MATLAB MANUAL
MAT231ET

Contents

RANDOM VARIABLES

Syntax and description:

- If (x, p) represents a probability mass function, then the mean and variance are given by mean = sum(x.*p), variance = sum(x.^2.*p)-mean^2
- To find the probabilities of x between x1 and x2 (i.e., P(x1<= x <= x2)), which are at positions i and j of the vector x, is given by sum(p(i:j))
- If f represents the probability density function of x defined on the interval (a, b), then the mean and variance are given by mean = int(x*f, a, b), variance =int(x^2*f, a, b) – mean^2
- To find the probabilities of x between x1 and x2: int(f, x1, x2)
- If (x, y, p) represents the joint probability mass function, where x(i), y(j) and p(i, j) are values taken by x, y and p,
  Marginal distribution of x is: px = sum(p(i,j) for fixed i
  Marginal distribution of y is: py = sum(p(i,j) for fixed j
  Mean of x series is: mx = sum(x.*px)
  Variance of x series is: vx = sum(x.^2.*px) - mx^2
  Standard deviation of x series is: sdx = sqrt(vx)
  Mean of y series is: my = sum(y.*py)
  Variance of y series is: vy = sum(y.^2.*py) - my^2
  Standard deviation of y series is: sdy = sqrt(vy)
- If f represents the joint probability density function, of the random variables x and y, defined on a < x < y < b
  Marginal density function of x: px=int(f, y, x, b)
  Marginal density function of y: py=int(f, x, a, y)
  Mean of x series is: mx = int(x*px, x, a, b)
  Variance of x series is: vx = int(x^2*px, x, a, b) - mx^2
  Standard deviation of x series is: sdx = sqrt(vx)
  Mean of y series is: my = int(y*py, a, b)
  Variance of y series is: vy = int(y^2*py, a, b) – my^2
  Standard deviation of y series is: sdy = sqrt(vy)
  Expected value of (x, y) mxy = int(int(x*y*f,y,x,b),x,a,b)
  Covariance of (x, y) is: cv = mxy – mx*my
  Correlation coefficient of (x, y) is: rh = cv/(sdx*sdy)

Example 1: The following table gives the probability distribution of the discrete random variable X. Find the mean and standard deviation.

X	0	1	2	3	4	5
P	1/32	5/32	5/16	5/16	5/32	1/32

```
x=[0 1 2 3 4 5]

p=[1/32 5/32 5/16 5/16 5/32 1/32]

m=sum(x.*p)
```

```
v=sum(x.^2.*p)-m^2

sd=sqrt(v)
```

```
x =
    0     1     2     3     4     5
p =

    0.0312    0.1562    0.3125    0.3125    0.1562    0.0312
m =
    2.5000
v =
    1.2500
sd =
    1.1180
```

Example 2: The probability mass function of the random variable X is given by the following table. Find $P(X \geq 5), P(3 \leq X \leq 6)$.

X	0	1	2	3		4	5	6
P	1/49	3/49	5/49	7/49		9/49	11/49	13/49

```
x=[0 1 2 3 4 5 6]

p=[1/49 3/49 5/49 7/49 9/49 11/49 13/49]

sum(p(6:7))

sum(p(4:7))
```

```
x =
     0     1     2     3     4     5     6

p =
    0.0204    0.0612    0.1020    0.1429    0.1837    0.2245    0.2653

p1 =
    0.4898

p2 =
    0.8163
```

Example 3: Suppose X is a continuous random variable with the following probability density function, $f(x) = 3x^2 \; for \; 0 < x < 1$. Find the mean and variance of X.

```
syms x

f=3*x^2

m=int(x*f,0,1)

v=int(x^2*f,0,1)-m^2
```

```
sd=sqrt(v)
```

```
f =
   3*x^2
m =
   3/4
v =
   3/80
sd =
   (3^(1/2)*5^(1/2))/20
```

