**15-453 Formal Languages, Automata, and Computation**

# Homework Assignment 4

Due Friday, February 18

## Problem 1 (30 points)

Give context-free grammars that generate the following languages. In all parts the alphabet $\Sigma$ is $\{0, 1\}$. [This is a modified version of Exercise 2.4 a–e in the textbook.]

a. $\{w \mid w$ constains at least three 1s$\}$. regular

b. $\{w \mid w$ starts and ends with the same symbol$\}$. regular

c. $\{w \mid$ the length of $w$ is odd$\}$. regular

d. $\{w \mid$ the length of $w$ is odd and its middle symbols is a 0$\}$. 1^k01^k, y is 1^k, if i is increased, 0 wont be the middle symbol anymore

e. $\{w \mid w$ contains more 1s than 0s$\}$. not regular, y=0^k and z=1^(k+1), at some i no of 0s is more than the no. of 1s

Indicate for each language if it is regular or not. For those languages which are **not** regular, prove that they are not regular.

## Problem 2 (30 points)

Give informal descriptions and state diagrams of pushdown automata for the languages in Problem 1. Note that even if the language is regular, you may be able to give a smaller PDA than NFA recognizing the language. [This is Exercise 2.5 a–e in the textbook.]

## Problem 3 (20 points)

This problem illustrates a technique for proving that languages are not context-free without directly using the pumping lemma. [This is Problem 2.17 in the textbook.]

a. Let $C$ be a context-free language and $R$ be a regular language. Prove that the language $C \cap R$ is context-free.

b. Use part (a) and one of examples 2.20, 2.21, or 2.22 in the textbook to show that the language $L = \{w \mid w \in \{a, b, c\}^*$ and contains equal numbers of $a$'s, $b$'s and $c$'s$\}$ is not context-free.

## Problem 4 (20 points)

Use the pumping lemma for context-free languages to show that $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$ is not context-free. Carry out your argument in detail and make clear which conditions of the pumping lemma you use and where. [This is adapted from Problem 2.18 (a) in the textbook.]

## Problem 5 (20 points extra credit)

Let $C = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x \neq y\}$. Show that $C$ is a context-free language. [This is Problem 2.26 in the textbook.]