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RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
III Semester B.E. April -2024 Examinations
DEPARTMENT OF MATHEMATICS
MATHEMATICS FOR ARTIFICIAL INTELLIGENCE & MACHINE LEARNING (AIML)
(2022 SCHEME)
MODEL QUESTION PAPER

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.
3.

PART-A

1	1.1	A computer assembling company receives 40% of parts from supplier A and 60% of parts from supplier B. 6% of parts supplied by A and 10% supplied by B are defective. The probability of receiving a defective part is ___.	02
	1.2	It takes an average of 40 seconds to download a certain file, with a standard deviation of 5 seconds. The actual distribution of the download time is unknown. Using Chebyshev's inequality, what is the probability of spending more than 1 minute for this download?	02
	1.3	After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files. Compute the probability that the manager has to check at least 6 files in order to find the first undamaged file.	02
	1.4	The amount of time, in minutes, that a person must wait for a metro is uniformly distributed between 0 and 10 minutes, inclusive. What is the probability that a person waits fewer than 6.5 minutes?	02
	1.5	A ballpoint pen is selected at random from a box that contains 3 blue pens, 2 red pens, and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, construct the joint probability distribution table.	02
	1.6	The time, in minutes, it takes to reboot a certain system is a continuous variable with the density function $f(x) = c(10-x)^2, if\ 0 < x < 10$ and $f(x) = 0, otherwise$. Compute the constant c.	02
	1.7	If $\mathbb{R}^+$, the set of all positive real numbers is a vector space over the field $\mathbb{R}$, defined under (i) $\alpha + \beta = \alpha\beta$ and (ii) $c \cdot \alpha = \alpha^c$, then the zero vector is ___ and the inverse vector of α is ___.	02
	1.8	Suppose the vector $(2,3)$ is rotated by an angle of 30^0 clockwise, then the resultant vector is ___.	02
	1.9	The orthogonal projection of $y = (4,3)$ on to the vector $u = (2,1)$ is ___ and the vector orthogonal to u is ___.	02
	1.10	Suppose the sum and product of eigenvalues of a 2×2 diagonal matrix are 7 and 12 respectively, then the matrix is ___.	02

PART-B

		UNIT-I	
2	a	A disk drive manufacturer sells storage devices with capacities of one terabyte, 500 gigabytes, and 100 gigabytes with probabilities 0.5, 0.3, and	04

