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DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	IMPROVEMENT CIE	Maximum marks: 50
Course code: 22MA21C	Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C	Time: 02:00PM-3:30PM Date: 06-09-2023

Sl. No.	Questions	Marks
1. (a)	$\frac{d\vec{r}}{dt} = 3t^2\hat{\imath} + 2t\hat{\jmath} + 2\hat{k} \Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{t=1} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$	2
	$\frac{d^2\vec{r}}{dt^2} = 6t\hat{\imath} + 2t\hat{\jmath} + 0\hat{k} \Rightarrow \left(\frac{d^2\vec{r}}{dt^2}\right)_{t=1} = 6\hat{\imath} + 2\hat{\jmath}$	1
	Component of velocity = $\frac{11}{\sqrt{11}}$	1
	Component of acceleration = $\frac{8}{\sqrt{11}}$	1
1. (b)	$\nabla \phi = 6x^2y^2z^4\hat{i} + 4x^3yz^4\hat{j} + 8x^3y^2z^3\hat{k}$	2
	$div(\vec{f}) = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$	2
	$div(\vec{f})_{(1,2,-1)} = 48 + 4 + 96 = 148$	1
2. (a)	$\nabla \phi = (ay^2 + 3cz^2x^2)\hat{i} + (2axy + bz)\hat{j} + (by + 2czx^3)\hat{k}$	2
	$\nabla \phi_{(1,2,-1)} = (4a + 3c)\hat{i} + (4a - b)\hat{j} + (2b - 2c)\hat{k}$	1
	Directional derivative of ϕ at $(1,2,-1)$ in the direction parallel to z axis is given by	
	$\nabla \phi. \hat{k} = 64$ $\Rightarrow 2b - 2c = 64 \text{ and } 4a + 3c = 0, 4a - b = 0$	
	$\Rightarrow 2b - 2c = 64$ and $4a + 3c = 0$, $4a - b = 0$ Solving $a = 6$, $b = 24$, $c = -8$.	1+1+1
	Solving $u = 0$, $v = 2$ i, $v = 0$.	1+1+1
2. (b)	$r^2 = x^2 + y^2 + z^2$ and $\frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial y} = \frac{y}{r}, \frac{\partial r}{\partial z} = \frac{z}{r}$	1
	$\nabla r^{n} = nr^{n-1} \frac{x}{r} \hat{i} + nr^{n-1} \frac{y}{r} \hat{j} + nr^{n-1} \frac{z}{r} \hat{k} = nr^{n-2} \vec{r}$	1+2
3	$curl \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix} = (c+1)\hat{\imath} - (4-a)\hat{\jmath} + (b-2)\hat{k} = \vec{0}$	1
	a=4 , $b=2$ and $c=-1$	3
	$\vec{F} = (x + 2y + 4z)\hat{i} + (2x - 3y - z)\hat{j} + (4x - y + 2z)\hat{k}$	
	$\frac{\partial \phi}{\partial x} = x + 2y + 4z, \ \frac{\partial \phi}{\partial y} = 2x - 3y - z, \ \frac{\partial \phi}{\partial z} = 4x - y + 2z$	1

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	$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$	1	
	$d\phi = d\left(\frac{x^2}{2}\right) + d(2xy) + d(4xz) - d\left(\frac{3}{2}y^2\right) - d(yz) + d(z^2)$	3	
	$\phi = \frac{x^2}{2} + 2xy + 4xz - \frac{3}{2}y^2 - yz + z^2 + c$	1	
4	Difference table		
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
	-10.8		
	3 74.5 3.3		
	-7.5 -2.3 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	3	
	5 67 1 1.6 -6.5 -0.7		
	7 60.5 0.3		
	-6.2		
	$p = \frac{t - t_0}{h} = \frac{2 - 1}{2} = 0.5$ and $p = \frac{t - t_n}{h} = \frac{8 - 9}{2} = -0.5$		
	1t 2		
	Using Newton's forward interpolation formula, we get $n(n-1) = n(n-1)(n-2)$		
	$y = \theta_0 + p\Delta\theta_0 + \frac{p(p-1)}{2!}\Delta^2\theta_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3\theta_0 + \cdots$		
	$\theta(2) = 79.28125$		
	Using Newton's backward interpolation formula, we get		
	$y = \theta_n + p\nabla\theta_n + \frac{p(p+1)}{2!}\nabla^2\theta_n + \frac{p(p+1)(p+2)}{2!}\nabla^3\theta_n + \cdots$		
	$\theta(8) = 57.34375$		
	$\frac{d\theta}{dt} = \frac{1}{h} \left[\nabla \theta_n + \frac{1}{2} \nabla^2 \theta_n + \frac{1}{3} \nabla^3 \theta_n + \frac{1}{4} \nabla^4 \theta_n + \dots \right] = \frac{1}{2} \left[-6.2 + \frac{0.3}{2} - \frac{0.7}{3} + \frac{1.6}{4} \right] = -2.9416$ $= \frac{(x - 7)(x - 10)(x - 12)(x - 15)}{(6 - 7)(6 - 10)(6 - 12)(6 - 15)} 3 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(7 - 10)(7 - 12)(7 - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(7 - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)(x - 15)} 10 + \frac{(x - 6)(x - 10)(x - 12)(x - 15)}{(x - 6)(x - 10)(x - 12)($	2	
5. (a)	$(x-7)(x-10)(x-12)(x-15) \qquad (x-6)(x-10)(x-12)(x-15)$		
	$= \frac{1}{(6-7)(6-10)(6-12)(6-15)} + \frac{1}{(7-6)(7-10)(7-12)(7-15)} + \frac{1}{(7-6)(7-10)(7-12)(7-15)} = \frac{1}{(6-7)(6-10)(6-12)(6-15)} + \frac{1}{(7-6)(7-10)(7-12)(7-15)} = \frac{1}{(7-6)(7-10)(7-12)(7-1$		
	$\frac{(x-6)(x-7)(x-12)(x-15)}{(10-6)(10-7)(10-12)(10-15)}43 + \frac{(x-6)(x-7)(x-10)(x-15)}{(12-6)(12-7)(12-10)(12-15)}75$	3	
	(10-6)(10-7)(10-12)(10-15) $(12-6)(12-7)(12-10)(12-15)$		
	$+\frac{(x-6)(x-7)(x-10)(x-12)}{(15-6)(15-7)(15-10)(15-12)}138$	2	
	$y(x) = x^2 - 6x + 3$		
— ~ ~ ~	y(11) = 58	1	
5. (b)	$x = \frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)} + \frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)}3$		
	$ \begin{array}{c} (10-24)(10-54)(10-129) & (24-10)(24-54)(24-129) \\ + & (y-10)(y-24)(y-129) \\ + & (y-10)(y-24)(y-129) \end{array} \begin{array}{c} 2 \\ + & (y-10)(y-24)(y-129) \end{array} $		
	$y(11) = 58$ $x = \frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)} + \frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)} 3 + \frac{(y-10)(y-24)(y-129)}{(54-10)(54-24)(54-129)} 5 + \frac{(y-10)(y-24)(y-54)}{(129-10)(129-24)(129-54)} 8$		
	When $y = 100$, $x = 5.9199$	2	