Conductivity - Numerical Problems

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Example 1

What is the probability of a level lying 0.01 eV below the Fermi level is *being* and *not being* occupied by electrons at $T = 300$ K?

$k = 1.381 \times 10^{-23}\,\mathrm{JK}^{-1}$, $T = 300\,\mathrm{K}$

$$kT = \frac{1.381 \times 10^{-23}\,\mathrm{JK}^{-1} \times 300\mathrm{K}}{1.602 \times 10^{-19}\,\mathrm{J/eV}} = 0.026\mathrm{eV}$$

$$\frac{E - E_F}{kT} = \frac{-0.01\,\mathrm{eV}}{0.026\,\mathrm{eV}} = -0.3846$$

The probability of the level *being* occupied is,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{-0.3846} + 1} = 0.595$$

The probability of the level *not being* occupied is,

$$1 - f(E) = 1 - 0.595 = 0.405$$

Example 2

Find the temperature at which there is 1% occupancy probability of a state 0.5 eV above the Fermi energy.

Given, $f(E) = 0.01$, $E - E_F = 0.5\,\text{eV}$, $T = ?$

$$k = \frac{1.381 \times 10^{-23}\,\text{JK}^{-1}}{1.602 \times 10^{-19}\,\text{J/eV}} = 8.617 \times 10^{-5}\text{eV}\,\text{K}^{-1}$$

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$\text{or} \quad f(E)e^{(E-E_F)/kT} + f(E) = 1$$

$$\text{or} \quad e^{(E-E_F)/kT} = \frac{1 - f(E)}{f(E)}$$

Taking natural log on both sides,

$$\frac{(E - E_F)}{kT} = \ln\left(\frac{1 - f(E)}{f(E)}\right)$$

$$\therefore \quad T = \frac{(E - E_F)}{k\,\ln\left(\frac{1-f(E)}{f(E)}\right)} = \frac{0.5\text{eV}}{8.617 \times 10^{-5}\text{eV}\,\text{K}^{-1} \times \ln\left(\frac{1-0.01}{0.01}\right)} = 1262.7\,\text{K}$$

Example 3

If the effective mass of holes in an intrinsic semiconductor is 4 times that of electrons. At what temperature would the Fermi energy be shifted up by 10% of the band gap from the middle of the energy gap? Given, the energy gap is 1 eV.

The shifted Fermi energy will be, (10% of 1 eV is 0.1 eV)

$$E_F = \frac{E_C + E_V}{2} + 0.1\text{eV}$$

Substituting it into,

$$E_F = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

we get

$$\cancel{\frac{E_C + E_V}{2}} + 0.1\text{eV} = \cancel{\frac{E_C + E_V}{2}} + \frac{kT}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$\therefore \quad T = \frac{2 \times 0.1\text{eV}}{k \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2}} = \frac{2 \times 0.1\text{eV}}{8.617 \times 10^{-5}\text{eV K}^{-1} \times \ln \left(\frac{4m_e^*}{m_e^*} \right)^{3/2}} = 1116.12\,\text{K}$$

Example 4

The Fermi level in silver is 5.5 eV at 0 K. Calculate the number of free electrons per unit volume at this temperature.

The electron concentration for metals at 0 K is given by,

$$
\begin{aligned}
n &= \frac{8\pi}{3h^3}(2m)^{3/2}E_F^{3/2} \\
&= \frac{8 \times 3.142}{3 \times (6.626 \times 10^{-34}\,\mathrm{Js})^3}(2 \times 9.109 \times 10^{-31}\,\mathrm{kg})^{3/2}(5.5 \times 1.602 \times 10^{-19}\,\mathrm{J})^{3/2} \\
&= 5.858 \times 10^{28}\,\mathrm{m}^{-3}
\end{aligned}
$$

Example 5

Calculate the Fermi energy of sodium at 0 K assuming that it has one free electron per atom and density of sodium is $970\,\mathrm{kg/m^3}$ and atomic weight $23\,\mathrm{g/mol}$.

