RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

Mathematics for Artificial Intelligence & Machine Learning (MA231TE)
Test 2 - Scheme and Solution

<table>
<tr><th>Sl. No.</th><th></th><th>M</th></tr>
<tr><td>1a</td><td>$\mu = 15, \lambda = \frac{1}{15}, \qquad f(x) = \frac{1}{15}e^{-\frac{x}{15}}$
(i) $P(X > 30) = \int_{30}^{\infty}\frac{1}{15}e^{-\frac{x}{15}}\,dx = -e^{-\frac{x}{15}}\Big|_{30}^{\infty} = e^{-2} = 0.1353$
(ii) $P(x < 10) = \int_{0}^{10}\frac{1}{15}e^{-\frac{x}{15}}\,dx = -e^{-\frac{x}{15}}\Big|_{0}^{10} = -e^{-\frac{2}{3}} + 1 = 0.4866$
(iii) $P(5 < x < 10) = \int_{5}^{10}\frac{1}{15}e^{-\frac{x}{15}}\,dx = -e^{-\frac{x}{15}}\Big|_{5}^{10} = -e^{-\frac{2}{3}} + e^{-\frac{1}{3}} = 0.2031$
(iv) $P(x < t) = 0.90 \Rightarrow \int_{0}^{t}\frac{1}{15}e^{-\frac{x}{15}}\,dx = -e^{-\frac{x}{15}}\Big|_{0}^{t} = 0.90 \Rightarrow -e^{-\frac{t}{15}} + 1 = 0.90 \Rightarrow e^{-\frac{t}{15}} = 0.1 \Rightarrow -\frac{t}{15} = \ln(0.1) \Rightarrow \mathrm{t} = 34.54$</td><td>1
1
2
2</td></tr>
<tr><td>1b</td><td>Poisson mean = 25 $logs/hr$
X-time in hours from the start of the interval until the first log-on.
$\lambda = 25\ logs/hr,\ f(x) = 25e^{-25x}$
(i)$P(x > 6\text{min}) = \mathrm{P}\left(\mathrm{x} > \frac{6}{60}hr\right) = P(x > 0.1\ hr) = \int_{0.1}^{\infty}25e^{-25x}dx = -e^{-25x}\big|_{0.1}^{\infty} = -0 + e^{-2.5} = 0.0821$
(ii) $P(2min < x < 3\text{min}) = P\left(\frac{2}{60hr} < x < \frac{3}{60}hr\right) = \int_{1/30}^{1/20}25e^{-25x}dx = -e^{-25x}\big|_{1/30}^{1/20} = -0.2865 + 0.4345 = 0.148$</td><td>1
1
2</td></tr>
<tr><td>2a</td><td>$\mu = 60\ kbs/sec,\ \sigma = 4\ kbs/sec,\ z = \frac{x-\mu}{\sigma}$
(i) $P(x \geq 70) - P\left(z > \frac{70-60}{4}\right) = P(z \geq 2.5) = 0.0062$
(ii) $P(x < 58) = P\left(z < \frac{58-60}{4}\right) = P(z < -0.5) = 0.3085$
(iii) average time $= \frac{8000\ kbs}{60\ kbs/sec} = 133.33\ sec$</td><td>1
1
1
1</td></tr>
<tr><td>2b</td><td>$\mu = 310\ mil.gal./day,\ \sigma = 45\ mil.gal./day$, storage capacity = 350 $mil.gal.$
(i) $P(x > 350) = P\left(z > \frac{350-310}{45}\right) = P(z > 0.8889) = 0.187$
(ii) $P(x > ?) = 0.01, P(z > ?) = 0.01 \Rightarrow z = 2.327 \Rightarrow \frac{x-310}{45} = 2.327 \Rightarrow x = 414.715$
(iii) $P(x > ?) = 0.95, P(z > ?) = 0.95 \Rightarrow z = -1.645 \Rightarrow \frac{x-310}{45} = -1.645 \Rightarrow x = 235.975$
(iv) $P(\mu > ?) = 0.01, P(z > ?) = 0.01 \Rightarrow z = 2.327 \Rightarrow \frac{350-\mu}{45} = 2.327 \Rightarrow \mu = 245.285$
Mean daily consumption per person is $\frac{245.285\ mil.gal.}{1.4\ mil.} = 175.20\ gal./person$</td><td>1
2
1
1
1</td></tr>
<tr><td>3a</td><td>(i)
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>0.2</td><td>0.25</td><td>0.55</td></tr></table><table><tr><td>Y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(Y)</td><td>0.28</td><td>0.25</td><td>0.17</td><td>0.30</td></tr></table>(ii) $P(x \geq 2) = 0.8,$
(iii) $P(Y < 2) = 0.28,$
(iv) $P(X > 2, Y > 2) = 0.1$</td><td>1
2
1
1
1</td></tr>
</table>

P.T.O.

