Superconductivity

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Outline

Introduction

- Zero electrical resistance and perfect diamagnetism – does not allow magnetic fields to penetrate into its interior.
- Discovered in 1911 by Heike Kamerlingh Onnes – Mercury at critical temperature $(T_c) = 4.2$K.
- Conventional superconductors: $T_c < 30$K. They can be fully explained by BCS theory and Landau-Ginzburg theory.
- At higher temperatures they are normal metals and ordinarily not very good conductors – Pb, Ta, Sn become superconductors while Cu, Ag, Au do not.
- In the normal state some superconducting metals are weakly diamagnetic and some are paramagnetic.

Applications of superconductivity

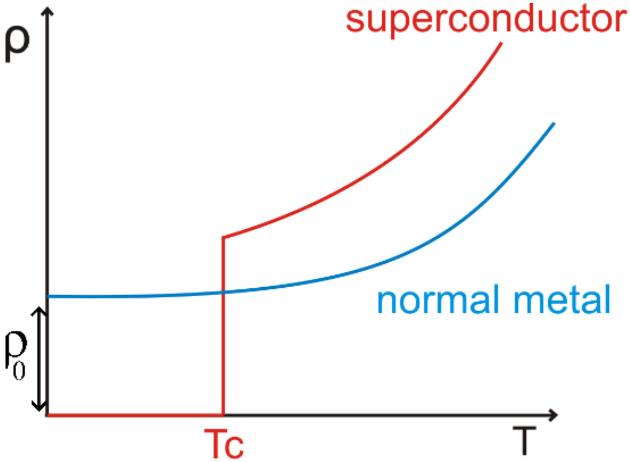
- To create strong magnetic fields – Magnetic Resonance Imaging (MRI), Particle Accelerators.
- Power Transmission.
- Magnetic levitation – Levitating trains – reducing friction and increasing efficiency.
- Energy storage – solenoids.
- Quantum computing – to create qubits.

Temperature dependence of resistivity

- Resistance of a metal is due to the scattering of free electrons by lattice vibrations.
- Temperature increases the lattice vibrations causing more frequent collisions with free electrons.
- However, at $T = 0$K, the resistivity is not zero due to the presence of impurities in the metal.
- Matthiessen's rule:

$$\text{Total resistivity} = \text{residual resistivity} + \text{temperature-dependent part}$$

$$\rho = \rho_0 + \rho(T)$$


superconductor
ρ
normal metal
ρ0
Tc
T

Superconductor	**Symbol**	**Critical Temperature** T_c (in K)
Mercury	Hg	4.2
Lead	Pb	7.2
Niobium	Nb	9.3
Tin	Sn	3.7
Gallium	Ga	1.5
Indium antimonide	InSb	1.9
Tungsten Carbide	WC	12.5
Magnesium Diboride	MgB2	39

Meissner effect

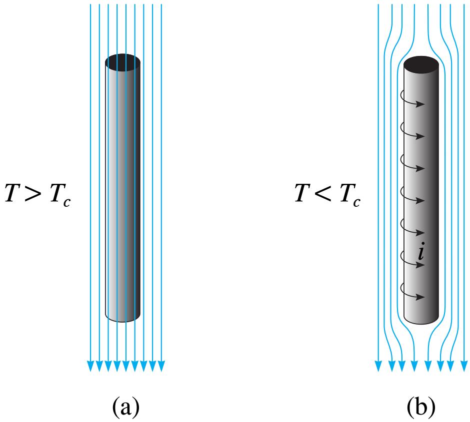


Figure: A superconductor in the form of a long cylinder in the presence of an external magnetic field. (a) At temperature above T_c, the field lines penetrate the cylinder because it is in its normal state. (b) When the cylinder is cooled to $T < Tc$ and becomes superconducting, magnetic flux is excluded from its interior by the induction of surface currents.

