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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

IV Semester B. E. Regular Examinations SEP/OCT – 2024

Computer Science and Engineering

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

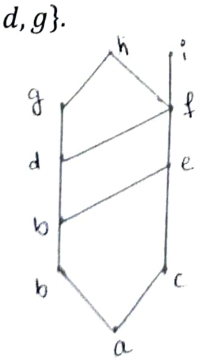
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of statistical tables and formula handbook permitted.

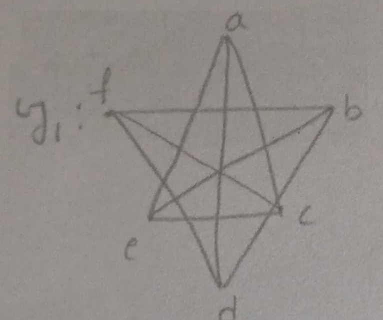
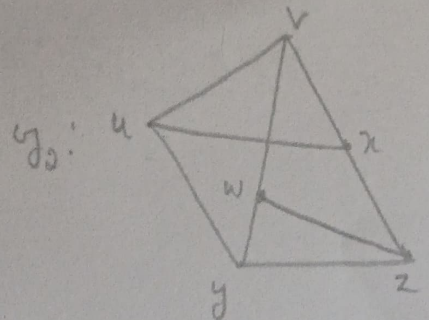
PART-A

			M	BT	CO
1	1.1	Fins the number of ways that the alphabets A, B, C, D, E, F, G are arranged such that A is not first position, B is not in second position, ... G is not in seventh position.	02	2	2
	1.2	Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}, n \geq 2$ subject to the initial conditions $a_1 = 1, a_2 = 3$.	02	2	2
	1.3	Simply using law of logic: $[(p \vee q)\, n(p \vee {\sim}q)] \vee q \Leftrightarrow p \vee q$	02	1	1
	1.4	Write down the converse, inverse of the following compound proposition. "A person is successful in life if he puts sincere efforts"	02	2	2
	1.5	Let $f: z \rightarrow z$ and $g: z \rightarrow z$, given by $f(x) = x - 1$ $g(x) = 2x$ find $f \circ g$ and $g \circ f$.	02	2	2
	1.6	If $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ is defined on the set $A = \{1,2,3,4\}$ determine the partition induced.	02	2	2
	1.7	Let $\boldsymbol{G} = \{q \in \theta \mid q \neq -1\}$. Identify the identify element of $\{G, 0\}$ where $xoy = x + y + xy$ for all $x, y \in G$.	02	2	2
	1.8	What is the hamming distance between the codes '11001011' and '10000111'.	02	1	1
	1.9	Let G be a simple graph of order n. If the size of G is 56 and size of $\bar{G}$ is 80, what is n?	02	2	2
	1.10	Given $V = \{(1,2,3,4,5,6)\}$ and $E = \{12, 13, 23, 35, 61, 66\}$ draw undirected and directed graph $G = (V, E)$. Also write down the order and size of G.	02	2	2

PART-B

			M	BT	CO
2	a	Find the number of proper divisors of 38808.	04	2	2
	b	A person inverse some amount at the rate of 11% annual compound interest. Determine the period for principal amount to get doubled.	04	2	3
	c	Find a generating function for the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$ for $n \geq 0, a_0 = 5, a_1 = 12$.	04	2	2
	d	In how many ways can 10 identical dimes be distributed among 5 children if i) There are no restrictions ii) Each child gets at least one dime.	04		

3	a	Let p, q and r be the propositions P: I Study; q: I will fail in the examination r: I watch TV in the evening Express each of these proposition as an English sentence (i) $p \to nq$ (ii) $q \to r$ (iii) $(p \to \sim r) \cup (q \to \sim r)$ (iv) $\sim p \to (q \cup r)$	04	1	1
	b	Prove that $[(\neg p \vee q) \wedge \{p \wedge (p \wedge q)\}] \Leftrightarrow p \wedge q$ using laws of logic.	06	2	2
	c	Write the following proposition in the symbolic form and find its negation. "If all triangles are right angles then no triangle is equiangular"	06	3	2
		OR			
4	a	For the following statement, state the converse inverse and contrapositive. The universe consist of all integers "If m divides n and n divides p, then m divides p"	04	2	2
	b	Prove the validity of the following argument $p \to (q \wedge g)$ $r \to s$ $\underline{\sim(g \wedge s)}$ $\therefore \sim p$	06	2	2
	c	Define open statement and find whether the following variable is valid. No engineering students of 1st or 2nd sem studies logic. Anil is an engineering student who studies logic. $\therefore$ Anil is not in second semester.	06	3	3
5	a	If $f: A \to B, g: B \to C, h: C \to D$ then show that $(h \circ g) of = h \circ (g \circ f)$.	04	2	2
	b	If $A = \{1,2,3,4\}$ and R, S are relations on A defined by $R = \{(1,2)(1,3)(2,4)(4,4)\}$ $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,3)(2,4)\}$. Find $R \circ S,\ S \circ R,\ R^2 and\ S^2$.	05	2	2
	c	Draw the Hasse diagram for all positive integer divisors of 72. Also write R.	07	3	3
		OR			
6	a	Find the lower and upper bounds of the subsets $\{a,b,c\}$; $\{i,h\}$ and $\{a,c,d,f\}$ in the poset with Hasse diagram shown in Fig 6a. Also find the glb and lub of $\{b,d,g\}$.  Fig 6a	08	3	2
	b	Let $f: R \to R$ be define by $f(x) = \begin{cases} 3x - 5 & if\ x > 0 \\ -3x + 1 & if\ x \le 0 \end{cases}$. Find $f^{-1}(1)$ and $f^{-1}(3)$.	04	3	3
	c	If R is a relation on $A = \{1,2,3,4\}$ define by xRy if x divides y. Prove that (A, R) is a poset.	04	2	2
7	a	If $\circ$ is an operation on Z define by $x \circ y = x + y + 1$. Prove that $(G, \circ)$ is an Abelian group.	05	3	3
	b	Prove that i) Identity element in a group is unique ii) Inverse of each element in a group is unique	04	2	2

	c	The encoding function $E = Z_2^2 \to Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ i) Find associated parity check matrix H ii) Determine all code words iii) Find decoded word for received msq [1 1 1 0 1] iv) What is error detection and correction capability.	07	4	4
		OR			
8	a	If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ Find $\alpha.\beta$ *and* α^{-1}.	04	2	2
	b	Define the encoding function $E: Z_2^3 \to Z_2^6$ by means of the parity – check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Determine all code words.	05	3	3
	c	State and prove Lagrange's theorem. Find the right cosets of $H = \{1, -1\}$ in multiplicative group of fourth root of unity.	07	2	3
9	a	Define Isomorphic and show that G_1 is isomorphic to G_2.   	08	3	3
	b	Explain the Konigsberg – Bridge problem.	08	4	4
		OR			
10	a	If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4 and one of degree 5, how many pendant vertices does it have?	08	2	2
	b	Prove the following for the graph $G = (V, E)$ i) $\sum_{vtv} \deg(V) = 2\|E\|$ ii) The number of vertices of odd degree must be even.	08	2	2