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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated toVTU

III Semester B. E. Examinations Nov/Dec-17

Computer Science and Engineering

DISCRETE MATHEMATICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

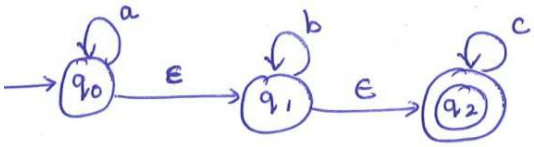
7. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
8. Answer FIVE full questions from Part B.In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	A six faced die is tossed four times and the numbers shown are recorded in a sequence. How many different sequences are there?	01
	1.2	Find the value of n, where $P(n,3) = 3P(n,2)$	02
	1.3	A box contains 15 IC chips of which 7 are defective and 8 are non-defective. In how many ways 5 chips can be chosen so that all are non-defective?	01
	1.4	Obtain a recursive definition for the sequence $a_n = 2 - (-1)^n$.	02
	1.5	Consider the following open statements with the set of all real numbers as the universe $p(x): \|x\| > 3, q(x): x > 3$. Find the truth value of the statement $\forall x, [p(x) \rightarrow q(x)]$	02
	1.6	Define $\in -NFA$.	02
	1.7	Obtain an NFA to accept strings of $0's$ and $1's$ such that left most symbol is different fromthe right most symbol.	02
	1.8	Let $A = \{1,2,3,4,5,6,7\}$ and $B = \{w,x,y,z\}$. Find the number of onto functions from A to B.	02
	1.9	Define Equivalence Relation with an example.	02
	1.10	Show that $(W_4,\times)$ is an abelian group where $W_4 = \{1,-1,i,-i\}$.	02
	1.11	A binary symmetric channel has probability $p = 0.05$ of incorrect transmission. If the word $c = 011011101$ is transmitted, what is the probability that triple error occurs.	02

PART-B

2	a	Find the number of arrangements of all the letters in $TALLAHASSEE$. How many of these arrangements have no adjacent $A's$?	04
	b	In how many ways can 10 identical pencils be distributed among 5 children in the following cases: i) There are no restrictions ii) Each child gets at least one pencil iii) The youngest child gets at least two pencils.	04

	c	Find the number of permutations of the English letters which contain i) Exactly two ii) At least two iii) Exactly three iv) At least three of the patterns CAR, DOG, PUN and $BYTE$.	08
3	a	Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 7^n$, for $n \geq 1$ given that $a_0 = 2$.	04
	b	Solve the recurrence relation $a_{n+2} - 4\,a_{n+1} + 3a_n = -200$, $n \geq 0$ $a_o = 3000$, and $a_1 = 3300$.	06
	c	A bank pays a certain percentage of annual interest on deposits, compounding the interest once in 3 months. If a deposit doubles in 6 years and 6 months, what is the annual percentage of interest paid by the bank?	06
		OR	
4	a	Prove that the following argument is valid, $\forall x, [p(x) \vee q(x)]$ $\exists x, \neg p(x)$ $\forall x, [\neg q(x) \vee r(x)]$ $\forall x, [s(x) \rightarrow \neg r(x)]$ _____ $\therefore \exists x, \neg s(x)$	06
	b	Prove that $R \rightarrow S$ is a valid conclusion from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q.	06
	c	Let m and n be integers, prove that $n^2 = m^2$ if and only if $m = n$ or $m = -n$.	04
5	a	Construct a minimal DFA which accepts set of all strings over {0,1,2} which when interpreted as a binary number is divisible by 4.	04
	b	Construct a NFA for strings $\{a, b\}$ in which 3^{rd} symbol from RHS is 'a'. Also convert the constructed NFA to DFA.	12
		OR	
6	a	Define the following with example: i) Languages accepted by DFA ii) Transition function of a DFA iii) DFA iv) Languages accepted by NFA.	08
	b	Convert the given $\in -NFA$ *to* DFA. (hint: convert $\in -NFA$ to NFA and then convert NFA to DFA).  	08
7	a	Write down the Hasse diagram for the positive divisors of 45.	04

	b	let $A = \{a, b, c\}$ and R and S be relations on A whose matrices are given as $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; $M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find the composite relations $R \circ S, S \circ R, R \circ R, S \circ S$ and their matrices.	06
	c	i) Let $A = \{1,2,3,4\}$ and f and g be functions from A to A given by $f = \{(1,4), (2,1), (3,2), (4,3)\}$ and $g = \{(1,2), (2,3), (3,4), (4,1)\}$. Prove that f and g are inverse of each other. ii) Let f, g, h be function from Z to Z defined by $f(x) = x - 1, g(x) = 3x$ $h(x) = \begin{cases} 0, & if\ x\ is\ even \\ 1, & if\ x\ is\ odd \end{cases}$ determine $(f \circ (g \circ h))(x)$and $((f \circ g) \circ h)(x)$.	06
8	a	State and prove Lagrange's theorem.	04
	b	Prove that $(Z_S^\star, \cdot)$ is a cycle group. Fond all its generators.	04
	c	The generator matrix for an encoding function $E: Z_2^3 \to Z_2^6$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ i) Find the code words assigned to 110 and 010 ii) Obtain the associate parity-check matrix iii) Hence decode the received words: $110110, 111101$ iv) Show that the decoding of 111111 is not possible by using H	08