



DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	Improvement Test	Maximum marks: 50
Course code: MA231TC	Third semester 2023-2024 Branch: CS, CD, CY	Time: 10:00AM-11:30AM Date: 19-03-2024

Instructions to candidates: Answer all questions.

Q.No.	QUESTIONS	M	BT	CO
1	Obtain the singular value decomposition of the matrix $\begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix}$.	10	3	4
2.	Obtain the QR -factorization of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix}$.	10	3	3
3.a	Find the projection of the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ on to the space $W = \text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 7 \end{bmatrix} \right\}$. Interpret the answer.	4	2	2
3.b	The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 400 hours. i) Determine the probability that an assembly operates for more than 500 hours before failure. ii) If an assembly has been on test for 400 hours without a failure, obtain the probability of a failure in the next 100 hours. iii) If 10 assemblies are tested, find the probability that at least one fails in less than 100 hours. Assume that the assemblies fail independently.	6	3	3
4.a	A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed. i) If the office opens at 9:00 A.M. and the lawyer leaves his house at 8:45 A.M. daily, determine the percentage of the time he is late for work. ii) Find the length of time above which we find the slowest 15% of the trips.	5	2	2
4.b	The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\mu = 3.2$ minutes and a standard deviation $\sigma = 1.6$ minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's window is i) More than 3.5 minutes; ii) At least 3.2 minutes but less than 3.4 minutes.	5	1	1
5.a	According to a recent study, 17.5% of the adult population of Canada are smokers. Suppose a random sample of 200 adult Canadians is taken. Determine i) The mean and standard deviation of the sample proportion. ii) The probability that more than 20% of the adults in the sample are smokers. iii) The probability that between 34 and 44 of the adults in the sample are smokers.	6	2	2
5.b	Electric CFL manufactured by company A have mean lifetime of 2400 hours with standard deviation 200 hours, while CFL manufactured by company B have mean lifetime of 2200 hours with standard deviation of 100 hours. If random samples of 125 electric CFL of each company are tested, find i) The mean and standard error of the sampling distribution of the difference of mean lifetime of electric CFLs. ii) The probability that the CFLs of company A will have a mean lifetime at least 160 hours more than the mean lifetime of the CFLs of company B.	4	2	2
