DEPARTMENT OF PHYSICS

SECOND SEMESTER BE PROGRAMS (CS, CD, CY, IS, AIML, & BT)

ACADEMIC YEAR 2022-2023

Dated	21st August 2023	Maximum Marks	50
Course Code	22PHY22C	Duration	90 min
Course	Quantum Physics For Engineers	**CIE-II (Test) - Long Scheme**	

1(a) *For a pure silicon semiconductor derive a general expression for electron concentration in the conduction band.* ⑦

The number of electrons in the conduction band is given by

$$n = \int_{E_C}^{\text{top of the band}} g_c(E) f(E)\, dE$$

where E_C is the bottom most energy level of the conduction band.

As $f(E)$ rapidly approaches zero for higher energies, we can write

$$n = \int_{E_C}^{\infty} g_c(E) f(E)\, dE \tag{1}$$

Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}\, dE \tag{2}$$

Any energy above E_C is the conduction band electron's kinetic energy ($= E - E_C$) relative to E_C. Electrons may gain energy by getting accelerated in an electric field and may lose energy through collisions with imperfections in the crystal.

Fermi factor is given by,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \tag{3}$$

Substituting eqns. (2) and (3) in eqn. (1),

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_C)^{1/2}}{e^{(E-E_F)/kT} + 1}\, dE$$

At sufficiently large E with $E > E_F$,

$$E - E_F \gg kT \quad \text{makes} \quad e^{(E-E_F)/kT} \gg 1$$

Hence we can approximate,

$$e^{(E-E_F)/kT} + 1 \approx e^{(E-E_F)/kT}$$

Therefore the above integral can be written as,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_C)^{1/2} e^{-(E-E_F)/kT}\, dE$$

Adding and subtracting E_c to the exponential term gives,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \int_{E_c}^{\infty} (E - E_C)^{1/2} e^{-(E-E_C)/kT}\, dE$$

To solve the integral, let $E - E_C = x$, then $dE = dx$ and

Lower limit: when $E = E_C$, $x = E_C - E_C = 0$

Upper limit: when $E = \infty$, $x = \infty - E_C = \infty$

Then the integral part will become, by letting $1/kT = a$,

$$\int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E-E_C)/kT}\, dE = \int_0^{\infty} x^{1/2} e^{-ax}\, dx$$

This form of integrals are known as a gamma functions. From literature,

$$\int_0^{\infty} x^{1/2} e^{-ax}\, dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

Substituting it back into the equation for n we get,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \left[\frac{\sqrt{\pi}}{2}(kT)^{3/2}\right]$$

By rearranging,

$$\boxed{n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_C - E_F)/kT}}$$

Or,

$$\boxed{n = N_C e^{-(E_C - E_F)/kT}} \qquad \text{with,} \; \boxed{N_C = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}$$

N_C is called the *effective density of states* of the conduction band.

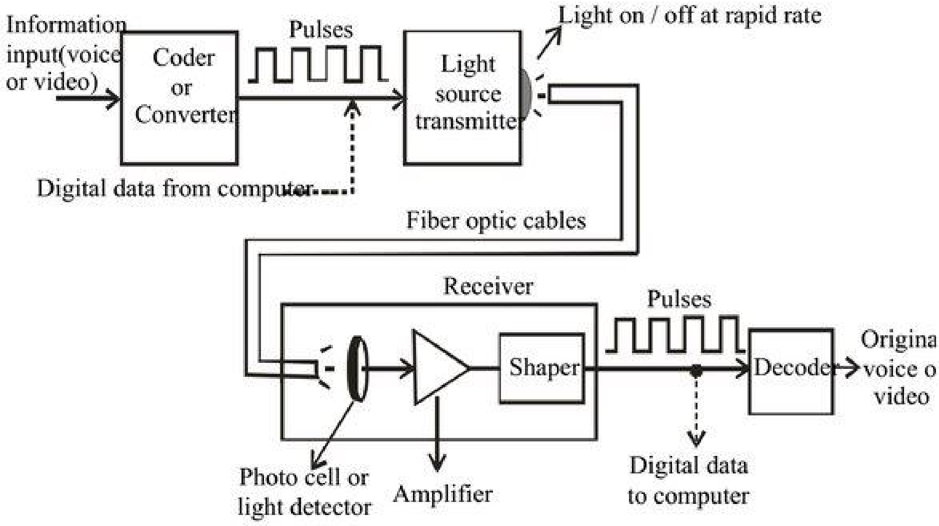
1(b) *A fiber surrounded by air has a numerical aperture of 0.369. Will the light entering the fiber at an angle of incidence of 25° remain in the fiber or will it escape? Why?* ③