<table>
<tr><td>3b</td><td>$2c+3c+4c+3c+4c+5c+4c+5c+6c=36c=1 \Rightarrow c=\frac{1}{36}$,
$E(X)=1\times 9c+2\times 12c+3\times 15c=\frac{78}{36}=\frac{13}{6}=2.1667$
$E(Y)=1\times 9c+2\times 12c+3\times 15c=\frac{78}{36}=\frac{13}{6}=2.1667$</td><td>2
1
1</td></tr>
<tr><td>4a</td><td>(i) $E(X)=\int_0^1\int_0^1\frac{2}{3}(x^2+2xy)\,\mathrm{dy\,dx}=\frac{2}{3}\int_0^1 x^2y+\frac{2xy^2}{2}\Big|_0^1 dx=\frac{2}{3}\int_0^1(x^2+x)dx=\frac{2}{3}\Big|\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^1=\frac{2}{3}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{9}=0.5556$
(ii) $E(Y)=\int_0^1\int_0^1\frac{2}{3}(xy+2y^2)\,\mathrm{dy\,dx}=\frac{2}{3}\int_0^1\frac{xy^2}{2}+\frac{2y^3}{3}\Big|_0^1 dx=\frac{2}{3}\int_0^1\left(\frac{x}{2}+\frac{2}{3}\right)dx=\frac{2}{3}\Big|\frac{x^2}{2}+\frac{2x}{3}\Big|_0^1=\frac{2}{3}\left(\frac{1}{4}+\frac{2}{3}\right)=\frac{11}{18}=0.6667$
(iii) $E(XY)=\int_0^1\int_0^1\frac{2}{3}(x^2y+2xy^2)\,\mathrm{dy\,dx}=\frac{2}{3}\int_0^1\frac{x^2y^2}{2}+\frac{2xy^3}{3}\Big|_0^1 dx=\frac{2}{3}\int_0^1\left(\frac{x^2}{2}+\frac{2x}{3}\right)dx=\frac{2}{3}\Big|\frac{x^3}{6}+\frac{2x^2}{6}\Big|_0^1=\frac{2}{3}\left(\frac{1}{6}+\frac{2}{6}\right)=\frac{1}{3}=0.3333$
$Cov[X,Y]=E(XY)-E(X)E(Y)=\frac{1}{3}-\frac{5}{9}\times\frac{11}{18}=-\frac{1}{162}=-0.0062$</td><td>2
2
2</td></tr>
<tr><td>4b</td><td>(i) $\int_{x=0}^{\infty}\int_{y=x}^{\infty}k\,e^{-0.001x-0.002y}dy\,dx=1\Rightarrow\int_{x=0}^{\infty}(\frac{-k}{0.002})e^{-0.001x-0.002y}dx\Big|_x^{\infty}=1\Rightarrow$
$\int_{x=0}^{\infty}\frac{k}{0.002}e^{-0.003x}dx=1\Rightarrow-\frac{k}{0.000006}e^{-0.003x}|_0^{\infty}=1\Rightarrow k=6\times10^{-6}$
$P(x<1000)=\int_{x=0}^{1000}\int_{y=x}^{\infty}6\times10^{-6}e^{-0.001x}e^{-0.002y}dy\,dx$
$=\int_{x=0}^{1000}-6\times10^{-6}e^{-0.001x}\frac{e^{-0.002y}}{0.002}\Big|_x^{\infty}dx$
$=\int_{x=0}^{1000}3\times10^{-3}e^{-0.003x}dx=-3\times10^{-3}\frac{e^{-0.003x}}{0.003}\Big|_0^{1000}=1-e^{-3}=0.9502$</td><td>2
2</td></tr>
<tr><td>5a</td><td>
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>0.1</td><td>0.35</td><td>0.55</td></tr></table>
<table><tr><td>Y</td><td>1</td><td>3</td><td>5</td></tr><tr><td>P(Y|X=2)</td><td>$\frac{0.05}{0.35}=\frac{1}{7}$</td><td>$\frac{0.10}{0.35}=\frac{2}{7}$</td><td>$\frac{0.20}{0.35}=\frac{4}{7}$</td></tr></table>
(ii)$E(Y|X=2)=\frac{1}{7}+\frac{6}{7}+\frac{20}{7}=\frac{27}{7}=3.8571,$
(iii)$Var(Y|X-2)=\frac{1}{7}+\frac{18}{7}+\frac{100}{7}-\left(\frac{27}{7}\right)^2=\frac{104}{49}=2.1228$</td><td>1
2
1
2</td></tr>
<tr><td>5b</td><td>$P_2(y)=\int_{x=0}^{y}10xy^2dx=\frac{10x^2y^2}{2}\Big|_0^y=5y^4$
$f(x|y)=\begin{cases}\frac{10xy^2}{5y^4}=\frac{2x}{y^2}, & 0<x<y<1\\ 0, & elsewhere\end{cases}$
$P\left(x>\frac{1}{2}\,\Big|\,y=0.5\right)=1-\int_0^{1/2}\frac{2x}{(0.5)^2}dx=1-\frac{x^2}{0.25}\Big|_0^{\frac{1}{2}}=1-4\left(\frac{1}{4}-0\right)=0$</td><td>1
1
2</td></tr>
</table>