Electrical Conductivity in Solids

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Outline

Outline . . .

- Quantum Mean Free Path of Electrons

- Phonons

- Matthiessen's rule

3. Band Theory of Solids

- Carrier Concentration in Metals at $0\,\mathrm{K}$

- Intrinsic Semiconductor

- Effective Mass of the Electrons and Holes

- Intrinsic Conductivity

- Electron Concentration in Conduction Band of a Semiconductor

- Hole Concentration in Valence Band of a Semiconductor

- Fermi Level in Intrinsic Semiconductor

- Intrinsic Carrier Concentration

Outline...

Classical Free Electron Theory

- The properties of metals such as *electrical conduction* and *thermal conduction* are due to the *free electrons* or conduction electrons in metals

- Classical free electron theory of metals was first proposed by Drude in 1900 and later developed by Lorentz.

- Hence classical free electron theory is also called as Drude-Lorentz theory.

Assumptions of Classical Free Electron Theory

- A metal is modeled as a three-dimensional ordered network of positive ions with the outermost electrons of the metal atoms *freely moving about the solid* colliding with the atoms in the lattice.

- The electric current in a metal, due to an applied field, is a consequence of *drift velocity of the free electrons* in a direction opposite to the direction of the field.

- The free electrons are treated like gas molecules and thus assumed to obey the laws of *kinetic theory of gases*. According to it, in the absence of the external electric field the average kinetic energy of a free electron at temperature T is,

$$E_{\text{av}} = \tfrac{3}{2}kT = \tfrac{1}{2}m\overline{v^2} \quad \left| \quad \begin{array}{l} k = \text{Boltzmann's constant} \\ \overline{v^2} = \text{average of velocity squares of the electrons} \end{array} \right.$$

- The electric potential due to the ionic core (lattice) is taken to be essentially constant throughout the metal.

- The attraction between the free electrons and the lattice ions and the repulsion between the electrons are considered insignificant.

Some Basic Terms

- **Drift Velocity:** (v_d)
  When electric field is applied there will be net displacement of randomly moving (with collisions with atoms in the metal) free electrons in a direction opposite to the direction of the field. This displacement per unit time is called *drift velocity* which will be constant for the free electrons in constant field. This accounts for the electric current.

  If E is the applied electric field, τ is mean collision time then drift velocity is given by,

  $$v_d = \frac{eE}{m}\tau$$

  where e and m are charge and mass of the electron.

Some Basic Terms...

- **Current density:** (J)
  It is the *current per unit area of cross section* of an imaginary plane
  held normal to the direction of current.

$$J = \frac{I}{A}$$

- **Mean Free Path:** (λ)
  It is the average distance traveled by the conduction electrons
  between successive collisions with lattice ions.

- **Mean Collision Time:** (τ)
  It is the average time that elapses between two consecutive collisions
  of an electron with the lattice ions.

Some Basic Terms...

- **Mobility of electrons:** (μ)
  It is defined as the magnitude of drift velocity acquired by the electron per unit field.

$$\mu = \frac{v_d}{E} = \frac{1}{E}\frac{eE}{m}\tau = \frac{e}{m}\tau$$

Ohm's Law

- Ohm's law was first established as an *experimental result* applicable to a wide range of metals and semiconductors.
- To make a current flow, you have to *push* the charges. How *fast* they move, in response to a given push, depends on the *nature of the material*. For most substances, the current density $\vec{J}$ is proportional to the electric field $\vec{E}$ (force per unit charge):

$$\vec{J} = \sigma \vec{E} \qquad \textbf{(Ohm's law)}$$

- The proportionality factor σ is an empirical constant that varies from one material to another and temperature; it's called the **conductivity** of the medium.
- The reciprocal of σ is called the **resistivity**:

$$\rho = \frac{1}{\sigma}$$

Ohm's Law which I knew

A cylindrical resistor of cross-sectional area A and length L is made from material with conductivity σ. (as indicated in the *Figure*, the cross section need not be circular, but I do assume it is the same all the way down.) If we stipulate that the potential is constant over each end, and the potential difference between the ends is V, what current flows?
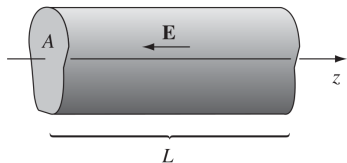
As it turns out, the electric field is uniform within the wire and $E = V/L$. It follows from $\vec{J} = \sigma\vec{E}$ that the current density is also uniform, so

$$I = JA = \sigma EA = \left(\frac{\sigma A}{L}\right)V = \frac{V}{R} \qquad \text{or} \qquad \boxed{V = IR} \qquad \text{where, } R = \frac{L}{\sigma A}$$

We call R as the resistance of the wire. R is a function of the geometry of the arrangement and the conductivity of the medium. This, of course, is the more familiar version of *Ohm's law*.

Let's derive Ohm's law by classical free electron theory:

- Conductor consist of a gas of classical particles (conduction electrons) moving through a background of immobile, heavy ions.

- The electrons in a metal move randomly along straight-line trajectories, which are constantly interrupted by collisions with lattice ions.

- The root mean square speed at room temperature is fairly high ($\sim$105 m/s) and may be calculated from the classical equipartition theorem:

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

- The root mean square speed may be found by finding the average of v^2, denoted $\overline{v^2}$, and then taking its square root.

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \left(\frac{3kT}{m}\right)^{1/2}$$

- Each step of the motion (the path between collisions) shown in *Figure* is a "free path"; the average free path, or **mean free path**, λ, is related to the **mean free time**, τ, the average time between collisions, and to v_{rms} by,
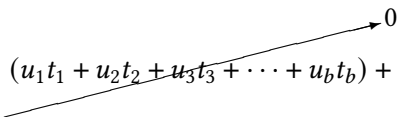
$$\lambda = v_{\text{rms}}\tau$$

- In Drude's original model, λ was taken to be several angstroms, consistent with the view that an electron generally travels one interatomic spacing before bumping into a large ion.

- When an electric field is applied to the sample, an electric force is exerted on an electron during each interval between collisions, resulting in a displacement that is small compared to the mean free path.

- The cumulative effect of these displacements may be viewed in terms of a small average **drift speed**, v_d, superimposed on the rather high random thermal speed, as shown in *Figure*.
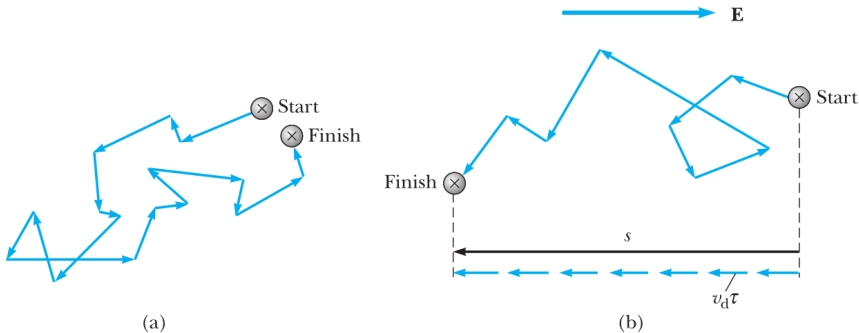
Figure: (a) Random successive displacements of an electron in a metal without an applied electric field. (b) A combination of random displacements and displacements produced by an external electric field. The net effect of the electric field is to add together multiple displacements of length $v_d\tau$ opposite the field direction. For purposes of illustration, this figure greatly exaggerates the size of v_d compared with v_{rms}.

- We may derive an expression for v_d by assuming that the total displacements, s, of an electron after b number of collisions is

$$s = (u_1 t_1 + u_2 t_2 + u_3 t_3 + \cdots + u_b t_b) \xrightarrow{\quad 0 \quad} + \frac{a}{2}(t_1^2 + t_2^2 + t_3^2 + \cdots + t_b^2)$$

where t_1, t_2, and so on are the successive times between collisions and a is the acceleration ($a = eE/m$) produced by the electric field. Note that we have ignored the random initial velocities of the electron since these average to zero.

- In terms of averages we may write,

$$s = \frac{a}{2} b \overline{t^2}, \qquad \text{where the average } \overline{t^2} = \frac{1}{b}(t_1^2 + t_2^2 + t_3^2 + \cdots + t_b^2)$$

- Average values: Consider a group of N electrons, all of which experience a collision with lattice ions at $t = 0$. One can show that the number of electrons that suffer their next collision between t and $t + dt$ follows the exponentially decreasing distribution

$$n(t)dt = \frac{Ne^{-t/\tau}}{\tau}dt$$

where τ is the mean free time. Then,

$$\int_0^\infty n(t)dt = N \qquad \text{(as expected)}$$

$$\bar{t} = \frac{n_1t_1 + n_2t_2 + \cdots + n_ft_f}{N} = \frac{1}{N}\int_0^\infty t\, n(t)\, dt = \tau$$

$$\overline{t^2} = \frac{n_1t_1^2 + n_2t_2^2 + \cdots + n_ft_f^2}{N} = \frac{1}{N}\int_0^\infty t^2\, n(t)\, dt = 2\tau^2$$

- Because the average value of t^2 is $\overline{t^2} = 2\tau^2$ this equation becomes

$$s = ab\tau^2$$

$$\text{or} \quad s = \frac{eE}{m}b\tau^2$$

- Comparing this equation to the expression for the displacement in terms of the drift speed, $s = bv_d\tau$, we find

$$v_d = \frac{eE\tau}{m}$$

- To find the magnitude of the current density, J, when n electrons *per unit volume* all move with speed v_d, we note that in a time dt the electrons move a distance $v_d dt$, so that $nAv_d\, dt$ electrons cross an area A perpendicular to the direction of electron flow (see *Figure*).
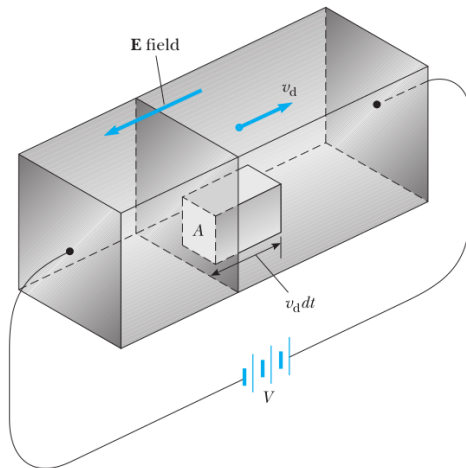
Figure: The connection between current density, J, and drift velocity, v_d. The charge that passes through A in time dt is the charge contained in the small parallelepiped, $neAv_ddt$.

