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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated to VTU

IIISemester B. E. Fast Track ExaminationsJuly-18

Computer Science and Engineering

DISCRETE MATHEMATICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

3. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
4. Answer FIVE full questions from Part B.In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART A

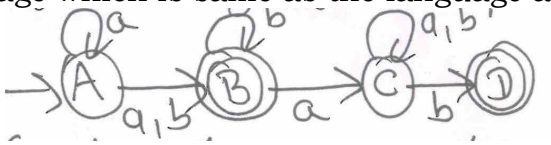
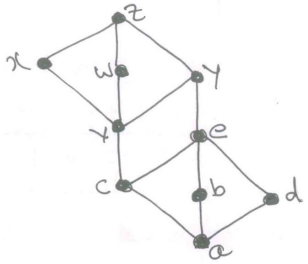
1	1.1	How many permutations of the eight letters a, c, f, g, i, t, w, x which starts with t and ends with c.	02
	1.2	how many arrangements of the letters in $MISSISSIPPI$ have no consecutive S's.	01
	1.3	Determine the co-efficient of $W^3X^2YZ^2$ in$(2W - X + 3Y - 2Z)^8$	02
	1.4	Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, where $x_1, x_2 \geq 5, x_3, x_4 \geq 7$.	02
	1.5	If p,qare primitive statements write the dual of the logical equivalence $(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow (p \wedge q)$	01
	1.6	Let P(x) be the open statement "$x^2 = 2x$" where the universe comprises all integers. Determine whether the following statements are true or false. a. p(2) b. fxp(x) c. $\forall x P(X)$ d. $P(-2)$.	02
	1.7	Determine the sets A,B where $A - B = \{1,2,4\}\, B - A = \{7,8\}$ and $A \cup B = \{1,2,4,5,7,8,9\}$.	02
	1.8	For each of the following functions $g: R \longrightarrow R$, determine whether the function is one to one and whether it is onto. If the function is not onto, determine the rage $g(R)$. i) $g(x) = 2x - 3$ ii) $g(x) = x^3$ iii) $g(x) = x^2$ iv) $g(x) = x^2 + x$	02
	1.9	For $A = \{1,2,3,4\}$, let $R = \{(1,1), (1,2), (2,3), (3,3), (3,4)\}$ be a relation on A. draw the directed graph G on A that is associated with R^2 and R^4.	02
	1.10	Determine whether the following collection of sets is partition for the given set A, if not explain why it fails to be. $A = \{1,2,3,4,5,6,7,8\};$ $A_1 = \{4,5\}$ $A_2 = \{1,3,4\}$ $A_3 = \{6,8\}$ $A_4 = \{2,7\}$	01

	1.11	Let C be a set of code words, where $C \subseteq Z_2^7$.In each of the following, two of e(error pattern), r(received word) and c(code word) are given , with $r = c + e$. Determine the third term. $C = 1010110, r = 1011111$.	01
	1.12	Let G be the group of complex numbers $\{1, -1, i, -i\}$under multiplication. Give its multiplication table. Show that it is a cyclic group.	02

PART B

2	a	i) How many arrangements are there of all letters in SOCIOLOGICAL? ii) In how many of the arrangements in part (i) are A and G adjacent? iii) In how many arrangements in part(i) are all vowels adjacent?	06
	b	What is the co-efficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$	04
	c	Find the number of integers between 1 and 1000 (both inclusive) that are divisible by none of 5,6 *and* 8.	06
3	a	Solve the recurrence relation. $2a_{n+2} - 11a_{n+1} + 5a_n = 0, \quad n \geq 0, a_0 = 2, a_{1=} - 8$	06
	b	Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200, \quad n \geq 0, a_0 = 3000, a_{1=}3300$	06
	c	Prove the following by mathematical induction. $1.3 + 2.4 + 3.5 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . n.(n+2) = \frac{n(n+1)(2n+7)}{6}$	04
		OR	
4	a	Define the following i) The rule of universal specification. ii) The rule of universal generalization.	04
	b	Provide the reasons for the steps verifying the following argument. $\forall x[p(x) \rightarrow (q(x) \wedge r(x))]$, $\forall x[p(x)] \wedge s(x)]$, $\therefore \forall x[r(x)] \wedge s(x)$	06
	c	Prove that the following argument is valid: "If the train arrives late and there are no trains at the station then John is late his meeting: John is not late for his meeting. The train did arrive late. Therefore there were taxis at the station".	06

	Steps	Reasons
1	$\forall x[p(x) \rightarrow (q(x) \wedge r(x))]$	
2	$\forall x[p(x)] \wedge s(x)]$	
3	$p(a) \rightarrow (q(a) \wedge r(a))]$	
4	$p(a) \wedge s(a)$	
5	$p(a)$	
6	$q(a) \wedge r(a)$	
7	$r(a)$	
8	$s(a)$	
9	$r(a) \wedge s(a)$	
10	$\therefore \forall x[r(x)] \wedge s(x)$	

5	a	Define the following i) DFA ii) Language of NFA iii) $\in$-closure (q), where $q \in Q$ of an automation.	06
	b	Give DFA's accepting the following languages over the alphabet $\sum = \{a, b\}$ i) The language of all strings that do not end with ab. ii) The language of all strings in which the number of a's is even.	10
		OR	
6	a	Consider the NFA below, using the lazy evaluation method, draw the DFA accepting the language which is same as the language accepted by NFA.  Fig . 6(a)	08
	b	Construct NFA-$\in$ to accept strings over $\sum = \{a, b, c\}$ such that the string contains any number of a's followed by any number of b's followed by nay number of c's. Convert this NFA-$\in$ to NFA.	08
7	a	For $A = \{1,2,3,4\}$,Let R and S be the relations on A desgined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$ and $S = \{(1,1), (1,2), (1,3), (2,3), (2,4)\}$ Find RoS, SoR, R^2, R^3, S^2 *and* S^3.	06
	b	Let $A = \{1,2,3,4,5,6,7\}$*and* $B = \{v, w, x, y, z\}$. Determine the number of onto functions $f: A \to B, where$ i) $f(A) = \{v, x\}$ ii) $f(A) = \{w, x, y\}$ iii) $\lvert f(A)\rvert = 2$ iv) $\lvert f(A)\rvert = 4$	06
	c	For $A = \{a, b, c, d, v, w, x, y, z\}$, consider the poset(A,R)whose Hasse diagram is shown below. Find glb{b,w}, lub{d,x}, least and greatest elements.  Fig. 7 (c)	04
8	a	Define the binary operation on Z by $xoy = x + y + 1$. Verify that $(Z, o)$is an abelian group.	08
	B	The encoding function $E: Z_2^2 \to Z_2^5$ is given by the generator matrix. $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i. Determine all code words. What can we say about the error detection capability of this code? What about error correction capability? ii. Find the associated parity check Matrix H. iii. Use H to decode each of the following received words. I. 11011 II. 10101 III. 11110	08