From the given numerical aperture NA = 0.369 acceptance angle can be calculated as,

$$\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1} 0.369 = 21.654^\circ$$

Given angle of incidence $\theta_i = 25^\circ$ is greater than the calculated acceptance angle $\theta_0 = 21.654^\circ$. Hence the ray will escape out into the cladding.

2(a) *Describe in detail the basics of point-to-point communication and describe the role of repeater.* ⑦



Point-to-point Communication The use of optical fibres in the field of communication has revolutionized the modern world. An optical fibre acts as the channel of communication (like electrical wires), but transmits the information in the form of optical waves. A simple p to p communication system using optical fibres is illustrated in the figure. The main components of p to p communication is:

1) An optical transmitter, i.e., the light source to transmit the signals/pulses

2) The communication medium (channel) i.e., optical fibre

3) An optical receiver, usually a photo cell or a light detector, to convert light pulses back into electrical signal.

The information in the form of voice or video to be transmitted will be in an analog electric signal format. This analog signal at first converted into digital electric (binary) signals in the form of electrical pulses using a Coder or converter and fed into the optical transmitter which converts digital electric signals into optic signals. An optical fibre can receive and transmit signals only in the form of optical pulses. The function of the light source is to work as an efficient transducer to convert the input electrical signals into suitable light pulses. An LED or laser is used as the light source for this purpose. Laser is more efficient because of its monochromatic and coherent nature. Hence semiconductor lasers are used for their compact size and higher efficiency.

The electrical signal is fed to the semiconductor laser system, and gets modulated to generate an equivalent digital sequence of pulses, which turn the laser on and off. This forms a series of optical pulses representing the input information, which is coupled into the optical fibre cable at an incidence angle less than that of acceptance cone half angle of the fibre.

Next the light pulses inside the fibre undergo total internal reflection and reach the other end of the cable. Good quality optical fibres with less attenuation to be chosen to receive good signals at the receiver end.

The final step in the communication system is to receive the optical signals at the end of the optical fibre and convert them into equivalent electrical signals. Semiconductor photodiodes are used as optical receivers. A typical optical receiver is made of a reverse biased junction, in which the received light pulses create electron-hole charge carriers. These carriers, in turn, create an electric field and induce a photocurrent in the external circuit in the form of electrical digital pulses. These digital pulses are amplified and re-gain their original form using suitable amplifier and shaper. The electrical digital pulses are further decoded into an analogues electrical signal and converted into the usable form like audio or video etc.,

As the signal propagates through the fibre it is subjected to two types of degradation. Namely attenuation and delay distortion. Attenuation is the reduction in the strength of the signal because power loss due to absorption and scattering of photons. Delay distortion is the reduction in the quality of the signal because of the spreading of pulses with time. These effects cause continuous degradation of the signal as the light propagates and may reach a limiting stage beyond which it may not be retrieve information from the light signal. At this stage repeater is needed in the transmission path.

Reciever	Amplifier	Transmitter

An optical repeater consists of a receiver and a transmitter arranged adjacently. The receiver section converts the optical signal into corresponding electrical signal. Further the electrical signal is amplified and recast in the original form and it is sent into an optical transmitter section where the electrical signal is again converted back to optical signal and then fed into an optical fibre.

Finally at the receiving end the optical signal from the fibre is fed into a photo detector where the signal is converted to pulses of electric current which is then fed to decoder which converts the sequence of binary data stream into an analog signal which will be the same information which was there at the transmitting end.