- Since each electron has a charge e, the charge crossing the area A in the time dt is $neAv_d\,dt$, and the current is

$$I = \frac{neAv_d\,dt}{dt} = neAv_d$$

and current density is

$$J = \frac{I}{A} = \frac{neAv_d}{A} = nev_d$$

- Substituting $v_d = eE\tau/m$ into the above equation yields

$$\boxed{J = nev_d = \frac{ne^2\tau}{m}E}$$

- The proportionality of J to E given by this equation shows that the *classical free electron model* predicts the observed Ohm's law. For this case, the conductivity is,

$$\sigma = \frac{ne^2\tau}{m}$$

- or, using $\tau = \lambda/v_{\text{rms}}$,

$$\sigma = \frac{ne^2\lambda}{mv_{\text{rms}}}$$

- Substituting the Maxwell – Boltzmann rms thermal speed $v_{\text{rms}} = \left(\frac{3kT}{m}\right)^{1/2}$ into this yields,

$$\boxed{\sigma = \frac{ne^2\lambda}{(3kTm)^{1/2}}} \qquad \text{and} \qquad \boxed{\rho = \frac{1}{\sigma} = \frac{(3kTm)^{1/2}}{ne^2\lambda}}$$

An Example

a. Show that the rms thermal speed of electrons at 300 K is many orders of magnitude higher than the drift speed, v_d. To find v_d, assume that a copper wire with a cross section of $2\,\text{mm} \times 2\text{mm}$ carries a current of $10\,\text{A}$, and that each copper atom contributes 1 free electron. The density of copper at room temperature is $8.96\,\text{g/cm}^3$.

b. Estimate, the average time between collisions for copper at room temperature, assuming that the mean free path is the interatomic distance, $2.6\,\text{Å}$.

c. Calculate the conductivity of copper at room temperature as a test of the classical free electron gas theory, and compare this to the measured value, which is $59 \times 10^6\,\Omega^{-1}\text{m}^{-1}$.

Solution

- We have,

$$v_{\text{rms}} = \left(\frac{3kT}{m}\right)^{1/2} = \left[\frac{3 \times 1.38 \times 10^{-23} \text{JK}^{-1} \times 300\text{K}}{9.11 \times 10^{-31}\text{kg}}\right]^{1/2} = 1.17 \times 10^5 \text{m/s}$$

To calculate v_d we use $J = nev_d$, or $v_d = J/ne$. The number of free (conduction) electrons per cubic centimeter, n, in copper is

$$n = \left(\frac{1 \text{ free electron}}{\text{atom}}\right)\left(\frac{6.02 \times 10^{23}\text{atoms}}{\text{mol}}\right)\left(\frac{8.96\text{g}}{\text{cm}^3}\right)\left(\frac{1 \text{ mol}}{63.5\text{g}}\right)$$
$$= 8.49 \times 10^{22}\text{electrons/cm}^3$$

Thus,

$$v_d = \frac{J}{ne} = \frac{10\text{A}/(4 \times 10^{-6}\text{m}^2)}{(8.49 \times 10^{28}\text{m}^{-3})(1.6 \times 10^{-19}\text{C})} = 1.8 \times 10^{-4}\text{m/s}$$

Solution . . .

The ratio of drift speed to rms speed is

$$\frac{v_d}{v_{rms}} = 1.5 \times 10^{-9}$$

b Using the following equation, we find for the average time between collisions,

$$\tau = \frac{\lambda}{v_{rms}} = \frac{2.6 \times 10^{-10}\text{m}}{1.2 \times 10^5 \text{m/s}} = 2.2 \times 10^{-15}\text{s}$$

Thus, in this model, the electrons experience several hundred trillion collisions per second!

Solution . . .

- Conductivity is given by, at $T = 300$K,

$$\sigma = \frac{ne^2\lambda}{(3kTm)^{1/2}}$$

$$\therefore \quad \sigma = \frac{8.49 \times 10^{28}\text{electrons m}^{-3} \times (1.6 \times 10^{-19}\text{C})^2 \times 2.6 \times 10^{-10}\text{m}}{(3 \times 1.38 \times 10^{-23}\text{JK}^{-1} \times 300\text{K} \times 9.11 \times 10^{-31}\text{kg})^{1/2}}$$

$$\therefore \quad \sigma = 5.3 \times 10^6 \ \Omega^{-1}\text{m}^{-1}$$

From experimental data we had $\sigma = 59 \times 10^6 \ \Omega^{-1}\text{m}^{-1}$. We see that the value of the conductivity obtained by our theory is about 10 times smaller than the measured value!

Failures of classical free electron theory

- Although the classical free electron gas model does predict Ohm's law, we see that it results in conductivity values that differ from measured values by an order of magnitude.
- The measured resistivity of most metals is found to be proportional to the absolute temperature ($\rho \propto T$), yet this model predicts a much weaker dependence of ρ on T, which is $\rho \propto \sqrt{T}$.
- According to the model electrical conductivity is directly proportional to the electron concentration ($\sigma \propto n$). But monovalent metal copper is found to have high electrical conductivity than the divalent & trivalent metals like Zinc and Aluminum. The model fails to explain this observation.

- Though metals are expected to exhibit negative Hall co-efficient since the charge carriers in them are electrons, some metals like zinc have positive Hall co-efficient. The free electron theory could not explain the positive Hall co-efficient of some metals.
- According to the model the heat capacity per mole is found to be $3R/2$, where R is universal gas constant. The experimental value shows that the specific heat is temperature dependent, whereas the model says that it is temperature independent.

Quantum Free Electron Theory

The deficiencies in classical free electron theory can be rectified by:

- replacing the Maxwell–Boltzmann distribution with the Fermi–Dirac distribution for the conduction electrons in the metal
- and by calculating the electron mean free path while explicitly taking the wave nature of the electron into account.

Assumptions of Quantum Free Electron Theory

- The energy values of free electrons are quantized. The allowed energy values are a set of energy levels instead of continuous values.
- Distribution of electrons follows Pauli's Exclusion Principle.
- Distribution of electrons obey Fermi-Dirac statistics.
- The free electrons travel in a constant potential inside the metal but stay confined within its boundaries.
- The attraction between the free electrons and lattice ions, the repulsion between the electrons are ignored.

Fermi-Dirac Statistics

- Particles that obey the Fermi–Dirac distribution are called *fermions*.
- Fermions are indistinguishable particles with half integral spin $(1/2)$. Eg: electrons, protons, neutrons
- They obey *Pauli's exclusion principle* – only one particle can occupy a given quantum state.
- *Fermi factor* $f(E)$ represents the probability of finding a particle with energy E, or in the language of statistical mechanics, the probability that a state with energy E is occupied at the absolute temperature T.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is called the Fermi energy and k is the Boltzmann's constant.

- If $g(E)$ is the *density of states* or the number of energy states per unit volume in the interval dE, then the number of particles per unit volume with energies between E and $E + dE$ is given by,

$$N(E)\, dE = g(E)f(E)\, dE$$

Dependence of Fermi factor on temperature

The probability of occupation for different cases:

- When $T = 0\,\mathrm{K}$ and $E < E_F$:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

- When $T = 0\,\mathrm{K}$ and $E > E_F$:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1} = 0$$

Fermi energy is the highest occupied energy level at absolute zero.
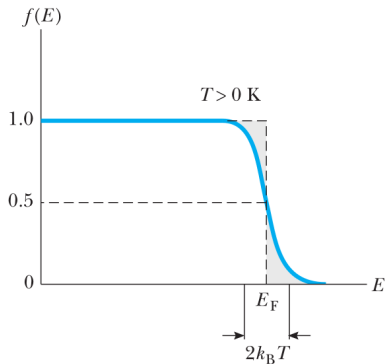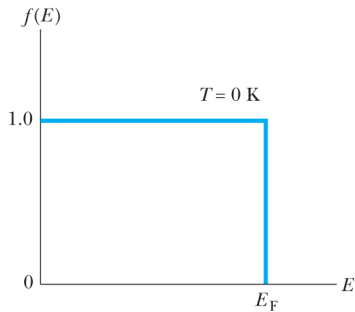
- At finite temperatures $T > 0\,\text{K}$:
    - For $E << E_F$, $f(E) = 1$
    - For $E < E_F$, and closer to E_F, $f(E)$ decreases from 1 to 0.5
    - For $E = E_F$,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^0 + 1} = \frac{1}{1+1} = \frac{1}{2}$$

    This expression shows the meaning of the Fermi energy: *The probability of finding an electron with an energy equal to the Fermi energy is exactly 1/2 at any temperature.*
    - For $E > E_F$, and closer to E_F, $f(E)$ decreases from 0.5 to 0
    - For $E >> E_F$, $f(E) = 0$

Also, Fermi energy is the *average energy* possessed by the conduction electrons in conductors at temperatures above $0\,\text{K}$.

