Dept. of AI&ML, RVCE, Bangalore

Practice Problems for Test 2

- Most of these problems require you to think. Am sure you are all good at it. Happy times solving the assignment!!! ☺

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1. The probability that a randomly chosen dog is a biter is 0.2. Dogs who are biters are three times as likely to be barkers as those who do not bite.What is the conditional probability that a randomly chosen dog is a biter, given that it is a barker? *Use Bayes' Theorem.*

2. Identify the set of possible values of the random variable in each case.
   (a) The number of breakdowns of city buses in a large city in one week.
   (b) The number of coins that match when three coins are tossed at once.
   (c) The distance a rental car rented on a daily rate is driven each day.
   (d) The number of words you speak in a day.
   (e) The calories that you burn on a single day.

3. In the following real world problems, only one random variable/distribution (Equally likely, Bernoulli, Binomial, Geometric, Poisson, Exponential, Uniform, Gaussian) best suits each case. Associate each of the real world problems below with exactly that random variable that best describes it.
   (a) Fraction of defective items in a production line.
   (b) Distance of a cell phone to the nearest base station.
   (c) Number of rides that an Ola/Uber driver does on a given day.
   (d) Number of active speakers in a collection of independent conversations.

4. An elevator starts with 7 passengers and stops on 10 floors. What is the probability that no two passengers leave on the same floor? (We assume that any passengers gets out on any floor with the same probability.)

5. On Thursday next week, the probability that a student exercises is 0.7, while the probability a student eats pizza is 0.2. Of those students who exercise next Tuesday, 15% will eat pizza. What is the probability that a randomly selected student does not exercise and does not eat pizza next Tuesday.

6. You are designing a video surveillance system on campus, using 6 cameras set up to view 6 independent locations. However, you only buy enough network bandwidth to deliver up to 4 videos at a time to your central monitoring location. So you design a method whereby a video is only sent to the central monitoring location if something "interesting" is happening. If the probability that something "interesting" happens is p, for each camera independently, what is the probability you will not have enough network bandwidth to deliver all "interesting" videos to the central location? (*Identify the distribution/random variable that best models the scenario given.*

7. You flip a fair coin repeatedly until you get a heads. Let X be the number of flips you made up to and including that first heads. What is the probability that $X = 1$ given that $X < 3$?

8. Undergraduate students from AI&ML Department of RVCE have designed a next generation Gen-Z phone along with their E&C friends. They have determined that the lifetime of the GenZ phone is given by a random variable Z (measured in hours), with the following PDF.

$$f_Z(z) = \begin{cases} \dfrac{10}{z^2} & z \geq 10 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Use the PDF to determine the probability that the GenZ phone will last more than 20hours.

   (b) Use the PDF to determine the probability that the GenZ phone will last less than 50hours. (*Note that the GenZ phone lasts at least 10hours. This is given by the 1st condition of the PDF).*

   (c) Find the mean life time of the GenZ phone.

9. Suppose each day that you drive to work a traffic light that you encounter is either green with probability 7/16, red with probability 7/16, or yellow with probability 1/8, independent of the status of the light on any other day. If over the course of five days, G, Y, and R denote the number of times the light is found to be green, yellow, or red, respectively, What is the probability that $P[G = 2, Y = 1, R = 2]$? Also , What is the probability $P[G = R]$?

10. On a given day, the expected number of times that you forget something important is 3. Suppose that this is described by a Poisson random variable,

    (a) What is the probability that on a given day you forget 5 things?

(b) What is the probability that on a given day you forget 5 things if we know that you have forgotten at least 4 things?

11. Suppose X is a random variable with PDF,

$$f_X(x) = \begin{cases} 2 - 2x & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the mean and variance of X.

12. When you go fishing, you attach 1n hooks to your line. Each time you cast your line, each hook will be swallowed by a fish with probability h, independent of whether any other hook is swallowed. What is the PMF of K, the number of fish that are hooked on a single cast of the line?

13. Any time a child throws a Frisbee, the child's dog catches the Frisbee with probability p, independent of whether the Frisbee is caught on any previous throw. When the dog catches the Frisbee, it runs away with the Frisbee, never to be seen again. The child continues to throw the Frisbee until the dog catches it. Let X denote the number of times the Frisbee is thrown.

(a) What is the PMF $p_X(x)$?

(b) If $p = 0.2$, what is the probability that the child will throw the Frisbee more than four times?

14. We listed out the properties of probability density function in class. With the help of those properties (*if you have taken down notes ;)*) identify which of the following can be valid PDFs? If a particular function you identify as not a valid PDF, justify as to why it is not a valid PDF, as in mention the property it violates.

(a)

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$f(x) = \begin{cases} |x| & |x| < 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

(c)

$$f(x) = \begin{cases} \frac{2}{3}(x-1) & 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

(d)

$$f(x) = \begin{cases} e^{-2x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(e)

$$f(x) = \begin{cases} \dfrac{x^3}{20} & x = -1, 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

15. Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} ax^2 + bx & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

where a and b are constants. Suppose $E[X] = 0.75$, find the values of $a, b, E[X^2], Var[X]$.

16. Mark each of the statements given below as *TRUE* or **FALSE**. Justify why you chose the specific option! (*Dear students, whenever you make a choice, you must have a logical reason or justification for the same. So please make you justification sensible.*)

   (a) If you add 3.97 to each entry on a list, that adds 3.97 to the average.

   (b) If you add 3.97 to each entry on a list, that adds 3.97 to the standard deviation.

   (c) If you double each entry on a list, that doubles the average.

   (d) If you double each entry on a list, that doubles the standard deviation.

   (e) If you change the sign of each entry on a list, that changes the sign of the average.

   (f) If you change the sign of each entry on a list, that changes the sign of the standard deviation.

17. A small data set is shown below; The correlation coefficient $r \approx 0.76$. If you switch the two columns, does this change r? Justify your answer. *You can explicitly calculate and see. However, you must be in a position to reason out even without calculation. You guys are capable of it... Give it a shot!*

x	y
1	2
2	3
3	1
4	5
5	6

Table 1: Small Data set