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RV COLLEGE OF ENGINEERING®
Autonomous Institution affiliated to VTU
First Semester B. E.

DEPARTMENT OF MATHEMATICS
Fundamentals of Linear Algebra, Calculus and Statistics
MODEL QUESTION PAPER-II
Branch: AI, BT, CD, CS, CY, IS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 & 4, 5 & 6, 7 & 8 and 9 & 10.

PART-A

1	1.1	The eigenvalues of $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ are the roots of the ______ equation.	1
	1.2	The system of linear equations $x + 2y = 1, 7x + 14y = 12$ possesses _______solution.	1
	1.3	Curvature of a circle with radius 3 is ______.	1
	1.4	The slope of the tangent of the curve $r = e^{k\theta}$ at $\theta = 0$ is ______.	1
	1.5	The normal equations to fit the straight line of the form $y = ax + b$ is ______.	1
	1.6	For a distribution, the mean is 12 and the variance is 19. Then the second moment of the mean is ______.	1
	1.7	The rank of the matrix $\begin{bmatrix} 1 & 0 & 9 \\ 1 & 2 & -3 \\ 0 & 2 & a \end{bmatrix}$ where a is the last digit of your Roll Number, is ______.	2
	1.8	The angle between the radius vector and tangent for the curve $r = ae^{\theta \cot\alpha}$ is ______.	2
	1.9	Determine $\frac{\partial(r,\theta)}{\partial(x,y)}$, when $x = r\cos\theta$, $y = r\sin\theta$.	2
	1.10	If $w(x,y) = \sin(xy)$, then $\frac{\partial^2 w}{\partial x \partial y}$ at the point $(\pi, 1)$ = ________.	2
	1.11	Evaluate $\int_0^2 \int_1^3 \int_1^2 dx\, dy\, dz$.	2
	1.12	Sketch the domain of integral $\int_2^5 \int_1^3 x(x^2 + y^2) dydx$.	2
	1.13	The following data regarding the heights (y) and the weights (x) of twelve college students are given $\sigma_x = 16.8, \sigma_y = 10.8, \sum(x - \bar{x})(y - \bar{y}) = 2020$. For the given data the regression coefficient of x on y is _______.	2

PART-B

2	a	Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.	5
	b	The temperatures u_1, u_2, u_3 of a metal plate follow the linear equations. $2u_1 + u_2 + 4u_3 = 12, 8u_1 - 3u_2 + 2u_3 = 20, 4u_1 + 11u_2 - u_3 = 33$. Compute u_1, u_2, u_3 using Gauss-Seidel iteration method. Perform four iterations.	5
	c	Determine the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$. Choose the initial vector as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Perform 5 iterations.	6

3	a	Show that for the curve $r = a\sin^3\left(\frac{\theta}{3}\right)$, the cube of radius of curvature varies as square of the radius vector.	8
	b	Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ upto terms containing $\left(x - \frac{\pi}{2}\right)^5$.	8
OR			
4	a	Find the circle of curvature of Folium $x^3 + y^3 = 3xy$ at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.	8
	b	Show that the lemniscate $r^2 = a^2\cos 2\theta$ and the cardioid $r = a(1 + \cos\theta)$ intersect at an angle $\frac{3}{2}\sec^{-1}\left(\frac{1}{1-\sqrt{3}}\right)$.	8

5	a	`(i) If $u = x\,log\,(xy)$ where $x^3 + y^3 + 3xy = 1$ find $\frac{dy}{dx}$ and hence find $\frac{\mathrm{du}}{\mathrm{dx}}$. (ii) If $z = f(y - 3x) + g(y + 2x) + sinx - ycosx$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = ycosx$.	8
	b	The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the unit sphere using Lagrange multiplier.	8
OR			
6	a	(i) The steady state temperature distribution of a rectangular metal plate is governed by $\psi_{xx} + \psi_{yy} = 0$. Show that $\psi(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ satisfies this equation. (ii) At a given instant the sides of a rectangle are 4ft. and 3ft. respectively and they are increasing at the rate of 1.5ft./sec and 0.5ft./sec. respectively. Find the rate at which the area is increasing at that instant.	8
	b	If $x = uv,\ \ y = \frac{u}{v}$, then prove that $JJ' = 1$.	8

7	a	Change the order of integration and hence evaluate $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \left[\frac{x}{\sqrt{x^2+y^2}}\right] dy\,dx$.	8
	b	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{(a^2-r^2)/a} r\,dz\,dr\,d\theta$.	8
OR			
8	a	Find the volume of the tetrahedron $x \geq 0,\ y \geq 0,\ z \geq 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \leq 1$.	8

	b	Using double integration, find the centre of gravity of a lamina in the shape of a quadrant of the curve $\left(\frac{x}{2}\right)^{\frac{2}{3}} + \left(\frac{y}{3}\right)^{\frac{2}{3}} = 1$ and the density being $\rho = 5xy$.	8

9	a	The following pair of observations was noted in an experimental work on cosmic rays. Find by the method of least squares, the best values of a and b for the equation $y = ax^b$ which fits the data	8

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Also extrapolate the value of y when $x = 7$.

	b	A computer while calculating correlation coefficient between two variables X and Y from 25 pairs of observations obtained the following results: $n = 25, \sum X = 125, \sum X^2 = 650, \sum Y = 100, \sum Y^2 = 460, \sum XY = 508$. If was, however, later discovered at the time of checking that he had copied down two pairs as (6,14) and (9,6) while the correct values were (8,12) and (6,8). Obtain the correct value of correlation coefficient for this data. Also find the regression lines of y on x and x on y.	8

OR

10	a	From the following frequency distribution compute first four moments about the mean and also find the measures β_1 and β_2.	8

x	4	8	12	16	20
f	2	2	1	4	1

	b	The method of least squares the best values of a, b, c for the equation $R = a + bV + cV^2$ which fits the data	8

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46

Also extrapolate the value of R when $V = 125$.