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RV College of Engineering

Mysore Road, RV Vidyaniketan Post, Bengaluru - 560059, Karnataka India

Academic year 2023-2024 (Even Sem)

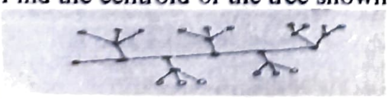
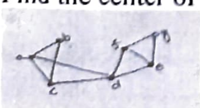
DEPARTMENT OF

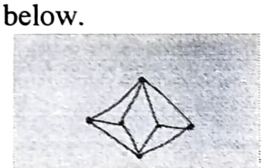
COMPUTER SCIENCE & ENGINEERING

Date	August 2024	Maximum Marks	60
Course Code	CS241AT	Duration	120 Min
Sem-IV	Improvement Test	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

(Common to CSE, ISE & AIML)

	PART-A	Marks	BT	CO
1.1	Let G be the set of real numbers not containing -1 and * be the binary operation defined by a*b=a+b+ab. What is the inverse of any number a $\in$ G.	1	1	1
1.2	If the binary operation * is associative, then complete the following table. <table><tr><td>*</td><td>a</td><td>b</td><td>c</td><td>d</td></tr><tr><td>a</td><td>a</td><td>b</td><td>c</td><td>d</td></tr><tr><td>b</td><td>b</td><td>a</td><td>c</td><td>d</td></tr><tr><td>c</td><td>c</td><td>d</td><td>c</td><td>d</td></tr><tr><td>d</td><td></td><td></td><td>c</td><td>d</td></tr></table>	1	1	1
1.3	Let G= $(Z_{12}, +)$ and H= {[0], [4], [8]}. What is the partition of G induced by the subgroup H.	1	3	2
1.4	A binary symmetric channel has probability p=0.05. What is the probability of sending the code word 110101101 and making at most 2 errors in the transmission?	1	2	2
1.5	Let E: $Z_2^3 \rightarrow Z_2^9$ be the encoding function for (9, 3) triple repetition code and D: $Z_2^9 \rightarrow Z_2^3$ is the corresponding decoding function. Find three different received words r for which D(r) =000.	1	2	1
1.6	Let G be the Peterson graph. Find $\chi(G)$.	1	2	2
1.7	What is the value of $\chi'(G)$ where G is $K_{3,2}$.?	1	2	3
1.8	If 5 colors are available, how many proper colorings are possible to color the graph $K_{3,3}$.	1	3	2
1.9	Find the centroid of the tree shown below. 	1	2	2
1.10	Find the center of the graph shown below. 	1	2	2

	PART-B			
2a.	In a group (G, *), prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$.	05	3	2
2b.	Show that $(Z_{12}, +)$ is a cyclic group and find all its generators.	05	4	3
3a.	Let G be a group and let a be any fixed element of G. Show that the function $f: G \rightarrow G$ defined by $f(x) = axa^{-1}$, for $x \in G$, is an isomorphism.	05	3	2
3b.	Let E: W□C be an encoding function with the set of messages $W \subseteq Z_2^m$ and the set of code words $E(W) = C \subseteq Z_2^n$, where m<n. For $k \in Z^+$, we can detect transmission errors of weight $\leq k$ iff the minimum distance between code words is at least k+1. Prove this.	05	2	3
4a.	Define the encoding function $E: Z_2^3 \rightarrow Z_2^6$ by means of the parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ i. Determine all code words. ii. What is the error detection and correction capability? iii. Decode the received words 000011, 111100.	06	2	2
4b.	**i.** If $x \in Z_2^{10}$, determine $\|S(x, 1)\|$, $\|S(x, 2)\|$, $\|S(x, 3)\|$. ii. For $n, k \in Z^+$ with $1 \leq k \leq n$, if $x \in Z_2^n$, what is $\|S(x, k)\|$?	04	3	2
5a.	Find $P(G, \lambda)$ for the graph shown below. 	06	3	4
5b.	Give an example of a connected graph that has i. Neither an Euler circuit nor a Hamilton cycle. ii. An Euler circuit but no Hamilton cycle. iii. No Euler circuit but has Hamilton cycle. iv. Both Euler circuit and a Hamilton cycle.	04	1	1
6a.	Prove that in every tree $\|V\| = \|E\| + 1$.	05	2	3
6b.	By applying the decomposition theorem, find the number of spanning trees for the graph shown below. 	05	3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5	L6
	Max Marks	7	26	16	11	-	6	22	27	5	-	-