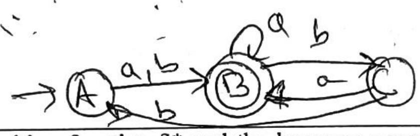
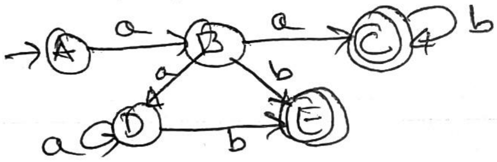
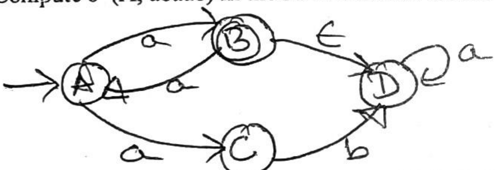
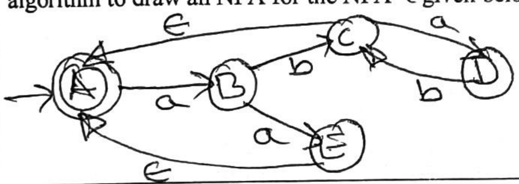
RV Educational Institutions ®
RV College of Engineering ®

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi | Approved by AICTE, New Delhi

Go, change the world

Academic year 2022-2023 (Odd Semester)

PART B

QNo.	Questions	M	BT	CO
1. a	Define DFA, the extended transition function δ* and the language accepted by DFA. Construct DFAs which generates the following languages over the alphabet $\Sigma=\{0, 1\}$. i. Set of all strings that do not end with 01. ii. Set of all strings that do not contain the substring 00.	07	L3	CO2
1. b	Find the language of the DFA shown below and compute δ*(A, ababb) and δ*(A, bbaaba). 	03	L2	CO1
2. a	Define NFA, the extended transition function δ* and the language accepted by NFA. Construct the NFA to accept the language $L=\{w \mid w \in \{a, b\}^*$ and w ends with ab or ba$\}$. Compute $\delta^*(q_0, bbaabba)$ where q_0 is the start state of the NFA constructed.	06	L2	CO1
2. b	Prove that for every NFA there exists a DFA accepting the same language. For the NFA shown below, using the subset construction draw the equivalent DFA. 	04	L3	CO2
3. a	Define NFA-ε, the extended transition function δ* and the language accepted by NFA-ε. Compute δ*(A, abaab) in the NFA-ε shown below. 	04	L2	CO2
3. b	Explain the algorithm to find an equivalent NFA from the given NFA- ϵ. Use this algorithm to draw an NFA for the NFA- ϵ given below. 	06	L1	CO1
4.a	Find all the subgroups of $(Z_{18}, +)$.	06	L4	CO3
4. b	Show that (U_{14}, x) is a cyclic group and find all its generators.	04	L2	CO1
5. a	Let $E: Z_2^3 \to Z_2^9$ be the encoding function for the (9, 3) triple repetition code. i. If $D: Z_2^9 \to Z_2^3$ is the corresponding decoding function, apply D to decode received words 111101100, 000100011, 010011111. ii. Find three different words r for which D(r)=000.	04	L2	CO2
5. b	The encoding function $E: Z_2^2 \to Z_2^5$ is given by the generator matrix $G=\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i. Determine all code words. What is the error detection and correction capability. ii. Find the associated parity check matrix H. iii. Use H to decode the received words 00111, 00110.	06	L4	CO3

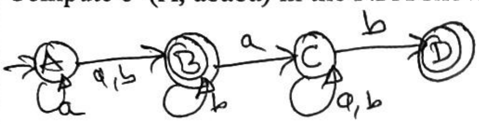
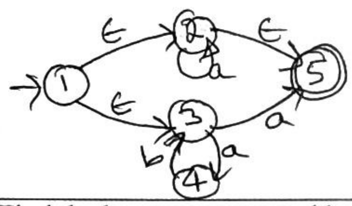
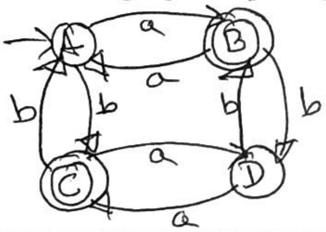
Go,

DMS 21CS

Academic year 2022-2023 (Odd Semester)

DEPARTMENT OF
CSE/ISE/AIML

Date	25^{th} Feb 2023	Maximum Marks	60
Course Code	21CS36	Duration	110 Mins
Sem	III	CIE - II	
DISCRETE MATHEMATICLA STRUCTURES			