Example 4: The length of time(in minutes) that a certain person speaks on telephone is found to be a random variable with probability density function $f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}}\, for\ x \geq 0 \\ 0\,,\ \ otherwise \end{cases}$. Find the probability that he speaks on the phone for (i) more than 10 minutes, (ii) less than 5 minutes, (iii) between 5 and 10 minutes.

```
syms x

f=(1/5)*exp(-x/5)

p1=int(f,10,inf)

p2=int(f,0,5)

p3=int(f,5,10)
```

```
f =
   exp(-x/5)/5
p1 =
   exp(-2)
p2 =
   1 - exp(-1)
p3 =
   exp(-1) - exp(-2)
```

Example 5: A joint distribution of two random variables X and Y is given by the following table:

Y X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine the marginal distribution of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

```
x=[1 5]

y=[-4 2 7]

p=[1/8 1/4 1/8;1/4 1/8 1/8]

px=[sum(p(1,:)) sum(p(2,:))]

py=[sum(p(:,1)) sum(p(:,2)) sum(p(:,3))]
```

```
mx=sum(x.*px)

vx=sum(x.^2.*px)-mx^2

sdx=sqrt(vx)

my=sum(y.*py)

vy=sum(y.^2.*py)-my^2

sdy=sqrt(vy)

mxy=0;

for i=1:length(x)

mxy=mxy+sum(x(i).*y.*p(i,:));

end

mxy

cvxy=mxy-mx*my

rhxy=cvxy/(sdx*sdy)
```

```
x =
    1     5

y =
   -4     2     7

p =
    0.1250    0.2500    0.1250
    0.2500    0.1250    0.1250

px =
    0.5000    0.5000

py =
    0.3750    0.3750    0.2500

mx =
     3

vx =
     4
sdx =
     2

my =
     1
vy =
   18.7500

sdy =
    4.3301

mxy =
    1.5000
cvxy =
   -1.5000
```

```
rhxy =
   -0.1732
```

Example 6: The joint probability density function of two random variables x and y is given by $p(x,y) = \begin{cases} 2, 0 < x < y < 1 \\ 0, otherwise \end{cases}$. Find the marginal density function of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

```
syms x y
f=2
px=int(f,y,x,1)
py=int(f,x,0,y)
mx=int(x*px,0,1)
vx=int(x^2*px,0,1)-mx^2
sdx=sqrt(vx)
my=int(y*py,0,1)
vy=int(y^2*py,0,1)-my^2
sdy=sqrt(vy)
mxy=int(int(x*y*f,y,x,1),x,0,1)
cv=mxy-mx*my
rh=cv/(sdx*sdy)
```

```
f =
   2
px =
   2 - 2*x
py =
   2*y
mx =
   1/3
vx =
   1/18
sdx =
   2^(1/2)/6
my =
   2/3
vy =
   1/18
sdy =
   2^(1/2)/6
mxy =
   1/4
cv =
   1/36
rh =
   1/2
```

Exercise:

1. The following table gives the probability distribution of the discrete random variable X. Find the mean and standard deviation.

X	0	1	2
P	1/4	2/4	1/4

2. The following table gives the probability distribution of the discrete random variable X. Find $P(X \leq 3), P(1 \leq X \leq 4)$.

X	0	1	2	3	4	5
P	1/32	5/32	5/16	5/16	5/32	1/32

3. Suppose X is a continuous random variable with the following probability density function, $f(x) = \frac{1}{18}x^2 \, for - 3 < x < 3$. Find the mean and variance of X.

4. In a certain city, the daily consumption of electric power (in million kw/hr) is random variable X having the probability density function $f(x) = \begin{cases} \frac{1}{9}xe^{-\frac{x}{3}}, x \geq 0 \\ 0, \ x < 0 \end{cases}$. If the cities power plant has a daily capacity of 12 million kw/hr, what is the probability that this power supply will be insufficient on any given day.

PROBABILTY DISTRIBUTIONS

Syntax and description:

A `Binomial Distribution` object consists of parameters, a model description, and sample data for a binomial probability distribution.

The binomial distribution models the total number of successes in repeated trials from an infinite population under the following conditions:

- Only two outcomes are possible for each of *n* trials.
- The probability of success for each trial is constant.
- All trials are independent of each other.

The binomial distribution uses the following parameters.

Parameter	Description	Support
N	Number of trials	positive integer
P	Probability of success	$0 \le p \le 1$

Distribution Parameters

N — Number of trials
positive integer value

p — Probability of success
positive scalar value in the range [0,1]

Creation

There are several ways to create a `Binomial Distribution` probability distribution object.

