Content:
Introduction to superconductors, temperature dependence of resistivity, Meissner effect, critical current, types of superconductors, temperature dependence of critical field.
BCS theory (qualitative), Quantum tunnelling, High temperature superconductivity, Josephson junction, DC and AC SQUIDs (qualitative), Applications in quantum computing, Numerical problems.

Introduction

Superconductors are materials that exhibit the state of superconductivity, in which they offer zero electrical resistance and do not permit magnetic fields to penetrate. As a result, an electric current flowing through a superconductor can continue to flow indefinitely without any energy loss or degradation.
Superconductivity was discovered on April 8, 1911 by Heike Kamerlingh Onnes. At the Mercury temperature of 4.2K, he observed that the resistance abruptly disappeared. In subsequent decades, superconductivity was observed in several other materials. In 1913, lead was found to be superconductor at 7 K, and in 1941 niobium nitride was found to superconduct at 16K. Before 1980, the critical temperature found cannot be higher than 30K. Those superconductor are called conventional superconductor. Theoretically, they can be fully explained by BCS theory and phenomenological Landau-Ginzburg theory.

A perfect superconductor is a material that exhibits two characteristic properties, namely zero electrical resistance and perfect diamagnetism, when it is cooled below a particular temperature Tc, called the critical temperature. At higher temperatures it is a normal metal, and ordinarily is not a very good conductor. For example, lead, tantalum, and tin become superconductors, while copper, silver, and gold, which are much better conductors, do not superconduct. In the normal state some superconducting metals are weakly diamagnetic and some are

paramagnetic. Below Tc they exhibit perfect electrical conductivity and also perfect or quite pronounced diamagnetism. Perfect diamagnetism, the second characteristic property, means that a superconducting material does not permit an externally applied magnetic field to penetrate into its interior.
Superconductivity has a wide range of applications in various fields of science and technology. Here are some of the applications of superconductivity:

1. Magnetic Resonance Imaging (MRI): Superconducting magnets are used in MRI machines to create strong magnetic fields. These magnets produce a more accurate and detailed image of internal body organs and tissues, making MRI an essential tool in medical diagnosis.

2. Particle Accelerators: Superconducting magnets are used in particle accelerators to produce high-energy particle beams. This is important in high-energy physics research and can help us understand the fundamental properties of matter.

3. Power Transmission: Superconducting wires can transmit electricity with zero resistance, which means less energy is lost during transmission. This makes superconducting power cables more efficient and cost-effective compared to traditional power cables.

4. Levitating Trains: Superconductors can be used to levitate trains above the track, reducing friction and increasing efficiency.

5. Energy Storage: Superconducting energy storage devices can store large amounts of energy for long periods without any significant loss. This technology is important for renewable energy systems, where energy storage is crucial to manage fluctuations in supply and demand.

6. Quantum Computing: Superconducting materials are used in quantum computers to create qubits, which are the basic building blocks of quantum computers. These computers have the potential to perform calculations much faster than traditional computers.

The electrical conductivity of metals arises from the presence of free electrons in their outermost shells that can move freely in response to an electric field. When a current flows through a metal, these free electrons move in a direction along the applied electric field. Meanwhile, the ions, which are positively charged, remain fixed in position and they vibrate in place around their equilibrium positions, known as lattice vibrations.

The resistance of a metal to electrical current flow is due to the scattering of these free electrons by lattice vibrations. As the temperature increases, the amplitude of these lattice vibrations also increases, causing more frequent collisions between the free electrons and lattice vibrations. Consequently, the resistance of the metal increases with temperature. The dependence of resistivity ρ on temperature for a typical metal is shown in Figure 1.

At low temperatures, the resistivity of a metal decreases as the temperature approaches absolute zero. This is because at low temperatures, the lattice vibrations are reduced, and the scattering of electrons is minimized. However, at T=0, the resistivity is not zero due to the presence of impurities in the metal. This residual resistivity is denoted by ρ_0.

The temperature dependence of resistivity can be expressed by the Matthiessen’s rule, which states that the total resistivity of a metal is the sum of its residual resistivity and its temperature-dependent part, as expressed in Equation 1.

$$\rho = \rho_0 + \rho(T) \text{ --- } (1)$$

Here, ρ is the resistivity of the given metal, ρ_0 is the residual resistivity, and $\rho(T)$ is the temperature-dependent part of the resistivity.

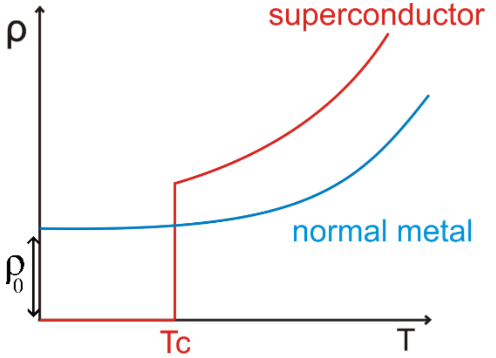


Figure 1: Temperature dependence of resistivity

Temperature dependence of resistivity

Figure 1 shows the temperature dependence of resistivity ρ in a superconductor. In the non-superconducting state, ρ decreases with decreasing temperature, similar to a normal metal, until it reaches a critical temperature Tc. At Tc, ρ abruptly drops to zero, indicating the transition from the normal state to the superconducting state of the material. The critical temperature varies depending on the superconductor under investigation. For example, mercury becomes a superconductor with zero resistance at 4.2 K. Superconductors and their critical temperature is listed in Table 1.

