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RV COLLEGE OF ENGINEERING*
Autonomous Institution affiliated to VTU
II Semester B. E. Examinations Oct-Nov 2023

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS
MODEL QUESTION PAPER
Branch: AI, BT, CD, CS, CY, IS

Time: 03 Hours**Maximum Marks: 100****Instructions to candidates:**

- Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
- Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART – A

1	1.1	For what value of b , the vector $\vec{F} = y(bx^2 + z)\hat{i} + x(y^2 - z^2)\hat{j} + 2xy(z - xy)\hat{k}$ is solenoidal.	01
	1.2	If $\phi = x^2y + y^2z + 4$, then $\nabla\phi =$ _____.	01
	1.3	Particular integral of $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = e^{3x}$ is_____.	01
	1.4	If $x = e^{-3t}$ is the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + kx = 0$ then value of k is _____.	01
	1.5	The value of $\Delta^3[(1 + 3x)(1 - 5x)(1 - 4x)]$ taking interval of differencing $h = 1$ is _____.	01
	1.6	Construct a forward difference table for the data $(0,2), (2,6), (4,7)$.	01
	1.7	If \vec{F} represents force acting on a particle then $\int_c \vec{F} \cdot d\vec{r}$ represents _____.	01
	1.8	If \vec{F} is velocity of fluid then $\iint_S \vec{F} \cdot \hat{n} ds$ represents _____.	01
	1.9	Calculate the sum of positive divisors of the integer 882.	02
	1.10	Find the remainder when $135 \times 74 \times 48$ is divided by 7.	02
	1.11	Find the directional derivative of $\phi = 3x^2 + 2y - 3z$ at $(1,1,1)$ in the direction of $2\hat{i} + 2\hat{j} - \hat{k}$.	02
	1.12	If $\vec{F} = (x^2 + y)\hat{i} + (3y - 5x)\hat{j}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$ along the straight line from $A(0,0)$ to $B(2,0)$.	02
	1.13	The complementary function of the differential equation is $c_1 \cos 3x + c_2 \sin 3x$, then Wronskian is _____.	02
	1.14	Given	02

		<table><tr><td>x</td><td>1</td><td>3</td><td>5</td><td>7</td></tr><tr><td>$f(x)$</td><td>2</td><td>7</td><td>16</td><td>29</td></tr></table>	x	1	3	5	7	$f(x)$	2	7	16	29	
x	1	3	5	7									
$f(x)$	2	7	16	29									
		The value of $f'(7) =$ _____.											

PART – B

2	a	Find the greatest common divisor d of the numbers 1819 and 3587 using Euclid's algorithm and then obtain the integers x and y to satisfy $1819x + 3587y = d$.	6
	b	Solve the linear congruence $6x \equiv 15 \pmod{21}$.	4
	c	Given the public key $(e, n) = (7, 51)$, encrypt plain text LIV , where the alphabets A, B, C, \dots, X, Y, Z are assigned the numbers $3, 4, 5, \dots, 26, 27, 28$. Give the cipher text and find the private key d .	6

3	a	Find the unit tangent vector s to the curve $\hat{r} = 4 \sin t \hat{i} + 4 \cos t \hat{j} + 3t \hat{k}$ at the points $t = \frac{\pi}{6}$ and $t = \frac{\pi}{4}$. Obtain the angle between these tangent vectors.	5
	b	If $\hat{F} = \nabla(x^2y + y^2z + z^2x - xyz)$, then compute $\text{div } \hat{F}$ and $\text{curl } \hat{F}$ at $(1, 2, 1)$.	5
	c	Find the constants a and b such that the surfaces $ax^2 - byz = ax + 2x$ and $4x^2y + z^3 = 4$ are orthogonal at the point $(1, -1, 2)$.	6

OR

4	a	A particle moves along the curve $C: x = (2t^2 + 1), y = (5t - 3)$ and $z = (t^2 - 5t)$; where t is the time. Find the components of velocity and acceleration at $t = 2$ in the direction of $\hat{i} + 2\hat{j} + 3\hat{k}$.	5
	b	Show that the vector $r^n \hat{r}$ is irrotational for any constant n and solenoidal only for $n = -3$.	5
	c	Compute the values of the constants a, b, c such that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is curl free vector. Determine the scalar potential function ϕ such that $\vec{F} = \nabla\phi$.	6

5	a	Verify Green's theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.	8
	b	Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$, if $\vec{F} = 4y\hat{i} + 18z\hat{j} - x\hat{k}$ and S is the surface of the plane $3x + 2y + 6z = 6$ contained in the first octant.	8

OR

6	a	If $\vec{F} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$, evaluate $\oint_C \vec{F} \cdot d\vec{r}$ around the triangle in the xy plane with vertices $(0, 0), (2, 0)$ and $(2, 1)$.	8
	b	Use Stokes theorem to evaluate $\int_C \sin z dx - \cos x dy + \sin y dz$ where c is the boundary of the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1, z = 3$.	8

7	a	Solve $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log_e x)$.	8
	b	Apply the method of variation of parameters to solve the ordinary differential equation $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = \frac{1}{1+e^x}$	8

OR

8	a	Obtain the general solution of the differential equation: $2 \frac{d^3y}{dt^3} - 3 \frac{d^2y}{dt^2} + 6 \frac{dy}{dt} - 9y = \cos^2\left(\frac{\sqrt{3}}{2}t\right) + t.$	8
	b	An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads connected in series. Find the charge q and the current i at any time t . Given that $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0$ at $t = 0$, $q = 0.05$ C, $i = \frac{dq}{dt} = 0$ when $t = 0$.	8

9	a	<p>The following table gives the relation between steam pressure and temperature.</p> <table><tr><td>$T^{\circ}\text{C}$</td><td>361</td><td>367</td><td>378</td><td>387</td><td>399</td></tr><tr><td>P</td><td>154.9</td><td>167.9</td><td>191</td><td>212.5</td><td>244.2</td></tr></table> <p>Using suitable interpolation formula find the pressure at the temperature 372 and 404.</p>	$T^{\circ}\text{C}$	361	367	378	387	399	P	154.9	167.9	191	212.5	244.2	8				
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P	154.9	167.9	191	212.5	244.2														
	b	<p>The following table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:</p> <table><tr><td>$x = \text{height}$</td><td>100</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td><td>400</td></tr><tr><td>$y = \text{distance}$</td><td>10.63</td><td>13.03</td><td>15.04</td><td>16.81</td><td>18.42</td><td>19.90</td><td>21.27</td></tr></table> <p>Estimate the value of y when $x = 180$ and $x = 410$.</p>	$x = \text{height}$	100	150	200	250	300	350	400	$y = \text{distance}$	10.63	13.03	15.04	16.81	18.42	19.90	21.27	8
$x = \text{height}$	100	150	200	250	300	350	400												
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OR

10	a	Given that <table><tr><td>x</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td><td>1.6</td></tr><tr><td>y</td><td>7.989</td><td>8.403</td><td>8.781</td><td>9.129</td><td>9.451</td><td>9.750</td><td>10.031</td></tr></table> Compute the velocity and acceleration at $x = 1.1$.	x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	y	7.989	8.403	8.781	9.129	9.451	9.750	10.031	8
	x	1.0	1.1	1.2	1.3	1.4	1.5	1.6											
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031												
	b	From the following data, estimate the number of students who obtained marks between 40 and 45 using Newton's interpolation method <table><tr><td>Marks:</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td></tr><tr><td>Number of Students:</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr></table>	Marks:	30-40	40-50	50-60	60-70	70-80	Number of Students:	31	42	51	35	31	8				
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