- Create a distribution with specified parameter values using `makedist`.
- Fit a distribution to data using `fitdist`.
- Interactively fit a distribution to data using the **Distribution Fitter** app.

`'Name'`	Distribution	Input Parameter `A`	Input Parameter `B`
`'Binomial'`	Binomial Distribution	n number of trials	p probability of success for each trial
`'Poisson'`	Poisson Distribution	λ mean	—
`'Exponential'`	Exponential Distribution	μ mean	—

'Name'	Distribution	Input Parameter A	Input Parameter B
'Normal'	Normal Distribution	μ mean	σ standard deviation

Example 1: The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now aged 60,

(i) at most 6,

(ii) at least 7

will live to be 70?

```
pd = makedist('Binomial','N',10,'p',0.65)

x = [6];

y = cdf(pd,x)

z=1-y
```

```
pd = BinomialDistribution

  Binomial distribution

    N =   10

    p = 0.65

y =

    0.4862

z =

    0.513
```

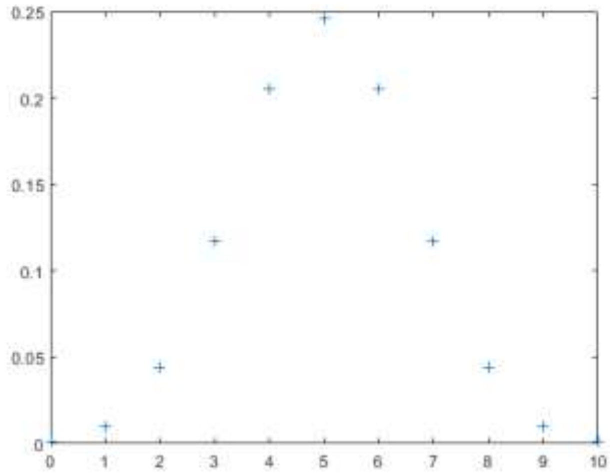
Example 2: Create a binomial distribution object by specifying the parameter values. Also compute the mean of the distribution.

```
pd = makedist('Binomial','N',30,'p',0.25)

m = mean(pd)
```

```
pd = Binomial distribution

    N =   30

    p = 0.25

m =

7.5000
```

Example 3: Generate a plot of the binomial pdf for n = 10 and p = 1/2.

```
x = 0:10;

y = binopdf(x,10,0.5);

plot(x,y,'+')
```



Example 4: Create a Poisson distribution object with the rate parameter, λ, equal to 2. Compute the pdf and cdf values for the Poisson distribution at the values in the input vector x = [0,1,2,3,4].

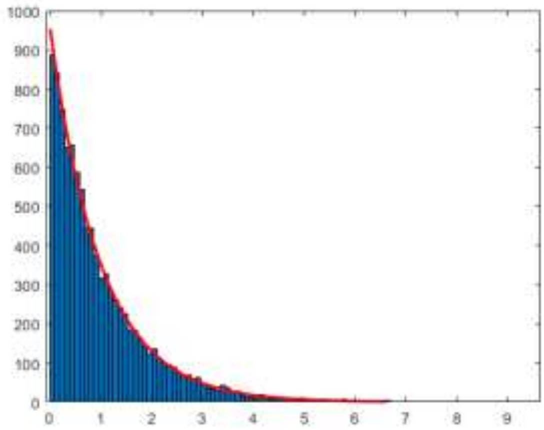
```
lambda = 2;

pd = makedist('Poisson','lambda',lambda);

x = [0,1,2,3,4];

y = cdf(pd,x)

z = pdf(pd,x)
```

```
y = 1×5

    0.1353    0.4060    0.6767    0.8571    0.9473

Z = 1×5

    0.1353    0.2707    0.2707    0.1804    0.0902
```

Example 5: Create an exponential probability distribution object using the default parameter values, generate random numbers from the distribution. Construct a histogram using 100 bins with Exponential distribution fit.

```
pd = makedist('Exponential')

rng('default')  % For reproducibility

r = random(pd,10000,1)

histfit(r,100,'Exponential')
```

```
pd = ExponentialDistribution

  Exponential distribution
    mu = 1

r = 10000×1

    0.2049
    0.0989
    2.0637
    0.0906
    0.4583
    2.3275
    1.2783
    0.6035
    0.0434
    0.0357
```