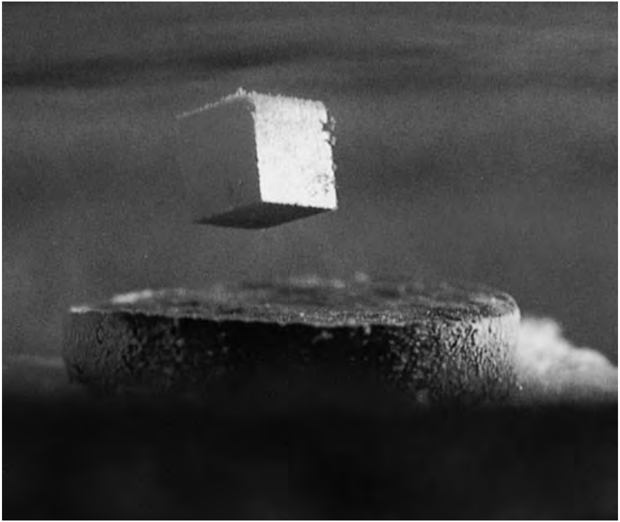


Figure: A small permanent magnet levitated above a pellet of the $YBa_2Cu_3O_{7\text{-}x}$ superconductor cooled to the temperature of liquid nitrogen, 77 K.

- The effect of magnetization is to establish *bound currents* $\vec{J}_b = \nabla \times \vec{M}$ within the material and $\vec{K}_b = \vec{M} \times \hat{n}$ on the surface.
- Where magnetization $\vec{M}$ = magnetic dipole moment per unit volume.
- The field due to magnetization of the medium is just the field produced by these *bound currents*.
- The field due to everything else we shall call the *free current* $\vec{J}_f$.
- The free current might flow through wires imbedded in the magnetized substance or, if the latter is a conductor, through the material itself.
- The total current can be written as

$$\vec{J} = \vec{J}_f + \vec{J}_b$$

- Ampère's law for magnetostatics can be written as,

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\text{or} \qquad \frac{1}{\mu_0}(\nabla \times \vec{B}) = \vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + (\nabla \times \vec{M})$$

or, collecting together the two curls:

$$\nabla \times \left(\frac{1}{\mu_0} \vec{B} - \vec{M} \right) = \vec{J}_f$$

By writing

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

in terms of $\vec{H}$, then, Ampère's law reads,

$$\nabla \times \vec{H} = \vec{J}_f$$

For most substances the magnetization is proportional to the field, provided the field is not too strong.

$$\vec{M} = \chi_m \vec{H}$$

The constant of proportionality χ_m is called the *magnetic susceptibility*. it is a dimensionless quantity that varies from one substance to another—positive for paramagnets and negative for diamagnets. Materials that obey above equation are called linear media. Therefore, for linear media

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \chi_m \vec{H}) = \mu_0(1 + \chi_m)\vec{H}$$

Thus $\vec{B}$ is also proportional to $\vec{H}$:

$$\vec{B} = \mu \vec{H} \qquad \text{where } \mu = \mu_0(1 + \chi_m)$$

μ is called the permeability of the material. In a vacuum, where there is no matter to magnetize, the susceptibility χ_m vanishes, and the permeability is μ_0. That's why μ_0 is called the permeability of free space.

- For superconducting materials it is observed that the magnetic field $\vec{B} = 0$ inside but $\vec{H} \neq 0$. Therefore,

$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = 0 \qquad \implies \chi_m = -1$$

  Thus, a superconductor is an ideal diamagnetic material.

- And $\vec{B} = 0$ in

$$\nabla \times \vec{B} = \mu_0 \vec{J} \qquad \implies \vec{J} = 0$$

  Thus, total current density $\vec{J} = 0$ in the interior of a superconductor. However, surface currents can exist.

- In a superconductor the magnetic flux is expelled from the material irrespective of its geometry.

- In reality, the induction field decays exponentially with distance from the surface of the sample. This characteristic depth is called the *penetration depth* and is generally estimated to be a few hundred angstroms (10^{-7}m).

