Probability and Distributions

Sample space and events

There are lots of phenomena in nature, like tossing a coin or tossing a die, whose outcomes cannot be predicted with certainity in advance, but the set of all ~~the~~ possible outcomes is known. These are what we call random phenomena or random experiments. Probability theory is concerned with such random phenomena or random experiments.

Consider a random experiment. The set of all possible outcomes is called the sample space of the experiment and is usually denoted by S. Any subset E of the samplespace S is called an event. Here are some examples

ex1: Tossing a coin.

The spample space is $S = \{H, T\}$.

$E = \{H\}$ is an event, i.e, occurance of a Head

ex 2: Tossing a die. ~~i.e. "occurrance of a Head"~~

The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

$E = \{2, 4, 6\}$ is an event, ie. "the number is even".

ex 3 : Tossing a coin twice.

The sample space is $S = \{HH, HT, TH, TT\}$

$E = \{HH, HT\}$ is an event,

which is "the first toss results in a Head."

ex 4 : Tossing a die twice.

The sample space is $S = \{(i, j); i, j = 1, 2, 3, 4, 5, 6\}$,

which contains 36 elements.

"The sum of the results of the two toss is equal to 10" is an event.

i.e, $E = \{(6,4), (4, 6), (5,5)\}$.

Since events are simply subsets of the sample space, we can talk about various set theoretic operations on events. In the following E, F, G are events.

$E \cup F$ denotes the union of E and F.

$E \cap F$ denotes the intersection of E and F.

E' stands for the complements of E,

i.e, $E = S - E$.

$E \subset F$ means that E is a subset of F.

If $E \cap F = \phi$, we say that E and F are disjoint.

Exhaustive events. A set of events is said to be exhaustive, if it includes all the possible events. For example, in tossing a coin there are two exhaustive cases either head or tail and there is no third possibility.

Mutally exclusive events. If the occurence of one of the events precludes the occurence of all other, then such a set of events is said to be mutually exclusive.

For example, in tossing a coin, either head comes up or the tail and both can't happen at the same time, i.e., these are two mutually exclusive cases.

Equally likely events. If one of the events cannot be expected to happen in preference to another then such events are said to be equally likely.

For example, in tossing a coin, the coming of the head or the tail is equally likely.

Probability.

If there are n exhaustive, mutually exclusive and equally likely cases of which m are favourable to an event A, then the probability (p) of the happening of A is

$$P(A) = \frac{m}{n}.$$

As there are n-m cases in which A will not happen (denoted by A'), the chance of A not happening is q or P(A') so that

$$q = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p$$

i.e, $P(A') = 1 - P(A)$, so that $P(A) + P(A') = 1$.

If an event is certain to happen then its probability is unity, while if it is certain not to happen, its probability is zero.

The definitions of probability fails when

(i) number of outcomes is infinite (not exhaustive)

(ii) outcomes are not equally likely.

* Example: Find the chance of throwing (a) four (b) an even number with an ordinary six faced die.

Solution:

(a) No of possible outcomes $= 6$

no of ways of throwing 4 $= 1$

Required chance $= \frac{1}{6}$

(b) no of possible outcomes $= 6$

no of ways of getting an even number (i.e, 2, 4, 6) $= 3$

Required chance $= \frac{3}{6} = \frac{1}{2}$

* Example: What is the chance that a leap year selected at random will contain 53 sundays?

Solution: A leap year has 366 days.

So there are 52 full weeks and extra two days

These two days can be

(i) M, T (ii) T, W (iii) W, Tr (iv) Tr Fr (v) Fr Sat (vi) Sat Sun (vii) Sun, Mon

Out of these 7 cases, the last two are favourable and hence the required probability

$= \frac{2}{7}$.

*A five figure number is formed by the digits 0, 1, 2, 3, 4 without repetition. Find the probability that the number formed is divisible by 4.

<u>Solution</u>: A five ~~figured~~ digit can be arranged in 5! ways.

Out of which 4! will begin with zero.

$\therefore$ Total no of 5-digit numbers $= 5! - 4! = 96$.

These numbers are divisible by 4, if they end with

04, 12, 20, 24, 32, 40.

