USN

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

II Semester B. E. Examinations Oct/Nov-2022

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

Common to all branches

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART A

1	1.1	If A is a square matrix of order n and A is a non-singular, then the rank of the matrix A is ___n___.	01
	1.2	The system of equations AX = B is inconsistent if ___$\rho(A) \neq \rho(A:B)$___.	01
	1.3	If $x = e^{-3t}$ is the solution of the differential equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + kx = 0$, then k = ___3___	01
	1.4	The particular integral of the differential equation $\frac{d^2x}{dt^2} + \frac{1}{a}(x - b) = 0$ is ___b___.	01
	1.5	The value of $\Delta^3[(1 + 3x)(1 - 5x)(1 - 4x)]$ taking interval of differencing h = 1 is ___360___.	01
	1.6	While applying Simpson's three-eight rule, the number of sub intervals should be taken as ___multiple of 3___	01
	1.7	The root of the equation $x\log_{10}(x) = 2$ lies in the interval ___(1, 4) or (3, 4)___.	01
	1.8	The coefficient of x^2 in the Maclaurin series expansion of cos(3x) is ___$-9/2$___.	01
	1.9	Find the eigenvalues of the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$. $\lambda = -1, -6$. & Ch. eq ①	02
	1.10	The complementary function of the differential equation is $(c_1 + c_2x)e^{2x}$, then the Wronskian is ________. 1 1 . e^{4x}	02
	1.11	One of the solution of the Lagrange's partial differential $(x + yz)\left(y\frac{\partial z}{\partial y} - xy\frac{\partial z}{\partial x}\right) = y^2$ is ________. A & ①, Sol. ① . $\log x + y = c$.	02
	1.12	The partial differential equation obtained by eliminating arbitrary constants a and b from the equation z = axy + b is ________. p & q ①,	02
	1.13	Construct the difference table of the polynomial $y = x^3 + 5x - 7$ for x = −1, 0, 1, 2. finding y ① & diff ①	02
	1.14	The approximate solution of $\frac{dy}{dx} = 3x + y^2$ with y(0) = 1 at x = 0.1 using Taylor series up to first degree term is ________. $y(x) = 1 + x$. ① $y(0.1) = 1.1$ ①	02

PART B

2	a	Apply Gauss Jordon method to solve the system of equations: $$a + 3b + 2c = 2$$ $$2a + 7b + 7c = -1$$ $$2a + 5b + 2c = 7.$$	05
	b	Find the dominant eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by Rayleigh power method taking the initial vector as $[1 \quad 0 \quad 0]^T$. Perform 4 iterations.	05
	c	Find the values of λ for which the system $$x + y + z = 1$$ $$x + 2y + 4z = \lambda$$ $$x + 4y + 10z = \lambda^2$$ has a solution. Solve the system for each possible case.	06
3	a	Obtain the general solution of the differential equation $2\frac{d^3x}{dt^3} - 3\frac{d^2x}{dt^2} + 6\frac{dx}{dt} - 9x = cos^2\left(\frac{\sqrt{3}\,t}{2}\right) + t.$	08
	b	Reduce the differential equation $x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + \frac{4}{x}y = x^2 \log_e x$, where $x > 0$, to a linear differential equation with constant coefficients and hence solve the same.	08
		OR	
4	a	Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}$ using the method of variation of parameters.	08
	b	The current in an LRC circuit is governed by the differential equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E(t)$. A circuit has in series an electromotive force given by $E(t) = 100\sin(60t)$ Volts, a resistor of 2 Ohms, an inductor of 0.1 Henry and a capacitor of $\frac{1}{260}$ Farads. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t > 0$.	08
5	a	Solve $x(x^2 + 3y^2)\frac{\partial z}{\partial x} - y(3x^2 + y^2)\frac{\partial z}{\partial y} = 2z(y^2 - x^2)$.	08
	b	Use the method of separation of variables to solve the partial differential equation $u_{xx} - 2u_x + u_y = 0$.	08
		OR	
6	a	Form the partial differential equations by: i) Eliminating arbitrary constants m and n from $$z = xy + y\sqrt{x^2 - m^2} + n$$ ii) Eliminating arbitrary function from $z = (x + y)f(x^2 - y^2)$	08
	b	Obtain various possible solutions of one-dimensional heat equation by the method of separation of variables and discuss the suitable solution.	08

for method 2 marks.

7	a	The following table gives the relation between steam pressure and temperature. | $T°C$ | 361 | 367 | 378 | 387 | 399 | | P | 154.9 | 167.9 | 191 | 212.5 | 244.2 | Using Suitable interpolation formula, find the pressure at the temperature 372 and 404.	08
	b	The following data was collected for the distance travelled versus time: | t(sec): | 0 | 25 | 50 | 75 | 100 | 125 | | y(km): | 0 | 32 | 59 | 78 | 92 | 100 | Use numerical differentiation to calculate velocity and acceleration at $t = 25$.	08
		OR	
8	a	From the following data, estimate the number of students who obtained marks between 40 and 45 using Newton's interpolation method | Marks: | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | | Number of Students: | 31 | 42 | 51 | 35 | 31 |	08
	b	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\, dx$ by dividing the interval into six equal parts using Simpson's one-three rule, Simpson's three-eight rule and Weddle's rule.	08
9	a	Using Newton- Raphson method, find the root of the equation $3x = \sqrt{1 + \sin(x)}$ correct to 4 decimal places choosing the initial guess $x_0 = 0.5$.	08
	b	Use the Runge-Kutta method of fourth order with $h = 0.1$ to find approximate value for the solution of the initial value problem $\frac{dy}{dx} + 2y = x^3 e^{-2x}$, $y(0) = 1$, at $x = 0.1$.	08
		OR	
10	a	Find a real root of $xe^x = \cos(x)$ correct to 4 decimal places by using Regula - Falsi method that lies between 0.4 and 0.6.	08
	b	Use Milne's predictor – corrector method to find the solution of the differential equation $\frac{dy}{dx} = x^2 - y$ at $x = 0.4,\ 0.5$, given that $y(0) = 1, y(0.1) = 0.9051, y(0.2) = 0.8212,\ y(0.3) = 0.7491$.	08