



**LINEAR ALGEBRA, CALCULUS AND STATISTICS (22MA11C)**  
**Multivariable Functions and Partial Differentiation**

**TUTORIAL SHEET-1**

1. 1. If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then  $\left(\frac{\partial y}{\partial \theta}\right)^2 + \left(\frac{\partial x}{\partial \theta}\right)^2 =$  \_\_\_\_\_ **Ans:**  $r$
2. If  $z = x \sin y + y \sin x$ , then  $\frac{\partial^2 z}{\partial x \partial y} =$  \_\_\_\_\_ **Ans:**  $\cos y + \cos x$
3. If  $z = e^{2x^2+xy}$ , then  $\frac{\partial z}{\partial y} =$  \_\_\_\_\_ **Ans:**  $xe^{2x^2+xy}$
4. The steady state temperature of a metal sheet is  $T(x, y) = x^2 - a^2 y^2$ . The values of 'a' for which  $T(x, y)$  satisfies the Laplace equation  $T_{xx} + T_{yy} = 0$  are \_\_\_\_\_. **Ans:**  $\pm a$
5. If  $u = y \cos(xy)$  then  $\frac{\partial u}{\partial y}$  at the point  $(1, \pi)$  is \_\_\_\_\_. **Ans:**  $-1$
6. If  $V = f(x - ct) + g(x + ct)$  where  $f$  and  $g$  are arbitrary functions of  $x - ct$  and  $x + ct$  respectively and  $c$  is a constant, then show that  $\frac{\partial^2 V}{\partial t^2} = c^2 \frac{\partial^2 V}{\partial x^2}$
7. If  $u = \frac{x+y}{x-y}$ , verify that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
8. If  $u = ae^{-gx} \sin(nt - gx)$ , where  $a, g$  and  $n$  are positive constants and  $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$ , show that  $g = \sqrt{\frac{n}{2\mu}}$ .
9. If  $V$  is the volume and  $S$  is the total surface area of rectangular box of length  $x$ , breadth  $y$  and height  $z$ , find
  - (i) the rate of change of  $V$  with respect to  $x$  if  $y=4$  and  $z=12$ ,
  - (ii) the rate of change of  $S$  with respect to  $z$  if  $x=3$  and  $y=4$ .**Ans:** (i) 48 (ii) 14
10. If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$ , then show that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ .



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**TUTORIAL SHEET-2**

1. Given  $z = xy^2 + x^3y$  where  $x$  and  $y$  are functions of  $t$  with  $x(1) = 1$ ,  $y(1) = 2$ ,  $x'(1) = 3$  and  $y'(1) = 4$ . The value of  $\frac{dz}{dt}$  at  $t = 1$  is \_\_\_\_\_. **Ans:** 16
2. For the implicit function  $(\cos x)^y = (\sin y)^x$ ,  $\frac{dy}{dx} =$  \_\_\_\_\_.  
**Ans:**  $-\left[\frac{y(\cos x)^{y-1}(-\sin x) - (\sin y)^x \log \sin y}{(\cos x)^y \log(\cos x) - x(\sin y)^{x-1} \cos y}\right]$
3. Given 't' represents time and  $u = x^2 - y^2$ ,  $x = \frac{1}{t}$ ,  $y = e^t$  then the rate of change of  $u$  with respect to 't' is \_\_\_\_\_. **Ans:**  $\frac{-2}{t^3} - 2e^{2t}$
4. For the implicit function  $e^x - e^y = 2xy$ ,  $\frac{dy}{dx} =$  \_\_\_\_\_. **Ans:**  $\left[\frac{e^x - 2y}{e^y + 2x}\right]$
5. Given,  $x^2 + y^2 + 3xz = 1$  and  $x + y = 1$ , then  $\frac{dz}{dx} =$  \_\_\_\_\_ **Ans:**  $\frac{-(2x+3)}{3} + \frac{2y}{3x}$
6. If  $z = z(x, y)$ ,  $x = e^u \sin v$ ,  $y = e^v \cos v$ , then  $\frac{\partial z}{\partial u} =$  \_\_\_\_\_. **Ans:**  $\frac{\partial z}{\partial x} e^u \sin v$
7. If  $u = xyz$  where  $x = e^{-t}$ ,  $y = e^{-t} \sin^2 t$ ,  $z = \sin t$ , then find  $\frac{du}{dt}$ .  
**Ans:**  $e^{-t} \sin^2 t (3 \cos t - 2 \sin t)$
8. If  $z = x^2 + 2xy + 4y^2$  and  $y = e^{3x}$ , find  $\frac{dz}{dx}$ . **Ans:**  $2(x + e^{3x}) + 2(x + 4e^{3x})3e^{3x}$
9. If  $z$  is a function of  $x$  and  $y$  and if  $x = e^u \sin v$ ,  $y = e^u \cos v$ , prove that  
(i)  $\frac{\partial z}{\partial u} = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$   
(ii)  $\frac{\partial z}{\partial x} = e^{-u} \left( \sin v \frac{\partial z}{\partial u} + \cos v \frac{\partial z}{\partial v} \right)$
10. If  $u = f(x - y, y - z, z - x)$ , show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .
11. If  $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ , determine  $\frac{\partial^2 u}{\partial x \partial y}$ .



