R V COLLEGE OF ENGINEERING
(An autonomous institution affiliated to VTU, Belgaum)
DEPARTMENT OF MATHEMATICS

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS (22MA21C)

UNIT 3: VECTOR INTEGRATION

TUTORIAL SHEET – 1

1. Find the total work done by the force represented by $\vec{F} = 3xy\hat{\imath} - y\hat{\jmath} + 2zx\hat{k}$ in moving a particle round the circle $x^2 + y^2 = 4,\ x = 2\cos\theta, y = 2\sin\theta\ and$ $z = 0, 0 \le \theta \le 2\pi$.
   $\boldsymbol{Ans}: \mathbf{0}$

2. Evaluate $\int_C (x + y)\,dx - (y - x)dy$ along the parabola $x = y^2$ from (1,1) to (4,2).
   $\boldsymbol{Ans}: \frac{\mathbf{34}}{\mathbf{3}}$.

3. Evaluate the line integral $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$, where $C: square: x = \pm 1,\ y = \pm 1$
   $\boldsymbol{Ans}: \mathbf{0}$

4. Verify Green's theorem for $\int_C (e^{-x}\sin y)dx + (e^{-x}\cos y)dy$, where C is the rectangle, whose vertices are $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right)\ and\ \left(0, \frac{\pi}{2}\right)$.
   Ans: $[\mathbf{2}(\boldsymbol{e}^{-\boldsymbol{\pi}} - \mathbf{1})]$

5. Using Green's theorem, evaluate $\oint_C (x^2 - \cosh y\,dx + (y + sinx)dy)$ where C is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1$.
   $\boldsymbol{Ans}: \boldsymbol{\pi}(\mathbf{cosh\,1} - \mathbf{1})$

6. Using the Green's theorem, evaluate $\oint_C (x^2 + xy)dx + (x^2 + y^2)dy)$ where C is the square formed by the lines $x = \pm 1$ and $y = \pm 1$.
   $\boldsymbol{Ans} = \frac{\mathbf{128}}{\mathbf{5}}$

7. If S is the surface of the sphere $x^2 + y^2 + z^2 = d^2$ and $\vec{A} = ax\hat{\imath} + by\hat{\jmath} + cz\hat{k}$, evaluate $\iint_S \vec{A} \cdot \hat{n}\,ds$.
   $\boldsymbol{Ans}: \frac{\mathbf{2}\boldsymbol{\pi}\boldsymbol{d}^{\mathbf{3}}}{\mathbf{3}}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$

8. If $\vec{F} = 2y\hat{\imath} - 3\hat{\jmath} + x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$ and $z = 6$, show that $\iint_S \vec{F} \cdot \hat{n}\,ds = 132$.

9. Find the surface integral over the parallelepiped $x = 0, y = 0, x = 1, y = 2, z = 3$ when $\vec{A} = 2xy\hat{\imath} + yz^2\hat{\jmath} + xz\hat{k}$
   $\boldsymbol{Ans}: \mathbf{33}$.

10. Using divergence theorem, evaluate $\iint_S \vec{r}.\hat{n}\,ds$ where S is the surface of the sphere $x^2+y^2+z^2=9$.
Ans: $\boldsymbol{108\pi}$

11. Verify divergence theorem for $\vec{F}=4xz\hat{\imath}-y^2\hat{\jmath}+yz\,\hat{k}$ taken over the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

$$\iint_S \vec{F}.\hat{n}\,ds=\frac{3}{2}$$

12. Using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}\,ds$ over the entire surface S of the region above xy plane bounded by the cone $x^2+y^2=z^2$ the plane $z=4$ where $\vec{F}=4xz\hat{\imath}-xyz^2\hat{\jmath}+3z\,\hat{k}$
Ans: $\boldsymbol{704\pi}$

13. Verify Stokes's theorem where $\vec{A}=(2x-y)\hat{\imath}-yz^2\hat{\jmath}-y^2z\,\hat{k}$ and S: upper half of the surface of the sphere $x^2+y^2+z^2=1$
Ans: $\boldsymbol{\pi}$

14. Evaluate $\oint_C xy\,dx+xy^2\,dy$ by Stoke's theorem where C is the square in the xy plane with vertices $(1,0)$ $(-1,0)$ $(0,1)$ $(0,-1)$.
$$\boldsymbol{Ans}=\frac{4}{3}$$

15. Evaluate $\oint_C 4z\,dx-2x\,dy+2x\,dz$ by Stoke's theorem where C is the ellipse $x^2+y^2=1,\ z=y+1$.
Ans: $\boldsymbol{-4\pi}$