Electron concentration will be,

$$n = \frac{N_A \rho}{M} = \frac{6.02 \times 10^{23} \mathrm{mol}^{-1} \times 970 \mathrm{kg\,m}^{-3}}{23 \times 10^{-3} \mathrm{kg\,mol}^{-1}} = 2.539\,8 \times 10^{28} \mathrm{m}^{-3}$$

Fermi energy for metals at 0 K is given by,

$$\begin{aligned} E_F &= \frac{h^2}{8m} \left(\frac{3}{\pi}\right)^{2/3} n^{2/3} \\ &= \frac{(6.626 \times 10^{-34} \mathrm{Js})^2}{8 \times 9.109 \times 10^{-31} \mathrm{kg}} \left(\frac{3}{3.142}\right)^{2/3} (2.539\,8 \times 10^{28} \mathrm{m}^{-3})^{2/3} \\ &= 5.048 \times 10^{-19} \mathrm{J} \\ &= \frac{5.048 \times 10^{-19} \mathrm{eV}}{1.602 \times 10^{-19}} \\ &= 3.151\,\mathrm{eV} \end{aligned}$$

Example 6

For silver, the density is 10.5×10^3kg m^{-3} and atomic weight is 107.9. If the conductivity of silver at 20°C is $6.8 \times 10^7 \Omega^{-1}$m^{-1} calculate electron mobility in silver at that temperature.

Conductivity is given by: $\sigma = ne\mu$.

Therefore, mobility is $\mu = \sigma/(ne)$

Electron concentration will be,

$$n = \frac{N_A \rho}{M} = \frac{6.02 \times 10^{23} \text{mol}^{-1} \times 10500 \text{kg m}^{-3}}{107.9 \times 10^{-3} \text{kg mol}^{-1}} = 5.860 \times 10^{28} \text{m}^{-3}$$

The mobility is,

$$\begin{aligned} \mu &= \frac{\sigma}{ne} \\ &= \frac{6.8 \times 10^{7} \Omega^{-1} \text{m}^{-1}}{5.86 \times 10^{28} \text{m}^{-3} \times 1.602 \times 10^{-19} \text{C}} \\ &= 7.2427 \, \text{ms}^{-1}/(\text{Vm}^{-1}) \end{aligned}$$

Example 7

Fermi energy of potassium is 2.1 eV. Calculate Fermi velocity.

$$E_F = \frac{1}{2} m v_F^2$$

$$\therefore \quad v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 2.1 \times 6.02 \times 10^{-19}\,\mathrm{J}}{9.109 \times 10^{-31}\,\mathrm{kg}}}$$

$$= 0.8595 \times 10^6 \ \mathrm{m/s}$$

Example 8

Show that the probability of finding an electron of energy ΔE above the Fermi level is same as probability of *not* finding an electron at energy ΔE below the Fermi level.

OR

Show that the probability that a state ΔE above the Fermi level E_F is filled equals the probability that a state ΔE below E_F is empty.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$f(E_F + \Delta E) = \frac{1}{e^{(E_F+\Delta E-E_F)/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1}$$

$$f(E_F - \Delta E) = \frac{1}{e^{(E_F-\Delta E-E_F)/kT} + 1} = \frac{1}{e^{-\Delta E/kT} + 1}$$

It is to show that,

$$f(E_F + \Delta E) = 1 - f(E_F - \Delta E)$$

$$\begin{aligned}
\therefore \quad \text{RHS} &= 1 - f(E_F - \Delta E) = 1 - \frac{1}{e^{-\Delta E/kT} + 1} \\
&= 1 - \frac{1}{\frac{1}{e^{\Delta E/kT}} + 1} = 1 - \frac{e^{\Delta E/kT}}{1 + e^{\Delta E/kT}} \\
&= \frac{1 + e^{\Delta E/kT} - e^{\Delta E/kT}}{1 + e^{\Delta E/kT}} = \frac{1}{e^{\Delta E/kT} + 1} \\
&= \text{LHS}
\end{aligned}$$

Example 9

Show that the occupancy probabilities of two states whose energies are equally spaced above and below the Fermi energy add up to one.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$f(E_F + \Delta E) = \frac{1}{e^{(E_F+\Delta E-E_F)/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1}$$

$$f(E_F - \Delta E) = \frac{1}{e^{(E_F-\Delta E-E_F)/kT} + 1} = \frac{1}{e^{-\Delta E/kT} + 1}$$

It is to show that,

$$f(E_F + \Delta E) + f(E_F - \Delta E) = 1$$

$$\begin{aligned}
\therefore \quad \text{LHS} &= \frac{1}{e^{\Delta E/kT} + 1} + \frac{1}{e^{-\Delta E/kT} + 1} \\
&= \frac{1}{e^{\Delta E/kT} + 1} + \frac{1}{\frac{1}{e^{\Delta E/kT}} + 1} = \frac{1}{e^{\Delta E/kT} + 1} + \frac{e^{\Delta E/kT}}{1 + e^{\Delta E/kT}} \\
&= \frac{e^{\Delta E/kT} + 1}{e^{\Delta E/kT} + 1} \\
&= 1 \\
&= \text{RHS}
\end{aligned}$$

Example 10

The forbidden gap in pure silicon is 1.1 eV. Compare the number of conduction electrons at temperatures 37°C and 27°C.

