Optical Fibres - Numerical Problems

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August 26, 2023

Example 1

Calculate the numerical aperture and maximum acceptance angle of a given optical fibre if the refractive indices of the core and cladding are 1.563 and 1.498 respectively.

$n_1 = 1.563$ and $n_2 = 1.498$.

$$\text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.563^2 - 1.498^2} = 0.446$$

$$\text{accepatnce angle} = \theta_0 = \sin^{-1}(0.446) = 26.49^\circ$$

Example 2

A fiber surrounded by air has a numerical aperture of 0.369. Will the light entering the fiber at an angle of incidence of 25° remain in the fiber or will it escape? Why?

From the given numerical aperture NA $= 0.369$ acceptance angle can be calculated as,

$$\theta_0 = \sin^{-1}(\mathsf{NA}) = \sin^{-1} 0.369 = 21.654^\circ$$

Given angle of incidence $\theta_i = 25^\circ$ is greater than the calculated acceptance angle $\theta_0 = 21.654^\circ$. Hence the ray will escape out into the cladding.

Example 3

An optical fibre in air has NA of 0.20 and a cladding refractive index of 1.59. Determine the acceptance angle for the fibre in water which has a refractive index of 1.33.

With $n_0 = 1$, NA $= 0.20$.

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad 0.20 = \sqrt{n_1^2 - 1.59^2}$$

$$\text{or} \quad n_1^2 = 0.2^2 + 1.59^2 = 2.5681$$

When the fibre is in water, $n_0 = 1.33$.

$$\text{NA} = \frac{1}{1.33}\sqrt{2.5681 - 1.59^2} = \frac{1}{1.33}\sqrt{0.04} = \frac{0.2}{1.33} = 0.1504$$

Therefore acceptance angle is,

$$\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1} 0.1504 = 8.65^\circ$$

Example 4

A cladding is manufactured with the same material as that of the core with refractive index 1.5. Then the cladding is doped to give a fractional index change of 0.0005. Find out a) the refractive index of final cladding, b) the critical internal reflection angle, c) the external critical angle (acceptance angle) with air as the surrounding medium, d) the numerical aperture.

Given $n_1 = 1.5$, $\Delta = 0.0005$

a) To get the refractive index of final cladding:

$$\Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

$$\therefore \quad n_2 = n_1(1 - \Delta) = 1.5(1 - 0.0005) = 1.49925$$

b) To get the critical angle, θ_c:

$$n_1 \sin\theta_c = n_2 \sin 90^\circ$$

or $$\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1.49925/1.5) = \sin^{-1}(0.9995) = 88.188^\circ$$

c) To get the acceptance angle, θ_0 with air as surrounding medium: $n_0 = 1$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \sqrt{1.5^2 - 1.49925^2} = 0.04743$$

and $$\theta_0 = \sin^{-1}(0.04743) = 2.7186^\circ$$

d) To get Numerical aperture, NA, with $n_0 = 1$:

$$\text{NA} = n_1\sqrt{2\Delta} = 1.5\sqrt{2 \times 0.0005} = 0.0474$$

Example 5

A step index fibre in air has a NA of 0.12, a core of refractive index of 1.42 and diameter 2mm. Determine the normalized frequency (V number) for the fibre when light of wavelength 0.8μm is transmitted.

Given: NA =0.12, $n_1 = 1.42$, $n_0 = 1$, diameter $d = 2\text{mm} = 2 \times 10^{-3}\text{m}$,
$\lambda = 0.8\mu\text{m} = 0.8 \times 10^{-6}\text{m}$

$$V = \frac{2\pi r}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi d}{\lambda}\text{NA} = \frac{3.142 \times 2 \times 10^{-3}\text{m}}{0.8 \times 10^{-6}\text{m}} \times 0.12 = 942.6$$

Example 6

Find the core radius necessary for a step index optical fibre to be operated in multi mode mode operation with number of modes equal to 2. Given that the wavelength of light used is 850μm, $n_1 = 1.48$ and $n_2 = 1.47$. Also find out numerical aperture and maximum acceptance angle of this fibre.

Given number of modes = 2, $\lambda = 850 \times 10^{-6}$m, $n_1 = 1.48$ and $n_2 = 1.47$.

For step index fibre, the number of modes $\approx V^2/2$.

$$\therefore \quad 2 = V^2/2 \qquad \text{or} \qquad V = \sqrt{2 \times 2} = 2$$

The V number is given by,

$$V = \frac{2\pi r}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad r = \frac{V\lambda}{2\pi\sqrt{n_1^2 - n_2^2}} = \frac{2 \times 850 \times 10^{-6}\text{m}}{2 \times 3.142 \times \sqrt{1.48^2 - 1.47^2}} = 1.575 \times 10^{-3}\text{m} = 1.575\,\text{mm}$$

NA is given by, with $n_0 = 1$:

$$\text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.47^2} = 0.172$$

Maximum acceptance angle is:

$$\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1}(0.172) = 9.904^\circ$$

Example 7

A graded index optical fibre has core diameter of 60m, core index of 1.48 and cladding index of 1.41. If the wavelength of the light source is 0.8µm determine the number of modes present in the fiber.

