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**RV COLLEGE OF ENGINEERING®**

Autonomous Institution affiliated to VTU

First Semester B. E.

**DEPARTMENT OF MATHEMATICS**

Fundamentals of Linear Algebra, Calculus and Statistics

**MODEL QUESTION PAPER-I**

Branch: AI, BT, CD, CS, CY, IS

**Time: 03 Hours****Maximum Marks: 100****Instructions to candidates:**

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 & 4, 5 & 6, 7 & 8 and 9 & 10.

**PART-A**

|   |      |   |   |
|---|------|---|---|
| 1 | 1.1  | In Gauss-Seidel method, the first approximate solution to the system of equations $4x - y = 3$ and $3x + 7y = -2$ with initial value $[0,0]$ is _____.  | 1 |
|   | 1.2  | If $\sqrt{5}$ is the eigenvalue of an orthogonal matrix $B$ , then _____ is also its eigenvalue.  | 1 |
|   | 1.3  | If rank of the matrix $A = \begin{bmatrix} 2 & 7 & 3 \\ 0 & 5 & 1 \\ k & 7 & 3 \end{bmatrix}$ is 2, then the value of $k =$ _____.  | 2 |
|   | 1.4  | If $(2, 3)$ are the coordinates of the centre of curvature whose curvature is $\sqrt{8}$ , then the equation of the circle of curvature is _____.   | 1 |
|   | 1.5  | The transformation of the polar curve $r = a(1 + \cos \theta)$ in cartesian system is _____.  | 1 |
|   | 1.6  | The Maclaurin series expansion for $\sinh x$ is _____.  | 2 |
|   | 1.7  | If $z = (\sin x)^y$ , then $\frac{\partial z}{\partial y} =$ _____.   | 2 |
|   | 1.8  | Given that $w = 2uv - 3u^2v$ and $u$ increases at the rate of 2cm/sec. Find the rate at which $v$ changes at the instant when $u = 3$ cm and $v = 1$ cm in order that $w$ shall be neither increasing nor decreasing. | 2 |
|   | 1.9  | The value of $\int_0^1 \int_{x^2}^{2-x} dy dx$ is _____.  | 2 |
|   | 1.10 | Sketch the region of integral for the function $f(x, y)$ taken over the area bounded by $x = 0$ to $x = a$ and $y = x$ to $y = a$ .   | 2 |
|   | 1.11 | For a distribution the mean is 10 and the variance is 16 then the second moment about the mean is _____.  | 1 |
|   | 1.12 | If the two regression coefficients are -0.4 and -0.9, then the correlation coefficient is _____.  | 1 |
|   | 1.13 | For a statistical data with $n = 5$ , $\sum x = 30$ , $\sum y = 40$ , $\sum x^2 = 220$ and $\sum xy = 266$ , the straight line of best fit by the method of least squares is _____.                                   | 2 |

## PART-B

|   |   |   |   |
|---|---|---|---|
| 2 | a | Estimate the values of $p$ and $q$ for which the system of linear equations $x + y + z = 1$ , $2x + y + 4z = 2$ , $4x + y + pz = q$ has (i) a unique solution (ii) no solution (iii) an infinite number of solutions.   | 8 |
|   | b | Google's page rank to find the most powerful page for a particular query is performed using eigen values and eigen vectors. Identify the dominant eigen value and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by taking the initial approximation $[1 \ 0 \ 0]^T$ . Perform 6 iterations. | 8 |

|   |   |   |   |
|---|---|---|---|
| 3 | a | For the curve $x = b(\cos \theta + \log(\tan \frac{\theta}{2}))$ , $y = b \sin \theta$ , show that the radius of curvature at any point $\theta$ is $b \cot \theta$ . | 8 |
|   | b | Find the angle between the radius vector and the tangent for the curve $r^m = a^m(\cos m\theta + \sin m\theta)$   | 8 |

**OR**

|   |   |   |   |
|---|---|---|---|
| 4 | a | Find the circle of curvature at the point $(a/4, a/4)$ for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ . Also show that sum of the co-ordinates of the centre of curvature at any point $(x, y)$ is $3(x + y)$ . | 8 |
|   | b | Expand $\log(1 + \sin^2 x)$ in ascending powers of $x$ as far as the term containing $x^6$ .  | 8 |

|   |   |   |   |
|---|---|---|---|
| 5 | a | The steady state temperature of a circular metal plate is $u = r^2 \cos(2\theta) - r \cos(\theta) + 2$ . Show that ' $u$ ' satisfies the Laplace equation $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$ . | 5 |
|   | b | Compute $J\left(\frac{u,v}{x,y}\right)$ , given $u + v = e^x \cos y$ & $u - v = e^x \sin y$ .   | 5 |
|   | c | A tank is in the form of a rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.  | 6 |

