**15-453: Formal Languages, Automata and Computability**
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**Homework # 3** **Due: February 4, 2014**

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**1**

Prove that for any $m$, there exists an NFA with $m$ states such that any equivalent DFA has at least $2^{m-1}$ states. Text

**2**

a) Consider the language of all binary strings with twice as many 0s as 1s. Give a CFG and a PDA for this language.

b) Prove that the following lanuage is Context-free:
$\{s_1 s_2 \dots s_n t_1 t_2 \dots t_n | s_i \in L_1, t_i \in L_2, n \in \mathbb{N}\}$ where $L_1$ and $L_2$ are Context-free languages.

S->S1SS2 I e where S1 and S2 are the start symbols of L1 and L2 respectively

**3**

Prove that the following languages are not context free by using the pumping lemma for context free grammars:

a) $\{a^{2^n} | n \in \mathbb{N}\}$ a^(2^k) where k is the pumping length, can increase by at max k, which is not sufficient

b) Set of all binary strings with a prime number of 1s.

a^m where m is a prime greater than k. suppose vx=1^p, let i equal m, then m+mp=m(1+p) not a prime

**4**

Say that a language is prefix-closed if the prefix of any string in the language is also in the language. Let C be an infinite, prefix-closed, context-free language. Show that C contains an infinite regular subset.

**5**

Include a References section. Cite all sources and people, including yourself, that you collaborated with on this assignment.