Figure: A comparison of the Fermi–Dirac distribution functions at (a) absolute zero and (b) finite temperature.

Density of states $g(E)$

- When a large number of atoms are brought together to form a solid, the various isolated-atom energy levels begin to split.

- If we consider the total number of atoms in a solid ($N \approx 10^{23}$ atoms/cm^3), we find a very large number of levels (determined by N) spaced within the width ΔE, so the levels may be regarded as a continuous band of energy levels.

- In general, a crystalline solid has numerous allowed energy bands, one band arising from each atomic energy level.

- The energy gaps, or forbidden energy regions, separate the allowed energy bands.

- Forbidden energy regions arise from the separation between different atomic levels and will always be present unless individual atomic levels broaden so much that they overlap, as do the $3s$ and $3p$ bands in sodium.

- If the solid contains N atoms, each energy band has N energy levels.
- Each energy level is occupied by two electrons: one with spin $+1/2$ and another with spin $-1/2$.
- *The density of states $g(E)$ is the number of available states per unit volume per unit energy interval.*
- For free electrons in a solid the number of states lying in the energy range E and $E + dE$ is given by,

$$g(E)dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}\,dE$$

Replacement of v_{rms} with Fermi velocity v_F

- The electrons in a metal must be described by Fermi–Dirac statistics.
- The net effect of the electric field is to leave an intact core of states for electrons with $E < E_F$ and to produce a displacement of those electrons near the Fermi level having $v \approx v_F$. Where,

$$E_F = \frac{1}{2}mv_F^2 \qquad \text{and} \qquad v_F = \left(\frac{2E_F}{m}\right)^{1/2}$$

- Thus only those electrons with $v \approx v_F$ are free to move and participate in electrical and thermal conduction, and we can presumably use the classical expressions for σ with the Maxwell–Boltzmann rms speed replaced by the Fermi speed; thus,

$$\sigma = \frac{ne^2\lambda}{mv_F}$$

Quantum Mean Free Path of Electrons

- Changes from the classical value of electron mean free path arise from the wave properties of the electron.

- If we simply use the interatomic distance between copper atoms as the mean free path λ for electrons, and v_F for v_{rms} in the classical expression for σ, we find a value for that is about 200 times smaller than the measured value of the conductivity.

- This discrepancy implies that we are using the wrong value for λ and that the scattering sites for electrons are not adjacent ion cores but more widely separated scattering centers.

- These scattering centers consist of departures from perfect lattice regularity, such as thermal displacements of ions from equilibrium lattice sites, dislocations, and impurity atoms.

Because quantum calculations of λ that include these effects are complicated, let us make a rough estimate of λ for copper at room temperature by using the measured value of $\sigma = 5.9 \times 10^7 \Omega^{-1} m^{-1}$. (Use n from previous problem.)

Solving $\sigma = \frac{ne^2\lambda}{mv_F}$ for λ yields,

$$\lambda = \frac{mv_F\sigma}{ne^2}$$

Using a Fermi energy of 7.05 eV for copper gives

$$v_F = \left(\frac{2E_F}{m}\right)^{1/2} = \left(\frac{2 \times 7.05 \times 1.6 \times 10^{-19} J}{9.11 \times 10^{-31} kg}\right)^{1/2} = 1.57 \times 10^6 m/s$$

Substituting this value into above equation gives

$$\lambda = \frac{(9.11 \times 10^{-31} kg) \times (1.57 \times 10^6 m/s) \times (5.9 \times 10^7 \Omega^{-1} m^{-1})}{(8.49 \times 10^{28} electrons/m^3) \times (1.6 \times 10^{-19} C)^2}$$
$$= 3.9 \times 10^{-8} = 390 \, \text{Å}$$

This is about 150 times the distance between copper atoms.

- We can account for the unexpectedly long electron mean free path by taking the wave nature of the electron into account.
- Quantum mechanical calculations show that electron waves with a broad range of energies can pass through a perfect lattice of ion cores unscattered, without resistance, and with an infinite mean free path.
- The actual resistance of a metal is due to the *random thermal displacements* (thermal vibrations) of ions about lattice points and to other deviations from a perfect lattice, such as *impurity atoms and defects* that scatter electron waves.
- The lack of electron scattering by a perfect lattice can be understood by noting that the electron wave generally travels through the metal unattenuated, just as does light through a transparent crystal.
- Strong reflections of electron waves are set up only for specific electron energies, and when this occurs, the electron wave cannot travel freely through the crystal. These strong reflections occur when the *lattice spacing is equal to an integral number of electronic wavelengths*, resulting in a discrete set of forbidden energy bands for electrons.

Phonons

The cause for temperature dependence of resistivity.

- Classical view: $\rho \propto T$ at high temperature because of *scattering of electrons by lattice ions* vibrating with larger amplitude at higher temperatures.

- Quantum view: lattice vibrations have a quantized energy $\hbar\omega$. These quantized lattice vibrations are called **phonons**, and for purposes of calculation, the vibrating lattice ions are replaced by phonons.

- The number of phonons n_p with energy $\hbar\omega$ that are available at a given temperature T is proportional to the Bose–Einstein distribution function:

$$n_p \propto \frac{1}{e^{\hbar\omega/kT} - 1}$$

At high temperatures, $kT \gg \hbar\omega$, and this becomes

$$e^{\hbar\omega/kT} = 1 + \frac{\hbar\omega}{kT} + \frac{1}{2!}\left(\frac{\hbar\omega}{kT}\right)^2 + \frac{1}{3!}\left(\frac{\hbar\omega}{kT}\right)^3 + \cdots \approx 1 + \frac{\hbar\omega}{kT}$$

$$\therefore \quad \frac{1}{e^{\hbar\omega/kT} - 1} \approx \frac{1}{1 + \frac{\hbar\omega}{kT} - 1} = \frac{kT}{\hbar\omega}$$

$$\therefore \quad n_p \propto \frac{kT}{\hbar\omega}$$

$$\text{or} \qquad n_p \propto T$$

Finally, since the *number of electron scatterers* is proportional to the *temperature*, so is the resistivity ρ.

Matthiessen's rule

The cause for temperature independent component of resistivity.
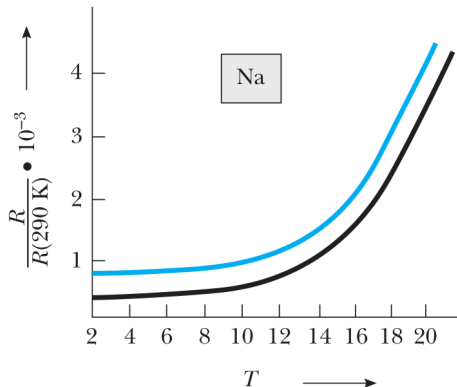
Figure: Resistance of sodium as a function of temperature. The colored curve is for a sodium sample with a higher concentration of imperfections.

Two mechanisms for scattering of conduction electrons leading to resistivity:

1. Resistivity ρ_L due to scattering of electrons by lattice vibrations (phonons) increases with temperature. It is the resistivity exhibited by a pure specimen that is free of all defects, and hence called the ideal resistivity.

2. Temperature-independent contribution to the resistivity of the metal clearly manifests itself for $T < 10\,K$. The residual resistivity ρ_i, which remains as $T \to 0\,K$, is produced by electron waves scattering from *impurities* and *structural imperfections* in a given sample.

The parallel nature of the curves shown in the *Figure* implies that

- the resistivity caused by thermal motion of the lattice, ρ_L, is independent of the impurity concentration and

- the resistivity caused by impurity concentration, ρ_i, is independent of temperature.

Matthiessen's rule: The resistivity of a metal may be written in the form

$$\rho = \rho_i + \rho_L$$

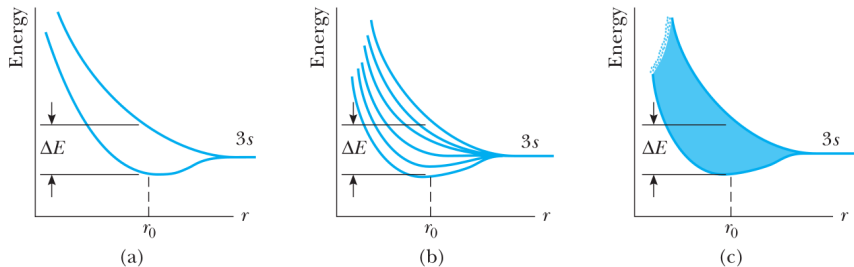where ρ_i depends only on the concentration of crystal imperfections and ρ_L depends only on T.

Using $\rho = 1/\sigma$ and $\sigma = ne^2\tau/m$, we can write,

$$\rho_L = \frac{m}{n\tau_L e^2} \qquad \text{and} \qquad \rho_i = \frac{m}{n\tau_i e^2}$$
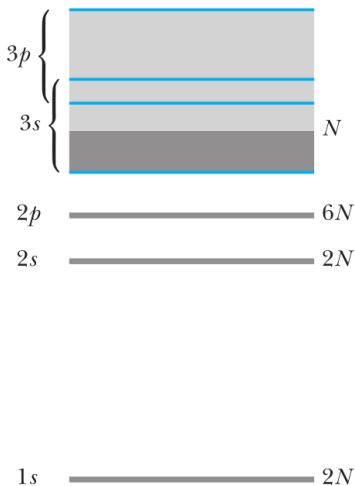
Substituting these into Matthiessen's rule,

$$\rho = \frac{m}{n\tau_L e^2} + \frac{m}{n\tau_i e^2}$$

Band Theory of Solids

Figure: (a) The splitting of the $3s$ levels when two sodium atoms are brought together. (b) The splitting when six sodium atoms are brought together. (c) The formation of a $3s$ band when a large number of sodium atoms are assembled to form a solid. Note that r_0 is the actual lattice constant.