<table>
<tr><th colspan="5">Part - A</th></tr>
<tr><th>Sl. No.</th><th>Questions</th><th>M</th><th>BT</th><th>CO</th></tr>
<tr><td>1</td><td>For the language L={ab, bc, a} over the alphabet $\sum$={a, b, c}, find L^3.</td><td>1</td><td>[illegible]</td><td>CO1</td></tr>
<tr><td>2</td><td>Let L_1 and L_2 are two languages over the alphabet $\sum$={a, b, c} as below.
Find $L_1 \cap L_2$. $L_1=\{a^n b^n c^m \mid n, m \geq 1\}$, $L_2=\{a^n b^m c^m \mid n, m \geq 1\}$.</td><td>[illegible]</td><td>[illegible]</td><td>CO1</td></tr>
<tr><td>3</td><td>Compute δ*(A, ababa) in the NFA shown below.
</td><td>1</td><td>L2</td><td>CO2</td></tr>
<tr><td>4</td><td>Find ε-closure ({1, 2, 3}) in the NFA-ε shown below.
</td><td>1</td><td>L2</td><td>CO2</td></tr>
<tr><td>5</td><td>Find the language accepted by the automat shown below.
</td><td>1</td><td>L1</td><td>CO2</td></tr>
<tr><td>6</td><td>If the binary operation * is associative, then complete the following table
<table>
<tr><td>*</td><td>a</td><td>b</td><td>c</td><td>d</td></tr>
<tr><td>a</td><td>a</td><td>b</td><td>c</td><td>d</td></tr>
<tr><td>b</td><td>b</td><td>a</td><td>c</td><td>d</td></tr>
<tr><td>c</td><td></td><td></td><td></td><td></td></tr>
<tr><td>d</td><td>d</td><td>c</td><td>c</td><td>d</td></tr>
</table></td><td>2</td><td>L3</td><td>CO2</td></tr>
<tr><td>7</td><td>If G is a group under the binary operation * then $(a*b)^{-1}$ maps to ----------- for all a, b ε G.</td><td>1</td><td>L2</td><td>CO1</td></tr>
<tr><td>8</td><td>For the following encoding function, find the minimum distance between the code words. What are the error detection and correction capabilities of these?
E: $Z_2^2 \rightarrow Z_2^{10}$
E(00)=0000000000, E(01)=0000011111, E(10)=1111100000,
E(11)=1111111111.</td><td>2</td><td>L2</td><td>CO3</td></tr>
</table>

Subject: DMS   CIE-II   Date: 25-02-2023
Sub. Code: 21CS36

Scheme and Solutions

Part-A:

1. L^3 = {aaa, aaba, abaa, abca, aaab, bcaa, ababa, abbca, bcaba, bcbca, aabab, aabbc, abcab, abcbc, abaab, ababc, ababab, ababbc, abbcab, abbcbc, bcaab, bcabc, bcabab, bcabbc, bcbcab, bcbcbc} —— 1 —— 1

2. $L_1 \cap L_2 = \{a^m b^m c^m \mid m \geq 1\}$

3. $\delta^*(A, ababa) = \delta^*(\delta(A,a), baba)$
$= \delta^*(\{A, B\}, baba)$
$= \delta^*(\delta(\{A,B\}, b), aba)$
$= \delta^*(\{B\}, aba)$
$= \delta^*(\delta(B,a), ba)$
$= \delta^*(\{C\}, ba)$
$= \delta^*(\delta(C,b), a)$
$= \delta^*(\{C,D\}, a)$ —— 1
$= \delta(\{C,D\}, a)$
$= \{C\}$

4. ϵ-closure$(\{1,2,3\})$
$= \epsilon$closure$(1) \cup \epsilon$-closure$(2) \cup \epsilon$-closure(3) —— 1
$= \{1,2,3,5\} \cup \{2,5\} \cup \{3\}$
$= \{1,2,3,5\}$

5. $L = \{w \mid w \in \{a,b\}^*$ and w has odd number of a's & even number of b's OR even no. of a's & odd number b's$\}$. —— 1

6.

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	$\boxed{c}$	$\boxed{d}$	$\boxed{c}$	$\boxed{d}$
d	d	c	c	d

——— 1

7. $\underline{b^{-1} * a^{-1}}$

8. Minimum distance between the code words = 5 ——— 2

It can detect all errors upto ≤ 4

It can correct all errors upto ≤ 2

Part - B :

1. a) Definition of DFA:

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA where

Q: finite set of states ——— 1

Σ: finite set of i/p symbols.

