

Solution to Number theory tutorial

1. $\gcd(143, 227) = 1$. Find x and y using Euclid Alg and verify.
2. $\gcd(272, 1479) = 17$
3. Remainder is 1.
4. Last digit is 9.
5. Last 2 digits is 56
6. Remainder is 6.
7. 4 and 1440
8. 14 and 16256
9. Solution, $x \equiv 14 \pmod{25}$
10. Solution, $x \equiv 18 \pmod{29}$
11. Solutions are congruent to 6, 13, 20 $\pmod{21}$
12. There are five solutions one of them is congruent to 11 mod 625 (find others)
13. 1009
14. Use RSA calculator and verify.

Soln 5) $36^6 \equiv 36 \pmod{100}$

$$\Rightarrow (36^6)^6 \equiv 36^6 \equiv 36 \pmod{100}$$

$$\Rightarrow (36^{36})^6 \equiv 36^6 \equiv 36 \pmod{100}$$

$$\Rightarrow 36^{216} \equiv 36 \pmod{100}$$

$$\begin{aligned}\therefore 36^{233} &= 36^{216} \cdot 36^{17} \equiv 36 \cdot 36^{17} \\ &= 36^{18} \\ &\equiv 36^3 \pmod{100} \\ &\equiv 56 \pmod{100}\end{aligned}$$

\therefore Last two digits : 56