RV College of Engineering
DEPARTMENT OF MATHEMATICS
Academic year 2023-2024 (Odd Semester 2023)

DEPARTMENT OF MATHEMATICS

Improvement Test - Scheme and Solution

Course Title: Mathematics for Artificial Intelligence & Machine Learning, Course Code: MAT231ET

Sl. No.	Solution	M
1a	Given $V=\{(x,y)\mid x,y\in\mathbb{R}\}$ Let $\alpha=(x_1,y_1),\beta=(x_2,y_2)\in V$ and $c,c'\in\mathbb{Q}$ (i)$(\alpha+\beta)+\gamma=\big((x_1+x_2)+x_3,(y_1+y_2)+y_3\big)=\big(x_1+(x_2+x_3),y_1+(y_2+y_3)\big)=\alpha+(\beta+\gamma)$, (ii) $\alpha+0=(x_1,y_1)+(0,0)=\alpha=0+\alpha,\therefore$ (0,0) is the zero vector, (iii) $\alpha+(-\alpha)=(x_1,y_1)+(-x_1,-y_1)=(0,0)=(-\alpha)+\alpha,\therefore -\alpha$ is the inverse of α, (iv) $\alpha+\beta=(x_1+x_2,y_1+y_2)=(x_2+x_1,y_2+y_1)=\beta+\alpha$, (v) $c\cdot(\alpha+\beta)=(cx_1+cx_2,cy_1+cy_2)=c\cdot(x_1,y_1)+c\cdot(x_2,y_2)=c\cdot\alpha+c\cdot\beta$, (vi) $(c+c')\cdot\alpha=(cx_1+c'x_1,cy_1+c'y_1)=c\cdot(x_1,y_1)+c'\cdot(x_1,y_1)=c\cdot\alpha+c'\cdot\alpha$, (vii) $(c\cdot c')\cdot\alpha=(cc'x_1,cc'y_1)=c\cdot(c'x_1,c'y_1)=c\cdot(c'\cdot\alpha)$, (viii) $1\cdot\alpha=(1\cdot x_1,1\cdot y_1)=(x_1,y_1)=\alpha$, where 1 is the unit element. $\therefore V$ is a vector space over the field $\mathbb{Q}$.	1 1 1 1 1 1
1b	$W=\left\{\begin{bmatrix}a&b\\b&d\end{bmatrix}\Big\vert\, a,b,d\in\mathbb{C}\right\}$. Let $\alpha=\begin{bmatrix}a_1&b_1\\b_1&d_1\end{bmatrix},\beta=\begin{bmatrix}a_2&b_2\\b_2&d_2\end{bmatrix}\in W,c\in\mathbb{C}$ (i) $\alpha+\beta=\begin{bmatrix}a_1+a_2&b_1+b_2\\b_1+b_2&d_1+d_2\end{bmatrix}\in W$, (ii) $c\cdot\alpha=\begin{bmatrix}ca_1&cb_1\\cb_1&cd_1\end{bmatrix}\in W$ $\therefore W$ is a subspace of $M_{2\times2}$.	2 2
2a	$T:M(\mathbb{R})\to M(\mathbb{R})$ defined by $T(A)=AC-CA$ Let $A,B\in M(\mathbb{R})$ and C be a fixed matrix $\in M(\mathbb{R})$ (i) $T(A+B)=(A+B)C-C(A+B)=(AC-CA)+(BC-CB)=T(A)+T(B)$ (ii) $T(c\cdot A)=(c\cdot A)C-C(c\cdot A)=c\cdot(AC-CA)=c\cdot T(A)$ $\therefore T$ is a Linear Transformation.	2 2
2b	Given $\{1+2t+t^2,2+2t-t^2,\ 1+3t+2t^2,1-t-3t^2\}$ Considering the constants, the coefficients of t and t^2 in matrix format implies $\begin{bmatrix}1&2&1\\2&2&-1\\1&3&2\\1&-1&-3\end{bmatrix}\sim\begin{matrix}R_2-2R_1\\R_3-R_1\\R_4-R_1\end{matrix}\begin{bmatrix}1&2&1\\0&-2&-3\\0&1&1\\0&-3&-4\end{bmatrix}\sim\begin{matrix}2R_3+R_2\\2R_4-3R_2\end{matrix}\begin{bmatrix}1&2&1\\0&-2&-3\\0&0&-1\\0&0&1\end{bmatrix}\sim R_4+R_3\begin{bmatrix}1&2&1\\0&-2&-3\\0&0&-1\\0&0&0\end{bmatrix}$ $\therefore\{1+2t+t^2,2+2t-t^2,1+3t+2t^2\}$ are L.I, and they form a basis, dimension of the subspace is 3.	3 1 1 1
3	$A=\begin{bmatrix}2&4&-2&1\\-2&-5&7&3\\1&3&-6&5\end{bmatrix}\sim\begin{matrix}R_2+R_1\\2R_3-R_1\end{matrix}\begin{bmatrix}2&4&-2&1\\0&-1&5&4\\0&2&-10&9\end{bmatrix}\sim R_3+2R_2\begin{bmatrix}2&4&-2&1\\0&-1&5&4\\0&0&0&17\end{bmatrix}$ Basis of $R(A)=\{(2,4,-2,1),(-2,-5,7,3),(1,3,-6,5)\}$ Basis of $C(A)=\{(2,-2,1),(4,-5,3),(1,3,5)\}$ $Ax=0\Rightarrow 2x_1+4x_2-2x_3+x_4=0,-x_2+5x_3+4x_4=0,17x_4=0,$ $n=4,r=3,\therefore$ choose 1 free variable. Let x_3 be the f.v. $x_4=0,x_2=5x_3,x_1=-9x_3$, Hence basis of $N(A)=\{(-9,5,1,0)\}$ $A^T=\begin{bmatrix}2&-2&1\\4&-5&3\\-2&7&-6\\1&3&5\end{bmatrix}\begin{matrix}R_2-2R_1\\R_3+R_1\\2R_4-R_1\end{matrix}\sim\begin{bmatrix}2&-2&1\\0&-1&1\\0&5&-5\\0&8&9\end{bmatrix}\begin{matrix}R_3+5R_2\\R_4+8R_2\end{matrix}\sim\begin{bmatrix}2&-2&1\\0&-1&1\\0&0&17\\0&0&0\end{bmatrix}$ $A^Ty=0\Rightarrow 2y_1-2y_2+y_3=0,-y_2+y_3=0,17y_3=0\Rightarrow y_3=0,y_2=0,y_1=0$ $\therefore N(A^T)=\{(0,0,0)\}$	2 1 1 1 1 1 2 1