Numbers ending in

$$
\begin{aligned}
04 &= 3! = 6 \\
12 &= 3! - 2! = 4 \\
20 &= 3! = 6 \\
24 &= 3! - 2! = 4 \\
32 &= 3! - 2! = 4 \\
40 &= 3! = 6.
\end{aligned}
$$

total number of favourable ways = 30

Hence the required probability is $\frac{30}{96} = \frac{5}{16}$

* A bag contains 40 tickets numbered 1, 2, 3, ... 40 of which four are drawn at random and arranged in ascending order ($t_1 < t_2 < t_3 < t_4$). Find the probability of t_3 being 25.?

Solution:

Here exhaustive number of cases = $^{40}C_4$

If $t_3 = 25$, then the tickets t_1 & t_2 must come out of 24 tickets numbered 1 to 24.

This can be done in $^{24}C_2$ ways.

Then t_4 must come out of the 15 tickets numbered 26 to 40 ~~whi~~

This can be done in $^{15}C_1$ ways

$\therefore$ probability of t_3 being 25 $= \dfrac{^{25}C_2 \times {}^{15}C_1}{^{40}C_4} = \dfrac{414}{9139}$

* An urn contains 5 red and 10 black balls. Eight of them are placed in another urn. What is the chance that the latter contains 2 red & 6 black balls.

Sol^n No of ways of drawing 8 out of 15 balls $^{15}C_8$.

No of ways of drawing 2 red = $^5C_2$ & corresponding to these there are $^{10}C_6$ ways of drawing 6 black balls.

Total no of ways of drawing 2 red & 6 black balls = $^5C_2 \times {}^{10}C_6$

Required probability = $\dfrac{^5C_2 \times {}^{10}C_6}{^{15}C_8} = \dfrac{140}{429}$

7. A has one share in a lottery in which there is 1 prize & 2 blanks. B has three shares in a lottery in which there are 3 prizes and 6 blanks. Compare the probability of A's success to that of B's success.

<u>Soln</u>:

A can draw a ticket in ${}^3C_1$ ways = 3

no. of cases in which A can get a prize = 1

∴ probability of A's success = $\frac{1}{3}$

B can draw a ticket in ${}^9C_3$ ways = 84.

no. of ways in which B gets all blanks = ${}^6C_3$ = 20

∴ no. of ways of B getting a prize = 84 − 20 = 64.

∴ probability of B's success = $\frac{64}{84} = \frac{16}{21}$.

A's probability of success : B's probability of success

$= \frac{1}{3} : \frac{16}{21}$

$= 7 : 16$

Properties of Probability

(i) $P(\phi) = 0$

(ii) If A and B are disjoint events/mutually exclusive events then $P(A \cup B) = P(A) + P(B)$

(iii) If $A \subset B$, then $P(A) < P(B)$

(iv) For any event A, $P(A') = 1 - P(A)$

(v) For any events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(vi) $P(S) = 1$, where S is the entire sample space.

(vii) For any event A, $0 \leq P(A) \leq 1$

(viii)

Addition Law of Probability

If two events A and B are mutually exclusive then

$$P(A \cup B) = P(A) + P(B)$$

However, when A and B are not mutually exclusive, $A \cap B \neq \phi$, then the general law is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

A bag contains 20 marbles, 3 are coloured red, 6 are coloured green, 4 are coloured blue, 2 are coloured white and 5 are coloured yellow. One ball is selected at random. Find the probabilities of the following events.

(a) the ball is either red or green

(b) the ball is not blue

(c) the ball is either red or white or blue.

(a) $P(R \cup G) = P(R) + P(G) = \frac{3}{20} + \frac{6}{20} = \frac{9}{20}$

(b) $P(B') = 1 - P(B) = 1 - \frac{4}{20} = \frac{16}{20} = \frac{4}{5}$

(c) $P(R \cup W \cup B)$

The complementary event is $G \cup Y$

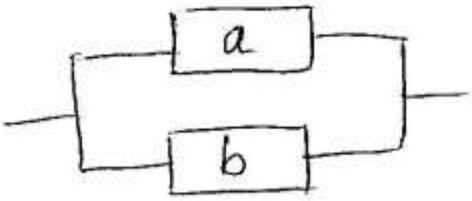
and $P(G \cup Y) = P(G) + P(Y) = \frac{6}{20} + \frac{5}{20} = \frac{11}{20}$