Superconductor	**Chemical Symbol**	**Critical Temperature (K)**
Mercury	Hg	4.2
Lead	Pb	7.2
Niobium	Nb	9.3
Tin	Sn	3.7
Gallium	Ga	1.5
Indium antimonide	In Sb	1.9
Tungsten Carbide	WC	12.5

Superconductor	Chemical Symbol	Critical Temperature (K)
Magnesium Diboride	MgB2	39

Table 1: Superconductors and its critical temperature

Meissner effect

It was observed by Meissner and Ochsenfield that, when a superconductor is cooled below its critical temperature (T_c), it undergoes a phase transition and enters a superconducting state. In this state, the superconductor exhibits perfect diamagnetism, meaning it repels any external magnetic field from its interior. This expulsion of magnetic flux is a characteristic feature of the Meissner effect.

In a perfect diamagnetic material, we know that

$$\vec{B} = \mu_0(\vec{H} + \overrightarrow{M})$$

B = μ_0 (H+M)

Where, $\vec{B}$ represents the magnetic induction, $\vec{H}$ the magnetic field intensity, $\vec{H}$ the magnetization vector and $\mu_0 = 4\pi \times \frac{10^{-7}H}{m}$ the permittivity of the free space. We can express magnetization vector $\vec{M}$ ($\vec{M}$ = magnetic dipole moment per unit volume) in the following form, for linear medium

$$\vec{M} = \chi\vec{H}$$

χ is + ve for paramagnetic like Aluminum, Platinum and –ve for diamagnetic like gold, silver, copper.

Where, χ is the magnetic susceptibility. Substituting the value of magnetization vector in eq. , one gets the following expression:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \chi\vec{H}) = \mu_0(1 + \chi)\vec{H}$$

As we have observed for a superconducting sample, the magnetic field is zero inside the specimen, that is,

$$0 = \mu_0(1+\chi)\vec{H}$$

$$\Rightarrow \chi = -1$$

Thus, a superconductor is an ideal diamagnetic material. For a normal diamagnetic material, $|\chi| \ll 1$

Also from $\vec{B} = \mu_0(\vec{H} + \vec{M}), \vec{B} = 0 \Rightarrow \vec{M} = -\vec{H}$

So magnetization is equal and opposite to that of $\vec{H}$

According to the Maxwell's equation,

$$\Delta \times \vec{B} = \mu_0 \vec{J}$$

$$\Rightarrow \vec{J} = 0$$

Thus, the total current density $\vec{J} = 0$ in the interior of a superconductor. Surface currents, however can exist in a superconductor.

In a superconducting pure metal, the magnetic flux is expelled from the sample, irrespective of its geometry and sequence in which the magnetic field is applied. In reality, the induction field decays exponentially with distance from the surface of the sample. This characteristic depth is called the penetration depth and is generally estimated to be a few hundred angstroms (10^{-7} m).

The Meissner effect occurs due to the formation of Cooper pairs in the superconductor. Cooper pairs are composed of two electrons with opposite spins and momenta that form a bound state due to the attractive interaction mediated by lattice vibrations (phonons). These Cooper pairs collectively behave as a single entity like Bosons, known as a macroscopic wavefunction, which extends throughout the entire superconductor.

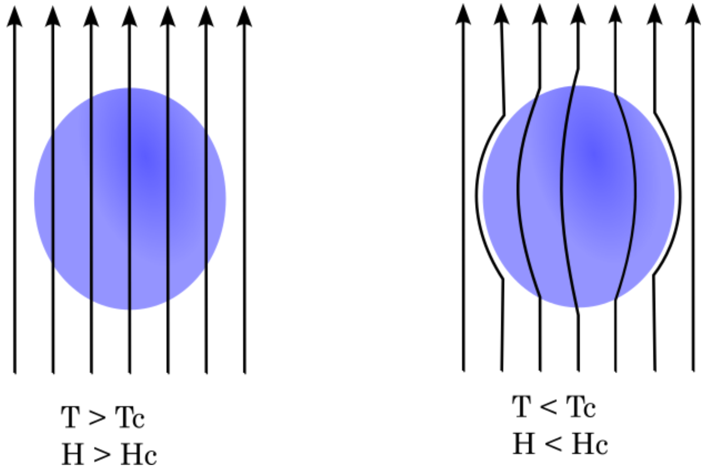


Figure 2. Meissner Effect

When an external magnetic field is applied to a superconductor, the magnetic flux tends to penetrate the material. However, the Cooper pairs actively respond to the magnetic field by forming currents that generate an opposing magnetic field. These induced currents create a magnetic field that perfectly cancels out the external magnetic field within the bulk of the superconductor. As a result, the magnetic field is expelled from the interior of the superconductor, confining it to the exterior regions as shown in figure 2.

It's important to note that the Meissner effect is only observed in type I superconductors, which have a single critical temperature and exhibit a complete expulsion of the magnetic field. Type II superconductors, on the other hand, exhibit a mixed state where magnetic flux penetrates the material in the form of quantized vortices. In type II superconductors, the Meissner effect is limited to the region between these vortices.