<table>
<tr><td>4a</td><td>Given $T(1,2,1)=(7,0,3,5),\ T(2,1,1)=(5,2,2,3)T(-1,2,1)=(5,-4,3,5)$,
$$T=\begin{bmatrix}7&5&5\\0&2&-4\\3&2&3\\5&3&5\end{bmatrix}\begin{bmatrix}1&2&-1\\2&1&2\\1&1&1\end{bmatrix}^{-1}=\begin{bmatrix}7&5&5\\0&2&-4\\3&2&3\\5&3&5\end{bmatrix}\begin{bmatrix}1/2&3/2&-5/2\\0&-1&2\\-1/2&-1/2&3/2\end{bmatrix}=\begin{bmatrix}1&3&0\\2&0&-2\\0&1&1\\0&2&1\end{bmatrix}$$
$$\therefore T(x,y,z)=(x+3y,2x-2z,y+z,2y+z)$$
$$\begin{bmatrix}1&3&0\\2&0&-2\\0&1&1\\0&2&1\end{bmatrix}R_2-2R_1\sim\begin{bmatrix}1&3&0\\0&-6&-2\\0&1&1\\0&2&1\end{bmatrix}\begin{matrix}6R_3+R_2\\3R_4+R_2\end{matrix}\sim\begin{bmatrix}1&3&0\\0&-6&-2\\0&0&4\\0&0&1\end{bmatrix}4R_4-R_3\sim\begin{bmatrix}1&3&0\\0&-6&-2\\0&0&4\\0&0&0\end{bmatrix}$$
Basis of $R(T)=\{(1,2,0,0),(3,0,1,2),(0,-2,1,1)\}$</td><td>1
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<tr><td>4b</td><td>$u_1=(2,5,-1),u_2=(-2,1,1)$ and $y=(1,2,3)$
$$\hat{y}=\frac{y\cdot u_1}{u_1\cdot u_1}u_1+\frac{y\cdot u_2}{u_2\cdot u_2}u_2=\frac{9}{30}(2,5,-1)+\frac{3}{6}(-2,1,1)=\left(\frac{-4}{10},\frac{20}{10},\frac{2}{10}\right)=\left(\frac{-2}{5},\frac{10}{5},\frac{1}{5}\right)$$
$z=y-\hat{y}=\left(\frac{7}{5},0,\frac{14}{5}\right)$, distance = $\sqrt{9.8}=3.13$</td><td>2
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<tr><td>5</td><td>$\begin{bmatrix}1&-2&-2\\2&-4&1\\-2&0&4\\1&0&-2\\2&-2&-2\end{bmatrix}$, $x_1=(1,2,-2,1,2)$, $x_2=(-2,-4,0,0,-2)$, $x_3=(-2,1,4,-2,-2)$
$u_1=x_1=(1,2,-2,1,2)$
$u_2=x_2-\frac{x_2\cdot u_1}{u_1\cdot u_1}u_1=(-2,-4,0,0,-2)-\frac{(-14)}{14}(1,2,-2,1,2)$
$u_2=(-1,-2,-2,1,0)$
$$u_3=x_3-\frac{x_3\cdot u_1}{u_1\cdot u_1}u_1-\frac{x_3\cdot u_2}{u_2\cdot u_2}u_2=(-2,1,4,-2,-2)-\frac{(-14)}{14}(1,2,-2,1,2)-\frac{(-10)}{10}(-1,-2,-2,1,0)$$
$$u_3=(-2,1,2,-1,0)$$
$$Q=\begin{bmatrix}1/\sqrt{14}&-1/\sqrt{10}&-2/\sqrt{5}\\2/\sqrt{14}&-2/\sqrt{10}&1/\sqrt{5}\\-2/\sqrt{14}&-2/\sqrt{10}&0\\1/\sqrt{14}&1/\sqrt{10}&0\\2/\sqrt{14}&0&0\end{bmatrix},\ R=Q^TA=\begin{bmatrix}\sqrt{14}&-\sqrt{14}&-\sqrt{14}\\0&\sqrt{10}&-\sqrt{10}\\0&0&\sqrt{5}\end{bmatrix}$$</td><td>2
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