RV Educational Institutions ®
RV College of Engineering ®

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi
Approved by AICTE, New Delhi

DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	**CIE-I**	**Maximum marks: 50**
Course code: MAT231CT	**Third semester 2023-2024** **Branch: CS, CD, CY**	**Time: 10:00AM-11:30AM** **Date: 08-01-2024**

Instructions to candidates:

Answer all questions.

<table>
<tr><th>Q.No</th><th>QUESTIONS</th><th>M</th><th>BT</th><th>CO</th></tr>
<tr><td>1</td><td>Let $\mathbb{R}^2, \mathbb{R}^3, P_4$ (set of all polynomials of degree 4 or less with real coefficients) and $M_{2\times 2}$ (set of all 2×2 real matrices) be vector spaces with usual addition and scalar multiplication. Verify whether the following sets forms a subspace or not. Justify your answer.
i) $S_1 = \left\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \middle| y = x^2\right\}$ of $\mathbb{R}^2$
ii) $S_2 = \left\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \middle| 2x = 3y\right\}$ of $\mathbb{R}^2$
iii) $S_3 = \left\{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \middle| x - 4y + 5z = 0\right\}$ of $\mathbb{R}^3$
iv) $S_4 = \{f(x) \in P_4 | f(1) \text{ is an integer}\}$
v) $S_5 = \{A \in M_{2\times 2} | det(A) \neq 0\}$.</td><td>10</td><td>2</td><td>2</td></tr>
<tr><td>2.a</td><td>Let $u = (1,3,2,1), v = (2,-2,-5,4), w = (2,-1,3,6)$ be vectors in $\mathbb{R}^4$. If possible express $t = (2,5,-4,0)$ as a linear combination of u, v and w.</td><td>5</td><td>2</td><td>2</td></tr>
<tr><td>2.b</td><td>Determine whether the set of vectors $S = \{1 + x - 2x^2, 2 + 5x - x^2, x + x^2\}$ in P_2 is linearly independent or linearly dependent.</td><td>5</td><td>2</td><td>2</td></tr>
<tr><td>3.a</td><td>Discrete random variable has the probability mass function as follows:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$p_X(x)$</td><td>0</td><td>k</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>$3k^2$</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr>
</table>
i) Determine the value of k
ii) Compute the probabilities $P(X \geq 5)$, $P(X < 3)$ and $P(2 < X \leq 5)$
iii) Find expectation of X.</td><td>6</td><td>1</td><td>1</td></tr>
<tr><td>3.b</td><td>A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability and cumulative distribution of X.</td><td>4</td><td>2</td><td>2</td></tr>
<tr><td>4.a</td><td>The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function
$$p(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & elsewhere. \end{cases}$$
Find the probability that over a period of one year, a family runs their vacuum cleaner
i) less than 120 hours;
ii) between 50 and 100 hours.</td><td>6</td><td>2</td><td>2</td></tr>
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<td>4.b</td>
<td>The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-8x}, & x \geq 0. \end{cases}$$

Find the probability density function and the probability of waiting less than 12 minutes between successive speeders.</td>
<td>4</td>
<td>2</td>
<td>2</td>
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<tr>
<td>5.a</td>
<td>Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their probability distribution is given as
<table>
<tr><td colspan="2" rowspan="2">$p(x,y)$</td><td colspan="3">x</td></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td rowspan="3">y</td><td>1</td><td>0.05</td><td>0.05</td><td>0.1</td></tr>
<tr><td>3</td><td>0.05</td><td>0.10</td><td>0.35</td></tr>
<tr><td>5</td><td>0.00</td><td>0.20</td><td>0.10</td></tr>
</table>
i) Evaluate the marginal distributions of X and Y
ii) Find $P(X > 1, Y \geq 3)$, $P(X < 3, Y = 3)$.
iii) Covariance of X and Y.</td>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5.b</td>
<td>A fair tetrahedral die (four faced die) is rolled twice. Let X denote the sum of two tosses less than 5 and let Y denote the maximum of the two tosses. Obtain the joint probability distribution function of X and Y.</td>
<td>4</td>
<td>2</td>
<td>2</td>
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