Critical current

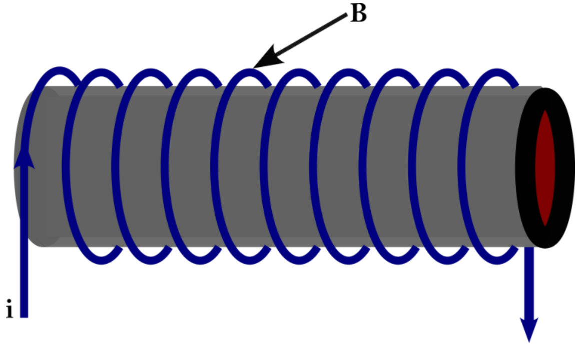


Figure 3. A superconducting coil of wire

Consider a current of ***i*** ampere is applied to a superconducting coil of wire as shown in figure 3. The application of the current induces a magnetic field and hence the superconducting property is destroyed. The critical current required to destroy the superconducting property is given by

$$i_c = 2\pi R H_c$$

Where, R is the radius of the superconducting wire, H_c is the critical magnetic field required to destroy the superconducting property. This property was discovered by Silsbee.

Example 1. The critical magnetic field at 5 *K* is 2×10^3 *A/m* in a superconductor ring of radius 0.02 m. Find the value of critical current.

Solution.

$I_c = 2\pi\ R\ H_C = 2\times3.14\times0.02\times2\times10^3$ *A/m* = **251.4 A**

Example 2. Calculate the critical current for a wire of lead having a diameter of 1 mm at 4.2 *K*. the critical temperature for lead is 7.18 *K* and $H_C(0)=6.5\times10^4$ *A/m*.

Solution.

$$H_c(T) = H_0(0)\left[1-\left(\frac{T}{T_C}\right)^2\right] = 6.5\times10^4\left[1-\left(\frac{4.2}{7.18}\right)^2\right]$$
$$= 4.28\times10^4\ A/m$$

The critical current $I_C = 2\pi r H_C = \pi d H_C = 1\times3.14\times10^{-3}\times4.28\times10^4$ = **134.5 *A***

Types of superconductors

Superconductors can be categorized into two groups based on their magnetization properties when subjected to an external magnetic field. They can be classified as Type 1 or Type 2 superconductors. *Soft superconductors* are referred to as Type 1 superconductors, while Type 2 superconductors are known as *hard superconductors*.

Type-1 Superconductors

In soft superconductors, the magnetic field gets totally expelled from the interior of the material, below a certain critical magnetic field H_c. At the critical magnetic field, there is an abrupt loss of superconductivity. A plot of magnetization versus magnetic field for a Type-1 or soft superconductor is shown in figure 4.

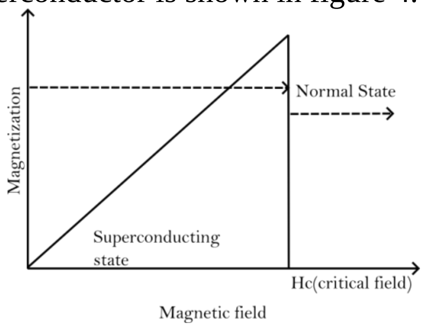


Figure 4. A plot of magnetization versus magnetic field

The figure 4 shows that the magnetic field can penetrate a material above the critical field H_c. The material above H_c is said to be in its normal state. This type of a superconductor demonstrates complete Meissner effect at magnetic fields below H_c where, it becomes an ideal diamagnetic material. Lead, tin and mercury are common examples of soft superconductors. The transition to the superconducting state at H_c is reversible. Most superconducting metals exhibit Type -1 superconductivity at very low H_c values (around 10^{-1} T).

Type-2 superconductors

Type-2 superconductors show two critical magnetic fields: H_{c1} and H_{c2}. Typical magnetization curves for hard superconductors are shown in figure 5.

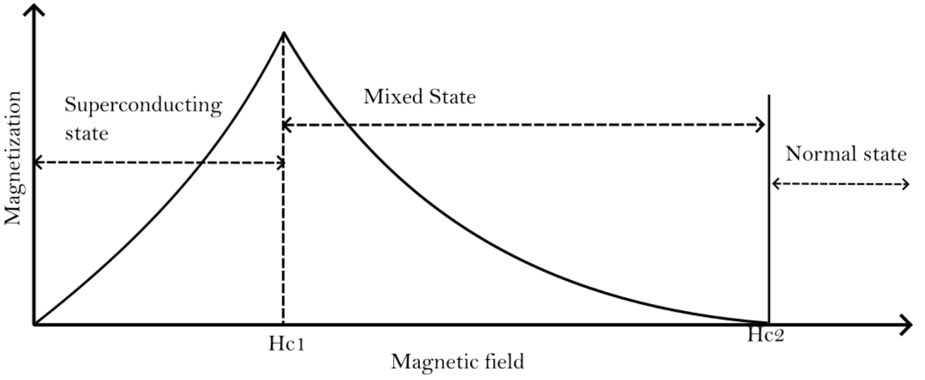


Figure 5. A plot of magnetization versus two critical magnetic fields

The first critical field H_{c1}, the material is superconducting, and hence exhibits ideal diamagnetic behaviour. Magnetic flux is expelled completely from within the material. Between the two critical fields H_{c1} and H_{c2} the material exists in a mixed state. The material still demonstrates superconducting property, but exclusion of flux from within the specimen is partial. The normal non-superconducting state is reached at the critical field H_{c2}.