$\delta: Q \times \Sigma \rightarrow Q$

$q_0 \in Q$ the start state

$F \subseteq Q$ the set of final states

Definition of extended transition function of DFA.

$\delta^*: Q \times \Sigma^* \rightarrow Q$ as follows

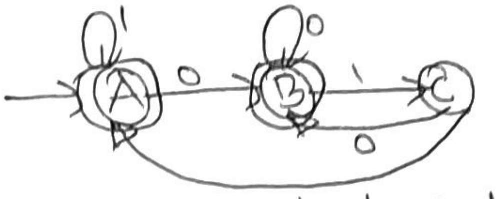
1. For any $q \in Q$ $\delta^*(q, \epsilon) = q$ ——— 1
2. For any $q \in Q$, $y \in \Sigma^*$, and $a \in \Sigma$

$\delta^*(q, ya) = \delta(\delta^*(q, y), a)$

Definition of language accepted by DFA.

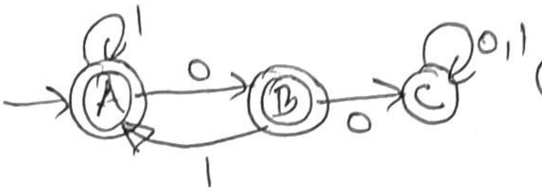
$L(M) = \{x \mid x \in \Sigma^* \text{ and } \delta^*(q_0, x) \in F\}$ ——— 1

i) DFA to accept set of all strings that do not end with 01



(or) any equivalent DFA ——— 2

ii) DFA to accept set of all strings that do not contain the substring 00.



(or) any equivalent DFA ——— 2

1.b) Language of the given DFA, M is

$L(M) = \{a, b, aa, aba, ba, baa, bba, aaa, abaa, abba, \ldots\}$ ——— 1

(or) any valid formal description

$\delta^*(A, ababb) = \delta^*(B, babb) = \delta^*(C, abb)$

$= \delta^*(B, bb) = \delta^*(C, b) = \delta(C, b) = A$ ——— 1

Since A is not final state the string ababb is not accepted by the given DFA ——— 1

2.a) Definition of NFA.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the NFA where ——— 1

Q: Finite set of states

Σ: finite set of i/p symbols.

$\delta : Q \times \Sigma \rightarrow 2^Q$

$q_0 \in Q$ the start state

$F \subseteq Q$ the set of final states.

Definition of extended transition function of NFA.

$\delta^* : Q \times \Sigma^* \rightarrow 2^Q$ is as follows ——— 1

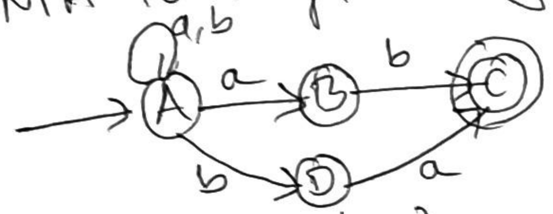
1. For any $q \in Q$, $\delta^*(q, \epsilon) = \{q\}$
2. For any $q \in Q$, $y \in \Sigma^*$ and $a \in \Sigma$

$$\delta^*(q, ya) = \bigcup_{r \in \delta^*(q, y)} \delta(r, a)$$

Definition of language accepted by NFA

$L(M) = \{ x \mid x \in \Sigma^* \text{ and } \delta^*(q_0, x) \cap F \neq \phi \}$

