numbers of* 1*'s is regular. Also draw the transition table for this DFA.*

- **Note:** A *run* of an alphabet is a continuous sequence of that alphabet, for example, here are few strings with runs of even numbers of 0's: $10011, 11100001, 11001000011$. You are required to give a DFA that accepts strings containing runs with even numbers of 0's **and** runs with odd numbers of 1's.

Problem 4. *Let* $L \subset \{0, 1\}^*$ *be the language of all strings such that there are two* 0*'s separated by a number of positions that is a* ***non-zero*** *multiple of* 5*. Construct an NFA for this language.*

- **Note:** Each position between the two 0's contains an arbitrary symbol (either 0 or 1). For example, 1001110 is not in L, but $1\underline{0}11111\underline{0}$ and $\underline{0}10101\underline{0}1$ are both in L.

Problem 5. *Consider an NFA* $N_1 = (Q, \Sigma, \delta_N, q_0, F)$ *with language* $L_1 = L(N_1)$*. Define a new NFA* $N_2 = (Q, \Sigma, \delta_N, q_0, Q - F)$*, i.e., going from* N_1 *to* N_2 *the final states become non-final, and vice-versa. Prove or disprove that the language* $L_2 = L(N_2)$ *is the* ***complement*** *of the language* L_1*, i.e.,* $L_2 = \Sigma^* - L_1$*.*

- **Note:** If the statement is true, you must provide a formal proof that applies to all NFA's N_1. However, if the statement is false, then all you need to do is describe a specific NFA N_1 and show that the statement is incorrect when applied to this N_1.