# DEPARTMENT OF MATHEMATICS

| Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS | IMPROVEMENT CIE | Maximum marks: 50 |
|---|---|---|
| Course code: 22MA21C | Second semester 2022-2023<br>Physics Cycle<br>Branch: AI, BT, CD, CS, CY, IS, SPARK-C | Time: 02:00PM-3:30PM<br>Date: 06-09-2023 |

| Sl. No. | Questions | Marks |
|---|---|---|
| 1. (a) | $\frac{d\vec{r}}{dt} = 3t^2\hat{\imath} + 2t\hat{\jmath} + 2\hat{k} \Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{t=1} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$<br>$\frac{d^2\vec{r}}{dt^2} = 6t\hat{\imath} + 2t\hat{\jmath} + 0\hat{k} \Rightarrow \left(\frac{d^2\vec{r}}{dt^2}\right)_{t=1} = 6\hat{\imath} + 2\hat{\jmath}$<br>Component of velocity $= \frac{11}{\sqrt{11}}$<br>Component of acceleration $= \frac{8}{\sqrt{11}}$ | 2<br>1<br>1<br>1 |
| 1. (b) | $\nabla\phi = 6x^2y^2z^4\hat{\imath} + 4x^3yz^4\hat{\jmath} + 8x^3y^2z^3\hat{k}$<br>$div(\vec{f}) = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$<br>$div(\vec{f})_{(1,2,-1)} = 48 + 4 + 96 = 148$ | 2<br>2<br>1 |
| 2. (a) | $\nabla\phi = (ay^2 + 3cz^2x^2)\hat{\imath} + (2axy + bz)\hat{\jmath} + (by + 2czx^3)\hat{k}$<br>$\nabla\phi_{(1,2,-1)} = (4a + 3c)\hat{\imath} + (4a - b)\hat{\jmath} + (2b - 2c)\hat{k}$<br>Directional derivative of $\phi$ at $(1,2,-1)$ in the direction parallel to $z$ axis is given by<br>$\nabla\phi.\hat{k} = 64$<br>$\Rightarrow 2b - 2c = 64$ and $4a + 3c = 0, \quad 4a - b = 0$<br>Solving $a = 6,\ b = 24, c = -8$ . | 2<br>1<br>1+1+1 |
| 2. (b) | $r^2 = x^2 + y^2 + z^2$ and $\frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial y} = \frac{y}{r}, \frac{\partial r}{\partial z} = \frac{z}{r}$<br>$\nabla r^n = nr^{n-1}\frac{x}{r}\hat{\imath} + nr^{n-1}\frac{y}{r}\hat{\jmath} + nr^{n-1}\frac{z}{r}\hat{k} = nr^{n-2}\vec{r}$ | 1<br>1+2 |
| 3 | $curl\ \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix} = (c+1)\hat{\imath} - (4-a)\hat{\jmath} + (b-2)\hat{k} = \vec{0}$<br>$a = 4\ , b = 2$ and $c = -1$<br>$\vec{F} = (x + 2y + 4z)\hat{\imath} + (2x - 3y - z)\hat{\jmath} + (4x - y + 2z)\hat{k}$<br>$\frac{\partial\phi}{\partial x} = x + 2y + 4z,\ \frac{\partial\phi}{\partial y} = 2x - 3y - z,\ \frac{\partial\phi}{\partial z} = 4x - y + 2z$ | 1<br>3<br>1 |

| | | |
|---|---|---|
| | $d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$ | 1 |
| | $d\phi = d\left(\frac{x^2}{2}\right) + d(2xy) + d(4xz) - d\left(\frac{3}{2}y^2\right) - d(yz) + d(z^2)$ | 3 |
| | $\phi = \frac{x^2}{2} + 2xy + 4xz - \frac{3}{2}y^2 - yz + z^2 + c$ | 1 |
| 4 | Difference table | 3 |

| $t$ | $\theta$ | $\Delta\theta$ | $\Delta^2\theta$ | $\Delta^3\theta$ | $\Delta^4\theta$ |
|---|---|---|---|---|---|
| 1 | 85.3 | | | | |
| | | **-10.8** | | | |
| 3 | 74.5 | | **3.3** | | |
| | | -7.5 | | **-2.3** | |
| 5 | 67 | | 1 | | **1.6** |
| | | -6.5 | | **-0.7** | |
| 7 | 60.5 | | **0.3** | | |
| | | **-6.2** | | | |
| 9 | 54.3 | | | | |

$p = \frac{t-t_0}{h} = \frac{2-1}{2} = 0.5$ and $p = \frac{t-t_n}{h} = \frac{8-9}{2} = -0.5$ — 1

Using Newton's forward interpolation formula, we get

$y = \theta_0 + p\Delta\theta_0 + \frac{p(p-1)}{2!}\Delta^2\theta_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3\theta_0 + \cdots$

$\theta(2) = 79.28125$ — 2

Using Newton's backward interpolation formula, we get

$y = \theta_n + p\nabla\theta_n + \frac{p(p+1)}{2!}\nabla^2\theta_n + \frac{p(p+1)(p+2)}{3!}\nabla^3\theta_n + \cdots$

$\theta(8) = 57.34375$ — 2

$\frac{d\theta}{dt} = \frac{1}{h}\left[\nabla\theta_n + \frac{1}{2}\nabla^2\theta_n + \frac{1}{3}\nabla^3\theta_n + \frac{1}{4}\nabla^4\theta_n + \cdots\right] = \frac{1}{2}\left[-6.2 + \frac{0.3}{2} - \frac{0.7}{3} + \frac{1.6}{4}\right] = -2.9416$ — 2

| | | |
|---|---|---|
| 5. (a) | $= \frac{(x-7)(x-10)(x-12)(x-15)}{(6-7)(6-10)(6-12)(6-15)}3 + \frac{(x-6)(x-10)(x-12)(x-15)}{(7-6)(7-10)(7-12)(7-15)}10 +$ $\frac{(x-6)(x-7)(x-12)(x-15)}{(10-6)(10-7)(10-12)(10-15)}43 + \frac{(x-6)(x-7)(x-10)(x-15)}{(12-6)(12-7)(12-10)(12-15)}75$ $+ \frac{(x-6)(x-7)(x-10)(x-12)}{(15-6)(15-7)(15-10)(15-12)}138$ | 3 |
| | $y(x) = x^2 - 6x + 3$ | 2 |
| | $y(11) = 58$ | 1 |
| 5. (b) | $x = \frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)} + \frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)}3$ $+ \frac{(y-10)(y-24)(y-129)}{(54-10)(54-24)(54-129)}5 + \frac{(y-10)(y-24)(y-54)}{(129-10)(129-24)(129-54)}8$ | 2 |
| | When $y = 100, x = 5.9199$ | 2 |