Alloys or transition metals with high electrical resistivity in the normal state generally demonstrate Type-2 superconductivity. The values of H_{c2} are 100 times or more than the values of Type-1 superconductors. H_{c2} values up to 50 T have been obtained in some materials. As the magnetic field is increased, magnetization observed in hard superconductors reduces gradually, whereas in soft superconductors, magnetization vanishes abruptly at the critical magnetic field. Hard superconductors are used as magnetizing coils to obtain high magnetic fields (10 T or higher).

Temperature dependence of critical field

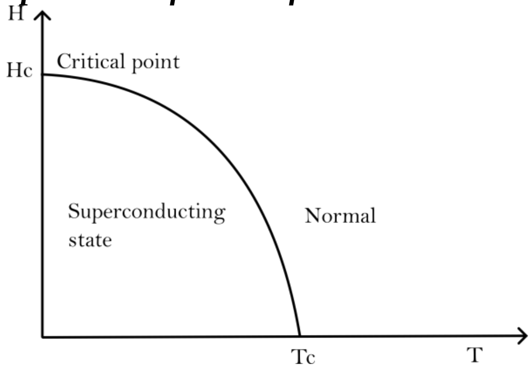


Figure 6. The temperature dependence of the critical field

The magnetic field plays a significant role in the observation of the phenomenon of superconductivity. Superconductivity disappears at magnetic fields greater than a critical field H_c. The minimum field required to destroy the superconducting property is given by

$$H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right]$$

Where, H_0 is the field required to destroy the superconducting property at 0 K, H_c the minimum field required to destroy the superconducting property at T K and T_c the transition temperature of the material.

The temperature dependence of the critical field is given by Tuyn's law. Variation of the magnetic field with temperature is shown in figure 6.

Example 3. The transition temperature for Pb is 7.2 *K*. However, at 5 *K* it loses the superconducting property if subjected to magnetic field of 3.3×10^4 *A/m*. find the maximum value of H which will allow the metal to retain its superconductivity at 0 *K*.

Solution.

$$H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right] = \frac{3.3\times10^4}{1-(25/51.28)} = 6.37\times10^4\ A/m$$

Example 4. The critical field of niobium is 1×10^5 *A/m* at 8 *K* and 2×10^5 *A/m* at 0 *K*. Calculate the transition temperature of the element.

Solution.

$$T_C = \frac{T}{\left[1 - \frac{H_C(T)}{H_C(0)}\right]^{1/2}} = \frac{8\ K}{\left[1 - \frac{1\times10^5\ A/m}{2\times10^5\ A/m}\right]^{1/2}} = 11.3\ K$$

Example 5. The transition temperature for lead is 7.26 *K*. The maximum critical field for the material is 8×10^5 *A/m*. Lead has to be used as a superconductor subjected to a magnetic field of 4×10^4 *A/m*. What precaution will have to be taken?

Solution.

$$T = T_C\left[1 - \frac{H_C(T)}{H_C(0)}\right]^{\frac{1}{2}} = 7.26\ K\left[1 - \frac{4\times10^4\ A/m}{8\times10^5\ A/m}\right]^{\frac{1}{2}} = 7.08\ K$$

Therefore, the temperature of the metal should be held below 7.08 *K*.

BCS theory

The BCS theory is based on the formation of ***Cooper pairs***. In 1957, Bardeen, Cooper and Schrieffer proposed a microscopic theory known as BCS theory. The BCS theory explains most of the phenomenon associated with superconductivity in a natural manner. This theory involves the electron interaction through phonon as mediators. In 1950, Froblich and Bardeen showed the existence of self energy of electron accompanied by virtual phonons when it moves through a crystal lattice. During the flow of current in a superconductor, when an electron approaches a positive ion of the metal lattice, there is Coulomb attraction between the electron and the lattice ion. This produces a distortion in the lattice i.e., the positive ion gets displaced from its mean position. The distortion gives rise to a ***phonon***. Smaller the mass of the positive ion core, the greater will be the distortion. This interaction called the **electron-phonon** interaction leads to a scattering of the electron and causes electrical resistivity. The distortion causes an increase in the density of ions in the region of distortion. The higher density of ions in the distorted regions attracts, in its turn another electron. Thus, a free electron exerts an attractive force another electron through phonons. Phonons are quanta of lattice vibrations. Suppose, an electron of wave vector K emits a virtual phonon ***q*** which is absorbed by another electron having wave vector K^1. Thus, K is scattered as K-q and K^1+q is as shown in figure 7. The resulting electron-electron interaction depends on the relative magnitude of the electronic energy change; the interaction becomes attractive interaction (V_{ph}). Thus, for attractive interaction, the wave vector and spin are represented as K↓ and K↑. Therefore, the two electrons interacting attractively in the phonon field are called cooper pair and the same is shown in figure 7.

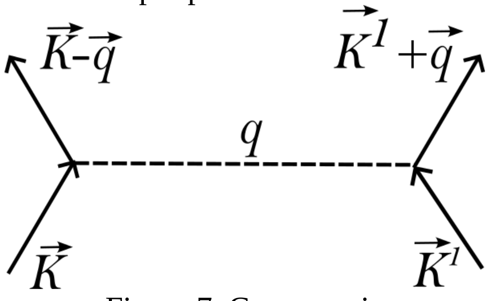


Figure 7. Cooper pair

At normal temperature, the attractive force is too small and pairing of electrons does not take place. Since the number of phonons increases with temperature, the resistivity is a sensitive function of temperature, particularly in the low temperature region when it varies at T^5.