Conduction electron concentration in intrinsic semiconductor (pure silicon) is given by,

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_C - E_F)/kT}$$

Since it is an intrinsic semiconductor E_F lies in middle of E_C and E_V. Therefore,

$$E_C - E_F = E_g/2$$

where $E_g = E_C - E_V$ is the energy gap. We can rewrite the expression for n for two different temperatures T_1 and T_2 as,

$$n_1 = 2\left[\frac{2\pi m_e^* kT_1}{h^2}\right]^{3/2} e^{-E_g/2kT_1} \qquad \text{and} \qquad n_2 = 2\left[\frac{2\pi m_e^* kT_2}{h^2}\right]^{3/2} e^{-E_g/2kT_2}$$

Their ratio will be,

$$\frac{n_2}{n_1} = \frac{T_2^{3/2}}{T_1^{3/2}} \frac{e^{-E_g/2kT_2}}{e^{-E_g/2kT_1}}$$

Substituting $T_2 = (27 + 273.15)\text{K} = 300.15\text{K}$, $T_2 = (37 + 273.15)\text{K} = 310.15\text{K}$, $E_g = 1.1\,\text{eV}$

$$k = \frac{1.381 \times 10^{-23}\,\text{JK}^{-1}}{1.602 \times 10^{-19}\,\text{J/eV}} = 8.617 \times 10^{-5}\text{eV K}^{-1}$$

$$\therefore \quad \frac{E_g}{2k} = \frac{1.1\,\text{eV}}{2 \times 8.617 \times 10^{-5}\text{eV K}^{-1}} = 6382.485\,\text{K}$$

$$\begin{aligned} \frac{n_2}{n_1} &= \frac{(310.15\text{K})^{3/2}}{(300.15\text{K})^{3/2}} \frac{e^{-6382.485\,\text{K}/310.15\text{K}}}{e^{-6382.485\,\text{K}/300.15\text{K}}} \\ &= 2.085 \end{aligned}$$

Therefore, conduction electron concentration of this semiconductor at 37°C is 2.085 times that of the concentration at 27°C.

Example 11

Compute the concentration of intrinsic charge carriers in a germanium crystal at 300 K. Given that the energy gap is 0.72 eV and assume that $m_e^* \approx m_e$.

Intrinsic charge carrier concentration in intrinsic semiconductor (pure germanium) is given by,

$$n_i = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_C - E_F)/kT}$$

Since it is an intrinsic semiconductor E_F lies in middle of E_C and E_V. Therefore,

$$E_C - E_F = E_g/2$$

where $E_g = E_C - E_V$ is the energy gap. We can rewrite the expression for n_i as,

$$n_i = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-E_g/2kT}$$

Substituting $m_e^* \approx m_e = 9.109 \times 10^{-31}$kg, $k = 1.38 \times 10^{-23}$JK^{-1}, $E_g = 0.72$eV $= 0.72 \times 1.602 \times 10^{-19}$J, $h = 6.626 \times 10^{-34}$Js and $T = 300$K into it, we get,

$$n_i = 2\left[\frac{2 \times 3.142 \times 9.109 \times 10^{-31}\text{kg} \times 1.38 \times 10^{-23}\text{JK}^{-1} \times 300\text{K}}{(6.626 \times 10^{-34}\text{Js})^2}\right]^{3/2}$$
$$\times \exp\left(\frac{-0.72 \times 1.602 \times 10^{-19}\text{J}}{2 \times 1.38 \times 10^{-23}\text{JK}^{-1} \times 300\text{K}}\right)$$
$$= 2.248 \times 10^{19}\,\text{m}^{-3}$$

Example 12

For silicon at $30°C$, calculate the number of states per unit energy per unit volume at an energy 26meV above the bottom of the conduction band $(m_e^* = 1.18m_e)$.

Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}\, dE$$

Substituting,

$E - E_C = 26\text{meV} = 26 \times 10^{-3} \times 1.602 \times 10^{-19}\,\text{J}$,
$m_e^* = 1.18 m_e = 1.18 \times 9.109 \times 10^{-31}\text{kg}$,
$h = 6.626 \times 10^{-34}\,\text{Js}$

into it, we get,

$$\begin{aligned}\frac{g_c(E)}{dE} &= \frac{4 \times 3.1416}{(6.626 \times 10^{-34}\,\text{Js})^3} \times (2 \times 1.18 \times 9.109 \times 10^{-31}\text{kg})^{3/2} \\ &\quad \times (26 \times 10^{-3} \times 1.602 \times 10^{-19}\,\text{J})^{1/2} \\ &= 8.788 \times 10^{45}\ \text{J}^{-1}\text{m}^{-3}\end{aligned}$$

Example 13

Determine the position of Fermi level in silicon semiconductor at 300 K relative to the top most level of the valence band E_V. Given that the band gap is 1.12 eV, $m_e^* = 0.12m_e$ and $m_h^* = 0.28m_e$.

Fermi level in intrinsic semiconductor is given by,

$$E_F = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

Energy gap is given by,

$$E_g = E_C - E_V \qquad \text{or} \qquad E_C = E_V + E_g$$

$$\begin{aligned}
\therefore \quad E_F &= E_V + \frac{E_g}{2} + \frac{kT}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2} \\
&= E_V + \frac{1.12\,\text{eV}}{2} + \frac{1.38 \times 10^{-23}\,\text{JK}^{-1} \times 300\text{K}}{2 \times 1.602 \times 10^{-19}\,\text{JeV}^{-1}} \ln \left(\frac{0.28 m_e}{0.12 m_e} \right)^{3/2} \\
&= E_V + 0.56\text{eV} + 0.016\,43\text{eV} \\
&= E_V + 0.5764\text{eV}
\end{aligned}$$

For the given semiconductor, at 300 K, Fermi level is 0.5764eV above the top most level of the valence band E_V.

Example 14

A copper strip 2 cm wide and 1 mm thick is placed in a magnetic field of 1.5 Wb m^{-2}. If a current of 200 A is set up in the strip calculate Hall voltage that appears across the strip. Assume $R_H = -6 \times 10^{-7}$m^{3}C^{-1}.

Hall voltage for metals is given by,

$$V_H = R_H \frac{BI}{t}$$

Substituting $t = 1 \times 10^{-3}$m, $I = 200$A, $R_H = -6 \times 10^{-7}$m^{3}C^{-1} and $B = 1.5$Wbm^{-2} $= 1.5$T into it,

$$\begin{aligned} V_H &= -6 \times 10^{-7}\text{m}^3\text{C}^{-1} \times \frac{1.5\text{T} \times 200\text{A}}{1 \times 10^{-3}\text{m}} \\ &= -0.18\text{V} \end{aligned}$$

The unit volt is shown to be arrived by,

$$1\frac{\text{TAm}^2}{\text{C}} = 1\frac{\text{N}}{\text{Am}}\frac{\text{Am}^2}{\text{C}} = 1\frac{\text{Nm}}{\text{C}} = 1\frac{\text{J}}{\text{C}} = 1\text{V}$$

Example 15

An n-type germanium semiconductor has donor density of $10^{21}\,\mathrm{m^{-3}}$. It is arranged in a Hall experiment having magnetic field 0.5 T and the current density is $500\,\mathrm{Am^{-2}}$. Find the Hall voltage if the sample is 3 mm wide.

Hall voltage for metals is given by,

$$V_H = R_H \frac{BI}{t}$$

By substituting $I = JA = Jwt$ we can write,

$$V_H = R_H \frac{BJwt}{t} = R_H BJw$$

The give n-type semiconductor has donor density $N_d = 10^{21}\text{m}^{-3}$. Assuming that the donors are completely ionized at room temperature and are the only contributors to electron density, we can substitute $n = N_d$ to get,

$$R_H = -\frac{1}{ne} = -\frac{1}{N_d e} \qquad \implies V_H = -\frac{1}{N_d e} BJw$$

