USN 1RV22AI007

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

II Semester B. E. Examinations Oct-2023

(Common to AI, BT, CS, CY, CD & IS)

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Formula book to be provided.

PART-A

1	1.1	General solution of $4x \equiv 7 \bmod 5$ is ________.	01
	1.2	The number of integers less than 181 that are relatively prime to 181 is ________.	01
	1.3	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $\lvert\vec{r}\rvert = r$, then $\nabla r =$ ________.	01
	1.4	If $\vec{F}(t)$ has a constant magnitude then $\vec{F}(t).\frac{d\vec{F}(t)}{dt}$ is ________.	01
	1.5	If $\vec{F}$ represent the velocity of fluid, the $\oint_c \vec{F}.d\vec{r}$ represents ________.	01
	1.6	If R is the projection of surface in XY –plane, then $ds =$______.	01
	1.7	Particular integral of $\frac{d^2x}{dt^2} + \frac{6}{7}\left(x - \sqrt{2}\right) = 0$ is ________.	01
	1.8	If the roots of the auxiliary equation are $\pm i$ and 2 then the corresponding differential equation is ______.	01
	1.9	The 3^{rd} order difference of 3^{rd} degree polynomial is ______.	01
	1.10	The value of $\Delta^3[(1-x)(1-3x)(1-5x)]$ taking the interval of differencing $h = 1$ is ________.	01
	1.11	The sum of all positive divisors of 8620 is ________.	02
	1.12	The velocity and acceleration of a particle along a curve $x = t^2$, $y = 3t^2$, $z = e^t$ at $t = 1$ is ________.	02
	1.13	If $\vec{F} = (x+y)\hat{\imath} + (2x-y)\hat{\jmath}$ evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line from $A(0,0)$ to $B(0,1)$.	02
	1.14	If $y = e^{-t}$ is the solution of the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + py = 0$, then the value of p is ________.	02
	1.15	Using suitable interpolation, fit a polynomial for the data.	02

x	-1	2	4
y	-5	4	0

PART-B

2	a	Find the greatest common divisor d of the number 4076 and 1024 using Euclid's algorithm and then obtain the integers x and y to satisfy $4076x + 1024y = d$.	06
	b	Given the public key $(e, n) = (7, 55)$, encrypt plain text $M\ I\ T$, where the alphabets $\{A, B, C, \ldots. X, Y, Z\}$ are assigned the numbers $\{5, 6, \ldots. 29, 30\}$. Find the cipher text and the private key d.	06
	c	Compute the remainder when 3^{247} is divided by 17.	04
3	a	Find the angle between the tangents to the curve $x = \left(t - \frac{t^2}{2}\right)$, $y = t^2$ and $z = \left(t + \frac{t^2}{2}\right)$ at $t = \pm 1$.	08
	b	Prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that $\vec{r}/r^3$ is solenoidal.	08

OR

4 a Show that an electromagnetic field $\vec{f} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative and find the scalar potential ϕ such that $\vec{f} = \nabla\phi$. Also show that $div\, \vec{f} = \nabla^2\phi$. 10

b Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $x\, log_e z - y^2 = -4$ at $(-1, 2, 1)$. 06

5 a Using line integral, compute the work done by a force $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ when it moves a particle from the point $(0,0,0)$ to $(2,1,3)$ along the curve $x = 2t^2,\ y = t,\ z = 4t^2 - t$ from $t = 0$ to 1. 08

b Evaluate using divergence theorem: $\iint_s \left((x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}\right).\hat{n}\, ds$ where S is the surface of the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. 08

OR

6 a Verify Green's theorem in the plane for $\oint\{(x^2 + y)dx - xy^2 dy\}$ taken around the boundary of the rectangle whose vertices are $(0,0)$, $(a,0)$, (a,b) and $(0,b)$. 08

b Evaluate by Stokes theorem $\oint_c ((x + y)dx + (2x - z)dy + (y + z)dz)$, C is the boundary of the triangular surface with vertices $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$. 08

7 a Obtain the radial displacement x in a rotating disc at a distance s from the axis, given by the differential equation $\frac{d^2x}{ds^2} + \frac{1}{s}\frac{dx}{ds} + \frac{x}{s^2} = \frac{\log_e s}{s^2}\sin(\log_e s)$ where $s > 0$. 08

b Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ using method of variation of parameters. 08

OR

8 a Obtain the solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$ given that $y(0) = \frac{1}{9},\ y'(0) = \frac{2}{9}$ 08

b Solve $(D^2 - 1)y = \sin x + (1 + x^2)e^x$ 08

9 a The following table gives the values of pressure P and specific volume V of saturated steam:

V	40	50	60	70	80
P	304	276	250	204	184

Find the rate of change of pressure with respect to volume at $V = 50$ and $\frac{d^2V}{dP^2}$ at $V = 70$. 08

b The velocity of a rocket as a function of time is given as follows: $v(1) = 2,\ v(3) = 10,\ v(4) = 17$. Obtain the functional representation of velocity as a function of time using Lagrange's interpolation formula. Also find the velocity at time 3.8 *units*. 08

OR

10 a The population of a town is given by the table

Years	1961	1971	1981	1991	2001
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using appropriate interpolation formula, calculate the increase in population from the year 1955 to 1985. 08

b Apply Lagrange's formula inversely to find a root of the equation $f(x) = 0$ given that $f(30) = -30,\ f(34) = -13,\ f(38) = 3,\ f(42) = 18$. 08