RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	**CIE-II**	**Maximum marks: 50**
Course code: MAT231TC	**Third semester 2023-2024** **Branch: CS, CD, CY**	**Time: 10:00AM-11:30AM** **Date: 20-02-2024**

Instructions to candidates: *Answer all questions.*

Q.No.	QUESTIONS	M	BT	CO
1	Determine bases and dimension for row space, column space and null space of the matrix, $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.	10	1	1
2.a	Consider a triangle with vertices $(0,0), (1,1)$ and $(1,-1)$. Determine the matrix representation of the linear transformation that i) Shrinks the triangle by a factor 0.5 ii) Reflects the triangle about $y-$ axis iii) Rotates the triangle by $90°$. Represent each transformation geometrically.	6	3	3
2.b	Verify whether the given transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is linear. $T(x, y, z) = (2x + y,\ 2y - z,\ 2x - y + z)$.	4	2	2
3	Obtain the orthogonal projection of the vector $(1, 1, -1, -1)^T$ onto the column space of the matrix $A = \begin{bmatrix} 1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$. Given that the columns of A are linearly independent.	10	2	2
4.a	Suppose that the error in the reaction temperature, in °C, and pressure, in kPa, for a controlled laboratory experiment are modelled as continuous random variables X and Y, respectively, having the joint density function, $f(x, y) = \begin{cases} c(x^2 + y^2), & 0 \le x \le 1, \quad 0 \le y \le 1; \\ 0, & \text{otherwise.} \end{cases}$ Determine: (i) The constant c (ii) The marginal density functions of X and Y (iii) $P(X + Y \le 1)$.	6	2	2
4.b	Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assume that a plane makes a safe flight if at least one-half of its engines run, determine whether a 4-engine plane or a 2-engine plane has the higher probability for a successful flight.	4	2	2
5.a	At a certain corporate company, the arrival of messages in the complaint inbox can be modelled as a Poisson process. Assume that on the average there are 10 messages per hour. i. Determine the probability that there are more than one, but less than 10 messages arrive within a span of two hours. ii. If no messages arrive in the first 30 min, determine the probability that no messages will arrive in the next 10 min. iii. Determine the interval of time such that the probability that no messages arrive in the interval is 0.8.	6	3	3
5.b	The compressive strength of samples of cement can be modelled by a normal distribution with a mean of 6000 kilograms per square centimetre and a standard deviation of 100 kilograms per square centimetre. i. Determine the probability that a sample's strength is between 5800 and 5900 kg/cm^2 ii. Find out the strength that is exceeded by 95% of the samples	4	2	2

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