$\therefore \quad P(R \cup W \cup B) = 1 - P(G \cup Y) = 1 - \frac{11}{20} = \frac{9}{20}$

Also, $P(R \cup W \cup B) = P(R) + P(W) + P(B)$

$= \frac{3}{20} + \frac{2}{20} + \frac{4}{20}$

$= \frac{9}{20}$



The figure shows a simplified circuit in which two components a and b are connected in parallel. The circuit functions if either or both of the components are operational. It is known that if A is the event 'component a is operating' and B is the event 'component b is operating' then $P(A) = 0.99$, $P(B) = 0.98$ and $P(A \cap B) = 0.9702$. Find the probability that the circuit is functioning.

Solⁿ:

The probability that the circuit is functioning is $P(A \cup B)$, i.e, either a or b must be functioning if the circuit is to function.

Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.99 + 0.98 - 0.9702$$

$$= 0.998$$

Find the probability of drawing an ace or a spade or both from a deck of cards.

Sol^n: The probability of drawing an ace from a deck of 52 cards $= \frac{4}{52}$ $P(A) = \frac{4}{52}$

The probability of drawing a card of spades $= \frac{13}{52}$

$$P(S) = \frac{13}{52}$$

The probability of drawing an ace of spades $= \frac{1}{52}$

$$P(A \cap S) = \frac{1}{52}$$

The probability of drawing an ace or a spade

ie, $P(A \cup S) = P(A) + P(S) - P(A \cap S)$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{4}{13}$$

There are 20 persons. 5 of them are graduates. 3 persons are randomly selected from these 20 persons. Find the probability that atleast one of the selected person is a graduate.

Sol^n

From 20 persons, 3 persons can be selected in $^{20}C_3$ ways. Thus there are $^{20}C_3$ equally likely, manually exclusive and exhaustive outcomes. Since there are 15 persons who are not graduates,

P[at least one is graduate] = 1 − P[none is graduate]

$$= 1 - \frac{^{15}C_3}{^{20}C_3} = 1 - \frac{91}{228} = \frac{137}{228} = 0.6$$

In a college, there are five lecturers. Among them, three are doctorates. If a committee consisting three lecturers is formed, what is the probability that atleast two of them are doctorates?

Sol^n From 5 lecturers, 3 lecturers can be selected in $^5C_3$ ways.

let A: Two of the selected lecturers are doctorates.

A has $^3C_2 \times {^2C_1}$ favourable outcomes. $P(A) = \frac{^3C_2 \times {^2C_1}}{^5C_3} = \frac{3 \times 2}{10} = \frac{3}{5}$

let B = All the three selected lecturers are doctorates.

B has $^3C_3$ favorable outcomes. $P(B) = \frac{^3C_3}{^5C_3} = \frac{1}{10}$

A & B are mutually exclusive

$\therefore$ P[at least 2 doctorates] = P[2 or 3 doctorates] = $P(A \cup B) = P(A) + P(B)$

$$= \frac{3}{5} + \frac{1}{10} = \frac{7}{10}$$

In a race, the odds in favour of the four horses H_1, H_2, H_3, H_4 are $1:4, 1:5, 1:6, 1:7$ respectively. Assuming that a dead heat is not possible, find the chance that one of them wins the race.

<u>Sol^n</u>:- Since it is not possible for all the horses to cover the same distance in the same time, the events are mutually exclusive.
If P_1, P_2, P_3, P_4 be the probabilities of winning of the horses H_1, H_2, H_3, H_4, respectively, then

$$P_1 = \frac{1}{1+4} = \frac{1}{5}, \quad P_2 = \frac{1}{1+5} = \frac{1}{6}$$

$$P_3 = \frac{1}{1+6} = \frac{1}{7}, \quad P_4 = \frac{1}{1+7} = \frac{1}{8}$$

Hence the chance that one of them wins

$$P(H_1 \cup H_2 \cup H_3 \cup H_4) = P(H_1) + P(H_2) + P(H_3) + P(H_4)$$

$$= P_1 + P_2 + P_3 + P_4$$

$$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$$

$$= \frac{533}{840}$$

Conditional Probability

Two events are said to be independent, if happening or failure of one does not affect the happening or failure of the other. Otherwise the events are said to be dependent.

