Discrete Mathematics and Algorithms (CSE 611)
Monsoon 2018
Assignment Set 1
Total Marks: 100

Deadline: August 21, 2018 (Tuesday), 1:00 PM, Class Room

1. Let us consider the structure $\langle \mathcal{P}(X), - \rangle$, where the operation $-$ (set difference) is defined by $A - B = \{x \mid x \in A \text{ and } x \notin B\} = A \cap B'$.
   (a) Show that the operation is neither commutative nor associative. [5]
   (b) Verify whether $A \cup (B \Delta C) = (A \cup B) \Delta (A \cup C)$ holds or not, where 'Δ' is the usual symmetric difference operator defined over sets. [5]
   (c) Show that the following properties hold for all $A, B, C \in \mathcal{P}(X)$:
   (i) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$,
   (ii) $A \Delta X = A'$,
   (iii) $A \Delta A' = X$, where $A \Delta B$ is the usual symmetric difference between A and B, and A' the complement of A. [3 × 5 = 15]
   (d) Given that $S - T = S \cap T'$, prove that $S \Delta T = (S \cup T) - (S \cap T)$. [5]

2. For a given set $\mathcal{A}$, consider the relation

   $$R = \{(x, y) \mid x \in \mathcal{P}(A), y \in \mathcal{P}(B), \text{ and } x \subseteq y\}.$$

   Show that R is a partial order relation. [5]

3. Let R and R' be two equivalence relations on a set A. Prove that

   (i) $R \cap R'$ is an equivalence relation in A. [5]

   (ii) $R \cup R'$ is not necessarily an equivalence relation in A. [5]

4. Let $\mathcal{A}$ be a set of English words and R be a binary relation on $\mathcal{A}$ such that two words w_1 and w_2 in $\mathcal{A}$ are related if they have one or more letters in common. Show that R is a compatible relation. [5]

5. Consider a function $f : X \to Y$. Determine the number of one-one, onto and bijective functions for the following cases:
   (a) when $|X| = 9, |Y| = 17$;
   (b) when $|X| = 10, |Y| = 5$. [5 + 5 = 10]

6. If $f : A \to B$ and $g : B \to C$ are both bijective, then prove that $g \cdot f$ is bijective, and $(g \cdot f)^{-1} = f^{-1} \cdot g^{-1}$. [5]

7. Prove that a function is right-invertible if and only if it is onto (surjective). [5]

8. Are the following functions from Z (set of all integers) to itself one-one, onto, a bijective function ? Accordingly, find their left, right or both sided inverses: $f(n) = 3n + 1$. [5]

9. Verify whether the mapping $f : R \to (-1, 1)$ defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$ is bijective, where R is the set of real numbers and $(-1, 1) = \{x \in R | -1 < x < 1\}$. [10]

10. Determine whether or not the set $A = \{a^2 | a \in N\}$ is countable, where N is the set of natural numbers. [5]

11. We know that the composite $g \cdot f$ of any two injections $f : S \to T$ and $g : T \to U$ is an injection. Let us now extend the definition of composite $g \cdot f$ to the case in which the domain of g contains the co-domain of f. Show that
    (i) the fact that the composite $g \cdot f$ of any two injections is an injection still holds, but
    (ii) the fact that the composite $g \cdot f$ of any two surjections is not a surjection. [5 + 5 = 10]

Submission Instructions

Copying in assignments leads to award ZERO marks in assignment marks. Also, the source from which you have copied, that source student will be treated under the same rule.

Please submit the assignment in hard copy stating the following at the top after the class on the deadline date only:

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submitted by
Name: XYZ
Roll No: abc