Example 6: Create a normal probability distribution object with mean 50 and SD 30. Generate a 2-by-3-by-2 array of random numbers from the distribution.

```
pd = makedist('Normal','mu',50,'sigma',30)

r = random(pd,[2,3,2])
```

```
pd = NormalDistribution

  Normal distribution
       mu = 50
    sigma = 30

r =

r(:,:,1) =

   31.4751   98.6682   42.3748
  122.9576   48.9985   25.4413
r(:,:,2) =
```

```
   96.1240   24.0135   17.3124
    1.8054   36.8490   13.3768
```

Exercise:

1. Execute the example questions for different distributions with different parameter values.

2. The probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001. Determine the probability that out of 2000 individuals

(i) exactly 3

(ii) more than 2

individuals will suffer a bad reaction. (use lambda = np)

3. The length of a telephone conversation on a cell phone has been an exponential distribution and found on an average to be 3 minutes. Find the probability that a random call made from this phone ends in less than 3 minutes.

4. In a test of 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for

(i) more than 1950 hours

(ii) more than 1920 and less than 2160 hours.

JOINT DISTRIBUTION OF RANDOM VARIABLES

Syntax and description:

- If (x, y, p) represents the joint probability mass function, where x(i), y(j) and p(i, j) are values taken by x, y and p,
  Marginal distribution of x is: px = sum(p(i,j) for fixed i
  Marginal distribution of y is: py = sum(p(i,j) for fixed j
  Mean of x series is: mx = sum(x.*px)
  Variance of x series is: vx = sum(x.^2.*px) - mx^2
  Standard deviation of x series is: sdx = sqrt(vx)
  Mean of y series is: my = sum(y.*py)
  Variance of y series is: vy = sum(y.^2.*py) - my^2
  Standard deviation of y series is: sdy = sqrt(vy)
- If f represents the joint probability density function, of the random variables x and y, defined on a < x < y < b
  Marginal density function of x: px=int(f, y, x, b)
  Marginal density function of y: py=int(f, x, a, y)
  Mean of x series is: mx = int(x*px, x, a, b)
  Variance of x series is: vx = int(x^2*px, x, a, b) - mx^2
  Standard deviation of x series is: sdx = sqrt(vx)
  Mean of y series is: my = int(y*py, a, b)
  Variance of y series is: vy = int(y^2*py, a, b) – my^2
  Standard deviation of y series is: sdy = sqrt(vy)
  Expected value of (x, y) mxy = int(int(x*y*f,y,x,b),x,a,b)
  Covariance of (x, y) is: cv = mxy – mx*my
  Correlation coefficient of (x, y) is: rh = cv/(sdx*sdy)

Example 1: A joint distribution of two random variables X and Y is given by the following table:

Y \ X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine the marginal distribution of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

```
x=[1 5]

y=[-4 2 7]

p=[1/8 1/4 1/8;1/4 1/8 1/8]

px=[sum(p(1,:)) sum(p(2,:))]

py=[sum(p(:,1)) sum(p(:,2)) sum(p(:,3))]

mx=sum(x.*px)

vx=sum(x.^2.*px)-mx^2
```

```
sdx=sqrt(vx)
my=sum(y.*py)
vy=sum(y.^2.*py)-my^2
sdy=sqrt(vy)
mxy=0;
for i=1:length(x)
mxy=mxy+sum(x(i).*y.*p(i,:));
end
mxy
cvxy=mxy-mx*my
rhxy=cvxy/(sdx*sdy)
```

```
x =
    1     5

y =
    -4     2     7

p =
    0.1250    0.2500    0.1250
    0.2500    0.1250    0.1250

px =
    0.5000    0.5000

py =
    0.3750    0.3750    0.2500

mx =
     3

vx =
     4
sdx =
     2

my =
     1
vy =
   18.7500

sdy =
    4.3301

mxy =
    1.5000
cvxy =
   -1.5000
rhxy =
   -0.1732
```

