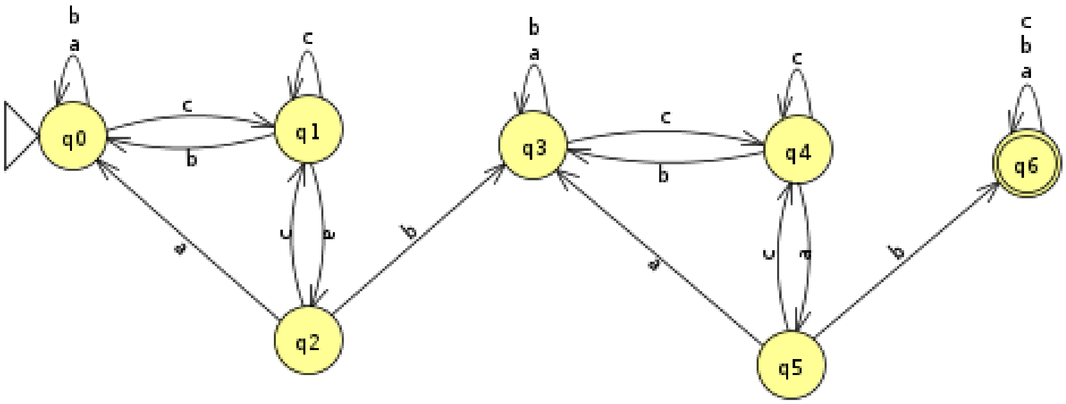
15-453: Formal Languages, Automata, and Computability **L. Blum, A. Jindia, A. Smith**
Solutions to Homework #1 **Due Date: January 21, 2014**

1

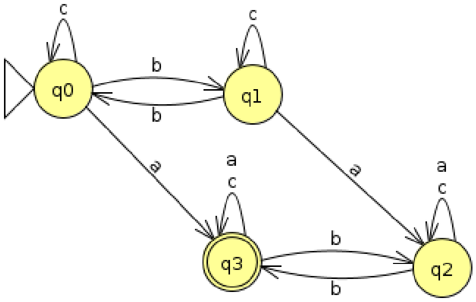
(a)



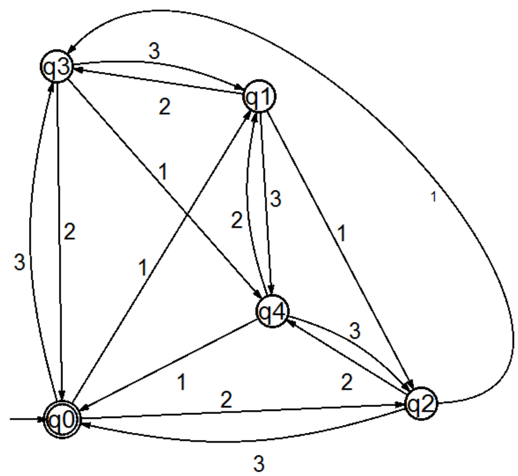
(b) One possible solution is to modify the DFA from part (a) by adding a third set of nonfinal states identical to $\{q_0, q_1, q_2\}$ or $\{q_3, q_4, q_5\}$. This would recognize strings with "*cab*" at least three times. Then by making every state *except* for the last one final, we make a DFA that recognizes the complement of this language – the strings with "*cab*" at most twice.

(c) The most obvious solution is a DFA with ten states $q_0, \ldots, q_9$ arranged in a circle, with each transitioning to the next for all symbols (i.e. $\delta(q_i, \sigma) = q_{i+1 \mod 10}$). The initial state is q_0 and states which are even or divisible by 5 are final.

(d)



2



We claim by induction on k that after k steps, this DFA is in state $q_{s(k)}$, where $s(k)$ is the sum of the first k characters of the inputs mod 5.

Suppose this claim is true for k. By construction for each $j = 1, 2$, or 3, $\delta(q_i, j) = q_{i+j \mod 5}$. Hence after $k+1$ steps it is at $q_{s(k)+j} = q_{s(k+1)}$.

Thus it accepts only if the sum of the entire input string is 0 mod 5.

3

Consider a DFA M for regular language A, given by $M = (Q, \Sigma, \delta, q_0, F)$.

Let $F' \subseteq F$ be the set of states q in F from which there is no path from q to any other final state.

Define M' to be the DFA $M' = (Q, \Sigma, \delta, q_0, F)$.

If M' accepts x, then x takes M to a final state q so $x \in A$, and there are no paths to final states from q, so no strings with x as a prefix are accepted by M. Thus $x \in \textsc{Max}(A)$.

If M' does not accept x, then either the state q_0 which x takes M' to is a nonfinal state, so $x \notin A$, or there is a path from q_0 to a final state. If the path is $q_0 q_1 \ldots q_n$ and $\delta(q_{i-1}, w_i) = q_i$, then the string $x w_1 \ldots w_n$ takes M to a final state and has x as a proper prefix, so $x \notin \textsc{Max}(A)$.

4

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing L_1. For each state q_i, let M_i be the DFA $(Q, \Sigma, \delta, q_i, F)$. Define $f_i = 1$ if $L(M_i) \cap L_2$ is nonempty, and $f_i = 0$ otherwise. Define $F' = \{q_i \in Q \mid f_i = 1\}$, and $M' = (Q, \Sigma, \delta, q_0, F')$. The strings which are accepted by M' are exactly those which take the DFA to a state from which some string in L_2 takes the DFA to a final state of M, which are exactly L_1/L_2.