

R V COLLEGE OF ENGINEERING

(An autonomous institution affiliated to VTU, Belgaum)

DEPARTMENT OF MATHEMATICS

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS (22MA21C)

UNIT 4: LINEAR ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER TUTORIAL SHEET – 1

1. Solve the following differential equations.

a.
$$2y'' - 5y' - 3y = 0$$

b.
$$(D^2 - 10D + 25)y = 0$$

c.
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y = 0$$

d.
$$y''' + 3y'' - 4y = 0$$

e.
$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$$

f.
$$3y''' + 5y'' + 10y' - 4y = 0$$

g.
$$\frac{d^3u}{dt^3} + \frac{d^2u}{dt^2} - 2u = 0$$

h.
$$y'' + 16y = 0, y(0) = 2, y'(0) = -2$$

i.
$$y'' - 2y' + y = 0$$
, $y(0) = 0$, $y(1) = 3$

j.
$$y'' + y = 0$$
, $y'(0) = 0$, $y'(\frac{\pi}{2}) = 0$

k.
$$y'' + 2ky' + (k^2 + w^2)y = 0$$
, $y(0) = 1$, $y'(0) = -k$

1.
$$y''' - 4y' = 0$$
, $y'(0) = y(0) = y''(0) = 1$

- 2. The roots of the cubic auxiliary equation are $m_1 = 4$, $m_2 = m_3 = -5$, what is the corresponding homogeneous linear differential equation?
- 3. Show that the solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 4y = 0 \text{ is } y = e^{-2x} [c_1 \cosh \sqrt{2} x + c_2 \sinh \sqrt{2} x].$$

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DEPARTMENT OF MATHEMATICS TUTORIAL SHEET - 2

1. Find the solution of the following differential equations:

(i)
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cosh x$$

(ii)
$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = (e^{3x} + 3)^2$$

2. Solve the following differential equations.

a.
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos^2 x + 2^x$$

b.
$$\frac{d^2x}{dt^2} + x = \sin t \sin 2t \sin 3t$$

c.
$$\frac{d^4y}{dx^4} - y = \cosh(x-1) + \sin x$$

d.
$$\frac{d^3y}{dx^3} + y = 65\cos(2x+1) + e^x$$

e.
$$(2D^2 + 2D + 3)y = x^2 + 2x - 1$$

f.
$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^3$$

g.
$$y'' - y = x, y(0) = 0, y(1) = 0$$

h.
$$\frac{d^2y}{dx^2} + 4y = \sin 2x$$
, $y(0) = 1$, $y(\frac{1}{2}) = 0$

3. Solve the following differential equations:

(i)
$$\frac{d^2y}{dt^2} + 4y = t^2 e^{3t}$$

(ii)
$$\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = e^{2x}(1+x)$$

(iii)
$$\frac{d^2y}{dx^2} + 4y = 2e^x \sin^2 x$$

(iv)
$$\frac{d^2x}{dt^2} - 4x = t^2 \cos t$$

$$(v) \qquad \frac{d^2u}{dx^2} - 4u = x \sinh x$$

(vi)
$$\frac{d^2y}{dx^2} - y = xe^x \sin x$$



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DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET - 3

1. Find the solution of the following differential equations:

(i)
$$2x \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - \frac{y}{x} = 5 - \frac{\sin(\log_e x)}{x}$$

(ii)
$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left[x + \frac{1}{x} \right]$$

(iii)
$$(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$$

(iv)
$$x^2 \frac{d^2y}{dx^2} - 2y + x = 0$$
, $y(2) = y(3) = 0$.

2. Solve the following differential equations by the method of variation of parameters:

(i)
$$y'' + y = \frac{1}{1 + \sin x}$$

(ii)
$$y'' + 2y' + y = 4e^{-x} \log_e x$$

(iii)
$$\frac{d^2y}{dx^2} - y = e^{-2x} \cos(e^{-x})$$

(iv)
$$xy'' - y' = (3 + x)x^2e^x$$

- 3. In a simple harmonic motion, the amplitude of motion is 5 meters and the period is 4 seconds. Find the time required by the particle in passing between the points which are at distance of 4 meters and 2 meters from the center of the force and are on same side of it. Also find the velocities at these points. (Equation of SHM is ^{d²x}/_{dt²} = -μ²x).
- 4. Find the current in the RLC circuit assuming zero initial current and charge and R=160 ohm, L=20 Henry, C= 2.10-3 Farad and E = 48.1 sin(10t) volts.

(The differential equation for the charge Q is $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{c} = E$)

Objective Questions:

- 1. The order and degree of the differential equation $y'' (y')^{3/2} = 2$ is ___and ____.
- 2. The complementary function of the differential equation is $c_1 cos3x + c_2 sin3x$, then Wronskian is _____.

3.
$$\frac{1}{D^2+1} \sin 2t =$$

- 4. Particular integral of $\frac{d^2x}{dt^2} + \frac{1}{b}(x-a) = 0$ is_____.
- 5. If $x = e^{2t}$ is the solution of the equation $\frac{d^2x}{dt^2} 5\frac{dx}{dt} + kx = 0$ then value of k is
- 6. Reduce the differential equation $x^2y'' xy' + y = \log_e x$ to a linear differential equation with constant coefficient.