The energy bands of sodium are shaded gray in this figure. The solid contains N atoms. Note the energy gaps between allowed bands and that the $3s$ and $3p$ bands overlap in sodium. The number of electrons in each energy band is indicated to the right of the band.
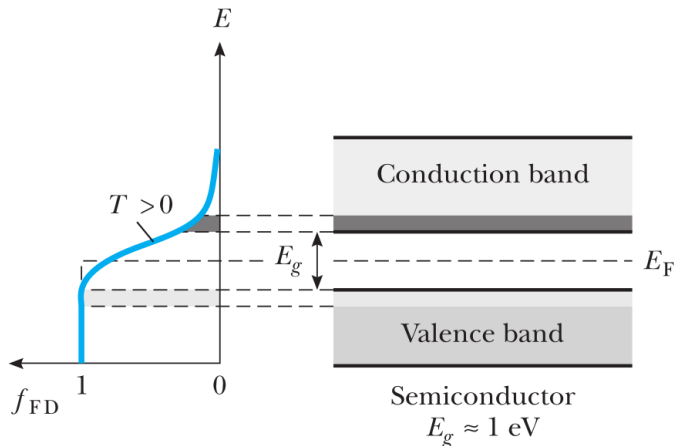
Conductor
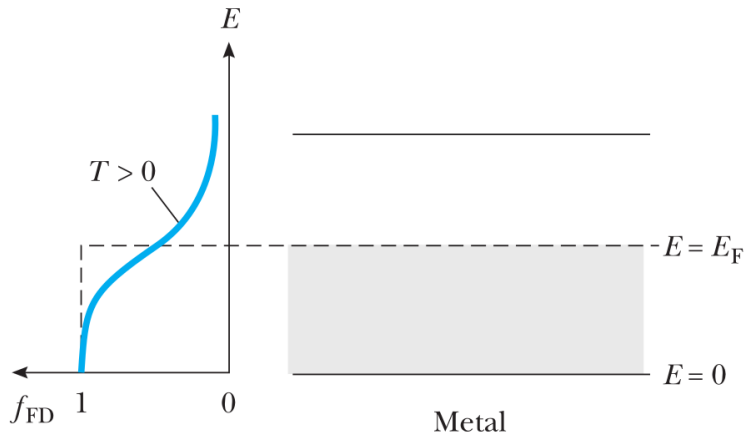
Figure: A half-filled band of a conductor such as the 3s band of sodium. At $T = 0$ K, the Fermi energy lies in the middle of the band. The Fermi–Dirac probability that an energy state E is occupied at $T > 0$ K is shown at the left.

Insulator
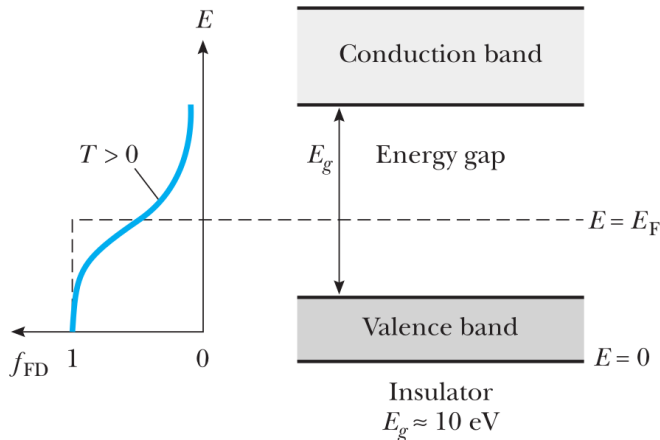
Figure: An insulator at $T = 0\,\text{K}$ has a filled valence band and an empty conduction band. The Fermi level lies midway between these bands. The Fermi–Dirac probability that an energy state E is occupied at $T > 0\,\text{K}$ is shown to the left.

Semicondutor

Figure: The band structure of a semiconductor at ordinary temperatures ($T \approx 300$ K). Note that the energy gap is much smaller than in an insulator and that many electrons occupy states in the conduction band.
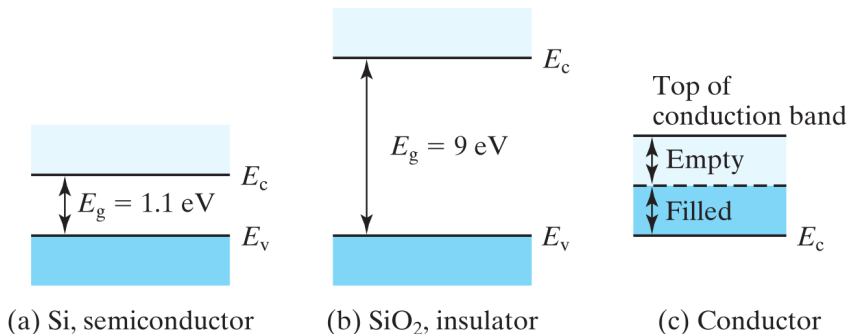
A comparison

(a) Si, semiconductor $\quad$ (b) SiO_2, insulator $\quad$ (c) Conductor

Figure: Energy band diagrams for a semiconductor, an insulator, and a conductor.

Carrier Concentration in Metals at 0 K

Number of free electrons per unit volume in the energy range E and $E + dE$ is given by,

$$N(E)\, dE = g(E) f(E)\, dE$$

The number of free electrons per unit volume of the material is equal to the total number of electrons that are distributed in various energy levels up to E_F. (No electrons exists above E_F at 0 K.) This number is given by,

$$n = \int_{E=0}^{E_F} N(E)\, dE$$

$$= \int_{E=0}^{E_F} g(E) f(E)\, dE$$

But $f(E) = 1$ at $T = 0$ K

$$\therefore \quad n = \int_{E=0}^{E_F} g(E)\, dE$$

Substituting

$$g(E)dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}\,dE$$

into it, we get,

$$n = \frac{4\pi}{h^3}(2m)^{3/2}\int_{E=0}^{E_F} E^{1/2}\,dE$$

$$= \frac{4\pi}{h^3}(2m)^{3/2}\frac{E_F^{3/2}}{3/2}$$

$$\boxed{n = \frac{8\pi}{3h^3}(2m)^{3/2}E_F^{3/2}}$$
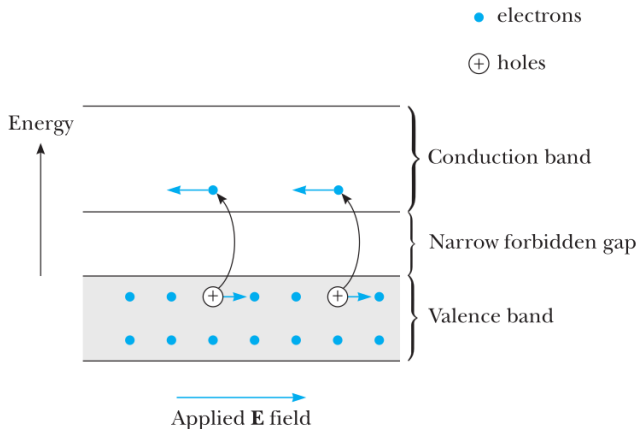
This gives concentration of electrons in a metal at 0 K.

Fermi energy in terms of n:

$$E_F = \left[\frac{3h^3}{8\pi(2m)^{3/2}}n\right]^{2/3} = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} n^{2/3} = Bn^{2/3}$$
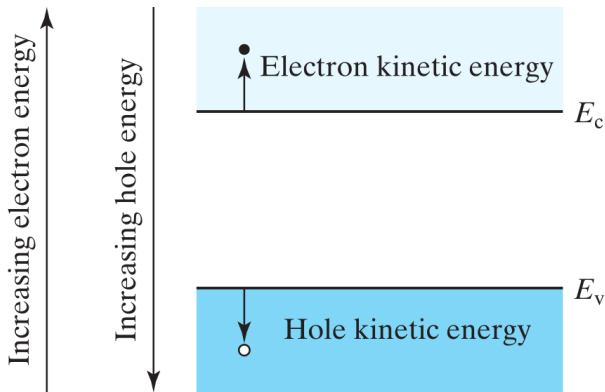
where

$$B = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} = 5.85 \times 10^{-38}\text{J}$$

Intrinsic Semiconductor

Figure: An intrinsic semiconductor. The electrons move toward the left and the holes move toward the right when the applied electric field is to the right as shown.

Electrons and Holes Energy Levels

Figure: Both electrons and holes tend to seek their lowest energy positions. Electrons tend to fall in the energy band diagram. Holes float up like bubbles in water.

Effective Mass of the Electrons and Holes

- A particle's effective mass (often denoted m^*) is the mass that it seems to have when responding to forces, or the mass that it seems to have when interacting with other identical particles in a thermal distribution.

- Under external electric field $\mathscr{E}$ an electron experiences a total force of

$$m_e a = -e\mathscr{E} + \text{force due to neighbouring ions and electrons}$$

- Since the latter force is not known quantitatively, we can write the above equation by modifying m as,

$$m_e^* a = -e\mathscr{E}$$

where m_e^* is called the effective mass of the electron within the crystal.

- For some purposes and some materials, the effective mass can be considered to be a simple constant of a material. In general, however, the value of effective mass depends on the purpose for which it is used, and can vary depending on a number of factors.
- Effective mass of an electron depends on its location in the energy band.
- Hole is also given the status of a particle with effective mass m_h^*.

