

## DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	TEST-II	Maximum marks: 50
Course code: 22MA21C	Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C	Time: 10:00AM-11:30AM Date: 21-08-2023

### Scheme and Solutions

Q.No	PART -B	Marks
1.	<p>The auxiliary equation is <math>m^2 - 2m + 2 = 0</math> and roots are <math>m = 1 \pm i</math></p> <p><math>CF = e^x(c_1 \cos(x) + c_2 \sin(x))</math> and <math>PI = Au + Bv</math></p> <p><math>W = \begin{vmatrix} e^x \cos(x) &amp; e^x \sin(x) \\ e^x(-\sin(x) + \cos(x)) &amp; e^x(\sin(x) + \cos(x)) \end{vmatrix} = e^{2x}</math></p> <p><math>A = -\int \frac{vf(x)}{W} dx = -\int \frac{e^x \sin(x) e^x \tan(x)}{e^{2x}} dx</math></p> <p><math>A = -\int \frac{\sin^2(x)}{\cos(x)} dx = -[\log_e(\sec(x)) + \tan(x)) - \sin(x)]</math></p> <p><math>B = \int \frac{uf(x)}{W} dx = \int \frac{e^x \cos(x) e^x \tan(x)}{e^{2x}} dx = \int \sin(x) dx = -\cos(x)</math></p> <p><math>y = e^x(c_1 \cos(x) + c_2 \sin(x)) - e^x \cos(x) \log_e(\sec(x) + \tan(x))</math></p>	<p>1</p> <p>1</p> <p>2</p> <p>3</p> <p>2</p> <p>1</p>
2.	<p>Given equation is converted to linear differential equation with constant coefficients by substituting <math>x = e^z</math> or <math>z = \log_e(x)</math> and <math>xD = D_1, x^2 D^2 = D_1(D_1 - 1)</math> where <math>D_1 = \frac{d}{dz}</math></p> <p><math>(D_1^2 + 4D_1 + 4)y = ze^z</math></p> <p>The auxiliary equation is <math>m^2 + 4m + 4 = 0</math> and roots are <math>m = -2, -2</math></p> <p><math>CF = \frac{(c_1 + c_2 \log_e(x))}{x^2}</math></p> <p><math>PI = \frac{1}{D_1^2 + 4D_1 + 4} e^z z = e^z \frac{1}{D_1^2 + 6D_1 + 9} z = \frac{e^z}{9} \left(1 + \frac{D_1}{3}\right)^{-2} z = \frac{e^z}{9} \left(z - \frac{2}{3}\right)</math></p> <p><math>PI = \frac{x}{9} \left(\log_e(x) - \frac{2}{3}\right)</math></p> <p><math>y = CF + PI</math></p>	<p>2</p> <p>2</p> <p>1+3+1</p> <p>1</p>
3(a)	<p><math>q'' + 40q' + 4000q = 0</math></p> <p>Auxiliary equation <math>m^2 + 40m + 4000 = 0</math></p> <p>Roots: <math>m = -20 \pm 60i</math></p> <p><math>q = e^{-20t}(c_1 \cos(60t) + c_2 \sin(60t))</math></p> <p><math>c_1 = 0, c_2 = 0</math></p> <p><math>q(t) = 0</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1+1</p> <p>1</p>