- External magnetic field applied to the superconductor induces currents in it. These induced currents create a magnetic field that perfectly cancels out the external magnetic field.
- Meissner effect is only observed in type I superconductors, which have a single critical temperature and exhibit a complete expulsion of the magnetic field. Type II superconductors, on the other hand, exhibit a mixed state where magnetic flux penetrates the material in the form of quantized vortices. In type II superconductors, the Meissner effect is limited to the region between these vortices.

Types of superconductors

- Categorized into two groups based on their magnetization properties.
- Classified as:
  - Type 1 superconductors (or soft superconductors)
  - Type 2 superconductors (or hard superconductors)

Type 1 superconductors

- Magnetic field gets totally expelled from the interior of the material below a certain critical magnetic field H_c.
- Superconductivity is abruptly destroyed via a first order phase transition when the strength of the applied field rises above a critical value H_c.
- The transition is reversible.
- Normally exhibited by pure metals, e.g. aluminium, lead, and mercury (H_c around 10^{-1}T).
- The only alloy known up to now which exhibits type I superconductivity is tantalum silicide ($TaSi_2$).

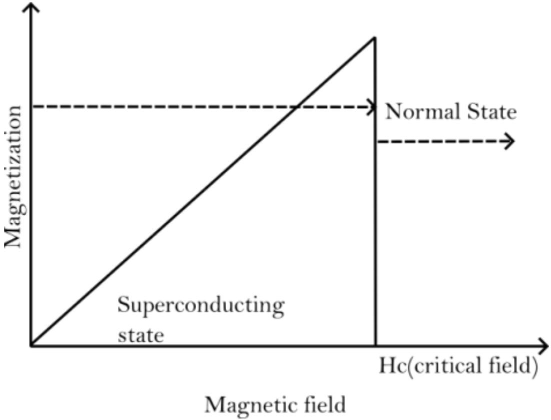


Figure: A plot of magnetization versus magnetic field for a Type-1 superconductor.

Type 2 superconductors

- Show two critical magnetic fields: H_{c1} and H_{c2}.
- Till H_{c1} the material is completely superconducting and exhibits ideal diamagnetism.
- Between $H_{c1}$and H_{c2} material exhibits a mixed state. Expelling of magnetic flux is partial.
- Above H_{c2} the material will not be superconducting.
- Alloys or transition metals with high electrical resistivity in the normal state generally demonstrate Type-2 superconductivity.
- H_{c2} is 100 times more than H_c of Type-1.
- H_{c2} up to 50T are obtained in some materials.
- Used as magnetizing coils to obtain high magnetic fields (10 T or higher).

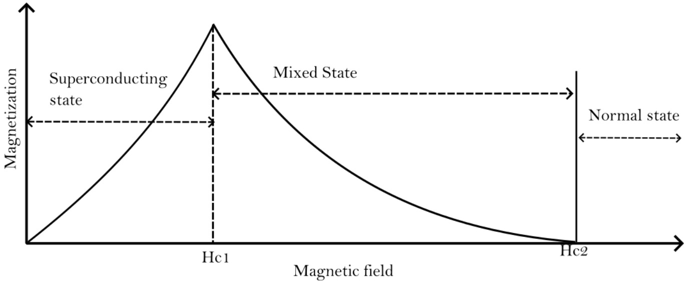
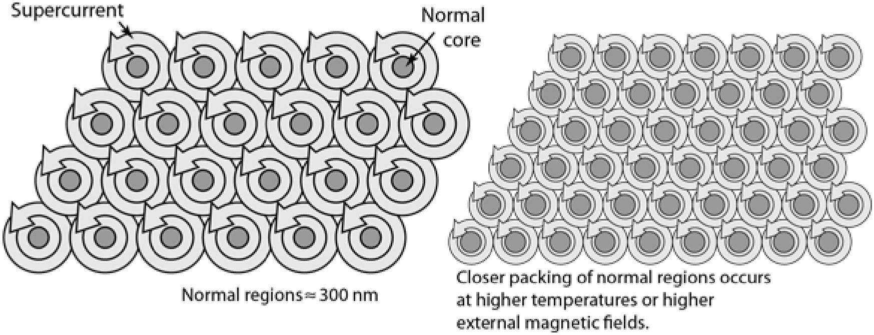
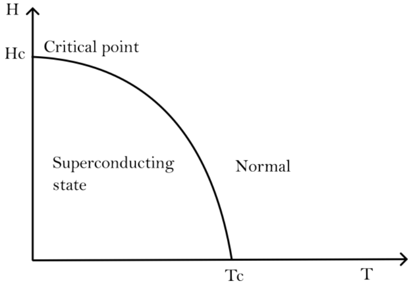