At lower temperature i.e., below the critical temperature the apparent force of attraction reaches a maximum value for any two electrons of equal and opposite spins and opposite momentum. This force of attraction exceeds the Columbian force of repulsion between two electrons and the electron stick together and move a pairs. These pairs of electrons of opposite momenta are called ***Cooper pairs***.

A pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field is called a Cooper pair.

The Cooper pair has a total spin of zero. As a result, the electron pairs in a superconductor are bosons.

At a temperature $\mathbf{T < T_c}$, the lattice electron interaction is stronger than electron-electron force of Coulomb. In a typical superconductor, the volume of a given pair encompasses as many 10^6 other pairs. This dense cloud of Cooper pairs from a collective state where strong correlations arise among the motions of all pairs because of which they drift cooperatively through the material. Thus, the superconducting state is an ordered state of the conduction electrons. The motion of all Cooper pairs is the same. Either they are at rest, or if the superconductor carries a current, they drift with identical velocity. Since the density of Cooper pairs is quite high, even large currents require only a small velocity. The small velocity of Cooper pairs combined with their precise ordering minimizes collision processes. The extremely rare collisions of Cooper pairs with the lattice leads to vanishing resistivity. At this stage, the Cooper pair of electrons smoothly sail over the lattice point without any exchange of energy. As a consequence of this, the substance possesses infinite electrical conductivity.

When the electrons flow in the form of Cooper pairs in materials, they do not encounter any scattering and thereby resistance reduces to zero or conductivity becomes infinity which is known as superconductivity.

Quantum tunnelling

The word 'tunnelling' came into use when, with the advent of quantum mechanics, it was observed that electrons possess the unique property of sneaking through nano-sized barriers to contribute to the minority carriers in semiconductors. The probability of tunnelling is small; of the order of 10^{-10}. The following question came to the mind of all scientists on observing and understanding superconductors. Do the Cooper pairs have this unique property of tunnelling? Josephson explored this probability of the newly-found cohesive pair of electrons that is the Cooper pair, to tunnel through, when two superconductors are joined together by a thin layer of oxide. This also led to many applications of superconductors.

Two superconductors are joined through a nano-dimensional insulating layer, forming coupled oscillators or coupled circuits. Josephson tunnelling can be explained in both DC and AC effects.

Josephson junction

In 1962, B.D Josephson predicted a number of remarkable phenomena about superconductivity, which are used to understand the superconducting properties.

DC Josephson Effect

An insulating material of thickness nearly 1 to 2 nm sandwiched between two different superconducting materials is known as Josephson device. In a Josephson device, a dc voltage is found to flow from a junction that has a higher density to the junction with lower density of superconducting electrons. This phenomenon is said to be DC Josephson effect. The dc current flowing through the device is equal to

$$J=J_0 \sin\delta$$

Where δ is the phase difference and J_0 is a constant and it is the maximum current density through the insulator. The Josephson device is shown in figure 8, where A and B are the two different superconductors.

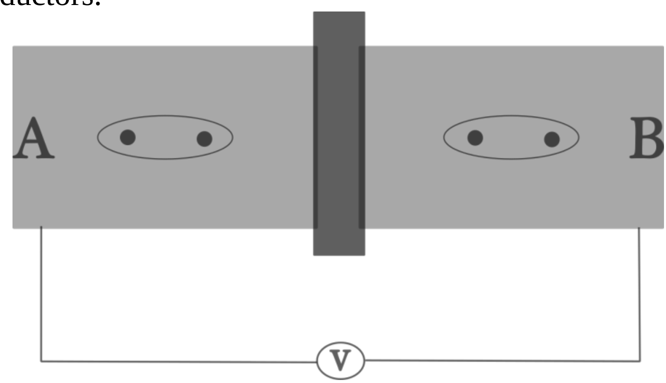


Figure 8. The Josephson device

The pair of electrons present in a superconducting material is in the same phase. Whenever a Josephson junction is formed by sandwiching an insulator in between two different superconductors, the superconducting electrons present in two different superconducting materials of a Josephson device need not be in the same phase. There will be a tunnelling of electrons from one superconducting material with higher electron density to another superconducting material with lower electron density. Due to this tunnelling of electrons, a dc voltage appears across the Josephson device, even though no field is applied there.

The insulating layer introduces phase difference (ϕ) between wave function of cooper pairs on either side and hence super current appears even though the applied voltage is zero. Super current (or junction current) is given by

$$I_J = I_C sin\phi$$

Here Ic is the maximum junction current that depends on the thickness of insulating layer. It is quite small (between 1μA to 1 mA).

There arises voltage V across the junction. It can be shown that the junction voltage is directly proportional to rate of change of super current I_J.

$$V \propto \frac{dI}{dt}$$

$$V = L_J \frac{dI}{dt}$$

Here L_J is known as Josephson Inductance which is very much similar to inductance of a coil. However Josephson inductance is nonlinear and can even be negative also. This is a crucial fact relevant to the operation of flux qubits.
Josephson junction consists of two superconducting electrodes separated by a thin insulator. Thus a Josephson junction has a capacity, which is indicated by the capacitor C.