Intrinsic Conductivity
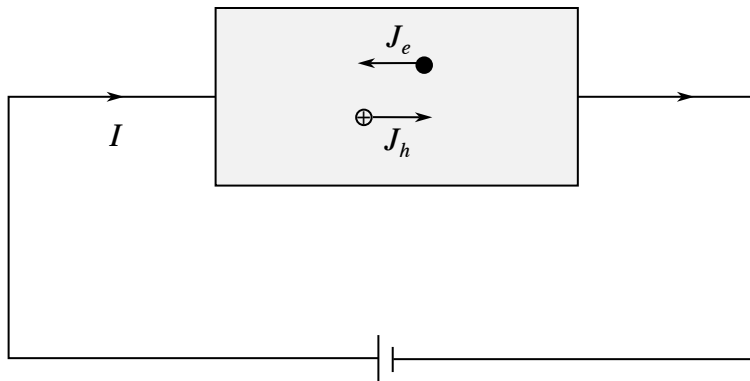
Mobility is defined as,

$$\mu = \frac{v_d}{\mathscr{E}} \implies v_d = \mu\mathscr{E}$$

where v_d is drift velocity and $\mathscr{E}$ is the applied electric field.

Current density due to electrons in conduction band is,

$$J_e = nev_{de} = ne\mu_e\mathscr{E} = \sigma_e\mathscr{E} \qquad \text{where } \sigma_e = ne\mu_e$$

where n is electron concentration, v_{de} is drift velocity of the electrons, μ_e is mobility of electrons and σ_e is conductivity due to electrons in conduction band.

Current density due to holes in valence band is,

$$J_h = nev_{dh} = pe\mu_h\mathscr{E} = \sigma_h\mathscr{E} \qquad \text{where } \sigma_h = pe\mu_h$$

where p is holes concentration, v_{dh} is drift velocity of the holes, μ_h is mobility of holes and σ_h is conductivity due to holes in valence band.

Total current density will be,

$$J = J_e + J_h = ne\mu_e\mathscr{E} + pe\mu_h\mathscr{E} = (ne\mu_e + pe\mu_h)\mathscr{E} = \sigma\mathscr{E}$$

where the conductivity is,

$$\boxed{\sigma = ne\mu_e + pe\mu_h = \sigma_e + \sigma_h}$$

Calling $n = p = n_i$ as *intrinsic carrier concentration* above expression for conductivity can be rewritten for intrinsic semiconductor as,

$$\boxed{\sigma_i = en_i(\mu_e + \mu_h)}$$

In general the variation of mobility with temperature is very small and the large variation in electrical conductivity in semiconductors is linked to the variation of electron concentration with temperature. As a first approximation we can write,

$$\sigma_i(T) \propto n_i(T)$$

Electron Concentration in Conduction Band of a Semiconductor
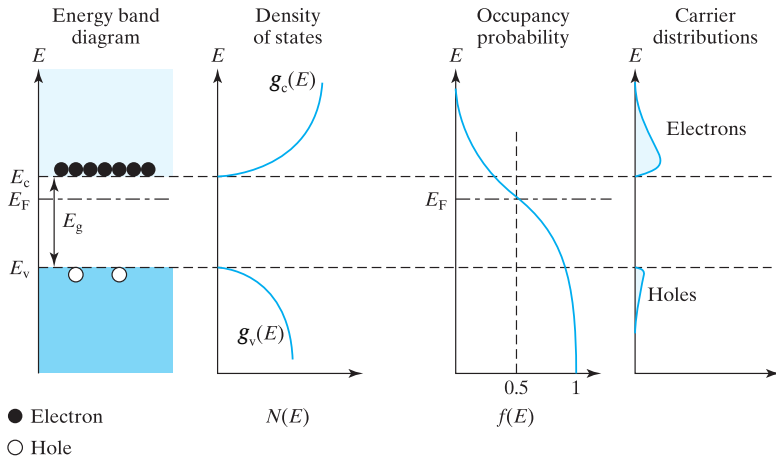
Figure: Schematic band diagram, density of states, Fermi–Dirac distribution, and carrier distributions versus energy.

The number of electrons in the conduction band is given by

$$n = \int_{E_C}^{\text{top of the band}} g_c(E) f(E) \, dE$$

where E_C is the bottom most energy level of the conduction band.

As $f(E)$ rapidly approaches zero for higher energies, we can write

$$n = \int_{E_C}^{\infty} g_c(E) f(E) \, dE \tag{1}$$

Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_C)^{1/2} \, dE \tag{2}$$

Any energy above E_C is the conduction band electron's kinetic energy ($= E - E_C$) relative to E_C. Electrons may gain energy by getting accelerated in an electric field and may lose energy through collisions with imperfections in the crystal.

Fermi factor is given by,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \tag{3}$$

Substituting eqns. (2) and (3) in eqn. (1),

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E-E_C)^{1/2}}{e^{(E-E_F)/kT} + 1}\, dE$$

At sufficiently large E with $E > E_F$,

$$E - E_F \gg kT \quad \text{makes} \quad e^{(E-E_F)/kT} \gg 1$$

Hence we can approximate,

$$e^{(E-E_F)/kT} + 1 \approx e^{(E-E_F)/kT}$$

Therefore the above integral can be written as,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_C)^{1/2} e^{-(E-E_F)/kT}\, dE$$

Adding and subtracting E_c to the exponential term gives,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}e^{(E_F-E_C)/kT}\int_{E_c}^{\infty}(E-E_C)^{1/2}e^{-(E-E_C)/kT}\,dE$$

To solve the integral, let $E - E_C = x$, then $dE = dx$ and
 Lower limit: when $E = E_C$, $x = E_C - E_C = 0$
 Upper limit: when $E = \infty$, $x = \infty - E_C = \infty$

Then the integral part will become, by letting $1/kT = a$,

$$\int_{E_C}^{\infty}(E-E_C)^{1/2}e^{-(E-E_C)/kT}\,dE = \int_0^{\infty}x^{1/2}e^{-ax}\,dx$$

This form of integrals are known as a gamma functions. From literature,

$$\int_0^{\infty}x^{1/2}e^{-ax}\,dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

Substituting it back into the equation for n we get,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}e^{(E_F - E_C)/kT}\left[\frac{\sqrt{\pi}}{2}(kT)^{3/2}\right]$$

By rearranging,

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}e^{-(E_C - E_F)/kT}$$

Or,

$$n = N_C e^{-(E_C - E_F)/kT}$$ with, $$N_C = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}$$

N_C is called the *effective density of states* of the conduction band.

Hole Concentration in Valence Band of a Semiconductor

The probability of an energy state being occupied by a hole is the probability of it *not* being occupied by an electron, i.e., $1 - f(E)$. The number of holes in the valence band is therefore given by

$$p = \int_{\text{bottom of the band}}^{E_v} g_v(E) \left[1 - f(E)\right] \, dE$$

where E_v is the top most energy level of the valence band. As $[1 - f(E)]$ rapidly approaches zero for lower energies, we can rewrite the lower limit as 0.

$$p = \int_0^{E_v} g_v(E) \left[1 - f(E)\right] \, dE$$

But further to solve the integral by gamma functions we will make the lower limit as $-\infty$ instead of 0. Though negative energy does n't exist, let us look at this integral in the eyes of mathematics. Having lower limit as $-\infty$ does not add any extra to the integral than having 0. It is because $[1 - f(E)]$ rapidly approaches zero for lower energies.

Therefore,

$$p = \int_{-\infty}^{E_v} g_v(E) \left[1 - f(E)\right] dE \tag{1}$$

Density of states for energies $E \leq E_v$ is given by,

$$g_v(E) = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2} dE \tag{2}$$

Any energy below E_V is the valence band hole's kinetic energy $(= E_V - E)$ relative to E_V. Holes may gain energy by getting accelerated in an electric field and may lose energy through collisions with imperfections in the crystal. When they gain energy they go downwards in the electronic energy levels since holes are positively charged.

Fermi factor is given by,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$\therefore \quad 1 - f(E) = 1 - \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{e^{(E-E_F)/kT} + 1 - 1}{e^{(E-E_F)/kT} + 1}$$

$$= \frac{e^{(E-E_F)/kT}}{e^{(E-E_F)/kT} + 1} = \frac{1}{1 + e^{-(E-E_F)/kT}} = \frac{1}{e^{(E_F-E)/kT} + 1} \quad (3)$$

Substituting eqns. (2) and (3) in eqn. (1),

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2} \int_{-\infty}^{E_V} \frac{(E_V - E)^{1/2}}{e^{(E_F-E)/kT} + 1}\, dE$$

At sufficiently small E with $E < E_F$,

$$E_F - E \gg kT \quad \text{makes} \quad e^{(E_F-E)/kT} \gg 1$$

Hence we can approximate,

$$e^{(E_F-E)/kT} + 1 \approx e^{(E_F-E)/kT}$$

Therefore the above integral can be written as,

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_F - E)/kT} \, dE$$

Adding and subtracting E_V to the exponential term gives,

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{(E_V - E_F)/kT} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_V - E)/kT} \, dE$$

To solve the integral, let $E_V - E = x$, then $-dE = dx$ and
Lower limit: when $E = -\infty$, $x = E_V - (-\infty) = \infty$
Upper limit: when $E = E_V$, $x = E_V - E_V = 0$

Then the integral part will become, by letting $1/kT = a$,

$$\int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_V - E)/kT} \, dE = -\int_{\infty}^{0} x^{1/2} e^{-ax} \, dx = \int_{0}^{\infty} x^{1/2} e^{-ax} \, dx$$

This form of integrals are known as a gamma functions. From literature,

$$\int_0^\infty x^{1/2} e^{-ax} \, dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

Substituting it back into the equation for p we get,

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{(E_V - E_F)/kT} \left[\frac{\sqrt{\pi}}{2} (kT)^{3/2} \right]$$

By rearranging,

$$p = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2} e^{-(E_F - E_V)/kT}$$

Or,

$$p = N_V e^{-(E_F - E_V)/kT} \qquad \text{with,} \qquad N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

N_V is called the *effective density of states* of the valence band.

