22MA11C

RVCE22BAI054

USN 1RV22AI

**RV COLLEGE OF ENGINEERING®**

(An Autonomous Institution Affiliated to VTU)

I Semester B. E. Examinations May-2023

**(Common to AI & ML, BT, CS, CY, CD and IS)**

**FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS**

*Time: 03 Hours* *Maximum Marks: 100*

*Instructions to candidates:*

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

## PART-A

| | | | |
|---|---|---|---|
| 1 | 1.1 | The reduced system of set of linear equations is $\begin{bmatrix}1&2&1\\0&1&-1\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}$. Then the solution for the systems is _________. | 01 |
| | 1.2 | If two characteristic roots of a singular matrix $A$ of order 3 are $4,5$ then the third characteristic root is _________. | 01 |
| | 1.3 | The circle $x^2+y^2-2ax=0$ in polar form is _________. | 01 |
| | 1.4 | The coefficient of $\left(x-\frac{\pi}{4}\right)$ in the Taylor's series expansion of $\sin x$ is _________. | 01 |
| | 1.5 | The curvature of the curve $y=e^x$ at the point where it crosses the $y$-axis is _________. | 02 |
| | 1.6 | The matrices taken for the computation are $A=\begin{bmatrix}2&2\\3&0\\-4&1\end{bmatrix}$ and $B=\begin{bmatrix}1&1\\2&3\\1&6\end{bmatrix}$, then the rank of the matrix $A-B$. | 02 |
| | 1.7 | If the temperature of a thin wire of finite length is $u=e^{-c^2p^2t}[a\cos(px)+b\sin(px)]$, where $a,b,p$ and $c$ are constants, then $u_{xx}=$ _________. | 02 |
| | 1.8 | For the implicit function $e^x-e^y=2xy, \frac{dy}{dx}$ using partial differentiation is _________. | 02 |
| | 1.9 | Evaluate the integral $\int_0^{\frac{\pi}{2}}\int_0^2 r^2\sin\theta\, dr d\theta$. | 02 |
| | 1.10 | Sketch the domain of integral $\int_0^1\int_x^{2-x}\frac{x}{y}dydx$ | 02 |
| | 1.11 | If the first three moments of a distribution about the value 2 of the variable are $3, 16$ and $-20$, then mean and variance of the distribution is _________. | 02 |
| | 1.12 | In a partially destroyed laboratory record of an analysis of a correlation data, the following results were noted: variance of $x=9$, equations of lines of regression of $y$ on $x$ is $4x-5y+33=0$ and $x$ on $y$ is $20x-9y=107$. For the given data the value of correlation coefficient is _________. | 02 |

PART-B

| | | | |
|---|---|---|---|
| 2 | a | Show that the equations<br>$$-2x + y + z = l$$ $$x - 2y + z = m$$ $$x + y - 2z = n$$ have a solution only if $l + m + n = 0$. Find all possible solutions when $l = 1, m = 1, n = -2$. | 05 |
| | b | Apply Gauss-Seidel iterative method, to solve the system of equations:<br>$$x + y + 54z = 110$$ $$27x + 6y - z = 85$$ $$6x + 15y + 2z = 72$$ Carry out three iterations using initial solution as $(0, 0, 0)$. | 05 |
| | c | Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by Rayleigh power method taking the initial vector as $[1\ 0\ 0]^T$. Perform four iterations. | 06 |
| 3 | a | Find the angle of intersection of the curves $r = a\cos\theta$ and $2r = a$. | 08 |
| | b | If $\rho_1$ and $\rho_2$ be the radii of curvature at the extremities of two conjugate semi-diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $(\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}})(ab)^{2/3} = a^2 + b^2$. | 08 |
| | | **OR** | |
| 4 | a | Find the curvature and the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $(\frac{a}{4}, \frac{a}{4})$. | 08 |
| | b | Obtain the Maclaurin's series expansion of $\log_e(1 + e^x)$ up to the term containing $x^4$ and hence deduce the series expansion of $\frac{e^x}{1+e^x}$. | 08 |
| 5 | a | i) If $u = x^2 + y^2$, where $x = at^2, y = 2at$, show that $\frac{du}{dt} = 4a^2t(t^2 + 2)$ using partial derivatives.<br>ii) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, find the value of $x^2u_x + y^2u_y + z^2u_z$. | 08 |
| | b | Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, using Lagrange's method of undetermined multipliers. | 08 |
| | | **OR** | |
| 6 | a | If $u$ and $v$ are the real and imaginary parts of a complex function $f(z) = u + iv$; where $u = e^{r\cos\theta}\cos(r\sin\theta), v = e^{r\cos\theta}\sin(r\sin\theta)$ then prove that $u$ and $v$ satisfy the Cauchy-Reimann equations $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. | 08 |
| | b | Prove that the functions $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ are functionally dependent using the concept of Jacobians and hence find the relation between them. | 08 |
| 7 | a | Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ by changing the order of integration. Represent the region of integration graphically. | 08 |
| | b | Compute the volume of the tetrahedron formed by the planes $x = 0, y = 0, z = 0$ and $6x + 4y + 3z = 12$ using triple integration. | 08 |

**OR**

8 a A plate is in the form of a positive quadrant of the circle $x^2 + y^2 = 1$, the thickness $\rho$ at any point is constant. Find the co-ordinates of the centre of gravity of the plate. 08

b Find the area bounded by the cardioid $r = a(1 + cos\theta)$ above the initial line using double integration. Represent the area graphically. 08

9 a The growth of bacteria ($y$) in a community after $x$ –hours is given by the following table.

| *Hours (x)* | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| *Number of bacteria (y)* | 32 | 47 | 65 | 92 | 132 | 190 |

Find the best value of $a$ and $b$ in the formula $y = ab^x$ to fit this data and estimate the number of bacteria $y$ at $x = 6$ hours by the method of least squares. 08

b Physiological tests of intelligence and of engineering ability were applied to 10 students. Here is a record of ungrouped data showing intelligence ratio $(I.R)$ and engineering ability $(E.R)$. Calculate the coefficient of correlation between intelligence ratio $(I.R)$ and engineering ability $(E.R)$. Also find the regression line of intelligence ratio $(I.R)$ on engineering ability $(E.R)$ and engineering ability $(E.R)$ on intelligence ratio $(I.R)$.

| *Student* | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *I* | *J* |
|---|---|---|---|---|---|---|---|---|---|---|
| *I.R* | 105 | 104 | 102 | 101 | 100 | 99 | 98 | 96 | 93 | 92 |
| *E.R* | 101 | 103 | 100 | 98 | 95 | 96 | 104 | 92 | 97 | 94 |

08

**OR**

10 a If the velocity $V$ (km/hr) and Resistance $R$ (kg/tonne) are related by a relation of the form $R = a + bV^2$, find $a$ and $b$ by the method of least squares with the use of the following data.

| *V* | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| *R* | 8 | 10 | 15 | 21 | 30 |

Compute the value of $R$ when $V = 35$. 08

b The following table gives the distribution of marks in Mathematics of 50 students in an examination. Compute $\mu_1, \mu_2, \mu_3, \mu_4$ for the following distribution. Also find $\beta_1$ and $\beta_2$.

| *Marks* | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| *Number of students* | 1 | 6 | 10 | 15 | 11 | 7 |

08