Figure: A plot of magnetization versus magnetic field for a Type-2 superconductor.

- Type II superconductors usually exist in a vortex state with normal cores surrounded by superconducting regions.
- This allows magnetic field penetration.
- As their critical temperatures are approached, the normal cores are more closely packed and eventually overlap as the superconducting state is lost.



Temperature dependence of critical field



The minimum field required to destroy the superconducting property at a given temperature T is given by,

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right] \qquad \text{(Tuyn's law)}$$

where H_0 is the field required to destroy superconductivity at 0 K and T_c is the critical temperature of the material at zero field.

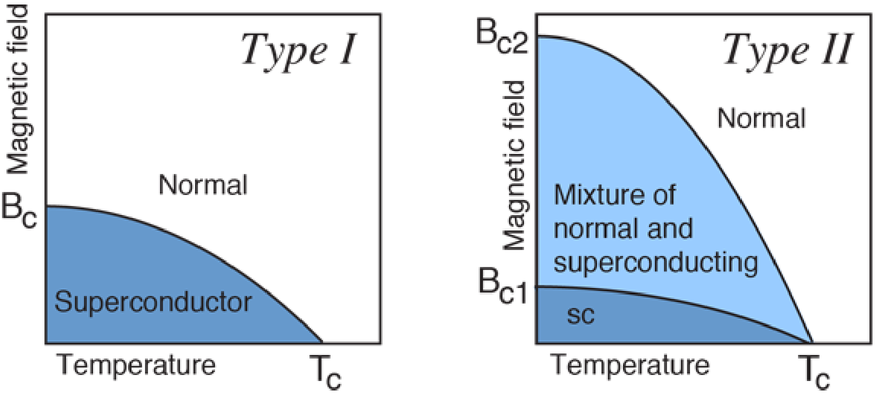


Figure: Phase Diagrams.

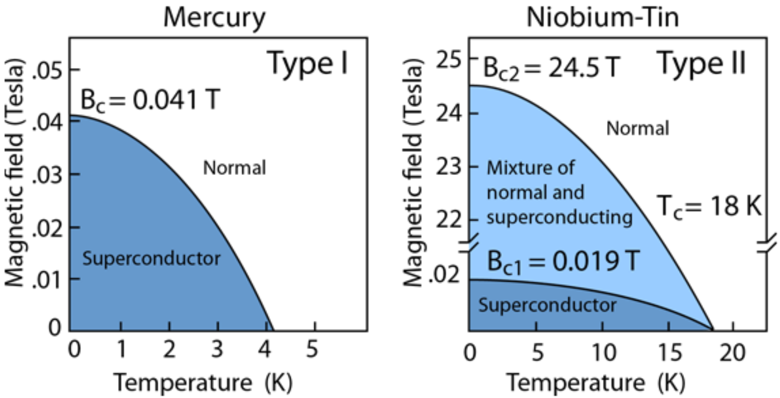


Figure: Phase Diagrams for mercury and niobium-tin.

Critical current

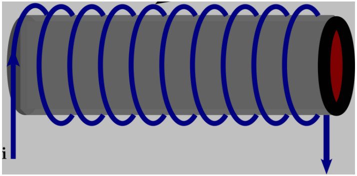


Figure: A superconducting coil of wire carrying current.