AC Josephson effect
If we apply DC voltage across the junction it introduces an additional phase difference between the Cooper pairs and an alternating current is generated. This is known as AC Josephson Effect.
The frequency of alternating current is directly proportional to applied voltage V and is given by

$$\nu = \frac{2eV}{h}$$

The photon energy of emission or absorption at the junction is hν = qV = 2eV

i.e. $\nu = \frac{qV}{h} = \frac{2eV}{h} = \frac{2\times1.6\times10^{-19}\times V}{6.625\times10^{-34}} = 483.5\times10^{12}V\ Hz$

for an applied voltage of 1μV, the frequency of the ac signal is

$$\nu = 483.5\times10^{12}\ V = 483.5\times10^{12}\times10^{-6} = 483.5\ M\ Hz$$

By measuring the frequency of the ac signal accurately, the value of e/h and the photon energy, 2eV are accurately measured.

Example 6. A Josephson junction with a voltage difference of 650 μV radiates electromagnetic radiation. Calculate its frequency.

Solution.

$$\nu = \frac{2\ eV}{h}\ Hz = \frac{2(1.6\times10^{-19})(650\times10^{-6})}{6.625\times10^{-34}} = 3\times10^{11}Hz$$

High-T_c Superconductors

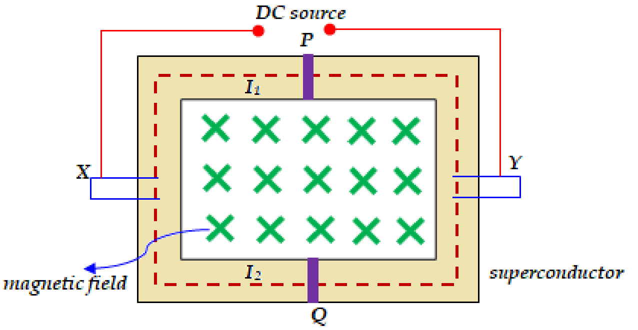
The temperature T_c below which a material shows superconductivity is an important criterion in deciding the application of a superconductor. The earliest known superconductors needed to be cooled to temperature close to 4 K (boiling temperature of helium) to display superconducting properties. Though such temperatures are achievable, cooling to such a low temperature involves enormous expenses. Continuous research is being carried out to discover superconductors with higher T_c values. Such superconductors are collectively referred to as high-T_c superconductors (HTSCs). In 1986, K.A Muller and J.G Bednorz reported superconducting properties of lanthanum, barium and copper oxides at temperatures higher than 30K. This was followed by the synthesis of yttrium compounds by C.W Chu in 1987. These yttrium compounds acquired the superconducting state with a T_c close to 90K. Several HTSCs have since been reported with T_c > 30K. The discovery of these HTSCs rekindled the interest in superconductivity from the application viewpoint.

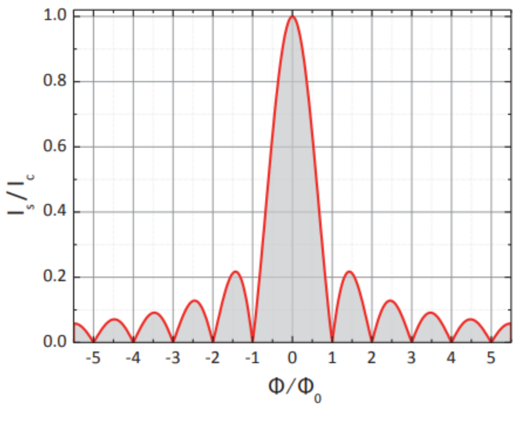
Material	**T_c(K)**
$La_{2-x}M_xCuO_4$ (M=Ca,Sr,Ba)	20-40
$LnBa_2Cu_3O_7$	90
$Bi_2(CaSr)_{n+1}Cu_nO_{2n+4}$(n=1-3)	90
$Tl_2Ca_{n-1}Cu_nO_{2n+4}$(n=1-4)	90-110

The highest Tc recorded till now is 138K, which has been obtained for a thallium-doped, mercuric-cuprate compound comprising elements such as mercury, thallium, barium, calcium, copper and oxygen.
Note: Supercondcutors.org here in reports that the 30c superconductor discovered in 2012 has been successfully reformulated to produce a high Tc of above 35°C (95°F, 308K). This was accompanied by a simple substitution of tetravalent silicon in the magnesium atomic sites. The chemical formula thus becomes $Tl_5Pb_2Ba_2Si_{2.5}Cu_{8.5}O_{17+}$.

SQUID

SQUID is an acronym for Superconducting Quantum Interference Device. It is a magnetometer. It is based on the principle of Josephson effect.
It is used to measure extremely weak magnetic field. It is basically a sensitive magnetometer. Fields as small as 10^{-21} T produced by biological currents in human heart and brain can be measured.
There are two types in SQUID namely DC SQUID and RF SQUID (or AC SQUID).
DC-SQUID





It consists of two Josephson junctions (P and Q) arranged in parallel and DC source is connected across X and Y. As a result biasing current enters at X and leaves at Y. Let I_1 and I_2 be the tunneling currents across P and Q. when magnetic field is applied perpendicular to the arrangement, a phase difference is introduced between I_1 and I_2 and they interfere and produce interference effect (very much similar to Young's double slit experiment).