Fermi Level in Intrinsic Semiconductor

In an intrinsic semiconductor electron and hole concentrations are equal. Therefore,

$$n = p$$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

Taking natural log on both sides,

$$\ln N_C - \frac{(E_C - E_F)}{kT} = \ln N_V - \frac{(E_F - E_V)}{kT}$$

By rearranging,

$$2E_F = E_C + E_V + kT \ln\left(\frac{N_V}{N_C}\right)$$

By substituting $N_V = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}$ and $N_C = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}$

$$E_F = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)^{3/2} = \frac{E_C + E_V}{2} + \frac{3}{4}kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

As kT is small and the effective masses $m_e^* \approx m_h^*$, the second term can be ignored. Therefore,

$$\boxed{E_F = \frac{E_C + E_V}{2}}$$

Here the energy gap is,

$$E_g = E_C - E_V$$

Or

$$E_C = E_V + E_g$$

Substituting this E_C into above expression for E_F,

$$E_F = \frac{E_V + E_g + E_V}{2} = E_V + \frac{E_g}{2}$$

Thus, Fermi level in intrinsic semiconductors lies at the center of the energy gap. See the following *Figure*.
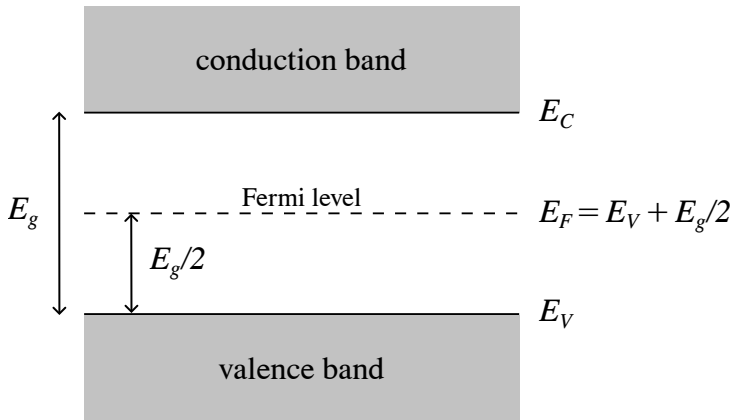
Intrinsic Carrier Concentration

Since E_F cannot be close to both E_C and E_V at the same time, n and p cannot both be large numbers at the same time. We had before,

$$n = N_C e^{-(E_C - E_F)/kT} \qquad \text{with, } N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$$

$$p = N_V e^{-(E_F - E_V)/kT} \qquad \text{with, } N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

When they are multiplied together, we obtain

$$\begin{aligned} np &= N_C e^{-(E_C - E_F)/kT} N_V e^{-(E_F - E_V)/kT} \\ &= N_C N_V e^{-(E_C - E_V)/kT} \\ &= N_C N_V e^{-E_g/kT} \qquad (\because E_C - E_V = E_g) \end{aligned}$$

This equation states that the np product is a constant for a given semiconductor and temperature T. Because np does n't depend on E_F, it is also independent of the dopant concentrations influencing position of E_F.

It is usually expressed in the following form:

$$np = n_i^2$$

$$n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$$

$$\text{or,} \quad n_i = 2\left[\frac{2\pi k}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4} T^{3/2} e^{-E_g/2kT}$$

In an intrinsic semiconductor, the nonzero n and p are the results of thermal excitation, which moves some electrons from the valence band into the conduction band. Since such movements create electrons and holes in pairs, $n = p$ in intrinsic semiconductors. This fact and $np = n_i^2$ immediately suggest that, in intrinsic semiconductors,

$$n = p = n_i$$

Therefore, n_i is called the **intrinsic carrier concentration**. n_i is a strong function of E_g and T.

One may explain why the np product is a constant this way: the electron–hole recombination rate is proportional to the np product. When $np = n_i^2$, the recombination rate happens to be equal to the rate of thermal generation of electron–hole pairs. This is the same **mass action principle** that keeps the product of the concentrations of $[H^+]$ and $[OH^-]$ constant in aqueous solutions, whether strongly acidic, strongly alkaline, or neutral.

Determining Band Gap of an Intrinsic Semiconductor

Conductivity of an intrinsic semiconductor is,

$$\sigma_i = e n_i (\mu_e + \mu_h)$$

Substituting,

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

$$\text{or,} \quad n_i = 2 \left[\frac{2\pi k}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4}\, T^{3/2} e^{-E_g/2kT}$$

into the above expression for σ_i, we get

$$\sigma_i = 2e(\mu_e + \mu_h) \left[\frac{2\pi k}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4}\, T^{3/2} e^{-E_g/2kT}$$

Let the constant part be,

$$2e(\mu_e + \mu_h) \left[\frac{2\pi k}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4} = K'$$

With this,

$$\sigma_i = K' T^{3/2} e^{-E_g/2kT}$$

Resistivity is given by,

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{K' T^{3/2} e^{-E_g/2kT}} = KT^{-3/2} e^{E_g/2kT}$$

where we just made $1/K' = K$. Further, the resistance is given by,

$$R_i = \rho_i \frac{L}{A} = \frac{L}{A} KT^{-3/2} e^{E_g/2kT}$$

where L and A are the length and area of cross section of the material.
Absorbing L/A into K and taking natural log on both sides, we get

$$\ln R_i = \ln K - \frac{3}{2} \ln T + \frac{E_g}{2kT}$$

Note that $\ln T$ remains almost constant over a small rage of temperature
within say 100K. For example,

$(3/2) \ln 300 = 8.5557$ at 27°C (room temperature)
$(3/2) \ln 350 = 8.7869$ at 77°C (hot water)
$(3/2) \ln 370 = 8.8703$ at 97°C (close to steam point)

Hence, for experiments involving small temperature ranges, $\frac{3}{2} \ln T$ term can also be absorbed into the constant term $\ln K$ leaving with,

$$\ln R_i = \ln K + \frac{E_g}{2kT}$$

In an experiment measuring resistance R_i of an intrinsic semiconductor as a function of temperature T, using a hot water or oil bath, a plot of $\ln R_i$ versus T results in a straight line with its

$$\text{slope} = \frac{E_g}{2k}$$
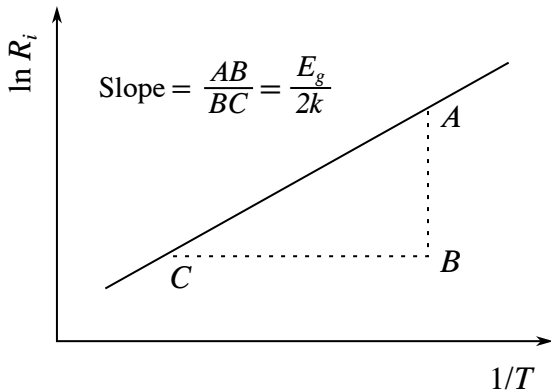
Figure: A plot of $\ln R_i$ versus $1/T$.

Extrinsic Semiconductors

- Intrinsic semiconductors have low conductivity.
- Introduction of impurities into an intrinsic semiconductor produces useful changes to its electrical conductivity.
- The method of introduction of controlled quantity of impurity into an intrinsic semiconductor is called *doping*.
- The added impurity is called *dopant*.
- Doped semiconductor is called *extrinsic semiconductor*.
- Doping with *pentavalent* atoms such as phosphorous, arsenic, etc., produces *n-type* semiconductors.
- Doping with *trivalent* atoms such as aluminum, boron, etc., produces *p-type* semiconductors.

If n = conduction band electrons density, p = valence band holes density, N_d = positive donor ions density, N_a = negative acceptor ions density, then:

Charge neutrality requires that the densities of the negative particles and positive particles are equal: $n + N_a = p + N_d$