Substituting, $e = 1.602 \times 10^{-19}\text{C}$, $N_d = 10^{21}\text{m}^{-3}$, $B = 0.5\text{T}$, $J = 500\text{Am}^{-2}$, $w = 3 \times 10^{-3}\text{m}$,

$$V_H = -\frac{0.5\text{T} \times 500\text{Am}^{-2} \times 3 \times 10^{-3}\text{m}}{10^{21}\text{m}^{-3} \times 1.602 \times 10^{-19}\text{C}}$$
$$= 0.004\,68\text{V} = 4.68\text{mV}$$

Example 16

An electric field of 100 V/m is applied to a sample of n-type semiconductor whose Hall coefficient is $-0.0125\text{m}^3\text{C}^{-1}$. Determine the current density in the sample assuming the mobility of electron $\mu_e = 0.6\text{m}^2/(\text{Vs})$.

Current density is given by,

$$J = nev_d$$

For n-type semiconductor $R_H = -1/(ne)$ and $\mu_e = v_d/\mathscr{E}$ where $\mathscr{E}$ is the applied electric field. Therefore,

$$J = nev_d = \frac{\mu_e \mathscr{E}}{-R_H}$$

Substituting $\mathscr{E} = 100\,\mathrm{Vm}^{-1}$, $R_H = -0.0125\mathrm{m}^3\mathrm{C}^{-1}$ and $\mu_e = 0.6\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$,

$$\begin{aligned} J &= \frac{0.6\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1} \times 100\,\mathrm{Vm}^{-1}}{-(-0.0125\mathrm{m}^3\mathrm{C}^{-1})} \\ &= 4800\mathrm{Am}^{-2} \end{aligned}$$

Example 17

In a Hall effect experiment a current of 0.25 A is sent through a metal strip having thickness 0.2 mm and width 5 mm. The Hall voltage is found to be -0.15 mV when a magnetic field 0f 2000 gauss is applied. (a) What is the carrier concentration in the sample? (b) What is the drift velocity of the carriers?

a) We have expression for Hall voltage as,

$$V_H = R_H \frac{BI}{t} = \frac{-1}{ne}\frac{BI}{t} \qquad \implies n = -\frac{IB}{V_H e t}$$

Substituting $I = 0.25\text{A}$, $t = 0.2 \times 10^{-3}\text{m}$, $V_H = -0.15 \times 10^{-3}\text{V}$, $B = 2000\text{gauss} = 0.2\text{T}$ ($1\text{T} = 10^4\text{gauss}$) and $e = 1.602 \times 10^{-19}\text{C}$ we get for carrier concentration,

$$\begin{aligned} n &= -\frac{0.25\text{A} \times 0.2\text{T}}{-0.15 \times 10^{-3}\text{V} \times 1.602 \times 10^{-19}\text{C} \times 0.2 \times 10^{-3}\text{m}} \\ &= 1.04025 \times 10^{25}\text{m}^{-3} \end{aligned}$$

b) Substituting $I = neAv_d = newtv_d$ into the expression for Hall voltage we get,

$$V_H = -\frac{BI}{net} = -\frac{Bnewtv_d}{net} = -Bwv_d \qquad \implies v_d = -\frac{V_H}{Bw}$$

Substituting $V_H = -0.15 \times 10^{-3}\text{V}$, $B = 0.2\text{T}$ and $w = 5 \times 10^{-3}\text{m}$ we get,

$$\begin{aligned} v_d &= -\frac{-0.15 \times 10^{-3}\text{V}}{0.2\text{T} \times 5 \times 10^{-3}\text{m}} \\ &= 0.15\text{ms}^{-1} \end{aligned}$$

Example 18

A bar of n-type germanium of dimension $1\text{cm} \times 0.1\text{cm} \times 0.1\text{cm}$ in the order of length, width and thickness is placed in a magnetic field of 0.2 T. If the drift velocity of the electrons is 4cm/s calculate the Hall voltage produced in the bar. Assume the magnetic field to be along the direction of width.

Expression for Hall voltage produced in a n-type semiconductor is, in which current $I = neAv_d$ is flowing is,

$$V_H = -\frac{BI}{net} = -\frac{BneAv_d}{net} = -\frac{Bnewtv_d}{net} = -Bwv_d$$

Substituting $B = 0.2\text{T}$, $w = 0.1\text{cm}$ and $v_d = 4\,\text{cm/s}$ we get,

$$V_H = -0.2\text{T} \times 0.1\text{cm} \times 4\,\text{cm/s} = -8 \times 10^{-6}\text{V}$$