Write every problem on a separate sheet of paper, write *legibly*, and write your name, your student ID, this Homework # and the problem # on top of each page.

The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.)

Make sure you are familiar with the collaboration policy, and read the overview in the class homepage theory.cs.stanford.edu/∼trevisan/cs154.

Recall that our late policy is that late homework is not accepted (however the lowest homework grade is dropped).

Refer to the class home page for instructions on how to submit the homework.

Homework 2

Due on Thursday, January 26, 2012, 9:30am

Updated 1/23/2012 to correct part (2) of problem 3

1. (3 points)
   Let $\Sigma = \{0, 1, @\}$. Use the Pumping Lemma to prove that the language

   $$L = \{x@y@z | x \in \{0,1\}^*, y \in \{0,1\}^*, z \in \{0,1\}^*, \text{bin}(x) + \text{bin}(y) = \text{bin}(z)\}$$

   is not regular. $\text{bin}(x)$ is the numerical value of x interpreted as a binary string, for example $\text{bin}(110) = 1 * 4 + 1 * 2 + 0 * 1 = 6$.

   Proceed as follows: for every integer $p \geq 1$,

   (a) (1 point) define a string $\{s\} \in L$ of length at least p such that $(\forall x, y, z)(s = xyz, |xy| \leq p, |y| > 0) \to (\exists i) xy^iz \notin L$. You don't have to prove this until the next part, but if your string fails to satisfy the stated conditions, you will receive 0 points.

   (b) (2 point) given any partition $s = xyz$ of s into three strings, with $|xy| \leq p$ and $|y| > 0$, prove that there is an i such that $xy^iz \notin L$.

2. (3 points)
   For $k \geq 1$, let the language L_k be all binary strings of length at least k that have a 1 in the k-th-to-last position. For example, L_2 would include 00010 and 110, but not 100.

(a) (1 point) Prove that L_k has a regular expression of length $O(k)$, where the length of a regular expression is the number of symbols in the expression.

(b) (1 point) Prove that L_k has an NFA with $O(k)$ states.

(c) (1 point) Find the number of states of the minimal DFA for L_k as a function of k and prove that it is minimal.

3. (3 points)
Define the reverse s^R of a string $s = s_1...s_n$ as the string $s^R = s_n...s_1$. For example $\text{won}^R = \text{now}$. If L is a language, define L^R to be the language $\{s : s^R \in L\}$.

(a) (1 point) Prove that if a language L is regular and is recognized by a k-state DFA, then the language L^R is also regular and is recognized by a DFA with at most 2^{k+1} states.

(b) (2 points) Prove that, for every k, there is a language L that is recognized by a DFA with $k+2$ states, but such that L^R needs at least 2^k states.