2(b) *A bar of n-type germanium of dimension* $1cm \times 0.1cm \times 0.1cm$ *in the order of length, width and thickness is placed in a magnetic field of* 0.2 T. *If the drift velocity of the electrons is* 4cm/s *calculate the Hall voltage produced in the bar. Assume the magnetic field to be along the direction of width.* ③

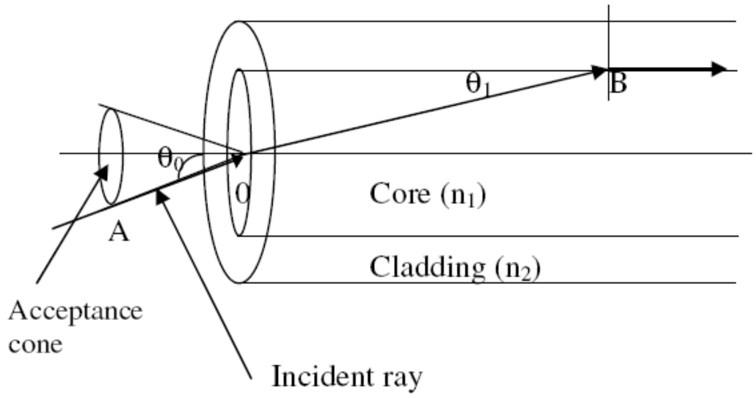
Expression for Hall voltage produced in a n-type semiconductor is, in which current $I = neAv_d$ is flowing is,

$$V_H = -\frac{BI}{net} = -\frac{BneAv_d}{net} = -\frac{Bnewtv_d}{net} = -Bwv_d$$

Substituting $B = 0.2\text{T}$, $w = 0.1\text{cm}$ and $v_d = 4\,\text{cm/s}$ we get,

$$V_H = -0.2\text{T} \times 0.1\text{cm} \times 4\,\text{cm/s} = -8 \times 10^{-6}\text{V}$$

3(a) *With a neat figure derive the expression for numerical aperture of an optical fibre and the fractional index change?* ⑦



Refractive index of surrounding = n_0
Refractive index of core = n_1
Refractive index of cladding = n_2

$$n_0 < n_2 < n_1$$

$90° - \theta_1$ is the critical angle of incidence for *total internal reflection (TIR)* to happen. Hence, OB further grazes along core and cladding interface. Incident angles $< \theta_0$ will further undergo TIR but $> \theta_0$ will simply refract into the cladding. Rotating OA forms a cone called *acceptance cone*. Only rays entering this cone will undergo TIR others refract into the cladding.

θ_0 is called the waveguide acceptance angle or the acceptance cone half angle which is the maximum angle from the axis of optical fibre at which light ray may enter the fibre and propagates in core by TIR.

$\sin\theta_0$ is called the numerical aperture (NA). It determines the light gathering aability of the fibre and purely depends on the refractive index of core, cladding and the surrounding.

$$\theta_0 = \theta_0(n_0, n_1, n_2)$$

By applying Snell's law at O,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$
$$\therefore \quad \sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \tag{1}$$

By applying Snell's law at B,

$$n_1 \sin(90° - \theta_1) = n_2 \sin 90°$$
$$\therefore \quad \cos\theta_1 = \frac{n_2}{n_1} \tag{2}$$

Using $\sin\theta_1 = \sqrt{1 - \cos^2\theta_1}$ and eqn. (2) in (1),

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_1} = \frac{n_1}{n_0}\sqrt{1 - \frac{n_2^2}{n_1^2}}$$
$$\therefore \quad \sin\theta_0 = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

If the surrounding medium is air, then $n_0 \approx 1$. Therefore numerical aperture (NA) is,

$$\therefore \quad \text{NA} = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

The condition for incident rays to be propagated in the fibre is,

$$\theta_i < \theta_0$$
$$\text{or} \qquad \sin\theta_i < \sin\theta_0$$
$$\text{or} \qquad \sin\theta_i < \text{NA}$$

Fraction index change (Δ) is defined as, the ratio of difference in refractive indices of the core and the cladding to the refractive index of the core.