For two dependent events A and B, the symbol $P(B|A)$ denotes the probability of occurrence of B, when A has already occurred. It is known as the conditional probability and is read as a 'probability of B given A.' and it is defined as $P(B|A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = P(A).P(B/A)$

If A and B are independent events, occurence of B will be independent of occurence of A. Therefore, the conditional and unconditional probabilites are equal. That is $P(B|A) = P(B)$.

then $P(B) = \frac{P(A\cap B)}{P(A)}$ ~~Th~~ $\Rightarrow P(A\cap B) = P(A).P(B)$

Multiplication Theorem

For two dependent events A and B,
[where occurence of B depends on A]
~~Then~~ $P(A\cap B) = P(A) . P(B/A)$
For two independent events A and B,
$P(A\cap B) = P(A) . P(B)$

* Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second a queen if the first card is (i) replaced (ii) not replaced.

Solution:-

(i) The probability of drawing a king $P(K) = \frac{4}{52} = \frac{1}{13}$.

If the card is replaced, the pack will again have 52 cards so that the probability of drawing a queen is $P(Q) = \frac{4}{52} = \frac{1}{13}$.

The two events being Independent, the probability of drawing both cards in succession

$$= P(K) \times P(Q)$$

$$= \frac{1}{13} \times \frac{1}{13}$$

$$= \frac{1}{169}$$

(ii) The probability of drawing a king $P(K) = \frac{4}{52} = \frac{1}{13}$.

If the card is not replaced, the pack will have 51 cards only, so that the chance of drawing a queen is $P(Q/K) = \frac{4}{51}$

Hence the probability of drawing both cards $= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$

$P(K) \times P(\frac{Q}{K})$

A pair of dice is tossed twice. Find the probability of scoring 7 points (a) once, (b) atleast once (c) twice.

Solution:

In a single toss of two dice, the sum 7 can be obtained as (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) in 6 ways, so that the probability of getting 7 $= \frac{6}{36} = \frac{1}{6}$

The probability of not getting 7 $= 1 - \frac{1}{6} = \frac{5}{6}$.

(a) The probability of getting 7 in the first toss and not getting 7 in the second toss $\Big\} = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

Similarly, the probability of not getting 7 in first toss and getting 7 in the second toss $\Big\} = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$\therefore$ The required probability of occurance of 7 once $\Big\} = \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$

(b) ~~The probability of not getting 7 atleast once~~ $\Rightarrow$

The probability of not getting 7 on either toss $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$\therefore$ The required probability of getting 7 atleast once $\Big\} = 1 - \frac{25}{36} = \frac{11}{36}$

(c) The probability of getting 7 in both tosses $\Big\} = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

A box A contains 2 white & 4 black balls. Another box B contains 5 white & 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white.

<u>Solution</u>:

The probability of drawing a white ball from box B will depend on whether the transferred ball is black or white.

If a black ball is transferred, its probability is $\frac{4}{6}$.

There are now 5 white & 8 black balls in the box B.

Then the probability of drawing white ball from box B = $\frac{5}{13}$

Thus the probability of drawing a white ball from urn B, if the transferred ball is black $= \frac{4}{6} \times \frac{5}{13} = \frac{10}{39}$

Similarly the probability of drawing a white ball from urn B, if the transferred ball is white $= \frac{2}{6} \times \frac{6}{13} = \frac{2}{13}$

Hence the required probability $= \frac{10}{39} + \frac{2}{13} = \frac{16}{39}$

A card is drawn at random from a pack of cards.
(i) what is the probability that it is a heart?
(ii) If it is known that the card drawn is red, what is the probability that it is a heart?

