12CS36 /12IS36

USN

R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated to VTU

III Semester B. E. Fast Track Examinations July-16

Common to CSE / ISE

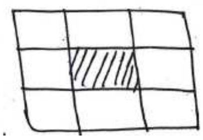
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

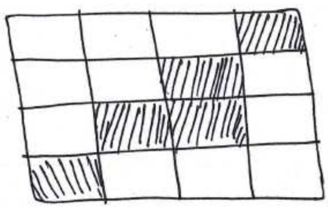
Instructions to candidates:

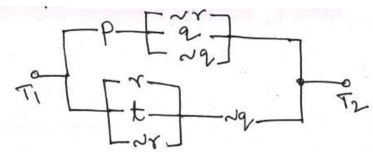
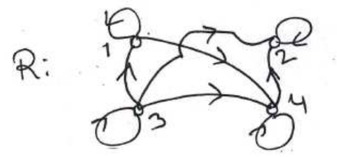
3. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
4. Answer FIVE full questions from Part B.

PART-A

1	1.1	How many positive integers between 100 and 999 inclusive, are not divisible by 4?	02
	1.2	How many arrangements of the letters $ABCDEFG$ contain the strings BA and GF?	02
	1.3	How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 7$, where x_1, x_2, x_3, x_4 are non-negative integers?	01
	1.4	For the positive integers $1,2,3,4, \ldots\ldots, n-1, n$ there are 11,660 derangements where 1,2,3,4 and 5 appear in the first five positions. What is the value of n?	01
	1.5	Find the rook polynomial for the chess board given below:  Fig 1.5	02
	1.6	Construct the truth table for: $((p \leftrightarrow q) \vee (p \rightarrow r)) \rightarrow (\sim q \wedge p)$	02
	1.7	Find the recurrence relation for the integer sequence given below: $2,10,50,250, \ldots \ldots$	02
	1.8	Let $x_1, x_2, x_3, x_4 \ldots \ldots \ldots \ldots, x_{20}$ be the list of distinct real numbers to be sorted by the bubble-sort technique. After how many comparisons will the 10 smallest numbers of the original list be arranged in ascending order?	02
	1.9	Let $A = \{1,2,3\}$, $B = \{w, x, y, z\}$, and $C = \{4,5,6\}$. Define the relations $R_1 \subseteq A \times B, R_2 \subseteq B \times C$, and $R_3 \subseteq B \times C$ where: $R_1 = \{(1, w), (2, x), (3, w), (1, y)\}$, $R_2 = \{(w, 5), (x, 6), (y, 4), (y, 6)\}$ and $R_3 = \{(w, 4), (w, 5), (y, 5)\}$. Determine $R_1 o (R_2 U R_3)$ and $(R_1 o R_2) U (R_1 o R_3)$.	02
	1.10	Let $A = \{1,2,3,,4,5\}$ and $B = \{a, b, c, d, e, f\}$. How many functions are there from A to B? How many of these are one-to-one? How many are onto?	02
	1.11	Show that $(Z_6, +)$ is an abelian group.	02

PART-B

2	a	i) How many permutations are there for the eight letters a, c, f, g, i, t, w, x? How many of these start with the letter f and end with the letter w? ii) How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000?	05
	b	i) Determine the co-efficient of $w^3x^2yz^2$ in the expansion of $(2w - x + 3y - 2z)^8$. ii) Determine the sum of all the coefficients in the expansion of $(x + y + z)^{10}$.	06
	c	Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where $x_i \geq 0, 1 \leq i \leq 4$.	05
		OR	
3	a	Determine the number of positive integers, n $1 \leq n \leq 2000$, that are not divisible by 2,3,5 *or* 7.	06
	b	i) How many permutations of 1,2,3,4,5,6,7 are not derangements? ii) List the derangements of 1,2,3,4,5,6 where the first three numbers are 1,2,3 in some order.	04
	c	By using the expansion formula, find the rook polynomial for the board shown below:  Fig 3c	06
4	a	i) Prove the following by mathematical induction: $1^2 + 3^2 + 5^2 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots .. + (2n-1)^2 = (n(2n-1)(2n+1))/3$. ii) Evaluate the following: $\sum_{i=11}^{33} i^2$	06
	b	Find the general solution for the following recurrence relation: $2a_n - 3a_{n-1} = 0, n \geq 1, a_4 = 81$.	04
		Solve the recurrence relation: $a_n - 6a_{n-1} + 9a_{n-2} = 0,\ \ n \geq 2$, $a_0 = 5, a_1 = 12$.	06
		OR	
5	a	Solve the following non-homogenous recurrence relation: $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, n \geq 0, a_0 = 0, a_1 = 1$.	08
	b	Find the generating function for the recurrence relation: $a_{n+2} - 5a_{n+1} + 6a_n = 2, n \geq 0, a_o = 3, a_1 = 7$. Hence solve the relation.	08
6	a	If p, q are primitive statements, prove that $\sim(p \vee (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q$ by using laws of logic. Write the dual of the logical equivalence and show that the resulting statements are equivalent.	06
	b	For the implication "If today is thanksgiving, then tomorrow is Friday", give the converse, inverse and contra positive of the implication.	04