Variation of Carrier Concentration with Temperature in Extrinsic Semiconductor
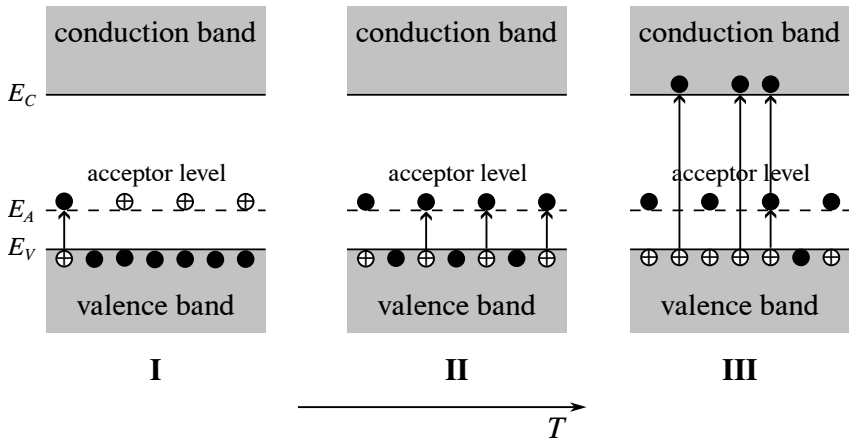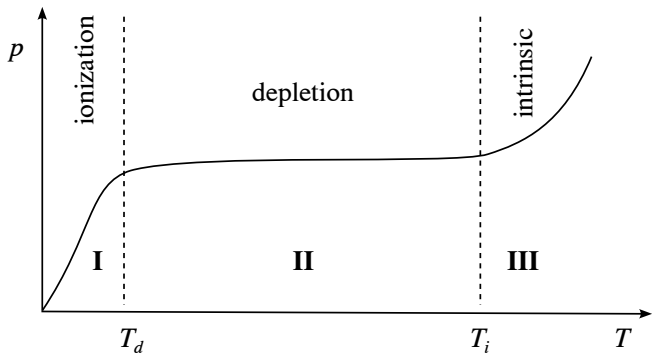
n-type:
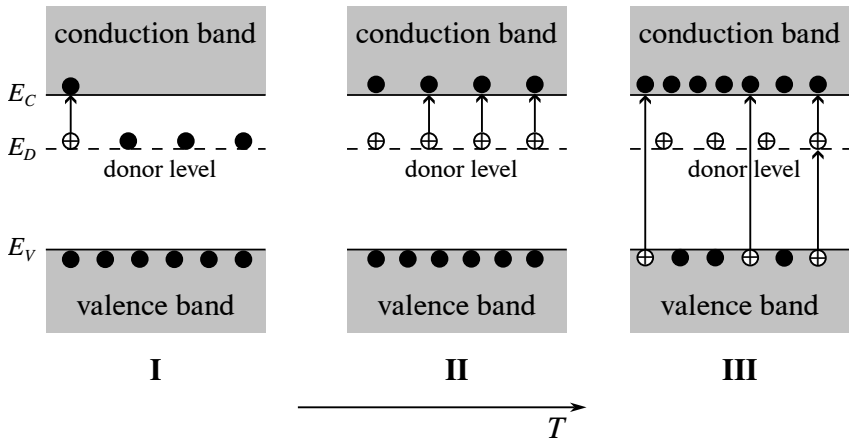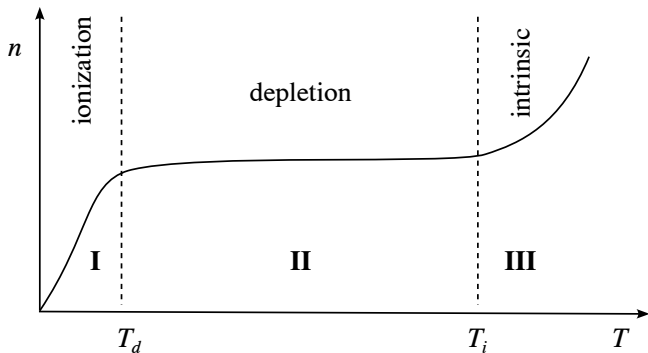
Figure: **I.** As T increases the donor atoms starts getting ionized. **II.** All donor atoms are ionized and all electrons from donor level are excited into conduction band $n = N_d$. **III.** Electron transitions from valence band to conduction band increases.
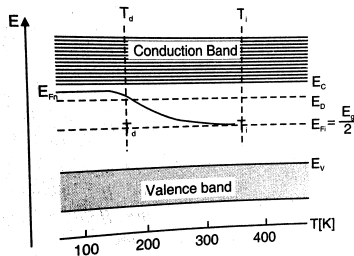
p-type:

Figure: **I.** As T increases the acceptor atoms starts getting ionized. **II.** All acceptor atoms are ionized and all holes from acceptor level are excited into valence band $p = N_a$. **III.** Electron transitions from valence band to conduction band increases, creating more and more hole in the valence band.

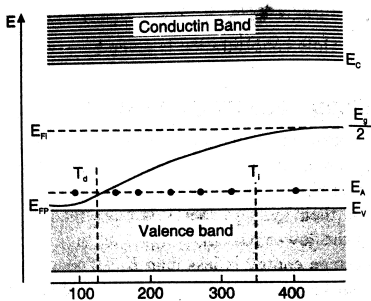# Variation of Fermi Energy with Temperature in Extrinsic Semiconductor

n-type:

Figure: Dependence of Fermi level on temperature in *n*-type semiconductor.

$$E_{Fn} = \frac{E_C + E_D}{2} - \frac{kT}{2} \ln \frac{N_C}{N_d} \quad \text{and} \quad E_{Fi} = \frac{E_C + E_V}{2}$$

p-type:

Figure: Dependence of Fermi level on temperature in p-type semiconductor. The Fermi level E_{Fp} rises with increasing temperature from below the acceptor level to intrinsic level E_{Fi}.

$$E_{Fp} = \frac{E_A + E_V}{2} + \frac{kT}{2} \ln \frac{N_V}{N_a} \quad \text{and} \quad E_{Fi} = \frac{E_C + E_V}{2}$$

This can be analyzed by the following general expression for Fermi energy of p-type semiconductor.

$$E_{Fp} = \frac{E_A + E_V}{2} + \frac{kT}{2} \ln \frac{N_V}{N_a} \quad \text{and} \quad E_{Fi} = \frac{E_C + E_V}{2}$$

where E_A is acceptor energy level, E_V is top most energy level in valence band, T is temperature, N_a is negative acceptor ions density and

$$N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

1. As the valence band is the source of electrons and the acceptor levels are the recipients for them, the Fermi level must lie between the top of the valence band and the acceptor levels. Also, for $T = 0\,\text{K}$ the above expression becomes,

$$E_{Fp} = \frac{E_A + E_V}{2}$$

Hence the Fermi level lies midway between the acceptor levels and the top of the valence band. In the low temperature region, holes in the valence band are only due to the transitions of electrons from the valence band to the acceptor levels.

2. As the temperature increases the acceptor levels gradually get filled and the Fermi level moves upward. At the temperature of saturation T_d (beginning of depleted region), the Fermi level coincides with the acceptor level E_A. Thus,

$$E_{FP} = E_A \qquad \text{at } T = T_d$$

③ As the temperature grows above T_d, the Fermi level shifts upward in an approximately linear fashion.

④ At temperature T_i intrinsic behaviour sets in. At higher temperatures, the p-type semiconductor loses its extrinsic character and behaves as an intrinsic semiconductor. In the intrinsic region, the hole concentration in the valence band increases exponentially and the Fermi level approaches the intrinsic value. Thus,

$$E_{Fp} = E_{Fi} = \frac{E_C + E_V}{2} \qquad \text{at } T = T_i$$

Variation of Fermi Energy with Impurity Concentration in Extrinsic Semiconductor
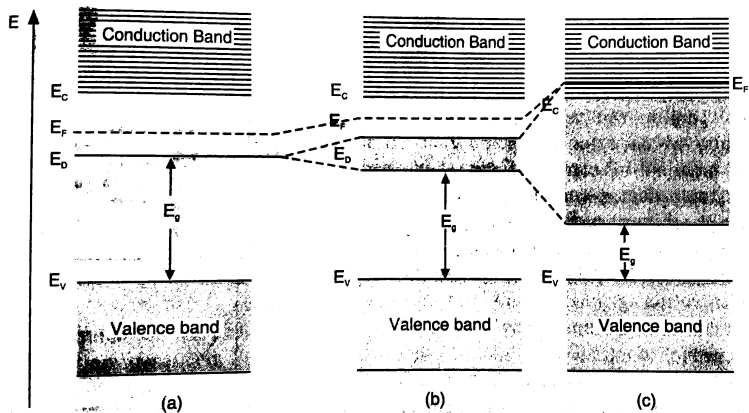
Figure: Energy band diagram of an *n*-type semiconductor at three different levels of doping. (a) low level doping, (b) medium doping, (c) heavy doping.
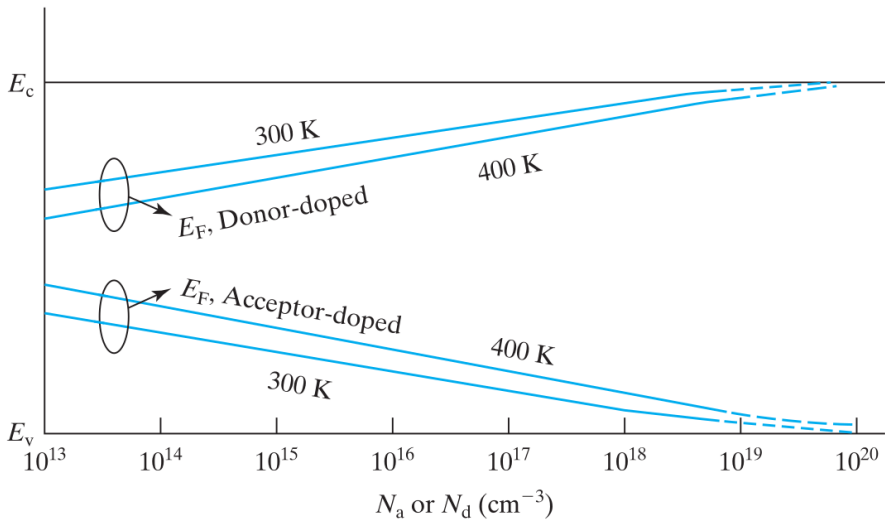
Figure: Location of Fermi level versus dopant concentration in Si at 300 and 400 K.

Hall Effect

When a transverse magnetic field is applied perpendicular to current carrying conductor, a potential difference is developed across the specimen in a direction perpendicular to both current and the magnetic field. This phenomenon is called as **Hall effect**. The voltage so developed is called as **Hall voltage**.