**OR**

|   |   |   |   |
|---|---|---|---|
| 6 | a | If $u = 3x + 2y - z$ , $v = x - 2y + z$ and $w = x(x + 2y - z)$ , then show that $u$ , $v$ , $w$ are functionally related   | 5 |
|   | b | If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ , then prove that $6 \frac{\partial u}{\partial x} + 4 \frac{\partial u}{\partial y} + 3 \frac{\partial u}{\partial z} = 0$ . | 5 |
|   | c | Show that by Lagrange's method of undetermined multipliers, the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.                    | 6 |

|   |   |  |   |
|---|---|--|---|
| 7 | a | Evaluate $\iint_R xy \, dx \, dy$ where $R$ is the triangular region bounded by the axes of coordinates and the line $\frac{x}{a} + \frac{y}{b} = 1$ . | 8 |
|   | b | Find the value of $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) \, dy \, dx \, dz$ .  | 8 |

**OR**

|   |   |   |   |
|---|---|---|---|
| 8 | a | Change the order of integration and hence evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y) \, dx \, dy$ | 8 |
|   | b | Using double integral, find the area enclosed by the curve $r = a(1 + \cos \theta)$ .                 | 8 |

9

a

In an experiment in which the growth of duckweed under certain conditions was measured, the following results were obtained

|                |    |    |     |     |     |     |
|----------------|----|----|-----|-----|-----|-----|
| Weeks ( $x$ )  | 2  | 3  | 4   | 5   | 6   | 7   |
| Weight ( $y$ ) | 52 | 77 | 135 | 211 | 326 | 550 |

Assuming the relationship of the form  $y = ae^{bx}$  find the best values of  $a$  and  $b$  by the method of least squares.

8

b

The distance (in km) of 60 engineers from residence to their place of work were found as follows:

|                  |     |      |       |       |       |       |
|------------------|-----|------|-------|-------|-------|-------|
| Distance (in km) | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 |
| No. of Engineers | 8   | 11   | 15    | 12    | 9     | 5     |

Compute first four moments about the mean. Also find the measures  $\beta_1$  and  $\beta_2$  for the distribution and comment on the nature of distribution.

8

**OR**

|     |      |  |      |      |      |      |      |      |      |     |      |      |      |      |      |      |   |
|-----|------|--|------|------|------|------|------|------|------|-----|------|------|------|------|------|------|---|
| 10  | a    | <p>Fit a second-degree polynomial <math>V = a + bT + cT^2</math> for the following data using least square method</p> <table><tr><td><math>T</math></td><td>1</td><td>2</td><td>3</td><td>5</td><td>7</td><td>10</td></tr><tr><td><math>V</math></td><td>2.31</td><td>2.01</td><td>3.80</td><td>1.55</td><td>1.41</td><td>2.78</td></tr></table> <p>Also find the value of <math>V</math> at <math>T = 12</math>.</p>  | $T$  | 1    | 2    | 3    | 5    | 7    | 10   | $V$ | 2.31 | 2.01 | 3.80 | 1.55 | 1.41 | 2.78 | 8 |
|     | $T$  | 1  | 2    | 3    | 5    | 7    | 10   |      |      |     |      |      |      |      |      |      |   |
| $V$ | 2.31 | 2.01   | 3.80 | 1.55 | 1.41 | 2.78 |      |      |      |     |      |      |      |      |      |      |   |
|     | b    | <p>The course average <math>x</math> just before a final exam and the score <math>y</math> on the final exam were recorded for six randomly selected students in a large Mathematics class results shown in the table</p> <table><tr><td><math>x</math></td><td>69.3</td><td>87.7</td><td>50.5</td><td>51.9</td><td>82.7</td><td>70.5</td></tr><tr><td><math>y</math></td><td>56</td><td>89</td><td>55</td><td>49</td><td>61</td><td>66</td></tr></table> <p>Compute the correlation coefficient for this data. Also find the regression lines of <math>y</math> on <math>x</math> and <math>x</math> on <math>y</math>.</p> | $x$  | 69.3 | 87.7 | 50.5 | 51.9 | 82.7 | 70.5 | $y$ | 56   | 89   | 55   | 49   | 61   | 66   | 8 |
| $x$ | 69.3 | 87.7   | 50.5 | 51.9 | 82.7 | 70.5 |      |      |      |     |      |      |      |      |      |      |   |
| $y$ | 56   | 89   | 55   | 49   | 61   | 66   |      |      |      |     |      |      |      |      |      |      |   |