Write every problem on a separate sheet of paper, write *legibly*, and write your name, your student ID, this Homework # and the problem # on top of each page.

The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.)

Make sure you are familiar with the collaboration policy, and read the overview in the class homepage theory.cs.stanford.edu/∼trevisan/cs154.

Recall that our late policy is that late homework is not accepted (however the lowest homework grade is dropped).

Refer to the class home page for instructions on how to submit the homework.

Homework 1

Due on Thursday, January 19, 2012, 9:30am

1. (3 points) For each of the following languages, provide both a regular expression and a DFA that accepts it. You should define each DFA by drawing it as a directed graph with accepting states marked by double concentric circles. You do not need to justify your answers.

   (a) (1 point) The language of strings over alphabet $\Sigma = \{0, 1, 2\}$ in which the first character of the string does not appear elsewhere in the string.

   (b) (1 point) The language of strings over alphabet $\Sigma = \{0, 1\}$ that contain 000 as a substring but not 111.

   (c) (1 point) The language $L = \{x | \text{bin}(x) \equiv 1 \mod 3\}$ with alphabet $\Sigma = \{0, 1\}$, where $\text{bin}(x)$ is the numerical value of interpreting x as a binary integer (e.g. $\text{bin}(1010) = 8 + 2 \equiv 1 \mod 3$).

2. (3 points) The following is the transition table of a two-state DFA:

	0	1
$\rightarrow A$	B	B
*B	A	A

Determine the language accepted by this DFA. Then, give a careful proof of your claim. Remember that you need to prove two directions: all strings in the language you specify must be accepted by the automaton, and all strings accepted by the automaton must be in the language. Each direction is an induction. What is the inductive statement in each case? On what parameter is the induction? What is the basis case? Prove it. What is the inductive statement? Prove it. While the ideas for this very simple example are easy, you will be graded on your ability to organize the proof as well as on the correctness of what you say.

3. (4 points) We say that $w = w_1 w_2 \dots w_n$ is a *final segment* of $v = v_1 v_2 \dots v_k$ if $n \leq k$ and for every $i \in \{1, 2, \dots, n\}$, $w_i = v_{i+k-n}$. In other words, w is some last part of v. For any language A, define:

$$A^{final} = \{y \mid y \text{ is a final segment of } x \text{ for some } x \in A\}$$

Show that if A is regular, then so is A^{final}.