RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Odd Semester 2023)

Date	**20th February 2024**	**Time**	**10:00 a.m. – 11:30 a.m.**	
TEST	**II**	**Maximum Marks**	**50**	
Course Title	**Mathematics for Artificial Intelligence & Machine Learning**	**Course Code**		**MA231TE**
Semester	**III**	**Programs**	**AIML**	

Instructions: i) Answer all questions.

Sl. No.	**Questions**	M	CO	BT
1a	The time between calls to a plumbing supply business is exponentially distributed with a mean time between calls of 15 minutes. What is the probability that (i) there are no calls within a 30-minute interval? (ii) at least one call arrives within a 10-minute interval? (iii) the first call arrives within 5 and 10 minutes after opening? (iv) Determine the length of an interval of time such that the probability of at least one call in the interval is 0.90.	06	1	2
1b	In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. If the same problem is treated as an Exponential process, what is the random variable X and the parameter λ, and hence find the probability that (i) there are no log-ons in an interval of six minutes? (ii) the time until the next log-on is between two and three minutes?	04	1	2
2a	The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of four kilobits per second. What is the probability that (i) the file will transfer at a speed of 70 kilobits per second or more? (ii) the file will transfer at a speed of less than 58 kilobits per second? (iii) If the file is one megabyte, what is the average time it will take to transfer the file? (Assume eight bits per byte.)	04	2	3
2b	The demand for water use in a city in 2003 hit a high of about 442 million gallons per day on June 27. Water use in the summer is normally distributed with a mean of 310 million gallons per day and a standard deviation of 45 million gallons per day. City reservoirs have a combined storage capacity of nearly 350 million gallons. (i) What is the probability that a day requires more water than is stored in city reservoirs? (ii) What reservoir capacity is needed so that the probability that it is exceeded is 1%? (iii) What amount of water use is exceeded with 95% probability? (iv) Water is provided to approximately 1.4 million people. What is the mean daily consumption per person at which the probability that the demand exceeds the current reservoir capacity is 1%? Assume that the standard deviation of demand remains the same.	06	4	4

3a	The response time is the speed of page downloads, and it is critical for a mobile website. Let X denote the number of bars of service, and let Y denote the response time (to the nearest second) for a particular user and site. <table><tr><td>$Y \downarrow X \rightarrow$</td><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>0.15</td><td>0.1</td><td>0.05</td></tr><tr><td>3</td><td>0.02</td><td>0.1</td><td>0.05</td></tr><tr><td>2</td><td>0.02</td><td>0.03</td><td>0.2</td></tr><tr><td>1</td><td>0.01</td><td>0.02</td><td>0.25</td></tr></table>Find (i) the marginal probability distributions of X and Y (ii) the probability that the number of bars of the service is at least 2, (iii) the response time is less than 2 seconds, (iv) both the number of bars of the service and the response time is greater than 2.	06	3	3
3b	Determine the value of c that makes the function f(x, y) = c(x + y) a joint probability mass function over the nine points with x = 1, 2, 3 and y = 1, 2, 3. Also find the expected values of X and Y.	04	3	3
4a	The function $f(x,y) = \frac{2}{3}(x+2y)$ is a joint probability density function over the range $0 \le x \le 1, 0 \le y \le 1$. Determine (i) $E[X]$, (ii) $E[Y]$, (iii) $Cov[X,Y]$.	06	3	4
4b	The random variable X denotes the time until a computer server connects to your machine (in milliseconds), and Y denotes the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. The joint probability density function for X and Y is given as $f(x,y) = k\, e^{-0.001x-0.002y}\ for\ 0 < x < y < \infty$. Determine the constant k and the probability that $x < 1000$.	04	4	4
5a	The joint probability distribution of two discrete random variables X and Y is given as below. Find (i) $P(Y\|X=2)$, (ii) $E(Y\|X=2)$, (iii) $Var(Y\|X=2)$. <table><tr><td>$Y \downarrow X \rightarrow$</td><td>1</td><td>2</td><td>3</td></tr><tr><td>1</td><td>0.05</td><td>0.05</td><td>0.10</td></tr><tr><td>3</td><td>0.05</td><td>0.10</td><td>0.35</td></tr><tr><td>5</td><td>0.00</td><td>0.20</td><td>0.10</td></tr></table>	06	2	3
5b	The joint density for the random variables (X,Y) is given as: $f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1 \\ 0, & elsewhere \end{cases}$. Find the conditional density $f(x\|y)$ and $P\left(x > \frac{1}{2} \middle\| y = 0.5\right)$.	04	2	2

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test Max Marks	10	14	16	10	-	16	20	14	-	-