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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated toVTU

III Semester B. E. Fast Track Examinations July-17

Common to CSE / ISE

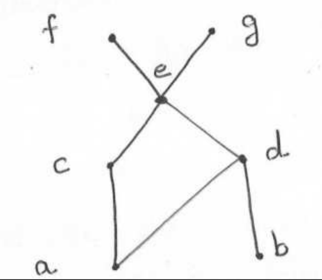
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

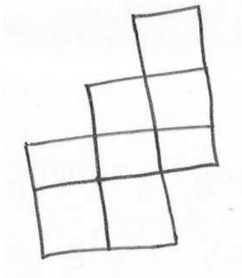
3. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
4. Answer FIVE full questions from Part B.

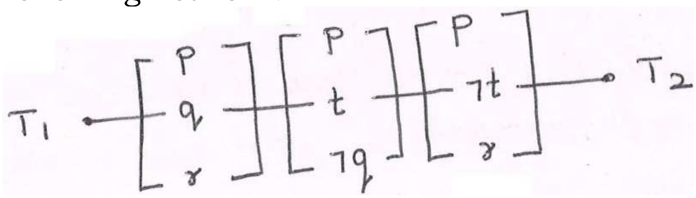
PART-A

1	1.1	Find d_5.	01
	1.2	If $A = \{2,3,4\}$ and $B = \{4,5\}$. Find: a) $B \times A$ b) $A - B$.	02
	1.3	Find the number of non-negative integer solutions of the equation: $x_1 + x_2 + x_3 + x_4 + x_5 = 8$.	01
	1.4	Prove the following statement by mathematical induction. For all $n \in z^+$, $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{(n)(n+1)}{2}$	02
	1.5	Obtain a recursive definition for the sequence $\{a_n\}$ in the following cases: a) $a_n = 5n$ b) $a_n = 2 - (-1)^n$	02
	1.6	Let P: Today is Thanksgiving Q: Tomorrow is Friday Write the statements for $P \rightarrow Q$ and its contrapositive, converse and inverse.	02
	1.7	Prove the following argument is valid using truth-table: $[(p \rightarrow r) \wedge (\neg q \rightarrow p) \wedge \neg r] \rightarrow q$	02
	1.8	Given that $S(8,4) = 1701$, $S(8,5) = 1050$ and $S(8,6) = 266$. Evaluate $S(10,6)$.	01
	1.9	Let $A = \{1,2,3\}$ and $B = \{w,x,y,z\}$. $f = \{(1,w),(2,x),(3,x)\}$ is a function. Find the co-domain and range of f.	01
	1.10	Consider the Poset whose Hasse diagram is shown below. Find Least Upper Bound and Greatest Lower Bound of $B = \{c,d,e\}$.  	02

1.11		Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$	02
1.12		The word $c = 1010110$ is transmitted through a binary symmetric channel. If $e = 0101101$ is the error pattern, find the word r received. If $p = 0.05$ is the probability that a signal is incorrectly received, find the probability with which r is received.	02

PART-B

2	a	Find the number of arrangements of the letters in $TALLAHASSEE$ which have no adjacent $A's$.	04
	b	Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by $2, 3$ or 5.	08
	c	Find the rook polynomial for the board C shown below: 	04
		OR	
3	a	Five teachers $T1, T2, T3, T4, T5$ are to be made class teachers for five classes, $C1, C2, C3, C4, C5$, one teacher for each class. $T1$ and $T2$ do not wish to become the class teachers for $C1$ or $C2$, $T3$ and $T4$ for $C4$ or $C5$ and $T5$ for $C3$ or $C4$ or $C5$. In how many ways can the teachers be assigned the work (without displeasing any teacher)?	06
	b	Determine the co-efficient of: i) $x^2y^2z^3$ in the expansion of $(x+y+z)^7$, ii) x^5y^2 in the expansion of $(x+y)^7$.	06
	c	A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting at least 2 questions from each part?	04
4	a	The number of bacteria in a culture is 1000(approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day.	06
	b	Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200, n \geq 0$ and $a_0 = 3000;\ a_1 = 3300$.	06
	c	Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 7^n\ for\ n \geq 1$, given that $a_0 = 2$.	04
		OR	
5	a	Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n\ for\ n \geq 0$, given $F_0 = 0, F_1 = 1$. (Hint: $F_0, F_1, F_2, \ldots. F_n$ represents Fibonacci sequence)	06
	b	Find a generating function for the recurrence relation $a_{n+1} - a_n = 3^n, n \geq 0$ and $a_0 = 1$. Hence solve the relation.	10

6	a	Simplify the following network: 	08
	b	Establish the validity of the following argument: If the band could not play rock music or the refreshments were not delivered on time, then the New year's party would never been canceled and Alicia would have been angry. If the party were canceled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music.	08
		OR	
7	a	Establish the validity of the following argument $\forall x\, [p(x) \vee q(x)]$ $\forall x\, \left[\left(\neg p(x) \wedge q(x)\right) \rightarrow r(x)\right]$ ____________________________ $\therefore \forall x\, [\neg r(x) \rightarrow p(x)]$	08
	b	Show that the following argument is invalid p $p \vee q$ $q \rightarrow (r \rightarrow s)$ $t \rightarrow r$ ____________________________ $\therefore \neg s \rightarrow \neg t$	05
	c	Prove the following statement in 3 different ways: i) Direct proof ii) Indirect proof and iii) Proof by contradiction. "If n is an odd integer, then $n + 11$ is an even integer"	03
8	a	If $A = \{1, 2, 3, 4\}$ and R, S are relations on A defined by $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}; S = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$ Find $RoS, SoR, R^2, S^2, R^c, S^c$.	06
	b	Draw the Hasse diagram representing the positive divisors of 36.	04
	c	Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ i) Determine how many functions are there from A to B. How many of these are one- to –one? How many are onto? ii) Determine how many functions are there from B to A. How many of these are onto? How many are one- to –one?	06
		OR	
9	a	Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5, & x > 0 \\ -3x + 1, & x \leq 0 \end{cases}$ Find: i) $f(0), f(-1)$ and $f\left(-\frac{5}{3}\right)$ ii) $f^{-1}(1), f^{-1}(3)$.	05

	b	Let $f, g, h: R \to R$, where $f(x) = x^2, g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Prove $\left((h \circ g) \circ f\right) = \left(h \circ (g \circ f)\right)$.	05
	c	Let $A = \{1, 2, 3, 4\}$. Give an example of a relation R on A that is: i) Reflexive and symmetric, but not transitive. ii) Reflexive and transitive, but not symmetric. iii) Symmetric and transitive, but not reflexive.	06
10	a	Determine the cyclic subgroups generated by the elements [2] and [3] of the group $(Z_6, +)$.	04
	b	For the group $G = (Z_{12}, +)$ and the subgroup $H = \{[0], [4], [8]\}$ of G, determine all the left cosets of H in G. Also, obtain the corresponding coset decomposition of G.	06
	c	If $G = (Z_6, +), H = (Z_3, +)$ and $K = (Z_2, +)$ prove that G and $H \times K$ are isomorphic.	06
		OR	
11	a	An encoding function $E: Z_2^2 \to Z_2^5$ is given by the generator matrix: $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i) Determine all the code-words. What can be said about the error-detection capability of this code? What about its error-correction capability? ii) Find the associated parity-check matrix H. iii) Use H to decode the received words: $11101, 11011$	12
	b	Define Group Code with example.	04