NFA to accept the given language is



(OR) Any equivalent NFA

$\delta^*(A, bbaabba)$

$= \delta^*(\delta(A,b), baabba)$

$= \delta^*(\{A,D\}, baabba)$

$= \delta^*(\delta(A,b) \cup \delta(D,b), aabba)$

$= \delta^*(\{A,D\} \cup \phi, aabba)$

$= \delta^*(\{A,D\}, aabba)$

$= \delta^*(\delta(\{A,D\},a), abba)$

$= \delta^*(\delta(A,a) \cup \delta(D,a), abba)$

$= \delta^*(\{A,B,C\}, abba) = \delta^*(\delta(\{A,B,C\},a), bba)$

$= \delta^*(\delta(A,a) \cup \delta(B,a) \cup \delta(C,a), bba)$

$= \delta^*(\{A,B\} \cup \phi \cup \phi, bba)$

$= \delta^*(\delta(\{A,B\},b), ba)$

$= \delta^*(\delta(A,b) \cup \delta(B,b), ba)$

$= \delta^*(\{A,D\} \cup \{C\}, ba)$

$= \delta^*(\delta(\{A,C,D\},b), a)$

$= \delta^*(\delta(A,a) \cup \delta(C,a) \cup \delta(D,a), a)$

$= \delta^*(\{A,B\} \cup \phi \cup \{C\}, a)$

$= \delta^*(\delta(\{A,B,C\}, a)$

$= \delta(A,a) \cup \delta(B,a) \cup \delta(C,a)$

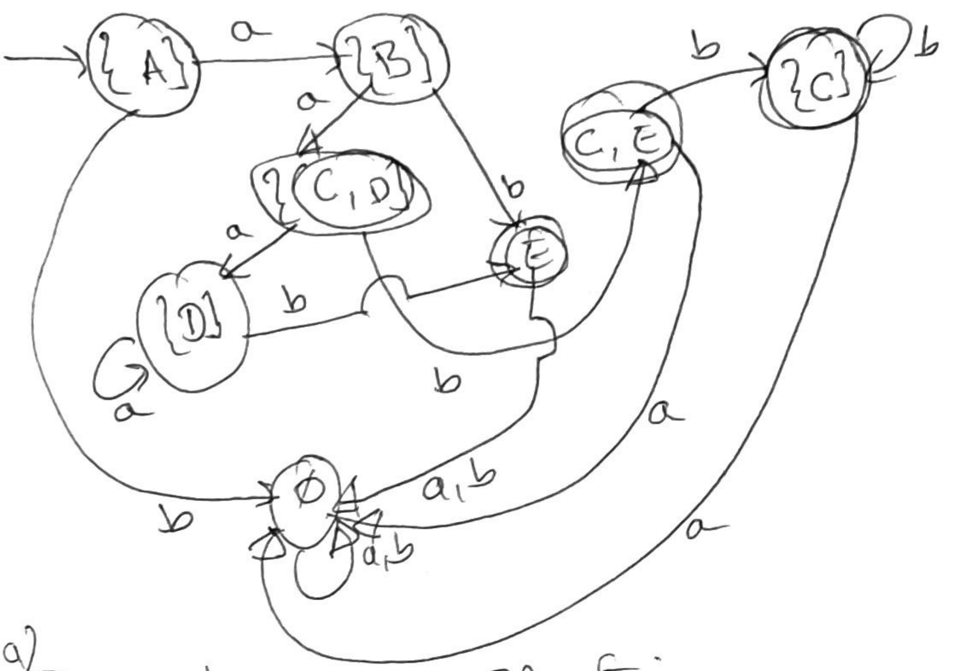
$= \{A,B\} \cup \phi \cup \phi = \{A,B\}$

Since the final state $C \in \{A,B\}$ the string not accepted.

2

List the steps to find the equivalent DFA from the given NFA — (2)

To draw the equivalent DFA for the NFA given — (2)



3.a) <u>Definition of NFA-ε:</u>

Let $M = (Q, \Sigma, \delta, q_0, F)$ where

Q: finite set of states

Σ: Finite set of input symbols — 1

$\delta: Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$

$q_0 \in Q$ the start state

$F \subseteq Q$ the set of final states.

<u>Definition of ~~extended~~ transition function δ^*</u>

$\delta^*: Q \times \Sigma^* \rightarrow 2^Q$ is as follows.

1. For any $q \in Q$, $\delta^*(q, \epsilon) = \epsilon\text{-closure}(\{q\})$. — 1
2. For any $q \in Q$, $y \in \Sigma^*$, & $a \in \Sigma$,

$$\delta^*(q, ya) = \epsilon\text{closure}\left(\bigcup_{r \in \delta^*(q, y)} \delta(r, a)\right)$$

<u>Language accepted by ENFA</u>

$L(M) = \{x \mid x \in \Sigma^* \text{ \& } \delta^*(q_0, x) \cap F \neq \emptyset\}$ — 1

$\delta^*(A, abaab) = \{B, D\}$ since B is final abaab is acc.

