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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

III Semester B. E. Examinations Apr-2024

Artificial Intelligence and Machine Learning

MATHEMATICS FOR ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of handbook for Mathematics permitted.

PART-A

			M	BT	CO
1	1.1	A computer assembling company receives 45% of parts from supplier A and 55% of parts from supplier B. 5% of parts supplied by A and 4% supplied by B are defective. Then the probability of receiving a defective part is _________.	02	2	2
	1.2	Suppose the number of errors in a new software has expectation 20 and standard deviation 4. According to the Chebyshev inequality, the upper bound on the probability that there are more than 25 errors is _________.	02	2	2
	1.3	An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 25% of the sites contain the keyword. The probability that the search engine has to visit at least 4 sites in order to find the first occurrence of the keyword is _________.	02	1	1
	1.4	The amount of time, in minutes, that a person must wait for a metro is uniformly distributed between 0 and 12 minutes, inclusive. Find the 90th percentile.	02	2	2
	1.5	Determine the value of c that makes the function $f(x,y) = c(x+y)$ a joint probability mass function over the points, $x = 1,2,3$ and $y = 1,2,3$.	02	1	1
	1.6	If the joint probability density function over the range $0 < x < 2$ and $0 < y < x$ is given by $f(x,y) = \frac{xy}{2}$, then the conditional probability distribution of Y given $X = 1$ is ____.	02	3	3
	1.7	The value of k such that the vectors $2+3t+t^2, 4+3t+kt^2, 1+2t+2t^2$ are linearly dependent is _________.	02	2	2
	1.8	The 2×2 rotation matrix through an angle of 30^0 clockwise is _________, and the vector $(2,1)$ rotated through an angle of 30^0 clockwise is ____.	02	1	1
	1.9	Given $y = (1,2,1)$ and $u_1 = (1,1,0), u_2 = (1,-1,0)$, then the orthogonal projection of y onto the space spanned by the orthogonal vectors $\{u_1, u_2\}$ is _________.	02	2	2
	1.10	If the sum and product of the eigen values of $A = \begin{bmatrix} a & 2 \\ 0 & b \end{bmatrix}$ are 2 and -8 respectively, then the matrix is _________.	02	3	3

PART-B

2	a	For the probability mass function X: −2, −1, 0, 1, 2 $P(X)$: 1/8, 2/8, 2/8, 2/8, 1/8 Determine: i) $P(X \leq 0), P(X > -2), P(-1 \leq X \leq 1), P(X < -1)$	04	1	1
	b	The number of messages sent per hour over a computer network has the following distribution: $x = number\ of\ messages$: 10, 11, 12, 13, 14, 15 $probabilities$: 0.08, 0.15, 0.30, 0.20, 0.20, 0.07 Determine the mean and standard deviation of the number of messages sent per hour.	04	2	2
	c	The lifetime, in years, of some electronic component is a continuous random variable with the density $f(x) = \begin{cases} \frac{3}{x^4}, & x \geq 1 \\ 0, & otherwise \end{cases}$ Find: i) probability for the lifetime to exceed 2 years, ii) probability that the lifetime is at most 3 years, iii) probability that the lifetime is between 2 years and 3 years, iv) the cumulative distribution function.	08	3	3
3	a	A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector randomly picks 20 items from a shipment. What is the probability that i) there will be at least one defective, ii) the number of defectives is between 3 and 5(both included).	04	2	2
	b	Suppose that the time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$. i) What proportion of the fans will last at least 10,000 hours?, ii) what proportion of the fans will last at most 7,000 hours?, iii) Determine the time to failure such that the probability of failure of fans before x hours is 0.75.	06	3	3
	c	The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes. i) what is the probability that a battery lasts more than 4 hours, ii) what is the probability that the battery lasts between 2 hours and 3 hours? **OR**	06	4	4
4	a	The probability that a patient recovers from a rare blood disease is 0.4. If 10 people are known to have contracted this disease, what is i) the probability that more than 8 survive, ii) the mean and variance?	04	2	2

