Dept. of C.S.E

Subject: DMS

Sub. Code: CS241AT Improvement CIE

Date: ~~Ag~~-2024

Scheme and Solutions

Part- A

1.1 $-a/(1+a)$ — 1

1.2 $d*a = d$, $d*b = c$ — 1

1.3 $[0]+H = \{0,4,8\}$
$[1]+H = \{1,5,9\}$
$[2]+H = \{2,6,10\}$
$[3]+H = \{3,7,11\}$
$\therefore$ partition of $G = \{\{0,4,8\}, \{1,5,9\}, \{2,6,10\}, \{3,7,11\}\}$ — 1

— 1

1.4 $(0.95)^9 + \binom{9}{2}(0.05)(0.95)^7$

1.5 000000000, 000 000 001, 100 000 000, 000 100 000, 000 010 000 Etc any three words — 1

— 1

1.6 $\chi(G) = 4$ — 1

1.7 $\chi'(K_{3,2}) = 4$

1.8 1440 proper colorings are possible — 1

1.9 Centroid $= \{12\}$ i-e the node with weight $= 12$. — 1

1.10 Center $= \{d\}$, the radius of G is 3 — 1

Part - B

2.a) **Proof**: Let $a, b \in G$, then $a^{-1}, b^{-1}, a*b, b^{-1}*a^{-1}$ all belogs to G. Let us consider

$(b^{-1}*a^{-1})*(a*b)$

Since $*$ is associative

$(b^{-1}*a^{-1})*(a*b) = [b^{-1}*(a^{-1}*a)]*b$ —— 2

$= (b^{-1}*e)*b$

$= b^{-1}*b = e$

Next, consider $(a*b)\circ(b^{-1}*a^{-1})$, since $*$ is associative

$(a*b)*(b^{-1}*a^{-1}) = a*(b*b^{-1})*a^{-1}$ —— 2

$= (a*e)*a^{-1}$

$= a*a^{-1} = e$

Therefore, $(b^{-1}*a^{-1})*(a*b) = (a*b)*(b^{-1}*a^{-1}) = e$ —— 1

This proves that $(b^{-1}*a^{-1})$ is the unique inverse of $a*b$.

2.b) To construct the multiplication table for $(Z_{12}, +)$ —— 1

To show $(Z_{12}, +)$ is a cyclic group. —— 2

To list all the generators —— 2

$\langle 1\rangle, \langle 5\rangle, \langle 7\rangle, \langle 11\rangle$ are the generators.

3.a) Let $x, y \in G$,

$f(xy) = axya^{-1} = axa^{-1}aya^{-1} = f(x)f(y)$ —— 1

$\therefore f$ is homomorphism.

Suppose $x \in G$, then

$f(a^{-1}xa) = aa^{-1}xaa^{-1} = x$, so f is onto. —— 2

Suppose $f(x) = f(y) \Rightarrow axa^{-1} = aya^{-1}$

$\Rightarrow a^{-1}(axa^{-1})a = a^{-1}(aya^{-1})a$

$\therefore x = y$. So f is one to one i.e isomorphism —— 2

3.b) <u>proof</u>: The set C is known to both sender and receiver, so if $w \in W$ is the message and $c = E(w)$ is transmitted. Let $c \neq T(c) = r$.
If the minimum distance b/w codewords is at least $k+1$, then the transmission of c can result in as many as k errors and r will not be listed in C. Hence we can detect all errors e where $wt(e) \leq k$. Conversely, let c_1, c_2 are code words with $d(c_1, c_2) < k+1$. Then $c_2 = c_1 + e$ where $wt(e) \leq k$. If we send c_1 and $T(c_1) = c_2$, then we feel that c_2 had been sent hence failing to detect an errors of weight $\leq k$. — 5

4.a) Given

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\therefore G = [I_3 \mid A] = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \quad — 1$$

i) Code words are

$[000][G] = [000\ 000]$, $[001][G] = [001\ 101]$,
$[010][G] = [010\ 010]$, $[011][G] = [011\ 111]$, — 2
$[100][G] = [100\ 111]$, $[101][G] = [101\ 010]$,
$[110][G] = [110\ 101]$, $[111][G] = [111\ 000]$.