$$\Delta = \frac{\text{Refractive index of core} - \text{Refractive index of cladding}}{\text{Refractive index of core}} = \frac{n_1 - n_2}{n_1}$$

It is also known as relative core clad index difference. That is,

$$\Delta = 1 - \frac{n_2}{n_1}$$

3(b) *With a neat sketch explain the structure and working of GRIN fibre.* ③

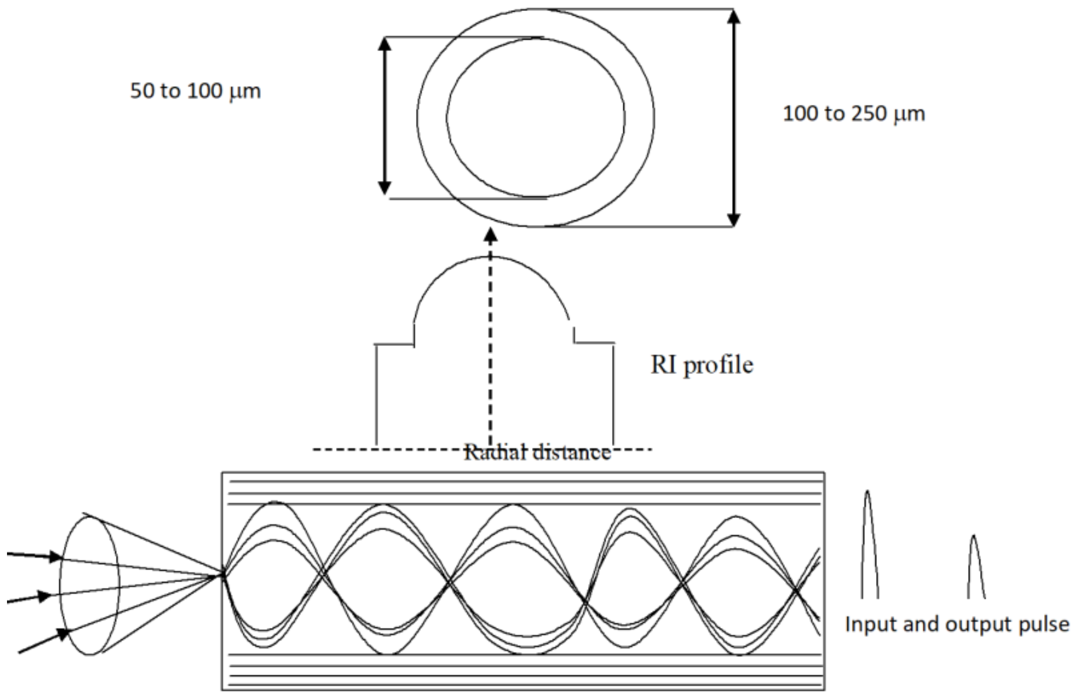
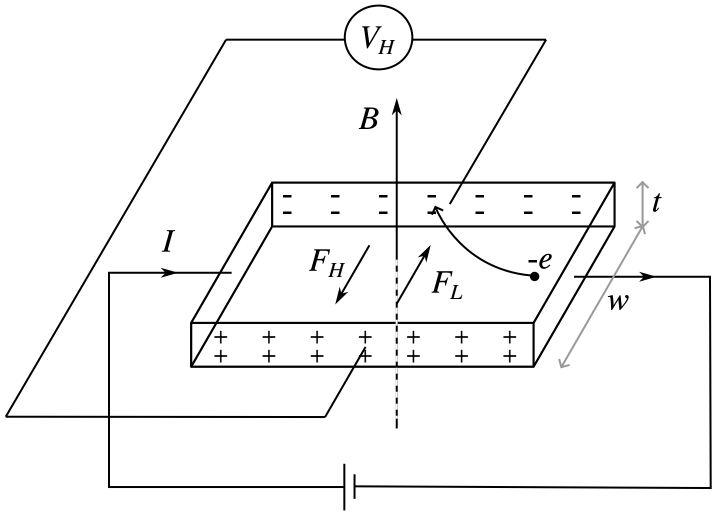
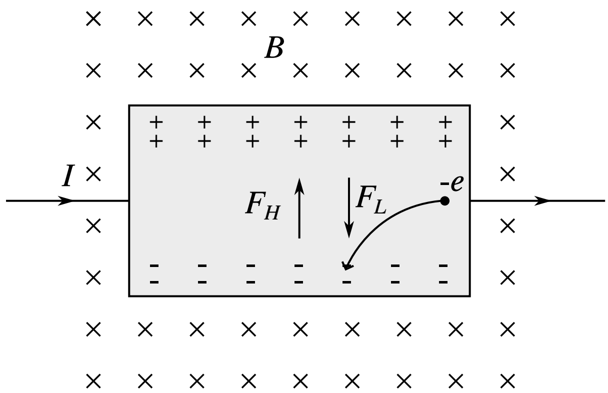