	b	The number of surface flaws in plastic panels used in the interior of automobiles has a Poisson distribution with a mean of 0.05 flaws per square foot of plastic panel. Assume an automobile interior contains 10 square feet of plastic panel. i) what is the probability that there are no surface flaws in the auto's interior?, ii) If 10 cars are sold to a rental company, what is the probability that none of the 10 cars has any surface flaws?, iii) If 10 cars are sold to a rental company, what is the probability that at most one car has any surface flaws?	06	3	3
	c	The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer. i) what is the probability that a line width is greater than 0.62 micrometer?, ii) the line width of 90% of samples is below what value?	06	4	4
5	a	Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, their joint probability distribution is given as $p(0,0)=\frac{3}{28}, p(0,1)=\frac{9}{28}, p(0,2)=\frac{3}{28}, p(1,0)=\frac{3}{14}, p(1,1)=\frac{3}{14}, p(1,2)=0,$ $p(2,0)=\frac{1}{28}, p(2,1)=0, p(2,2)=0$. Find: i) the marginal distributions of X and Y, ii) the expected values of X, Y and XY, the covariance of X and Y, iii) $P(Y<2\|X=1)$.	08	3	3
	b	Determine the value of c that makes the function $f(x,y)=ce^{-2x-3y}$ is a joint probability density function over the range $x>0$ and $0<y<x$. Also determine i) $P(y>1)$, ii) $P(x<1, y<2)$.	08	4	4
		OR			
6	a	If the joint probability mass function of X and Y is given by $p(x,y)=\frac{x+y}{30}, for\ x=0,1,2,3; y=0,1,2,$ construct the joint distribution table and hence find: i) $P(X\le 2, Y=1)$, ii) $P(X>Y)$, iii) $P(X\|Y=0)$.	08	3	3
	b	A popular clothing manufacturer receives internet orders via two different routing systems, operating independently. The time between orders for each routing system in a typical day is given by $f(x,y)=\lambda^2 e^{-\lambda(x+y)}, where\ \lambda=\frac{1}{3.2}$. what is i) the probability that no orders will be received in a 5-minute period?(i.e., $P(x>5, y>5)$, ii) the probability that an order will be received in a 10-minute period?, iii) the probability that both systems receive an order between 3.2 minutes and 6.4 minutes.	08	4	4

7	a	Show that W, the set of all 2×2 upper triangular matrices is a subspace of M_{2x2}, the set of all 2×2 matrices, over the Field $\mathbb{R}$.	04	2	2
	b	Find the basis and dimension of the row space and null space of the matrix $A = \begin{bmatrix} 1 & 1 & -2 & 1 \\ 3 & 1 & 2 & 4 \\ 5 & 3 & -2 & 6 \end{bmatrix}$.	06	3	3
	c	Find the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T(1,1,1) = (-1,1,0), T(1,2,-1) = (-3,5,2), T(2,-1,2) = (4,-4,0)$. Also find the basis and dimension of the range space.	06	3	3
		OR			
8	a	Show that the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (x\,cos\theta - y\,sin\theta,\ x\,sin\theta + y\,cos\theta)$ is a linear transformation.	04	2	2
	b	Find the basis and dimension of the column space and left null space of the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 2 & 6 \\ 2 & -1 & 3 \\ -1 & 1 & -1 \end{bmatrix}$.	06	3	3
	c	Show that the vectors $1 + t + 2t^2 - t^3, 2 + 3t + t^2 + t^3$, $2 + 4t + 4t^2 - 2t^3, 3 + 5t + 3t^2$ are linearly dependent. Extract the linearly independent subset and hence find the basis and dimension of the subspace spanned by them.	06	3	3
9	a	Applying the Gram-Schmidt process, obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 3 \\ 1 & 3 \\ 1 & 3 \\ -1 & -2 \\ -1 & 1 \end{bmatrix}$.	08	4	4
	b	Using the process of diagonalization decompose the matrix A as PDP^{-1}, where $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix}$.	08	3	3
		OR			
10	a	Obtain the orthogonal basis for the column space of the matrix $A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$	06	4	4
	b	Decompose the matrix A as $U\Sigma V^T$, using the singular value decomposition process, where $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	10	3	3