In superconductors the current is caused by the Cooper pairs. Cooper pairs are associated with de Broglie waves. Applied magnetic flux introduces phase shift between these waves and hence they interfere when arrive at Y. The resultant current is

$$I_r = \frac{I_c \sin\frac{\pi\phi}{\phi_0}}{\frac{\pi\phi}{\phi_0}}$$

Where ϕ is the flux linked with the SQUID and ϕ_0 = (h/2e = 2.06 ×10^{-15} wb/m^2) called fluxoid. A graph of $\frac{I_r}{Ic}$ v/s $\frac{\phi}{\phi_0}$ is as shown. The graph is exactly similar to intensity distribution in single slit diffraction of light.

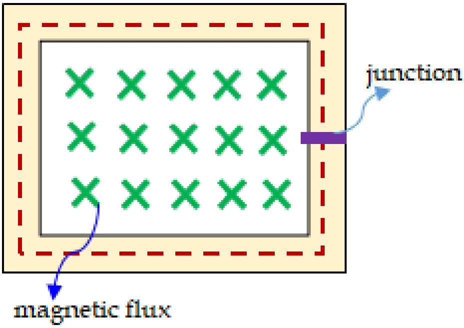
By measuring the resultant current one can measure the applied magnetic flux. It is possible to measure very small magnetic flux using this arrangements.

DC SQUIDS are used in

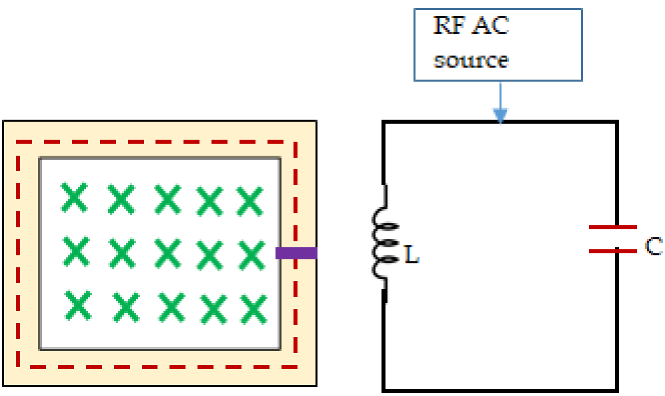
- ✓ Measurement of magnetic fields as small as 10^{-21} T produced by biological currents in human heart and brain can be measured.
- ✓ Geophysical measurements (connected with rocks, seismic waves etc)
- ✓ Nondestructive testing (testing the material without damaging it)
- ✓ fabrication of qubits
- ✓ Very precise and accurate measurement of voltage

RF-SQUID

The RF (Radio Frequency) SQUID is a one-junction SQUID loop and is used as a magnetic field detector. Although it is less sensitive than the DC SQUID, it is cheaper and easier to manufacture and is therefore more commonly used. It is as shown in fig (i)



(i)



(ii)

RF SQUID loop is placed near a LC circuit which is connected to RF AC source as shown in Fig (ii). The loop is immersed in a magnetic field whose flux is Φ (to be measured). Now pass an oscillating current (I) through LC circuit from the RF source. It induces magnetic flux Φ_{RF}. This flux is coupled with the loop. The total external flux is

$$\Phi_{ex} = \Phi + \Phi_{RF}$$

The loop is linked to circuit through mutual inductance. By chance if the flux in the loop (Φ) changes, there will be corresponding changes in Φ_{ex}. Any changes in the total flux Φ_{ex} will induce emf (according to Faraday's law) in the LC circuit and hence voltage (V) across LC changes. By measuring the change in V one can measure the external magnetic flux (Φ) and its variation w.r.t. time.

RF-SQUIDS are used in

- ✓ bio magnetism (ie, to measure magnetic field produced by different organs of our body)
- ✓ geophysical measurements (connected with rocks, seismic waves etc)
- ✓ non destructive testing (testing the material without damaging it)

Applications in quantum computing:
Superconducting electronic circuits using superconducting qubits as artificial atoms are among the most promising approaches to building quantum computers. For superconducting qubits, the two logic states are the ground state and the excited state, denoted $|g\rangle$ and $|e\rangle$ respectively. Superconductors are implemented due to the fact that at low temperatures they have almost infinite conductivity and almost zero resistance. Superconducting capacitors and inductors are used to produce a resonant circuit that dissipates almost no energy, as heat can disrupt quantum information. The superconducting resonant circuits are a class of artificial atoms that can be used as qubits.
Superconducting circuits are the basis of superconducting qubits, as they are built using superconducting elements. Superconducting circuits that behave quantum mechanically. For a circuit to behave quantum mechanically there must be no dissipation, that is, the circuit must have zero resistance at the (qubit) operating temperature. This is essential to preserve quantum coherence. Non-linearity is also essential, because we want to approximate our system to a two level qubit, meaning that the energy level cannot be uniformly spaced. The only element which is both non-dissipative and non-linear is a Josephson junction. This makes Josephson junctions the essential element of superconducting qubits.

Qubits

Introduction:

Quantum computation relies on quantum bits or ‘qubits’. A qubit is the physical carrier of quantum information and it is equivalent of the classical bits (0 and 1) used in today's computers.
The quantum state of qubits are written in terms of two levels namely $|\mathbf{0}\rangle$ and $|\mathbf{1}\rangle$. The abstract or theoretical notion of a qubit is as follows (in two dimensional vector form).