$\because$ ϵ-closure$(A) = \{A\}$

$\therefore \delta(A, a) = \{B, C\}$

ϵ-closure$\{B, C\} = \{B, C\}$

$$\delta(\{B, C\}, b) = \delta(B, b) \cup \delta(C, b)$$
$$= \phi \cup \{D\} = \{D\}$$

ϵ-closure$\{D\} = \{D, B\}$

$$\delta(\{D, B\}, a) = \delta(D, a) \cup \delta(B, a)$$
$$= \{D\} \cup \{A\} = \{A, D\}$$

ϵ-closure$(\{A, D\}) = \{A, B, D\}$

$$\delta(\{A, B, D\}, a) = \delta(A, a) \cup \delta(B, a) \cup \delta(D, a)$$
$$= \{B, C\} \cup \{A\} \cup \{D\}$$
$$= \{A, B, C, D\}$$

1

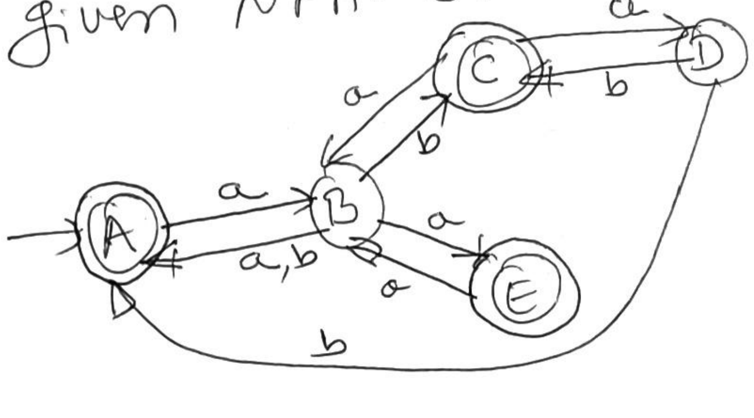
ϵ-closure$(\{A, B, C, D\}) = \{A, B, C, D\}$

$\delta(\{A, B, C, D\}, b) = \phi \cup \phi \cup \{D\} \cup \phi = \{D\}$

$\therefore$ ϵ-closure$(\{D\}) = \{B, D\}$ //

3.b) List all the steps to find equivalent NFA from the given NFA - ϵ. — 2

To draw the equivalent NFA for the given NFA - ϵ. — 4



+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0
2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1
3	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1	2
4	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1	2	3
5	5																	
6	6																	
7	7																	
8	8																	
9	9																	
10	10																	
11	11																	
12	12																	
13	13																	
14	14																	
15	15																	
16	16																	
17	17	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

The above table is optional students may write or not.

The subgroups of $(Z_{18}, +)$ are as below. $1+5 = 6$

1) $\{[2],[4],[6],[8],[10],[12],[14],[16],[0]\}$
2) $\{[3],[6],[9],[12],[15],[0]\}$
3) $\{[6],[12],[0]\}$
4) $\{[9],[0]\}$
5) $\{[0]\}$

4. b) The $\times$ is a multiplication modulo 14 in U_{14}.

$\therefore U_{14} = \{1, 3, 5, 9, 11, 13\}$

$\times$	1	3	5	9	11	13
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11	3	13	9
9	9	13	3	11	1	5
11	11	5	13	1	9	3
13	13	11	9	5	3	1

In U_{14}, $[3] = [3]^1$
$[3]^2 = [9]$
$[3]^3 = [27] = [13]$
$[3]^4 = [81] = [11]$
$[3]^5 = [243] = [5]$
$[3]^6 = [1]$

$\therefore (U_{14}, \times)$ is cyclic group and $[3]$ is the generator.

We can generate all the elements from $[5]$ also hence $[5]$ is a generator.

$[5]^1 = 5$, $[5]^2 = 11$, $[5]^3 = 13$, $[5]^4 = 9$, $[5]^5 = 3$, $[5]^6 = 1$

$1+1+1+1 = 4$.

5.a) i) ~~01010111~~ the decoded words.

D(111 101 100) = 101 — 2
D(000 100 011) = 000
D(010011 111) = 011 — 2

ii) 000 100 011
000 101 010
100 000010 etc.

5.b) i) The code words are — 2

$E(00) = [0\,0\,0\,0\,0]$, $E(10) = [1\,0\,1\,1\,0]$
$E(01) = [0\,1\,0\,1\,1]$, $E(11) = [1\,1\,1\,0\,1]$

The minimum weight is 3 hence errors upto weight ≤ 2 can be detected and single errors can be corrected. — 2

ii) Parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

iii) $H[0\,0\,1\,1\,1]^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ decoding not possible — 2

$H[0\,0\,1\,1\,0]^T = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ same as 1st column
$\therefore$ 10110 is code word.