DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	TEST-I	Maximum marks: 50
Course code: 22MA21C	Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C	Time: 9:30AM-11:00AM Date: 10-07-2023

Scheme and Solutions

Q.No	PART -B	Marks
1.	Cumulative frequency table Marks less than (x): 30, 40, 50, 60, 70, 80 Number of students (y): 35, 84, 146, 220, 260, 280 Difference table x | y | Δy | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ 30 | 35 | | | | | | | **49** | | | | 40 | 84 | | **13** | | | | | 62 | | **-1** | | 50 | 146 | | 12 | | **-45** | | | 74 | | -46 | | **105** 60 | 220 | | -34 | | **60** | | | 40 | | **14** | | 70 | 260 | | **-20** | | | | | **20** | | | | 80 | 280 | | | | | $p = \frac{x-x_0}{h} = \frac{45-30}{10} = 1.5$ and $p = \frac{x-x_n}{h} = \frac{65-80}{10} = -1.5$ Using Newton's forward interpolation formula, we get $y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$ $y(45) = 111.15 \approx 111$ Using Newton's backward interpolation formula, we get $y = y_0 + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$ $y(65) = 246.0117 \approx 246$ The number of students with marks between 45 and 65 is $246 - 111 = 135$.	1 3 1 2 2 1
2(a)	$y = \frac{(x-367)(x-378)(x-387)(x-399)}{(361-367)(361-378)(361-387)(361-399)}154.9 + \frac{(x-361)(x-378)(x-387)(x-399)}{(367-361)(367-378)(367-387)(367-399)}167.9 + \frac{(x-361)(x-367)(x-387)(x-399)}{(378-361)(378-367)(378-387)(378-399)}191 + \frac{(x-361)(x-367)(x-378)(x-399)}{(387-361)(387-367)(387-378)(387-399)}212.5 + \frac{(x-361)(x-367)(x-378)(x-387)}{(399-361)(399-367)(399-378)(399-387)}244.2$ $y(x) = -0.00002x^4 + 0.0314x^3 - 17.9744x^2 + 4568.21x - 435168 = 178.1827$	2 4

2(b)	$x = \frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)} + \frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)}3 + \frac{(y-10)(y-24)(y-129)}{(54-10)(54-24)(54-129)}5 + \frac{(y-10)(y-24)(y-54)}{(129-10)(129-24)(129-54)}8$ When $y = 100, x = 5.9199$	2 2
3(a)	Difference table $\left(\frac{dp}{dv}\right)_{v=4} = \frac{1}{h}\left[\Delta p_0 - \frac{1}{2}\Delta^2 p_0 + \frac{1}{3}\Delta^3 p_0 - \frac{1}{4}\Delta^4 p_0 + \cdots\right] = \frac{1}{2}\left[-17.4 - \frac{8.8}{2} - \frac{3.9}{3}\right] = -11.55$ $\left(\frac{dp}{dv}\right)_{v=8} = \frac{1}{h}\left[\nabla p_n + \frac{1}{2}\nabla^2 p_n + \frac{1}{3}\nabla^3 p_n + \frac{1}{4}\nabla^4 p_n + \cdots\right] = \frac{1}{2}\left[-8.6 + \frac{8.8}{2} - \frac{36.1}{3}\right] = -8.11$	2 2 2
3(b)	Auxiliary equations $m^3 - 8m^2 + 5m + 50 = 0$ Roots $m = -2, 5, 5$ $x = c_1 e^{-2t} + (c_2 + c_3 t)e^{5t}$	1 1 2
4	Auxiliary equations $2m^2 + m - 1 = 0$, Roots $m = \frac{1}{2}, -1$ $C.F = c_1 e^{x/2} + c_2 e^{-x}$ $PI = \frac{xe^{x/2}}{3} - \frac{1}{2} + \frac{33\cos(4x) - 4\sin(4x)}{2210} - (x^2 + 2x + 6)$ $y = CF + PI$	1 1 2+3+2 1
5(a)	Auxiliary equations $m^2 - 4m + 13 = 0$, Roots $m = 2 \pm 3i$ $C.F = e^{2x}(c_1 \cos(3x) + c_2 \sin(3x))$ $PI = \frac{1}{D^2 - 4D + 13} e^{2x}\sin(3x) = e^{2x}\frac{1}{D^2 + 9}\sin(3x) = -\frac{xe^{2x}\cos(3x)}{6}$ $y = CF + PI = e^{2x}(c_1 \cos(3x) + c_2 \sin(3x)) - \frac{xe^{2x}\cos(3x)}{6}$	1 1 2 1
5(b)	Auxiliary equations $m^2 + \mu = 0$, Roots $m = \pm\sqrt{\mu}i$ $x = c_1 \cos(\sqrt{\mu}t) + c_2 \sin(\sqrt{\mu}t)$ $x' = -c_1\sqrt{\mu}\sin(\sqrt{\mu}t) + c_2\sqrt{\mu}\cos(\sqrt{\mu}t)$ $c_1 = -a$ and $c_2 = 0$ $x = -a\cos(\sqrt{\mu}t)$	1 1 2 1

v	p	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
2	105				
		-62.3			
4	42.7		44.9		
		-17.4		-36.1	
6	25.3		**8.8**		32.2
		-8.6		**-3.9**	
8	16.7		4.9		
		-3.7			
10	13				