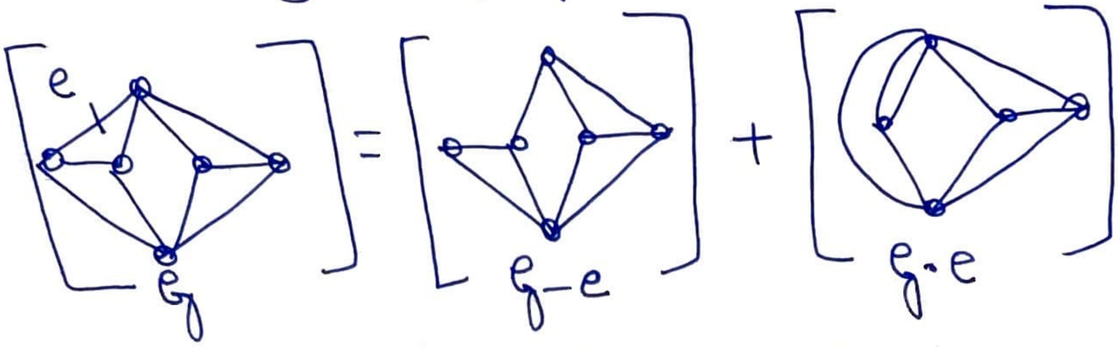
ii) Minimum distance between the codewords is 2
$\therefore$ All errors of single bit are detectable — 1
And No errors can be corrected. — 2

iii) $[H]\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $(H)\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

4.b) i) $|S(x,1)| = 11$, $|S(x,2)| = 56$, $|S(x,3)| = 176$ $\rightarrow 1\times 3 = 3$

ii) $|S(x,k)| = \sum_{i=0}^{k} \binom{n}{i}$ — 1

5.a) To show the working of decomposition theorem on the given graph — 4



$P(G-e,\lambda) = 2\lambda^6 - 3\lambda^5 - 2\lambda^4 + 2\lambda^3 + 16\lambda^2 - 15\lambda$ —
$P(G\cdot e,\lambda) = \lambda^6 - 2\lambda^5 + 2\lambda^4 - 5\lambda^3 + 14\lambda^2 - 10\lambda$. —
$P(G,\lambda) = 3\lambda^6 - 5\lambda^5 - 3\lambda^3 + 30\lambda^2 - 25\lambda$ — 2

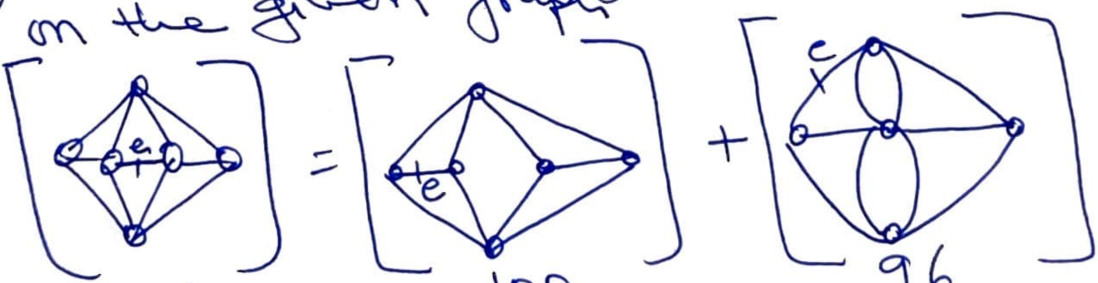
5.b) (i) ii) iii) iv)

$1\times 4 = 4$

OR Any example graphs

6.a) Using mathematical induction on $|E|$ this is to be proved. — 1
For Basis — 1
for Hypothesis — 3
for Inductive proof

6.b) To show the working of decomposition theorem on the given graph — 3



196 = 100 + 96 — 2