Example 2: The joint probability density function of two random variables x and y is given by $p(x,y) = \begin{cases} 2, 0 < x < y < 1 \\ 0, otherwise \end{cases}$. Find the marginal density function of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

```
syms x y

f=2

px=int(f,y,x,1)

py=int(f,x,0,y)

mx=int(x*px,0,1)

vx=int(x^2*px,0,1)-mx^2

sdx=sqrt(vx)

my=int(y*py,0,1)

vy=int(y^2*py,0,1)-my^2

sdy=sqrt(vy)

mxy=int(int(x*y*f,y,x,1),x,0,1)

cv=mxy-mx*my

rh=cv/(sdx*sdy)
```

```
f =
   2
px =
   2 - 2*x
py =
   2*y
mx =
   1/3
vx =
   1/18
sdx =
   2^(1/2)/6
my =
   2/3
vy =
   1/18
sdy =
   2^(1/2)/6
mxy =
   1/4
cv =
   1/36
rh =
   1/2
```

Exercise:

1. A joint distribution of two random variables X and Y is given by the following table:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Determine the marginal distribution of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

2. The joint probability density function of two random variables x and y is given by

$p(x, y) = \begin{cases} \frac{8}{9}xy, & 0 \le x < y \le 2 \\ 0, & otherwise \end{cases}$. Find the marginal density function of X and Y, E(X), E(Y), E(XY), Cov(X, Y), correlation coefficient of (X, Y).

LINEAR ALGEBRA I

Syntax and description:

- `sym()` -Stores the values in symbolic math toolbox notation.
- `null(A)`–Finds the basis for the nullspace of the matrix A.
- `colspace(A)`–Finds the basis for the columnspace of the matrix A.
- `null(A')`–Finds the basis for the left nullspace of the matrix A.
- `colspace(A')`– Finds the basis for the rowspace of the matrix A.
- `B*inv(A)`– Finds the matrix representation of Linear transformation, where A is the matrix having the basis vectors of the domain as its columns, B is the matrix having the images of the basis vectors as its columns.
- `null(LT)`– Finds the basis for the nullspace of the Linear transformation, where LT is the matrix representation of the Linear transformation.
- `colspace(B)`– Finds the basis for the columnspace of the Linear transformation, where B is the matrix having the images of the Linear transformation as its columns.
- `rank(colspace(B))`– Finds the rank of the Linear transformation
- `rank(null(LT))`– Finds the nullity of the Linear transformation.

Example 1: Obtain the bases for the Four Fundamental Subspaces of the matrix A, by storing the matrix using symbolic math toolbox notation.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

```
A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])
```

A =

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

```
nsA=null(A)
```

nsA =

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
csA=colspace(A)
```

csA =

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
lnsA=null(A')
```

lnsA =

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
rsA=colspace(A')
```

rsA =

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

Note:

One can also find the Four Fundamental Subspaces using .m file.

(i) Create a function file with the name ffss.

```
function[fourfundamentalsubspaces]=ffss(A)
nsA=null(A)
csA=colspace(A)
lnsA=null(A')
rsA=colspace(A')
end
```

(ii)Enter the matrix A in command window using symbolic math toolbox notation as:

```
A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])
```

A =

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

(iii) call the m file in the command window as ffss(A)

```
ffss(A)
```

nsA =

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
csA =
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
rsA =
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
lnsA =
```

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Example 2: Find the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ such that $T(1,1) = (0,1,2), T(-1,1) = (2,1,0)$.

```
A=[1,-1;1,1]
```

```
A = 2×2

     1    -1
     1     1
```

```
B=[0,2;1,1;2,0]
```

```
B = 3×2

     0     2
     1     1
     2     0
```

```
LT=B*inv(A)
```

```
LT = 3×2

    -1     1
     0     1
     1     1
```

Example 3: Find the range space, null space, rank and nullity of T, where $T : V_3(\mathbb{R}) \longrightarrow V_4(\mathbb{R})$, defined by $T(e_1) = (0, 1, 0, 2), T(e_2) = (0, 1, 1, 0), T(e_3) = (0, 1, -1, 4)$.

```
A= sym([1,0,0;0,1,0;0,0,1])
```

