

CS154: Homework #2

Due: Wednesday, July 18, 2012 by 5PM

Problem 1. For any DFA, we extend the transition function δ by breaking the input string $\mathbf{w} = \mathbf{xa}$ during the inductive step, where \mathbf{x} is any string followed by a single symbol \mathbf{a} . However, we informally think of δ as describing what happens along a path with a certain string of symbols, and if so, then it should not matter how we break the input string. Show that in fact,

$$\delta(q, \mathbf{xy}) = \delta(\delta(q, \mathbf{x}), \mathbf{y})$$

for any state q and strings \mathbf{x} and \mathbf{y} .

Problem 2. Give DFA's accepting the following languages over the alphabet $\{0, 1\}$.

- The set of all strings such that each block of **four consecutive** symbols contains **at least two** 0's.
- The set of strings such that the number of 0's is divisible by 3, **and** the number of 1's is divisible by 3.

Problem 3. Consider the following ε -NFA.

	ε	a	b	c
$\rightarrow p$	$\{q, r\}$	\emptyset	$\{q\}$	$\{r\}$
q	\emptyset	$\{p\}$	$\{r\}$	$\{p, q\}$
r^*	\emptyset	\emptyset	\emptyset	\emptyset

Recall that starred states denote accept/final states.

- Compute the ε -closure of each state.
- Give all the strings of length three or less accepted by the automaton.
- Convert the automaton to a DFA.

Problem 4. Write regular expressions for the following languages. In all parts the alphabet is $\{0, 1\}$.

- $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$.
- $\{w \mid w \text{ has at most one pair of consecutive } 1\text{'s}\}$.
- $\{w \mid \text{the number of } 0\text{'s in } w \text{ is divisible by } 3\}$.
- $\{w \mid \text{every pair of adjacent } 0\text{'s in } w \text{ appears before any pair of adjacent } 1\text{'s}\}$.
- $\{w \mid \text{every odd position of } w \text{ is a } 1\}$.

Problem 5. Prove that the language $\mathbf{L} = \{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$ is not regular.