Given: core diameter $d = 60\mu m$, $n_1 = 1.48$, $n_2 = 1.41$, $\lambda = 0.8\mu m$, number of modes = ?

V number for the fibre is,

$$V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{3.142 \times 60\mu m}{0.8\mu m}\sqrt{1.48^2 - 1.41^2} = 105.976$$

Number of modes for a graded index fibre is,
$\approx V^2/4 = 105.976^2/4 = 2807.748$

Example 8

The angle of acceptance of an optical fibre is 30° when kept in air. Find the angle of acceptance when it is in kept in water of refractive index 1.33.

Given: acceptance angle for air, $\theta_0 = 30^\circ, n_0' = 1.33$.

We had, numerical aperture in a medium as,

$$\sin\theta_0 = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

a) For air:

$$\sin 30^\circ = \frac{1}{1}\sqrt{n_1^2 - n_2^2}$$

b) For water:

$$\sin\theta_0' = \frac{1}{1.33}\sqrt{n_1^2 - n_2^2}$$

Dividing the above equations b) from a),

$$\frac{\sin\theta_0'}{\sin 30^\circ} = \frac{1}{1.33}$$

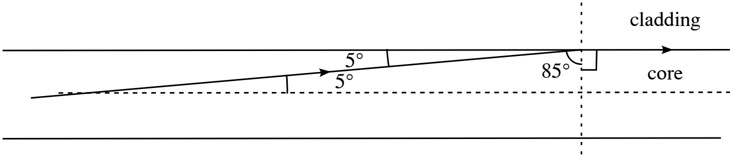
$$\text{or} \qquad \sin\theta_0' = \frac{\sin 30^\circ}{1.33} = \frac{0.5}{1.33} = 0.3759$$

$$\therefore \qquad \theta_0' = \sin^{-1}(0.375) = 22.08^\circ$$

Example 9

An optical fibre of refractive index 1.45 is to be cladded with another glass to ensure internal reflection that will contain traveling within 5° of the fiber axis, what maximum index of refraction is allowed for the cladding?

$n_1 = 1.45$, angle between the ray and the axis = 5°.



For incidence at critical angle: Angle of incidence = $90^\circ - 5^\circ = 85^\circ$

$$n_1 \sin 85^\circ = n_2 \sin 90^\circ$$

$$\therefore \qquad n_2 = n_1 \frac{\sin 85^\circ}{\sin 90^\circ} = 1.4445$$

Example 10

The attenuation of light in an optical fibre is estimated to be 2.2 db/km. What fraction of the initial intensity remains after 2 km?

Given $\alpha = 2.2\,\mathrm{db/km}$, $L = 2\,\mathrm{km}$.

The attenuation coefficient is given by,

$$\alpha = -\frac{10}{L}\log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \qquad \text{(in dB/km)}$$

$$\text{or} \qquad \log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) = -\frac{\alpha L}{10\text{db}}$$

$$\text{or} \qquad \frac{P_{\text{out}}}{P_{\text{in}}} = 10^{-\alpha L/10} = 10^{-2.2\,\text{db km}^{-1}\times 2\,\text{km}/10\,\text{db}} = 10^{-0.44} = 0.363$$

Example 11

Find the attenuation in an optical fibre of length 200m when a light signal of power 100mW emerges out of the fibre with a power of 90mW.

Attenuation is given by,

$$\alpha = -\frac{10}{L}\log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \qquad \text{(in dB/km)}$$

$$\alpha = -\frac{10\text{db}}{200\text{m}}\log\left(\frac{90\text{mW}}{100\text{mW}}\right) = -\frac{10\text{db}}{0.2\text{km}}\log 0.9 = 2.288\,\text{db/km}$$

Example 12

An optical signal lost 15% of its power after traveling 500m in an optical fibre. What is the attenuation in this fibre?

Given that for $L = 500$m the power lost is 15%. Therefore,
$P_{\text{out}} = (100 - 15)\%$ of $P_{\text{in}} = 85\%$ of $P_{\text{in}} = 0.85P_{\text{in}}$.

$$\alpha = -\frac{10}{L}\log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \qquad \text{(in dB/km)}$$

$$\alpha = -\frac{10\text{db}}{500\text{m}}\log\left(\frac{0.85P_{\text{in}}}{P_{\text{in}}}\right) = -\frac{10\text{db}}{0.5\text{km}}\log 0.85 = 1.4116\,\text{db/km}$$