	c	Simplify the following electrical network:  Fig. 6c	06
		OR	
7	a	Establish the validity of the following argument: $P, P \rightarrow q, (s \vee r), \quad r \rightarrow \sim q \vdash s \vee t$	04
	b	For the universe of integers, let $p(x), q(x), r(x), s(x)$ and $t(x)$ be the following open statements: $P(x): x > 0, q(x): x$ is even, $r(x): x$ is a perfect square, $s(x): x$ is divisble by 4, $t(x): x\ is\ divislble\ by$ 5. Write the following statements in symbolic form: i) Atleast one integer is even; ii) If x is even, then x is not divisible by 5; iii) No even integer is divisible by 5; iv) If x is even and x is perfect square then x is divisible by 4.	04
	c	Explain the rule of universal generalization and universal specification. Using these rules establish the validity of the argument: $K_x\left(p(x) \rightarrow \left(q(x) \wedge r(x)\right)\right), K_x(p(x) \wedge s(x)) + K_x r(x) \wedge s(x))$.	08
8	a	For $A = \{1,2,3,4,5,6\}$, and $R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (4,5), (5,4), (5,5), (6,6)\}$ is a relation on A. Is R an equivalence relation? If yes, find $\frac{A}{R}$.	05
	b	Let $S = \{a, b, c\}$ and $P(s)$ be the power set of S. On $P(s)$, define the relation R by xRy if and only if $x \subseteq y$. Show that $(P(S), \subseteq)$ is a partial order on $P(S)$. Draw its Hasse diagram.	06
	c	Consider the relation represented by the diagraph shown below:  Fig 8c Write R as a collection of ordered pairs. Compute reflexive closure, symmetric closure and transitive closure of R.	05
		OR	
9	a	For each of the following functions: $f: R \rightarrow R$, determine whether the function is one-to-one and whether the function is onto. If the function is not onto, determine the range $f(R)$. i) $f(x) = x + 7$ ii) $f(x) = 2x - 3$ iii) $f(x) = x^2$ iv) $f(x) = x^2 + x$	04

	b	Prove that composition of functions is not commutative but composition of function is associative.	06
	c	Prove the following: i) A function $f: A \to B$ is invertible if and only if it is one to one and onto. ii) If $f: A \to B, g: B \to C$ are invertible functions then $g \circ f: A \to C$ is invertible and $(gof)^{-1} = f^{-1} \circ g^{-1}$.	06
10	a	Define the following : i) Semigroup ii) Monoid iii) Group iv) Abelian group.	04
	b	If f is a homomorphism from G_1 to G_2 and if f is onto, prove the following: i) If G_1 is abelian then G_2 is also abelian. ii) If e_1 is the identity of G_1 and e_2 is the identity of G_2, then we have $f(e) = e_2$. iii) $f(a^{-1}) = (f(a))^{-1}$ for all $a \in G_1$.	06
	c	State and prove Langrange's theorem.	06
		OR	
11	a	Let $E: Z_2^3 \to Z_2^9$ be the encoding function for the(9,3) triple repetition code. i) If $D: Z_2^9 \to Z_2^3$ is the corresponding decoding function, apply D to decode the received words 111101100; 000100011; 010011111. ii) Find three different received words r for which $D(r) = 000$. iii) For each $\omega \in Z_2^3$, what is $\|D^{-1}(w)\|$?	06
	b	For each of the following encoding functions, find the minimum distance between the code words. Discuss the error detecting and correcting capabilities of each code. i) $E: Z_2^2 \to Z_2^5$ m) $00 \to 00001; 01 \to 01010$ n) $11 \to 11111; 10 \to 10100.$ ii) $E: Z_2^2 \to Z_2^{10}$ m) $00 \to 0000000000, 01 \to 0000011111$ n) $10 \to 1111100000, 11 \to 1111111111$	04
	c	The encoding function $E: Z_2^2 \to Z_2^5$ is given by the generator matrix: $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i) Determine all code words. What can we say about the error detection capability of this code? What about error correction capability? ii) Find the associated parity check matrix H. iii) Use H to decode each of the following received words. m) 11011 n) 00110.	06