Fig. Graded index multimode fibre

- Refractive index of the core varies across the core diameter (radially graded) as shown in figure, while the refractive index of the cladding is fixed.
- Many modes can be transmitted without intermodal dispersion.
- The rays move in a curved path through the core.
- Light travels at lower speed in the high-index region – fastest components of the ray take the longer path and the slower components take the shorter path in the core.
- Hence the travel time of the different modes will be almost same.
- Losses are minimum. Very little pulse broadening.
- Suitable for large bandwidth, medium distance and medium bit rate communication systems.
- Either a laser or LED source can be used.
- Joining the optical fibres is relatively easy.

4(a) *Define Hall effect and with a neat figure arrive at an expression for hall coefficient for a pentavalent doped semiconductor.* ⑦





Consider a rectangular plate of n-type semiconductor (n-type is pentavalent doped) having width w and thickness t. When a potential difference is applied across its ends a current I flows through it opposite to the direction of flow of electrons. The current passing through the semiconductor is given by,

$$I = -neAv_d \qquad \text{or} \qquad v_d = \frac{-I}{neA} = \frac{-I}{newt} \tag{1}$$

where n is concentration of electrons, $A = wt$ is area of cross section of end face, e is electronic charge, v_d is drift velocity of electrons.

If magnetic field is applied perpendicular to the current flow the Lorentz force deflects the electrons to sideways. The magnitude of this force on an electron is,

$$F_L = eBv_d \tag{2}$$

As electrons pile up on one side an equivalent amount of positive charges are left on the other opposite side. As a result an electric field E_H is developed between these separated unlike charges. This transverse electric field E_H is known as Hall field. The magnitude of the corresponding electrostatic force on an electron will be,

$$F_H = eE_H = e\frac{V_H}{w} \tag{3}$$

where V_H is the corresponding voltage for the field E_H and it is called Hall voltage.

A condition of equilibrium is reached when the force F_H due to transverse electric field balances the Lorentz force F_L.

$$F_L = F_H$$

Substituting the forces from eqns. (2) and (3),

$$e\frac{V_H}{w} = eBv_d$$

Substituting v_d from eqn. (1) into it and rearranging we get expression for **Hall voltage** as,

$$\boxed{V_H = -\frac{BI}{net}} \tag{4}$$

Reciprocal of carrier charge density is called Hall coefficient R_H. In case of electrons,

$$\boxed{R_H = \frac{-1}{ne}}$$

With this eqn. (4) can be written as,

$$\boxed{V_H = R_H \frac{BI}{t}} \qquad \text{or} \qquad \boxed{R_H = V_H \frac{t}{BI}}$$

The Hall voltage can be measured with a voltmeter. **For conductors and *n*-type semiconductors Hall voltage is** conventionally taken as **negative**.

