Academic year 2023-2024 (Odd Semester 2023)

DEPARTMENT OF MATHEMATICS
Test 1- Scheme and Solution
Course Title: Mathematics for Artificial Intelligence & Machine Learning, Course Code:MAT231ET

Sl. No.	Answers	M
1a		

	1	2	3		M
$P(B_i)$	0.5	0.3	0.2	sum=1	1
$P(W\|B_i)$	0.75	0.8	0.85		
$P(W\|B_i)P(B_i)$	0.375	0.24	0.17	sum=0.785	1
$P(B_i\|W)$	$\frac{0.375}{0.785} = 0.4777$	$\frac{0.24}{0.785} = 0.3057$	$\frac{0.17}{0.785} = 0.2166$		

(i) $P(W) = 0.785$, (ii) $P(A|W) = 0.4777, P(B|W) = 0.3057, P(C|W) = 0.2166.$ — 1,3

1b

X	1	3	5	7	9
P(X)	k	2k	2k	3k	k

(i) $\sum P(X) = 1 \implies 9k = 1 \implies k = 1/9$, (ii) $P[1 < X \leq 7] = 7k = 7/9$ — 2, 2

2a

X	0	1	2
P(X)	0.7	0.2	0.1

(i) $E[X] = 1 \times 0.2 + 2 \times 0.1 = 0.4$, — 1

expectation of the company's daily loss $=0.4 \times 4000 = Rs. 1{,}600$ — 1

(ii) $Var[X] = E[X^2] - \{E[X]\}^2 = 1^2 \times 0.2 + 2^2 \times 0.1 - 0.4^2 = 0.44$, — 1

variance of the company's daily loss $=0.44 \times 4000 = Rs. 1{,}760$. — 1

2b

Given $f(x) = e^{-|x|}$, should have been $f(x) = \frac{1}{2}e^{-|x|}$

$$E[X] = \frac{1}{2}\left\{\int_{-\infty}^{0} xe^{x}dx + \int_{0}^{\infty} xe^{-x}dx\right\} = \frac{1}{2}\{xe^x - e^x|_{-\infty}^{0} + -xe^{-x} - e^{-x}|_0^{\infty}\} \quad 1$$

$$= \frac{1}{2}\big((0-1) - (0-0) + (-0-0) - (0-1)\big) = 0 \quad 1$$

$$Var[X] = E[X^2] - [E[X]]^2 = \frac{1}{2}\left\{\int_{-\infty}^{0} x^2e^{x}dx + \int_{0}^{\infty} x^2e^{-x}dx\right\} - 0^2 \quad 1$$

$$= \frac{1}{2}\{x^2e^x - 2xe^x + 2e^x|_{-\infty}^{0} + -x^2e^{-x} - 2xe^{-x} - 2e^{-x}|_0^{\infty}\} \quad 2$$

$$= \frac{1}{2}\big((0-0+2) - (0-0+0) + (-0-0-0) - (-0-0-2)\big) = 2 \quad 1$$

(Credit will be given if the mean(0) and variance(4) found using the given function

3a

$f(x) = k(1 - x^3)$ for $0 < x < 1$

(i) $\int_0^1 k(1-x^3)dx = 1 \implies k\left[x - \frac{x^4}{4}\right] = 1 \implies k\left[1 - \frac{1}{4}\right] \implies k = \frac{4}{3}$ — 2

(ii) $P\left(X < \frac{1}{2}\right) = \int_0^{1/2} \frac{4}{3}(1-x^3)dx = \frac{4}{3}\left[x - \frac{x^4}{4}\right]_0^{1/2} = \frac{4}{3}\left[\frac{1}{2} - \frac{1}{64}\right] = \frac{31}{48}$ — 2

(iii) $P\left(X > \frac{3}{4}\right) = \int_{3/4}^{1} \frac{4}{3}(1-x^3)dx = \frac{4}{3}\left[x - \frac{x^4}{4}\right]_{3/4}^{1} = \frac{4}{3}\left[\left(1 - \frac{1}{4}\right) - (\frac{3}{4} - \frac{81}{1024}\right] = \frac{27}{256}$ — 2

3b	$f(x)=\begin{cases}\frac{3}{x^4} & for\ \ x\geq 1\\ 0 & for\ x<1\end{cases}$. CDF=$\begin{cases}0, & t<1\\ \int_1^t \frac{3}{x^4}dx=-\frac{1}{x^3}\Big\vert_1^t=1-\frac{1}{t^3}, & t\geq 1\end{cases}$ $P(x>2)=1-P(x\leq 2)=1-F(x=2)=1-\left(1-\frac{1}{8}\right)=\frac{1}{8}$	1 2 1
4a	$p=0.25,q=0.75,n=20$ (i)$P(X\geq 4)=1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)]$ $=1-[(0.75)^{20}+20\times(0.25)\times(0.75)^{19}+190\times(0.25)^2(0.75)^{18}$ $+1140\times(0.25)^3\times(0.75)^{17}]$ $=0.7748$ (ii) $P(X=4)=4845\times(0.25)^4\times(0.75)^{16}=0.1897$ (iii) $P(X\leq 4)=P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$ $(0.75)^{20}+20\times(0.25)\times(0.75)^{19}+190\times(0.25)^2(0.75)^{18}\quad+1140\times(0.25)^3\times(0.75)^{17}$ $+4845\times(0.25)^4\times(0.75)^{16}=0.4149$	3 1 2
4b	$n=5,P(X=1)=0.2592,P(X=2)=0.3456$ $5pq^4=0.2592,10p^2q^3=0.3456\Rightarrow\frac{1}{2}\frac{q}{p}=0.75\Rightarrow q=\frac{3}{2}p$ $\Rightarrow p+\frac{3}{2}p=1\Rightarrow\frac{5}{2}p=1\Rightarrow p=\frac{2}{5},q=\frac{3}{5}$	2 2
5a	$\mu=10\,,f(x)=\frac{e^{-\mu}\mu^x}{x!}$ (i) $P(X>4)=1-P(X\leq 4)$ $=1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)]$ $=1-e^{-10}\left[1+\frac{10}{1}+\frac{10^2}{2}+\frac{10^3}{6}+\frac{10^4}{24}\right]=0.9707$ (ii) $P(X<6)=P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$ $=e^{-10}[1+\frac{10}{1}+\frac{10^2}{2}+\frac{10^3}{6}+\frac{10^4}{24}+\frac{10^5}{120}=0.0671$ (iii) $P(X=5)=\frac{e^{-10}10^5}{5!}=0.0378$	3 2 1
5b	$\mu\ =\ 0.3\times\ 12=3.6,\ f(x)=\frac{e^{-3.6}\,3.6^x}{x!}$ (i) $P(X\geq 3)=1-P(X<3)=1-[P(X=0)+P(X=1)+P(X=2)]$ $=1-e^{-3.6}\left[1+\frac{3.6}{1}+\frac{3.6^2}{2}\right]=0.6973$ (ii) $P(X\leq 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$ $=e^{-3.6}\left[1+\frac{3.6}{1}+\frac{3.6^2}{2}+\frac{3.6^3}{6}\right]=0.5152$	2 2