Sol^n: Let A: card drawn is red

B: card drawn is heart.

$$P(A) = \frac{26}{52}$$

$$P(B) = \frac{13}{52}$$

$$P(A \cap B) = \frac{13}{52}.$$

(i) The unconditional probability of drawing a heart is

$$P(B) = \frac{13}{52}$$

(ii) The conditional probability of drawing a heart given that it is red card is

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{13/52}{26/52} = \frac{1}{2}$$

* Given $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{3}$ & $P(A \cup B) = \frac{1}{2}$, evaluate $P(A|B)$, $P(B|A)$, $P(A \cap B')$ & $P(A|B')$

Solⁿ: $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = -\left(\frac{1}{2} - \frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{1}{12}.$$

(i) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/12}{1/3} = \dfrac{1}{4}$

(ii) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1/12}{1/4} = \dfrac{1}{3}$

(iii) $P(A \cap B') = P(A) - P(A \cap B)$

$$= \frac{1}{4} - \frac{1}{12}$$

$$= \frac{1}{6}$$

(iv) $P(A|B') = \dfrac{P(A \cap B')}{P(B')}$

$$= \frac{1/6}{1 - P(B)}$$

$$= \frac{1/6}{1 - 1/3}$$

$$= \frac{1}{4}$$

* If $P(A) = 0.8$, $P(B) = 0.5$ and $P(A \cup B) = 0.9$, find $P(A|B)$. Are A and B independent events?

Sol^n:-

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.9 = 0.8 + 0.5 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = -(0.9 - 0.8 - 0.5)$$

$$= 0.4$$

Also $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{0.4}{0.5}$$

$$P(A|B) = 0.8$$

$\because \; P(A|B) = 0.8 = P(A)$, we can say that A and B are independent.

Baye's Theorem:

Suppose that $A, B_1, B_2, \ldots, B_n$ are events from a sample space S. Suppose that $\bigcup_{i=1}^{n} B_i = S$ and that $B_i \cap B_j = \phi$ for all $i \neq j$. Suppose $P(A) > 0$, and $P(B_j) > 0$ for all j. Then for all j:

$$P(B_j|A) = \frac{P(A|B_j)\,P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

Proof :

$$P(B_j|A) = \frac{P(B_j \cap A)}{P(A)}$$

$$= \frac{P(A|B_j)P(B_j)}{P(A)} \quad \left[\text{By } P(A|B_j) = \frac{P(A\cap B_j)}{P(B_j)}\right] \quad - (1)$$

A can be written as $A = \bigcup_{i=1}^{n}(A \cap B_i)$ $\left[\because \bigcup_{i=1}^{n} B_i = S \text{ \& } B_i \cap B_j = \phi\right]$

Then $P(A) = P\left(\bigcup_{i=1}^{n}(A\cap B_i)\right)$

$$= P(A\cap B_1) + P(A\cap B_2) + \cdots + P(A\cap B_n)$$

[By addition theorem]

$$= \sum_{i=1}^{n} P(A\cap B_i)$$

$$= \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

$$\therefore (1) \Rightarrow P(B_j|A) = \frac{P(A|B_j)\,P(B_j)}{\sum_{i=1}^{n} P(A|B_i)\,P(B_i)}$$

Example 61

Three machines M_1, M_2 and M_3 produce identical items. Of their respective output 5%, 4% & 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output?

Soln:

Let the event of drawing a faulty item from any of the machines be A, and the event that an item drawn at random was produced by M_i be B_i. We have to find $P(B_i/A)$ for which we proceed as follows:

	M_1	M_2	M_3	
$P(B_i)$	0.25	0.30	0.45	∵ Sum = 1
$P(A/B_i)$	0.05	0.04	0.03	
$P(B_i)P(A/B_i)$	0.0125	0.012	0.0135	sum = 0.38
$P(B_i/A)$	$\frac{0.0125}{0.038}$	$\frac{0.012}{0.038}$	$\frac{0.0135}{0.038}$	by Baye's Theorem.

The highest output being from M_3, the required probability $= \frac{0.0135}{0.038} = 0.355$

* There are three bags: first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.

<u>Sol</u>

Sol^n:

Let B_1, B_2, B_3 pertain to the first, second, third bags chosen and A - the two balls are white & red.

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$P(A|B_1) = P$ (a white and a red ball are drawn from first bag)

$$= \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{2}{15}$$

$|||^{ly}$

$$P(A|B_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = \frac{2}{5}$$

$$P(A|B_3) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$$

By Baye's theorem, $P(B_2|A) = \dfrac{P(B_2)\,P(A|B_2)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)\,P(A|B_3)}$

$$= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{3} \times \frac{2}{15} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{1}{5}}$$

$$= \frac{6}{11}$$

* You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for swine flue, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10,000 people in Australia. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. You test positive. What is the new probability that you have a swine flue?