Hall effect helps to:

- determine the sign of charge carrier in the material
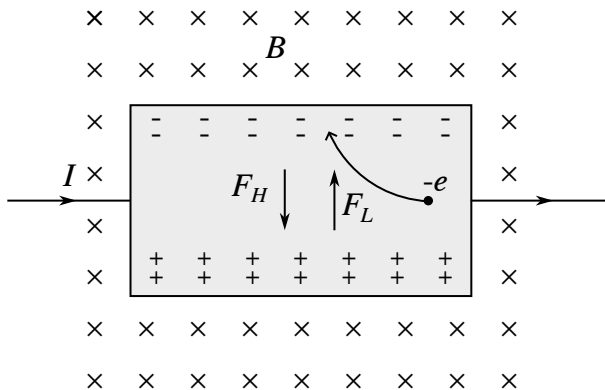- determine the charge carrier concentration
- determine the mobility of charge
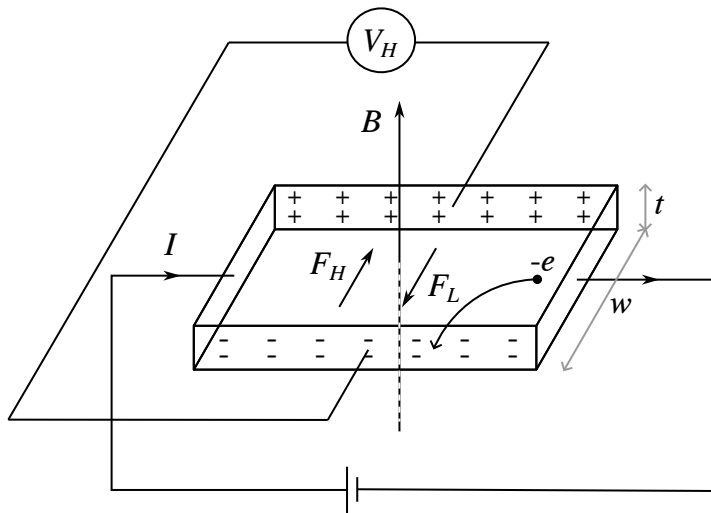
Figure: Cross-sectional view. Magnetic field is into the paper.

Consider a rectangular metal plate having width w and thickness t. When a potential difference is applied across its ends a current I flows through it opposite to the direction of flow of electrons. The current passing through the metal is given by,

$$I = -neAv_d \qquad \text{or} \qquad v_d = \frac{-I}{neA} = \frac{-I}{newt} \qquad (1)$$

where n is concentration of electrons, $A = wt$ is area of cross section of end face, e is electronic charge, v_d is drift velocity of electrons.

If magnetic field is applied perpendicular to the current flow the Lorentz force deflects the electrons to sideways. The magnitude of this force on an electron is,

$$F_L = eBv_d \qquad (2)$$

As electrons pile up on one side an equivalent amount of positive charges are left on the other opposite side. As a result an electric field E_H is developed between these separated unlike charges. This transverse electric

field E_H is known as Hall field. The magnitude of the corresponding electrostatic force on an electron will be,

$$F_H = eE_H = e\frac{V_H}{w} \qquad (3)$$

where V_H is the corresponding voltage for the field E_H and it is called Hall voltage.

A condition of equilibrium is reached when the force F_H due to transverse electric field balances the Lorentz force F_L.

$$F_L = F_H$$

Substituting the forces from eqns. (2) and (3),

$$e\frac{V_H}{w} = eBv_d$$

Substituting v_d from eqn. (1) into it and rearranging we get expression for **Hall voltage** as,

$$V_H = -\frac{BI}{net} \tag{4}$$

Reciprocal of carrier charge density is called Hall coefficient R_H. In case of electrons,

$$R_H = \frac{-1}{ne}$$

With this eqn. (4) can be written as,

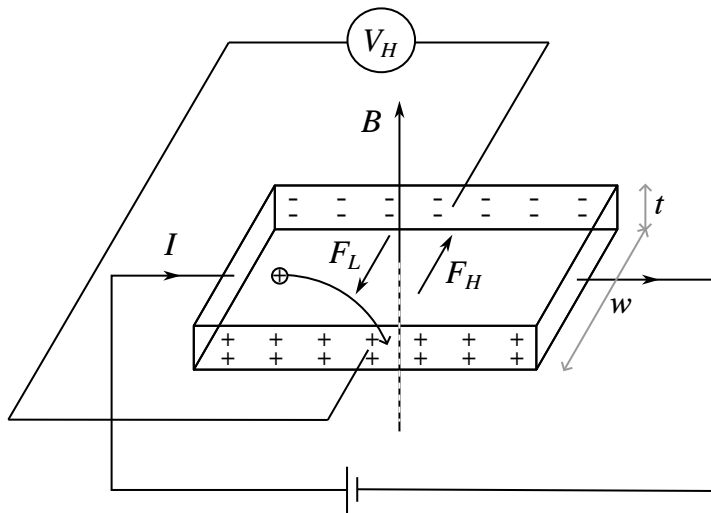$$V_H = R_H \frac{BI}{t} \qquad \text{or} \qquad R_H = V_H \frac{t}{BI}$$

The Hall voltage can be measured with a voltmeter. **For conductors Hall voltage is** conventionally taken as **negative**.

Once the Hall voltage is measured the carrier concentration can be calculated:

$$n = \frac{-1}{R_H e} = \frac{-BI}{et V_H}$$

In case of metals and n-type semiconductors the majority charge carriers are electrons and hence the same expressions holds for both.
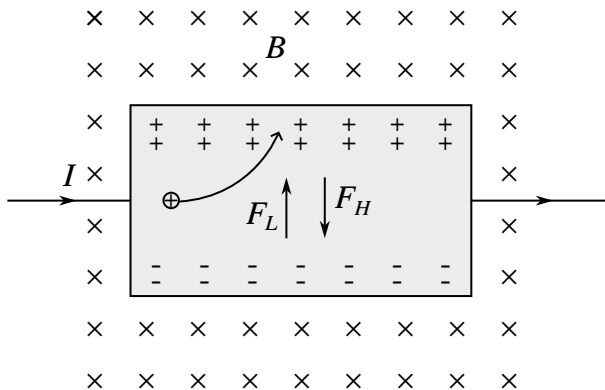
Figure: Cross-sectional view. Magnetic field is into the paper.

Consider a rectangular p-type semiconductor plate having width w and thickness t. When a potential difference is applied across its ends a current I flows through it along the direction of flow of holes. The current passing through the metal is given by,

$$I = peAv_d \qquad \text{or} \qquad v_d = \frac{I}{peA} = \frac{I}{pewt} \qquad (5)$$

where p is concentration of holes, $A = wt$ is area of cross section of end face, e is electronic charge, v_d is drift velocity of holes.

If magnetic field is applied perpendicular to the current flow the Lorentz force deflects the holes to sideways. The magnitude of this force on a hole is,

$$F_L = eBv_d \qquad (6)$$

As the holes pile up on one side an equivalent amount of negative charges are left on the other opposite side. As a result an electric field E_H is developed between these separated unlike charges. This transverse electric

field E_H is known as Hall field. The magnitude of the corresponding electrostatic force on a hole will be,

$$F_H = eE_H = e\frac{V_H}{w} \tag{7}$$

where V_H is the corresponding voltage for the field E_H and it is called Hall voltage.

A condition of equilibrium is reached when the force F_H due to transverse electric field balances the Lorentz force F_L.

$$F_L = F_H$$

Substituting the forces from eqns. (6) and (7),

$$e\frac{V_H}{w} = eBv_d$$

Substituting v_d from eqn. (5) into it and rearranging we get expression for **Hall voltage** as,

$$V_H = \frac{BI}{pet} \tag{8}$$

Reciprocal of carrier charge density is called Hall coefficient R_H.

$$R_H = \frac{1}{pe}$$

With this eqn. (8) can be written as,

$$V_H = R_H \frac{BI}{t} \qquad \text{or} \qquad R_H = V_H \frac{t}{BI}$$

The Hall voltage can be measured with a voltmeter. For p-type semiconductors Hall voltage will be **positive**.

Once the Hall voltage is measured the carrier concentration in p-type semiconductor can be calculated:

$$p = \frac{1}{R_H e} = \frac{BI}{etV_H}$$

For n-type semiconductor (like for metals):

$$n = \frac{-1}{R_H e} = \frac{-BI}{etV_H}$$

When both types of charge carriers are considered,

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

where μ_h and μ_e are mobilities ($\mu = v_d/E$) of holes and electrons respectively.

Negative and Positive Hall Voltages

The hall voltage is positive if the number of positive charges is more than the negative charges. Similarly, it is negative when electrons are more than holes.
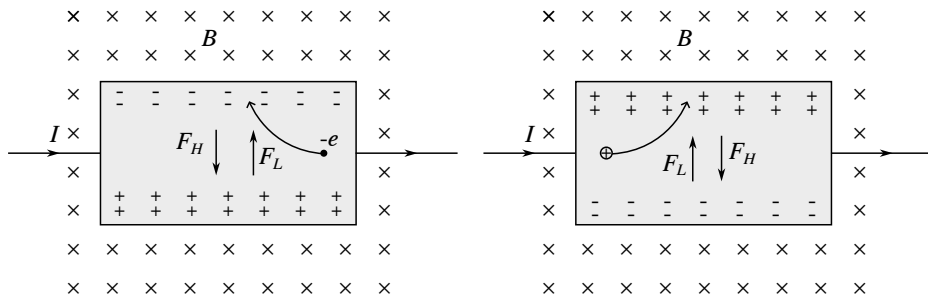

Figure: a) Metal or *n*-type semiconductor. b) *p*-type semiconductor.

Further Reading

📄 *Modern Physics*, Raymond A Serway, Clement J Moses and Curt A Moyer, 3rd edition, Thomson Learning, Inc., 2005.

📄 *Modern Semiconductor Devices for Integrated Circuits*, Chenming Calvin Hu, Pearson Education, Inc., 2010. This book is freely available in PDF at the author's web page: https://www.chu.berkeley.edu

📄 *A Textbook of Engineering Physics*, M N Avadhanulu, P G Kshirsagar, S Chand & Company Ltd., New Delhi, 2013.