A =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
B=sym([0,0,0;1,1,1;0,1,-1;2,0,4])
```

B =

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

```
LT=B*inv(A)
```

LT =

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

```
nsLT=null(LT)
```

nsLT =

$$\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

```
rsLT=colspace(B)
```

rsLT =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}$$

```
rLT=rank(rsLT)
```

rLT = 2

```
nLT=rank(nsLT)
```

nLT = 1

Exercise:

1. Compute the bases for the four fundamental subspace of the following matrix

   $$A = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{pmatrix}.$$

2. Compute the bases for the four fundamental subspace of the following matrix

   $$B = \begin{pmatrix} 3 & 4 & -2 & -5 \\ 4 & 3 & 2 & 4 \\ 2 & 5 & -6 & -14 \end{pmatrix}.$$

3. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x) = Ax$, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{pmatrix}$.

4. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x) = Ax$, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ -2 & 1 & -3 \end{bmatrix}$.

LINEAR ALGEBRA II

Syntax and description:

- `onv = gramschmidt(A)`- computes the orthonormal vector of the column vectors of A.
- `[Q,R]=qr(A)`-returns an upper triangular matrix R and a unitary matrix Q such that A = Q*R.
- The orthogonal or QR, factorization expresses any rectangular matrix as the product of an orthogonal or unitary matrix and an upper triangular matrix. An orthogonal matrix or a matrix with orthonormal columns, is a real matrix whose columns all have unit length and are perpendicular to each other. If Q is orthogonal then $Q^T Q = I$
- `e=eig(A)` -returns a column vector containing the eigenvalues of square matrix A.
- `[V,D]=eig(A)`–returns diagonal matrix D of eigenvalues and matrix V whose columns are the corresponding right eigenvectors, so that A*V = V*D.
- The eigenvalue problem is to determine the solution to the equation Av = λv, where A is an n-by-n matrix, v is a column vector of length n and λ is a scalar. The values of λ that satisfy the equation are the eigenvalues. The corresponding values of v that satisfy the equation are the right eigenvectors.
- `s=svd(A)` -returns the singular values of matrix A in descending order.
- `[U,S,V]=svd(A)`–performs a singular value decomposition of matrix A such that A = U*S*V.

Singular value decomposition is a process through which any m by n matrix A can be factored into A = U*S*V = (orthonormal matrix) (diagonal matrix) (orthonormal matrix). The columns of U (m by m) are eigenvectors of $A\,A^T$ and the columns of V (n by n) are eigenvectors of $A^T A$. The r singular values on the diagonal of S (m by n) are the square roots of the non-zero eigenvalues of both $A\,A^T$ and $A^T A$.

Example 1: Extract the set of orthonormal vectors from the vectors $(1,2,-2), (-1,3,1), (1,-2,5), (4,3,0)$.

A matlab file can be written for the Gram-Schmidt process as below, which is saved as gramschmidt.m file, and then run the file.
The matlab file is:

```
function [V] = gramschmidt(A)
% A = a matrix of size mxn containing n vectors.
% The dimension of each vector is m.
% V = output matrix: Columns of V form an orthonormal set.
[m,n] = size(A);
V = [A(:,1)/norm(A(:,1))];
for j = 2:1:n
v = A(:,j);
for i = 1:size(V,2)
a = v'*V(:,i);
v = v- a*V(:,i);
```

```
end
if (norm(v))^4 >= eps
V = [V v/norm(v)];
else
end
end
```

```
A=[1,2,-2;-1,3,1;1,-2,5;4,3,0]'
```

A = 3×4

```
     1    -1     1     4
     2     3    -2     3
    -2     1     5     0
```

```
ov1=gramschmidt(A)
```

ov1 = 3×3

```
    0.3333   -0.4216    0.8433
    0.6667    0.7379    0.1054
   -0.6667    0.5270    0.5270
```

```
B=sym([1,2,-2;-1,3,1;1,-2,5;4,3,0]')
```

B =

$$\begin{pmatrix} 1 & -1 & 1 & 4 \\ 2 & 3 & -2 & 3 \\ -2 & 1 & 5 & 0 \end{pmatrix}$$