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**TUTORIAL SHEET-3**

1. Match the following:

i)	If $x = e^u \sin v, y = e^v \cos v$ then $J\left(\frac{x,y}{u,v}\right) = \underline{\hspace{2cm}}$ .	a)	$\frac{1}{4v \sin 2u}$
ii)	If (a,b) is a stationary point of $f(x,y)$ and $f_{xx} = 3, f_{xy} = 2$ and $f_{yy} = 2$ at this point then the nature of (a,b) is ____.	b)	minimum
iii)	The nature of the point (1, -1) to the function $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ is ____.	c)	$e^{u+v}(\sin v \cos v - \sin^2 v)$
		d)	$\frac{v \sin 2u}{4}$
iv)	If $\frac{\partial(x,y)}{\partial(u,v)} = v \sin 2u$ and $\frac{\partial(x,y)}{\partial(r,\theta)} = \frac{1}{4}$ then $\frac{\partial(u,v)}{\partial(r,\theta)} = \underline{\hspace{2cm}}$ .	e)	Neither maximum nor minimum
		f)	Saddle point
		g)	maximum
		h)	$e^{u+v}(\sin v \cos v - \sin^2 v) - e^u \cos v$

**Ans:** (i) - (c) (ii) - (g) (iii) - (e) (iv) - (a)

- Find the extreme values of  $\sin x + \sin y + \sin(x + y)$ . **Ans:** Maximum value at  $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ , maximum value is  $\frac{3\sqrt{3}}{2}$
- Find the volume of largest rectangular parallelepiped that can be inscribed in an ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ . **Ans:** Maximum volume  $= \frac{8abc}{3\sqrt{3}}$
- Find the maximum and minimum distances of the point (1, 2, 3) from the sphere  $x^2 + y^2 + z^2 = 56$  using Lagrange's Method of undetermined multipliers. **Ans:** Minimum distance at (2, 4, 6)  $= \sqrt{14}$ , Maximum distance at (-2, -4, -6), maximum distance  $= \sqrt{126}$
- Show that  $u = \frac{x^2 - y^2}{x^2 + y^2}, v = \frac{2xy}{x^2 + y^2}$  are functionally dependent and find the relation between them. **Ans:**  $u^2 + v^2 = 1$ .
- For  $u = xyz, v = yz + zx + xy, w = x + y + z$ , find  $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ , **Ans:**  $(y - z)(z - x)(x - y)$ .
- If  $x = e^v \sec u, y = e^v \tan u$ , then verify that  $JJ' = 1$ .