- At certain value of applied magnetic field superconductivity of the material is destroyed. This field is called the critical magnetic field H_c.
- If we consider a superconducting coil then the critical current i_c required to produce H_c in the coil so that the superconductivity is destroyed is

$$i_c = 2\pi R H_c$$

Where, R is the radius of the superconducting coil and H_c is the critical magnetic field.

BCS theory

- The properties of Type I superconductors were modeled successfully by the efforts of John Bardeen, Leon Cooper, and Robert Schrieffer in what is commonly called the BCS theory.
- This theory involves the electron interaction through phonon as mediators.
- During the flow of current in a superconductor, when an electron approaches a positive ion of the metal lattice, there is Coulomb attraction between the electron and the lattice ion. This produces a distortion in the lattice i.e., the positive ion gets displaced from its mean position. The distortion gives rise to a phonon.
- At normal temperature, the attractive force is too small and pairing of electrons does not take place.
- At lower temperature i.e., below the critical temperature the apparent force of attraction reaches a maximum value for any two electrons of equal and opposite spins and opposite momentum.

BCS theory ...

- This force of attraction exceeds the coulomb force of repulsion between two electrons and the electron stick together and move as a pairs. These pairs of electrons of opposite momenta are called *Cooper pairs*.
- The Cooper pair has a total spin of zero. As a result, the electron pairs in a superconductor behaves as bosons.
- All these bosons condense to occupy lowest state. The next possible state will have a energy gap for which the electrons will not have sufficient energy to make transition.
- The motion of all Cooper pairs is the same. Either they are at rest, or if the superconductor carries a current, they drift with identical velocity.

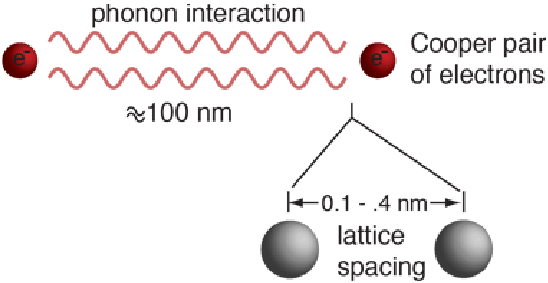


Figure: The behavior of superconductors suggests that electron pairs are coupling over a range of hundreds of nanometers, three orders of magnitude larger than the lattice spacing. Called Cooper pairs, these coupled electrons can take the character of a boson and condense into the ground state. This pair condensation is the basis for the BCS theory of superconductivity. The effective net attraction between the normally repulsive electrons produces a pair binding energy on the order of milli-electron volts, enough to keep them paired at extremely low temperatures.

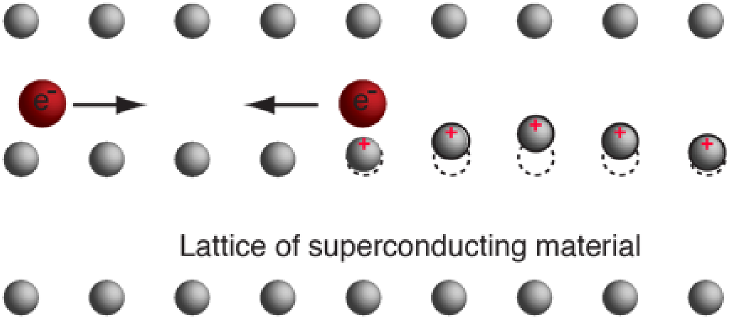


Figure: A passing electron attracts the lattice, causing a slight ripple toward its path.

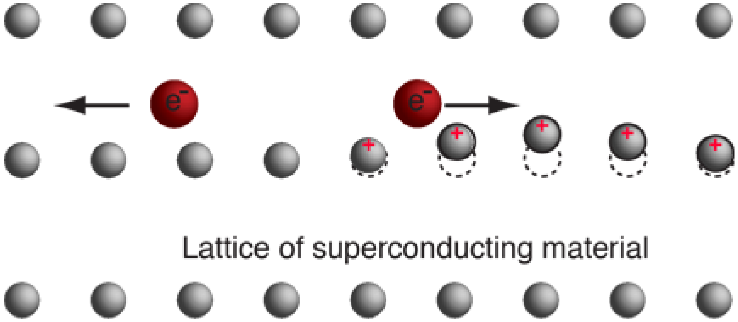


Figure: Another electron passing in the opposite direction is attracted to that displacement.

