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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated to VTU

III Semester B. E. Examinations Nov/Dec-16

Common to CSE / ISE

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

3. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
4. Answer FIVE full questions from Part B.

PART-A

1	1.1	How many arrangements are there of all letters in $SOCIOLOGICAL$?	01
	1.2	Rewrite the following statement as an implication in if-then form "Soumya will be allowed on Suraj's motorcycle only if she wears helmet".	01
	1.3	Define an Injective function.	01
	1.4	Define group code.	01
	1.5	A student is to answer seven out of ten questions on an examination. In how many ways can he make his selection if: i) There are no restrictions ii) He must answer the first two questions.	02
	1.6	Let p(x) be the open statement "$x^2 = 2x$", where the universe comprise of all integers. Determine whether each of the following statements is true or false: a) $p(0)$ b) $p(1)$ c) $p(2)$ d) $\exists x p(x)$.	02
	1.7	If there are 2187 functions $f: A \rightarrow B$ and $\|B\| = 3$, what is $\|A\|$?	02
	1.8	Find a recurrence relation with initial condition, that uniquely determines the following geometric progression: $6, -18, 54, -162, \ldots$	02
	1.9	Determine whether $R = \{(x, y) \| x, y \in Z, y = x^2 + 7\}$, a relation from Z to Z, is a function. Find its range if it is a function.	02
	1.10	Let C be a set of code words, where $C \subseteq Z_2^7$. In the following , two of e(error pattern), r (received word) and c (code word) are given with $r = C + e$. Determine the third term: i) $c = 1010110, r = 1011111$ ii) $c = 1010110, e = 0101101$.	02
	1.11	Define cyclic group with an example.	02
	1.12	Show that $n^3 - n$ is divisible by 3, where n is positive integer.	02

PART-B

2	a	Three students write an examination. Their chances of passing are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, respectively. find the probability that: i) All of them pass ii) Atleast one of them pass iii) Atmost two of them pass.	06
	b	Consider the following program segment, where $i, j,$ and k are integer variables: $for\ i \leftarrow 1\ to\ 20\ do$ $for\ j \leftarrow 1\ to\ i\ do$ $for\ k \leftarrow 1\ to\ j\ do$ $print\ (i * j + k)$. How many times is the print statement executed in this program segment?	06
	c	Find and list all the dearrangements of 1,2 3,4.	04
		OR	
3	a	In a survey of 260 college students the following data were obtained: 64 had taken machine learning course, 94 had taken cloud computing, 58 had taken big data, 26 had taken both machine learning and cloud computing , 28 had taken machine learning and big data, 22 had taken cloud computing and big data and 14 had taken all the three courses. In the survey: i) How many students had taken none of the three courses? ii) How many had taken only cloud computing course?	06
	b	Define the cartesian product of two sets. For non empty sets A, B and C, prove that $A \times (B - C) = (A \times B) - (A \times C)$.	06
	c	Find the rook polynomial for the standard 8×8 chess board. State for $n \times n$ chess board too.	04
4	a	State the induction principle. Prove by induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.	06
	b	For the sequence $\{a_n\}$ defined recursively by $a_1 = 8, a_2 = 22$, $a_n = 4(a_{n-1} - a_{n-2})$ for $n \geq 3$, prove that $a_n = (5 + 3n)2^{n-1}$ for $n \geq 1$.	06
	c	Find a_{12} if $a_{n+1}^2 = 5a_n^2$.	04
		OR	
5	a	If $F_0, F_1, F_2,..$ are fibonacci numbers, prove that $\sum_{i=0}^{n} F_i = F_{n+2} - 1$.	06
	b	Solve the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2}$; $a_0 = 2, a_1 = 5$.	06
	c	Show that $2^n > n^2$ whenever n is a positive integer greater than 4.	04
6	a	Define the converse and the inverse of a conditional. State the converse and inverse of the following statement "If Suraj can solve then puzzle ten Suraj can solve the problem".	06
	b	Prove that $\big(p \to (q \vee r)\big) \leftrightarrow \big((p \wedge \sim q) \to r\big)$is a tautology.	06
	c	Establish the validity of the argument $(q \vee \sim r) \vee s$ $\underline{\sim q \vee (r \wedge \sim q)}$ therefore $r \to s$	04
		OR	

7	a	Test the validity of the argument: All employers pay their employees <u>Mukesh is an employer</u> therefore Anil pays his employees.	06
	b	Prove by contradiction that " if $n^2$is an odd integer then n is odd ".	06
	c	Prove that for all real numbers x and y if $x + y \geq 100$ then $x \geq 50$ or $y \geq 50$.	04
8	a	If $A = \{1,2,3,4\}$ and R is a relation on A defined by $R = \{(1,2),(1,3),(2,4),(3,2),(3,3),(3,4)\}$ find R^2 and R^3. Also draw their digraphs.	06
	b	Find the number of equivalence relations that can be defined on a finite set A with $\|A\| = 6$.	06
	c	Let $A = \{1,2,3,4\}$, f and g be functions from A to A given by: $f = \{(1,4),(2,1),(3,2),(4,3)\}$ $g = \{(1,2),(2,3),(3,4),(4,1)\}$ Prove that f and g are inverses of each other.	04
		OR	
9	a	Let $A = R, B = \{x \| x\ is\ real\ and\ x \geq 0\}$. Is the function $f: A \to B$ defined by $f(a) = a^2$ an onto function? a one to one function?	06
	b	Let $A = \{1,2,3,4,5,6,7\}$ and R be an equivalence relation on A that induces the partition: $A = \{1,2\} \cup \{3\} \cup \{4,5,7\} \cup \{6\}$. find R.	06
	c	Draw the Hasse diagram representing the positive divisors of 36.	04
10	a	The parity check matrix for an encoding function $E: Z_2^3 \to Z_2^6$ is given by $H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$. i) Determine the associated generator matrix. ii) Does this code correct all single errors in transmission?	06
	b	i) Define cyclic group. ii) Prove that the group $(Z_4, +)$ is cyclic. Find all its generators.	06
	c	State and prove the Lagrange's theorem.	04
		OR	
11	a	A binary symmetric channel has probability $p = 0.05$ of incorrect transmission. If the word $c = 011011101$ is transmitted, what is the probability that i) A single error occurs ii) A double error occurs iii) A triple error occurs.	06
	b	Show that any group G is abelian iff $(ab)^2 = a^2b^2$ for all $a, b \in G$.	06
	c	Prove that every subgroup of a cyclic group is cyclic.	04