Soln:

Let $P(S)$ be the probability you have swine flu.

Let $P(T)$ be the probability of a positive test.

We want to know $P(S|T)$

From Baye's theorem $P(S|T) = \dfrac{P(T|S)\,P(S)}{P(T)}$

$$= \frac{P(T|S)\,P(S)}{P(T|S)\,P(S) + P(T|NS)\,P(NS)}$$

where $P(NS)$ is the probability of not having swine flu

$P(S) = \dfrac{1}{10{,}000} = 0.0001$ (a priori probability you have swine flu)

$P(NS) = 1 - 0.0001 = 0.9999$

$P(T|S) = 1$ (if you have swineflu the test is always positive)

$P(T|NS) = 0.01$ (1% chance of a false positive)

$\therefore P(S|T) = \dfrac{1 \times 0.0001}{1 \times 0.0001 + 0.01 \times 0.9999} \simeq 0.01$

ie. even though the test was positive your chance of having swineflue is only 1%.

* A factory uses three machines X, Y, Z to produce certain items. (14) (15)

(i) Machine X produces 50 percent of the items of which 3 percent are defective.

(ii) Machine Y produces 30 percent of the items of which 4 percent are defective.

(iii) Machine Z produces 20 percent of the items of which 5 percent are defective.

Suppose a defective item is found among the output. Find the probability that it came from each of the machines.

	X	Y	Z	
$P(P_i)$	0.50	0.30	0.20	sum = 1
$P(D/P_i)$	0.03	0.04	0.05	
$P(P_i)P(D/P_i)$	0.015	0.012	0.010	sum = 0.037
$P(P_i/D)$	$\frac{0.015}{0.037}$	$\frac{0.012}{0.037}$	$\frac{0.010}{0.037}$	
	40.5%	32.5%	27.0%	

* Of the 3 men, the chances that a politician, a businessman and an academician will be appointed as a vice-chancellor of a university are 0.50, 0.30 and 0.20 respectively. Probability that research is promoted by these people if they are appointed as V.C are 0.3, 0.7 & 0.8 respectively.

(a) Determine the probability that research is promoted in the university,

(b) If research is promoted in the university, what is the probability that the V.C is an (i) academician? (ii) businessmann.

Soln:

	P	B	A	
$P(A_i)$	0.50	0.30	0.20	sum = 1
$P(R/A_i)$	0.3	0.7	0.8	
$P(A_i)P(R/A_i)$	0.15	0.21	0.16	sum = 0.52
$P(A_i/R)$	$\frac{0.15}{0.52}$	$\frac{0.21}{0.52}$	$\frac{0.16}{0.52}$	
	0.28846	0.4038	0.30769	

(a) $P(R) = 0.52$

(b) (i) $P(A_i/R) = 0.30769$

(ii) $P(A_i/R) = 0.4038$

* In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. 15 ~~16~~

(a) what is the probability that mathematics is being studied?

(b) If a student is selected at random and is found to be studying Mathematics, find the probability that the student is a (i) girl (ii) a boy.

Sol^n

	B	G	
$P(B_i)$	0.4	$\cancel{\frac{60}{100}}$ 0.6	sum = 1
$P(M/B_i)$	0.25	0.10	
$P(B_i)P(M/B_i)$	0.1	0.06	sum = 0.16
$P(B_i/M)$	$\frac{0.10}{0.16}$	$\frac{0.06}{0.16}$	
	0.625	0.375	

(a) $P(M) = 0.16$

(b) (i) $P(B_i/M) = 0.375$

(ii) $P(B_i/M) = 0.625$

* For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (i) a '1' is received (ii) a '1' was transmitted given that a '1' was received.

	0	1	
$P(B_i)$	0.4	0.6	sum = 1
$P(R/B_i)$	~~0.95~~ 0.05	0.90	sum = 0.56
$P(B_i)P(R/B_i)$	~~0.38~~ 0.02	0.54	
$P(B_i/R)$	$\frac{0.02}{0.56}$	$\frac{0.54}{0.56}$	
	0.0357	0.9643	

(i) ~~p(a) = 0.9643~~ $P(R) = 0.56$

(ii) $P(B_i/R) = 0.9643$