```
ov2=gramschmidt(B)
```

ov2 =

$$\begin{pmatrix} \frac{1}{3} & -\frac{2\sqrt{10}}{15} & \frac{4\sqrt{90}}{45} \\ \frac{2}{3} & \frac{7\sqrt{10}}{30} & \frac{\sqrt{90}}{90} \\ -\frac{2}{3} & \frac{\sqrt{10}}{6} & \frac{\sqrt{90}}{18} \end{pmatrix}$$

```
orth(A)
```

ans = 3×3

```
    0.2247    0.7082    0.6693
    0.6829    0.3755   -0.6267
   -0.6951    0.5979   -0.3992
```

Example 2: Orthonormalize the matrix $A = \begin{pmatrix} 2 & -3 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$.

```
A1 = [2 -3 -1; 1 1 -1; 0 1 -1];
onvA1=gramschmidt(A1)
```

```
onvA1 = 3×3

    0.8944   -0.4082   -0.1826
    0.4472    0.8165    0.3651
         0    0.4082   -0.9129
```

```
A2=sym([2,-3,-1;1,1,-1;0,1,-1])
```

A2 =

$$\begin{pmatrix} 2 & -3 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

```
onvA2=gramschmidt(A2)
```

onvA2 =

$$\begin{pmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{8}\sqrt{15}}{60} \\ \frac{\sqrt{5}}{5} & \frac{\sqrt{6}}{3} & \frac{\sqrt{8}\sqrt{15}}{30} \\ 0 & \frac{\sqrt{6}}{6} & -\frac{\sqrt{8}\sqrt{15}}{12} \end{pmatrix}$$

Example 3: If the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$, find Q and R matrices representing the QR decomposition of A.

```
A=[1,1,1;1,2,3;1,3,6]
```

```
A = 3×3

     1     1     1
     1     2     3
     1     3     6
```

```
[Q,R]=qr(A)
```

```
Q = 3×3

   -0.5774    0.7071    0.4082
   -0.5774   -0.0000   -0.8165
   -0.5774   -0.7071    0.4082
R = 3×3

   -1.7321   -3.4641   -5.7735
         0   -1.4142   -3.5355
         0         0    0.4082
```

Example 4: If the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$, find the characteristic equation, the eigenvalues and the eigenvectors of A.

```
A = [8 -6 2; -6 7 -4; 2 -4 3]
```

```
A = 3×3

     8    -6     2
    -6     7    -4
     2    -4     3
```

```
p=poly(A)
```

```
p = 1×4

1.0000  -18.0000   45.0000   -0.0000
```

```
e=eig(A)
```

```
e = 3×1

    0.0000
    3.0000
   15.0000
```

```
[V,D]=eig(A)
```

```
V = 3×3

    0.3333    0.6667   -0.6667
    0.6667    0.3333    0.6667
    0.6667   -0.6667   -0.3333
D = 3×3

    0.0000         0         0
         0    3.0000         0
         0         0   15.0000
```

Example 5: Compute the singular values of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$.

```
A = [1 0 1; -1 -2 0; 0 1 -1]
```

```
A = 3×3

     1     0     1
    -1    -2     0
     0     1    -1
```

```
s = svd(A)
```

```
s = 3×1

    2.4605
    1.6996
    0.2391
```

Example 6: Find the singular value decomposition of a rectangular matrix $A = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

```
A = [-1; 2; 2]
```

```
A = 3×1

    -1
     2
     2
```

```
[U,S,V] = svd(A)
```

```
U = 3×3

   -0.3333    0.6667    0.6667
    0.6667    0.6667   -0.3333
    0.6667   -0.3333    0.6667
S = 3×1

     3
     0
     0
V = 1
```

Exercise:

1. If $v_1 = (0, 1, 2), v_2 = (1, 1, 2), v_3 = (1, 0, 1)$ construct an orthonormal basis.
2. Find the QR factorization of A $= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
3. Consider the matrix D $= \begin{pmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$, find a factorization D = QR.
4. Find the characteristic equation, eigenvalues of the matrix B $= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Also diagonalize it.
5. Diagonalize the matrix A $= \begin{pmatrix} 5 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 9 \end{pmatrix}$. Also find its characteristic equation and its eigenvalues.
6. Obtain the SVD of matrix A $= \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$.