<table>
<tr><td></td><td></td><td>0.2, respectively. The revenues associated with the sales in that year are estimated to be $50 million, $25 million, and $10 million, respectively. Let X denote the revenue of storage devices during that year. Determine the probability mass function of X and hence find the average revenue and the standard deviation.</td><td></td></tr>
<tr><td></td><td>b</td><td>The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric is given by
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X)</td><td>0.41</td><td>0.37</td><td>0.16</td><td>0.05</td><td>0.01</td></tr>
</table>
Construct the cumulative distribution function of X and hence using $F(X)$, find (i) $P(X = 1)$, (ii) $P(0 < X \leq 2)$.</td><td>04</td></tr>
<tr><td></td><td>c</td><td>The diameter of a particle of contamination (in micrometers) is modeled with the probability density function $f(x) = \frac{2}{x^3}$ $for\ x > 1$. Determine the following (i) $P(x < 2)$, (ii) $P(x > 5)$, (iii) $P(4 < x < 8)$, (iv) $P(x < 4\ or\ x > 8)$, (v) c such that $P(x < c) = 0.95$, (vi) the cumulative density function.</td><td>08</td></tr>
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<table>
<tr><td colspan="4">UNIT-II</td></tr>
<tr><td>3</td><td>a</td><td>There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let X be the number of errors in these three blocks. Using Binomial distribution, construct the distribution and hence find the mean and variance.</td><td>06</td></tr>
<tr><td></td><td>b</td><td>The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce. (i) What is the probability that a shoe weighs more than 13 ounces? (ii) What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes weighs less than 13 ounces? (iii) If the standard deviation remains at 0.5 ounce, what must the mean weight be for the company to state that 99.9% of its shoes weighs less than 13 ounces?</td><td>06</td></tr>
<tr><td></td><td>c</td><td>When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with $\lambda = 0.2$. (i) What is the expected number of errors per test area?, (ii) What percentage of test areas have two or fewer errors?</td><td>04</td></tr>
<tr><td></td><td></td><td>OR</td><td></td></tr>
<tr><td>4</td><td>a</td><td>Suppose that the counts recorded by a Geiger counter follow a Poisson process with an average of two counts per minute. (i) What is the probability that there are no counts in a 30-second interval? (ii) What is the probability that the first count occurs in less than 10 seconds? (iii) What is the probability that the first count occurs between one and two minutes after start-up?</td><td>06</td></tr>
<tr><td></td><td>b</td><td>The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours. (i) What is the probability that you do not receive a message during a two-hour period? (ii) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours? (iii) What is the expected time between your fifth and sixth messages?</td><td>06</td></tr>
<tr><td></td><td>c</td><td>The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch and a standard deviation of 0.0004 inch.
(i) What is the probability that a diameter is between 0.0014 and 0.0026?
(ii) What standard deviation of diameters is needed so that the probability in part (i) is 0.995?</td><td>04</td></tr>
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		UNIT-III	
5	a	Determine the value of c that makes the function $f(x,y) = c(x+y)$ a joint probability mass function over the nine points with $x = 1,2,3$ and $y = 1,2,3$. Determine (i) the marginal distributions of X and Y, (ii)$E(X), E(Y), E(XY), Cov(X,Y)$.	08
	b	Let the random variable X denote the time until a computer server connects to your machine (in milliseconds), and let Y denote the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. Assume that the joint probability density function for X and Y is $f(x,y) = 2e^{-x-y}\ for\ x < y$. (i) Verify that $f(x,y)$ is a valid joint density function, (ii) compute the probability that Y exceeds 2 milliseconds, the marginal density function of X, the conditional probability density function for Y, given that $X = x$.	08
		OR	
6	a	Let X and Y be the number of hardware failures in two computer labs in a given month. The joint distribution of X and Y is given by $P(X,Y)$, where $P(0,0) = 0.52, P(0,1) = 0.20, P(0,2) = 0.04, P(1,0) = 0.14, P(1,1) = 0.02, P(1,2) = 0.01, P(2,0) = 0.06, P(2,1) = 0.01, P(2,2) = 0$. Compute (i) the marginal probability distributions of X and Y, (ii) the conditional probability of Y, given $X = 1$, (iii) the conditional probability of X, given $Y = 2$, (iv) $E(Y\|X = 1), E(X\|Y = 2)$.	08
	b	Determine the value of c such that the function $f(x,y) = cxy$ for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function. Determine (i) $E(X), E(Y), E(XY)$.	08

		UNIT-IV	
7	a	Show that $\mathbb{P}_3$ the set of all polynomials of degree at most 3 over the field $\mathbb{R}$ is a subspace of $\mathbb{P}_n$ the set of polynomials of degree at most n.	04
	b	Find the bases and dimension of the row space and null space of the matrix $A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 5 & 4 & 9 & -1 \\ 6 & 3 & 9 & -3 \\ 2 & 2 & 4 & 2 \end{bmatrix}$	06
	c	Find the Linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^4$, such that $T(1,1,0) = (-1,5,-1,1), T(0,1,1) = (-2,2,1,2), T(1,1,1) = (-1,4,1,2)$. Also find the basis of the range space of the Linear transformation.	06
		OR	
8	a	Let $T: \mathbb{R}^2 \to \mathbb{R}^4$ defined by $T(a,b) = (a-2b, b+2a, 3a, 2b)$. Show that T is a linear transformation.	04
	b	Find the bases and dimension of the column space and left-null space of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 3 \\ 5 & 0 & 2 & 3 \end{bmatrix}$.	06
	c	Show that the matrices $\begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 5 & -5 \\ 2 & 10 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ are linearly dependent in $M_{2\times2}$. Extract a linearly independent subset. Also find the basis and dimension of the subspace spanned by them.	06

UNIT-V

9	a	Obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 3 \\ -1 & -3 \\ 0 & 2 \\ 1 & 5 \\ 1 & 5 \end{bmatrix}$.	06
	b	Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 4 & 8 & 4 \end{bmatrix}$	10
		OR	
10	a	Obtain an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$.	06
	b	Obtain the SVD of the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	10