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DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	**Improvement Test**	**Maximum marks: 50**
Course code: MA231TC	**Third semester 2023-2024** **Branch: CS, CD, CY, IS**	**Time: 10:00AM-11:30AM** **Date: 20-03-2024**

SCHEME AND SOLUTION

Q.No	Solutions	Marks
1.	$AA^T = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$, Eigenvalues of AA^T are 90 and 0 $A^TA = \begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$ Eigenvalues of A^TA are 90, 0 and 0 Eigenvector of AA^T for 90 is $[-3 \quad 1]^T$ and for 0 is $[1 \quad 3]^T$ Eigenvector of A^TA for 90 is $[-1 \quad 2 \quad 2]^T$ and for 0 is $c_1[2 \quad 1 \quad 0]^T + c_2[2 \quad 0 \quad 1]^T$ Orthogonal eigenvectors of A^TA are $[-1 \quad 2 \quad 2]^T$, $[2 \quad 1 \quad 0]^T$ and $[-2 \quad 4 \quad -5]^T$ $U = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} \Sigma = \begin{bmatrix} \sqrt{90} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, V = \begin{bmatrix} -1/3 & 2/\sqrt{5} & -2/3\sqrt{5} \\ 2/3 & 1/\sqrt{5} & 4/3\sqrt{5} \\ 2/3 & 0 & -\sqrt{5}/3 \end{bmatrix}$	1+1 1+1 1+1 1+1 1 1
2.	$v_1 = [1 \quad -1 \quad 0 \quad 1]^T, v_2 = [3 \quad 1 \quad 2 \quad 1]^T, v_3 = [2 \quad 0 \quad 1 \quad 4]^T$ $u_1 = v_1, u_2 = v_2 - \frac{(u_1 \cdot v_2)}{u_1 \cdot u_1} u_1 = [2 \quad 2 \quad 2 \quad 0]^T$ $u_3 = v_3 - \frac{(u_1 \cdot v_3)}{u_1 \cdot u_1} u_1 - \frac{(u_2 \cdot v_3)}{u_2 \cdot u_2} u_3 = [-1 \quad 1 \quad 0 \quad 2]^T$ $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 & 2/\sqrt{6} \end{bmatrix}, R = Q^TA = \begin{bmatrix} \sqrt{3} & \sqrt{3} & 2\sqrt{3} \\ 0 & 2\sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{bmatrix}$	1+2 3 2+2
3.a	Let $u = [1 \quad 2 \quad 3]^T, v_1 = [1 \quad -1 \quad 1]^T, v_2 = [1 \quad 8 \quad 7]^T$ $\vec{p} = \frac{u \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{u \cdot v_2}{v_2 \cdot v_2} v_2 = \frac{2}{3} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $[1 \quad 2 \quad 3]^T$ is in W	2+1 1
3.b	Given $\mu = 400hr \Rightarrow \lambda = 1/400$. i) $P(X > 500) = e^{-\lambda 500} = e^{-500/400} = 0.2865$ ii) $P(X < 500 \mid X > 400) = P(X < 100) = 1 - e^{-\lambda 100} = 1 - e^{-\frac{100}{400}} = 0.2212$ iii) Required probability $= \sum_{x=1}^{10} b(x; p.n) = 1 - b(0; p, n) = 1 - (1-p)^{10} = 1 - e^{-\frac{10}{4}} = 0.9179$	2 2 2
4.a	Given $\mu = 24min$ and $\sigma = 3.8min$ i) $P(X > 15) = P(Z > -2.37) = 1 - \phi(-2.37) = 0.9911$ ii) $P(X > x) = 0.15 \Rightarrow P\left(Z > \frac{x-24}{3.8}\right) = 0.15 \Rightarrow P\left(Z < \frac{x-24}{3.8}\right) = 0.85 \Rightarrow \frac{x-24}{3.8} = 1.04$ or $x =$ 27.952min	2 3
4.b	Given $\mu = 3.2min,\ \sigma = 1.6min,\ n = 64$. Therefore, $\mu_{\bar{X}} = \mu = 3.2min,\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 0.2$. i) $P(\bar{X} > 3.5) = P\left(Z > \frac{3.5 - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right) = P(Z > 1.5) = 1 - \phi(1.5) = 0.0668$	1 2 2

	ii) $P(3.2 < \bar{X} < 3.4) = P(0 < Z < 1) = \phi(1) - 0.5 = 0.3413$	
5.a	Given $p = 0.175,\ n = 200.$ i) $\mu_{\hat{P}} = p = 0.175,\ \sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}} = 0.02687$ ii) $P(\hat{P} > 0.2) = P\left(Z > \frac{0.2-0.175}{0.02687}\right) = P(Z > 0.93) = 1 - \phi(0.93) = 0.1761$ iii) $P\left(\frac{34}{200} < X < \frac{44}{200}\right) = P(-0.186 < Z < 1.67) = 0.95254 - 0.424655 = 0.527885$	2 2 2
5.b	Population 1: $\mu_1 = 2400,\ \sigma_1 = 200,\ n_1 = 125$ Population 2: $\mu_2 = 2200,\ \sigma_2 = 100,\ n_2 = 125$ i) $\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2 = 200$ and $\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 20.$ ii) $P(\bar{X}_1 - \bar{X}_2 > 160) = P\left(Z > \frac{160-200}{20}\right) = 1 - \phi(-2) = 0.97725.$	2 2