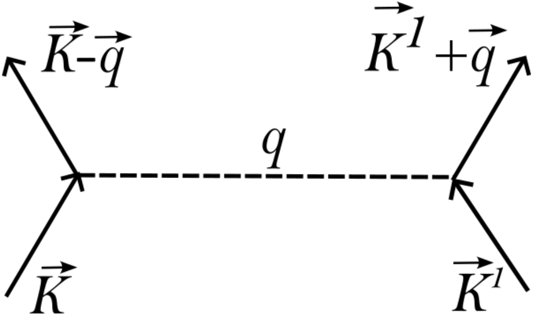


Figure: Feynman diagram for Cooper pair.

High temperature superconductors

- Ceramic materials are expected to be insulators – certainly not superconductors.
- But that is just what Georg Bednorz and Alex Muller found when they studied the conductivity of a lanthanum-barium-copper oxide ceramic in 1986.
- Its critical temperature of 30K was the highest which had been measured to date, but their discovery started a surge of activity which discovered superconducting behavior as high as 125K.
- The high temperature superconductors are ceramic materials with layers of copper-oxide spaced by layers containing barium and other atoms.

High temperature superconductors . . .

- The formula for the 1-2-3 superconductor is $YBa_2Cu_3O_{7-x}$, where x is about 0.1 for samples that superconduct at about 95 K. If $x \approx 1.0$, giving a formula of $YBa_2Cu_3O_6$, the material is an electrical insulator. The superconducting phase is thus with a fixed ratio of metal atoms but a variable oxygen content.

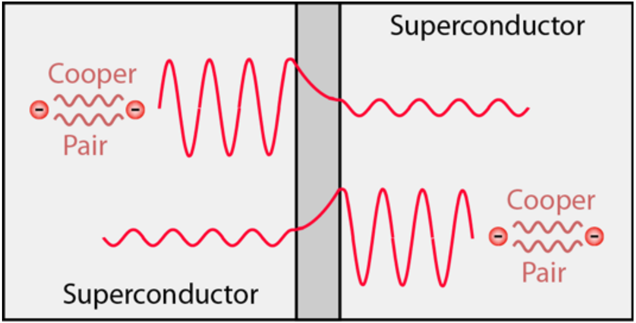
Material	T_c(K)
$La_{2-x}M_xCuO_4$ (M=Ca,Sr,Ba)	20-40
$Bi_2(CaSr)_{n+1}Cu_nO_{2n+4}$ (n=1-3)	90
$YBa_2Cu_3O_{7-x}$	92
$Tl_2Ca_{n-1}Cu_nO_{2n+4}$ (n=1-4)	90-110

Josephson Junction

- Two superconductors separated by a thin insulating layer can experience tunneling of Cooper pairs of electrons through the junction.
- The Cooper pairs on each side of the junction can be represented by a wavefunction similar to a free particle wavefunction.
- In fact, all the Cooper pairs in a superconductor can be described by a single wavefunction in the absence of a current because all the pairs have the same phase - they are said to be “phase coherent”.
- If two superconductors are separated by a thin insulating layer, then quantum mechanical tunneling can occur for the Cooper pairs without breaking up the pairs.
- The wavefuntions for Cooper pairs on each side of the junction penetrating into the insulating region and “locking together” in phase. Under these conditions, a current will flow through the junction in the absence of an applied voltage (the DC Josephson effect).

$$I_J = I_c \sin \phi$$

where ϕ is the phase difference between the wave functions on either side.



AC Josephson effect

- When a DC voltage is applied to a Josephson junction, an oscillation of frequency of

$$f = \frac{2e\Delta V}{h}$$

occurs at the junction.

- Since this relationship of voltage to frequency involves only fundamental constants and since frequency can be measured with extreme accuracy, the Josephson junction has become the standard voltage measurement.