4(b) *For silicon at* $30°C$*, calculate the number of states per unit energy per unit volume at an energy* $26 meV$ *above the bottom of the conduction band (*$m_e^* = 1.18 m_e$*).* ③

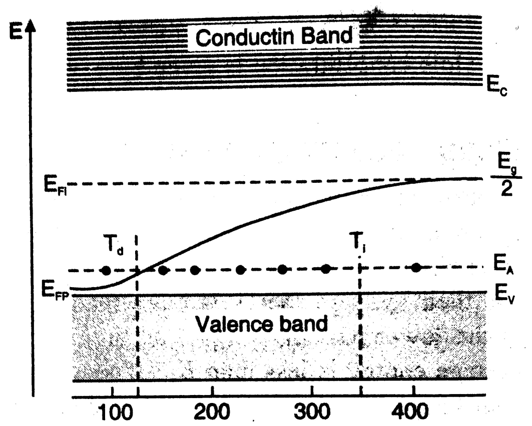
Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}\, dE$$

Substituting $E - E_C = 26\text{meV} = 26 \times 10^{-3} \times 1.602 \times 10^{-19}\text{J}$, $m_e^* = 1.18 m_e = 1.18 \times 9.109 \times 10^{-31}\text{kg}$, $h = 6.626 \times 10^{-34}\text{Js}$ into it, we get,

$$\frac{g_c(E)}{dE} = \frac{4 \times 3.1416}{(6.626 \times 10^{-34}\text{Js})^3}(2 \times 1.18 \times 9.109 \times 10^{-31}\text{kg})^{3/2}(26 \times 10^{-3} \times 1.602 \times 10^{-19}\text{J})^{1/2}$$
$$= 8.788 \times 10^{45}\, \text{J}^{-1}\text{m}^{-3}$$

5(a) *With a neat figure describe the variation of Fermi level with respect to temperature for an intrinsic semiconductor doped with a trivalent impurity.* ⑦



In case of p-type semiconductor the Fermi level E_{Fp} rises with increasing temperature from below the acceptor level to intrinsic level E_{Fi} as shown in the figure.

This can be analyzed by the following general expression for Fermi energy of p-type semiconductor.

$$E_{Fp} = \frac{E_A + E_V}{2} + \frac{kT}{2}\ln\frac{N_V}{N_a} \quad \text{and} \quad E_{Fi} = \frac{E_C + E_V}{2}$$

where E_A is acceptor energy level, E_V is top most energy level in valence band, T is temperature, N_a is negative acceptor ions density and

$$N_V = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}$$

1. As the valence band is the source of electrons and the acceptor levels are the recipients for them, the Fermi level must lie between the top of the valence band and the acceptor levels. Also, for $T = 0\,\mathrm{K}$ the above expression becomes,

$$E_{Fp} = \frac{E_A + E_V}{2}$$

Hence the Fermi level lies midway between the acceptor levels and the top of the valence band. In the low temperature region, holes in the valence band are only due to the transitions of electrons from the valence band to the acceptor levels.

2. As the temperature increases the acceptor levels gradually get filled and the Fermi level moves upward. At the temperature of saturation T_d (beginning of depleted region), the Fermi level coincides with the acceptor level E_A. Thus,

$$E_{FP} = E_A \qquad \text{at } T = T_d$$

3. As the temperature grows above T_d, the Fermi level shifts upward in an approximately linear fashion.

4. At temperature T_i intrinsic behaviour sets in. At higher temperatures, the p-type semiconductor loses its extrinsic character and behaves as an intrinsic semiconductor. In the intrinsic region, the hole concentration in the valence band increases exponentially and the Fermi level approaches the intrinsic value. Thus,

$$E_{Fp} = E_{Fi} = \frac{E_C + E_V}{2} \qquad \text{at } T = T_i$$

5(b) *Define Fermi factor, Fermi energy and sketch the variation of Fermi level (factor?) when $T \neq 0K$.* ③

Fermi factor $f(E)$ represents the probability of finding a particle with energy E, or in the language of statistical mechanics, the probability that a state with energy E is occupied at the absolute temperature T.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is called the Fermi energy and k is the Boltzmann's constant.

Fermi energy:

- Fermi energy is the highest occupied energy level at absolute zero.
- The probability of finding an electron with an energy equal to the Fermi energy is exactly 1/2 at any temperature.
- Also, Fermi energy is the *average energy* possessed by the conduction electrons in conductors at temperatures above 0 K.