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

But the physical type of qubit is made from SQUIDS.

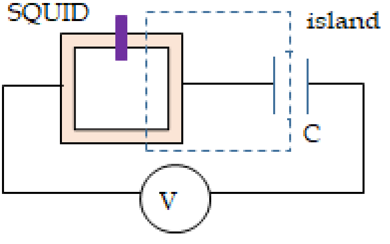
Among all types of qubits, superconducting qubits will be the chosen ones to be studied here. Superconducting qubits are promising candidates for implementing quantum computers. As we will now see, they have demonstrated large coherence times of the order of nanoseconds and promising scalability. One of the most important features that differentiates them from the other types of qubits, however, is their macroscopic dimension and the easy manufacture. Their size is of the order of micrometres and can be built using electron beam lithography, the same process used in microelectronics. In this section the basic construction and operation of superconducting qubits will be introduced. We will study different types of superconducting qubits, depending on their quantized states: charge, flux or phase. We will study some options of how each qubit can be measured and also how they are affected by external noise and their ability to maintain coherence.
Principle of SQUID as qubit:
The main principle behind the designing of qubit is ‘discreteness’ or’ quantization’. We all know that charge, magnetic flux and phase are

quantized (discrete in nature). In SQUIDS we have charges (Cooper pairs), magnetic flux and phase (of matter wave associated with Cooper pairs). By controlling these parameters across the junction using electromagnetic pulses we can operate the quantum states and thereby generate ‘bits’.

Charge qubit

The basic idea of charge qubit is to create a small superconducting area called island which is connected to a circuit in such a way that we can control the number of charge carriers (cooper pairs) that are located on the island.

Construction and working

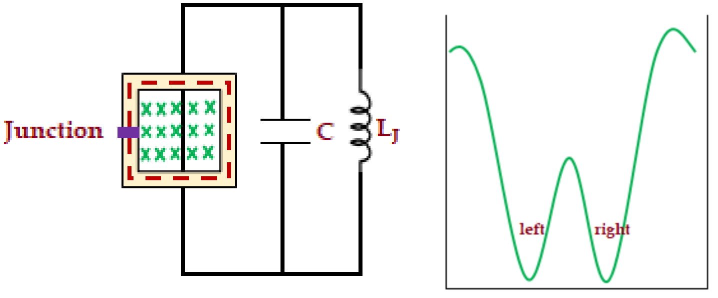


The circuit consists of RF SQUID, a capacitor C and external voltage source V. The dotted region is *island* and it encloses SQUID and capacitor. Cooper pairs tunnel through the junction and enter the island or they may leave the island and enter the junction. This entry and exit of Cooper pair is controlled by external voltage source. The number of Cooper pairs entering and leaving the *island* leads to variation in the energy levels. We can use any two of these energy levels to represent qubit.

Flux qubit

Construction and working

A flux qubit is a superconducting loop having single Josephson junction and immersed in an external magnetic field. The Josephson junction has an intrinsic inductance L_J and the capacitance C. the equivalent circuit is as shown. Cross marks represent the external magnetic flux (Φ).



At a given point of time this system can be described by two quantities – the *charge stored* in the capacitor C and the *magnetic flux* through the loop. Using quantum mechanics one can show that this system is equivalent to a particle moving in a double potential well as shown in fig).The lowest point of each well is treated as one energy state. Hence this system has two basic states, one corresponding to the left minimum and the other corresponding to the right minimum. There is a probability for the particle to cross the potential wall by means of **tunneling** and to flip from one state into the other state. This two-state system property is used in the operation of flux qubit.
NOTE: This Arrangement is linked with a LC tank circuit by means of mutual induction (as discussed in the working of RF SQUID). The LC circuit is driven by periodically varying current (I) having a definite frequency (ω). Hence the magnetic flux linked with the loop (SQUID) as well as voltage across LC will change periodically. This property of RF-SQUID is used to design the qubit.

Phase qubit
Superconducting phase qubit is also known as current-biased Josephson junctions. It is nothing but current biased Josephson junction operating at zero voltage state (ie, DC SQUID). The applied magnetic flux introduces the phase difference between the cooper pairs across the junctions. Hence the resultant super currents in the junctions (due to flow of Cooper pairs) interfere producing maxima and minima. These

maxima and minima are treated as two states and this two-state property is used in the operation of phase qubit.

Questions:

1. The dc resistance of a superconductor is practically zero. What about its ac resistance?
2. How can you change a superconductor from type I to type II?
3. What are Cooper pairs?
4. How is Josephson tunnelling different from single particle tunnelling?
5. A superconducting wire and a copper wire are connected in parallel. Does the copper wire carry current when a potential difference is applied?
6. What is the significance of critical temperature, critical magnetic field and critical current density for superconductors?
7. What is Meissner effect?
8. Compare the dependence of resistance on temperature of a superconductor with that of a normal conductor.
9. What do you mean by "perfect diamagnetic" of a superconductor?
10. Describe how Cooper pairs are formed and explain the salient features of superconductivity?
11. Explain the term high temperature superconductivity. Give the various applications of superconductors.
12. What are type I and type II superconductors? Explain B.C.S theory with key note of Cooper pairs.
13. Why are type I superconductors are poor current carrying conductors?
14. Explain ac and dc Joesphson's effect.
15. Describe the principle of SQUID and mention its applications.
16. Discuss the principle and working of SQUID.