AC Josephson effect . . .

- Josephson's equation for the supercurrent through a superconductive tunnel junction is given by

$$I = I_c \sin\left(\frac{4\pi e}{h}\int V dt\right)$$

where I is the junction current, I_c is the critical current, V is the junction voltage as a function of time. I_c is a function of the junction geometry, the temperature, and any residual magnetic field inside the magnetic shields that are used with voltage standard devices.

- When a dc voltage is applied across the junction:

$$I = I_c \sin\left(\frac{4\pi e}{h}\int V dt\right) = I_c \sin\left(\frac{4\pi e}{h} V t\right) \qquad \text{(for DCV)}$$

Comparing it with an AC equation,

$$I = I_c \sin(\omega t) = I_c \sin(2\pi f_J t)$$

AC Josephson effect . . .

we get

$$2\pi f_J t = \frac{4\pi e}{h} V t$$

This equation upon shows that the current will oscillate at a frequency

$$f_J = 2eV/h$$

where $2e/h$ is approximately equal to 483.6 GHz/mV.

AC Josephson effect . . .

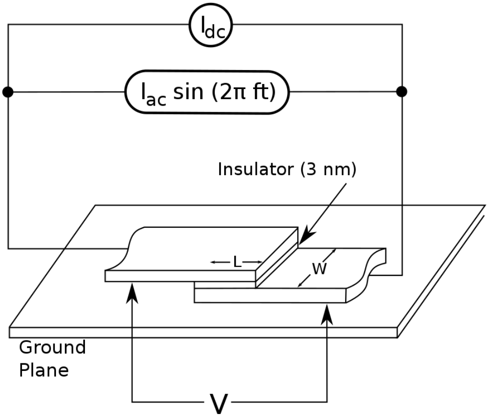
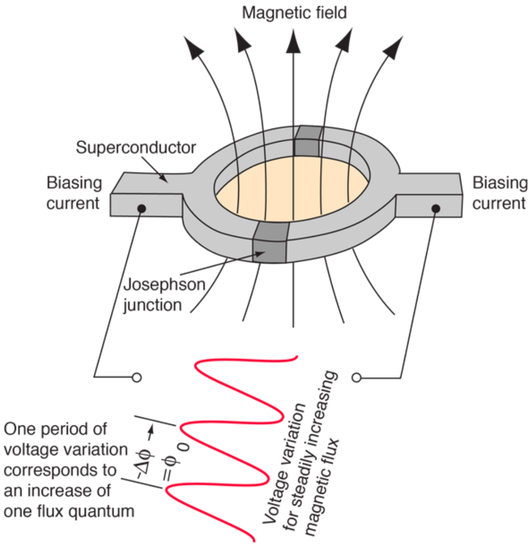


Figure: The structure of a superconductor–insulator–superconductor Josephson junction typically used in dc voltage standards.

SQUID - Superconducting QUantum Interference Device



- The superconducting quantum interference device (SQUID) consists of two superconductors separated by thin insulating layers to form two parallel Josephson junctions.
- The device may be configured as a magnetometer to detect incredibly small magnetic fields – small enough to measure the magnetic fields in living organisms.
- Squids have been used to measure the magnetic fields in mouse brains to test whether there might be enough magnetism to attribute their navigational ability to an internal compass.
    - Threshold for SQUID: 10^{-14} T
    - Magnetic field of heart: 10^{-10} T
    - Magnetic field of brain: 10^{-13} T
- The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field associated with one flux quantum. One of the discoveries associated with Josephson junctions was that flux is quantized in units

$$\Phi_0 = \frac{2\pi\hbar}{2e} = 2.0678 \times 10^{-15}\mathrm{Tm}^2$$

- If a constant biasing current is maintained in the SQUID device, the measured voltage oscillates with the changes in phase at the two junctions, which depends upon the change in the magnetic flux. Counting the oscillations allows you to evaluate the flux change which has occurred.

References

http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/supcon.html